





### ANDREA PUHM

#### AMPLITUDES @ CERN

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### **S-matrix**

has holographic flavour can we make it more manifest?







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Exploit powerful CFT toolkit and other methods such as twistors.







has holographic flavour can we make it more manifest?

Exploit powerful CFT toolkit and other methods such as twistors.

bottom-up top-down

#### **understand universal properties** of

quantum gravity that are independent of short distance microphysics



explore toy models and (super) symmetry to **construct dual pairs**





\*extends to general spacetime dimensions

# Celestial Holography



time

#### **Some recent progress:**

- Bulk  $O \rightarrow$  boundary  $O$
- CCFT building blocks
- universal features of observables



\*extends to general spacetime dimensions

# Celestial Holography



time

#### **Some recent progress:**

- Bulk  $O \rightarrow$  boundary  $O$
- CCFT building blocks
- universal features of observables

novel type of holography



\*extends to general spacetime dimensions

From bulk operators to boundary operators:







#### Carrollian primaries

null boundary: degenerate metric,  $c \rightarrow 0$  field theory



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### conformal primaries

bulk Lorentz acts as boundary global conformal which in gravity is enhanced to local conformal



Carrollian primaries

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▶ ``Extrapolate" dictionary for celestial holography.

[Pasterski,AP,Trevisani'21]

### in asymptotically flat spacetimes Observables energy basis  $|p_i\rangle = | \omega_i, x_i\rangle$ *Sd*

### **Scattering amplitudes:**

basic observables in flat space

 $\mathscr{A}(p_1,...p_n) \equiv \langle out | \mathscr{S} | in \rangle$ 



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### **Celestial amplitudes:**

natural observables in CCFT:

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\langle \mathcal{O}_{\Delta_1}(x_1) \dots \mathcal{O}_{\Delta_n}(x_n) \rangle
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transform nicely under<br>conformal transformations!



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translation symmetry and the contract of the c

plane wave 
$$
\Phi_{\omega}(X; x) = e^{ip(\omega, x) \cdot X}
$$
  

$$
p^{\mu} = \pm \omega q^{\mu}(x)
$$

### **Celestial amplitudes:**

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$$
\Phi_{\Delta}(X; x) = \frac{1}{(-q(x) \cdot X)^{\Delta}} \text{ conformal primarydisprescripton}
$$

[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

## 3 bases

Δ boost weight

*ω u*

energy null time

[Donnay, Pasterski,AP'22]



[Donnay, Pasterski,AP'22]





Together with an *iε* prescription for well-definedness of transforms. [Donnay, Pasterski,AP'22]



<sup>[</sup>Strominger'13+…]

 $\langle \text{Out} | S | \text{in} \rangle$ <sub>boost</sub> =  $\langle \mathcal{O}_{\Delta_1, \ell_1}(x_1) ... \mathcal{O}_{\Delta_n, \ell_n}(x_n) \rangle$ <sub>CCFT</sub>

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• no Wilsonian decoupling since integration over all energies

[Arkani-Hamed,Pate,Raclariu,Strominger'20]

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 $\delta^{(d+2)}(\sum_{i=1}^{N} p_i^{\mu}(\omega_i, x_i))$ 

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→ *scattering on backgrounds: more standard correlators*

• spectrum  $(\Delta)$  complex  $\rightarrow$  non-unitary CFT



## Outline

Introduction

- I. Celestial string amplitudes @ tree and 1-loop
- II. Symmetries of celestial CFT *d*
- III. Celestial amplitudes on (particle-like) backgrounds

Summary & outlook

based on

I. [2307.03551](https://arxiv.org/abs/2307.03551) with **Laura Donnay, Gaston Giribet, Hernán Gonzáles** & **Francisco Rojas**  II. 2302.10222 with **Yorgo Pano** & **Emilio Trevisani**  III. 2207.13719 with **Riccardo Gonzo** & **Tristan McLoughlin** 

No Wilsonian decoupling: integration over all energies potentially problematic in field theory but not string theory

## I. Celestial string amplitudes

@ tree and 1-loop

### Strings on the celestial sphere

Focus: 4d scattering processes of 4 gluons in open string theory

 $4$ d momenta  $p_i^\mu = (p_i^0, p_i^1, p_i^2, p_i^3, \overline{0})$  & 10d loop momenta  $\ell^\mu$  $\ddot{\phantom{a}}$  $p_i^{\mu} = \eta_i \omega_i q_i^{\mu}(z_i, \bar{z}_i)$  &  $q_i^{\mu} = (1 + z_i \bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i \bar{z}_i)$  from vector pointing

null vector pointing<br>to celestial sphere

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Celestial amplitude:

$$
\mathcal{M}[\mathcal{A}_4](\{\Delta_i, z_i, \bar{z}_i\}) = \int_0^\infty \prod_{i=1}^4 d\omega_i \omega_i^{\Delta_i - 1} \delta^{(4)} \left( \sum_{i=1}^4 p_i^\mu \right) A(\{\omega_i, z_i, \bar{z}_i\})
$$

$$
A_{\text{string}}(p_1, p_2, p_3, p_4) = A_{YM}^{(0)}(\{p_i\})(f^{(0)} + f^{(1)} + \dots)
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#### Strings on the celestial sphere  $\mathbf{r}$ nnoro  $\nu$ Yang-Mills theory in the worldline formalism. Second, we perform the field theory limit  $t_{\rm{1}}$  at 1-loop following two routes. Figures how the field theory limit of string theory limit of str amplitudes of four gluons, corresponding to the appropriate sector of the moduli space of the genus-1 integrated correlators, matches the expression for the 4-gluon amplitude in the high and low energy regimes. At loop-level the natural field theory limit arises from a limit on the moduli space regardless of the value of the conformal cross ratios of celestial amplitudes. We compute this field theory limit for celestial 4-gluon amplitudes in string theory at 1-loop following two routes. First, we review how the field theory limit of string theory limit

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 $s = -(p_1 + p_2)^2$ *s*  $z_{12}z_{34}$   $t = -(p_2 + p_3)^2$ cross ratio  $t \nvert z_{23}z_{41} \nvert u = -(p_1 + p_3)^2.$ boost weight met boost weight  $\quad \& \quad \quad \text{cross ratio} \quad \quad s = -(p_1 + p_2)^2$ invariant *s*, the external particle momenta  $t = -(p_0 + p_2)^2$  $\beta = -\frac{i}{2} \sum_{i=1}^{4} \text{Im}\Delta_i$ <br>  $r = -\frac{s}{t} = \frac{z_{12}z_{34}}{z_{23}z_{41}}$ <br>  $u = -(p_1 + p_3)^2$ 2  $\overline{4}$ ∑ *i*=1  $\text{Im}\Delta_i$   $r = -\frac{s}{t}$ *t* = *z*12*z*<sup>34</sup> *z*23*z*<sup>41</sup>  $\Delta_i \in 1 + i\mathbb{R}$   $\longrightarrow$ Define:

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#### Celestial strings @ tree *A*string(*p*1*, p*2*, p*3*, p*4) = *A*(0) *Y M*(*{pi, Ji}*) *f*(0) + *f*(1) + *... ,* (3.2) with *f*(0)*, f*(1) *, ...* being contributions from tree level and different loops in the expansion.

[Stieberger,Taylor'18]

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\tilde{f}^{(0)}(r,\beta) = -\alpha'^{\beta} r \Gamma(1-\beta) \int_0^1 \frac{dx}{x} \left[ r \log x - \log(1-x) \right]^{\beta-1}
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 $\rightarrow$  Does α' dependence still factor out @ loop?  $\rightarrow$  Does  $\alpha'$  dependence *a* (*F*<sub>1</sub>)<sup>2</sup> *d*<sub>1</sub> *a* (*F*<sub>1</sub>)<sup>2</sup> *d*<sub>1</sub> ut C  $\frac{1}{2}$ 3 *d* 

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→ *Recover field theory limit @ loop?*

#### $\overline{\phantom{a}}$  $\overline{\phantom{0}}$  $\sim$   $^+$ ⇥ *G*(*q*<sup>2</sup> ) *<sup>G</sup>*(*q*<sup>2</sup> ) ⇤ + *f*(1) *NP* (*s, t*)*.* (4.2) *NP* is the orientable, non-planar contribution, while the planar (orientable) contri*<sup>f</sup>*(1)(*s, t*) = <sup>16</sup>⇡<sup>3</sup>*g*<sup>2</sup> 10 ↵0  $\overline{\phantom{a}}$ 0 *q* ⇥ *G*(*q*<sup>2</sup> ) *<sup>G</sup>*(*q*<sup>2</sup> ) ⇤ + *f*(1) *NP* (*s, t*)*.* (4.2) Here *f*(1) *NP* is the orientable, non-planar contribution, while the planar (orientable) contri*<sup>f</sup>*(1)(*s, t*) = <sup>16</sup>⇡<sup>3</sup>*g*<sup>2</sup> ↵0  $\overline{\phantom{a}}$ ⇥ *G*(*q*<sup>2</sup> ) *<sup>G</sup>*(*q*<sup>2</sup> ) ⇤ + *f*(1) *NP* (*s, t*)*.* (4.2) Here *f*(1) *NP* is the orientable, non-planar contribution, while the planar (orientable) contribution which we will focus on its contract will focus on its contract on its contract on its contract on its c Celestial strings @ 1-loop

1-loop stringy form factor (planar, orientable): [Donnay,Giribet,Gonzáles,AP,Rojas'23] *p* (*e*):  $\frac{1}{2}$ 

10 onnay, )*,* (4.3)

$$
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spectrum  $(\Delta)$  complex  $\rightarrow$  non-unitary CFT

### II. Symmetries in celestial CFT<sub>d</sub>

Conservation equations of operators  $O$  define symmetries in QFT.

Noether currents  $\mathcal{J}^a$  from contraction of  $\mathcal O$  with parameter  $\epsilon$ .

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up to contact terms:

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\langle \partial_a \mathcal{J}^a(x) \mathcal{O}_1(x_1) \dots \mathcal{O}_N(x_N) \rangle = \sum_{i=1}^N \delta^{(d)}(x - x_i) \langle \mathcal{O}_1(x_1) \dots \delta \mathcal{O}_i(x_i) \dots \mathcal{O}_N(x_N) \rangle \qquad \text{Ward identity}
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**Topological surface charge**  $Q_{\Sigma} = \int_{\Sigma} dS^a \mathcal{J}_a$  **... conserved upon deformations**  $\Sigma \to \Sigma'$ 



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When  $\Sigma$  contains all insertions we can deform the integral to infinity and get

$$
0 = \sum_{i=1}^{N} \langle \mathcal{O}_1(x_1) \dots \delta \mathcal{O}_i(x_i) \dots \mathcal{O}_N(x_N) \rangle
$$

which defines a **symmetry** transformation.



surface element x 
$$
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#### **What are all the symmetries (of nature)?**

Key for any holographic dual construction.

Important in its own right.



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(+ a bit more advanced stuff)

Noether current  $\rightarrow$  charge  $\rightarrow$  symmetry group.







#### Conformally soft theorems :

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$$
  
\n
$$
\Delta = 1, 0, -1, \dots
$$

gauge theory: [Fan,Faotopoulos,Taylor'19][[Nandan,Schreiber,Volovich,Zlotnikov'19]Pate,Raclariu,Strominger'19] gravity: [Adamo,Mason,Sharma'19][AP'19] [Guevara'19]



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#### Classify all conformally soft operators in gauge theory and gravity.

 $C$  $CFT$ <sub>2</sub> → *x* = (*z*,*z*̄)  $\overline{z,z}$ 



Conformally soft primary  ${\mathscr O}_{\mathbf{\Delta}}$  operators have descendant operators that are primaries themselves. They organize into multiplets.

 $C$ CFT $_2$ conservation laws for asymptotic symmetry charges.  $\rightarrow$   $x = (z, \bar{z})$   $\left(\frac{z, \bar{z}}{\cdots}\right)$ 



celestial diamonds @ ⌘ @*z*¯. The global conformal multiplets for the conformal soft operators in CCFT2 take the form of diamonds of the form of diamonds of diamonds of diamonds of the form of diamonds of the for



types I, II, III: spin of descendant >,<,= spin of parent primary

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Bottom of a diamond: conservation equation for  $\mathcal{O}_{\Lambda}$ .

$$
C C F T_2 \rightarrow x = (z, \bar{z})
$$



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[Pasterski,AP,Trevisani'21]

Leading soft photon theorem:

$$
\bar{\partial} \lim_{\Delta \to 1} (\Delta - 1) \mathcal{O}_{\Delta, \ell = +1} = 0
$$

Leading soft graviton theorem:

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\bar{\partial}^2 \lim_{\Delta \to 1} (\Delta - 1) \mathcal{O}_{\Delta, \ell = +2} = 0
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large gauge transformations  $J$  with  $\bar{\partial}\epsilon = 0$ 

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 $\bar{\partial}^3 \tilde{T}$ 

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Stress tensor?  $\tilde{T}$  is shadow transform of  $T$  which satisfies  $\partial \overline{T} = \overline{\partial}^3 \tilde{T} = 0$   
integral transform on  $S^2$ :  $\delta h(\mathcal{O}_{\Delta}) = \tilde{\mathcal{O}}_{2-\Delta}$   $\mathcal{J} \text{ with } \partial \overline{\ell} = 0$
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\nFrom OPE of type I, II, III: find algebra  $w_{1+\infty}$ !

\n[Surevara, Himwich, Pate, Strominger'21]

\n[Strominger'21]

### From  $d = 2$  to  $d > 2$

[Pano,AP,Trevisani'23]

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● 3 more types of primary descendants:

**even**  $d$ : I, II, III vs **odd**  $d$ : I, II, P, S parity shadow

#### From  $d = 2$  to  $d > 2$  $d<sub>l</sub>$  $\overline{\phantom{0}}$ > <u>/</u>  $-$

Expired, Indiana, AP, Trevisani' 23 This can be used to differentiate the vectors each  $[Pano, AP, Trevisani'23]$ 

• ∃ more types of primary descendants:

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- $SO(d)$  spin more complicated: semplicated.
- $\rightarrow$  to capture all soft theorems:

 go beyond traceless and symmetric  $T$  is defined in such a way that if we contract its indices bit with a tensor that its indices bit with a tensor t (note that if  $\alpha$ 

 $\rightarrow$  refinement of types I, II, III  $\rightarrow$  I<sub>k</sub>, II<sub>k</sub>, III<sub>k</sub>  $\text{III} \rightarrow \text{I}_{\text{L}}$ .  $\text{II}_{\text{L}}$ .  $\text{III}_{\text{L}}$  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6, \mathbf{v}_7, \mathbf{v}_8, \mathbf{v}_9, \mathbf{v}_9, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6, \mathbf{v}_7, \mathbf{v}_8, \mathbf{v}_9, \mathbf{v}_1, \mathbf{v}_2, \mathbf{$ 



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- → interpretation inℝ<sup>1,d+1</sup> ?  $\bullet$  No non-trivial charges without **shadows** = integral transform on  $S^d$ ! !a1...a! = π! <sup>a</sup><sup>1</sup> ··· <sup>a</sup>! ; <sup>b</sup><sup>1</sup> ··· <sup>b</sup>! symmetric, these representations are sometimes called mixed-symmetric tensor representations.  $\rightarrow$  interpretation in $\mathbb{R}^{1,d+1}$  ?



polarization vectors <sup>e</sup><sup>i</sup> with i = 1,...k, such that the indices a<sup>i</sup>

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Summarizing 50+ pages… 27<br>27 Juni row Summarizing 50+ pages...



# CCFT*<sup>d</sup>*

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[Pano,AP,Trevisani'23] We schematically in figure 2 the structure  $\Gamma$  conformal multiplet in  $C$  conformal multiplet in  $C$ 

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Bottom of a necklace: conservation equation for  ${\mathscr O}_{\Delta}.$ Bottom or a necklace. Conservation equation for  $\mathcal{O}_4$ 

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[Pano,AP,Trevisani'23] related work by [Kapec,Mitra'17,'21]

#### Leading soft photon & graviton, subleading soft graviton:

Ward ID for type  $I_2$  operators: trivial charges

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[Brust,Hinterbichler'16]

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\mathcal{J}^{\epsilon a} = O^{aa_1...a_{\ell-1}} \partial_{a_1} \cdots \partial_{a_{n-1}} \epsilon_{a_n...a_{\ell-1}} \n- \partial_{a_1} O^{aa_1...a_{\ell-1}} \partial_{a_2} \cdots \partial_{a_{n-1}} \epsilon_{a_n...a_{\ell-1}} \n+ \partial_{a_1} \partial_{a_2} O^{aa_1...a_{\ell-1}} \partial_{a_3} \cdots \partial_{a_{n-1}} \epsilon_{a_n...a_{\ell-1}} \n\vdots \n+ (-1)^{n-1} \partial_{a_1} \cdots \partial_{a_{n-1}} O^{aa_1...a_{\ell-1}} \epsilon_{a_n...a_{\ell-1}}
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\n
$$
+ \partial_{a_1} \partial_{a_2} \mathcal{O}^{aa_1 \dots a_{\ell-1}} \partial_{a_3} \dots \partial_{a_{n-1}} \epsilon_{a_n \dots a_{\ell-1}}
$$
\n
$$
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Noether current  $\rightarrow$  charge  $\rightarrow$  symmetry group:

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 $CCFT$ infinite-dimensional symmetry group **-** superrotations BMS supertranslations 2D local conformal symmetry

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CCFT*d*>2



▶ Role of higher-dimensional BMS ?

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CCFT*d*>2



▶ Role of higher-dimensional BMS ?

compare to [Kapec,Lysov,Pasterski,Strominger'15] vs [Hollands,Ishibashi,Wald'16]

interesting interpretation: [Kapec'22]

 ${\sf distributional}$  support on the sphere from bulk  $\delta^{(d+2)}(\sum_{i=1}^N p_i^\mu)$ can be smeared out if translation symmetry broken

### III. Celestial amplitudes on backgrounds

## Scattering on backgrounds

To study wave scattering on classical backgrounds we use the method of [Boulware,Brown'68] which amounts to solving the classical equations of motion for  $\Phi(x)$  in the presence of a source  $J(x)$ .

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Solve for  $\bar{\Phi}(p) = \sum \bar{\Phi}^{(n)}(p)$  in momentum-space order-by-order in coupling. ∞ ∑  $n=0$  $\bar{\Phi}^{(n)}(p)$ order *n* in coupling

The *n*-point amplitude is:

$$
\mathscr{A}_n(p_1,\ldots,p_n) = i^n \prod_{k=1}^n \left( \lim_{p_k^2 \to 0} p_k^2 \right) \frac{\delta}{\delta \bar{J}(p_1)} \cdots \frac{\delta}{\delta \bar{J}(p_{n-1})} \bar{\Phi}(-p_n) \Big|_{J=0}
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$$

Consider scattering scalars on particle-like backgrounds (sources of mass and charge) & interpret them in celestial CFT.

[Gonzo, McLoughlin, AP'22]

Coulomb field of static and spinning point charges & ultraboost limits. Schwarzschild, Kerr & ultraboost limits: Aichelburg-Sexl and gyraton metrics.

Focus on 2-point amplitudes:

$$
\mathcal{A}_2(p_1, p_2) = -\prod_{k=1}^2 \left( \lim_{p_k^2 \to 0} p_k^2 \right) \frac{\delta}{\delta \bar{J}(p_1)} \bar{\phi}(-p_2) \Big|_{J=0}
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Wave equation for complex scalar field minimally coupled to gravity in presence of a source:

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\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi(x)) = J(x)
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$$
\mathcal{A}_2(p_1, p_2) = (2\pi)^4 \lim_{p_1^2 \to 0} \lim_{p_2^2 \to 0} p_1^2 \delta^{(4)}(p_1 + p_2) - \left[ (p_1)_{\mu}(p_2)_{\nu} - \frac{1}{2} \eta_{\mu\nu} p_1 \cdot p_2 \right] \bar{h}^{\mu\nu}(p_1 + p_2) + \dots
$$
  
\n
$$
n = 0
$$

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### Black hole avatars in CCFT

[Gonzo, McLoughlin, AP'22]

Compute celestial scattering on **Schwarzschild** & **Kerr**:


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▶ Celestial amplitudes on backgrounds: nicer features than in flat space

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 $\delta$ -function support on  $S^2$ !

ℳellin integrals UV divergent.

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Celestial amplitudes on backgrounds: nicer features than in flat space



Supported everywhere on the  $S^2$ . vs power-law in *z* ! *ij* <sup>=</sup> *zi* <sup>−</sup> *zj*

Classical spin acts as UV regulator.

non-spinning spinning ∫ ∞  $\theta$  $d\omega \omega^{\Delta-1} \rightarrow$ ∞  $\theta$  $d\omega \omega^{\Delta} H_{-1}^{(2)}(a\omega)$  Hankel fct 2*πδ*(*i*Δ) finite support

 $\delta$ -function support on  $S^2$ !

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Ultraboost limit of black holes is special:



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Shockwaves are generated by conformal primaries in CCFT! [Pasterski,AP'20]

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Scalar shockwaves:

$$
\phi_{sw}(X) = -\log(X^2)\delta(q \cdot X) \qquad (\Delta, \ell) = (1,0) \text{ scalar primary}
$$
\n
$$
q^{\mu} = (1 + |z_{sw}|^2, z_{sw} + \bar{z}_{sw}, i(\bar{z}_{sw} - z_{sw}), 1 - |z_{sw}|^2)
$$
\n
$$
\text{null vector pointing at } (z_{sw}, \bar{z}_{sw}) \text{ on celestial sphere}
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\n
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\nKerr-Schild double copy

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#### Celestial shockwave correlators

[Gonzo,McLoughlin,AP'22]

electromagnetic:

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gravitational:

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 $Y$ *)* $q$ <sub>*μ*</sub> ( $\Delta$ ,  $\ell$ ) = (0,0) vector primary

$$
\mathcal{M}_3(\Delta_1, \Delta_2, \Delta_{sw}) = \frac{e(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 2))}{|z_{12}|^{\Delta_1 + \Delta_2} |z_{1sw}|^{\Delta_1 - \Delta_2} |z_{2sw}|^{\Delta_2 - \Delta_1}}
$$
  
 looks like 3-point correlator in standard CFT<sub>2</sub> !

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$$

$$
\mathcal{M}_3(\Delta_1, \Delta_2, \Delta_{sw}) = \frac{r_0(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 1))}{\left|z_{12}\right|^{\Delta_1 + \Delta_2 + 1} \left|z_{1sw}\right|^{\Delta_1 - \Delta_2 - 1} \left|z_{2sw}\right|^{\Delta_2 - \Delta_1 - 1}}
$$
  
-1

Looks like 3-point correlator in standard  $\mathsf{CFT}_2$ ( after continuing off the principal series  $\text{Re}(\Delta_1 + \Delta_2) = 1$  )!

#### Correlators from on-shell action

 $S =$  eom  $S_{\text{bdy}}$ 

In AdS/CFT on-shell action generates CFT correlators.

Celestial holography: on-shell action generates CCFT correlators?







 $S =$  eom +  $S_{\mathscr{F}^{-} \cup \mathscr{F}^{+}}$ 

\* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

$$
S = \text{com} + S_{\mathcal{J} - \cup \mathcal{J}^+}
$$

$$
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
$$

Complex massless scalar *ϕ* minimally coupled to gravity

1  $\frac{1}{\sqrt{g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi(x)) = J(x)$ 

\* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

ℐ−

*v*

*i* 0

 $\mathscr{I}^+$ 

*u*

*S*2

*<sup>r</sup>* <sup>=</sup>

*r*

 $\overline{\prime}$ 

∞

∞

*i* +

*i* −

 $S =$  eom +  $S_{\mathscr{J}^{-} \cup \mathscr{J}^{+}}$ 

 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ 

Complex massless scalar *ϕ* minimally coupled to gravity

$$
\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi(x)) = J(x) \quad \to \quad \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}\phi(x) = J_{eff}(x)
$$

effective source



\* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

 $S = \text{com} + S_{\mathscr{I}^{-} \cup \mathscr{I}^{+}}$ 

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$$

effective source

$$
\phi = \phi_{in} + \phi_{out} \qquad \phi_{in} = e^{ip \cdot X} \otimes \phi_{out}
$$
 solved via Green's fct

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ℐ−

*v*

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*u*

*S*2

*<sup>r</sup>* <sup>=</sup>

*r*

 $\overline{\prime}$ 

∞

∞

*i* +

*i* −

 $S = \text{com } + S_{\varphi- \cup \varphi} + \varphi$ 

 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ 

Complex massless scalar *ϕ* minimally coupled to gravity

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\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\phi(x)) = J(x) \quad \to \quad \eta^{\mu\nu}\partial_{\mu}\partial_{\nu}\phi(x) = J_{eff}(x)
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$$
 solved via Green's fct

At large *r* at fixed  $v = t + r$  and  $u = t - r$ :

 $S_{\mathcal{J}^-\cup\mathcal{J}^+}(p) = #\bar{J}_{eff}(p)$ [Gonzo, McLoughlin, AP'22]

On-shell action localizes on the boundary onto the Fourier transform of effective source evaluated along the incoming momentum.

\* on-shell action also studied in [Fabbrichesi,Pettorini,Veneziano,Vilkovisky'93]

ℐ−

*v*

*i* 0

 $\mathscr{I}^+$ 

*u*

*S*2

*<sup>r</sup>* <sup>=</sup>

*r*

 $\overline{\prime}$ 

∞

∞

*i* +

*i* −

Boulware-Brown

$$
\mathcal{A}_2(p_1, p_2) = -\lim_{p_1^2 \to 0} \lim_{p_2^2 \to 0} p_1^2 p_2^2 \frac{\delta \bar{\phi}_{out}(-p_1)}{\delta \bar{J}(p_2)}
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$$

$$
= \lim_{p_1^2 \to 0} \lim_{p_2^2 \to 0} p_2^2 \frac{\delta \bar{J}_{eff}(-p_1)}{\delta \bar{J}(p_2)}
$$

solving the eom to leading order in coupling

$$
\bar{\phi}_{out}(p) = -\frac{\bar{J}_{eff}(p)}{p^2}
$$

[Gonzo, McLoughlin, AP'22]

Boulware-Brown

$$
\mathscr{A}_{2}(p_{1},p_{2}) = -\lim_{p_{1}^{2} \to 0} \lim_{p_{2}^{2} \to 0} p_{1}^{2} p_{2}^{2} \frac{\delta \bar{\phi}_{out}(-p_{1})}{\delta \bar{J}(p_{2})}
$$
\n
$$
= \lim_{p_{1}^{2} \to 0} \lim_{p_{2}^{2} \to 0} p_{2}^{2} \frac{\delta \bar{J}_{eff}(-p_{1})}{\delta \bar{J}(p_{2})}
$$
\n
$$
= \lim_{p_{1}^{2} \to 0} \lim_{p_{2}^{2} \to 0} p_{2}^{2} \frac{\delta \bar{J}_{eff}(-p_{1})}{\delta \bar{J}(p_{2})}
$$
\n
$$
= \frac{1}{\#} \lim_{p_{1}^{2} \to 0} \lim_{p_{2}^{2} \to 0} p_{2}^{2} \frac{\delta S_{\mathcal{J}^{-} \cup \mathcal{J}^{+}}(-p_{1})}{\delta \bar{J}(p_{2})}
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= \frac{1}{\#} \lim_{p_{1}^{2} \to 0} \lim_{p_{2}^{2} \to 0} p_{2}^{2} \frac{\delta S_{\mathcal{J}^{-} \cup \mathcal{J}^{+}}(-p_{1})}{\delta \bar{J}(p_{2})}
$$
\napproximation

 $_2(p_1, p_2) = - \lim_{n \to \infty}$  $p_1^2$   $\rightarrow$  0 lim  $p_2^2 \rightarrow 0$  $p_1^2 p_2^2$  $\delta \bar{\phi}_{out}(-p_1)$  $\delta\bar{J}(p_{2})$ = 1 lim lim  $p_2^2$  $\delta S$ <sub>I</sub>−<sub>UI</sub>+(− $p$ <sub>1</sub>) large  $r$  limit + saddle point approximation *r*  $S_{\mathcal{J}^-\cup\mathcal{J}^+}(p) = #\bar{J}_{eff}(p)$  $=$   $\lim$  $p_1^2 \rightarrow 0$ lim  $p_2^2 \rightarrow 0$  $p_2^2$  $\delta \bar{J}$  $\epsilon_{eff}(-p_1)$  $\delta \bar{J}(p_2)$  $\bar{\phi}_{out}(p) = -\frac{\bar{J}_{eff}(p)}{p^2}$ solving the eom to leading order in coupling Boulware-Brown

Boundary on-shell action generates CCFT correlators!

 $\delta \bar{J}(p_{2})$ 

#

 $p_1^2 \rightarrow 0$ 

 $p_2^2 \rightarrow 0$ 

[Gonzo, McLoughlin, AP'22]

*p*2

## Summary and outlook

We classified the soft symmetries of celestial CFT<sub>d</sub>.

In  $d = 2$  they include infinite BMS-type enhancements which obey  $w_{1+\infty}$  algebra.

In  $d > 2$  there are only finite-dimensional global symmetries.

Celestial string amplitudes: well-defined Mellin integrals, ∃ natural loop expansion parameter and get field theory limit regardless of cross ratio.

Celestial amplitudes on backgrounds: smear  $\delta$ -fct support on sphere, well-defined Mellin integrals for classical spin.

**Future:** identify CCFTs and all its properties!

 $d+2$ 

CCFT*<sup>d</sup>*

Quantum

Gravity

Axioms? Toy models? More string theory! More loops! Non-perturbative physics?…