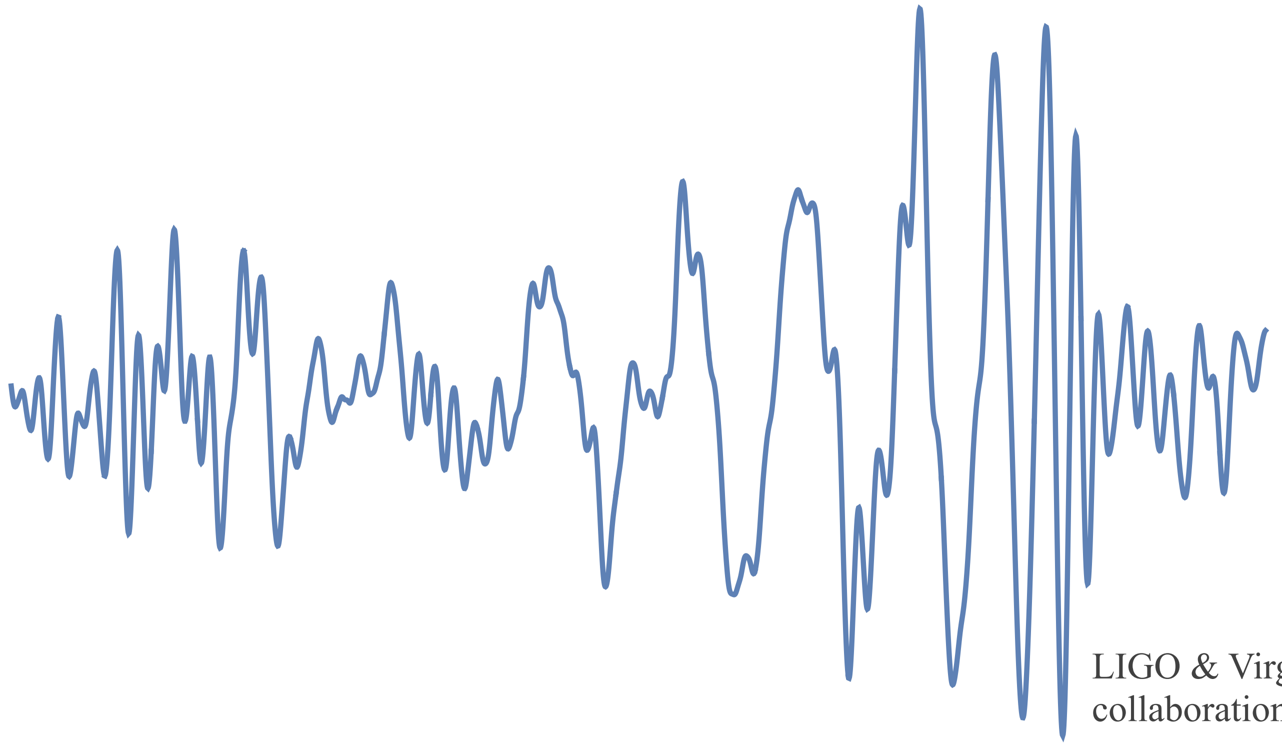
Amplitudes 2023, CERN

Gravitational Radiation from Amplitudes

Dónal O'Connell
Higgs Centre, Edinburgh

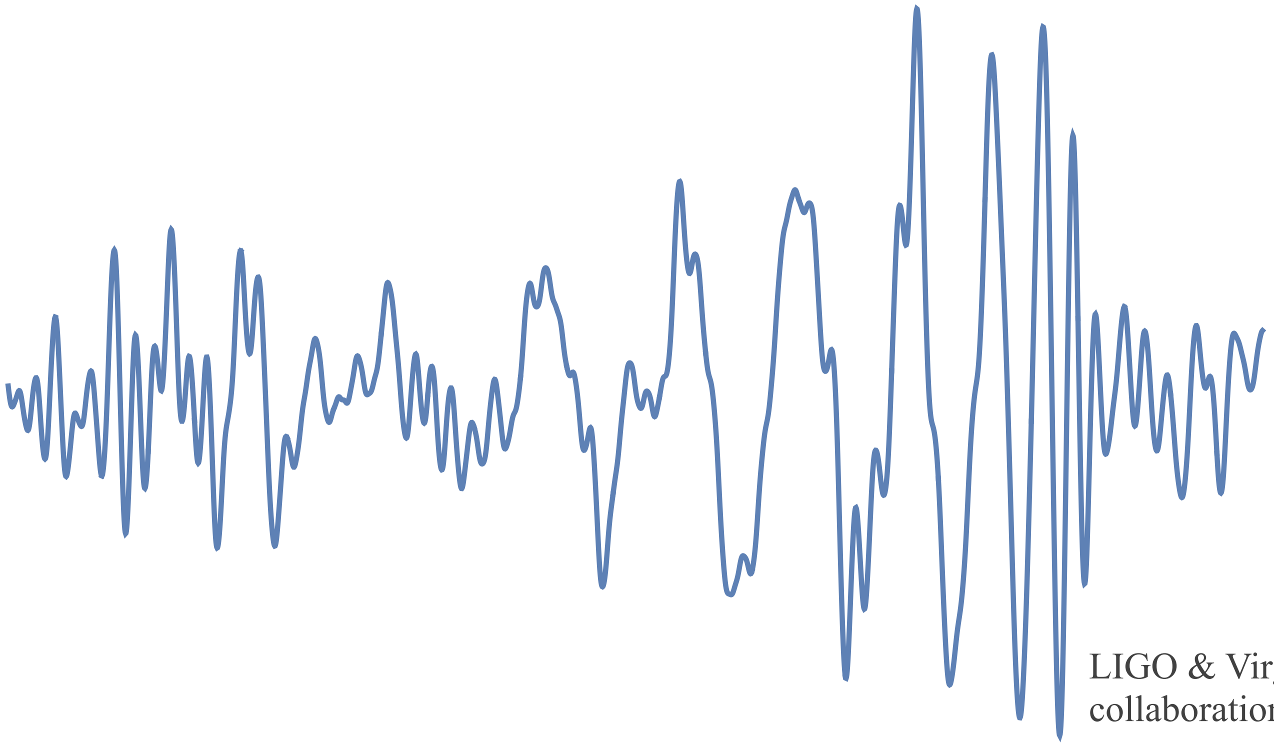
Motivation

Motivation

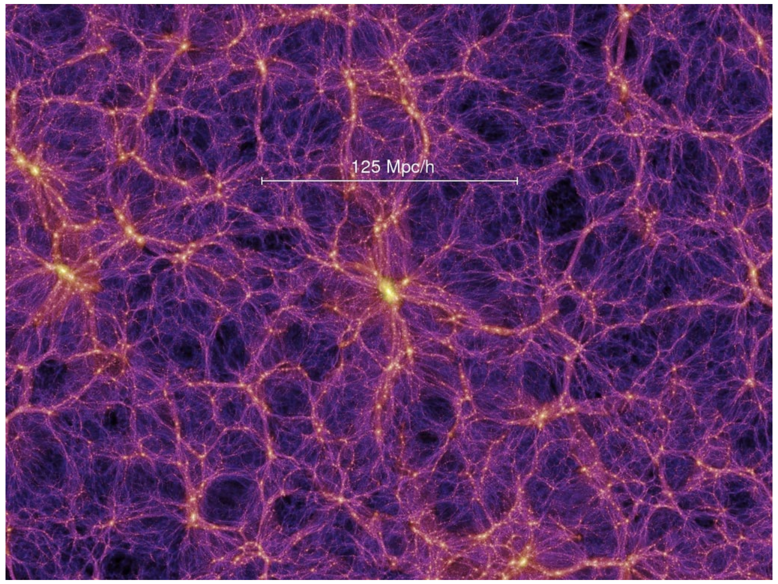


LIGO & Virgo
collaborations

Motivation

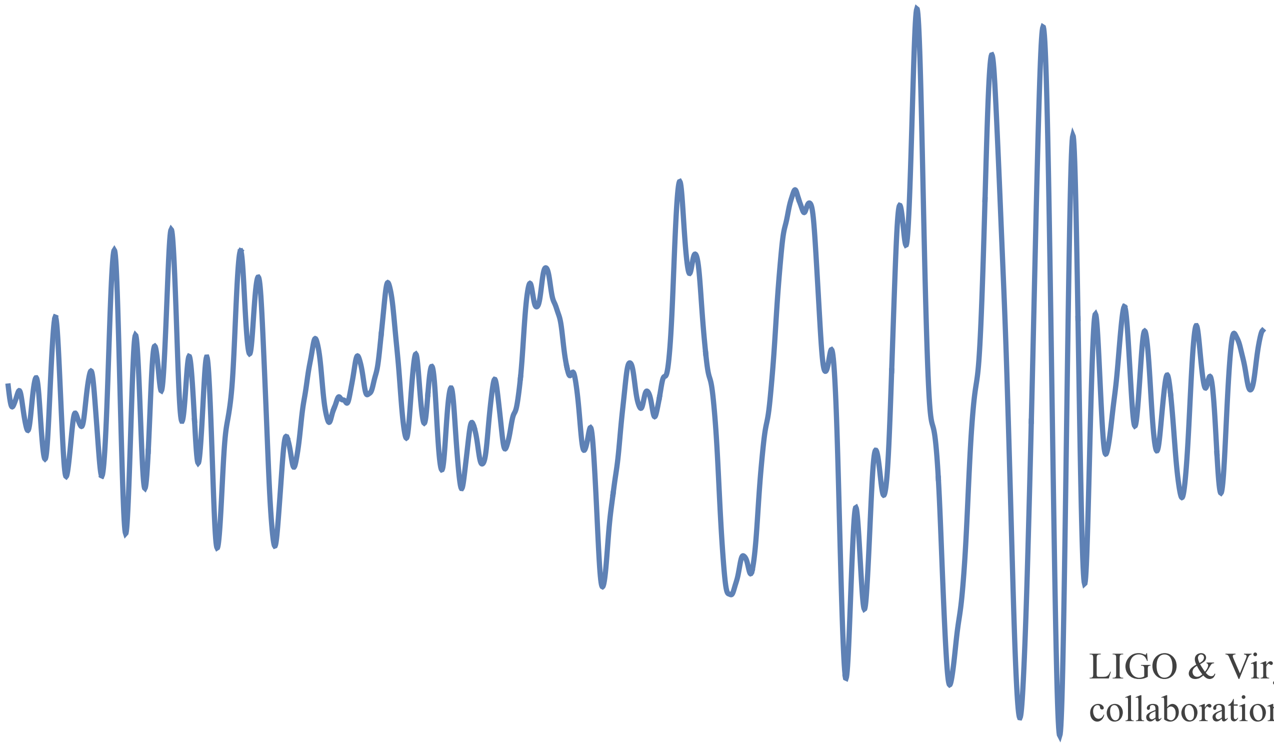


LIGO & Virgo
collaborations

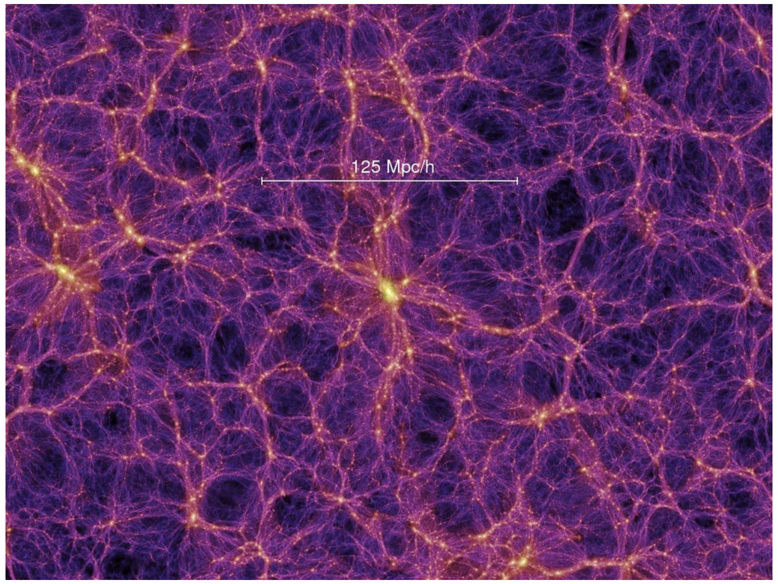


Millenium Simulation Project

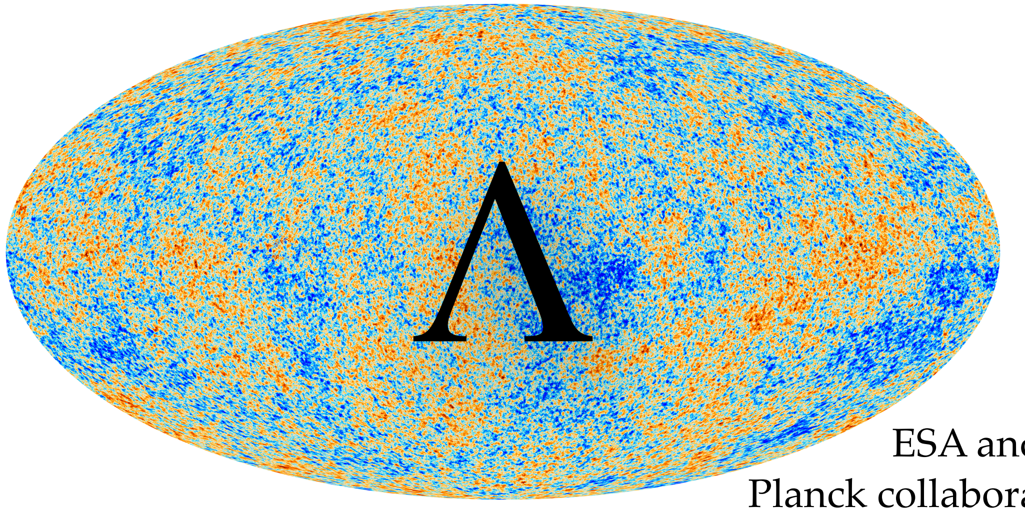
Motivation



LIGO & Virgo
collaborations



Millenium Simulation Project



ESA and the
Planck collaboration

Motivation

1. What can we compute with amplitudes?
2. What can we learn *about* amplitudes?

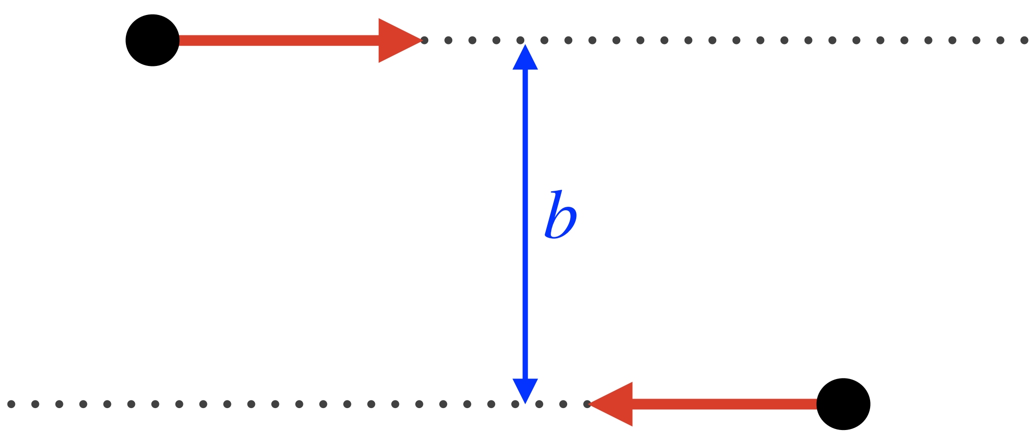
This talk

1. Gravitational waveform due to scattering event

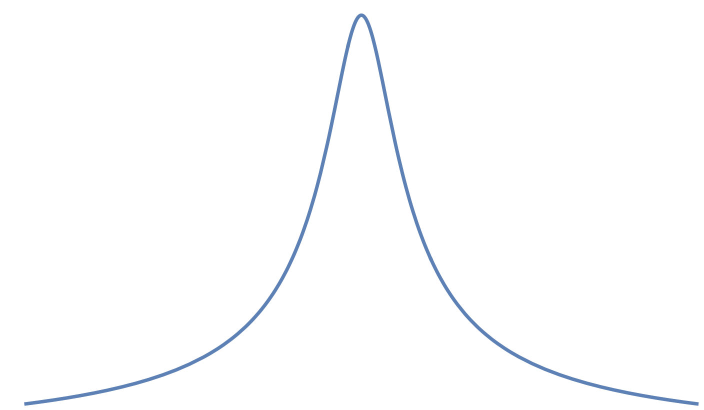
2. Variances from amplitudes

Scattering waveforms

Waves from Amplitudes



$$\text{waveform}(t) =$$



Kosower, Maybee & DOC

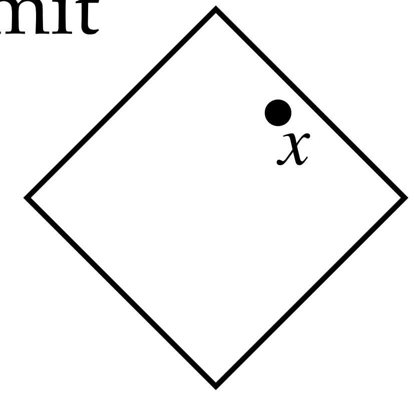
Cristofoli, Gonzo, Kosower & DOC

Asymptotically Minkowski

Classical point particle approximation. Absorption: *Aoude & Ochirov*

Waves from Amplitudes

Measure expectation of curvature component in classical limit

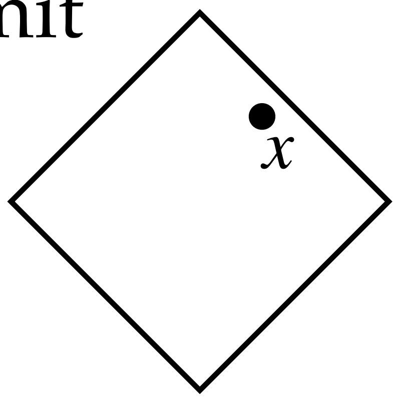


TT gauge: $\ddot{h}_{ij} \sim R_{0i0j}$

$$\text{waveform} \equiv \langle \psi | S^\dagger \mathbb{R}_{....}(x) S | \psi \rangle$$

Waves from Amplitudes

Measure expectation of curvature component in classical limit



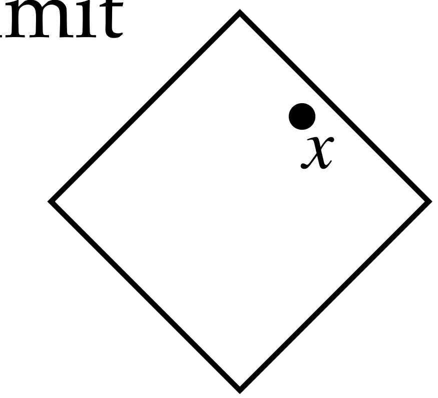
TT gauge: $\ddot{h}_{ij} \sim R_{0i0j}$

Curvature operator
(NP scalar Ψ_4)

$$\text{waveform} \equiv \langle \psi | S^\dagger \mathbb{R}_{....}(x) S | \psi \rangle$$

Waves from Amplitudes

Measure expectation of curvature component in classical limit



TT gauge: $\ddot{h}_{ij} \sim R_{0i0j}$

Curvature operator
(NP scalar Ψ_4)

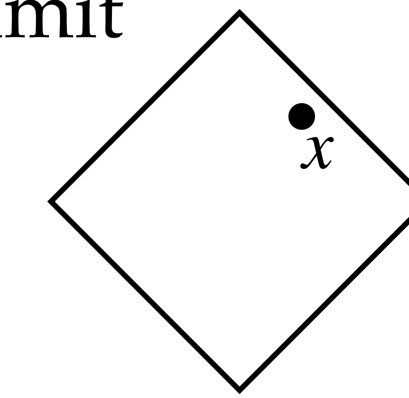
$$\text{waveform} \equiv \langle \psi | S^\dagger \mathbb{R}\ldots(x) S | \psi \rangle$$

$$|\psi\rangle \sim \int \bigwedge \bigwedge e^{ip_1 \cdot b} |p_1\, p_2\rangle$$

Classical Cauchy data

Waves from Amplitudes

Measure expectation of curvature component in classical limit



TT gauge: $\ddot{h}_{ij} \sim R_{0i0j}$

Curvature operator
(NP scalar Ψ_4)

$$\text{waveform} \equiv \langle \psi | S^\dagger \mathbb{R}\ldots(x) S | \psi \rangle$$

Final state
Amplitudes!

$$|\psi\rangle \sim \int \bigwedge \bigwedge e^{ip_1 \cdot b} |p_1 \, p_2\rangle$$

Classical Cauchy data

Waves from Amplitudes

Measure expectation of curvature component in classical limit

$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

$$\text{waveform} = \int \widetilde{\mathrm{d}k}\, \left[k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \langle\psi|S^\dagger a(k) S|\psi\rangle + \text{c.c.}\right]$$

$$= i \int \widetilde{\mathrm{d}k}\, k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\langle\psi|a(k) T|\psi\rangle - i\langle\psi|T^\dagger a(k) T|\psi\rangle\Big] + \text{c.c.}$$

Waves from Amplitudes

Measure expectation of curvature component in classical limit

$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

iT

$$\text{waveform} = \int \widetilde{\mathrm{d}k} \left[k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \langle\psi|S^\dagger a(k) S|\psi\rangle + \text{c.c.}\right]$$

$$= i \int \widetilde{\mathrm{d}k}\, k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k) T|\psi\rangle\Big] + \text{c.c.}$$

Waves from Amplitudes

Measure expectation of curvature component in classical limit

$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

iT

$$\text{waveform} = \int \widetilde{\mathrm{d}k}\,\left[k k\,\varepsilon\varepsilon\, e^{-ik\cdot x}\langle\psi|S^\dagger a(k) S|\psi\rangle + \text{c.c.}\right]$$

$$= i\int \widetilde{\mathrm{d}k}\, k k\,\varepsilon\varepsilon\, e^{-ik\cdot x}\left[\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k)T|\psi\rangle\right] + \text{c.c.}$$

"waveshape"

Waves from Amplitudes

Measure expectation of curvature component in classical limit

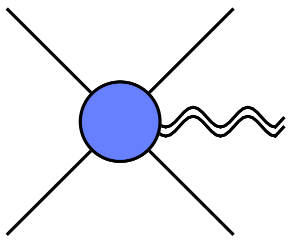
$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

$$\text{waveform} = \int \widetilde{\mathrm{d}k}\, \left[k k\, \varepsilon \varepsilon\, e^{-ik\cdot x} \langle \psi | S^\dagger a(k) S | \psi \rangle + \text{c.c.}\right]$$

$$= i \int \widetilde{\mathrm{d}k}\, k k\, \varepsilon \varepsilon\, e^{-ik\cdot x} \left[\boxed{\langle \psi | a(k) T | \psi \rangle} - i \langle \psi | T^\dagger a(k) T | \psi \rangle\right] + \text{c.c.}$$

5 point amplitude



Waves from Amplitudes

Measure expectation of curvature component in classical limit

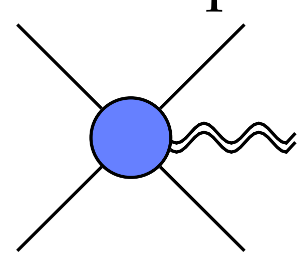
$$\mathbb{R}_{....}(x) = \partial.\partial.\mathbb{h}_{..}(x)$$

Graviton polarisation

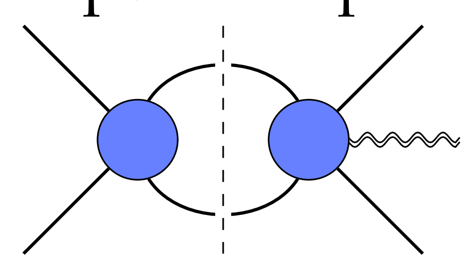
$$\text{waveform} = \int \widetilde{\mathrm{d}k}\,\left[k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \langle\psi|S^\dagger a(k) S|\psi\rangle + \text{c.c.}\right]$$

$$= i\int \widetilde{\mathrm{d}k}\, k k\, \varepsilon\varepsilon\, e^{-ik\cdot x} \left[\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k) T|\psi\rangle\right] + \text{c.c.}$$

5 point amplitude



Cut 5 point amplitudes



$+\cdots$

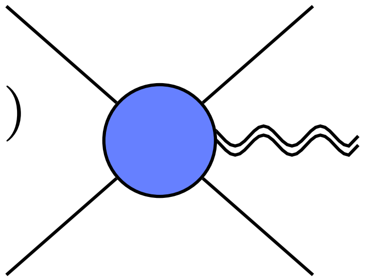
Waves from Amplitudes

Leading order:

Frequency space

$$\text{waveform}(\omega) =$$

$$\frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2 \, \delta(p_1 \cdot q_1) \delta(p_2 \cdot q_2) \delta^4(k - q_1 - q_2) \, e^{ib\cdot(q_1 - q_2)}$$



Cristofoli, Gonzo, Kosower & DOC

Recover *Kovacs & Thorne* (1977/1978) waveform

Computed with spin: *Jakobsen, Mogull, Plefka & Steinhoff*

Waves from Amplitudes

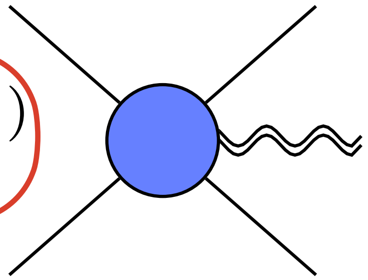
Leading order:

Frequency space

$$\text{waveform}(\omega) =$$

Fourier integrals: always present, from state $|\psi\rangle$

$$\frac{1}{\text{distance}} \int \mathrm{d}^4 q_1 \mathrm{d}^4 q_2 \, \delta(p_1 \cdot q_1) \delta(p_2 \cdot q_2) \delta^4(k - q_1 - q_2) \, e^{ib\cdot(q_1 - q_2)}$$



Cristofoli, Gonzo, Kosower & DOC

Recover *Kovacs & Thorne* (1977 / 1978) waveform

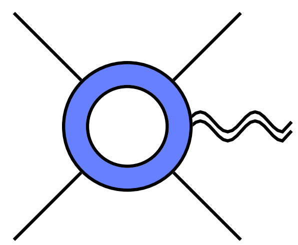
Computed with spin: *Jakobsen, Mogull, Plefka & Steinhoff*

Waves at NLO

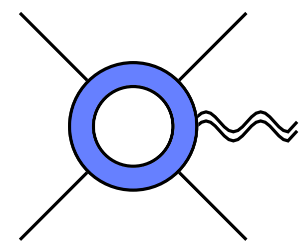
$$\text{waveform} = i \int \widetilde{\mathrm{d}k}\, kk\, \varepsilon\varepsilon\, e^{-ik\cdot x} \Big[\underbrace{\langle\psi|a(k)T|\psi\rangle - i\langle\psi|T^\dagger a(k)T|\psi\rangle}_{\text{waveshape } \alpha} \Big] + \text{c.c.}$$

Second term in waveshape: cut aka imaginary part of one-loop 5-point

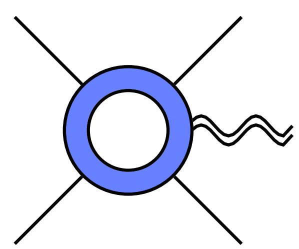
$\alpha|_{\mathrm{NLO}} =$



$- \, i \; \mathrm{Im}$



$\sim \mathrm{Re}$



Waves at NLO

Incomplete cancellation of Im part: *radiation reaction*

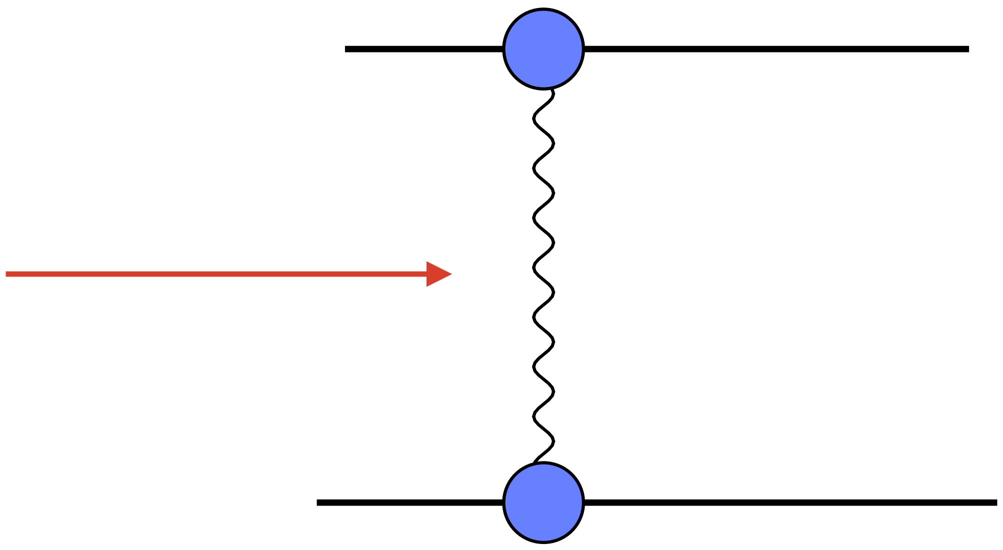
Waves at NLO

Incomplete cancellation of Im part: *radiation reaction*

Easy to understand in QED:

Lorentz force

$$\frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} \sim Q_1 Q_2 F_1$$



Waves at NLO

Incomplete cancellation of Im part: *radiation reaction*

Easy to understand in QED:

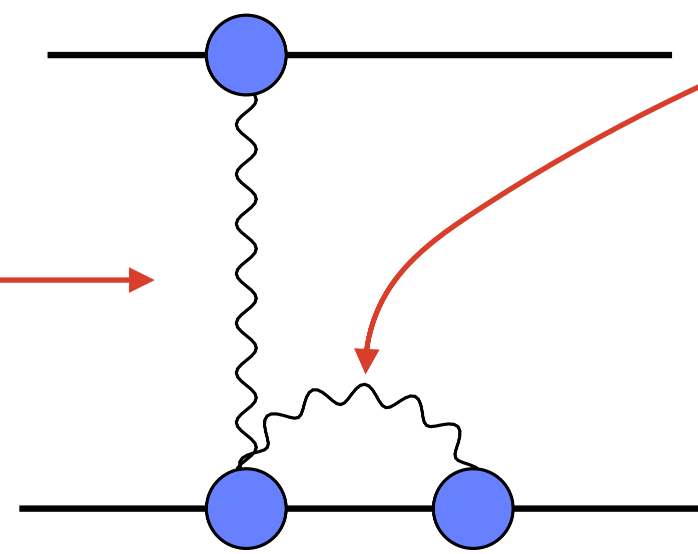
Self force (ALD)

$$\frac{\mathrm{d}\Delta p_2^\mu}{\mathrm{d}\tau} \sim Q_2^2 \frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2} + \cdots$$

Schott term

Lorentz force

$$\frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} \sim Q_1 Q_2 F_1$$



Waves at NLO

Incomplete cancellation of Im part: *radiation reaction*

Easy to understand in QED:

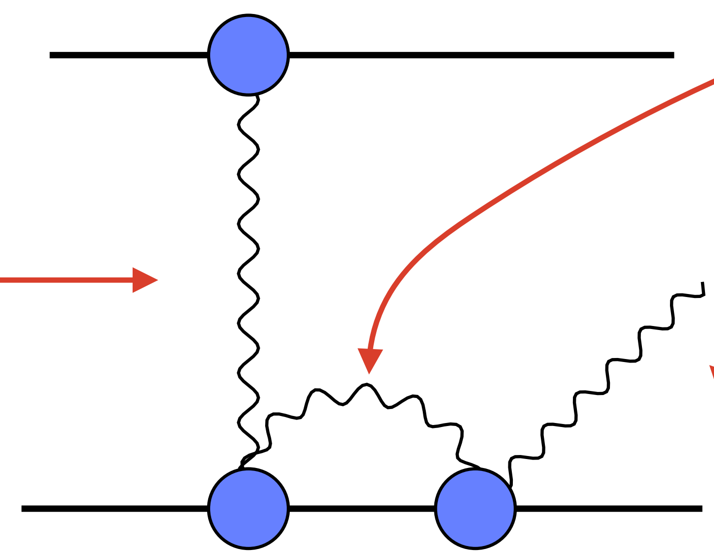
Self force (ALD)

$$\frac{\mathrm{d}\Delta p_2^\mu}{\mathrm{d}\tau} \sim Q_2^2 \frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2} + \cdots$$

Schott term

Lorentz force

$$\frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} \sim Q_1 Q_2 F_1$$



Corrected radiation field

$$\partial^2 \Delta A^\mu \sim Q_2 \int \Delta p_2^\mu$$

Waves at NLO

Incomplete cancellation of Im part: *radiation reaction*

Easy to understand in QED:

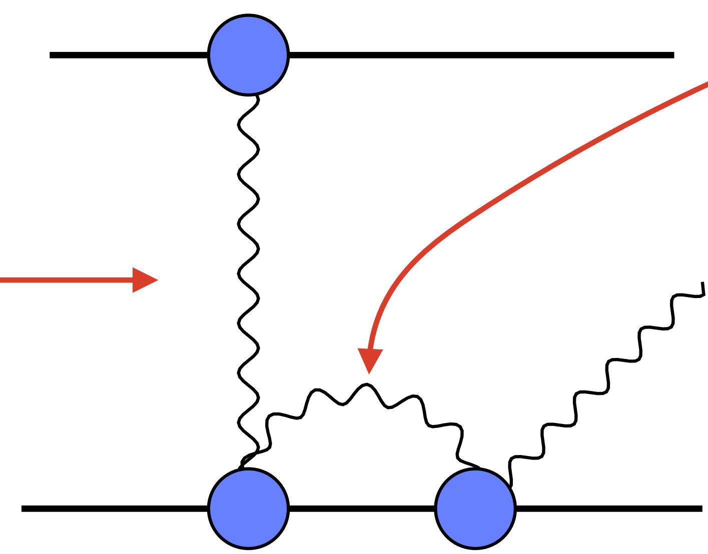
Self force (ALD)

$$\frac{\mathrm{d}\Delta p_2^\mu}{\mathrm{d}\tau} \sim Q_2^2 \frac{\mathrm{d}^2 p_2^\mu}{\mathrm{d}\tau^2} + \cdots$$

Schott term

Lorentz force

$$\frac{\mathrm{d}p_2^\mu}{\mathrm{d}\tau} \sim Q_1 Q_2 F_1$$



Corrected radiation field

$$\partial^2 \Delta A^\mu \sim Q_2 \int \Delta p_2^\mu$$

Net order $Q_1 Q_2^4$ radiation perturbation

"Radiation from radiation-reaction"

Waves at NLO

Gravity Im part, integrand level in

- *Elkhidir, Sergola, Vazquez-Holm, DOC*

Integrated NLO waveforms in

- *Herderschee, Roiban, Teng;*
- *Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini;*
- *Georgoudis, Heissenberg, Vazquez-Holm*

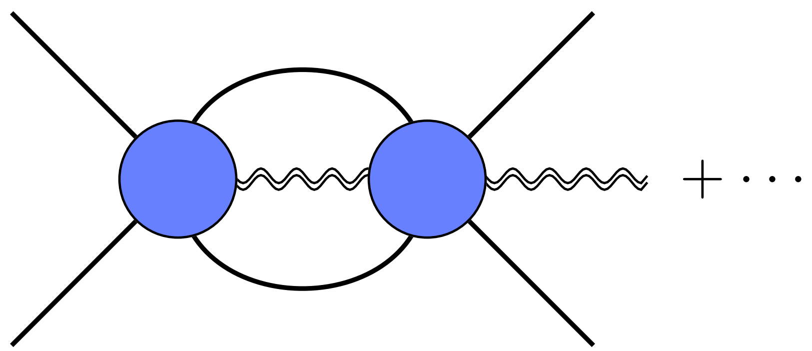
→ Poster!

Further discussion of implications of $i\epsilon$ yesterday by

- *Caron-Huot, Giroux, Hofie Hannesdóttir, Mizera*

Frontiers

NNLO waveform



Fourier integrals

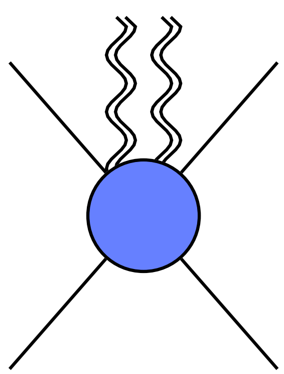
- Closely related to loop integrals
- Simpler in positions space?
- Numerical at one loop

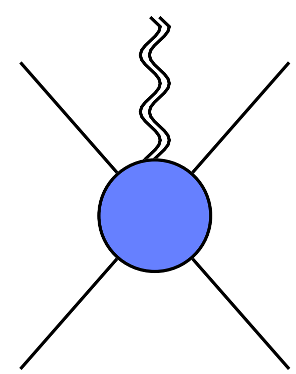
Variances

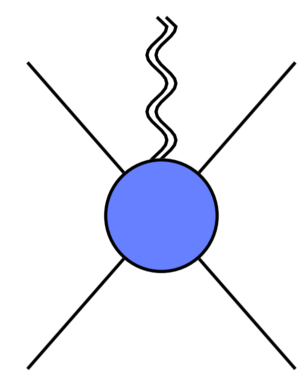
Waveform Variance

Classical

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(x)S|\psi\rangle = \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger\mathbb{R}_{\alpha\beta\gamma\delta}(x)S|\psi\rangle$$

 $+\cdots$





$$\kappa^4\mathcal{M}_{6,\text{tree}} + \kappa^6\mathcal{M}_{6,\text{1loop}} + \cdots \qquad \kappa^3\mathcal{M}_{5,\text{tree}} \qquad \kappa^3\mathcal{M}_{5,\text{tree}}$$

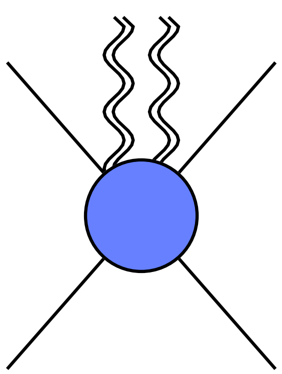
Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC

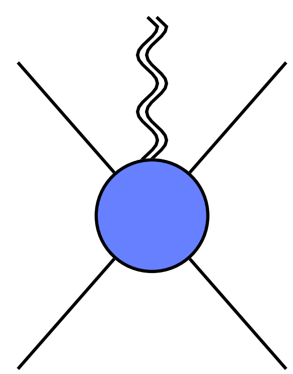
Britto, Gonzo, Jehu

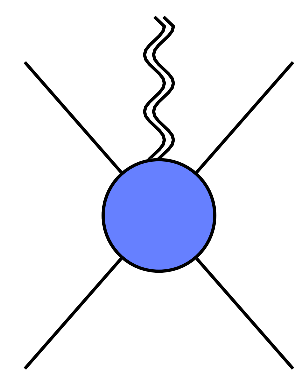
Waveform Variance

Classical

$$\langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{R}_{\alpha\beta\gamma\delta}(x)S|\psi\rangle = \langle\psi|S^\dagger \mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger \mathbb{R}_{\alpha\beta\gamma\delta}(x)S|\psi\rangle$$

 $+\cdots$





$\kappa^4\mathcal{M}_{6,\text{tree}} + \kappa^6\mathcal{M}_{6,\text{1loop}} + \cdots$ $\quad$ $\kappa^3\mathcal{M}_{5,\text{tree}}$ $\quad$ $\kappa^3\mathcal{M}_{5,\text{tree}}$

Quantum!

Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC

Britto, Gonzo, Jehu

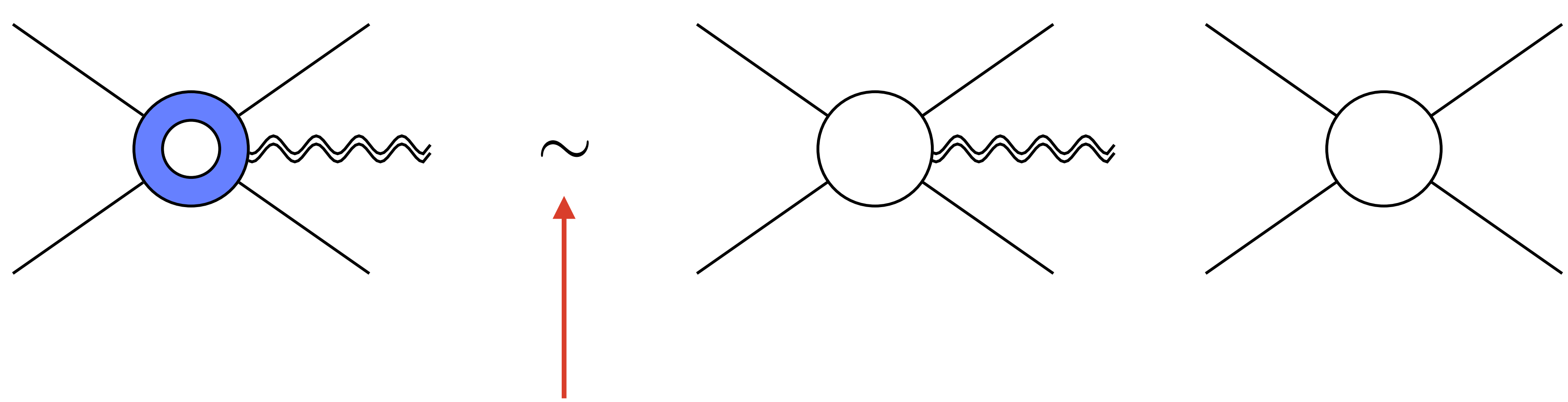
Zero-Variance Relations

There are an infinite number of relations. Eg

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{P}_\alpha S|\psi\rangle = \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger\mathbb{P}_\alpha S|\psi\rangle$$

Five points — Five points — Four points

$$\mathcal{M}_{5,\mathrm{1loop}} \sim \mathcal{M}_{5,\mathrm{tree}} \times \mathcal{M}_{4,\mathrm{tree}}$$



Dominant part

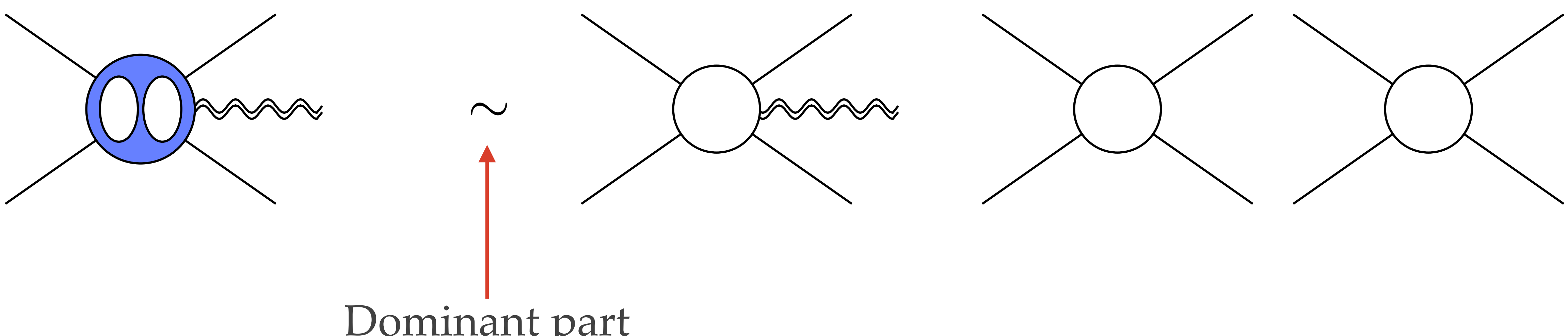
Zero-Variance Relations

There are an infinite number of relations. Eg

$$\langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)\mathbb{P}_\alpha\mathbb{P}_\beta S|\psi\rangle = \langle\psi|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}(x)S|\psi\rangle\langle\psi|S^\dagger\mathbb{P}_\alpha S|\psi\rangle\langle\psi|S^\dagger\mathbb{P}_\beta S|\psi\rangle$$

Five points Five points Four points Four points

$$\mathcal{M}_{5,\text{2loop}} \sim \mathcal{M}_{5,\text{tree}} \times \mathcal{M}_{4,\text{tree}} \times \mathcal{M}_{4,\text{tree}}$$



Zero-Variance Relations

Some structure in scattering amplitudes admits classical limit

Zero-Variance Relations

Some structure in scattering amplitudes admits classical limit

1. It would be nice to prove these hold/understand details

Zero-Variance Relations

Some structure in scattering amplitudes admits classical limit

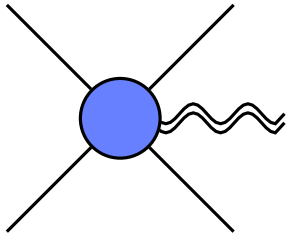
1. It would be nice to prove these hold / understand details
2. Suggests eikonal-like exponentiation in classical region

$$S|\psi\rangle \sim \int \exp\left(i\tilde{\mathcal{M}}_4(x,q)\right)\exp\left(\int \widetilde{\mathrm{d}k}\ \left(\mathcal{M}_5(x_1,x_2,k)+\cdots\right)a^\dagger(k)\right)|p_1',p_2'\rangle$$

Ciafaloni, Colferai, Veneziano
Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC
Di Vecchia, Heissenberg, Russo, Veneziano

Exponentiation

Contrary to intuition?

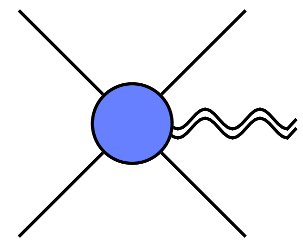
$$\text{waveform} = \frac{1}{\text{distance}} \int$$  $$+ \cdots$$

One graviton $\neq$ classical field

Exponentiation

Contrary to intuition?

$$\text{waveform} = \frac{1}{\text{distance}} \int$$



$$+ \cdots$$

One graviton $\neq$ classical field

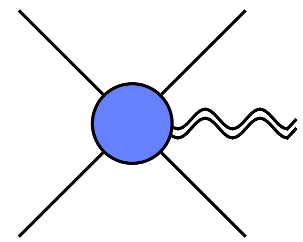
Classical field: expectation of a coherent state

$$\exp\left(\int \widetilde{\mathrm{d}k}\, \alpha(k) a^\dagger(k) \right) |0\rangle$$

Exponentiation

Seems contrary to intuition:

$$\text{waveform} = \frac{1}{\text{distance}} \int$$



$$+ \cdots$$

Exponentiation would resolve

$$S|\psi\rangle \sim \int \exp\left(i\tilde{\mathcal{M}}_4(x,q)\right) \exp\left(\int \widetilde{\mathrm{d}k}\; \left(\mathcal{M}_5(x_1,x_2,k) + \cdots\right) a^\dagger(k)\right) |p_1', p_2'\rangle$$

Generalisation of eikonal to dissipative case

Ciafaloni, Colferai, Veneziano

Cristofoli, Gonzo, Moynihan, Ross, Sergola, White, DOC

Di Vecchia, Heissenberg, Russo, Veneziano

→ Eikonal observable poster!

Conclusions

- Amplitudes compute a wealth of interesting observables
- Interesting structure of amplitudes in classical limit
- Bound-state waveform from amplitudes?