Physical thresholds and cluster decomposition

Zeno Capatti
ETH Zürich

Amplitudes 2023
07/06/2023, CERN, Switzerland

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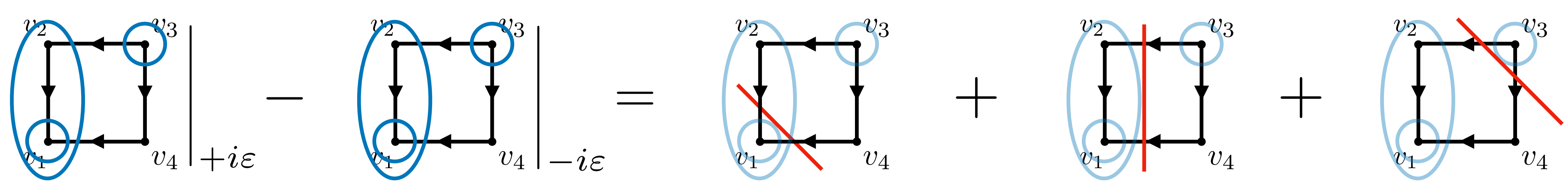
First part: Cross-Free Families

- Sketch the derivation of the representation

$$f^{3\mathrm{d}}_{G_u} = \int \left[\prod_{i=1}^{L} \frac{\mathrm{d}k_i^0}{2\pi}\right] \frac{\mathcal{N}(\{q_e^0\}_{e\in\mathcal{E}})}{\prod_{e\in\mathcal{E}}(q_e^0)^2 - E_e^2}$$

- Highlight role of connectedness by comparison with Time-Ordered Perturbation Theory



(spurious singularities in TOPT)

Acyclic graphs and edge contraction

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We start with an arbitrary diagram integrated over loop energies only

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$$q_e^0 = p_e^0 + \sum_{i=1}^{L} s_{ei} k_i^0$$

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G_u undirected graph

$\mathcal{E}$ set of edges of graph

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Notation

Energy conservation

Acyclic graphs

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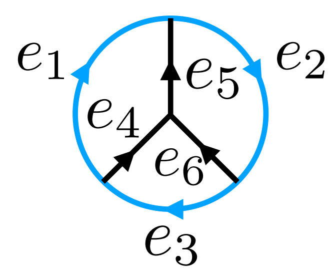

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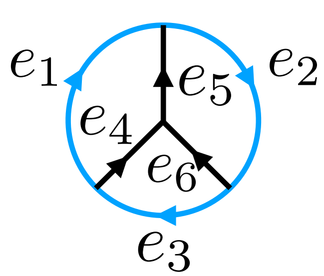

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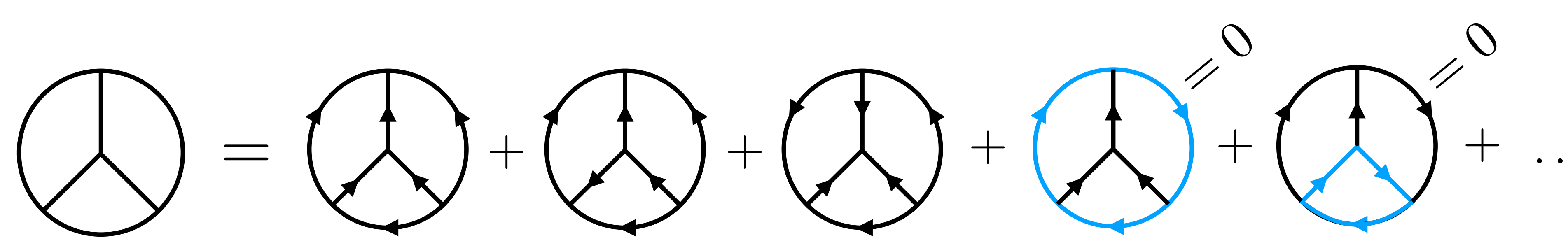
Diagrammatically



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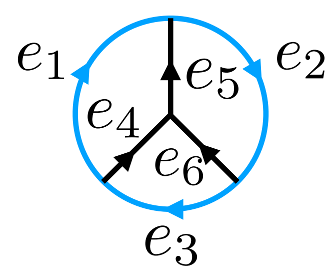

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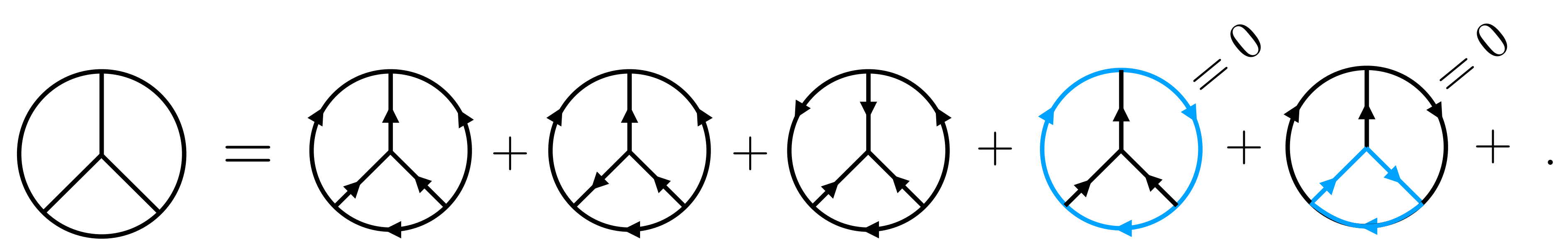

Position space: Fourier duality maps acyclic graphs to strongly-connected graphs

(Borinsky, ZC, Laenen, Salas-Bernardez [arXiv:2210.05532])

Edge contraction

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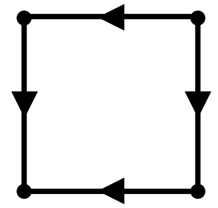

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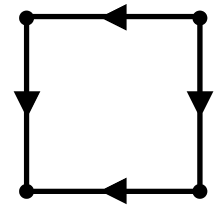

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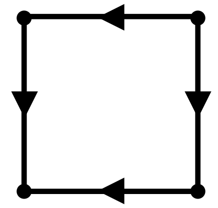

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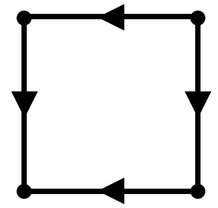
Triangulation introduces spurious singularities or spurious intersections

Edge contraction



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The cone is non-simplicial
Needs triangulation!

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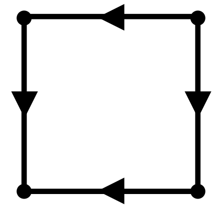

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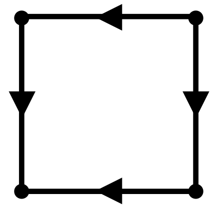
Triangulation introduces spurious singularities or spurious intersections

Edge contraction

How do we perform the remaining integrations (**one for each edge**)? Edge-contraction



$$= \int \left[\prod_{j=1}^{4} \frac{\mathrm{d}\tau_j}{2E_j} e^{i\tau_j (E_j^0 - \sigma_j p_j^0)} \Theta(\tau_j)\right] \delta\left(-\tau_1 - \tau_2 + \tau_3 + \tau_4\right)$$

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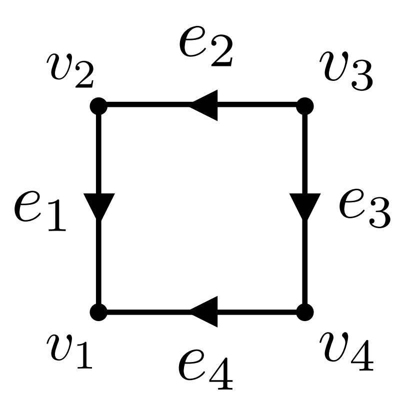

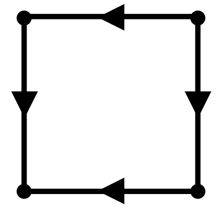

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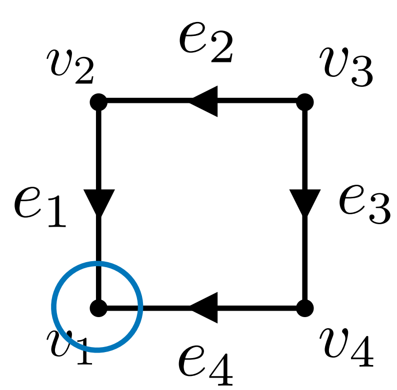

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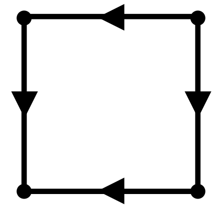

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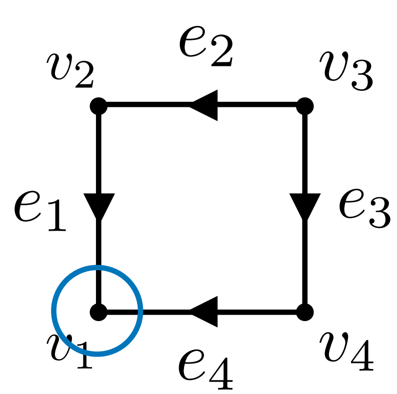
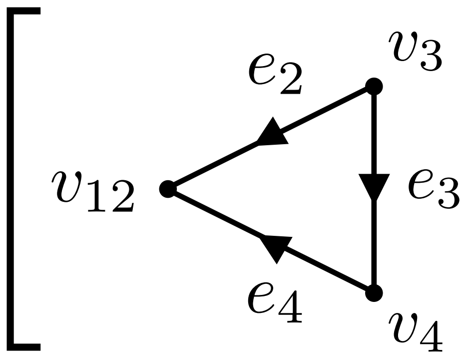
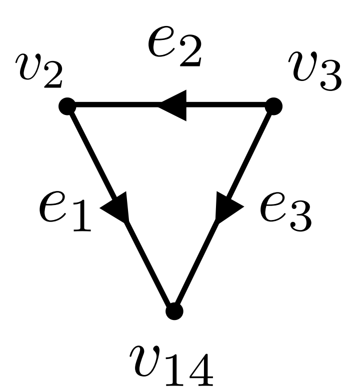


$$= \frac{i}{E_1 + E_4 - p_1^0} \Bigg[$$



$$+$$



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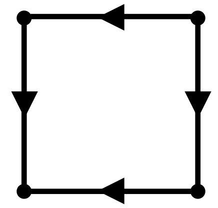

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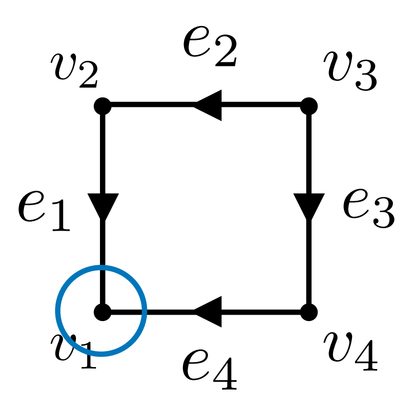

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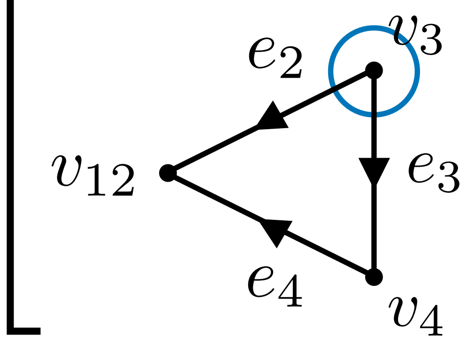

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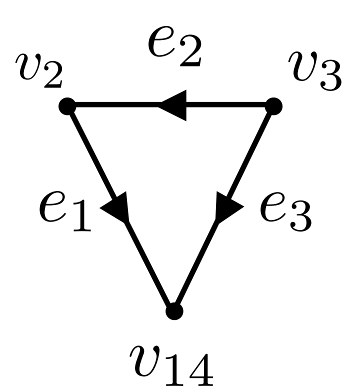

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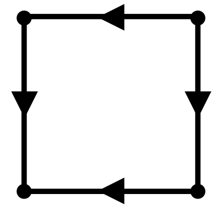

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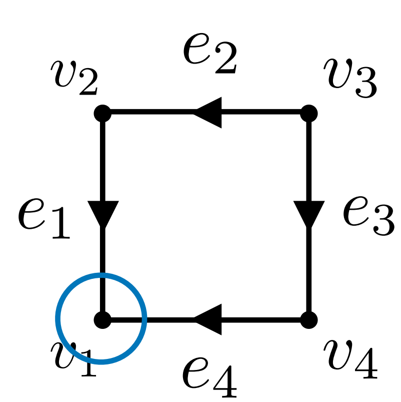

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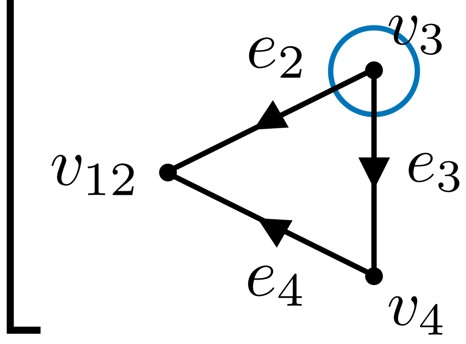

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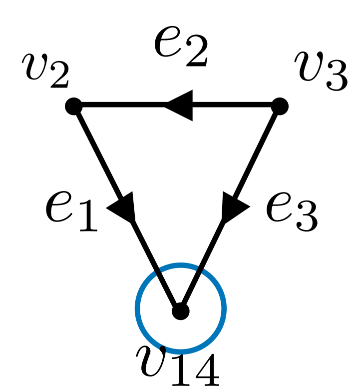

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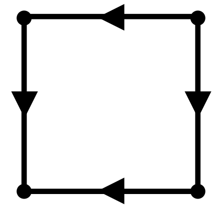

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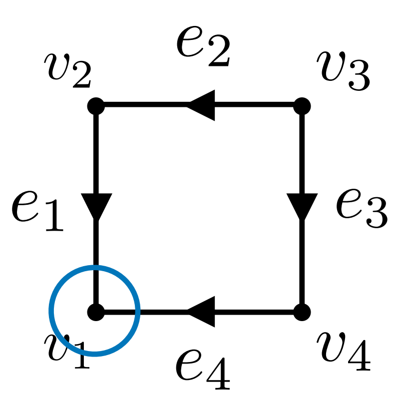

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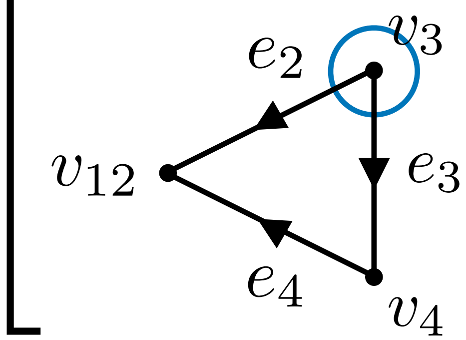

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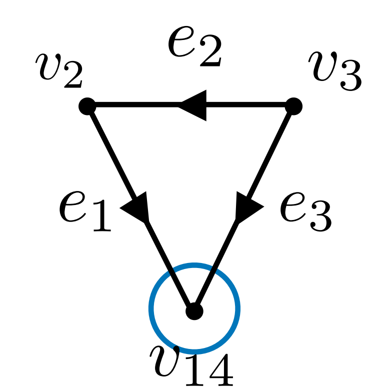

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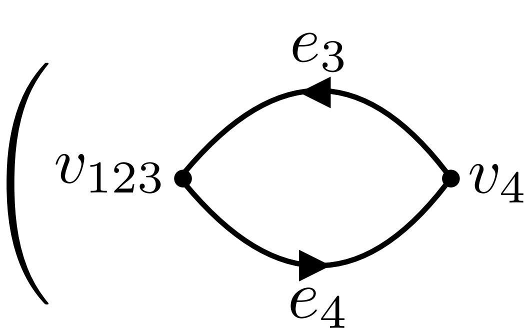

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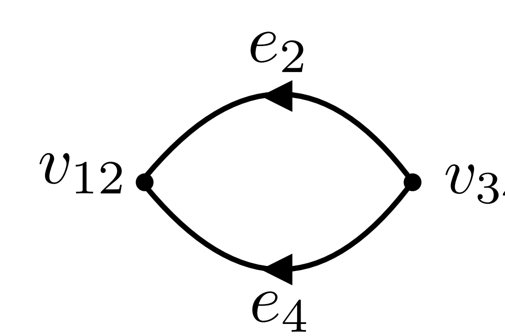

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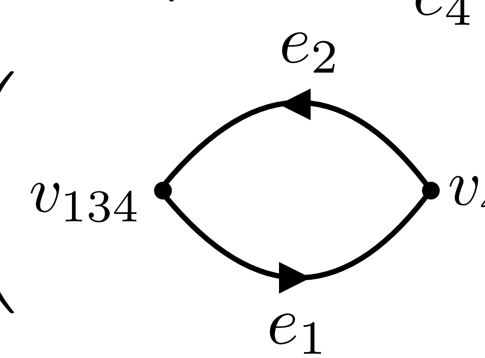

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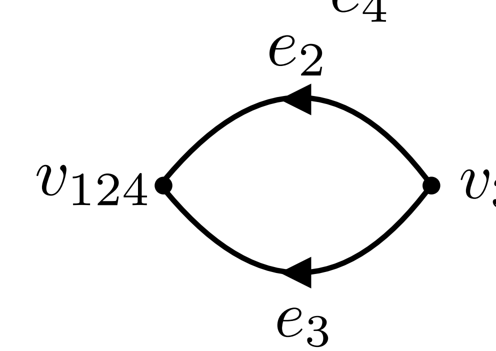

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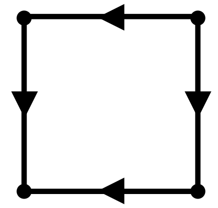

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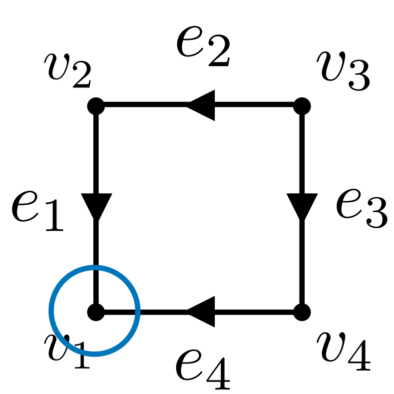

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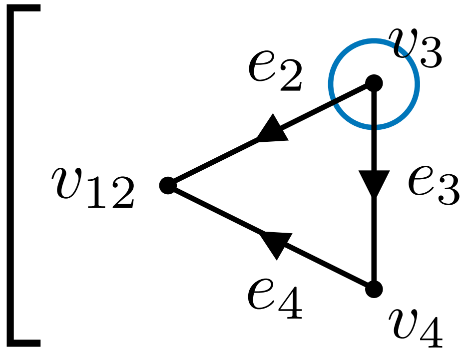

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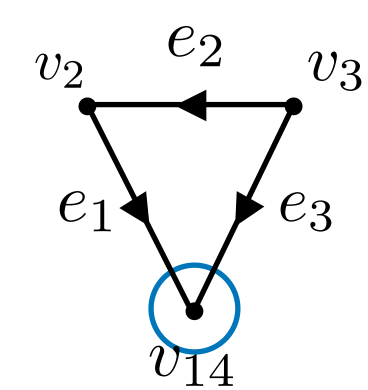

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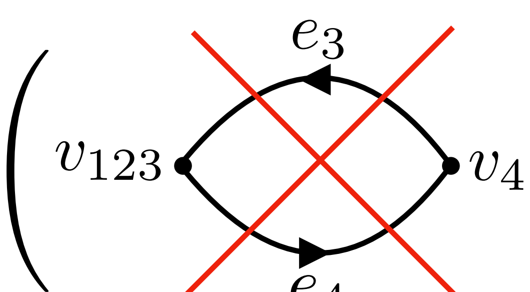

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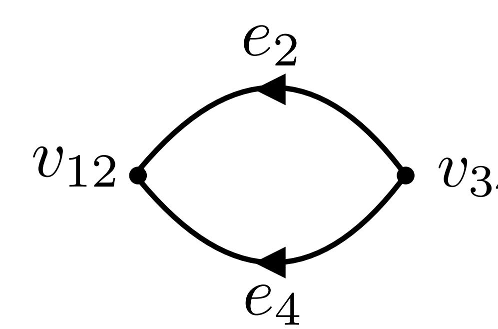

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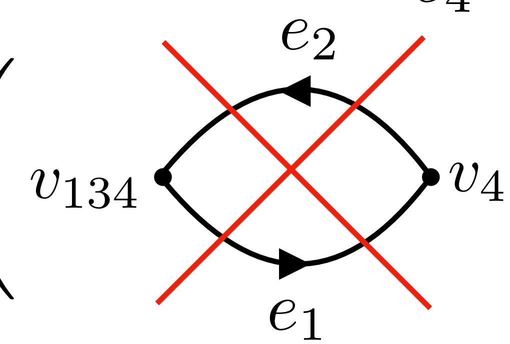

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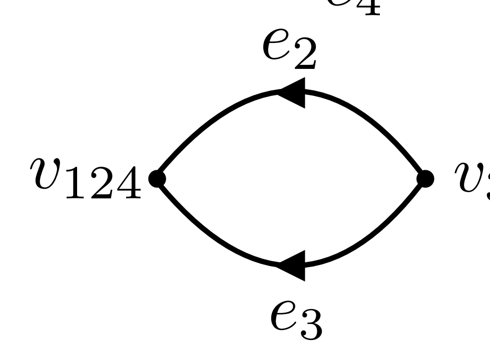
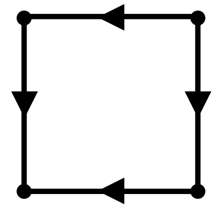


$\left. \Bigg) \right]$

4. Throw out non-acyclic graphs



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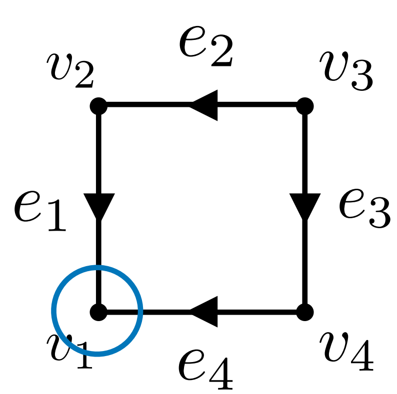

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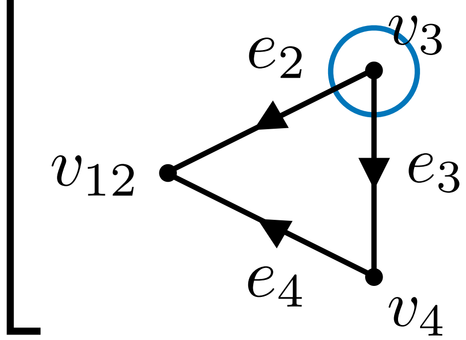

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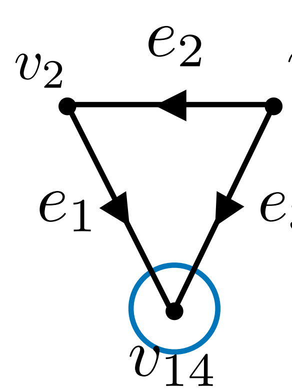

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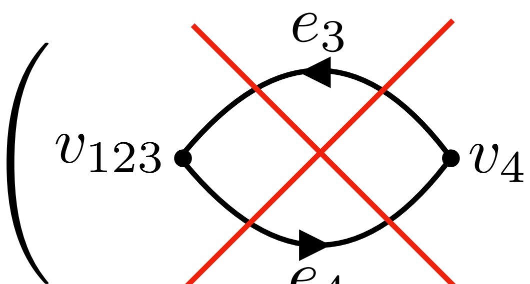

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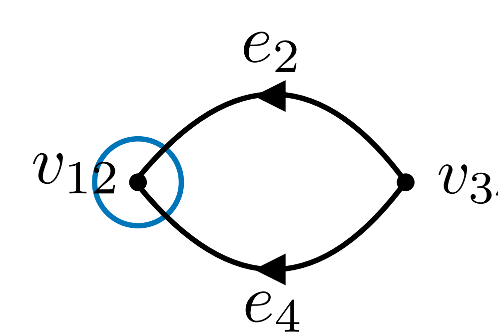

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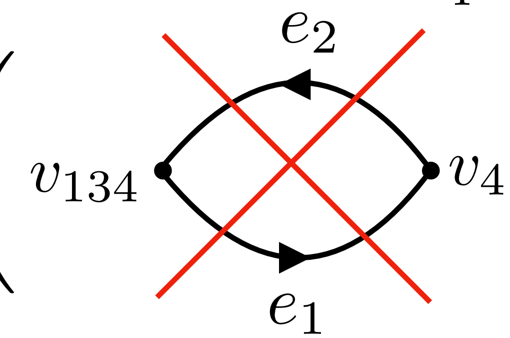

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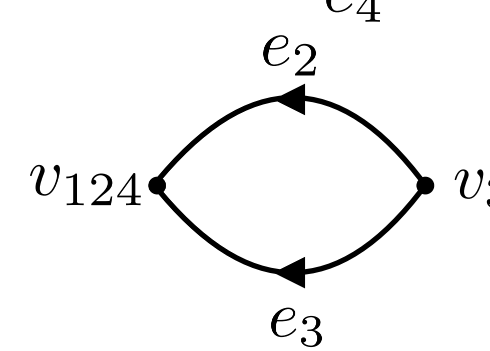

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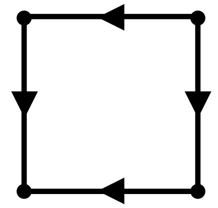

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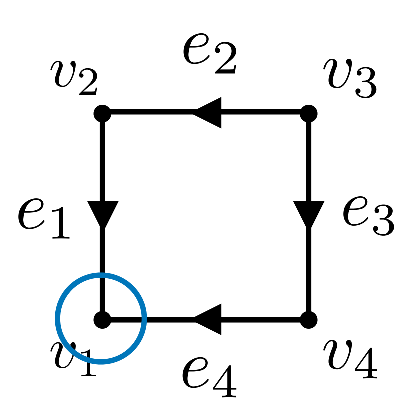

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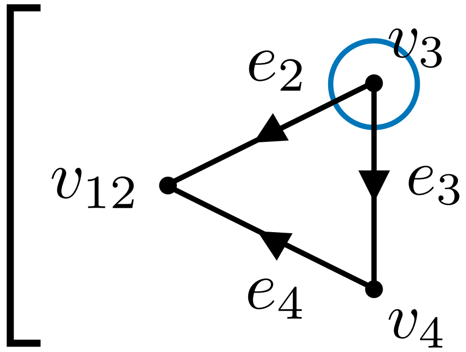

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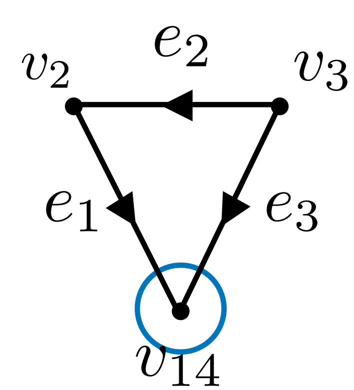

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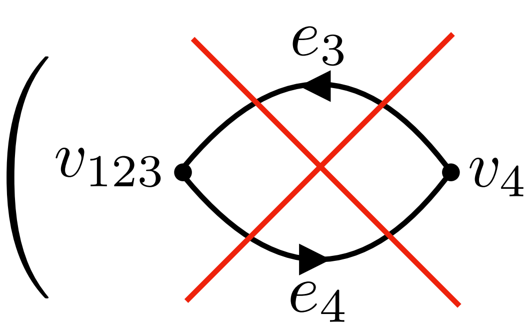

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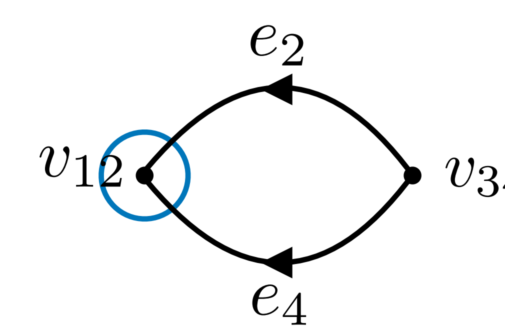

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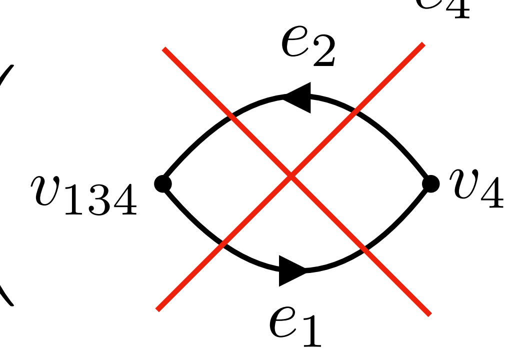

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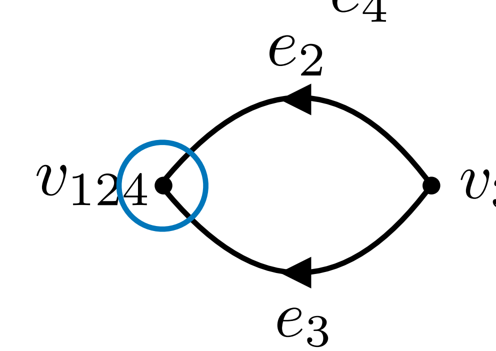

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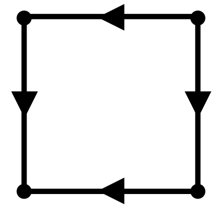

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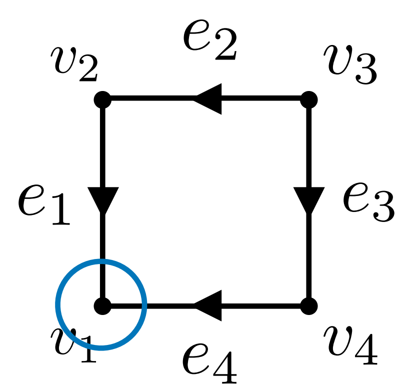
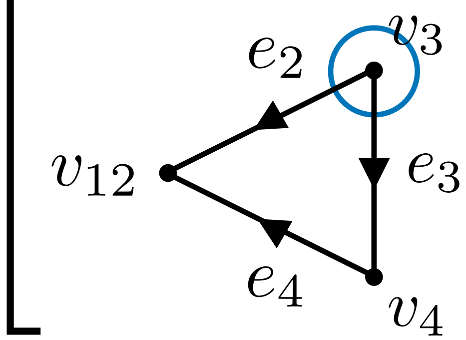


$$= \frac{i}{E_1 + E_4 - p_1^0} \Bigg[$$



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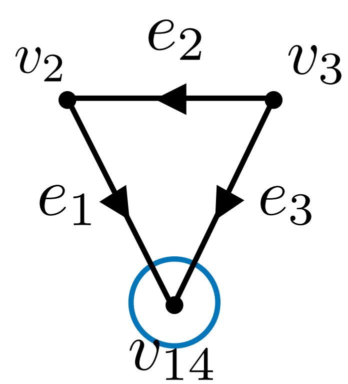

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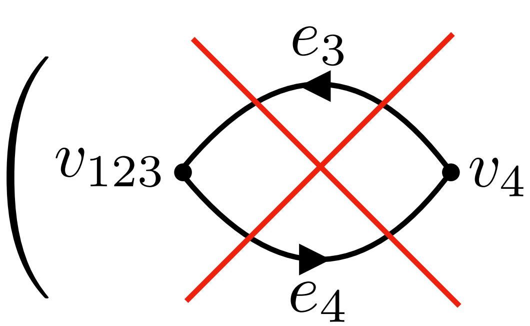

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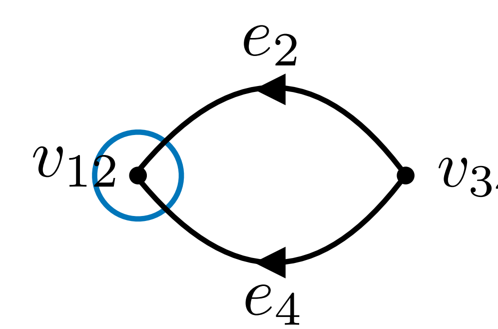

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4. Throw out non-acyclic graphs

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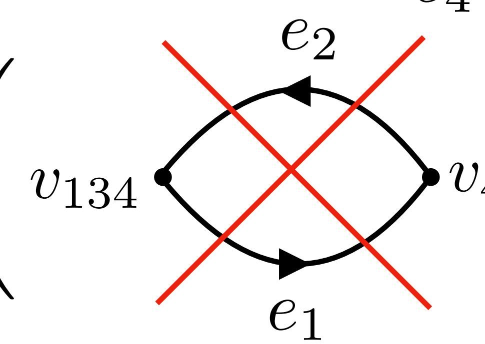

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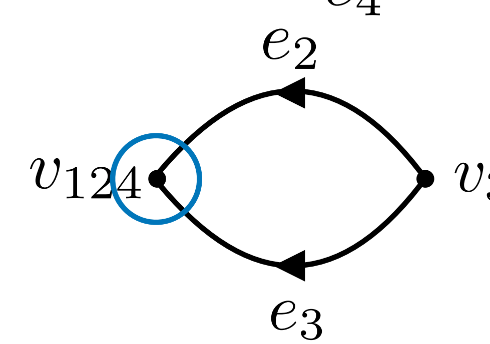

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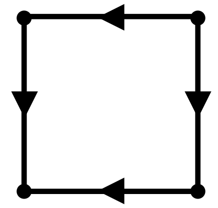

$$= \int \left[\prod_{j=1}^{4} \frac{\mathrm{d}\tau_j}{2E_j} e^{i\tau_j (E_j^0 - \sigma_j p_j^0)} \Theta(\tau_j) \right] \delta\left(-\tau_1 - \tau_2 + \tau_3 + \tau_4\right)$$

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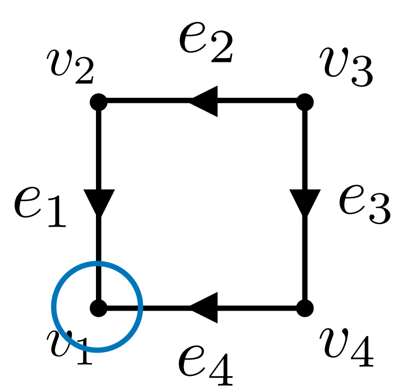

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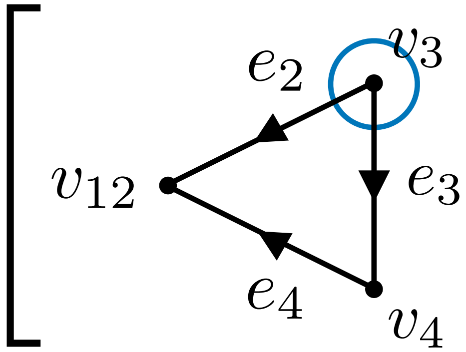

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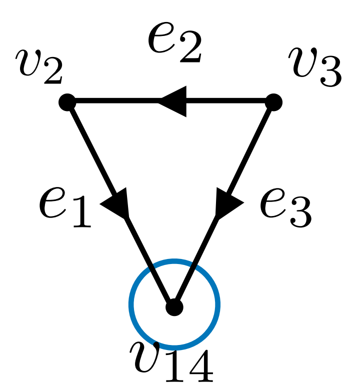

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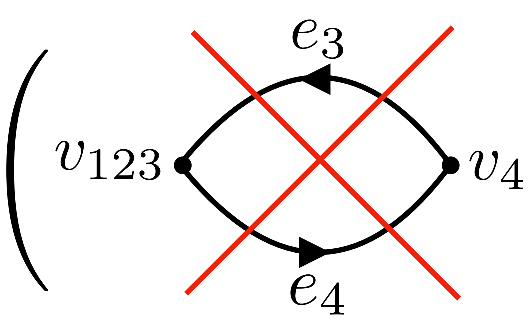

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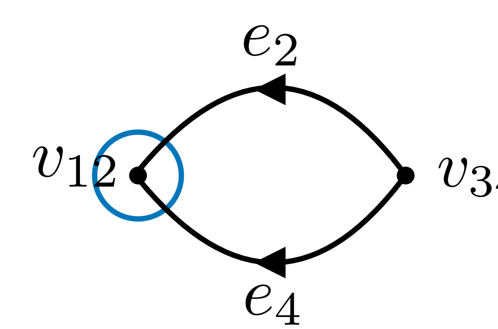

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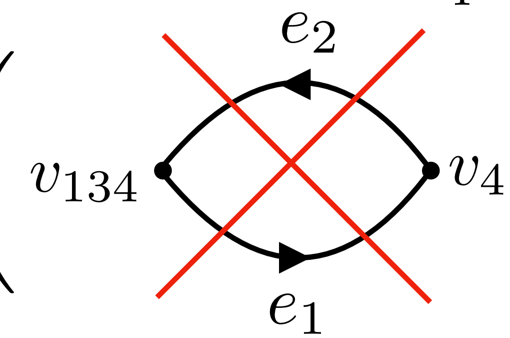

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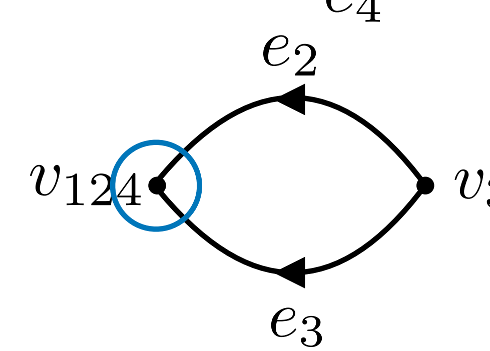

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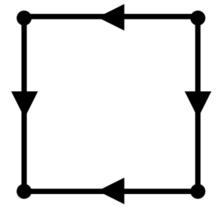

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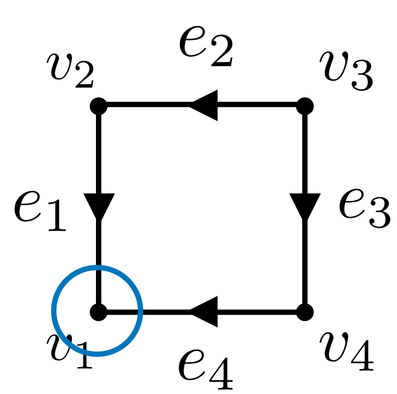

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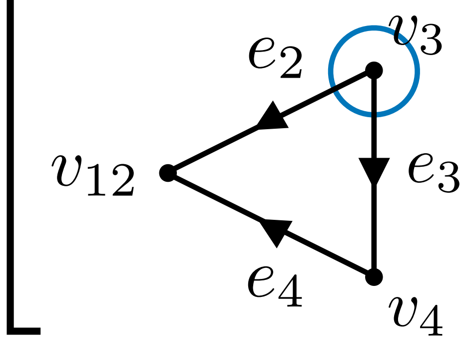

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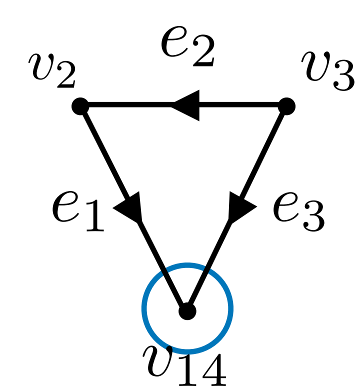

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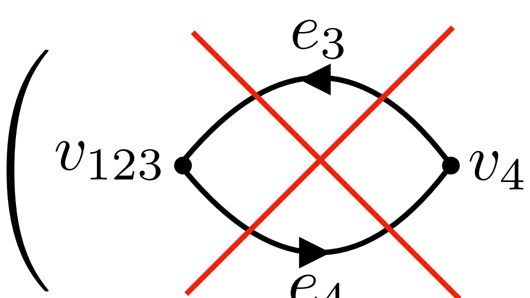

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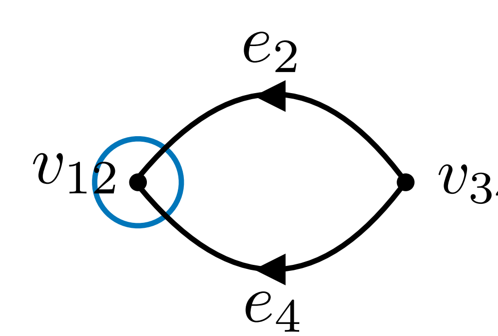

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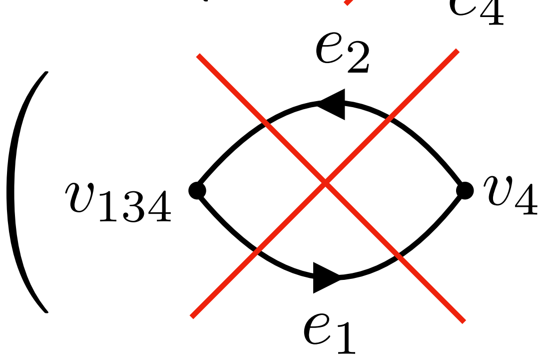

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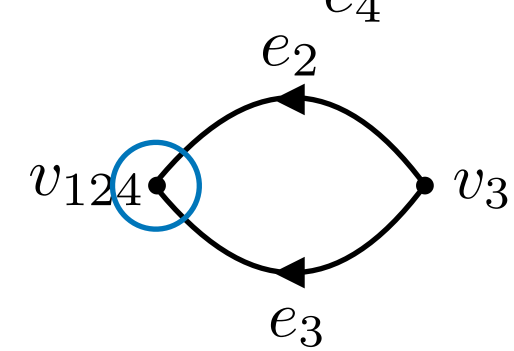

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All time integrations are performed diagrammatically!

Cross-Free Families

Collecting the chosen vertices and collecting them according to order of choice, we get a decision tree, whose root is the first chosen vertex

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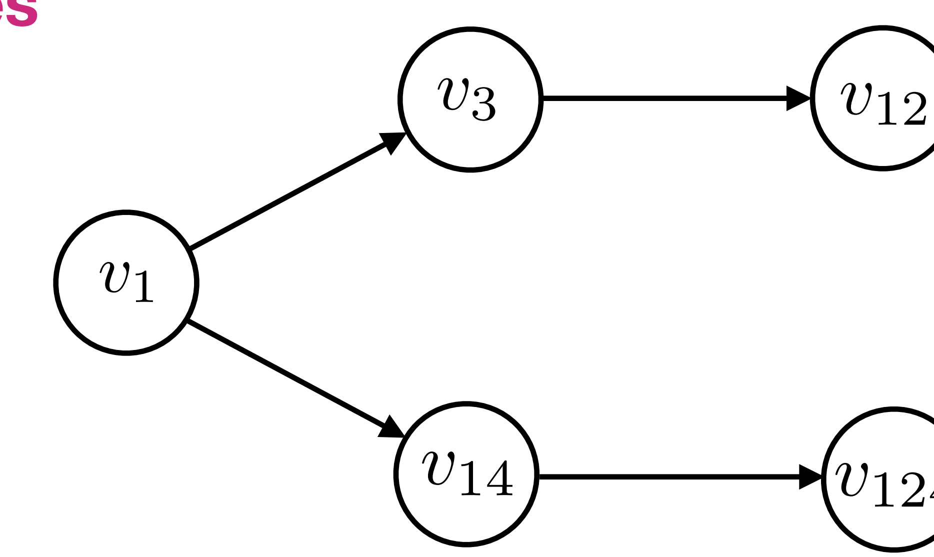

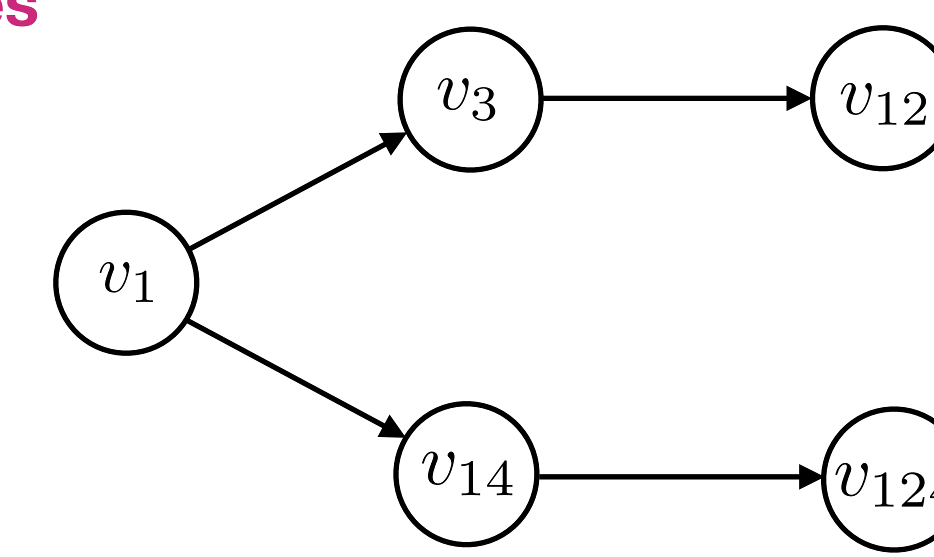

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Tracing the route from the leaves to the root gives sets of vertices

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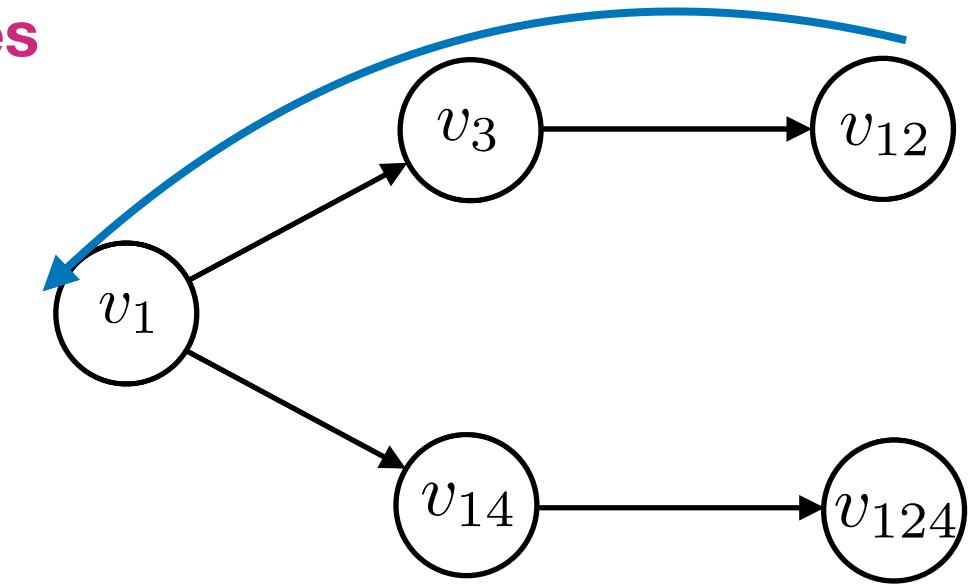

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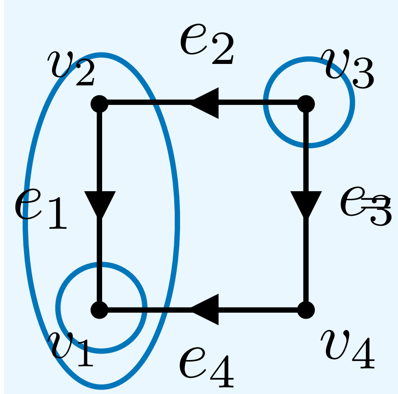

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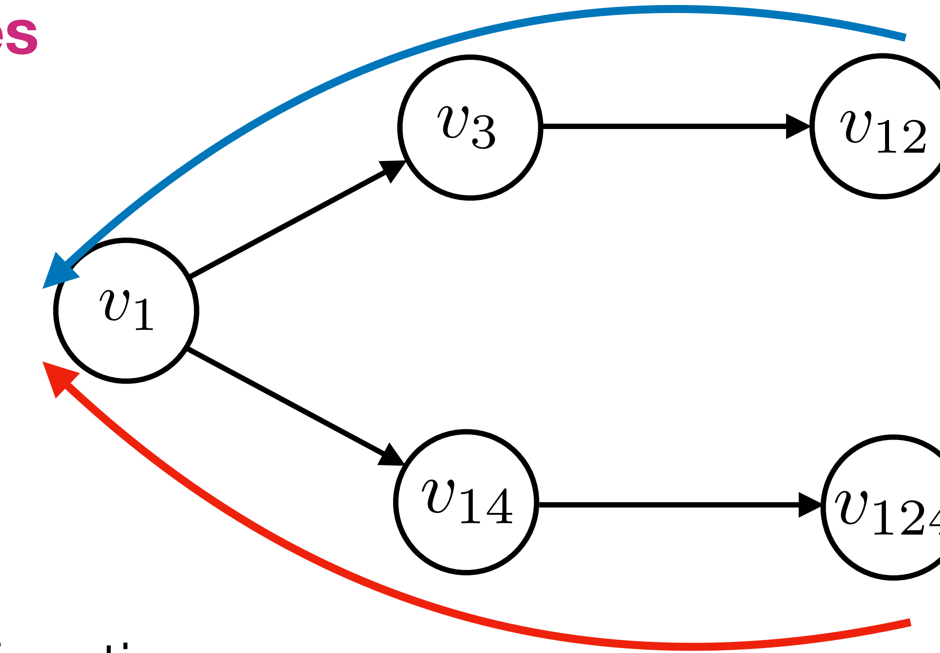

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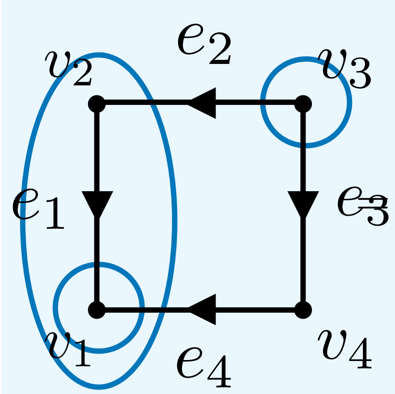

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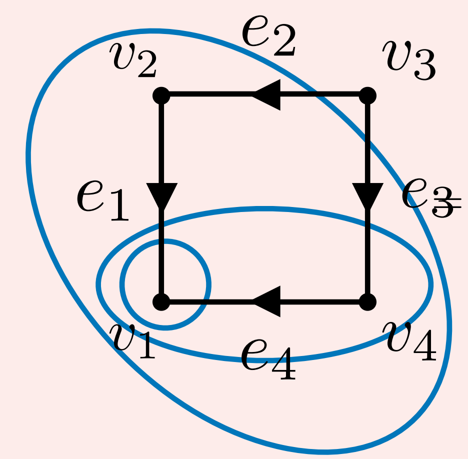

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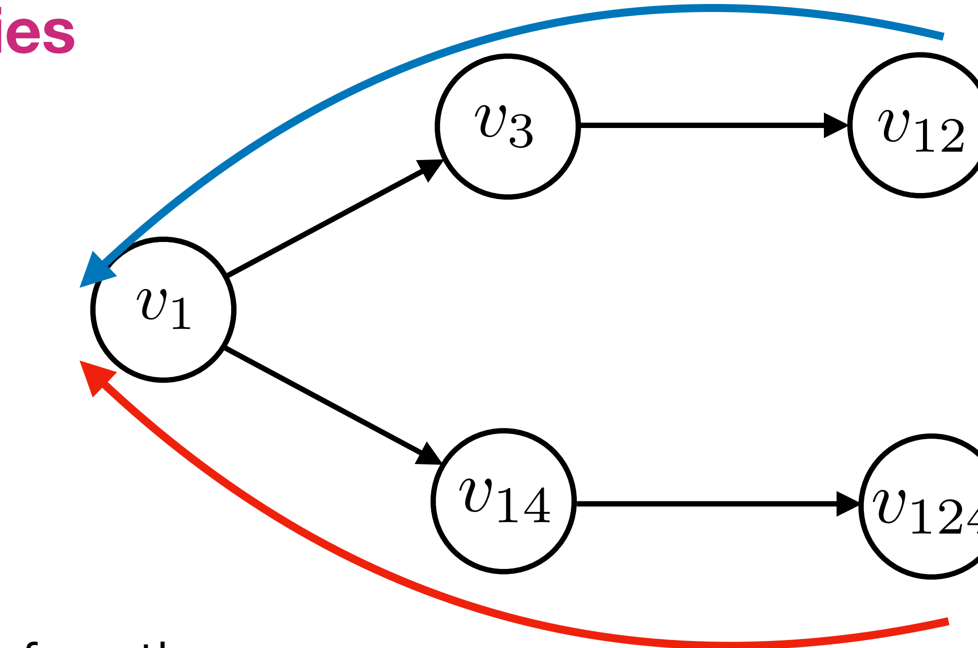

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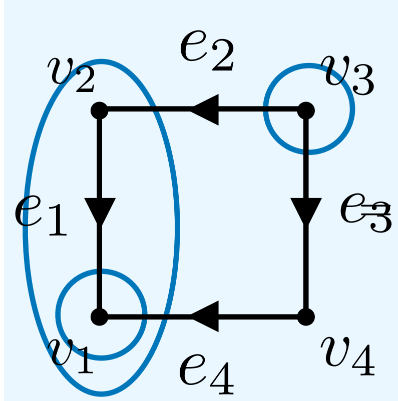

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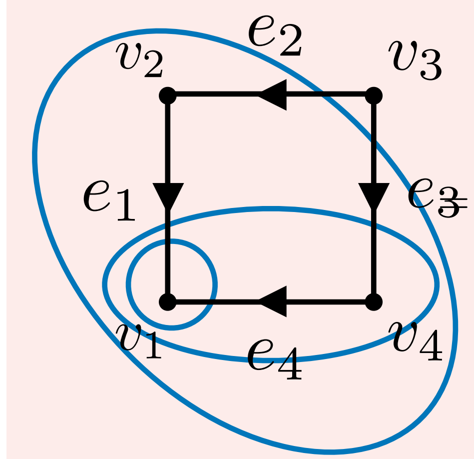

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Boundary operator provides nexus e.g. $\partial(\{v_1, v_2\}) = \{e_2, e_4\}$ $\Rightarrow$ $\dfrac{i}{E_2 + E_4 - p_1^0 - p_2^0}$

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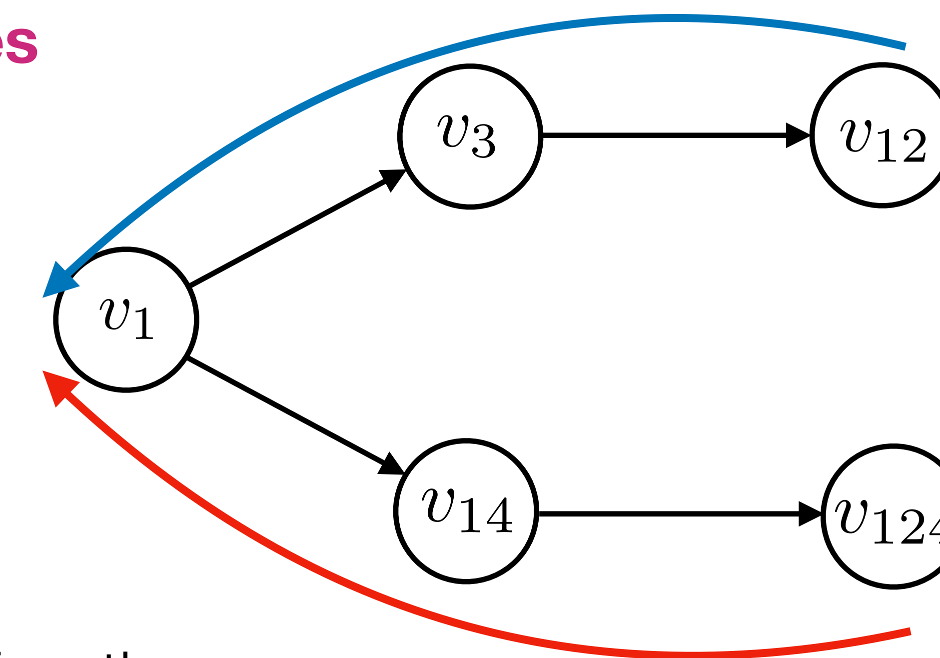

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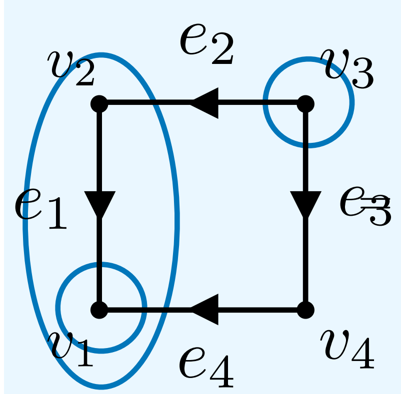

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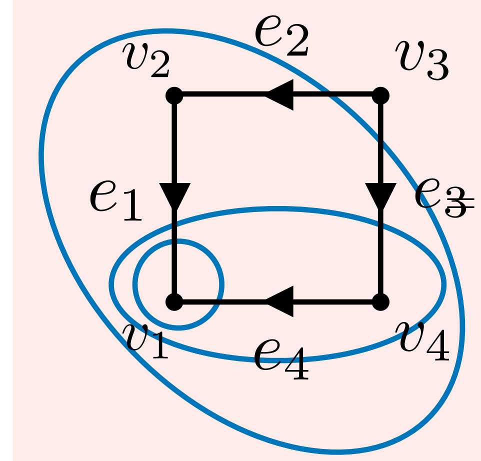

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We notice some regularities… these families of cuts satisfy

$$S \in F \quad \Rightarrow \quad S,\ V\setminus S \text{ are connected}$$

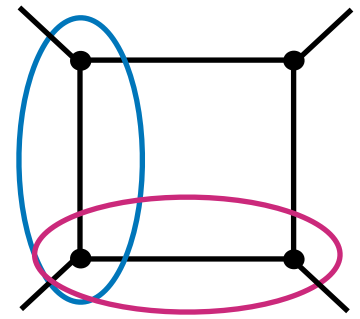
$$S_1,S_2 \in F \quad \Rightarrow \quad S_1\subset S_2 \text{ or } S_2\subset S_1 \text{ or } S_1\cap S_2=\emptyset$$

$$S \in F \quad \Rightarrow \quad S \text{ cannot be written as union of other sets in } F$$

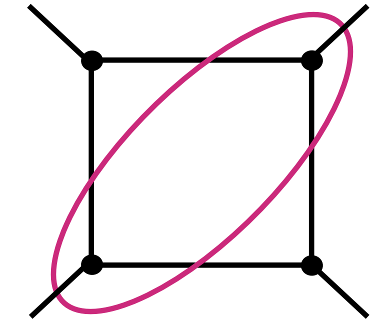
Abreu, Britto, Duhr, Gardi arXiv:2010.01068 (2014)
Bloch, Kreimer arXiv:1512.01705 (2015)
Arkani-Hamed, Benincasa, Postnikov arXiv:1709.02813 (2017)
Benincasa, McLeod, Vergu arXiv:2009.03047 (2020)
Capatti, Hirschi, Pelloni, Ruijl, arXiv:2010.01068 (2020)

Forbidden configurations

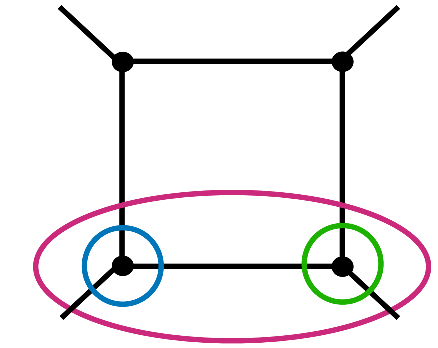
General formula



crossing



connectedness



obstruction

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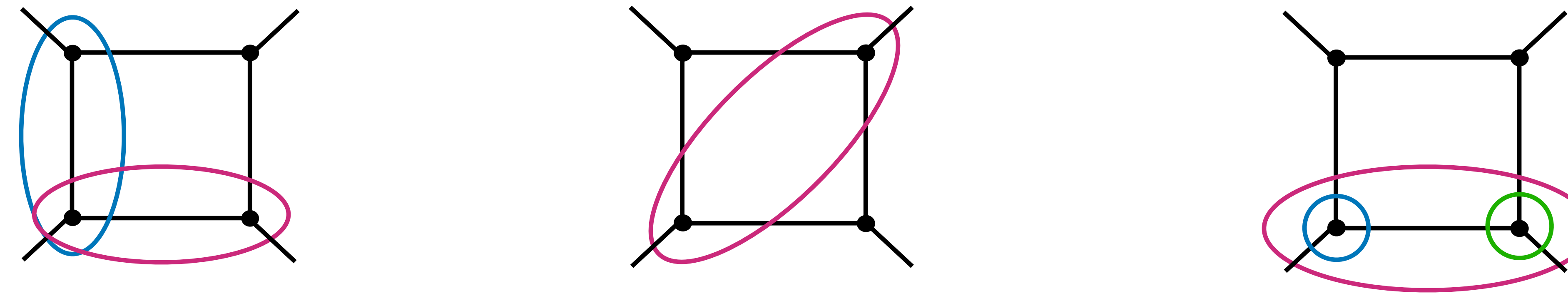

crossing | connectedness | obstruction

General formula

Repeating the same edge-contraction procedure for all acyclic graphs

$$f^{\mathrm{3d}}_{G_u} = \sum_{\substack{\text{acyclic}\\ \text{graph } G}} \frac{\mathcal{N}_G}{\prod_e 2E_e} \sum_{F \in \mathcal{F}_G} \frac{1}{\prod_{S\in F}\left[\boldsymbol{E}\cdot\mathbf{1}^{\partial(S)} - \sum_{v\in S} p_v^0\right]}$$

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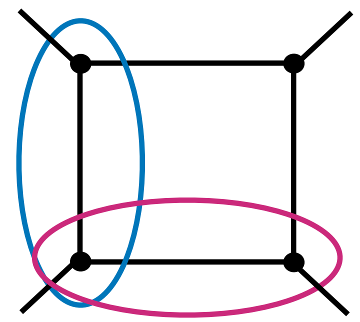

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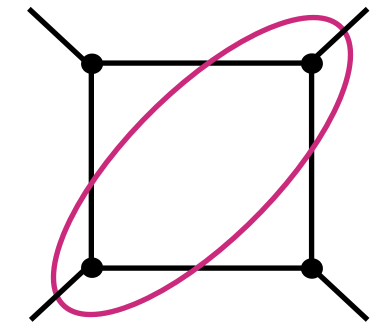

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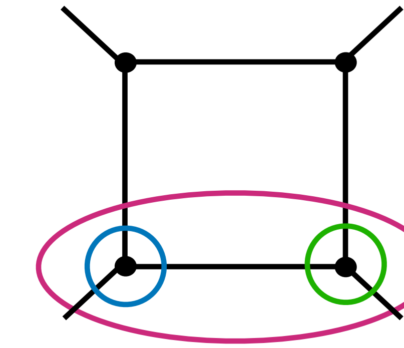

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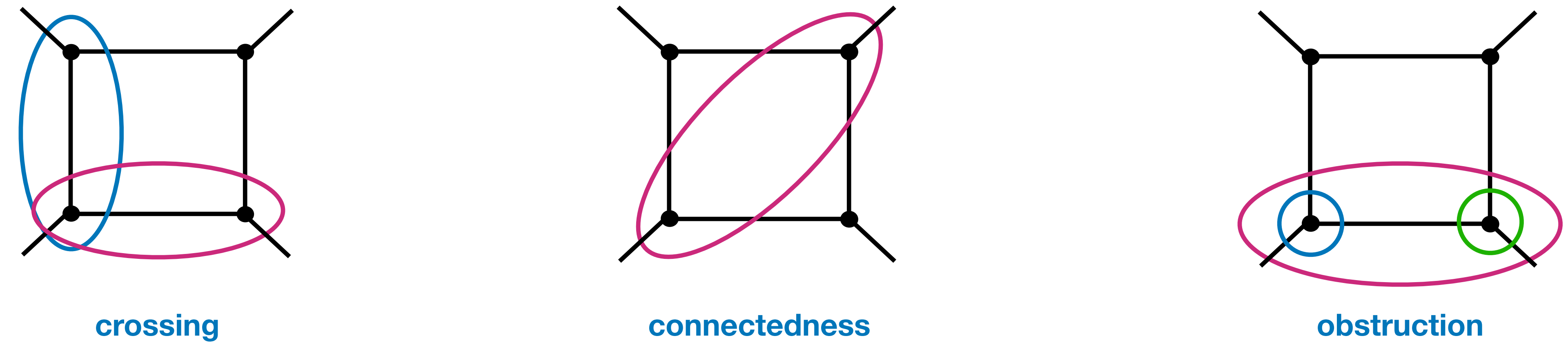

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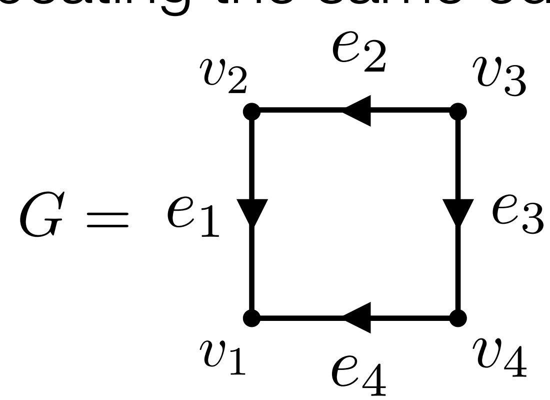

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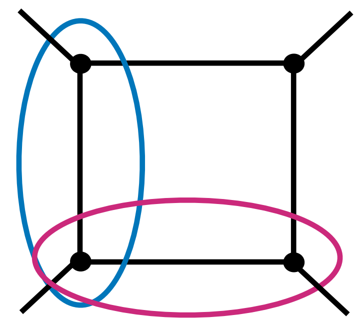
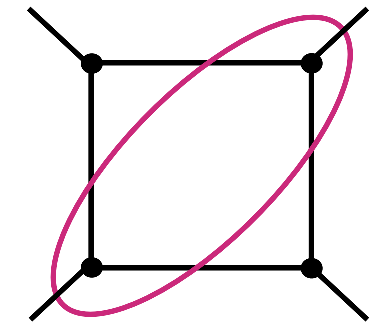
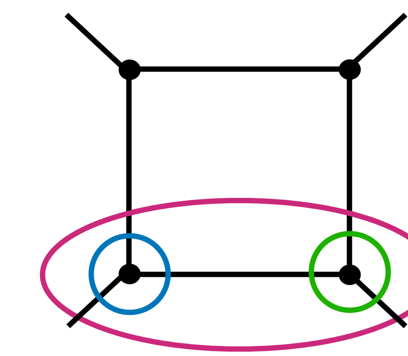


crossing



connectedness



obstruction

General formula

Repeating the same edge-contraction procedure for all acyclic graphs

$$f_{G_u}^{3\mathrm{d}} = \sum_{\substack{\text{acyclic} \\ \text{graph } G}} \frac{\mathcal{N}_G}{\prod_e 2E_e} \sum_{F \in \mathcal{F}_G} \frac{1}{\prod_{S \in F} \left[\boldsymbol{E} \cdot \mathbf{1}^{\partial(S)} - \sum_{v \in S} p_v^0\right]}$$

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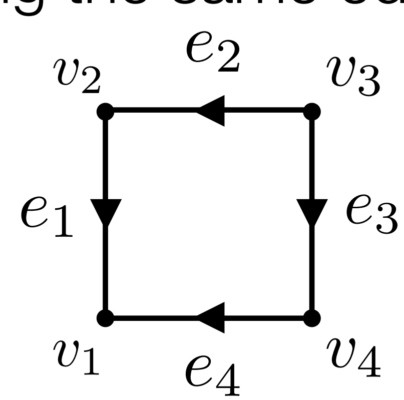
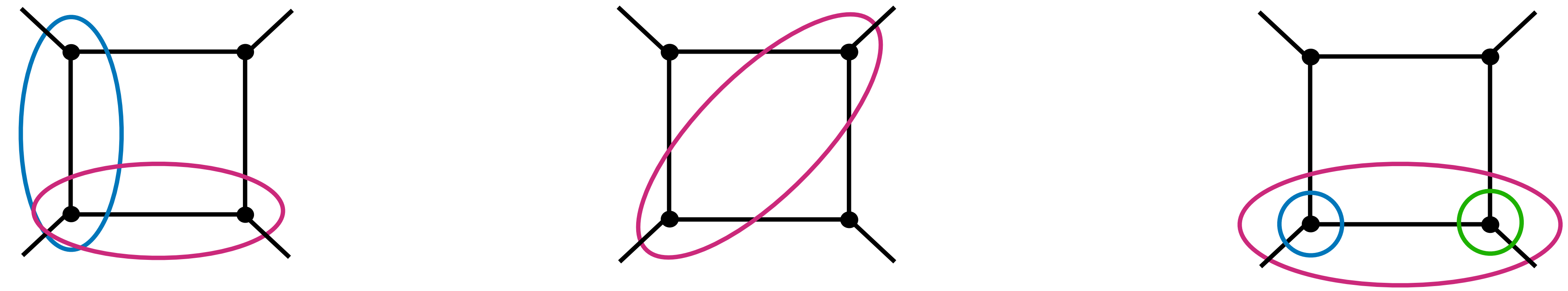
$G =$ 

Forbidden configurations



crossing — connectedness — obstruction

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$$f^{\text{3d}}_{G_u} = \sum_{\substack{\text{acyclic} \\ \text{graph } G}} \frac{\mathcal{N}_G}{\prod_e 2E_e} \sum_{F \in \mathcal{F}_G} \frac{1}{\prod_{S \in F} \left[\boldsymbol{E} \cdot \mathbf{1}^{\partial(S)} - \sum_{v \in S} p^0_v \right]}$$

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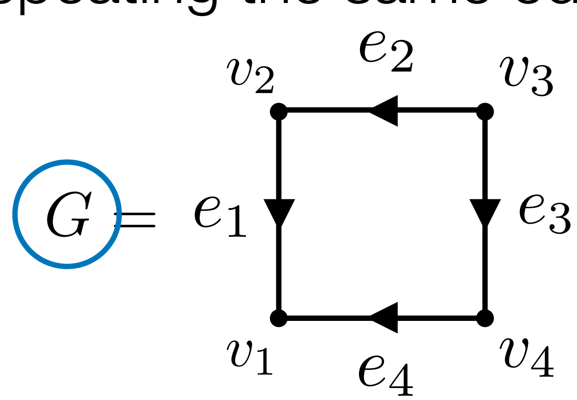

$$\mathcal{F}_G = \{F_1, F_2\}$$

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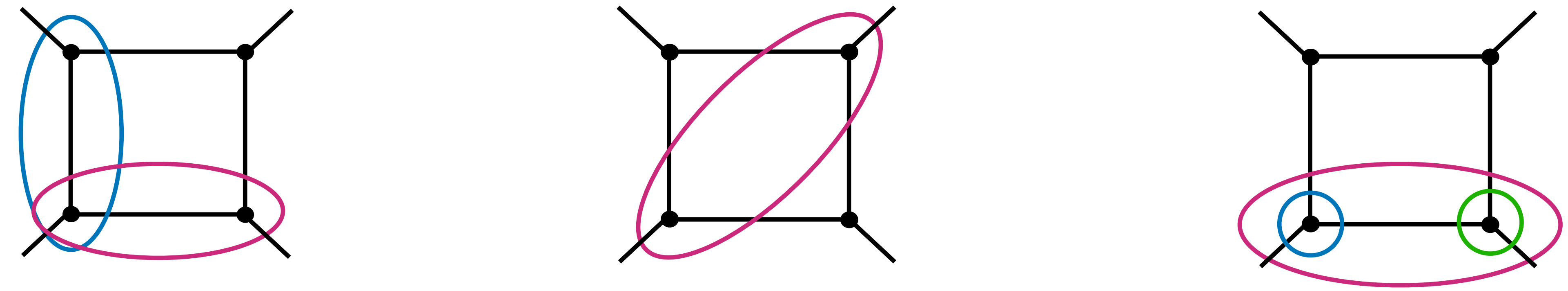

crossing — connectedness — obstruction

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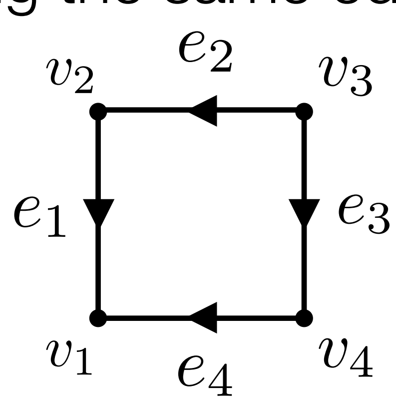
$G =$ 

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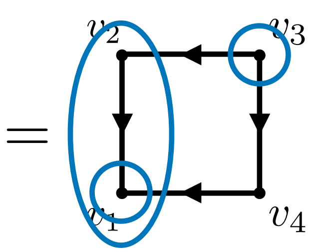
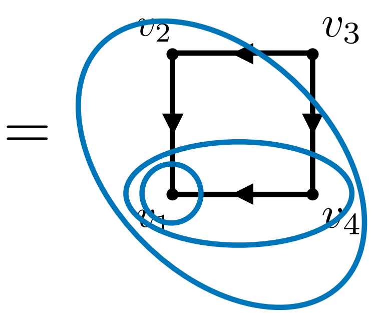
$F_1 =$ 

$F_2 =$ 

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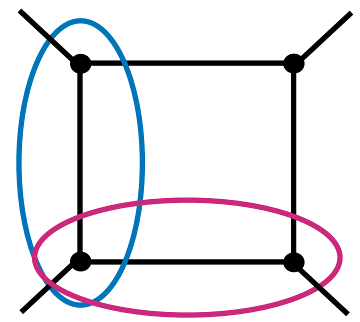

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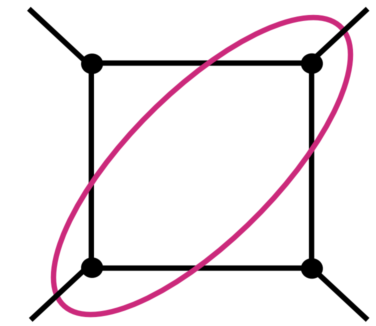
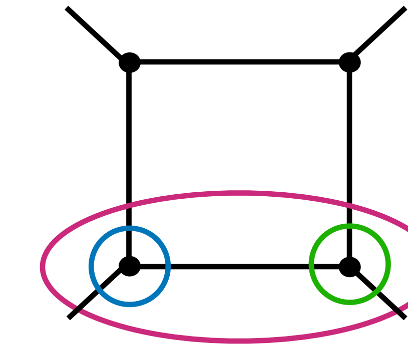


connectedness



obstruction

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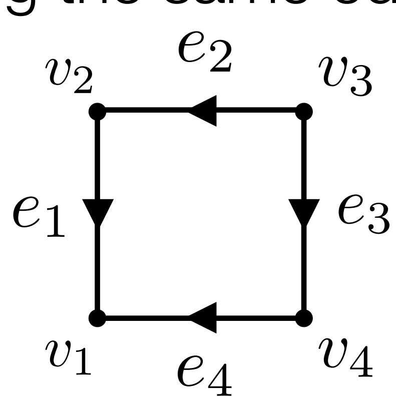
$G =$ 

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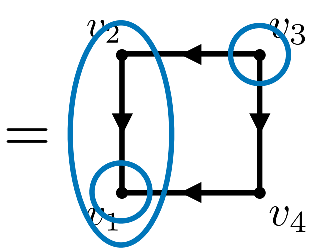
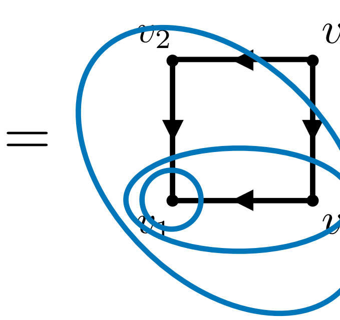
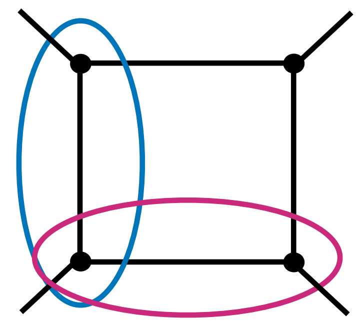
$F_1 =$ 

$F_2 =$ 

Forbidden configurations



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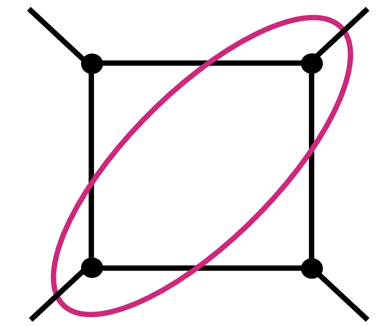

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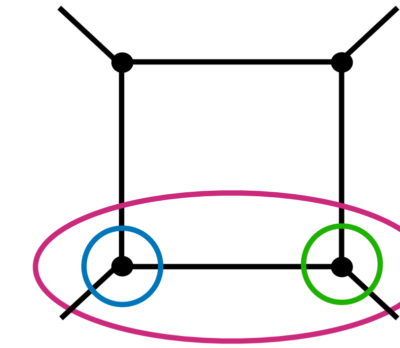

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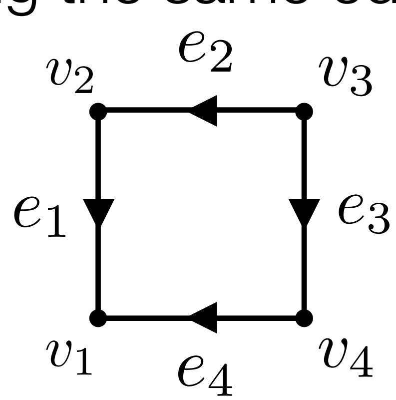

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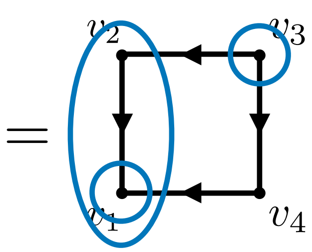
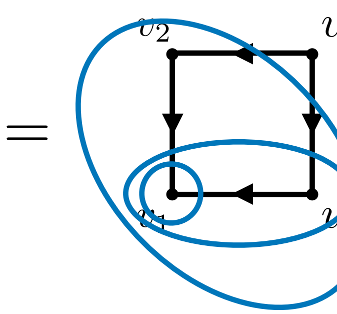


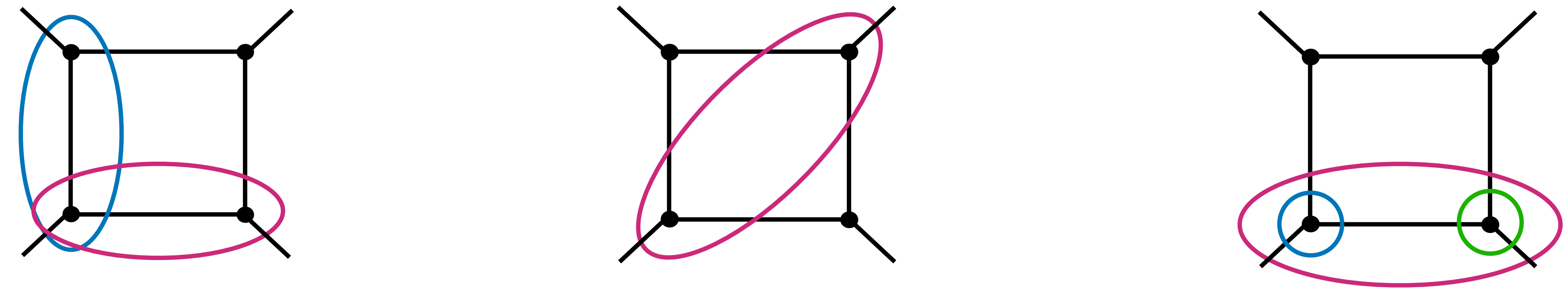
$F_2 =$



Each element of a cross-free family corresponds to a threshold

$$F_1 = \{\{v_1\}, \{v_3\}, \{v_1, v_2\}\}$$

Forbidden configurations



crossing connectedness obstruction

General formula

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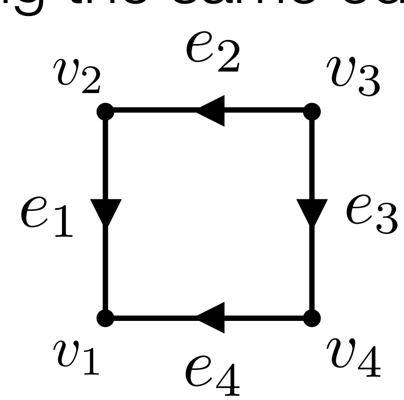
$G =$ 

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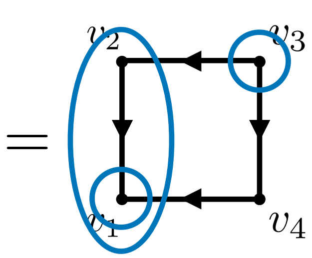
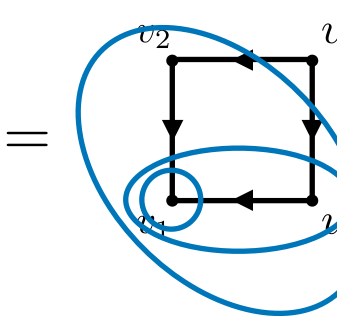
$F_1 =$ 

$F_2 =$ 

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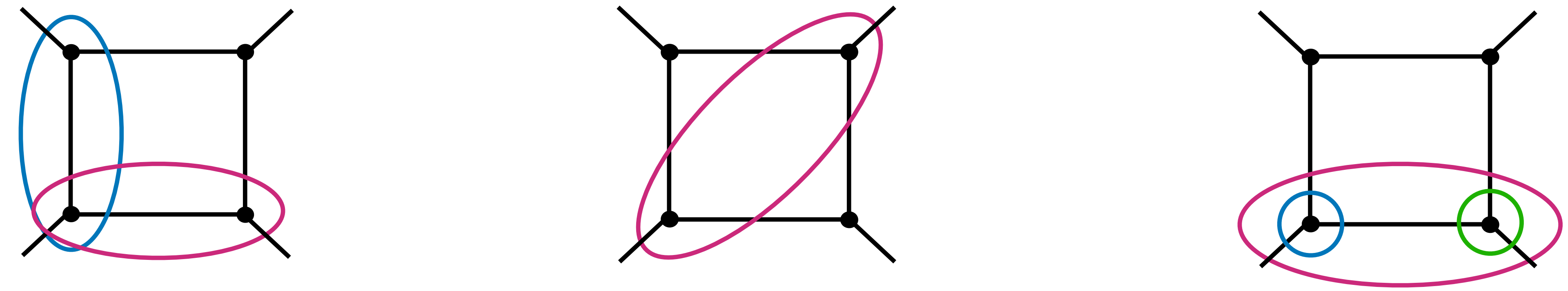

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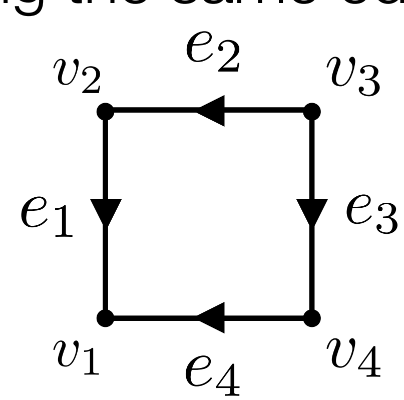
$G =$ 

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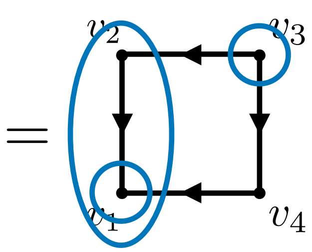
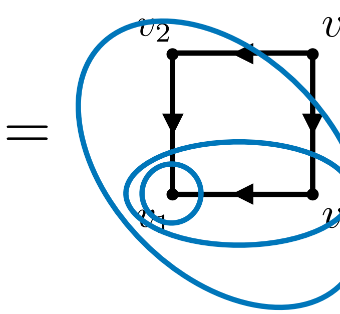
$F_1 =$ 

$F_2 =$ 

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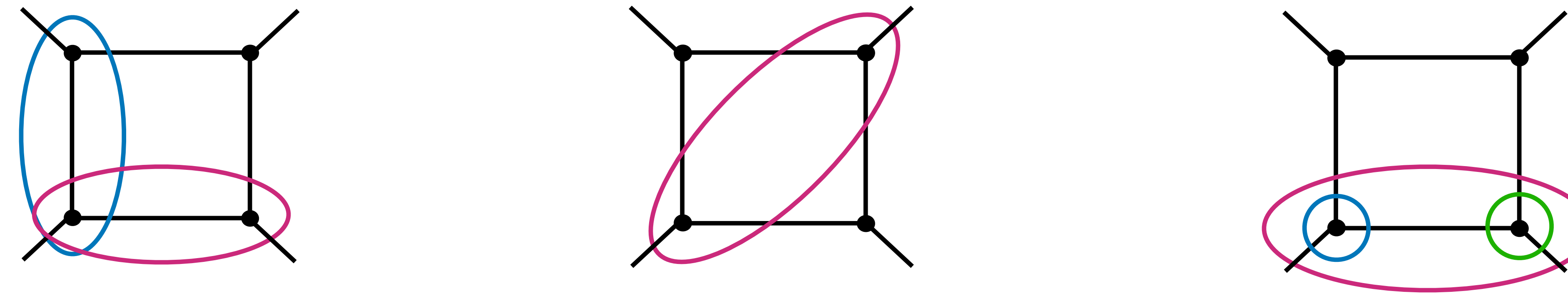

crossing connectedness obstruction

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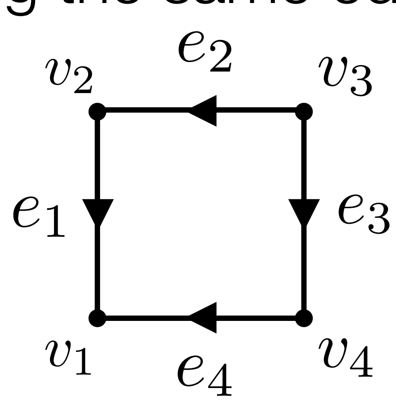
$G =$ 

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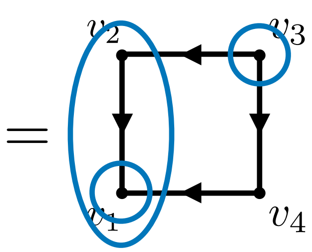
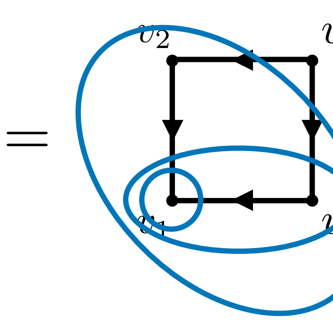
$F_1 =$ 

$F_2 =$ 

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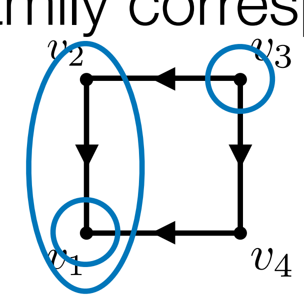
And the cross-free family corresponds to a product of thresholds



$$= \frac{i}{E_1 + E_4 - p_1^0} \frac{i}{E_2 + E_3 + p_3^0} \frac{i}{E_2 + E_4 - p_1^0 - p_2^0}$$

Discontinuities

Discontinuities

Local discontinuities

We can compute discontinuities (Bourjaily, Hannesdottir, McLeod, Schwartz, Vergu [arXiv:2007.13747])

$$\frac{1}{\prod_{S\in F}\left[\boldsymbol{E}\cdot\mathbf{1}^{\partial(S)}-\sum_{v\in S}p_v^0+i\varepsilon\right]}-\frac{1}{\prod_{S\in F}\left[\boldsymbol{E}\cdot\mathbf{1}^{\partial(S)}-\sum_{v\in S}p_v^0-i\varepsilon\right]}=\sum_{S\in F}\frac{\delta\left(\boldsymbol{E}\cdot\mathbf{1}^{\partial(S)}-\sum_{v\in S}p_v^0\right)}{\prod_{S'\in F\setminus\{S\}}\left[\boldsymbol{E}\cdot\mathbf{1}^{\partial(S')}-\sum_{v\in S'}p_v^0\right]}$$

Spurious singularities in TOPT

Discontinuities

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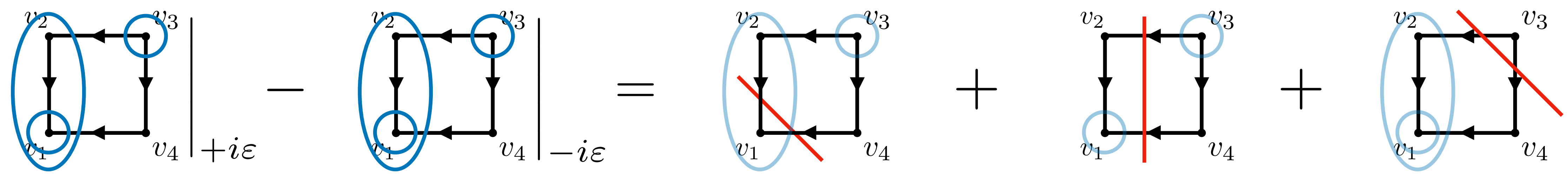

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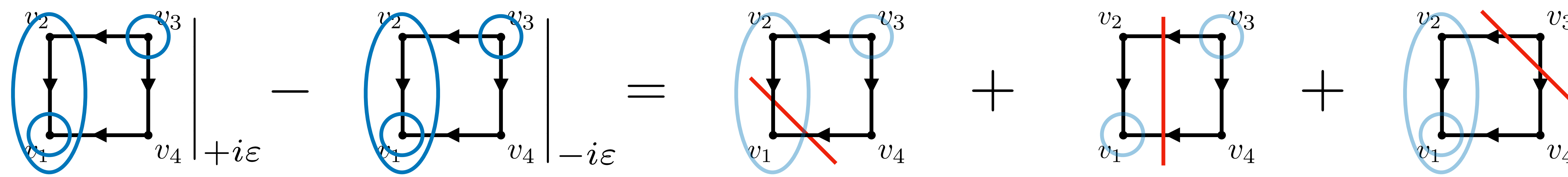
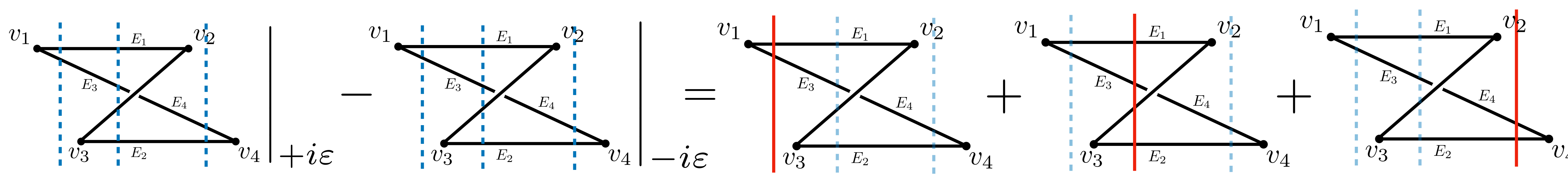


Why use the CFF rep. and not TOPT? Focus on the TOPT term ordering $\{v_1, v_3, v_2, v_4\}$



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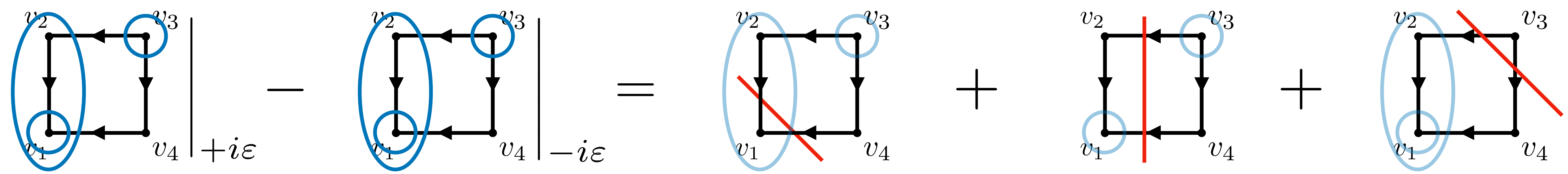

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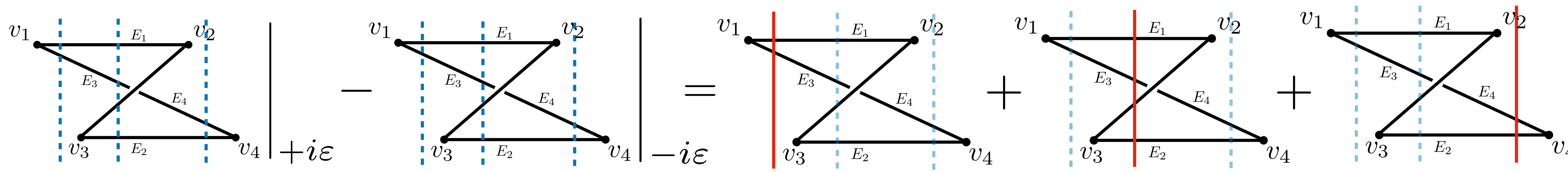
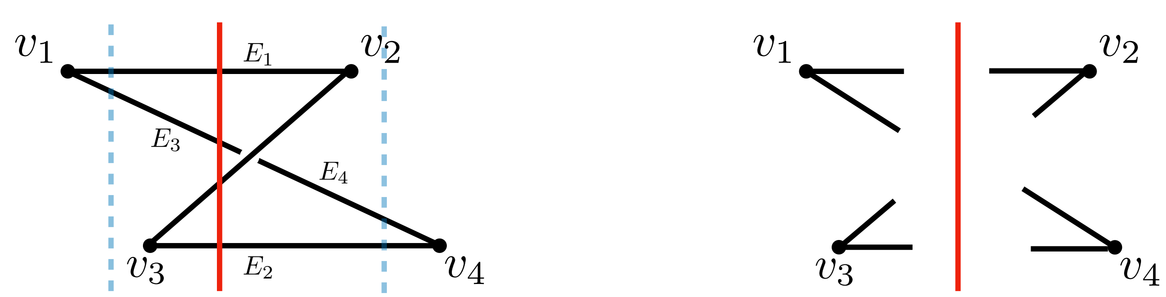


Looking at the second cut



$\delta(E_1+E_2+E_3+E_4-p_1^0-p_3^0)$

Divides the graph in four connected components, but the CFF representation tells us this is not possible! It is a **spurious threshold**

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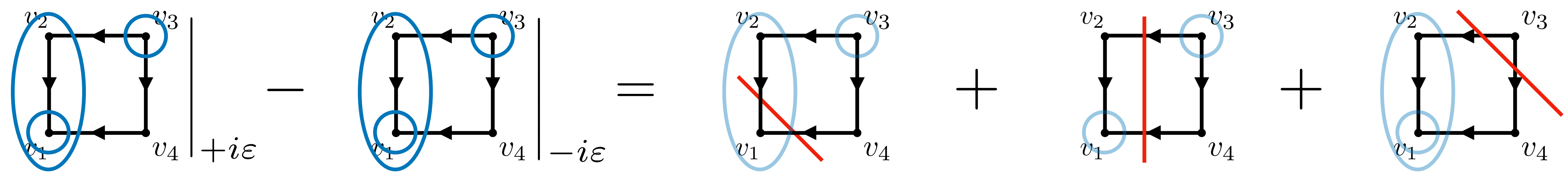

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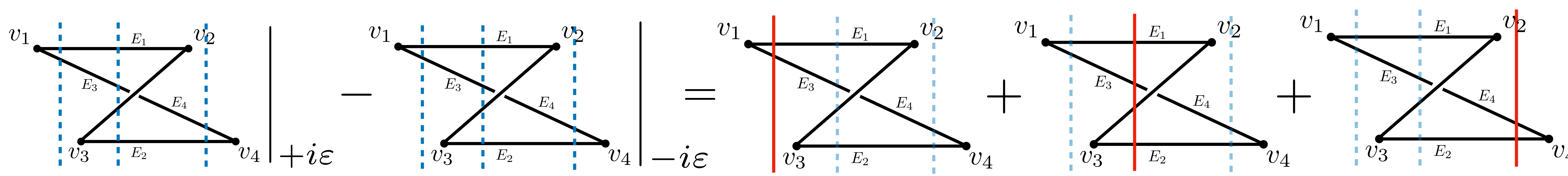

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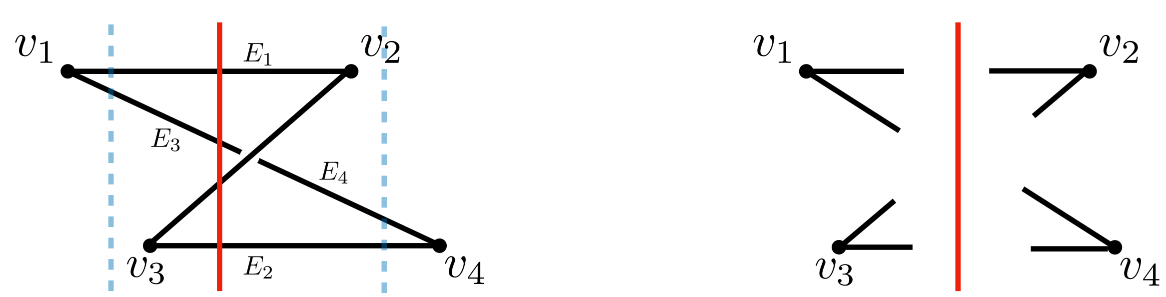

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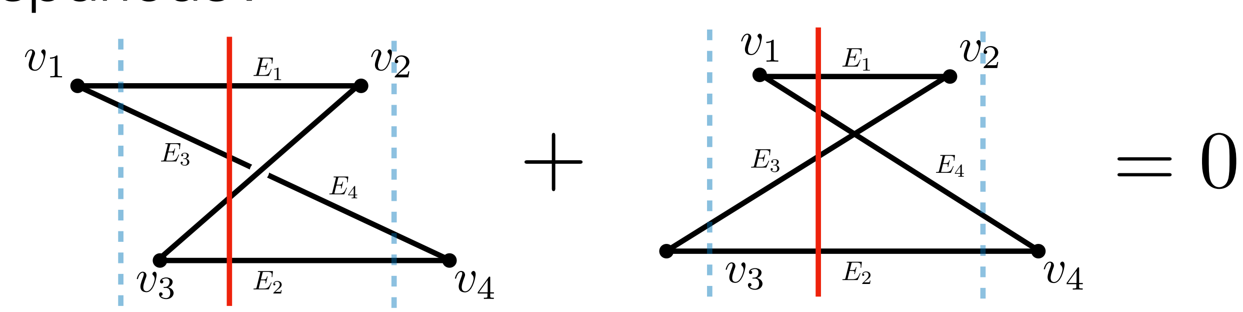
$$\delta(E_1+E_2+E_3+E_4-p_1^0-p_3^0)$$

How do we see that it is spurious?



Factorisation formula

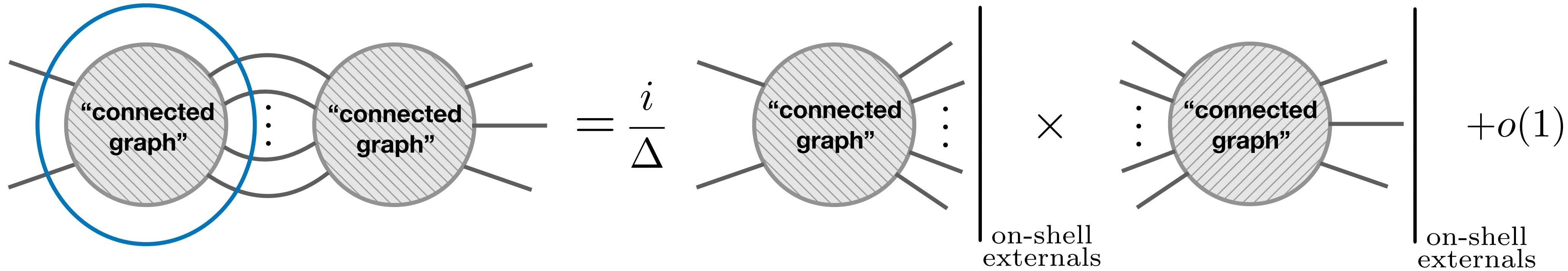
We can see that in general from a diagram-level factorisation formula

Spectators

Factorisation formula

Spectators

We can see that in general from a diagram-level factorisation formula



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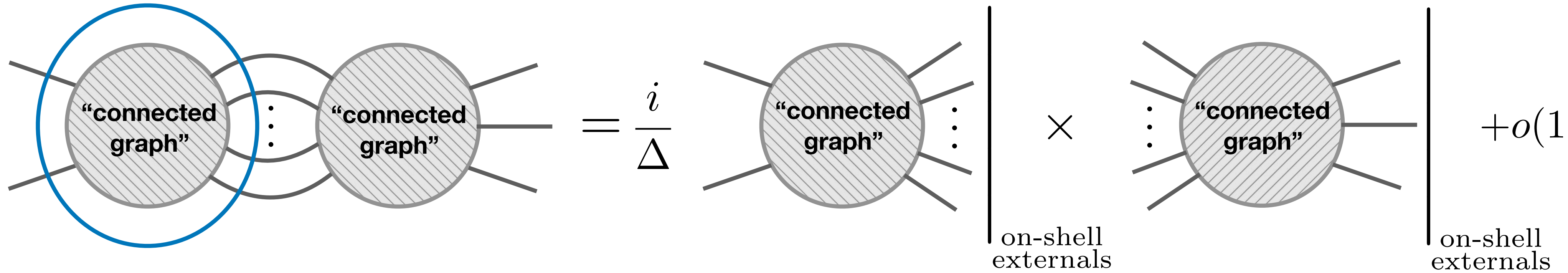

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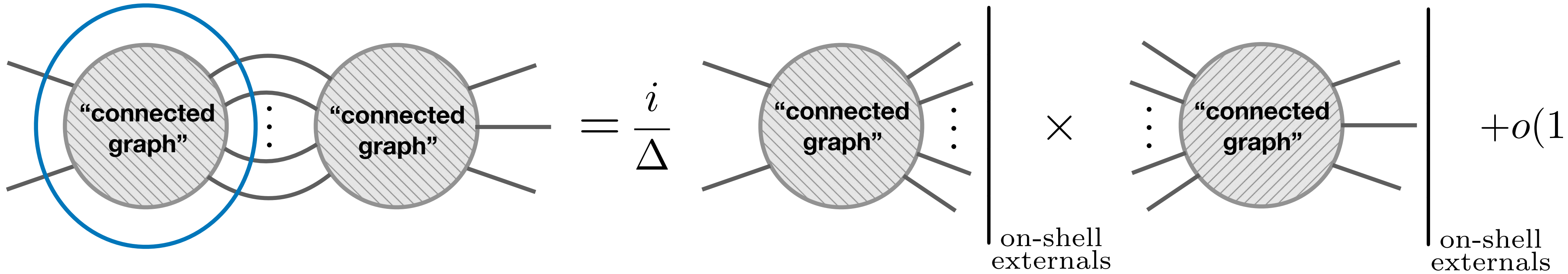

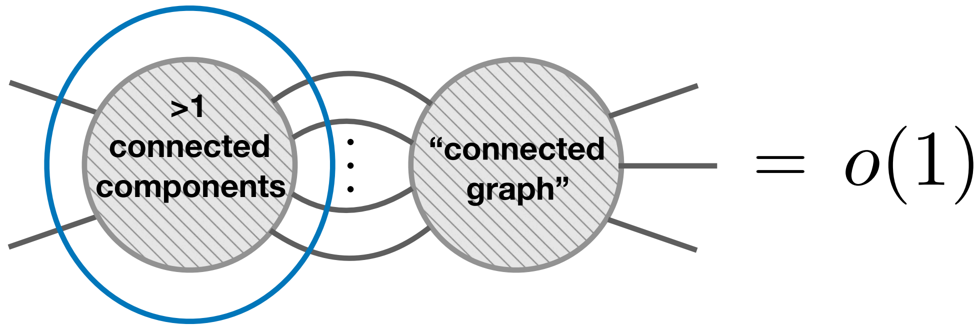

If more than two disconnected components, it is of order one

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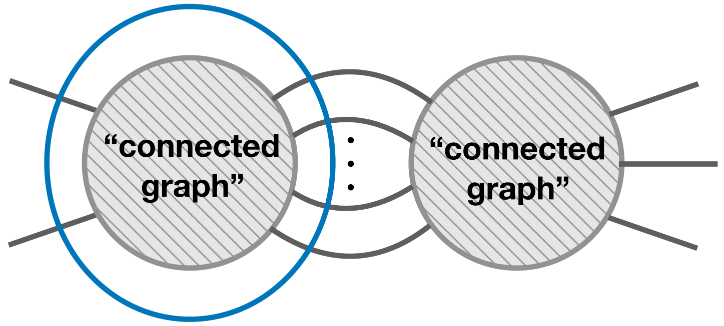
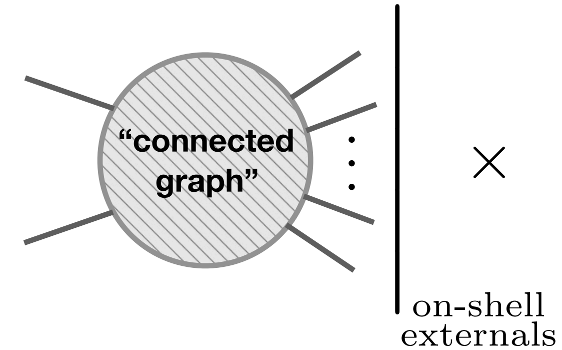
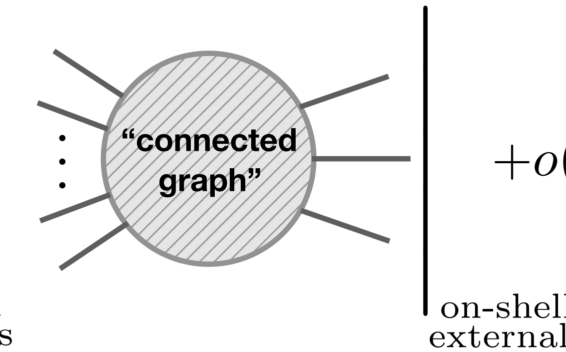


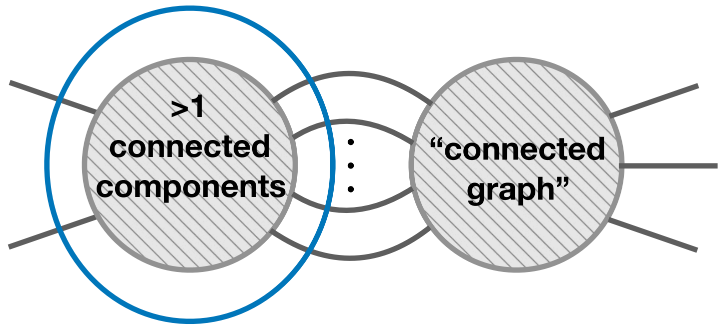
$$= \frac{i}{\Delta}$$



$$\times$$



$$+o(1)$$



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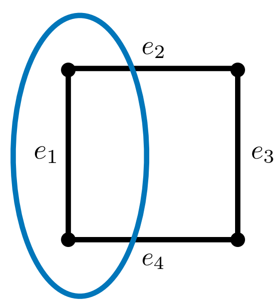
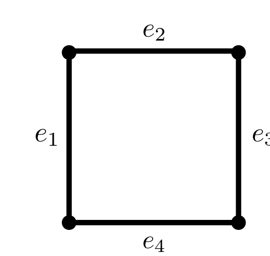


$$\Delta = E_2 + E_4 - p_1^0 - p_4^0 \to 0$$



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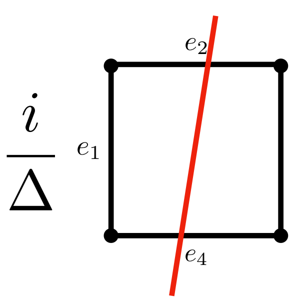

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Factorisation formula

Spectators

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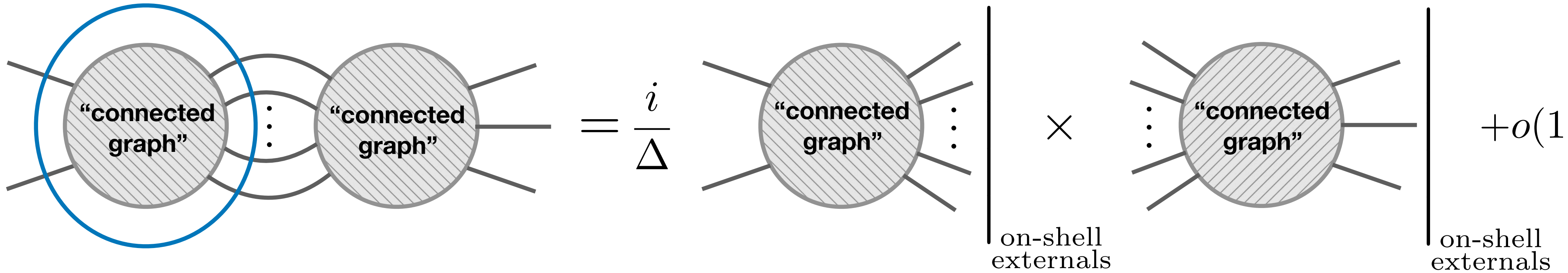

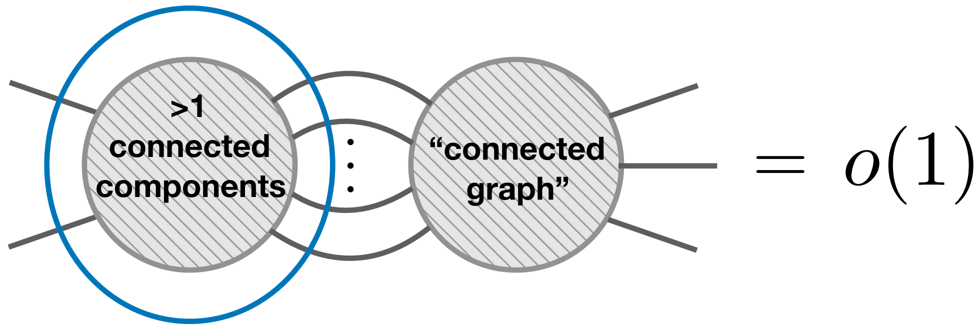

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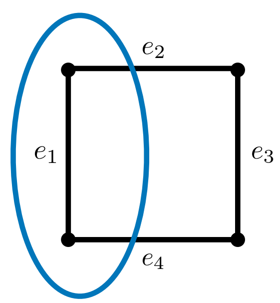

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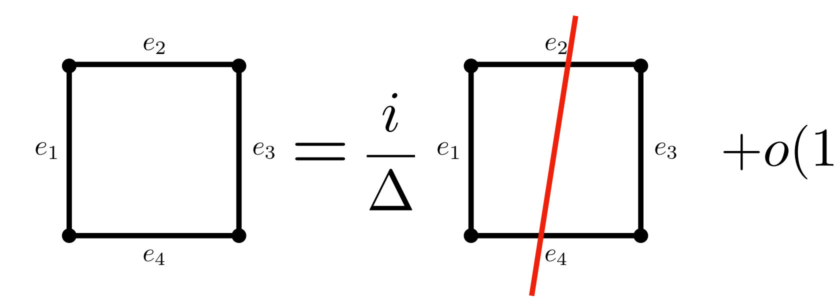

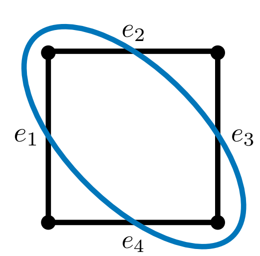

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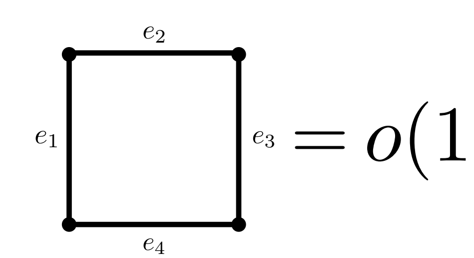

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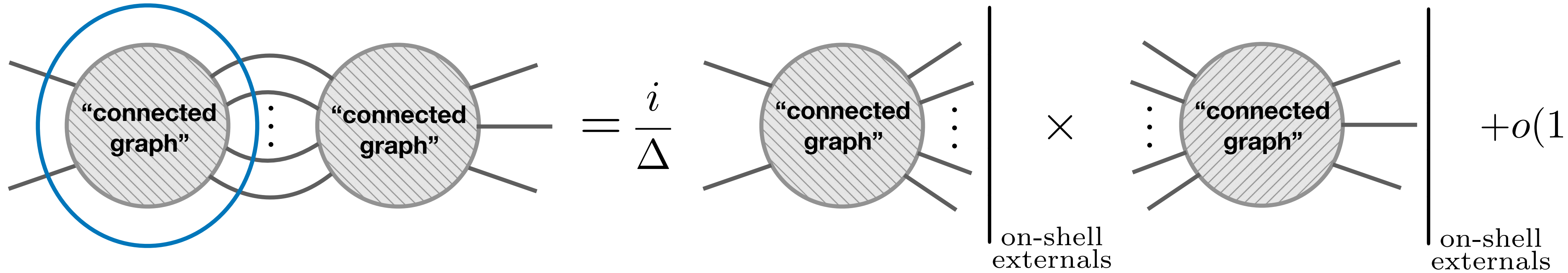

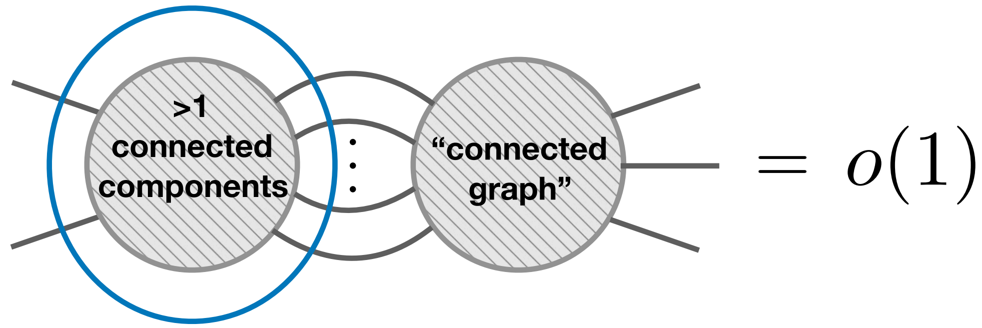

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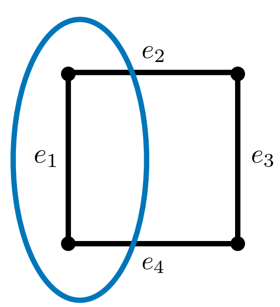

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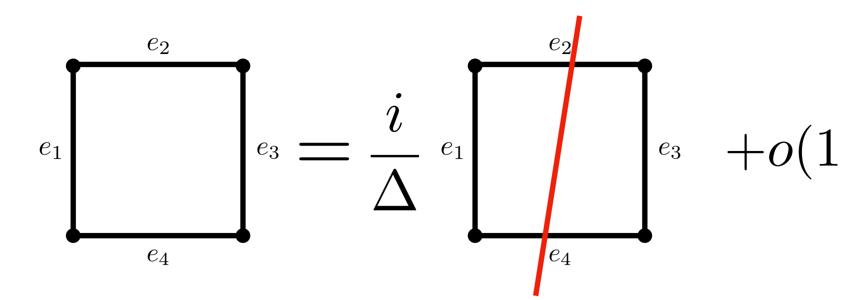

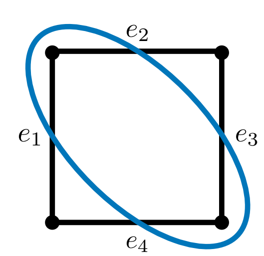

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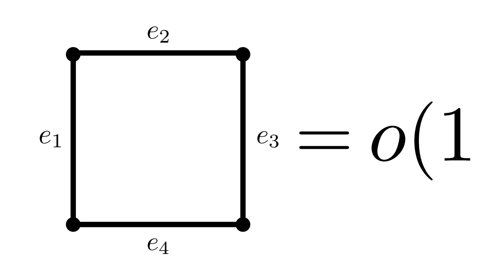
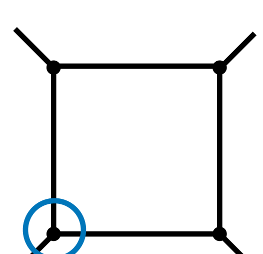


Spectators

What do we really mean by "connected graph"?



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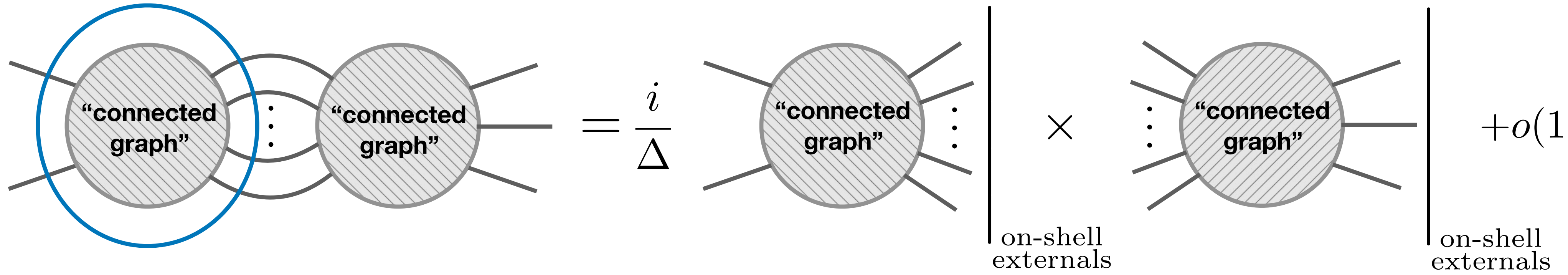

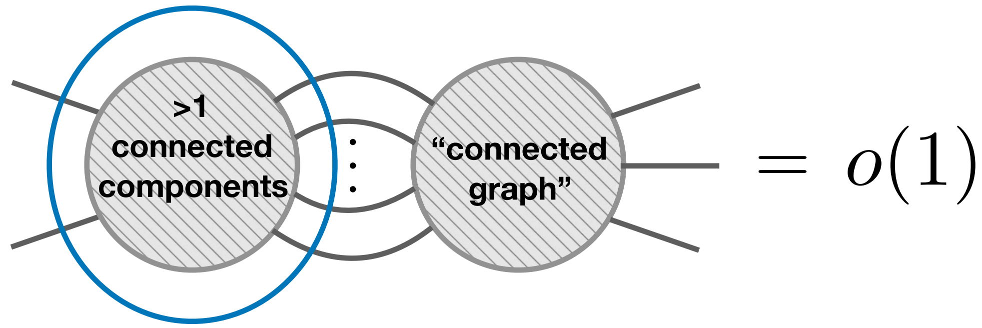

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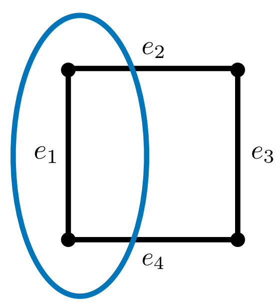

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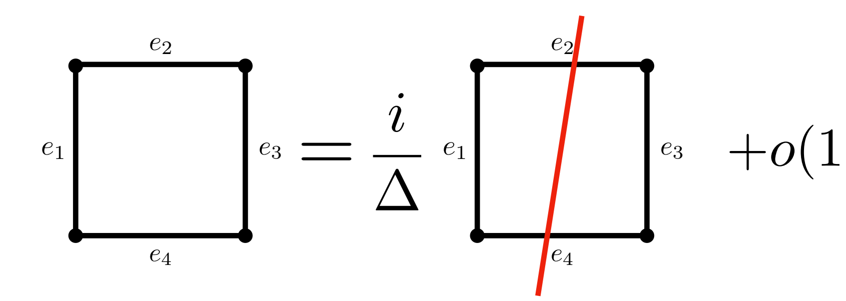

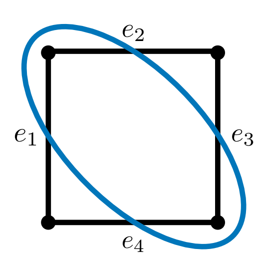

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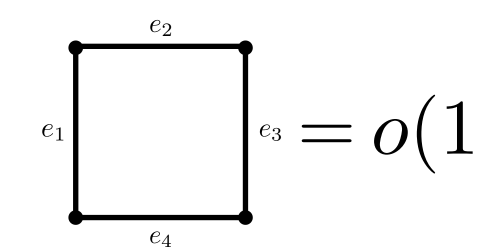

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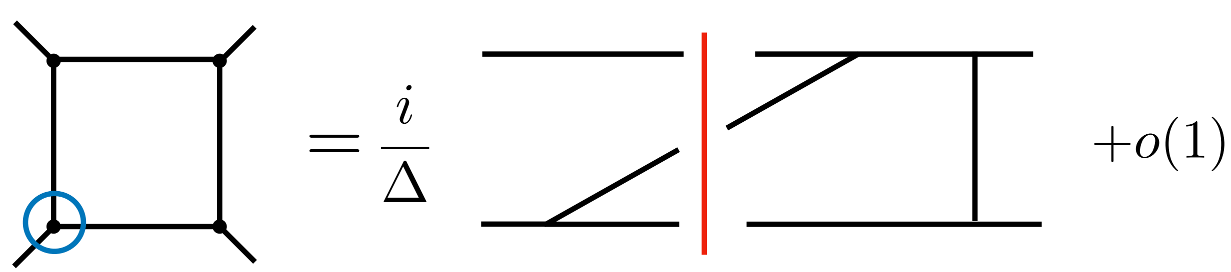

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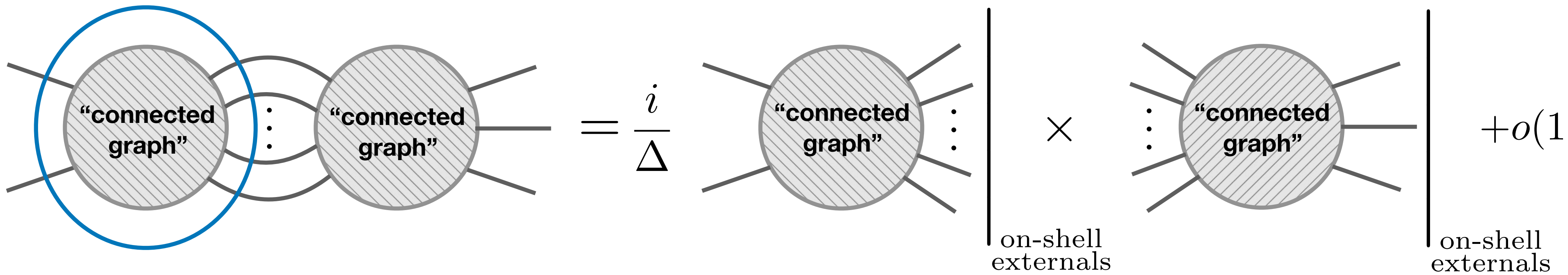

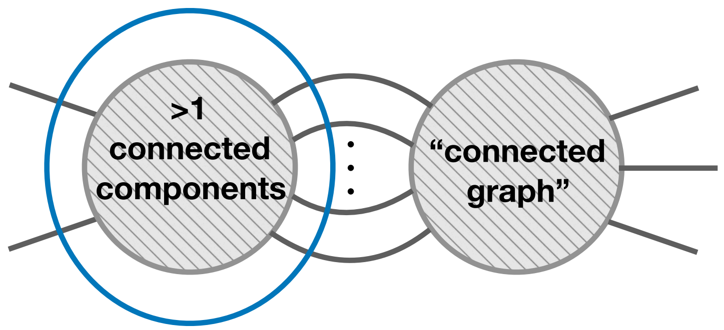

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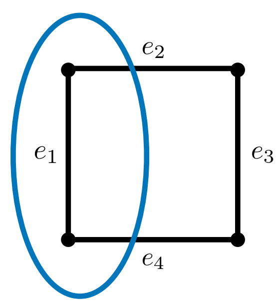

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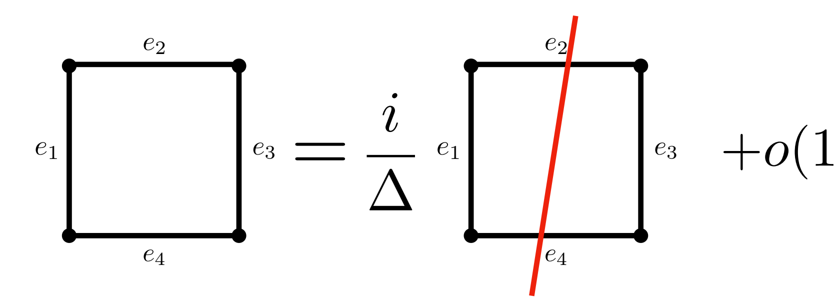

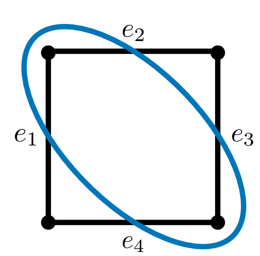

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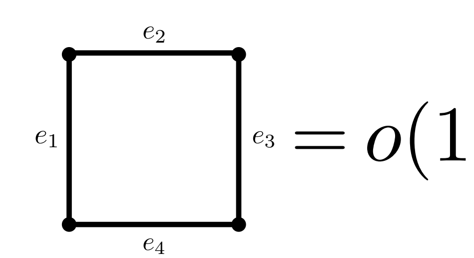

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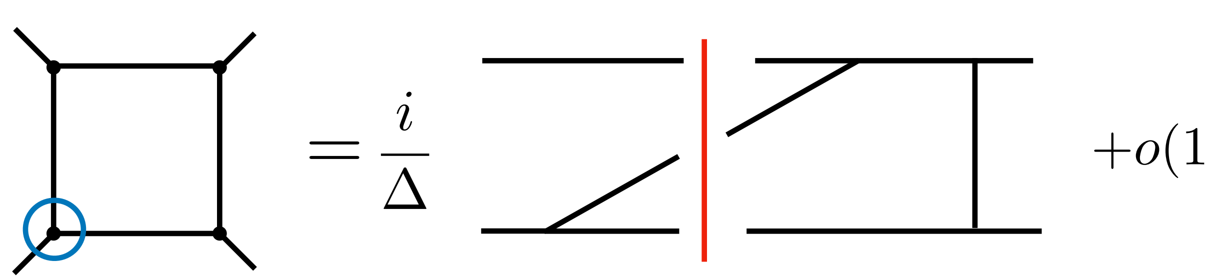
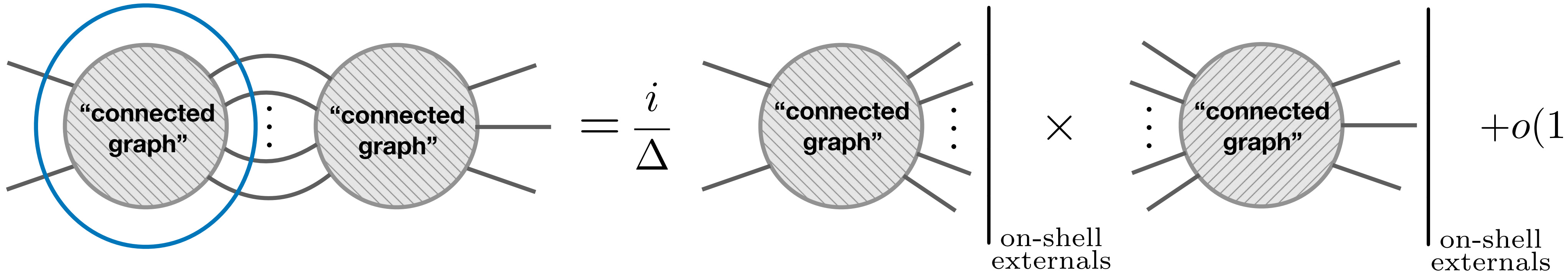
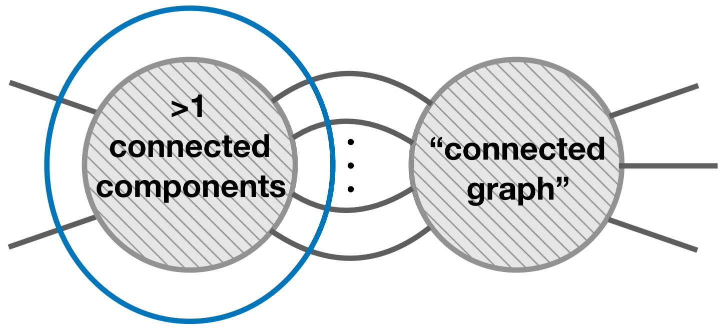


Connected up to spectator particles!

Factorisation formula

We can see that in general from a diagram-level factorisation formula





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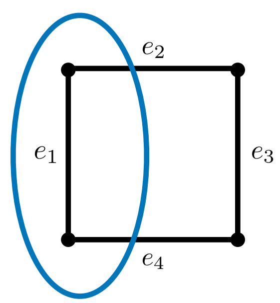

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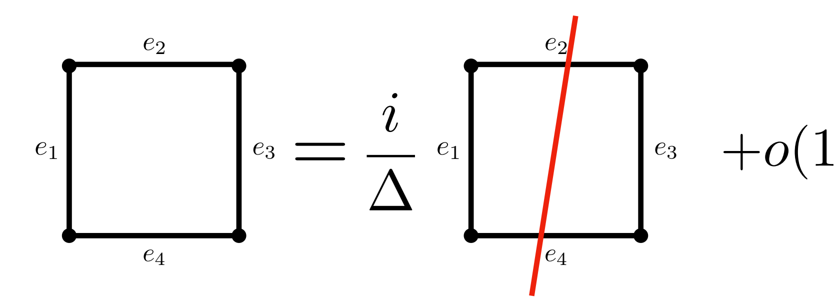

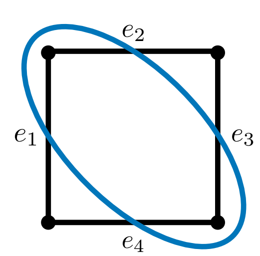

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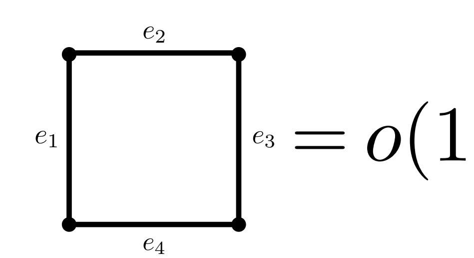

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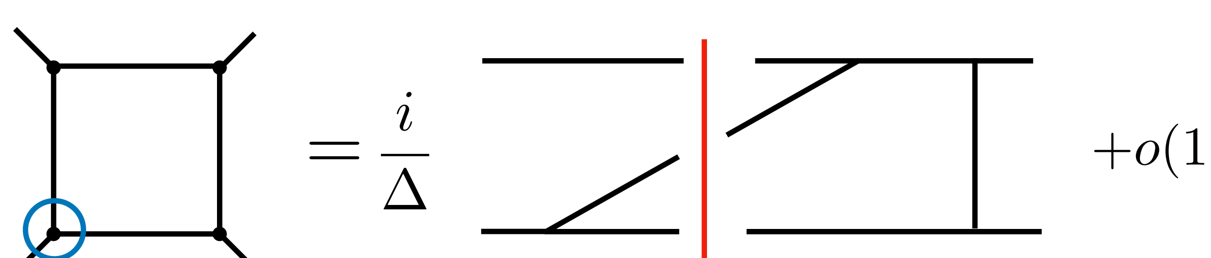

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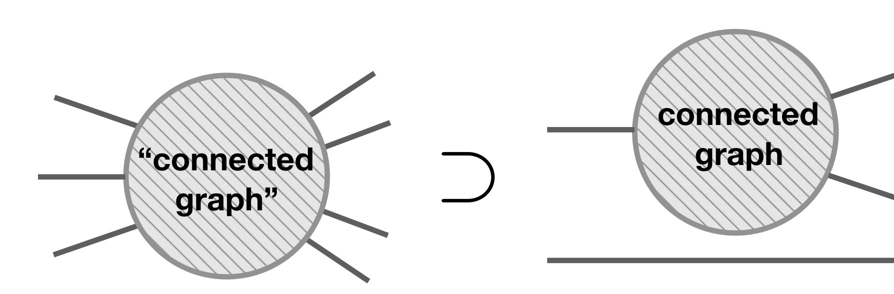
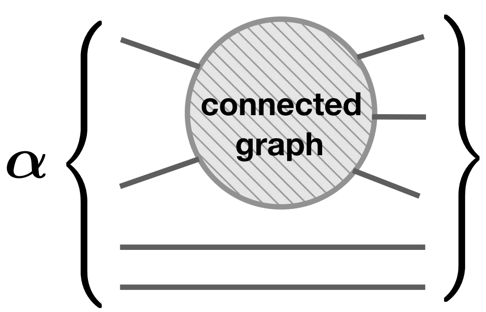


Second part: cluster decomposition and infrared finiteness

- Highlight role of connectedness at the operator level

$$\langle \boldsymbol{\alpha} | \mathbf{T}_{\mathrm{c}} | \boldsymbol{\beta} \rangle \supset \boldsymbol{\alpha}$$



$$\boldsymbol{\beta}$$

Reconstruct unitarity, cluster decomposition principle and infrared finiteness from the diagrammatic analysis

- Use Local Unitarity methods to take advantage of this analysis to numerically evaluate cross-sections

Connected transition matrix

S-matrix

“Unitarity”

Cluster Decomposition

Connected transition matrix

S-matrix

“Unitarity”

Cluster Decomposition

Can we express the role of connectedness at the operator level? Define the connected transition matrix

Cluster Decomposition

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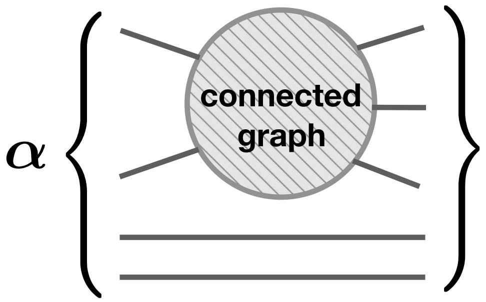

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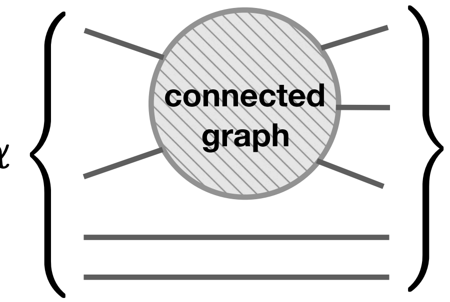

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The connected transition matrix is not unitary (expected). What is the relationship with S-matrix?

Connected transition matrix

S-matrix

“Unitarity”

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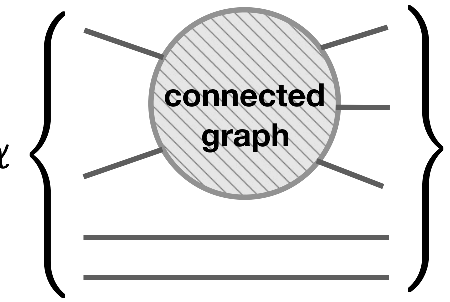

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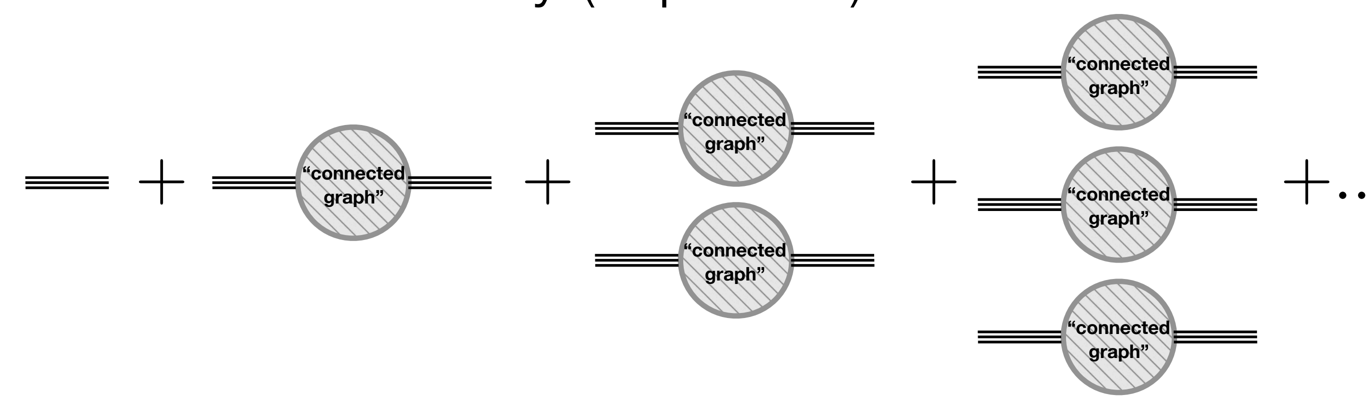
$$\mathbf{S} =$$



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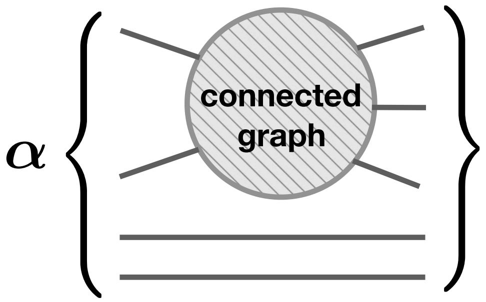

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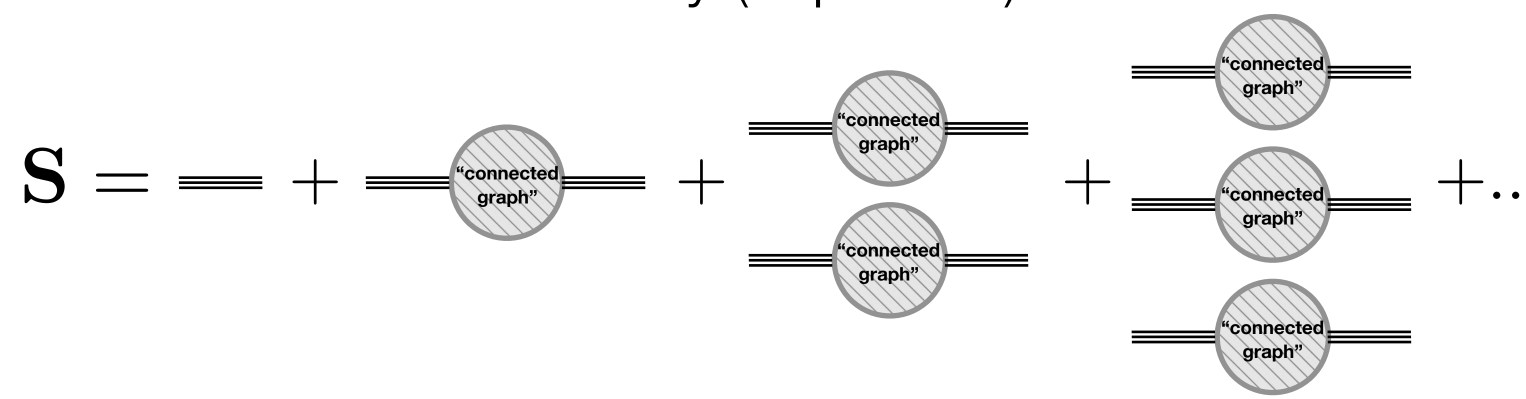

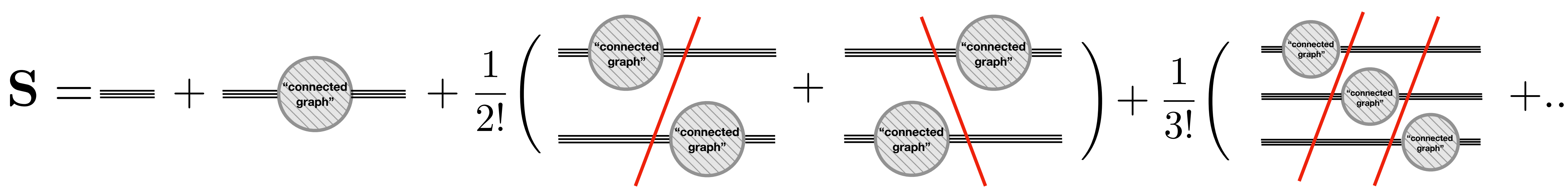

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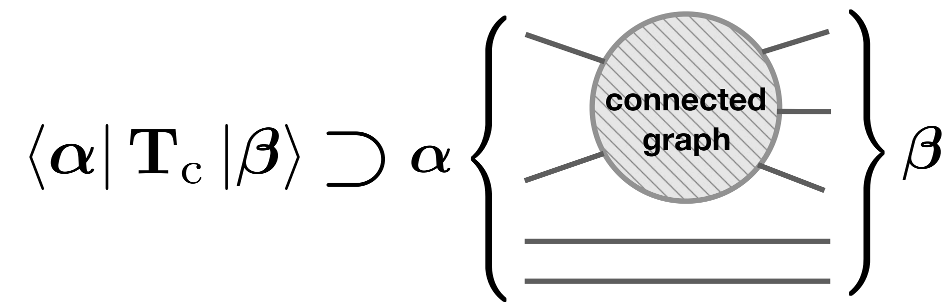

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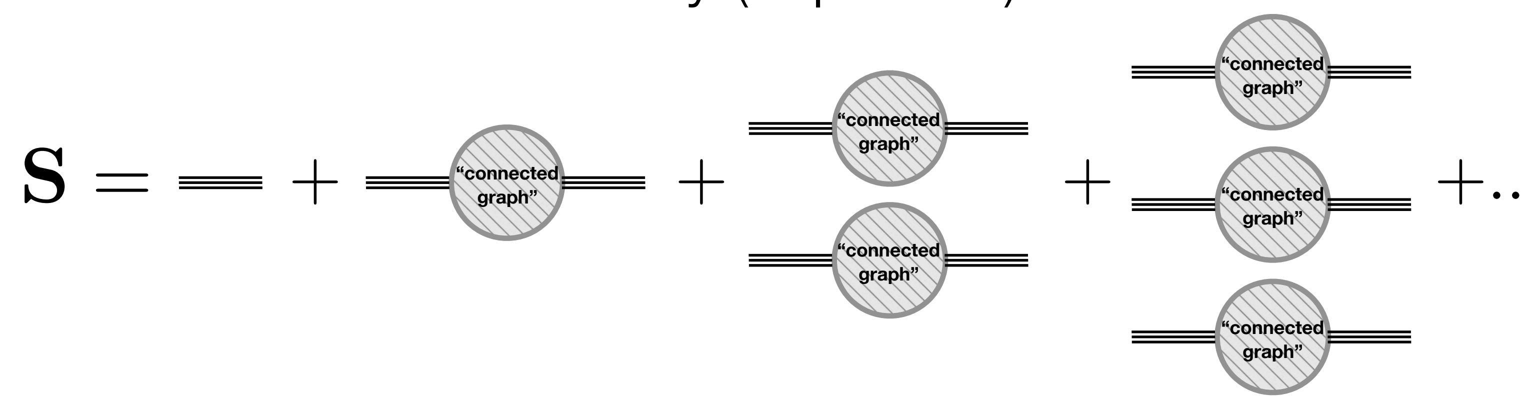

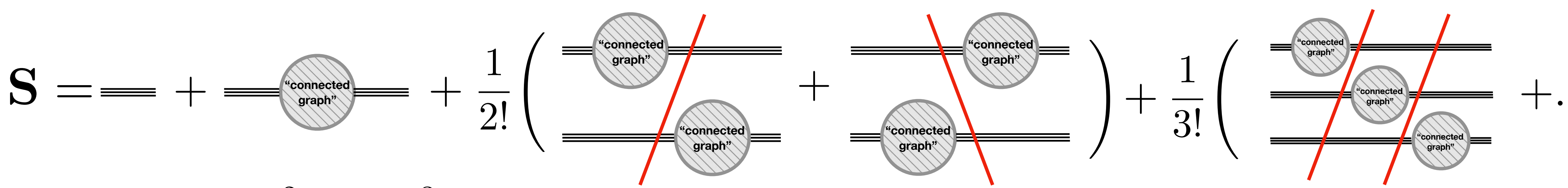

$$\mathbf{S} = 1 + i\mathbf{T}_c + \frac{i^2}{2!}\mathbf{T}_c^2 + \frac{i^3}{3!}\mathbf{T}_c^3 + \ldots = e^{i\mathbf{T}_c}$$

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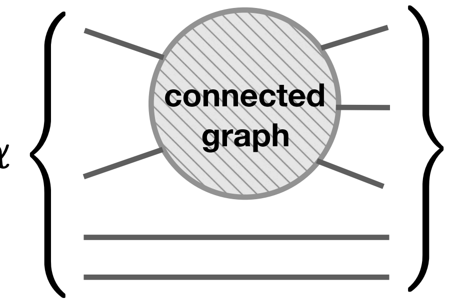
$$\langle \boldsymbol{\alpha} | \, \mathbf{T}_{\rm c} \, | \boldsymbol{\beta} \rangle \supset \boldsymbol{\alpha}$$  $$\boldsymbol{\beta}$$

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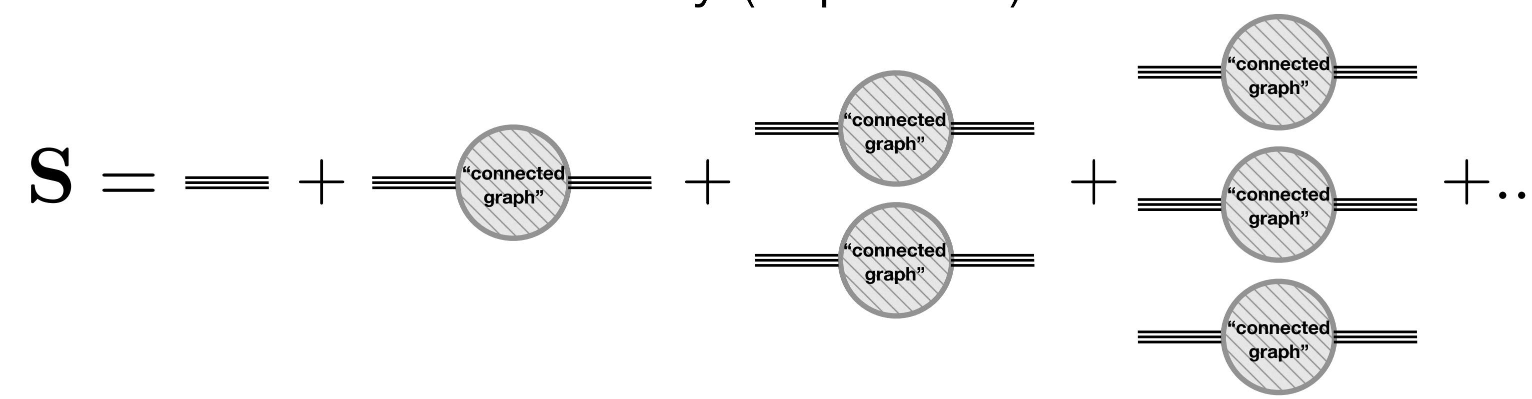

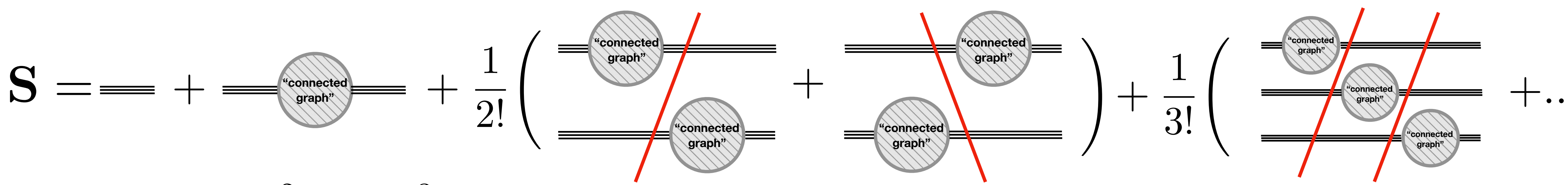

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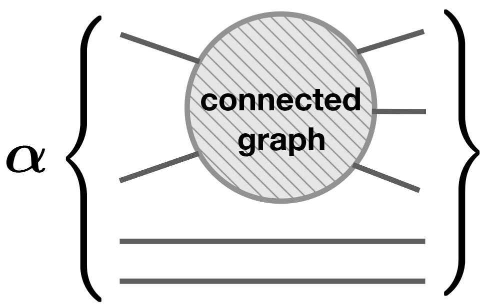

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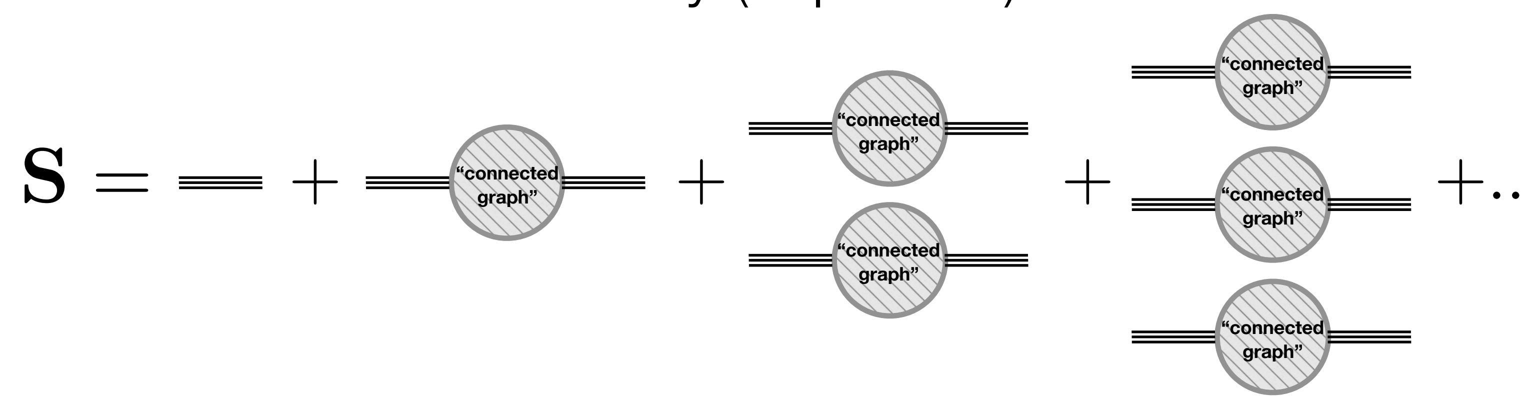

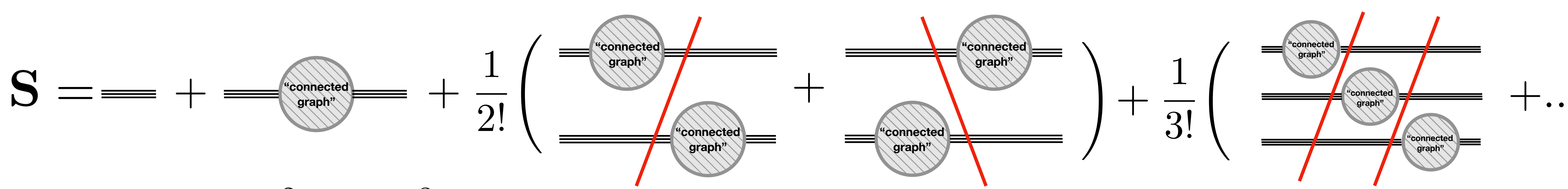

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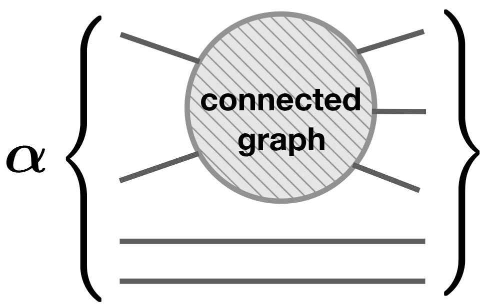
$\langle \boldsymbol{\alpha} | \, \mathbf{T}_{\rm c} \, | \boldsymbol{\beta} \rangle \supset \boldsymbol{\alpha}$  $\boldsymbol{\beta}$

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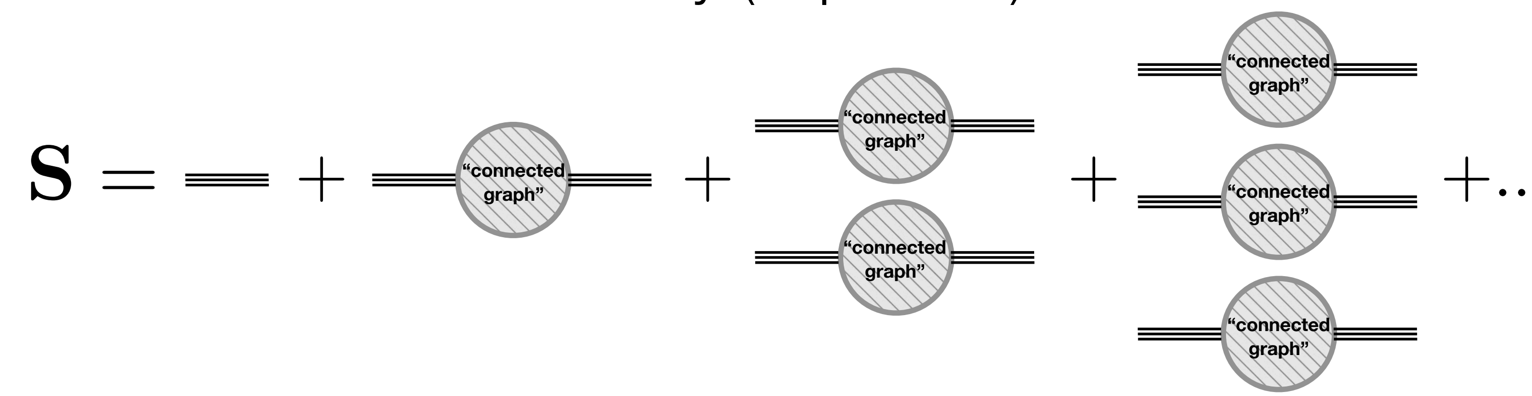

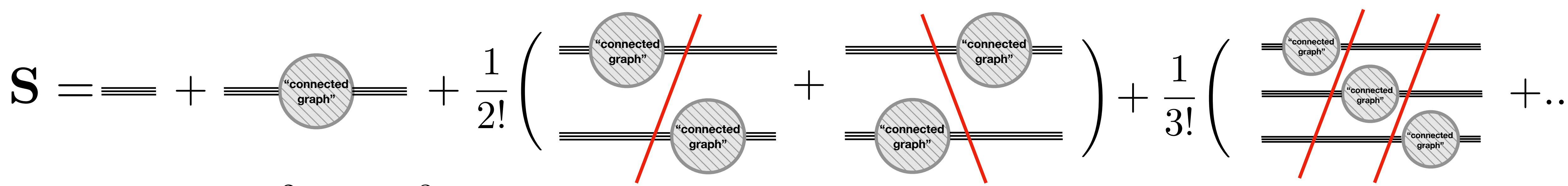

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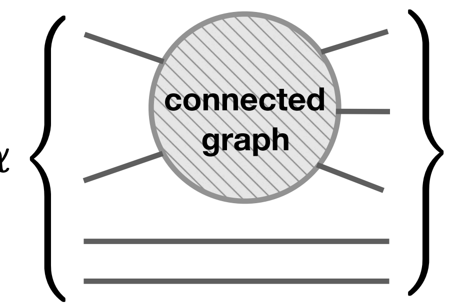
$$\langle \boldsymbol{\alpha} | \, \mathbf{T}_{\rm c} \, | \boldsymbol{\beta} \rangle \supset \boldsymbol{\alpha}$$ 

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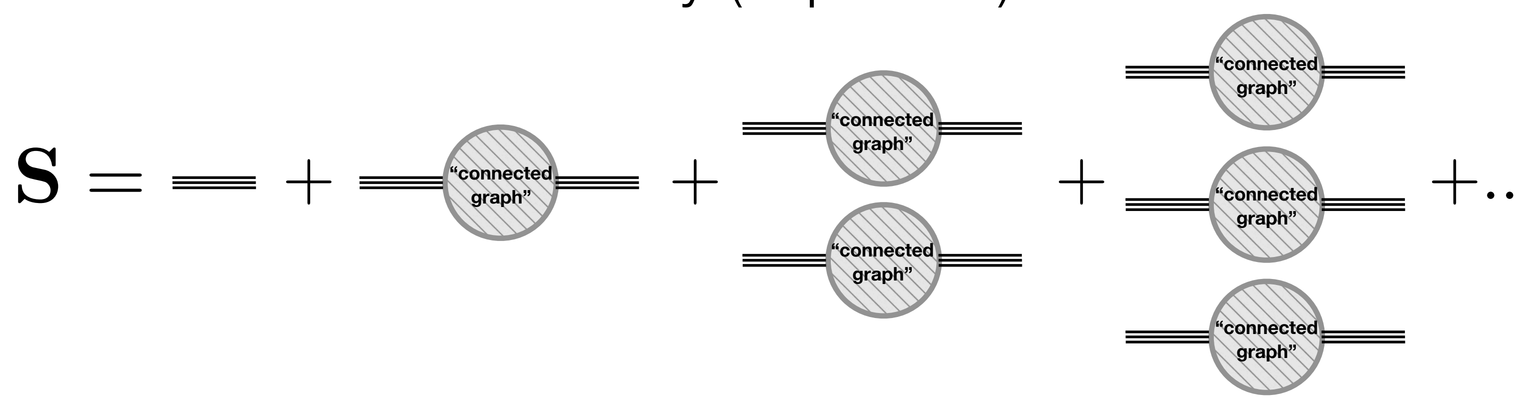

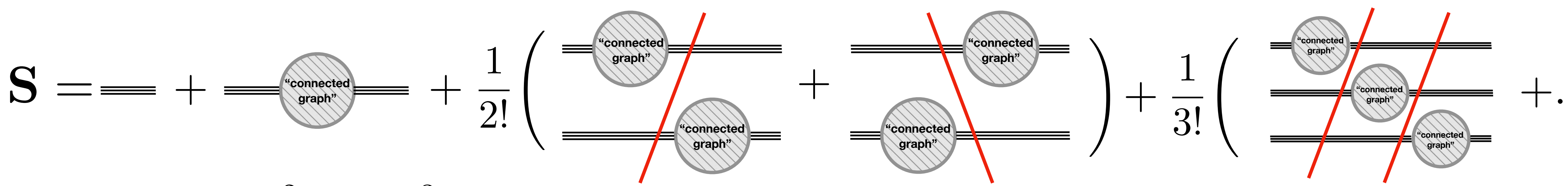

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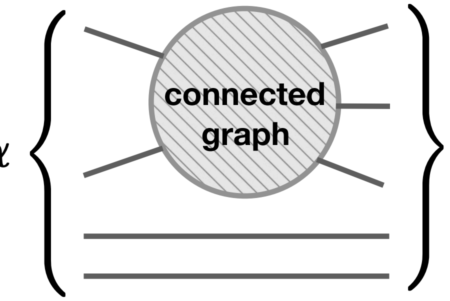

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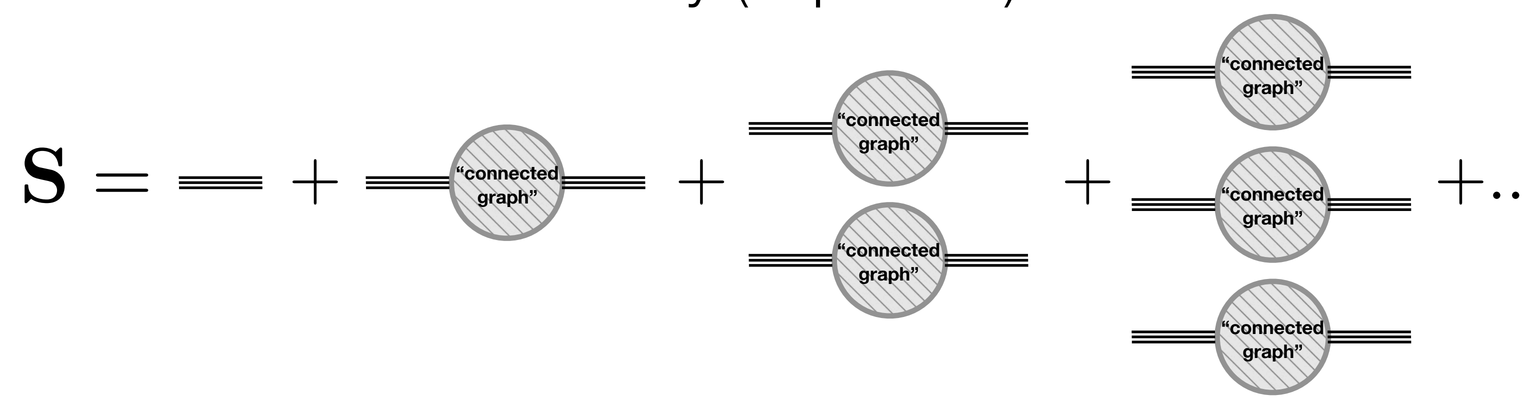

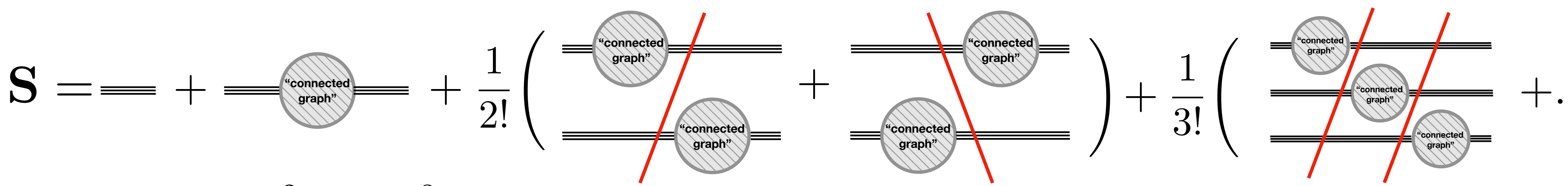

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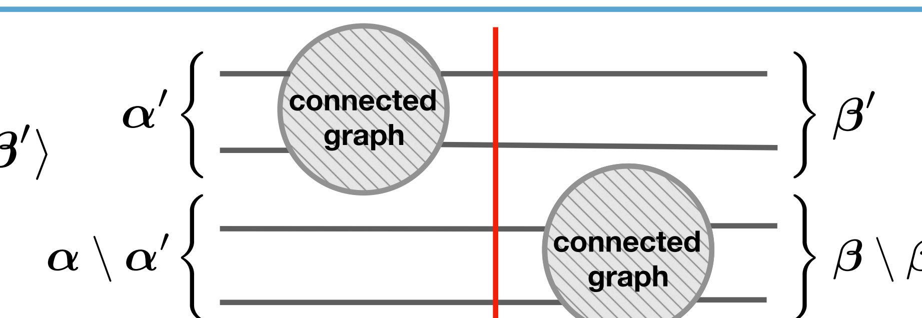

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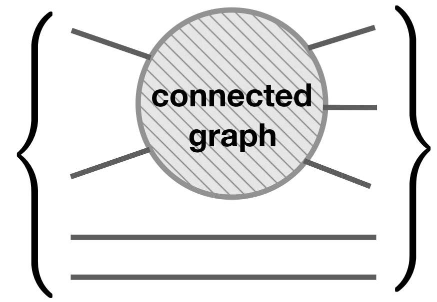

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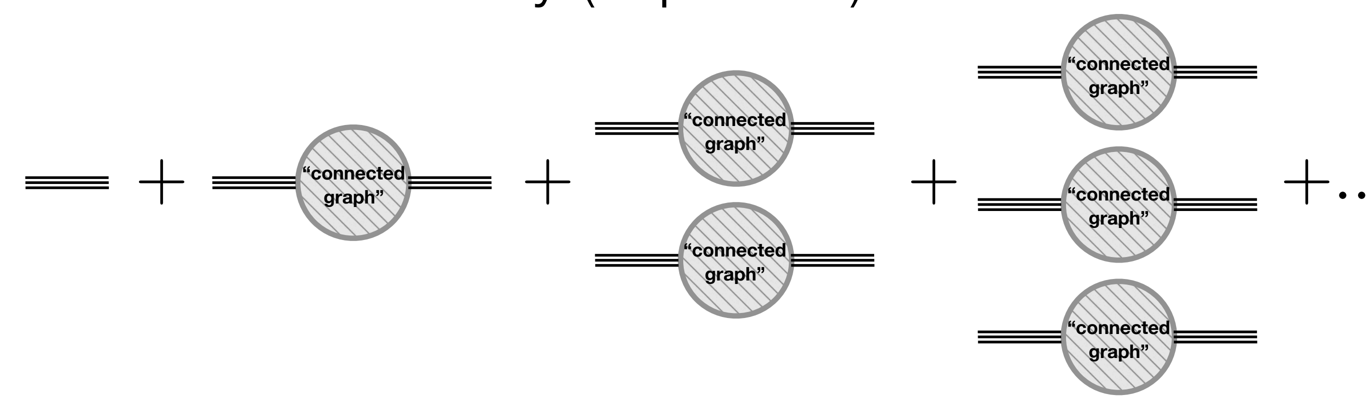

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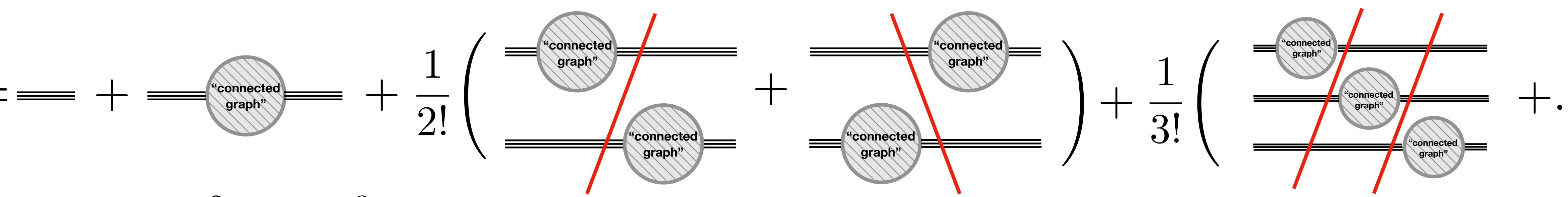

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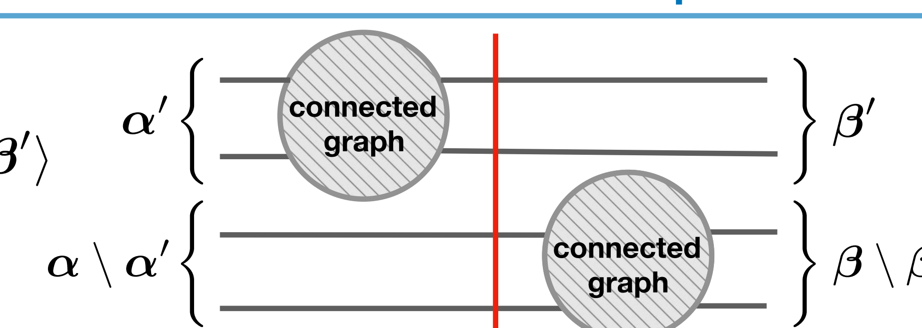

Factorisation formula expressed at the operator level!

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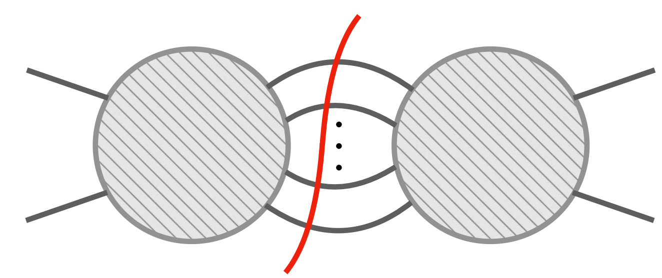

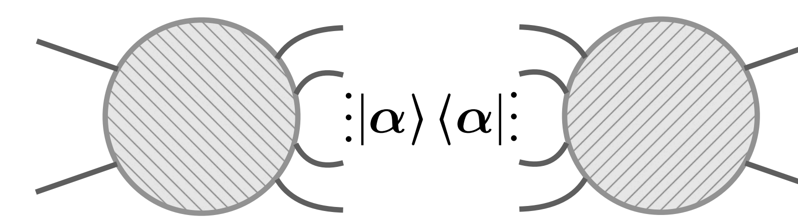

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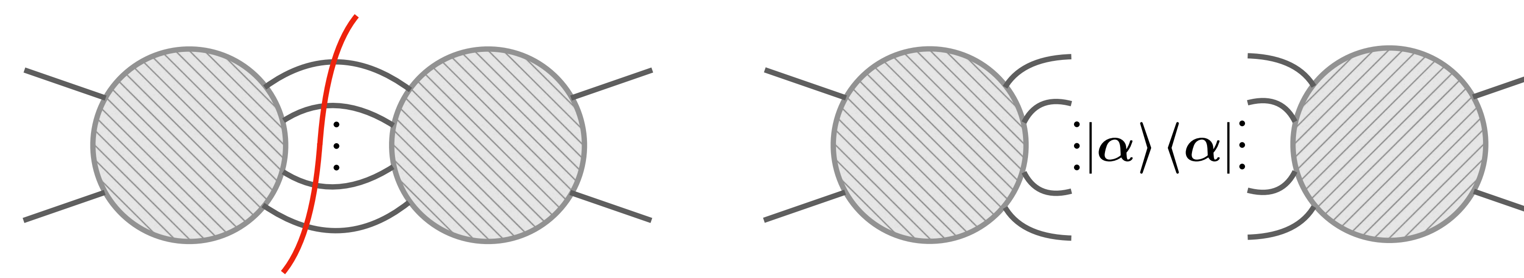

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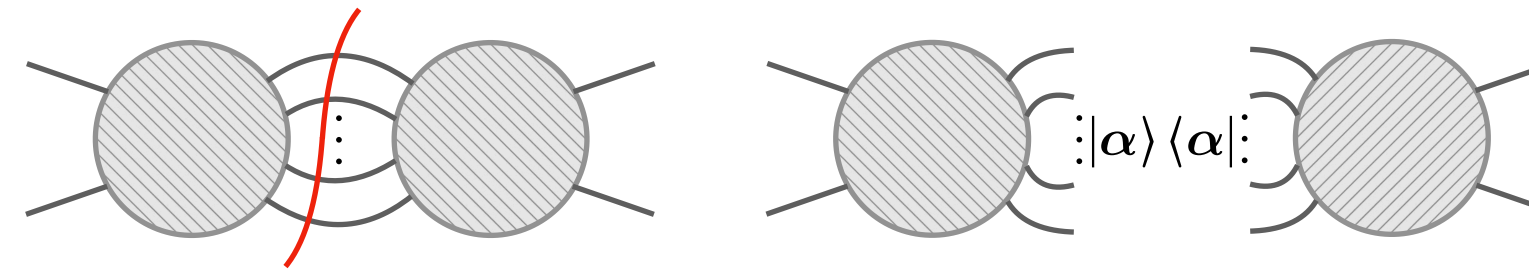

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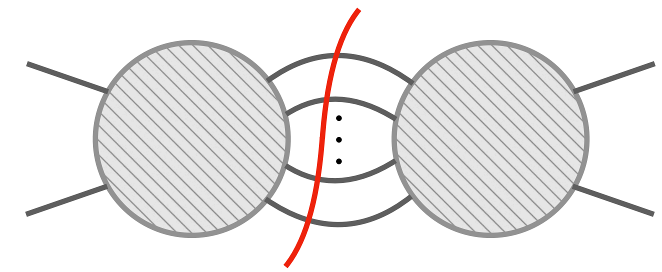

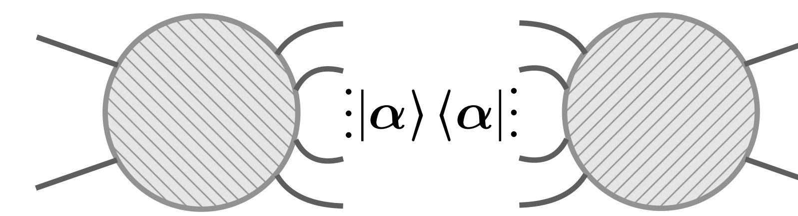

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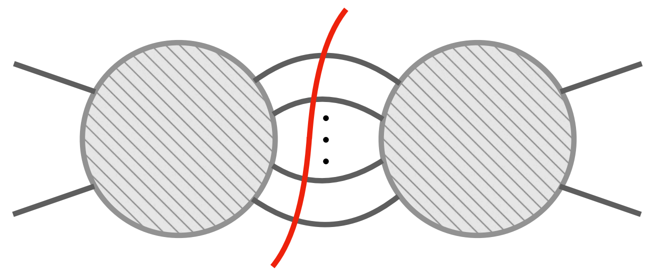

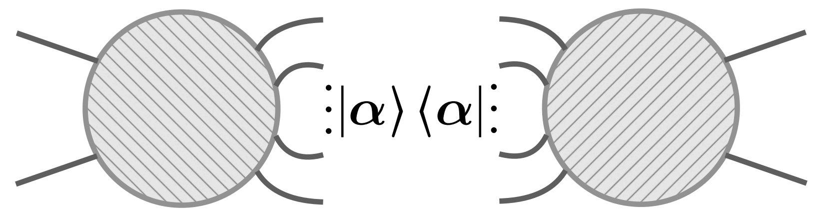

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Infrared-finiteness follows from the unitarity relation we showed in the previous slide. But we can also look at it at a diagrammatic level.

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(Capatti, Hirschi, Ruijl, Pelloni [arXiv:2010.01068])

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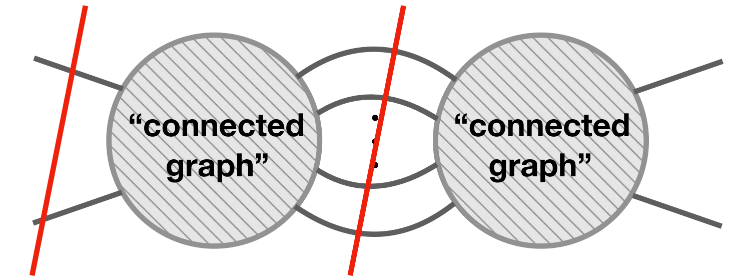

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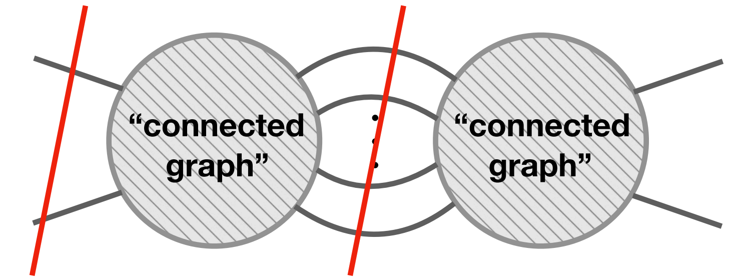

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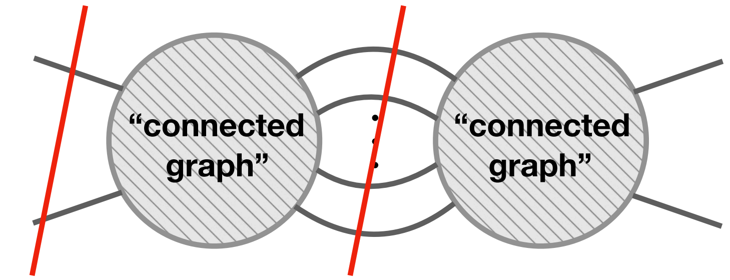

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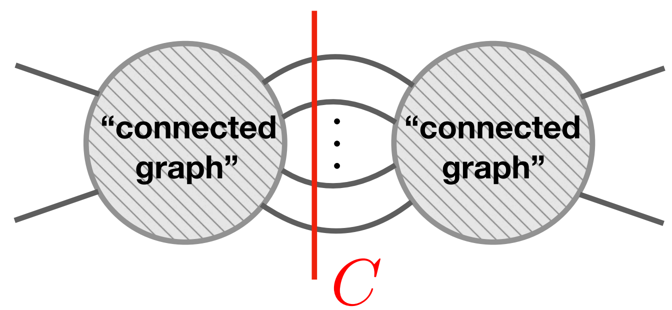
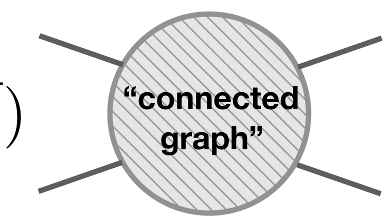


$$= \lim_{\sqrt{s}\to E_C} (E_C - \sqrt{s})$$



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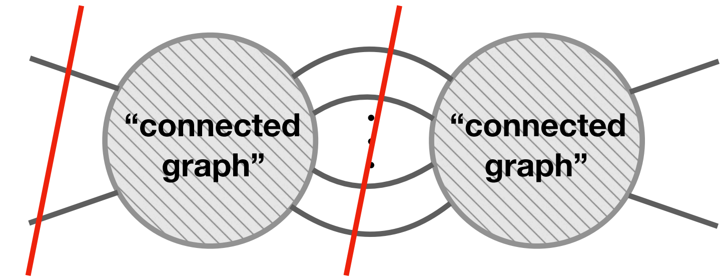

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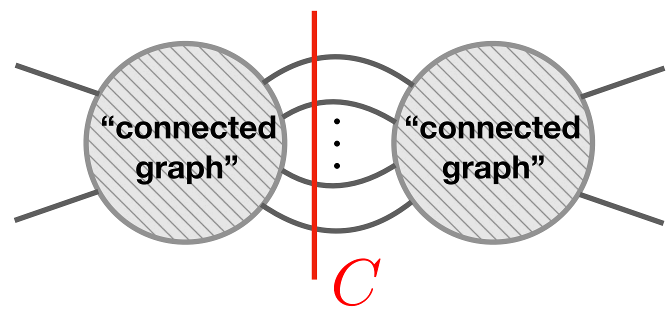

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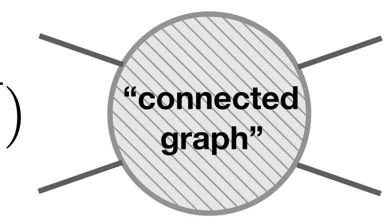

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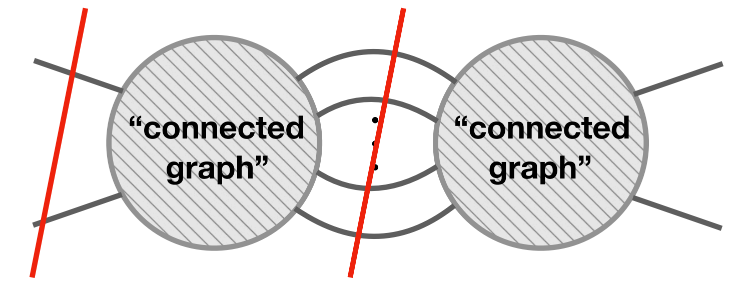

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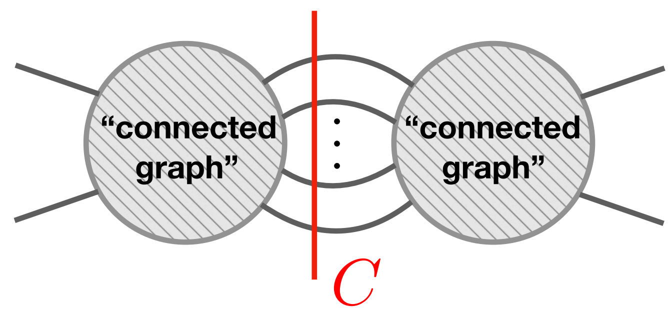

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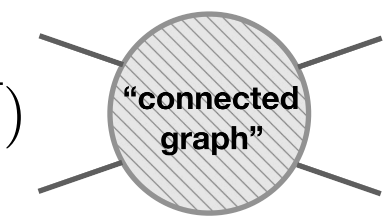

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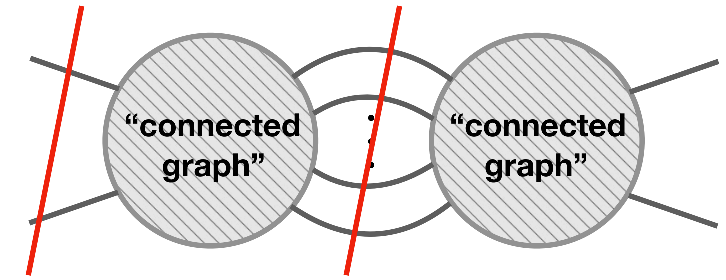

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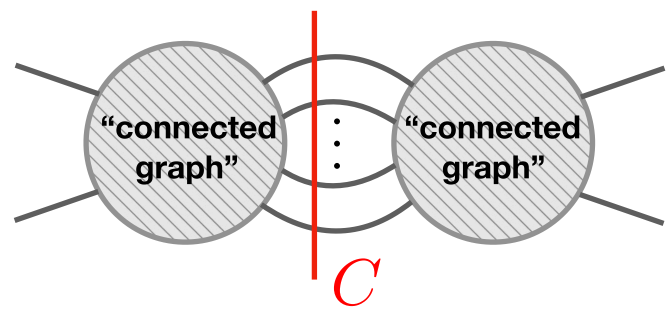

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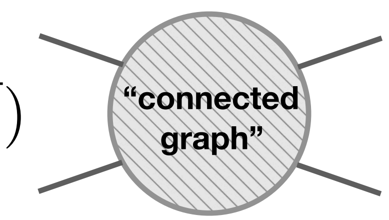

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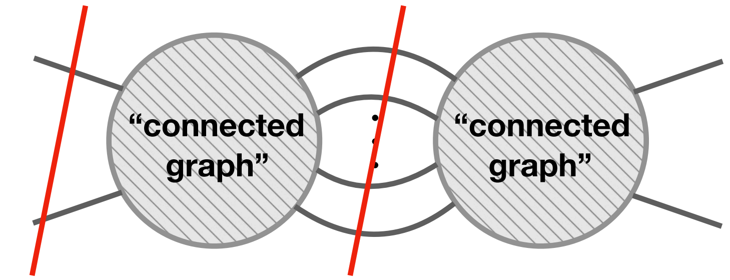

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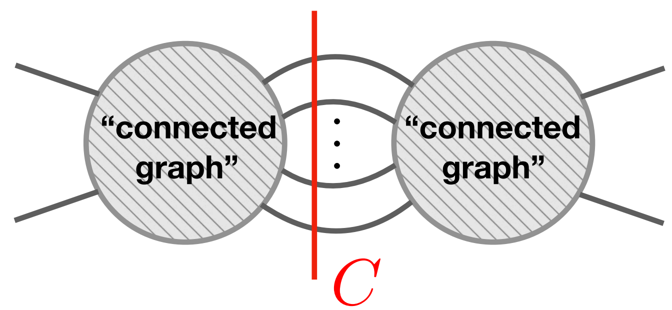

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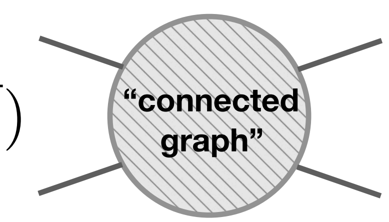

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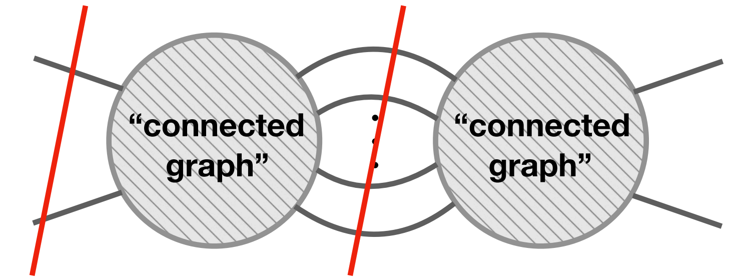

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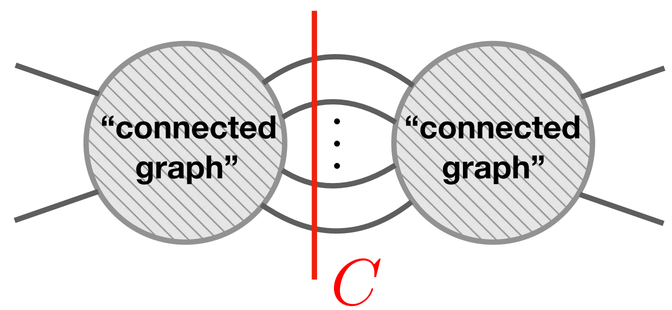

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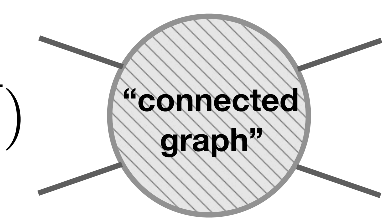

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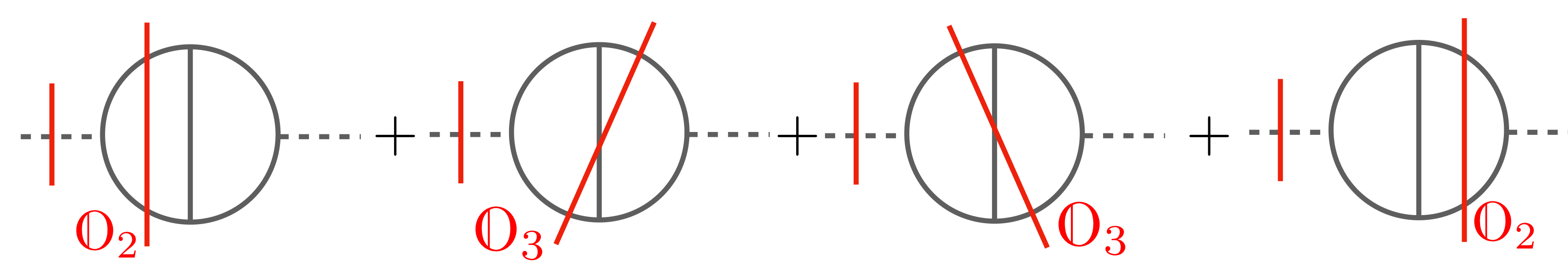

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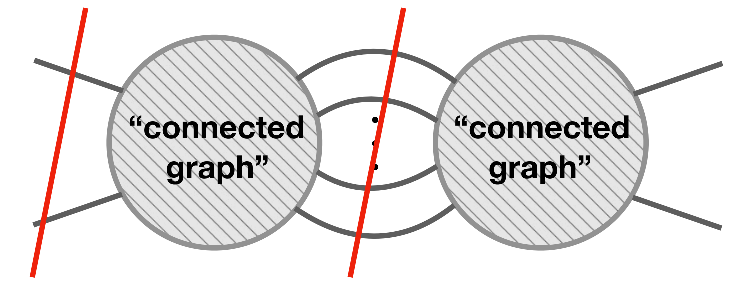

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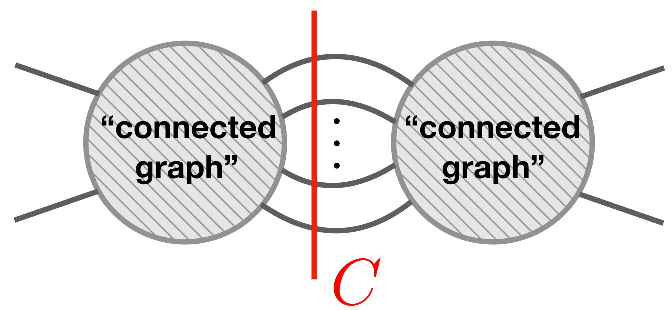

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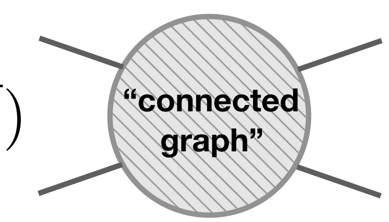

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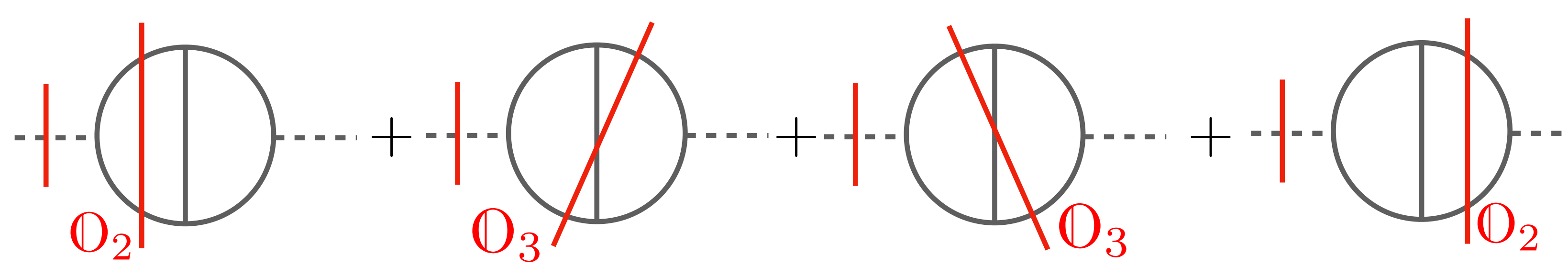

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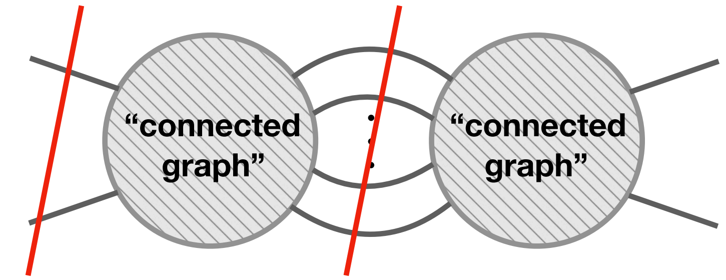

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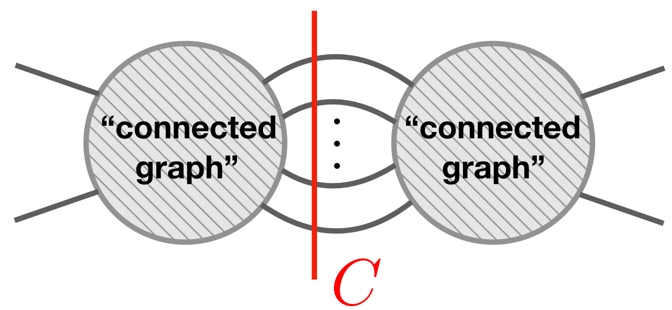

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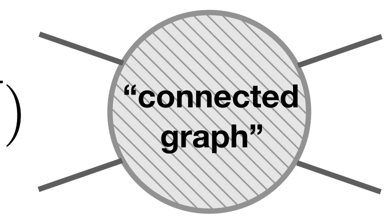

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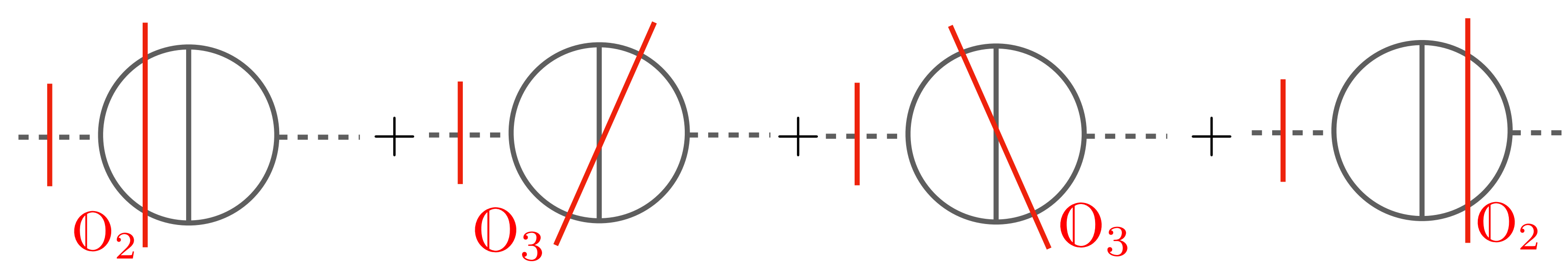

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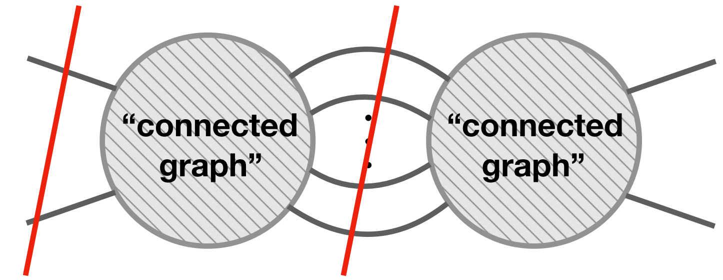

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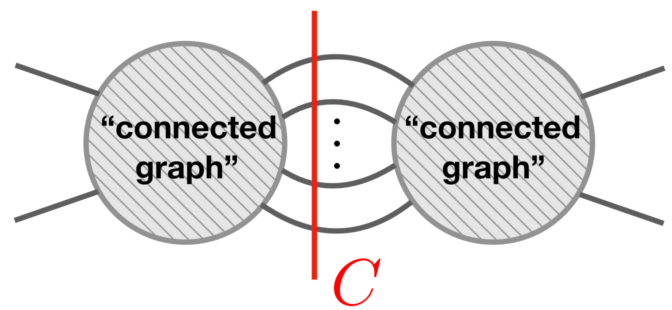

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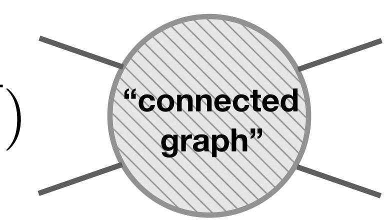

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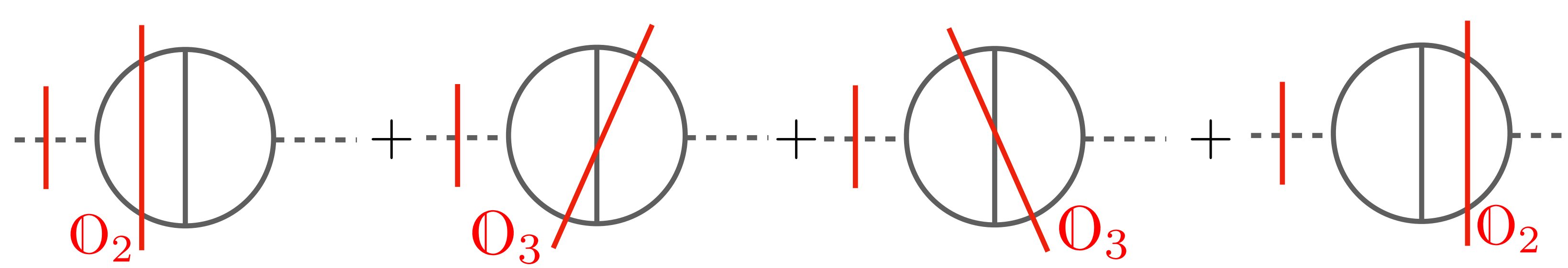

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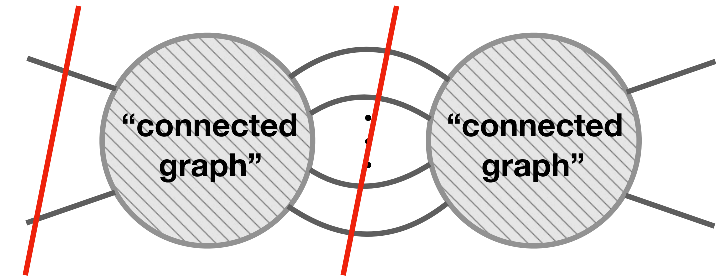

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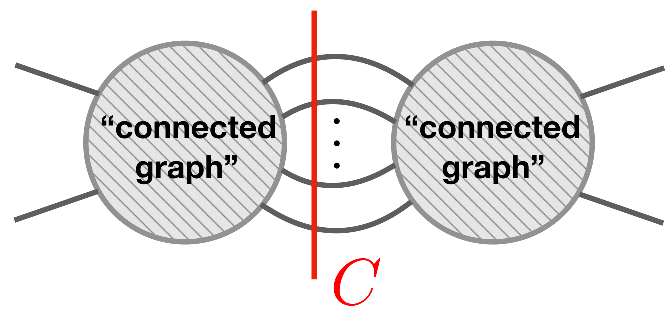

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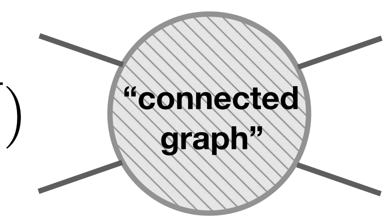

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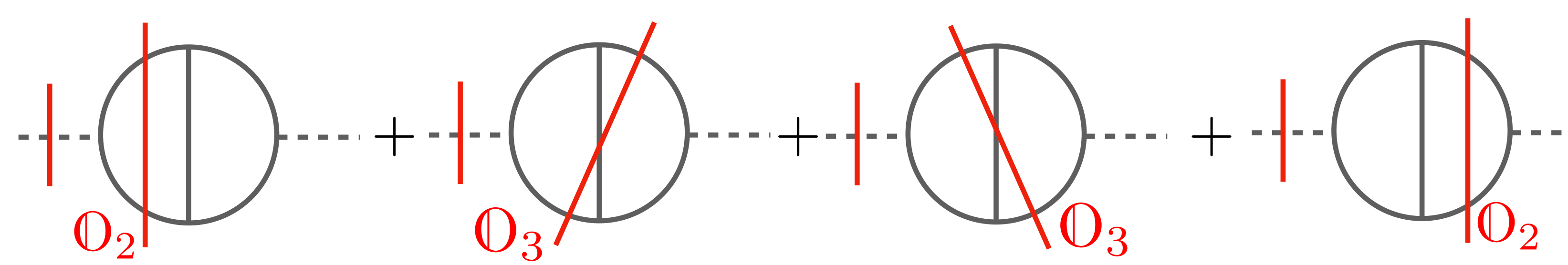

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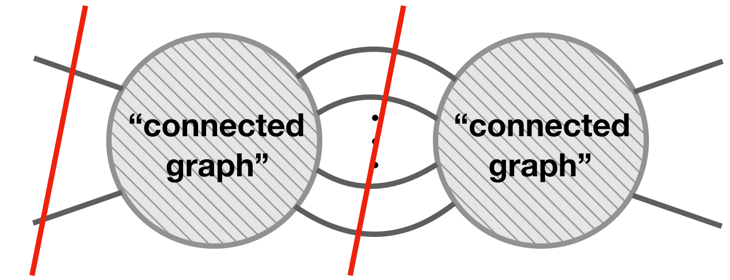

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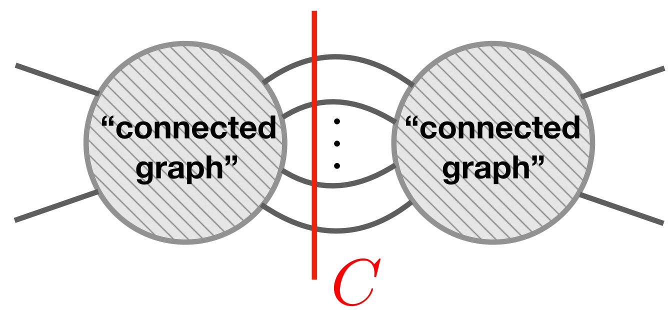

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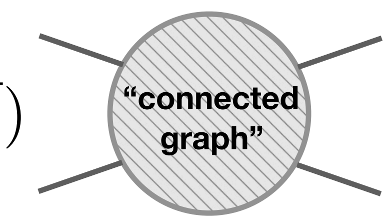

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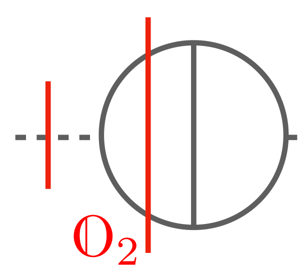

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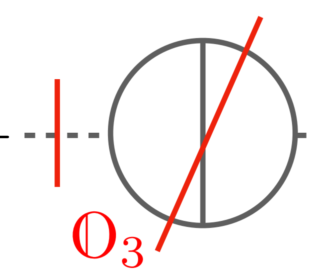

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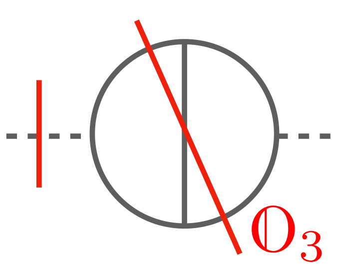

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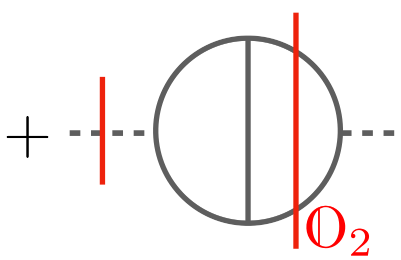

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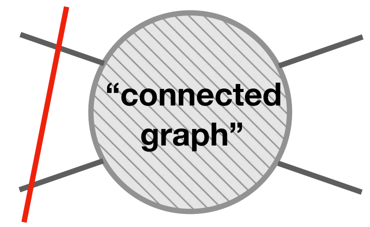

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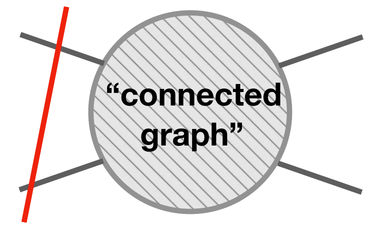

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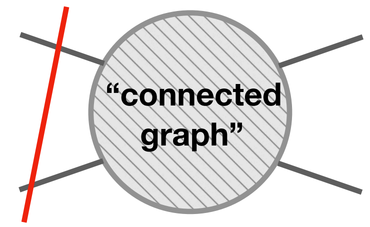

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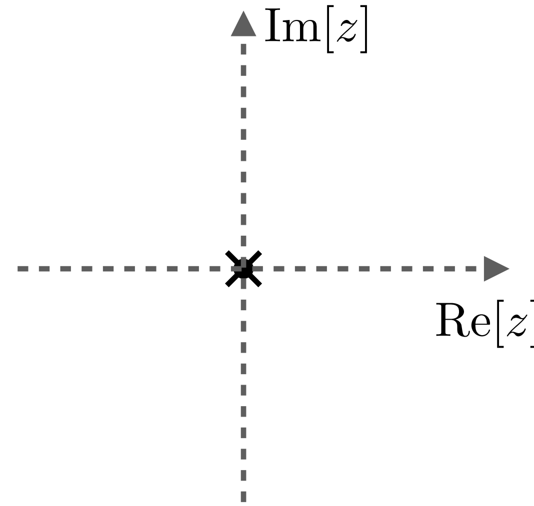

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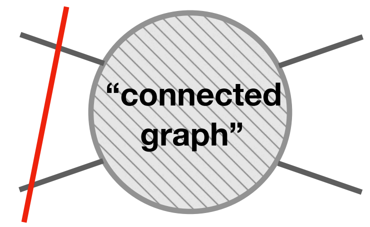

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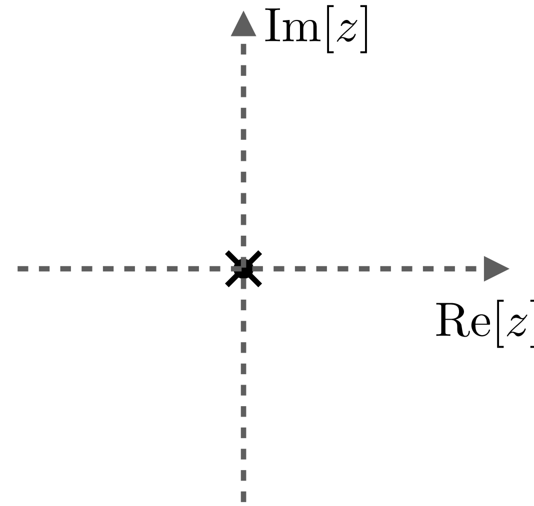

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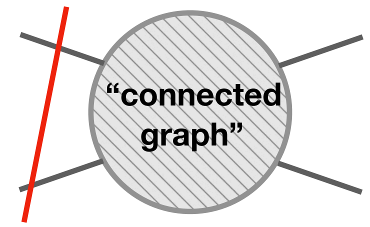

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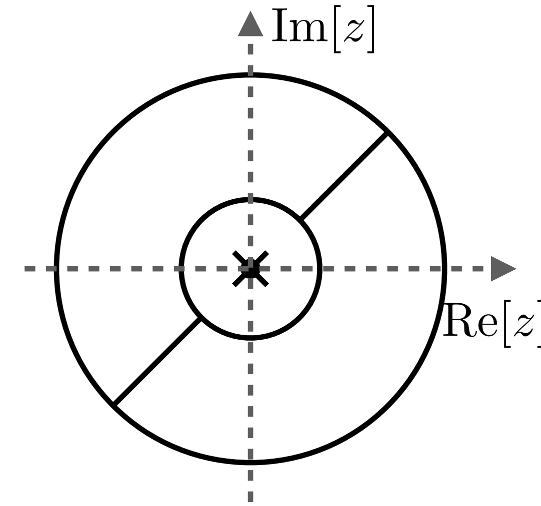

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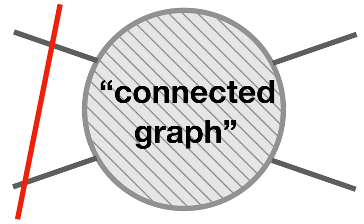

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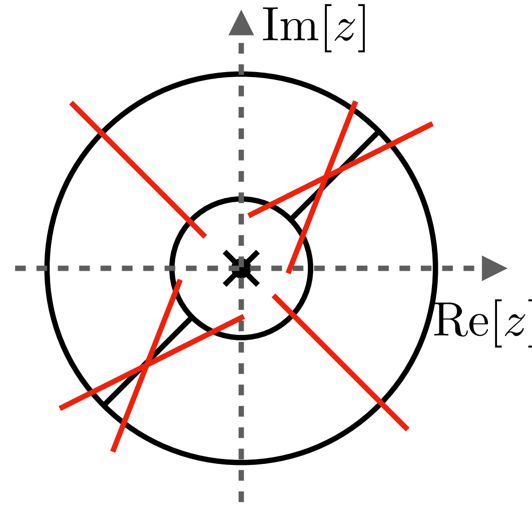

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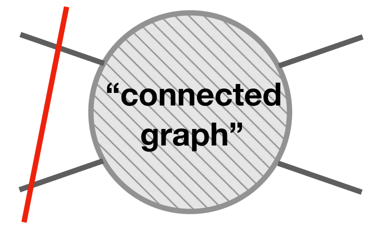

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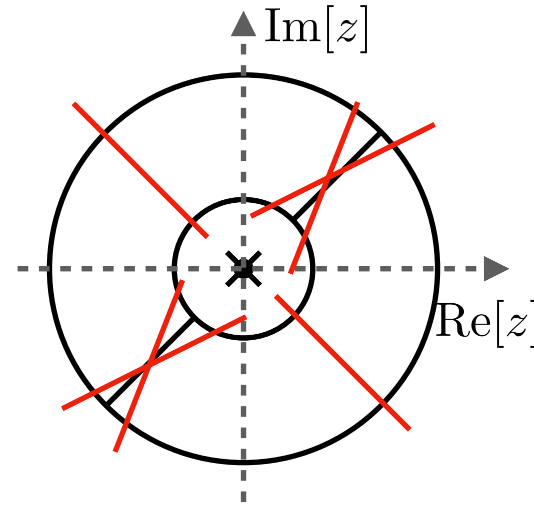

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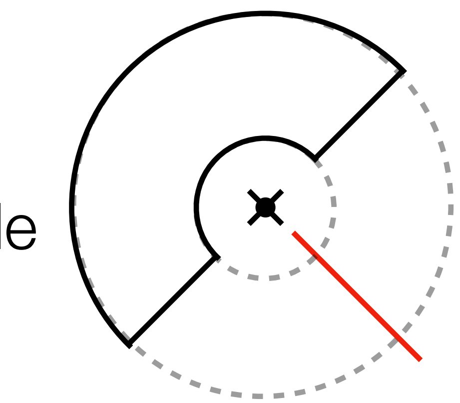

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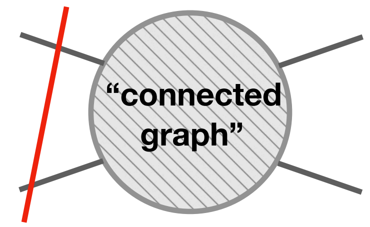
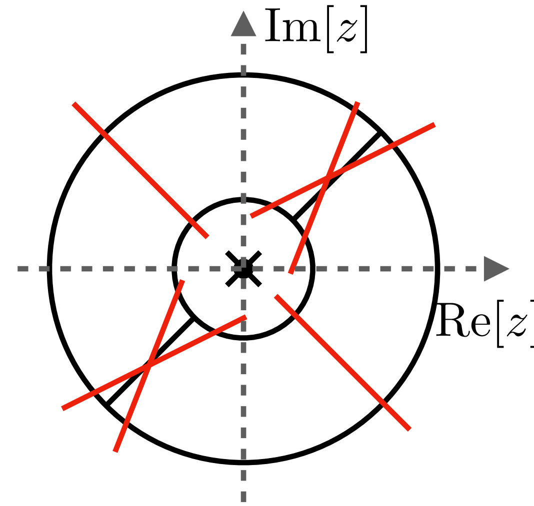


We need to write forward-scattering diagrams as residues of something!

How do we extend to initial-state sums? In other words what are forward-scattering diagrams residues of

Embedding vacuum graph in $\mathbb{C} \setminus \{0\}$
$x^0 \to z$



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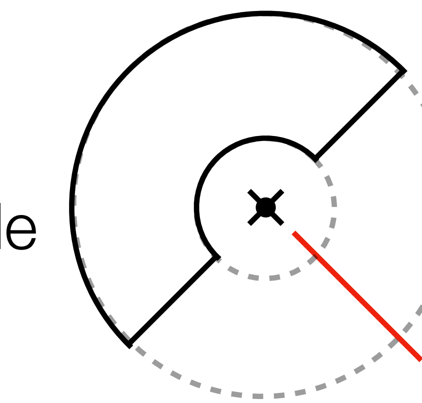
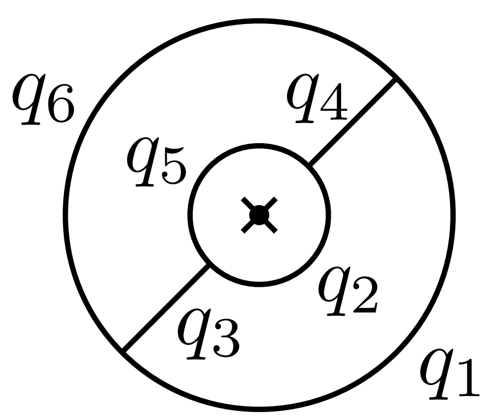


We can construct a three-dimensional representation for embeddings



$$= \int \left[\prod_{i=1}^{6} \frac{\mathrm{d}q_i^0}{q_i^2}\right] \delta(q_1+q_2-p)\delta(q_1+q_2-p)\delta(q_2+q_3-q_6-p)\delta(q_5+q_6-p)\delta(q_2+q_4+q_6-p)$$

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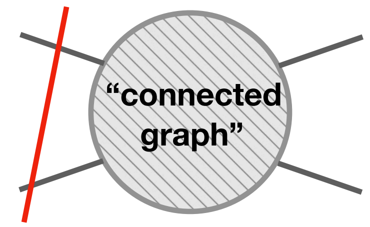

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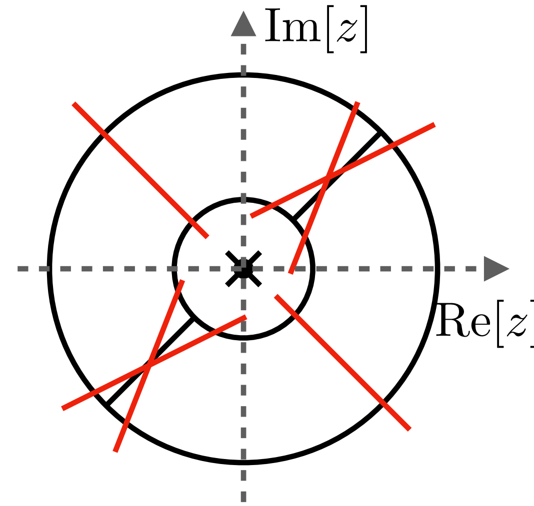

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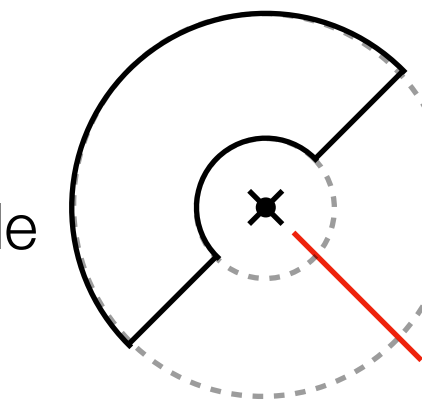

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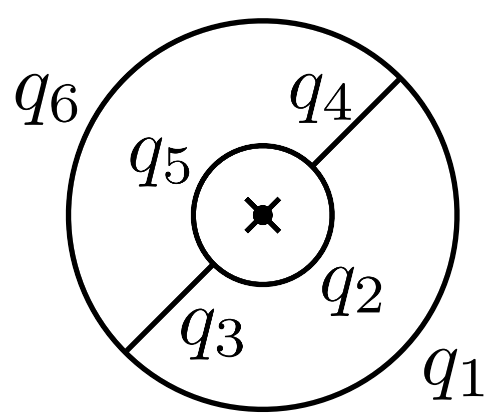
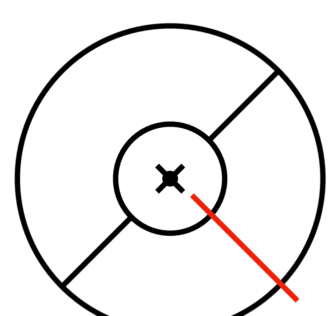
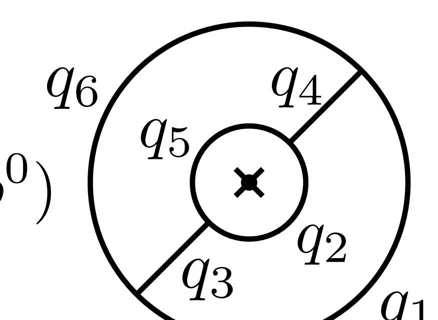


$$= \int \left[\prod_{i=1}^{6} \frac{\mathrm{d}q_i^0}{q_i^2}\right] \delta(q_1 + q_2 - p)\delta(q_1 + q_2 - p)\delta(q_2 + q_3 - q_6 - p)\delta(q_5 + q_6 - p)\delta(q_2 + q_4 + q_6 - p)$$

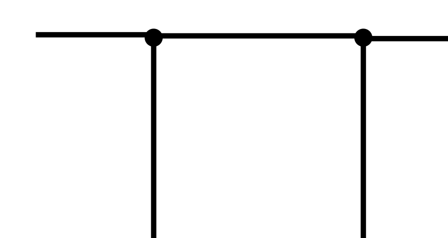
Embeddings have thresholds associated with their Cutkosky cuts



$$= \lim_{p^0 \to E_1 + E_2} (E_1 + E_2 - p^0)$$



$$=$$



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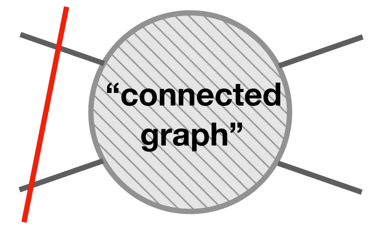

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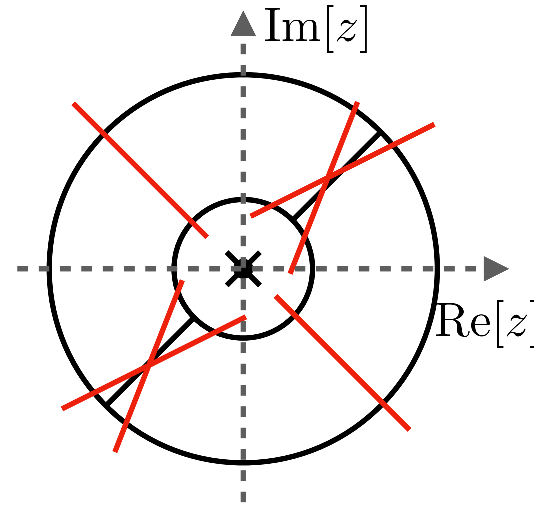

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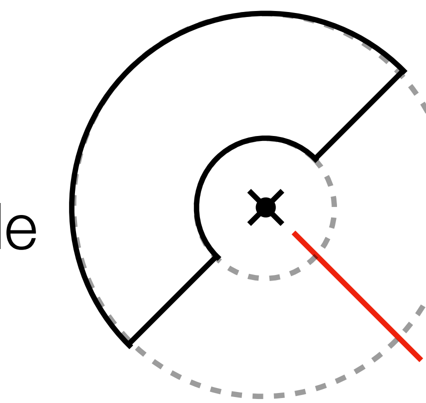

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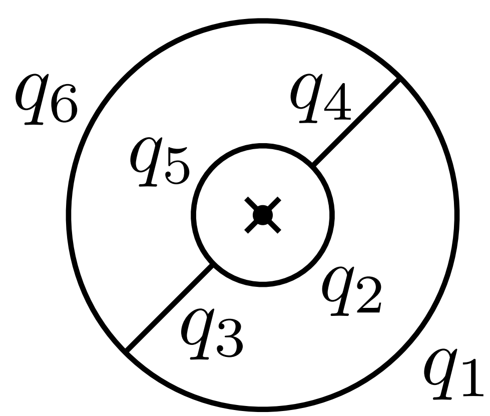

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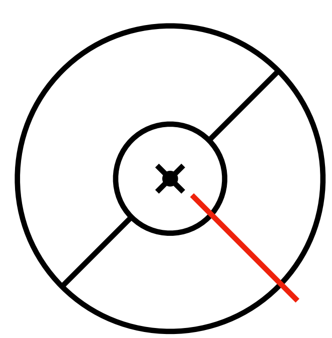

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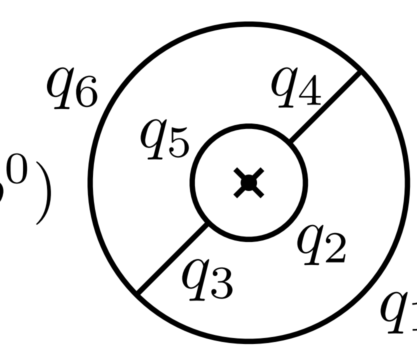

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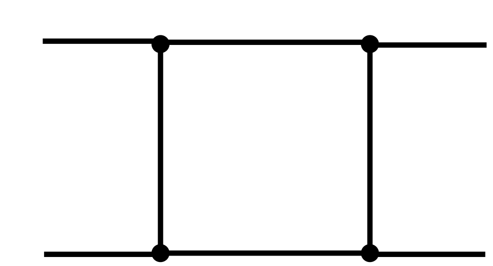

This observation allows to extend the Local Unitarity representation to initial states!

Conclusion

- Energy conservation implies a rigid diagrammatic structure for threshold singularities
    - Connectedness
    - Absence of crossing
    - Obstruction-freedom

- These principles are manifest in a novel 3D-representation that holds for any theory (independent of numerator) and at any loop

    Implemented in Mathematica package ⟶ https://github.com/apelloni/cLTD

- The presence of connectedness suggests that, in order to understand IR-finiteness, one should decompose cross-sections according to the degree of connectedness

- 3D representations can be used to express interference diagrams as local residues of forward-scattering diagrams, and forward-scattering diagrams as local residues of vacuum embeddings

- In turn, these local residues can be used to write cross-sections in a way that manifests the KLN cancellation mechanism at the local level (Local Unitarity)

- Local Unitarity can be used to numerically evaluate cross-sections