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Gravitational Amplitudes: from Gedanken* to Real** Experiments

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CERN

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A common denominator:

Semiclassical
Gravitational Scattering

High-energy vs. large mass

- Dealing with gravitational scattering in general looks prohibitively hard. Are there limits in which the problem simplifies?
- One such limit is the semiclassical one to which we shall limit our attention here. It can be used, fortunately, in (at least) two interesting physical situations:

  1. The transplackian-energy regime for light particle/ string collisions; NB: necessarily ultra-relativistic
  2. The collision of transplanckian-mass (e.g. heavy black holes) objects at generic v/c (c=1 hereafter)

- They share the property that the total action of the system is much larger than h (in case 1. because of the large energy; in case 2. because of the large mass(es))

A possible misconception

- Of course one can study classically the gravitational scattering of light objects @ low (subplanckian) energy
- CRG has no characteristic mass (M_P) or length (l_P) scale
- The classical result will be the "same" (up to rescaling) for heavy/energetic or for light/soft collisions.
- But in the latter case the classical result is not reliable because the real process is quantum (e.g. forming a subplanckian-mass classical BH doesn't make sense)

- Q: Is the semiclassical limit interesting theoretically and/or phenomenologically?
- In case # 2 it is clearly phenomenologically interesting: it can help understanding classical BH encounters, mergers; also a theoretically challenging (hence interesting) problem.
- In case # 1 the phenomenological interest is more doubtful (early Universe collisions?). But it is certainly theoretically interesting since it represents the cleanest situation in which one can address fundamental questions such as the information puzzle. (NB: Hawking's spectrum contains a semiclassical Boltzmann factor ~ $\exp(-GM^2/h)$).
- A common tool to tackle both situation is given by the gravitational eikonal. Recent review: DHRV,2306.16488

The semiclassical S-matrix

- In non-relativistic QM the semiclassical limit is associated with the WKB approx. in which the wave function is written in terms of a rapidly-varying phase O(1/h) and a pre-factor expanded in powers of h
- A general way to generalize this to the relativistic case is the one proposed by Damgaard-Planté-Vanhove (2107.12891) in which the S-operator is written as

  S = exp(i χ), with χ = hermitian operator

  & χ has a leading term O(1/h). What makes life a little complicated is that amplitudes give the exponential rather than the exponent. Some tricks are needed...
- Exponentiation of "super-classical" terms must hold if the classical limit has to be well defined.

The gravitational eikonal

- The S-operator connects different channels. It is useful to diagonalize it as much as possible using exact, or approximate, conservation laws.
- An obvious one is energy-momentum. A less trivial one is angular momentum. For J >> h we can use b ~ J/p
- Finally, there are effective conservation laws due to the semiclassical approx. and/or to the particular kinematics under consideration.
- In the most favorable cases the S-operator becomes a

c-number function of the quantities that characterize a given channel. This is the traditional eikonal.

Outline

- Semiclassical gravitational scattering
- Gedanken gravitational collisions
  - Deflection, time delay, tidal excitations
  - Stringy effects at short distance
  - Radiation. Resummation & E-loss spectrum
- Real gravitational collisions
  - A 3PM puzzle and radiation reaction
  - The J-loss puzzle continues…
  - E-loss at 3PM and an "energy crisis"
  - Eikonal operator for soft & not-so-soft rad.
  - A rich UR frontier & non-analyticity in G

I. Gedanken Gravitational Collisions

of (light) particles and strings:
(a quick reminder of ACV)

Motivations in late eighties were purely theoretical:

- Recovering perturbative unitarity, emergence of classical and quantum/string gravity from amplitude calculations in flat spacetime (quite successful)
- Checking unitarity even in regimes where the process is expected to lead, classically, to black-hole formation (not quite as successful so far)

Parameter-space, regimes (D=4)

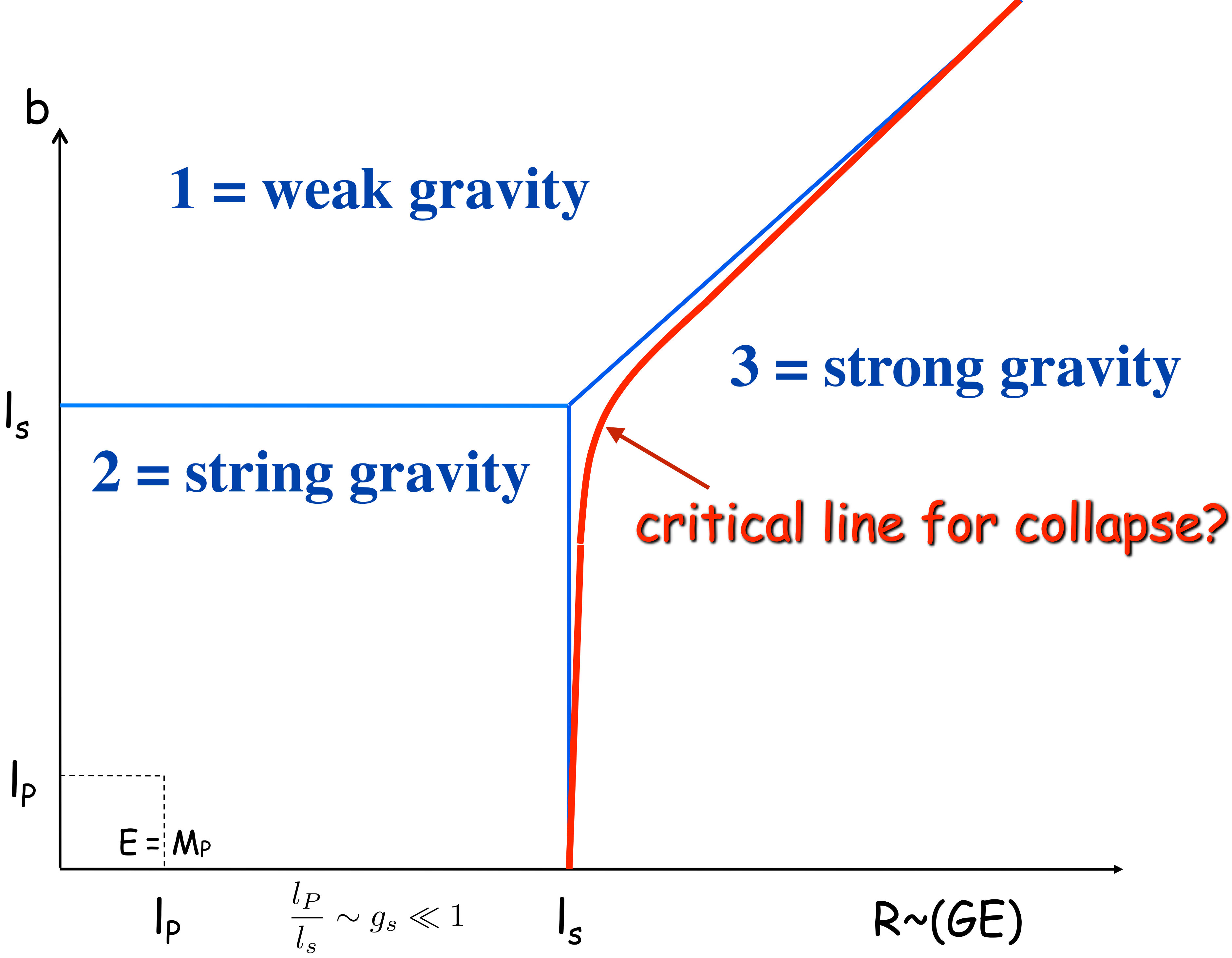
$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R \sim G\sqrt{s} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^2 \sim g_s^2 l_s^2$$

- 3 relevant length scales (neglecting l_P @ $g_s \ll 1$)

i.e. 2 relevant ratios, 2-dimensional phase diagram.

- Different regimes emerge.

Basic technique: eikonal resummation in b-space plus saddle-point approx. for h -> 0



b
1 = weak gravity
3 = strong gravity
l_s
2 = string gravity
critical line for collapse?
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)

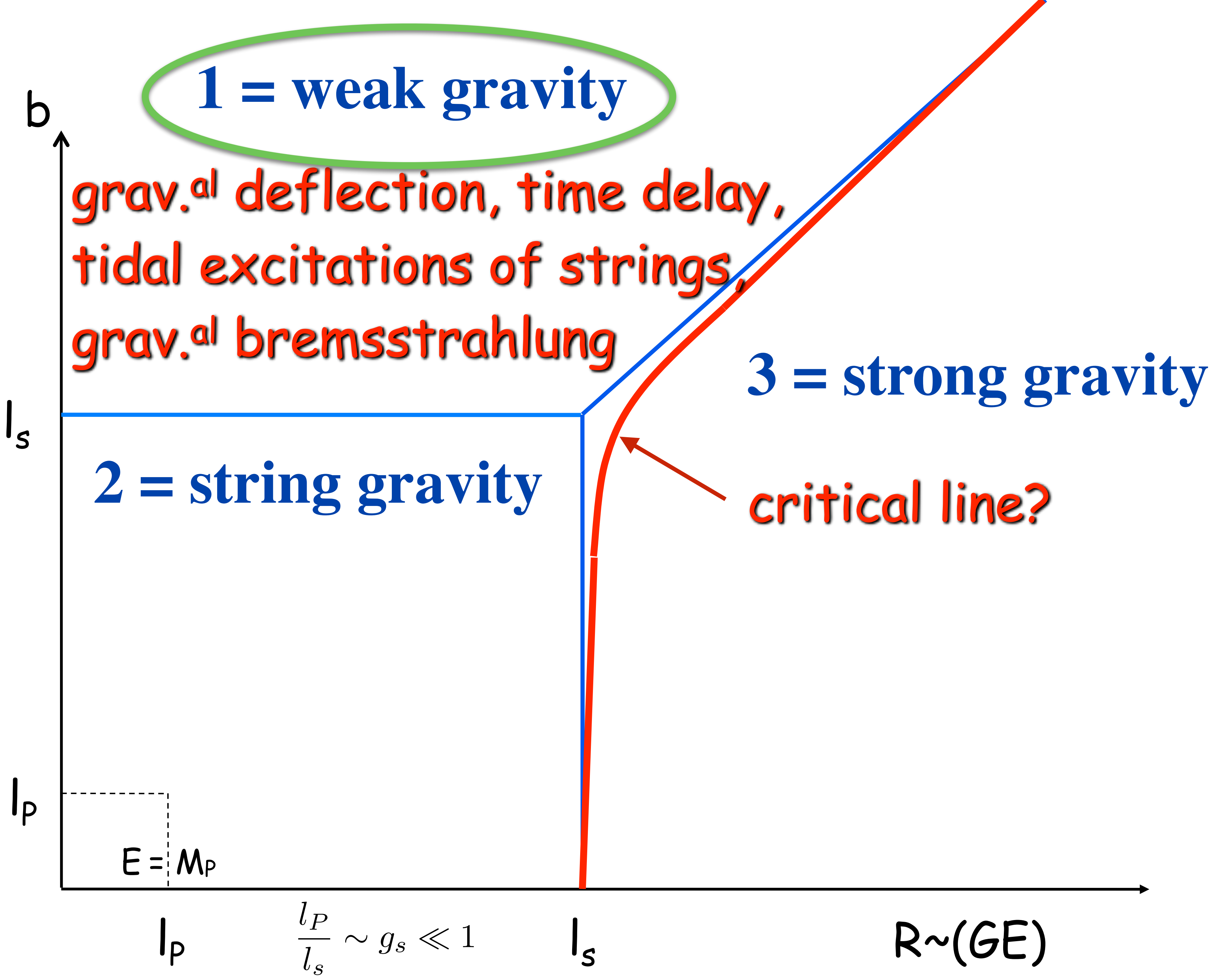
The elastic eikonal "phase" defined by

$$S(E, b) \sim e^{2i\delta} \; ; \; \delta = \delta_0 + \delta_1 + \ldots \; ; \; \delta_n = \mathcal{O}\left(\frac{G^{n+1}}{\hbar}\right)$$

gives the scattering angle and time delay as derivatives of Re 2δ w.r.t. impact parameter and energy, respectively.

On the other hand, Im $2\delta > 0$ is related to the opening of inelastic channels and to a corresponding suppression of the elastic one.

Inelastic unitarity should hold!


1 = weak gravity
b
grav.al deflection, time delay,
tidal excitations of strings,
grav.al bremsstrahlung
3 = strong gravity
ls
2 = string gravity
critical line?
lP
E = MP
lP
$\frac{l_P}{l_s} \sim g_s \ll 1$
ls
R~(GE)

- Restoration of (elastic) unitarity via eikonal resummation of s-channel ladders (unlike QCD!)
- Gravitational deflection & time delay: emerging shock-wave metric at O(G) (Cf. 'tHooft 1987); extension up to $O(G^3)$ (see below)
- t-channel "fractionation": hard scattering (large Q) from large-distance (large b) physics

$$S(E,b) \sim exp(-i\frac{Gs}{\hbar}\log b^2)$$

$$Q \sim \theta_s \frac{\sqrt{s}}{2} \sim -\hbar \frac{\partial(2\delta)}{\partial b} = 2\frac{Gs}{b} \Rightarrow b \sim \frac{G\sqrt{s}}{\theta_s}$$

Deflection angle @ 1, 2 & 3PM

ACV90 results up to 3PM (D=4, GR, m=0)

1PM

$$2\delta_0 = -\frac{Gs}{\hbar}\log b^2$$

classical

****

2PM

$$2Re\delta_1 = \frac{12G^2 s}{\pi b^2}\log s \; ; \; Im\delta_1 = 0$$

quantum and non-universal

=> URL -> 0 @ 2PM in classical limit

****

3PM

deflection

$$2Re\delta_2 = \frac{4G^3 s^2}{\hbar b^2}$$

classical, finite

radiation

$$Im\delta_2 \sim \frac{G^3 s^2}{\hbar b^2}\log s \log \frac{b^2}{\lambda^2}$$

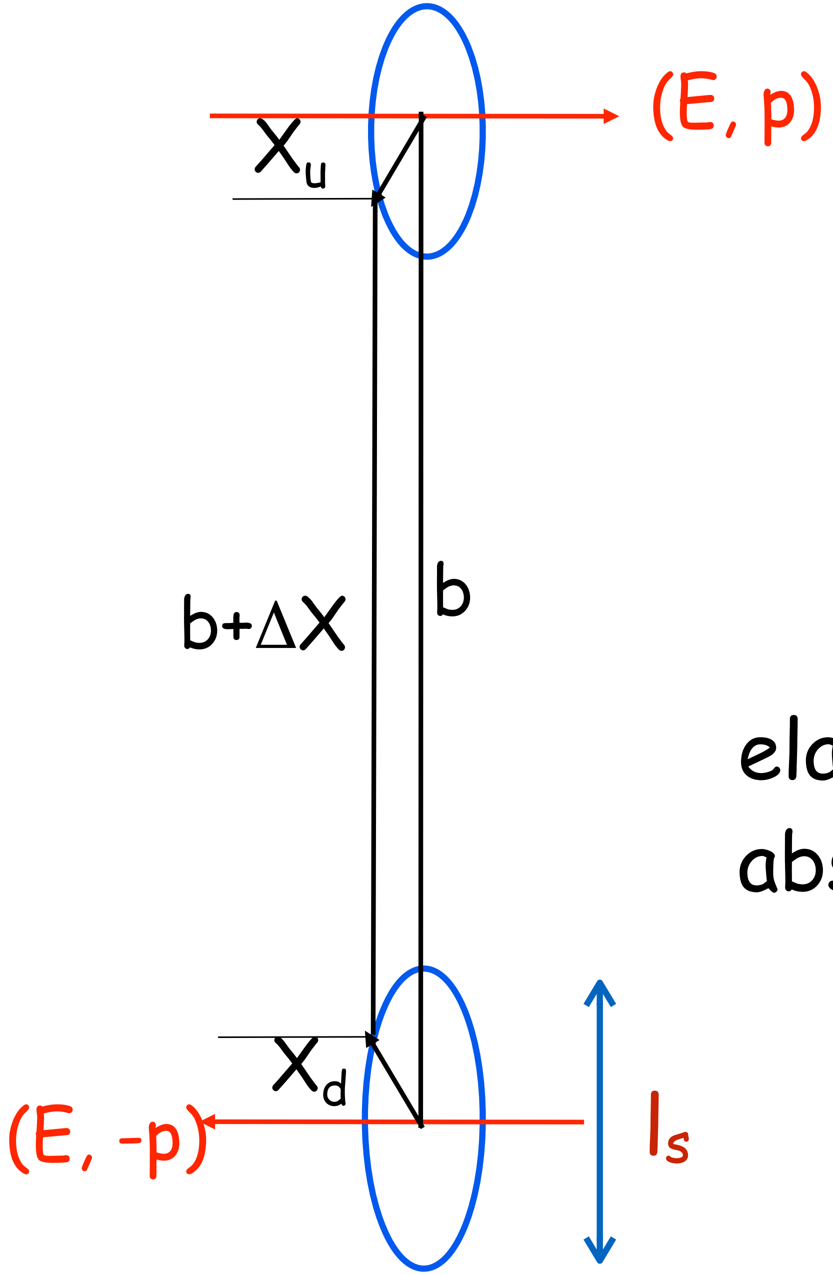
classical, divergent

- Tidal excitation of colliding strings, inelastic unitarity via unitary eikonal operator

$$S(b, \dots) = \exp\left(2i\delta(b, \dots)\right) \Rightarrow \hat{S}(b, \dots) = \exp\left(2i\hat{\delta}(b, \dots)\right)$$

$$\hat{\delta}(b, \dots) = \frac{1}{4\pi^2} : \int_0^{2\pi} d\sigma_u d\sigma_d \; \delta(b + \hat{X}_u(\sigma_u, 0) - \hat{X}_d(\sigma_d, 0), \dots) := \hat{\delta}^\dagger$$

… with a nice physical interpretation



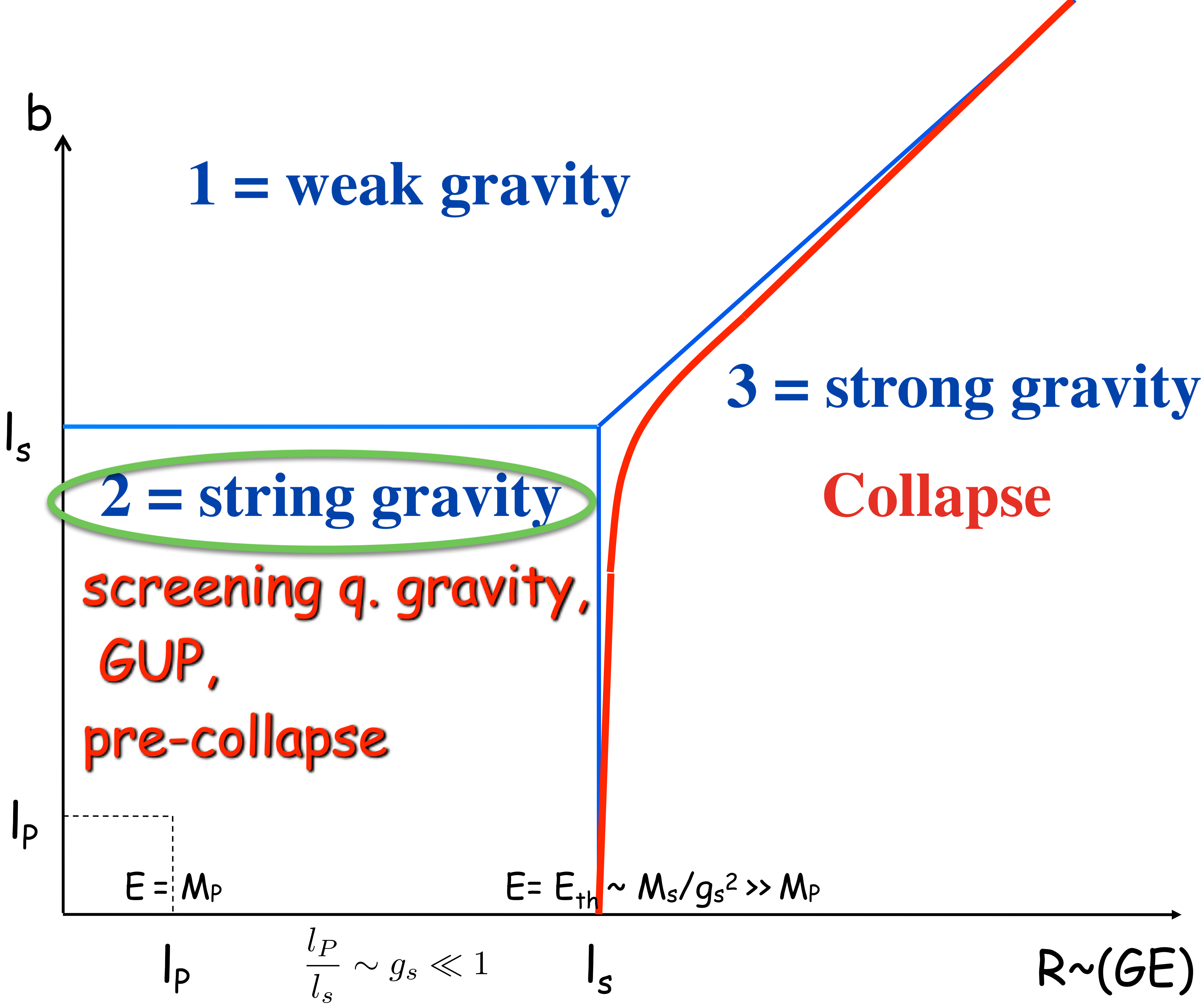
kicks in @ $b = b_t$

w/ $b_t^2 \sim (Gs/h)\, l_s^2 \gg l_s^2$

elastic amplitude strongly absorbed below b_t

$$S(b,\dots) = \exp\left(2i\delta(b,\dots)\right) \Rightarrow \hat{S}(b,\dots) = \exp\left(2i\hat{\delta}(b,\dots)\right)$$

$$\hat{\delta}(b,\dots) = \frac{1}{4\pi^2} : \int_0^{2\pi} d\sigma_u d\sigma_d \; \delta(b + \hat{X}_u(\sigma_u, 0) - \hat{X}_d(\sigma_d, 0), \dots) :$$



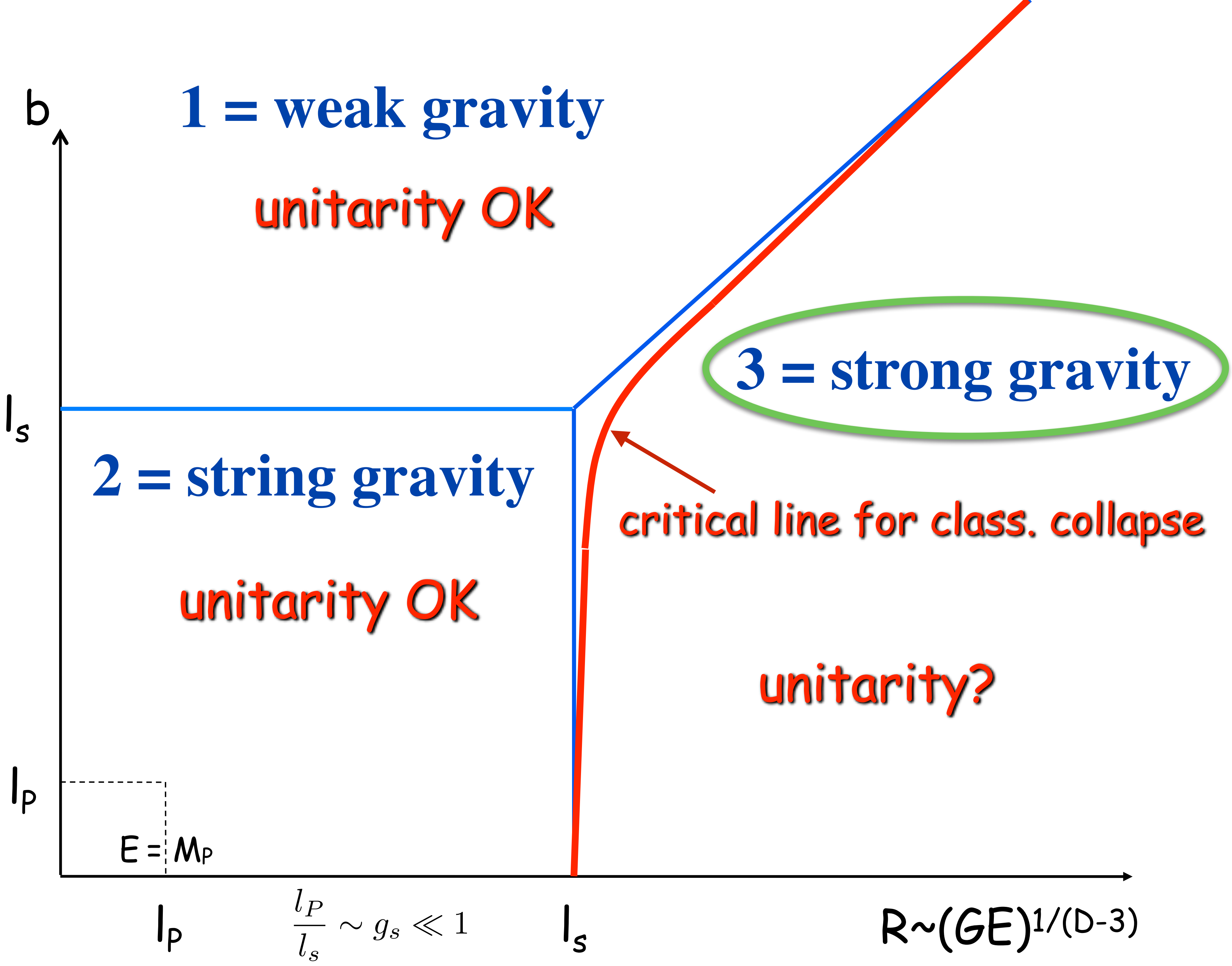
b
1 = weak gravity
3 = strong gravity
l_s
2 = string gravity
Collapse
screening q. gravity,
GUP,
pre-collapse
l_P
E = M_P
E= E_{th} ~ M_s/g_s^2 >> M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)

- String softening of quantum gravity @ small b: solving a causality problem (Edelstein et al.)
  - Maximal classical deflection, comparison/ agreement w/ Gross-Mende-Ooguri above it
  - Generalized uncertainty principle (ACV, Gross-Mende)

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

- s-channel "fractionation", anti-scaling, and precocious black-hole-like behavior:

$$\langle E_f \rangle = \frac{E}{\langle n_f \rangle} \sim \frac{E}{Gs/\hbar} \sim \frac{\hbar}{GE}$$



b
1 = weak gravity
unitarity OK
l_s
2 = string gravity
unitarity OK
3 = strong gravity
critical line for class. collapse
unitarity?
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R~(GE)^{1/(D-3)}

- Identifying (semi) classical contributions as connected trees

- An effective 2D field theory to re-sum them after some "truncation"

- Emergence of critical surfaces in good agreement with CGR collapse criteria. But...

- Unitarity beyond critical surface still problematic...(work by Ciafaloni and Colferai)

We(I) switched to simpler problems

Gravitational Radiation in the ultra-rel. limit (URL)

Also: string-brane collisions (DDRV*, 2010-'15)

NB. A second form of absorption when a closed string is captured by a brane system leading to a closed -> open transition. Not for today.

see DHRV 2306.16488 for a review

* D'Appollonio, Di Vecchia, Russo, GV

Two approaches

1. A classical GR approach
   (A. Gruzinov & GV, 1409.4555)

2. An amplitude-based (quantum) approach
   (CC Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)

NB: 2. goes over to 1. in the classical limit in spite of their completely different methodologies!
Both limited to small θ_s (deflection angle) and θ (GW direction w.r.t. initial momenta).
In soft limit agreement with Sen et al..

The D'Eath (Kovacs-Thorne) bound

- Before embarking in those non-trivial calculations of the URL we (G&V) checked the literature and asked some experts, including NR guys.
- Each time, after some initial optimism, the feedback was disappointing…
- Instead, we found Kovacs & Thorne's warning (based on D'Eath's work) on the limit of validity of their 1977 result, and decided to try go beyond it.

THE GENERATION OF GRAVITATIONAL WAVES. IV. BREMSSTRAHLUNG*†‡

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ABSTRACT

This paper attempts a definitive treatment of "classical gravitational bremsstrahlung"—i.e., of the gravitational waves produced when two stars of arbitrary relative mass fly past each other with arbitrary relative velocity v, but with large enough impact parameter that

$$(\text{angle of gravitational deflection of stars' orbits}) \ll (1 - v^2/c^2)^{1/2} .$$

$\theta_S\, \sigma^{1/2} \ll 1$ in our notations

I will refer to $\theta_S\, \sigma^{1/2} = 1$ as the DKT bound

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High-speed black-hole encounters and gravitational radiation

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Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge, England
(Received 15 March 1977)

Encounters between black holes are considered in the limit that the approach velocity tends to the speed of light. At high speeds, the incoming gravitational fields are concentrated in two plane-fronted shock regions, which become distorted and deflected as they pass through each other. The structure of the resulting curved shocks is analyzed in some detail, using perturbation methods. This leads to calculations of the gravitational radiation emitted near the forward and backward directions. These methods can be applied when the impact parameter is comparable to $Gc^{-2}M\gamma^2$, where M is a typical black-hole mass and γ is a typical Lorentz factor (measured in a center-of-mass frame) of an incoming black hole. Then the radiation carries power/solid angle of the characteristic strong-field magnitude c^5G^{-1} within two beams occupying a solid angle of order γ^{-2}. But the methods are still valid when the black holes undergo a collision or clos encounter, where the impact parameter is comparable to $Gc^{-2}M\gamma$. In this case the radiation is apparently not beamed, and the calculations describe detailed structure in the radiation pattern close to the forward and backward directions. The analytic expressions for strong-field gravitational radiation indicate that a significant fraction of the collision energy can be radiated as gravitational waves.

below DKT bound $M\gamma \sim \sqrt{s}$; $\gamma \sim \sqrt{\sigma}$ above DKT bound!

The classical limit (NB: a resummation in G!)

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2\ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s\ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega\mathbf{x}\cdot\tilde{\boldsymbol{\theta}}}\left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad\qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

Re ζ^2 and Im ζ^2 correspond to the usual (+, x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones.

Analytic results: a Hawking knee

& (not for today) an unexpected

bump @ $\omega\, b \sim 0.5$

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log(\omega R)^{-2}$$

Below $\omega = b^{-1}$ it "freezes", giving the expected zero-frequency limit (ZFL) (Smarr 1977)

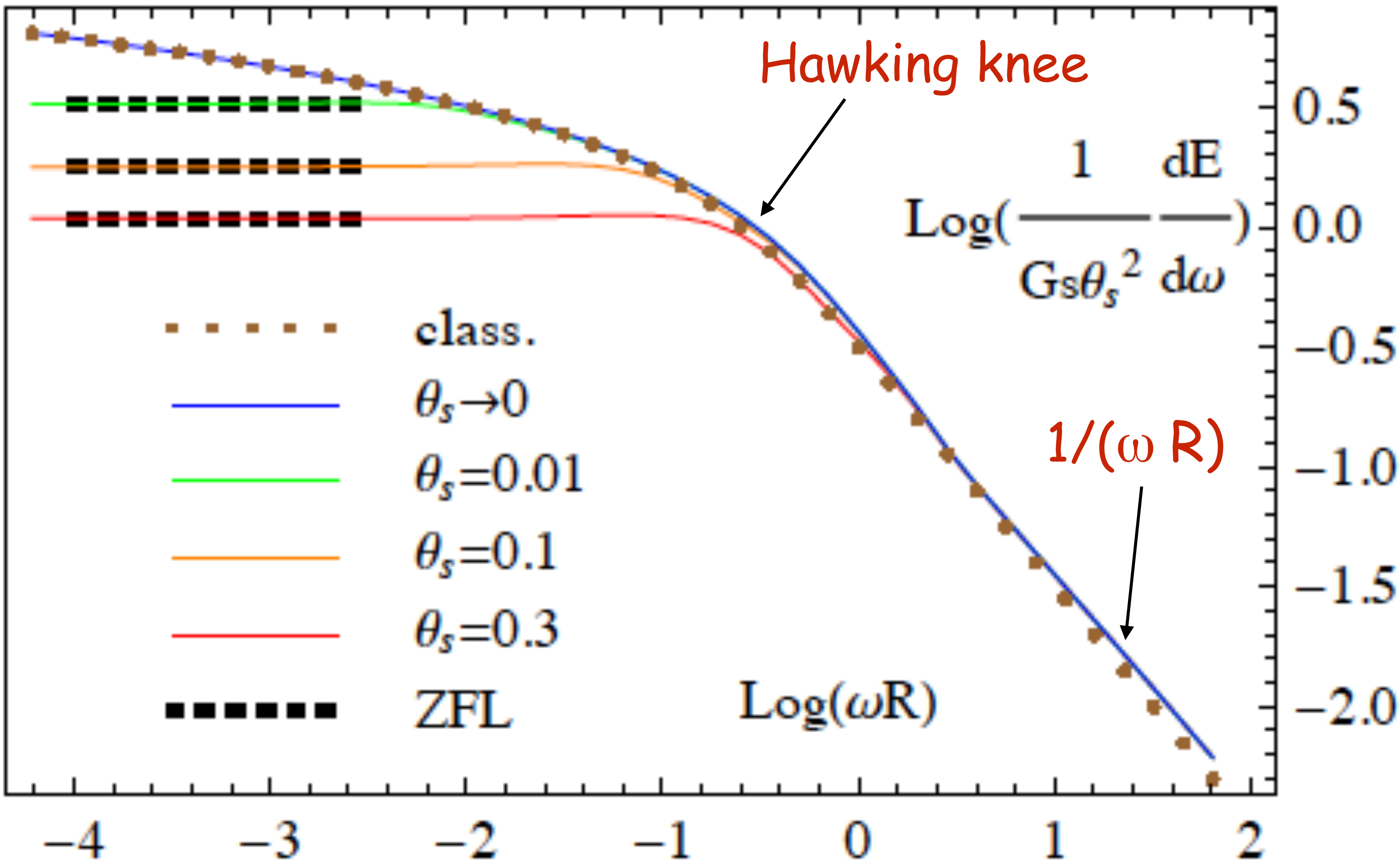
$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\ \theta_s^2 E^2\ \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ drops, becomes "scale-invariant"

Hawking knee!

$$\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}$$

(CCCV 1512.00281)



The "scale-invariant" spectrum gives a log ω^* sensitivity in the total radiated energy for a cutoff at $\omega = \omega^*$

Using, with some motivations, $\omega^* \sim R^{-1}\,\theta_s^{-2}$ G&V predict (to leading-log accuracy):

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\,\theta_s^2\,\log(\theta_s^{-2})$$

Note "low" order in G!

To get the full answer: we need to go beyond some approximations made in G&V or CCCV, find the actual value of ω^*, extend the method to arbitrary θ.
We will come back to this at the end of the talk.

II. Real Gravitational Collisions

Black Hole-Black Hole scattering

(bound-orbit problem connected by analytic continuation? R. Porto…)

Q: How does one formulate BH-BH coalescence in an S-matrix framework?

A 3PM Puzzle and Radiation Reaction
(DHRV 2008.12743)

- In 1901.04424/1908.01493 an impressive calculation by BCRSSZ led to the first 3PM (i.e. 2-loop) result (in GR for two massive scalars).
- Checked to be consistent up to "6PN" (integer) order but presented a puzzle.

- The high-energy (or just the massless) limit of the BCRSSZ result exhibited a logarithmic divergence in contrast with the finite result by ACV90 shown earlier.

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BCRSSZ = Bern, Cheung, Roiban, Shen, Solon, Zeng

The resolution of the puzzle took 1.5 y.

- In 2008.12743 DHRV extended the ACV90 argument to massive UR case & obtained the same finite result as in the massless case.
- They then confirmed it by computing the full amplitude in massive N=8 SUGRA at arbitrary energy including contributions from the full soft (rather than just the potential) integration region.
- The result was simple and illuminating

3PM eikonal in N=8 SUGRA

$$\mathrm{Re}(\delta_2) = \frac{2G^3(2m_1m_2\sigma)^2}{\hbar b^2}\left[\frac{\sigma^4}{(\sigma^2-1)^2} - \cosh^{-1}(\sigma)\left(\frac{\sigma^2}{\sigma^2-1} - \frac{\sigma^3(\sigma^2-2)}{(\sigma^2-1)^{5/2}}\right)\right]$$

P-MRZ/BCRSSZ

ACV-limit

New

cancel @ large σ

$$2m_1m_2\sigma = s - m_1^2 - m_2^2$$

$$\cosh^{-1}(\sigma) \sim \log\sigma \text{ as } \sigma \to \infty$$

NB: old and new terms behave quite differently in the NR limit, $\sigma \to 1$ ($s \to (m_1+m_2)^2$): even vs odd powers of v

- When we presented this result at a workshop in Aug. 2020, Damour immediately grasped the physical meaning of what we had found:
- Our half-integer PN terms meant that we had added to the conservative dynamics of Bern et al's calculation the effect of radiation on the eikonal phase, the so-called radiation reaction. The full, physical, deflection angle, unlike the conservative one, has a smooth UR limit.
- A couple of months later, using a smart shortcut, Damour extended the result to GR (see below).
- Yet a bit later, using a different shortcut, DHRV gave another derivation of both the GR and the N=8 result based on analyticity and the ZFL.

- Several confirmations of the full 3PM result were given later through full-fledged two-loop calculations*.

* DHRV 2104.03256; Herrmann, Parra-Martinez, Ruf, Zeng 2104.03957; Bjerrum-Bohr, Damgaard, Planté, Vanhove 2105.05218; Brandhuber, Chen, Travaglini, Wen, 2108.04216; …

3PM eikonal for GR (2010.01641)

1PN

2PN(BCRSSZ)

2.5PN

$$2Re\delta_2 = \frac{2G^3 m_1 m_2 s}{\hbar b^2 (\sigma^2-1)^{3/2}}\left(12\sigma^4 - 10\sigma^2 + 1\right)$$
$$-\frac{4G^3 m_1^2 m_2^2}{\hbar b^2 (\sigma^2-1)^{1/2}}\left(\frac{\sigma(14\sigma^2+25)}{3} + (4\sigma^4 - 12\sigma^2 - 3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$
$$+\frac{2G^3 m_1^2 m_2^2 (2\sigma^2-1)^2}{\hbar b^2 (\sigma^2-1)^2}\left(\frac{8-5\sigma^2}{3} + \sigma(2\sigma^2-3)\frac{cosh^{-1}(\sigma)}{(\sigma^2-1)^{1/2}}\right)$$

UR-limit: log s terms become again subleading &

$(48 - 56/3 - 40/3)\sigma^2 = 16\sigma^2$ => ACV90!

UNIVERSALITY OF THE MASSLESS LIMIT?

Smooth connection with the relativistic massive case?

Radiation and an "energy crisis" ?

- An $O(G^3)$ calculation of the total E^{rad} (HP-MRZ*, 2101.07255) has confirmed KT's result leading to an "energy crisis" similar to (and worse than) the one we have discussed for massless scattering.
- Indeed E^{rad}/E grows like $\theta_s^3\,\sigma^{1/2}$ violating E-cons. as σ goes to infinity @ fixed θ_s
- Remember KT's warning on limit of validity of their result: $\theta_s\,\sigma^{1/2} < 1$. In that situation $E^{rad}/E < \theta_s^2$ and there is no crisis.

HP-MRZ= Hermann, Parra-Martinez, Ruf, Zeng

HP-MRZ, 2101.07255 (confirmed in DHRV, 2104.03256)

$$E^{rad} = \frac{\pi G^3 m_1^2 m_2^2 (m_1 + m_2)}{b^3 \sqrt{s}} \left[f_1(\sigma) + f_2(\sigma) \log \frac{\sigma + 1}{2} + f_3(\sigma) \frac{\sigma \cosh^{-1} \sigma}{2\sqrt{\sigma^2 - 1}} \right]$$

where in $\mathcal{N} = 8$

$$f_1 = \frac{8\sigma^6}{(\sigma^2-1)^{\frac{3}{2}}}, \qquad f_2 = -\frac{8\sigma^4}{\sqrt{\sigma^2-1}}, \qquad f_3 = \frac{16\sigma^4(\sigma^2-2)}{(\sigma^2-1)^{\frac{3}{2}}},$$

while in GR

$$f_1 = \frac{210\sigma^6 - 552\sigma^5 + 339\sigma^4 - 912\sigma^3 + 3148\sigma^2 - 3336\sigma + 1151}{48(\sigma^2-1)^{\frac{3}{2}}},$$

$$f_2 = -\frac{35\sigma^4 + 60\sigma^3 - 150\sigma^2 + 76\sigma - 5}{8\sqrt{\sigma^2-1}},$$

$$f_3 = \frac{(2\sigma^2-3)(35\sigma^4 - 30\sigma^2 + 11)}{8(\sigma^2-1)^{\frac{3}{2}}},$$

$$\nu \equiv \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \sqrt{\frac{\sigma}{\nu}} \; ; \; \text{for } \sigma \to \infty$$

Another "energy crisis" above DKT

- Amusingly, a warning can already be found in the ZFL
Integrating the 3PM ZFL up to $\omega \sim 1/b$:

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\sigma) \Rightarrow \frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \log(\sigma)$$

- In this case, however, Weinberg tells us how to cure the problem.
- One can look at the exact ZFL for massless case: the result is quite different (and finite, no coll.div.!):

$$\frac{dE^{rad}}{d\omega} \to \frac{Gs}{\pi}\theta_s^2 \log(\theta_s^{-2}) \Rightarrow \frac{E^{rad}}{\sqrt{s}} \sim \theta_s^3 \log(\theta_s^{-2})$$

- It is perfectly finite but non-analytic in G (as in the (G+V) and CCCV "solution")! We'll come back to this towards the end of the talk.

Radiation from an eikonal operator

(DHRV, 2210.12118)

A better (the correct) framework to discuss both conservative and dissipative phenomena (and their interplay) is to upgrade the eikonal "phase" to an hermitian eikonal operator.

This was recognized long ago by ACV (starting w/ tidal excitations), CC(C)V (for grav. rad.), etc.

Actually, the much appreciated KMOC* formalism is already using in a crucial way the existence of a unitary S-matrix(operator).

* Kosower, Maybee, O'Connell (1811.10950)

From $S^+S = 1$ and $|\psi, \mathrm{out}\rangle = S|\psi, \mathrm{in}\rangle$

$$\langle \mathcal{O} \rangle_{\mathrm{out}} = \frac{\langle \psi, \mathrm{out} | \hat{\mathcal{O}} | \psi, \mathrm{out} \rangle}{\langle \psi, \mathrm{out} | \psi, \mathrm{out} \rangle} = \frac{\langle \psi, \mathrm{in} | S^\dagger \hat{\mathcal{O}} S | \psi, \mathrm{in} \rangle}{\langle \psi, \mathrm{in} | \psi, \mathrm{in} \rangle}$$

as well as

$$\langle \mathcal{O} \rangle_{\mathrm{out}} - \langle \mathcal{O} \rangle_{\mathrm{in}} = \frac{\langle \psi, \mathrm{in} | S^\dagger [\hat{\mathcal{O}}, S] | \psi, \mathrm{in} \rangle}{\langle \psi, \mathrm{in} | \psi, \mathrm{in} \rangle}$$

Can we actually compute such observables in the classical limit? With $S = \exp(i\,\chi)$ (DPV, 2107.12891, Damgaard, Hansen, Planté, Vanhove, 2307.04746)

the classical limit will be determined by χ at the leading order in h (to be carefully defined/extracted).

(terms in $S \sim h^{-n}$, $n > 1$ should exp. as $\exp(i/h\, I_{cl})$)

A further simplification occurs if χ is fixed by the exponentiation of some low order calculation.

This leads to a coherent state (the closest possible to a classical field) representation w/ χ a linear function of creation and destruction operators

- Such an approximation is known to be valid in the soft-graviton limit but how soft is soft?

- Individual gravitons are very soft. They typically carry a classical frequency (related to some classical length in the problem like b, GE) i.e. a quantum energy. However, all together, they can carry, a fraction of the energy of the process, hence at least energy conservation has to be implemented in the coherent state formalism.

- A challenge is to reconcile energy-momentum conservation with unitarity.
- Recently, Cristofoli et al. (2112.07556) have addressed and (partially?) answered this question.
- DHRV's claim (2210.12118): at leading-order (in h, G?) inelastic unitarity looks OK.
- We were then able to reliably compute various radiative observables: waveforms, memory, energy and linear/angular momentum losses by each particle.
- Some already known (e.g. in Mougiakakos Riva Vernizzi 2102.08339, RV 2110.10140) others are new.

In the rest of my time, if any, I will discuss what we did for soft radiation at arbitrary σ and end up with some unpublished results and/or speculations on what may happen, at arbitrary σ, beyond the soft limit.

Features of the ZFL in the URL

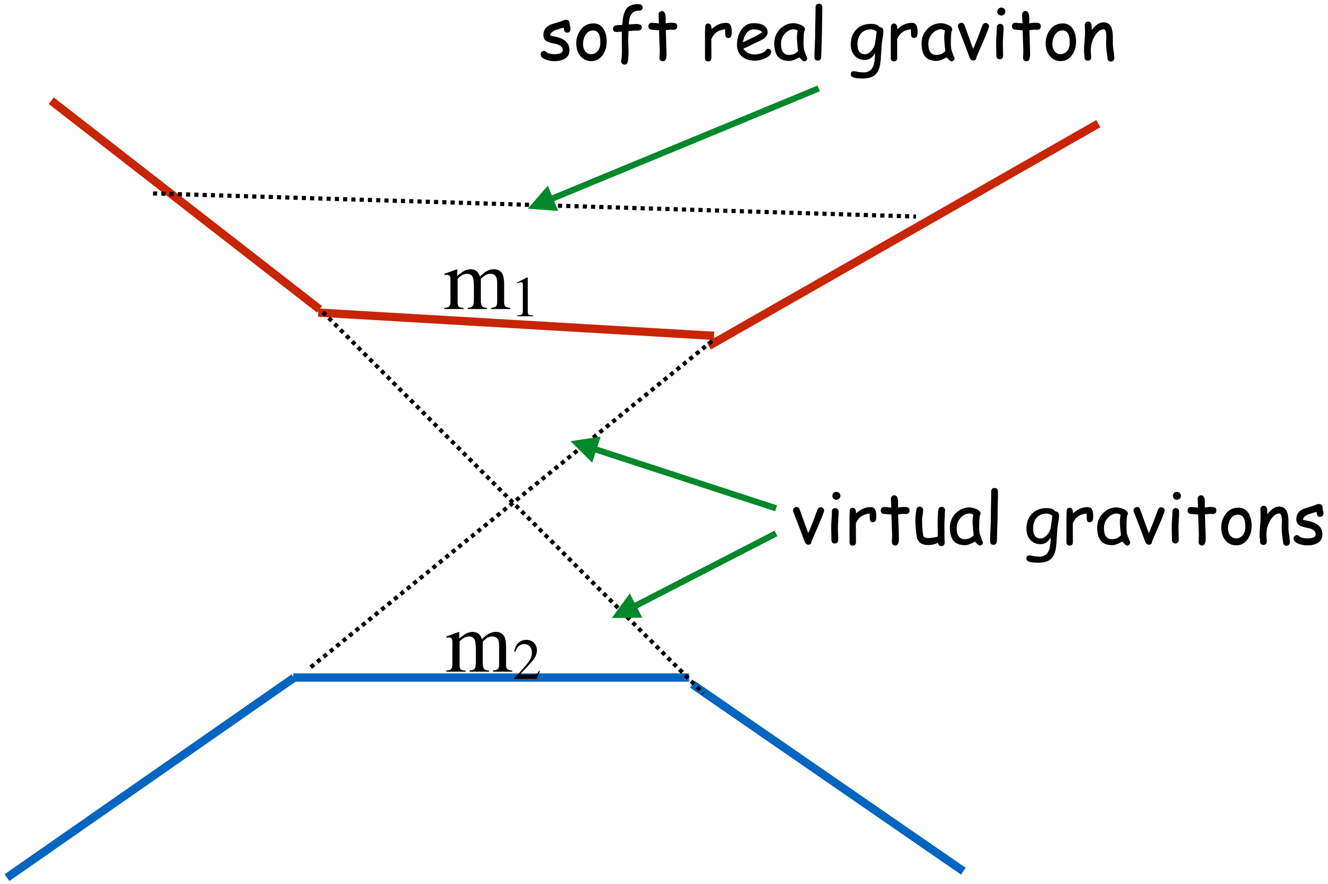
- Rich structure of URL emerging
- In URL the ZFL depends non trivially on two "scaling variables": $x_i = Q/2m_i$. One combination is of course related to ν, the other is related to DKT's $\theta_s^2\,\sigma$
- Dependence is non-analytic in G and a PM expansion in powers of G (or in the x_i) has a finite radius of convergence, given precisely by $x_1 = 1$ and $x_2 = 1$.
- Reason: a singularity at the unphysical points

$$x_i^2 = -1 : Q^2 = -4\,m_i^2$$

corresponding to t-channel thresholds

- This defines quantitatively the DKT bound!
- Open problem: does this phenomenon occur also elsewhere (e.g. deflection angle) above 3PM?

Diagram giving a branch point at $Q^2 = 4\ m_1^2$



An interesting use of QFT's crossing symmetry:
the same analytic function describes BH-BH annihilation!

Explicit results in the ZFL allow to have full control of the various regimes (from NR to UR)

GR

$$\sigma_Q = \sigma - \frac{Q^2}{2m_1 m_2} = -\frac{u - m_1^2 - m_2^2}{2m_1 m_2}$$

$$\lim_{\omega\to 0} \frac{dE^{\mathrm{gr}}}{d\omega} = \frac{4G}{\pi}\Bigg[2m_1 m_2\left(\sigma^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}} - 2m_1 m_2\left(\sigma_Q^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$

$$+\frac{m_1^2}{2} - m_1^2\left(\left(1+\tfrac{Q^2}{2m_1^2}\right)^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}$$

$$+\frac{m_2^2}{2} - m_2^2\left(\left(1+\frac{Q^2}{2m_2^2}\right)^2 - \tfrac{1}{2}\right)\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

in URL

$$\frac{4G}{\pi}\Bigg[1 + \frac{1}{8x_1^2} + \frac{1}{8x_2^2} + \log(\theta_s^{-2}) + \log(16x_1x_2)$$

$$-\frac{\left(1+x_1^2+\frac{1}{8x_1^2}\right)\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}} - \frac{\left(1+x_2^2+\frac{1}{8x_2^2}\right)\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\Bigg]$$

and in N=8-SUGRA

$$\lim_{\omega\to 0}\frac{dE^{\mathcal{N}=8}}{d\omega}=\frac{4G}{\pi}\Bigg[2m_1m_2\sigma^2\frac{\operatorname{arccosh}\sigma}{\sqrt{\sigma^2-1}}-2m_1m_2\sigma_Q^2\frac{\operatorname{arccosh}\sigma_Q}{\sqrt{\sigma_Q^2-1}}$$

$$-\frac{(Q^2)^2}{4m_1^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_1^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_1^2}\right)^2-1}}-\frac{(Q^2)^2}{4m_2^2}\frac{\operatorname{arccosh}\left(1+\frac{Q^2}{2m_2^2}\right)}{\sqrt{\left(1+\frac{Q^2}{2m_2^2}\right)^2-1}}\Bigg]_{Q=2p\sin\frac{\Theta_s}{2}}$$

becoming in URL

$$\frac{4G}{\pi}\left[1+\log(\theta_s^{-2})+\log(16x_1x_2)-x_1^2\frac{\cosh^{-1}(1+2x_1^2)}{\sqrt{(1+2x_1^2)^2-1}}-x_2^2\frac{\cosh^{-1}(1+2x_2^2)}{\sqrt{(1+2x_2^2)^2-1}}\right]$$

Universality broken at finite x_i, recovered only for x_i going to infinity (above DKT)

From below to above DKT @ all ω

(DHRV, in preparation)

- We have just considered leading order in $\theta_s \ll 1$.
- Up to $\omega \sim b^{-1}\sigma^{1/2}$ (resp. $b^{-1}\theta_s^{-1}$) one goes qualitatively from below to above DKT simply by $\sigma^{1/2} \to \theta_s^{-1}$
- Below DKT, and for $b^{-1}\sigma^{1/2} < \omega < b^{-1}\sigma^{3/2}$, we confirm a power law behavior but with an exponent definitely larger than 1 (~ 2.2).
- If that's true also above DKT (in the corresponding window) the log ω^* of G&V & CC(C)V for (E^{rad}/E) would be the result of some unjustified approximation.
- Guess by G&V about strong fall-off above $\omega^* \sim b^{-1}\theta_s^{-3}$ ($\sim b^{-1}\sigma^{-3/2}$) appears to be confirmed.
- A preliminary table summarizing the situation:

UR limits @ different ω (prelim.)

	soft ($\omega\, b < 1$)	interm. ($1 < \omega\, b < \sigma^{1/2}$) ($1 < \omega\, b < 1/\theta_s$)	hard ($\sigma^{1/2} < \omega\, b < \sigma^{3/2}$) ($\theta_s^{-1} < \omega\, b < \theta_s^{-3}$)
below DKT	$\theta_s^3 \ \log\sigma$ (same)	$\theta_s^3 \ \log\left(\frac{\sigma}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^3\sqrt{\sigma})$	$\theta_s^3\sqrt{\sigma}\ (\omega b)^{-1-\Delta}$ $\Delta E/\sqrt{s} = \theta_s^3\sqrt{\sigma}$ confirmed w/ $\Delta \sim 1$
above DKT	$\theta_s^3 \ \log\theta_s^{-2}$ (same)	$\theta_s^3 \ \log\left(\frac{\theta_s^{-2}}{\omega^2 b^2}\right)$ $(\Delta E/\sqrt{s} = \theta_s^2)$	$\theta_s^2\ (\omega b)^{-1}$ $\Delta E/\sqrt{s} = \theta_s^2 \log\theta_s^{-2}$ G&V/CCCV to be checked

$$\frac{1}{\sqrt{s}}\frac{dE^{rad}}{d\omega b}\ ; \quad \frac{\Delta E^{rad}}{\sqrt{s}}$$

Take-Home Conclusions

- Semiclassical gravitational scattering is an interesting topic for at least two reasons:

1. For large masses it complements other, purely classical, methods for extracting GW signals from astrophysical phenomena (BH mergers, encounters, …)
2. At high energy it can address (via gedanken experiments involving elementary particles/strings) fundamental quantum gravity issues for which the semiclassical approximation may already be sufficient (BH formation, unitarity, information paradox…)

These two regimes are somehow connected via the PM expansion at large σ, but the connection appears now to be more subtle than previously thought (breakdown of G-expansion in the URL for certain observables?).

Thank you for your attention!

The angular momentum puzzle

- Damour's shortcut uses a linear response formula (Bini-Damour '12) relating radiation reaction to the loss of E and J. In order to get a non-zero RR at 3PM one needs an E or J loss at 2PM (1 loop)
- Everybody agrees that E-loss starts at 3PM. So one needs a 2PM J-loss. But how can this be if radiation reaction effects start at 3PM?
- There is now some consensus that the 2PM J-loss is due to J-transfer from the two-particle system to the static gr. field (=strictly zero frequency modes)
- Damour talks about a "mechanical" J loss. How is this related to the J_B defined in terms of the Bondi-Sachs form of the metric?

- J_B has an infamous gauge ambiguity
- Usually solved by choosing a "canonical gauge" in which J_B -> J_{ADM} as u -> - \infty.
- The loss of that J_B starts at 3PM and looks to be the one carried by true radiation.
- How can we identify (in BS-coordinates) the J_{mech} needed in Damour's argument?

- According to Vilkovisky and myself (2201.11607, also Javadinezhad & Porrati, 2211.06538) it corresponds to choosing a different ("intrinsic") Bondi gauge
- Another proposal (Yau et al. 2102. ; Riva et al. 2302.) identifies J_{mech} at both u = -\infty & + \infty by enforcing the canonical gauge at both ends.
- NB: This is NOT J_B in ANY gauge! All agree @ 2PM…