Knowns and Unknowns in $B \to D^*$ FFs

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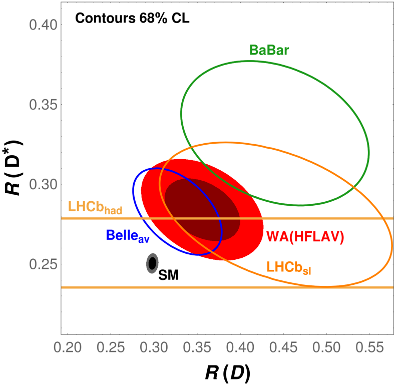
SEZIONE DI TORINO

Lepton-non-Universality in $b \to c\tau\nu$

$$R(X) \equiv \frac{\mathrm{Br}(B \to X\tau\nu)}{\mathrm{Br}(B \to X\ell\nu)}$$

- Partial cancellation of uncertainties

➥ Precise predictions (and measurements)



- $R(D^{(*)})$: BaBar, Belle, LHCb
  ➥ average $\sim 3-4\sigma$ from SM

More flavour $b \to c\tau\nu$ observables:

- τ-polarization ($\tau \to$ had) [1608.06391]
- $B_c \to J/\psi\tau\nu$ [1711.05623] : huge
- Differential rates from Belle, BaBar
- Total width of B_c
- $b \to X_c\tau\nu$ by LEP
- D^* polarization (Belle)
- $R(\Lambda_c) \to$ below SM

Note: only 1 result $\geq 3\sigma$ from SM

In the following: discuss SM + NP predictions

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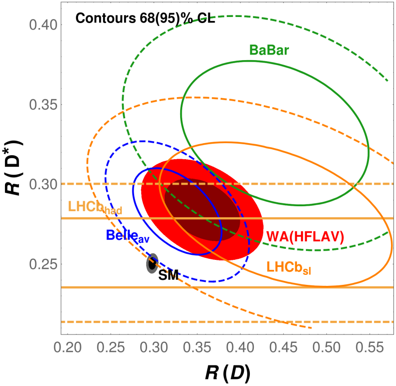

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- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points
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- ➡ Most $B \to D^*$ data not usable due to model dependence!

Give as much information as possible independently of this choice!

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Even with FF-model-dependent data:

Consistent HFLAV $B \to D^*$ fit in CLN
➥ Experimental w-dependence well established!

In the following: mostly BGL and HQE ($\to$ CLN) parametrizations

HQE parametrization

Heavy-Quark Expansion (HQE) employs additional information:

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
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CLN parametrization [Caprini+'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
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Solution: Include systematically $1/m_c^2$ corrections
[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] , using [Falk/Neubert'92]
[Bernlochner+'22] : model for $1/m_c^2$ corrections $\to$ fewer parameters

Theory determination of $b \to c$ Form Factors

[Bordone/MJ/vanDyk'19,Bordone/Gubernari/MJ/vanDyk'20]

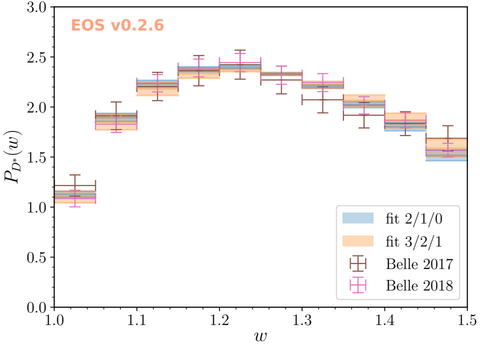
For general NP analysis, FF shapes needed from theory!

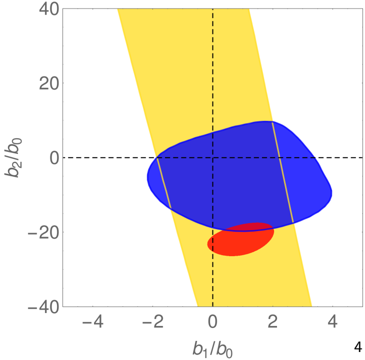
Fit to all $B \to D^{(*)}$ FFs, using lattice, LCSR, QCDSR and unitarity

[CLN,BGL,HPQCD'15'17,FNAL/MILC'14'15,Gubernari+'18,Ligeti+'92'93]

k/l/m: order in z for leading/subleading/subsubleading IW functions

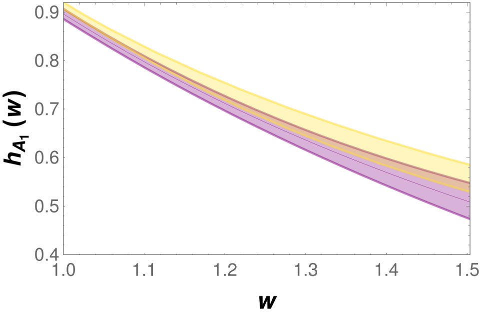
- 2/1/0 works, but only 3/2/1 captures uncertainties
- Consistent V_{cb} value from Belle'17+'18
- Predictions for diff. rates, perfectly confirmed by data





Comparison with new lattice calculations

Major improvement: $B_{(s)} \to D^*_{(s)}$ FFs@$w > 1$! (B_s: [Harrison+'22])



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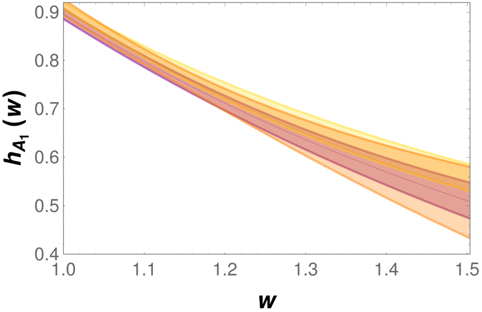

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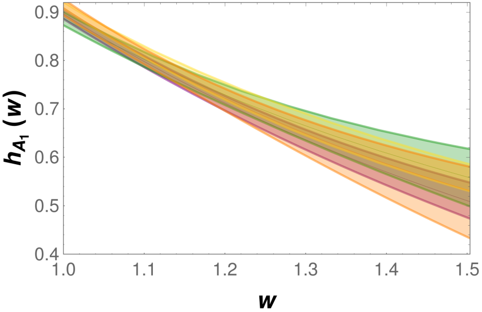

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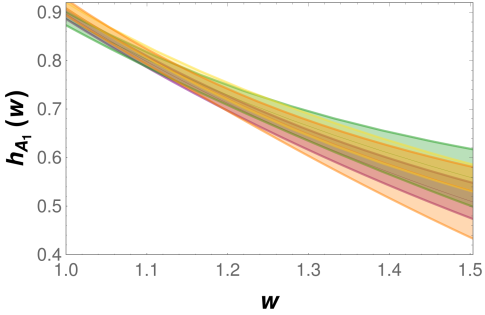

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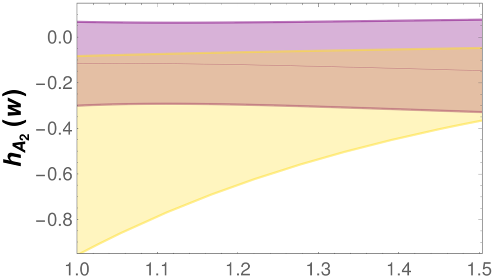

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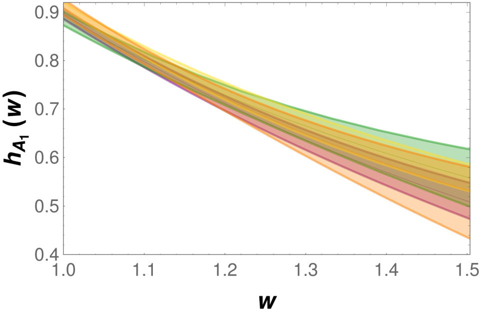

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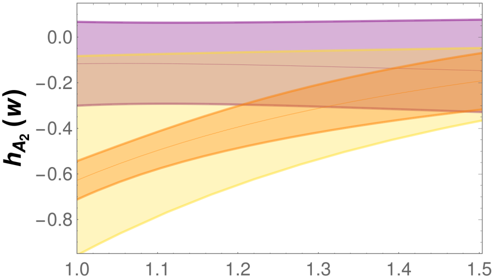

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- Slope HPQCD-FNAL/MILC?

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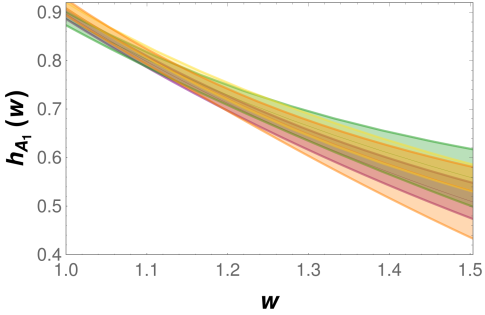

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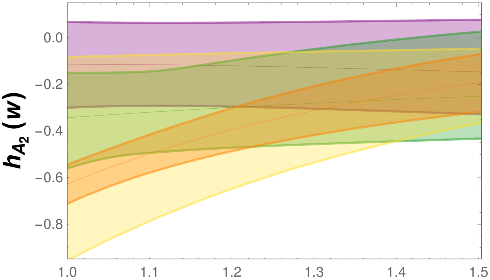

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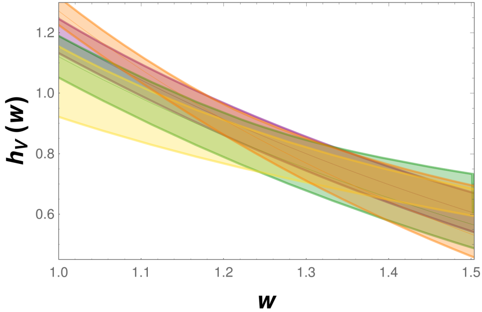

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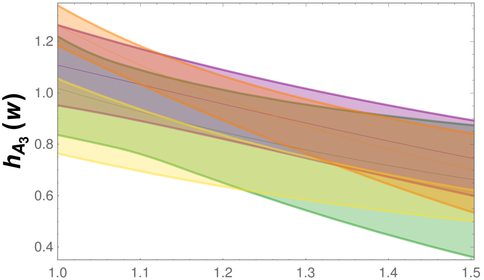

- Similar pattern in h_V and h_{A_3}
- Tension between BGJvD and FNAL/MILC in h_{A_2}

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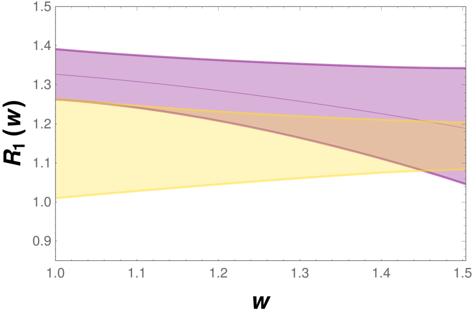

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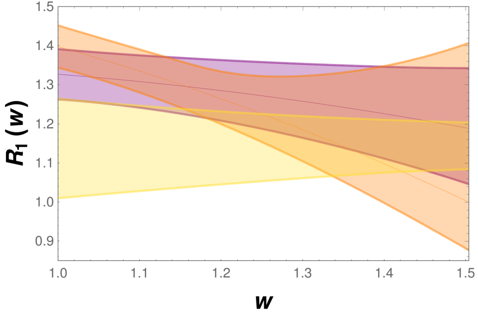

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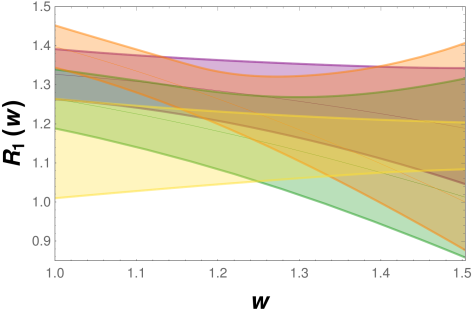

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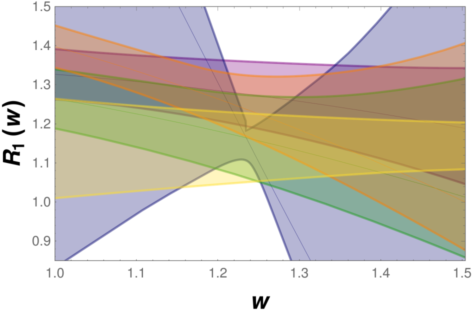

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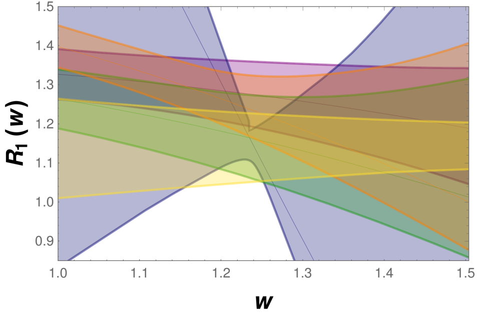

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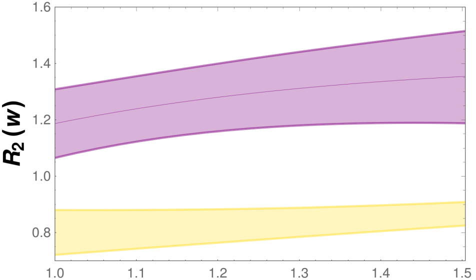

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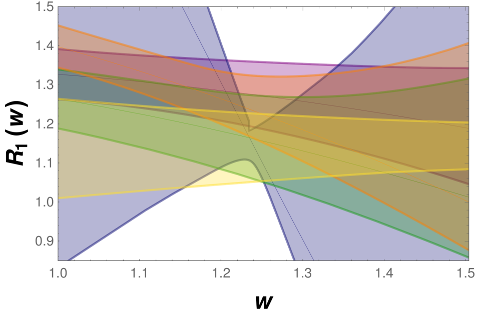

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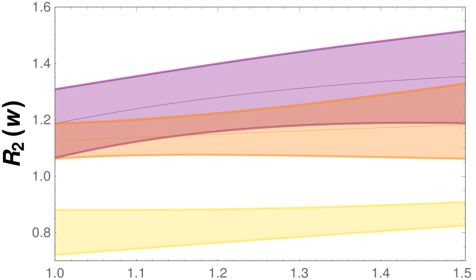

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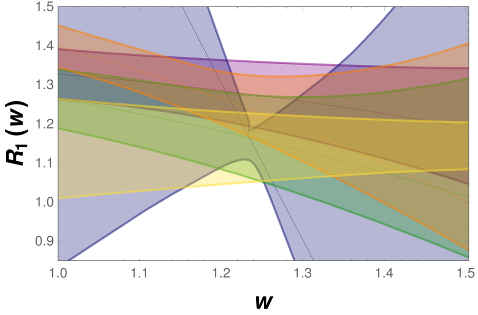

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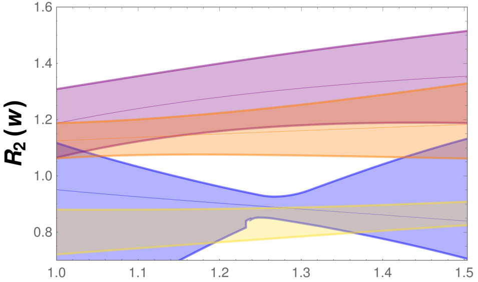

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- Deviation wrt experiment ($R_2^{\rm HFLAV}(1) = 0.853(17)$)

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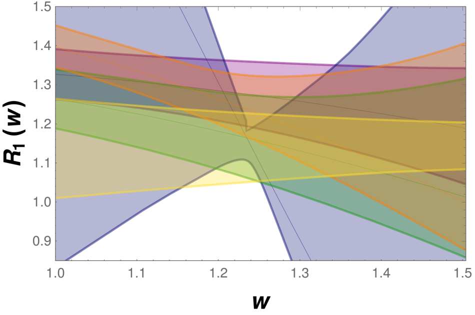

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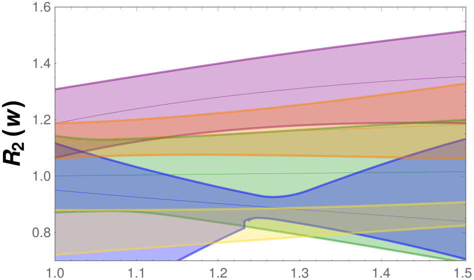

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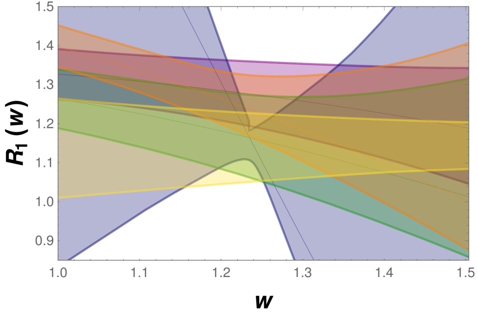

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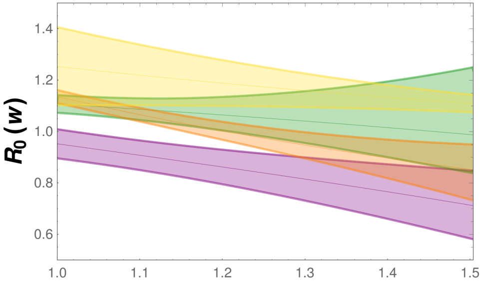
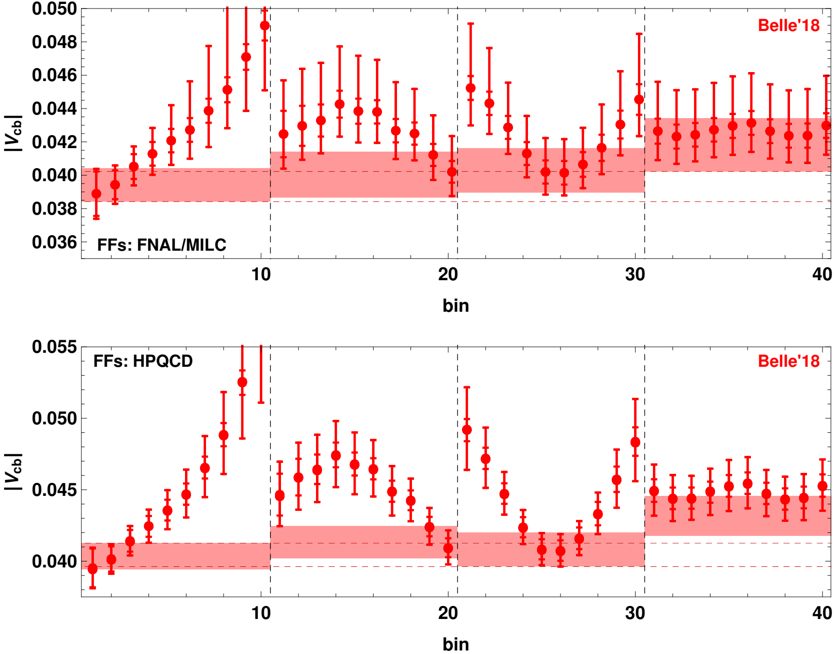


- Also in R_0 deviation wrt BGJvD
- JLQCD again "diplomatic"

➥ Requires further investigation!

➥ Correlations?

Binned V_{cb} from Belle'18 data: FNAL/MILC vs HPQCD



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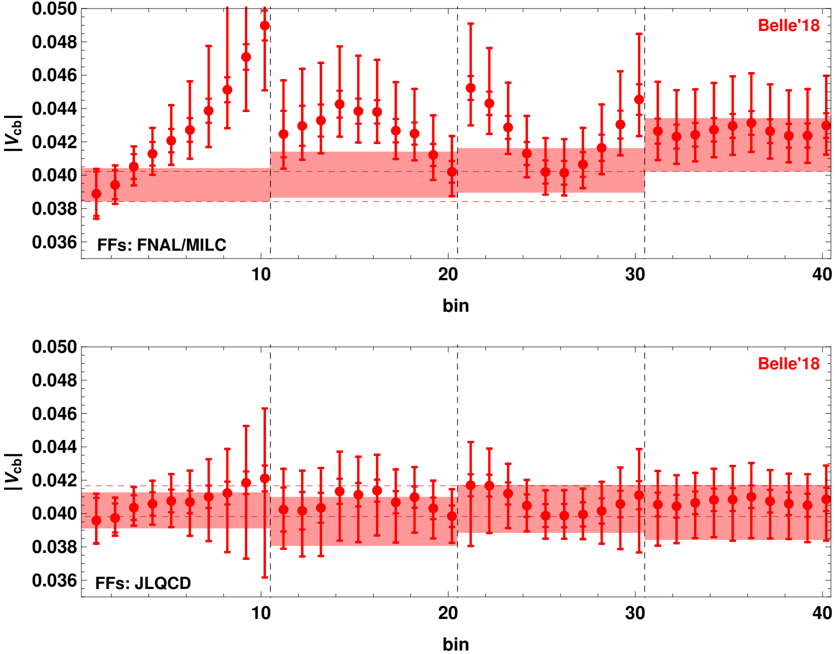
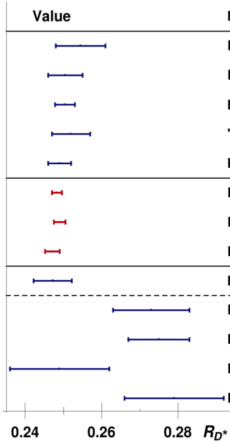


Overview over predictions for $R(D^*)$

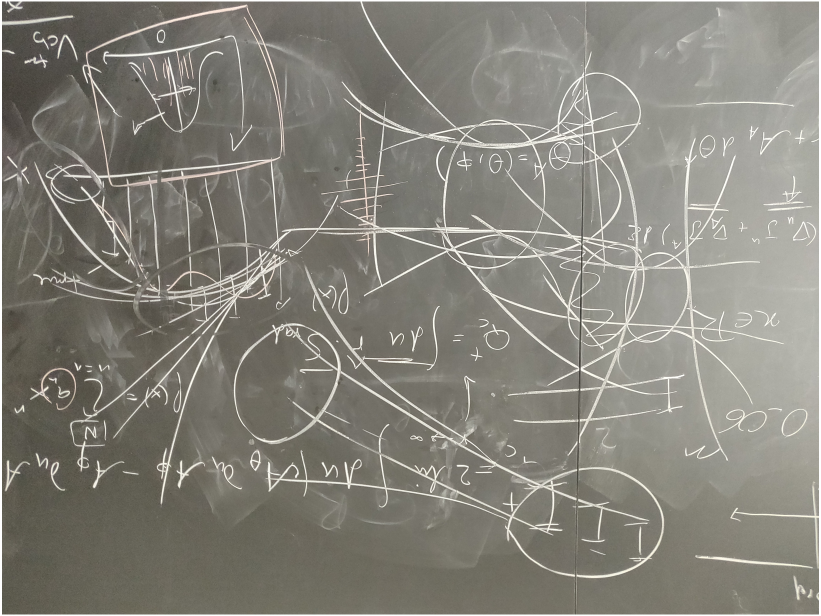


Method	Input Theo	Input Exp	Reference
BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
"Average"			HFLAV'21
HQET$_{RC}$@$1/m^2,\alpha_s^{(2)}$	Belle'17'18	Lattice	Bernlochner et al.'22
BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
BGL	Lattice	Belle'18	JLQCD prel. (MJ)
BGL	Lattice	Belle'18	Davies, Harrison'23
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
BGL	Lattice	---	Vaquero et al.'21v2
DM	Lattice	---	Martinelli et al.
BGL	Lattice	---	JLQCD prel. (MJ)
BGL	Lattice	---	Davies, Harrison'23

Lattice $B \to D^*$: $h_{A_1}(w=1)$ [FNAL/MILC'14,HPQCD'17] , [FNAL/MILC'21]
Other lattice: $f_{+,0}^{B\to D}(q^2)$ [FNAL/MILC,HPQCD'15]
QCDSR: [Ligeti/Neubert/Nir'93,'94] , LCSR: [Gubernari/Kokulu/vDyk'18]

Overall consistent SM predictions!
"Explaining" $R(D^*)$ by FM/HPQCD $\to$ NP in $B \to D^*(e,\mu)\nu$!

Form-factor truncation



Form-factor truncation

Key question: Where do we truncate our expansions?

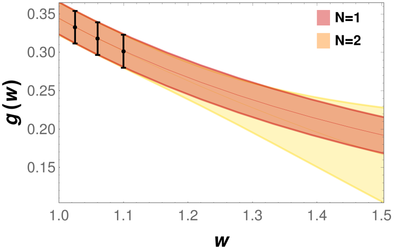
- A [Bernlochner+'19] : include parameter only if χ^2 decreases significantly
- B (GJS, BGJvD): include one "unnecessary" order

Comments:

- Large difference, $\sim 50\%$ difference in uncertainty
- Motivation for A: convergence, avoid overfitting
- Motivation for B: avoid underestimating uncertainties
- Different perspectives: only describing data, A is ok.
  However: we extrapolate to regions where we lack sensitivity

Example: $g(w)$ from FNAL/MILC

- perfect description at $\mathcal{O}(z)$
- large impact from $\mathcal{O}(z^2)$
- Nevertheless: $\mathcal{O}(z^2) \leq 6\% \times \mathcal{O}(z)$
  - overfitting limited



Just because you're not sensitive, doesn't mean it's not there!

Priors and potential biases I

Common error estimate:

$$\delta X \sim \mathcal{O}(1) \times \text{ known factor}$$

➥ What's "$\mathcal{O}(1)$"?

- Answer seems to be community-dependent
- Often in lattice analyses: gaussian around 0, width 1
- BGJvD HQET FFs: flat range $[-20, 20]$
- JLQCD: $d_w(h_{A_2}) = 10.3(7.1)$

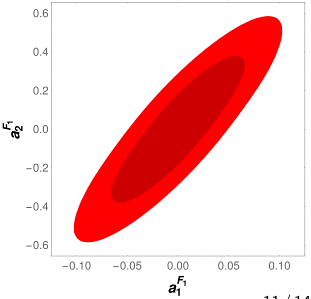
➥ potentially large differences

➥ needs to be checked and communicated

Similarly: treatment of BGL coefficients

- FNAL/MILC and HPQCD: series in $(w-1)^n$
- Priors can be strong in BGL space
- Plot: prior information, only (HPQCD)

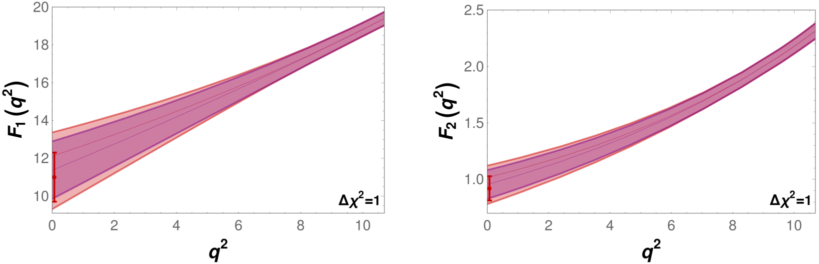
➥ Comparable to final result



Priors and potential biases II

Different conclusions starting from identical information

Example: $R(D^*)$ extraction from FNAL/MILC data



$R(D^*)$ including kinematical identities and weak unitarity

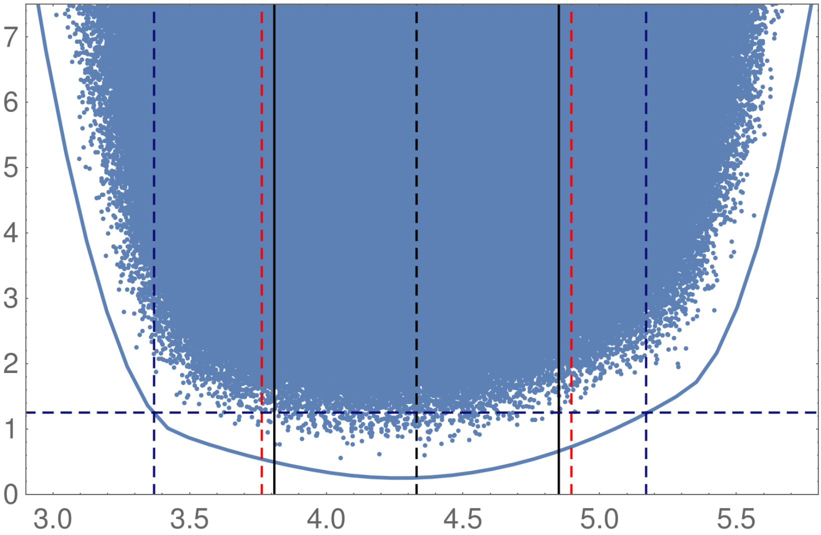
$$R(D^*) \stackrel{\rm WU}{=} 0.269^{+\mathbf{0.020}}_{-0.008} \quad \stackrel{\rm FM}{=} 0.274 \pm 0.010 \quad \stackrel{\rm Rome}{=} 0.275 \pm 0.008\,.$$

Difference WU-FM: FM apply prior on BGL coefficients

Difference WU-Rome (educated guess): iterated "unitarity filter" + different error estimate

Applying data: $R(D^*) = 0.249 \pm 0.001$(!) **universally**.

Uncertainty determination

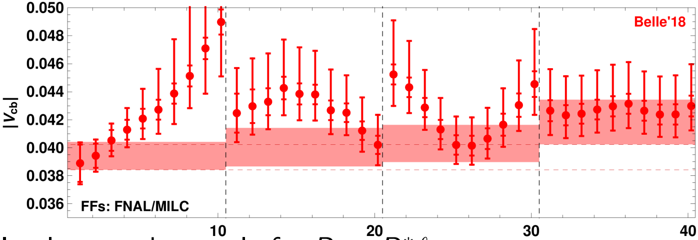


MC points together with χ^2 profile (minimizing for each FF value)
Vertical: CV MC, "1σ" MC, symmetric 68.3% interval MC, $\Delta\chi^2 = 1$

The Dispersive Matrix (DM) Method

Alternative implementation of unitarity [Bourrely+'81,Lellouch'95] :

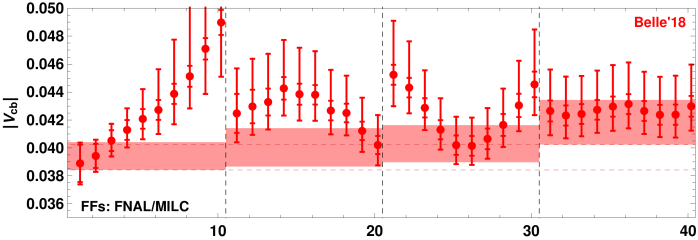
- Identical starting point as BGL: dispersion relation
- Known information in a matrix with positive determinant
  ➥ Form-factor bounds, equivalent to GJS [Caprini'19]
- Enables parametrization-free analysis



Implemented recently for $B \to D^* \ell \nu$ [DiCarlo+'21,Martinelli+'21,22] :

- Use DM w/ new FNAL/MILC data to obtain FF bands
- Calculate V_{cb} bin-wise, combine $d\Gamma/dx$ bins ($x = q^2, \cos\theta, \ldots$) (including experimental and theoretical correlations)
  ➥ 2×4 V_{cb} values. Claim: 0.5σ to $V_{cb}^{\rm incl}$, 1.3σ to $R(D^*)$

The Dispersive Matrix (DM) Method



Differences between DM and GJS [Gambino/MJ/Schacht'19] :

- GJS: Combined fit of lattice and experiment, imposing unitarity
- DM: Unweighted, uncorrelated average of the 4 V_{cb} values:

$$\mu = \frac{1}{N}\sum_{k=1}^{N} x_k\,, \quad \sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu_x)^2$$

➥ $V_{cb}^{\rm GJS} = \left(39.2^{+1.4}_{-1.2}\right) \times 10^{-3}, \qquad V_{cb}^{\rm DM} = (40.8 \pm 1.7) \times 10^{-3}$

Conclusions

We have work ahead of us!

1. q^2 dependence of FFs critical
   - ➥ Need parametrization-independent data
2. Inclusion of higher-order (theory) uncertainties essential
3. HQE: systematic expansion in $1/m, \alpha_s$, relates FFs
   - ➥ $\mathcal{O}(1/m_c)$ ($\rightarrow$ CLN) not sufficient anymore
4. Important first LQCD analyses in $B_{(s)} \rightarrow D^*_{(s)}$ @ finite recoil

➥ HPQCD: First 2+1+1 results, full q^2 range!

➥ Tensions in ratios – correlations?

5. Despite complications: $R(D^{(*)})$ SM prediction robust!

Central lesson:
Experiment and theory (lattice + pheno) need to work closely together!

Exclusive decays: Form factors

In exclusive decays, hadronic information encoded in Form Factors
They parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks)
vs.
Experiment with hadrons

$$\left\langle D_q(p')|\bar{c}\gamma^\mu b|\bar{B}_q(p)\right\rangle = (p+p')^\mu f_+^q(q^2) + (p-p')^\mu f_-^q(q^2)\,,\; q^2=(p-p')^2$$

Most general matrix element parametrization, given symmetries:
Lorentz symmetry plus P- and T-symmetry of QCD
$f_\pm(q^2)$: real, scalar functions of one kinematic variable

How to obtain these functions?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...)
  Precision?
- Measurable e.g. in semileptonic transitions
  Normalization? Suppressed FFs? NP?

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD symmetries (unitarity, crossing)
   ➥ dispersion relation
4. Calculate partonic part (mostly) perturbatively

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Result: Model-independent parametrization

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t,t_0)]^n \,.$$

- a_n: real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_+
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$

Series in z with bounded coefficients (each $|a_n| \leq 1$)!
➥ Uncertainty related to truncation is calculable!

$B \to D\ell\nu$

$B \to D\ell\nu$, aka "The teacher's pet":

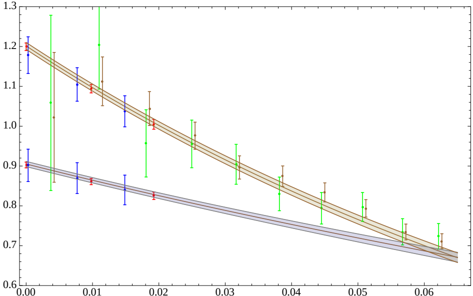
- Excellent agreement between experiments [BaBar'09,Belle'16]
- Excellent agreement between two lattice determinations [FNAL/MILC'15,HPQCD'16]

➥ Lattice data inconsistent with CLN parametrization!
(but consistent w/ HQE@1/m, discussed later)

- BGL fit [Bigi/Gambino'16] :

$$R(D) = 0.299(3)\,.$$

See also [Jaiswal+,Berlochner+'17,MJ/Straub'18,Bordone/MJ/vanDyk'19]



$f_{+,0}(z)$, inputs:

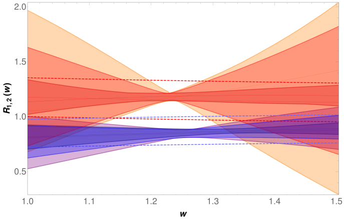
- FNAL/MILC'15
- HPQCD'16
- BaBar'09
- Belle'16

$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'18(+'17) provide FF-independent data for 4 single-differential rates

BGL analysis:

- Datasets compatible
- d'Agostini bias + syst. important
- Expand FFs to z^2
  ➥ 50% increased uncertainties



- Belle'18: no parametrization dependence
- Belle'17 never published → replace w/ Belle'23, not available yet
- Tension w/ inclusive reduced, but not removed

$$R(D^*) = 0.253^{+0.007}_{-0.006} \quad \text{(including LCSR point)}$$

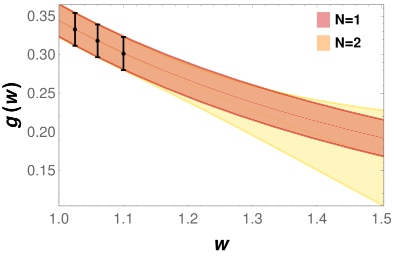
Comparison to Bernlochner+'22

Bernlochner et al. also perform HQE analysis @$1/m_c^2$. Differences:

- Postulate different counting within HQET
  - ➥ Highly constraining model for higher-order corrections
- Avoid use of LCSR (and mostly QCDSR) results
- Include partial α_s^2 corrections
- Include FNAL/MILC results partially
- Expansion in z: 2/1/0 (justified in [Bernlochner+'19])

Observations:

- $1/m_c^2$ corrections necessary
- Overall small uncertainties
- $V_{cb} = (38.7 \pm 0.6) \times 10^{-3}$
  - ➥ smaller due to larger $\mathcal{F}(1)$
- $R(D^*)$: agreement w/ BGJvD
- $R(D) \sim 3\sigma$ from GJS + BGJvD
  - ➥ In my opinion due to model

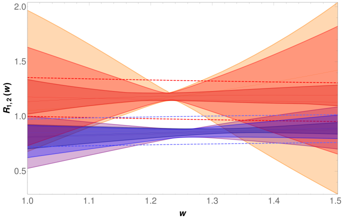


$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

Belle'17+'18 provide FF-independent data for 4 single-differential rates

Analysis of these data with BGL form factors:

- Datasets roughly compatible
- d'Agostini bias + syst. important
- All FFs to z^2 to include uncertainties
  ➥ 50% increased uncertainties
- 2018: no parametrization dependence



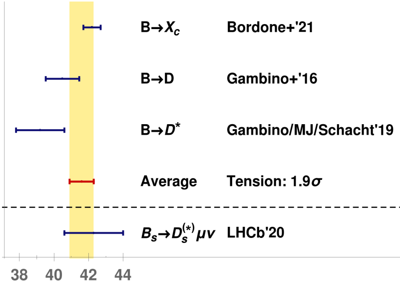
$$|V_{cb}^{D^*}| = 39.6^{+1.1}_{-1.0}\,[39.2^{+1.4}_{-1.2}] \times 10^{-3}$$

$$R(D^*) = 0.254^{+0.007}_{-0.006}\,[0.253^{+0.007}_{-0.006}]$$

In brackets: 2018 only ($\Delta V_{cb}^{\rm Belle} = 0.9$)

Updating the $|V_{cb}|$ puzzle:

- Tension 1.9σ (larger $\delta V_{cb}^{B\to D^*}$)
- $B_s \to D_s^{(*)}$ reduces tension further
- $V_{cb}^{B\to D^*}$ vs. $V_{cb}^{\rm incl}$ still problematic



See also [Bigi+,Bernlocher+,Grinstein+'17,Jaiswal+'17'19,MJ/Straub'18,Bordone+'19'20]

Theory determination of $b \to c$ Form Factors

SM: BGL fit to data + FF normalization $\to |V_{cb}|$

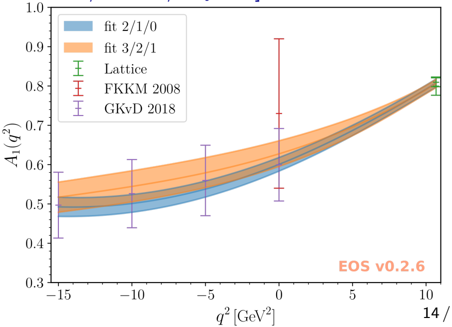
NP: can affect the q^2-dependence, introduces additional FFs

➥ To determine general NP, FF shapes needed from theory

[MJ/Straub'18,Bordone/MJ/vDyk'19] used all available theory input:

- Unitarity bounds (using results from [CLN, BGL])
  ➥ non-trivial $1/m$ vs. z expansions
- LQCD for $f_{+,0}(q^2)$ $(B \to D)$, $h_{A_1}(q^2_{\max})$ $(B \to D^*)$ [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod f_T) [Gubernari/Kokulu/vDyk'18]
- QCDSR results for $1/m$ IW functions [Ligeti+'92'93]
- HQET expansion to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$

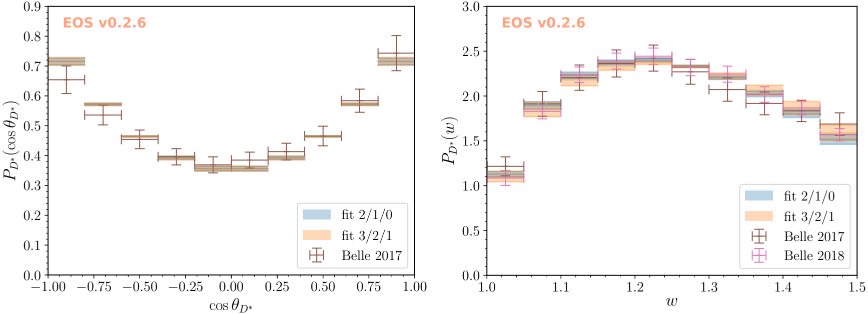
FFs under control;
$R(D^*) = 0.247(6)$
[Bordone/MJ/vDyk'19]



Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- Fits 3/2/1 and 2/1/0 are theory-only fits(!)
- $k/l/m$ denotes orders in z at $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- w-distribution yields information on FF shape $\to V_{cb}$
- Angular distributions more strongly constrained by theory, only

➥ Predicted shapes perfectly confirmed by $B \to D^{(*)}\ell\nu$ data

➥ V_{cb} from Belle'17 compatible between HQE and BGL!

Robustness of the HQE expansion up to $1/m_c^2$

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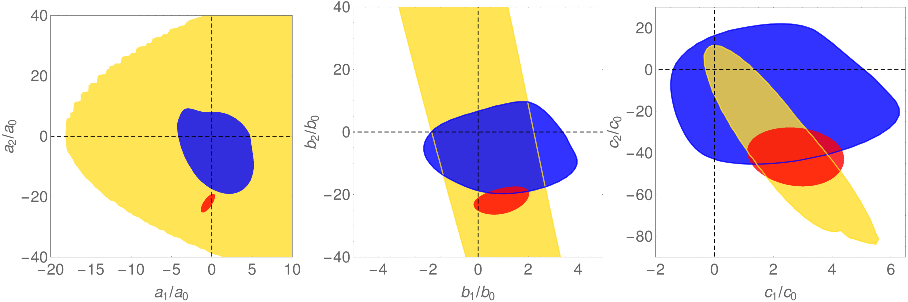


- $B \to D^*$ BGL coefficient ratios from:
  1. Data (Belle'17+'18) + weak unitarity (yellow)
  2. HQE theory fit 2/1/0 (red)
  3. HQE theory fit 3/2/1 (blue)

➥ Again compatibility of theory with data

➥ 2/1/0 underestimates the uncertainties massively

➥ For b_i, c_i ($\to f, \mathcal{F}_1$) data and theory complementary

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors [Bordone/Gubernari/MJ/vDyk'20]

Dispersion relation *sums* over hadronic intermediate states

- Includes $B_s D_s^{(*)}$, included via SU(3) + conservative breaking
- Explicit treatment can improve also $\bar{B} \to D^{(*)}\ell\nu$

Experimental progress in $\bar{B}_s \to D_s^{(*)}\ell\nu$:
2 new LHCb measurements [2001.03225, 2003.08453]

Improved theory determinations required, especially for NP

We extend our $1/m_c^2$ analysis by including:

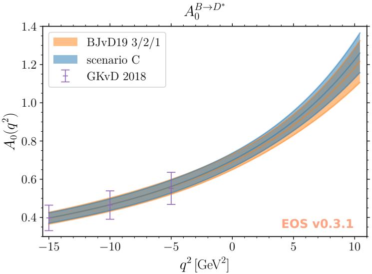
- Available lattice data:
  (2 $\bar{B}_s \to D_s$ FFs (q^2 dependent), 1 $\bar{B}_s \to D^*$ FF (only $q^2_{\max}$))
- Adaptation of existing QCDSR results [Ligeti/Neubert/Nir'93'94] , including SU(3) breaking
- New LCSR results extending [Gubernari+'18] to B_s, including SU(3) breaking

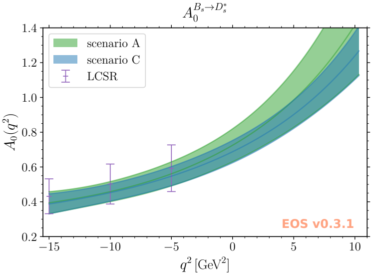
- Fully correlated fit to $\bar{B} \to D^{(*)}$, $\bar{B}_s \to D_s^{(*)}$ FFs

Including $\bar{B}_s \to D_s^{(*)}$ Form Factors, Results

We observe the following:

- Theory constraints fitted consistently in an HQE framework
- $\mathcal{O}(1/m_c^2)$ power corrections have $\mathcal{O}(1)$ coefficients
- No indication of sizable SU(3) breaking
- Slight influence of strengthened unitarity bounds
- Improved determination of $\bar{B}_s \to D_s^{(*)}$ FFs





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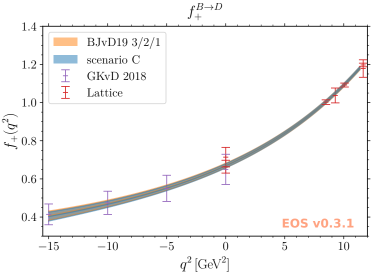

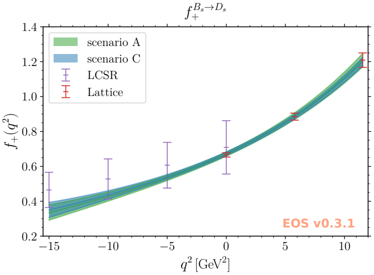

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Theory-only predictions:

$$R(D) = 0.299(3) \qquad R(D^*) = 0.247(5)$$
$$R(D_s) = 0.297(3) \qquad R(D_s^*) = 0.245(8)$$

Theory+Experiment (Belle'17) predictions:

$$R(D) = 0.298(3) \qquad R(D^*) = 0.250(3)$$
$$R(D_s) = 0.297(3) \qquad R(D_s^*) = 0.247(8)$$

Application: Flavour universality in $B \to D^*(e,\mu)\nu$

[Bobeth/Bordone/Gubernari/MJ/vDyk'21]

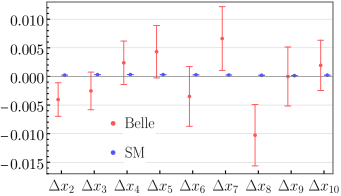
So far: Belle'18 data used in SM fits, flavour-averaged

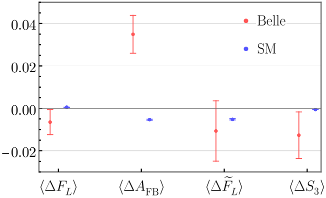
However: Bins 40×40 covariances given separately for $\ell = e, \mu$

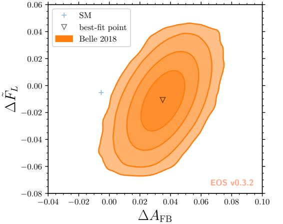
- Belle'18: $R_{e/\mu}(D^*) = 1.01 \pm 0.01 \pm 0.03$
- What can we learn about flavour-non-universality? $\to$ 2 issues:
  1. $e - \mu$ correlations not given, but constructible from Belle'18
  2. 3 bins linearly dependent, but covariances not singular

Two-step analysis:

1. 2×4 angular observables suffice for 2×30 angular bins
   - Model-independent description including NP!
2. Compare with SM predictions, using FFs@$1/m_c^2$ [Bordone+'19]
   - $\sim 4\sigma$ discrepancy in $\Delta A_{\rm FB} = A_{\rm FB}^{\mu} - A_{\rm FB}^{e}$







Generalities regarding this anomaly

$\sim$ 15% of a SM tree decay $\sim V_{cb}$: This is a huge effect!
➥ Need contribution of $\sim 5-10\%$ (w/ interference) or $\gtrsim 40\%$ (w/o interference) of SM

What do we do about this?

- Check the SM prediction!
  [→ Bigi+,Bordone+,Gambino+,Grinstein+,Bernlochner+]
  ➥ $\delta R(D^*)$ larger, anomaly remains
- Combined analysis of all $b \to c\tau\nu$ observables [100+ papers]
  ➥ First model discrimination
- Related indirect bounds (partly model-dependent)
  ➥ High p_T searches, lepton decays, LFV, EDMs, ...
- Analyze flavour structure of potential NP contributions
  ➥ quark flavour structure, e.g. $b \to u$
  ➥ lepton flavour structure, e.g. $b \to c\ell(= e, \mu)\nu$

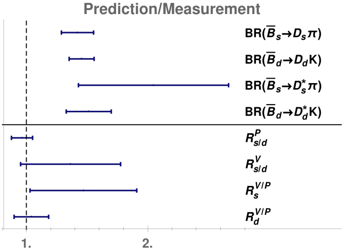
A puzzle in non-leptonic $b \to c$ transitions

[Bordone/Gubernari/Huber/MJ/vDyk'20]

FFs also of central importance in non-leptonic decays:

- Complicated in general, $B \to M_1 M_2$ dynamics
- Simplest cases: $\bar{B}_d \to D_d^{(*)} \bar{K}$ and $\bar{B}_s \to D_s^{(*)} \pi$ (5 diff. quarks)
  - ➥ Colour-allowed tree, $1/m_b^0 @ \mathcal{O}(\alpha_s^2)$ [Huber+'16] , factorizes at $1/m_b$
  - ➥ Amplitudes dominantly $\sim \bar{B}_q \to D_q^{(*)}$ FFs
  - ➥ Used to determine f_s/f_d at hadron colliders [Fleischer+'11]

Updated and extended calculation: tension of 4.4σ w.r.t. exp.!



- Large effect, $\sim -30\%$ for BRs
- Ratios of BRs ok
- QCDf uncertainty $\mathcal{O}(1/m_b^2, \alpha_s^3)$
- Data consistent (too few abs. BRs)
- NP? $\Delta_P \sim \Delta_V \sim -20\%$ possible

➥ We will learn something important!