Heavy to light form factors

☞ *discussion topics*

Seeds for discussion

- How to combine form factors which are somewhat **inconsistent**
  - Error inflation? e.g. PDG rescaling
  - Example: combination of $B \to \pi$ form factors from FNAL/MILC, RBC/UKQCD and JLQCD
- Systematic uncertainties that take into account the **choice of parameterization**?
  - Criteria to choose an optimal number of z-parameters
  - Do we trust standard error estimations for these parameters to be “accurate”?
  - Do we need to adopt alternative approaches?
    Gubernari, **van Dyk**, **Virto**, 2011.09813
    Di Carlo, **Martinelli**, Naviglio, Sanfilippo, Simula, Vittorio, 2105.02497
    Martinelli, Simula, Vittorio, 2202.10285
    Gubernari, **Reboud**, van Dyk, **Virto**, 2206.03797
    Blake, Meinel, Rahimi, **van Dyk**, 2205.06041
    Flynn, **Jüttner**, **Tsang**, 2303.11285
- How to properly account for uncertainties in **extrapolations** outside of the region directly covered by lattice calculations:
  - Example: $B_s \to K$ form factors
- Issues raised by the recent Belle analysis which considers simultaneously exclusive ($B \to (\pi, \rho, \omega, \eta, \eta')$ and inclusive $B \to X_u \ell \nu$ decays

$B \to \pi$ form factors

FLAG5 Web update

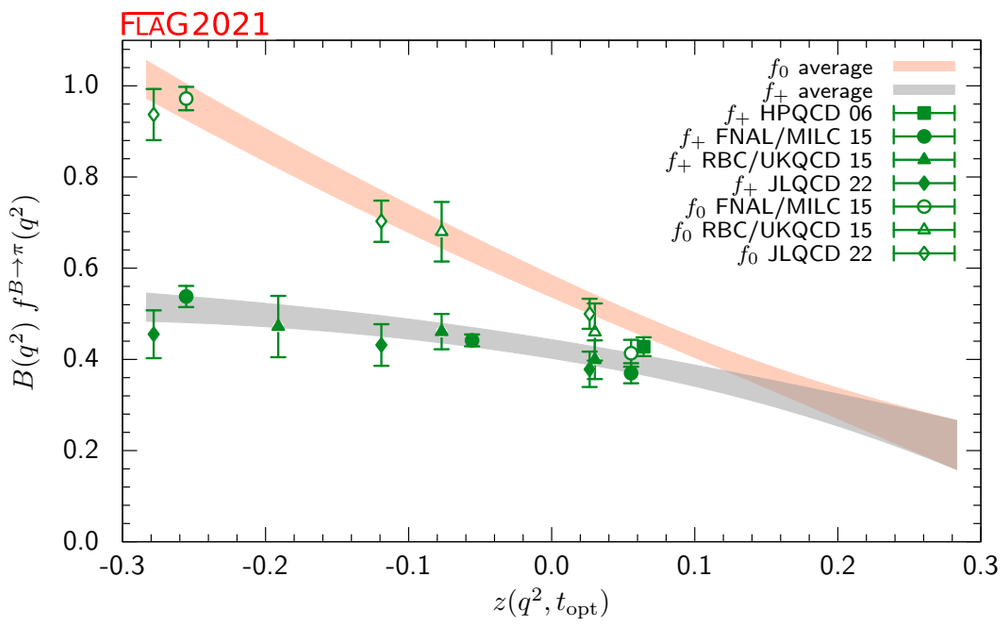
http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf

FNAL/MILC (15) + RBC/UKQCD (15) + JLQCD (22)

$B \to \pi$ $(N_f = 2+1)$

	Central Values	Correlation Matrix				
a_0^+	0.423 (21)	1	-0.00506	-0.0740	0.402	0.0923
a_1^+	−0.508 (93)	-0.00506	1	0.497	-0.0557	0.659
a_2^+	−0.74 (34)	-0.0740	0.497	1	-0.152	0.677
a_0^0	0.561 (24)	0.402	-0.0557	-0.152	1	-0.548
a_1^0	−1.42 (11)	0.0923	0.659	0.677	-0.548	1

$\chi^2/\text{dof} = 43.6/12$: error rescaled by $\sqrt{\chi^2/\text{dof}} = 1.9$



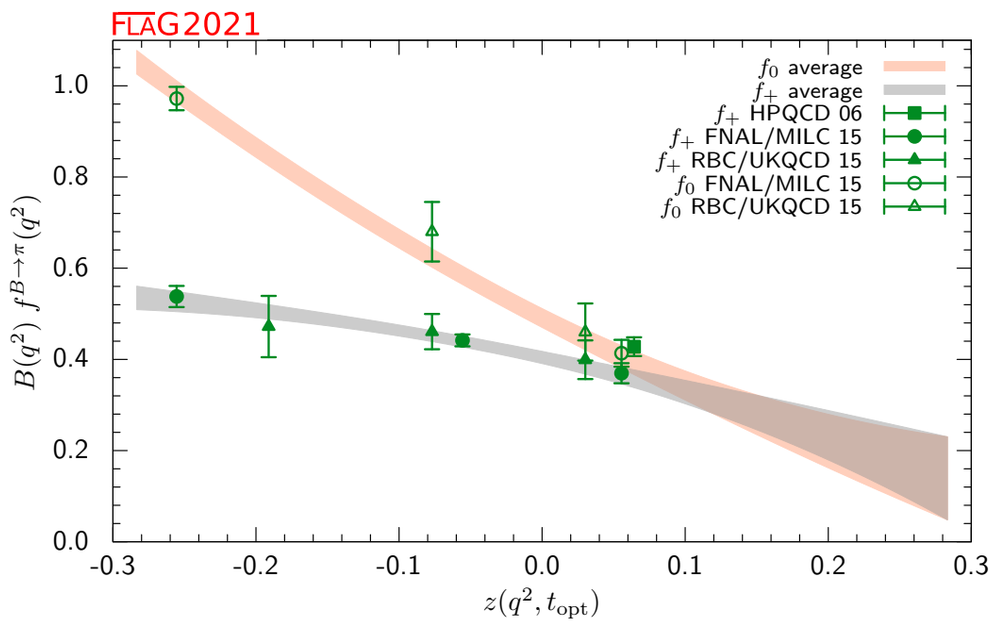
FLAG5 (2111.09849)

FNAL/MILC (15) + RBC/UKQCD (15)

$B \to \pi$ $(N_f = 2+1)$

	Central Values	Correlation Matrix				
a_0^+	0.404 (13)	1	0.404	0.118	0.327	0.344
a_1^+	−0.68 (13)	0.404	1	0.741	0.310	0.900
a_2^+	−0.86 (61)	0.118	0.741	1	0.363	0.886
a_0^0	0.490 (21)	0.327	0.310	0.363	1	0.233
a_1^0	−1.61 (16)	0.344	0.900	0.886	0.233	1

$\chi^2/\text{dof} = 0.82$: no error rescaling



$B \to \pi$ form factors

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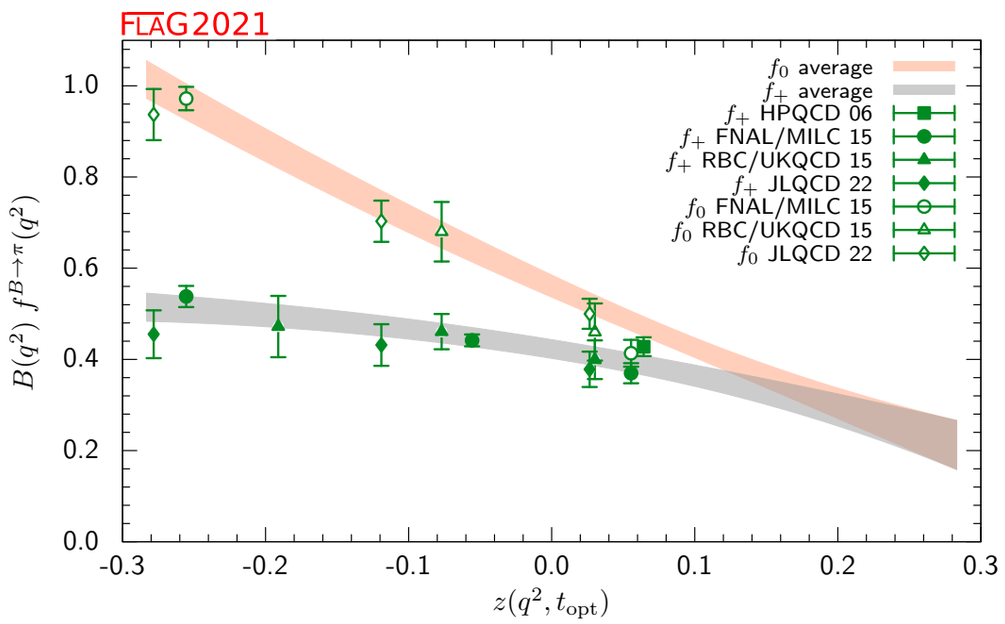

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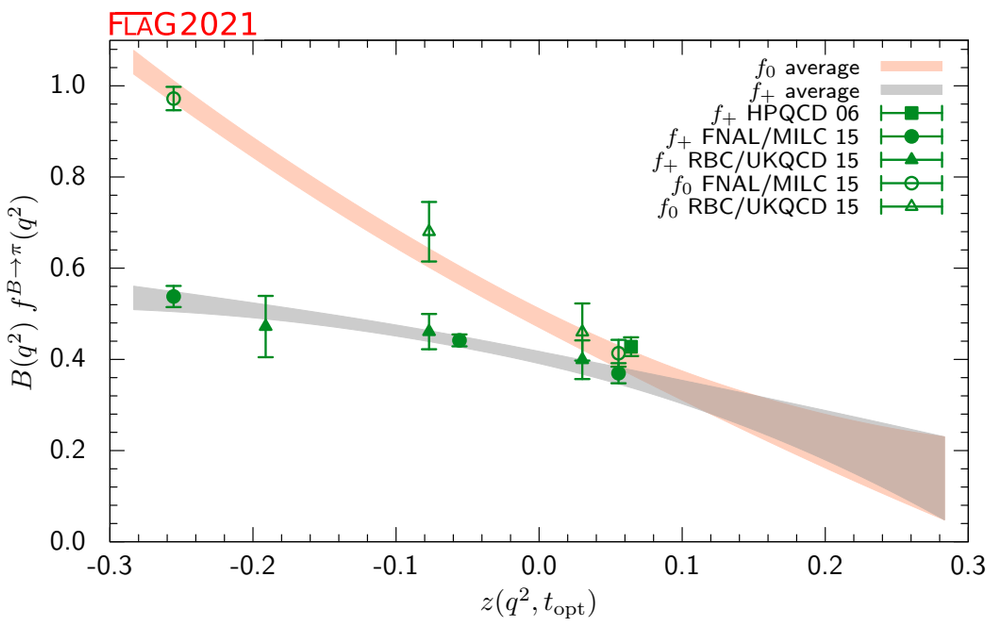

- Error on normalization increased $(a_0^{+,0})$

$B \to \pi$ form factors

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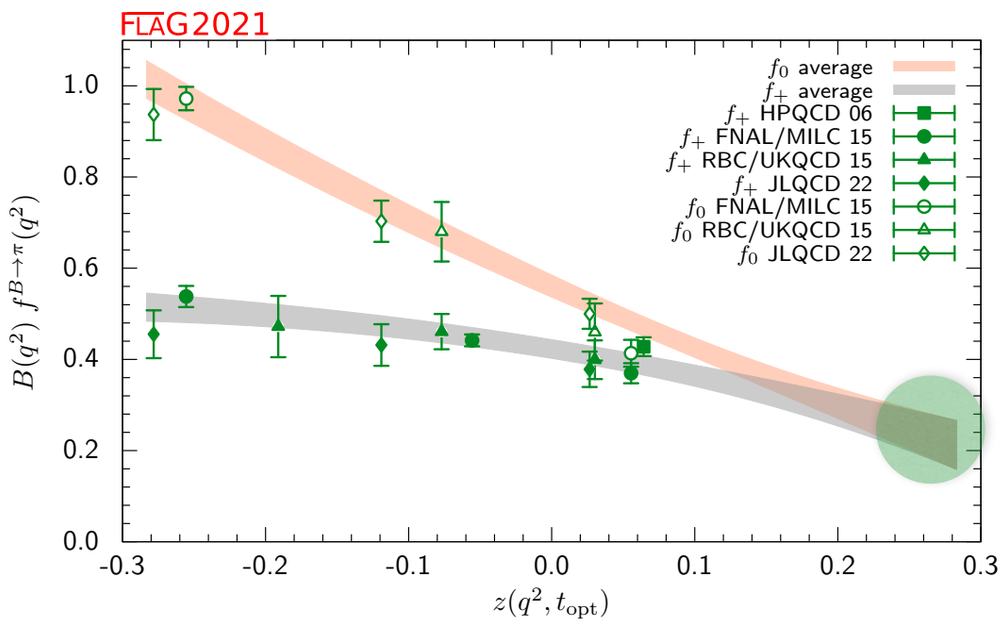

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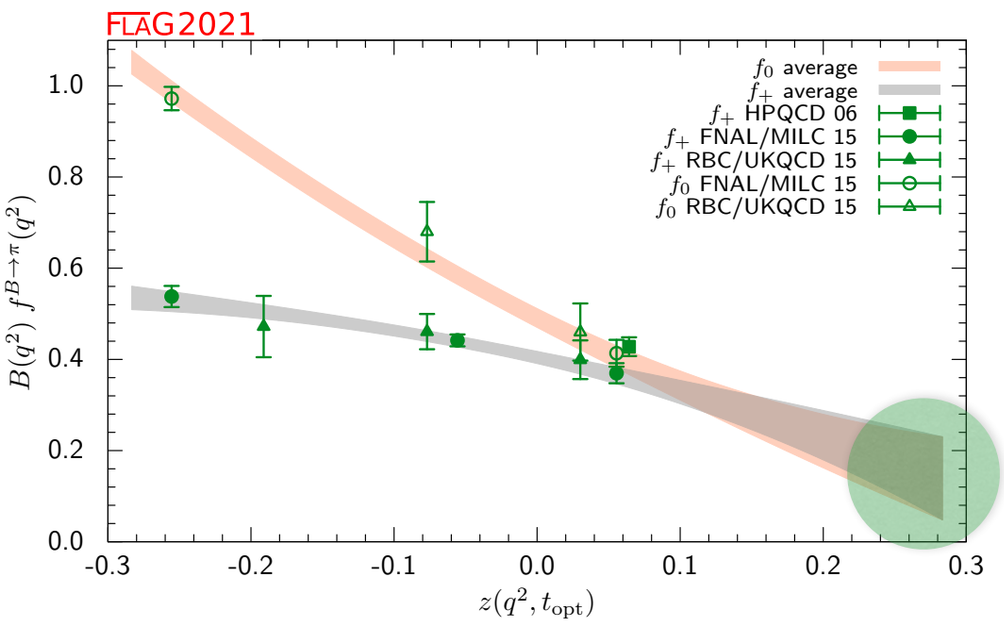


- Error on slope parameters decreased ($a_{1,2}^+$ and a_1^0)

$B \to \pi$ form factors

Small impact on $|V_{ub}|$ after including experimental data (information at small q^2 / large z)

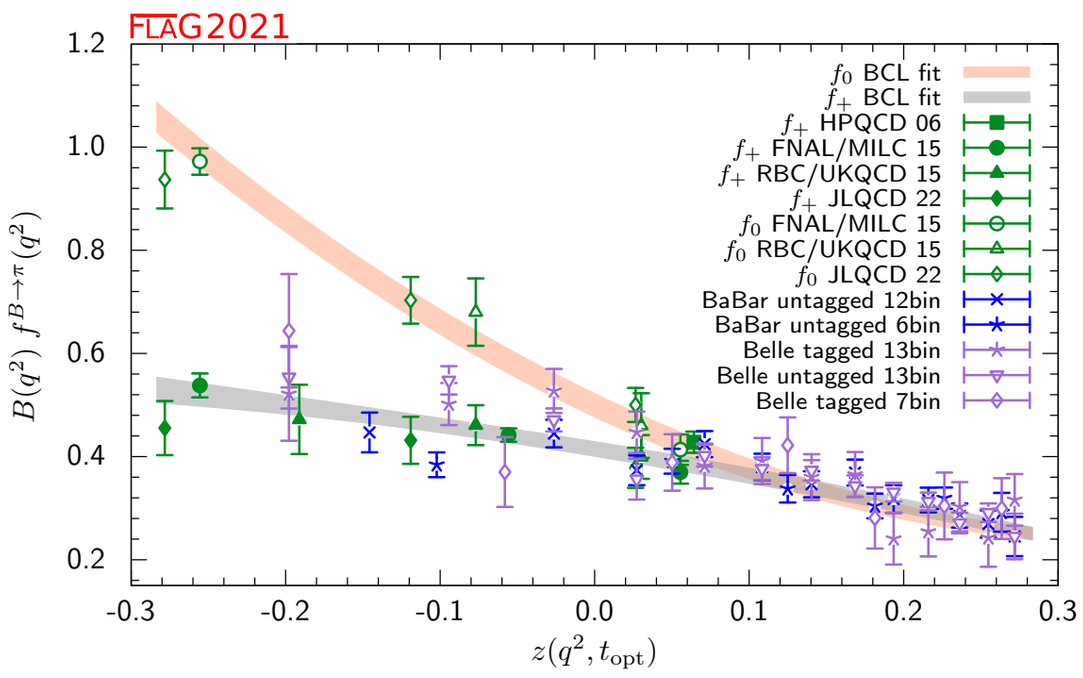
FLAG5 Web update

http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf

$B \to \pi \ell \nu$ $(N_f = 2+1)$

	Central Values	Correlation Matrix					
$\|V_{ub}\| \times 10^3$	3.64 (16)	1	-0.812	-0.107	0.127	-0.325	-0.151
a_0^+	0.425 (15)	-0.812	1	-0.189	-0.308	0.409	0.00937
a_1^+	-0.443 (39)	-0.107	-0.189	1	-0.499	-0.0345	0.150
a_2^+	-0.51 (13)	0.127	-0.308	-0.499	1	-0.189	0.128
a_0^0	0.560 (17)	-0.325	0.409	-0.0345	-0.189	1	-0.772
a_1^0	-1.346 (53)	-0.151	0.00937	0.150	0.128	-0.772	1

$\chi^2/\text{dof} = 116.6/62$: error rescaled by $\sqrt{\chi^2/\text{dof}} = 1.37$

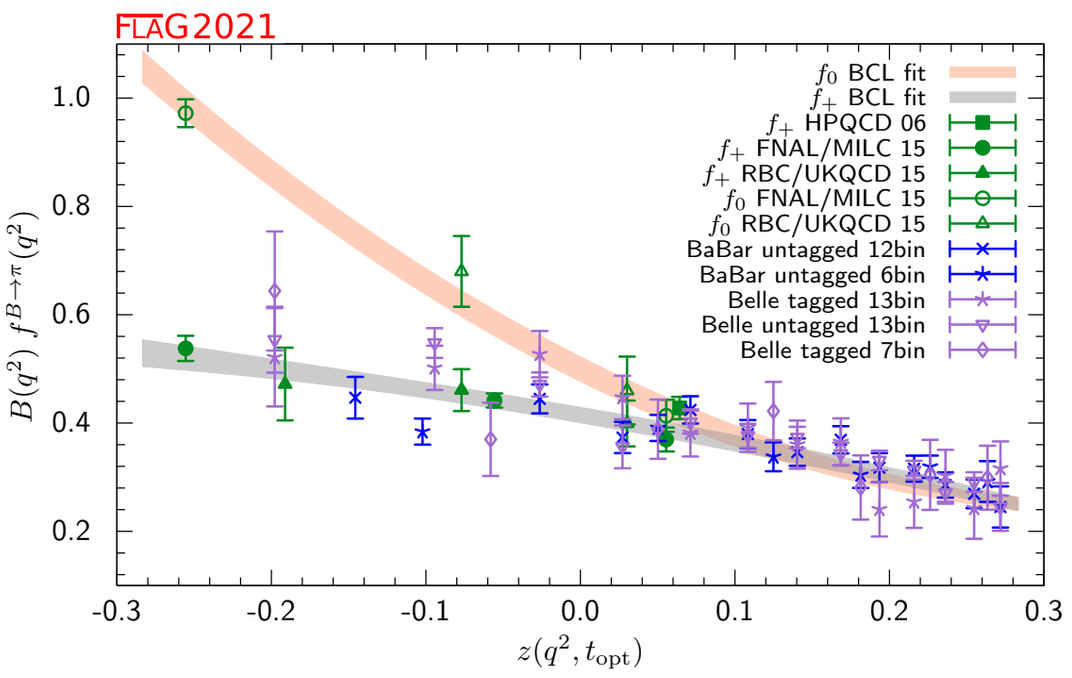


FLAG5 (2111.09849)

$B \to \pi \ell \nu$ $(N_f = 2+1)$

	Central Values	Correlation Matrix					
$\|V_{ub}\| \times 10^3$	3.74 (17)	1	−0.851	−0.349	0.375	−0.211	−0.246
a_0^+	0.415 (14)	−0.851	1	0.155	−0.454	0.260	0.144
a_1^+	−0.488 (53)	−0.349	0.155	1	−0.802	−0.0962	0.220
a_2^+	−0.31 (18)	0.375	−0.454	−0.802	1	0.0131	−0.100
a_0^0	0.500 (23)	−0.211	0.260	−0.0962	0.0131	1	−0.453
a_1^0	−1.424 (54)	−0.246	0.144	0.220	−0.100	−0.453	1

$\chi^2/\text{dof} = 78.7/56 = 1.41$: error rescale by $\sqrt{\chi^2/\text{dof}} = 1.19$



$B \to \pi$ form factors

- Dependence on number of z-parameters

	(Nz,Nz0)={2, 2}
chi2	139.
dof	14
p-value	0.
PDG factor	3.1473
b[0]	0.381 ± 0.031
b[1]	-0.768 ± 0.093
b0[0]	0.563 ± 0.040

	(Nz,Nz0)={2, 3}
chi2	52.2
dof	13
p-value	1.23439×10^{-6}
PDG factor	2.00423
b[0]	0.420 ± 0.022
b[1]	-0.451 ± 0.090
b0[0]	0.555 ± 0.025
b0[1]	-1.308 ± 0.092

	(Nz,Nz0)={3, 3}
chi2	43.6
dof	12
p-value	0.0000178496
PDG factor	1.90617
b[0]	0.423 ± 0.021
b[1]	-0.507 ± 0.093
b[2]	-0.75 ± 0.34
b0[0]	0.561 ± 0.024
b0[1]	-1.42 ± 0.11

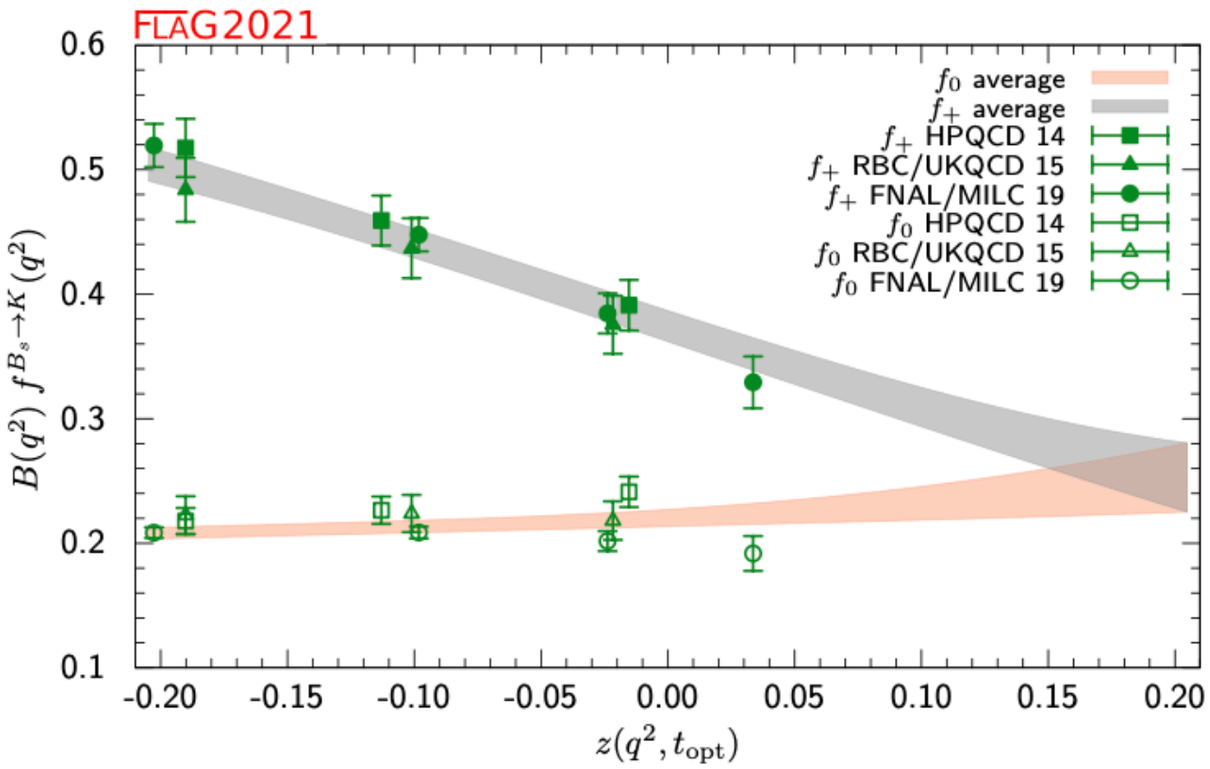
	(Nz,Nz0)={3, 4}
chi2	43.2
dof	11
p-value	0.0000101179
PDG factor	1.9812
b[0]	0.423 ± 0.022
b[1]	-0.50 ± 0.10
b[2]	-0.71 ± 0.38
b0[0]	0.564 ± 0.027
b0[1]	-1.44 ± 0.13
b0[2]	0.65 ± 0.40

	(Nz,Nz0)={4, 4}
chi2	29.5
dof	10
p-value	0.00102425
PDG factor	1.71827
b[0]	0.418 ± 0.019
b[1]	-0.22 ± 0.15
b[2]	4.8 ± 2.6
b[3]	18.8 ± 8.9
b0[0]	0.549 ± 0.024
b0[1]	-1.05 ± 0.21
b0[2]	7.1 ± 3.0

	(Nz,Nz0)={4, 5}
chi2	29.1
dof	9
p-value	0.000623945
PDG factor	1.79807
b[0]	0.418 ± 0.020
b[1]	-0.24 ± 0.17
b[2]	4.5 ± 2.9
b[3]	17.6 ± 9.8
b0[0]	0.543 ± 0.031
b0[1]	-0.95 ± 0.36
b0[2]	8.0 ± 4.0
b0[3]	$16. \pm 11.$

$B_s \to K$ form factors and $|V_{ub}/V_{cb}|$

- FLAG5 combined form factors:



$$\frac{1}{|V_{ub}|^2}\int_{q^2_{\rm min}=m_\mu^2}^{7\ {\rm GeV}^2}\frac{{\rm d}\Gamma(B_s\to K^-\mu^+\nu_\mu)}{{\rm d}q^2} = (2.26\pm0.38)\ {\rm ps}^{-1}$$

$$\frac{1}{|V_{ub}|^2}\int_{7\ {\rm GeV}^2}^{q^2_{\rm max}=(m_{B_s}-m_K)^2}\frac{{\rm d}\Gamma(B_s\to K^-\mu^+\nu_\mu)}{{\rm d}q^2} = (4.02\pm0.31)\ {\rm ps}^{-1}$$

$$\frac{|V_{ub}|}{|V_{cb}|}({\rm low}) = 0.0819 \pm 0.0072_{\rm lat.} \pm 0.0029_{\rm exp}$$

$$\frac{|V_{ub}|}{|V_{cb}|}({\rm high}) = 0.0860 \pm 0.0037_{\rm lat.} \pm 0.0038_{\rm exp}$$

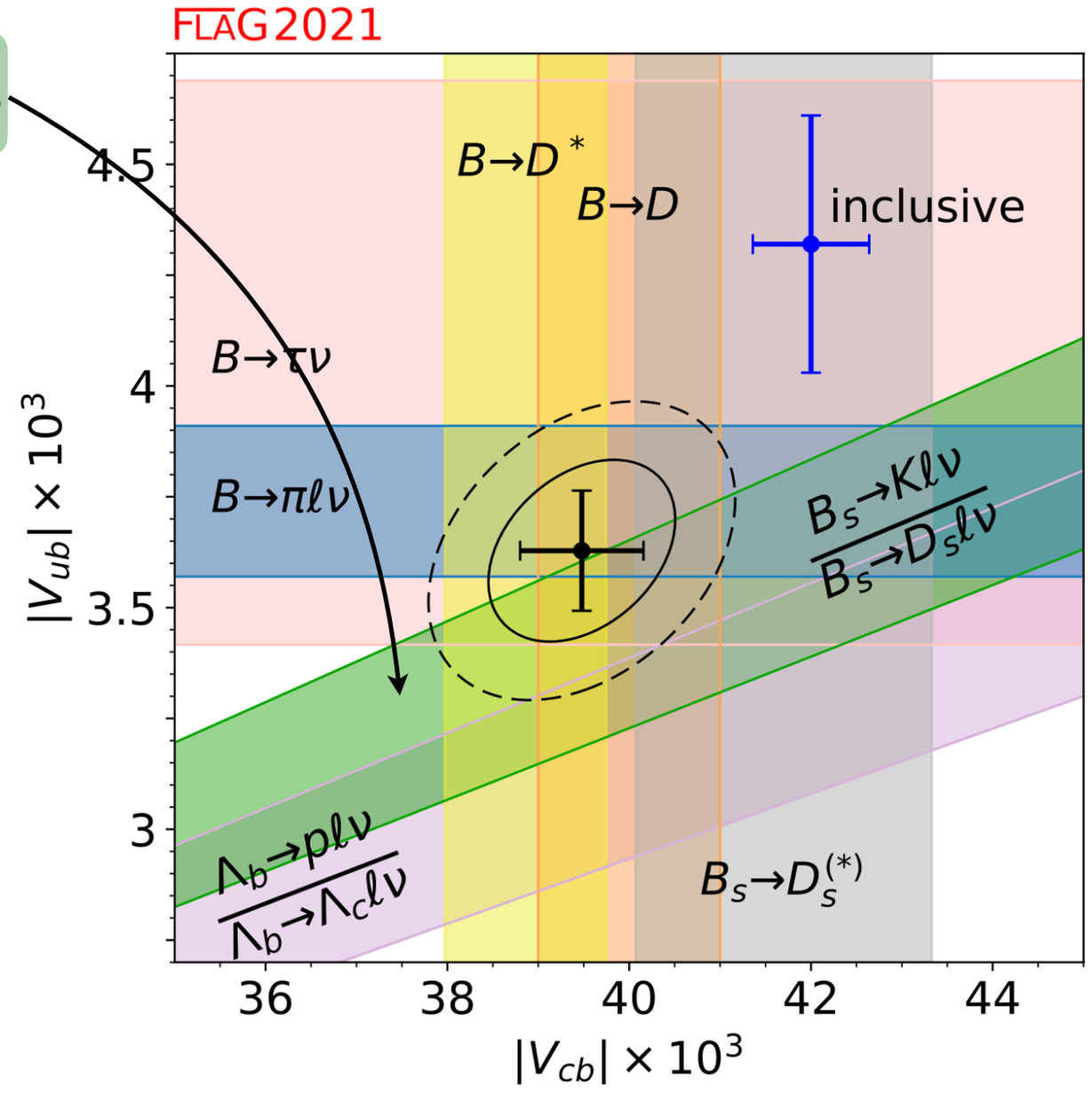
Note: RBC/UKQCD provides synthetic data points, HPQCD and FNAL/MILC only z-fit results

$B_s \to K$ form factors and $|V_{ub}/V_{cb}|$

- FLAG $|V_{ub}/V_{cb}|$ from $B_s \to K$ (only high-q^2):

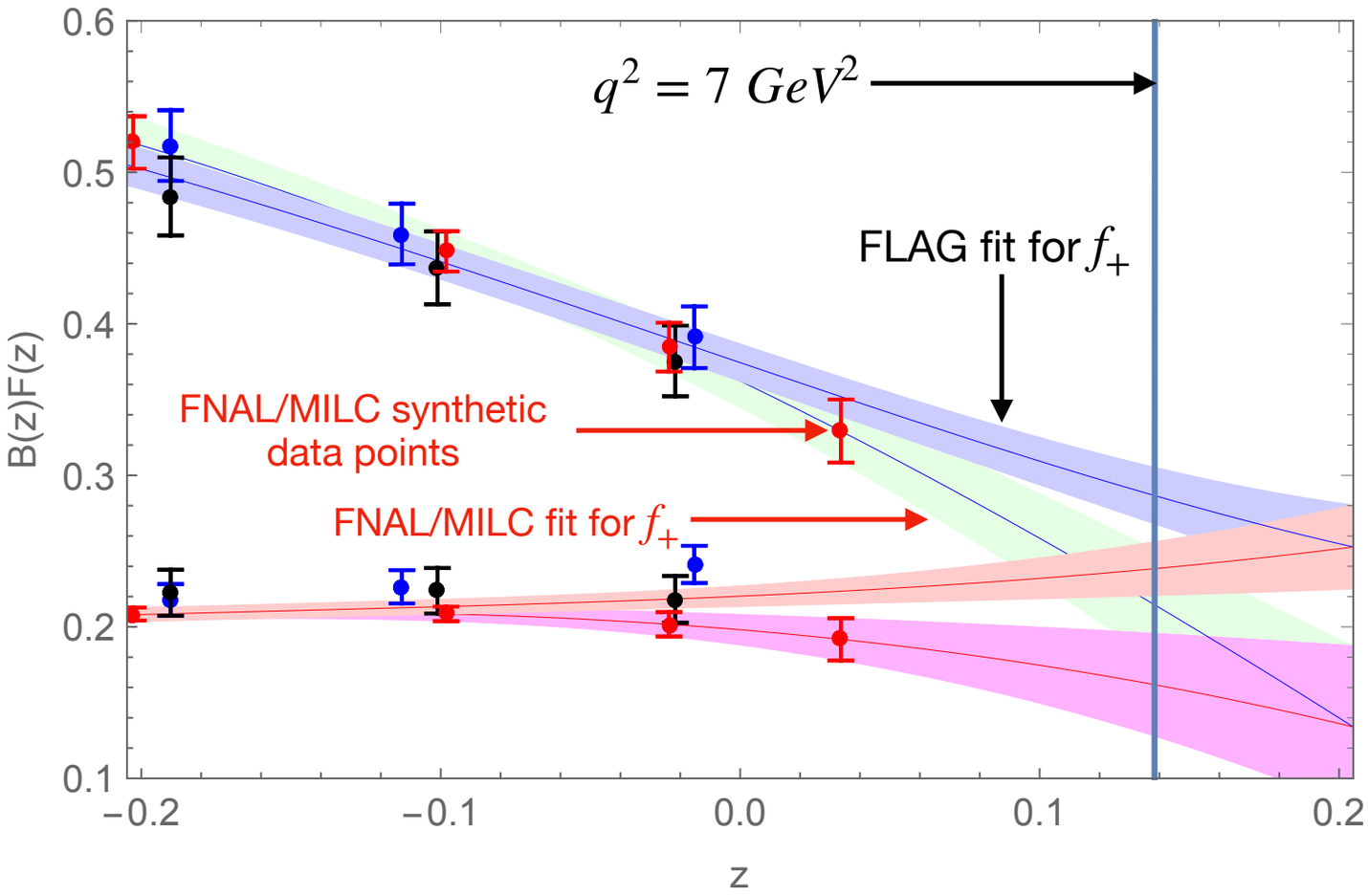
$$\frac{|V_{ub}|}{|V_{cb}|}(\text{low}) = 0.0819 \pm 0.0072_{\text{lat.}} \pm 0.0029_{\text{exp}}$$

$$\frac{|V_{ub}|}{|V_{cb}|}(\text{high}) = 0.0860 \pm 0.0037_{\text{lat.}} \pm 0.0038_{\text{exp}}$$



$B_s \to K$ form factors and $|V_{ub}/V_{cb}|$

- Using only FNAL/MILC:

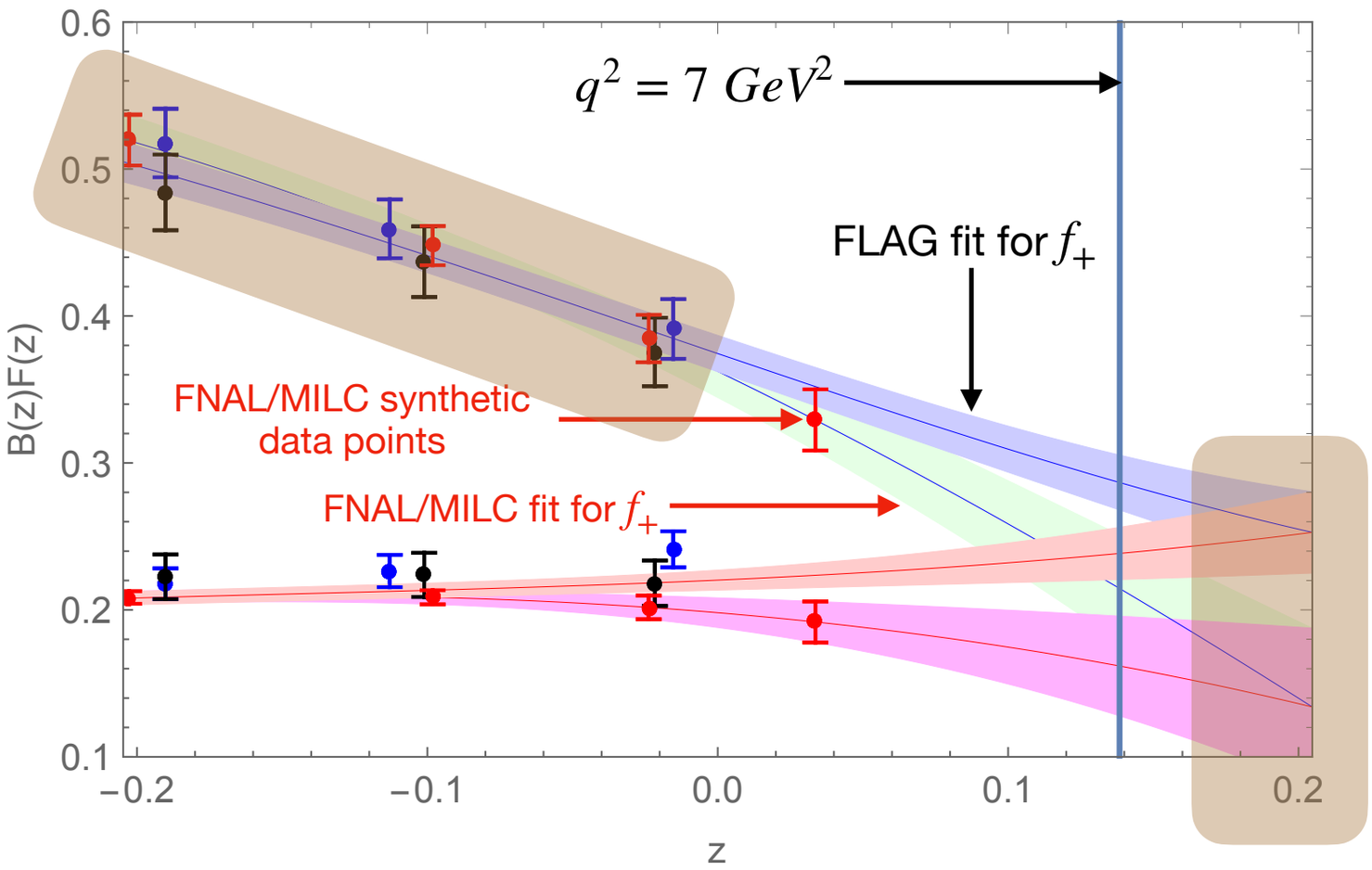


$$\frac{1}{|V_{ub}|^2}\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(B_s \to K^-\mu^+\nu_\mu}{dq^2} = (3.32 \pm 0.49)\ \mathrm{ps}^{-1}$$

This is the value quoted by LHCb [LHCb 2012.05143]

$B_s \to K$ form factors and $|V_{ub}/V_{cb}|$

- Using only FNAL/MILC:



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This is the value quoted by LHCb [LHCb 2012.05143]

The inclusion of data points from different lattice collaborations lead to huge extrapolation differences even though the points are perfectly compatible with the three highest q^2 FNAL/MILC ones

Simultaneous extraction of $|V_{ub}|_{\rm excl}$ and $|V_{ub}|_{\rm incl}$

- Monte Carlo simulations combine theoretical information on the shape of exclusive and inclusive triple differential distributions (q^2, E_ℓ^B and M_X):
  lattice/LCSR/exp information on the shape of $B \to (\pi, \rho, \omega, \eta, \eta')$ and inclusive shape function calculations are used to produce a triple differential distribution which reproduces that contains the exclusive peaks at low M_X while reproducing all inclusive partial rates (in ~500 bins)

- In the past these samples have been used to study acceptances/efficiencies/… for inclusive analysis only

- In the most recent Belle analysis (2303.17309) based on 711 fb^{-1} this combined simulation is used to extract:

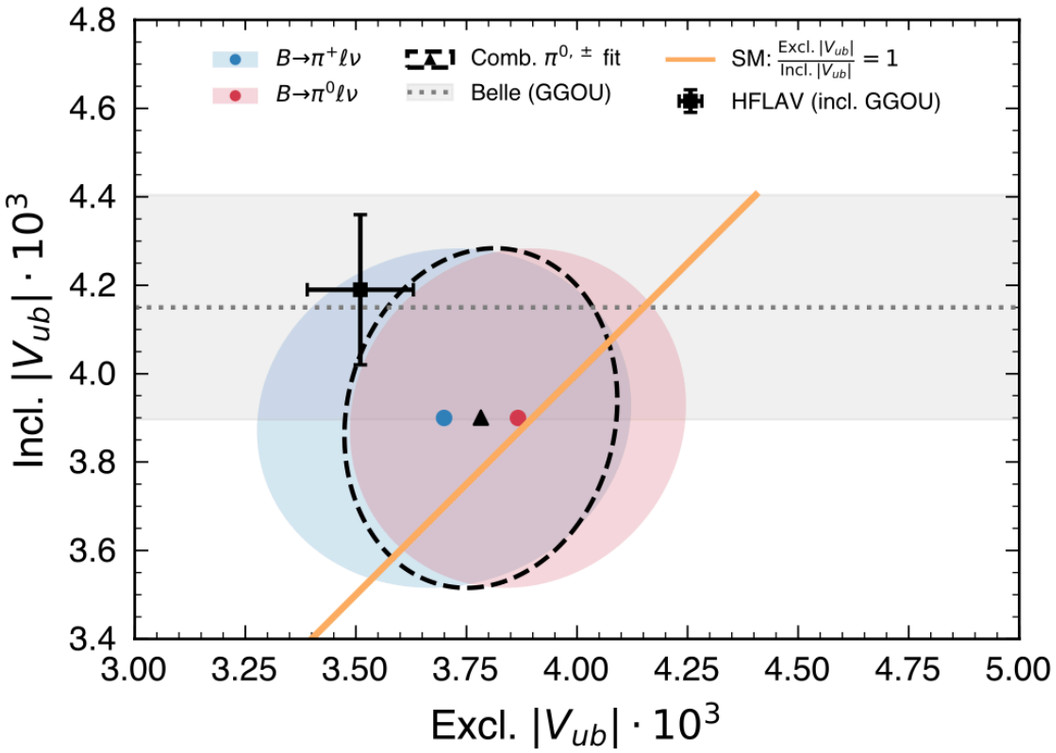
$$\mathcal{B}(\bar{B}^0 \to \pi^+ \ell \nu) = \begin{cases} (1.43 \pm 0.19 \pm 0.13) \times 10^{-4} & \text{latt/LCSR+inclusive} \\ (1.53 \pm 0.18 \pm 0.12) \times 10^{-4} & \text{latt/LCSR/exp+inclusive} \end{cases}$$

$$\Delta\mathcal{B}(B \to X_u \ell \nu) = (1.40 \pm 0.14 \pm 0.23) \times 10^{-3}$$

- In a previous analysis with 711 fb^{-1} (2102.00020) Belle found:
  $\Delta\mathcal{B}(B \to X_u \ell \nu) = (1.59 \pm 0.07 \pm 0.16) \times 10^{-3}$

Seeds for discussion

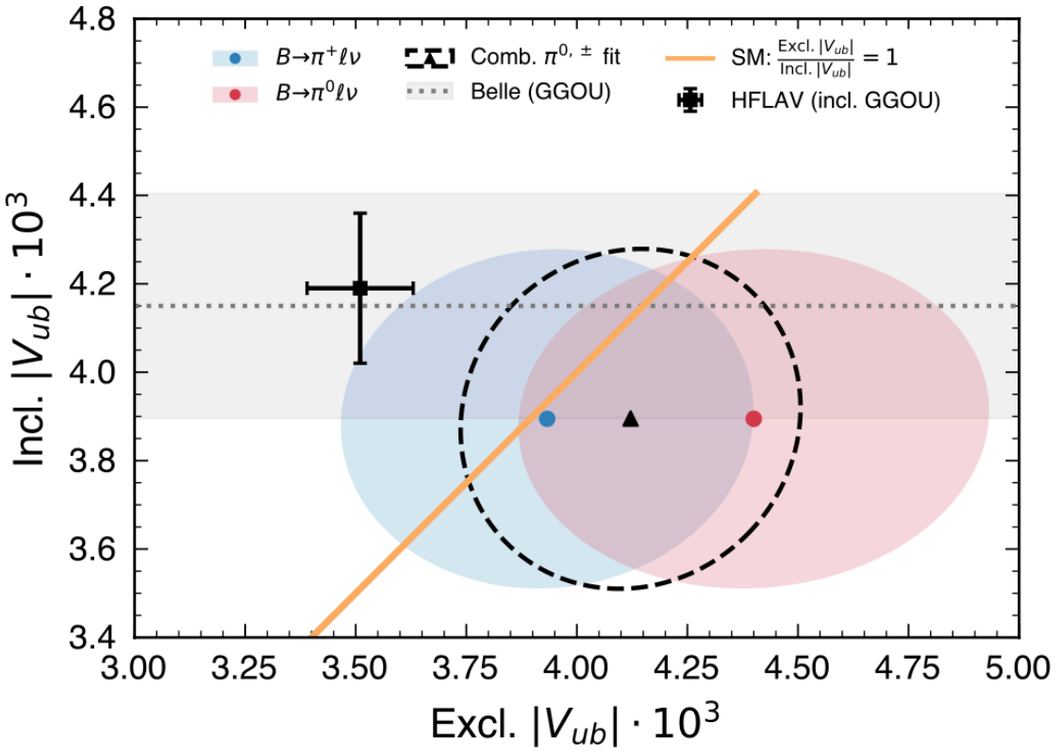
Latt/LCSR + inclusive



$$\left|V_{ub}^{\text{excl.}}\right| = (4.12 \pm 0.30 \pm 0.18 \pm 0.16) \times 10^{-3}$$
$$\left|V_{ub}^{\text{incl.}}\right| = (3.90 \pm 0.20 \pm 0.32 \pm 0.09) \times 10^{-3}$$

correlation = 0.07

Latt/LCSR/exp + inclusive



$$\left|V_{ub}^{\text{excl.}}\right| = (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$$
$$\left|V_{ub}^{\text{incl.}}\right| = (3.90 \pm 0.20 \pm 0.32 \pm 0.09) \times 10^{-3}$$

correlation = 0.10

- Question: is this procedure somehow “imposing” a degree of consistency between inclusive and exclusive V_{ub}?