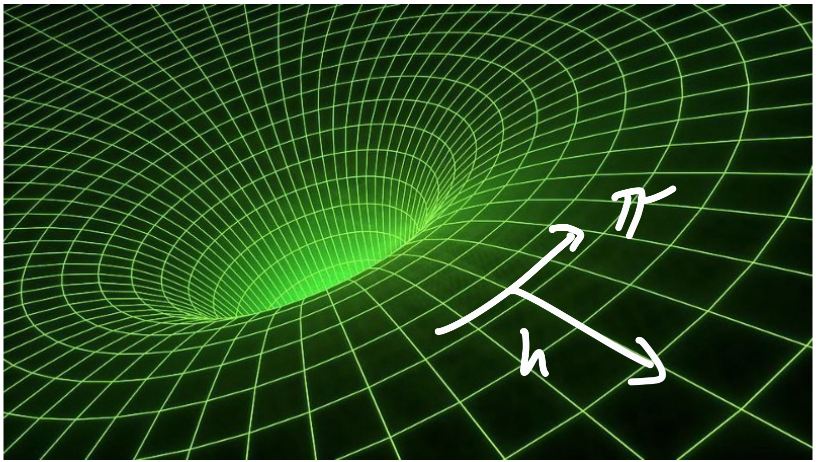
On-Shell Covariance of QFT Amplitudes



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CERN 8 Aug

The Standard Model as EFT

"Heavy physics decouples"
(or does it??)

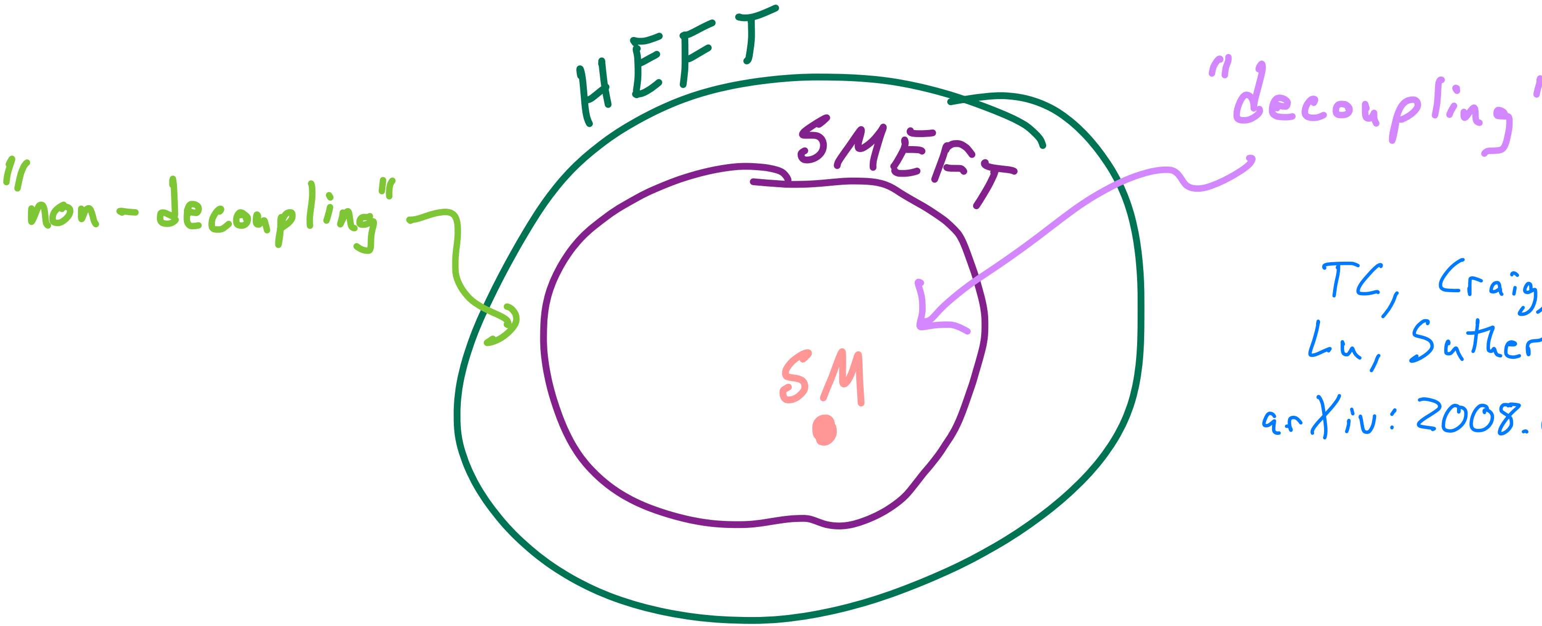
- Only SM dofs
- Symmetries: Lorentz + $SU(3) \times SU(2) \times U(1)$

Realize electroweak symmetry

linearly or non-linearly

SMEFT HEFT

Is SMEFT Enough?



TC, Craig, Lu, Sutherland
arXiv: 2008.08597

Field redefinitions obscure relationship between EFTs

The Geometric Higgs

Scope (for now)

EFT Lagrangian includes up to two derivatives $\Rightarrow$ defines metric + potential

Field redefinitions without derivatives

Interpret Higgs dofs as coordinates on a manifold:

Cartesian $\Rightarrow$ SMEFT

Polar $\Rightarrow$ HEFT

Curvature Invariants

Analog w/ GR: define metric on moduli space

Note $$(\partial \vec{n})^2 = \left(\delta_{ij} + \frac{n_i n_j}{1-n^2}\right)(\partial^\mu n_i)(\partial_\mu n_j)$$

$$\Rightarrow \quad \mathcal{L}_{HEFT} \supset \frac{1}{2}\left[\underline{K}(h)\right]^2 (\partial h)^2 + \frac{1}{2}\left[v F(h)\right]^2 (\partial \vec{n})^2$$

$\Rightarrow$ metric

$$g_{hh} = \underline{K}^2$$

$$g_{ij} = v F^2 \left(\delta_{ij} + \frac{n_i n_j}{1-n^2}\right)$$

Alonso, Jenkins, Manohar arXiv:1605.03602

Curvature Invariants

metric $\Rightarrow$ Christoffel symbols: $\Gamma_{ab}^{\ \ c}$

$$\Gamma_{hh}^{\ \ h} = \frac{\mathcal{K}'}{\mathcal{K}}, \quad \Gamma_{ij}^{\ \ h} = -\frac{F'}{F\mathcal{K}^2} g_{ij}, \quad \Gamma_{jh}^{\ \ i} = \Gamma_{hj}^{\ \ i} = \frac{F'}{F}\delta_j^{\ i}$$

$$\Gamma_{jk}^{\ \ i} = \frac{\pi_i}{(vF)^2} g_{jk}$$

$(i = \pi_i, \ j = \pi_j)$

$\Rightarrow$ Ricci scalar:

$$R = -\frac{2N_\varphi}{\mathcal{K}^2 F}\left[(\partial_h^2 F) - (\partial_h \mathcal{K})\left(\frac{1}{\mathcal{K}}\partial_h F\right)\right] + \frac{N_\varphi(N_\varphi - 1)}{v^2 F^2}\left[1 - \left(\frac{v}{\mathcal{K}}\partial_h F\right)^2\right]$$

Sectional Curvatures

The components of R are

$$R_{ihhj} = -g_{hh}\, g_{ij}\, \lambda_h \qquad (i = \pi_i)$$

$$R_{iklj} = (g_{il}\, g_{kj} - g_{ij}\, g_{kl})\, \lambda_\pi$$

$$\Rightarrow \quad R = 6(\lambda_h + \lambda_\pi)$$

sectional curvature

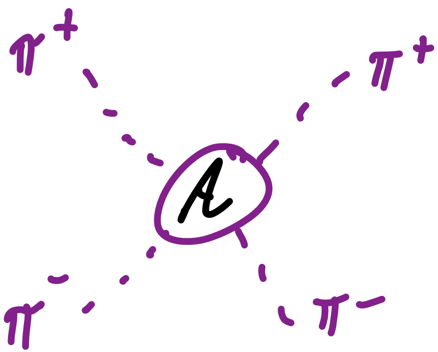
Geometric Amplitudes

TC, Craig, Lu, Sutherland
arXiv: 2108.03240

Ex: $W_L^+ W_L^- \to W_L^+ W_L^-$

$$\mathcal{L}_{HEFT} \supset \frac{1}{2v^2}\left[\partial(\pi^+\pi^-)\right]^2 + 2\bar{F}' h\, \partial\pi^+\partial\pi^-$$

'bar' denotes eval at our vac: $h=0,\ \pi=0$



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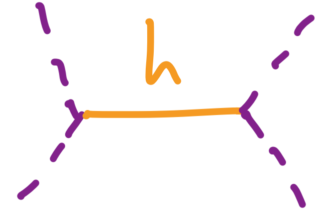
$+$



$+$ t-ch

$$= -\frac{1}{v^2}(s+t) + \bar{F}'^2\left[\frac{s^2}{s-m_h^2} + \frac{t^2}{t-m_h^2}\right]$$

$$= \left(\bar{F}'^2 - \frac{1}{v^2}\right)(s+t) + \bar{F}'^2\left[2m_h^2 + \frac{m_h^4}{s-m_h^2} + \frac{m_h^4}{t-m_h^2}\right]$$

Geometric Amplitudes: $W_L^+ W_L^- \to W_L^+ W_L^-$

$$\bar{R}_{+--+} = -\bar{g}_{+-,+-} + \bar{\Gamma}^h_{+-}\bar{\Gamma}_{-+h} = \frac{1}{v^2} - \bar{F'}^2$$

Using $\bar{V}_{;(h+-)} = -\bar{V}_{,hh}\bar{\Gamma}^h_{+-} = -m_h^2\bar{F'}^2$

$$\bar{V}_{;(++--)} = 2\bar{V}_{,hh}\bar{\Gamma}^h_{+-}\bar{\Gamma}^h_{+-} = 2m_h^2\bar{F'}^2$$

we can express our amplitude geometrically:

$$\mathcal{A} = \left(\bar{F'}^2 - \frac{1}{v^2}\right)(s+t) + \bar{F'}^2\left[2m_h^2 + \frac{m_h^4}{s-m_h^2} + \frac{m_h^4}{t-m_h^2}\right]$$

$$= -\bar{R}_{+--+}(s+t) + \bar{V}_{;(++--)}$$

$$+ \bar{V}_{;(h+-)}\,\bar{g}^{hh}\,\bar{V}_{;(h+-)}\left[\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right]$$

Geometric Feynman Rules

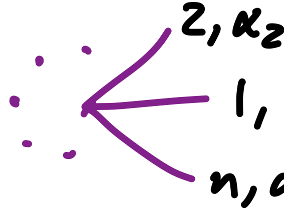
Expand the Lagrangian:

$$\mathcal{L} = \frac{1}{2} g_{\alpha\beta}(\varphi)\, \partial\varphi^{\alpha}\, \partial\varphi_{\beta} - V(\varphi)$$

$$= \sum_n \frac{1}{n!} \varphi^{\gamma_1} \cdots \varphi^{\gamma_n} \left(\bar{g}_{\alpha\beta,\gamma_1\ldots\gamma_n} \frac{1}{2} \partial\varphi^{\alpha} \partial\varphi^{\beta} - \bar{V}_{,\gamma_1\ldots\gamma_n} \right)$$

propagator: $\alpha \text{———} \beta = \dfrac{i\bar{g}^{\alpha\beta}}{p^2 - m_\alpha^2}$ w/ $m_\alpha^2 \delta_\alpha^{\gamma} = V_{,\alpha\beta}\, \bar{g}^{\alpha\gamma}$

vertices:



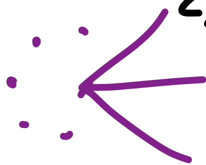
$$= -i \bar{V}_{,\alpha_1\ldots\alpha_n}$$

omit

$$- i \sum_{1 \le i < j \le n} p_i \cdot p_j\, \bar{g}_{\alpha_i \alpha_j, \alpha_1 \ldots \hat{\alpha}_i \ldots \hat{\alpha}_j \ldots \alpha_n}$$

Geometric Feynman Rules in Normal Coordinates

propagator: α —— β $= \dfrac{i\,\bar{g}^{\alpha\beta}}{p^2 - m_\alpha^2}$

vertices: 

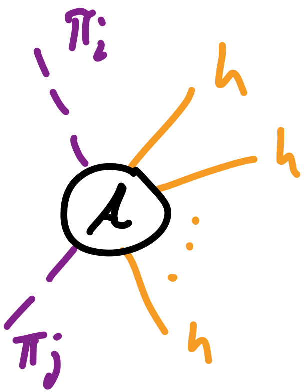
$= -i\,\bar{V}_{;\alpha_1 \ldots \alpha_n}$

becomes covariant

$$-i \sum_{1 \le i < j \le n} s_{ij} \left(\frac{n-3}{n-1}\right)\left[\bar{R}_{\alpha_i(\alpha_1 \alpha_2 | \alpha_j ; | \alpha_3 \ldots \hat{\alpha}_i \ldots \hat{\alpha}_j \ldots \alpha_n)} + \mathcal{O}(R^2)\right]$$

$$+\, i \sum_{1 \le i \le n} (n-1)\, m_i^2 \,\cancelto{0}{\bar{g}_{\alpha_i(\alpha_1 \ldots \hat{\alpha}_i \ldots \alpha_n)}} + i \sum_{1 \le i \le n} (n-1)(p_i^2 - m_i^2)\,\cancelto{0}{\bar{g}} \ldots$$

Geometric $\mathcal{A}(\pi_i \pi_j \to h^{n-2})$



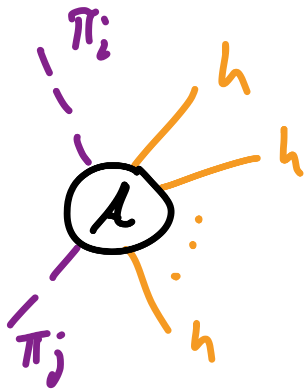
$$= \frac{1}{3}\delta_{ij}\, \partial_h^{n-2}\overline{\left(\nabla^2 V - \partial_h^2 V\right)}$$

$$- \delta_{ij}\, \overline{\partial_h^{n-4}}\partial_h \left(s_{12} - \frac{2m_h^2}{n-1} \right)$$

Dominates as $E \to \infty$

$$+\, \mathcal{O}(\bar{R}^2) + \text{factorizable pieces}$$

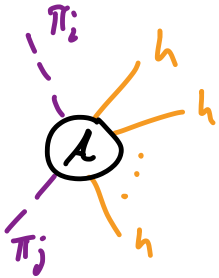
Perturbative Unitarity Violation



$$\xrightarrow{E\to\infty} \; - E^2 \delta_{ij}\, \overline{\partial_h^{n-4} \chi_h}$$

Cauchy-Hadamard Thm relates radius of convergence v_* to size of successive derivatives. Radius of convergence set by closest pole in complex plane.

Perturbative Unitarity Violation



$$S^\dagger S = 1 \Rightarrow E \lesssim 4\pi \left| \frac{\partial_h^{n-2} \lambda_h}{n!} \right|^{-1/n} (n!)^{1/n}$$

$$\xrightarrow{n \gtrsim \text{few}} \quad E \lesssim 4\pi v_* (n!)^{1/n}$$

$2 \to 2 \sim |\lambda_h|^{-1/2}$ measures curvature at our vacuum

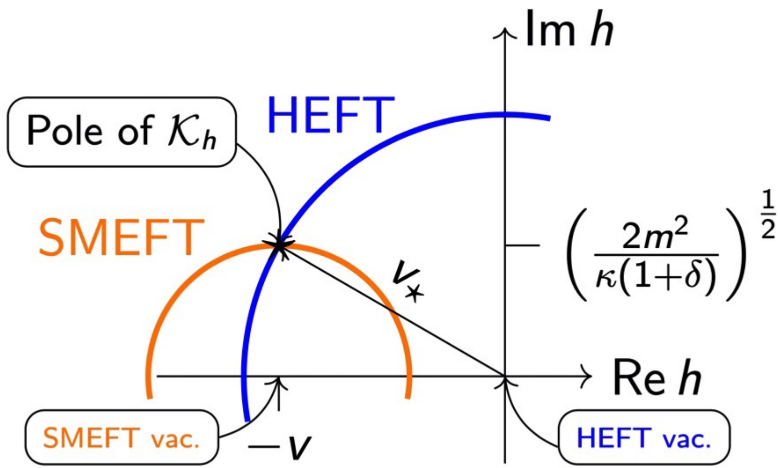
$2 \to n \sim v_*$ measures radius of convergence

Many perturbative examples w/ $v_*^2 |\lambda_h| \ll 1$

See also: Falkowski + Rattazzi arXiv:1902.05936; Abu-Ajamieh, Chang, Chen, Luty arXiv:1902.05556 arXiv:2009.11293

Example: $\mathcal{L}_{UV} \supset \frac{1}{2} S\left(-\partial^2 - m^2 - \kappa |H|^2\right) S$

Integrate out S: $\kappa_h = \delta \, \frac{\kappa}{2} \, \frac{m^2}{\left(m^2 + \frac{1}{2}\kappa(1+\delta)(v+h)^2\right)^2}$



$\delta = \frac{\kappa}{96\pi^2}$

non-decoupling

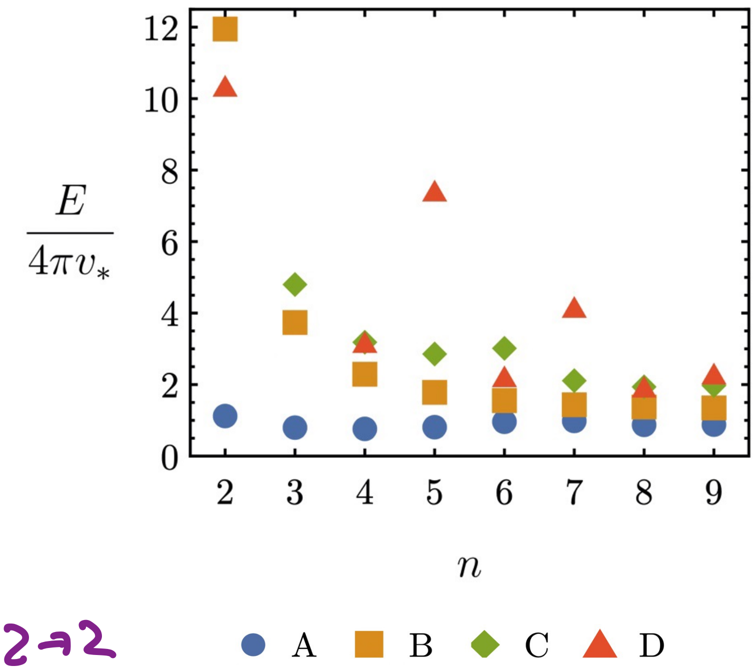
$m^2 \to 0$: most of S mass from v $\Rightarrow$ HEFT

$v_* \to v \Rightarrow \quad E \lesssim 4\pi v$

Example: $\mathcal{L}_{UV} \supset \frac{1}{2} S(-\partial^2 - m^2 - \kappa|H|^2)S$

Benchmarks

	m^2/v^2	$\kappa/2$	$v_\star/v$	$v_\star^2\,\overline{\mathcal{K}}_h$	$v_\star^2\,\overline{\mathcal{K}}_\pi$
A	$(4\pi)^2$	$(4\pi)^2$	2	0.2	0.4
B	1	$(4\pi)^2$	1	1×10^{-3}	0.3
C	1	1	1	1×10^{-3}	2×10^{-3}
D	10^6	1	1000	2×10^{-3}	2×10^{-3}



Features

- B, C, D: lower cutoff than $2\to2$
- A, B, C are non-decoupling ($v \sim v_*$); D is SMEFTy

Summary

Scalar EFT (up to 2 derivatives) can be "geometrized"

- Operator coeffs are ∂_μ^n of $g_{\mu\nu}(\varphi)$ and $V(\varphi)$
- Amplitudes built using D_μ^n of $R_{\mu\nu\rho\sigma}(\varphi)$ and $V(\varphi)$

$\Rightarrow$ Amplitudes are covariant!

Summary

Scalar EFT (up to 2 derivatives) can be "geometrized"

- Applied to HEFT to show $2 \to n$ scattering probes distance to pole in curvature invariant v_*
- Scale of perturbative unitarity violation distinguishes HEFT from SMEFT.

What about field redefinitions <u>with</u> derivatives?!?

TC, Craig, Lu, Sutherland
arXiv: 2202.06965 + 23xx.xxxxx

(see also Cheung, Helset, Parra-Martinez arXiv: 2202.06972)

They Are Unavoidable

$$
\begin{tikzcd}
\boxed{\mathcal{L}_{UV}} \arrow[rr, "\text{Redef without } \partial"] \arrow[d, "\text{Match}"] & & \boxed{\tilde{\mathcal{L}}_{UV}} \arrow[d, "\text{Match}"] \\
\boxed{\mathcal{L}_{EFT}} \arrow[rr, "\text{Redef } \underline{\text{with}} \ \partial"] & & \boxed{\tilde{\mathcal{L}}_{EFT}}
\end{tikzcd}
$$

Mess Up Geometric Interpretation

$$\mathcal{L} = -V(\varphi) + \frac{1}{2} g_{\alpha\beta}(\varphi)\, \partial\varphi^\alpha \partial\varphi^\beta + \mathcal{O}(\partial^4)$$

Let $\varphi^\alpha \to \tilde{\varphi}^\alpha + \frac{1}{2} h^\alpha_{\gamma\delta}(\tilde{\varphi})\, \partial\tilde{\varphi}^\gamma \partial\tilde{\varphi}^\delta$

$$\Rightarrow \mathcal{L} \to -V(\tilde{\varphi}) + \frac{1}{2} \overbrace{\Big(g_{\alpha\beta}(\tilde{\varphi}) - V_{,\gamma}(\tilde{\varphi})\, h^\gamma_{\alpha\beta}(\tilde{\varphi}) \Big)}^{\tilde{g}(\tilde{\varphi})} \partial\tilde{\varphi}^\alpha \partial\tilde{\varphi}^\beta + \mathcal{O}(\partial^4)$$

$g(\varphi) \neq \tilde{g}(\tilde{\varphi}) \Rightarrow$ Curvature invariants change

Want to Generalize Geometry

State of the art

$\mathcal{A} \sim \text{geometry} \times \text{kinematics}$

Our approach

Formalism that puts kinematics and geometry on equal footing $\Rightarrow$ "functional geometry"

Generating Functionals

Compute amplitudes using generating functional $W[J]$

$J(x)$ is source for field $\eta(x)$

We also need 1PI effective action

$$-\Gamma = J_x \phi^x - W$$

$$\left(J_x \phi^x = \int d^4x\, J(x)\, \phi(x) \right)$$

Covariant Amplitudes

Take amplitude $\mathcal{A}$ and strip LSZ residues

$$\overline{\mathcal{M}}(p_1 \cdots p_n) \equiv -Z^{-n/2}\mathcal{A}(p_1 \cdots p_n)$$

Go to position space

$$(2\pi)^4 \delta^4(p_1 \cdots p_n)\overline{\mathcal{M}} = \int \left(\prod_i d^4x_i \, e^{ip_i \cdot x_i}\right) \mathcal{M}_{x_1 \cdots x_n}\Big|_{J=0}$$

w/ $$\underbrace{\mathcal{M}_{x_1 \cdots x_n}}_{\text{"Covariant amplitude"}} = -\left(-iD^{-1}_{x_1y_1}\right) \cdots \left(-i\underbrace{D^{-1}_{x_ny_n}}_{\text{inverse propagator}}\right) \frac{\delta^n W}{\delta J_{y_1} \cdots \delta J_{y_n}}$$

On-shell Conditions

To go from $\mathcal{M} \to \mathcal{A}$, must impose

$J = 0$ and $p_i^2 = m_i^2$ for external lines

This ensures that only physical pieces of $\mathcal{M}$ contribute

Equations of motion: $$\frac{\delta(-\Gamma)}{\delta\varphi^x} = J_x = 0$$

On-shell legs: $$\left.\frac{\delta^2(-\Gamma)}{\delta\varphi^x\,\delta\varphi^y}\right|_{J=0} = -i\,D_{xy}^{-1}\Big|_{J=0} = 0$$

Off-Shell Recursion

Will argue that $\mathcal{M}$ satisfy

$$\mathcal{M}_{x_1 \ldots x_n x} = \frac{\delta}{\delta \varphi_x} \mathcal{M}_{x_1 \ldots x_n} - \sum_{i=1}^{n} G^{y}{}_{x x_i} \mathcal{M}_{x_1 \ldots \underbrace{\hat{x}_i y}_{\text{replace}} \ldots x_n}$$

$$\equiv \nabla_x \mathcal{M}_{x_1 \ldots x_n}$$

"increase rank of tensor by acting w/ covariant derivative"

$J \neq 0 \Rightarrow$ encodes all-pts

Functional Christoffel Symbol

$$G^{y}{}_{x_1 x_2} = i D^{yz} \mathcal{M}_{z x_1 x_2} = i D^{yz} \frac{\delta^3 (-\Gamma)}{\delta \varphi^z \delta \varphi^{x_1} \delta \varphi^{x_2}}$$

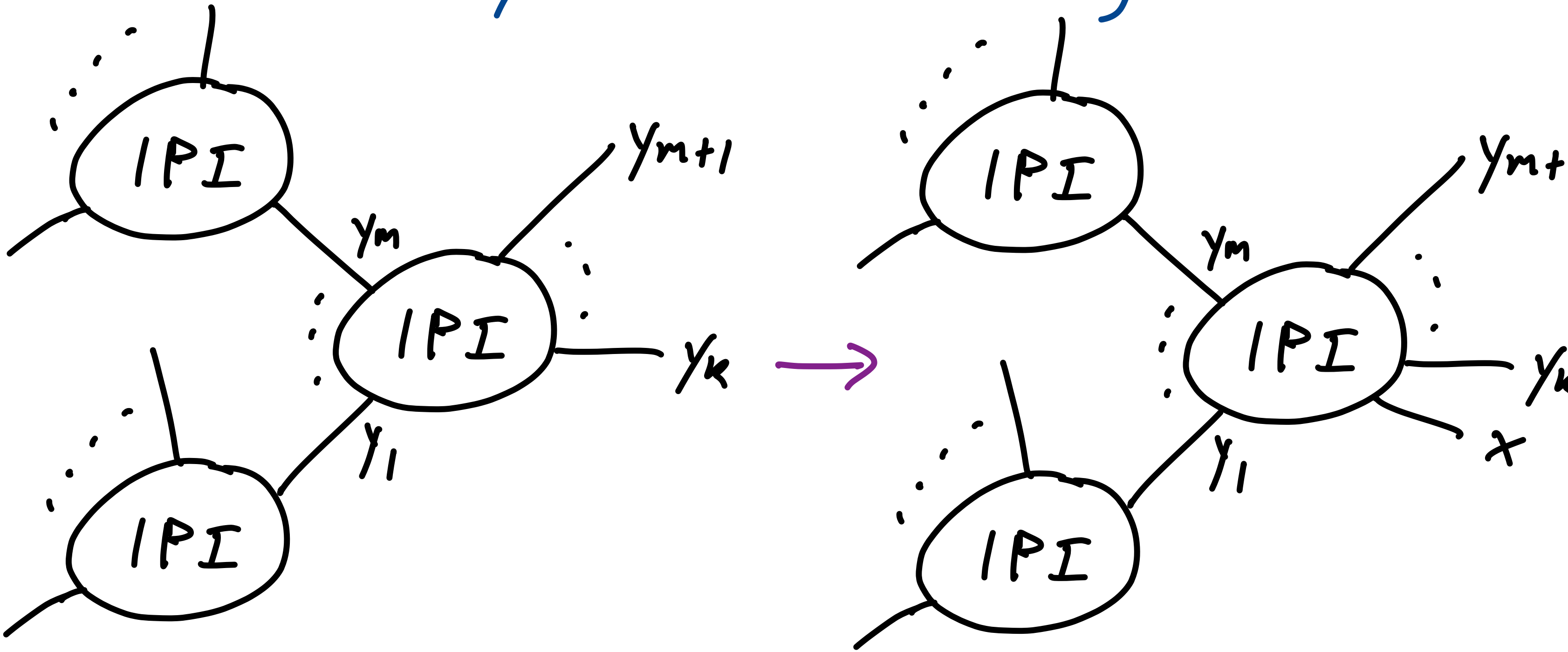
Off-Shell Recursion

Construct amplitude out of

k-point 1PI vertices: $-i\,\dfrac{\delta^k(-\Gamma)}{\delta\varphi^{y_1}\cdots\delta\varphi^{y_k}}$

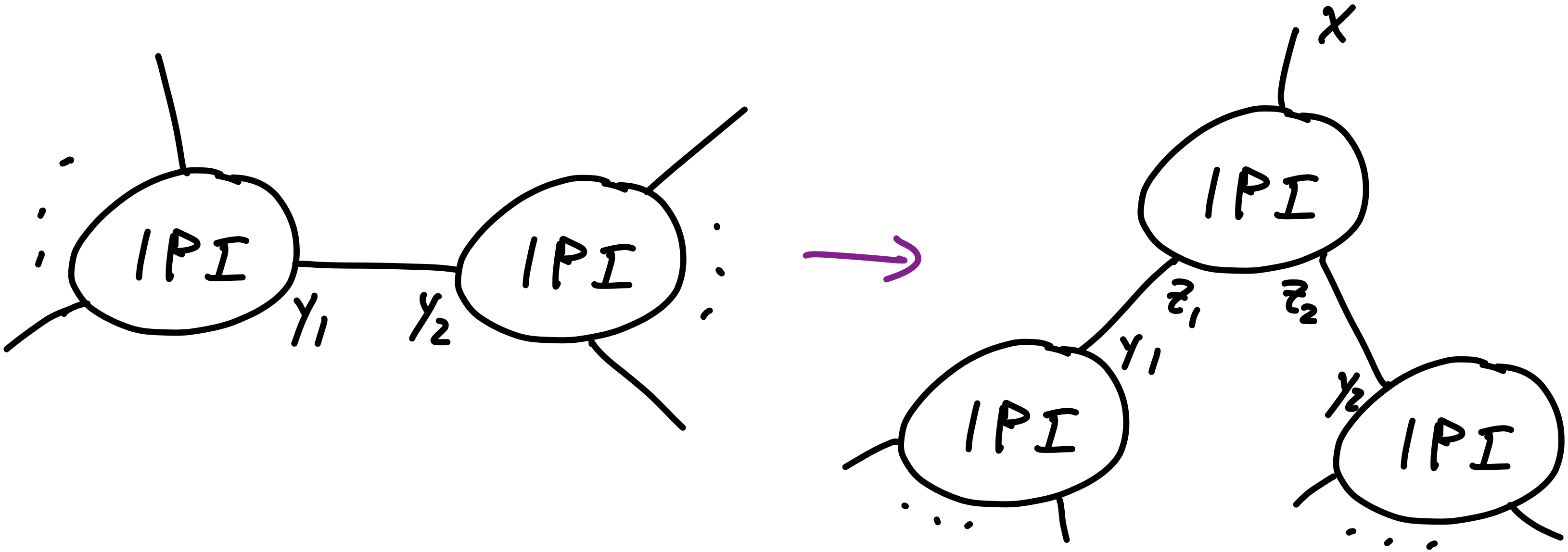
(full) propagators: $D^{y_1 y_2}$

3 Ways to Add a Leg: 1



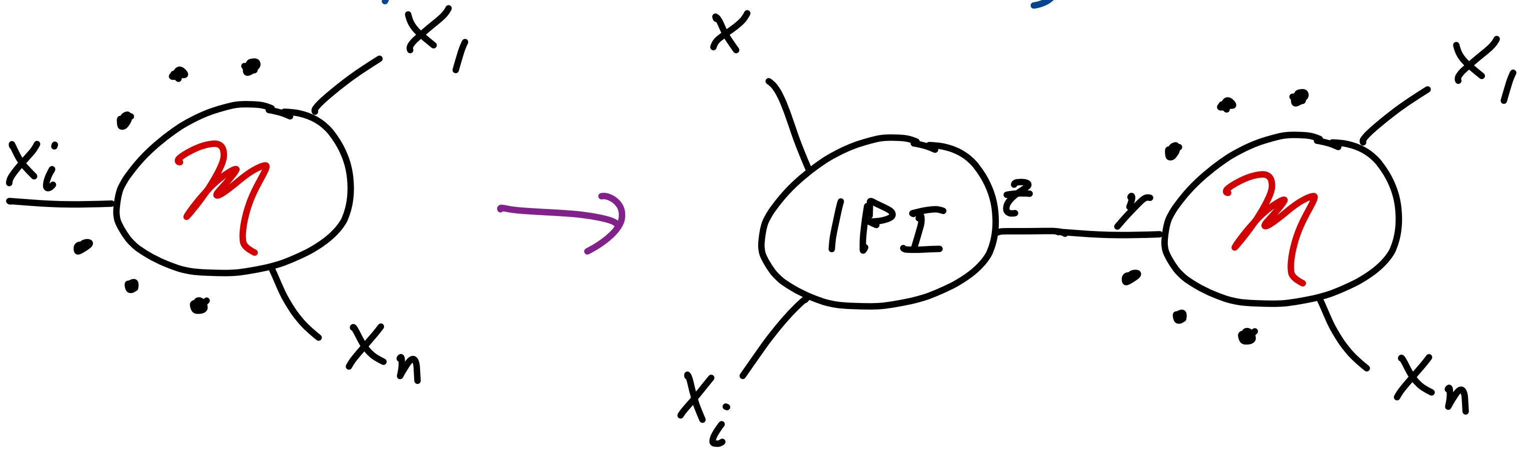
$$-i\frac{\delta^k(-\Gamma)}{\delta\phi^{y_1}\cdots\delta\phi^{y_k}} \rightarrow -i\frac{\delta^{k+1}(-\Gamma)}{\delta\phi^{y_1}\cdots\delta\phi^{y_k}\delta\phi^{x}} = \frac{\delta}{\delta\phi^{x}}\left[-i\frac{\delta^k(-\Gamma)}{\delta\phi^{y_1}\cdots\delta\phi^{y_k}}\right]$$

3 Ways to Add a Leg: 2



$$D^{y_1 y_2} \longrightarrow D^{y_1 z_1}\left[-i\,\frac{\delta^3(-\Gamma)}{\delta\varphi^{z_1}\,\delta\varphi^{x}\,\delta\varphi^{z_2}}\right] = \frac{\delta}{\delta\varphi^{x}} D^{y_1 y_2}$$

3 Ways to Add a Leg: 3



$$\mathcal{M}_{x_1\cdots x_i\cdots x_n} \longrightarrow -i\,\frac{\delta^3(-\Gamma)}{\delta\varphi^{x}\,\delta\varphi^{x_i}\,\delta\varphi^{z}}\,D^{zy}\,\mathcal{M}_{x_1\cdots \overbrace{\hat{x}_i y}^{\text{replace}}\cdots x_n}$$

$$= -G^{y}_{x x_i}\,\mathcal{M}_{x_1\cdots\hat{x}_i y\cdots x_n}$$

Berends - Giele Recursion

Nucl. Phys. B 306 (1988)

Enforce $J = 0$

Define $\quad G^{y}_{x_1 \cdots x_n} = i D^{xy} \mathcal{M}_{x x_1 \cdots x_n}$

which satisfy $\quad G^{y}_{x_1 \cdots x_n x} = \nabla_x G^{y}_{x_1 \cdots x_n}$

Berends - Giele recursion follows from

$$\varphi^y = \hat{J}^y - \sum_{n=2}^{\infty} \frac{1}{n!} \left(G^{y}_{x_1 \cdots x_n} \Big|_{J=0} \right) \hat{J}^{x_1} \cdots \hat{J}^{x_n}$$

w/ $\quad \hat{J}^x = \left(i D^{xy} \big|_{J=0} \right) J_y$

On-Shell Covariance

At tree-level, $\Gamma[\varphi]$ transforms as a scalar under field redefinitions $\varphi(x) \to \tilde{\varphi}(x)$.

$$\tilde{\Gamma}[\tilde{\varphi}] = \tilde{S}[\tilde{\varphi}] = S[\varphi[\tilde{\varphi}]] = \Gamma[\varphi[\tilde{\varphi}]]$$

$$\Rightarrow \tilde{\mathcal{M}}_{x_1 \ldots x_n} = \left(\frac{\delta \varphi^{y_1}}{\delta \tilde{\varphi}^{x_1}} \cdots \frac{\delta \varphi^{y_n}}{\delta \tilde{\varphi}^{x_n}} \right) \mathcal{M}_{y_1 \ldots y_n} + U_{x_1 \ldots x_n}$$

where
$$U_{x_1 \ldots x_n} = a_{x_1 \ldots x_n y_1} \frac{\delta(-\Gamma)}{\delta \varphi^{y_1}} + \sum_{i=1}^{n} b_{x_1 \ldots \hat{x}_i \ldots x_n y} \frac{\delta \varphi^{y_2}}{\delta \tilde{\varphi}^{x_1}} \frac{\delta^2(-\Gamma)}{\delta \varphi^{y_1} \delta \varphi^{y_2}}$$

vanish on-shell

On-Shell Covariance

Functional metric: $-i D^{-1}_{xy} = \dfrac{\delta^2(-\Gamma)}{\delta\varphi^x \delta\varphi^y}$

Functional inverse metric: $i D^{xy}$

Functional Christoffel symbol:

$$G^{y}{}_{x_1 x_2} = i D^{yz} \frac{\delta^3(-\Gamma)}{\delta\varphi^z \delta\varphi^{x_1} \delta\varphi^{x_2}}$$

Connection to Field Space Geometry

Let $\mathcal{L} = -V + \frac{1}{2} g_{ab}\, \partial\phi^a \partial\phi^b$

Then
$$\lim_{q^2\to\infty} \int d^4x_1\, d^4x_2\, d^4y\; e^{ip_1x_1 + ip_2x_2}\, e^{-iqy} \times \left[G^c_{ab}(x_1, x_2, y)\Big|_{\partial_\mu\varphi_i = 0} \right]$$
$$= (2\pi)^4 \delta^4(p_1 + p_2 - q)\, \frac{1}{2} g^{cd} \left(g_{da,b} + g_{db,a} - g_{ab,d} \right)$$

$\Rightarrow G^c_{ab}$ reduces to Christoffel symbol

(Similar story for functional metric.)

But

all curvature invariants evaluate to zero...

Does functional manifold exist???

Stay Tuned

Formal proof of on-shell covariance

Extension to one-loop

Reproduce geometric soft theorems

Cheung, Helset, Parra-Martinez
arXiv:2111.03045

Geometric interpretation?

Summary

We have presented an interpretation of field redefinition invariance as an "on-shell covariance."

We demonstrated how one can recursively construct amplitudes by acting with a covariant derivative.

LSZ stripped amplitudes transform covariantly up to term that vanish on-shell.

Outlook

What is "Functional Geometry?"

Connection to jet bundles?

Craig, Lee arXiv: 2307.15742
Alminawi, Brivio, Davighi arXiv: 2308.00017

Insight into finding optimal basis choice?

Charictorize allowed space of field redefinitions?

Backup Slides

SMEFT $(v=0)$

Let $\vec{\varphi}$ be an $O(4)$ vector

$$\vec{\varphi} \rightarrow O\,\vec{\varphi}$$

$SU(2) \times U(1)$ + custodial symmetry

Identify $$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_4 + i\varphi_3 \end{pmatrix}$$

s.t. $\langle H \rangle \neq 0 \Leftrightarrow \langle \varphi_4 \rangle \neq 0$

HEFT ($v \neq 0$)

Non-linearly realized sym breaking

$O(4)/O(3)$ Calan, Coleman, Wess, Zumino (1969)

h (physical Higgs)

$\vec{n}$ (Goldstone bosons)

$$\vec{\varphi} = (v_0 + h)\,\vec{n}$$

$$\vec{n} \in S^3 \qquad \vec{n}\cdot\vec{n} = 1$$

$$\vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_i^2} \end{pmatrix}$$

HEFT $(v \neq 0)$

$O(4)$ transformation: $h \to h$, $\vec{n} \to O\vec{n}$

$\Rightarrow \vec{n}$ in non-linear rep

$$\mathcal{L}_{HEFT} = \frac{1}{2}\left[\mathbb{K}(h)\right]^2 (\partial h)^2 + \frac{1}{2}\left[v F(h)\right]^2 (\partial \vec{n})^2$$

$$- V(h) + \mathcal{O}(\partial^4)$$

$\langle h \rangle = 0$

$(\mathbb{K}(0) = 1$ is canonical norm$)$

HEFT $\longrightarrow$ SMEFT ?

Map:

$$|H|^2 = \frac{1}{2}\vec{\varphi}\cdot\vec{\varphi} = \frac{1}{2}(v+h)^2$$

$$|\partial H|^2 = \frac{1}{2}(\partial\vec{\varphi})^2 = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(v+h)^2(\partial\vec{n})^2$$

$$\left(\partial|H|^2\right)^2 = (\vec{\varphi}\cdot\partial\vec{\varphi})^2 = (v+h)^2(\partial h)^2$$

Naively:

$$\mathcal{L}_{HEFT} = \frac{v^2 F}{2|H|^2}|\partial H|^2 + \frac{1}{2}\left(\partial|H|^2\right)^2\frac{1}{2|H|^2}\left(K^2 - \frac{v^2F^2}{2|H|^2}\right)$$

$$+\, \tilde{V}(|H|^2) + \mathcal{O}(\partial^4)$$

Analytic @ $|H|=0$?

Field Redefinitions of h

Let

$$\mathcal{L} = \frac{1}{2}\left(1+\frac{h}{2v}\right)^2(\partial h)^2 + \frac{1}{2}(v+h)^2\left(\frac{3}{4}+\frac{h}{4v}\right)^2(\partial\vec{n})^2 - V$$

$$= \frac{1}{4}\left(1+\frac{\sqrt{2|H|^2}}{v}+\frac{|H|^2}{2v^2}\right)|\partial H|^2$$

$$+\frac{1}{4v^2}\left(\frac{v}{\sqrt{2|H|^2}}+\frac{3}{4}\right)\frac{1}{2}\left(\partial|H|^2\right)^2 - \tilde{V}$$

w/ $V = V(h)$

$V'(0)=0$

V analytic

Looks like no SMEFT expansion...

Field Redefinitions of h

But let $h_1 = h + \frac{1}{4v} h^2$ (no shift in min of V)

$$\Rightarrow \partial_\mu h_1 = \left(1 + \frac{h}{2v}\right) \partial_\mu h$$

and $(v_1 + h_1)^2 = (v+h)^2 \left(\frac{3}{4} + \frac{h}{4v}\right)$ $\quad v_1 = \frac{3}{4} v$

$$\Rightarrow \mathcal{L} = \frac{1}{2} (\partial_\mu h_1)^2 + \frac{1}{2} (v_1 + h_1)^2 (\partial \vec{n})^2 + V$$

$$= |\partial H_1|^2 + \tilde{V} \Rightarrow \text{SMEFT!}$$

Field Redefinitions of h

We learned that analytic field redefs of h can obscure analyticity in terms of H.

Field redefs within HEFT can obscure SMEFT

Can we make field redef invariance of Observables manifest?

- EFT is useful for parametrizing BSM
- SMEFT: linear realized EW sym
  decoupling manifest
- HEFT: non-linear realized EW sym
  useful when new physics scale
  is near v
- HEFT required
  - BSM state gets all mass from H
  - BSM source of sym breaking
- HEFT violates unitarity @ $s \sim (4\pi v)^2$
- Viable Loryon parameter space exists!

HEFT $(v \neq 0)$

Does HEFT know that $\langle H \rangle = v$?

AJM $\Rightarrow$ There might be special place on manifold $h_* = -v$ where $O(4)$ symmetry is manifest

Determined by $F(h_*) = 0$

If $h = h_*$ exists $\Rightarrow$ HEFT $\rightarrow$ SMEFT possible

Curvature Criterion

See paper for proof

A HEFT can be expressed as SMEFT iff

1) $F(h=h_*)=0$: Candidate $O(4)$ invariant point

2) The metric is analytic @ h_*
   - F + $\underline{\overline{K}}$ have convergent Taylor exp @ h_*
   - Curvature invariants $(\nabla^{2n})R$ are finite @ h_*

3) The potential is analytic @ h_*
   - V has convergent Taylor exp @ h_*
   - $(\nabla^{2n})V$ are finite @ h_*

HEFT is a Black Hole

Conjecture: Checking finiteness of R & V is sufficient.

Two classes of models need HEFT:

Conical singularity: BSM state gets all of its mass from H

Horizon: BSM sources of symmetry breaking

Conical Singularity

Ex: Singlet w/ $S|H|^2$ + S^3 $\Rightarrow$ tree level

$$\Rightarrow R(h=-v) = \frac{a^2}{m^4} N_\varphi (N_\varphi + 1)$$

finite w/ $m^2 \neq 0$ but diverges as $m^2 \to 0$

Ex: Singlet w/ $S^2|H|^2$ $\Rightarrow$ loop level

$$\Rightarrow R(h=-v) = \frac{1}{192\pi^2} \frac{\lambda}{3m^2} N_\varphi (N_\varphi + 1)$$

but $$R\Big|_{m^2=0} = \frac{N_\varphi(N_\varphi - 1)}{(v+h)^2} \frac{\lambda}{96\pi^2 + \lambda} \xrightarrow[h \to -v]{} \infty$$

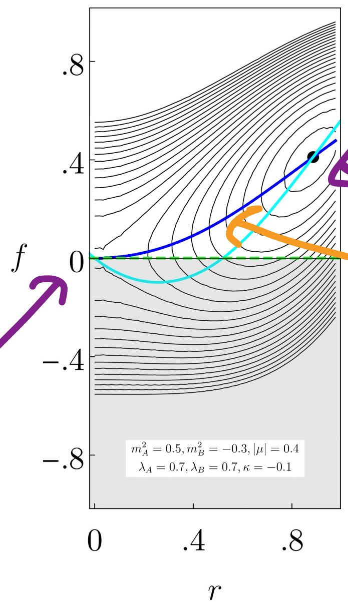
Horizon

We provide three examples in paper.

Rely on "EFT submanifold" picture

Ex: Abelian toy model w/ vevs f & r

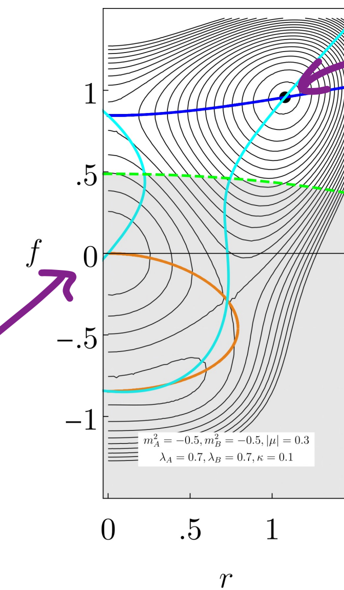
$m^2_{BSM} > 0$



physical vac

SMEFT trajectory

fixed point



physical vac

$m^2_{BSM} < 0$

fixed point

no SMEFT trajectory $\Rightarrow$ HEFT

No Curvature for Functional Geometry

Connection: $\quad G^{y}_{x_1 x_2} = -iD^{yz}\,\Gamma_{,z x_1 x_2}$

$$\Rightarrow G^{y}_{x_1 x_2, x_3} = -\left[iD^{yz}\right]_{,x_3}\Gamma_{,z x_1 x_2} - iD^{yz}\,\Gamma_{,z x_1 x_2 x_3}$$

$$= -\left[iD^{wz}\right]\Gamma_{,wt x_3}\left[iD^{ty}\right]\Gamma_{,z x_1 x_2} - iD^{yz}\,\Gamma_{,z x_1 x_2 x_3}$$

and $\quad G^{y}_{w x_3}\, G^{w}_{x_1 x_2} = \left[iD^{wz}\right]\Gamma_{,wt x_3}\left[iD^{ty}\right]\Gamma_{,z x_1 x_2}$

$$\Rightarrow R^{y}_{x_1 x_2 x_3} = G^{y}_{x_1 x_2, x_3} + G^{y}_{w x_3}\, G^{w}_{x_1 x_2} - \left[x_2 \Leftrightarrow x_3\right] = -iD^{yz}\,\Gamma_{,z x_1 x_2 x_3} + \left[x_2 \Leftrightarrow x_3\right] = 0$$