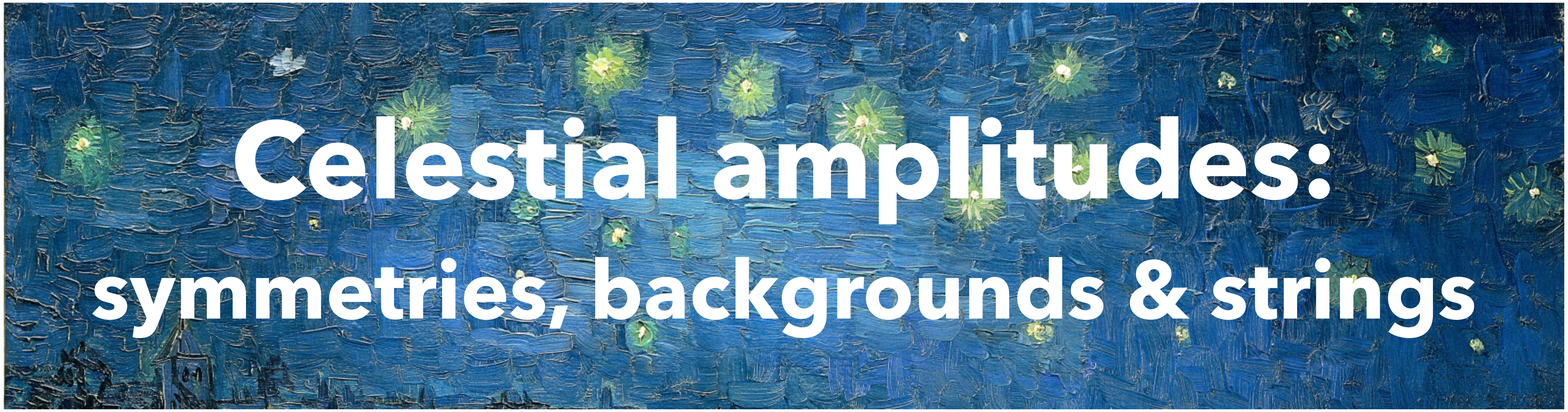


Celestial amplitudes:

symmetries, backgrounds & strings

ANDREA PUHM

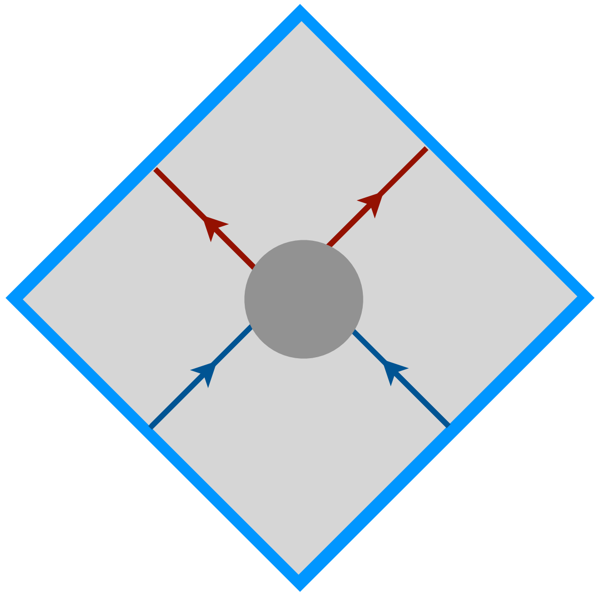
AMPLITUDES @ CERN

10 AUGUST 2023

European Research Council
Established by the European Commission

ERC STARTING GRANT HOLOHAIR 852386

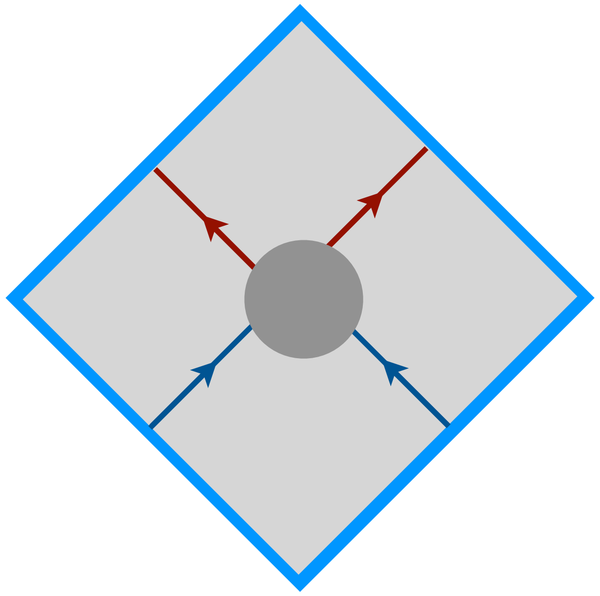
A celestial hologram?



S-matrix

has holographic flavour -
can we make it more
manifest?

A celestial hologram?



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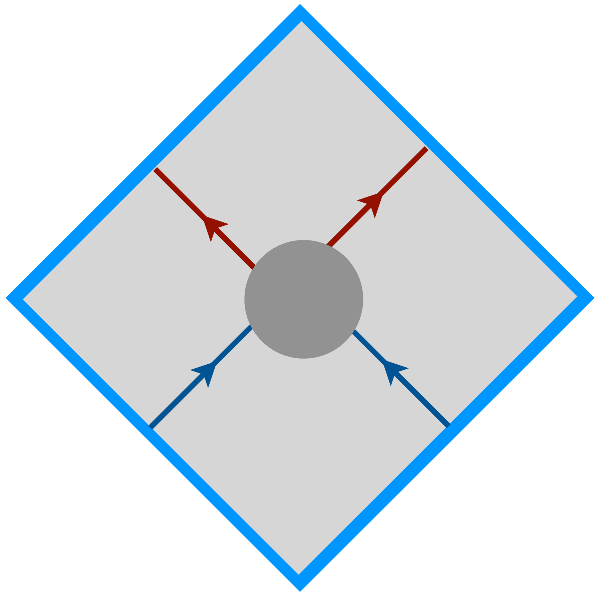

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Exploit powerful CFT toolkit and other methods such as twistors.

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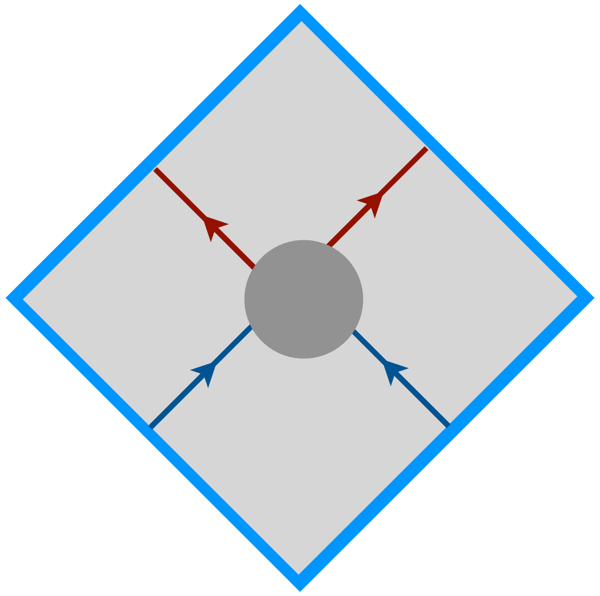


S-matrix

has holographic flavour -
can we make it more
manifest?

Exploit powerful CFT toolkit and
other methods such as twistors.

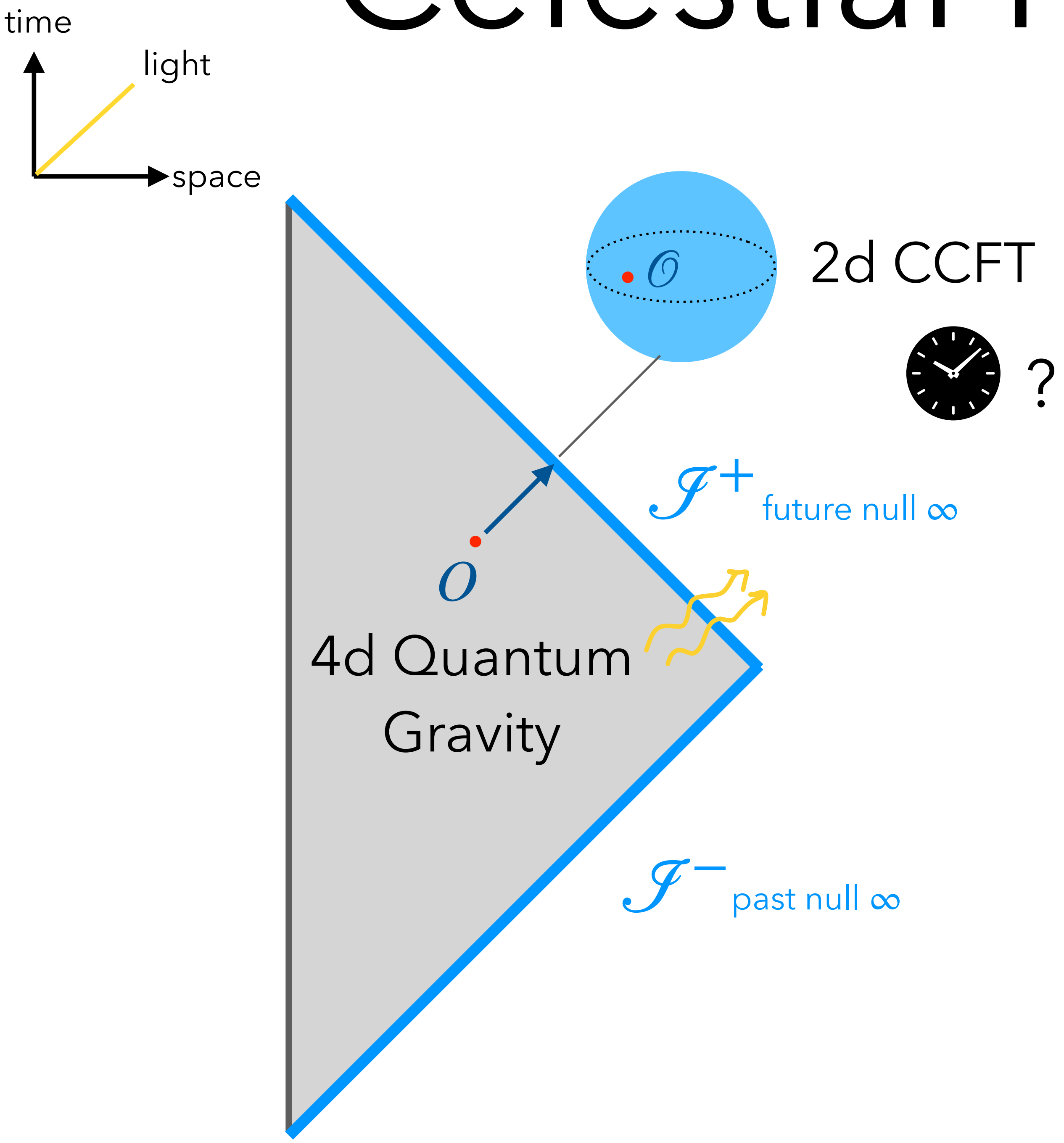
bottom-up

understand universal properties of quantum gravity that are independent of short distance microphysics

top-down

explore toy models and (super) symmetry to **construct dual pairs**

Celestial Holography

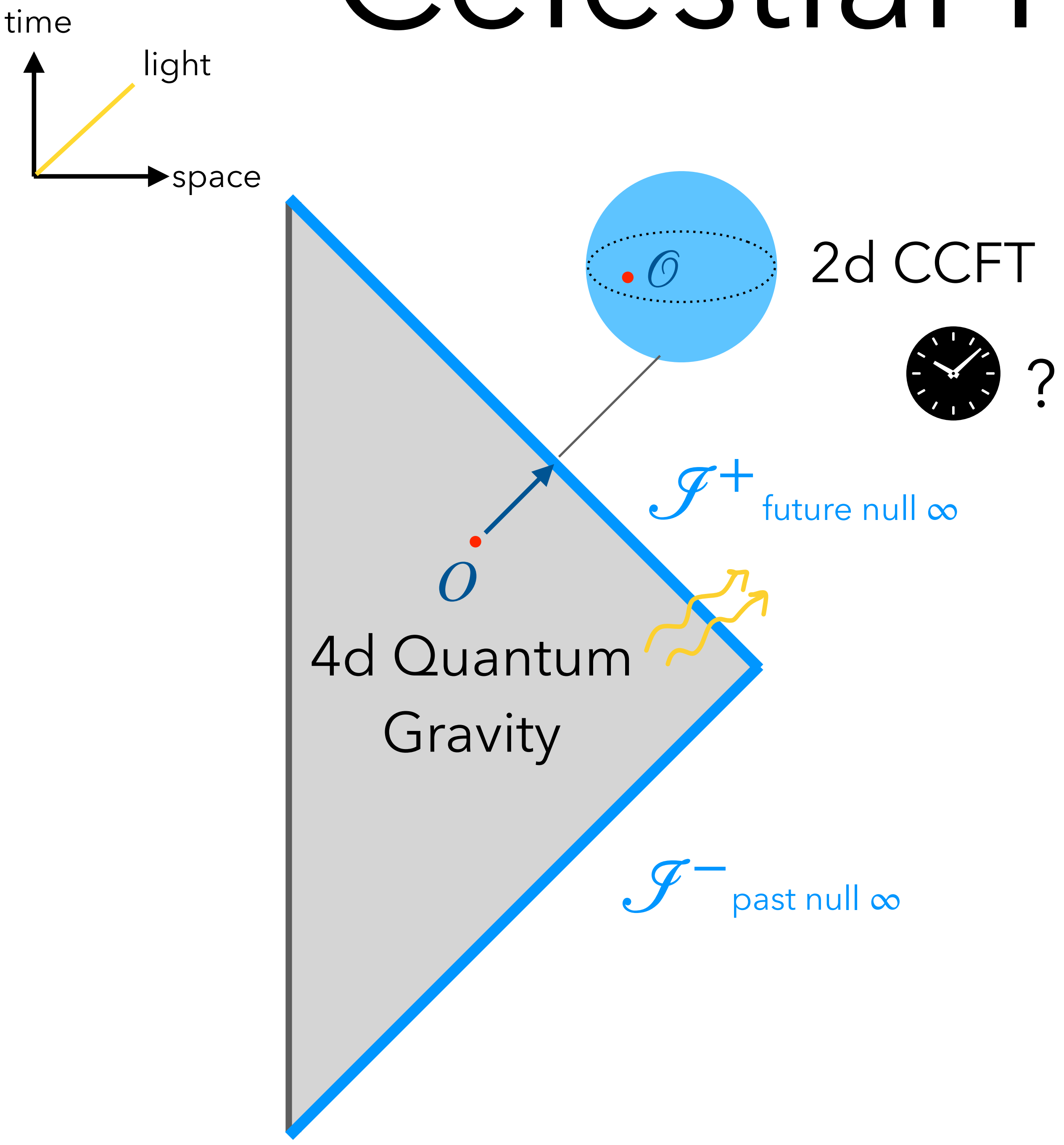


4d Quantum Gravity = 2d Celestial CFT

in asymptotically flat spacetimes

or **CCFT** for short

Celestial Holography



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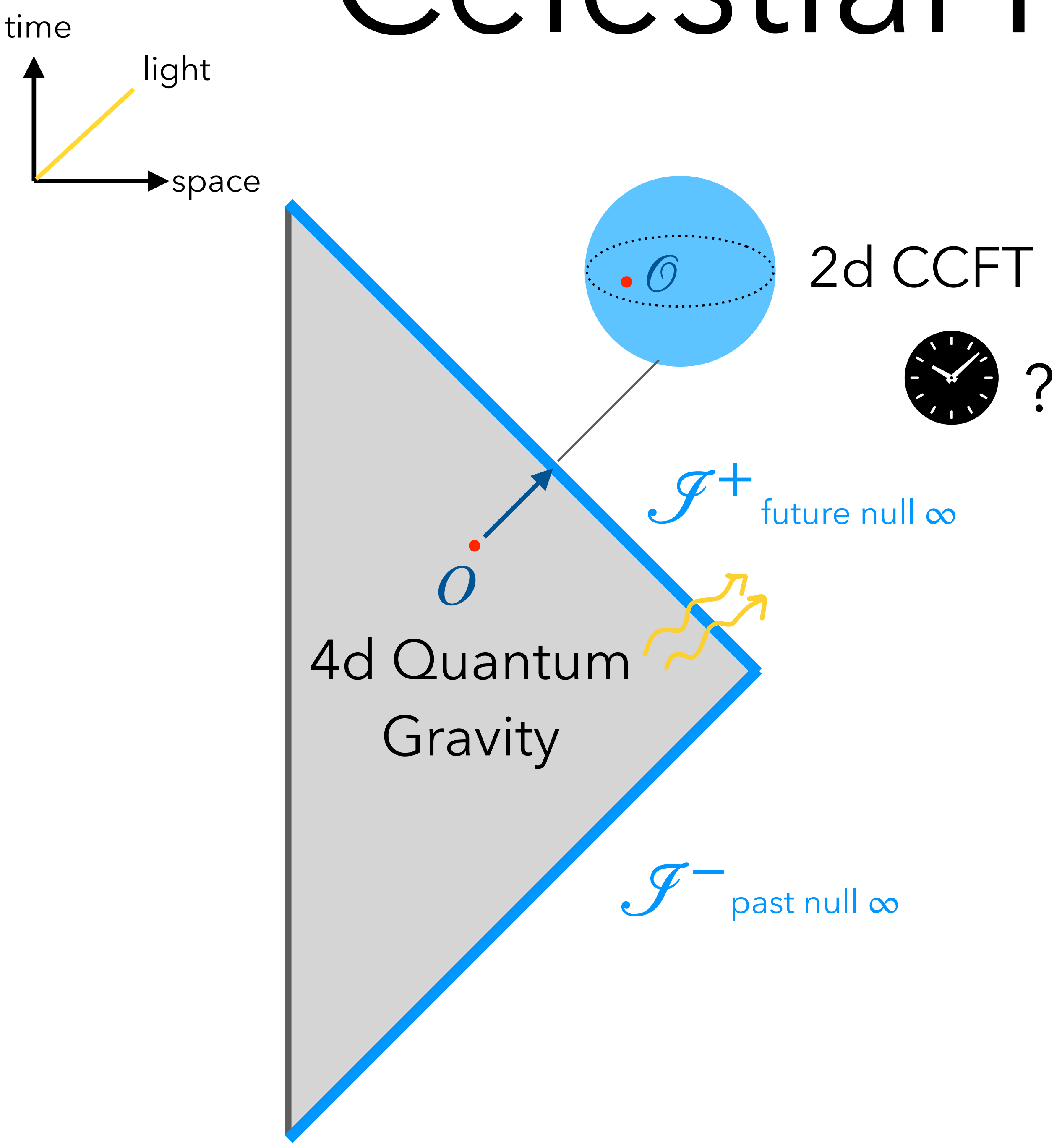
*extends to general spacetime dimensions

Celestial Holography



Some recent progress:

- Bulk O → boundary $\mathscr{O}$
- CCFT building blocks
- universal features of observables

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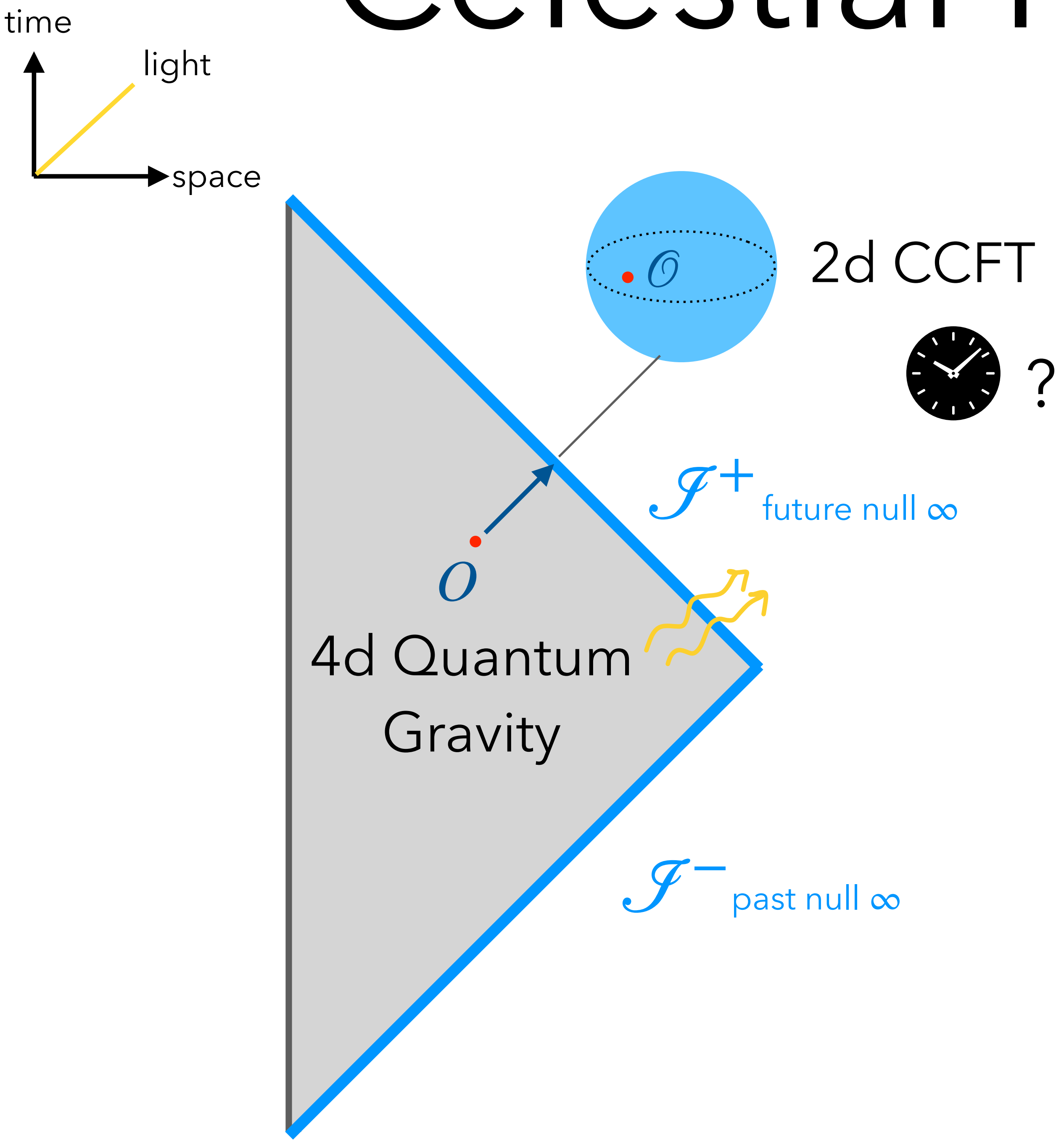
Celestial Holography



Some recent progress:

- Bulk O → boundary $\mathcal{O}$
- CCFT building blocks
- universal features of observables

↓

novel type of holography

4d Quantum Gravity = 2d Celestial CFT*

in asymptotically flat spacetimes

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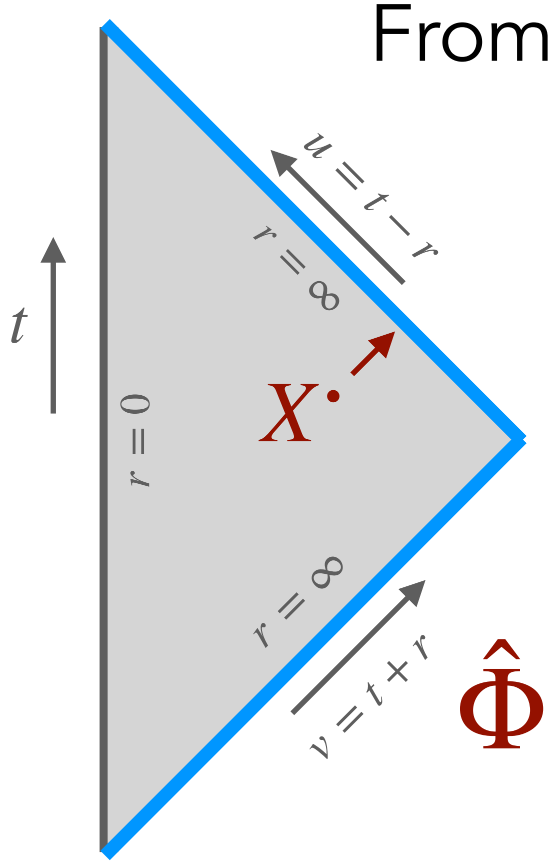
*extends to general spacetime dimensions

Bulk–boundary dictionary

From bulk operators to boundary operators:

Bulk–boundary dictionary

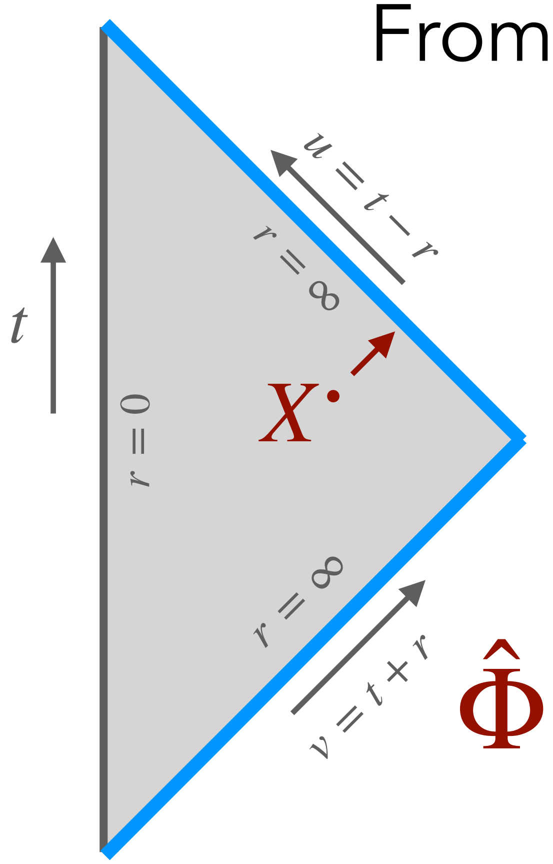
From bulk operators to boundary operators:



$$\hat{\Phi}(X)$$

Bulk–boundary dictionary

From bulk operators to boundary operators:



$$\hat{\Phi}(X) \xrightarrow[\lim\limits_{r\to\infty} r^{1-|\ell|}]{\text{large } r \text{ limit}}$$

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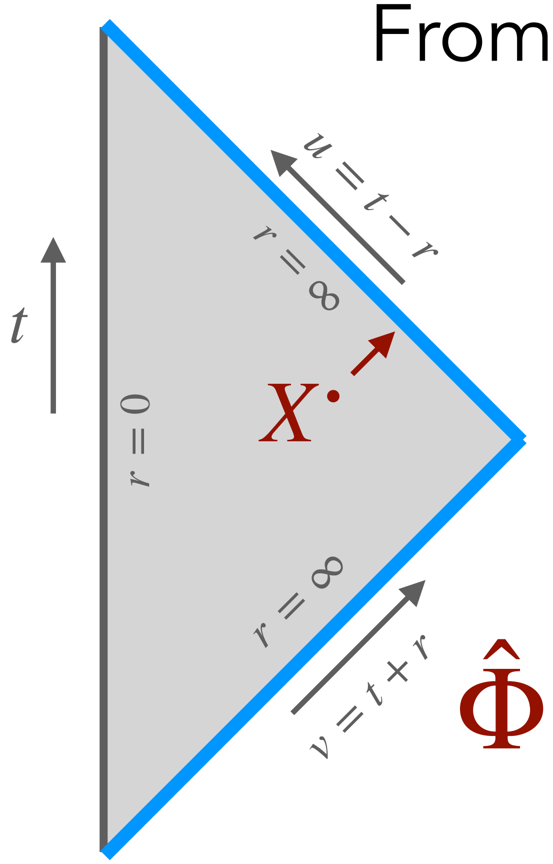

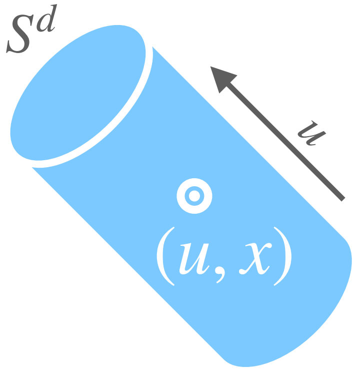

$$\hat{\Phi}(X) \xrightarrow[\lim_{r\to\infty} r^{1-|\ell|}]{\text{large } r \text{ limit}} \Phi_\ell(u, x)$$

spin

Carrollian primaries

null boundary:
degenerate metric,
$c \to 0$ field theory

Bulk–boundary dictionary

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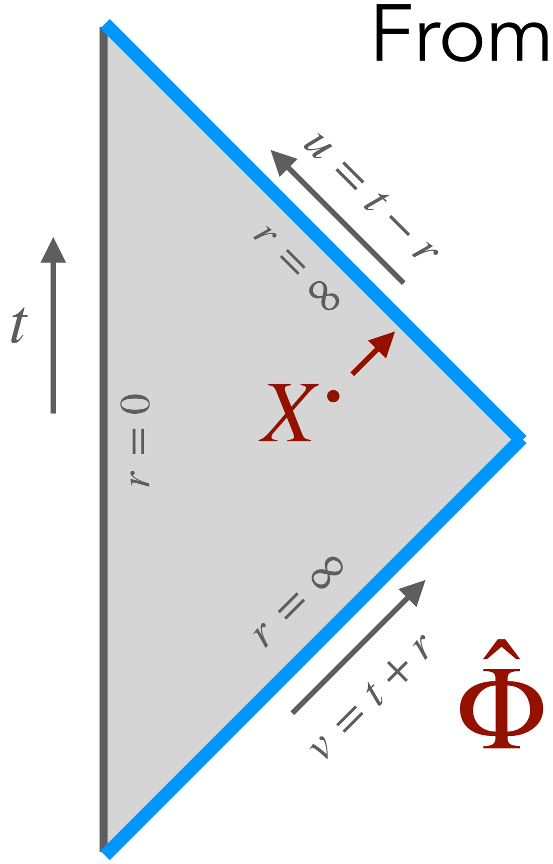

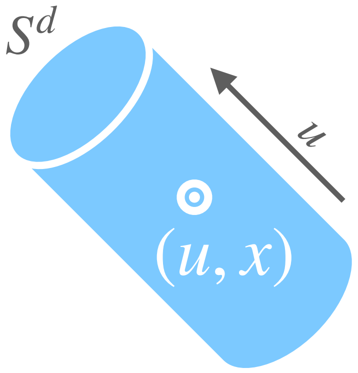

$$\hat{\Phi}(X) \xrightarrow[\lim\limits_{r\to\infty} r^{1-|\ell|}]{\text{large } r \text{ limit}} \Phi_{\ell}(u,x) \xrightarrow[\int_{-\infty}^{+\infty} du u^{-\Delta}]{\mathscr{L}\text{ight-ray}}$$

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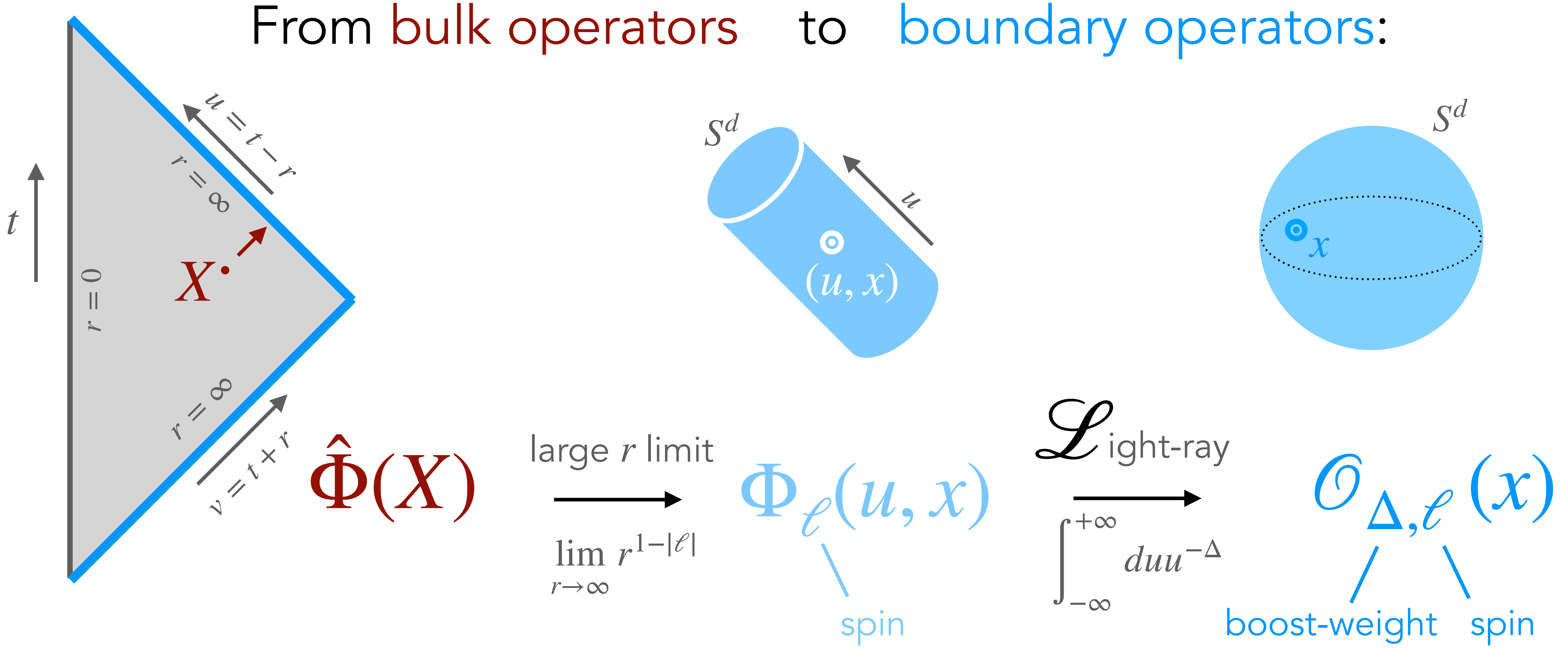

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spin

boost-weight spin

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conformal primaries

bulk Lorentz acts as
boundary global conformal
which in gravity is enhanced
to local conformal

Bulk–boundary dictionary

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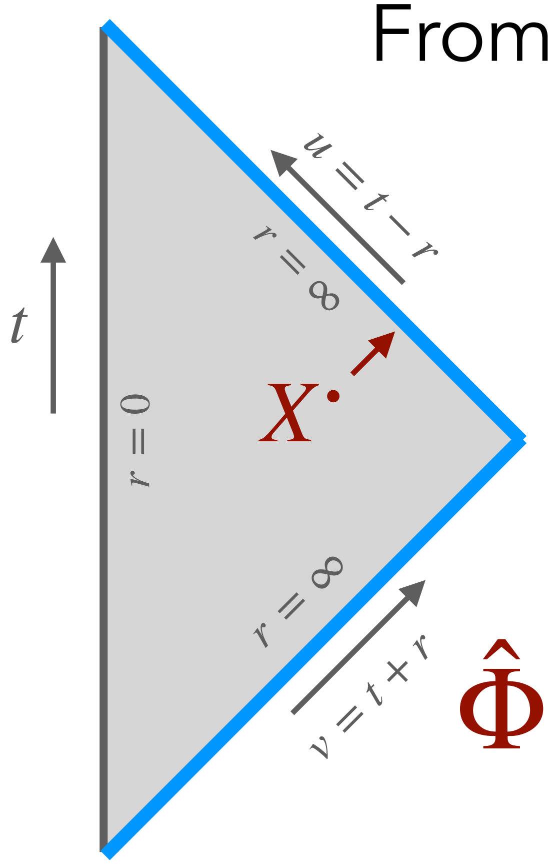

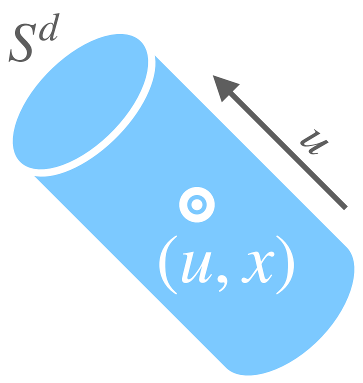

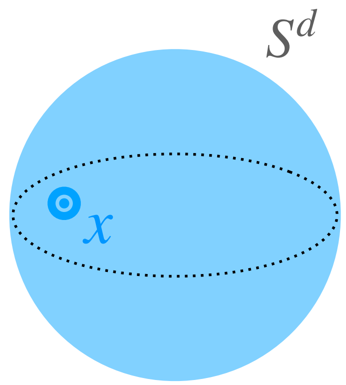

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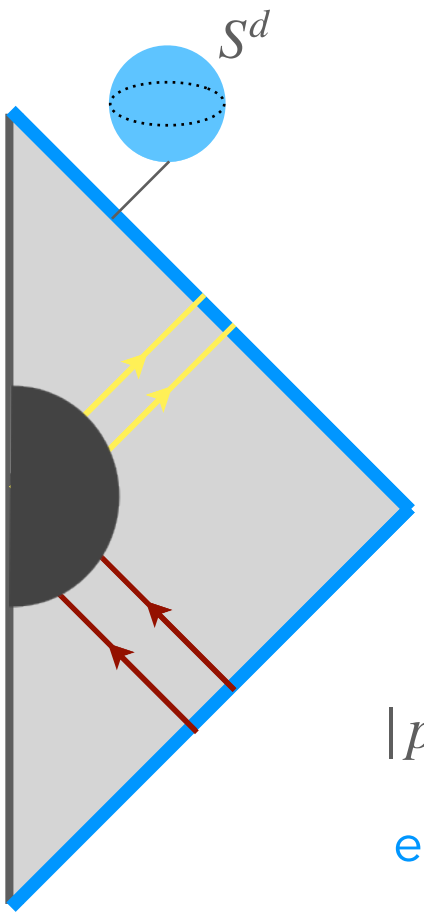
- ``Extrapolate" dictionary for celestial holography.

[Pasterski,AP,Trevisani'21]

Observables

in asymptotically flat spacetimes



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

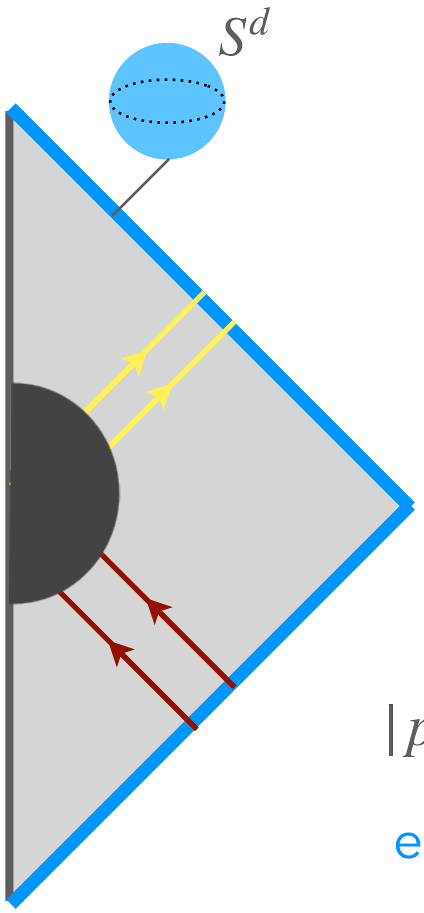
Scattering amplitudes:

basic observables in flat space

$$\mathscr{A}(p_1, \ldots p_n) \equiv \langle out \,|\, \mathcal{S} \,|\, in \rangle$$

Observables

in asymptotically flat spacetimes



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

$$\xrightarrow{\mathscr{M}\text{ellin}}$$

integrate over all energies

$$\int_0^\infty d\omega\omega^{\Delta-1}$$

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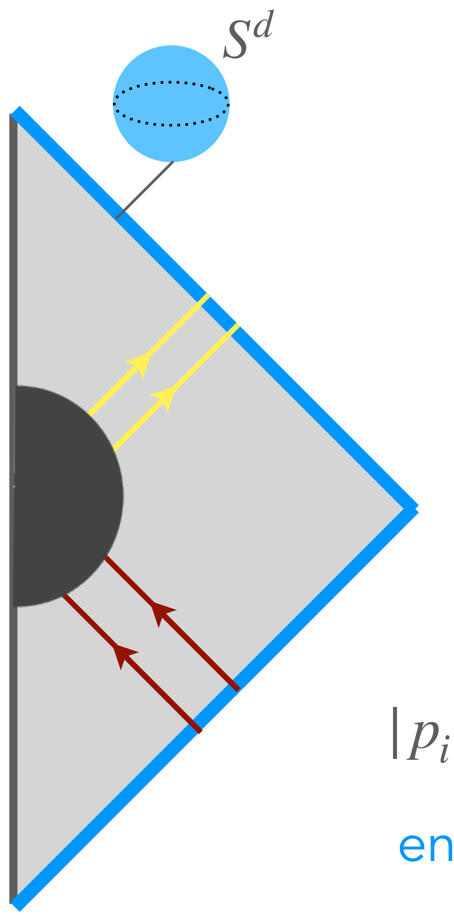


$\mathscr{M}$ellin

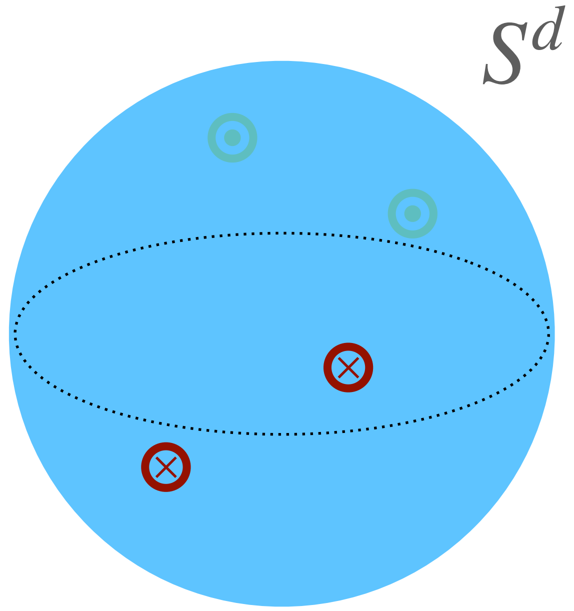
$\longrightarrow$

integrate over all energies

$|p_i\rangle = |\omega_i, x_i\rangle$ $\quad \int_0^\infty d\omega \omega^{\Delta-1}$ $\quad |\Delta_i, x_i\rangle$

energy basis

boost-weight basis



Scattering amplitudes:

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Celestial amplitudes:

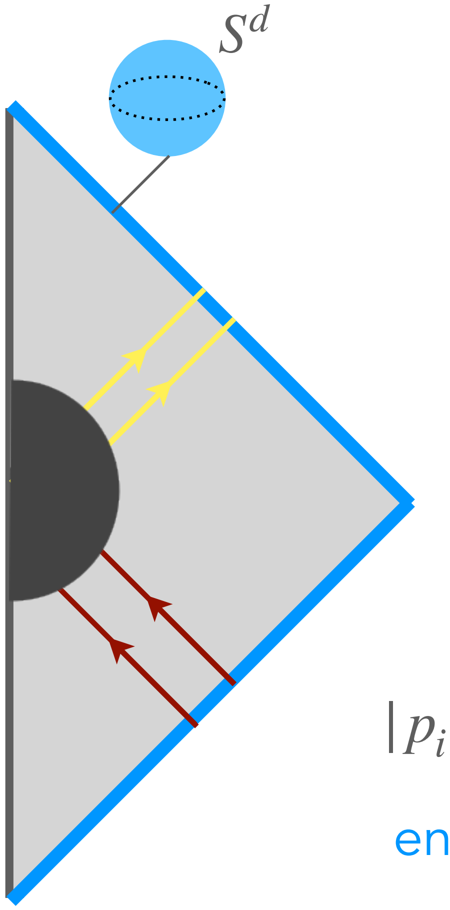
natural observables in CCFT:

$$\langle \mathcal{O}_{\Delta_1}(x_1) \ldots \mathcal{O}_{\Delta_n}(x_n) \rangle$$

transform nicely under conformal transformations!

Observables

in asymptotically flat spacetimes



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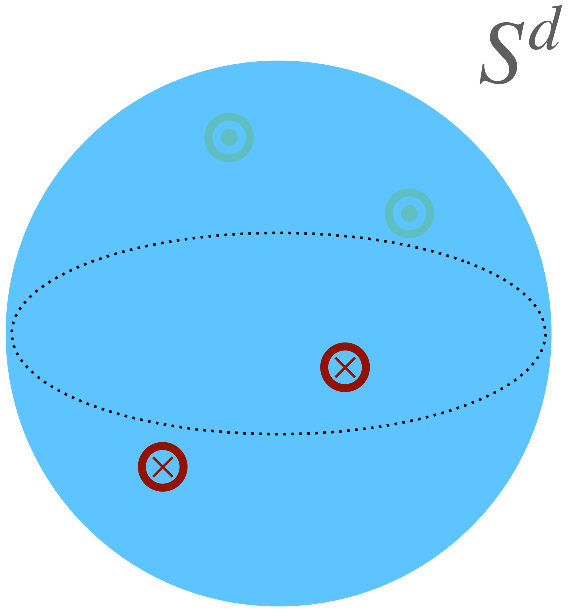
energy basis

Mellin

integrate over all energies

$\int_0^\infty d\omega \omega^{\Delta-1}$



$|\Delta_i, x_i\rangle$

boost-weight basis

Scattering amplitudes:

basic observables in flat space

$$\mathscr{A}(p_1, \dots p_n) \equiv \langle out|\mathscr{S}|in\rangle$$

translation symmetry

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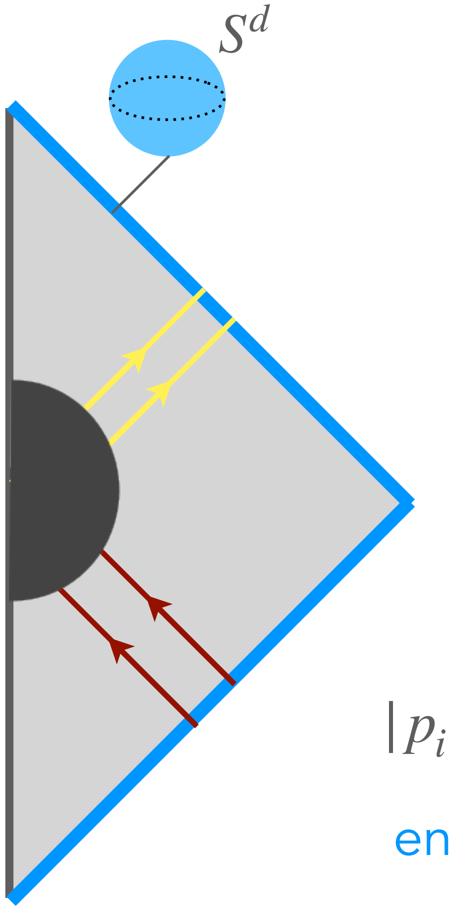
Lorentz symmetry

Observables

in asymptotically flat spacetimes



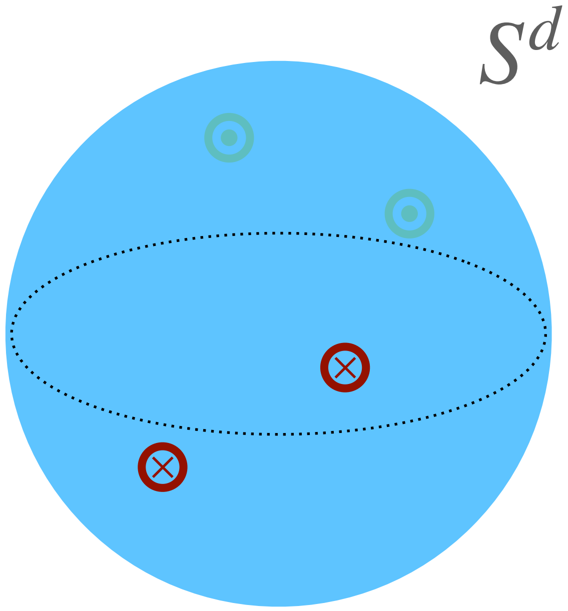
$$\mathscr{M}\text{ellin} \longrightarrow$$

integrate over all energies

$$|p_i\rangle = |\omega_i, x_i\rangle \qquad \int_0^\infty d\omega\,\omega^{\Delta-1} \qquad |\,\Delta_i, x_i\rangle$$

energy basis

boost-weight basis



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Lorentz symmetry

plane wave $$\Phi_\omega(X;x) = e^{ip(\omega,x)\cdot X}$$

$p^\mu = \pm\,\omega q^\mu(x)$

$$\Phi_\Delta(X;x) = \frac{1}{(-q(x)\cdot X)^\Delta}$$ conformal primary wavefunction

$\pm i\varepsilon$ prescription

[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

3 bases

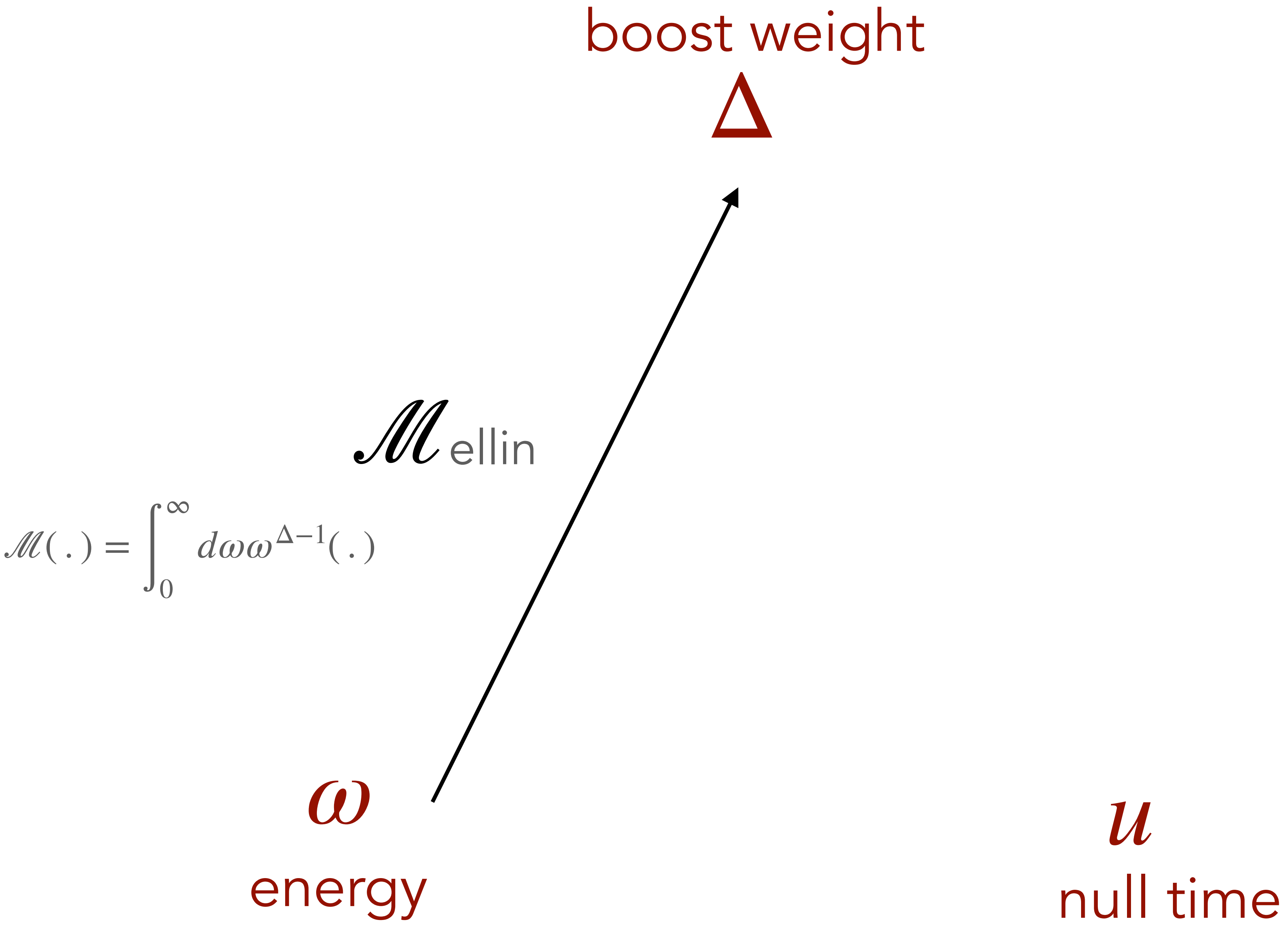
boost weight
Δ

ω
energy

u
null time

[Donnay, Pasterski,AP’22]

3 bases



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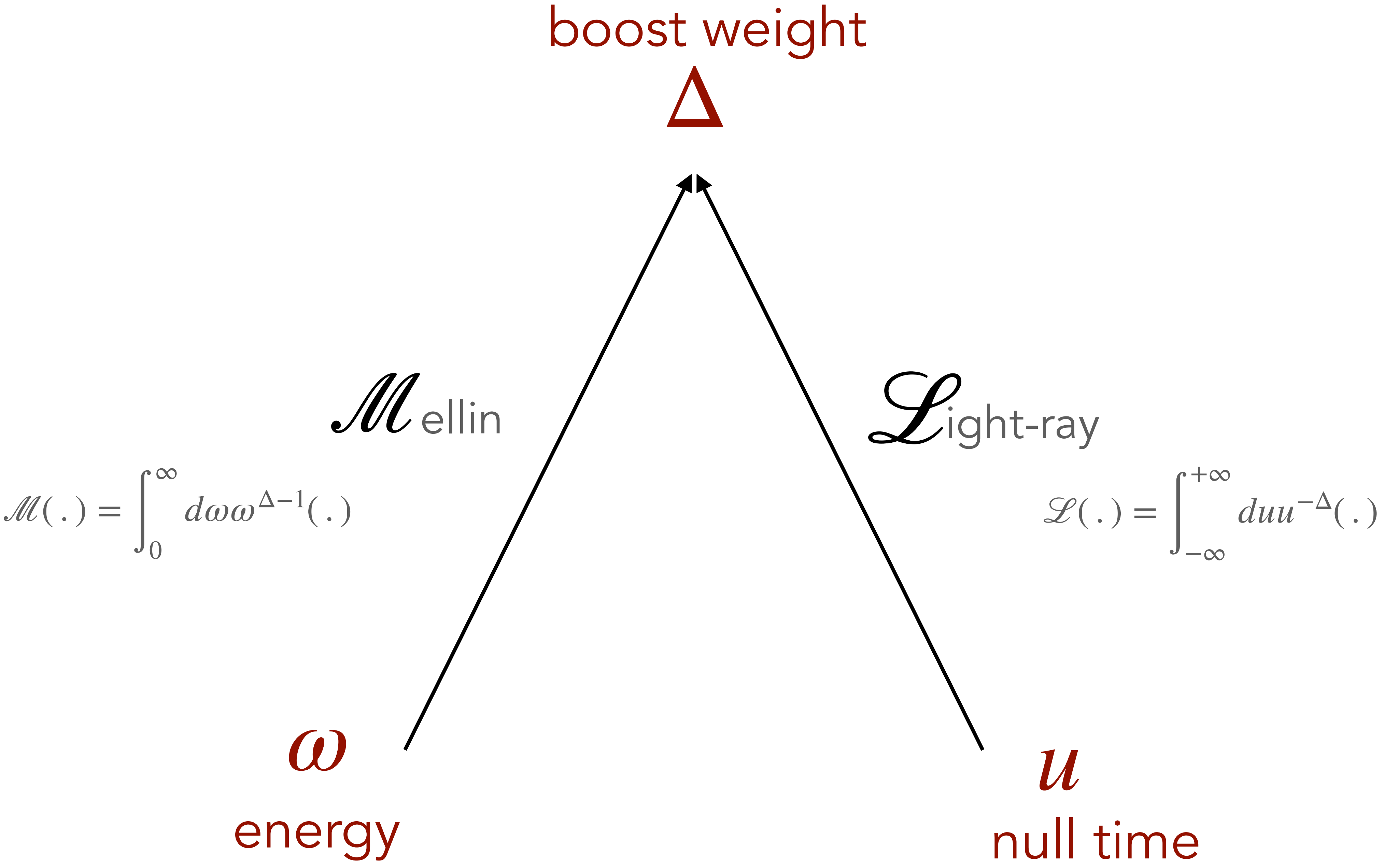

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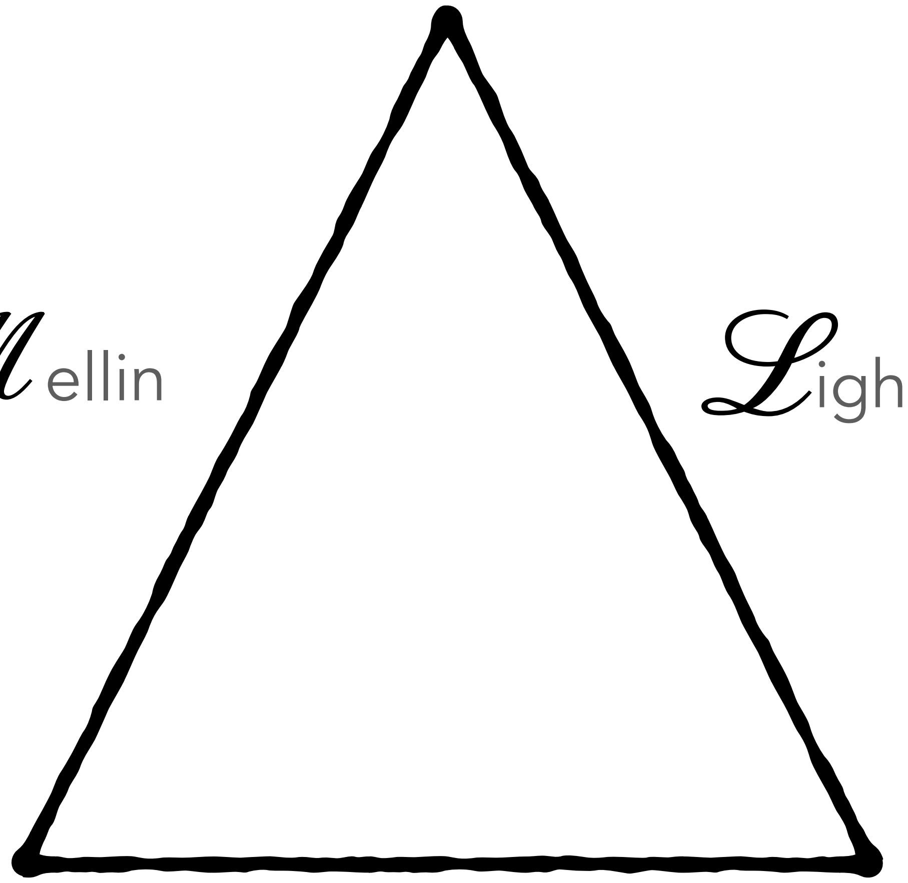

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ω

energy

$\mathscr{F}$ourier

u

null time

$$\mathscr{F}(\,.\,) = \int_{-\infty}^{+\infty} du e^{i\omega u}(\,.\,)$$

Together with an $i\varepsilon$ prescription for well-definedness of transforms.

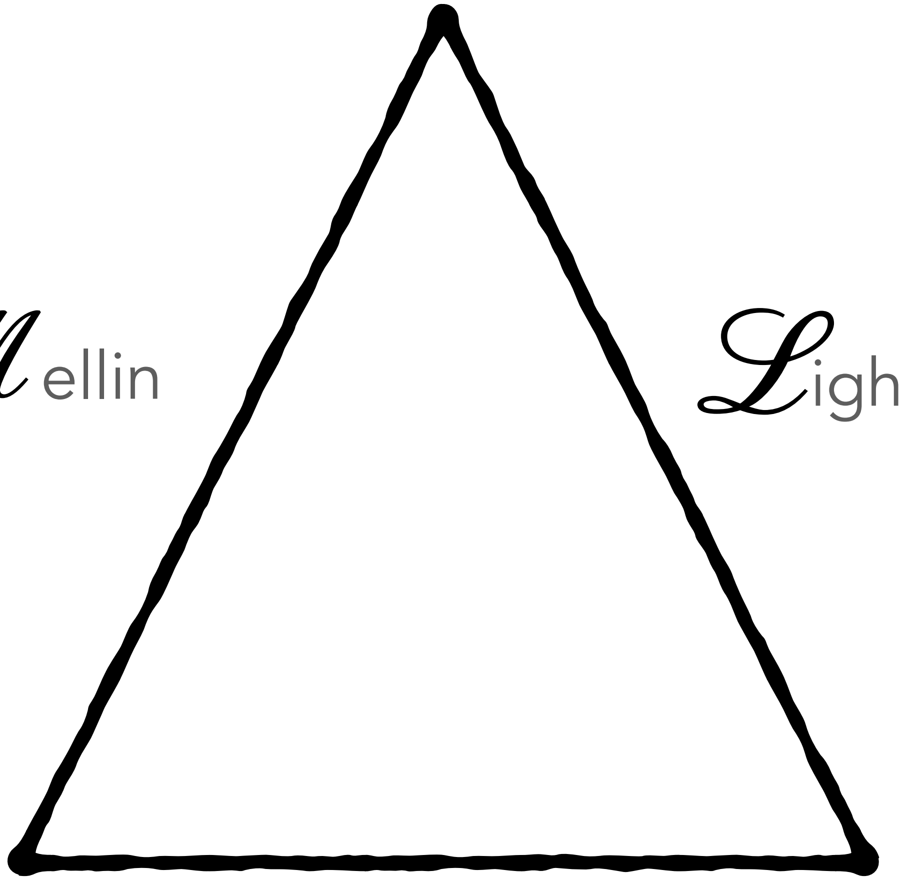
[Donnay, Pasterski,AP’22]

IR triangle

symmetries

boost weight

Δ



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ω

energy

$\mathscr{F}$ourier

u

null time

soft theorems

$$\mathscr{F}(\,.\,) = \int_{-\infty}^{+\infty} due^{i\omega u}(\,.\,)$$

memory effect

[Strominger'13+...]

Features of celestial amplitudes

$$_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}_{\Delta_1,\ell_1}(x_1) \ldots \mathcal{O}_{\Delta_n,\ell_n}(x_n) \rangle_{\text{CCFT}}$$

Features of celestial amplitudes

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- **no Wilsonian decoupling** since integration over all energies

  [Arkani-Hamed,Pate,Raclariu,Strominger'20]

  → *potentially problematic in field theory but not in string theory*

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- **distributional support** on the sphere from translation symmetry in bulk

  [Pasterski,Shao,Strominger'17]

  $\delta^{(d+2)}(\sum_{i=1}^{N} p_i^{\mu}(\omega_i, x_i))$

  → *scattering on backgrounds: more standard correlators*

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  $$\delta^{(d+2)}\big(\textstyle\sum_{i=1}^{N} p_i^{\mu}(\omega_i, x_i)\big)$$

  → *scattering on backgrounds: more standard correlators*

- spectrum (Δ) complex → **non-unitary CFT**

  $\omega > 0 \quad \longleftrightarrow \quad \Delta \in 1 + i\mathbb{R}$ [Pasterski,Shao'17]

  $\omega = 0 \qquad\qquad\ \ \Delta \in 1 - \mathbb{Z}_{\geq}$

  → *symmetries?*

  [Donnay,AP,Strominger'18] [Donnay,Pasterski,AP'22]
  [Cheung,de la Fuente,Sundrum'16]...

Outline

Introduction

I. Celestial string amplitudes @ tree and 1-loop

II. Symmetries of celestial CFT$_d$

III. Celestial amplitudes on (particle-like) backgrounds

Summary & outlook

based on

I. 2307.03551 with **Laura Donnay, Gaston Giribet, Hernán Gonzáles** & **Francisco Rojas**

II. 2302.10222 with **Yorgo Pano** & **Emilio Trevisani**

III. 2207.13719 with **Riccardo Gonzo** & **Tristan McLoughlin**

No Wilsonian decoupling: integration over all energies potentially problematic in field theory but not string theory

↓

I. Celestial string amplitudes

@ tree and 1-loop

Strings on the celestial sphere

Focus: 4d scattering processes of 4 gluons in open string theory

4d momenta $p_i^\mu = (p_i^0, p_i^1, p_i^2, p_i^3, \overrightarrow{0})$ & 10d loop momenta ℓ^μ

$p_i^\mu = \eta_i \omega_i q_i^\mu(z_i, \bar{z}_i)$ & $q_i^\mu = (1 + z_i\bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i\bar{z}_i)$

null vector pointing
to celestial sphere

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null vector pointing to celestial sphere

Celestial amplitude:

$$\mathscr{M}[\mathscr{A}_4](\{\Delta_i, z_i, \bar{z}_i\}) = \int_0^\infty \prod_{i=1}^4 d\omega_i \omega_i^{\Delta_i - 1} \delta^{(4)}\left(\sum_{i=1}^4 p_i^\mu\right) A(\{\omega_i, z_i, \bar{z}_i\})$$

$$A_{\text{string}}(p_1, p_2, p_3, p_4) = A_{YM}^{(0)}(\{p_i\})(f^{(0)} + f^{(1)} + \ldots)$$

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$$\uparrow$$

$$A_{\text{string}}(p_1, p_2, p_3, p_4) = A_{YM}^{(0)}(\{p_i\})(f^{(0)} + f^{(1)} + \ldots)$$

Define:

boost weight: $\Delta_i \in 1 + i\mathbb{R}$ $\longrightarrow$ net boost weight: $\beta = -\frac{i}{2}\sum_{i=1}^4 \text{Im}\Delta_i$

& cross ratio: $r = -\frac{s}{t} = \frac{z_{12}z_{34}}{z_{23}z_{41}}$

$s = -(p_1 + p_2)^2$

$t = -(p_2 + p_3)^2$

$u = -(p_1 + p_3)^2$

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$p_i^\mu = \eta_i \omega_i q_i^\mu(z_i, \bar{z}_i)$ & $q_i^\mu = (1 + z_i\bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i\bar{z}_i)$ null vector pointing to celestial sphere

Celestial amplitude:

$$\mathcal{M}[\mathcal{A}_4](\{\Delta_i, z_i, \bar{z}_i\}) = \int_0^\infty \prod_{i=1}^4 d\omega_i \omega_i^{\Delta_i - 1} \delta^{(4)}\left(\sum_{i=1}^4 p_i^\mu\right) A(\{\omega_i, z_i, \bar{z}_i\})$$

$$A_{\text{string}}(p_1, p_2, p_3, p_4) = A_{YM}^{(0)}(\{p_i\})(f^{(0)} + f^{(1)} + \dots)$$

Define:

boost weight $\Delta_i \in 1 + i\mathbb{R}$ $\longrightarrow$ net boost weight $\beta = -\frac{i}{2}\sum_{i=1}^4 \mathrm{Im}\Delta_i$ & cross ratio $r = -\frac{s}{t} = \frac{z_{12}z_{34}}{z_{23}z_{41}}$

$s = -(p_1 + p_2)^2$

$t = -(p_2 + p_3)^2$

$u = -(p_1 + p_3)^2$

using $\delta^{(4)}(\sum_{i=1}^4 p_i^\mu)$

Then $A_{YM}^{(0)} = g_{10}^2 \frac{\omega_1\omega_2}{\omega_3\omega_4}\frac{z_{12}^3}{z_{23}z_{34}z_{41}} \overset{\downarrow}{=} g_{10}^2 r \frac{z_{12}\bar{z}_{34}}{\bar{z}_{12}z_{34}}$ independent of $\omega_i \rightarrow$ only need $\mathcal{M}[f^{(i)}]$!

Celestial strings @ tree

[Stieberger,Taylor'18]

Veneziano amplitude: $f^{(0)}(s,t) = \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$

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Celestial tree amplitude:

$$\tilde{f}^{(0)}(r,\beta) = -\alpha'^{\beta} r\,\Gamma(1-\beta)\int_0^1 \frac{dx}{x}\,\left[r\log x - \log(1-x)\right]^{\beta-1}$$

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  → *Does α' dependence still factor out @ loop?*

- ✳Low-energy field theory limit from limit cross ratio $r = \to \infty$

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  → *Recover field theory limit @ loop?*

Celestial strings @ 1-loop

1-loop stringy form factor (planar, orientable): [Donnay,Giribet,Gonzáles,AP,Rojas'23]

$$f_P^{(1)}(s,t) = \frac{16\pi^3 g_{10}^2}{\alpha'} st \int_0^1 \frac{dq}{q} G(q^2)$$

↑

$$G(q^2) = \int_{\mathcal{D}} \prod_{i=2}^{4} d\theta_i \prod_{i<j} \psi(\theta_{ji}, q)^{2\alpha' p_i \cdot p_j}$$

↑

$$\psi(\theta, q) = \sin\theta \prod_{n=1}^{\infty} \frac{1 - 2q^{2n}\cos 2\theta + q^{4n}}{(1-q^{2n})^2}$$

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Celestial 1-loop amplitude:

$$\tilde{f}_P^{(1)}(r,\beta) = -16\pi^3 g_{10}^2 (\alpha')^{\beta-3} \Gamma(2-\beta)\, r \int_0^1 \frac{dq}{q} \int_{\mathcal{D}} \prod_{i=2}^{4} d\theta_i \, [rV - W]^{\beta-2}$$

$$V \equiv \log\left(\frac{\psi(\theta_{42},q)\psi(\theta_3,q)}{\psi(\theta_{43},q)\psi(\theta_2,q)}\right) \quad , \quad W \equiv \log\left(\frac{\psi(\theta_{42},q)\psi(\theta_3,q)}{\psi(\theta_4,q)\psi(\theta_{32},q)}\right)$$

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∗ Overall $\alpha'^{\beta+3}$ combines with g_{10}^2 to loop expansion parameter g_{10}^2/α'^3 !

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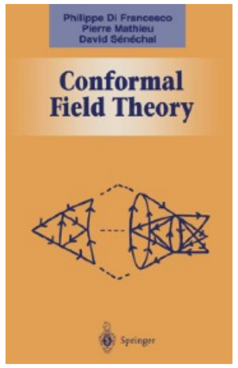
- ✳ Like @ tree: dependence on α' is overall factor - no scale in flat space!
- ✳ Overall $\alpha'^{\beta+3}$ combines with g_{10}^2 to loop expansion parameter g_{10}^2/α'^3 !
- ✳ The field theory limit, dominated by $q \to 1$ region, as $\alpha' \to 0$ regardless of cross ratio r and commutes with the Mellin transform.

spectrum (Δ) complex → **non-unitary CFT**

↓

II. Symmetries in celestial CFT_d

Symmetries in QFT

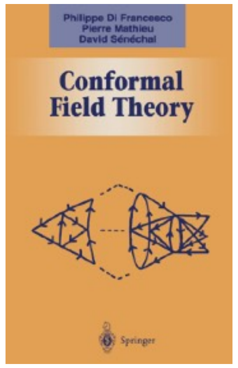


Conservation equations of operators $\mathscr{O}$ define symmetries in QFT.

Noether currents $\mathscr{J}^a$ from contraction of $\mathscr{O}$ with parameter ϵ.

Conservation $\partial_a \mathscr{J}^a = 0$ imposes condition on ϵ.

Symmetries in QFT



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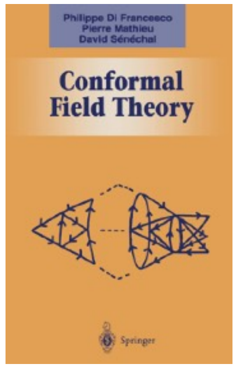
up to contact terms:

$$\langle \partial_a \mathcal{J}^a(x) \mathcal{O}_1(x_1) \ldots \mathcal{O}_N(x_N) \rangle = \sum_{i=1}^{N} \delta^{(d)}(x - x_i) \langle \mathcal{O}_1(x_1) \ldots \delta\mathcal{O}_i(x_i) \ldots \mathcal{O}_N(x_N) \rangle$$

Ward identity

Symmetries in QFT



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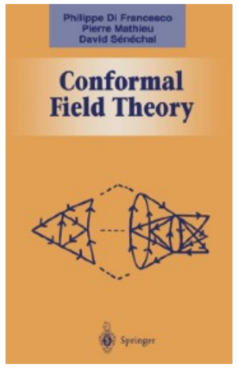
Ward identity

surface element x n^a normal to Σ = $d-1$ dimensional surface in $\mathbb{R}^d$

Topological surface charge $Q_\Sigma = \int_\Sigma dS^a \mathcal{J}_a$ … conserved upon deformations $\Sigma \to \Sigma'$

Symmetries in QFT


Philippe Di Francesco
Pierre Mathieu
David Sénéchal

Conformal Field Theory

Springer

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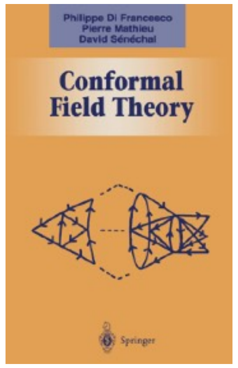

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… acts as variation on enclosed operators

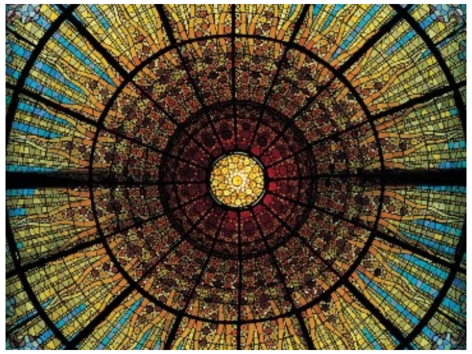
When Σ contains all insertions we can deform the integral to infinity and get

$$0 = \sum_{i=1}^{N} \langle \mathcal{O}_1(x_1) \ldots \delta \mathcal{O}_i(x_i) \ldots \mathcal{O}_N(x_N) \rangle$$

which defines a **symmetry** transformation.

Symmetries

What are all the symmetries (of nature)?



Key for any holographic dual construction.

Important in its own right.

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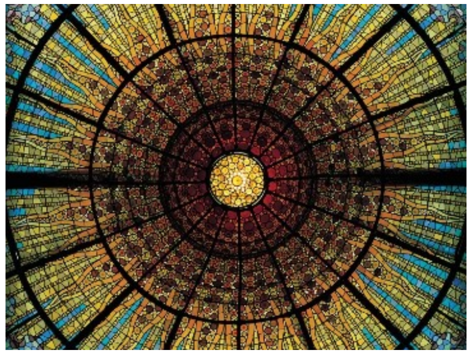

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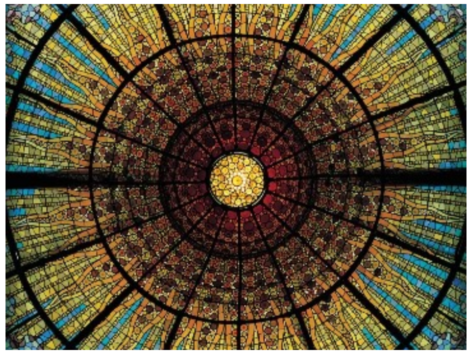

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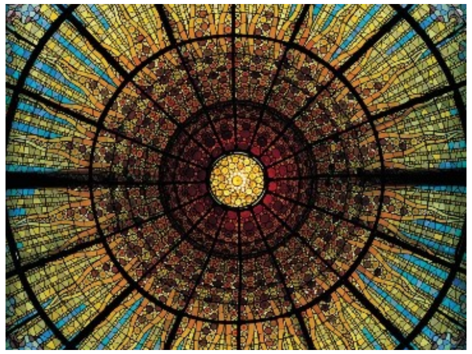
Soft theorems $\xrightarrow{\mathscr{M}\text{ellin}}$ CFT correlator for $\mathcal{O}_\Delta$ with conformally soft $\Delta \in \mathbb{Z}$

Symmetries



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$$\begin{tikzcd}
\text{Soft theorems} \arrow[r, "\mathscr{M}\text{ellin}"] & \text{CFT correlator for } \mathcal{O}_\Delta \text{ with conformally soft } \Delta \in \mathbb{Z} \arrow[d, "\substack{\text{CFT tool:} \\ \text{conformal representation theory}}"] \\
& \text{Ward identity for } \mathcal{O}_\Delta
\end{tikzcd}$$

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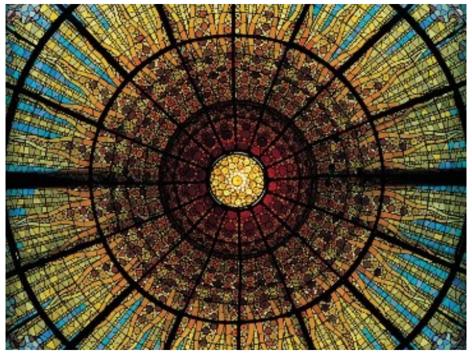

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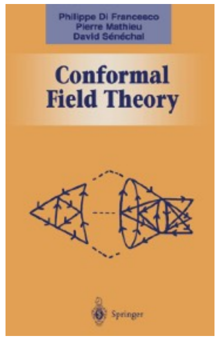
$\downarrow$ CFT tool:

conformal representation theory

Ward identity for $\mathcal{O}_\Delta$

$\downarrow$



(+ a bit more advanced stuff)

Noether current → charge → symmetry group.

Soft Ward identities

(Energetically) soft theorems : $\frac{1}{\omega}, 1, \omega, \dots$

$$\lim_{\omega \to 0} \mathscr{A}_{N+1}(\omega q, \varepsilon) = \sum_k S^{(1-k)}(\omega q, \varepsilon) \mathscr{A}_N$$

propagation direction of soft particle

polarization

Soft Ward identities

(Energetically) soft theorems :

$$\lim_{\omega\to 0}\mathscr{A}_{N+1}(\omega \underset{\substack{\big\uparrow\\ \text{propagation direction of soft particle}}}{q},\varepsilon)=\sum_k \overset{\substack{\frac{1}{\omega},1,\omega,\dots\\ \big\downarrow}}{S^{(1-k)}}(\omega q,\underset{\substack{\big\uparrow\\ \text{polarization}}}{\varepsilon})\mathscr{A}_N$$

$\mathscr{M}$ellin

$$\int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta} \quad \Bigg\downarrow$$

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$$\lim_{\Delta\to k}(\Delta-k)\langle\mathscr{O}_{\Delta,\ell}(q,\varepsilon)\mathscr{O}_{\Delta_1,\ell_1}\dots\mathscr{O}_{\Delta_N,\ell_N}\rangle=\sum_{i=1}^{N}\hat{S}_i^{1-k}(q,\varepsilon)\langle\mathscr{O}_{\Delta_1,\ell_1}\dots\delta\mathscr{O}_{\Delta_i,\ell_i}\dots\mathscr{O}_{\Delta_N,\ell_N}\rangle$$

soft operator

$\Delta = 1, 0, -1, \dots$

gauge theory: [Fan,Faotopoulos,Taylor'19][[Nandan,Schreiber,Volovich,Zlotnikov'19]Pate,Raclariu,Strominger'19]

gravity: [Adamo,Mason,Sharma'19][AP'19] [Guevara'19]

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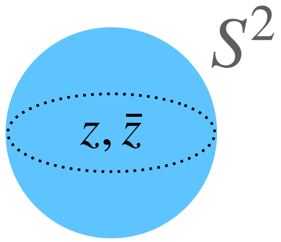
Classify all conformally soft operators in gauge theory and gravity.

CCFT$_2$ $\rightarrow$ $x = (z, \bar{z})$



Conformally soft primary $\mathcal{O}_\Delta$ operators have descendant operators that are primaries themselves. They organize into **multiplets**.

[Pasterski,AP,Trevisani'21]

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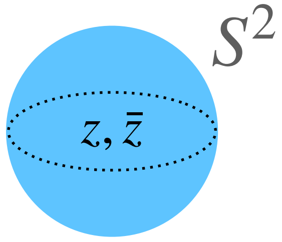

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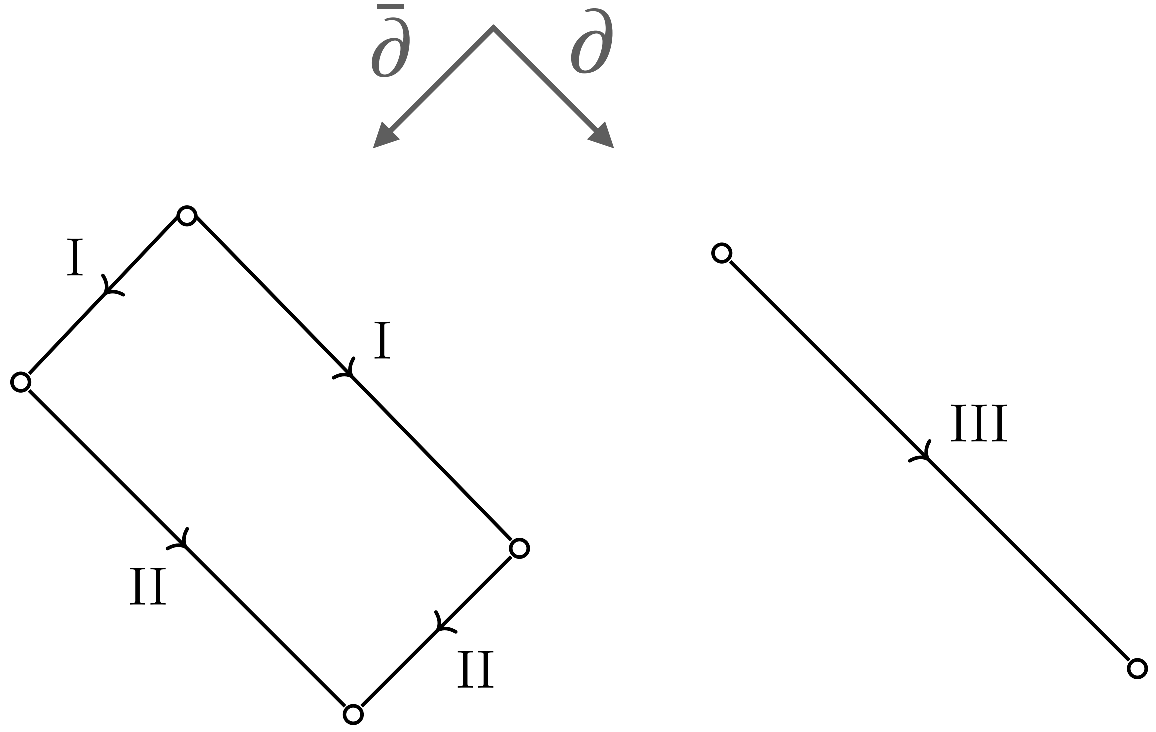
celestial diamonds



types I, II, III: spin of descendant >,<,= spin of parent primary

[Pasterski,AP,Trevisani'21]

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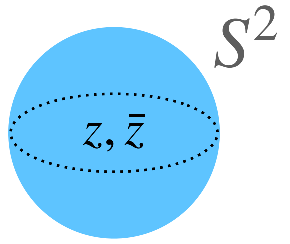

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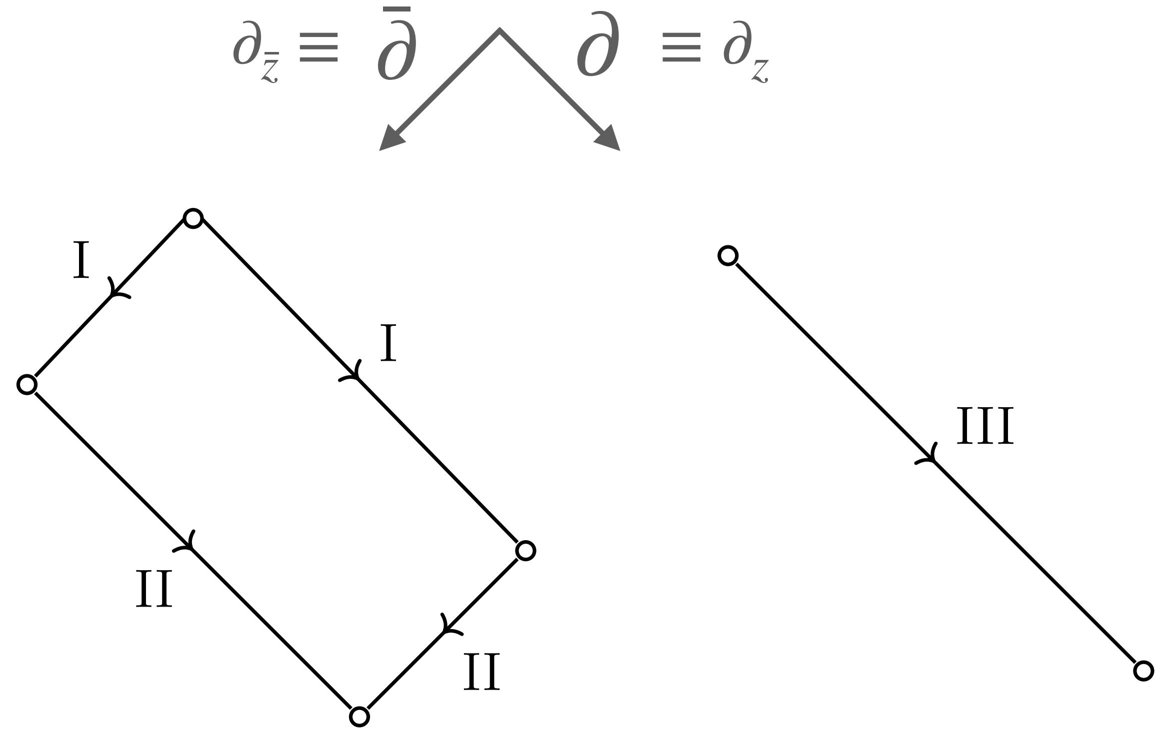
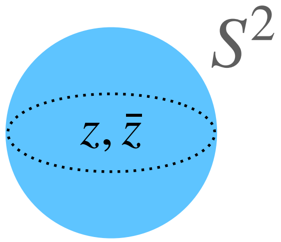


types I, II, III: spin of descendant >,<,= spin of parent primary

[Pasterski,AP,Trevisani'21]

CCFT$_2$ $\rightarrow$ $x = (z, \bar{z})$



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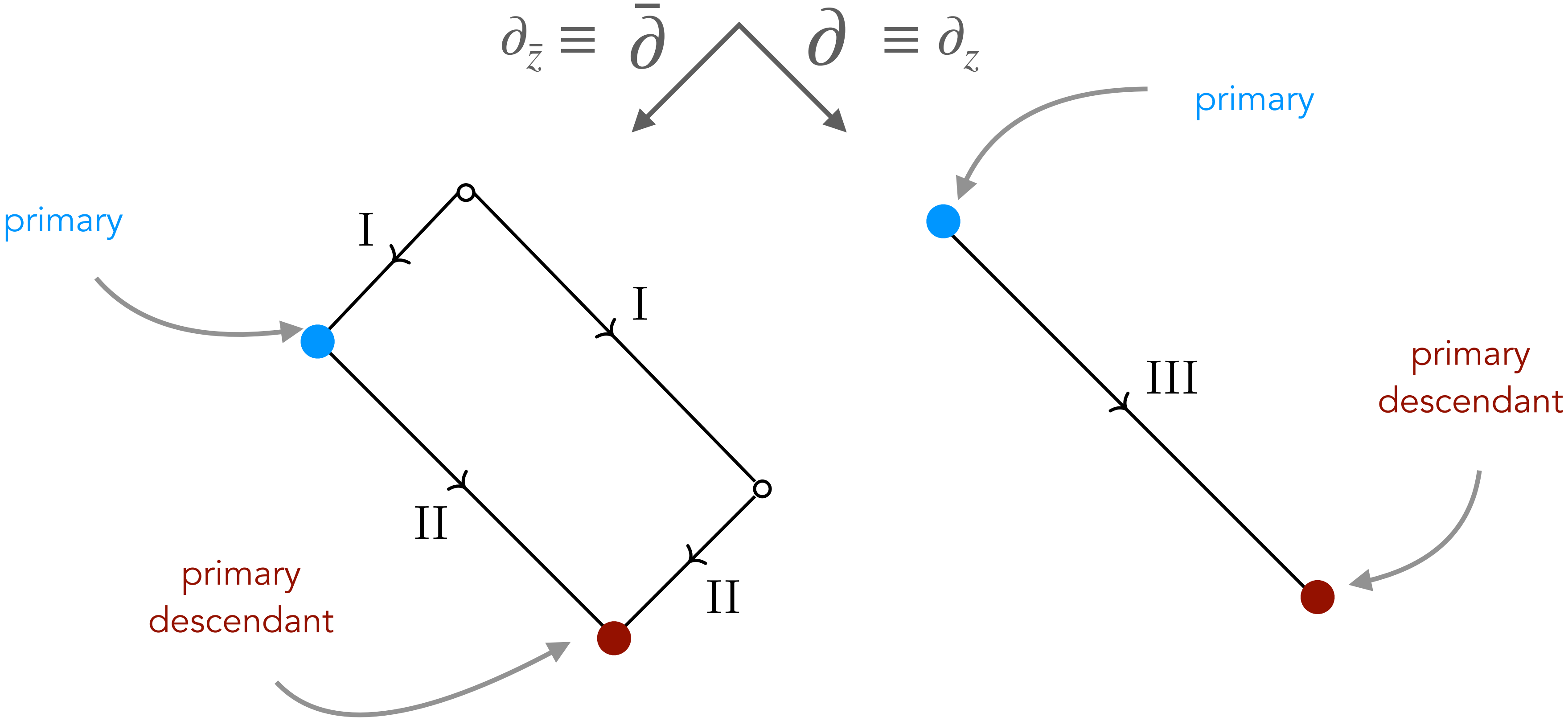

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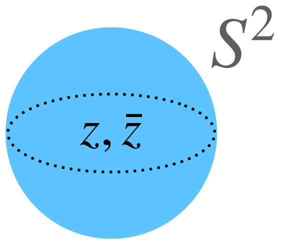

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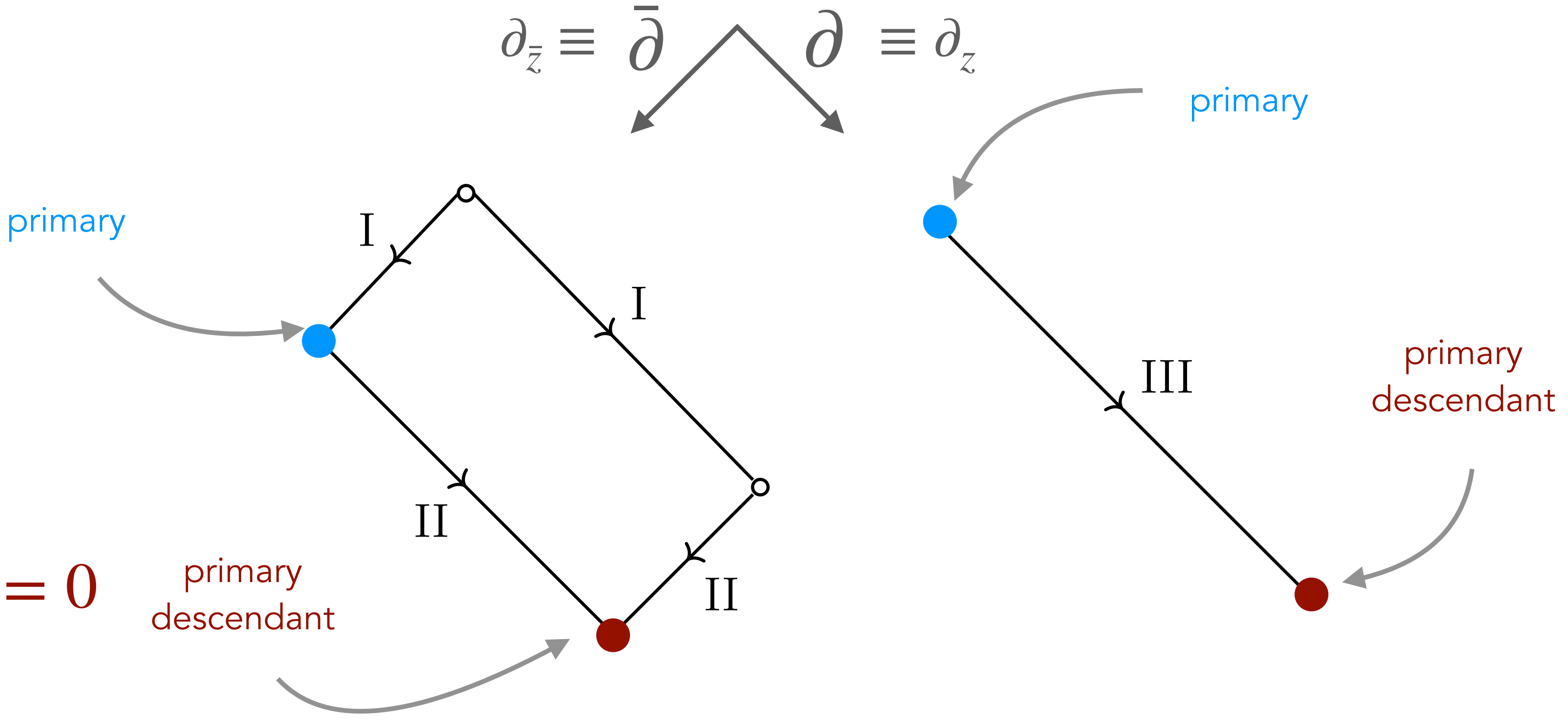

$$\bar{\partial}^{\#} \mathcal{O}(z, \bar{z}) = 0$$

$\uparrow$

\# depends on

types I, II, III: spin of descendant >,<,= spin of parent primary

Bottom of a diamond: conservation equation for $\mathcal{O}_\Delta$.

[Pasterski,AP,Trevisani'21]

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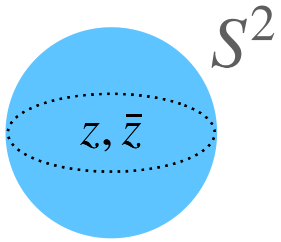

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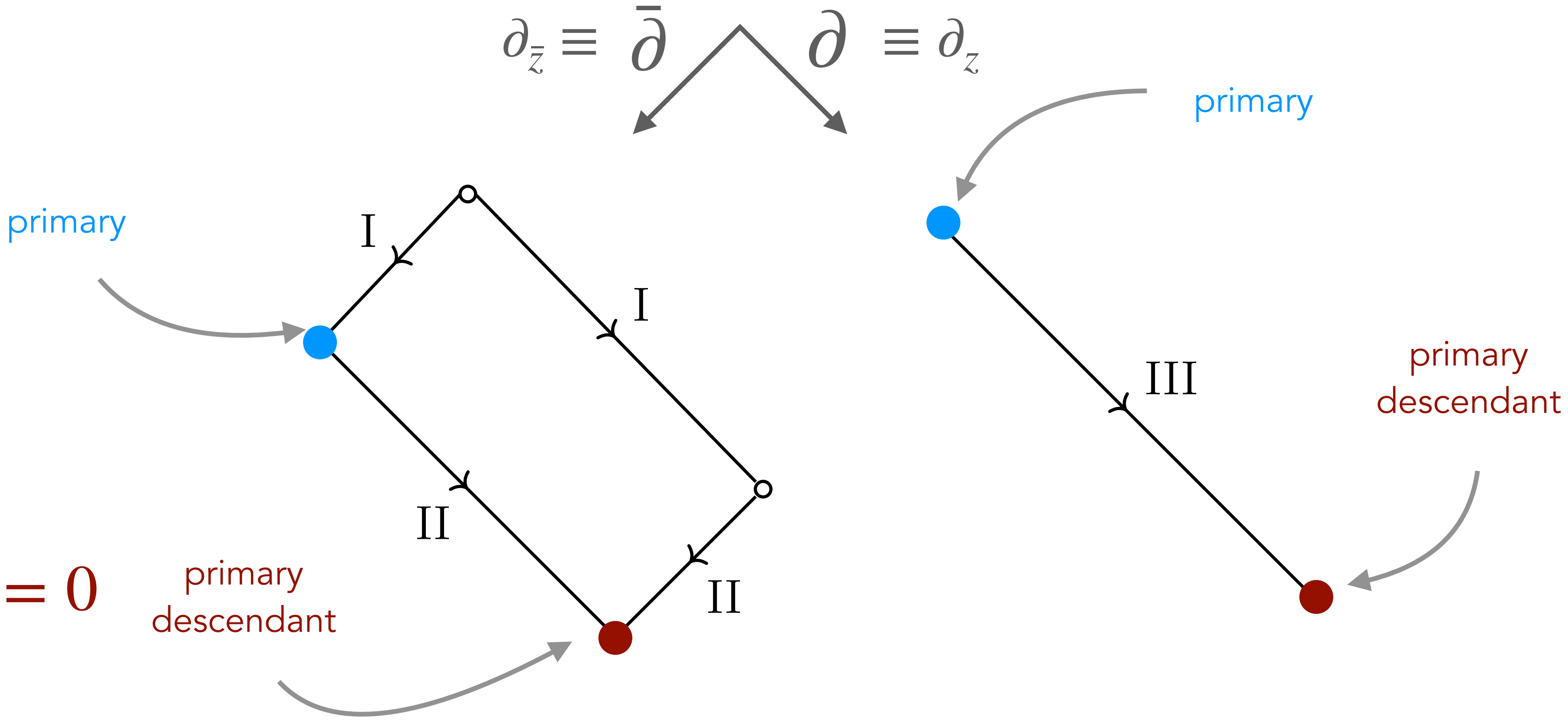

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Noether current: $$\mathcal{J} = \sum_{m=0}^{\#-1} (-1)^m \bar{\partial}^m \epsilon(z, \bar{z}) \bar{\partial}^{\#-m-1} \mathcal{O}(z, \bar{z})$$

[Pasterski,AP,Trevisani'21]

Soft symmetries

Leading soft photon theorem:

$$\bar{\partial} \lim_{\Delta \to 1} (\Delta - 1)\mathcal{O}_{\Delta,\ell=+1} = 0$$

Leading soft graviton theorem:

$$\bar{\partial}^2 \lim_{\Delta \to 1} (\Delta - 1)\mathcal{O}_{\Delta,\ell=+2} = 0$$

Subleading soft graviton theorem:

$$\bar{\partial}^3 \lim_{\Delta \to 0} \Delta\mathcal{O}_{\Delta,\ell=+2} = 0$$

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$$\underbrace{\bar{\partial} \lim_{\Delta \to 1} (\Delta - 1) \mathcal{O}_{\Delta, \ell = +1}}_{\bar{\partial} J} = 0$$

→ large gauge transformations

$\mathcal{J}$ with $\bar{\partial}\epsilon = 0$

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From OPE of **type** I, **II**, III: find algebra $w_{1+\infty}$!

[Guevara,Himwich,Pate,Strominger'21]
[Strominger'21]

From $d = 2$ to $d > 2$

[Pano,AP,Trevisani'23]

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- ∃ more types of primary descendants:

even d**:** I, II, III vs **odd** d**:** I, II, P, S

parity shadow

From $d = 2$ to $d > 2$

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- ∃ more types of primary descendants:

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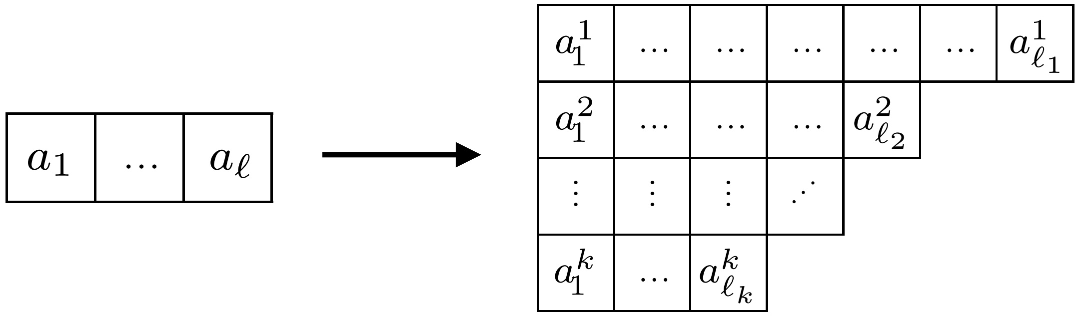
parity shadow

- $SO(d)$ spin more complicated:

→ to capture all soft theorems:

go **beyond traceless and symmetric**



→ **refinement** of types $\mathrm{I}, \mathrm{II}, \mathrm{III} \rightarrow \mathrm{I}_\mathrm{k}, \mathrm{II}_\mathrm{k}, \mathrm{III}_\mathrm{k}$

From $d = 2$ to $d > 2$

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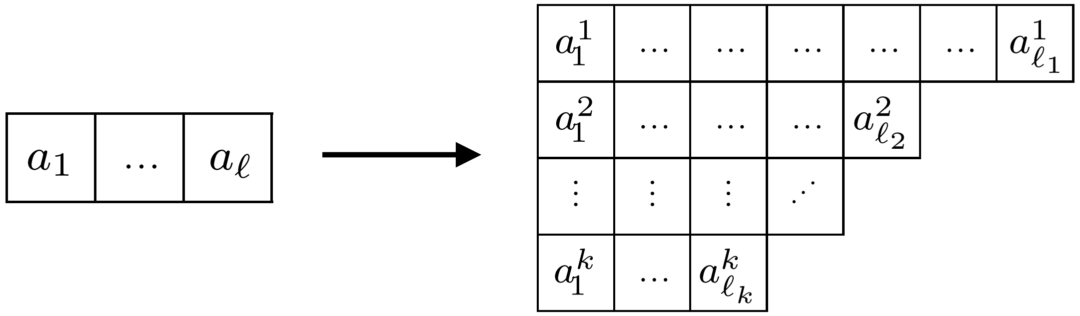

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→ interpretation in $\mathbb{R}^{1,d+1}$?

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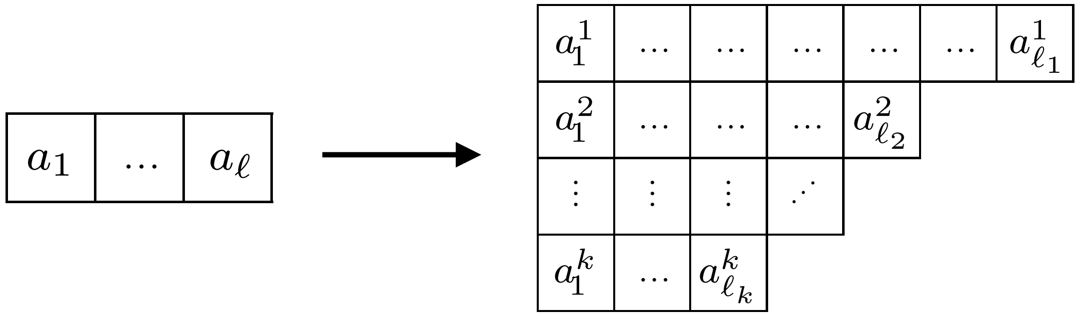

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Summarizing 50+ pages…

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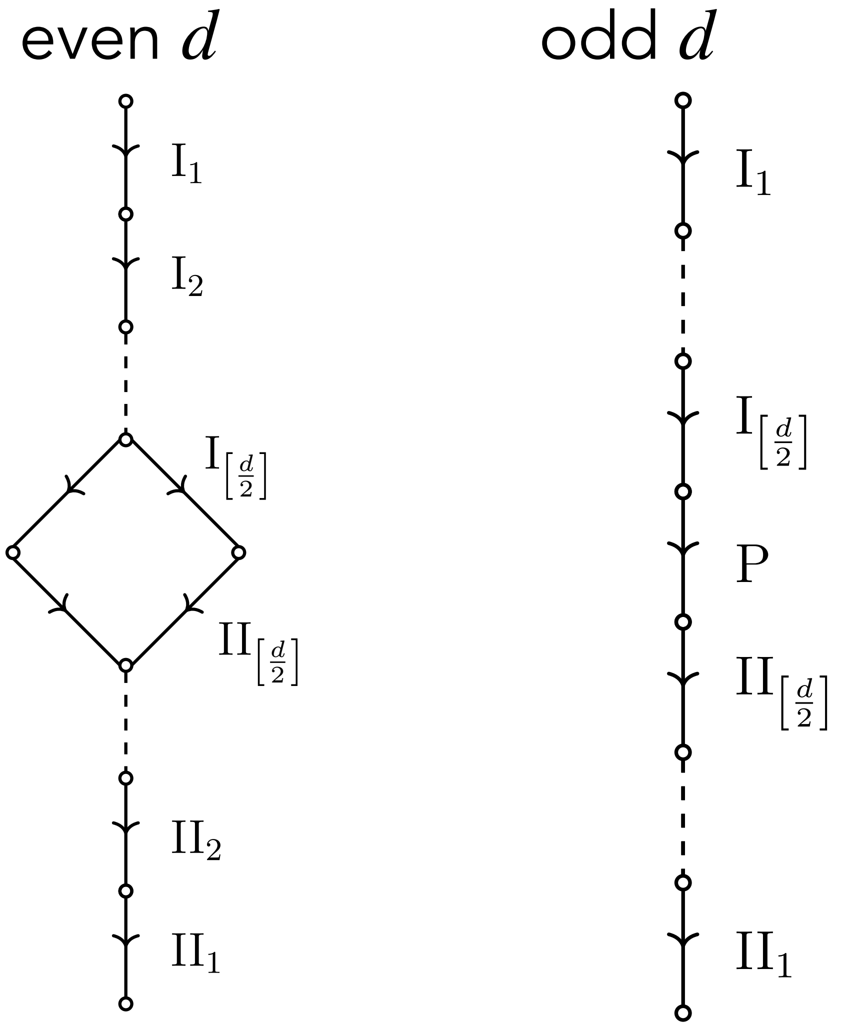

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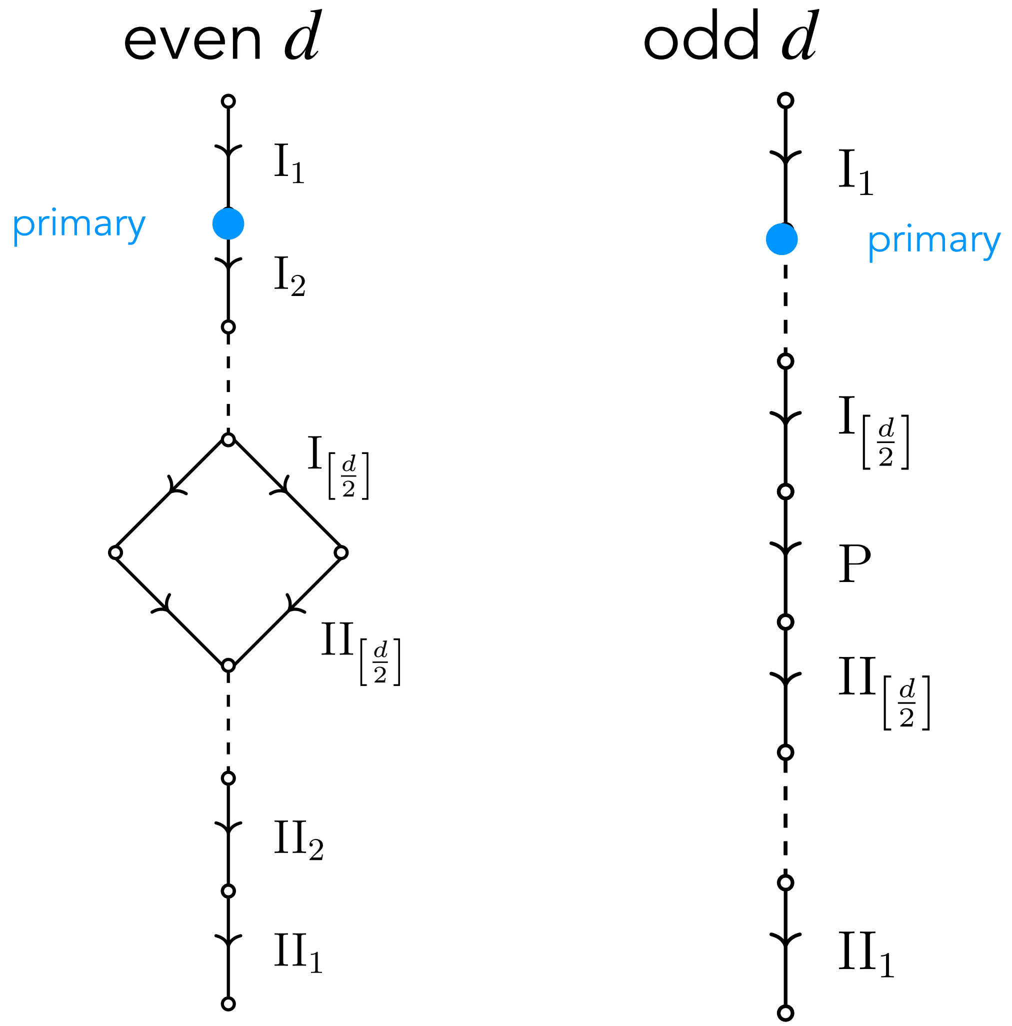

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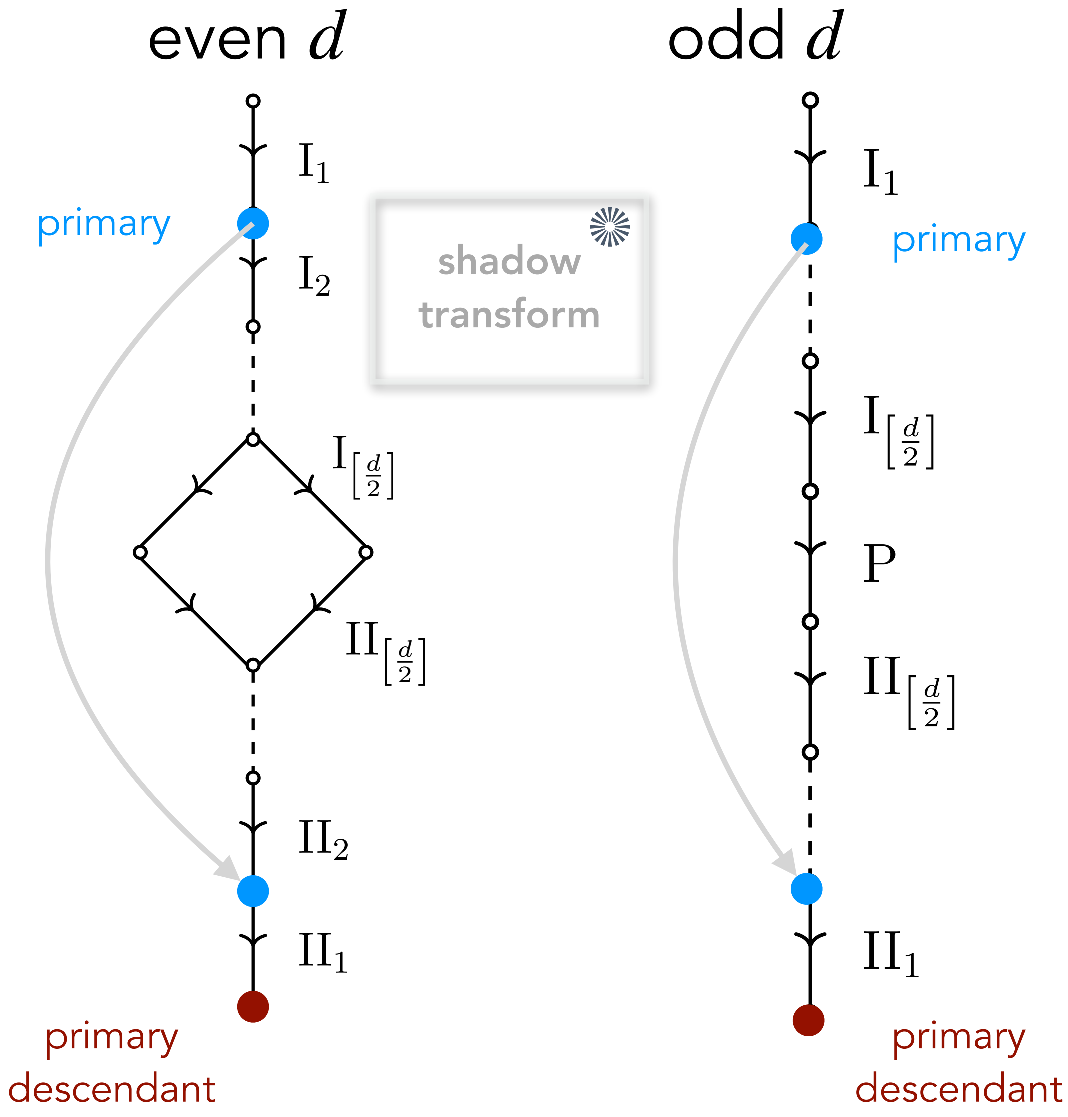

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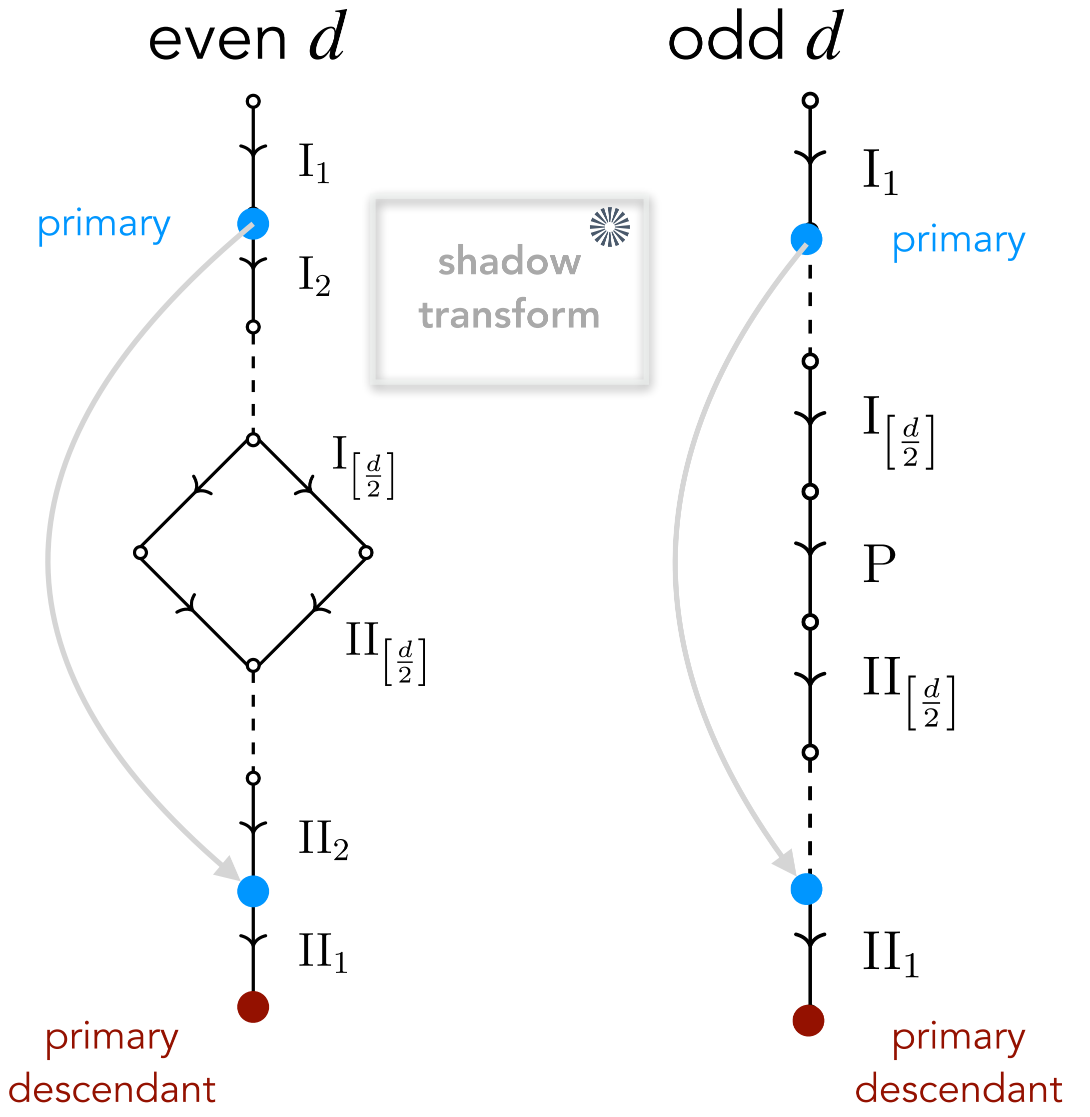

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Bottom of a necklace: conservation equation for $\mathcal{O}_\Delta$.

[Pano,AP,Trevisani'23]

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↓

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very constraining!

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CCFT$_d$: J, T, P

$d+2$ bulk: U(1), Lorentz, translations

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Charges for **global symmetries!**

$$Q^{\epsilon}_{\Sigma} = \int_R d^d x\, \epsilon_{a_{n+1}\dots a_\ell}(x)\partial_{a_1}\dots\partial_{a_n}\mathcal{O}^{a_1\dots a_n\dots a_\ell}(x)$$

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Noether current → charge → symmetry group:

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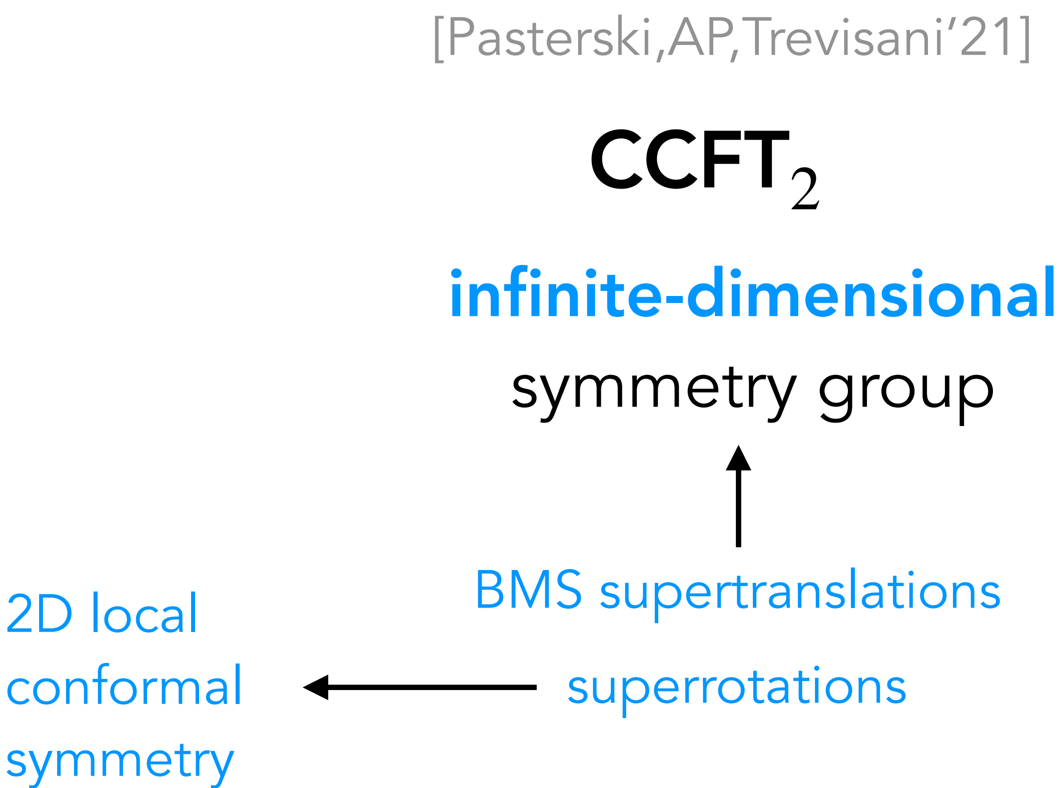

[Pasterski,AP,Trevisani'21]

CCFT$_2$

infinite-dimensional

symmetry group

↑

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CCFT$_{d>2}$

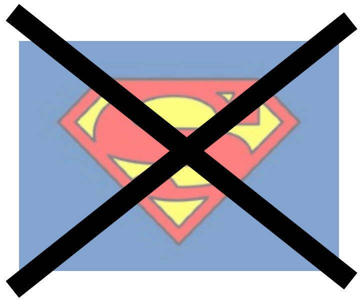
finite-dimensional

symmetry group

↑

translations

rotations



- Role of higher-dimensional BMS ?

Soft symmetries

Noether current → charge → symmetry group:

[Pasterski,AP,Trevisani'21]

CCFT$_2$

infinite-dimensional

symmetry group

↑

BMS supertranslations

superrotations ⟶ 2D local conformal symmetry

[Pano,AP,Trevisani'23]

CCFT$_{d>2}$

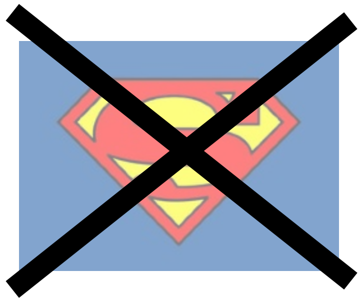
finite-dimensional

symmetry group

↑

translations

rotations



- Role of higher-dimensional BMS ?

compare to [Kapec,Lysov,Pasterski,Strominger'15]
vs [Hollands,Ishibashi,Wald'16]

interesting interpretation: [Kapec'22]

distributional support on the sphere from bulk $\delta^{(d+2)}(\sum_{i=1}^{N} p_i^{\mu})$
can be **smeared out** if translation symmetry broken

↓

III. Celestial amplitudes on backgrounds

Scattering on backgrounds

To study wave scattering on classical backgrounds we use the method of [Boulware,Brown'68] which amounts to solving the classical equations of motion for $\Phi(x)$ in the presence of a source $J(x)$.

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order n in coupling

Solve for $\bar{\Phi}(p) = \sum_{n=0}^{\infty} \bar{\Phi}^{(n)}(p)$ in momentum-space order-by-order in coupling.

The n-point amplitude is:

$$\mathscr{A}_n(p_1, \ldots, p_n) = i^n \prod_{k=1}^{n} (\lim_{p_k^2 \to 0} p_k^2) \frac{\delta}{\delta \bar{J}(p_1)} \cdots \frac{\delta}{\delta \bar{J}(p_{n-1})} \bar{\Phi}(-p_n) \Big|_{J=0}$$

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Consider scattering scalars on particle-like backgrounds (sources of mass and charge) & interpret them in celestial CFT.

↓

[Gonzo, McLoughlin, AP'22]

Coulomb field of static and spinning point charges & ultraboost limits.
Schwarzschild, Kerr & ultraboost limits: Aichelburg-Sexl and gyraton metrics.

Scattering on particle-like backgrounds

Focus on 2-point amplitudes:

$$\mathscr{A}_2(p_1, p_2) = -\prod_{k=1}^{2}(\lim_{p_k^2 \to 0} p_k^2)\frac{\delta}{\delta \bar{J}(p_1)}\bar{\phi}(-p_2)\Big|_{J=0}$$

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Solve for $\bar{\phi}(p) = \sum_{n=0}^{\infty}\bar{\phi}^{(n)}(p)$ perturbatively in G and to linear order in h and plug into

$$\mathcal{A}_2(p_1,p_2) = (2\pi)^4\lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_1^2\delta^{(4)}(p_1+p_2) - [(p_1)_\mu(p_2)_\nu - \frac{1}{2}\eta_{\mu\nu}p_1\cdot p_2]\bar{h}^{\mu\nu}(p_1+p_2) + \dots$$

$n=0$

$n=1$

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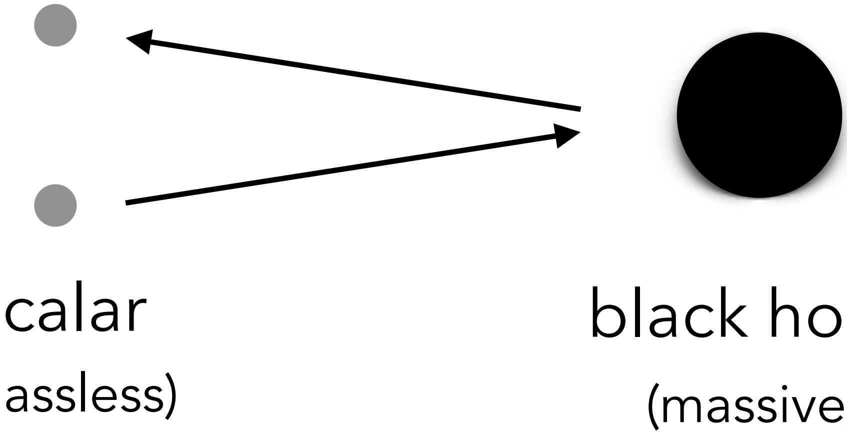
Black hole avatars in CCFT

[Gonzo, McLoughlin, AP'22]

Compute celestial scattering on **Schwarzschild** & **Kerr**:



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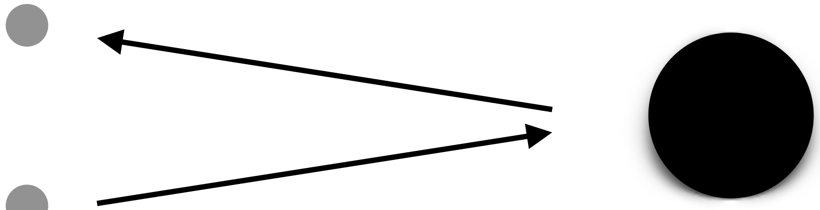

scalar

(massless)

black holes

(massive)

- Celestial amplitudes on **backgrounds**: <u>nicer features</u> than in **flat** space

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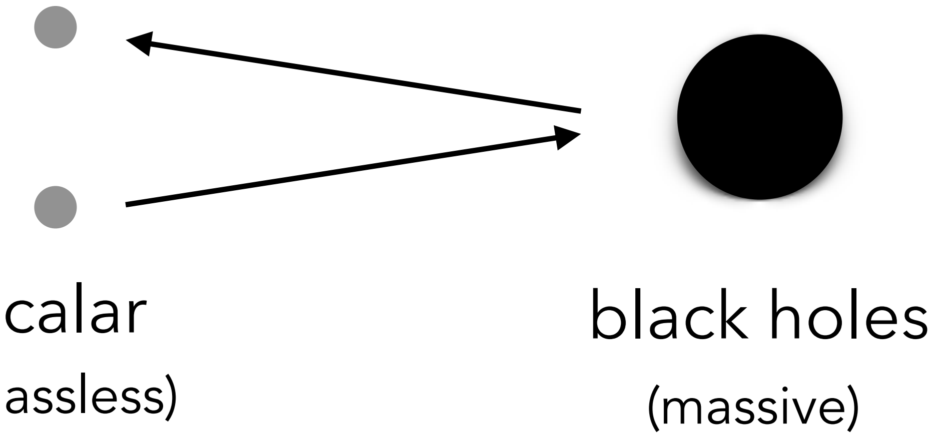

- Celestial amplitudes on **backgrounds**: nicer features than in **flat** space

δ-function support on S^2 !

$\mathscr{M}$ellin integrals UV divergent.

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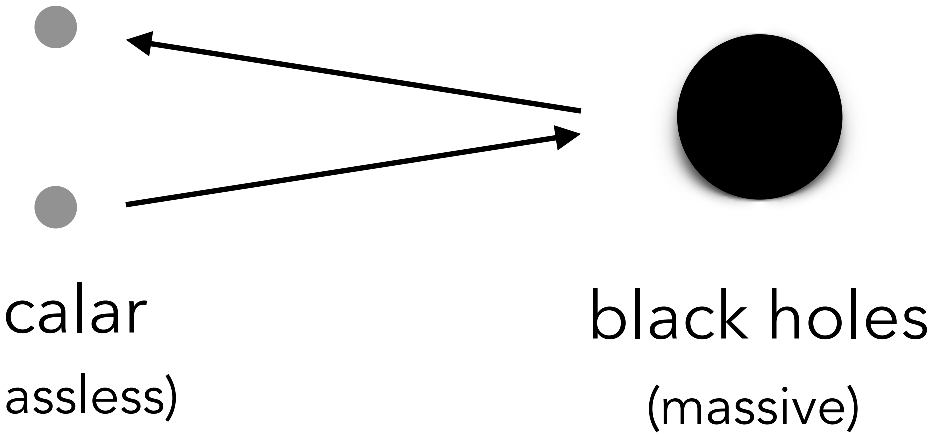

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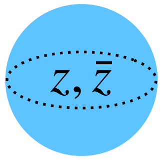


Supported everywhere on the S^2.

power-law in $z_{ij} = z_i - z_j$!

vs

δ-function support on S^2 !

Classical spin acts as UV regulator.

$\mathscr{M}$ellin integrals UV divergent.

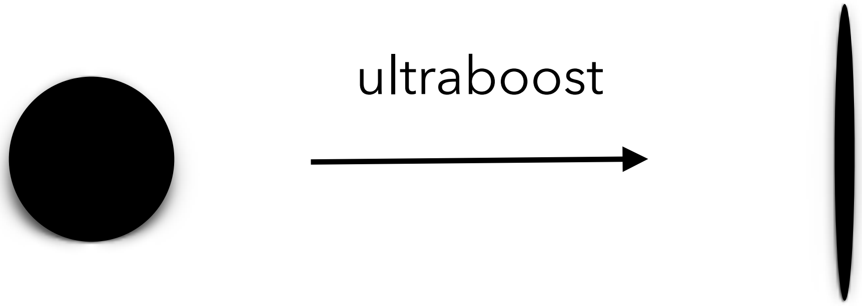
non-spinning spinning

$$\begin{tikzcd} \int_0^\infty d\omega\omega^{\Delta-1} \arrow[r] \arrow[d] & \int_0^\infty d\omega\omega^{\Delta}H^{(2)}_{-1}(a\omega) \arrow[d] \\ 2\pi\delta(i\Delta) & \text{finite support} \end{tikzcd}$$

Hankel fct

Shocks in CCFT

Ultraboost limit of black holes is special:



ultraboost

black holes

Schwarzschild metric

shockwaves

Aichelburg-Sexl metric

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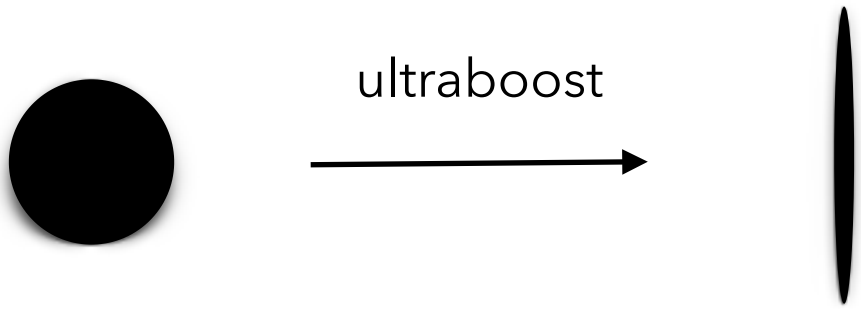

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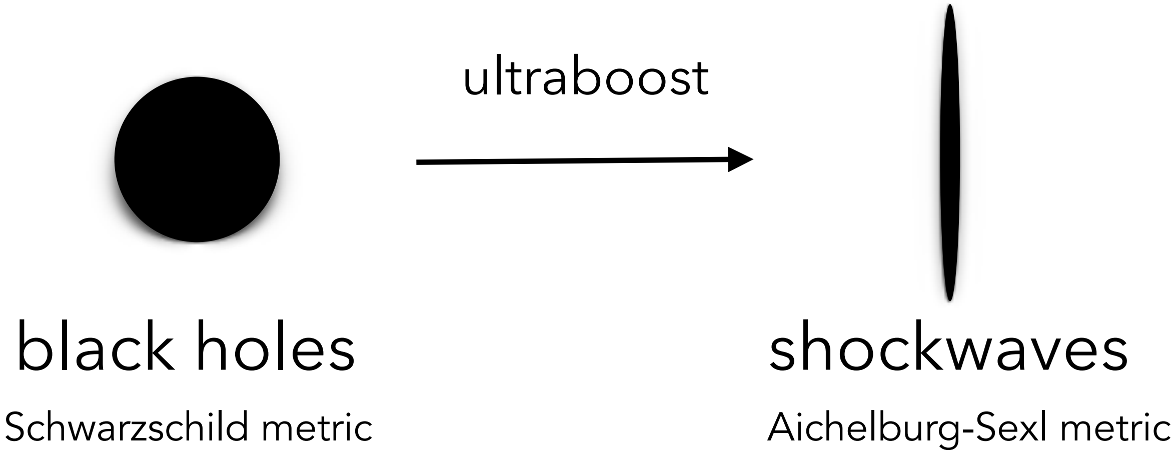

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Scalar shockwaves: $\phi_{sw}(X) = -\log(X^2)\delta(q \cdot X)$ $(\Delta, \ell) = (1{,}0)$ scalar primary

$$\uparrow$$

$$q^\mu = (1 + |z_{sw}|^2, z_{sw} + \bar{z}_{sw}, i(\bar{z}_{sw} - z_{sw}), 1 - |z_{sw}|^2)$$

null vector pointing at $(z_{sw}, \bar{z}_{sw})$ on celestial sphere

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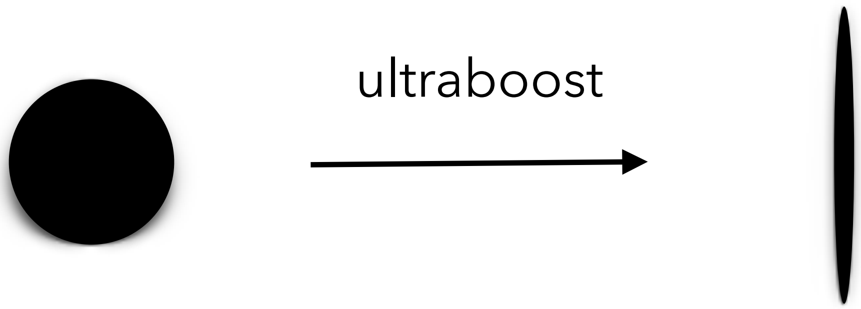

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Kerr-Schild double copy $\downarrow$

Spinning shockwaves:

$A_\mu(X) = r_0\phi_{sw}(X)q_\mu$ $(\Delta, \ell) = (0{,}0)$ vector primary

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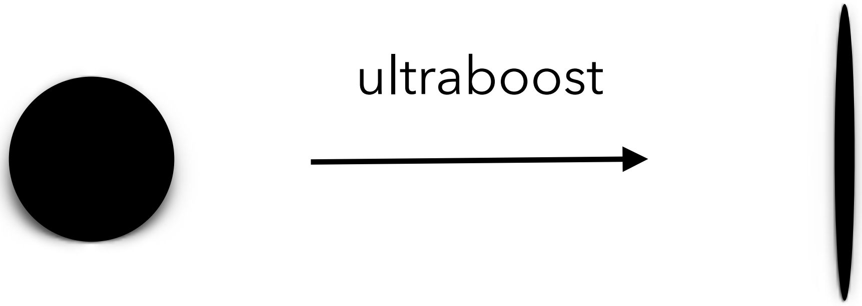

black holes

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Aichelburg-Sexl metric $g_{\mu\nu} = \eta_{\mu\nu} + r_0 \log(X^2)\delta(q \cdot X) q_\mu q_\nu$

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Celestial shockwave correlators

[Gonzo,McLoughlin,AP'22]

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$$\mathcal{M}_3(\Delta_1, \Delta_2, \underset{\underset{0}{\parallel}}{\Delta_{sw}}) = \frac{e(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 2))}{|z_{12}|^{\Delta_1+\Delta_2} |z_{1sw}|^{\Delta_1-\Delta_2} |z_{2sw}|^{\Delta_2-\Delta_1}}$$

Looks like 3-point correlator in standard CFT$_2$!

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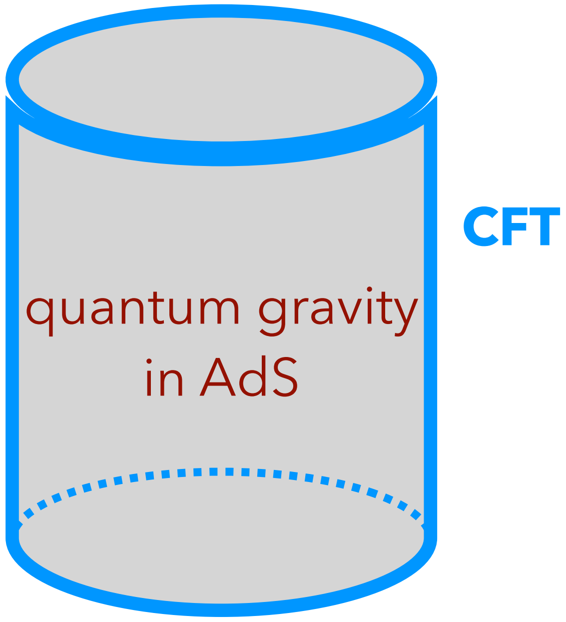
(after continuing off the principal series $\mathrm{Re}(\Delta_1 + \Delta_2) = 1$) !

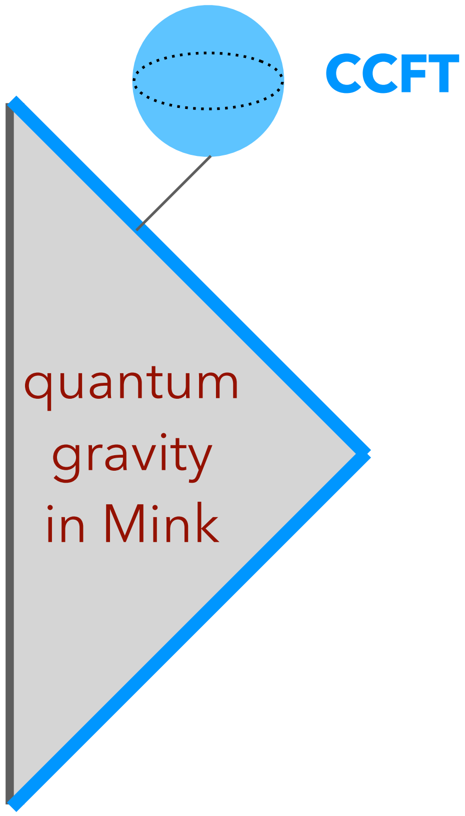
Correlators from on-shell action

$$S = \text{eom} \ + S_{\text{bdy}}$$

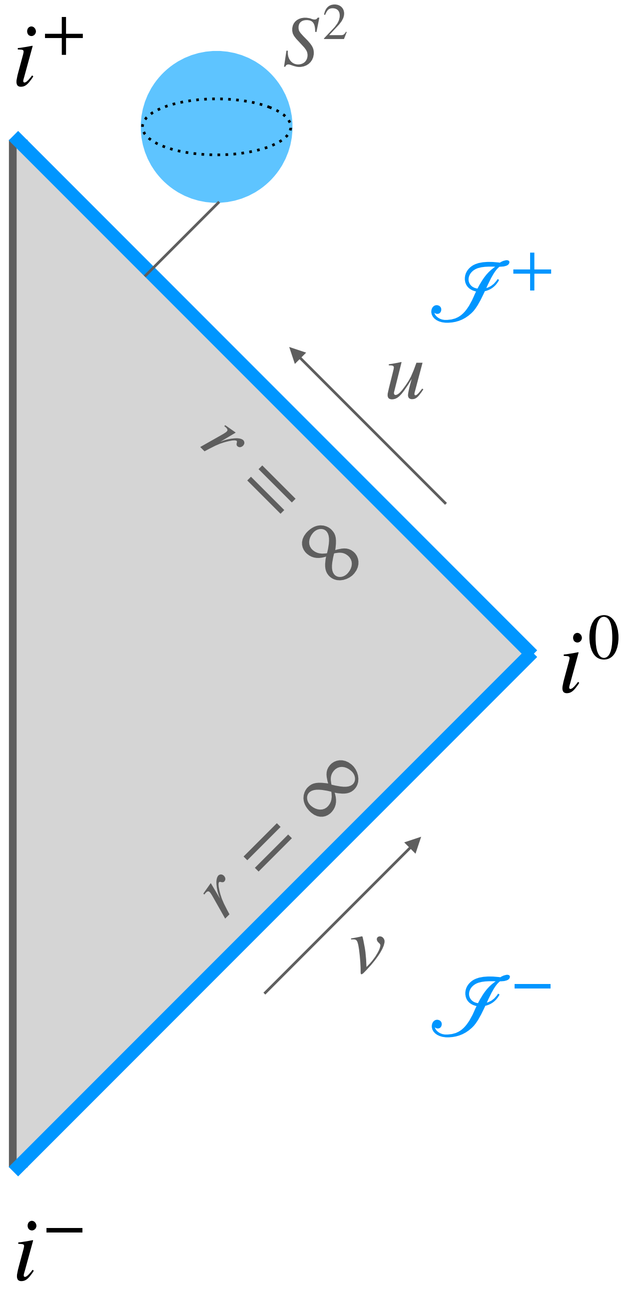
In AdS/CFT on-shell action generates CFT correlators.

Celestial holography: on-shell action generates CCFT correlators?





On-shell action



$$S = \text{eom} \;+ S_{\mathscr{I}^- \cup \mathscr{I}^+}$$

* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky’93]

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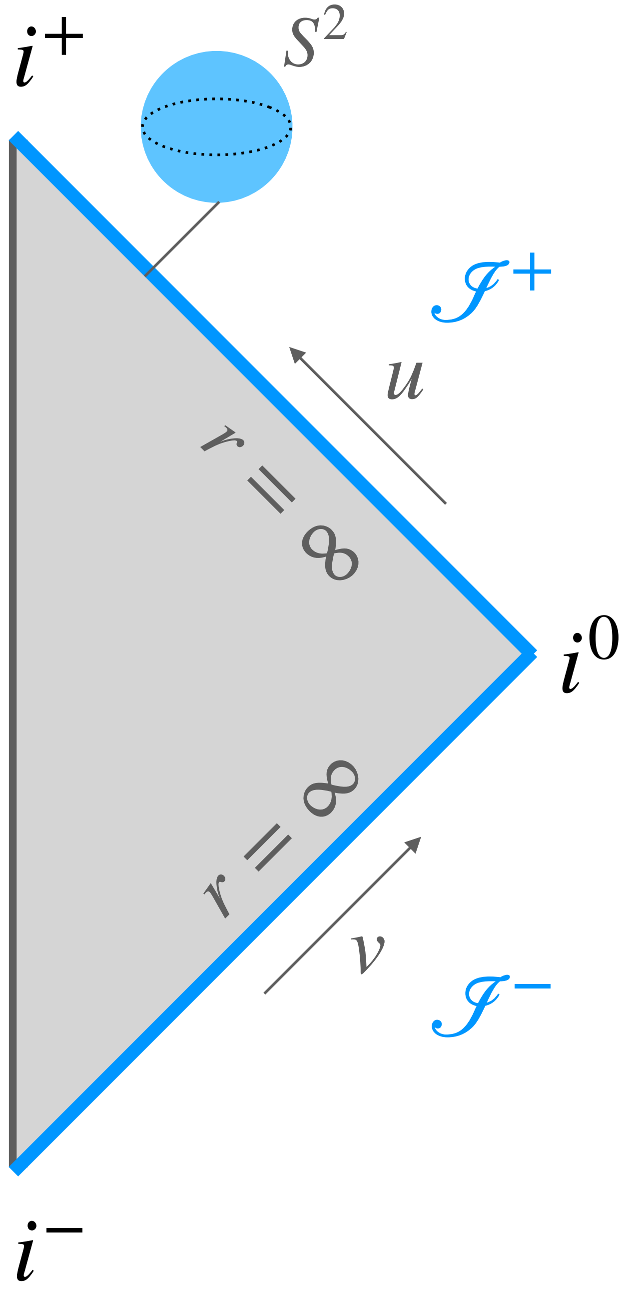

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Complex massless scalar ϕ minimally coupled to gravity

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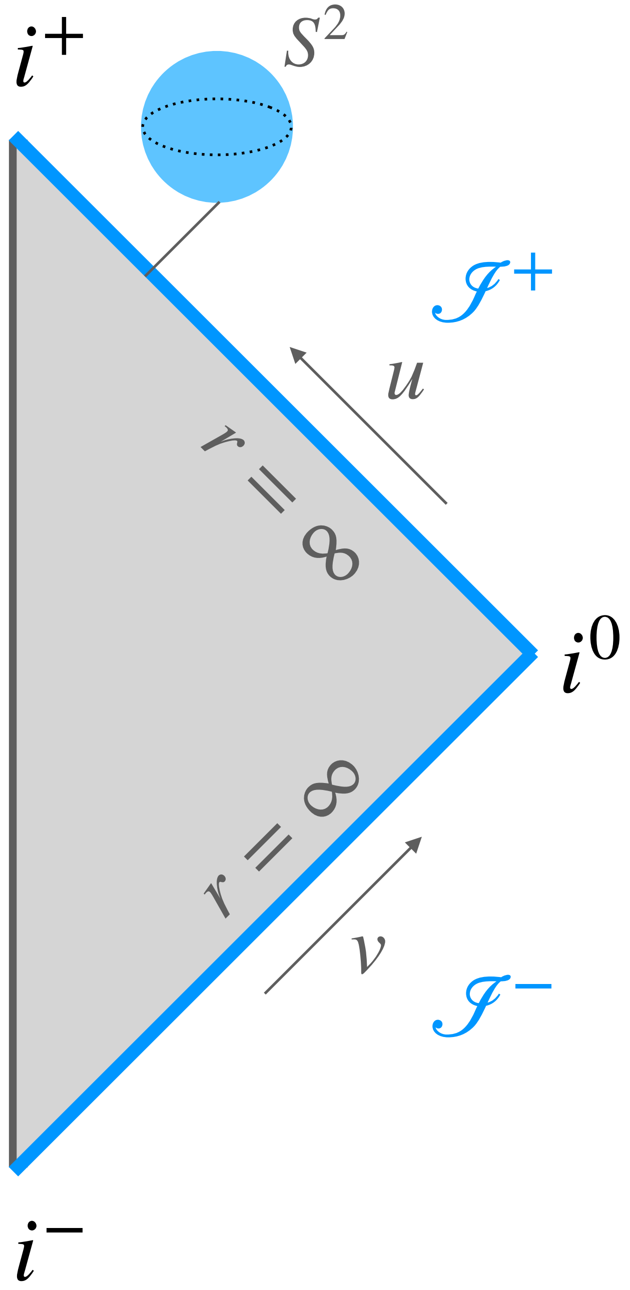

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effective source

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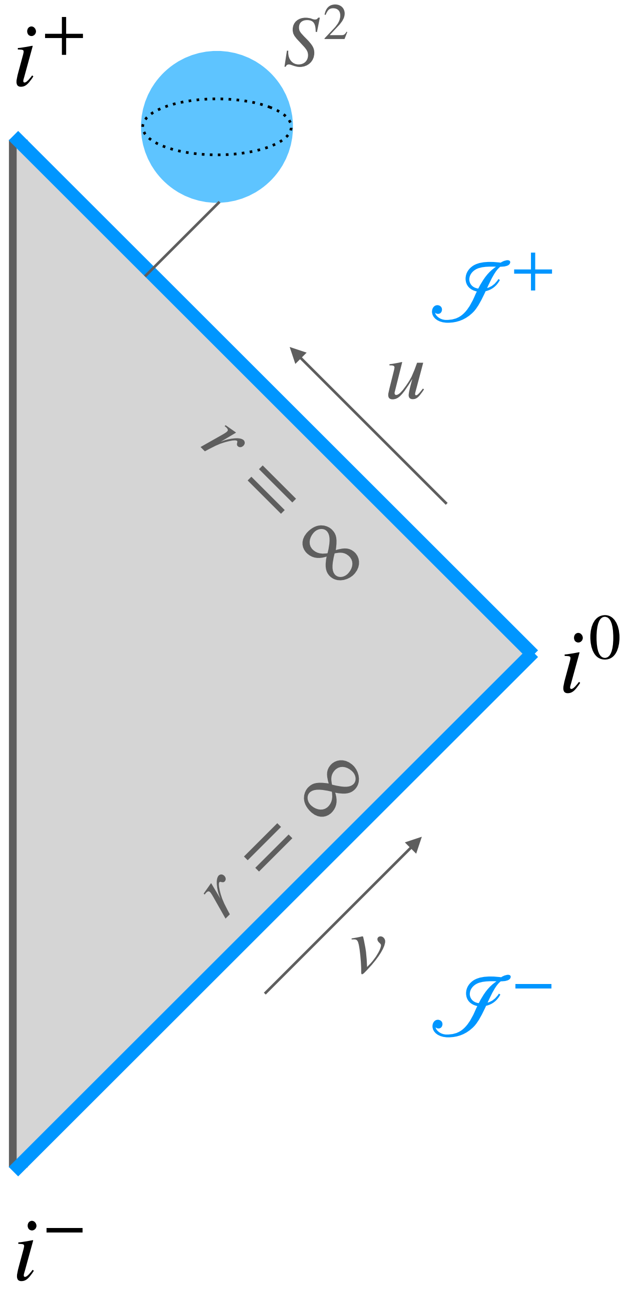
On-shell action



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$\phi = \phi_{in} + \phi_{out}$ $\qquad$ $\phi_{in} = e^{ip\cdot X}$ & ϕ_{out} solved via Green's fct

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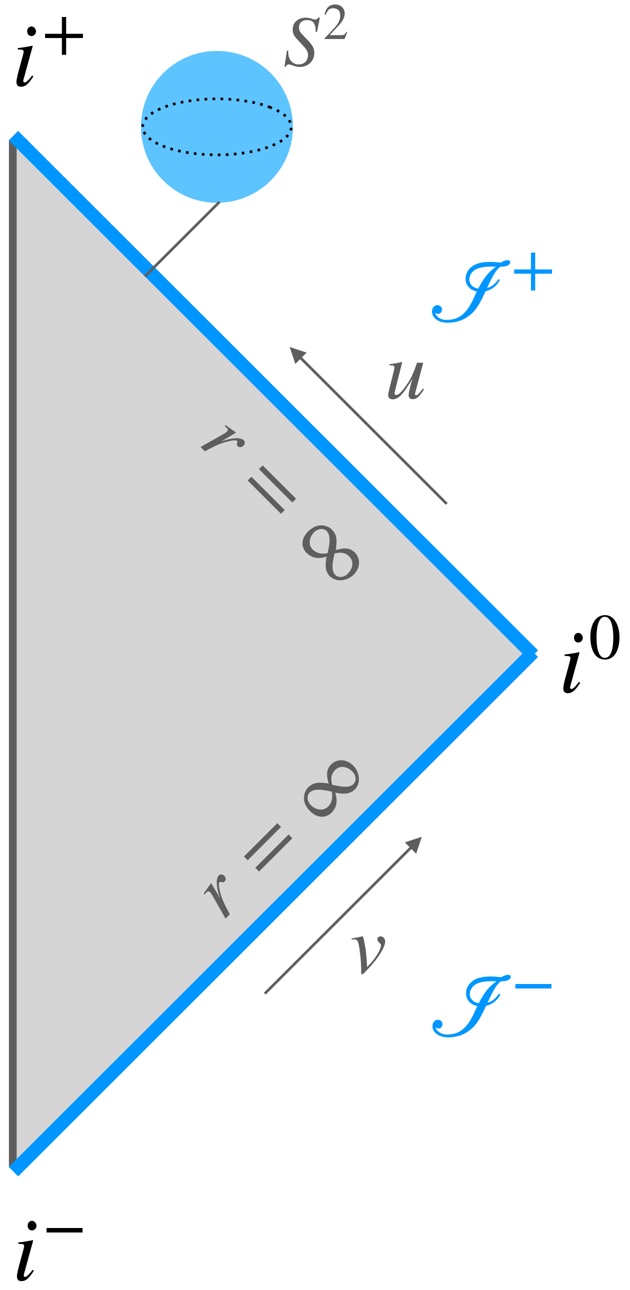

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At large r at fixed $v = t + r$ and $u = t - r$:

$$S_{\mathscr{I}^- \cup \mathscr{I}^+}(p) = \# \bar{J}_{eff}(p)$$

[Gonzo, McLoughlin, AP'22]

On-shell action localizes on the boundary onto the Fourier transform of effective source evaluated along the incoming momentum.

* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

Generating CCFT correlators

$$\mathscr{A}_2(p_1,p_2) \overset{\text{Boulware-Brown}}{\overset{\downarrow}{=}} -\lim_{p_1^2\to 0}\lim_{p_2^2\to 0} p_1^2 p_2^2 \frac{\delta\bar{\phi}_{out}(-p_1)}{\delta\bar{J}(p_2)}$$

[Gonzo, McLoughlin, AP'22]

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solving the eom
to leading order
in coupling

$$\bar{\phi}_{out}(p) = -\frac{\bar{J}_{eff}(p)}{p^2}$$

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large *r* limit +
saddle point
approximation

$$S_{\mathscr{I}^-\cup\mathscr{I}^+}(p) = \#\bar{J}_{eff}(p)$$

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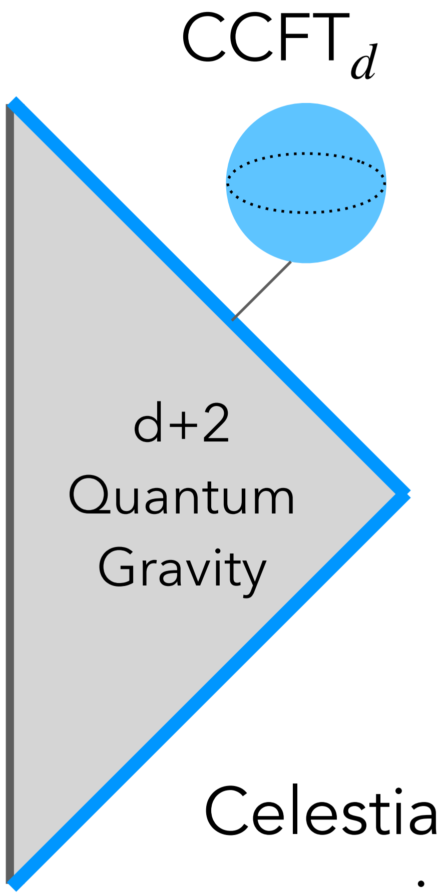
Boundary on-shell action generates CCFT correlators!

[Gonzo, McLoughlin, AP'22]

Summary and outlook



We classified the soft symmetries of celestial CFT$_d$.

In $d=2$ they include infinite BMS-type enhancements which obey $w_{1+\infty}$ algebra.

In $d>2$ there are only finite-dimensional global symmetries.

Celestial string amplitudes: well-defined Mellin integrals, ∃ natural loop expansion parameter and get field theory limit regardless of cross ratio.

Celestial amplitudes on backgrounds: smear δ-fct support on sphere, well-defined Mellin integrals for classical spin.

Future: identify CCFTs and all its properties!

Axioms? Toy models? More string theory! More loops! Non-perturbative physics?…