Positivity made simple

Amplitudes 2023

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2108.05896 [B. Bellazzini, GI, M. Lewandowski, F. Sgarlata]

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8 August 2023

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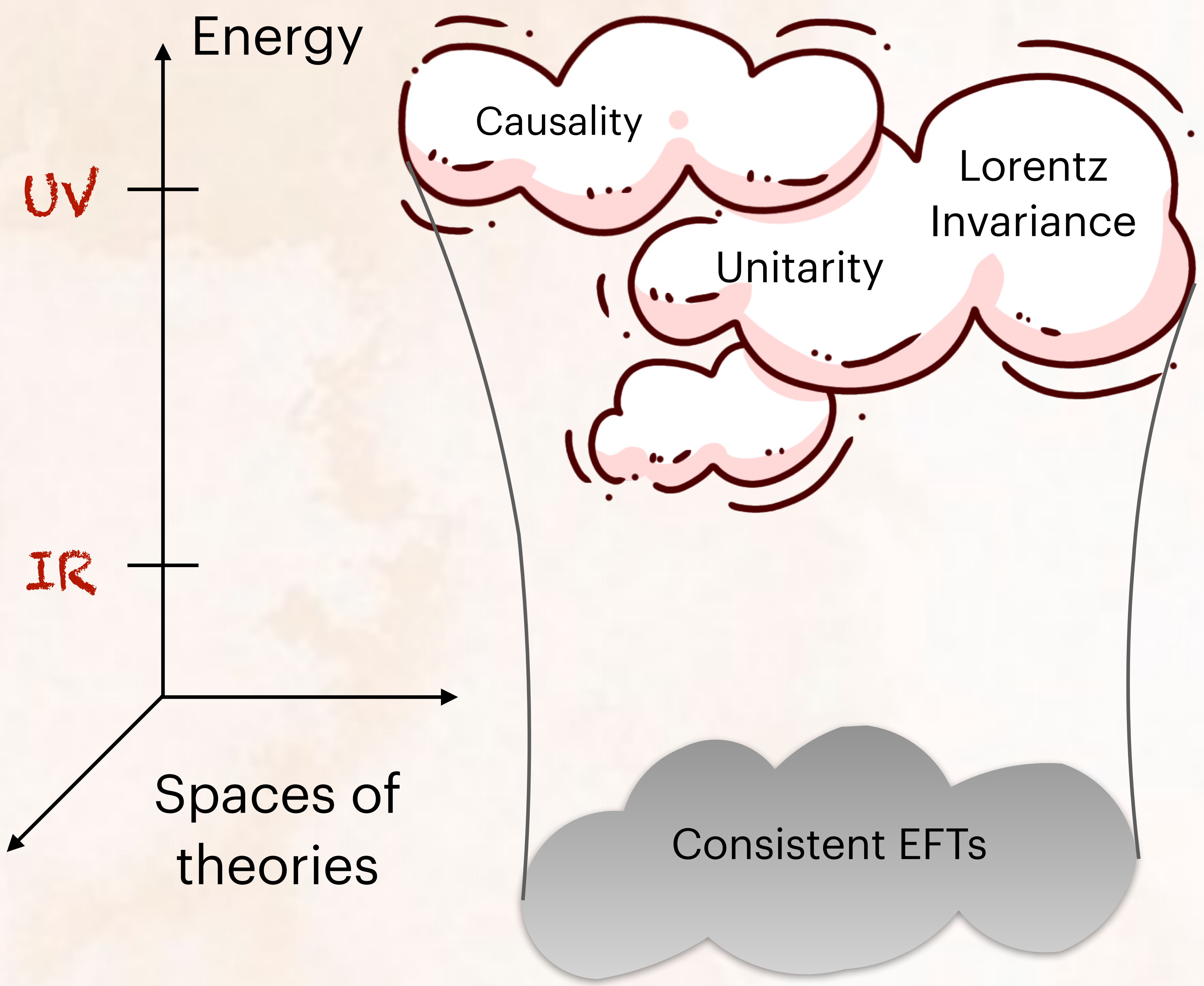
UNIVERSITÉ DE GENÈVE

Positivity constraints: the philosophy



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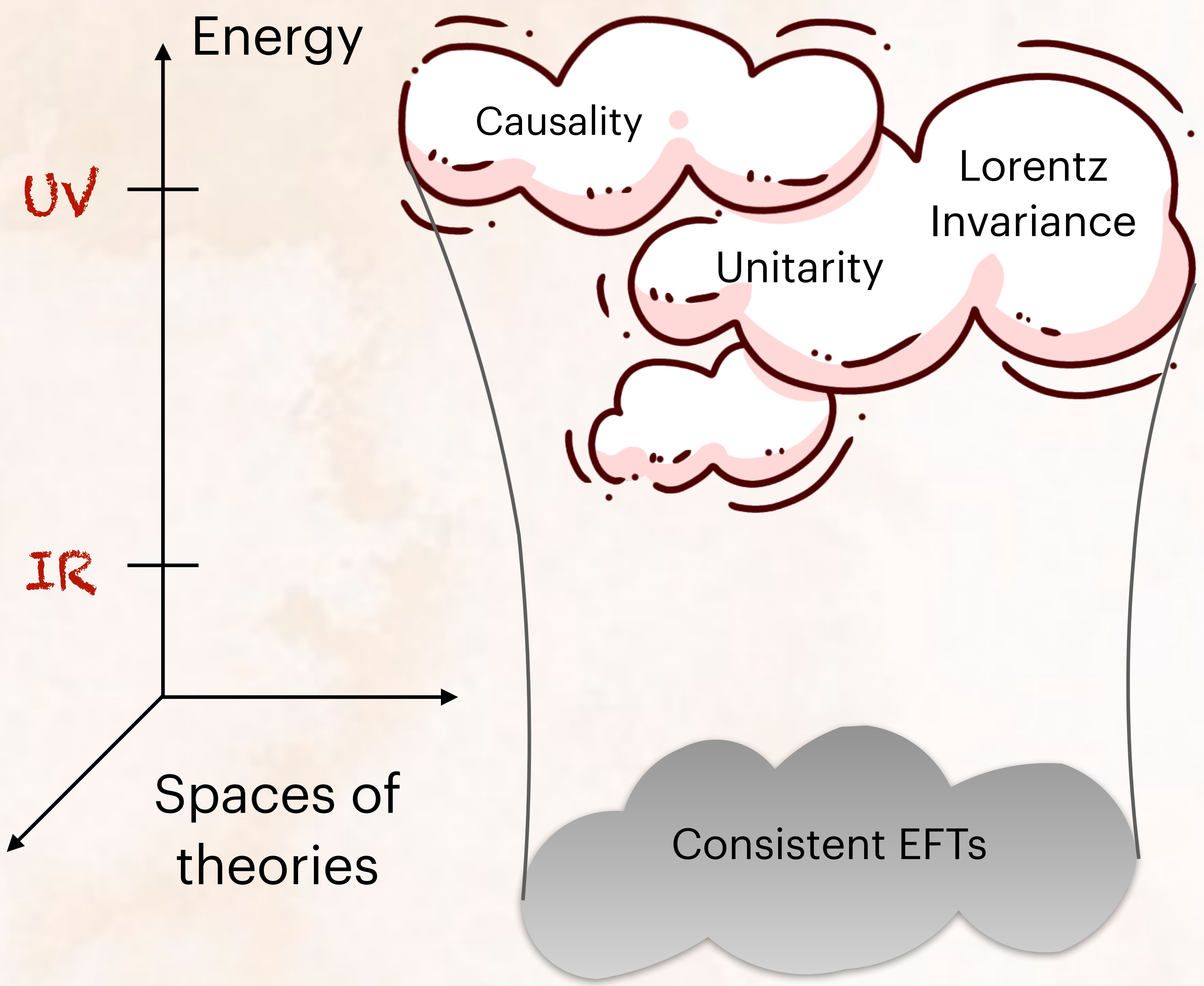

Assumptions

- Lorentz Invariance
- Unitarity
- Crossing Symmetry
- Micro-causality/Analyticity
- Polynomial boundedness

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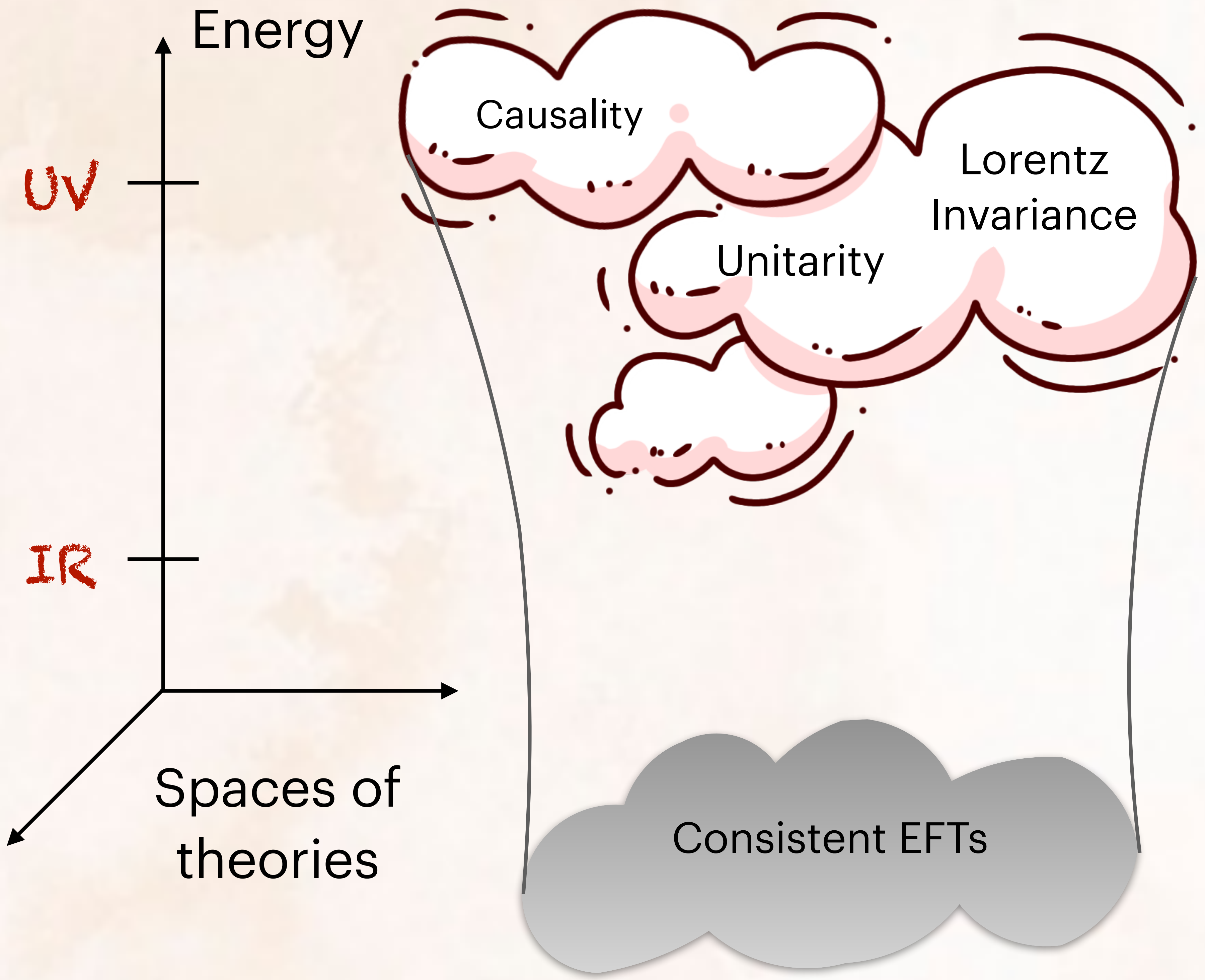

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Scattering amplitudes

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

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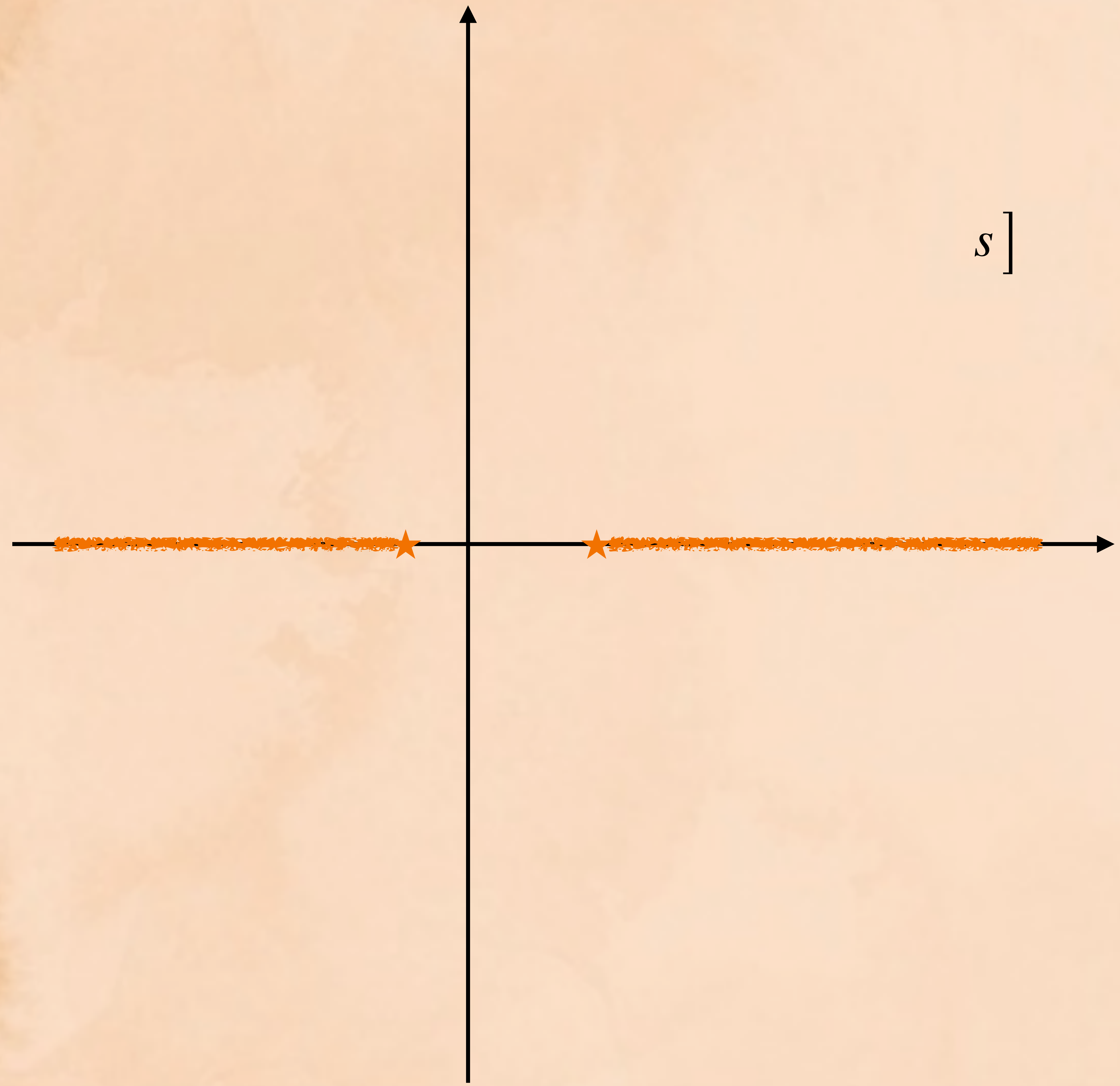

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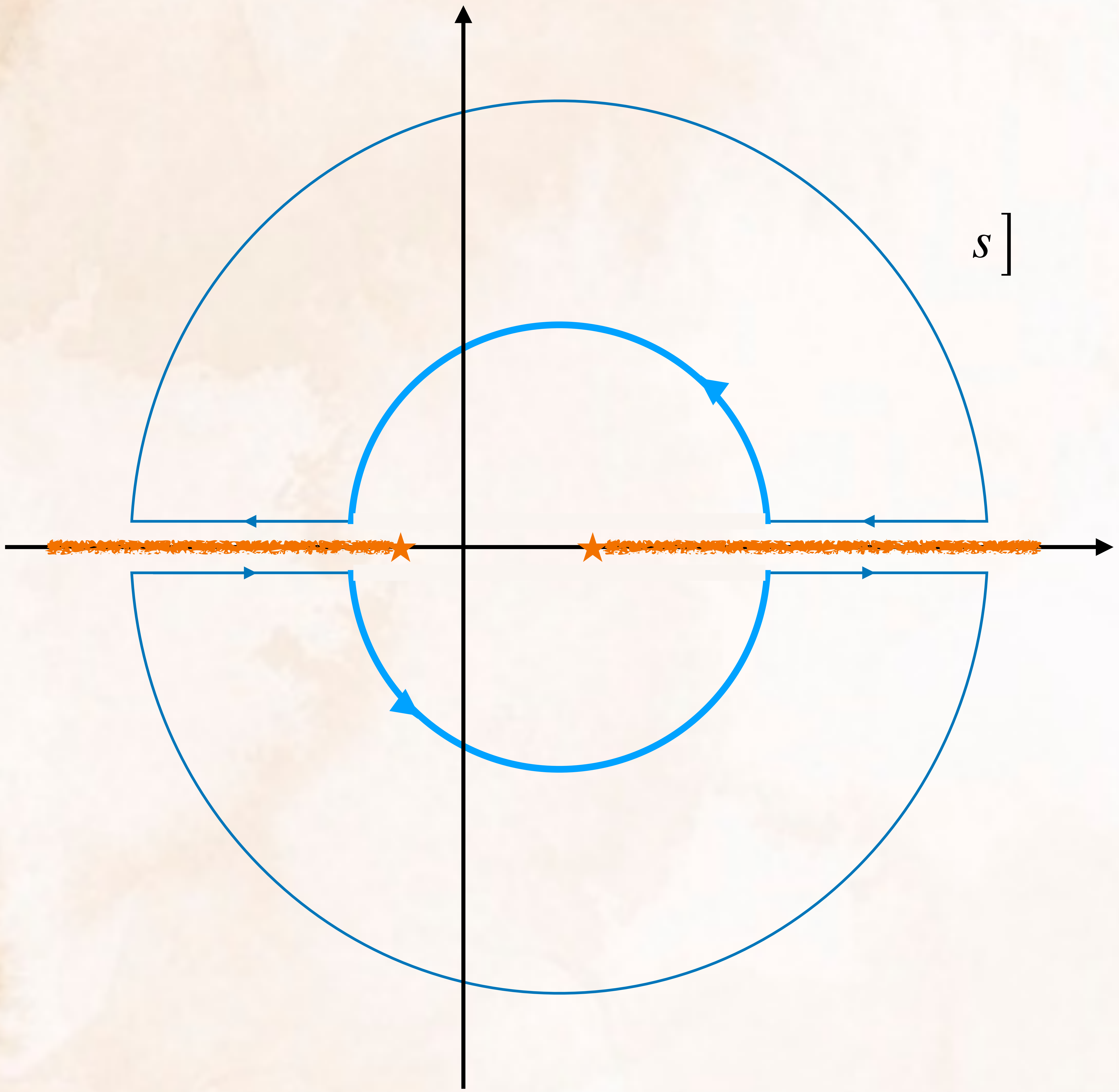

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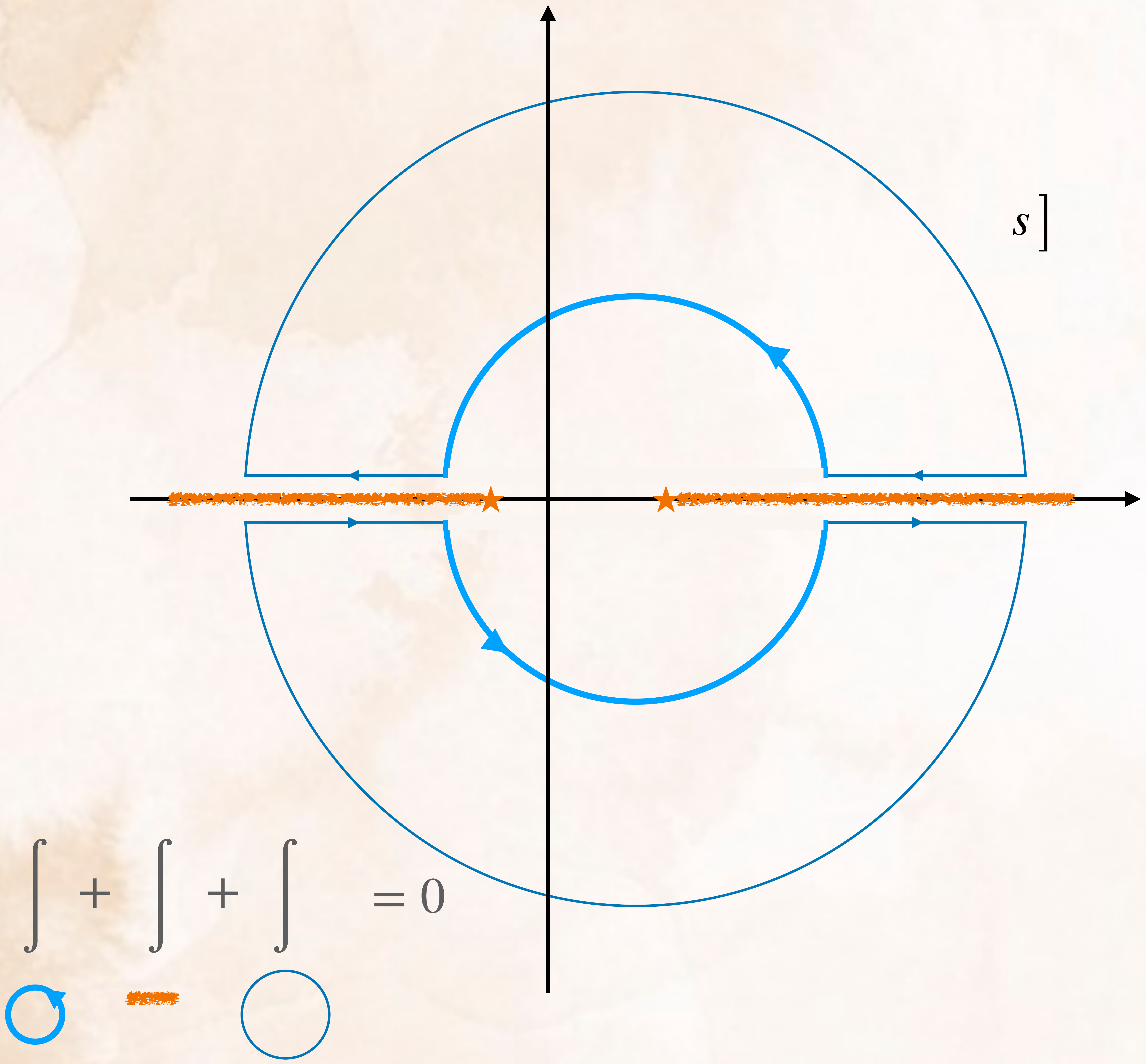

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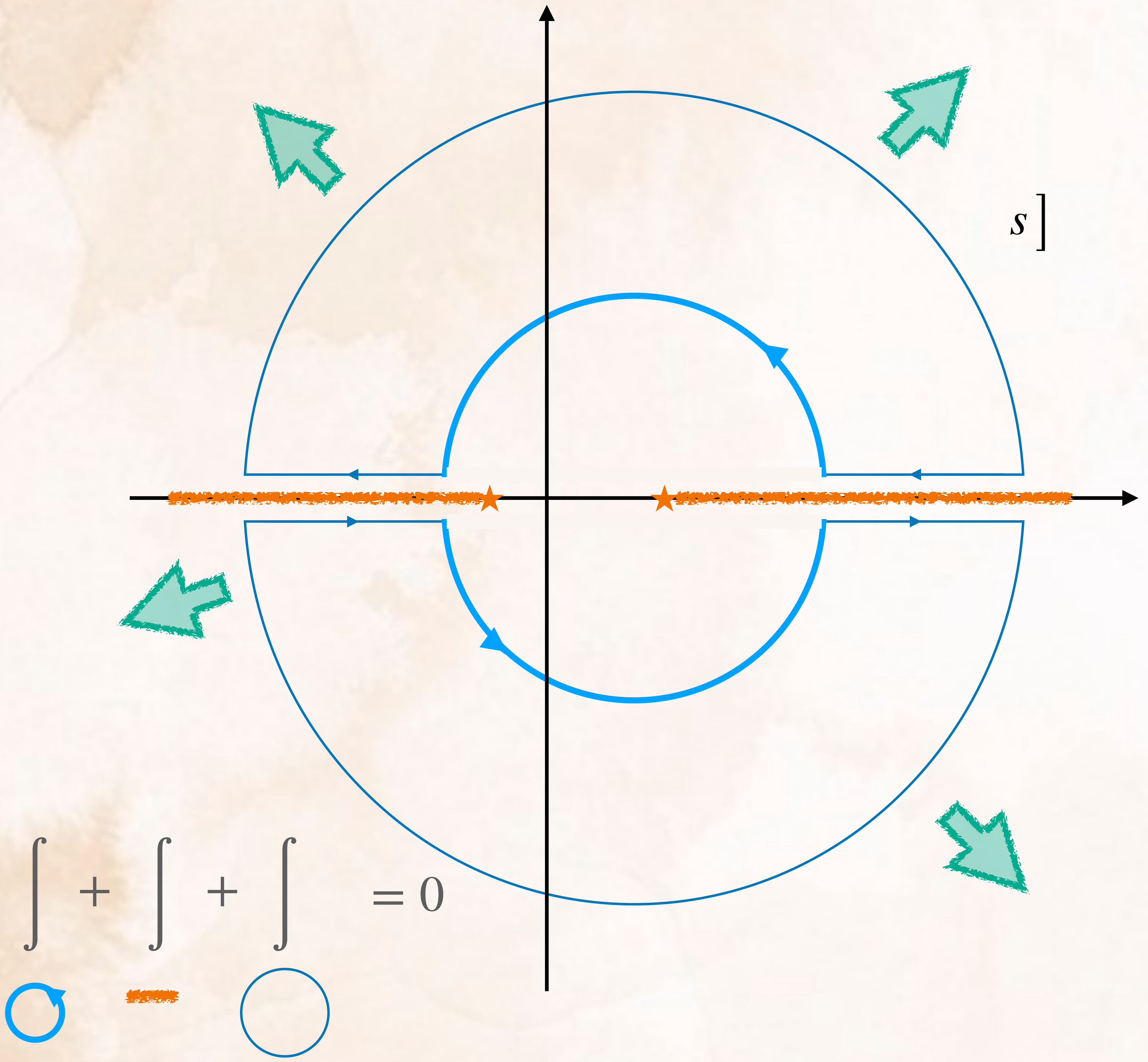

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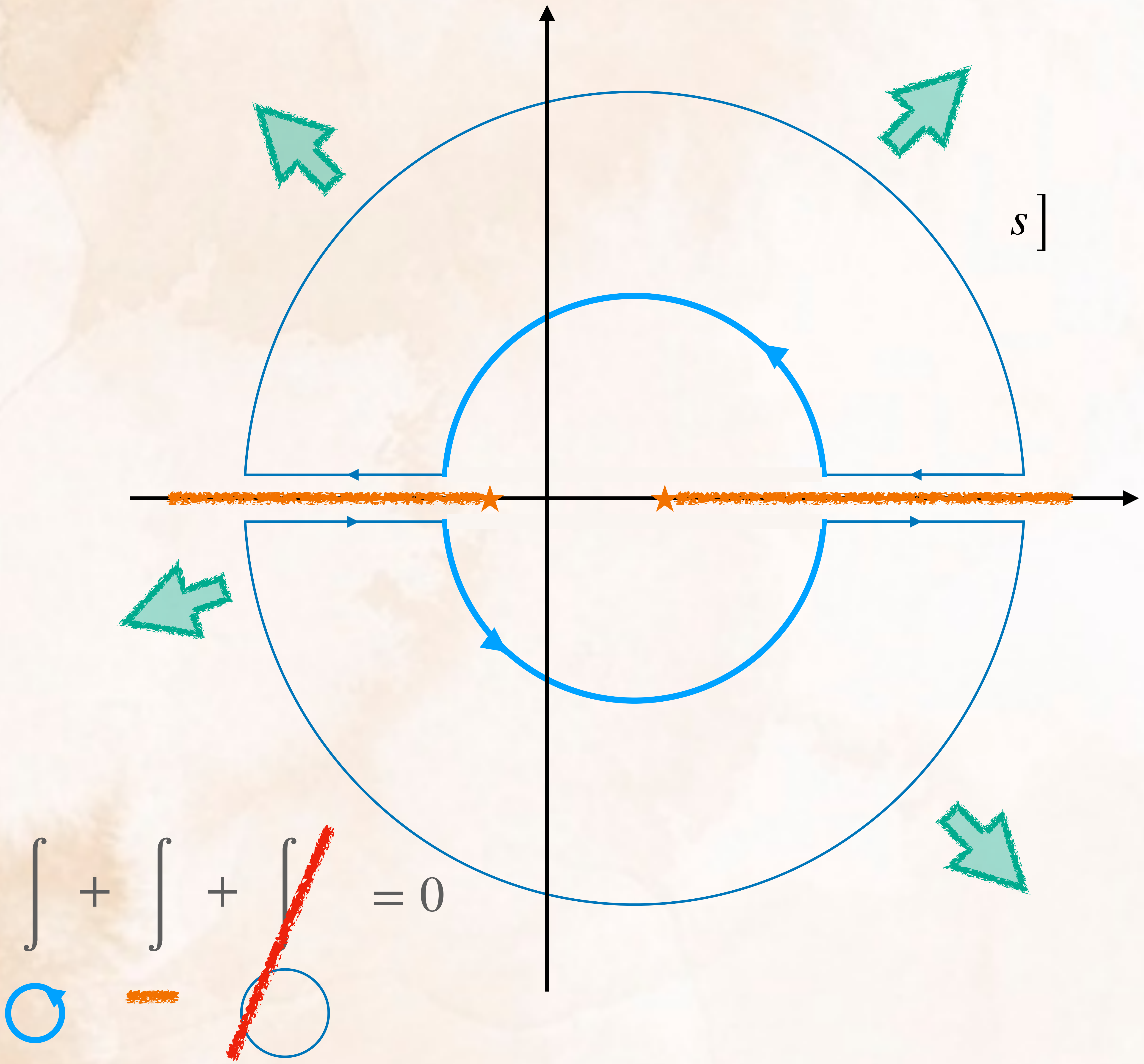
$$\frac{M}{|s|^2} \xrightarrow[s \to \infty]{} 0$$

Froissart bound

Positivity constraints: the philosophy



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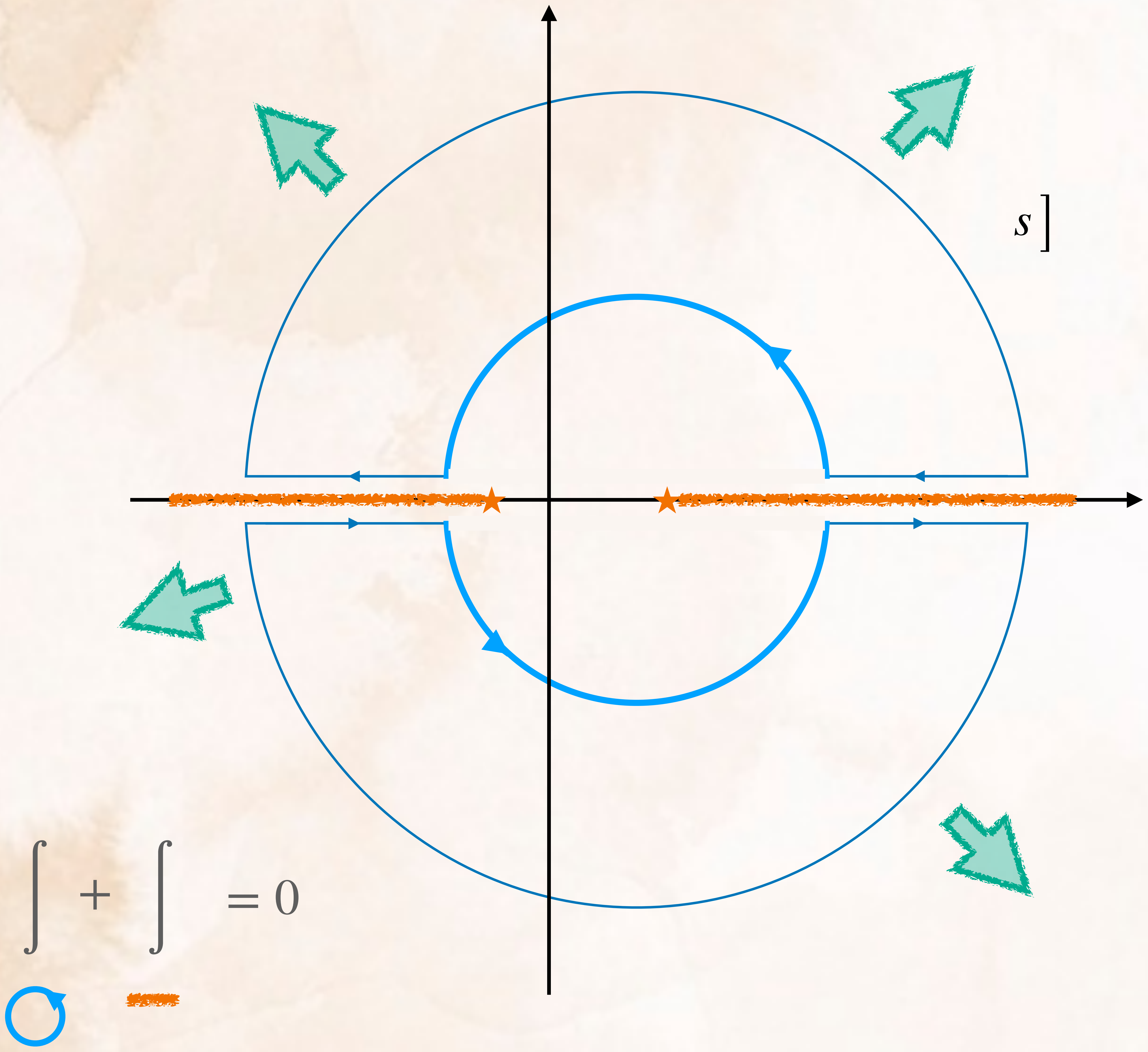
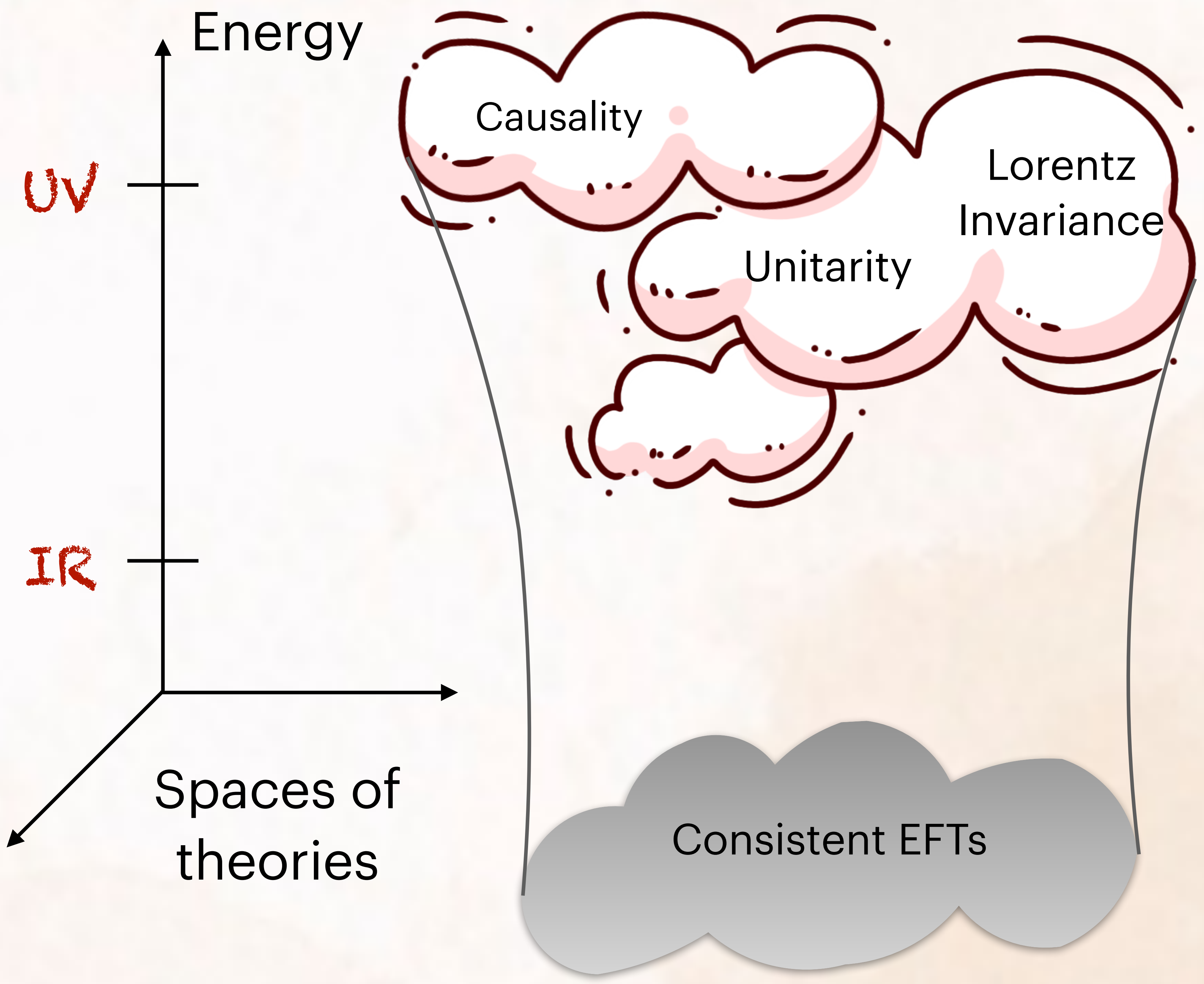
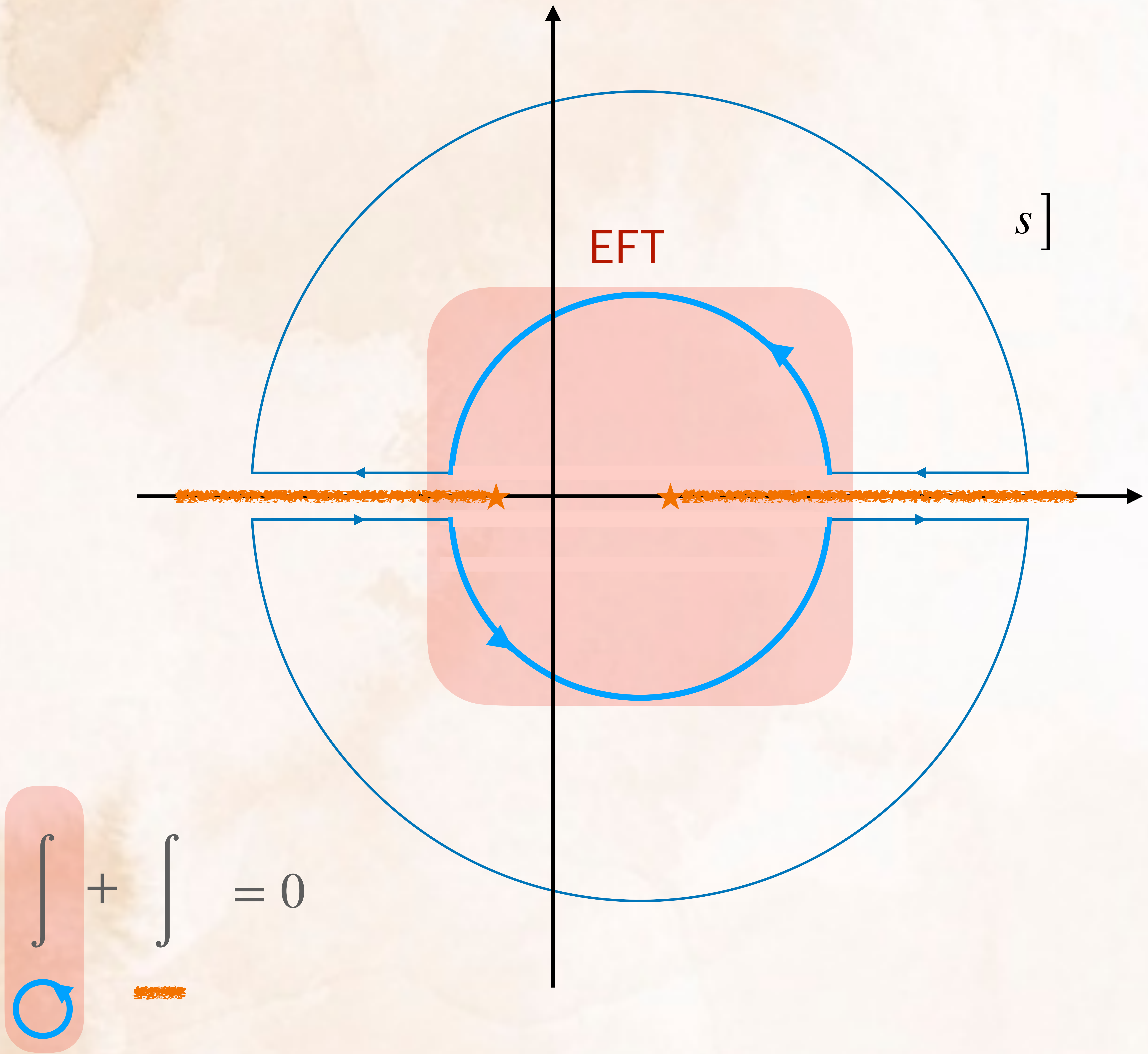
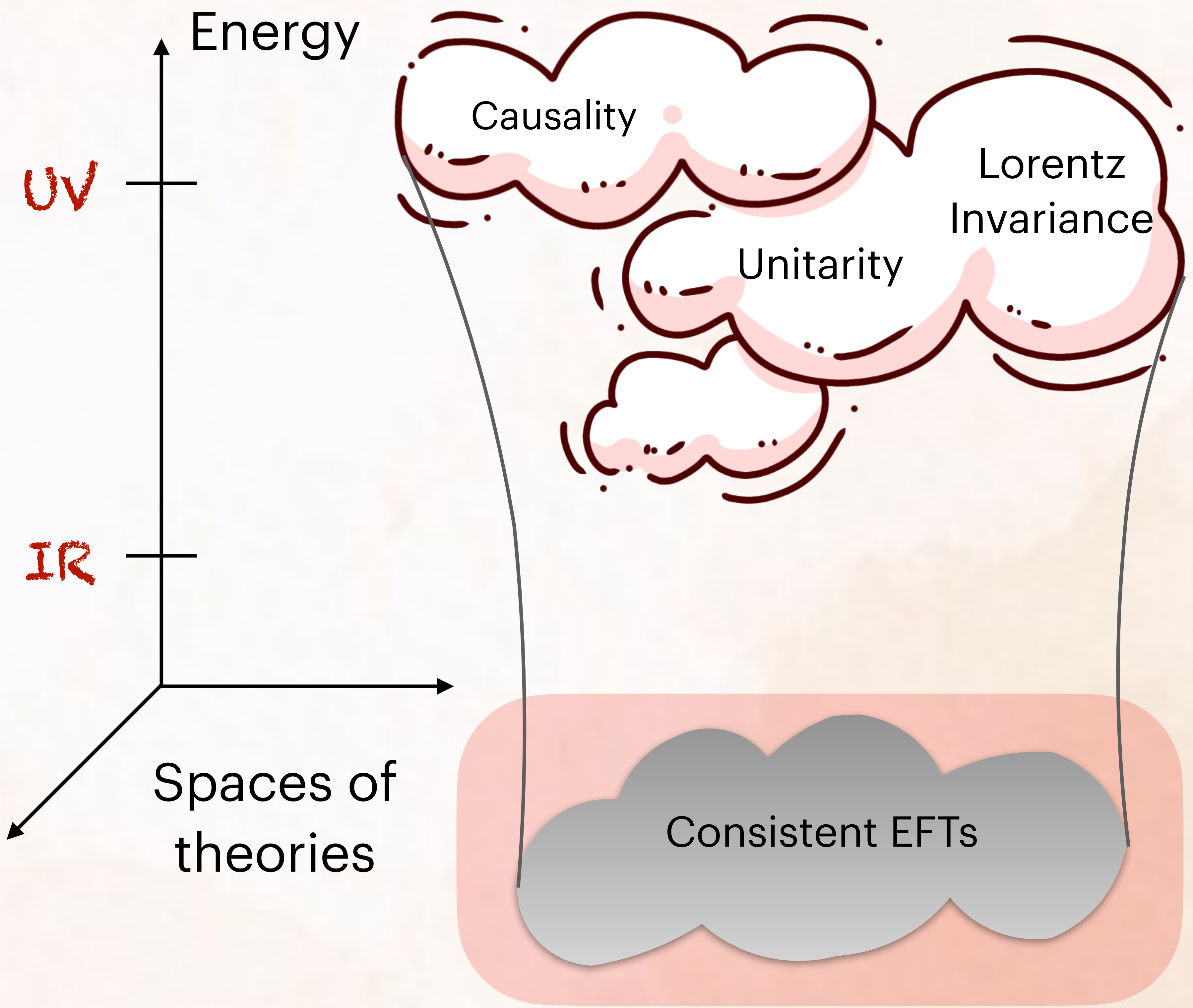




Positivity constraints: the philosophy





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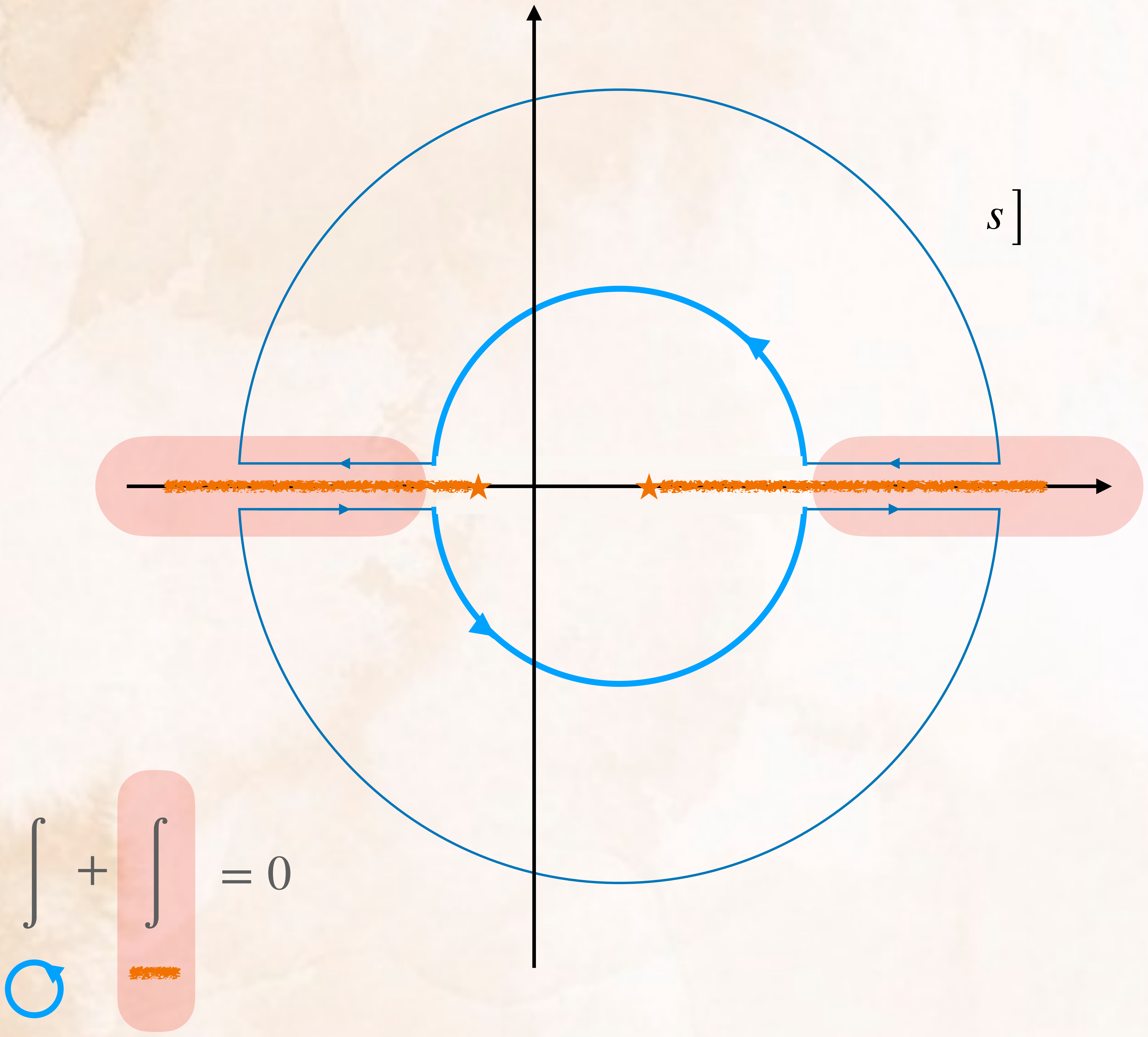

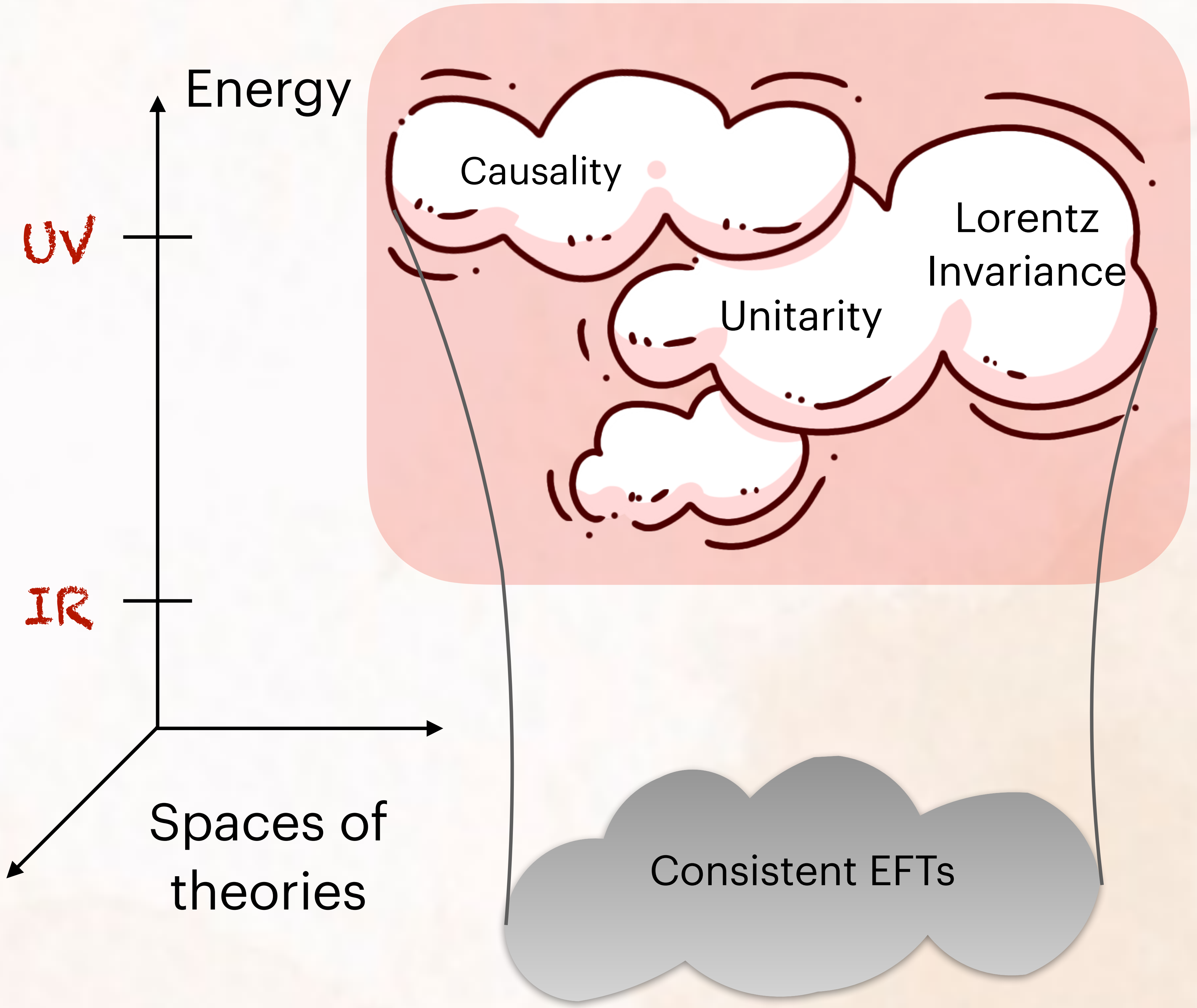

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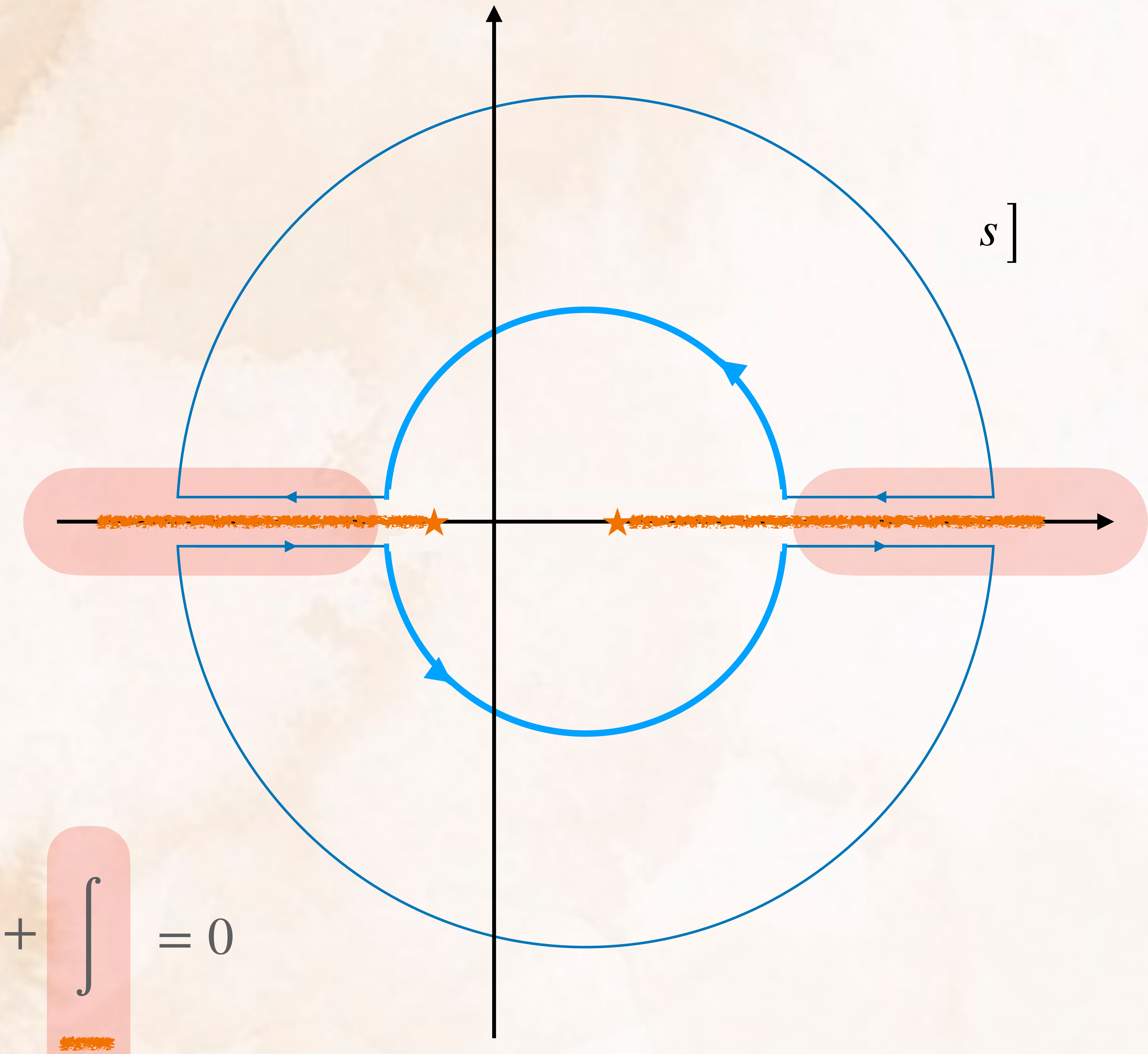
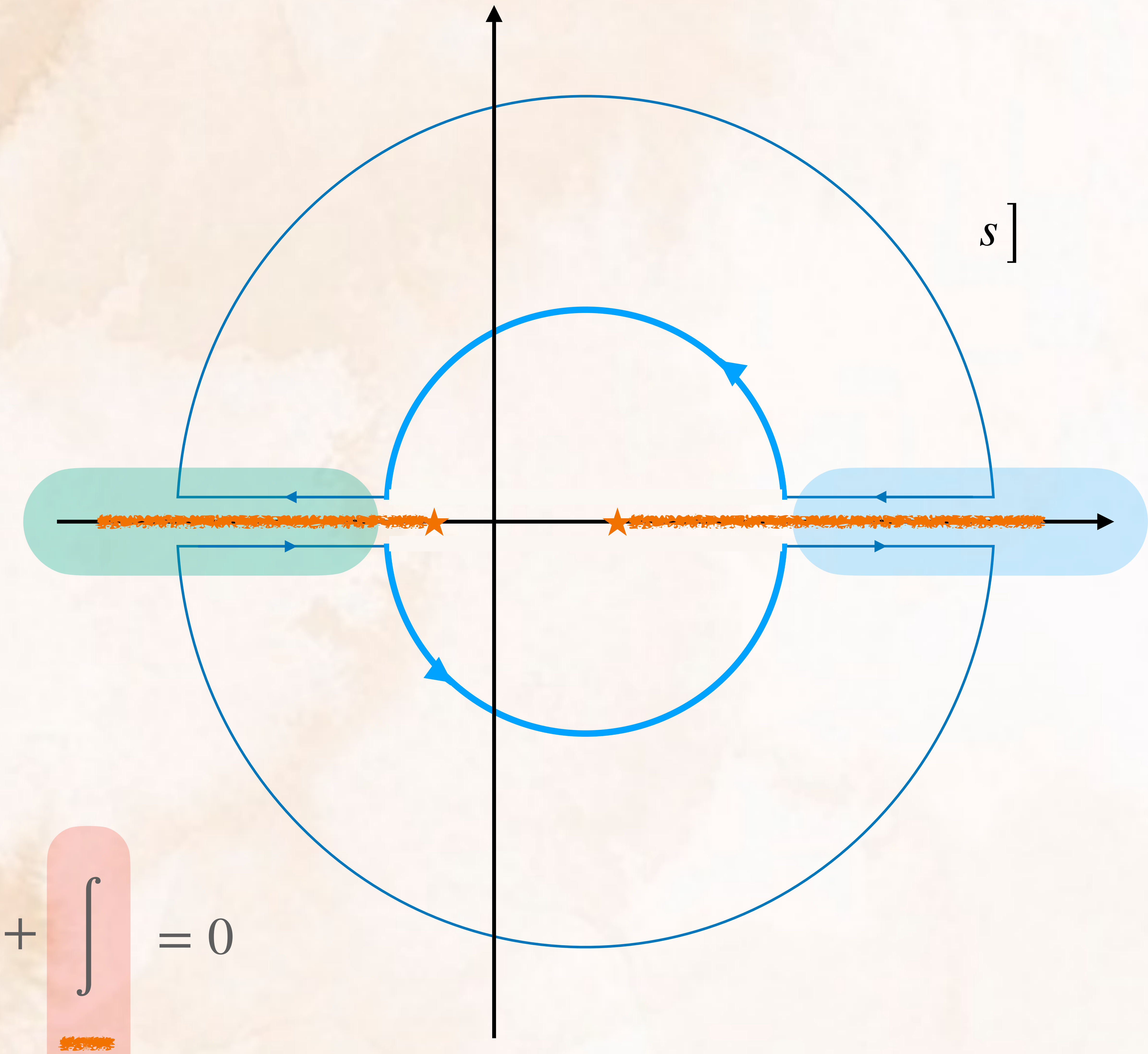


$$\int = $$

$$\int + \int = 0$$

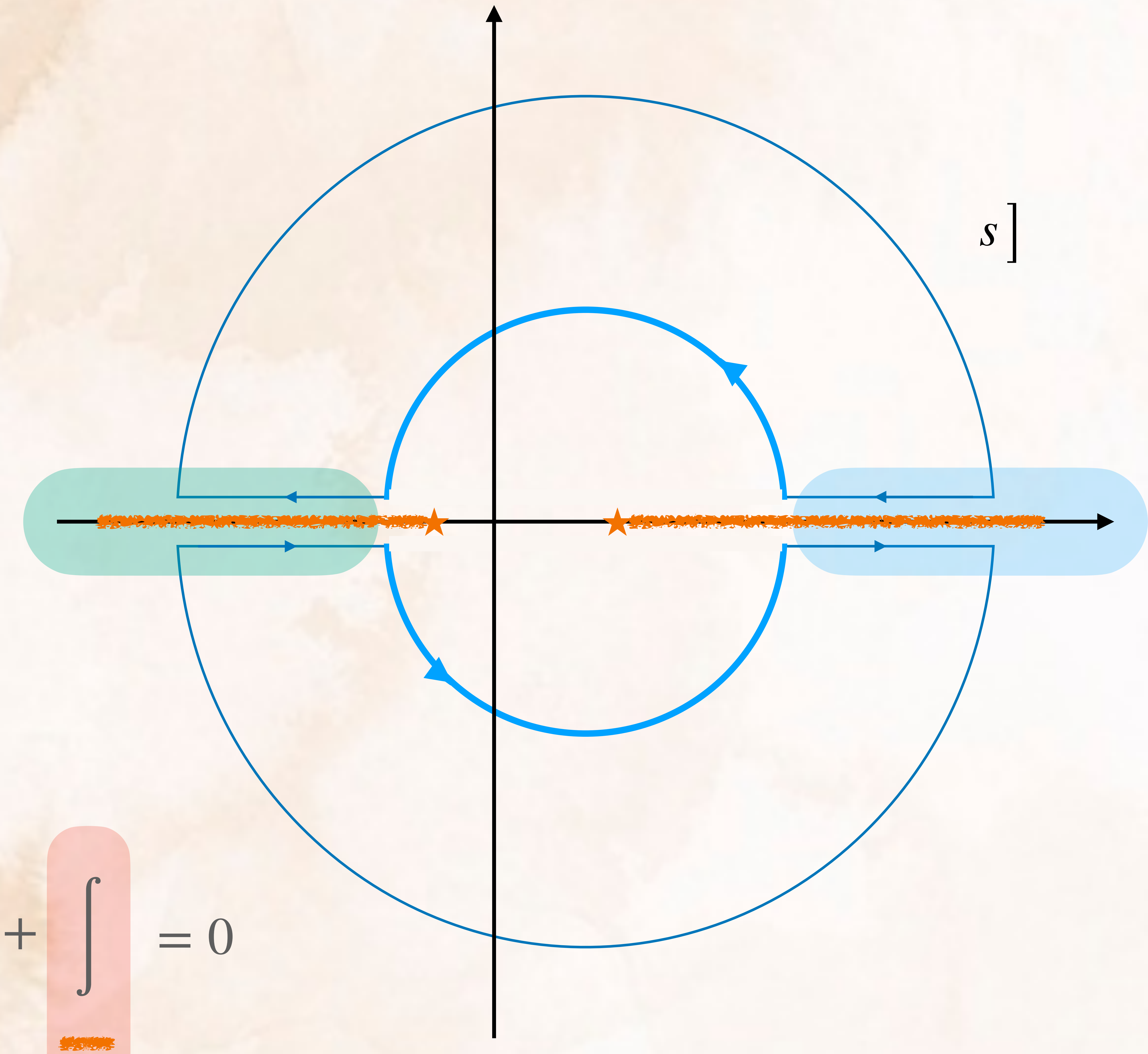
Positivity constraints: the philosophy



$$\int = \left[\int + \int \right] \frac{ds}{2\pi i} \frac{M(\alpha \to \alpha)}{|s|^n}$$

$$\int + \int = 0$$

Positivity constraints: the philosophy



Crossing Symmetry $\quad M(u, t=0) = M(s, t=0)$

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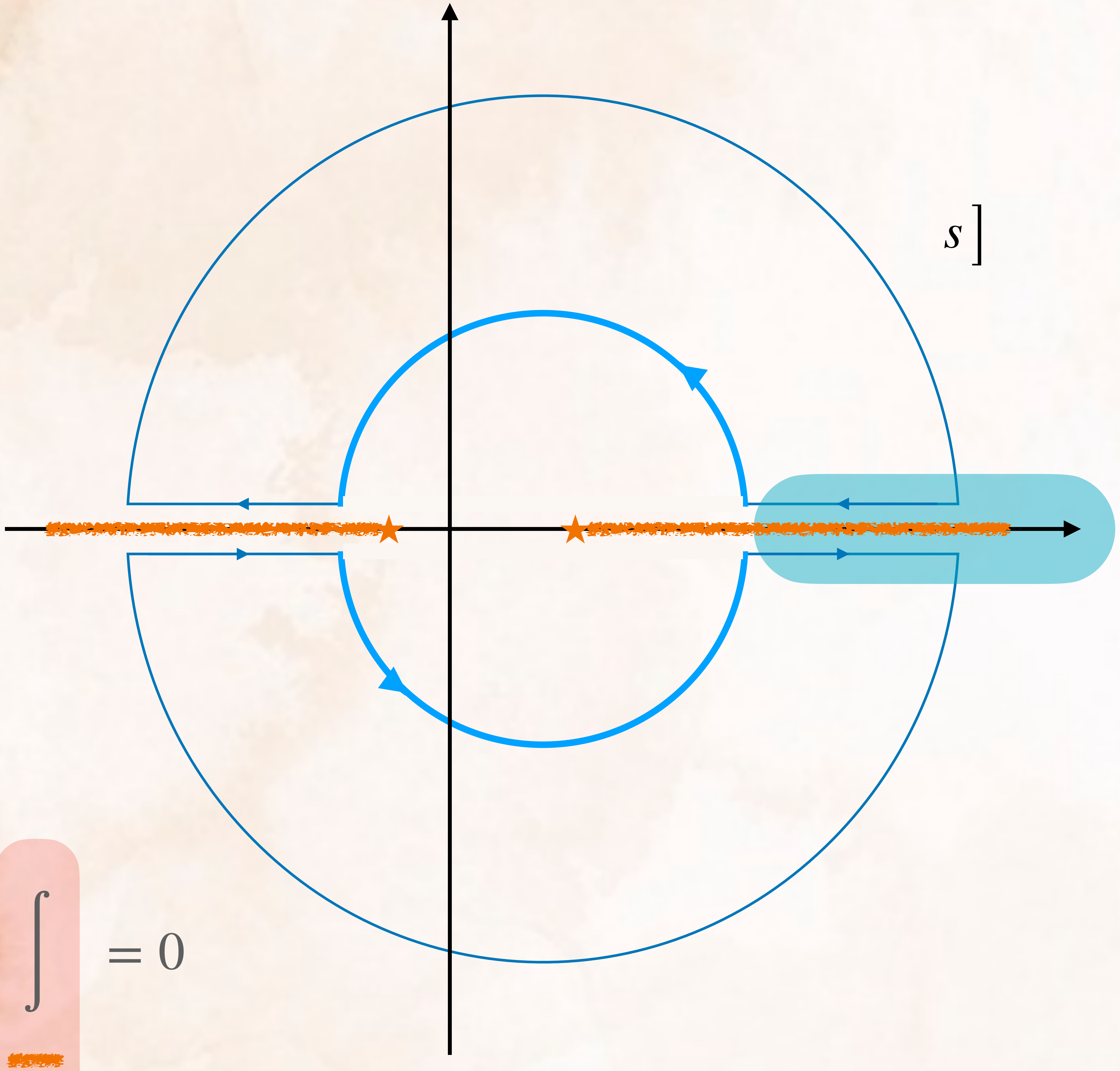

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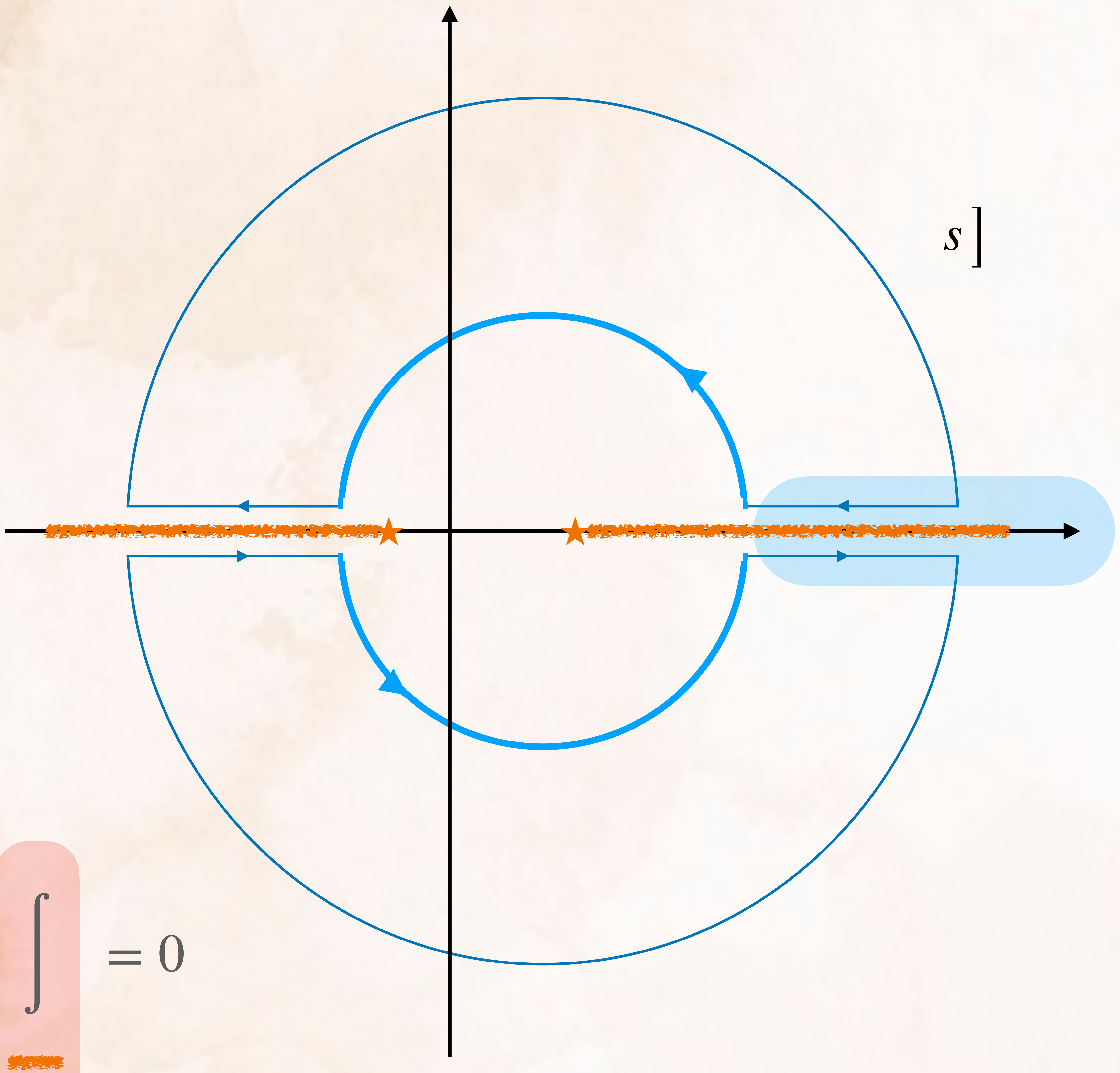

$$\int = \left[\int + \int \right] \frac{ds}{2\pi i} \frac{M(\alpha \to \alpha)}{|s|^n}$$

$$= 2 \times \int_{\Lambda^2}^{\infty} \frac{ds}{2\pi i} \frac{\mathrm{Disc} M}{|s|^n}$$

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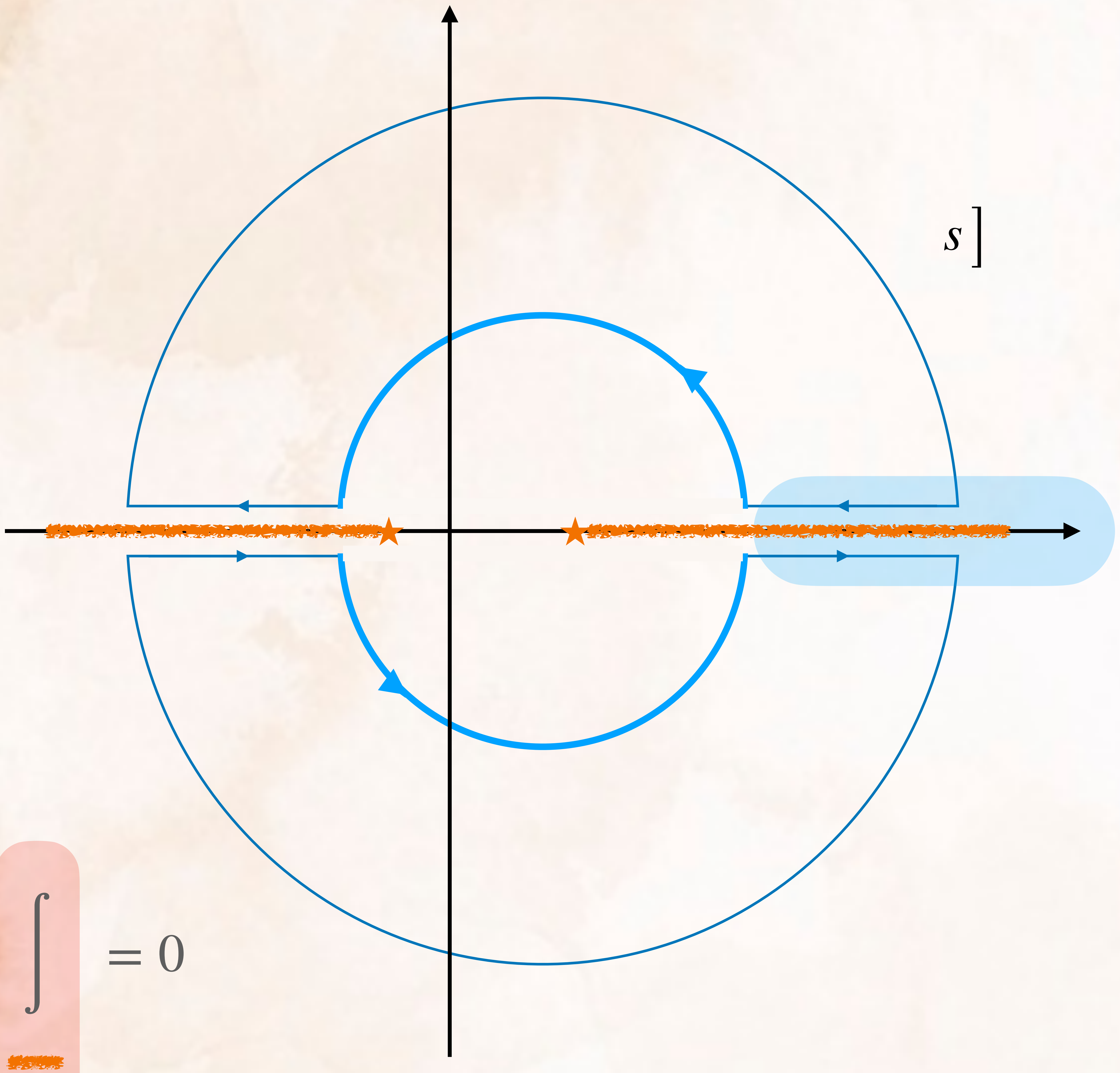
Positivity constraints: the philosophy



$$\int_{\circlearrowleft} + \int_{\text{cut}} = 0$$

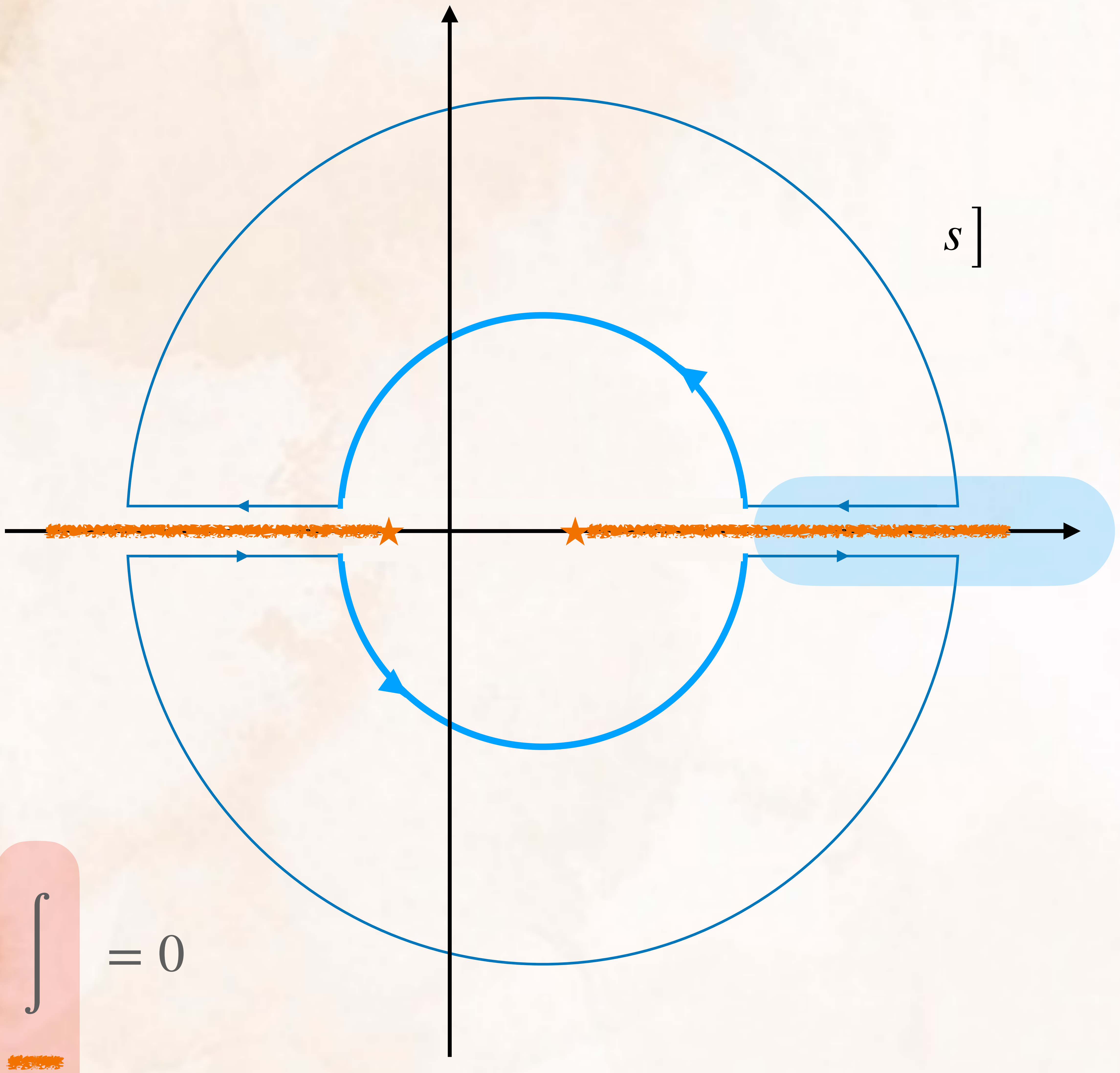
Unitarity $M - M^{\dagger} = iM^{\dagger}M$

$$\int_{\text{cut}} = \left[\int_{\text{cut}} + \int_{\text{cut}} \right] \frac{ds}{2\pi i} \frac{M(\alpha \to \alpha)}{|s|^n}$$

$$= 2 \times \int_{\Lambda^2}^{\infty} \frac{ds}{2\pi i} \frac{\mathrm{Disc}M}{|s|^n}$$

$$= \int_{\Lambda^2}^{\infty} \frac{ds}{\pi} \frac{\langle \alpha | M^{\dagger} M | \alpha \rangle}{|s|^n}$$

Positivity constraints: the philosophy



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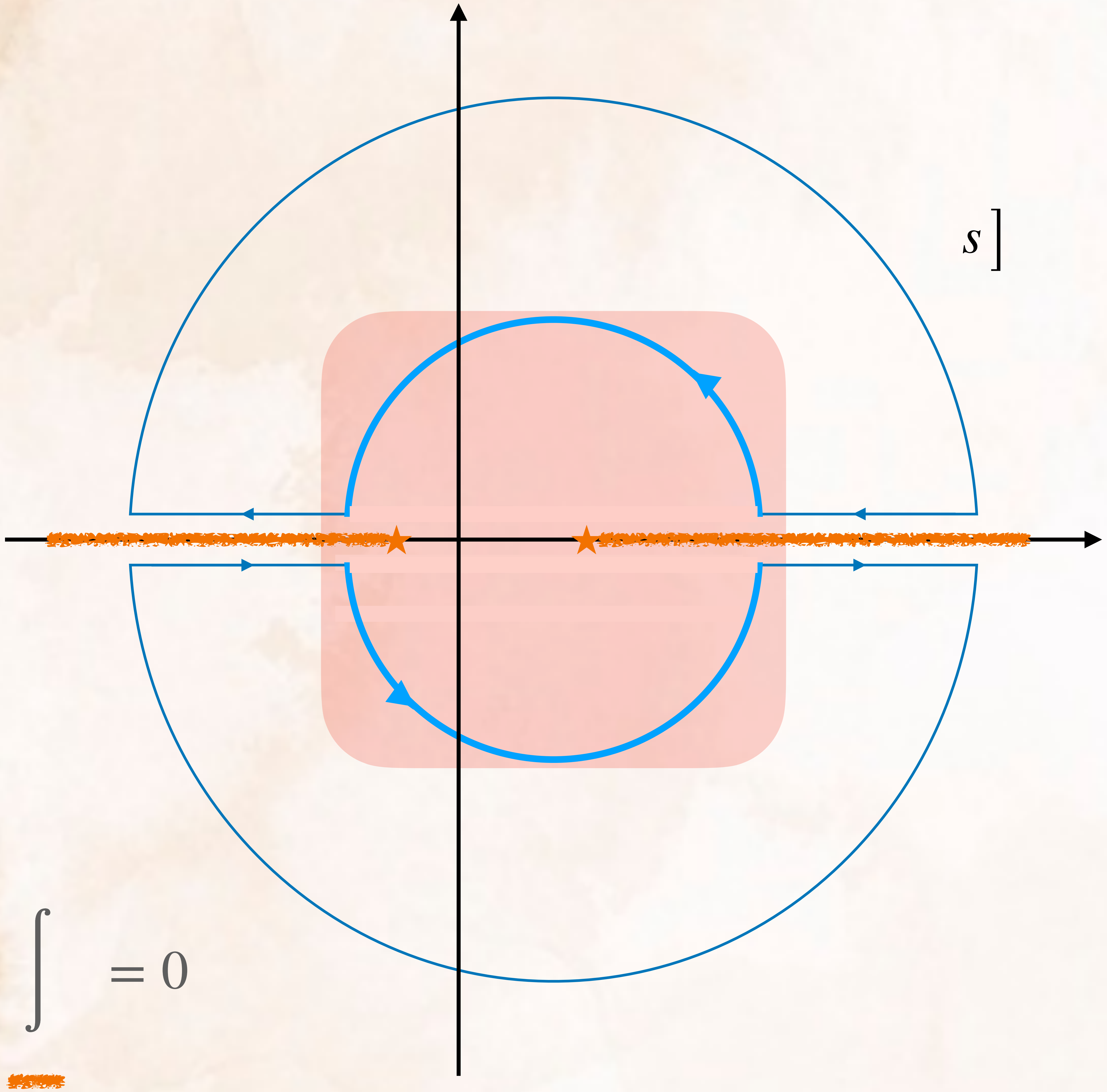
Positivity constraints: the philosophy



$$\int_{\circlearrowleft} \frac{ds}{2\pi i}\,\frac{M(\alpha \to \alpha)}{|\,s\,|^n} > 0$$

$$\int_{\circlearrowleft} + \int_{\text{cut}} = 0$$

Moving away from $t = 0$

$$|\beta\rangle = R(\theta)\,|\alpha\rangle$$

$$\int = \left[\int + \int \right] \frac{ds}{2\pi i} \frac{M(\alpha \to \beta)}{|s|^n}$$

Moving away from $t = 0$

Challenges

- Crossing for massive spinning states

$$M_{\lambda_1\lambda_2}(u,t) = \sum X^{\lambda_1'\lambda_2'\lambda_3'\lambda_4'}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\, M^{\lambda_3'\lambda_4'}_{\lambda_1'\lambda_2'}(s,t)$$

Crossing Symmetry ~~$M(u,t) = M(s,t)$~~

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- Non-forward discontinuities are not positive

$$\langle \alpha | M^\dagger M | \beta \rangle \not> 0$$

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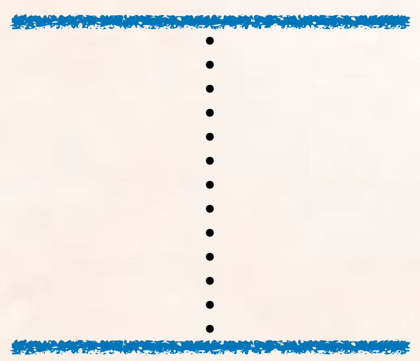

$$\sim \frac{Gs^2}{t}$$

Moving away from $t = 0$

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Solutions

- Consider all helicities contributions

Moving away from $t = 0$

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Spin 1: 17 amplitudes

Spin 2: 97 amplitudes

Moving away from $t = 0$

Challenges

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Spin 1: 17 amplitudes

Spin 2: 97 amplitudes

- Find positive functional to obtain positivity and convergence

$$\int dt\, \Psi(t) \langle \alpha | M^\dagger M | \beta \rangle \geq 0$$

Moving away from $t = 0$

Challenges

- Crossing for massive spinning states

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$$\int dt\, \underbrace{\Psi(t)}_{\text{SDPB}} \langle \alpha | M^\dagger M | \beta \rangle \geq 0$$

SDPB heavy numerics

Moving away from $t = 0$

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$$\int dt \underbrace{\Psi(t)}\langle \alpha | M^\dagger M | \beta \rangle \geq 0$$

SDPB heavy numerics

Great and optimal bounds!

Moving away from $t = 0$

Challenges

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- Non-forward discontinuities are not positive

$$\langle\alpha|M^\dagger M|\beta\rangle \not\succ 0$$

- Arcs diverging at $t = 0$

Simpler solutions?

- Consider all helicities contributions

Spin 1: 17 amplitudes

Spin 2: 97 amplitudes

- Find positive functional to obtain positivity and convergence

$$\int dt\, \underbrace{\Psi(t)}_{\text{SDPB}}\langle\alpha|M^\dagger M|\beta\rangle \geq 0$$

SDPB heavy numerics

Great and optimal bounds!

...but a lot of hard work

Moving away from $t = 0$

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Part I

- $|t| \gg m^2$

Application to massive gravity

Part II

- $\ell \gg 1$

Bounding classical observables

Part I

Large t : Bounding Massive Gravity

Gravity as an EFT

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,[R$$

⇒ **2 d.o.f**

$$\mathcal{M}(hh \to hh) \sim \frac{s}{M_{Pl}^2}$$

$\frac{1}{M_{Pl}}$ $\frac{1}{H_0}$

Massive Gravity as an EFT

dRGT gravity

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,[\,R + m^2 V\,]$$

⇨ ~~2~~ d.o.f

2 + 3

$$\mathcal{M}(hh \to hh) \sim \cancel{\frac{s}{M_{Pl}^2}} \sim \frac{s^3}{\Lambda_3^6} f(c_3, d_5)$$

$$\Lambda_3 = \left(m^2 M_{Pl}\right)^{1/3}$$

$\frac{1}{M_{Pl}}$ $\qquad$ $\frac{1}{\Lambda_3}$ $\qquad$ $\frac{1}{H_0} \sim \frac{1}{m}$

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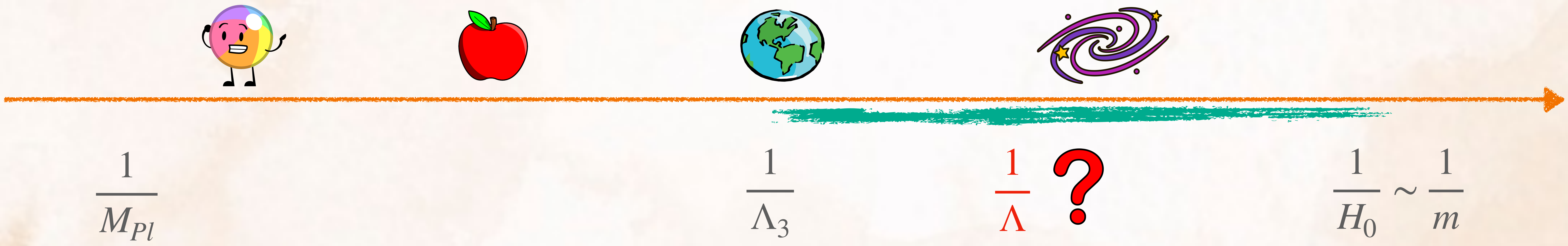

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$$\Lambda_3 = \left(m^2 M_{Pl}\right)^{1/3}$$

$$\frac{1}{M_{Pl}} \qquad \frac{1}{\Lambda_3} \qquad \frac{1}{\Lambda}\,\textbf{?} \qquad \frac{1}{H_0} \sim \frac{1}{m}$$

Positivity to constrain $\left\{\begin{array}{ll}\text{Phase-space of} & (c_3, d_5) \\ \text{Physical cutoff} & \Lambda\end{array}\right.$

Positivity in dRGT gravity

Challenges

- Crossing for massive spinning states

$$M_{\lambda_1\lambda_2}(u,t) = \sum X^{\lambda_1'\lambda_2'\lambda_3'\lambda_4'}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\, M^{\lambda_3'\lambda_4'}_{\lambda_1'\lambda_2'}(s,t)$$

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$$\langle \alpha \,|\, M^\dagger M \,|\, \beta \rangle \ngtr 0$$

- Arcs diverging at $t = 0$

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Key Ideas

- $m^2 \ll |t| \ll s$ Crossing simplifies!

$$M_{\lambda_1\lambda_2}(u,t) = M_{-\lambda_1\lambda_2}(s,t) + \mathcal{O}\left(\sqrt{t}m/s\right)$$

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- Unitarity is more than forward positivity!

$$|\langle 1^{\lambda_1} 2^{\lambda_2} | M^\dagger M | 3^{\lambda_1} 4^{\lambda_2} \rangle| \le \langle 1^{\lambda_1} 2^{\lambda_2} | M^\dagger M | 1^{\lambda_1} 2^{\lambda_2} \rangle$$

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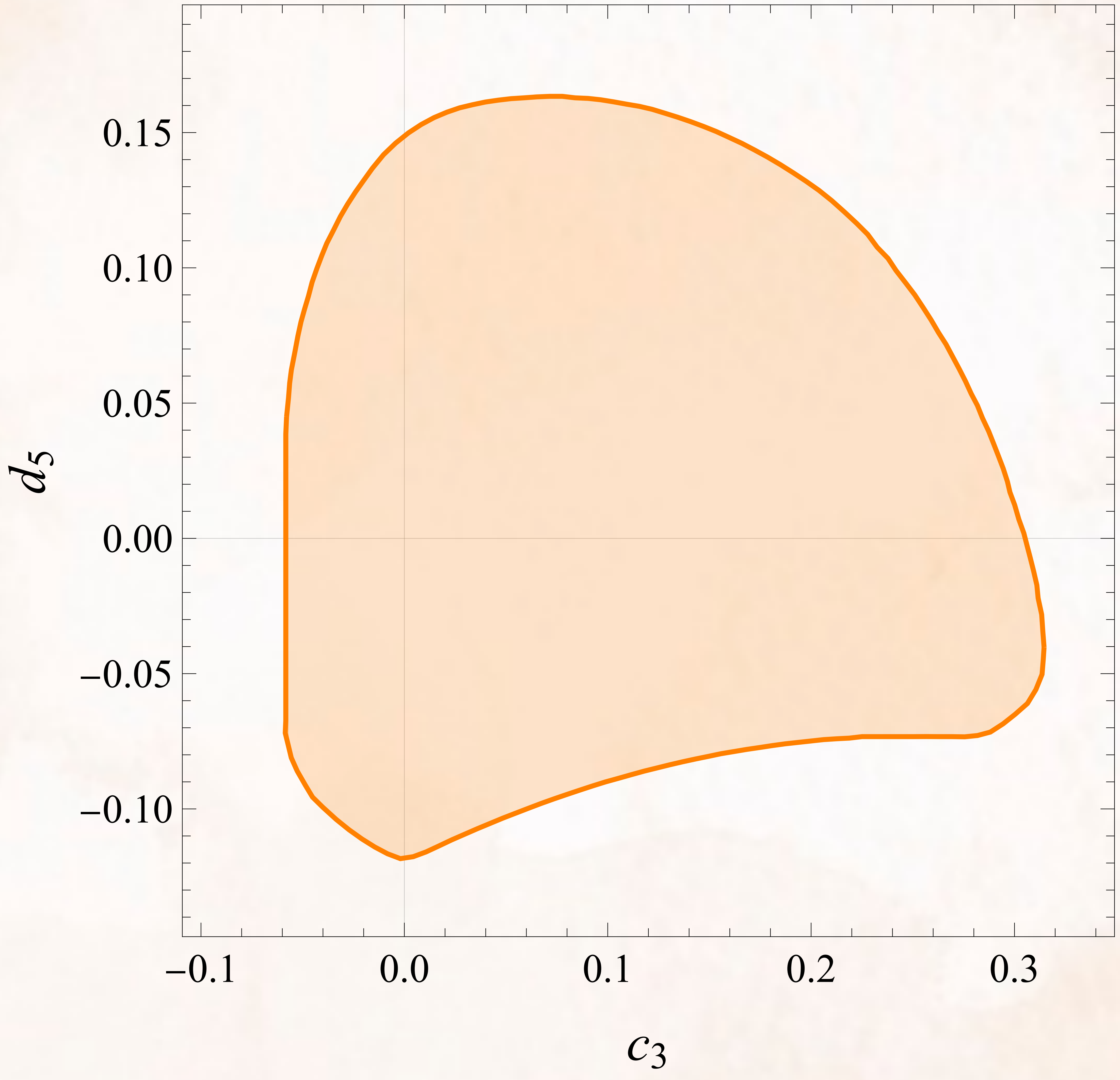
$$A_{\lambda_1\lambda_3}(t) \sim \oint$$

$$\frac{|A_{\lambda_1\lambda_2}(t)|}{A_{\lambda_1\lambda_2}(0)} < 1 + \mathcal{O}\left(\frac{\sqrt{tm}}{s}\right)$$

Phase-space constraints at $t = 0$

$$A_{\lambda_1\lambda_2}(0) > 0$$



'16, Cheung, Remmen

Phase-space constraints at large t

$$m^2 \ll |t| \ll s$$

$$\frac{|A_{\lambda_1\lambda_2}(t)|}{A_{\lambda_1\lambda_2}(0)} < 1 + \mathcal{O}\left(\frac{\sqrt{t}m}{s}\right)$$

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$$A_{\lambda_1\lambda_2}(t) \longrightarrow \frac{t}{\Lambda_3^6} g_{\lambda_1\lambda_2}(c_3, d_5)$$

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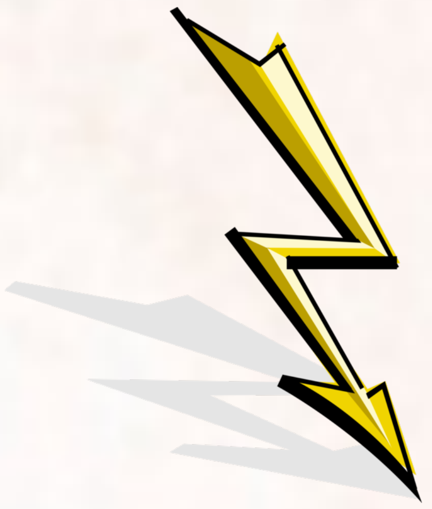

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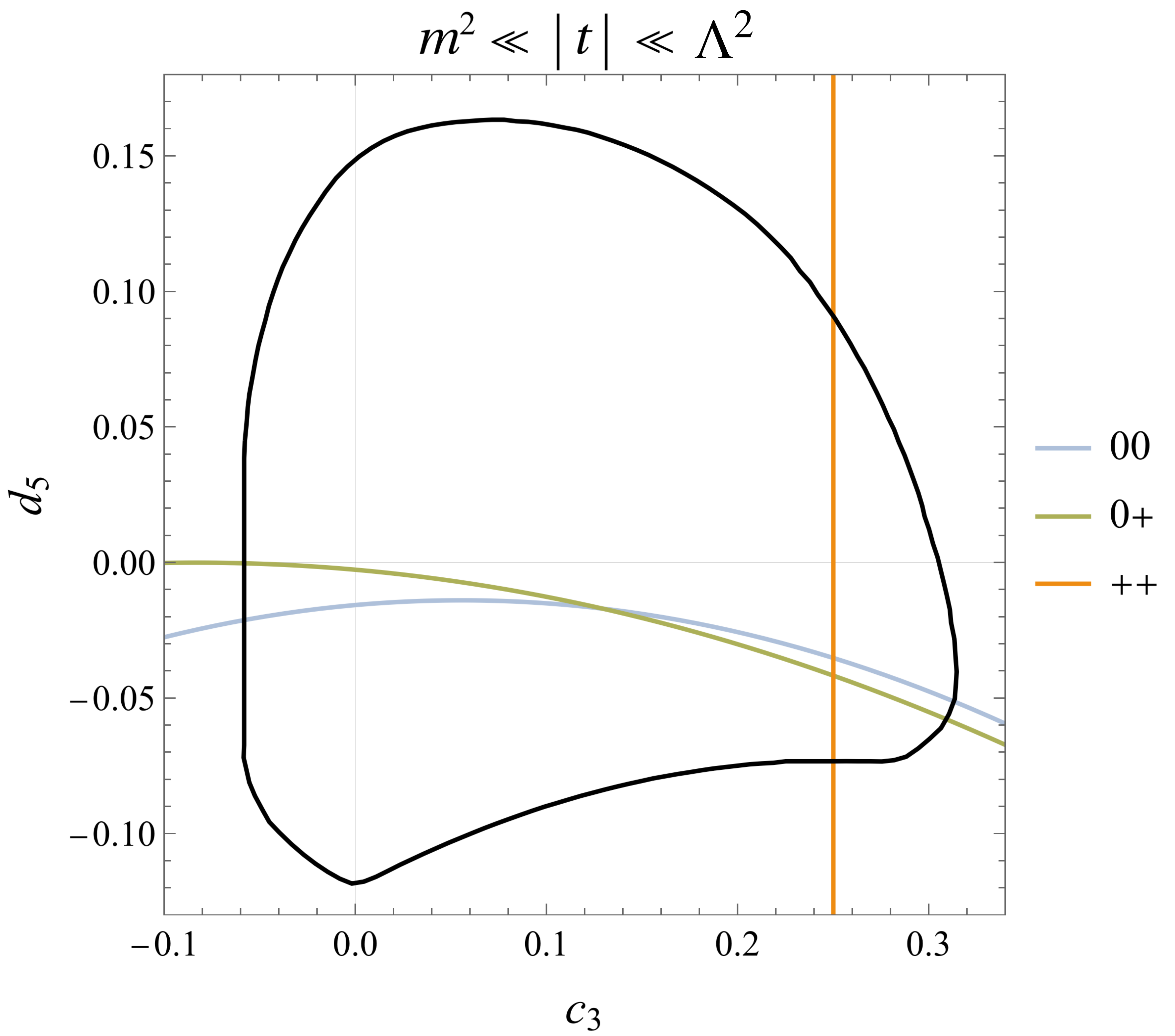

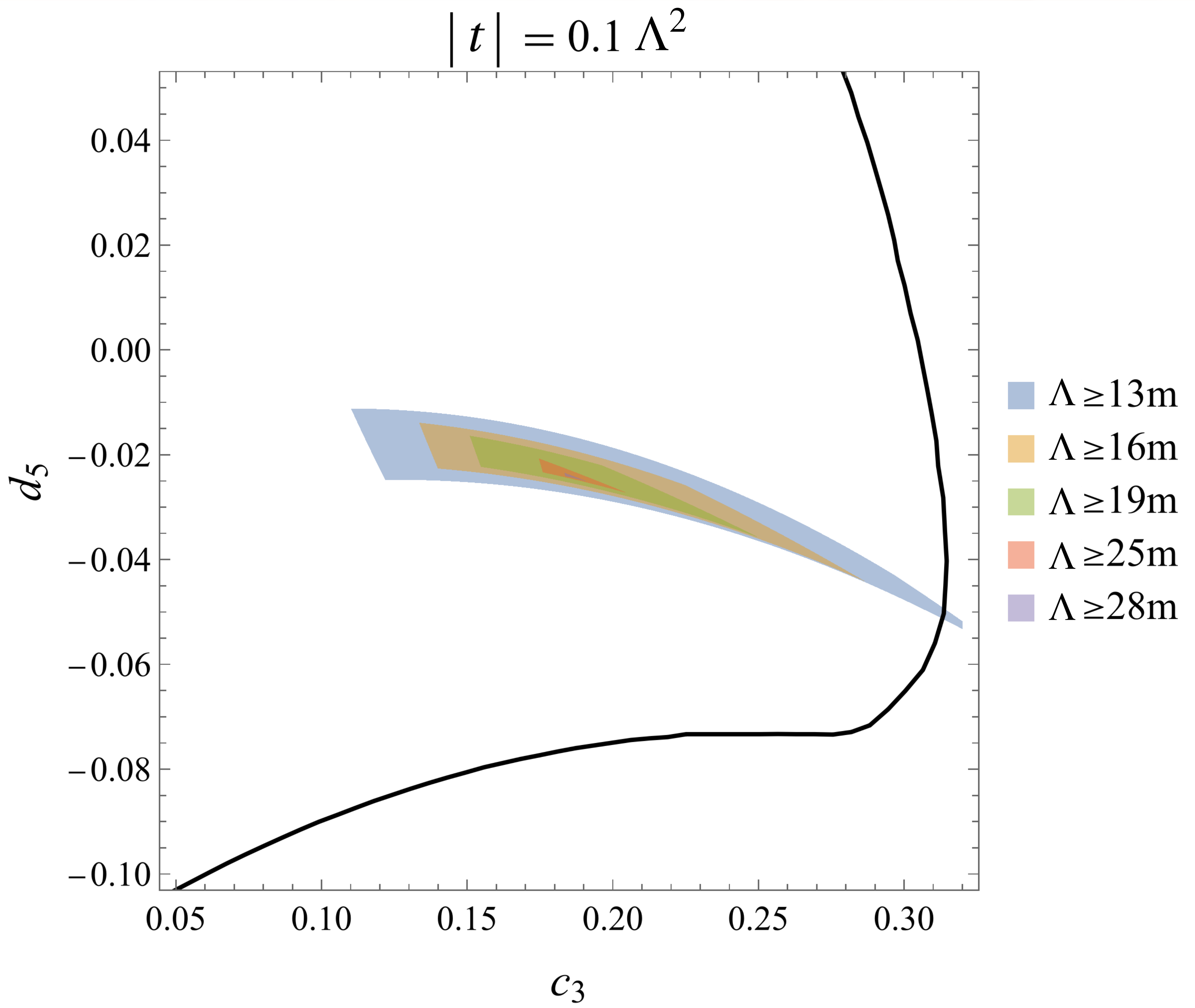

Phase-space constraints at large t

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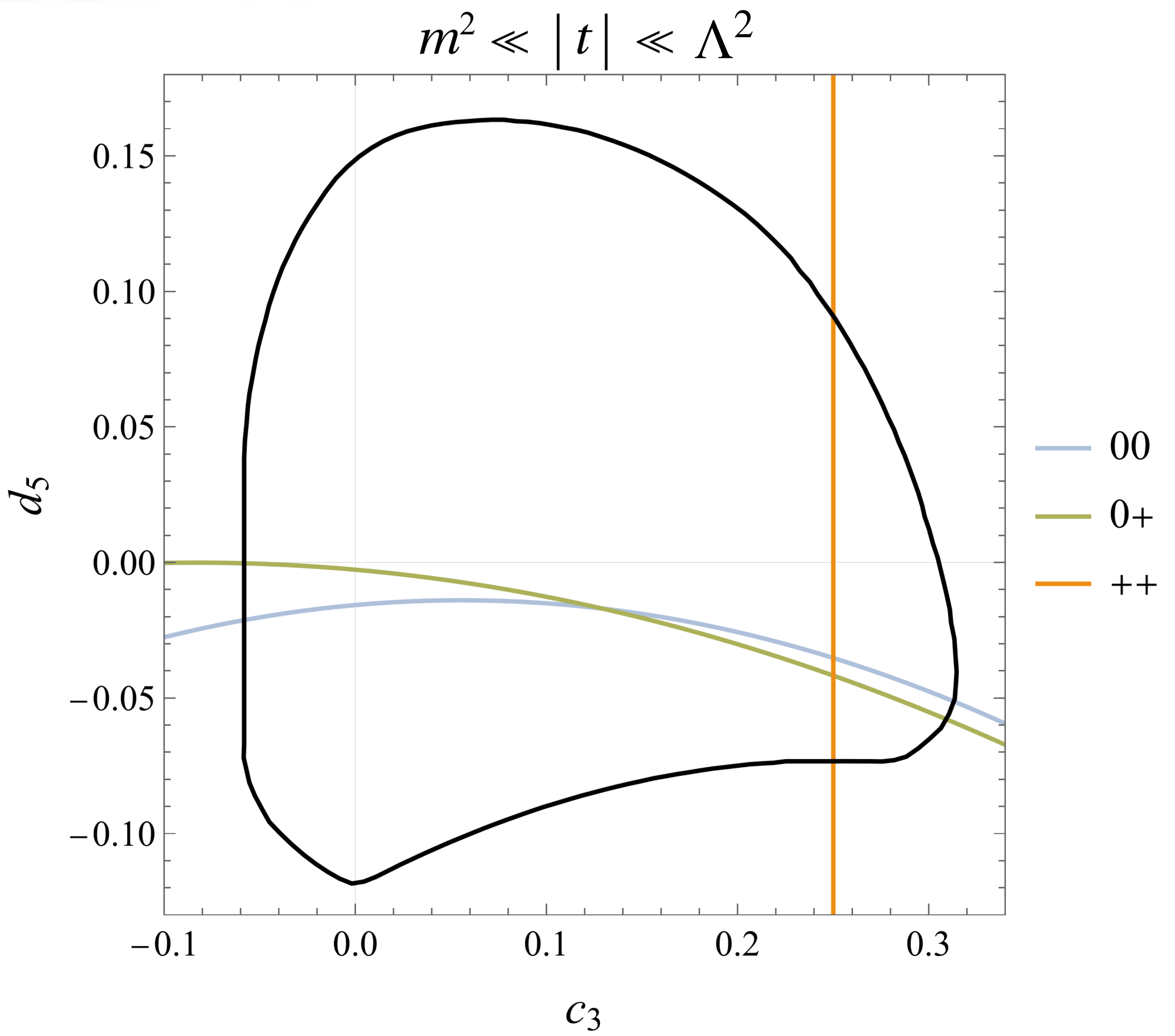
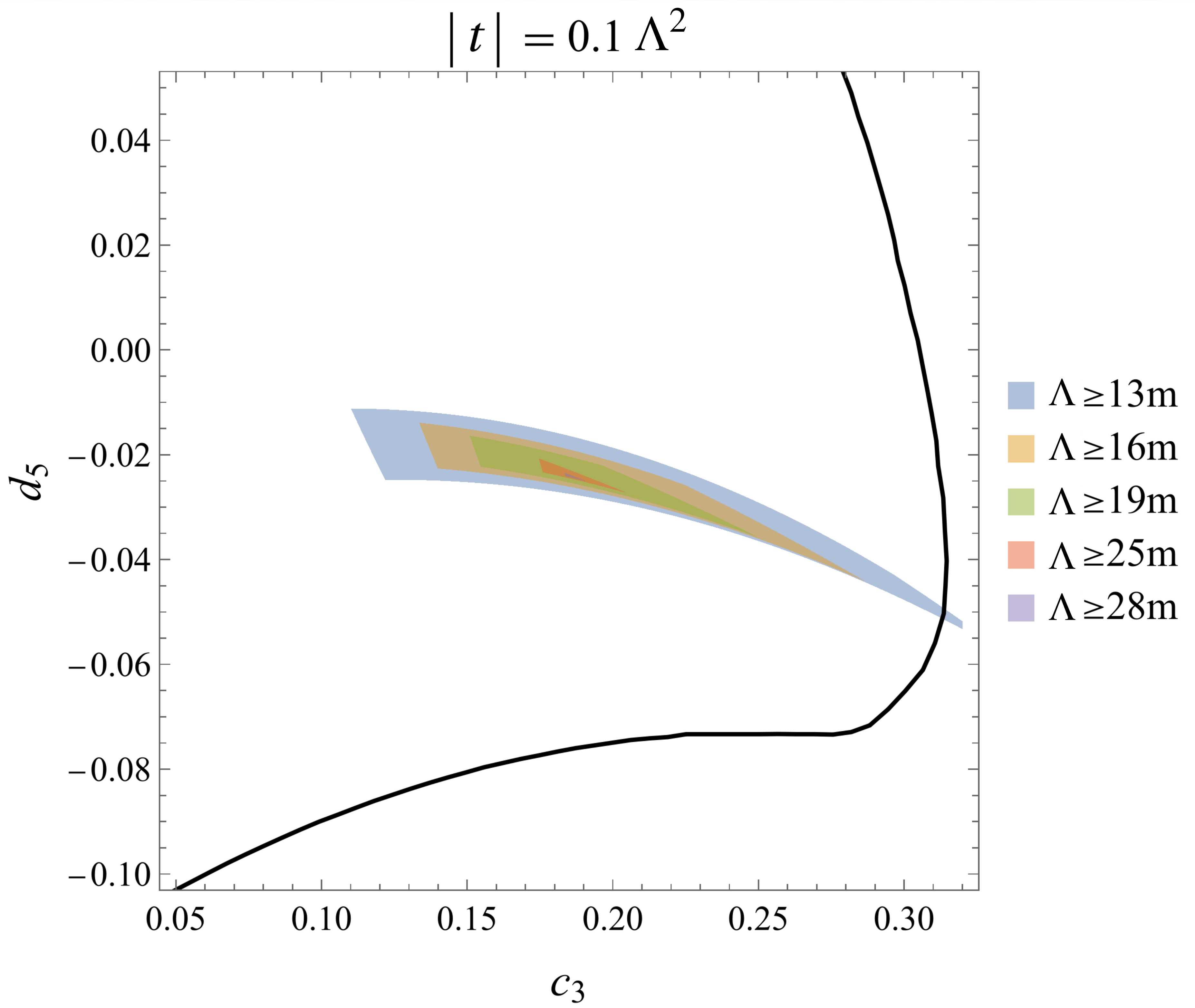
$$\frac{|A_{\lambda_1\lambda_2}(t)|}{A_{\lambda_1\lambda_2}(0)} < 1 + \mathcal{O}\left(\frac{\sqrt{t}m}{s}\right)$$

$$\Lambda \leq 30\, m \left(\frac{0.1}{-t/\Lambda^2}\right)^{1/2}$$





What is the regime of validity of dRGT gravity?

dRGT gravity

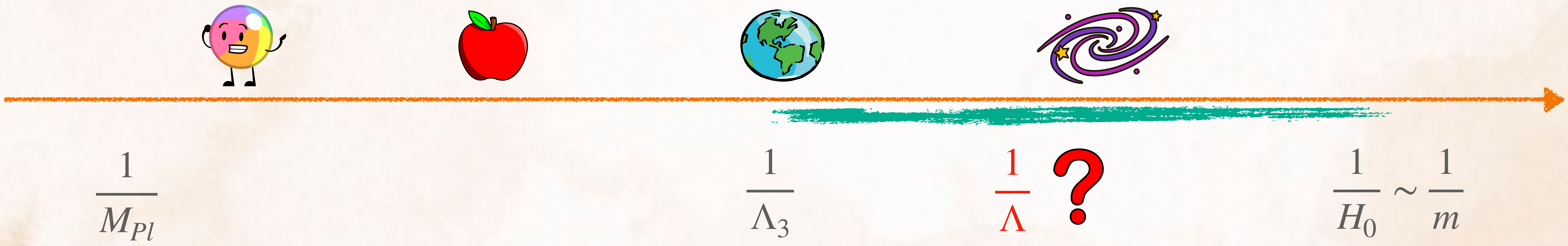
$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,[\,R + m^2 V\,]$$

~~2~~ **d.o.f**

2 + 3

$$\mathcal{M}(hh \to hh) \sim \frac{s}{M_{Pl}^2} \sim \frac{s^3}{\Lambda_3^6} f(c_3, d_5)$$

$$\Lambda_3 = \left(m^2 M_{Pl}\right)^{1/3}$$



Positivity to constrain {
- Phase-space of (c_3, d_5)
- Physical cutoff Λ

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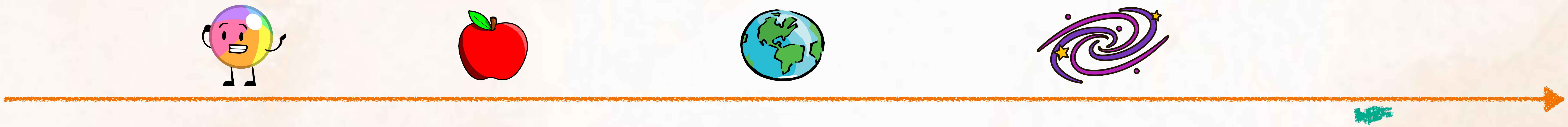

$\frac{1}{M_{Pl}}$ $\frac{1}{\Lambda_3}$ $\frac{1}{H_0}$

Positivity to constrain {
- Phase-space of (c_3, d_5)
- Physical cutoff $\Lambda \leq O(10)m$

“Massive gravity does not exist!”

Discussion at Strings 2023 “The future of the S-matrix”

Simon Caron-Huot and Sebastian Mizera

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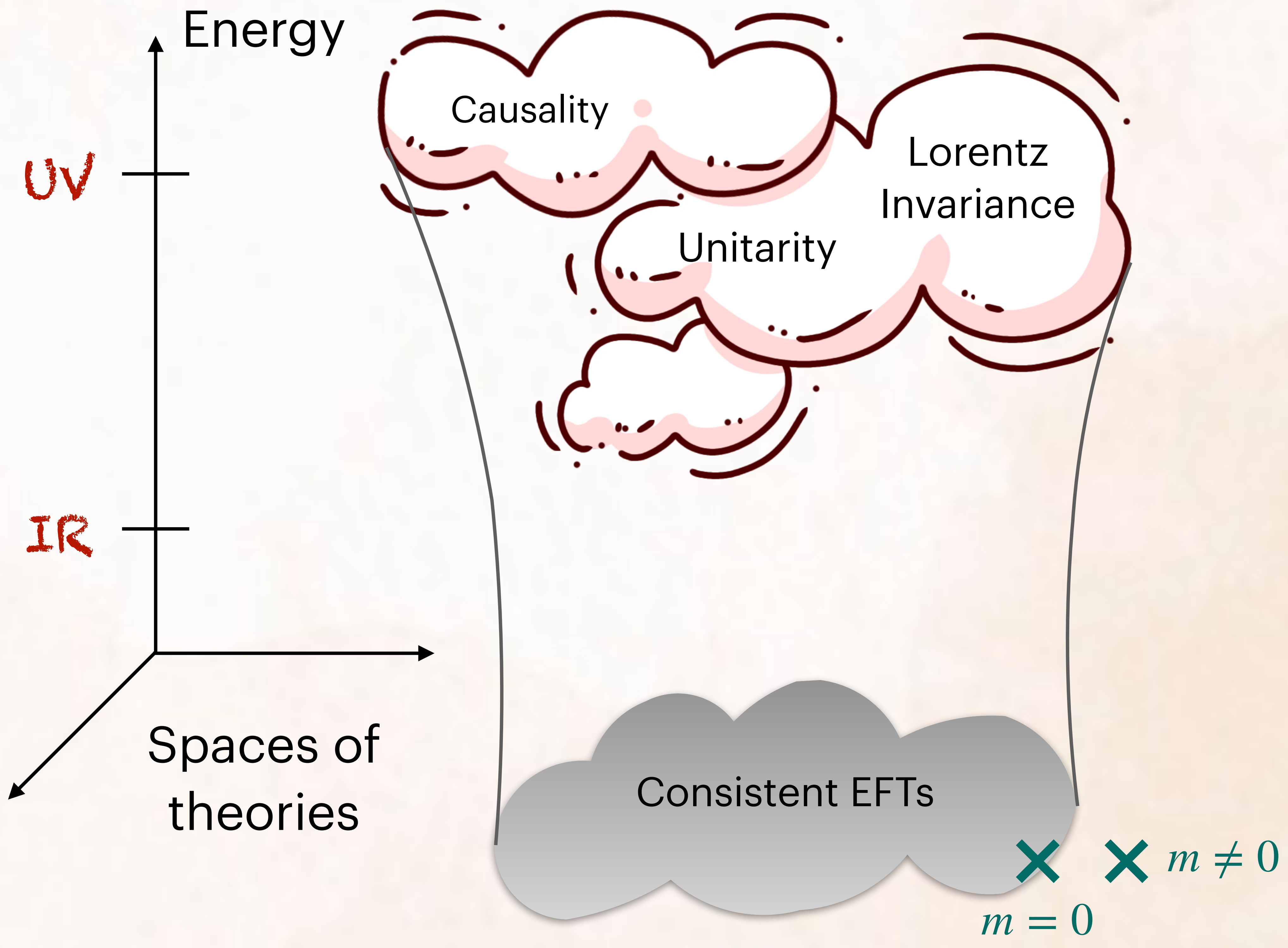


Part II

Large ℓ : Bounding classical observables

The large angular momentum limit

Challenges

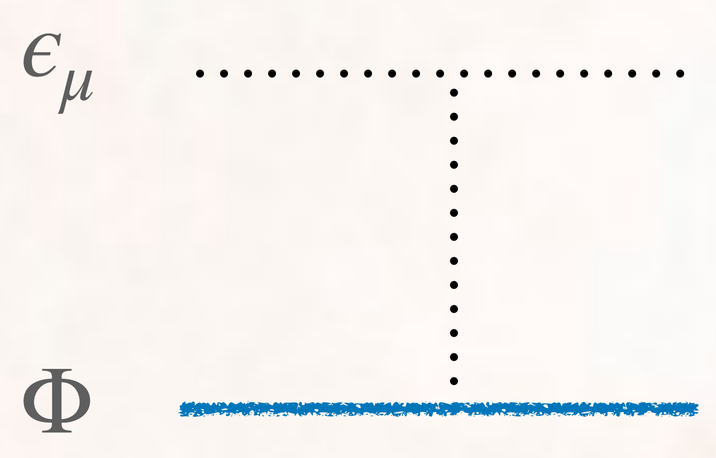
- Crossing for massive spinning states

$$M_{\lambda_1\lambda_2}(u,t) = \sum X^{\lambda_1'\lambda_2'\lambda_3'\lambda_4'}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\, M^{\lambda_3'\lambda_4'}_{\lambda_1'\lambda_2'}(s,t)$$

- Non-forward discontinuities are not positive

$$\langle \alpha | M^\dagger M | \beta \rangle \ngtr 0$$

- Arcs diverging at $t = 0$



The large angular momentum limit

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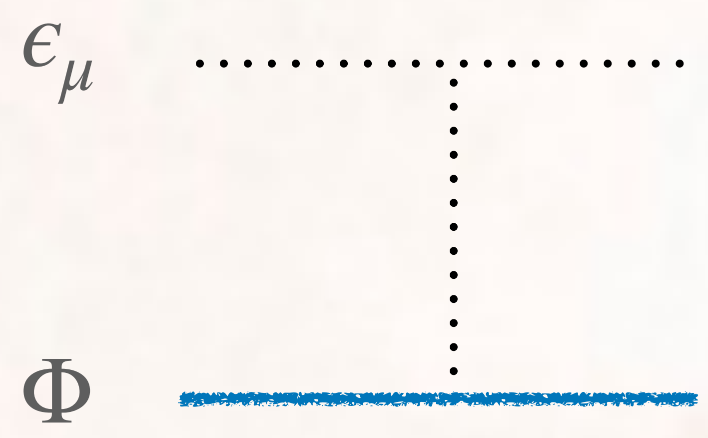

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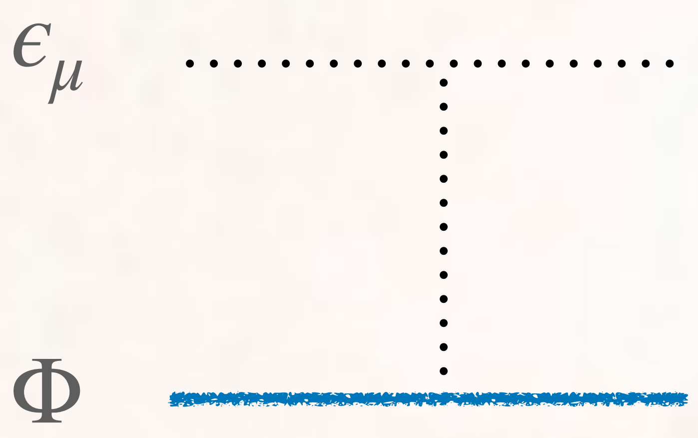
Key Ideas



- Large ℓ limit of partial waves

$$\langle \alpha | M^\dagger M | \beta \rangle = \mathcal{N} \sum (2\ell + 1) \langle \ell \alpha | M^\dagger M | \ell \beta \rangle d_\ell^{\alpha\beta}(\theta)$$

The large angular momentum limit

Challenges

- Crossing for massive spinning states

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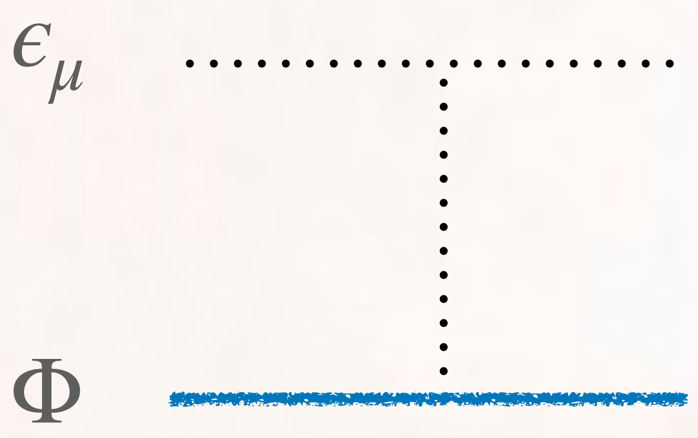

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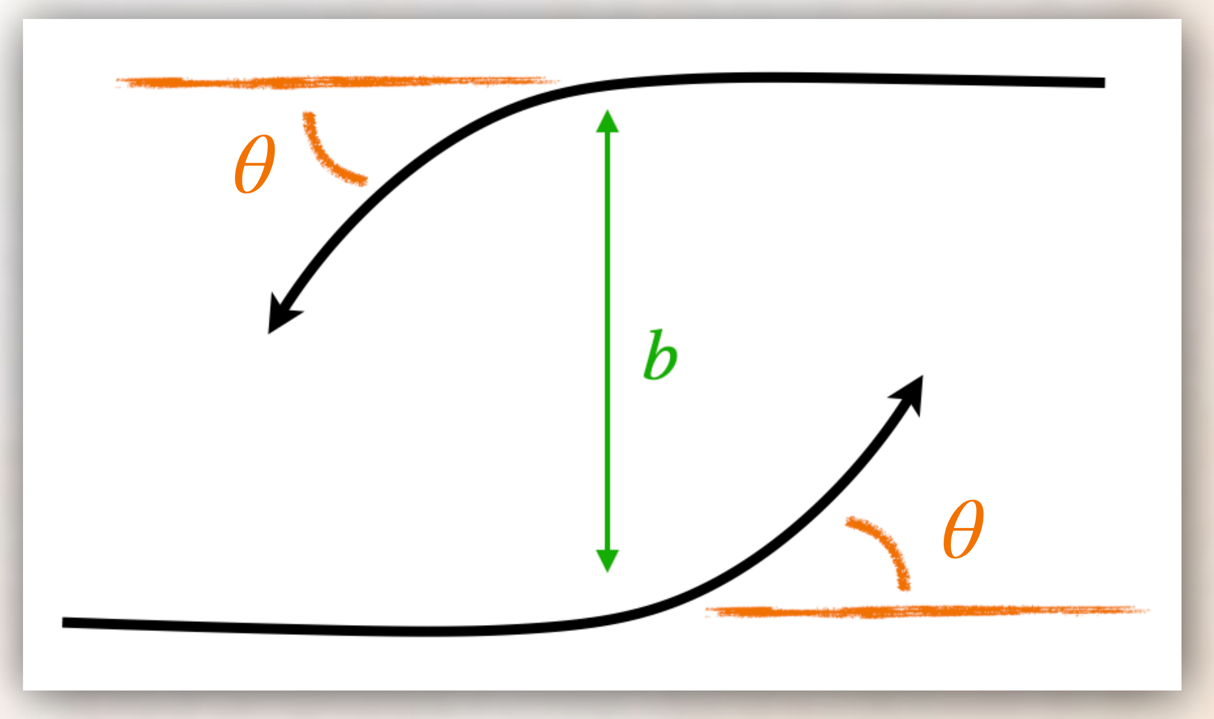
Recover long distance semi-classical scattering



$$\ell \sim b\sqrt{s}$$

The large angular momentum limit

Challenges

- Crossing for massive spinning states

$$M_{\lambda_1\lambda_2}(u,t) = \sum X^{\lambda_1'\lambda_2'\lambda_3'\lambda_4'}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\, M^{\lambda_3'\lambda_4'}_{\lambda_1'\lambda_2'}(s,t)$$

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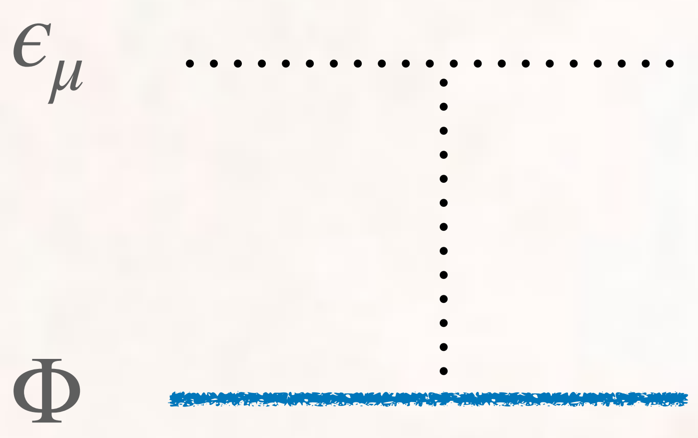

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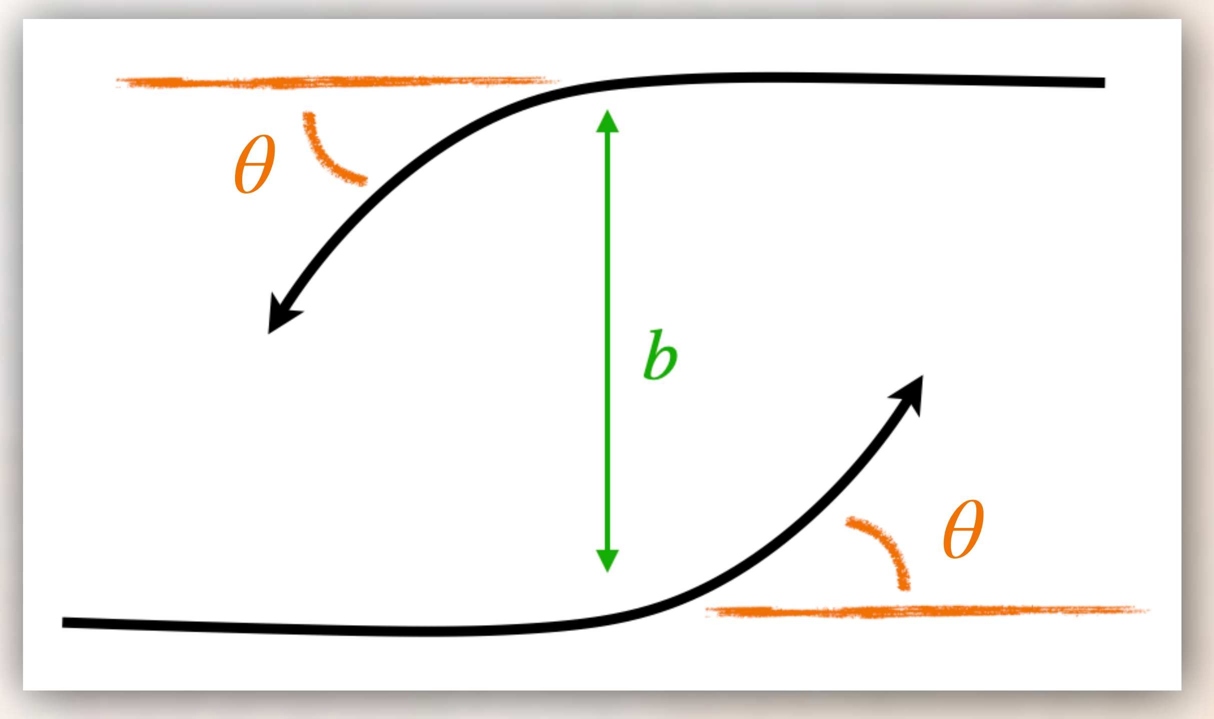

$\ell \sim b\sqrt{s}$

Fourier transform $\sim$ Smearing in t

The large angular momentum limit

Challenges

- Crossing for massive spinning states

$$M_{\lambda_1\lambda_2}(u,t) = \sum X^{\lambda'_1\lambda'_2\lambda'_3\lambda'_4}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\, M^{\lambda'_3\lambda'_4}_{\lambda'_1\lambda'_2}(s,t)$$

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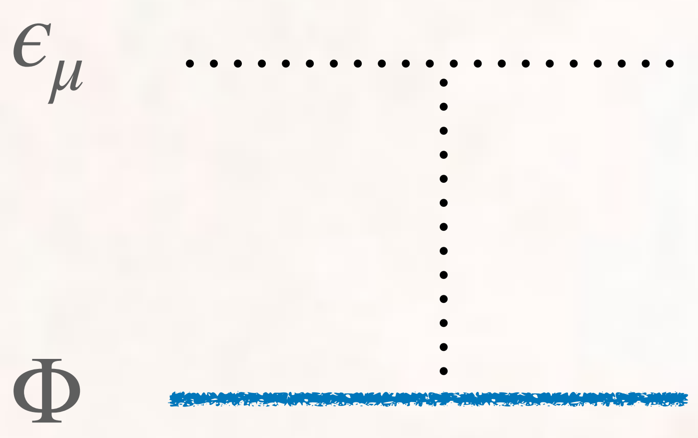

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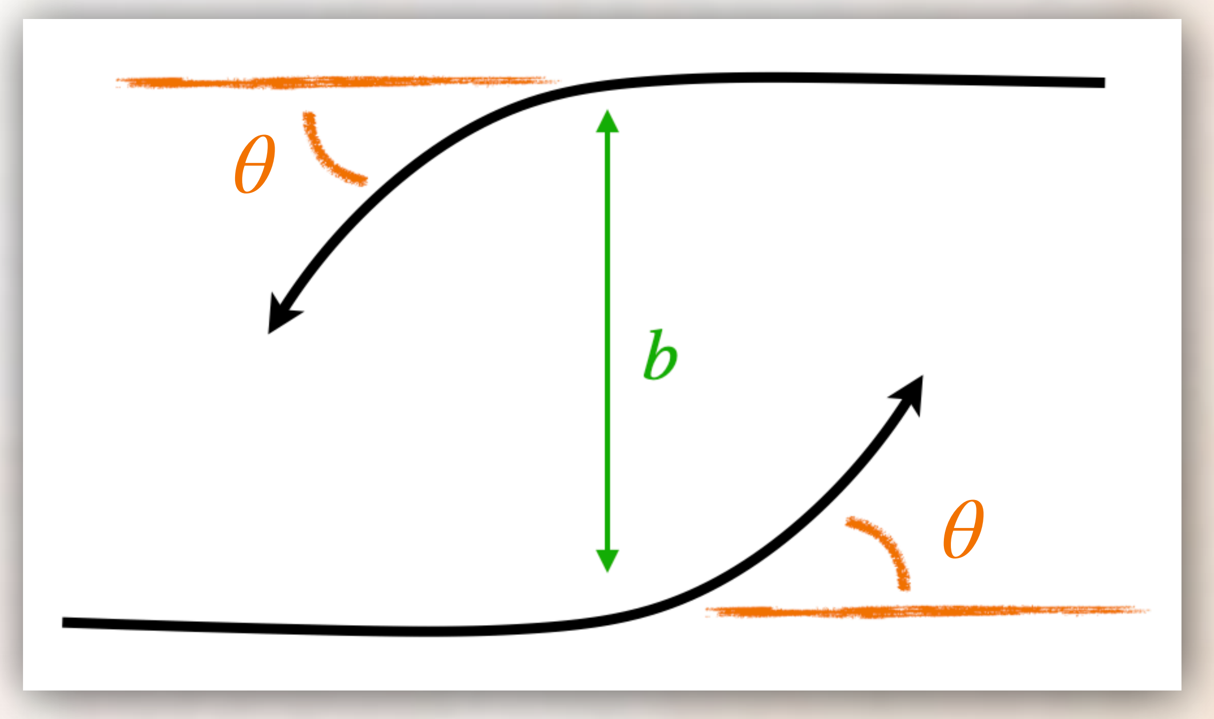
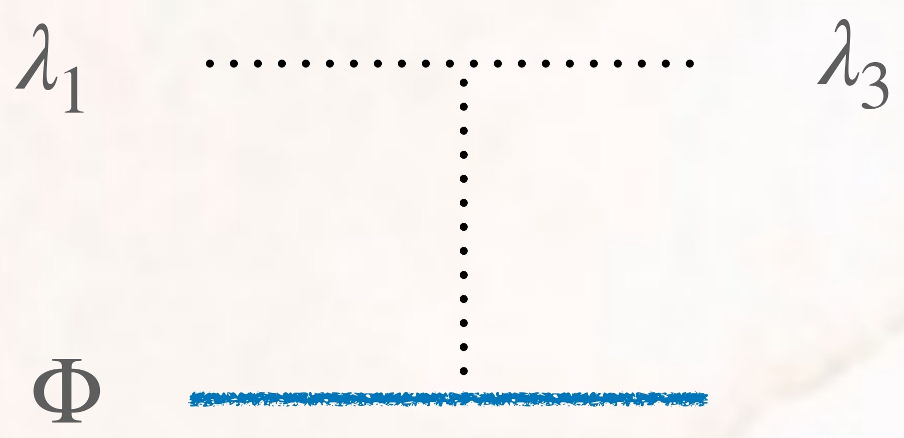


$$\ell \sim b\sqrt{s}$$

Fourier transform $\sim$ Smearing in t

'22, Caron-Huot, Li, Parra-Martinez, Simons-Duffins

From dispersion relations to eikonal scattering

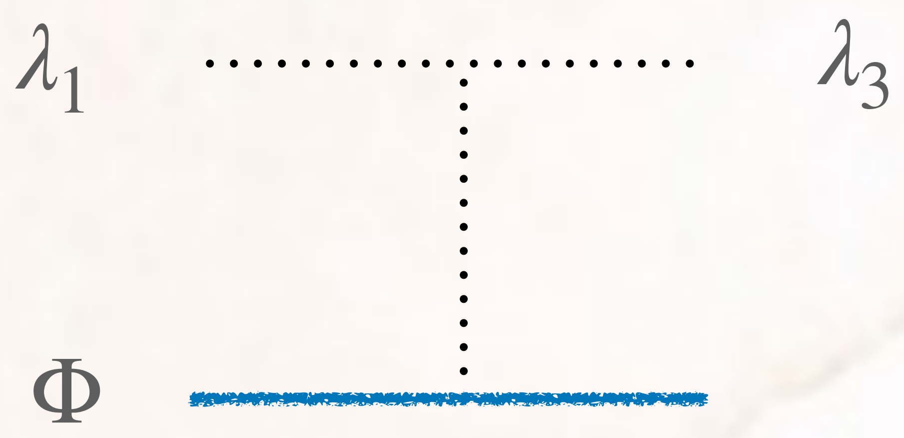


$$\int \quad = \quad \int$$

From dispersion relations to eikonal scattering

Project on definite angular momentum

$$\int dt\, \Psi_\ell(t) \int \; = \; \int dt\, \Psi_\ell(t) \int$$



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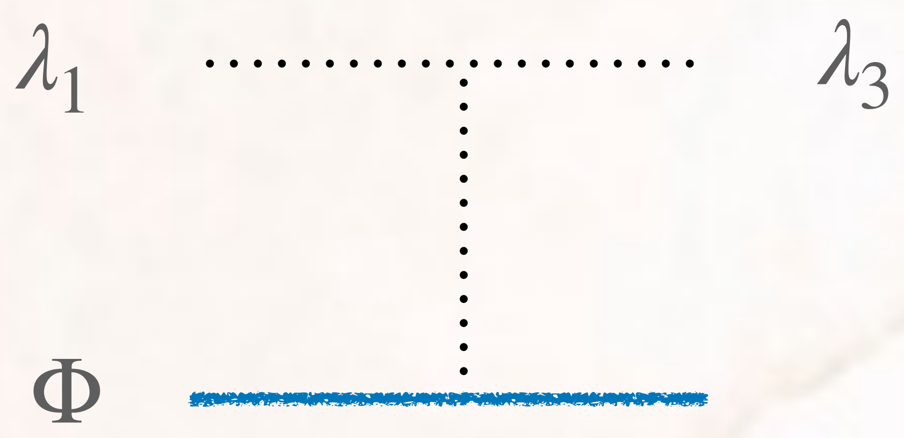

Project on definite
angular momentum

$\ell \to \infty$

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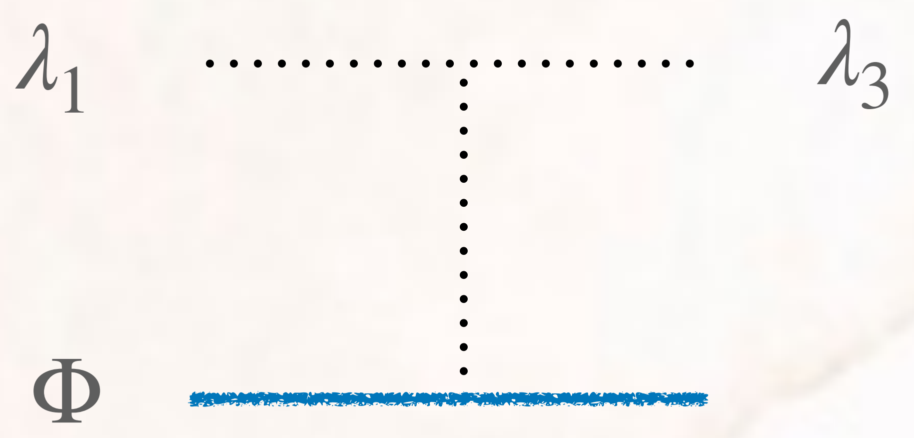
From dispersion relations to eikonal scattering



Project on definite
angular momentum

$\ell \to \infty$

$$\int dt\, \Psi_\ell(t) \int \quad = \quad \int dt\, \Psi_\ell(t) \int$$

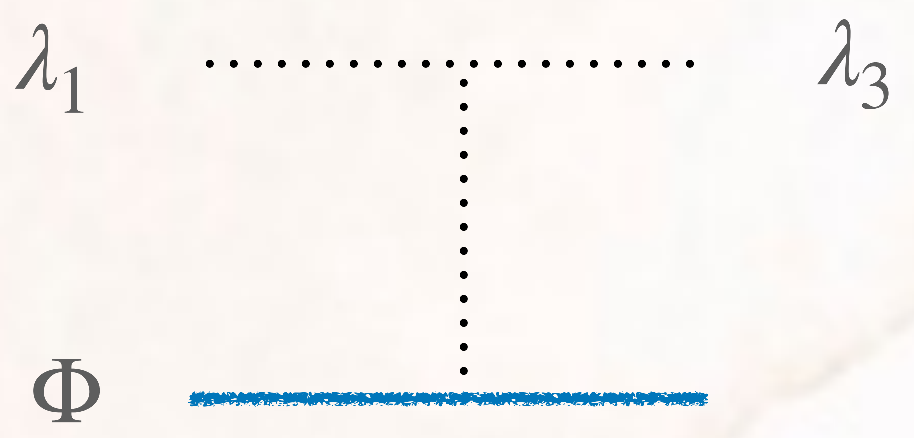
$$\int \frac{ds}{(s-m^2)^n} \int d^2q\, M_{\lambda_1 \lambda_3}(s, q^2)\, e^{iqb}$$

Fourier transform

$$t = q^2$$

$$\ell = b\sqrt{s}$$

From dispersion relations to eikonal scattering



Project on definite angular momentum

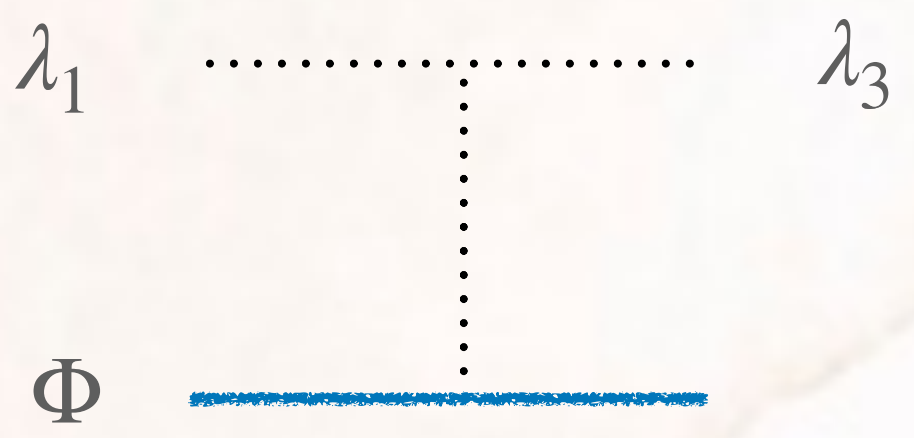
$\ell \to \infty$

$$\int dt\, \Psi_\ell(t) \int_{\circlearrowleft} = \int dt\, \Psi_\ell(t) \int$$

Amplitude in the eikonal regime

$$\int_{\circlearrowleft} ds \frac{M_{\lambda_1\lambda_3}(s,b)}{(s-m^2)^n}$$

From dispersion relations to eikonal scattering



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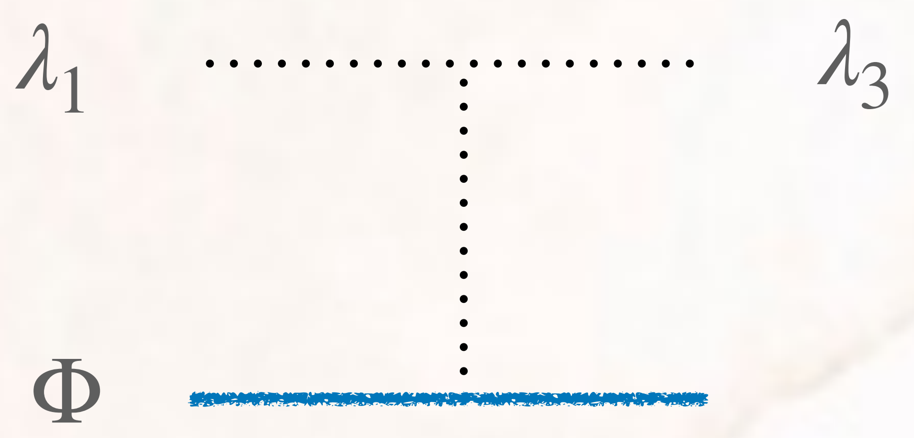
$$\succ 0$$

From dispersion relations to eikonal scattering

Positive Eikonal arcs

$$\oint ds \frac{M_{\lambda_1 \lambda_3}(s,b)}{\left(s - m^2\right)^n} > 0$$



From dispersion relations to eikonal scattering

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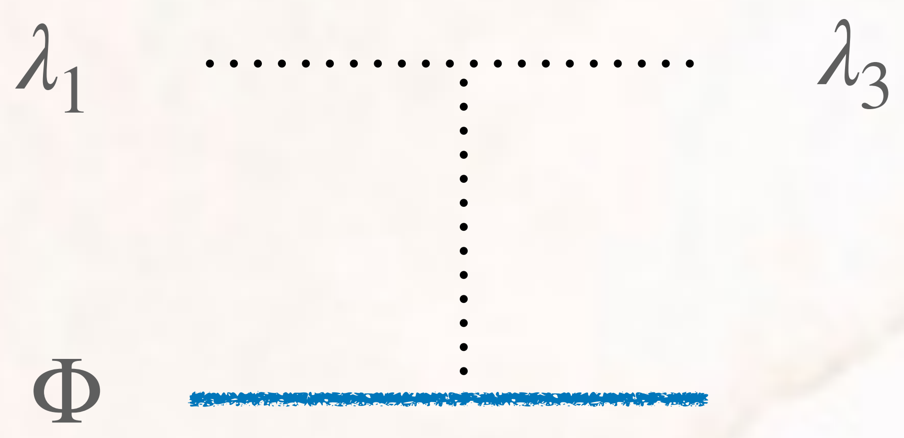
$$\oint ds \frac{M_{\lambda_1 \lambda_3}(s,b)}{\left(s-m^2\right)^n} > 0$$

$$M(s,b) \sim e^{2i\delta(s,b)} - 1$$



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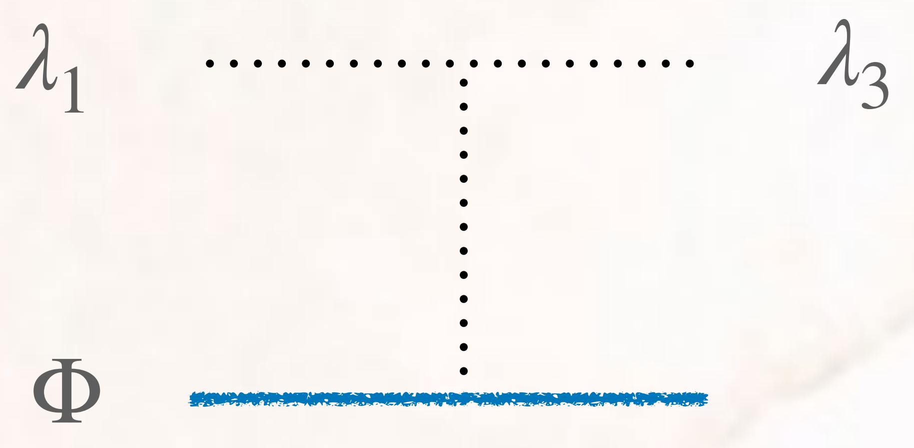

Positive Eikonal arcs

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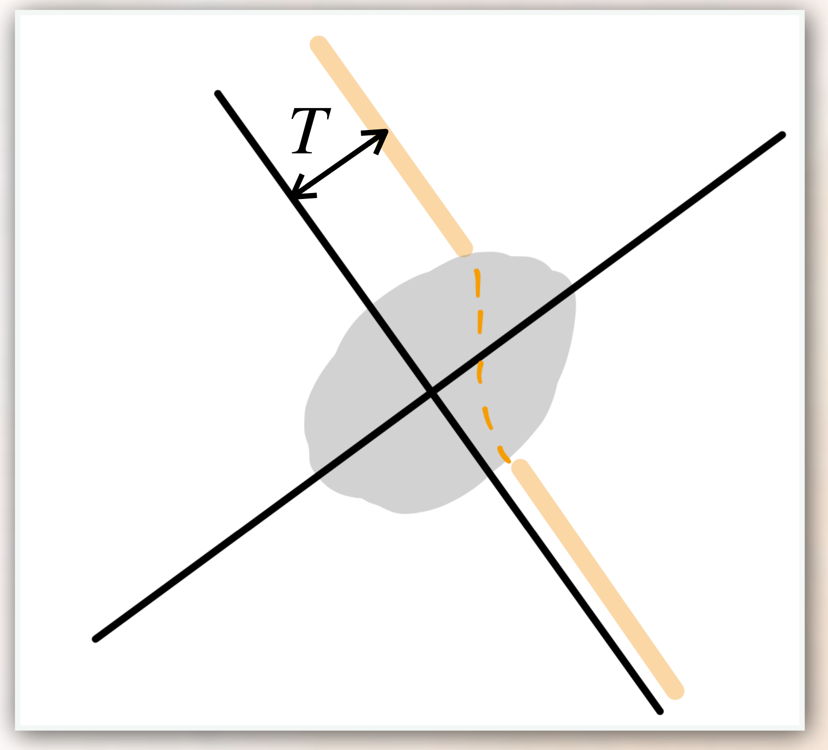
$$M(s,b) \sim e^{2i\delta(s,b)} - 1$$

$$n = 2 \ : \qquad T_{\lambda_1\lambda_3}(\omega,b) = 2\frac{\partial}{\partial\omega}\delta_{\lambda_1\lambda_3}(s,b) > 0$$

Positivity of the time-delay



S-Matrix principles ⇒ Asymptotic Causality

Analyticity + Unitarity

Example: asymptotic causality at tree level

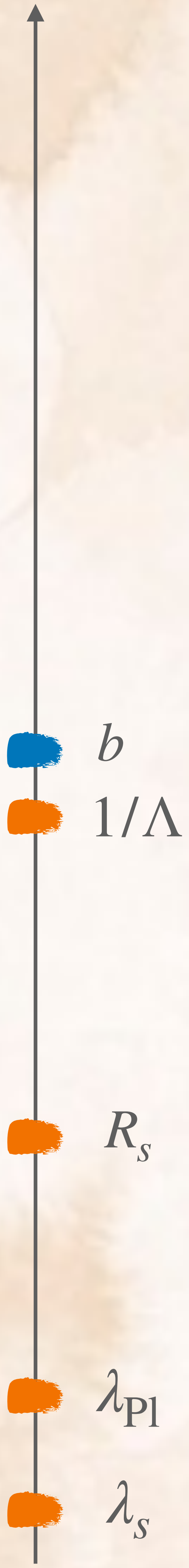
$$T_{\pm}(s,b) \sim GM \left(-\log \frac{b}{b_{IR}} \pm \frac{\beta}{b^2} \right) > 0$$

Causality violation at : $b \sim 1/\Lambda$

Bound on β : $|\beta| \lesssim 1/\Lambda^2 \log \frac{1/\Lambda}{b_{IR}}$

'14, Camanho, Edelstein, Maldacena, Zhiboedov

Example: asymptotic causality at tree level



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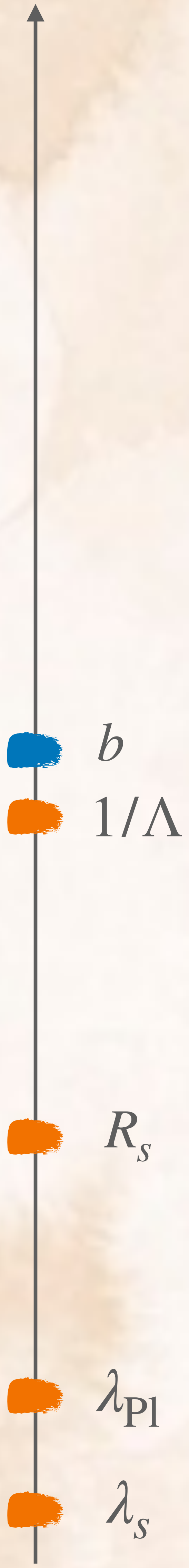

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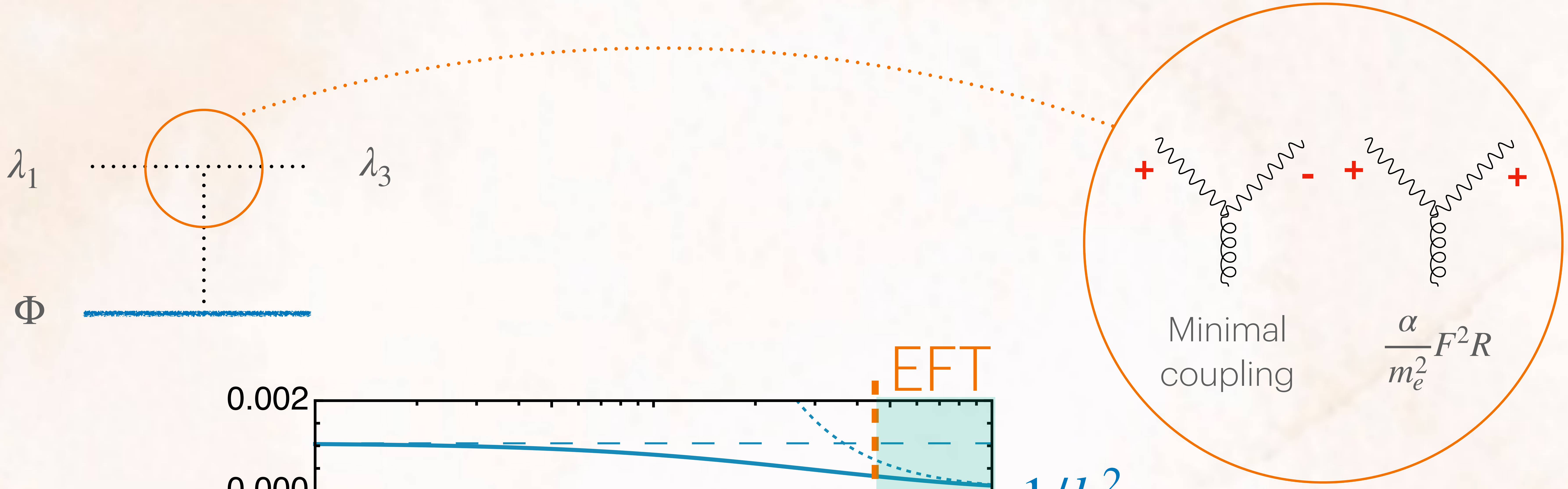
Tree-level solution to causality violation requires tower of higher spin

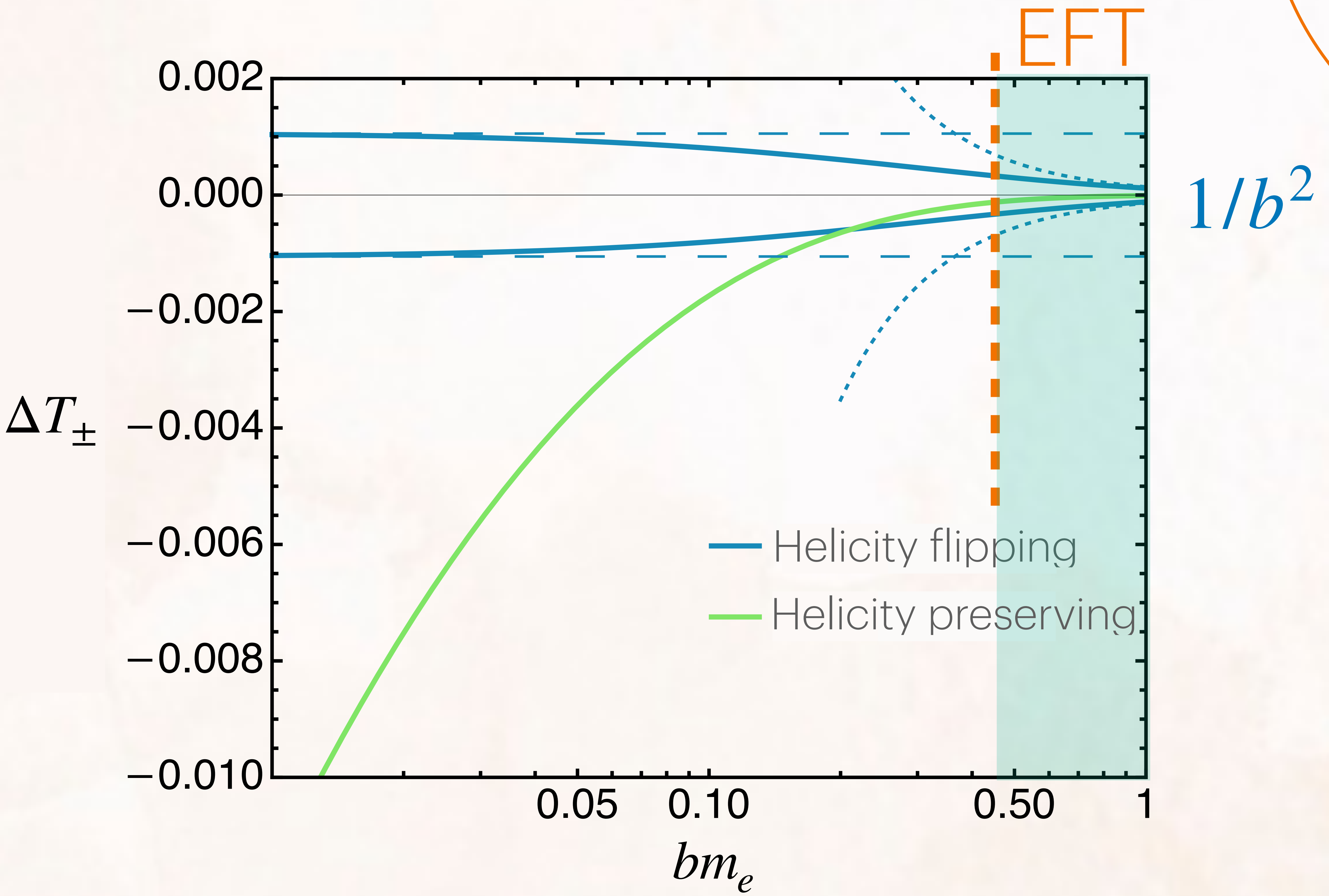
’14, Camanho, Edelstein, Maldacena, Zhiboedov

QED docet

b
$1/m_e$
R_s
$\lambda_{\rm Pl}$
λ_s

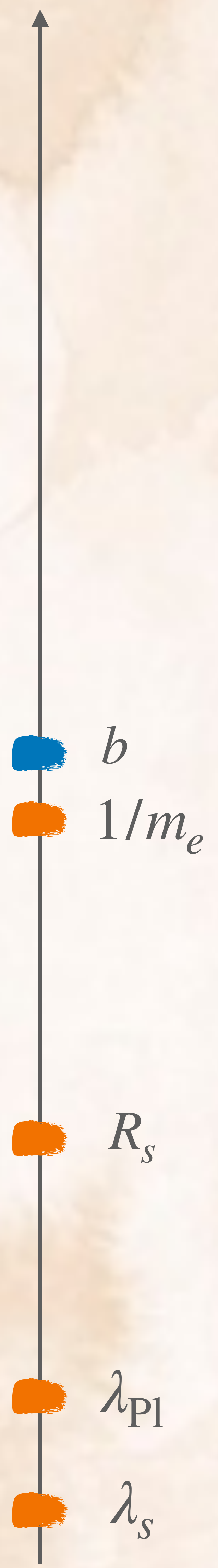


λ_1
λ_3
Φ
Minimal coupling
$\frac{\alpha}{m_e^2}F^2R$

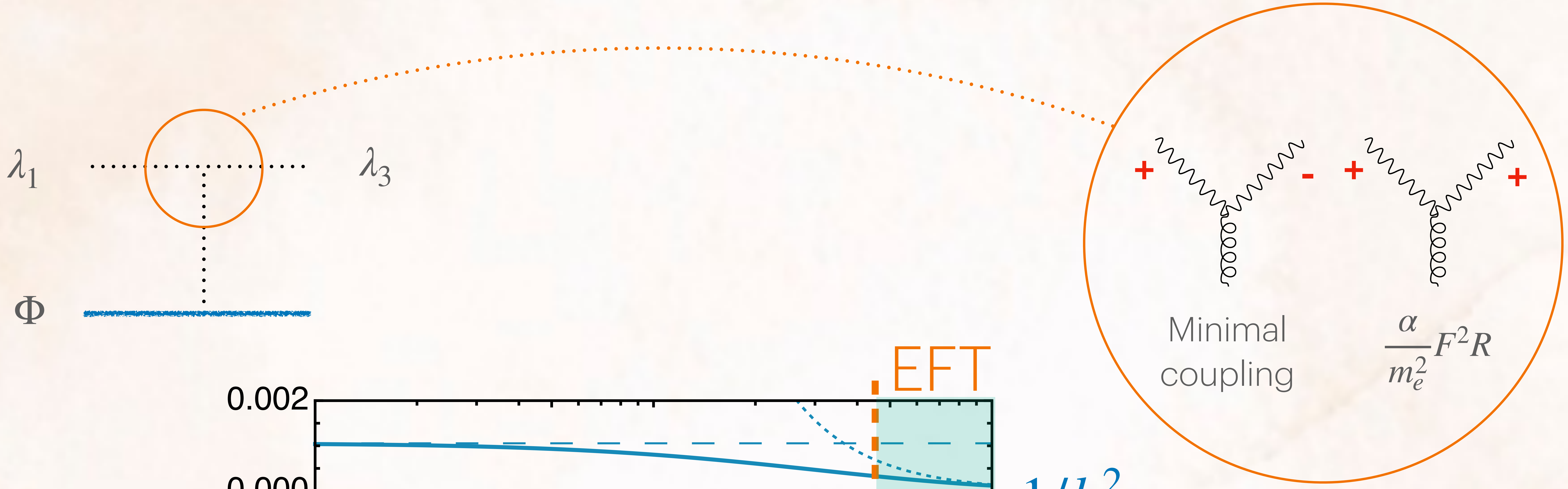


EFT
$1/b^2$
$\Delta T_\pm$
0.002
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
0.05
0.10
0.50
1
bm_e
Helicity flipping
Helicity preserving

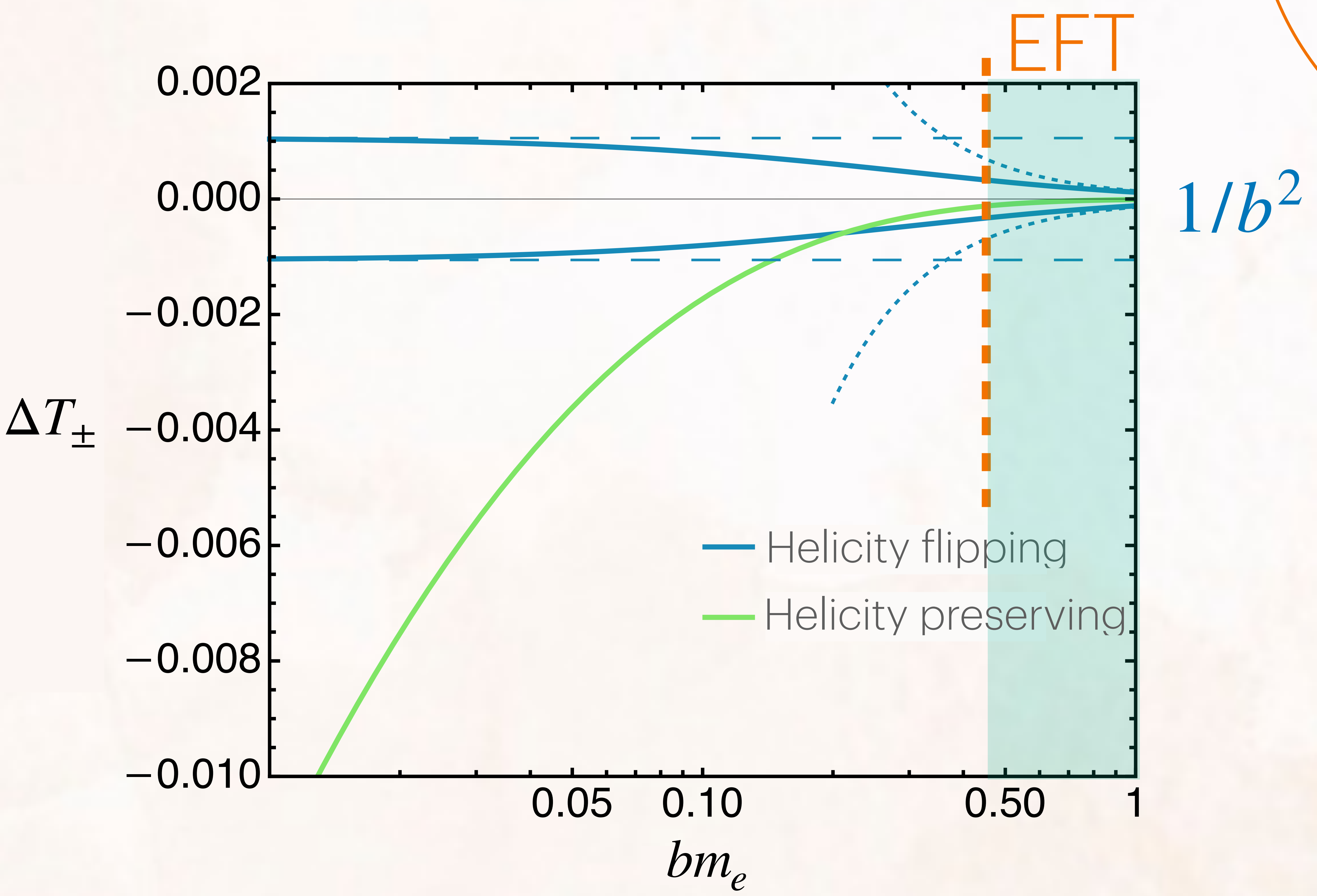
QED docet



b
$1/m_e$
R_s
$\lambda_{\rm Pl}$
λ_s



λ_1
λ_3
Φ
+
-
+
+
Minimal coupling
$\frac{\alpha}{m_e^2}F^2R$



EFT
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$\Delta T_\pm$
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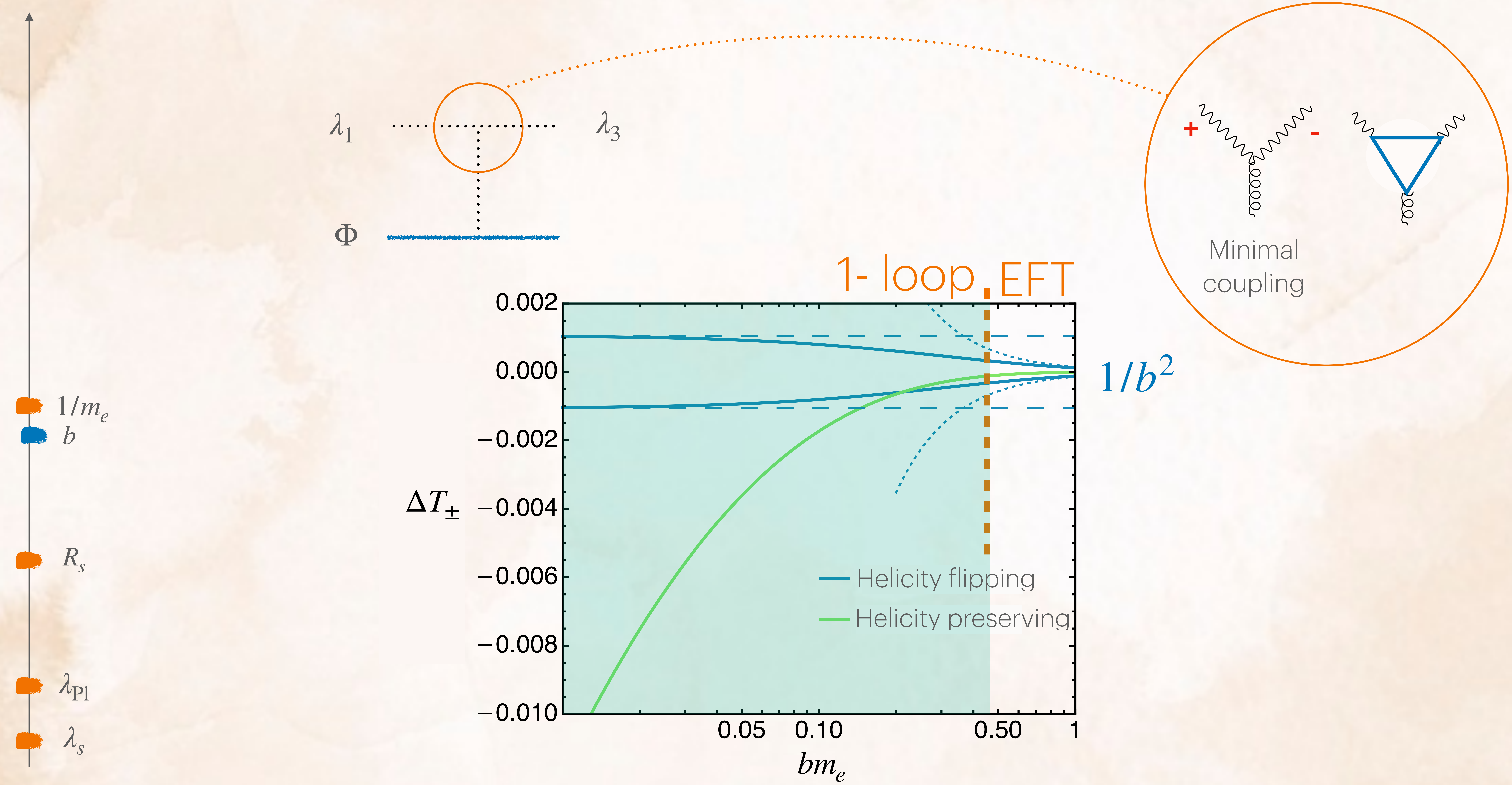

$1/m_e$
b
R_s
$\lambda_{\rm Pl}$
λ_s
λ_1
λ_3
Φ
+
-
Minimal
coupling
1- loop
EFT
$1/b^2$
$\Delta T_\pm$
0.002
0.000
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1
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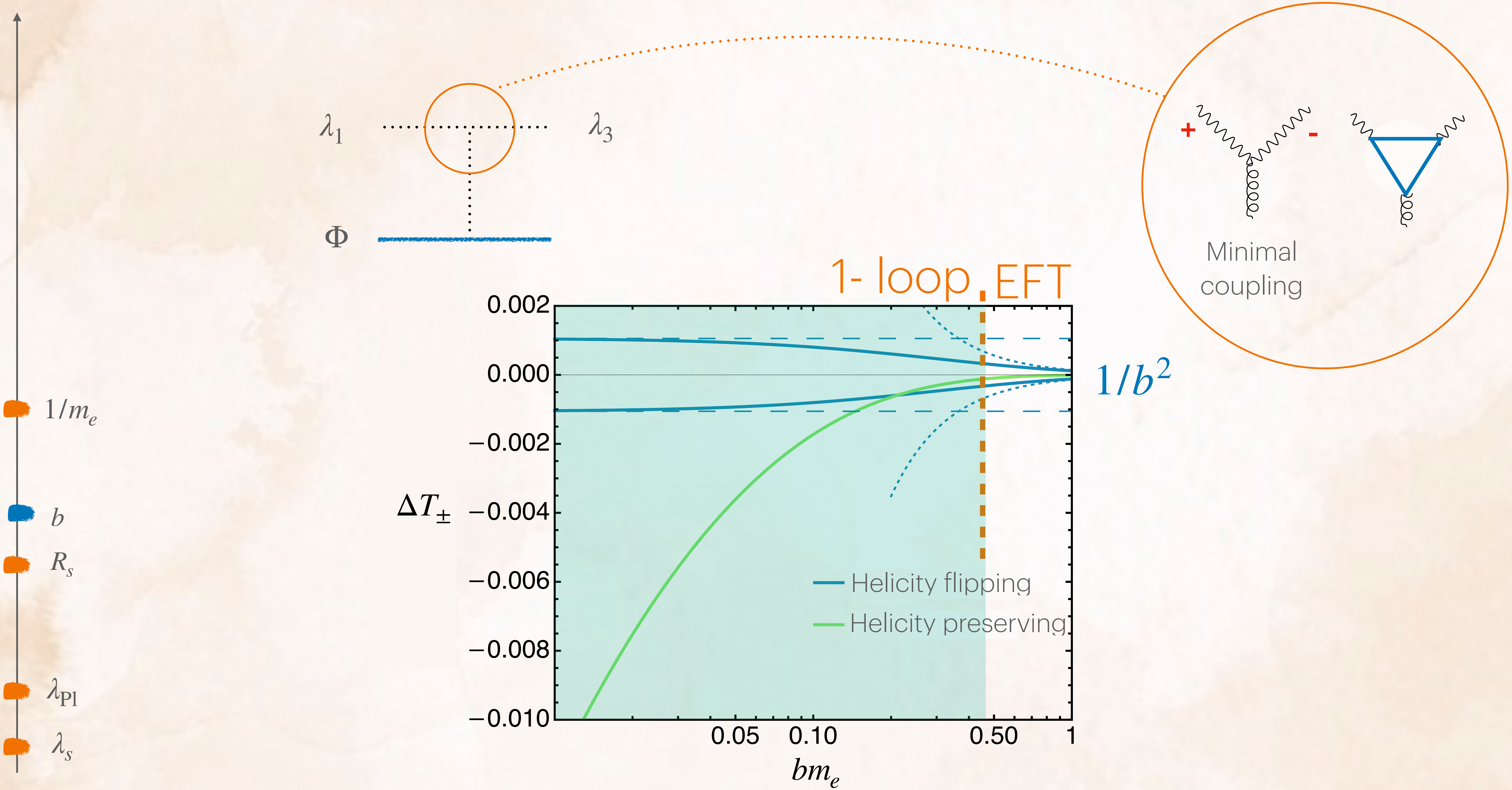
QED docet



$1/m_e$
b
R_s
λ_{Pl}
λ_s
λ_1
λ_3
Φ
+
-
Minimal coupling
1- loop
EFT
$1/b^2$
0.002
0.000
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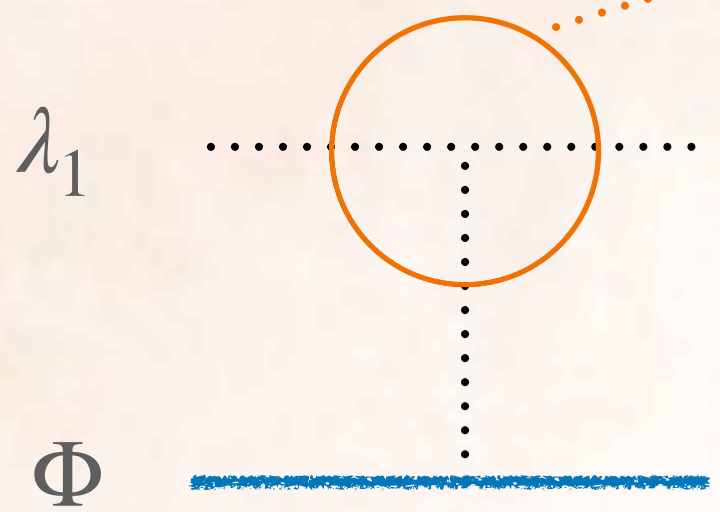
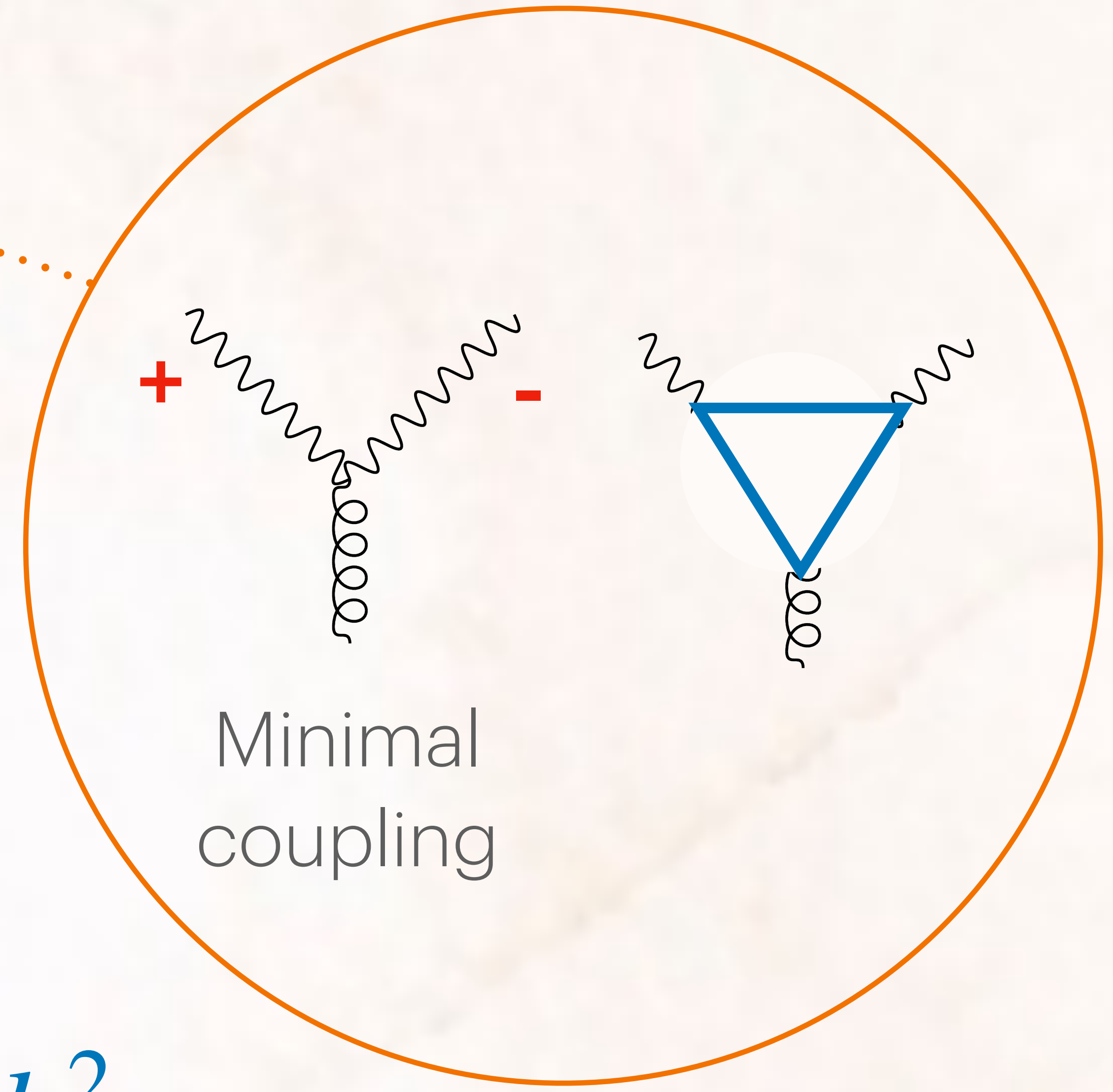
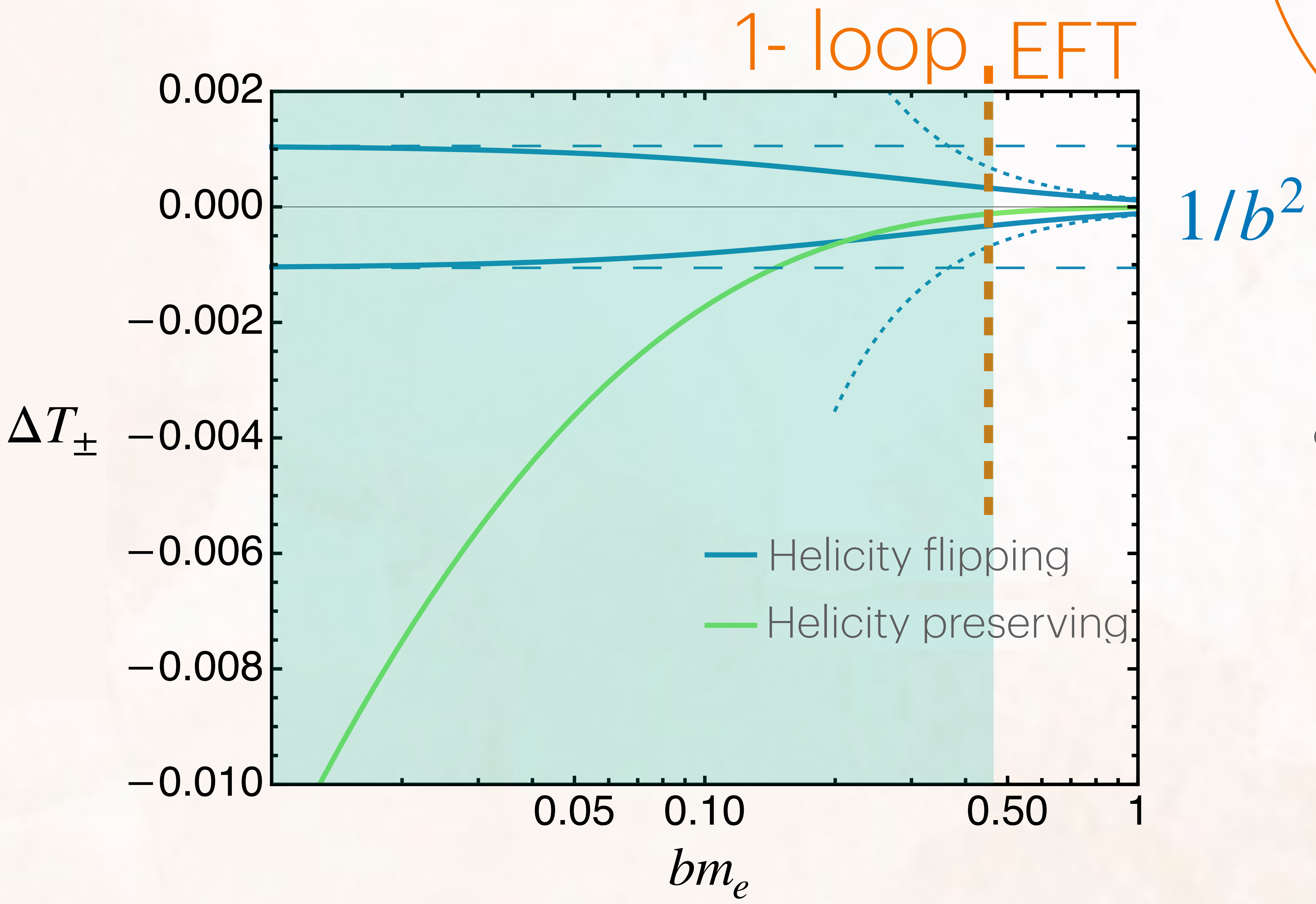
QED docet







- Causality detects the presence of a Landau pole

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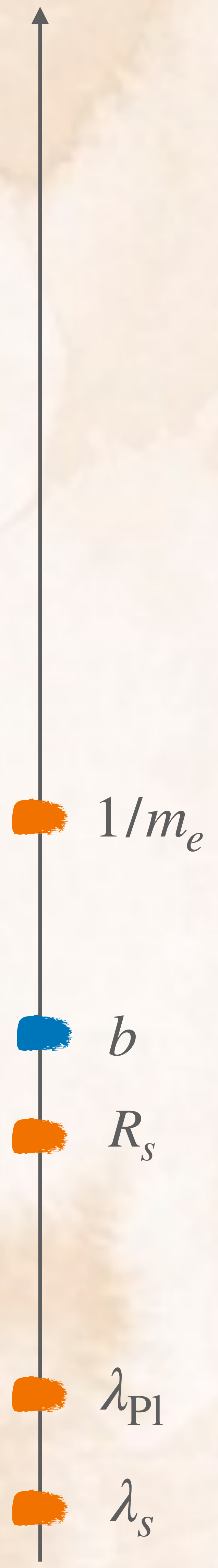

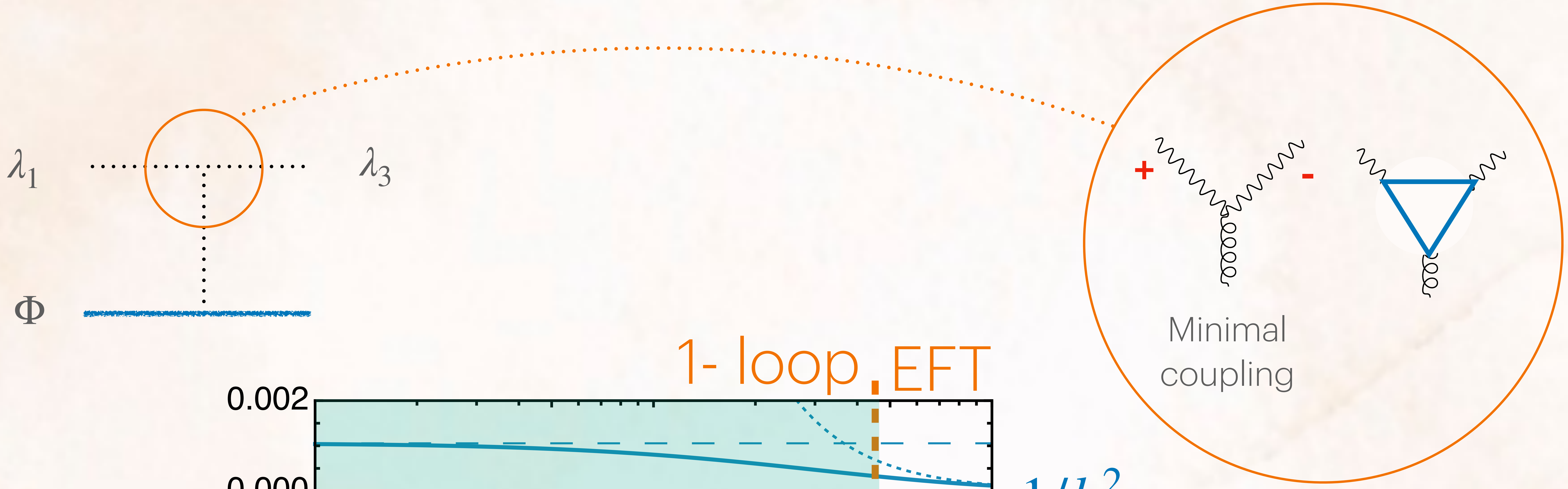

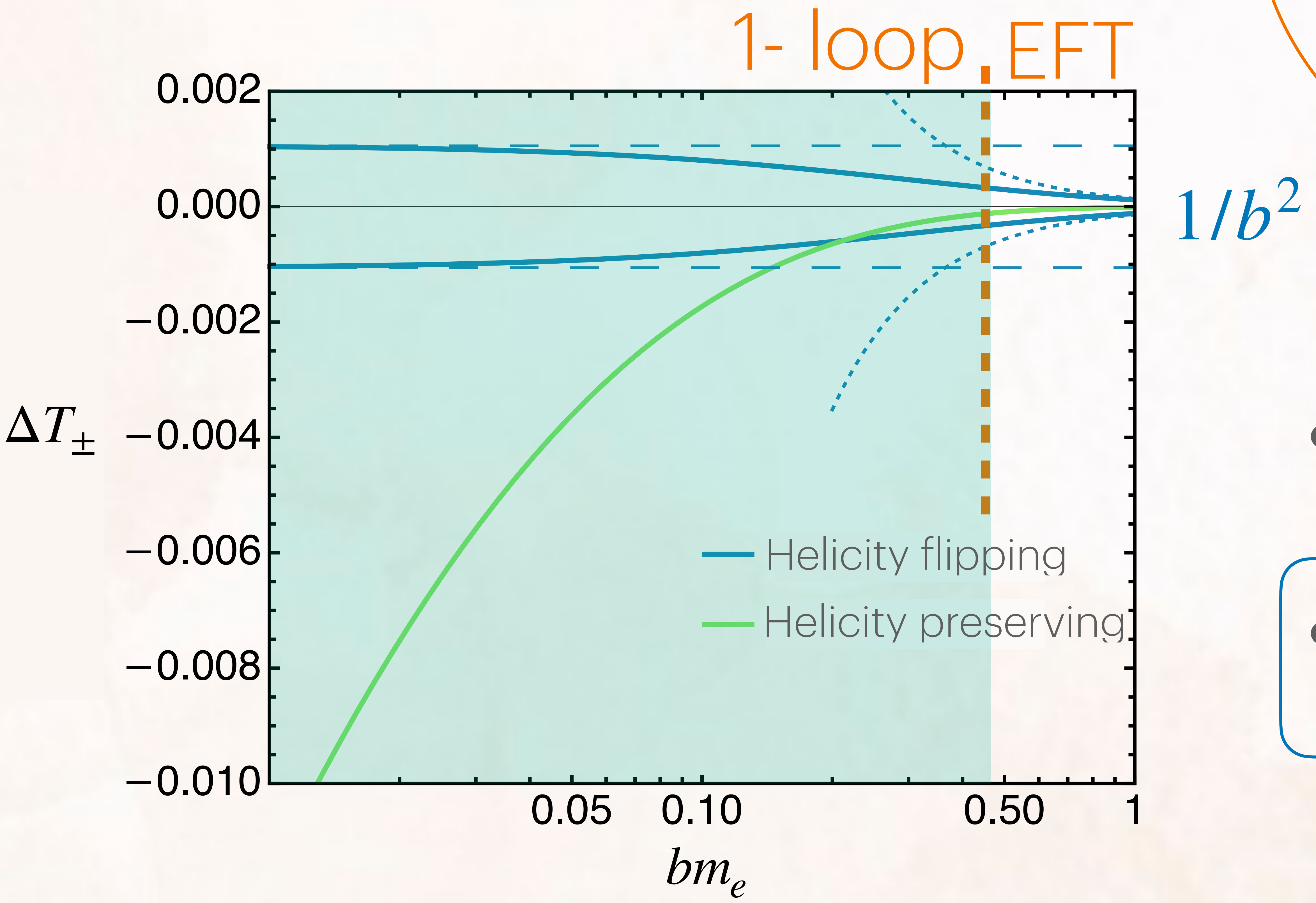

- Causality detects the presence of a Landau pole

- Causality violations solved at loop level

What could we learn?

Positive Eikonal arcs

$$\oint ds \frac{M_{\lambda_1\lambda_3}(s,b)}{\left(s-m^2\right)^n} \succ 0$$

$$M(s,b) \sim e^{2i\delta(s,b)} - 1$$

Many phase-shifts are known for tidal/spins effects in the context of GW

Non optimal bounds, but a much cheaper lunch...

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Can we learn something about neutron stars physics?

Part III

Conclusions

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Positivity is hard, but there are limits where things simplify!

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$$|t| \gg m^2$$

- The cutoff of massive gravity must satisfy $\Lambda < O(10)m$

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Tree level → Higher spins

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Thank you!

Backup

Beyond dRGT

In the decoupling limit

$$\langle 3^0 4^0 | \mathcal{M} | 1^0 2^0 \rangle = H(s,t),$$
$$\langle 3^+ 4^- | \mathcal{M} | 1^+ 2^- \rangle = \langle 32 \rangle^2 [14]^2 G_{+-}(s,t),$$
$$\langle 3^0 4^+ | \mathcal{M} | 1^0 2^+ \rangle = \langle 41 \rangle^2 [12]^2 G_{0+}(s,t),$$

$$H(s,t) = h_0(s^2+t^2+u^2)/2 + h_1 stu + \dots$$
$$G_{+-}(s,t) = f_0 + f_1(s+t) + f_2(s^2+t^2) + \dots$$
$$G_{0+}(s,t) = g_0 + g_1 t + g_2(s^2+u^2) + g_2' su + \dots$$

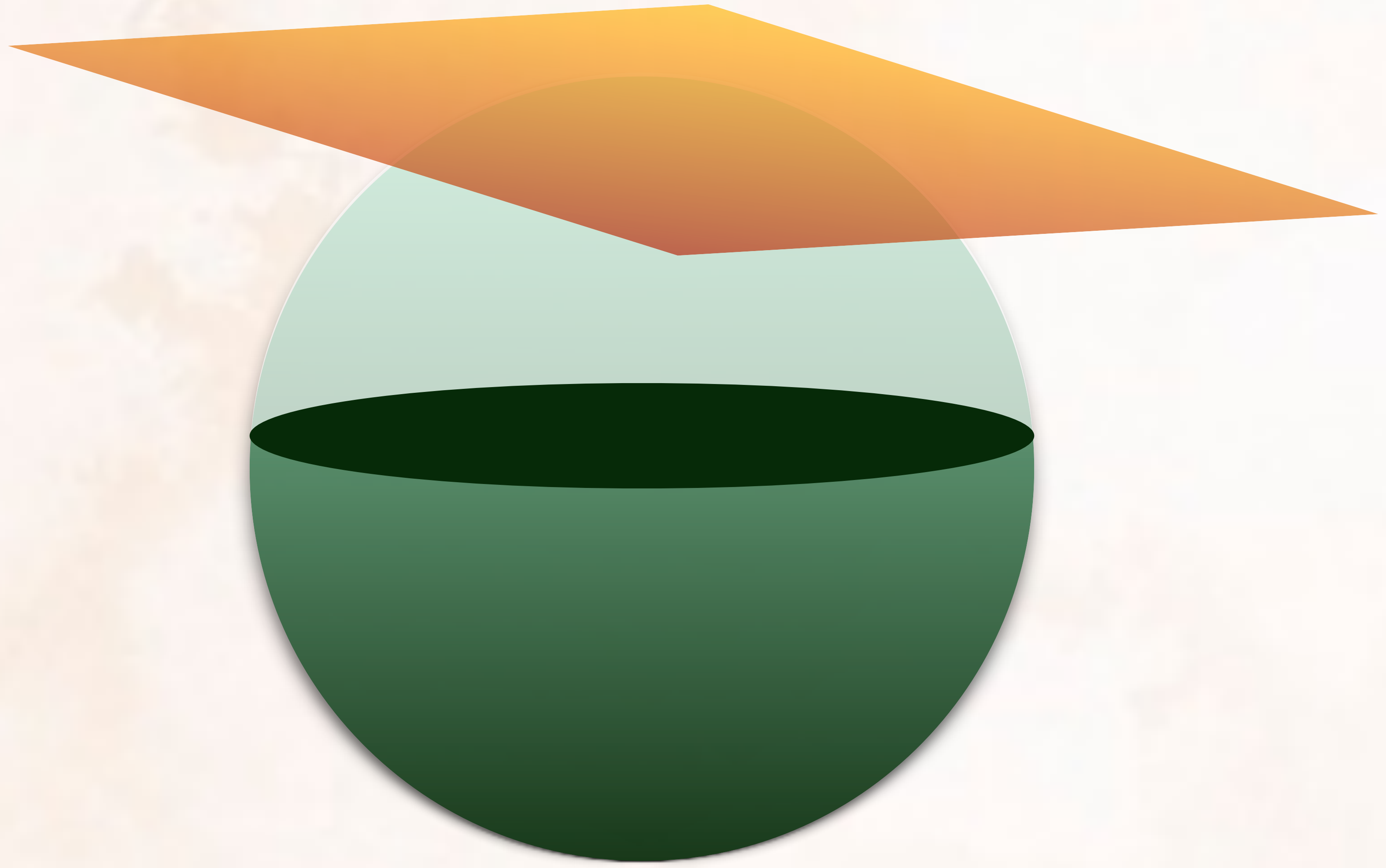
$$-8h_0 \le h_1 M^2 \le \frac{3}{2} h_0,$$
$$-f_0 \le f_1 M^2 \le f_0,$$
$$-\frac{5}{2} g_0 \le g_1 M^2 \le \frac{1}{3}(10 g_0 + 4 h_0 + 7 f_0)$$

(lower bounds) ↓ 0 (upper bounds) ↓ 0

Same conclusion, independent of higher derivatives

Emergence of classical ℓ

$$d^{\ell}_{\lambda,\lambda'}(\theta) \Rightarrow \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{i(\lambda'-\lambda)\varphi} e^{i\theta\ell \sin\varphi}$$



$$SO(3) \underset{\ell \to \infty}{\Rightarrow} ISO(2)$$

Compact
Finite dim irreps

Non-compact
Continuous irreps