Global fits to $b \to s\ell\ell$ data

Flavour@TH 2023 – 11/05/2023

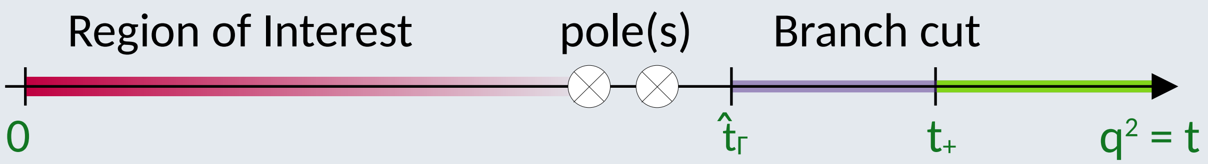
Méril Reboud

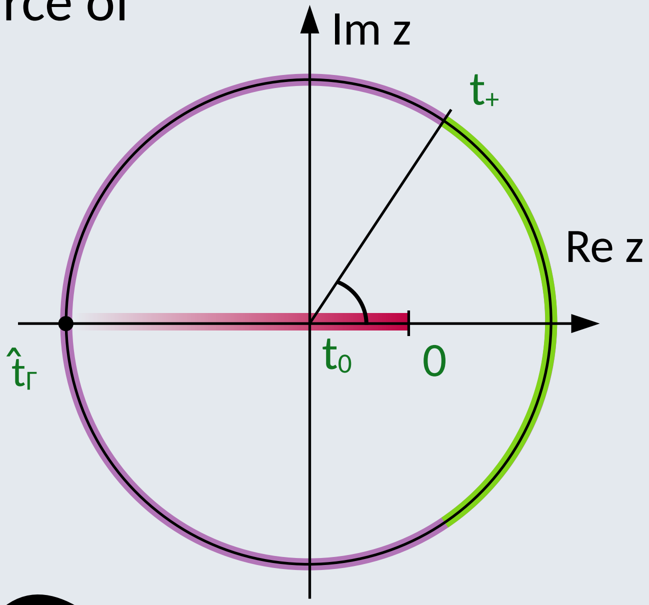
Based on:

- Gubernari, van Dyk, JV 2011.09813
- Gubernari, Reboud, van Dyk, JV 2206.03797
- Ahmis, Bordone, Reboud 2208.08937
- Gubernari, Reboud, van Dyk, JV 2305.06301

Introduction

- Previously in Flavour@TH 2023:
  - **Local** and **non-local form factors** are the main source of uncertainties in $b \to s\ell\ell$ decays
  - Both follow the same analytic structure:





  - The GRvDV parametrization diagonalizes the dispersive bounds:

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}} \qquad \widehat{\mathcal{H}}(\hat{z}) = \sum_{n=0}^{\infty} \beta_n \, p_n(\hat{z})$$

Orthonormal polynomials of the arc of the unit circle

Introduction and Outline

- I will cover three types of global $b \to s\ell\ell$ fits:
  - The fit of the local form factors using dispersive bounds
  - The fit of the non local contributions charm loops
  - The fit of WET coefficients based on experimental data

- Given the discussion we had so far [esp. during Jonathan's, Paolo's and Martin's talk], I will start with discussing the parametrization in practice with $\Lambda_b \to \Lambda^* \ell\ell$

I. The method in practice

Example with $\Lambda_b \to \Lambda(1520)\ell\ell$

- Inputs:
  - **LQCD** estimates at q^2 = 16.3 and 16.5 GeV^2 [Meinel, Rendon '21]
  - no LCSR available
    → use (loose) **SCET relations** [Descotes-Genon, M. Novoa-Brunet '19]

$$
\begin{aligned}
f_{\perp'}(0) &= 0 \pm 0.2\,, & g_{\perp'}(0) &= 0 \pm 0.2\,, & h_{\perp'}(0) &= 0 \pm 0.2\,,\\
\tilde{h}_{\perp'}(0) &= 0 \pm 0.2\,, & f_+(0)/f_\perp(0) &= 1 \pm 0.2\,, & f_\perp(0)/g_0(0) &= 1 \pm 0.2\,,\\
g_\perp(0)/g_+(0) &= 1 \pm 0.2\,, & h_+(0)/h_\perp(0) &= 1 \pm 0.2\,, & f_+(0)/h_+(0) &= 1 \pm 0.2\,,
\end{aligned}
$$

$O(\alpha_s/\pi, \Lambda_{QCD}/m_b)$

- 14 form factors: **17 parameters (N = 1), 31 parameters (N = 2)**

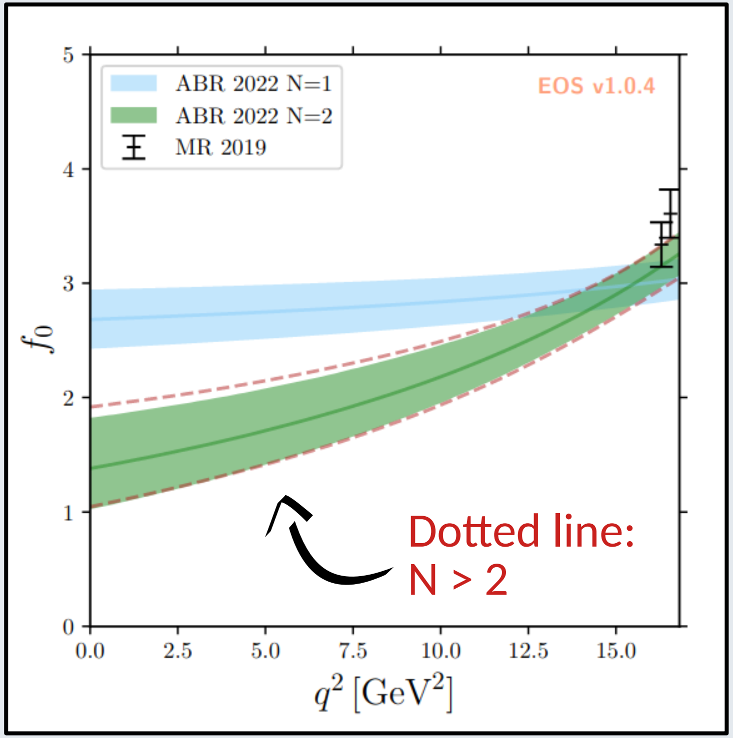
  21 LQCD inputs + 9 SCET relations: **30 constraints**

  2 * 14 – 7 endpoint relations at q^2_{max}

Example with $\Lambda_b \to \Lambda(1520)\ell\ell$

- N = 1 does not give a good fit (p value ~ 0)
- Use an **under-constrained fit** (N>1) and allows for saturation of the dispersive bound

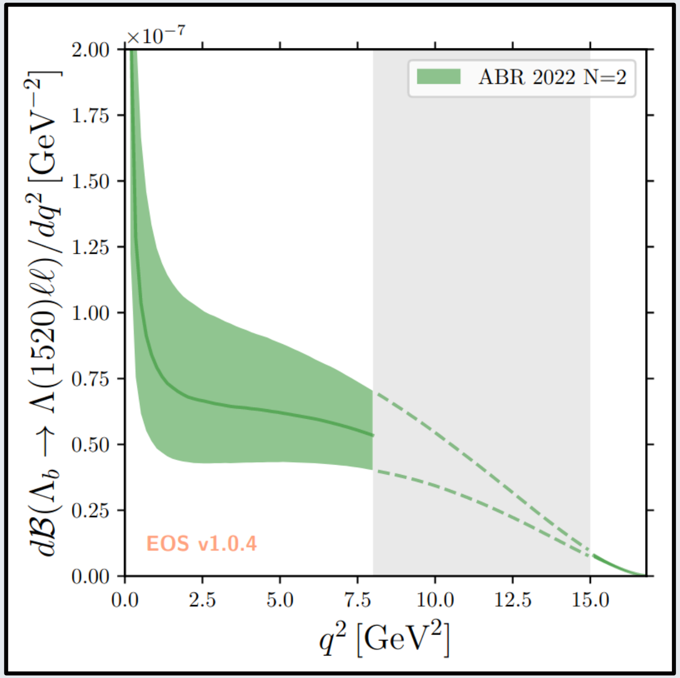
  → The uncertainties are truncation order independent: increasing the order does not change their size
- Same conclusions were found for $\Lambda_b \to \Lambda$ form factors [Blake, Meinel, Rahimi, van Dyk '22]

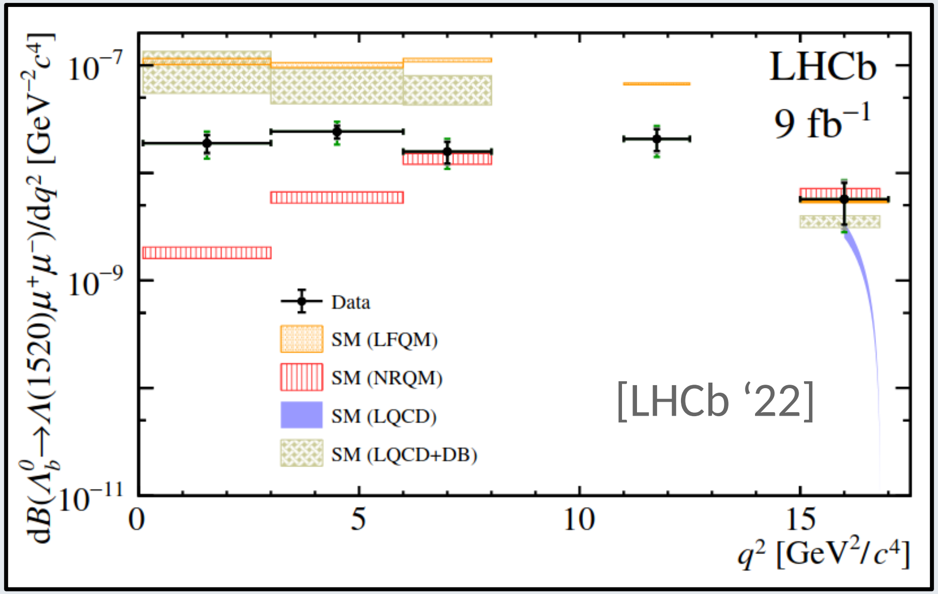


[Ahmis, MR, Bordone '22]

Phenomenology

- Uncertainties are large but **under control** and **systematically improvable**
- LHCb analysis confirmed the usual $b \to s\ell\ell$ tension at low q^2



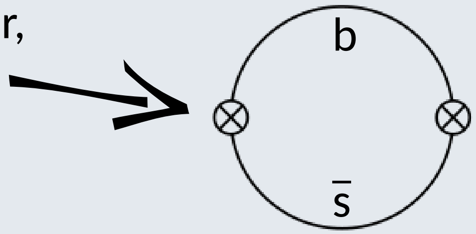


II. Improved dispersive bounds

Correlator and Helicities

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



+ other diagrams: loops, quark and gluon condensates…

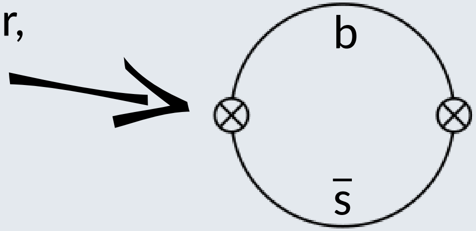
- Usually, the correlator $\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{ J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0) \right\}|0\rangle$

  is decomposed as:

$$\Pi_\Gamma^{\mu\nu}(q) \equiv \frac{q^\mu q^\nu}{q^2} \Pi_\Gamma^{(J=0)} + \frac{1}{D-1}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_\Gamma^{(J=1)}$$

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

Insertion of a scalar, vector or tensor current



+ other diagrams: loops, quark and gluon condensates…

- We suggest the more generic decomposition:

$$\Pi_\Gamma^{\mu\nu}(q) \equiv \sum_{\lambda,\lambda'} \epsilon^\mu(\lambda)\epsilon^{\nu*}(\lambda')\,\Pi_\Gamma^{(\lambda,\lambda')}(q^2)$$

Polarization vectors

Correlator and Helicities

- **Main advantage:**
  - The OPE calculation is independent of the helicities:

$$\Pi_\Gamma^{(J=1)}\big|_{\text{OPE}} = \Pi_\Gamma^{(0)}\big|_{\text{OPE}} = \Pi_\Gamma^{(\parallel)}\big|_{\text{OPE}} = \Pi_\Gamma^{(\perp)}\big|_{\text{OPE}}$$

    → The calculation of Ref. [Bharucha, Feldmann, Wick '10] still applies!

  - Remove spurious correlations between form factors:
    - e.g. A_1 and A_{12} now fulfill different bounds
    - decorrelate completely $B \to K$ from ($B \to K^*$, $B_s \to \varphi$)

- **In equations:**
  - This is the bound used in the literature:

$$\chi_A^{(J=1)}\Big|_{BK^*} = \frac{\eta^{B\to K^*}}{24\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds\,\frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3}\left[s\,(M_B+M_{K^*})^2\,|A_1^{B\to K^*}|^2 + 32\,M_B^2 M_{K^*}^2\,|A_{12}^{B\to K^*}|^2\right]$$

  - And this is what we propose:

$$\chi_A^{(0)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds\,\frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3}\,4\,M_B^2 M_{K^*}^2\,|A_{12}^{B\to K^*}|^2,$$

$$\chi_A^{(\|)}\Big|_{\bar{B}K^*} = \frac{\eta^{B\to K^*}}{8\pi^2}\int_{(M_B+M_{K^*})^2}^{\infty} ds\,\frac{\lambda_{\rm kin}^{1/2}}{s^2(s-Q^2)^3}\,s\,(M_B+M_{K^*})^2\,|A_1^{B\to K^*}|^2,$$

Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Baryonic decays should be added, but there are currently only few constraints

Setup

- Bayesian analysis using EOS
- Implementation of the dispersive bound:

$$-2\log P(r) = \begin{cases} 0 & \text{if } r < 1\,, \\ \frac{(r-1)^2}{\sigma^2} & \text{otherwise}\,, \end{cases}$$

10% uncertainty on the OPE calculation
[Bharucha, Feldmann, Wick '10]

- To many constraint to perform an under-constrained fit
  → **Stability criterion**: truncate the series expansion to N = 2, 3, 4 and compare the form factor uncertainties

Setup

- All the samples are considered to be **correlated only via the dispersive bounds**
- Since $B \to K$ and ($B \to K^*$, $B_s \to \varphi$) are decoupled, we perform **3 separated fits**
- $B \to K^*$ and $B_s \to \varphi$ samples are combined with a weighting procedure:

$$w(r_{\Gamma,\lambda}^{B\to K^*}) = \int dr\, p_{\Gamma,\lambda}^{B_s\to\phi}(r) \times P\left(\text{dispersive bound for } \Gamma,\lambda \,|\, r_{\Gamma,\lambda}^{\text{1pt}} + r_{\Gamma,\lambda}^{B\to K^*} + r\right)$$

Current-specific weight

Integration over the $B_s \to \varphi$ saturations PDF

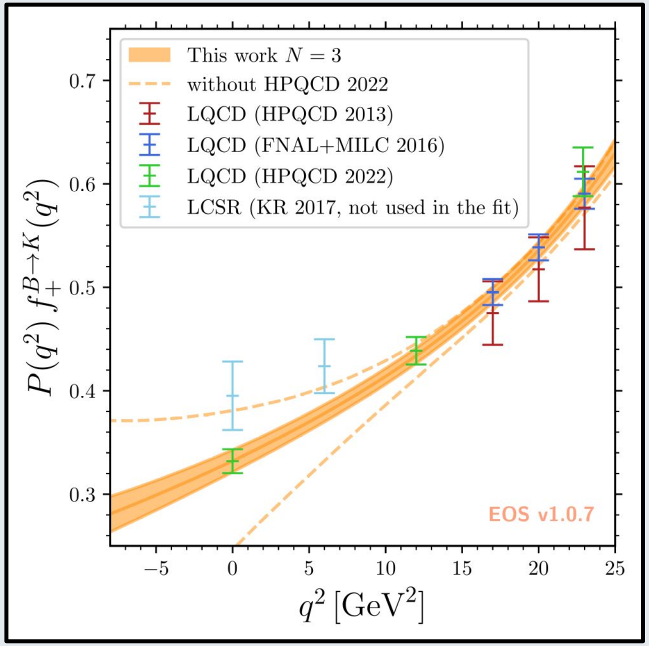
Dispersive bound of the previous slide

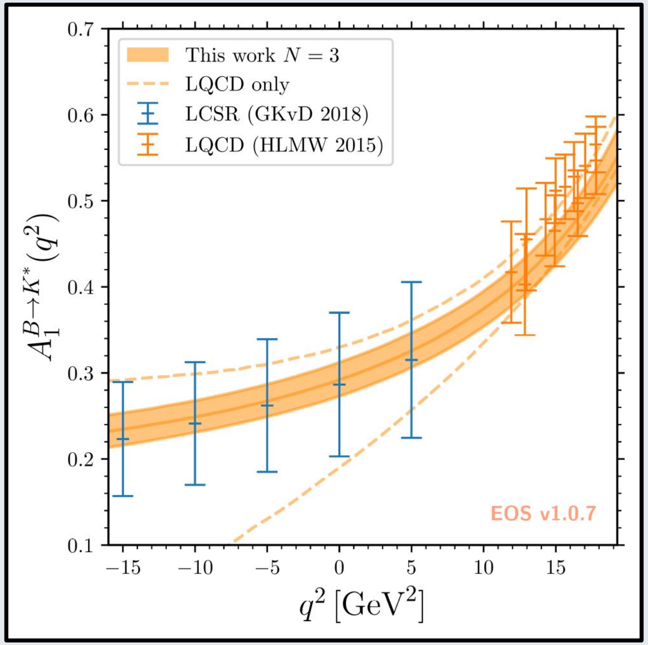
$$w^{B\to K^*} = \prod w(r_{\Gamma,\lambda}^{B\to K^*})$$

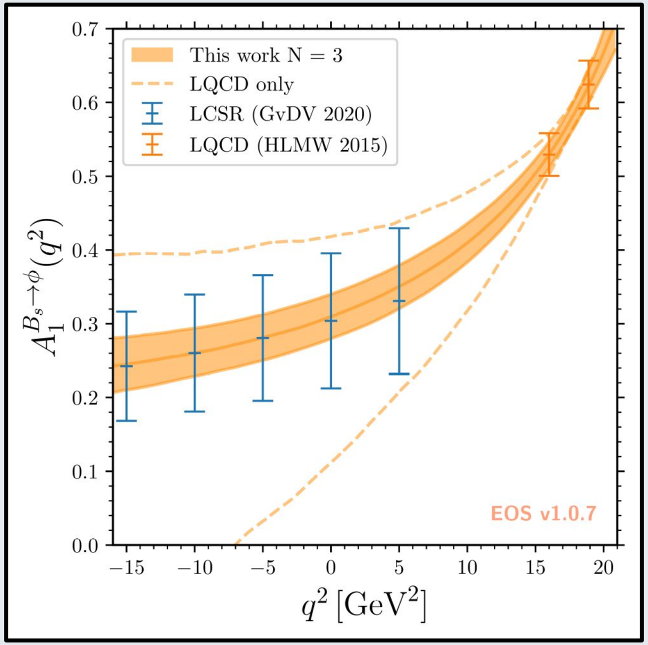
Results

Main conclusions:

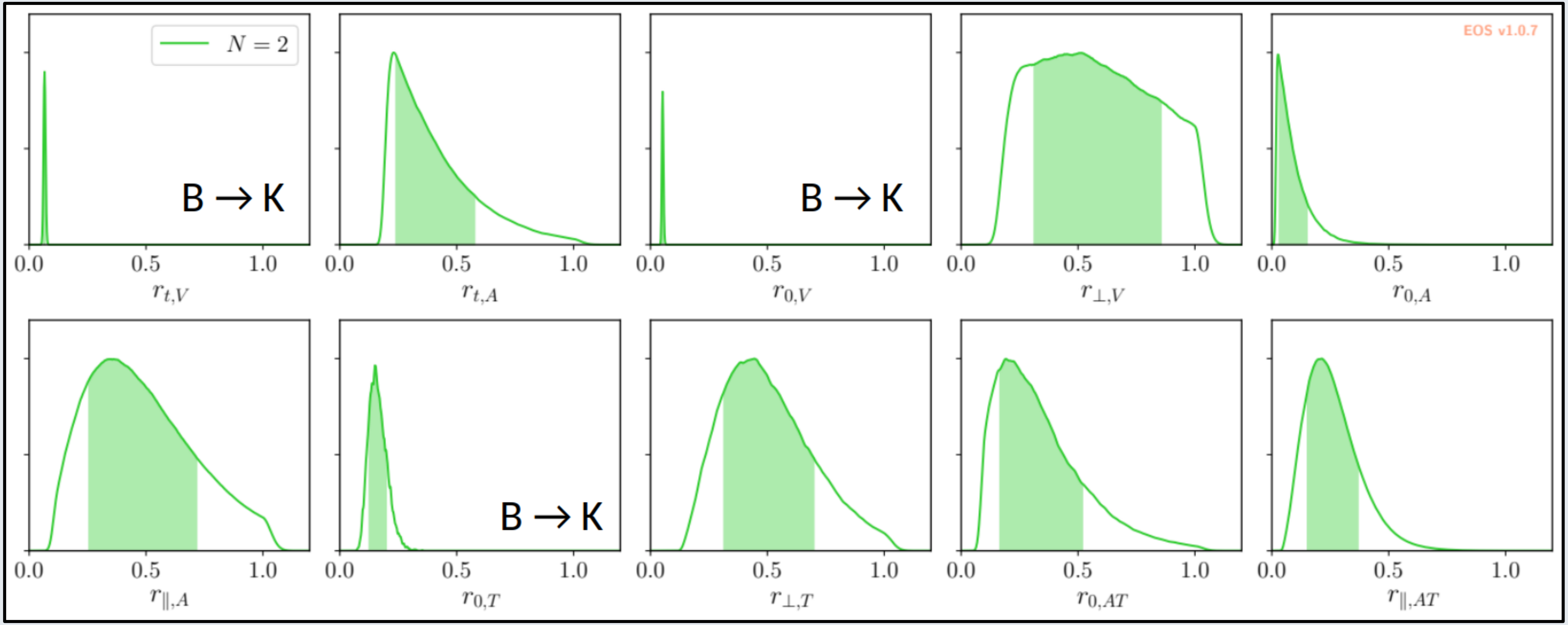
- Fits are very good already at N = 2 (p-values > 77%)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will eventually make LCSR irrelevant (?)





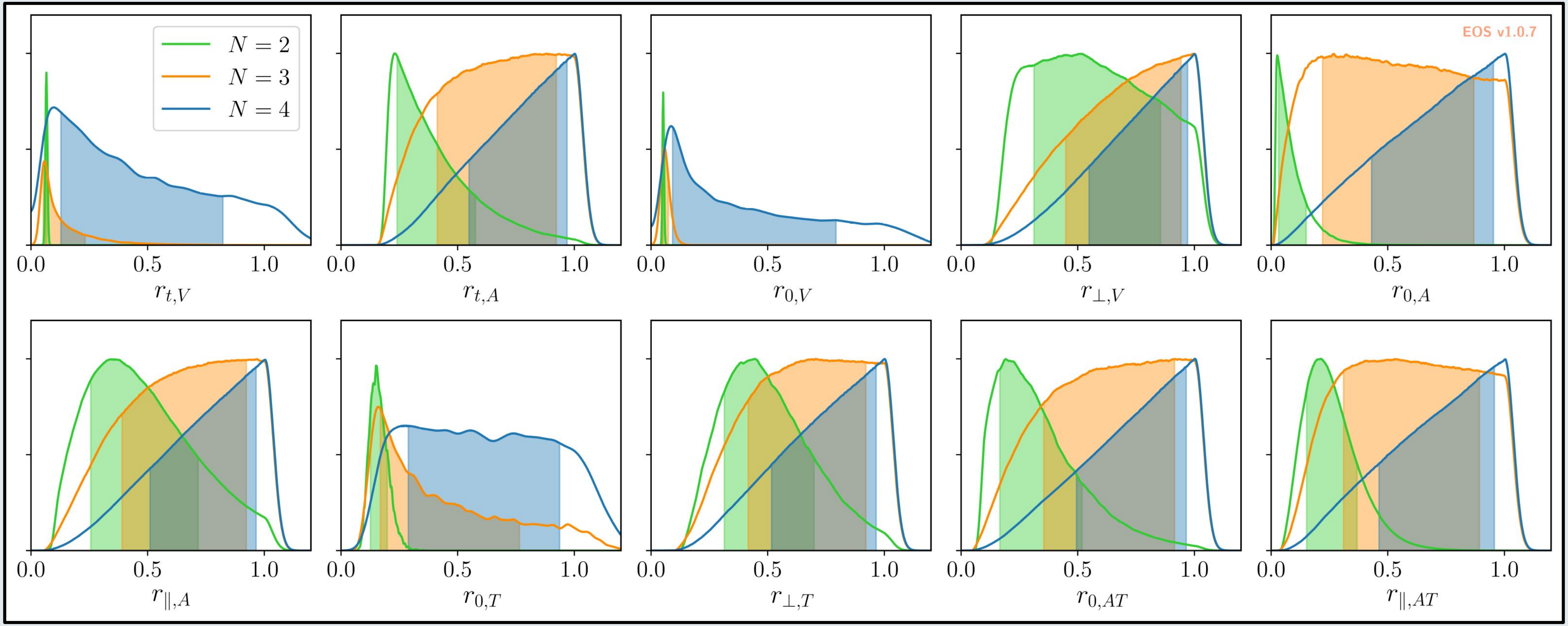


Saturations



- Saturations are small for N = 2, in agreement with [Bharucha, Feldmann, Wick '10]

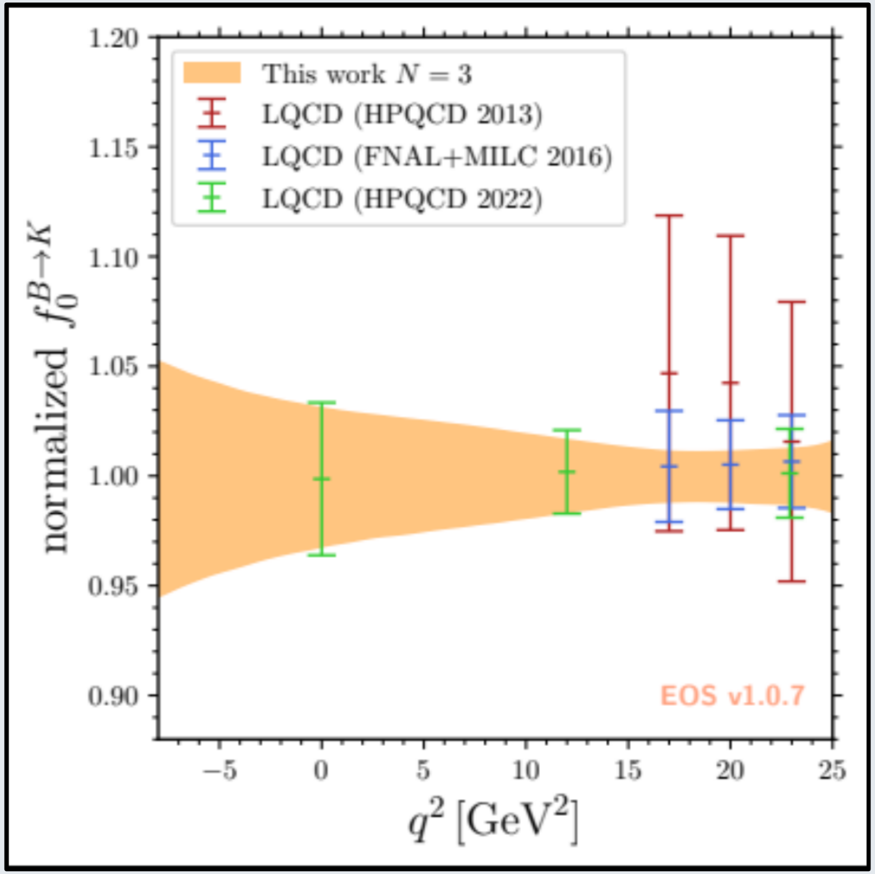
Saturations



- The low saturations in $r_{t,V}$ and $r_{0,V}$ are probably due to large contributions in the baryonic decays, as discussed in the first part of this talk

Comparison plots

- For comparison purposes I normalize the form factors to our N = 3 best-fit point
- Uncertainties for B → K are now well below 5% in the physical region



Comparison plots

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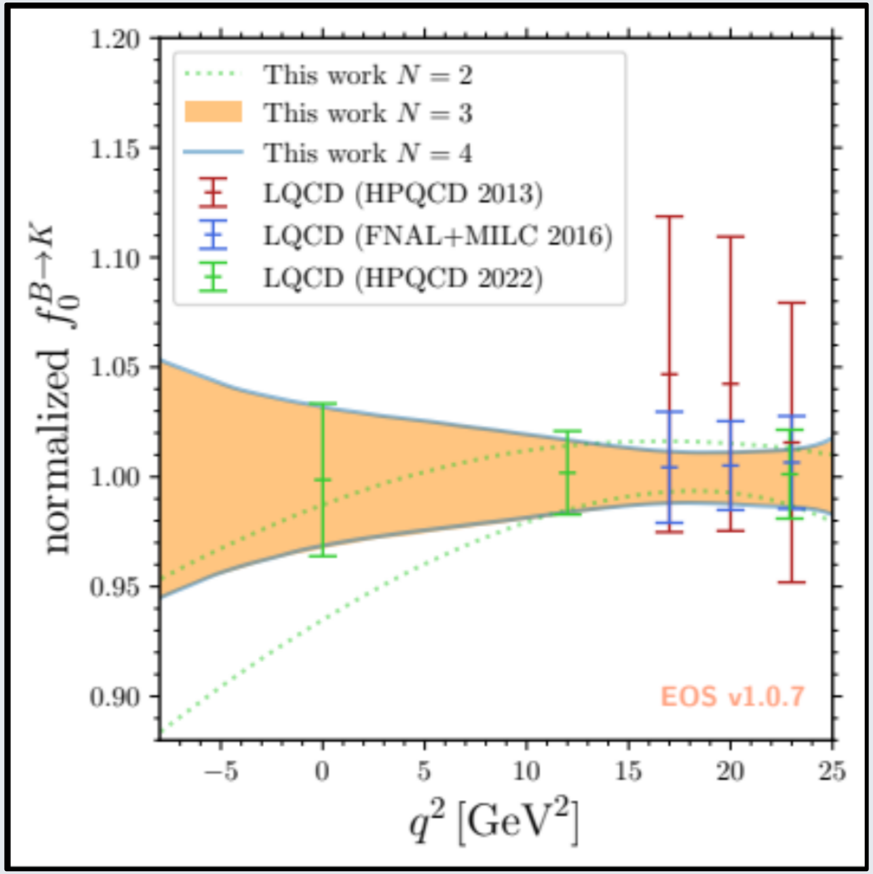
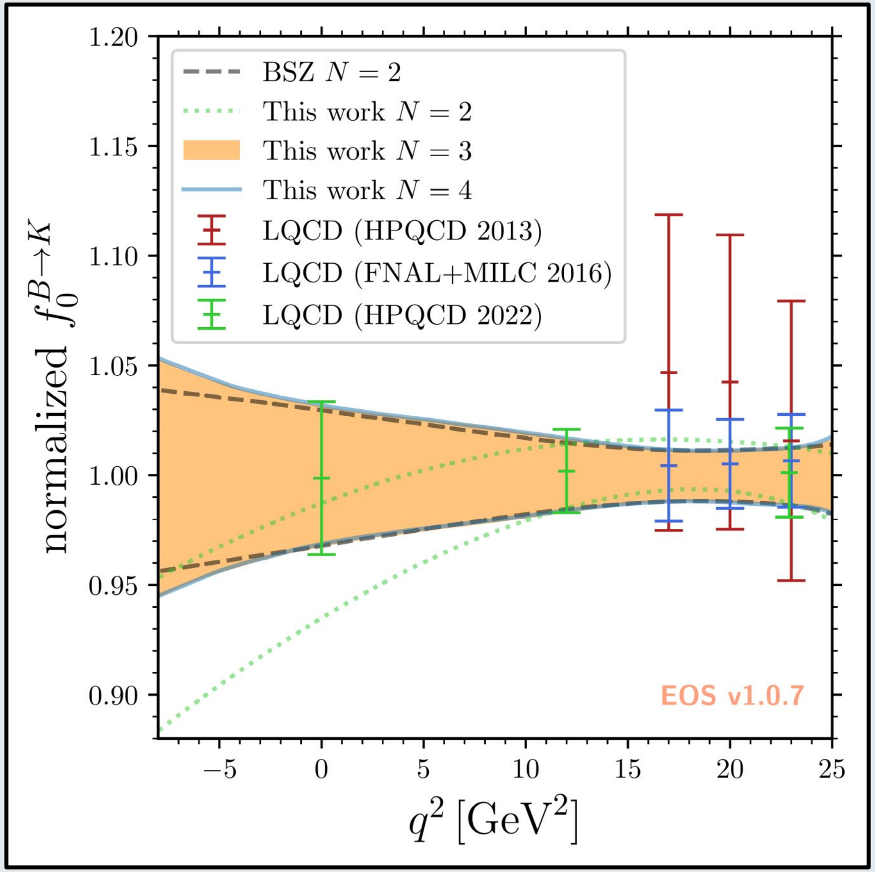
- We compare the different values of the truncation order N



Comparison plots

- For comparison purposes I normalize the form factors to our N = 3 best-fit point
- Uncertainties for $B \to K$ are now well below 5% in the physical region
- We compare the different values of the truncation order N
- I also add the result of a usual Simplified Series Expansion *à la* [Bharucha, Feldmann, Wick ‘10; Bharucha, Straub, Zwicky ‘15]

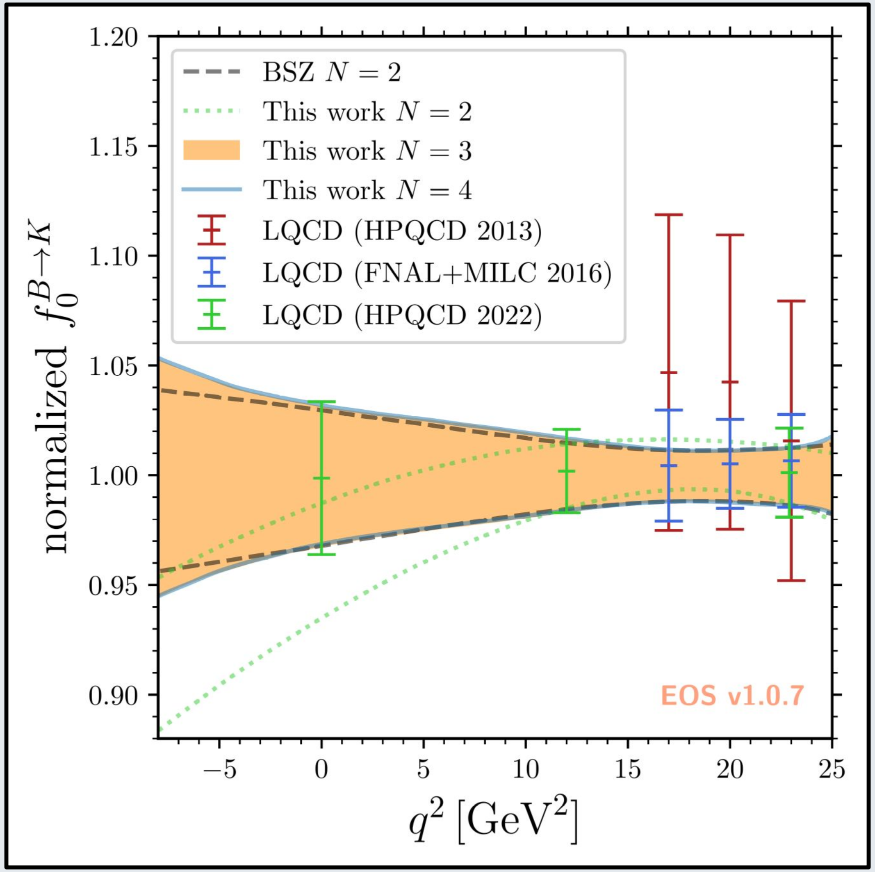
$$f(t) = \frac{1}{P(t)} \sum_k \tilde{\alpha}_k \, z^k(t, t_0)$$



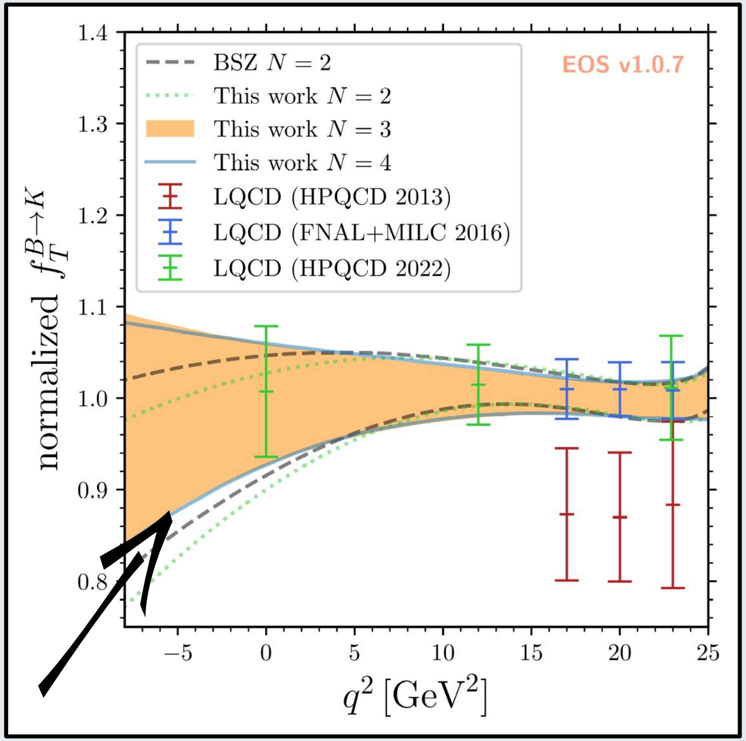
Generic result

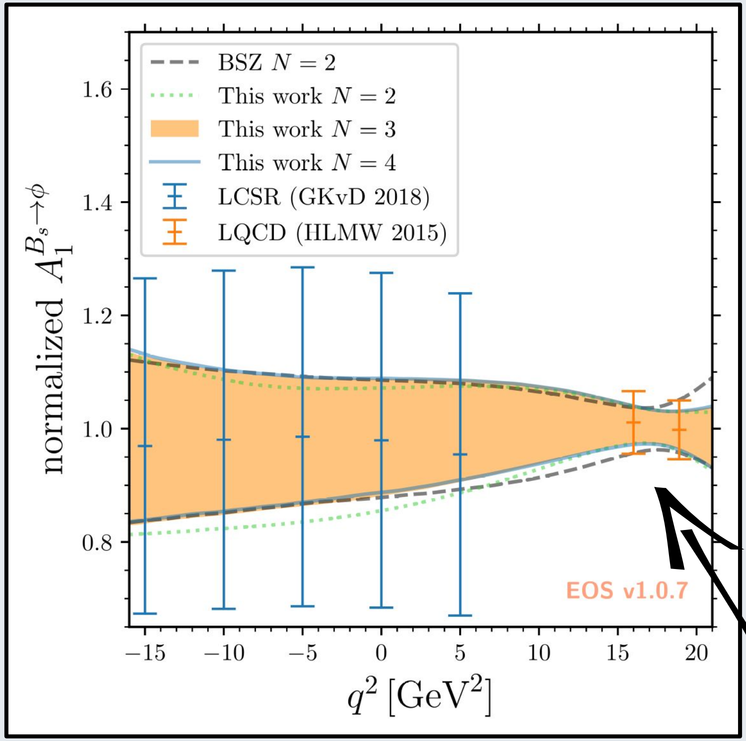
This is the generic result, namely:

- N = 2 shows a peculiar behaviour
- For N > 2 the uncertainties are stable
- BSZ is a good approximation in the physical range, but underestimates the uncertainties at negative q^2



Specific cases



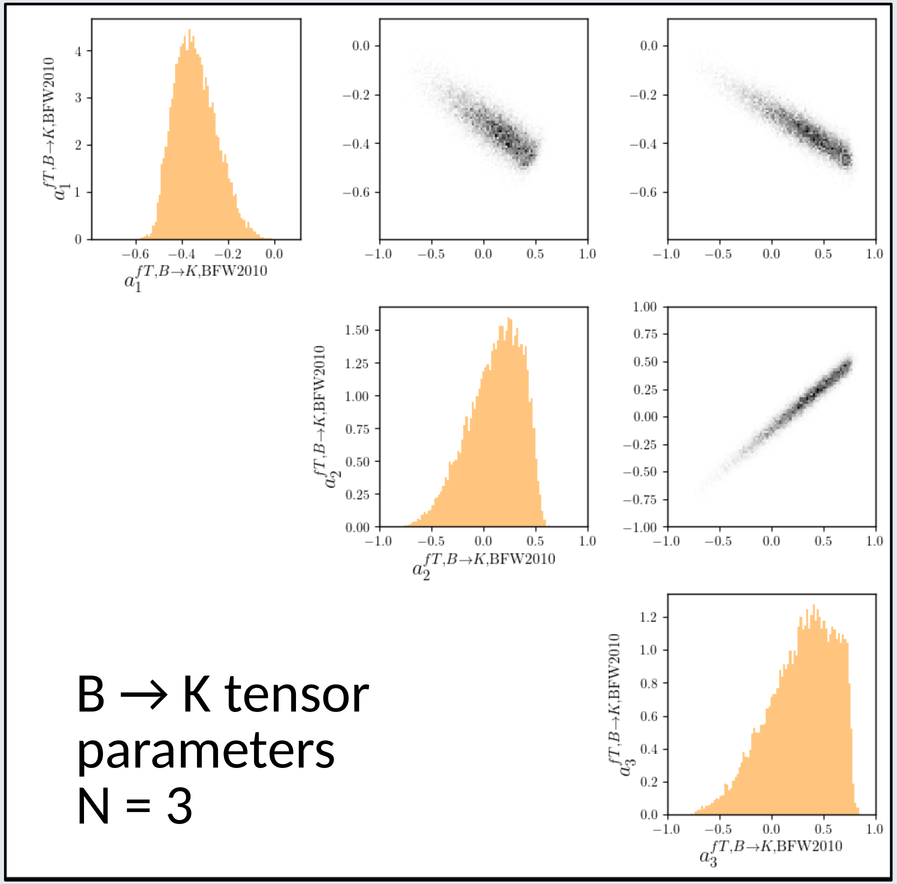


→ The dispersive bounds stabilizes regions of the phase space with few theory constraints

→ This is particularly useful at negative q^2 to estimate the non-local form factors

Gaussian and non-Gaussian behaviours

- For N = 2, the bounds are not saturated and the parameters follow Gaussian distributions to a good approximation (perplexities > 95%)
- Already at N = 3, distortions of the distribution are clearly visible



Where to find our results

- All the plots are available here: github.com/eos/analysis-2023-02
- We also added
  - the updated posterior distributions for N = 2 in our parametrization and using a SSE as YAML files
  - All the tools/documentation to reproduce our results
- These results are also available in **EOS v1.0.7**:
  - /eos/constraints/B-to-P-P-form-factors.yaml
  - /eos/constraints/B-to-P-P-form-factors.yaml

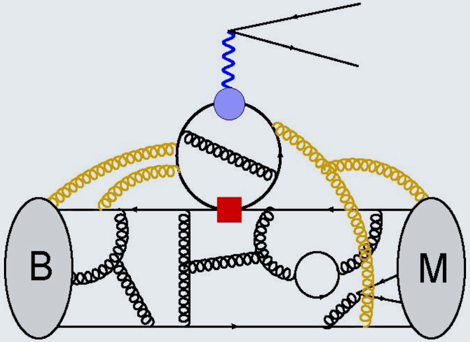
III. Parametrization of non-local form factors

→ I will stick to the conclusions of our paper and defer all discussion to this afternoon's session

Non-local form factors

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k, q) = i \int d^4x \, e^{iq\cdot x} \langle \bar{M}(k) | T\{ \mathcal{J}_\mu^{\rm em}(x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

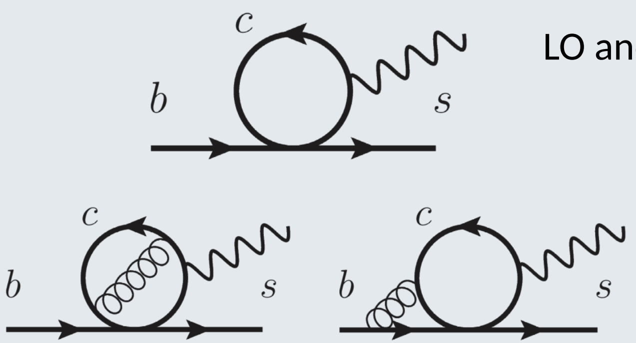


- Problematic because **they can mimic a BSM signal!**
  - $\mathcal{H}_\lambda$ can be interpreted as a shift to C_9 and C_7
  - This shift is lepton-flavour universal (as now seen in the data)
- Notably **harder to estimate**, no lattice computation so far
- **Different parametrizations** are suggested

Theory inputs

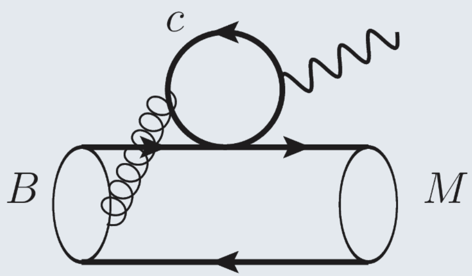
$\mathcal{H}_\lambda$ can still be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



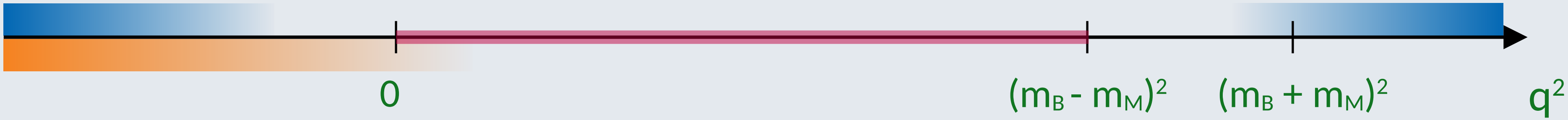
LO and α_s corrections

[Asatarian, Greub, Virto '19]



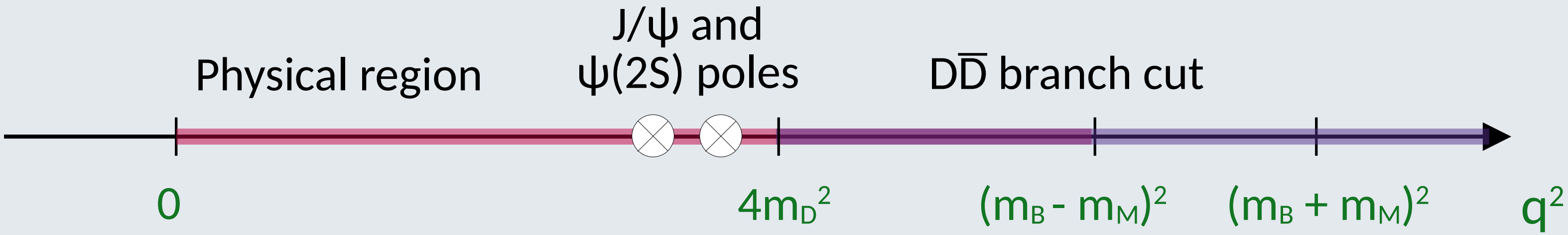
Non-perturbative soft gluon corrections

[Gubernari, van Dyk, Virto '20]



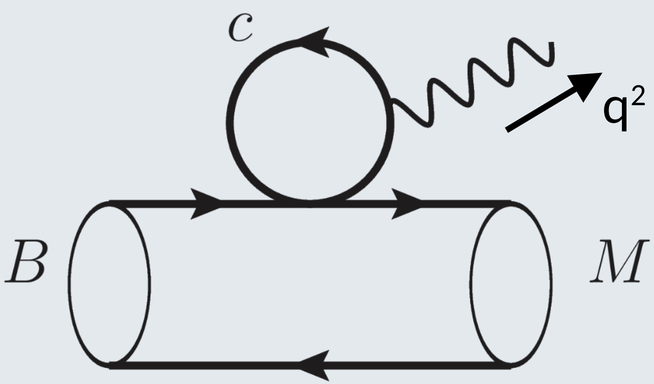
Analyticity properties

$$\mathcal{H}_\mu(k,q) = i\int d^4x\, e^{iq\cdot x}\langle \bar{M}(k)|T\{\mathcal{J}_\mu^{\rm em}(x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$



Analyticity properties of the Q_c dependent part:

- Poles due to **charmonium state**
- **Branch cut** in the physical range due to on-shell D meson production: $B \to MD\bar{D}$
- The branch cut in k^2 makes the coefficients of the z-expansion **complex-valued**



Parametrization of the charm loop

- Still focusing on $B \to K$, $B \to K^*$ and $B_s \to \varphi$

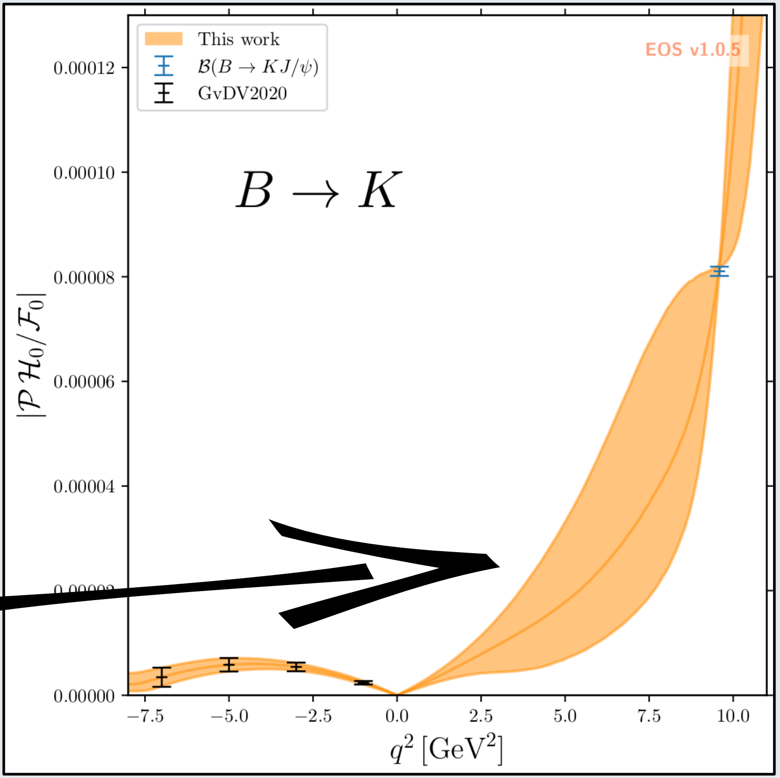
  Inputs:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ (we keep $\psi(2S)$ for future work)

- Use again an under-constrained fit (N = 5) and allows for saturation of the dispersive bound

  → The uncertainties are **truncation order independent**, increasing the expansion order does not change their size

  → All p-values are larger than 11%

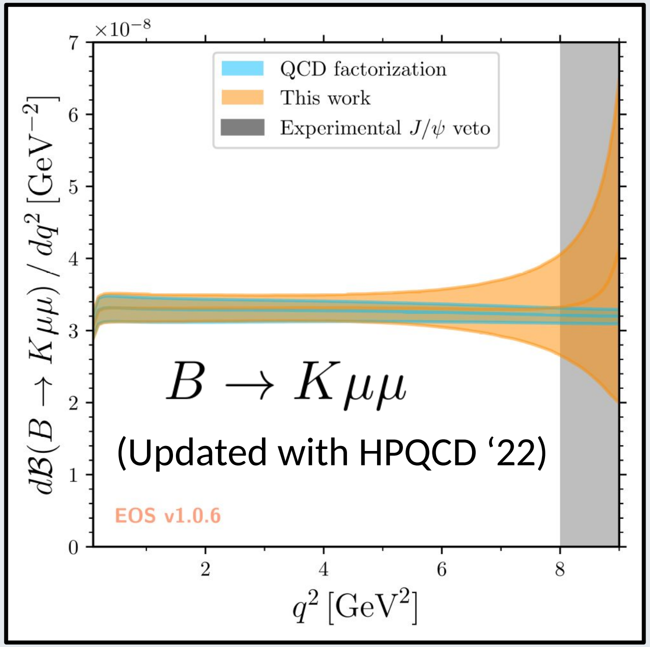
[Gubernari, MR, van Dyk, Virto '22]

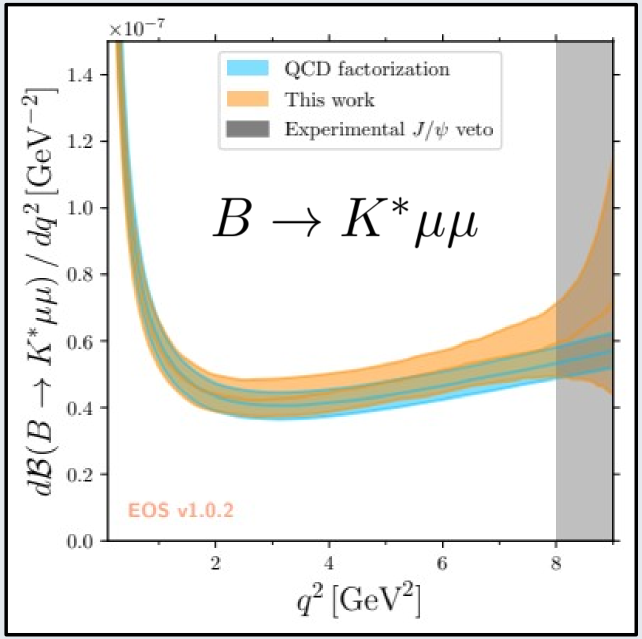


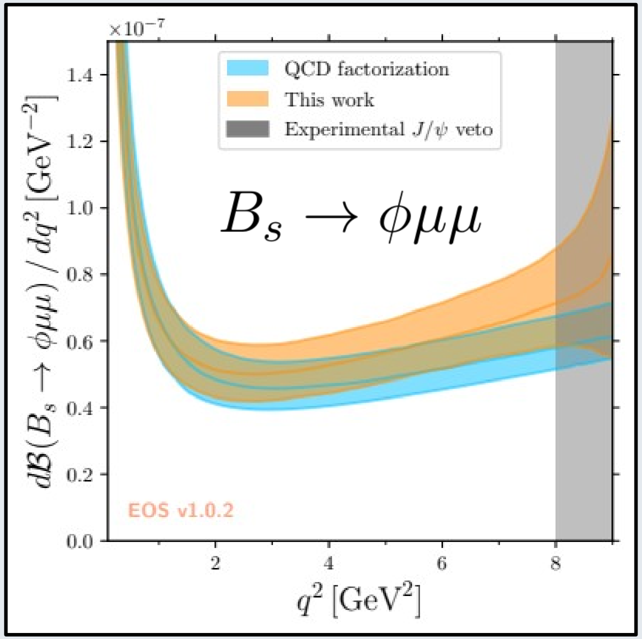
SM predictions

- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel ‘01 & ‘04]
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ

  → The approach is **systematically improvable** (new channels, ψ(2S) data…)



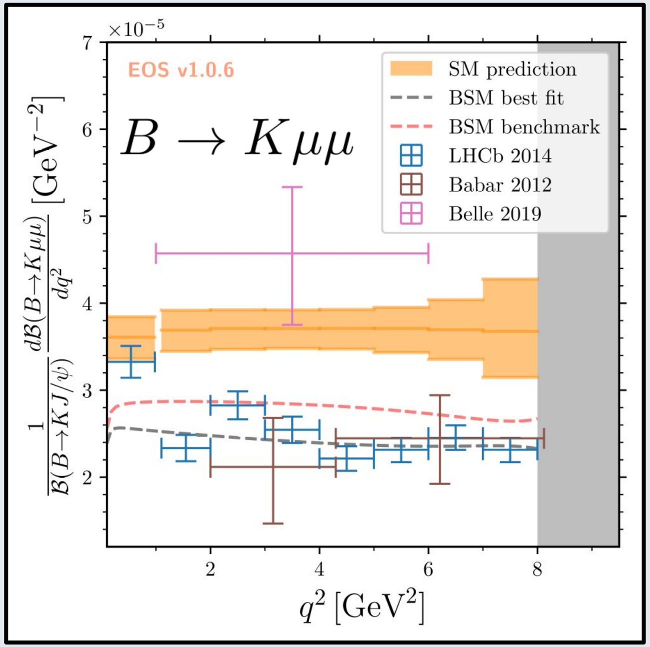


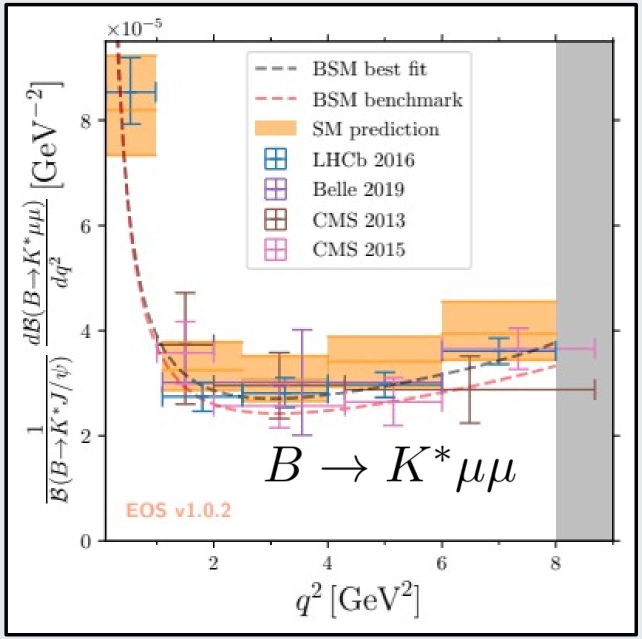


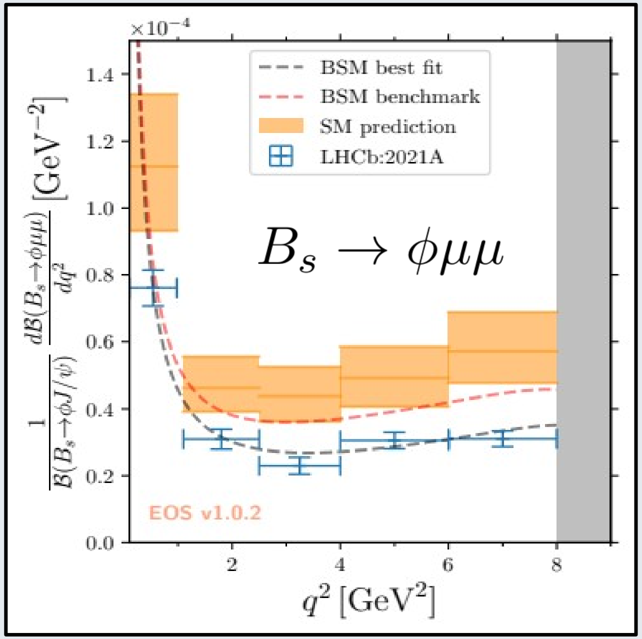
Confrontation with data

- Conservatively accounting for the non-local form factors does not solve the $b \to s\mu\mu$ anomalies
- The largest source of theoretical uncertainty at low q^2 still comes from local form factors

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]

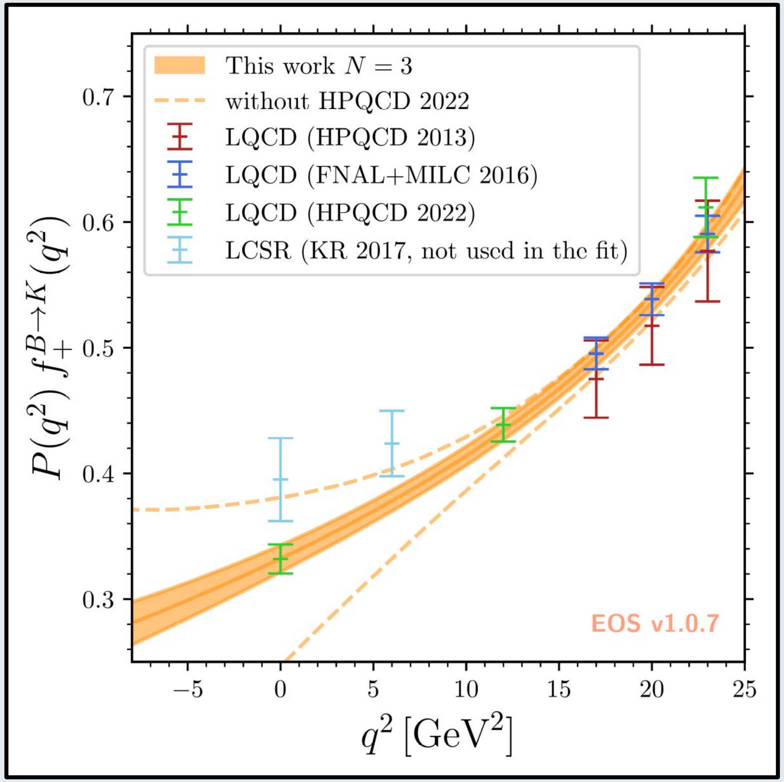




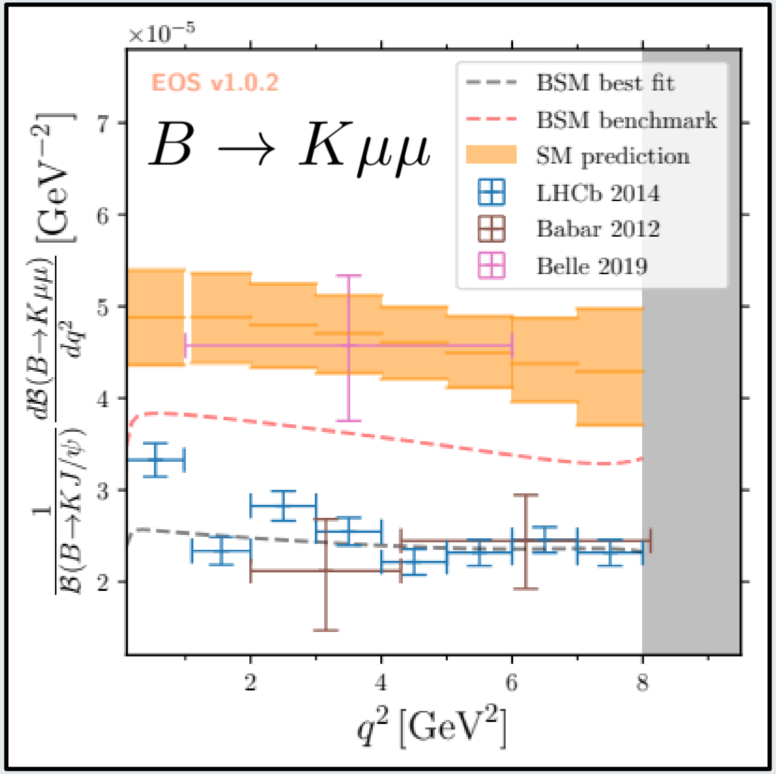


Additional plots can be found in the paper: 2206.03797

Effect of HPQCD 2022



With Khodjamirian-Rusov 2017



[Gubernari, MR, van Dyk, Virto '22]

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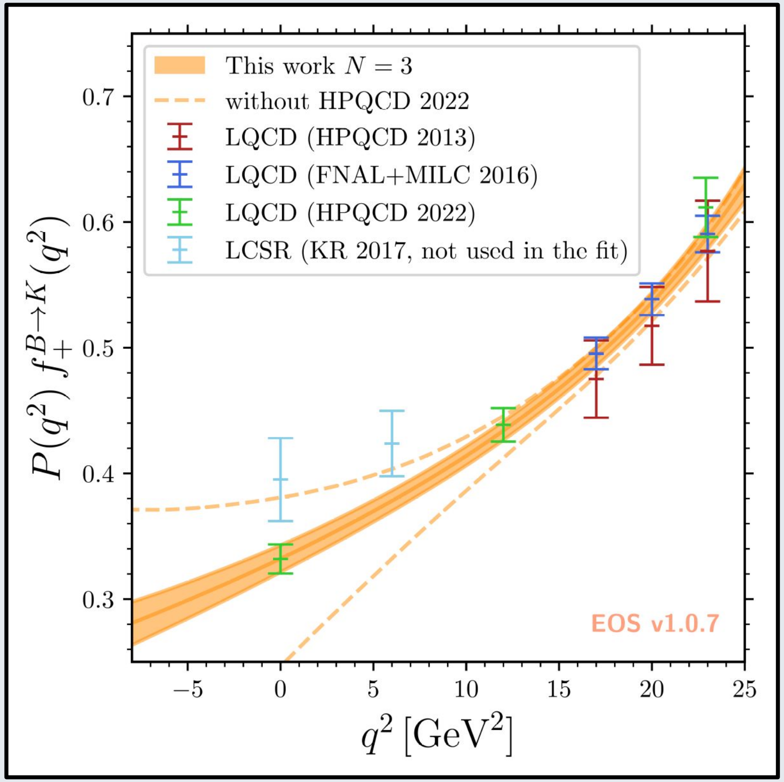
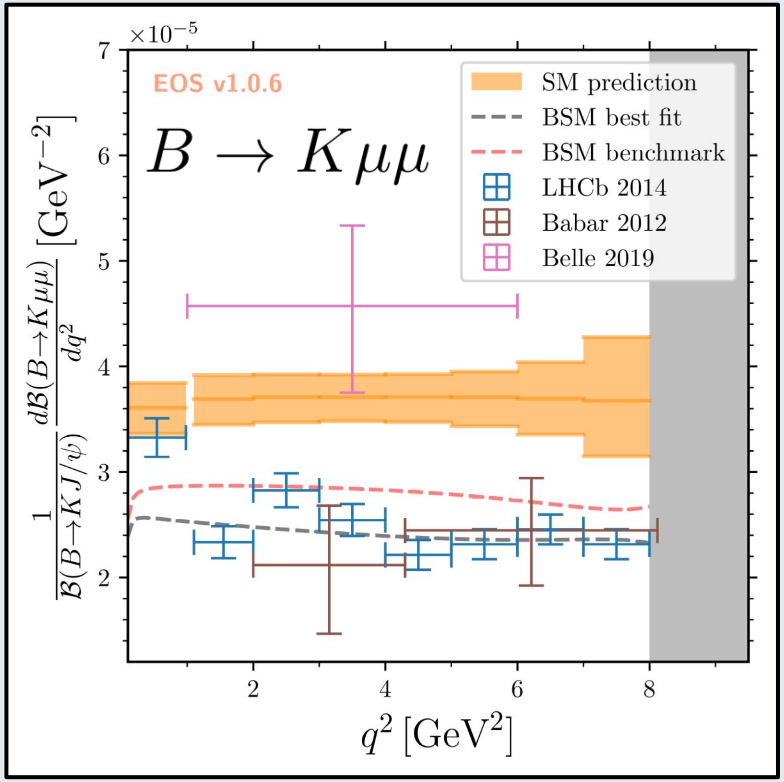


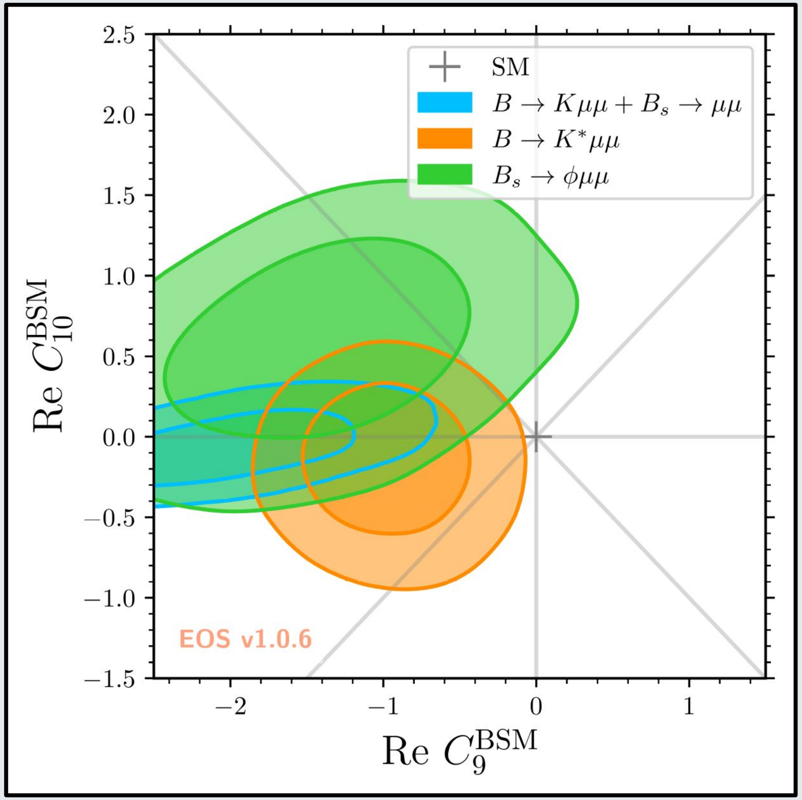
With HPQCD 2022



IV. BSM analysis: proof of concept

BSM 'proof-of-concept' analysis

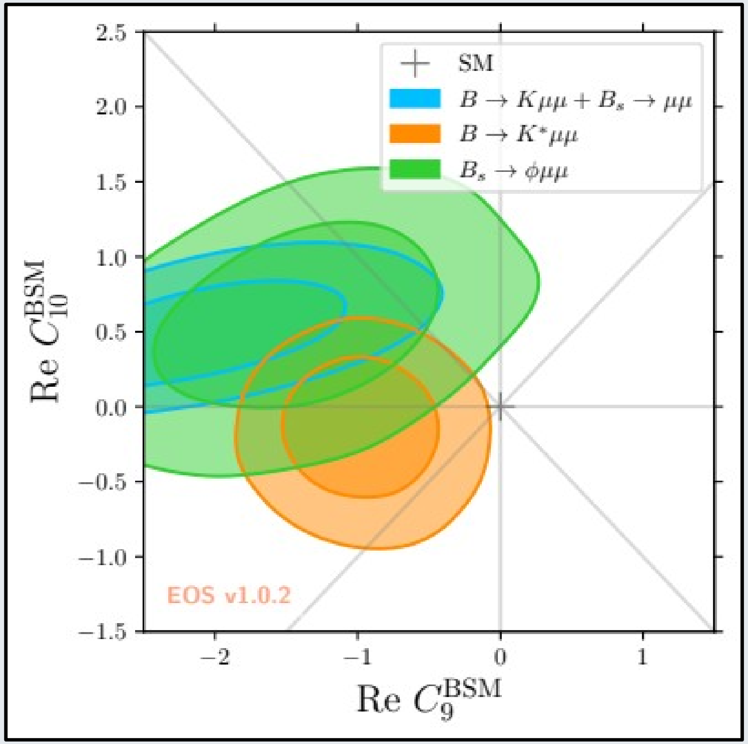
- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit C_9 and C_{10} **separately** for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$

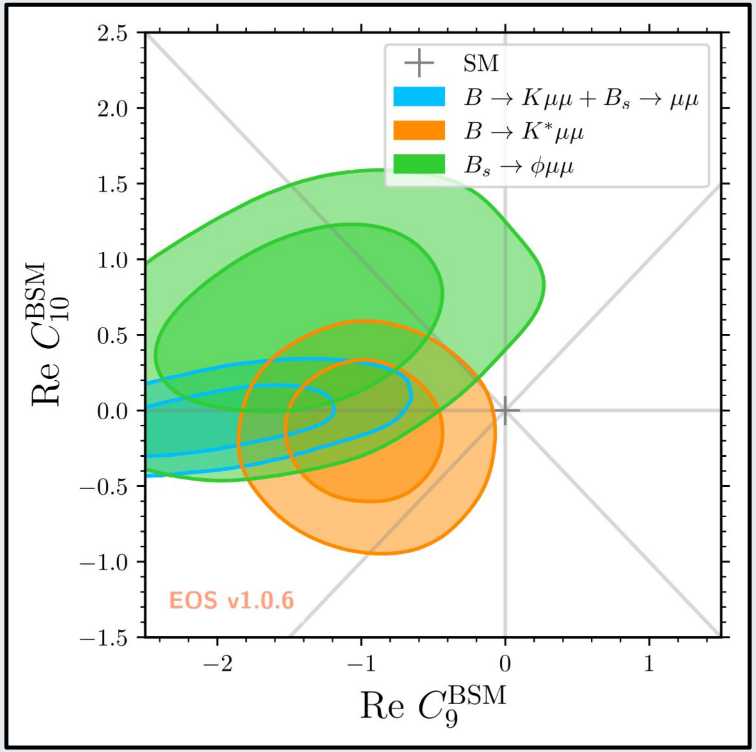


Effect of HPQCD 2022

Accounting for:

- CMS' $B_s \to \mu^+\mu^-$ measurement [2212.10311] → SM-like, $C_{10}^{BSM} \to 0$
- HPQCD '22 $B \to K$ form factors





Conclusion

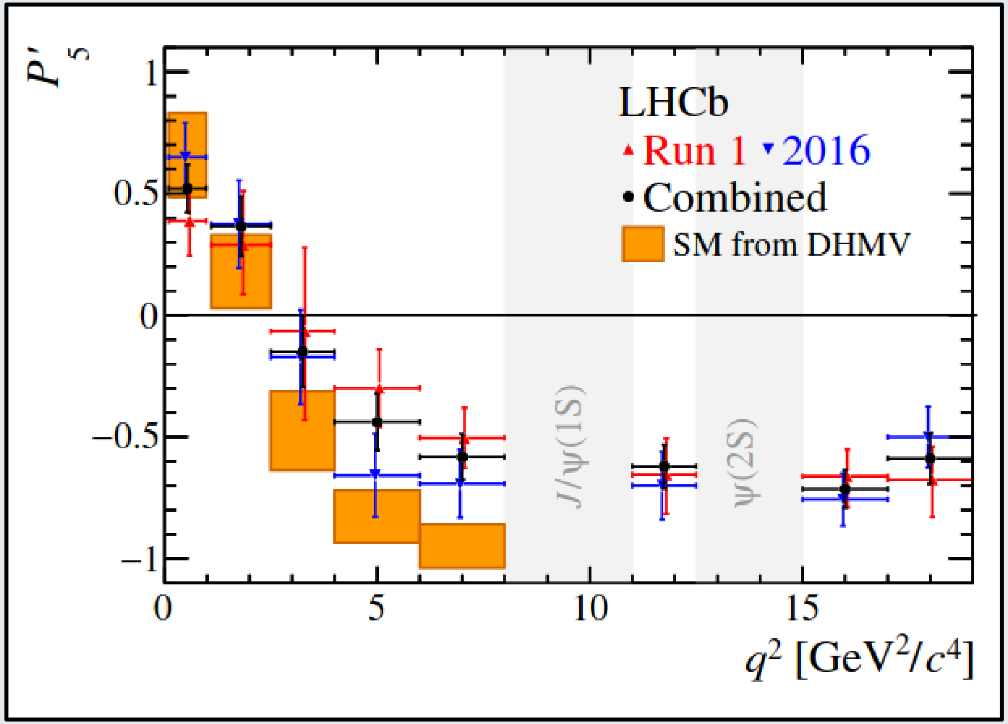
Discussing BSM models requires a solid understanding of the hadronic physics:

- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
  - This is the first global analysis of $b \to s$ form factors
  - It is reassuring as it confirms channel-specific analyses…
  - … and promising as dispersive effects start to be visible
- **Non-local form factors** can also be constrained by theory calculation and experimental measurements

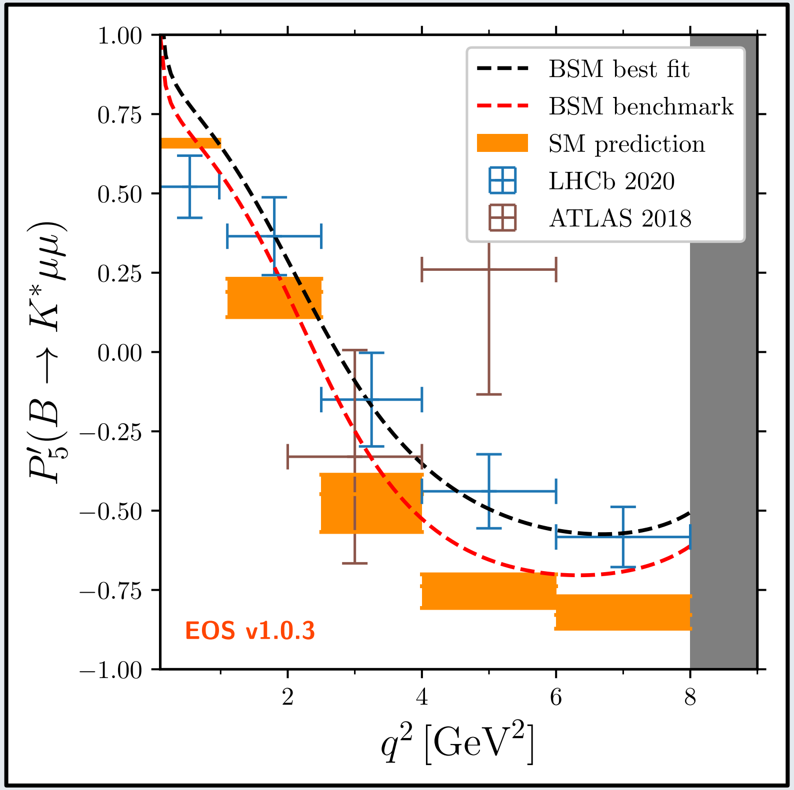
  → In both cases:
  - Uncertainties are still large, but controlled by dispersive bounds
  - Our approach is systematically improvable

Back-up

B → K* P'$_5$



LHCb
Run 1
2016
Combined
SM from DHMV
$J/\psi(1S)$
$\psi(2S)$
P'_5
q^2 [GeV^2/c^4]



BSM best fit
BSM benchmark
SM prediction
LHCb 2020
ATLAS 2018
EOS v1.0.3
$P'_5(B \to K^* \mu\mu)$
q^2 [GeV^2]