

Lattice determinations of $B \rightarrow D^* \ell \nu$ form factors

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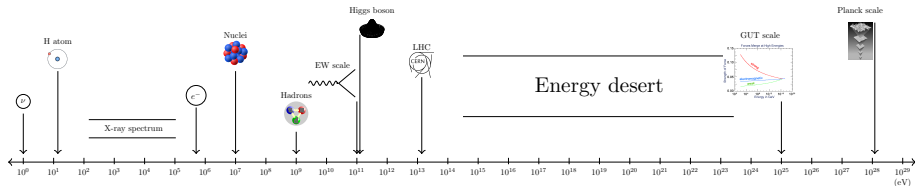


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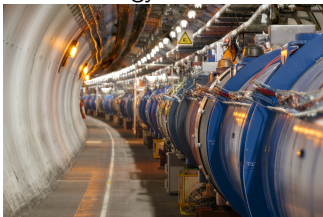
Motivation: Searches for new physics

- The Standard Model (SM) describes phenomena in a wide range of scales
- Yet, we expect it to fail at some point
 - Hierarchy problem, too many parameters, absence of gravity, dark matter/energy, neutrino mixing...
 - SM regarded as an Effective Field Theory (EFT)
- New physics searches more important than ever



Motivation: Searches for new physics

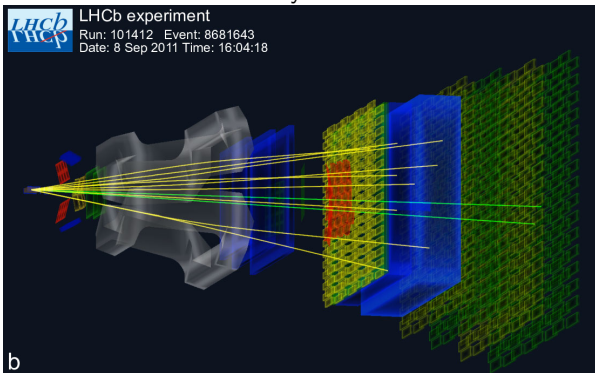
Energy frontier



Cosmology frontier



Intensity frontier



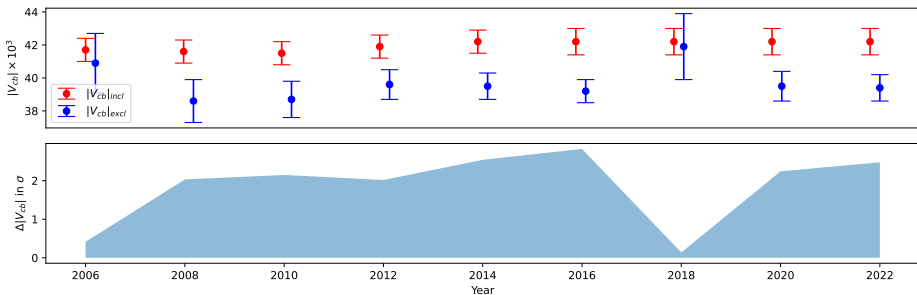
- High expectations with the LHC
- Intensity frontier becoming increasingly important

Motivation: New physics in the flavor sector of the SM

The CKM matrix

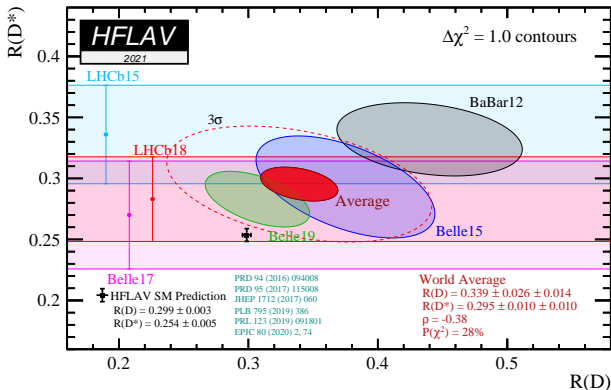
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Matrix must be unitary (preserve the norm)
- Tensions have been there for a long time
- Evolution of the tensions according to PDG



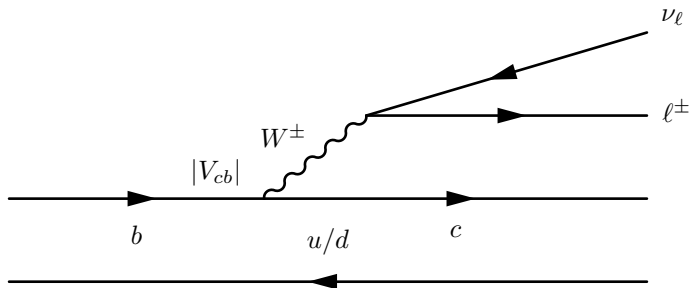
Motivation: Tensions in lepton universality ratios

$$R(D^*) = \frac{\mathcal{B}(B \rightarrow D^* \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^* \ell \nu_\ell)}$$



- Current $\approx 3.3\sigma$ tension with the SM (HFLAV)

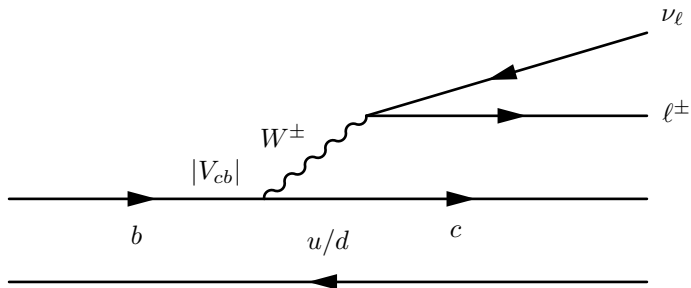
Semileptonic B decays on the lattice: Exclusive $|V_{cb}|$



$$\underbrace{\frac{d\Gamma}{dw}(\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell)}_{\text{Experiment}} = \underbrace{K_{D^*}(w, m_\ell)}_{\text{Known factors}} \underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} \times |V_{cb}|^2, \quad w = v_{D^*} \cdot v_B$$

- The amplitude \mathcal{F} must be calculated in LQCD
 - Data more precise at w close to 1
- $K_{D^*}(w, m_\ell) \propto (w^2 - 1)^{\frac{1}{2}}$ requires extrapolation of experimental data

Semileptonic B decays on the lattice: Universality ratios



$$R(D^*) = \frac{\int_1^{w_{\text{Max},\tau}} dw K_{D^*}(w, m_\tau) |\mathcal{F}(w)|^2 \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\text{Max}}} dw K_{D^*}(w, 0) |\mathcal{F}(w)|^2 \times \cancel{|V_{cb}|^2}}$$

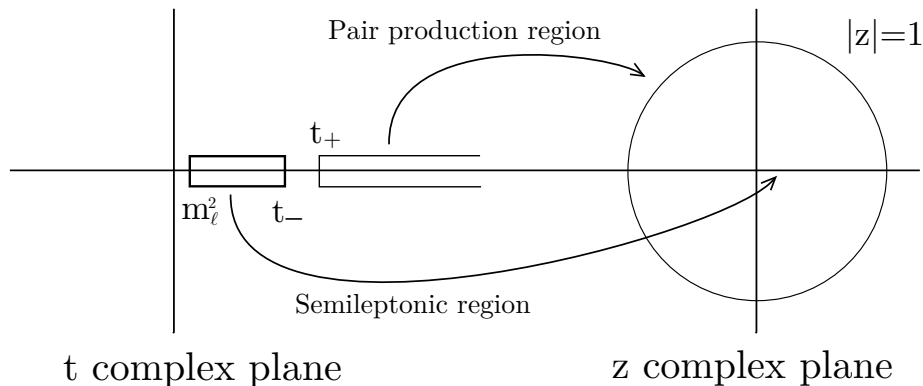
- The universality ratio depends only on the form factors
- It is possible to extract $R(D^*)$ without experimental data!

Semileptonic B decays on the lattice: Parametrizations

Most parametrizations perform an expansion in the z parameter

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}, \quad z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

with $t_{\pm} = (m_B \pm m_{D^*})^2$, $t = (p_B - p_{D^*})^2$, $w = v_B \cdot v_{D^*}$



Semileptonic B decays on the lattice: Parametrizations

- Boyd-Grinstein-Lebed (BGL)

Phys. Rev. Lett. 74 (1995) 4603-4606

Phys.Rev. D56 (1997) 6895-6911

Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

- B_{f_X} Blaschke factors, includes contributions from the poles
- ϕ_{f_X} is called *outer function* and must be computed for each form factor
- Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN)

Nucl. Phys. B530 (1998) 153-181

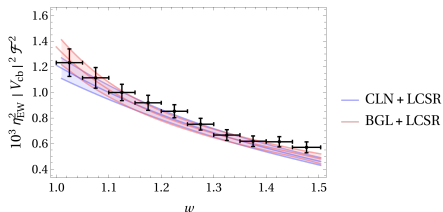
$$F(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho), d = f_d(\rho)$$

- Relies strongly on HQET, spin symmetry and (old) inputs
- Tightly constrains $F(w)$: four independent parameters, one relevant at $w = 1$
- Current consensus: abandon CLN
 - Spiritual successors of CLN

Bernlochner et al. *Phys.Rev.D* 95 (2017) 115008, *Phys.Rev.D* 97 (2018) 059902

Bordone, Gubernari, Jung, Straub, Van Dyk... *Eur.Phys.J.C* 80 (2020) 74, *Eur.Phys.J.C* 80 (2020) 347, *JHEP* 01 (2019) 009

Semileptonic B decays on the lattice: Parametrizations



From *Phys. Lett. B* 769 (2017) 441-445 using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
 - From Babar's paper arXiv:1903.10002 **BGL is compatible with CLN and far from the inclusive value**
 - Belle's paper arXiv:1809.03290v3 finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

Semileptonic B decays on the lattice: Parametrizations

- Dispersive approach

Bourrely et al. *Nucl.Phys.B* 189 (1981) 157, Lellouch *Nucl.Phys.B* 479 (1996) 353

Di Carlo et al. *Phys.Rev.D* 104 (2021) 054502

- Express unitarity bounds as a norm, define an inner product

$$\langle \phi f | \phi f \rangle = \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} |\phi(z, q_0^2) f(z)|^2 \leq \chi(q_0^2), \quad \langle g | h \rangle \equiv \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \bar{g}(z) h(z)$$

- Use Cauchy integral theorem to test unitarity in synthetic data at

$$z = z_{t_1}, z_{t_2} \dots$$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}_t z}$$

$$\langle g_t | \phi f \rangle = \phi(z_t, q_0^2) f(z_t)$$

$$\det \mathcal{M} = \begin{vmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_{t_1} \rangle & \langle \phi f | g_{t_2} \rangle & \dots \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_{t_1} \rangle & \langle g_{t_1} | g_{t_2} \rangle & \dots \\ \langle g_{t_2} | \phi f \rangle & \langle g_{t_2} | g_{t_1} \rangle & \langle g_{t_2} | g_{t_2} \rangle & \dots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix} \geq 0$$

Matrix \mathcal{M} positive semidefinite

Semileptonic B decays on the lattice: Parametrizations

- Bayesian inference
- Calculate the form factor $f(\mathbf{a})$ as a function of the vector of coefficients \mathbf{a} in the presence of *prior knowledge*

$$\langle f(\mathbf{a}) \rangle = \frac{1}{Z} \int d\mathbf{a} f(\mathbf{a}) \pi(\mathbf{a}|B)$$

- Modify the prior knowledge to include the unitarity constraint

$$\pi(\mathbf{a}|B) = \theta(1 - |a_F|^2) e^{-\frac{1}{2}(\mathbf{a}-\tilde{\mathbf{a}})^T C_{\tilde{\mathbf{a}}}^{-1}(\mathbf{a}-\tilde{\mathbf{a}})}$$

- Agrees very consistently with the DM approach

J. Flynn, A. Jüttner and T. Tsang, arXiv:2303.11285

Semileptonic B decays on the lattice: Parametrizations

- Non-perturbative calculation of the susceptibilities

Di Carlo et al. *Phys.Rev.D* 104 (2021) 054502

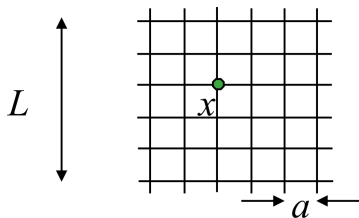
| Channel | LQCD | NNLO PT |
|-------------------------------|----------|-----------|
| $0^+(10^{-3})$ | 7.58(59) | 6.204(81) |
| $1^-(10^{-4}\text{GeV}^{-2})$ | 5.88(44) | 5.131(48) |
| $0^-(10^{-2})$ | 2.19(19) | 1.94 |
| $1^+(10^{-4}\text{GeV}^{-2})$ | 4.69(30) | 3.894 |

Taken from S. Simula slides at Barolo's WS *Challenges in Semileptonic B decays*

- The perturbative evaluation is more precise
- The non-perturbative evaluation is systematically improvable
 - High-precision implementation of bounds possible

Semileptonic B decays on the lattice: Introduction to Lattice QCD

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (\gamma^\mu D_\mu + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$



- Discretize space-time in a computer
 - Finite lattice spacing a
 - Finite spatial volume L
 - Finite time extent T

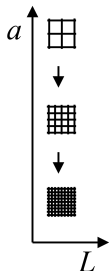
- Perform simulations in an unphysical setup and approach the physical limit
 - Enlarge the volume and reduce a
 - Quark masses \implies Pion masses (hadrons are matched)
 - Number of sea quarks $n_f = 2 + 1, \quad 2 + 1 + 1, \quad 1 + 1 + 1 + 1 \dots$

Semileptonic B decays on the lattice: Introduction to Lattice QCD

The systematic error analysis is based on **EFT** descriptions of QCD

The EFT description:

- provides functional form for different extrapolations (or interpolations)
- can be used to construct improved actions
- can estimate the size of the systematic errors



In order to keep the systematic errors under control we must repeat the calculation for several lattice spacings, volumes, light quark masses... and use the EFT to extrapolate to the physical theory

Semileptonic B decays on the lattice: Heavy quarks

- Heavy quark treatment in Lattice QCD
 - For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
 - For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
- Need special actions to describe the bottom quark, difficult renormalization
 - Relativistic HQ actions (f.i. FermiLab)
 - Non-Relativistic QCD (NRQCD)
- If the action is improved enough, one can treat the bottom as a light quark
 - Highly improved action AND small lattice spacing
 - Use unphysical values for m_b and extrapolate

The discretization errors needn't disappear **as long as we keep them under control**

Semileptonic B decays on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{V}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} = \frac{1}{2} \epsilon^{\nu*} \varepsilon^{\mu\nu\rho\sigma} v_B^\rho v_{D^*}^\sigma \mathbf{h}_V(w)$$

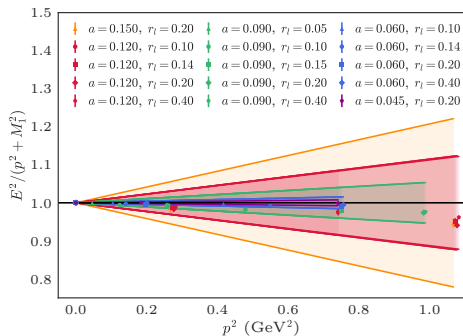
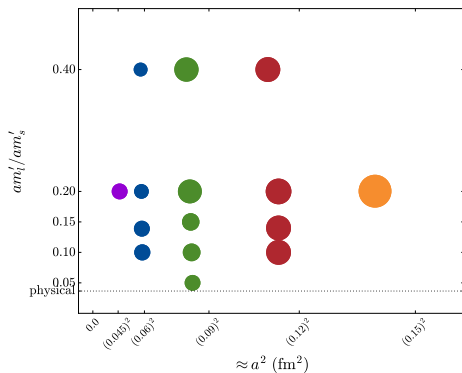
$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{A}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} =$$

$$\frac{i}{2} \epsilon^{\nu*} [g^{\mu\nu} (1+w) \mathbf{h}_{A_1}(w) - v_B^\nu (v_B^\mu \mathbf{h}_{A_2}(w) + v_{D^*}^\mu \mathbf{h}_{A_3}(w))]$$

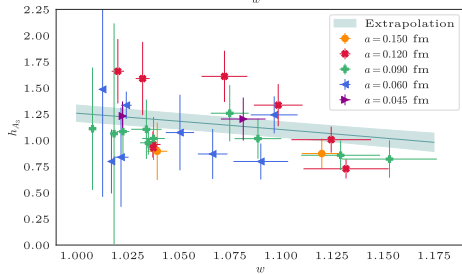
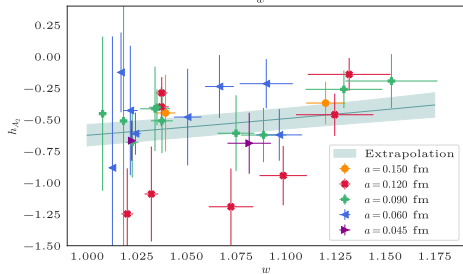
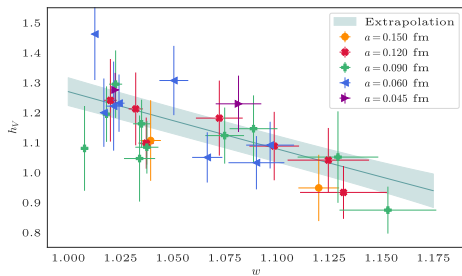
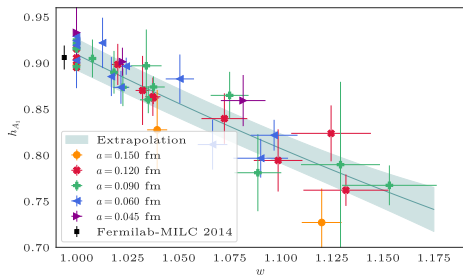
- \mathcal{V} and \mathcal{A} are the vector/axial currents in the continuum
- The h_X enter in the definition of the decay amplitudes
- We can calculate h_X directly from the lattice

Semileptonic B decays on the lattice: Fermilab/MILC

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action
- Lightest $m_\pi \approx 180$ MeV

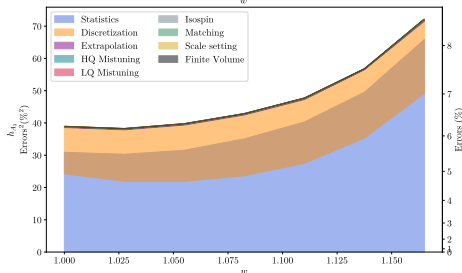
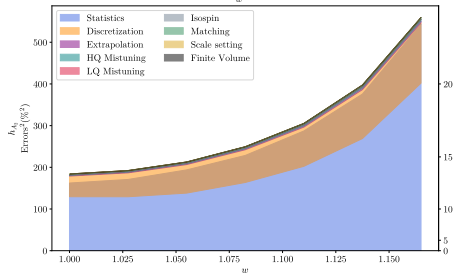
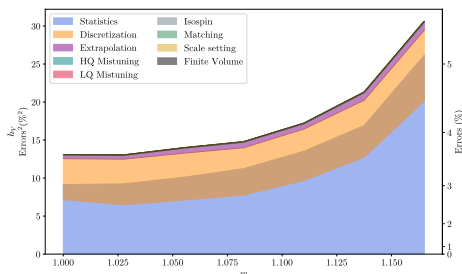
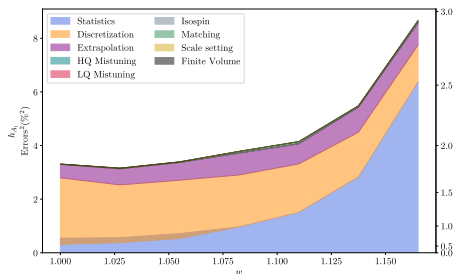


Semileptonic B decays on the lattice: Fermilab/MILC



Combined fit $\chi^2/\text{dof} = 85.2/95$

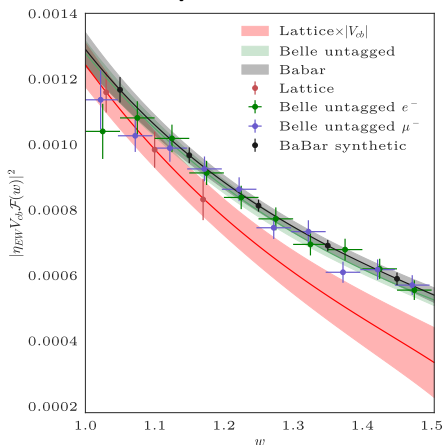
Semileptonic B decays on the lattice: Fermilab/MILC



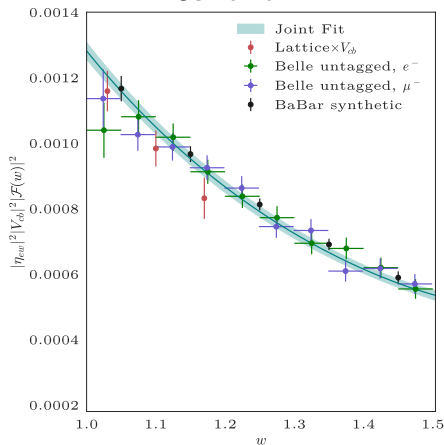
Largest systematic errors come from discretization

Semileptonic B decays on the lattice: Fermilab/MILC

Separate fits



Joint fit



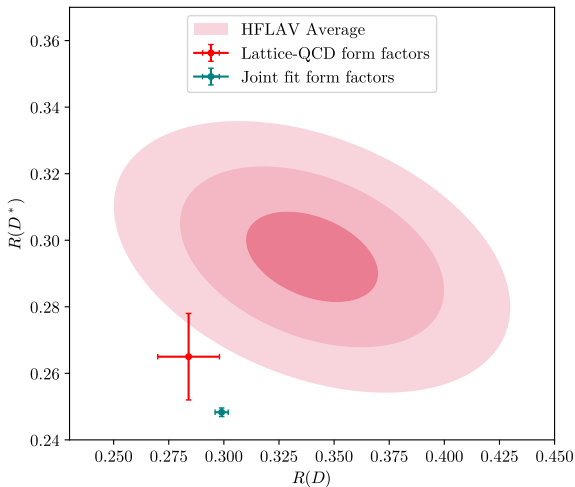
| Fit | Lattice | Exp | Lat + Belle | Lat + BaBar | Lat + Exp |
|---------------------|---------|--------|-------------|-------------|-----------|
| χ^2/dof | 0.63/1 | 104/76 | 111/79 | 8.50/4 | 126/84 |

Unblinded, final result $|V_{cb}| = 38.40(78) \times 10^{-3}$

Semileptonic B decays on the lattice: Fermilab/MILC

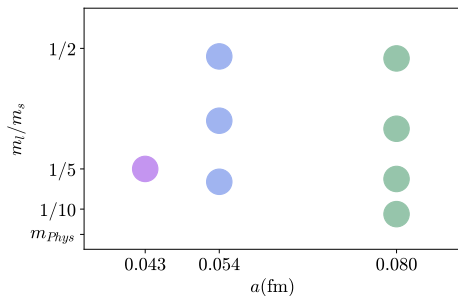
$$R(D^*)_{\text{Lat}} = 0.265(13) \quad R(D^*)_{\text{Lat+Exp}} = 0.2483(13)$$

Phys.Rev.D92 (2015), 034506; Phys.Rev.D100 (2019), 052007; Phys.Rev.D103 (2021), 079901; Phys.Rev.Lett. 123 (2019), 091801

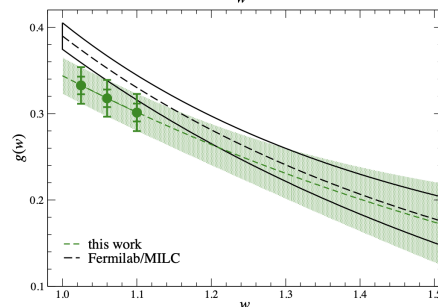
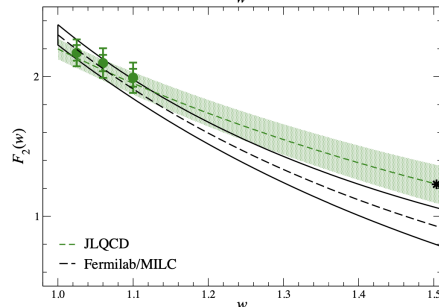
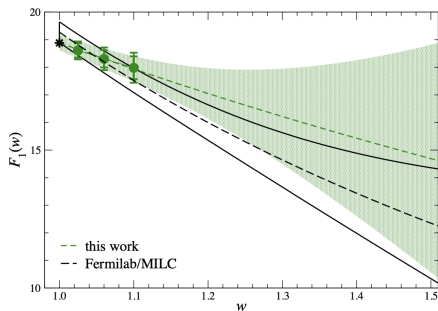
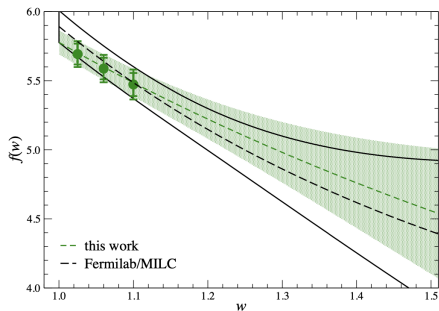


Semileptonic B decays on the lattice: JLQCD

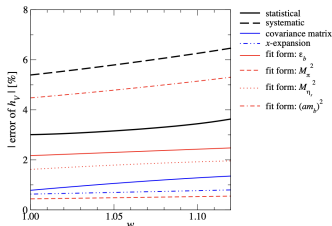
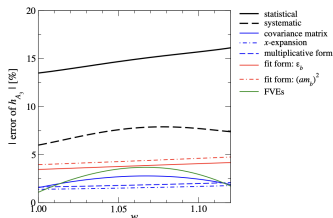
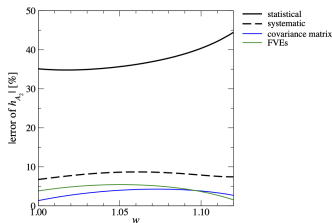
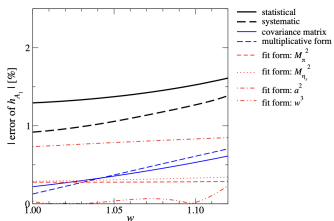
- Using 9 $N_f = 2 + 1$ ensembles of sea DW quarks
- The heavy quarks use the same DW action
 - Simulations at unphysical b masses $m_b \lesssim 0.7a$
 - Requires extrapolation
 - Easier and more precise renormalization
- m_π in the range 230 – 500 MeV
 - Stable D^*



Semileptonic B decays on the lattice: JLQCD

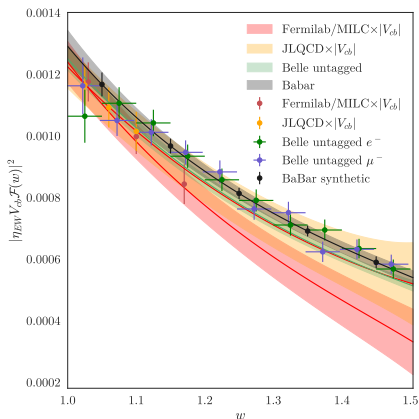


Semileptonic B decays on the lattice: JLQCD



- Discretization errors dominate the systematic contributions

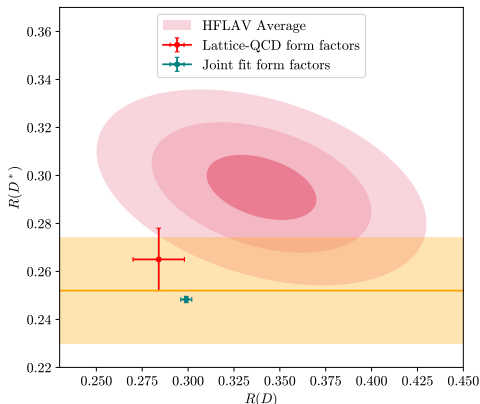
Semileptonic B decays on the lattice: JLQCD



$$|V_{cb}|^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\text{FerMILC}} = 38.60(86) \times 10^{-3}$$

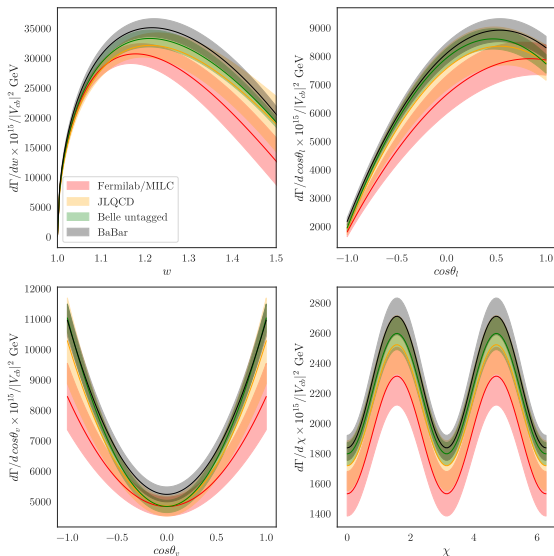
- Fit to Belle dataset, no Coulomb factor
- Combined fit $\chi^2/\text{dof} \sim 0.90$



$$R(D^*)^{\text{JLQCD}} = 0.252(22)$$

$$R(D^*)^{\text{FerMILC}} = 0.265(13)$$

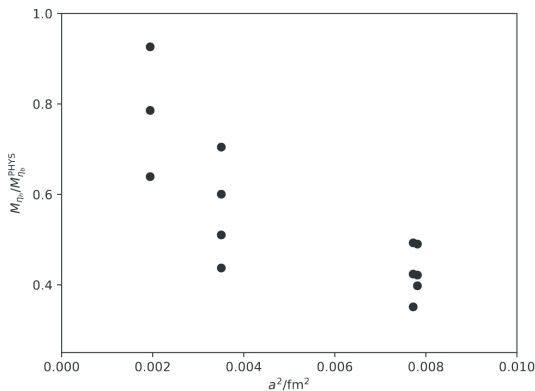
Semileptonic B decays on the lattice: JLQCD



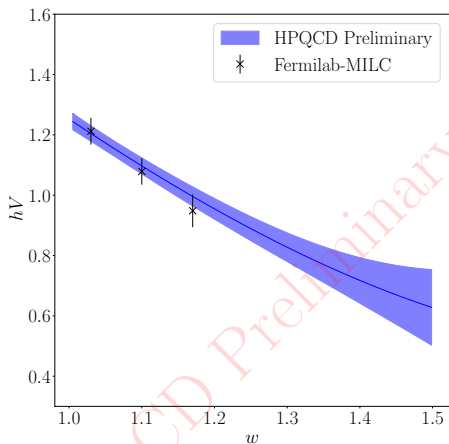
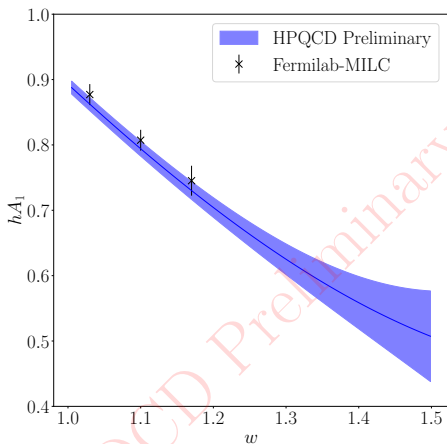
- $|V_{cb}|$ extracted from the branching fraction higher (L. Vittorio, LHCb Open WS Frascati 2023; HPQCD 2023)

Semileptonic B decays on the lattice: HPQCD

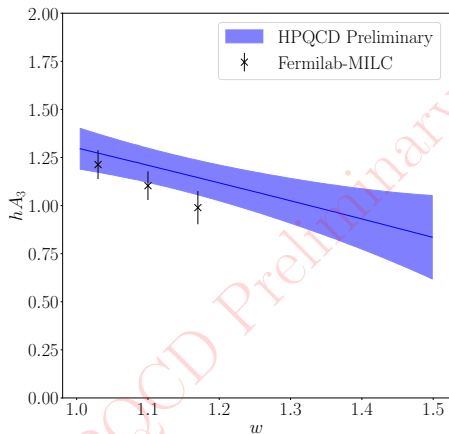
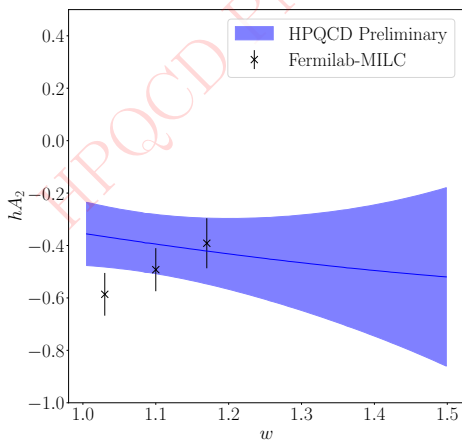
- Using 4 $N_f = 2 + 1 + 1$ MILC ensembles of sea HISQ quarks
- The b quark uses the HISQ action and unphysical masses
- m_π ranges from 330 MeV to 129 MeV



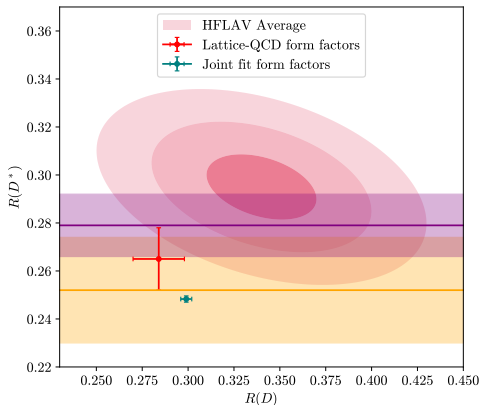
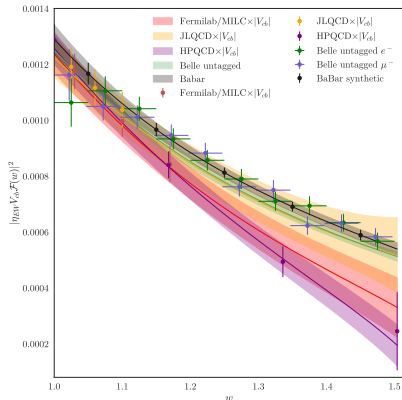
Semileptonic B decays on the lattice: HPQCD



Semileptonic B decays on the lattice: HPQCD



Semileptonic B decays on the lattice: HPQCD



$$|V_{cb}|^{\text{HPQCD}} = 39.31(74) \times 10^{-3}$$

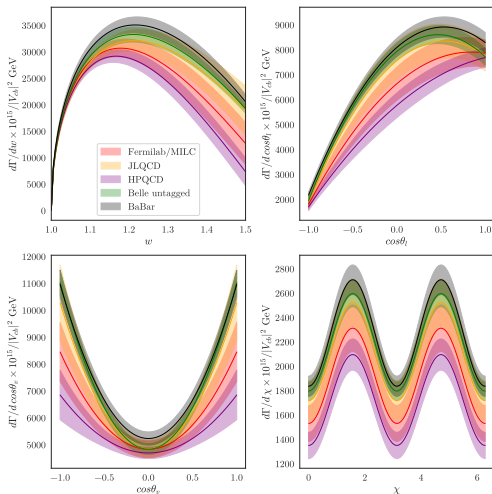
$$|V_{cb}|^{\text{FerMILC}} = 38.17(85) \times 10^{-3}$$

$$R(D^*)^{\text{HPQCD}} = 0.279(13)$$

$$R(D^*)^{\text{FerMILC}} = 0.265(13)$$

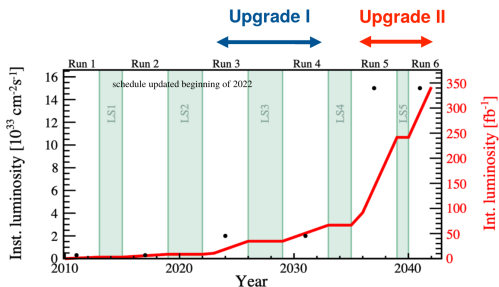
- Fit to Belle dataset WITH the Coulomb factor

Semileptonic B decays on the lattice: JLQCD



- From total decay rate $|V_{cb}| = 44.2(1.8) \times 10^{-3}$

Semileptonic B decays on the lattice: Experimental data



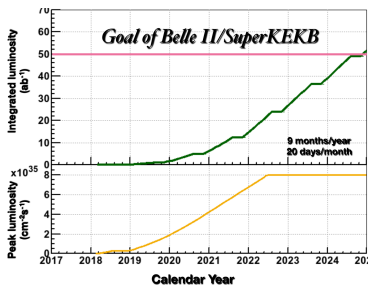
- Belle II IL 424 fb^{-1}
 - Target 50 ab^{-1}
 - Results at 190 fb^{-1}

ICHEP 2022

$$|V_{cb}|_{B \rightarrow D \ell \nu}^{\text{Untag}} = 38.28 \pm 1.16$$

$$|V_{cb}|_{B \rightarrow D^* \ell \nu}^{\text{Tag}} = 37.9 \pm 2.9$$

$$\eta_{\text{EW}} = 1.0066 \pm 0.0050$$



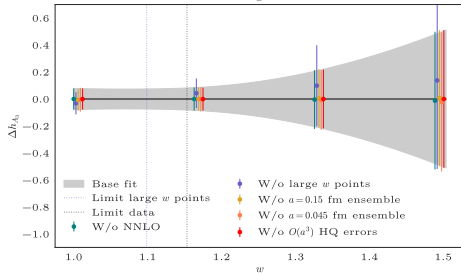
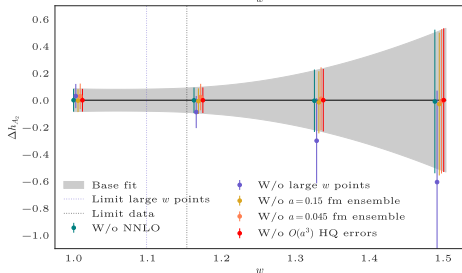
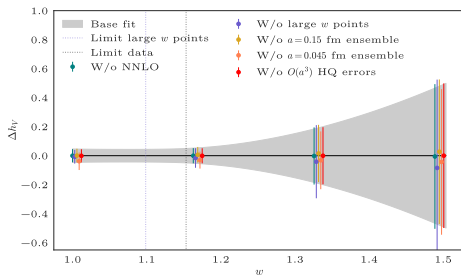
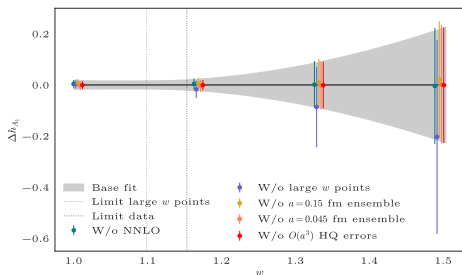
Summary

- Exciting times in flavor physics
 - Good progress, both in theoretical and experimental fronts
- Current results are not conclusive:
 - $|V_{cb}|$ agrees with previous determinations and the inclusive-exclusive tension remains unsolved
 - Results show $R(D^*)$ very close to **phenomenological expectations**, still in tension with experiment
- As we reduce our errors, new problems arise
 - Stability of the D^* meson
 - QED effects (Coulomb factor and beyond)
- Expect interesting results from the flavor sector in the next years

THANK YOU

BACKUP SLIDES

Semileptonic B decays on the lattice: Fermilab/MILC



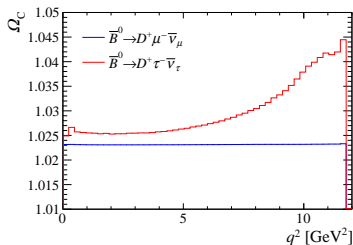
For all fits $\chi^2/\text{dof} \lesssim 1$

Semileptonic B decays on the lattice: QED effects

- Most important correction: Coulomb factor
 $(1 + \alpha\pi) = 1.023$

D. Atwood, W. Marciano, Phys.Rev.D41 (1990), 1736

- **Not** included in PHOTOS
- Applies to decays with a charged D^*
- Experiments should distinguish between both decays
- Structure-dependent corrections
 $\approx (1 + \alpha/\pi)$
- Velocity-dependent correction, but \approx constant for light leptons
- Current consensus (Barolo) is to include it as much as possible



S. Cali, S. Klaver, M. Rotondo, B. Sciascia, Eur.Phys.J.C79
(2019), 744