Energy correlations in multi-particle states

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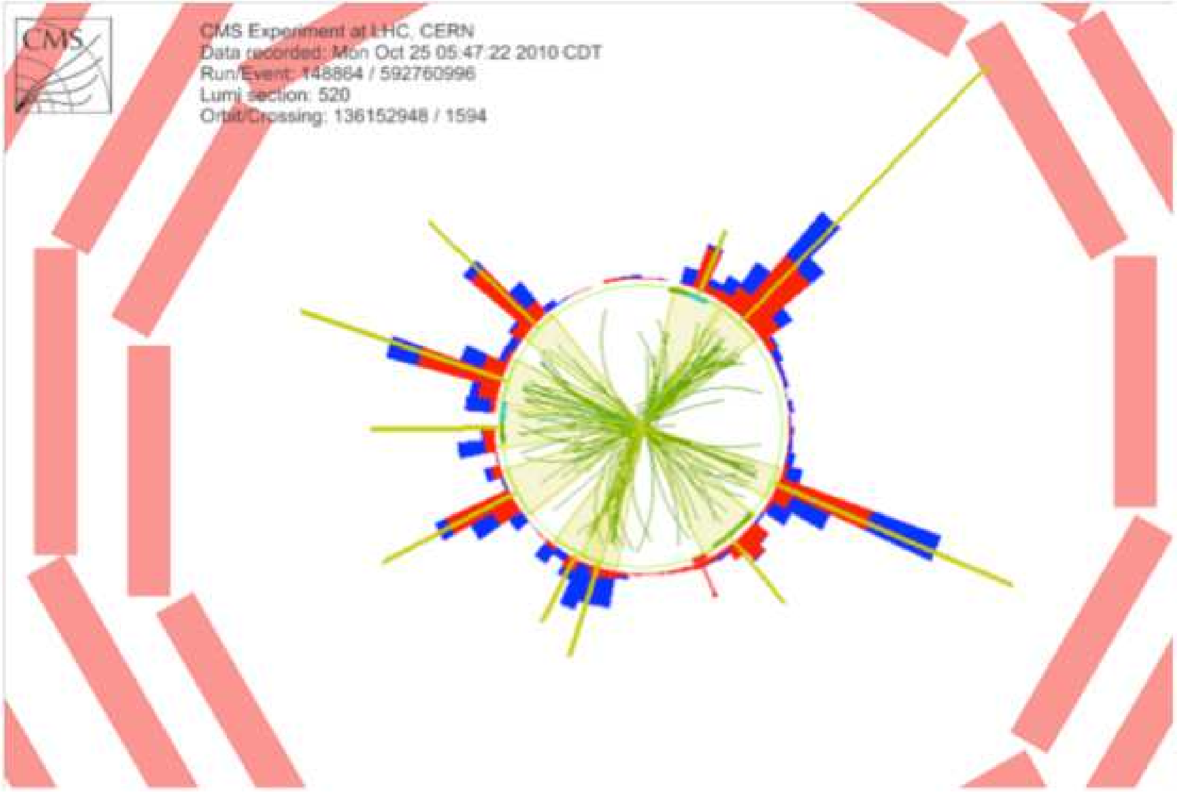
Amplitudes 23
August 9, 2023

Why multi-particle states

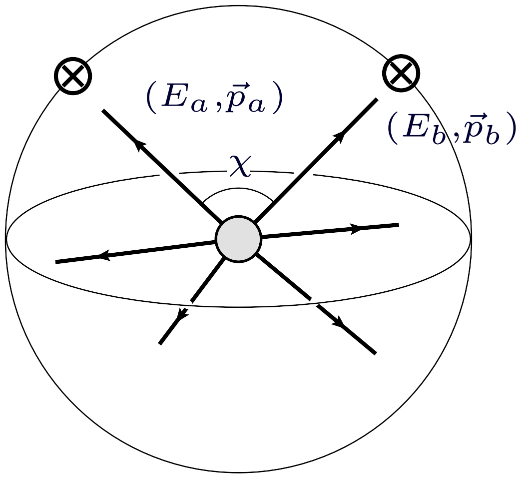
Final states at LHC:



- ✔ A lot of particles produced
- ✔ Energy of particles is deposit at the calorimeters
- ✔ Admit description in terms of the energy distribution on the celestial sphere

Energy-energy correlation

- ✔ Function of the angle $0 \leq \chi \leq \pi$ between detected particles [Basham,Brown,Ellis,Love'78]



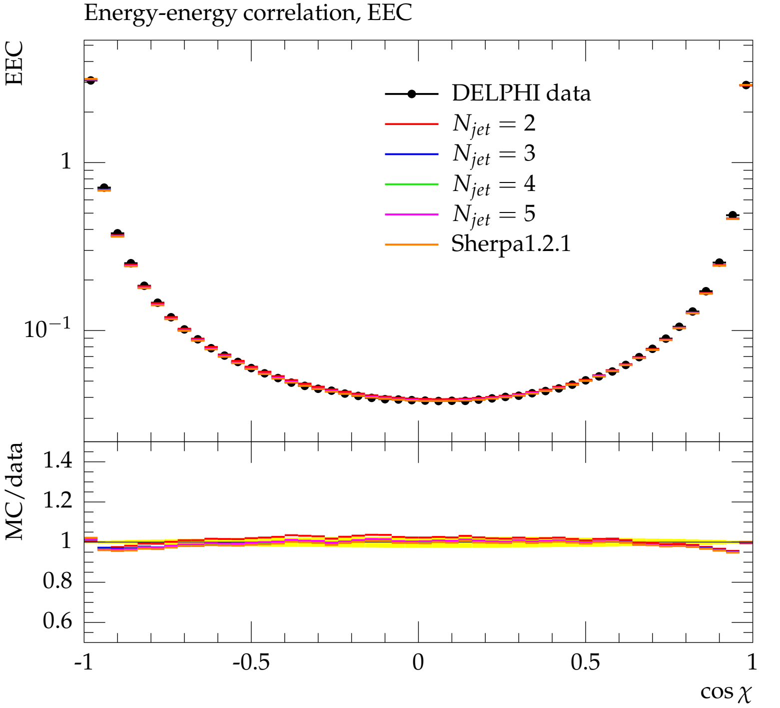
$$\mathrm{EEC}(\chi) = \sum_{a,b} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)$$

Total energy $\sum_a E_a = Q$

- ✔ EEC in e^+e^- final states (1978 – today):
  - ✗ Very precise experimental data
  - ✗ Slow progress on the theory side

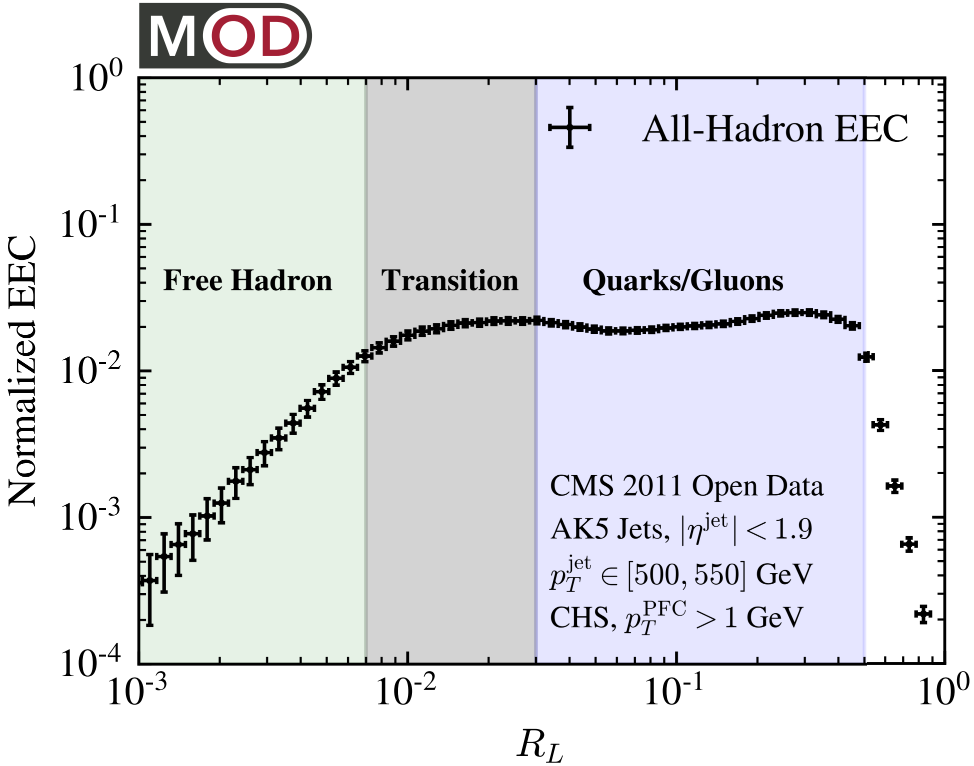
$$\mathrm{EEC}(\chi) = \underbrace{\alpha_s(Q)A(\chi)}_{\text{Basham et al'78}} + \underbrace{\alpha_s^2(Q)B(\chi)}_{\text{Dixon et al'18}} + O(\alpha_s^3)$$

  - ✗ Much faster progress in MSYM theory (3 loops + strong coupling)



Energy correlations at LHC

Recent analysis of CMS Open Data [Komiske,Moult,Thaler,Zhu'23]



χ^2EEC(χ) versus χ

For small angle χ, the EEC describes the correlation between particles belonging to the same jet

Behaves differently at small angles depending on how χ compares with $\Lambda_{\rm QCD}/Q$

$$\textbf{Flat}: \quad \text{EEC} \sim \text{const}, \qquad\qquad \textbf{OPE}: \quad \text{EEC} \sim 1/\chi^{2-\gamma}$$

What is physics mechanism of the transitioning between the two regimes?

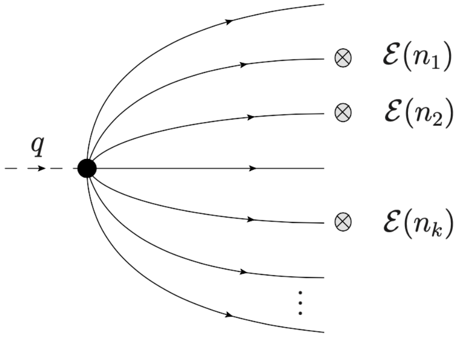
Warm up example: free scalar theory

Multi-particle final state containing K massless scalar particles

$$|H(q)\rangle = \int d^4x\, e^{iqx} O_K(x)|0\rangle\,, \qquad O_K(x) = \phi^K(x)$$

Energy correlations at finite K (= new parameter of the expansion)

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_k)\rangle =$$



$$= \int d\sigma_K\, w_{n_1}(p_1)\dots w_{n_k}(p_k)$$

The weight factor $w_n(p)$ selects the particle in the final state moving along $n^\mu = (1, \vec{n})$

$$w_n(p) = p^0 \delta^{(2)}\left(\frac{\vec{p}}{p^0} - \vec{n}\right)$$

The differential cross-section

$$d\sigma_K = (2\pi)^4 \delta^{(4)}(q - \sum_{i=1}^{K} p_i) \prod_{n=1}^{K} d\text{LIPS}(p_n)\,, \qquad d\text{LIPS}(p) = \frac{d^4p}{(2\pi)^4} 2\pi\delta_+(p^2)$$

Large K limit in a free theory

Energies of detected particle scale as $p_i^0/Q = \varepsilon_i/K$

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_k)\rangle = Q^k \int_0^\infty \prod_{i=1}^k \frac{d\varepsilon_i\,\varepsilon_i^2}{8\pi}\,e^{-\varepsilon_i}\left[1-\frac{1}{K}\sum_{i<j}\varepsilon_i\varepsilon_j(1-z_{ij})+\dots\right]$$

Ensemble of k noninteracting particles with energies distributed according to $dP(\varepsilon) = d\varepsilon\,\varepsilon^2 e^{-\varepsilon}$

Dimensionless angular variables

$$z_{ij} = \frac{q^2(n_in_j)}{2(qn_i)(qn_j)} = \frac{1}{2}(1-\cos\theta_{ij})\,, \qquad q^\mu = (Q,\vec{0})$$

✔ The leading term is independent of the angular variables

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_k)\rangle_{\rm free} = \left(\frac{Q}{4\pi}\right)^K \left[1+\frac{1}{K}\sum_{1\le i<j\le k} 9z_{ij}+\dots\right]$$

Homogeneous distribution of the energy, the angular dependence is suppressed by $1/K$

✔ The same behaviour was observed in $\mathcal{N}=4$ SYM at strong coupling $\lambda\to\infty$ [Hofman,Maldacena]

$$\langle \mathcal{E}(n_1)\dots\mathcal{E}(n_k)\rangle_{\mathcal{N}=4} = \left(\frac{Q}{4\pi}\right)^K [1+O(1/\lambda)]$$

In both cases, the flat regime is associated with a large number of soft particles in the final state

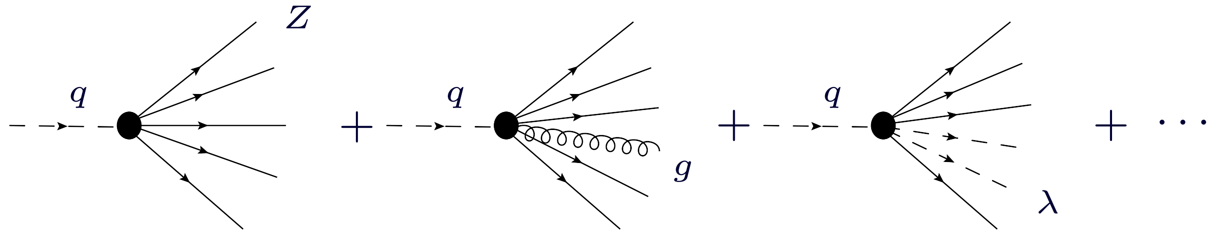
Large K limit in interacting theory

Energy correlation in multi-particle final state is flat in a free theory

How does the interaction between the particles in the final state affect this result?

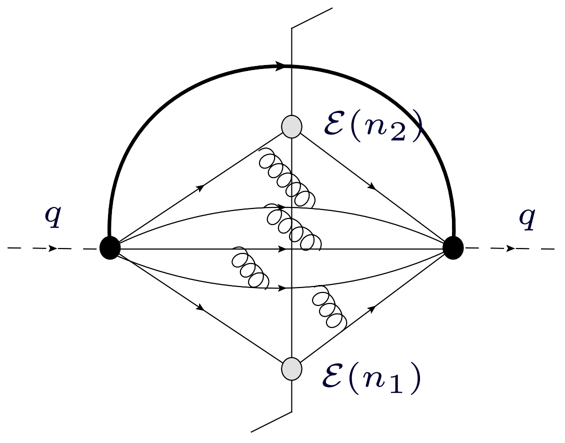
Consider the EEC in $\mathcal{N}=4$ SYM in the final state created by a half-BPS operator $O_K = \mathrm{tr}[Z^K(x)]$

$$|H(q)\rangle = \int d^4x\, e^{iqx} O_K(x)|0\rangle =$$



Energy-energy correlation

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_K = \frac{(q^2)^4}{2(qn_1)^3(qn_2)^3}\,\frac{\mathcal{F}_K(z)}{(4\pi)^2} =$$



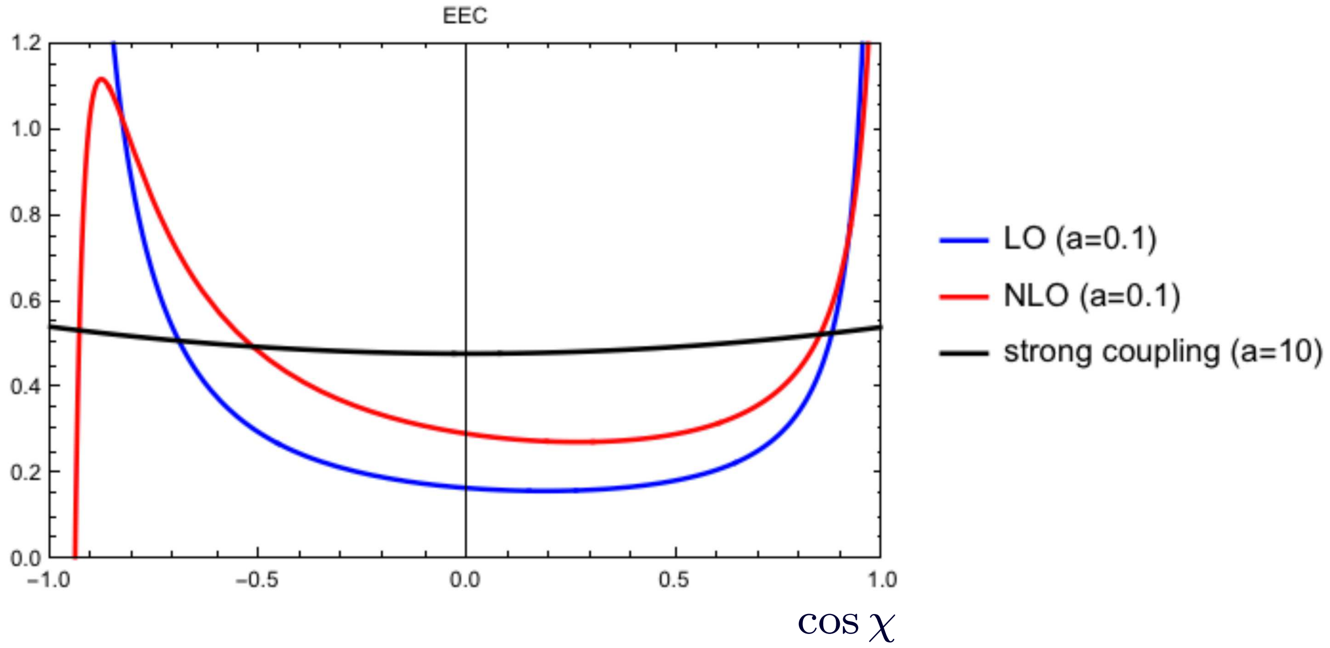
Sum over all possible final states containing many on-shell particles

Depends on 't Hooft coupling λ and angle between the detectors

$$z = \frac{q^2(n_1 n_2)}{2(qn_1)(qn_2)} = \frac{1}{2}(1-(\vec{n}_1\vec{n}_2)) = \frac{1}{2}(1-\cos\theta_{12})$$

What we expect for $\mathcal{F}_K(z,\lambda)$

For $K=2$ the EEC is peaked around $z=0$ and $z=1$ at weak coupling (two jet final state!)



The EEC becomes flat at strong coupling $\lambda\to\infty$ and K fixed [Hofman,Maldacena]

$$\mathcal{F}_K(z) \overset{\lambda\to\infty}{\sim} 2+\frac{4\pi^2}{\lambda}(1-6z(1-z))+O(\lambda^{-3/2})$$

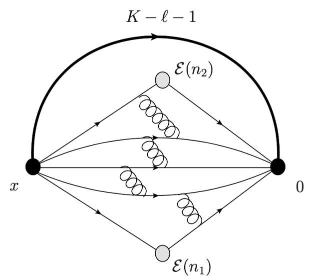
For $K\gg 1$ and finite λ we expect the EEC to be flat as well [Chicherin,GK,Sokatchev,Zhiboedov]

$$\mathcal{F}_{K\gg 1}(z)=2+O(1/K)$$

Examine the transition from $K=2$ to $K\to\infty$ at weak coupling

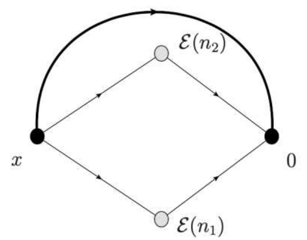
Weak coupling

$$\mathcal{F}_K(z,\lambda) = \mathcal{F}_K^{(0)}(z) + \sum_{\ell=1}^{\infty}\left(\frac{\lambda}{4\pi^2}\right)^{\ell}\mathcal{F}_K^{(\ell)}(z) =$$

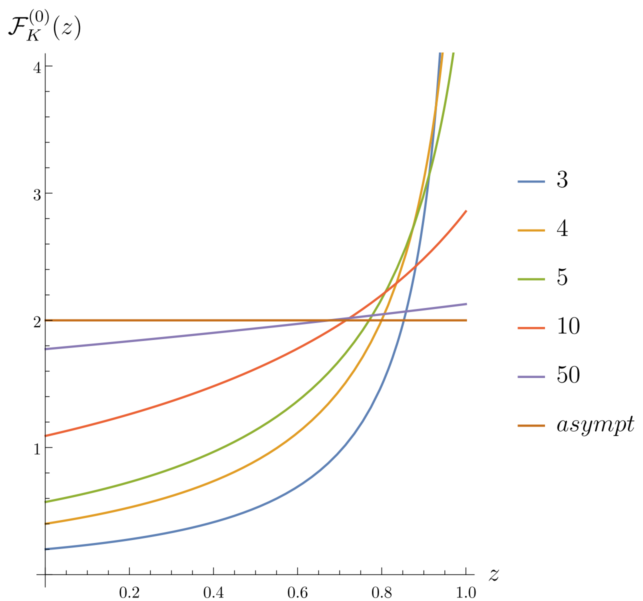


Born approximation for arbitrary weight K

$$\mathcal{F}_K^{(0)}(z) = \frac{2(K-2)(K-1)}{(K+1)(K+2)}\,{}_2F_1\left(3,3;K+3|z\right) =$$



The angular dependence flattens out for $K \to \infty$



$$\mathcal{F}_K^{(0)}(z) \overset{K\gg 1}{=} 2 + \frac{6}{K}(3z-2) + O(1/K^2)$$

One and two loops

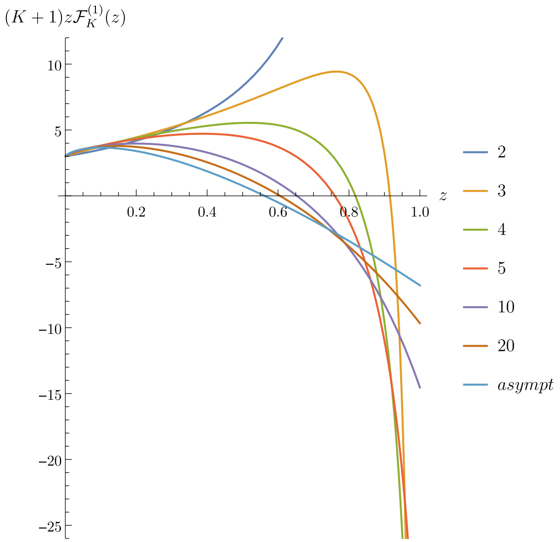
$$\mathcal{F}_K^{(1)}(z) = \frac{1}{z^{K+2}}\Big[c_1^{[K]}(z)L(z) + c_2^{[K]}(z)\text{Li}_2(z) + c_3^{[K]}(z)\log(z)\log(1-z)$$
$$+ c_4^{[K]}(z)\log^2(1-z) + c_5^{[K]}(z)\log(1-z) + c_6^{[K]}(z)\log(z) + c_7^{[K]}(z)\Big]$$

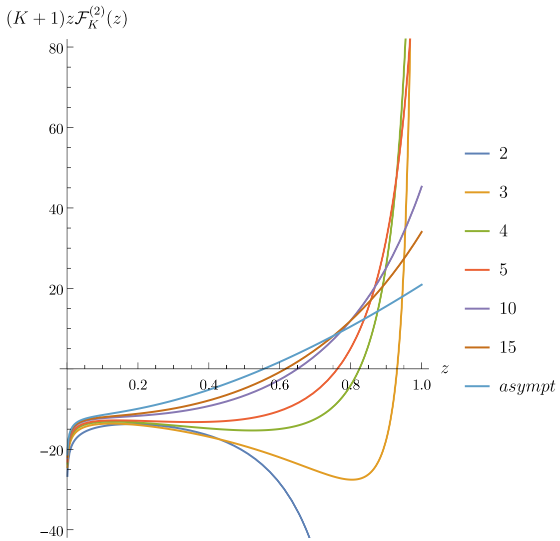
$$\mathcal{F}_K^{(2)}(z) = \frac{1}{z^{K+2}}\Big[\text{Multi-linear combinations of HPLs with argument } \sqrt{z} \text{ of weight } w \leq 5\Big]$$

$c_m^{[K]}(z)$ are polynomials of degree $K-1$

$$L(z) := \text{Li}_3(1-z) + \frac{1}{2}\text{Li}_2(z)\log(1-z) - \frac{1}{12}\log^3(1-z) + \frac{1}{2}\log^2(1-z)\log(z) - \zeta_2\log(1-z) - \zeta_3$$

The angular dependence of $(K+1)z\mathcal{F}_K^{(\ell)}(z)$ flattens out for $K \to \infty$





$$\mathcal{F}_{K\gg 1}^{(2)}(z) = O(1/K)$$

Back to LHC

Energy correlations for small angles $z \to 0$ and weak coupling $a = \lambda/(4\pi^2) < 1$

$$\begin{aligned}\mathcal{F}_K(z) &= \mathcal{F}_K^{(0)}(z) + a\mathcal{F}_K^{(1)}(z) + a^2\mathcal{F}_K^{(2)}(z) + \dots \\ &= 2 + \frac{3}{K}\left[\frac{a}{z} + \frac{a^2}{z}\log z + \dots\right] \\ &= 2 + \frac{3a}{K} z^{-1+a}\end{aligned}$$

Agrees with the leading OPE contribution at small angles $z = \sin^2(\chi/2) \ll 1$ [Kologlu et al]

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle \sim 1 + \frac{\langle \mathbb{O}_3^+\rangle_H}{z^{1-\gamma_3^+/2}}\,, \qquad \langle \mathbb{O}_3^+\rangle_H = O(1/K)$$

$\mathbb{O}_3^+$ is the spin three light-ray operator of positive signature with anomalous dimension γ_3^+

Two regimes at small z / large K:

For fixed z and $K \to \infty$:	$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle = 1 + O(1/K)$	*Flat regime*
For $z \to 0$ and fixed K :	$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle \sim \dfrac{1}{z^{1-\gamma_3^+/2}}$	*OPE regime*

The transition happens for $K \sim 1/z^{1-\gamma_3^+/2}$

Conclusions

- ✔ The energy correlations in multi-particle states exhibit two characteristic regimes:

$$\mathbf{Flat:}\quad \text{EEC} \sim \text{const}, \qquad\qquad \mathbf{OPE:}\quad \text{EEC} \sim 1/\chi^{2-\gamma}$$

- ✔ The transition between the two regimes is controlled by the particle multiplicity K and the dynamics of the theory

$$\text{EEC} \sim 1 + \frac{C}{K\chi^{2-\gamma}}$$

- ✔ An analogous transition was previously observed in QCD in the measurement of the angular energy distribution of particles belonging to the same energetic jet

- ✔ Scattering amplitudes/cross sections have interesting properties in the limit of large number of particles/legs