Flattening the EFT-hedron

Positivity Bounds with Maximal Supersymmetry

Based on work with

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to appear

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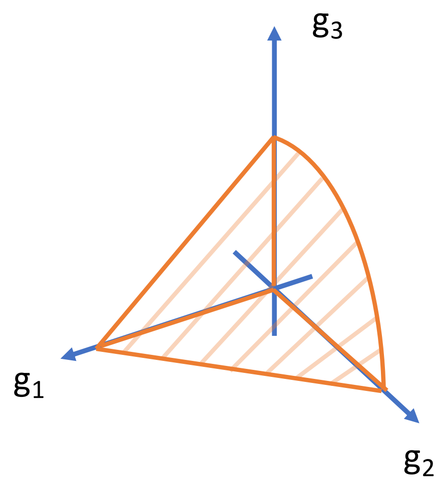
Positivity bounds for Effective Field Theories

Constrain EFTs using 2-to-2 scattering processes and basic assumptions of locality, unitarity, analyticity, and large-energy behavior

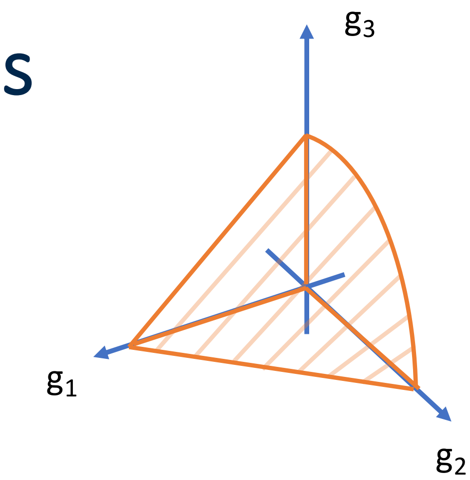
e.g. toolkit overview Correia, Sever, Zhiboedov (2020)

$\Rightarrow$ **low-energy EFT Wilson coefficients given by a high-energy dispersive representation**

$\Rightarrow$ **Bounds on EFT Wilson coefficients**



Positivity bounds for Effective Field Theories



Examples

Explore abelian single-scalar theory [Caron-Huot, Van Duong]

EFT-hedron (geometric perspective) [Arkani-Hamed, T-C Huang, Y-t Huang]
[Chiang, Huang, Li, Rodina, Weng]

Pions & photons [Albert, Rastelli;
Fernandez Ma, Pomarol,Riva, Sciotti;
Häring, Hebbar, Karateev, Meineri, Penedones]

Gravity [Caron-Huot, Li, Parra-Martinez, Simmons-Duffin; Bern, Herrmann, Kosmopoulos, Roiban; Huang, Remmen]

Relation to causality [Gonzales, De Rahm, Jaitly, Poszgay, Tokareva, Tolley, …]

… and much more [Buric, F. Russo, Vichi; Bellazzini, Isabella, Ricossa, Riva; Bachu, Hillman; Tourkine;
Mazac; Guerrieri; Vierira; Trott,….]

This talk: Combine positivity constraints with maximal supersymmetry in 4d.

Consider N=4 super Yang-Mills + all local 4-field higher-derivative operators compatible with N=4 supersymmetry:

$$\mathcal{L} = \mathcal{L}_{\rm SYM} + \frac{b_0}{\Lambda^4}\mathrm{tr}(F^4) + \frac{b_1}{\Lambda^6}\mathrm{tr}(D^2F^4) + \ldots$$

This class of EFTs include the tree-level open superstring amplitude (Veneziano amplitude)

This talk: Combine positivity constraints with maximal supersymmetry in 4d.

Consider N=4 super Yang-Mills + all local 4-field higher-derivative operators compatible with N=4 supersymmetry: "N=4 SYM+h.d.", e.g.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SYM}} + \frac{b_0}{\Lambda^4}\mathrm{tr}(F^4) + \frac{b_1}{\Lambda^6}\mathrm{tr}(D^2F^4) + \ldots$$

This class of EFTs include the tree-level open superstring amplitude (Veneziano amplitude)

GOALS

- What are the most general bounds on the coefficients in max SUSY EFTs?
- Where is string theory?
- How does the "EFT-hedron" behave at increasing order in the derivative expansion?

Constraints from N=4 supersymmetry

Setup: 4-point amplitudes in N=4 SYM + hd

Constraints of maximal supersymmetry: start with the color-ordered 4-point on-shell superamplitude

$$\mathcal{A}_4 = \delta^8(\tilde{Q})\frac{[12]^2}{\langle 34\rangle^2}\, f(s,u) \quad \text{where N=4 SUSY requires} \quad f(s,u) = f(u,s)$$

Project out gluons: $A_4[++--] = [12]^2\langle 34\rangle^2 f(s,u)$

Leading order: gluon amplitude is Parke-Taylor: $A_4^{\rm YM}[++--] = \dfrac{\langle 34\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} = -\dfrac{[12]^2\langle 34\rangle^2}{su}$

So pure YM corresponds to $f(s,u) = -\dfrac{1}{su}$

Setup: 4-point amplitudes in N=4 SYM + hd

On-shell local higher-derivative terms with 4 fields are in 1-1 correspondence with Mandelstam polynomials in *s, t, u* subject to s+t+u = 0:

Ansatz: $$f(s,u) = \underbrace{-\frac{1}{su}}_{\text{YM}} + \sum_{0\le q\le k} \underbrace{a_{k,q}}_{\text{Wilson coefficients}} s^{k-q}u^q\,, \qquad \text{with} \qquad a_{k,k-q} \overset{\text{Crossing / SUSY}}{=} a_{k,q}$$

$a_{0,0}$ is the coefficient of the N=4 SUSY operator tr(F^4)

$$A_4[++--] = [12]^2\langle 34\rangle^2 f(s,u)$$

$a_{1,0} = a_{1,1}$ is the coefficient of the N=4 SUSY operator tr(D^2F^4)

$a_{2,0} = a_{2,2}$ and $a_{2,1}$ are the two coeffs of the two independent N=4 SUSY operators tr(D^4F^4)…. etc

Setup: 4-point amplitudes in N=4 SYM + hd

The S-matrix bootstrap is simpler for external scalar states, so consider a pair of complex conjugate N=4 scalars, say

$$z = z^{12} \qquad \bar{z} = z^{34}$$

Project from the superamplitude to find

$$A_4[zz\bar{z}\bar{z}] = s^2 f(s,u)$$

$$\boxed{A(s,u) = A[zz\bar{z}\bar{z}] = -\frac{s}{u} + s^2 \sum_{0\leq q\leq k} a_{k,q}\, s^{k-q}\, u^q \qquad \text{with} \qquad a_{k,k-q} = a_{k,q}}$$

This amplitude is the focus of our analysis; we'll place bounds on the $a_{k,q}$'s

Veneziano in this framework

One interesting theory within this framework is the open string. Specifically, the 4-point open string tree amplitude.

In the scalar sector, the Veneziano amplitude takes the form

$$A^{\mathrm{str}}[zz\bar{z}\bar{z}] = -(\alpha' s)^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' u)}{\Gamma(1-\alpha'(s+u))}$$

Upon expanding in small $\alpha' s$ and $\alpha' u$ we find

$$A^{\mathrm{str}}[zz\bar{z}\bar{z}] = -\frac{s}{u} + s^2\left(\zeta_2\alpha'^2 + \zeta_3\alpha'^3(s+u) + \zeta_4\alpha'^4\left(s^2+u^2\right) + \frac{1}{4}\zeta_4\alpha'\, su + \ldots\right)$$

$a_{0,0}$ $\quad$ $a_{1,0} = a_{1,1}$ $\quad$ $a_{2,0} = a_{2,2}$ $\quad$ $a_{2,1}$

Dispersive representations

Assumptions $A(s,u) = A[zz\bar{z}\bar{z}]$

1) **Large N** => color-ordered single-trace amplitude has no *t*-pole or *t*-channel discontinuities.

2) **Weak-coupling** at all energy scales => ensures we can disregard loops of massless states.

3) **Mass gap:** other than the massless N=4 supermultiplet, there are no other states with mass less than M_{gap}.

4) **Partial wave decomposition** $A(s,u) = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell\big(\cos(\theta)\big)$

   Legendre

   Unitarity => $\mathrm{Im}(a_\ell(s)) > 0$

5) Amplitude **analytic** away from the real *s*-axis.

6) **Froissart-Martin-like bounds** $\text{fixed } u < 0: \lim_{s\to\infty} \frac{A(s,u)}{s^2} = 0$ and $\text{fixed } t < 0: \lim_{s\to\infty} \frac{A(s,-s-t)}{s^2} = 0$

Dispersive representation

important

At low-energy, we have $A(s,u) = A[zz\bar{z}\bar{z}] = -\frac{s}{u} + s^2 \sum_{0\leq q\leq k} a_{k,q}\, s^{k-q}\, u^q$

So we can extract each Wilson coefficient via a contour integral: $a_{k,q} = \frac{1}{q!}\frac{\partial^q}{\partial u^q}\int_{\mathcal{C}^\star}\frac{ds'}{2\pi i}\frac{A(s',u)}{s'^{k-q+3}}\Bigg|_{u=0}$

(fixed u<0)

All *k* and *q*

Contour around s=0

Dispersive representation

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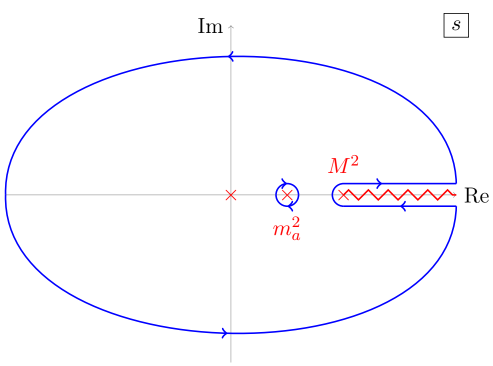
(fixed u<0)

All k and q

Contour around s=0

Deform the contour: pick up pole-terms and branch cuts on the real s-axis and



$$a_{k,q} = \frac{1}{q!}\frac{\partial^q}{\partial u^q}\left(\frac{1}{\pi}\int \frac{ds'}{s'^{k-q+3}}\mathrm{Im}A(s',u)\right)\bigg|_{u=0}.$$

Then use the partial wave expansion and $M^2 = s'$ to get

Dispersive representation

$$a_{k,q} = \sum_{\ell=0} \int_{M_{\rm gap}^2}^{\infty} dM^2\, \rho_\ell(M^2) \left(\frac{1}{M^2}\right)^{k+3} v_{\ell,q}$$

where $\rho_\ell(M^2) = 16(2\ell+1)\,\mathrm{Im}\big(a_\ell(M^2)\big) \; > 0$

and we have expanded the Legendre polynomials as

$$P_\ell(1+2\delta) = \sum_{q=0}^{\ell} v_{\ell,q}\delta^q \quad \text{with} \quad v_{\ell,q} = \frac{\prod_{a=1}^{q}\left[\ell(\ell+1) - a(a-1)\right]}{(q!)^2} \quad \geq 0$$

Finally, rescale the $a_{k,q}$'s by $(M_{gap})^{2(k+2)}$ (so they are all dimensionless)

and change the integration variable M^2 to $x = \frac{M_{\rm gap}^2}{M^2}$: then

$$\boxed{a_{k,q} = \sum_{\ell=0} \int_0^1 dx\, p_\ell(x)\, x^k\, v_{\ell,q}, \qquad \text{with} \qquad p_\ell(x) = x\,\rho_\ell\big(M_{\rm gap}^2/x\big) > 0}$$

Dispersive representation

Result

$$a_{k,q} = \sum_{\ell=0} \int_0^1 dx\, p_\ell(x)\, x^k\, v_{\ell,q}, \qquad p_\ell(x) > 0$$

$$\text{all } 0 \leq q \leq k$$

"moment map"

Low-energy EFT

Integral over the high-energy spectrum

Immediate consequences:

1) $a_{k,q} \geq 0$

2) $a_{k',q} \leq a_{k,q}, \quad k \leq k'$ (b/c 0 < x < 1)

Dispersive representation

Result

$$\boxed{a_{k,q} = \sum_{\ell=0} \int_0^1 dx\, p_\ell(x)\, x^k\, v_{\ell,q}, \qquad p_\ell(x) > 0 \qquad \text{all } 0 \leq q \leq k}$$

"moment map"

Low-energy EFT

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Immediate consequences:

1) $a_{k,q} \geq 0$

2) $a_{k',q} \leq a_{k,q}, \quad k \leq k'$ (b/c 0 < x < 1)

3) Using SUSY crossing $a_{k,k-q} = a_{k,q}$ =>

Biggest

$$\left\{\begin{array}{llll} a_{0,0} \geq & a_{1,0} \geq & a_{2,0} \geq & a_{3,0} \dots \\ & \rotatebox{90}{=} & & \\ & a_{1,1} \geq & a_{2,1} \geq & a_{3,1} \dots \\ & & & \rotatebox{90}{=} \\ & & & a_{3,2} \dots \\ & & & \vdots \end{array}\right.$$

So

$$\boxed{0 \leq \frac{a_{k,q}}{a_{0,0}} \leq 1}$$

for all *k* and *q*

Then what?

From the moment maps, one can compute analytic bounds (Hankel, cyclic polytope, product Hankel)

[Arkani-Hamed, T-C Huang, Y-t Huang 2012.15849]

[Chiang, Huang, Li, Rodina, Weng 2105.02862]

However, at higher *k* (higher derivative terms) this quickly becomes unwieldy.

Instead: formulate as an optimization problem and using numerical methods, such as semi-definite programming (SDPB package by Simmons-Duffin)

written for the conformal bootstrap

[Caron-Huot, Van Duong 2011.02957]

[Albert, Rastelli 2203.11950 + 2307.01246]

Numerical analysis

We use two different complementary numerical methods

SDPB semi-definite programming

CPLEX linear optimization by IBM

Following standard set up, we reformulate the setup as an optimization problem with two sets of null constraints (aka sum rules)

- SUSY Crossing $a_{k,k-q} = a_{k,q}$
- ST channel sum rule

Our $A(s,u) = s^2 f(s,u)$ is not crossing symmetric, but $f(s,u)$ is and that is sufficient to derive constant-t crossing relations like those in the analysis of the pion amplitudes of [Albert, Rastelli 2203.11950]

Numerical analysis practicalities

- k_{max} **max derivative order included**

  The higher k_{max}, the stronger bounds.

  e.g. k_{max} = 4 means null constraints are imposed up to 12-derivative order.

$$A[zz\bar{z}\bar{z}] = -\frac{s}{u} + s^2 \sum_{0\leq q\leq k} a_{k,q}\, s^{k-q}\, u^q$$

- l_{max} **truncate the sum over all spin**

$$a_{k,q} = \sum_{\ell=0} \int_0^1 dx\, p_\ell(x)\, x^k\, v_{\ell,q},$$

  Chosen to ensure that bounds are converging (typically 200 to 1000)

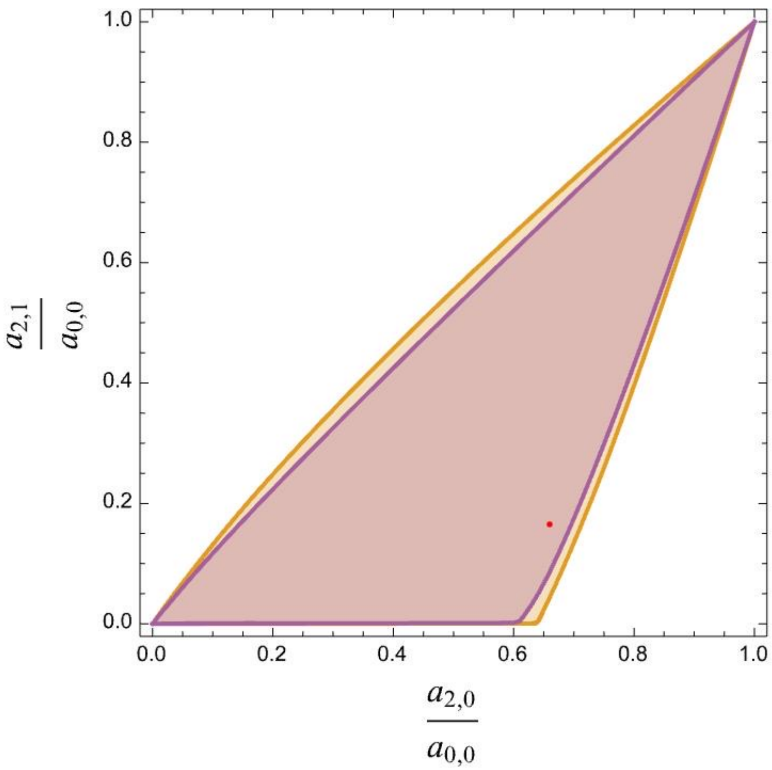
- x_{max} **Implementation in CPLEX requires discretization**

  discretize the integral over x (i.e. the mass-spectrum). We use x_{max} to denote the number of discretization points of the interval 0 to 1 (typically between 200 and 1000).

Bounds

Justin Berman, Aidan Herderschee, HE (in progress)

Example of SDPB Results: (a_{20},a_{21}) plane (two tr(D^4F^4) N=4 SUSY ops)



Orange: k_{max} = 4, l_{max} = 200 allowed region

Rosa: k_{max} = 10, l_{max} = 300 allowed region

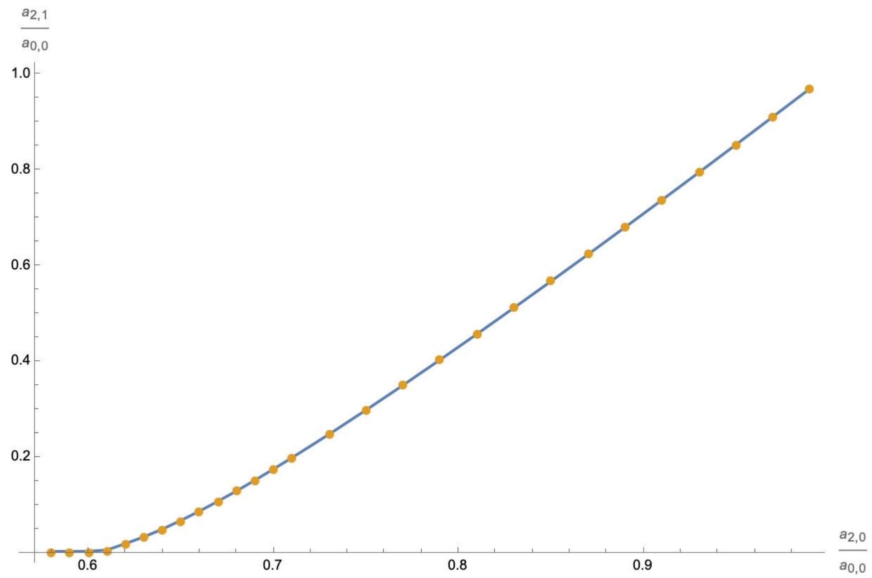
Red dot: Veneziano w/ choice $\alpha' M_{\text{gap}}^2 = 1$

- (0,0) includes theory with tr(F^4) as the only h.d. interaction.
- (1,1) includes theory with all $a_{k,q}$=$a_{0,0}$. Can be re-summed to

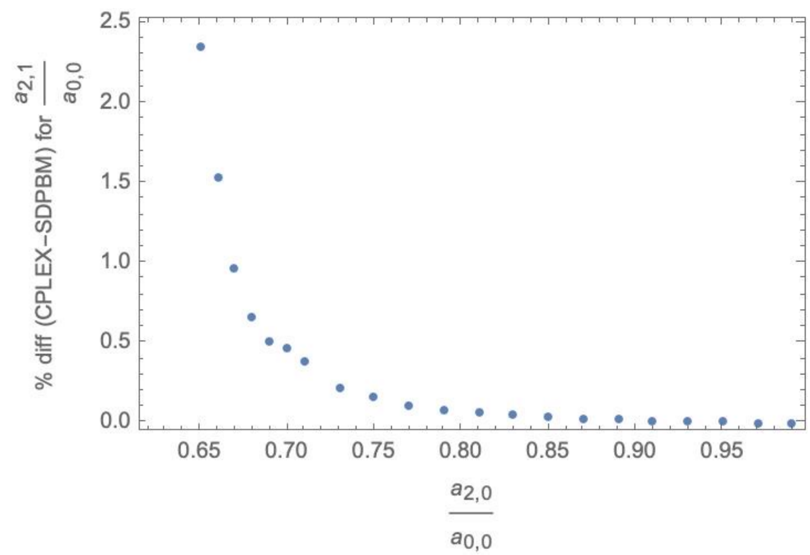
$$A(s,u) = -\frac{s}{u} + s^2 \frac{1}{(s-1)(u-1)}$$

Tends to show up in bootstraps;
Probably not a sensible theory.

Comparison: SDPB and CPLEX k_{max}=10 and l_{max} = 300



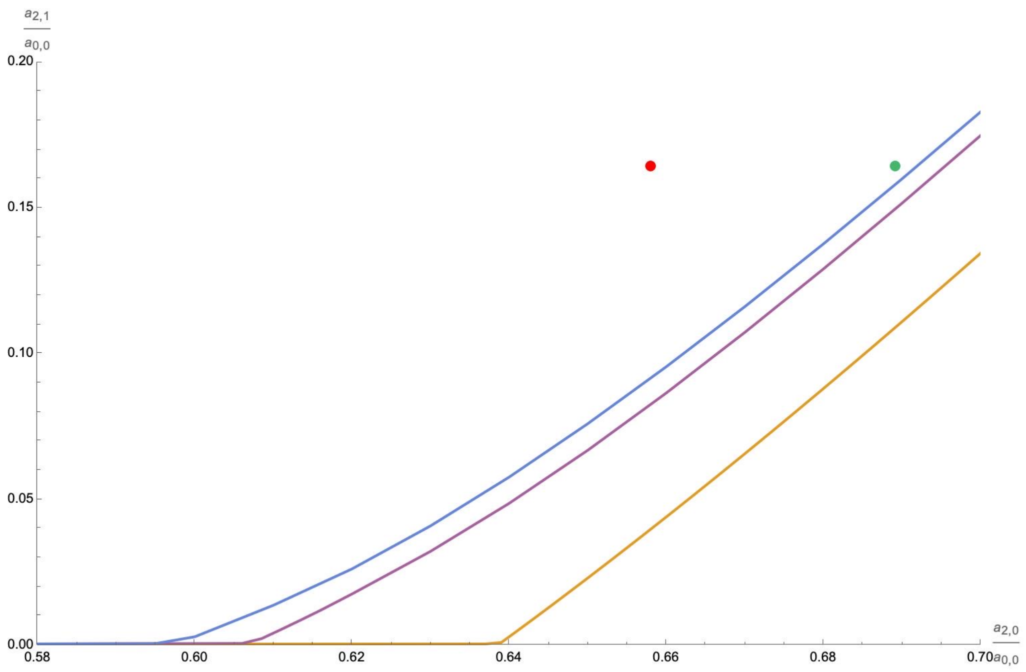
Orange: CPLEX w/ x_{max} = 300
Blue SDPB



Percentile difference CPLEX vs. SDPB.

- CPLEX discretization error larger on the a21 = 0 axis.
- CPLEX faster by ~ factor of 5 for these runs, but high precision requires higher x_{max}

What happens near the string?



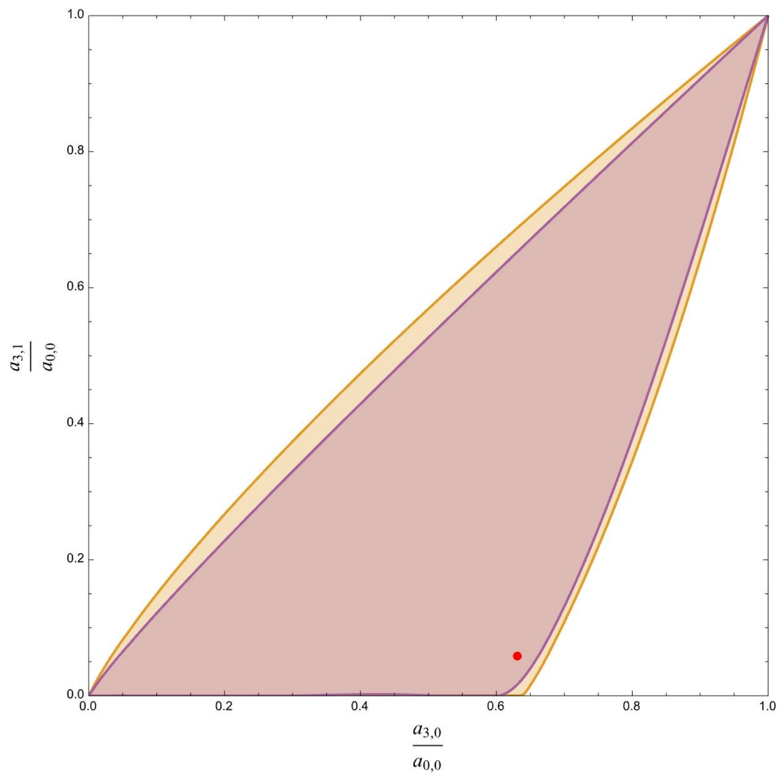
Orange: $k_{max} = 4$ $l_{max} = 200$ allowed region
Rosa: $k_{max} = 10$ $l_{max} = 300$ allowed region
Blue: $k_{max} = 15$ $l_{max} = 800$ allowed region
Green dot: $k_{max} = 20$ $l_{max} = 600$ at $a_{2,0}$ string value

Red dot: Veneziano

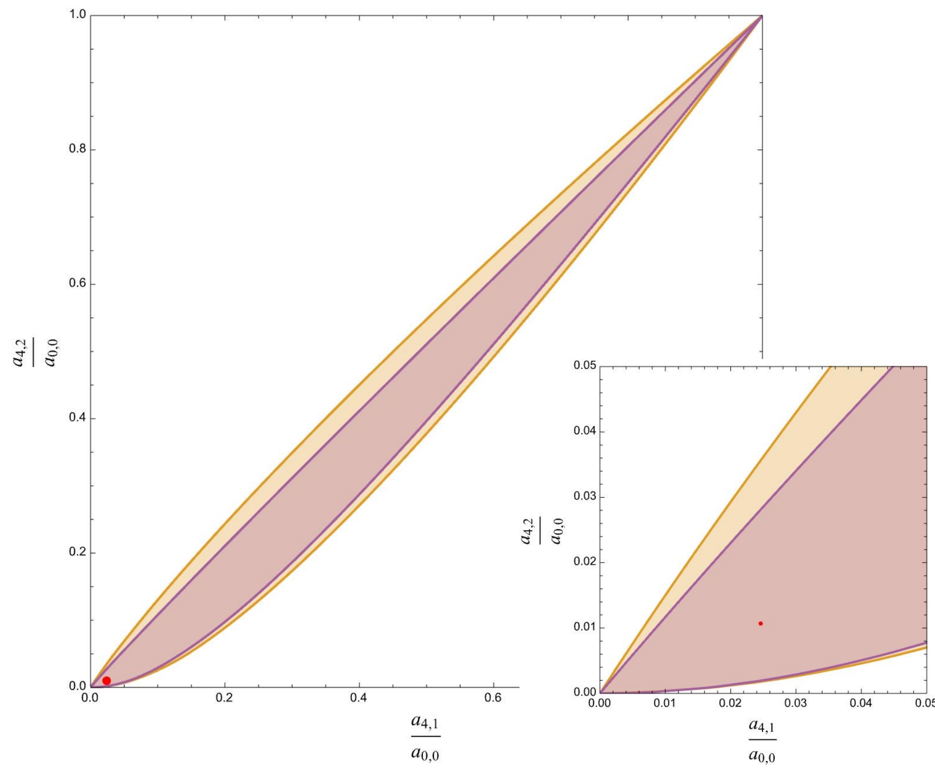
What happens as k_{max} increases?

Other projections

$(a_{3,0}, a_{3,1})$ tr(D^{6}F^4)



$(a_{4,1}, a_{4,2})$ tr(D^{8}F^4)



Monodromies

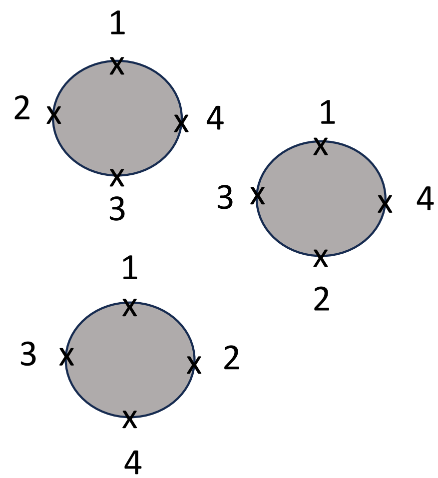
Inspired by

Yu-tin Huang, Jin-Yu Liu, Laurentiu Rodina, Yihong Wang [2008.02293]

Monodromy relations

String disk amplitudes

$$A[1234] = \frac{-s^2}{t} \times \int_0^1 dz z^{-\alpha' s - 1}(1-z)^{-\alpha' u - 1} \; ,$$
$$A[1324] = \frac{s^2}{t} \times \int_1^\infty dz z^{-\alpha' s - 1}(z-1)^{-\alpha' u - 1} \; ,$$
$$A[2134] = \frac{s^2}{t} \times \int_{-\infty}^0 dz (-z)^{-\alpha' s - 1}(1-z)^{-\alpha' u - 1} \; .$$



Contour deformation relates the different color-orderings while picking up monodromies at x=0 and x=1.

Stieberger (2009)
Bjerrum-Bohr, Damgaard, Vanhove (2009)

String monodromy relations $\quad 0 = A[2134] + e^{i\pi\alpha' s} A[1234] + e^{-i\pi\alpha' t} A[1324]$

SUSY Ansatz w/ Monodromy Imposed

linear combination fixed	string value
$a_{0,0}$	$= \zeta_2 = \frac{\pi^2}{6}$
$a_{2,0}$	$= \zeta_4 = \frac{\pi^4}{90}\,,$
$a_{2,1}$	$= \frac{1}{4}\zeta_4 = \frac{\pi^4}{360}\,,$
$a_{3,1} - 2a_{3,0} + \zeta_2\, a_{1,0}$	$= 0\,,$
$a_{4,0}$	$= \zeta_6 = \frac{\pi^6}{945}\,,$
$a_{4,2} - 2a_{4,1}$	$= -\frac{1}{16}\zeta_6 = -\frac{\pi^6}{15120}\,,$
$a_{5,1} - 3a_{5,0} + \zeta_2 a_{3,0} + \zeta_4 a_{1,0}$	$= 0\,,$
$a_{5,2} - 5a_{5,0} + 2\zeta_2\, a_{3,0} + \frac{5}{4}\zeta_4\, a_{1,0}$	$= 0\,,$

etc

Unfixed coefficients and their string values

$$a^{\rm str}_{1,0} = \zeta_3 = 1.202057$$
$$a^{\rm str}_{3,0} = \zeta_5 = 1.036928$$
$$a^{\rm str}_{4,1} = \frac{\left(\pi^6 - 630\zeta_3^2\right)}{1260} = 0.040537\,,$$
$$a^{\rm str}_{5,0} = \zeta_7 = 1.00835\,.$$

etc

Note: monodromy ``knows” π only, so cannot access any information about coefficients with $\zeta(\mathrm{odd})$ in the low-energy expansion of the open string amplitude.

Monodromy + EFT-hedron

Yu-tin Huang, Jin-Yu Liu, Laurentiu Rodina, Yihong Wang [2008.02293]

Monodromy + EFT-hedron ``carves out the open string"

Using all **Hankel + Cyclic Polytope + Product Hankel** constraints to $k_{\text{max}} = 4$ and one from $k_{\text{max}} = 5$, they found

$$\frac{x^{max} - x^{min}}{x^{string}} = \frac{1.20667 - 1.18890}{\zeta(3)} \approx 1.5\%,$$
$$\frac{y^{max} - y^{min}}{y^{string}} = \frac{1.03808 - 1.03594}{\zeta(5)} \approx 0.2\%,$$
$$\frac{z^{max} - z^{min}}{z^{string}} = \frac{0.05699 - 0.03560}{(\pi^6 - 630\zeta(3)^2)/1260} \approx 52.8\%$$

$$a^{\text{str}}_{1,0} = \zeta_3 = 1.202057$$
$$a^{\text{str}}_{3,0} = \zeta_5 = 1.036928$$
$$a^{\text{str}}_{4,1} = \frac{(\pi^6 - 630\zeta_3^2)}{1260} = 0.040537$$

Indicates that the intersection of the monodromy subspace and EFT-hedron may be a point

Monodromy + SUSY Positivity

Justin Berman, Aidan Herderschee, HE (in progress)

- ST sum rules cannot be expressed in terms of the $a_{k,q}$ so cannot be captured by the (product) Hankel / cyclic polytope constraints or their enhancements [Chiang, Huang, Li, Rodina, Weng 2105.02862]
- Implementation in SDPB and CPLEX allows going to higher orders

Goals:

1) Further test if positivity bounds and monodromy isolate the string

2) Understand in what sense string theory is then on the boundary of the SUSY EFT-hedron

Monodromy + SUSY Positivity

Justin Berman, Aidan Herderschee, HE (in progress)

The two lowest Wilson coefficients unfixed by string monodromy are $\mathbf{a_{1,0}}$ **and** $\mathbf{a_{3,0}}$

Without monodromy:

Exact upper and lower bounds
(indep. of $k_{max} > 2$ and l_{max})

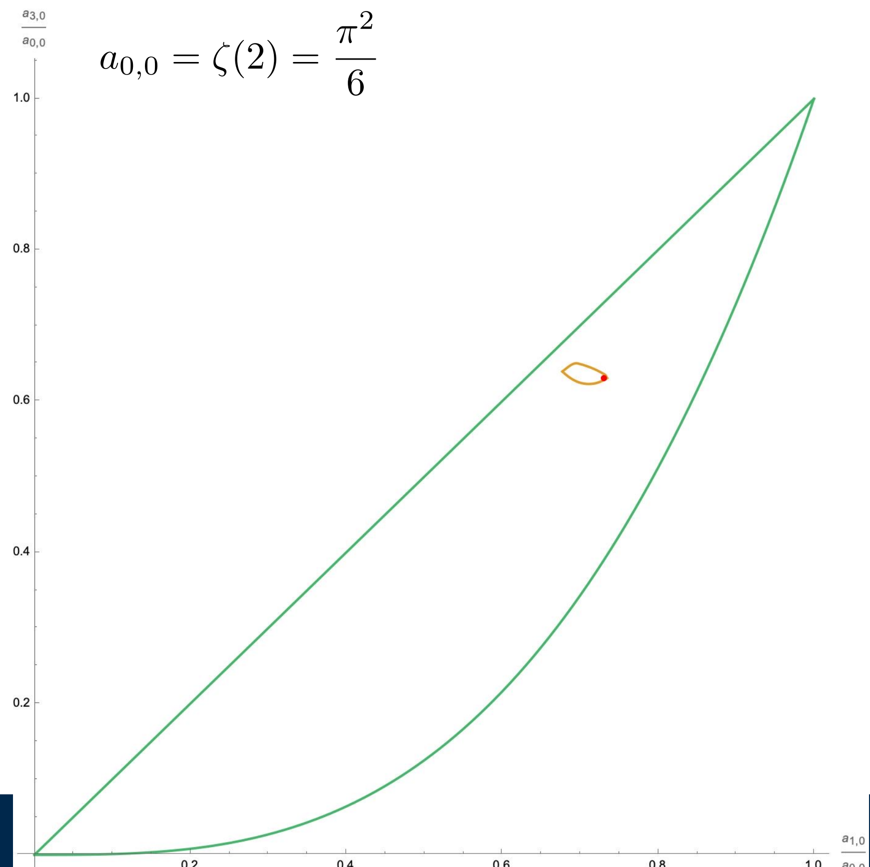
$$\left(\frac{a_{1,0}}{a_{0,0}}\right)^3 \leq \frac{a_{3,0}}{a_{0,0}} \leq \frac{a_{1,0}}{a_{0,0}}$$

With monodromy:

Orange:
k_{max} = 3 bounds with monodromy imposed as null constraints

Red: Veneziano

$$a_{0,0} = \zeta(2) = \frac{\pi^2}{6}$$



Monodromy + SUSY Positivity

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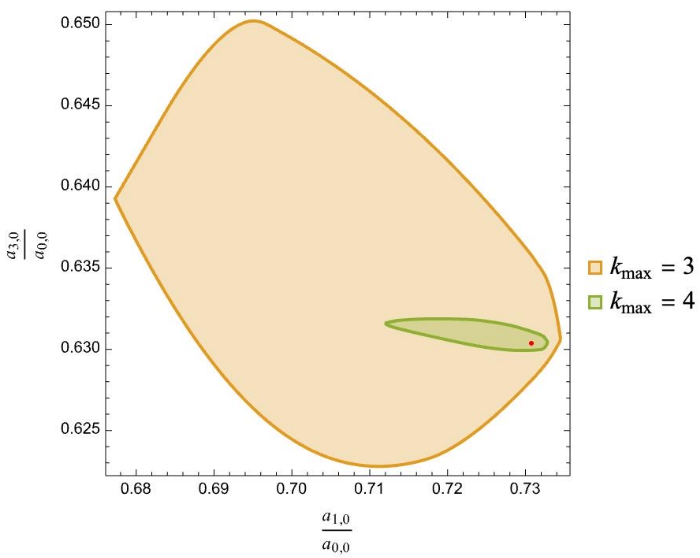
Increase k_{max}



Zoom in on green region

Monodromy + SUSY Positivity

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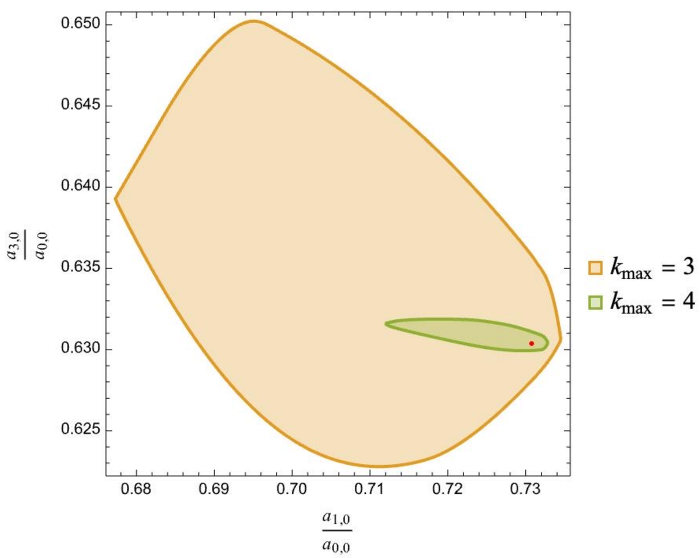

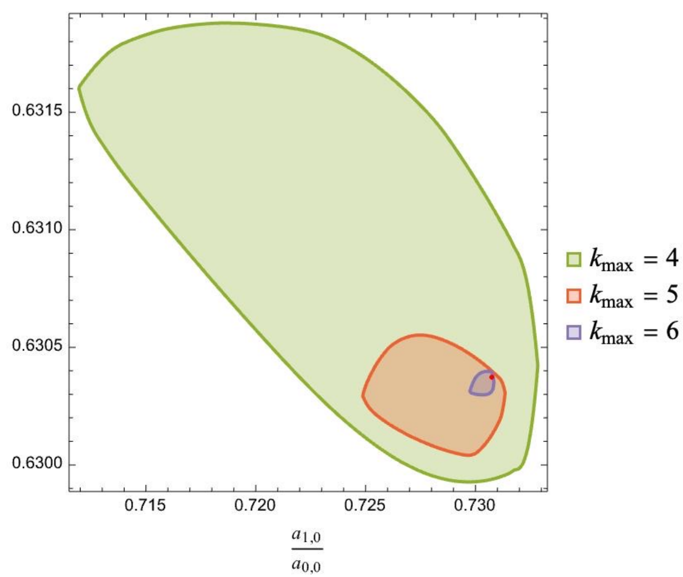

Monodromy + SUSY Positivity

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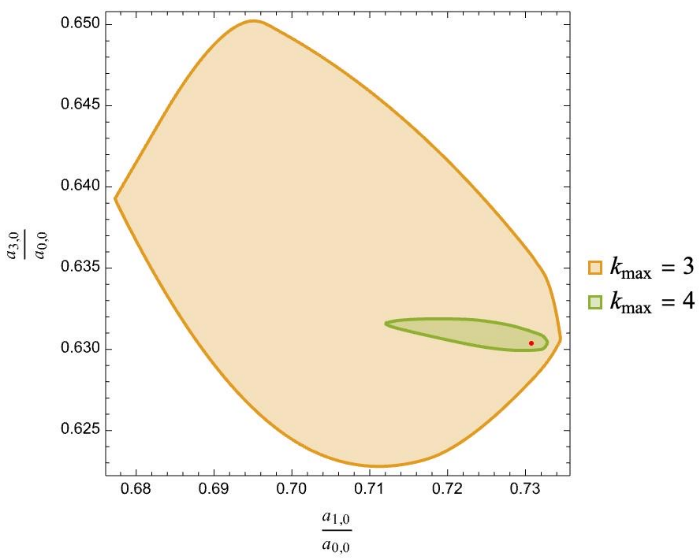
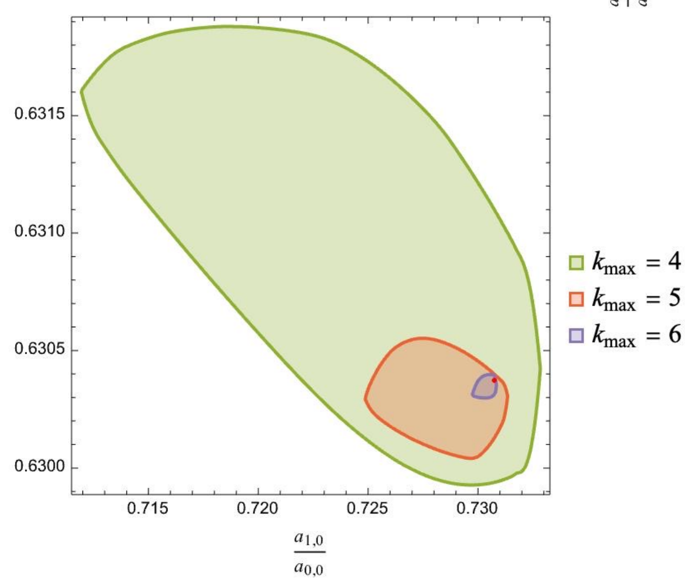
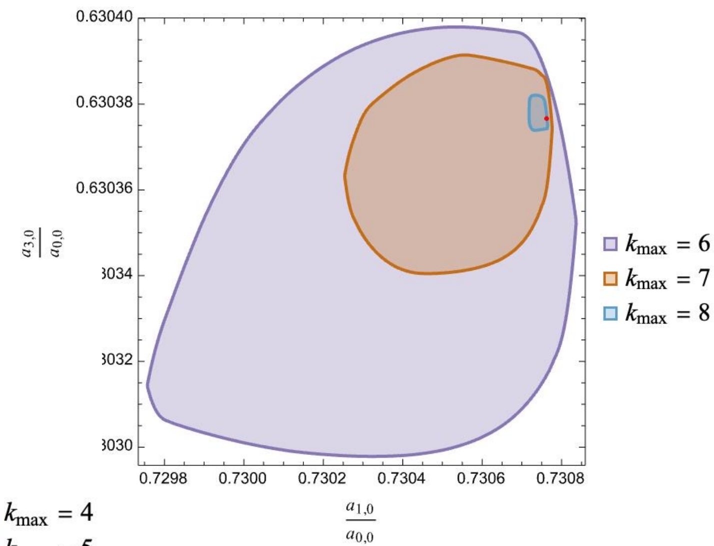






Monodromy + SUSY Positivity

Justin Berman, Aidan Herderschee, HE (in progress)

Overall bounds for k_{max} = 8 with l_{max} = 800

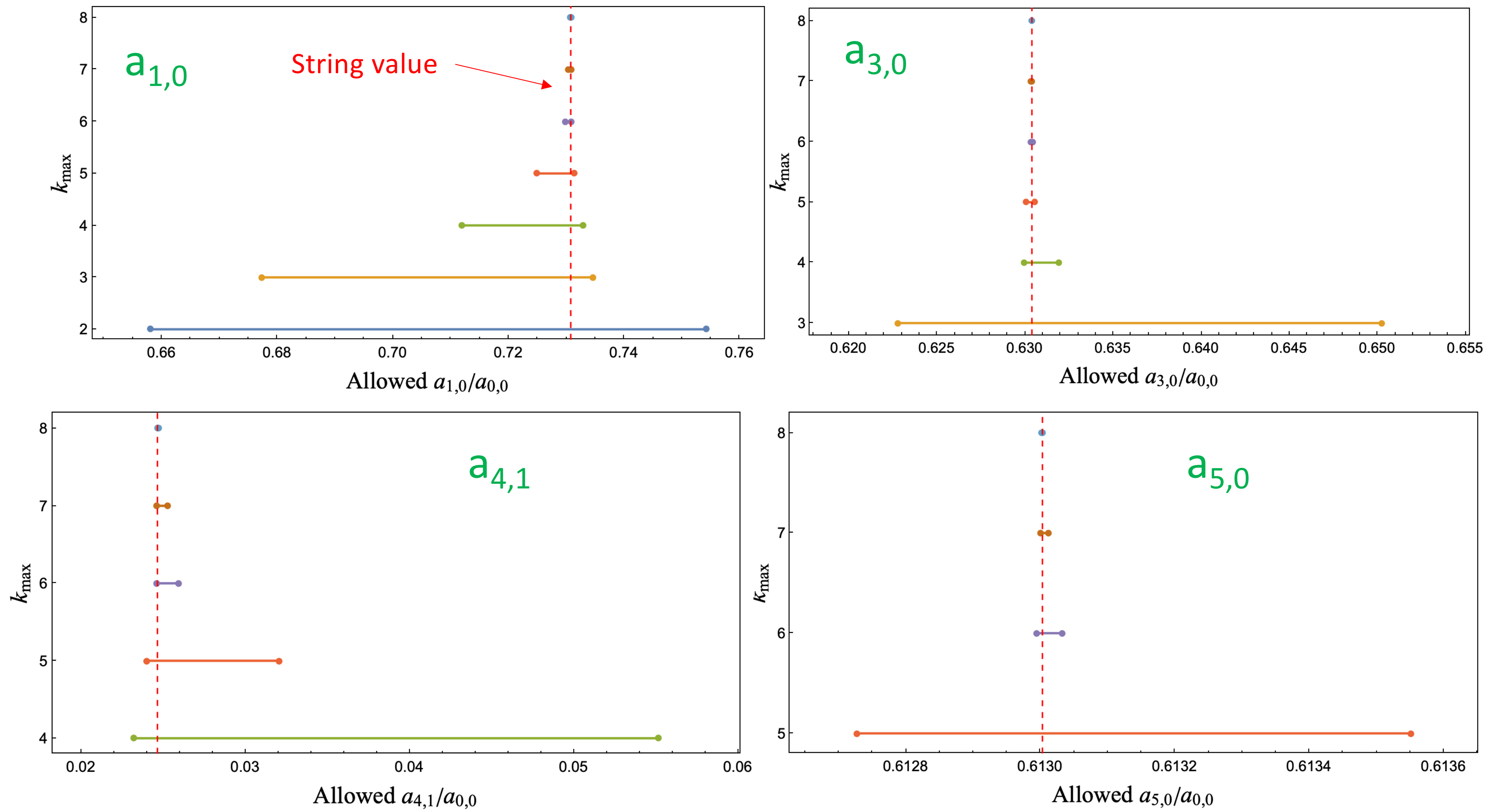
$$1.20198 < a_{1,0} < 1.20206 \quad \text{within} \quad 0.0066\% \quad \text{of} \qquad a^{\text{str}}_{1,0} = \zeta_3 = 1.202057$$

$$1.03692 < a_{3,0} < 1.03694 \quad \text{within} \quad 0.0014\% \quad \text{of} \qquad a^{\text{str}}_{3,0} = \zeta_5 = 1.036928$$

vs. ~1.5% and 0.2% in Huang, Liu, Rodina, Wang [2008.02293]

Monodromy + SUSY Positivity

Justin Berman, Aidan Herderschee, HE (in progress)



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Overall bounds for k_{max} = 8 with l_{max} = 800

$$1.20198 < a_{1,0} < 1.20206 \ \text{ within } \ 0.0066\% \ \text{ of } \quad a^{\text{str}}_{1,0} = \zeta_3 = 1.202057$$

$$1.03692 < a_{3,0} < 1.03694 \ \text{ within } \ 0.0014\% \ \text{ of } \quad a^{\text{str}}_{3,0} = \zeta_5 = 1.036928$$

vs. ~1.5% and 0.2% in Huang, Liu, Rodina, Wang [2008.02293]

$$0.0405345 < a_{4,1} < 0.0406249 \ \text{ within } \ 0.22\% \ \text{ of } \quad a^{\text{str}}_{4,1} = \frac{(\pi^6 - 630\zeta_3^2)}{1260} = 0.040537$$

vs. ~52% in Huang, Liu, Rodina, Wang [2008.02293]

	$a_{5,0}$	$a_{6,1}$	$a_{7,0}$	$a_{7,2}$	$a_{8,1}$
Within	0.0001%	0.9%	0.00006%	27%	3.8%

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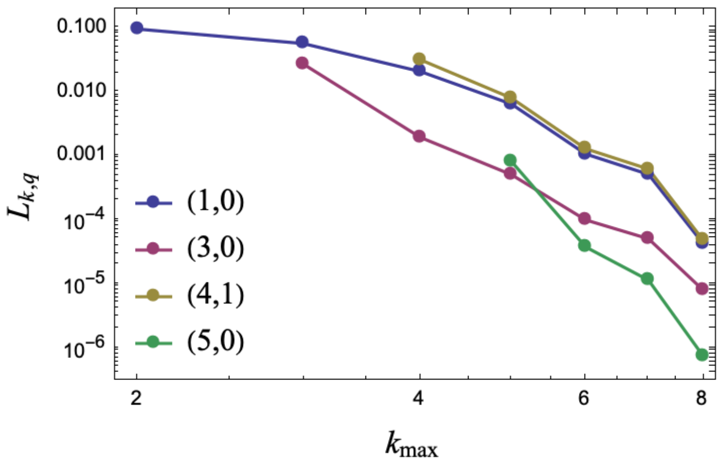
Define $L_{k,q} = \max\left(\frac{a_{k,q}}{a_{0,0}}\right) - \min\left(\frac{a_{k,q}}{a_{0,0}}\right)$ measure of how well each coefficient is fixed for given k_{max}.

Log-Log plot shows convergence faster than powers of $1/k_{\text{max}}$

$\alpha' M_{\text{gap}}^2 = 1$



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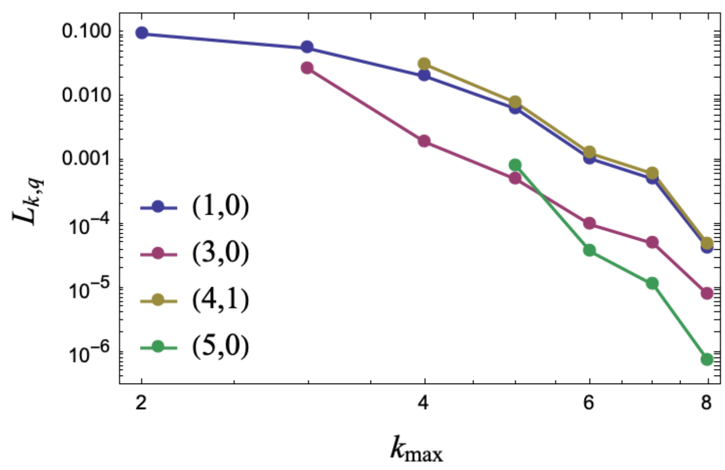
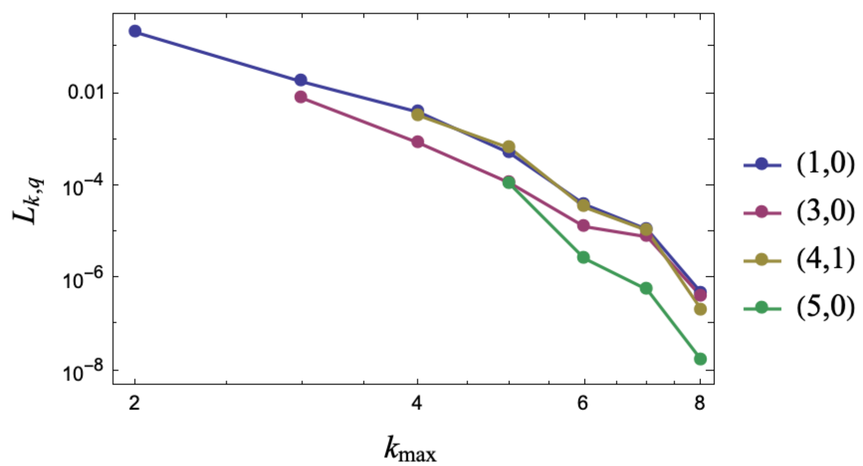


$\alpha' M_{\rm gap}^2 = \frac{1}{2}$



Where is string theory?

So, the tree-level open string (i.e. the Veneziano amplitude) does indeed appear to be on the intersection of the convex space carved out by the positivity constraints and the monodromy constraints. But **how?**

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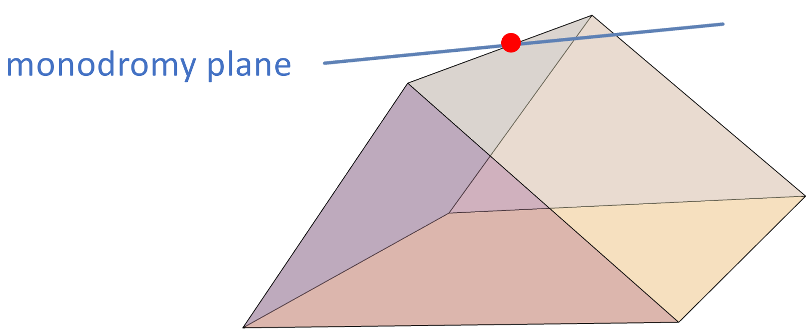
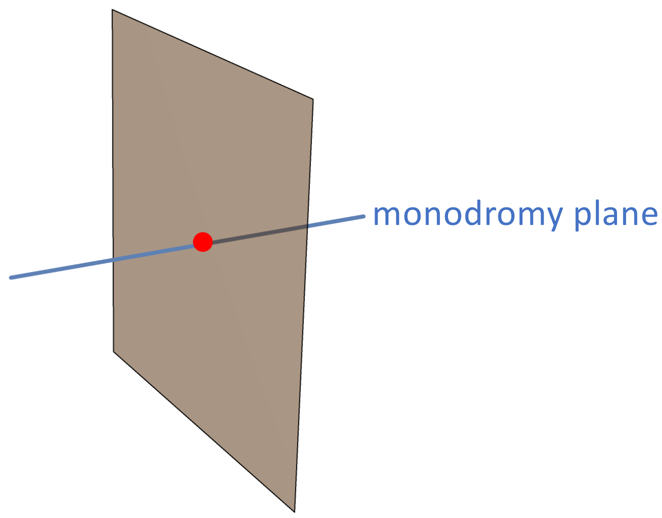
Is the *string on the boundary* of the convex positive region?



Or *does the convex space shrink* in such a way that the intersection shrinks to a point as k_{max} increases?



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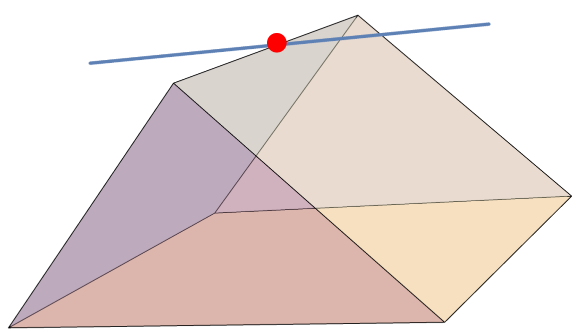
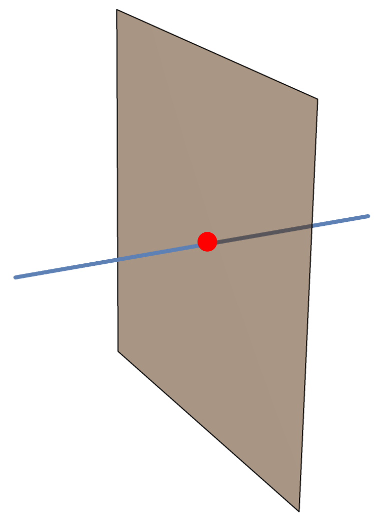


Or does the convex space shrink in such a way that the intersection shrinks to a point as $k_{\max}$ increases?



To assess,
vary the monodromy
plane and bootstrap

Testing the geometry

Monovariables

Recall string monodromies

linear combination fixed	string value
$a_{0,0}$	$= \zeta_2 = \frac{\pi^2}{6}$
$a_{2,0}$	$= \zeta_4 = \frac{\pi^4}{90}\,,$
$a_{2,1}$	$= \frac{1}{4}\zeta_4 = \frac{\pi^4}{360}\,,$
$a_{3,1} - 2a_{3,0} + \zeta_2\, a_{1,0}$	$= 0\,,$
$a_{4,0}$	$= \zeta_6 = \frac{\pi^6}{945}\,,$
$a_{4,2} - 2a_{4,1}$	$= -\frac{1}{16}\zeta_6 = -\frac{\pi^6}{15120}\,,$
$a_{5,1} - 3a_{5,0} + \zeta_2 a_{3,0} + \zeta_4 a_{1,0}$	$= 0\,,$
$a_{5,2} - 5a_{5,0} + 2\zeta_2\, a_{3,0} + \frac{5}{4}\zeta_4\, a_{1,0}$	$= 0\,,$

Monovariables

Recall string monodromies

linear combination fixed	string value	
$a_{0,0}$	$= \zeta_2 = \frac{\pi^2}{6}$	$= r_0$
$a_{2,0}$	$= \zeta_4 = \frac{\pi^4}{90}\,,$	$= r_1$
$a_{2,1}$	$= \frac{1}{4}\zeta_4 = \frac{\pi^4}{360}\,,$	$= r_2$
$a_{3,1} - 2a_{3,0} + \zeta_2\, a_{1,0}$	$= 0\,,$	=> $= r_3$
$a_{4,0}$	$= \zeta_6 = \frac{\pi^6}{945}\,,$	$= r_4$
$a_{4,2} - 2a_{4,1}$	$= -\frac{1}{16}\zeta_6 = -\frac{\pi^6}{15120}\,,$	$= r_5$
$a_{5,1} - 3a_{5,0} + \zeta_2 a_{3,0} + \zeta_4 a_{1,0}$	$= 0\,,$	$= r_6$
$a_{5,2} - 5a_{5,0} + 2\zeta_2\, a_{3,0} + \frac{5}{4}\zeta_4\, a_{1,0}$	$= 0\,,$	$= r_7$

Monovariables

Recall string monodromies

linear combination fixed	"Monovariables"
$a_{0,0}$	$= r_0$
$a_{2,0}$	$= r_1$
$a_{2,1}$	$= r_2$
$a_{3,1} - 2a_{3,0} + \zeta_2\, a_{1,0}$	$= r_3$
$a_{4,0}$	$= r_4$
$a_{4,2} - 2a_{4,1}$	$= r_5$
$a_{5,1} - 3a_{5,0} + \zeta_2 a_{3,0} + \zeta_4 a_{1,0}$	$= r_6$
$a_{5,2} - 5a_{5,0} + 2\zeta_2\, a_{3,0} + \frac{5}{4}\zeta_4\, a_{1,0}$	$= r_7$

Monovariables

Then construct theories that obey the positivity bounds

"(1,1) theory"

$$A(s,u) = \frac{-s}{u} + s^2 \left(\int_{M_{\rm gap}^2}^{\infty} dm^2 \rho^{(1)}_{m^2} \left[\frac{1}{(m^2-s)(m^2-u)} \right] + \int_{M_{\rm gap}^2}^{\infty} dm^2 \frac{\rho^{(2)}_{m^2}}{m^4} \left[\frac{m^4}{su} - \frac{\Gamma(-s/m^2)\Gamma(-u/m^2)}{\Gamma(1+t/m^2)} \right] \right)$$

Subtract off massless poles

Veneziano

e.g. picking $\rho^{(1)}_{m^2} = \rho^{(2)}_{m^2} = \frac{1}{2}\delta(m^2-1)$ gives $a_{k,q} = \frac{1}{2}\left(a^{\rm str}_{k,q} + a^{(1,1)}_{0,0}\right)$

Generally, we can pick the densities to be sums over delta-functions at various masses.
These will be the **"test theories"**.

Test theories

For a given test-theory, we compute the corresponding $a_{k,q}$ and thus the monovariables r_i.

Monovariable constraints are null constraints for SDPB/CPLEX.

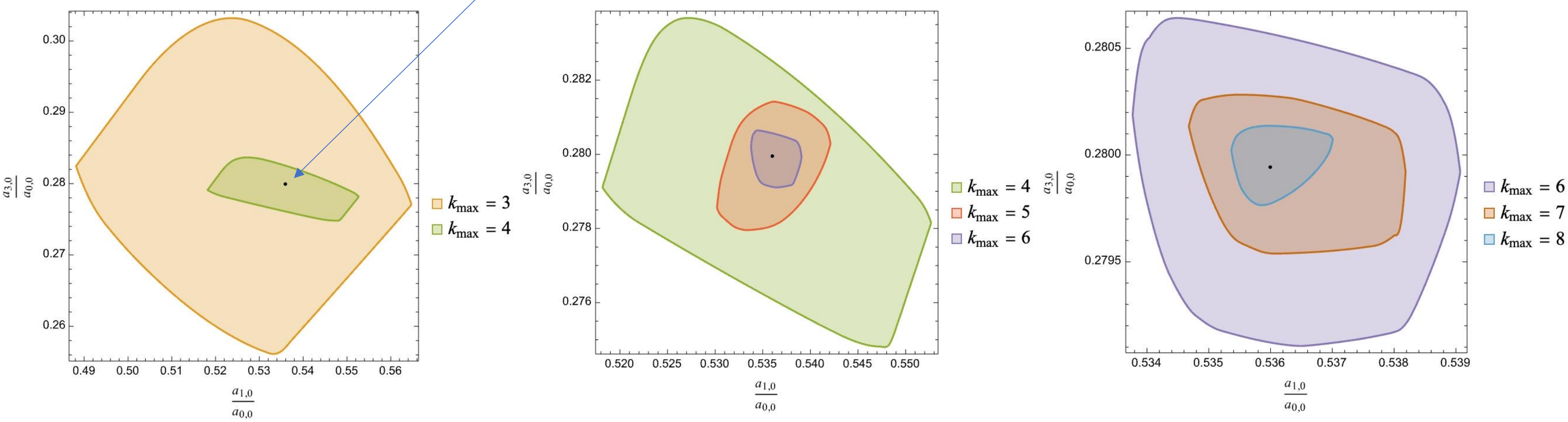
We test if SDPB/CPLEX narrows in on the Wilson coefficients unfixed by the monovariable constraints (just as it did for Veneziano).

Because we know all $a_{k,q}$ by construction, we know if SDPB/CPLEX gets it right.

Example ->

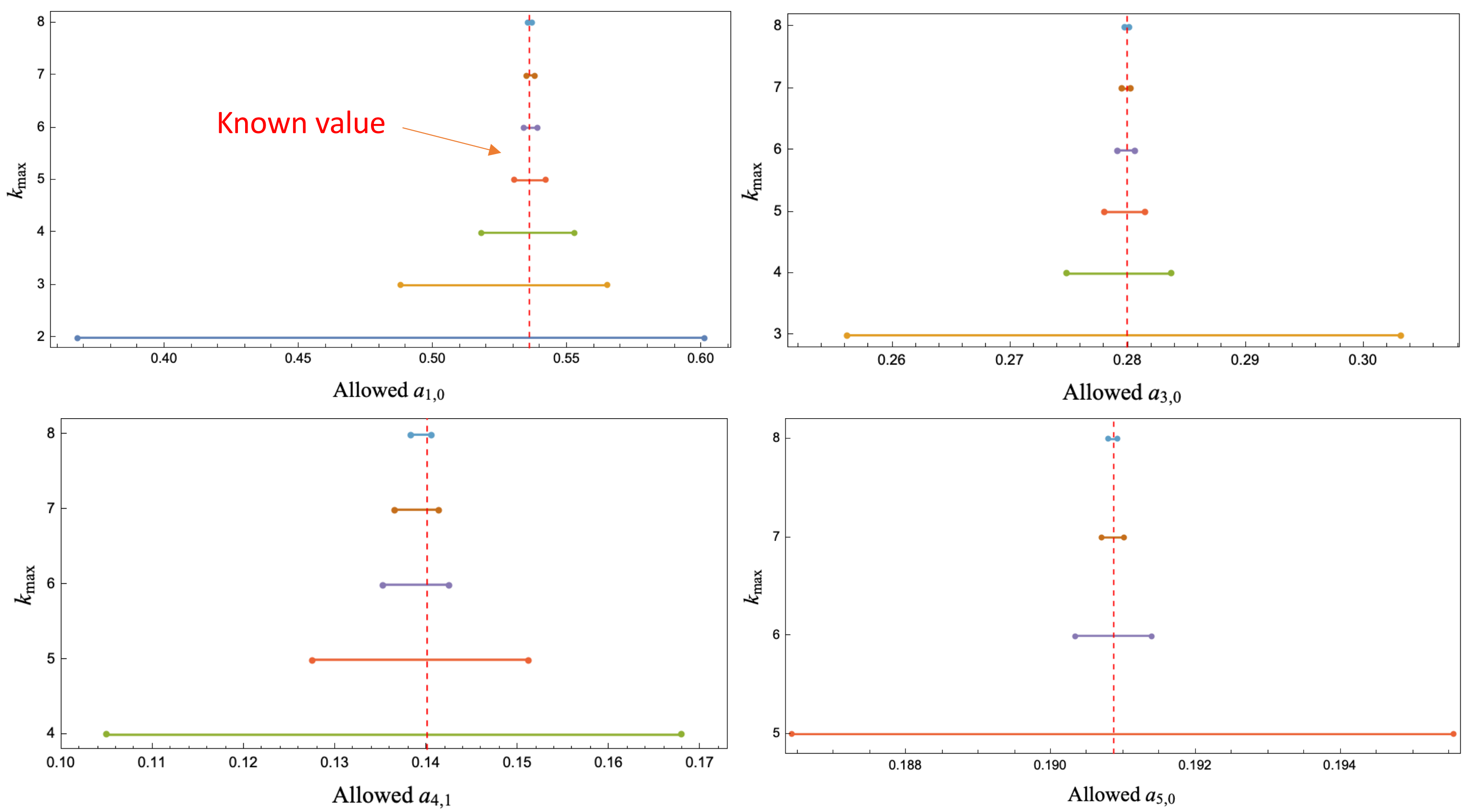
Test theory example

Known (by construction) values of $a_{1,0}$ and $a_{3,0}$



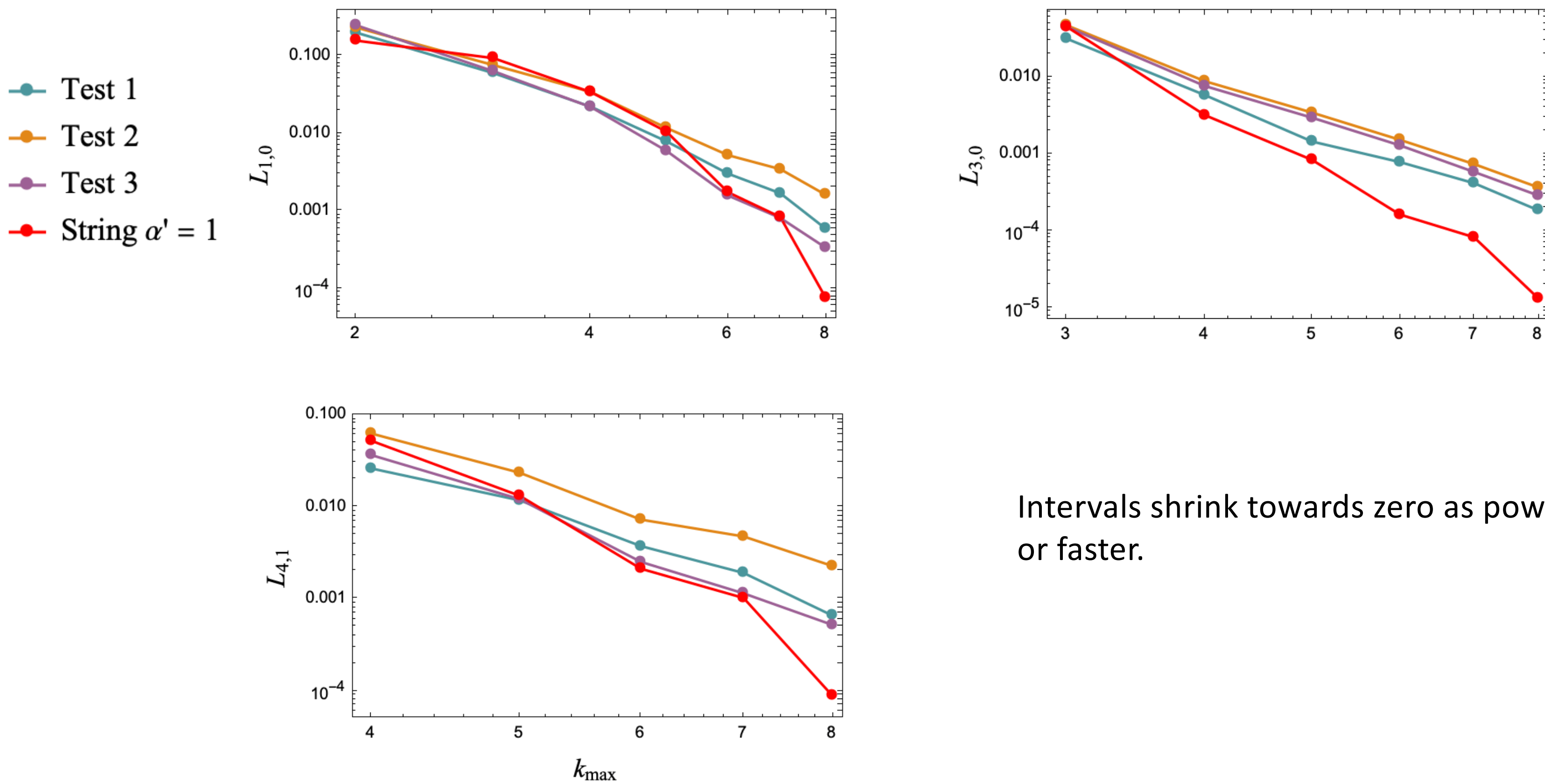
Similarly to the string: closing in on narrower and narrower allowed regions, shrinking toward the point of the constructed (known) values of the Wilson coefficients.

Test Theory: ranges shrinking with increasing $k_{\max}$



Three sample theories

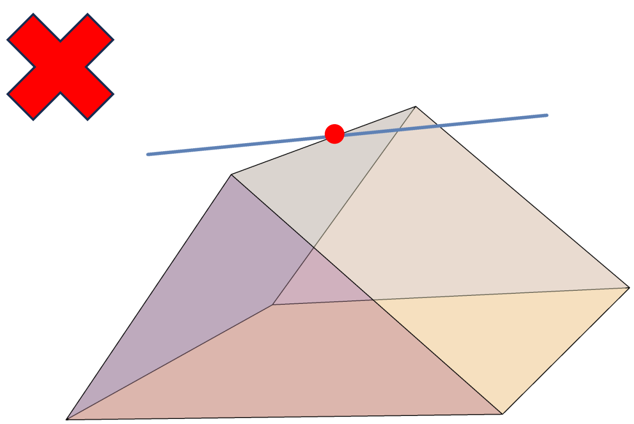
$$L_{k,q} = \max\left(\frac{a_{k,q}}{a_{0,0}}\right) - \min\left(\frac{a_{k,q}}{a_{0,0}}\right)$$

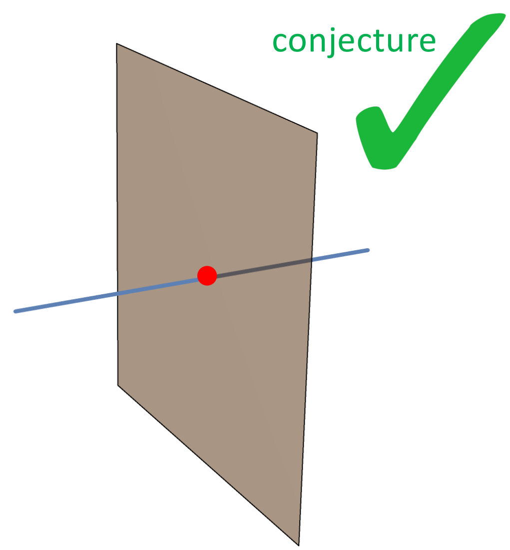


Intervals shrink towards zero as power laws or faster.

Flattening of the EFT-hedron

This indicates that as k_{max} grows, the allowed convex region *flattens* in certain directions





Flattening of the EFT-hedron

To do:

- Higher k_{max} with monovariables and other ways of testing the flattening conjecture
- Understand how the flattening happens. Which directions? How generic is this (also for other S-matrix bootstraps?)
- Flattening => **correlations between linear combinations of EFT coefficients**. Can this be exploited to extract information about the UV theory?

Flattening of the EFT-hedron

- Also, assuming as stringy a relation as the monodromy to isolate the string is not quite satisfactory. It would be interesting to understand what minimal conditions (purely field theoretical) can be used instead to isolate the string in the space of EFTs.

- Note: string monodromies do show up in low-energy physics as shown in for the bi-adjoint scalar with higerh-derivative interactions and in the context of generalizing the double copy.

[Alan (S-K) Chen, Aidan Herderschee, HE 2212.13998 + 2302.04895]

But that's another story for another time.

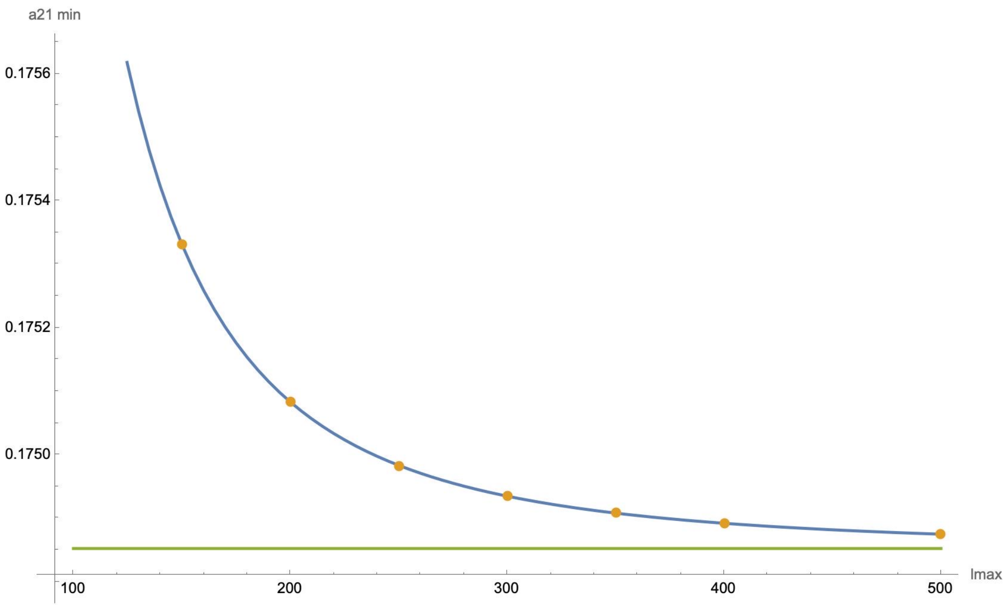
Thank you

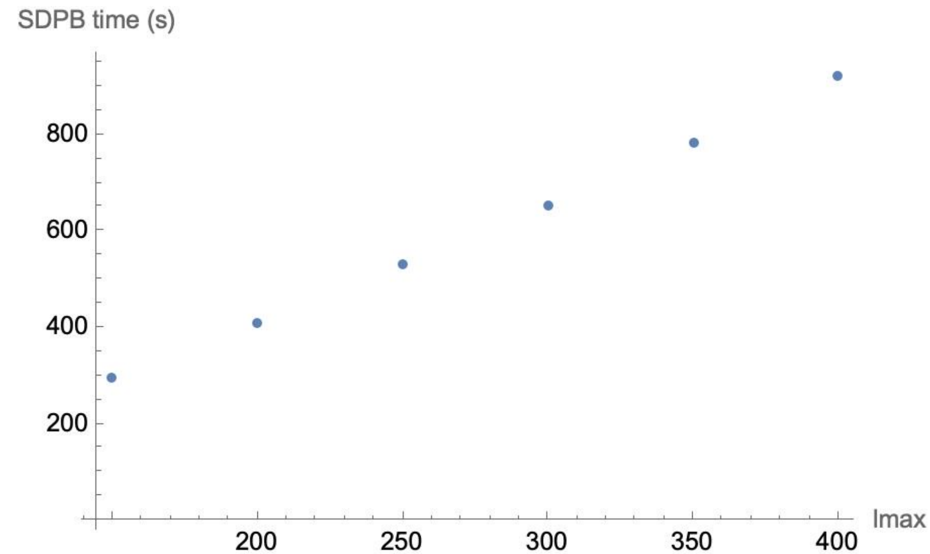
Justin Berman

Aidan Herderschee

Extras

Example of dependence of on l_{max} (k_{max} = 10 for $a_{2,0}$ = 0.7)





Orange: k_{max} = 10 lower bound on a21 min vs l_{max}
Blue: fit to $A/(l_{max})^c + B$
Green: asymptotic fit value B = 0.17485…
l_{max}=300 value: 0.17493..

Difference between l_{max} 300 and fitted asymptotic value is ~ 8 x10^{-5}
=> OK for plots
But for precision bounds, need higher l_{max}