

Flattening the EFT-hedron

Positivity Bounds with Maximal Supersymmetry

Based on work with

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Positivity bounds for Effective Field Theories

Constrain EFTs using 2-to-2 scattering processes and basic assumptions of locality, unitarity, analyticity, and large-energy behavior

- Þ **low-energy EFT Wilson coefficients given by a high-energy dispersive representation**
- Þ **Bounds on EFT Wilson coefficients**

e.g. toolkit overview Correia, Sever, Zhiboedov (2020)

Positivity bounds for Effective Field Theories

Examples

Explore abelian single-scalar theory [Caron-Huot, Van Duong]

EFT-hedron (geometric perspective) [Arkani-Hamed, T-C Huang, Y-t Huang] [Chiang, Huang, Li, Rodina, Weng]

[Albert, Rastelli; Fernandez Ma, Pomarol,Riva, Sciotti; Häring, Hebbar, Karateev, Meineri, Penedones] Pions & photons

Gravity [Caron-Huot, Li, Parra-Martinez, Simmons-Duffin; Bern, Herrmann, Kosmopoulos, Roiban; Huang, Remmen]

Relation to causality [Gonzales, De Rahm, Jaitly, Poszgay, Tokareva, Tolley, ...]

... and much more [Buric, F. Russo, Vichi; Bellazzini, Isabella, Ricossa, Riva; Bachu, Hillman; Tourkine; Mazac; Guerrieri; Vierira; Trott,….]

This talk: Combine positivity constraints with maximal supersymmetry in 4d.

Consider N=4 super Yang-Mills + all local 4-field higher-derivative operators compatible with N=4 supersymmetry:

$$
\mathcal{L} = \mathcal{L}_{\text{SYM}} + \frac{b_0}{\Lambda^4} \text{tr}(F^4) + \frac{b_1}{\Lambda^6} \text{tr}(D^2 F^4) + \dots
$$

This class of EFTs include the tree-level open superstring amplitude (Veneziano amplitude)

This talk: Combine positivity constraints with maximal supersymmetry in 4d.

Consider N=4 super Yang-Mills + all local 4-field higher-derivative operators compatible with N=4 supersymmetry: "N=4 SYM+h.d.", e.g.

$$
\mathcal{L} = \mathcal{L}_{\text{SYM}} + \frac{b_0}{\Lambda^4} \text{tr}(F^4) + \frac{b_1}{\Lambda^6} \text{tr}(D^2 F^4) + \dots
$$

This class of EFTs include the tree-level open superstring amplitude (Veneziano amplitude)

GOALS

- What are the most general bounds on the coefficients in max SUSY EFTs?
- Where is string theory?
- How does the "EFT-hedron" behave at increasing order in the derivative expansion?

Constraints from N=4 supersymmetry

Setup: 4-point amplitudes in N=4 SYM + hd ^h34i² *^f*(*s, u*) with 8(*Q*˜) = ¹ The ordering of the external states is understood to be 1234 unless otherwise specified. The on-shell superspace formalism with the Grassmann variables ⌘*iA* can be found in Chapter three pairs of complex scalars *zAB*. Here *A, B, C* = 1*,* 2*,* 3*,* 4 are R-indices *SU*(4)*^R* of which Setup: 4-point amplitudes in N=4 SYI

Constraints of maximal supersymmetry: start with the color-ordered 4-point on-shell superamplitude derivatives with the Color-Ordered 4-point on-shen superamplitude $\begin{array}{l} \textbf{Construct} \textbf{or} \textbf{$ constraints of maximums

$$
A_4 = \delta^8(\tilde{Q}) \frac{[12]^2}{\langle 34 \rangle^2} f(s, u) \quad \text{where N=4 SUSY requires} \quad f(s, u) = f(u, s)
$$

the *N* = 4 SYM amplitudes to be on the Coulomb branch, but asymptotically close to the origin [17–21].

Project out gluons: $A_4[+ + - -] = [12]^2 \langle 34 \rangle^2 f(s,u)$ Parke-Taylor gluon amplitude is Project out gluons: $A_4[+ + - -] = [12]^2 \langle 34 \rangle^2 f(s, u)$

find

 λ is shown that σ is the gluon λ^{YM} theory, the tree-level λ^{YM} theory, the tree-level λ^{34} Leading order: gluon amplitude is Parke-Taylor: $A_4^{\text{YM}}[+ + - -] = \frac{\langle 34 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} = -\frac{[12]^2 \langle 34 \rangle^2}{su}$ $\langle 12\rangle \langle 20\rangle \langle 94\rangle \langle 41\rangle$ su

²*f*(*s, u*) = *A*4[¯*zzzz*¯] *.* (2.4)

Cyclicity requires *A*4[2341] = *A*4[1234] so together with the supersymmetry requirement

 $\textsf{So pure YM}$ corresponds to $\ \ f(s,u) = -\frac{1}{su}.$ $\overline{\mathcal{L}}$ *z* $\overline{\mathcal{L}}$ indices whereas the negative helicity gluon corresponds to the singlet with all four R-indices, ϵ ²The amplitudes we study are technically not well defined because perturbative *^N* = 4 SYM amplitudes

so this corresponds to *f*(*s, u*) = 1*/*(*su*).

Setup: 4-point amplitudes in N=4 SYM + hd 2.2 ^h34i² *^f*(*s, u*) with 8(*Q*˜) = ¹

On-shell local higher-derivative terms with 4 fields are in 1-1 correspondence with Mandelstam polynomials in *s, t, u* subject to s+t+u = 0: On-shell local higher-derivative terms with 4 fields are in 1-1 correspondence with Mandelstam are compatible with *N* = 4 supersymmetry, and that is reflected in constraints on the $\text{supject to } \text{S+t+u} = 0.$

Answers / SUSY	
Answers / SUSY	
5u	
$f(s, u) = -\frac{1}{su} + \sum_{0 \le q \le k} a_{k,q} s^{k-q} u^q$, with $a_{k,k-q} = a_{k,q}$	
VW	Wilson coefficients

 $a_{0,0}$ is the coefficient of the N=4 SUSY operator tr(F⁴) $A_4[+ + - -] =$

$$
A_4[+ + - -] = [12]^2 \langle 34 \rangle^2 f(s, u)
$$

Consider a pair of conjugate scalars *^z* ⁼ *^z*¹² and *^z*¯ ⁼ *^z*³⁴ of the massless *^N* = 4 super-

¹ This is simply the statement that tr(*F*4)

 $a_{1,0}$ = $a_{1,1}$ is the coefficient of the N=4 SUSY operator tr(D²F⁴) *^A*[*zzz*¯*z*¯] = *^s*

 $a_{2,0}$ = $a_{2,2}$ and $a_{2,1}$ are the two coeffs of the two independent N=4 SUSY operators tr(D⁴F⁴).... etc a ^t is shown the gluon helicity state is shown in pure the gluon b is shown in dependent N=4 SUSY operators ${\rm tr}(D^4F^4)....$ etc Parke-Taylor gluon amplitude is

one.

so this corresponds to *f*(*s, u*) = 1*/*(*su*).

than four-derivatives contributes to this amplitude, i.e. there are no *N* = 4 compatible

Setup: 4-point amplitudes in N=4 SYM + hd $\frac{1}{2}$ $\frac{1}{2}$ Satup: <u>4 paint amplitudes in N-4 SVM the</u> **Setup: 4-point amplit** Local higher-derivative corrections are in 1-1 correspondence with terms of *d* that are poly*a* des in N=4 SYM + hd

than four-derivatives contributes to this amplitude, i.e. there are no *N* = 4 compatible

The S-matrix bootstrap is simpler for external scalar states, so consider a pair of complex

coniugate N=4 scalars, sav conjugate N=4 scalars, say **contained under the equations of motion** and integrations of motion and integrations o so this corresponds to *f*(*s, u*) = 1*/*(*su*). so this corresponds to *f*(*s, u*) = 1*/*(*su*). *nsider a pair of complex* nomials *s, t, u*, subject to momentum conservation *s* + *t* + *u* = 0. Hence, in the low-energy mplemor externative The S-matrix bootstrap is simpler for external scalar states, so consider a pair of comp

$$
z = z^{12} \qquad \bar{z} = z^{34}
$$

multiplet. Projecting three different 4-scalar amplitudes from the superamplitude (2.1), we multiplet. Projecting three different 4-scalar amplitudes from the superamplitude (2.1), we Project from the superamplitude to find
and the crossing relation (2.1), we have a *a*¹/₂, namely relation (2.5), namely relat where the *ak,q* are the Wilson coefficients of linear combinations of the set of on-shell local operators truck that are independent under the equations of motion and integrations of motion and integrations of \overline{C}

further bounds from 4-point supersymmetry alone.

$$
A_4[zz\overline{z}\overline{z}] = s^2 f(s, u)
$$

for us of our analysis;
we'll place bounds on
the $a_{k,q}$'s

$$
A(s, u) = A[zz\overline{z}\overline{z}] = -\frac{s}{u} + s^2 \sum_{0 \le q \le k} a_{k,q} s^{k-q} u^q
$$
 with $a_{k,k-q} = a_{k,q}$

 U sing (2.4), we write the low-energy expansion of the $4-$ scalar amplitude as \mathcal{U} amplitude as \mathcal{U}

+ *s*² X

ak,q sk^q u^q

focus of our analysis;

, (2.8)

^A[*zzz*¯*z*¯] = *^s*

$$
\mathbf{M}^{\mathbf{r}}\left(\mathbf{M}^{\mathbf{r}}\right) =\mathbf{M}^{\mathbf{r}}\left(\mathbf{M}^{\mathbf{r}}\right) =\mathbf{M}^{\mathbf{r}}\left(\mathbf{M}^{\mathbf{r}}\right)
$$

Veneziano in this framework

One interesting theory within this framework is the open string. Specifically, the 4-point open string tree amplitude. compatible with *N* = 4 supersymmetry upon restriction to 4d, and obeys the criteria in

In the scalar sector, the Veneziano amplitude takes the form Projecting to two pairs of massless external scalars, the Veneziano amplitude is [AH: Need In the scalar sector, the veneziano amplitude takes the form Section 2.3, so it must 'live' within the boundaries of our supersymmetric EFT-hedron. Projecting to two pairs of massless external scalars, the Veneziano amplitude is [AH: Need

$$
A^{\text{str}}[zz\bar{z}\bar{z}] = -(\alpha' s)^2 \frac{\Gamma(-\alpha' s)\Gamma(-\alpha' u)}{\Gamma(1 - \alpha' (s + u))}
$$

Expanding this in small ↵0 *s* and ↵0 *u* gives Upon expanding in small $\alpha's$ and $\alpha'u$ we find

$$
A^{\text{str}}[zz\bar{z}\bar{z}] = -\frac{s}{u} + s^2 \left(\zeta_2 \alpha'^2 + \zeta_3 \alpha'^3 (s+u) + \zeta_4 \alpha'^4 (s^2+u^2) + \frac{1}{4} \zeta_4 \alpha' s u + \dots \right)
$$

$$
\uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow
$$

$$
a_{0,0} \qquad a_{1,0} = a_{1,1} \qquad a_{2,0} = a_{2,2} \qquad a_{2,1}
$$

⁶The vertex representation of polytopes in projective space is reviewed in Appendix [].

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Dispersive representations

Assumptions $A(s, u) = A[zz\overline{z}\overline{z}]$ **ASSUITIPLIONS** $A(s, u) = A[zzzz]$ 2. The theory has a mass gap, *M*gap, such that there are no states with nonzero mass $\mathbf{B} = A$

- 1) Large $N \Rightarrow$ color-ordered single-trace amplitude has no t -pole or t -channel discontinuities. z color order \mathcal{A}_1 . The amplitude admits admits a partial wave decomposition \mathcal{A}_2
- **2) Weak-coupling** at all energy scales => ensures we can disregard loops of massless states. 3. Weak coupling at all anargy scales. The ansures we can disregard loops of massless states discontinuities. $2)$ oupling at all energy scales => ensures we can disregard loops of massless states.
	- 3) Mass gap: other than the massless N=4 supermultiplet, there are no other states with mass less than M_{gap}. ϵ mass less than ϵ $M_{\rm gap}$ expansion of the amplitude to be amplitude bept office than the massicss in a supermalifyice, there a
local than M ssless N=4 supe ermultiplet, there are no other states with
	- **4) Partial wave decomposition** 2. The theory has a mass gap, *M*gap, such that there are no states with nonzero mass $A(s,u)=16\pi\sum^{\infty}_{ }% \left[\frac{A(s,u)}{s}\right] ^{s}$ $_{\ell=0}$ $\left(2\ell + 1 \right) a_{\ell}(s) \, P_{\ell}\big(\cos(\theta) \big)$ Legendre *A*(*s, u*) = 16⇡ X 1 JSIL
. (2` + 1) *a*`(*s*) *P*` cos(✓) *,* (2.10) where cos(*Σ*)=1+2*u/s* and the Legendre polynomials P are labeled by the spin P are labeled by the spin P we decomposition $A(s, y) = 16\pi \sum_{\ell}(2\ell+1) a_{\ell}(s) P_{\ell}(cos(\theta))$ $\sum_{\ell=0}^{N}$ $16\pi \sum (2\ell+1) a_{\ell}(s) P_{\ell}(\cos(\theta))$ $\ell = 0$

6. The amplitude obeys a Froissart-Martin-like bound:

*^s*² = 0 *,*

fixed *t <* 0 : lim

s!1

 $$ Crucially, unitarity requires Im(*a*`(*s*)) *>* 0. $\mathbf{F}_{\mathbf{r}}$, $\mathbf{F}_{\mathbf{r}}$

- 5) Amplitude analytic away from the real s-axis.
- **6)** Froissart-Martin-like bounds $\frac{\text{fixed } u < 0 : \lim_{n \to \infty} \frac{1}{n}$ = 0 and \mathcal{L} implicate analysis and product and the contract \mathcal{L} $\overline{6}$ **a** - \mathbf{M} artin-like hounds \mathbf{f} ixed \mathbf{v} fixed $u < 0$: lim $s \rightarrow \infty$ *A*(*s, u*) $\frac{\partial}{\partial s^2}$ = 0 **and** fixed $t < 0$: $\lim_{s \to \infty}$ *A*(*s, s t*) fixed $t < 0$: lim $s \rightarrow \infty$ $A(s, -s-t)$ $\frac{1}{s^2}$ = 0

*^s*² = 0 *,*

*^s*² = 0 *.*

(2.11)
(2.11)
(2.11)

A rigorous derivation of P rigorous derivation of P for general theories is not currently known, but it does

hold at all orders in perturbation theory \mathbb{S}^2 , \mathbb{S}^2 , \mathbb{S}^2 can be shown to hold with \mathbb{S}^2

fixed *u <* 0 : lim

A rigorous derivation of Property 5 for general theories is not currently known, but it does not currently known, but it d

index structure, for example *z*¹²*z*²³*z*³⁴*z*⁴¹; i.e. they involve two different pairs of conjugate scalars, not just

limit.3 Using the partial wave decomposition, the discontinuity of the amplitude can be amplitude can be ampli

 \mathbf{z}

2*u* ◆

Deform the contour: pick up pole-terms and branch cuts
on the real s-axis and
on the real s-axis and interactions of the form tr(*z*2*z*¯2) and tr(*D*2*z*2*z*¯2). with the contour deformation deformation deformation described in \mathcal{A} on the real s-axis and \mathbf{S} and \mathbf{S} , the formula formula formula formula formula formula formula formula for the Wilson coefficient \mathbf{S}

All *k* and *q*

✓

limit.3 Using the partial wave decomposition, the discontinuity of the amplitude can be amplitude can be ampli

2*u* →
→
→

 \mathbf{z}

. (2.13)

2*u* ◆

$$
a_{k,q} = \frac{1}{q!} \frac{\partial^q}{\partial u^q} \left(\frac{1}{\pi} \int \frac{ds'}{s'^{k-q+3}} \text{Im}A(s',u) \right) \Bigg|_{u=0}.
$$

limit.³ Using the partial wave decomposition, the discontinuity of the amplitude can be

index structure, for example *z*¹²*z*²³*z*³⁴*z*⁴¹; i.e. they involve two different pairs of conjugate scalars, not just

 $\overline{}$

 $\begin{array}{ccc} m_a^* & \nearrow & \end{array}$ Then use the nartial wave expansion and $M^2 = s'$ to get t_{max} that the discontinuity of the amplitude is $\frac{2}{\sqrt{3}}$ is $\frac{2}{\sqrt{3}}$ Then use the partial wave expansion and $M^2 = s'$ to get

Dispersive representation limit.³ Using the partial wave decomposition, the discontinuity of the amplitude can be polynomials in (2.14), so, after a change of integration variable, (2.13) becomes of integration variable, (2.

z
Z 1910
Z 1910

$$
a_{k,q}=\sum_{\ell=0}\int_{M^2_{\rm gap}}^\infty dM^2\,\rho_\ell(M^2)\left(\frac{1}{M^2}\right)^{k+3}v_{\ell,q}\qquad\qquad\text{where}\qquad \rho_\ell(M^2)=
$$

 $v_{\ell,q}$ where $\rho_{\ell}(M^2) = 16(2\ell+1)\,\mathrm{Im}\big(a_{\ell}(M^2)\big) > 0$ \mathcal{M} , \mathcal{M} *a*`(*M*2) w here ρ_{ℓ} $= 16(2\ell + 1)$ $\text{Im}(a_{\ell}(M^2))$ >

A similar version of the crossing symmetric EFT-hedron (also called *s*-channel EFT-hedron)

was examined in Refs. μ and μ but without the supersymmetric context. μ

dx p`(*x*) *x^k v*`*,q, p*`(*x*) *>* 0 *.* (2.20)

and we have expanded the Legendre polynomials as **the high energy spectrum. The manufactures is an**d the high energy spectrum. Unitarity requires non-*.* (2.17) terms of the high energy spectrum. Unitarity requires $\frac{1}{\sqrt{M}}$ and the places nontrivial restrictions on the *ak,q*. written as de and we have expanded the Legendre polynomials as To make the Wilson coefficients dimensionless, we multiply (2.16) by (*M*²

$$
P_{\ell}(1+2\delta) = \sum_{q=0}^{\ell} v_{\ell,q} \delta^q \quad \text{with} \quad v_{\ell,q} = \frac{\prod_{a=1}^q \left[\ell(\ell+1) - a(a-1) \right]}{(q!)^2} \ge 0
$$

Finally, rescale the $a_{k,q}$'s by $(M_{gap})^{2(k+2)}$ (so they are all dimensionless) *^x* ⁼ *^M*² Finally, rescale the $a_{k,q}$'s by $(M_{\text{gap}})^{2(k+2)}$ (so they are all dimensionless) e the a_{k,q}'s by (M_{gap})^{2(k+2)} (so they are all dimensionless)
and 2 T make $\frac{1}{2}$ by (M ²⁾ $\frac{1}{2(k+2)}$ (2.2¹hours and all dimensionless) μ and $a_{k,q}$ and μ (*wigap)*

–5–

(*M*² \cdot and M ² \cdot a The dispersive representation (2.16) of the Wilson coefficients can then be written as written as well as the
The Wilson coefficients can then be written as well as $M_{\sigma \rm an}^2$ and change the integration variable M^2 to $x=\frac{3}{M^2}$: then and change the integration variable M^2 to $x = \frac{M_{\text{gap}}^2}{M^2}$: then $\frac{M_{\rm gap}}{M^2}$: then $M =$ $M = a_{k,q}$ $M = w_{\text{gap}}$ $M = 1$ and change the integration variable *W* it to $x = M^2$ and then $\frac{dM_{\rm gap}}{M^2}$: then

$$
\left(a_{k,q} = \sum_{\ell=0}^{\infty} \int_0^1 dx \, p_{\ell}(x) \, x^k \, v_{\ell,q}, \qquad \text{with} \qquad p_{\ell}(x) = x \, \rho_{\ell}(M_{\text{gap}}^2/x) > 0\right)
$$

^ak,q ⁼ ^X

was examined in Refs. μ and μ but without the supersymmetric context. μ

The dispersive representation (2.16) of the Wilson coefficients can then be written as written as α A similar version of the crossing symmetric EFT-hedron (also called *s*-channel EFT-hedron) was examined in Refs. \mathbb{R}^n but with the supersymmetric context. \mathbb{R}^n but without the supersymmetric context.

Result

Immediate consequences: **Immediate consequences:** $Immediate \; conconuence,$

- 1) $a_{k,q} \geq 0$ further, $a_{k} > 0$
- Further, $a_{k',q} \leq a_{k,q}, \quad k \leq k' \quad \text{(b/c 0 < x < 1)}$ $\frac{1}{p^2}$, $\frac{1}{q^2}$ $\frac{1}{q^3}$ $\frac{1}{q^4}$ 2) $a_{k',q} \le a_{k,q}, \quad k \le k'$ (b/c 0 < x < 1)

*a*0*,*⁰ *a*1*,*⁰ *a*2*,*⁰ *a*3*,*⁰ *...*

0*qk*

*a*0*,*⁰ *a*1*,*⁰ *a*2*,*⁰ *a*3*,*⁰ *...*

Then what?

From the moment maps, one can compute analytic bounds (Hankel, cyclic polytope, product Hankel)

[Arkani-Hamed, T-C Huang, Y-t Huang 2012.15849]

[Chiang, Huang, Li, Rodina, Weng 2105.02862]

However, at higher *k* (higher derivative terms) this quickly becomes unwieldy.

Instead: formulate as an optimization problem and using numerical methods, such as semi-definite programming (SDPB package by Simmons-Duffin) [Albert, Rastelli 2203.11950 + 2307.01246] [Caron-Huot, Van Duong 2011.02957] written for the conformal bootstrap

nomials *nominals analysis* structure to momentum conservation \mathbf{v} + $\mathbf{$

We use two different complementary numerical methods su

0*qk* where the a^{*k*}, q^2 are the set of on-shell local SDPB semi-definite programming

CPLEX linear optimization by IBM and integration and integrati

Following standard set up, we reformulate the setup as an optimization problem with two sets of null constraints (aka sum rules)

- SUSY Crossing $a_{k,k-q} = a_{k,q}$
- ST channel sum rule

f A(s,u) = s² J(s,u)
ssing relations lik + *s*² X *ak,q sk^q u^q ,* (2.8) *crossing relations like those in the analysis of the pion amplitudes of [Albert, Rastelli 2203.11950] Our A(s,u) = s*² *f(s,u) is not crossing symmetric, but f(s,u) is and that is sufficient to derive constant-t*

Numerical analysis practicalities **ability of crossing relations** are compatible with *N* = 4 supersymmetry, and that is reflected in constraints on the To make the Wilson coefficients dimensionless, we multiply (2.16) by (*M*² define the *ak,q* as⁴

The higher *k*max, the stronger bounds. \mathcal{L} the low-energy expansion of the \mathcal{L} the 4-scalar amplitude as \mathcal{L} and the 4-scalar amplitude as \mathcal{L} and \mathcal{L} are the 4-scalar amplitude as \mathcal{L} and \mathcal{L} and \mathcal{L} are the 4-scalar am and define *<i>x* $\frac{1}{2}$

e.g. k_{max} = 4 means null constraints are imposed up $A[zz\bar{z}\bar{z}] = -\frac{s}{z} + \frac{z\bar{z}}{z}$ to 12-derivative order.

^A[*zzz*¯*z*¯] = *^s u* + *s*² X 0*qk ak,q sk^q u^q ,* (2.8) *M*² gap*/x >* 0 *.* (2.19) The dispersive representation (2.16) of the Wilson coefficients can then be written as

•
$$
\text{I}_{\text{max}} \quad \text{truncated the sum over all spin} \qquad a_{k,q} = \sum_{\ell=0} \int_0^1 dx \, p_\ell(x) \, x^k \, v_{\ell,q},
$$

Chosen to ensure that bounds are converging (typically 200 to 1000) A similar version of the crossing symmetric EFT-hedron (also called *s*-channel EFT-hedron)

this paper.

is the lowest-dimensional λ supersymmetric higher-derivative operator λ supersymmetric operator λ vector sector. was examined in Refs. [4, 8] but with the supersymmetric context. \mathcal{A} but without the supersymmetric context. It is immediately clear from (2.20) that all Wilson coefficients have to be non-negative, • *x***max Implementation in CPLEX requires discretization**

discretize the integral over *x* (i.e. the mass-spectrum). We use x_{max} to denote the number of discretization points of the interval 0 to 1 (typically between 200 and 1000).

Bounds

Justin Berman, Aidan Herderschee, HE (in progress)

Example of SDPB Results: (a_{20},a_{21}) plane (two tr(D^4F^4) N=4 SUSY ops)

Orange: $k_{\text{max}} = 4$, $l_{\text{max}} = 200$ allowed region Rosa: $k_{\text{max}} = 10$, $l_{\text{max}} = 300$ allowed region

Red dot: Veneziano w/ choice $\alpha'M_{\rm gap}^2=1$

- \cdot (0,0) includes theory with tr(F^4) as the only h.d. interaction.
- (1,1) includes theory with all $a_{k,q}=a_{0,0}$. Can be re-summed to

$$
A(s, u) = -\frac{s}{u} + s^2 \frac{1}{(s-1)(u-1)}
$$

Tends to show up in bootstraps; Probably not a sensible theory.

Comparison: SDPB and CPLEX $k_{max}=10$ and $l_{max}=300$

• CPLEX faster by \sim factor of 5 for these runs, but high precision requires higher x_{max}

What happens near the string?

Orange: *k*max = 4 *l*max= 200 allowed region Rosa: $k_{\text{max}} = 10$ $l_{\text{max}} = 300$ allowed region Blue: $k_{\text{max}} = 15$ $l_{\text{max}} = 800$ allowed region Green dot: k_{max} = 20 l_{max} = 600 at $a_{2,0}$ string value

Red dot: Veneziano

What happens as k_{max} increases?

Other projections

 $(a_{3,0},a_{3,1})$ tr(D⁶F

4) $(a_{4,1}, a_{4,2})$ tr(D⁸F⁴)

Monodromies

Inspired by

Yu-tin Huang, Jin-Yu Liu, Laurentiu Rodina, Yihong Wang [2008.02293]

Monodromy relations extending the powers in the integrands.

String disk amplitudes a universal pre-factor. Specifically at 4-point, we have $\frac{4}{1}$

Stirling disk simplitudes

\n
$$
A[1234] = \frac{-s^{2}}{t} \times \int_{0}^{1} dz z^{-\alpha' s - 1} (1 - z)^{-\alpha' u - 1},
$$
\n
$$
A[1324] = \frac{s^{2}}{t} \times \int_{1}^{\infty} dz z^{-\alpha' s - 1} (z - 1)^{-\alpha' u - 1},
$$
\n
$$
A[2134] = \frac{s^{2}}{t} \times \int_{-\infty}^{0} dz (-z)^{-\alpha' s - 1} (1 - z)^{-\alpha' u - 1}.
$$
\n3

\n2

Contour deformation relates the different color-orderings Stieberger (2009) while picking up monodromies at x=0 and x=1. Bjerrum-Bohr, D. whilo picking un monodromios at $x=0$ and $x=1$ subsets and $x=1$ $\frac{1}{\sqrt{2}}$ put in the string monodromy relations $\frac{1}{\sqrt{2}}$

Here and below, we assume states 1 and 2 to be *z* and 3 and 4 to be *z*¯, with the complex

When the $\overline{}$ and $\overline{}$ and $\overline{}$ for the 4-point amplitude is plugged into the string into

The Wilson coefficients unfixed by monodromy for *k* 5 are *a*1*,*0, *a*3*,*0, *a*4*,*1, *a*5*,*0. For the

Stieberger (2009) Bjerrum-Bohr, Damgaard, Vanhove (2009)

 \star

scalars introduced in Section 2.1. Because the integrands are the same, the three amplitudes

When the *N* = 4 SUSY ansatz (2.8) for the 4-point amplitude is plugged into the string

monodromy relation (5.2) and solved order by order by order by order by order in the low-energy expansion, particularly expansion, μ

4

String monodromy relations
$$
0 = A[2134] + e^{i\pi\alpha's}A[1234] + e^{-i\pi\alpha't}A[1324]
$$

4

SUSY Ansatz w/ Monodromy Imposed **SUSY** Ansatz w/

Veneziano amplitude their values are

Table 1: The string monodromy relation (5.2) fixes particular linear combination of the Note: monodromy ``knows" π only, so cannot access any information about coefficients with $\zeta(\text{odd})$ in the low-energy expansion of the open string amplitude.

> The authors of Γ found numerical evidence that the coefficients unit Γ found by monodromy monodromy monodromy were closed to the values (5.3) when the monodromy constraints were combined with a monodromy constraints were

M onodromy + EFT-hedron *^a*4*,*² ²*a*4*,*¹ ⁼ ¹ *a*4*,*⁰ = ⇣⁶ = ⇡⁶

Yu-tin Huang, Jin-Yu Liu, Laurentiu Rodina, Yihong Wang [2008.02293] Rodina, Yihong Wang [2008.02293] *^a*5*,*² ⁵*a*5*,*⁰ + 2⇣² *^a*3*,*⁰ ⁺ ⁵ $16 - 3$

> Monodromy + EFT-hedron "carves out the open string" Table 1: The string monodromy relation (5.2) fixes particular linear combination of the

Using all **Hankel + Cyclic Polytope + Product Hankel** constraints to k_{max} = 4 and one from k_{max} = 5, they found **Product Hankel** constraints to $\kappa_{\text{max}} = 4$ and on

5.2 Numerical Analysis

indicates that the intersection of the monodromy subspace and EFT-nedron Indicates that the intersection of the monodromy subspace and EFT-hedron may be a point

point in the *k* = 6 allowed region of Figure 9 or Figure 10a is compatible with *k* = 7*,* 8 Hankel

- ST sum rules cannot be expressed in terms of the $a_{k,q}$ so cannot be captured by the (product) Hankel / cyclic polytope constraints or their enhancements [Chiang, Huang, Li, Rodina, Weng 2105.02862]
- Implementation in SDPB and CPLEX allows going to higher orders

Goals:

- **1) Further test if positivity bounds and monodromy isolate the string**
- **2) Understand in what sense string theory is then on the boundary of the SUSY EFT-hedron**

The two lowest Wilson coefficients unfixed by string monodromy are $a_{1,0}$ and $a_{3,0}$

Increase k_{max}

Zoom in on green region

Increase k_{max}

Increase k_{max}

Monodromy + SUSY Positivity Justin Berman, Aidan Herderschee, HE (in progress) Overall bounds for $k_{\text{max}} = 8$ with $l_{\text{max}} = 800$ $a_{1,0}^{\text{str}} = \zeta_3 = 1.202057$ $a^{\text{str}} = \zeta_5 = 1.036928$ % in Huang, Liu, Rodina, Wang [2008.02293] $50 \t - 100002$ $-$ 8 WILIT I_{max} – 800 $a_{1,0}$ – ζ_3 – 1.202057 $1.03692 < a_{3,0} < 1.03694$ within 0.0014% of $a_{3,0}^{\text{str}} = \zeta_5 = 1.036928$, Liu, Rodina, War ¹²⁶⁰ = 0*.*⁰⁴⁰⁵³⁷ *,* vs. ~1.5% and 0.2% in Huang, Liu, Rodina, Wang [2008.02293] $1.20198 < a_{1,0} < 1.20206$ within 0.0066\% of

5.2 Numerical Analysis

Monodromy + SUSY Positivity Overall bounds for $k_{\text{max}} = 8$ with $l_{\text{max}} = 800$ $a_{1,0}^{\text{str}} = \zeta_3 = 1.202057$ $a^{\text{str}} = \zeta_5 = 1.036928$ % in Huang, Liu, Rodina, Wang [2008.02293] $50 \t - 100002$ $a_{4,1}^{\text{str}} = \frac{(\pi^6 - 630\zeta_3^2)}{8.0083}$ $-$ 8 WILIT I_{max} – 800 $a_{1,0}$ – ζ_3 – 1.202057 $1.03692 < a_{3,0} < 1.03694$ within 0.0014% of $a_{3,0}^{\text{str}} = \zeta_5 = 1.036928$, Liu, Rodina, War ²0dina, Wang [2008.02293]
- 6.30/2 *a*str ⁵*,*⁰ = ⇣⁷ = 1*.*00835 *.* vs. ~1.5% and 0.2% in Huang, Liu, Rodina, Wang [2008.02293] $1.20198 < a_{1,0} < 1.20206$ within 0.0066% of $a_{1,0}^{\text{str}} = \zeta_3$ $0.0405345 < a_{4,1} < 0.0406249 \,\,\,\hbox{within} \,\,\,\, 0.22\% \,\,\,\hbox{of} \,\,\,\,\,\, a^{\rm str}_{4,1} =$ Justin Berman, Aidan Herderschee, HE (in progress) linear combinations of Wilson coefficients are fixed as shown in Table 1. The Wilson coefficients unfixed by monodromy for *k* 5 are *a*1*,*0, *a*3*,*0, *a*4*,*1, *a*5*,*0. For the $(\pi^6 - 630\zeta_3^2)$ $\overline{)}$ $\frac{1260}{1260} = 0.040537$

> vs. ~52% in Huang, Liu, Rodina, Wang [2008.02293] $B_{\rm eff}$ they they they they they they coefficients depend on \sim

5.2 Numerical Analysis

Where is string theory?

So, the tree-level open string (i.e. the Veneziano amplitude) does indeed appear to be on the intersection of the convex space carved out by the positivity constraints and the monodromy constraints. But **how?**

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three dimensions with codimensions \mathcal{C} or $\mathcal{C$

Testing the geometry

Monovariables

Recall string monodromies

Table 1: The string monodromy relation (5.2) fixes particular linear combination (5.2) fixes particular linear combination of the string \sim

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Monovariables that are inside that are inside the monovariables that are inside the EFThedron. To generate such monovariables ables, we considered linear combinations of the corner theories in Section ?? and Veneziano

Then construct theories that obey the positivity bounds amplitudes with varying mass gap. To be specific, we considered an ansatz of the form α

$$
A(s, u) = \frac{-s}{u} + s^2 \left(\int_{M_{\text{gap}}^2}^{\infty} dm^2 \rho_{m^2}^{(1)} \left[\frac{1}{(m^2 - s)(m^2 - u)} \right]^{m^2 + s^2} \left[\frac{1}{(m^2 - s)(m^2 - u)} \right]
$$

+
$$
\int_{M_{\text{gap}}^2}^{\infty} dm^2 \frac{\rho_{m^2}^{(2)}}{m^4} \left[\frac{m^4}{su} - \frac{\Gamma(-s/m^2)\Gamma(-u/m^2)}{\Gamma(1 + t/m^2)} \right] \right)
$$

Subtract off massless poles Veneziano

e.g. picking
$$
\rho_{m^2}^{(1)} = \rho_{m^2}^{(2)} = \frac{1}{2}\delta(m^2 - 1)
$$
 gives $a_{k,q} = \frac{1}{2}(a_{k,q}^{\text{str}} + a_{0,0}^{(1,1)})$

Generally, we can pick the densities to be sums over delta-functions at various masses. These will be the **"test theories"**.

Test theories

For a given test-theory, we compute the corresponding $a_{k,q}$ and thus the monovariables r_i.

Monovariable constraints are null constraints for SDPB/CPLEX.

We test if SDPB/CPLEX narrows in on the Wilson coefficients unfixed by the monovariable constraints (just as it did for Veneziano).

Because we know all $a_{k,q}$ by construction, we know if SDPB/CPLEX gets it right.

Example ->

Similarly to the string: closing in on narrower and narrower allowed regions, shrinking toward the point of the constructed (known) values of the Wilson coefficients.

Test Theory: ranges shrinking with increasing k_{max}

Flattening of the EFT-hedron

 $1.0 \square$

 0.8

This indicates that as k_{max} grows, the allowed convex region *flattens* in certain directions

 1.0

 $0.8\,$

three dimensions with codimensions \mathcal{C} or $\mathcal{C$

Flattening of the EFT-hedron

To do:

- Higher k_{max} with monovariables and other ways of testing the flattening conjecture
- Understand how the flattening happens. Which directions? How generic is this (also for other S-matrix bootstraps?)
- Flattening => **correlations between linear combinations of EFT coefficients**. Can this be exploited to extract information about the UV theory?

Flattening of the EFT-hedron

- Also, assuming as stringy a relation as the monodromy to isolate the string is not quite satisfactory. It would be interesting to understand what minimal conditions (purely field theoretical) can be used instead to isolate the string in the space of EFTs.
- Note: string monodromies do show up in low-energy physics as shown in for the bi-adjoint scalar with higerh-derivative interactions and in the context of generalizing the double copy.

[Alan (S-K) Chen, Aidan Herderschee, HE 2212.13998 + 2302.04895]

But that's another story for another time.

Thank you

Justin Berman **Aidan Herderschee**

Extras

Example of dependence of on I_{max} ($k_{\text{max}} = 10$ for $a_{2,0} = 0.7$)

Orange: $k_{\text{max}} = 10$ lower bound on a21 min vs l_{max}
Blue: fit to $A/(l_{\text{max}})^c + B$ fit to $A/(I_{max})^c$ + B Green: asymptotic fit value B = 0.17485… *l*max=300 value: 0.17493..

Difference between I_{max} 300 and fitted asymptotic value is $\approx 8 \times 10^{-5}$ => OK for plots But for precision bounds, need higher I_{max} Imax