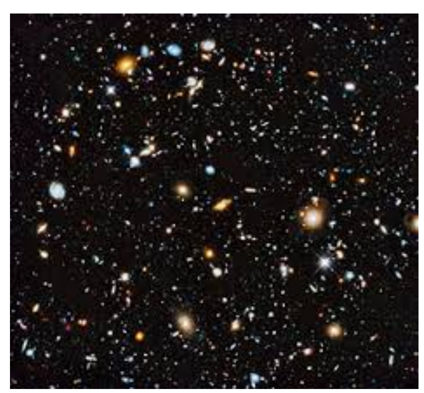
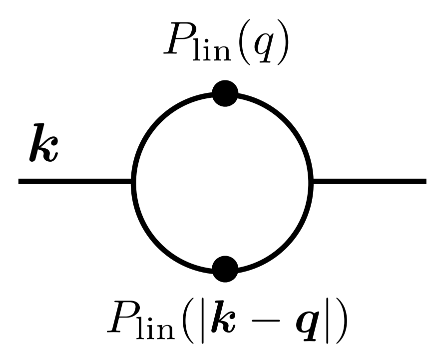
Amplitudes meet galaxy clustering

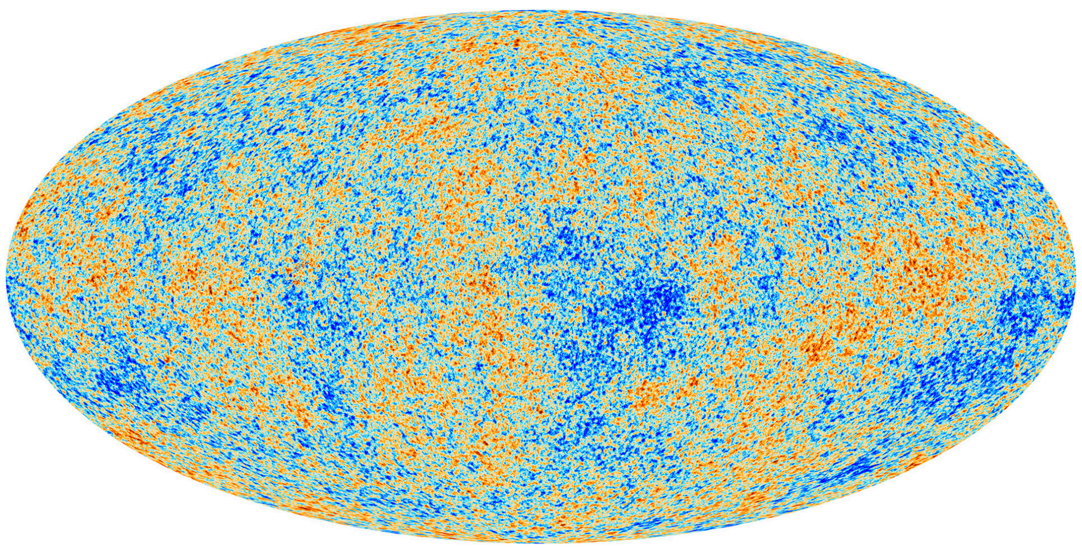
Marko Simonović (CERN)

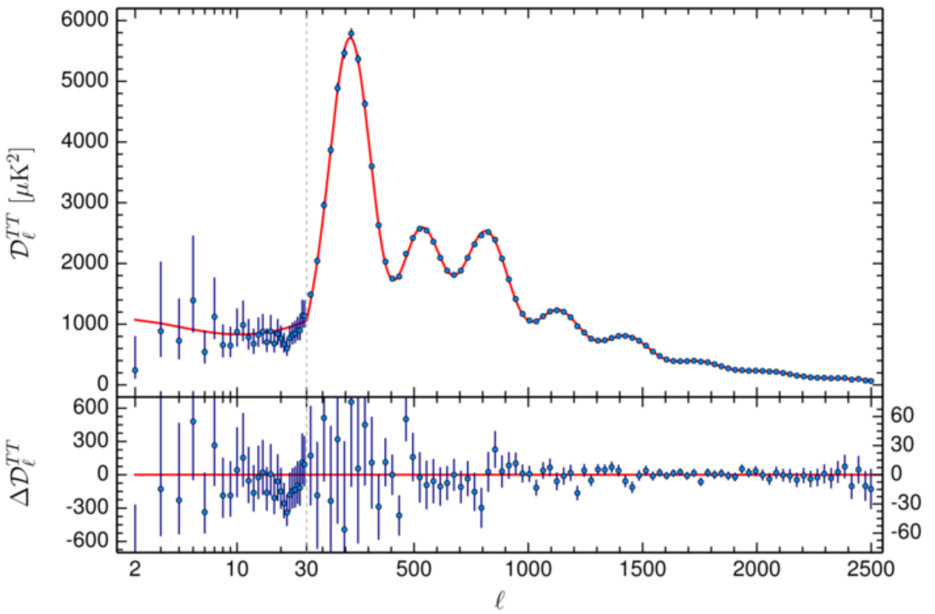




Amplitudes 2023

Cosmic Microwave Background

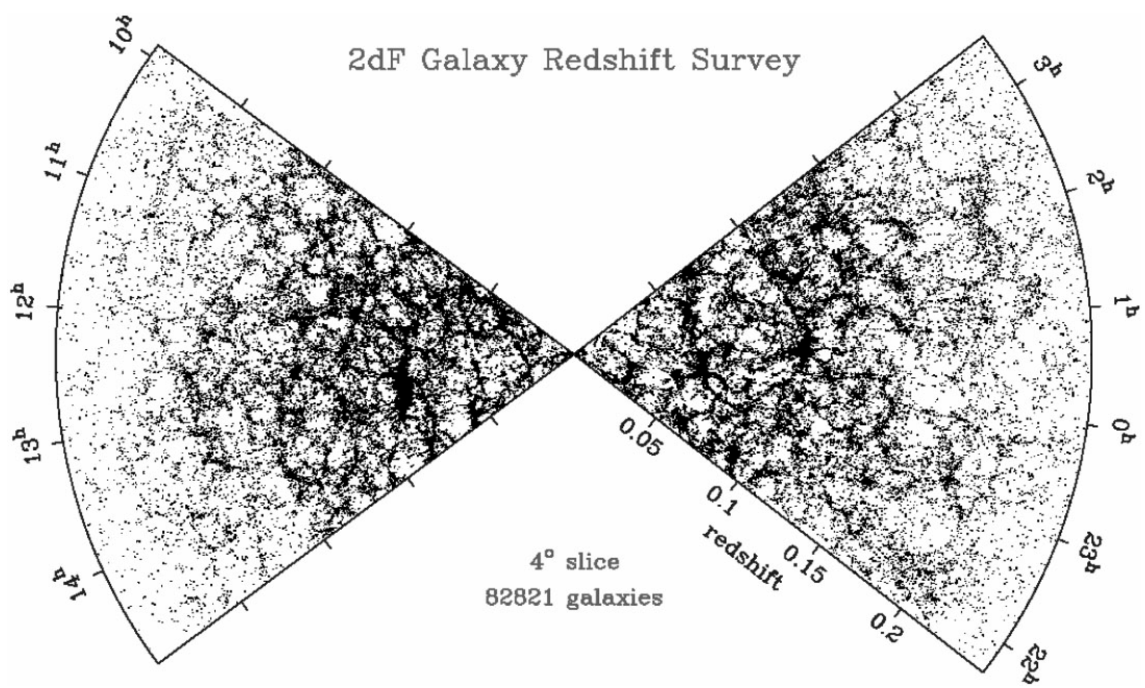


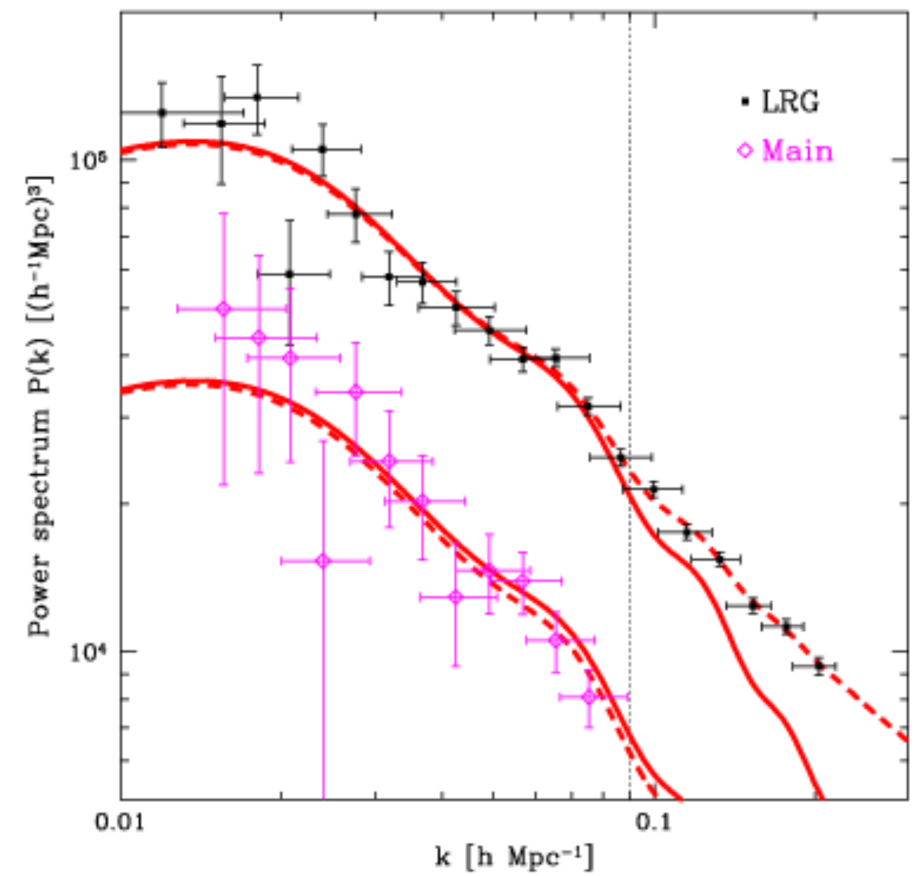


CMB will reach its limits with upcoming polarization experiments

Many open questions in cosmology regarding inflation, history of the early universe, dark matter, late-time acceleration and structure formation require new data

Spectroscopic Galaxy Surveys





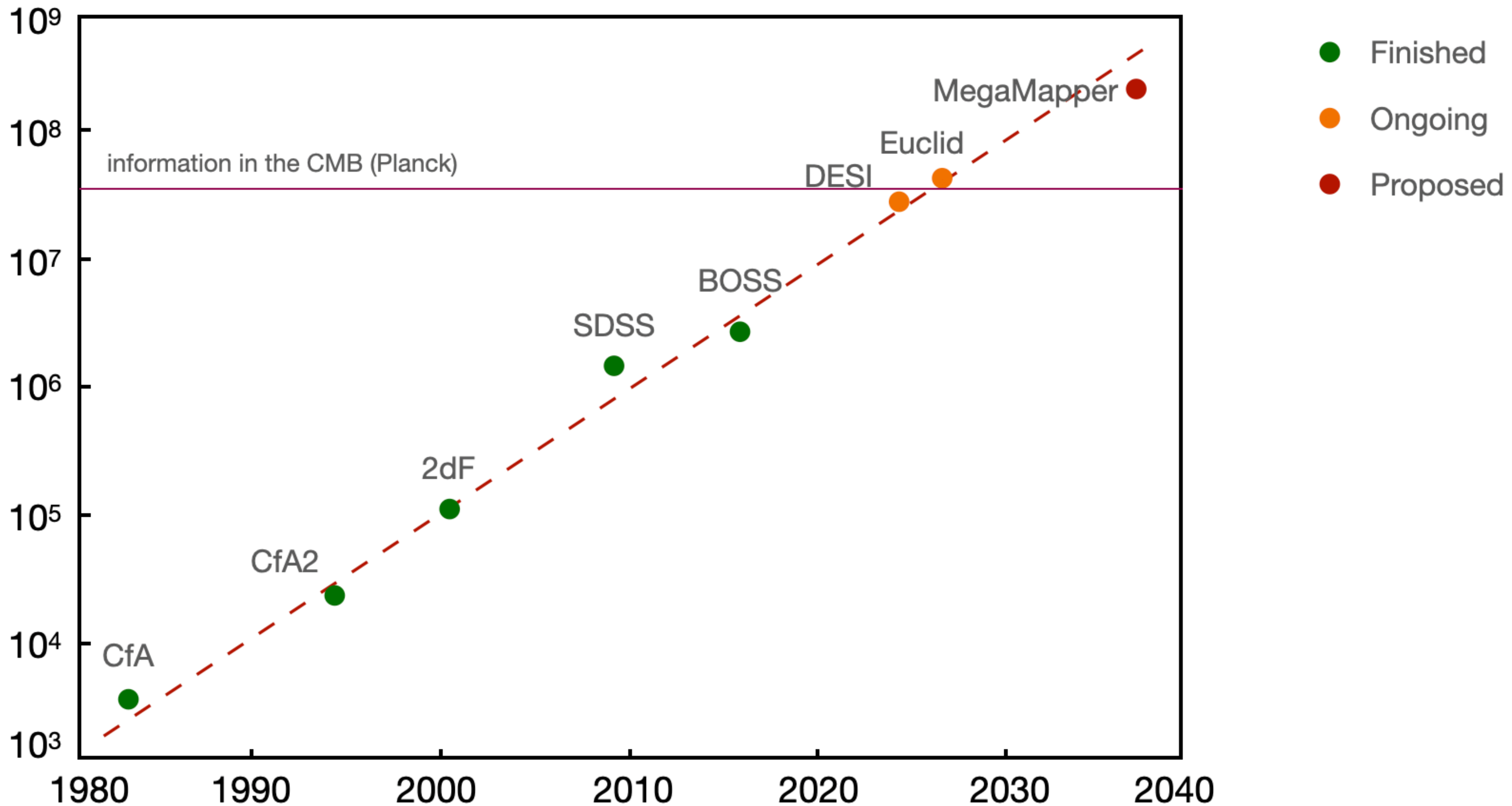
$$\delta(\boldsymbol{x}) \equiv \frac{\rho(\boldsymbol{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\delta(\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \delta_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \qquad \langle \delta_{\boldsymbol{k}} \delta_{\boldsymbol{k}'} \rangle = (2\pi)^3 \delta^D(\boldsymbol{k} + \boldsymbol{k}') P(k)$$

Make maps → measure n-point functions → fit the data

Large volume and more information in principle, but hard due to nonlinearities

Spectroscopic Galaxy Surveys

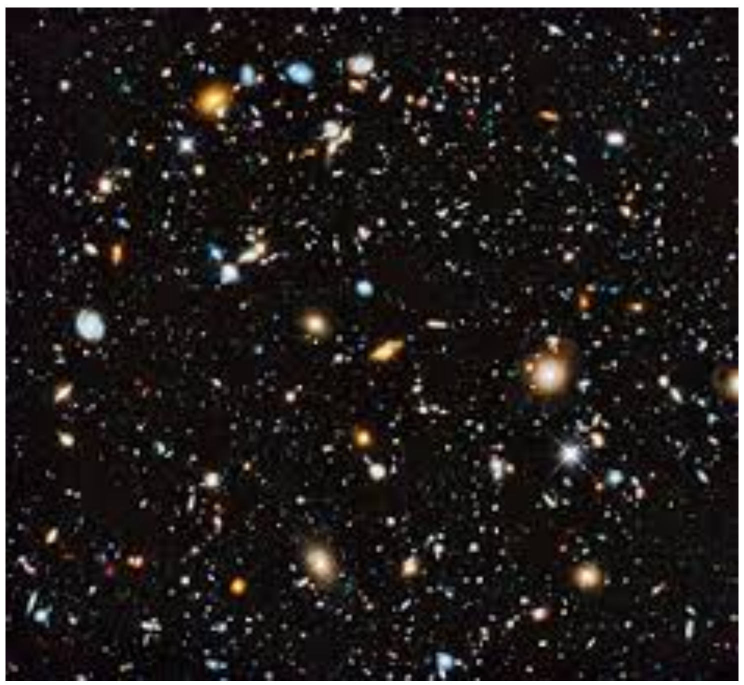


Effective Field Theory of Large-Scale Structure

Baumann, Nicolis, Senatore, Zaldarriaga (2010)

Carrasco, Hertzberg, Senatore (2012)

...



Large distance dof: δ_g

EoM are fluid-like, including gravity

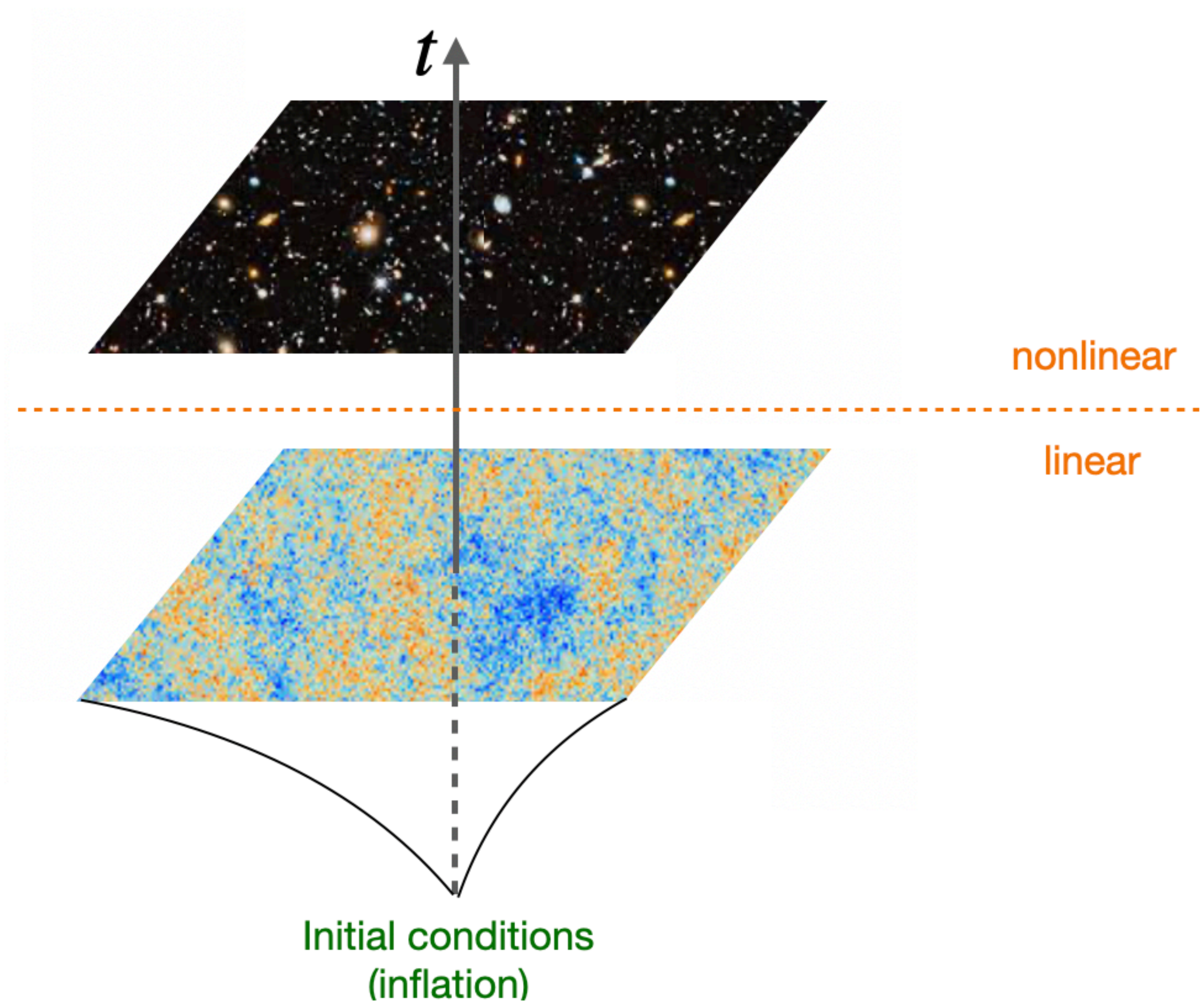
Symmetries, Equivalence Principle

Expansion in variance of $\delta_{\rm lin}$ and derivatives $\partial/k_{\rm NL}$

"UV" dependence is in a handful of free parameters

There is a universal description of galaxy clustering on scales larger than $1/k_{\rm NL}$

Chudaykin, Ivanov, Philcox, MS (2019)
D’Amico, Senatore, Zhang (2019)
Chen, Vlah, Castorina, White (2020)



CLASS-PT
PyBird
velocileptors

MCMC made possible

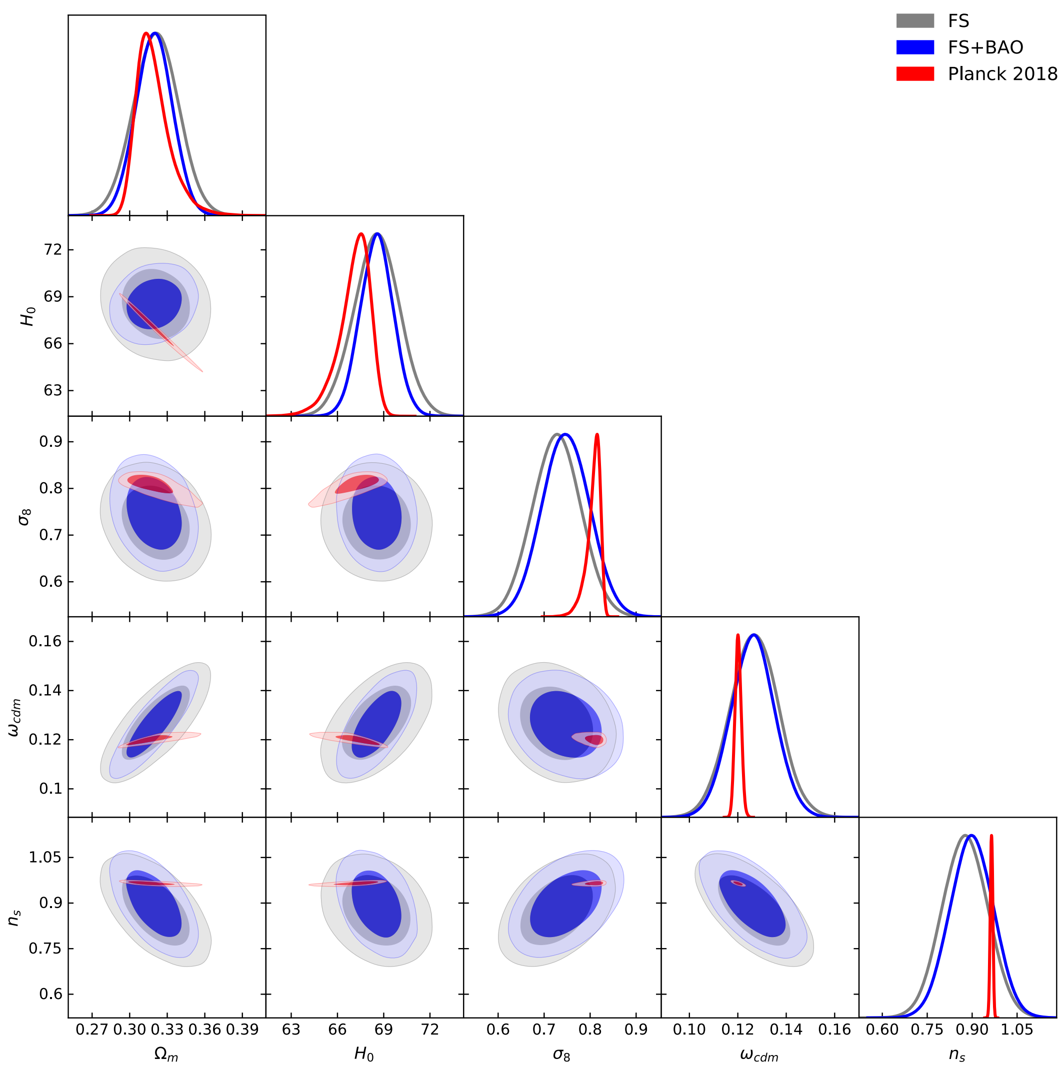
CMBFAST
CAMB
CLASS

MCMC done routinely

Complete evolution of the vacuum state from inflation to redshift zero

Application to BOSS Data

Ivanov, MS, Zaldarriaga (2019)
d'Amico, Gleyzes, Kokron, Markovic, Senatore, Zhang, Beutler, Gil Marin (2019)
Philcox, Ivanov, MS, Zaldarriaga (2020)



(fixing the tilt)

$$H_0 = 67.8 \pm 0.7 \ \mathrm{km/s/Mpc}$$

CMB-independent

+

many extensions of ΛCDM

These results will dramatically improve with data from ongoing surveys

Challenges in performing an analysis

In an MCMC chain we typically need $\sim 10^7$ evaluations of the likelihood

To be practical, each evaluation of the likelihood should last at most a few sec

With the conventional numerical approach this is hard

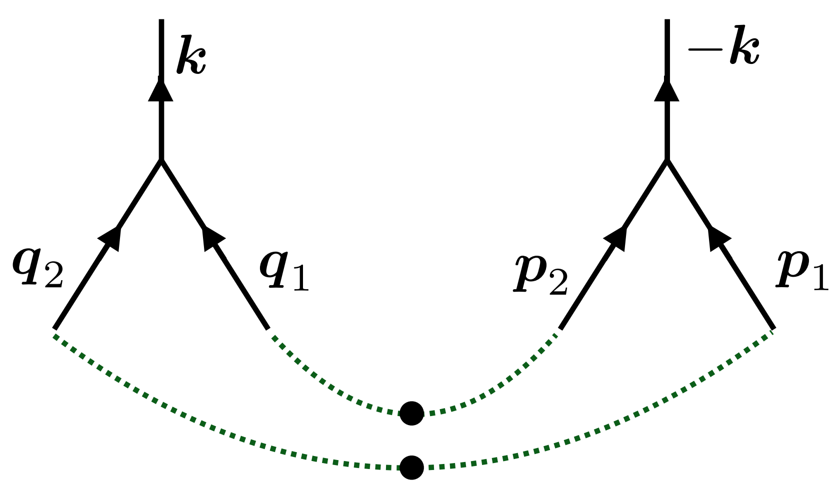
The problem can be rewritten in terms of QFT amplitudes

This leads to a dramatic improvements in efficiency

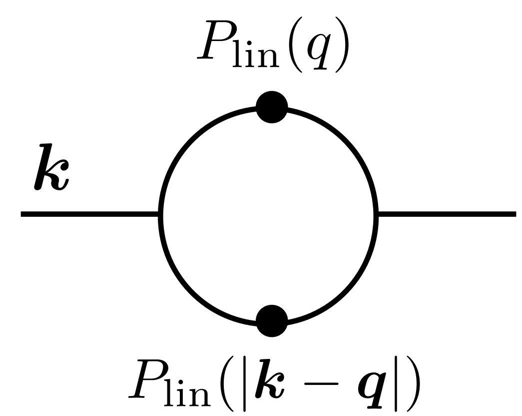
An example: nonlinear dark matter at one-loop

$$\delta^{(2)}_{\boldsymbol{k}}(\eta) = D^2(\eta) \int_{\boldsymbol{q}_1} \int_{\boldsymbol{q}_2} F_2(\boldsymbol{q}_1, \boldsymbol{q}_2) \delta^0_{\boldsymbol{q}_1} \delta^0_{\boldsymbol{q}_2} (2\pi)^3 \delta^D(\boldsymbol{k} - \boldsymbol{q}_1 - \boldsymbol{q}_2)$$

$$F_2(\boldsymbol{q}_1, \boldsymbol{q}_2) = \frac{5}{7} + \frac{1}{2} \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1 q_2} \left(\frac{q_1}{q_2} + \frac{q_2}{q_1} \right) + \frac{2}{7} \left(\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1 q_2} \right)^2$$



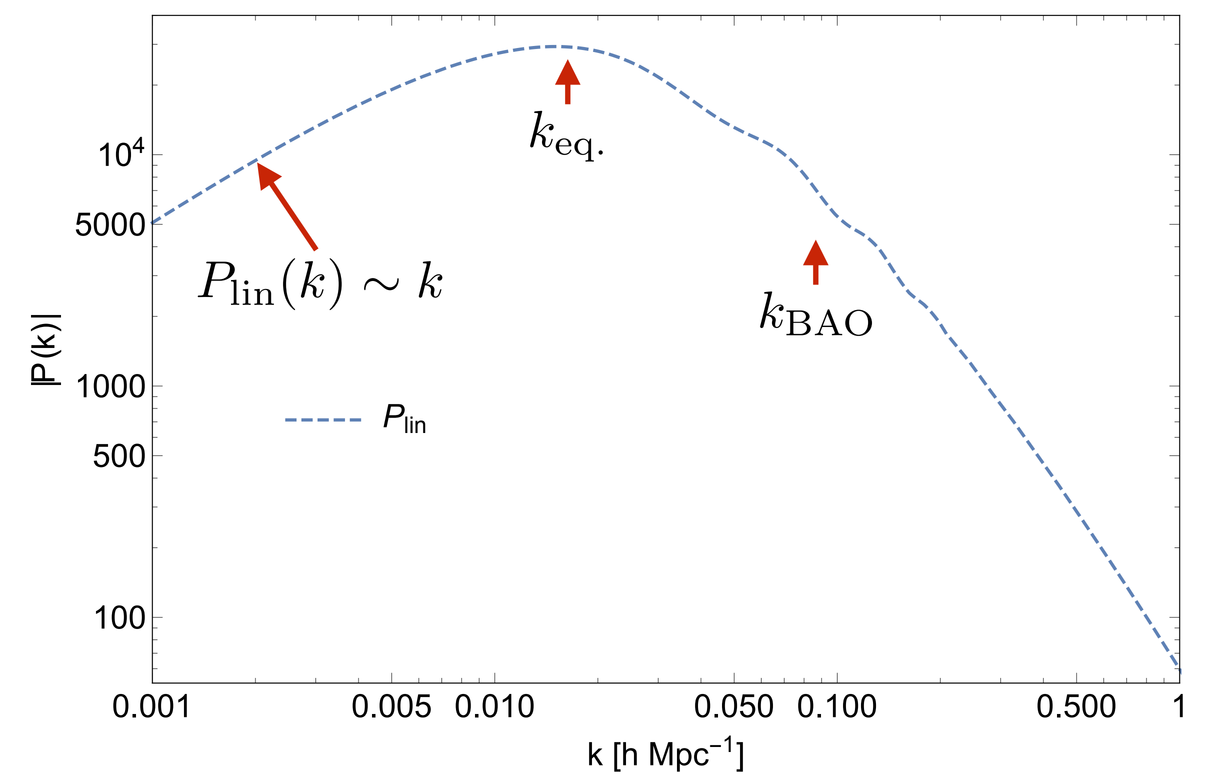
Average over
initial conditions



$$P_{22}(k) = 2 \int_{\boldsymbol{q}} F_2^2(\boldsymbol{q}, \boldsymbol{k} - \boldsymbol{q}) P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k} - \boldsymbol{q}|)$$

An example: nonlinear dark matter at one-loop

$$P_{22}(k) = 2\int_{\boldsymbol{q}} F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\boldsymbol{k}-\boldsymbol{q}|)$$



Rather than solving numerically, expand P_{lin} using simple functions

$$\bar{P}_{\text{lin}}(k_n) = \sum_{m=-N/2}^{m=N/2} c_m\, k_n^{\nu + i\eta_m}$$

using FFT Log

all cosmology dependence in coefficients

In practice N~100 and η_{max}~50 → Very large imaginary parts!

Connection to QFT loop diagrams

MS, Baldauf, Zaldarriaga, Carrasco, Kollmeier (2017)

Basic integral

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1}|\boldsymbol{k}-\boldsymbol{q}|^{2\nu_2}} \equiv k^{3-2\nu_{12}}\, I(\nu_1,\nu_2)$$

Euclidean and 3D

"off-shell", since $k^2 > 0$

complex exponents in propagators

$$I(\nu_1,\nu_2) = \frac{1}{8\pi^{3/2}} \frac{\Gamma(\frac{3}{2}-\nu_1)\Gamma(\frac{3}{2}-\nu_2)\Gamma(\nu_{12}-\frac{3}{2})}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(3-\nu_{12})}$$

The full answer can be written as

$$P_{22}(k) = k^3 \sum_{m_1,m_2} c_{m_1} k^{-2\nu_1} \cdot M_{22}(\nu_1,\nu_2) \cdot c_{m_2} k^{-2\nu_2}$$

~1000 times faster!

$$M_{22}(\nu_1,\nu_2) = \frac{(\frac{3}{2}-\nu_{12})(\frac{1}{2}-\nu_{12})[\nu_1\nu_2(98\nu_{12}^2-14\nu_{12}+36)-91\nu_{12}^2+3\nu_{12}+58]}{196\,\nu_1(1+\nu_1)(\frac{1}{2}-\nu_1)\,\nu_2(1+\nu_2)(\frac{1}{2}-\nu_2)}\, I(\nu_1,\nu_2).$$

Connection to QFT loop diagrams

MS, Baldauf, Zaldarriaga, Carrasco, Kollmeier (2017)

This idea generalizes to higher order n-point functions or higher loops

one-loop bispectrum

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1}|\boldsymbol{k}_1 - \boldsymbol{q}|^{2\nu_2}|\boldsymbol{k}_2 + \boldsymbol{q}|^{2\nu_3}} \equiv k_1^{3-2\nu_{123}}\, J(\nu_1, \nu_2, \nu_3; x, y) \qquad x \equiv k_3^2/k_1^2 \qquad y \equiv k_2^2/k_1^2$$

two-loop power spectrum

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_4}|\boldsymbol{k} - \boldsymbol{q}|^{2\nu_5}} \int_{\boldsymbol{p}} \frac{1}{p^{2\nu_1}|\boldsymbol{k} - \boldsymbol{p}|^{2\nu_2}|\boldsymbol{q} - \boldsymbol{p}|^{2\nu_3}} \equiv k^{6-2\nu_{12345}}\, K(\nu_1, \ldots, \nu_5)$$

The main challenge is to find “simple” expressions for J, K…

Connection to QFT loop diagrams

In general, function K is very hard to compute

We know many properties: recursion relations, $Z_2 \mathrm{x} S_6$ symmetry etc.

Luckily, we only need a special case where two parameters are equal 1

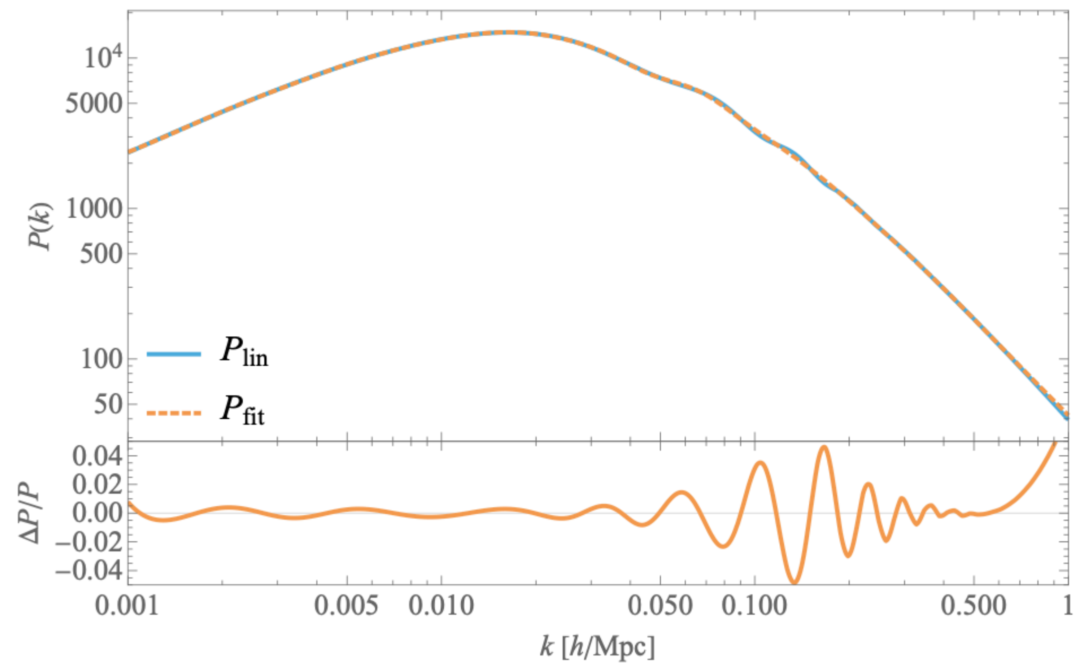
$K(\nu_1, 1, \nu_3, \nu_4, 1) \longrightarrow$ can be written in terms of ${}_3F_2$ functions

$K(1, \nu_2, \nu_3, \nu_4, 1) \longrightarrow$ no known simple closed form expression

Connection to QFT loop diagrams

Anastasiou, Bragança, Senatore, Zheng (2023)

A similar approach with massive propagators



$$P_{\rm fit}(k) = \sum_{n=0}^{N-1} \alpha_n f_n(k^2)$$

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1 + \frac{(k^2 - k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j}$$

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) = \sum_{n=1}^{j} k_{\rm UV}^{2(n-i)}\, k^{2i} \left(\frac{\kappa_n}{(k^2 + M)^n} + \frac{\kappa_n^*}{(k^2 + M^*)^n} \right)$$

Advantage: only integer powers of propagators —> use recursion relations

Particularly useful for the one-loop bispectrum

Conclusions

Using amplitudes in galaxy clustering has been very fruitful

It made running MCMC possible and lead to a small revolution in the field

However, going to higher loops/n-point functions is still a challenge

New ideas are needed!