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# THE $V_{cb}$ PUZZLE

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# PRECISION FLAVOUR PHYSICS

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Tests of the **flavour structure** of the SM: 3 generations of up and down quarks with different masses, mixing with each other via charged current.

The unitary 3x3 Cabibbo-Kobayashi-Maskawa (CKM) parametrises the mixing and leads to CP violation in the SM.

$$\hat{V}_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \hat{V}_{\text{CKM}}^\dagger \hat{V}_{\text{CKM}} = 1$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

first row

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

second row    etc.

***New Physics could manifest itself as violation of unitarity, or shift Flavour Changing Neutral Current (loop induced in the SM) like  $b \rightarrow s\gamma$ , B and K mixing, etc***

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# The importance of $|V_{cb}|$

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$V_{cb}$  plays an important role in UT

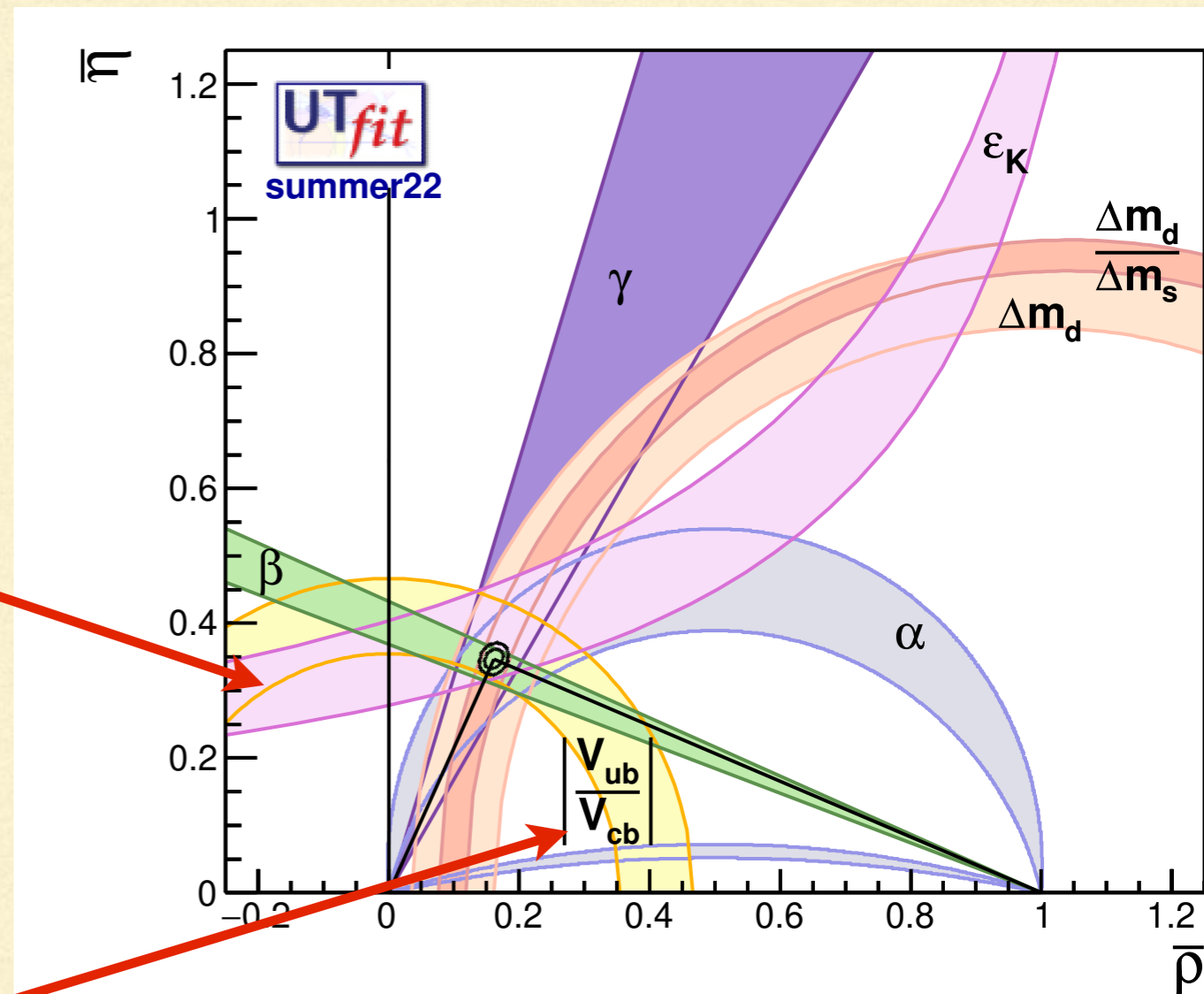
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[ 1 + O(\lambda^2) \right]$$

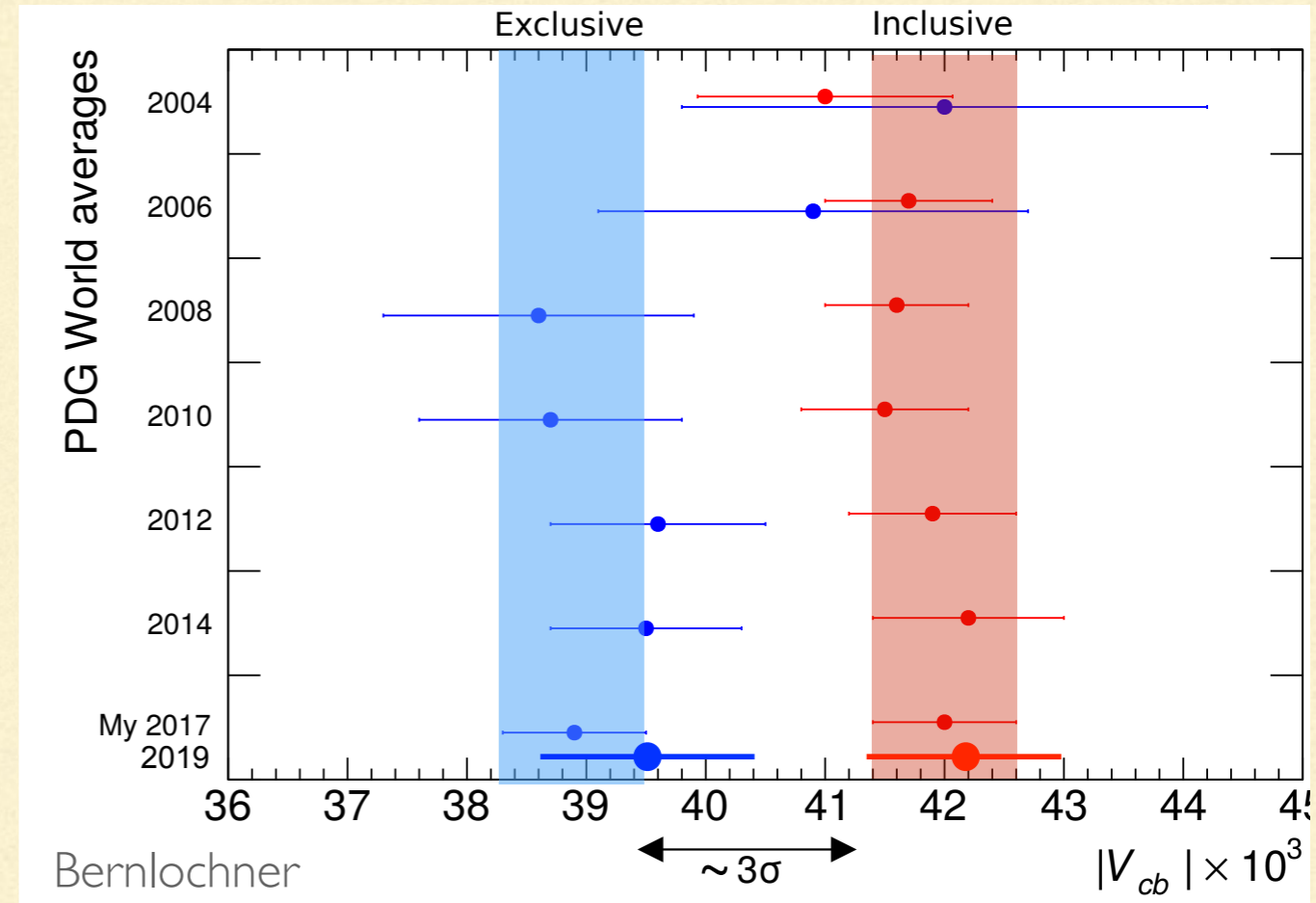
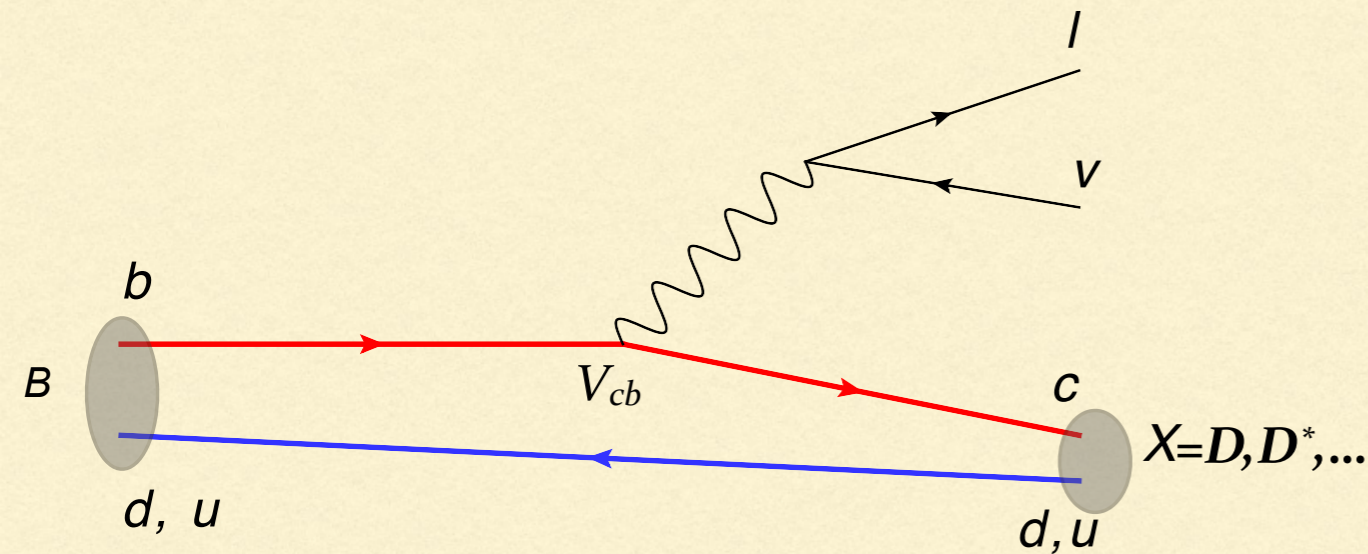
where it often dominates the theoretical uncertainty.

$V_{ub}/V_{cb}$  constrains directly the UT



**Our ability to determine precisely  $V_{cb}$  is crucial for indirect NP searches**

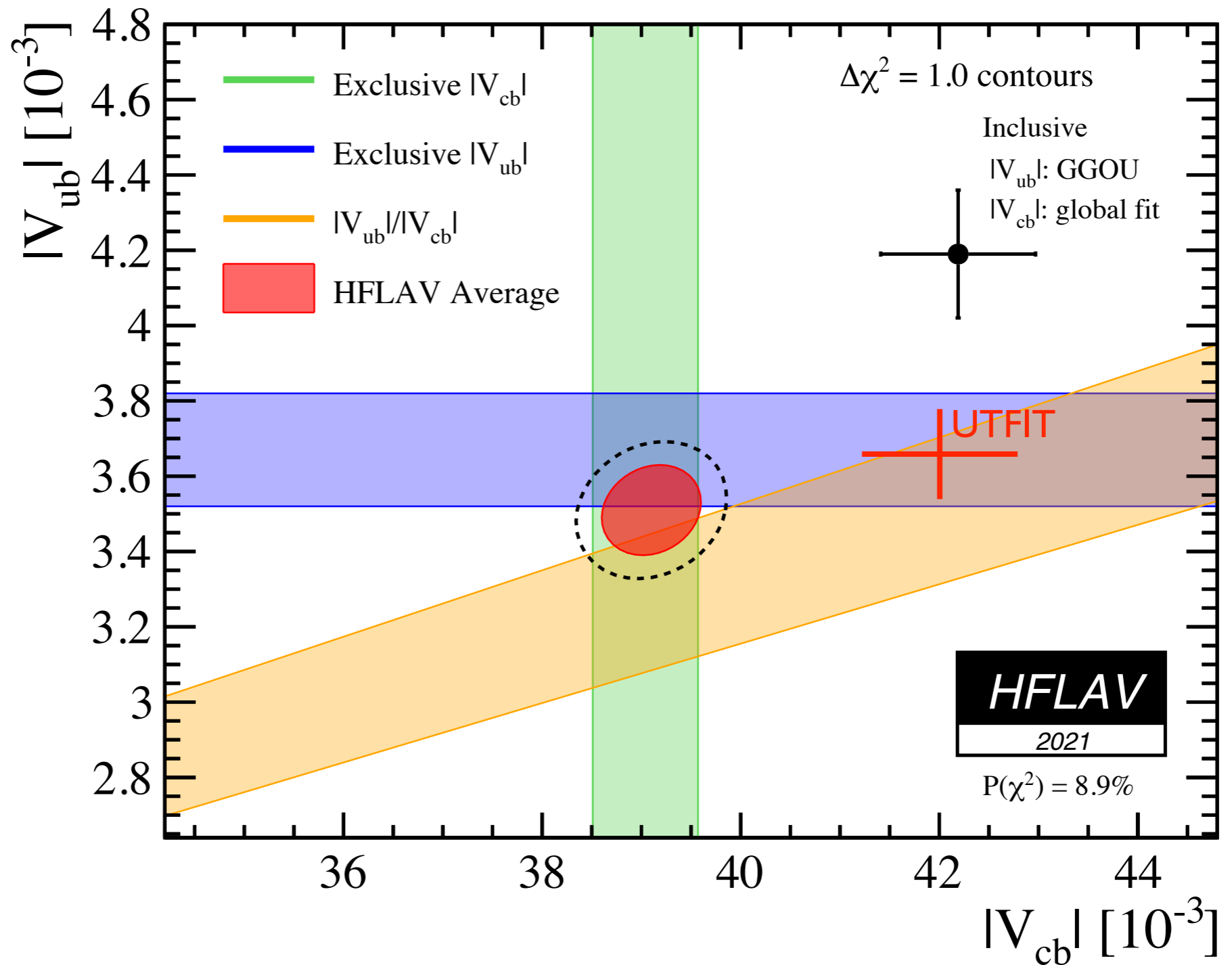
# A LONG-STANDING TENSION



Semileptonic B decays measure the magnitude of the CKM matrix elements  $V_{cb}$  and  $V_{ub}$

Their determinations from inclusive and exclusive decays differ since many years. Intense experimental and theoretical activity.

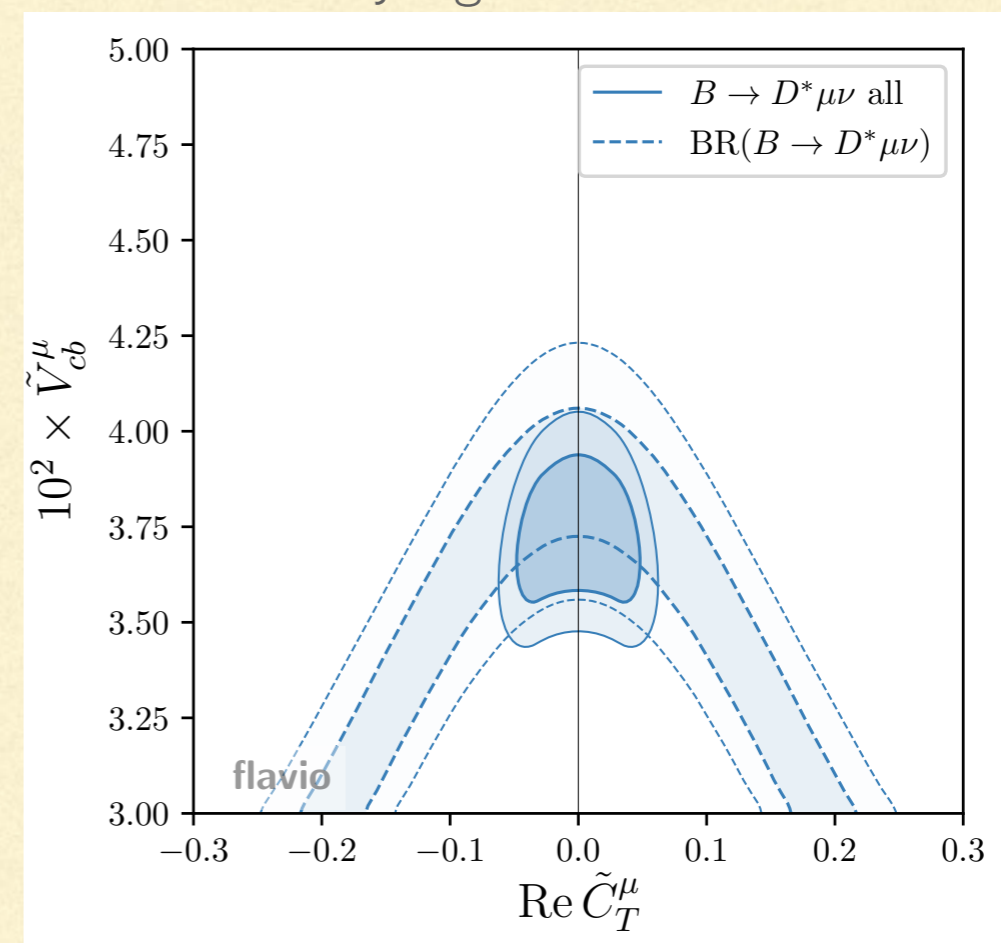
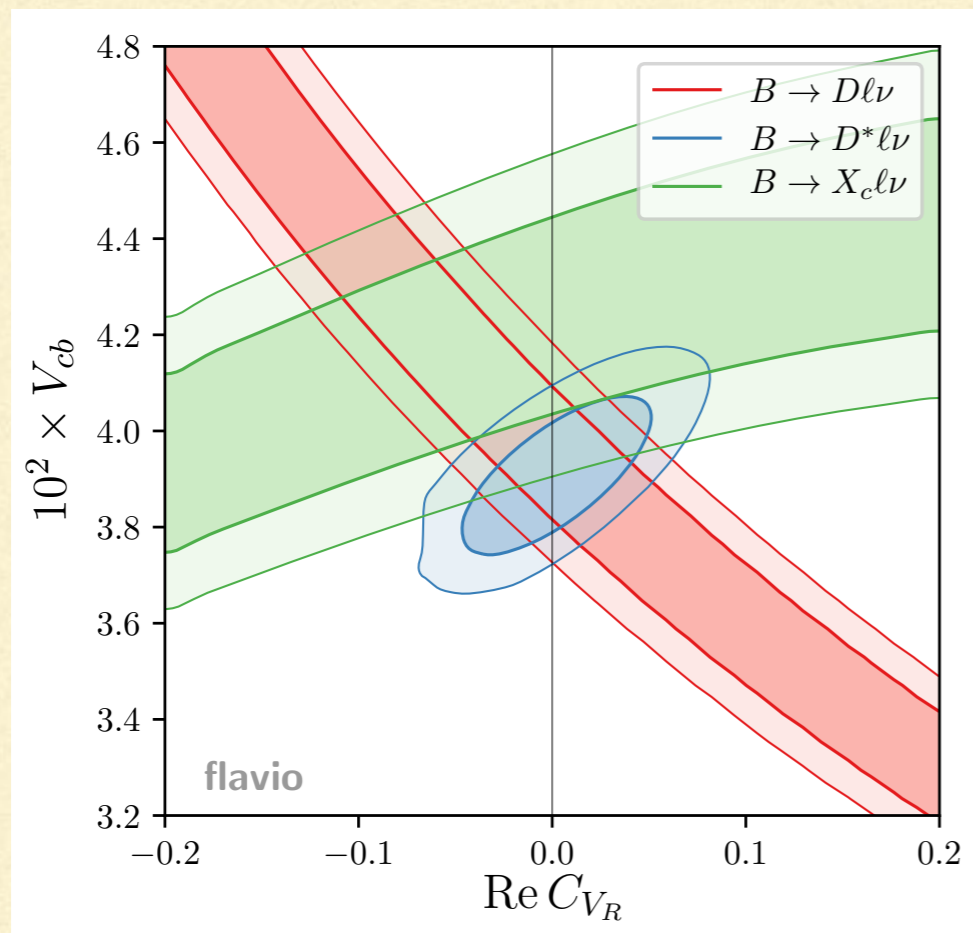
latest exp  
results  
suggest  $V_{ub}$   
discrepancy  
may be  
fading away



**Recently:** new calculations of FFs by several lattice collaborations and with light-cone sum rules, new perturbative calculations, all facing the challenges of precision measurements... and several new measurements as well!

# NEW PHYSICS?

Jung & Straub, 1801.01112

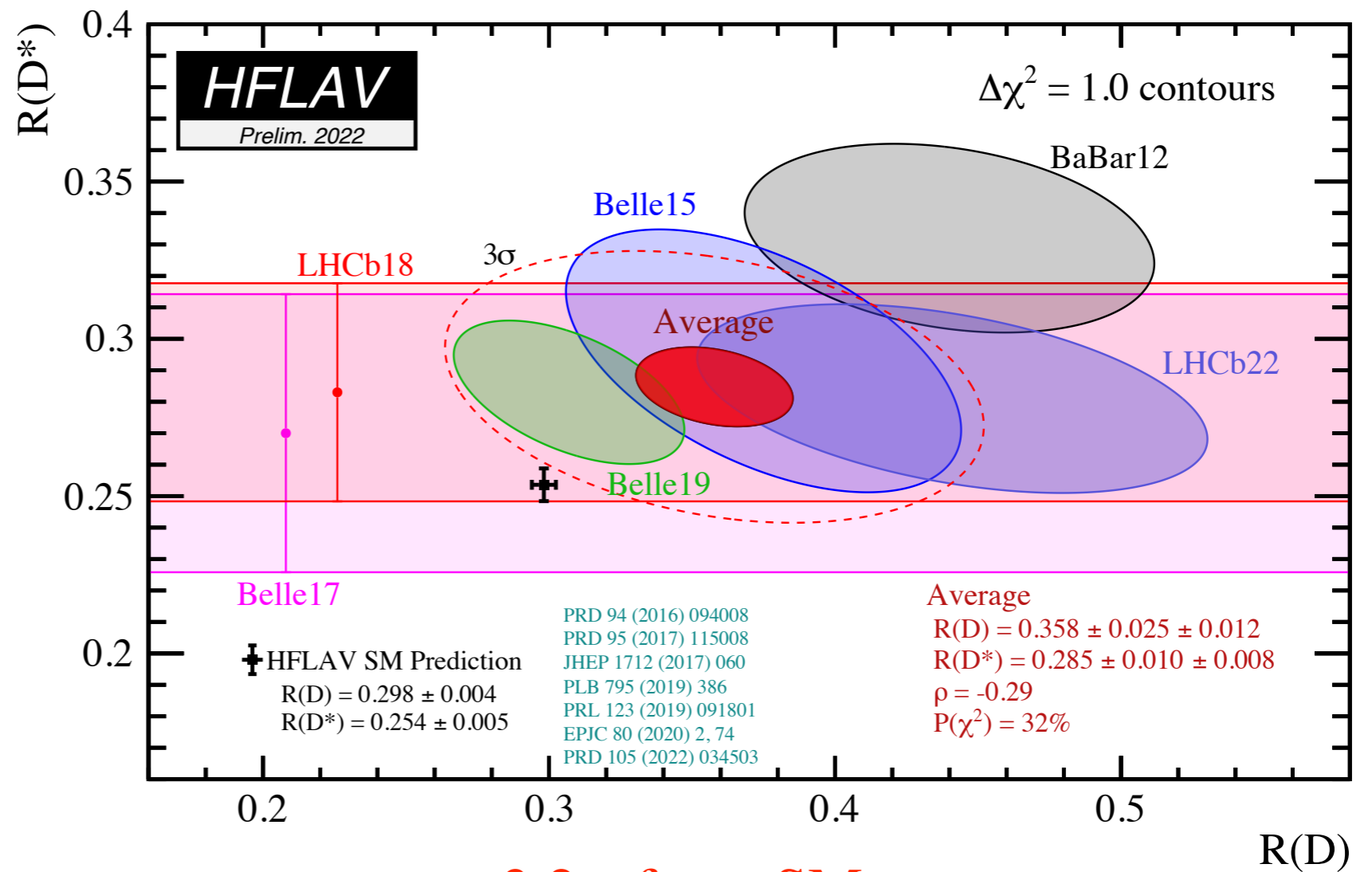


Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub...

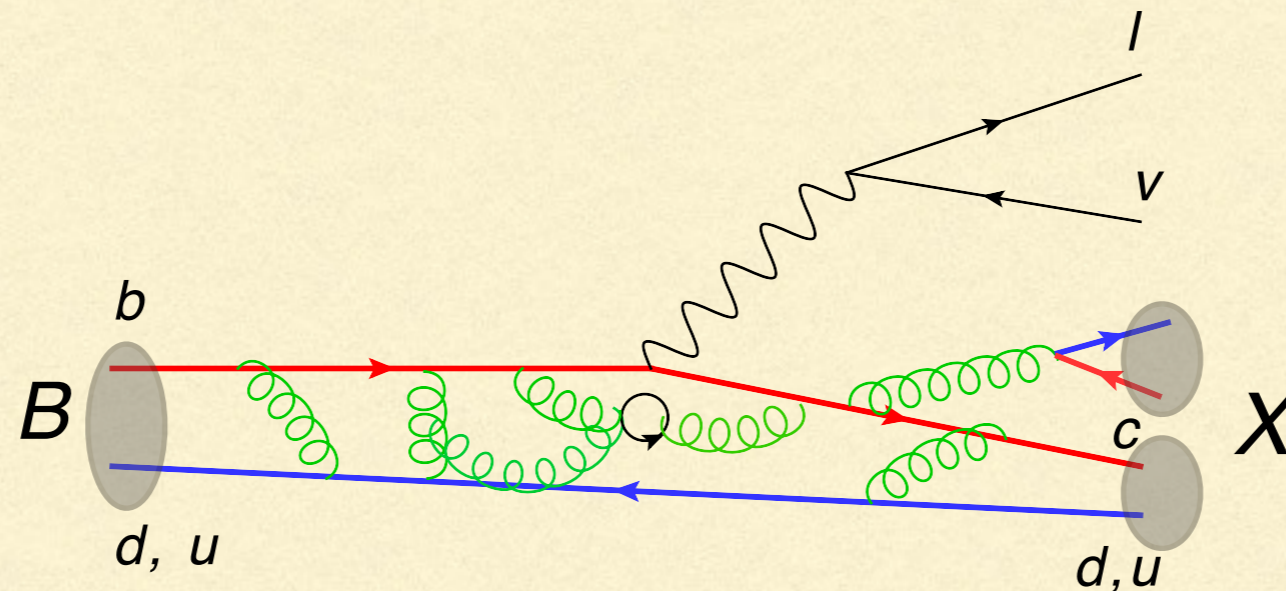
# VIOLATION of LFU with TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \rightarrow D^{(*)} \tau \nu_{\tau}\right)}{\mathcal{B}\left(B \rightarrow D^{(*)} \ell \nu_{\ell}\right)}$$

SM predictions based on same theory as  $V_{cb}$  extraction



# INCLUSIVE DECAYS: BASICS



- **Simple idea:** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators.
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: **double series in  $\alpha_s, \Lambda/m_b$**
- Lowest order: decay of a free  $b$ , linear  $\Lambda/m_b$  absent. Depends on  $m_{b,c}$ , two parameters at  $O(\Lambda^2/m_b^2)$ , 2 more at  $O(\Lambda^3/m_b^3)$  ... Many higher order effects have been computed.



# INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in  $\Lambda_{QCD}/m_b$  and  $\alpha_s$

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2} \\ + \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Global **shape** parameters (first moments of the distributions, with various lower cuts on  $E_l$ ) tell us about  $m_b, m_c$  and the B structure, total **rate** about  $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays,  $V_{ub}, \dots$ )

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest itself as inconsistency in the fit.

**Kinetic scheme** provides short distance definition of  $m_b$  and OPE parameters with hard cutoff  $\mu_{kin} \sim 1\text{GeV}$ . Fit includes all corrections  $\mathcal{O}(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$ ,  $m_c$  constraint from sum rules/lattice, and recent  $\mathcal{O}(\alpha_s^3)$  contribution to width.

# 3 LOOP CALCULATIONS

Fael, Schoenwald, Steinhauser, 2011.11655, 2011.13654, 2205.03410

3loop and 2loop charm mass effects in relation between kinetic and  $\overline{\text{MS}}$   $b$  mass

$$m_b^{\text{kin}}(1\text{GeV}) = \left[ 4163 + 259\alpha_s + 78\alpha_s^2 + 26\alpha_s^3 \right] \text{MeV} = (4526 \pm 15) \text{MeV}$$

Using FLAG  $\overline{m}_b(\overline{m}_b) = 4.198(12)\text{GeV}$  one gets  $m_b^{\text{kin}}(1\text{GeV}) = 4.565(19) \text{GeV}$

3loop correction to **total semileptonic width and moments without cuts**  
(asymptotic expansion around  $m_c = m_b$ )

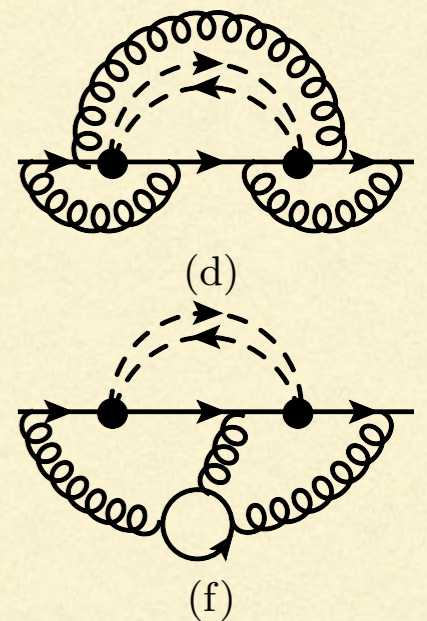
$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[ 0.9255 - 0.1162\alpha_s - 0.0350\alpha_s^2 - 0.0097\alpha_s^3 \right]$$

in the kin scheme with  $\mu = 1\text{GeV}$  and  $\overline{m}_c(3\text{GeV}) = 0.987 \text{GeV}$ ,  $\mu_{\alpha_s} = m_b^{\text{kin}}$

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[ 0.9255 - 0.1140\alpha_s - 0.0011\alpha_s^2 + 0.0103\alpha_s^3 \right]$$

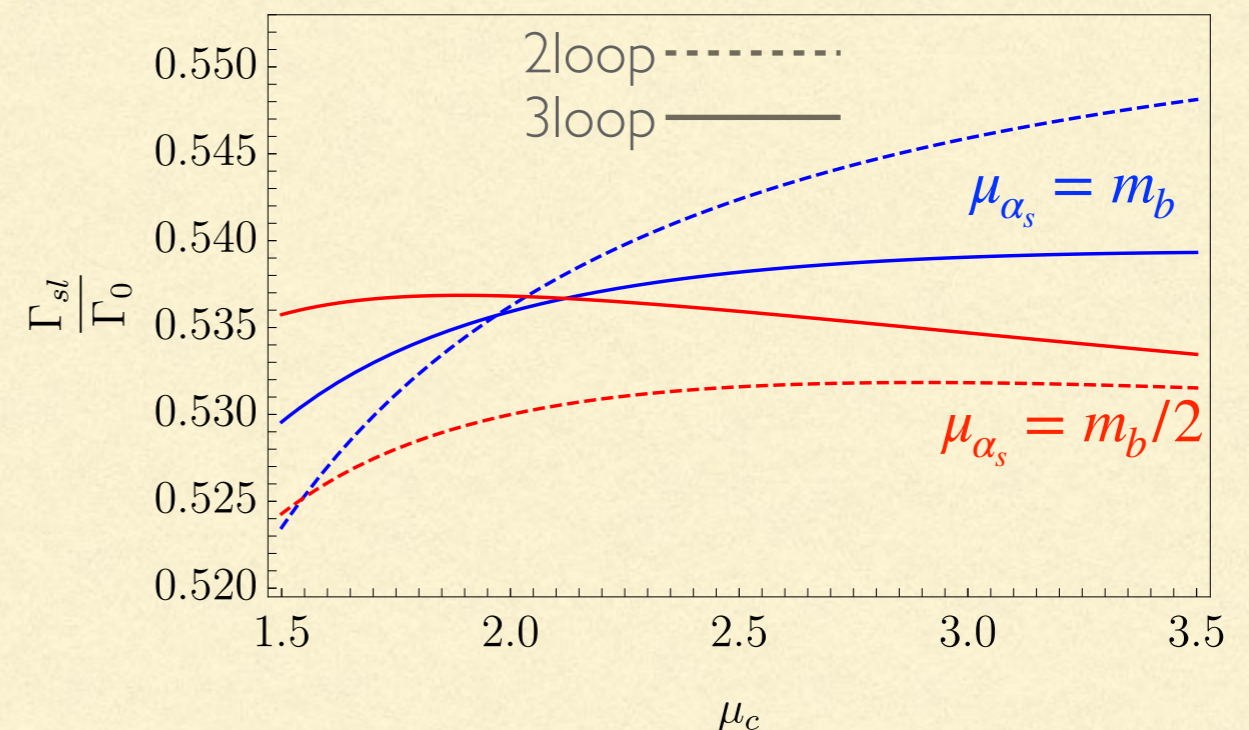
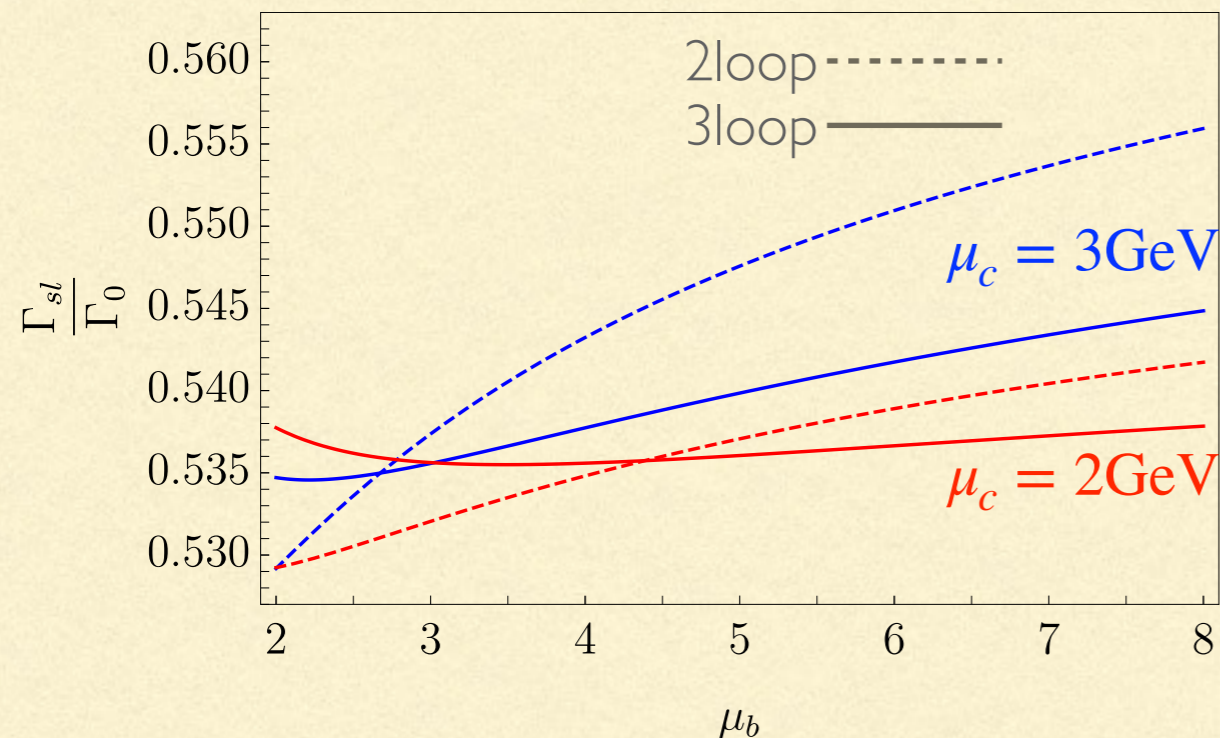
in the kin scheme with  $\mu = 1\text{GeV}$  and  $\overline{m}_c(2\text{GeV}) = 1.091 \text{GeV}$ ,  $\mu_{\alpha_s} = m_b^{\text{kin}}/2$

**3loop correction to  $\Gamma_{sl}$  around 1%, pushes  $|V_{cb}|$  slightly up or down ( $\sim 0.5\%$ )**



# RESIDUAL UNCERTAINTY on $\Gamma_{sl}$

Bordone, Capdevila, PG, 2107.00604



Similar reduction in  $\mu_{kin}$  dependence. Purely perturbative uncertainty  $\pm 0.7\%$  (max spread), central values at  $\mu_c = 2\text{ GeV}, \mu_{\alpha_s} = m_b/2$ .

$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$  effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of  $O(\alpha_s/m_b^3 m_c)$ , duality violation.

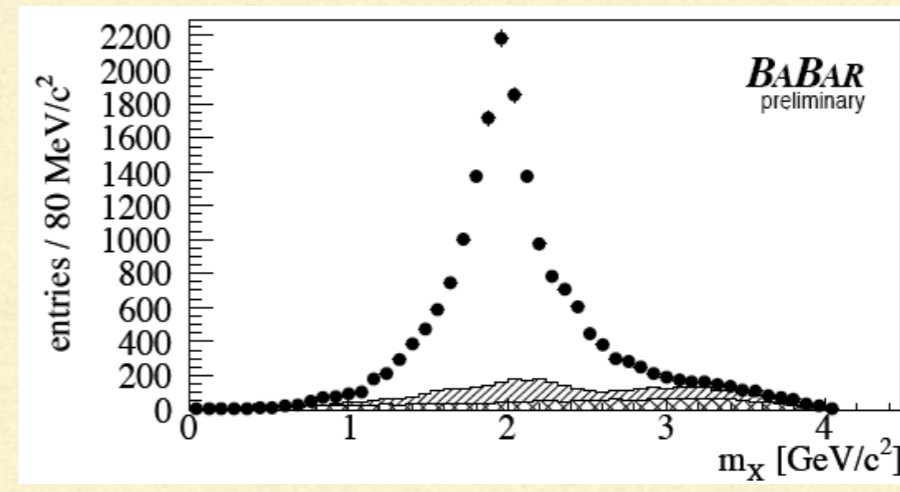
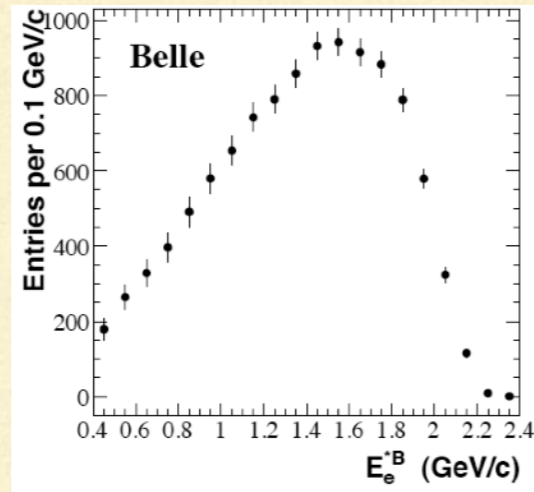
**Conservatively: 1.2% overall theory uncertainty in  $\Gamma_{sl}$  (a ~50% reduction)**

Interplay with fit to semileptonic moments, known only to  $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$

# INCLUSIVE SEMILEPTONIC FITS

Bordone, Capdevila, PG, 2107.00604

Electron energy  
and invariant  
hadronic mass  
spectra



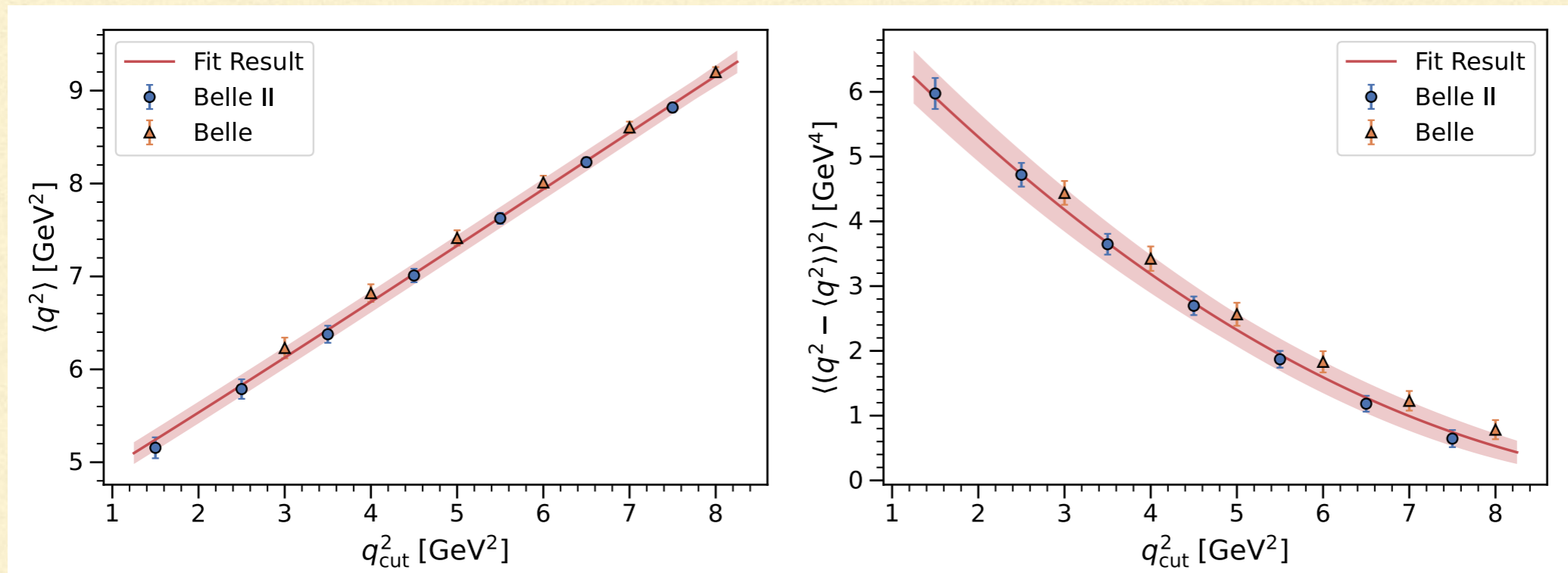
$m_b^{kin}$	$\bar{m}_c(2\text{GeV})$	$\mu_\pi^2$	$\rho_D^3$	$\mu_G^2(m_b)$	$\rho_{LS}^3$	$\text{BR}_{cl\nu}$	$10^3  V_{cb} $
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

**Higher power corrections** see a proliferation of parameters but Wilson coefficients are known at LO. We use the Lowest Lying State Saturation Approximation (Mannel, Turczyk, Uraltsev 1009.4622) as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

Update of 1606.06174  
similar results in IS scheme Bauer et al.

# $q^2$ MOMENTS



2205.10274

New measurements of the  $q^2$  moments by Belle (2109.01685) and Belle II (2205.06372) not yet included in our fit (work in progress).

**Reparametrisation invariance** implies that  $q^2$  moments and total width depend on a smaller set of HQE parameters (Fael, Mannel, Vos), 8 at  $O(1/m_b^4)$ , but *using only the  $q^2$  moments*:  $|V_{cb}| = 41.99(65) \cdot 10^{-3}$  using the same BR input we employ (Bernlochner et al. 2205.10274)

It would be useful to measure also the FB asymmetry as proposed by Turzcyk

# QED CORRECTIONS

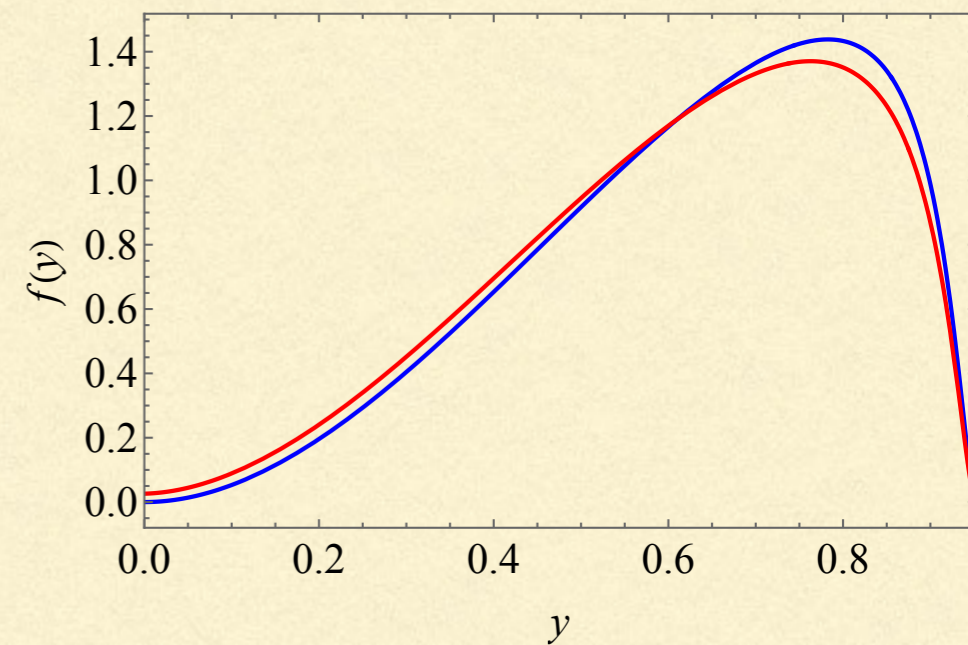
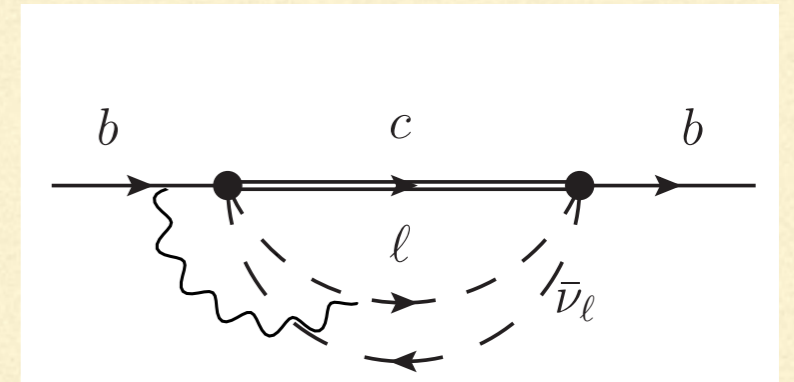
D. Bigi, Bordone, Haisch, PG

In the presence of photons, **OPE valid only for total width** and moments that do not resolve lepton properties ( $E_\ell, q^2$ ). Expect mass singularities and  $O(\alpha\Lambda/m_b)$  corrections.

**Leading logs**  $\alpha \ln m_e/m_b$  can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)}\left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$

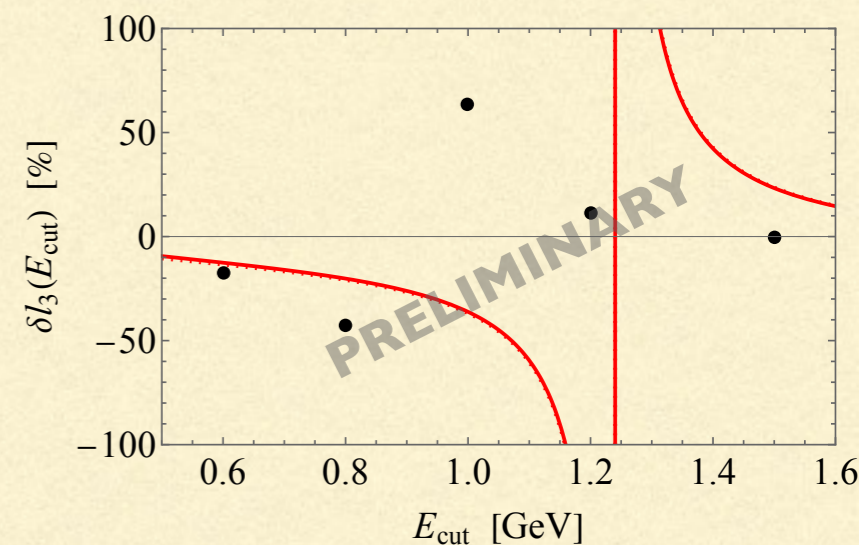
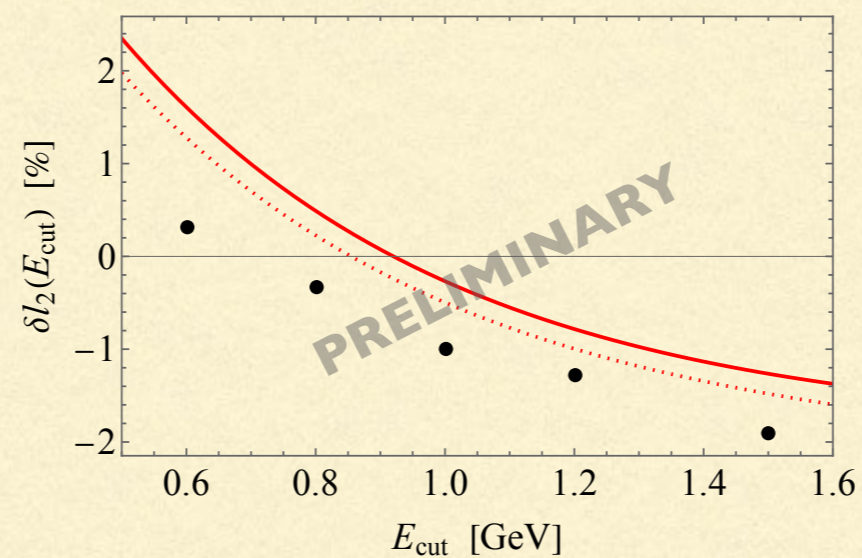
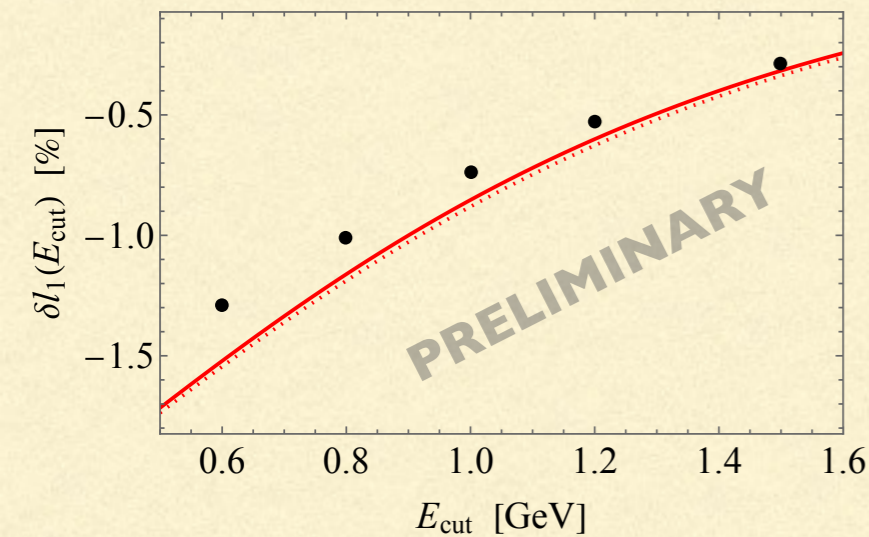
$$P_{\ell\ell}^{(0)}(z) = \left[ \frac{1+z^2}{1-z} \right]_+$$



Electron energy spectrum

# LEPTONIC MOMENTS

D. Bigi, Bordone, Haisch, PG



Typically exp measurements are completely inclusive,  $B \rightarrow X_c \ell \nu(\gamma)$ , but QED radiation is **subtracted** by experiments using **Photos** (soft-collinear photon radiation to MC final states).

BaBar hep-ex/0403030 provides both uncorrected and corrected lepton moments, allowing for comparison with our inclusive LL calculation. Shifts are  $0.2-0.7\sigma_{\text{exp}}$  but NLO logs and effects on power corrections can be included.

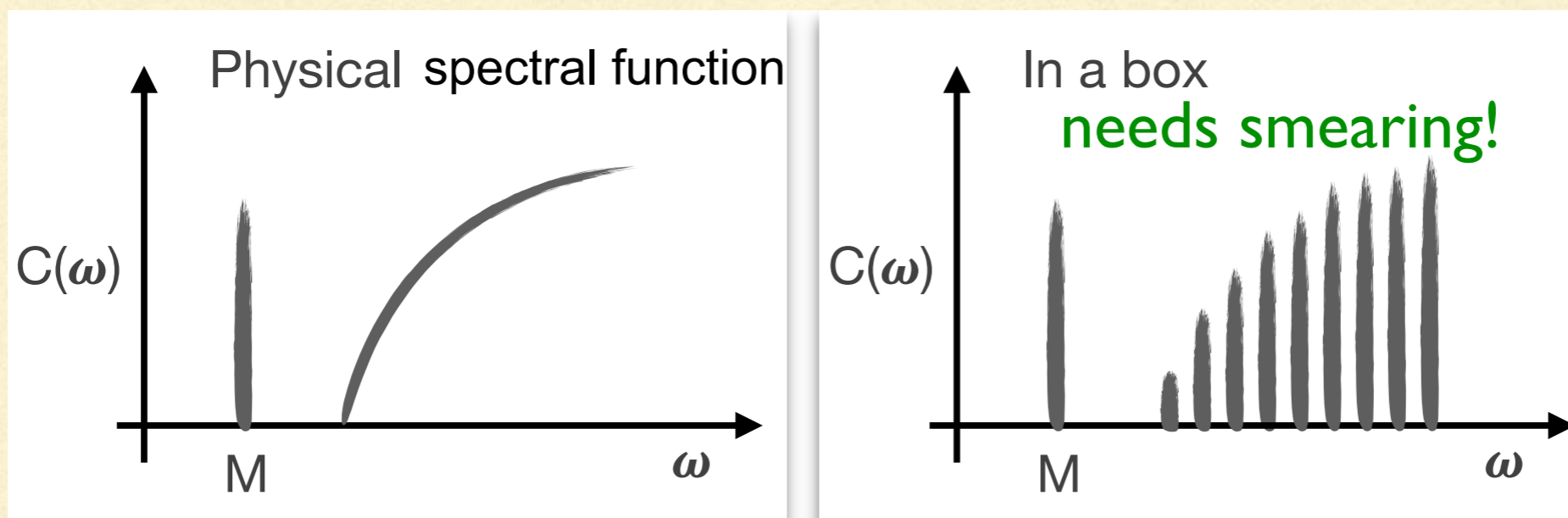
Complete  $\mathcal{O}(\alpha)$  calculation checks subleading terms and other moments.

# INCLUSIVE DECAYS ON THE LATTICE

Inclusive processes impractical to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to  $e^+e^- \rightarrow$  hadrons or  $\tau$  decay via analyticity. In our case the correlators have to be computed in the  $B$  meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the cut associated to  $B$  semileptonic decays.

While the lattice calculation of the spectral density of hadronic correlators is an **ill-posed problem**, the spectral density is accessible after smearing

Hansen, Meyer, Robaina, Hansen, Lupo, Tantaló, Bailas, Hashimoto, Ishikawa



W. Jay @Snowmass workshop

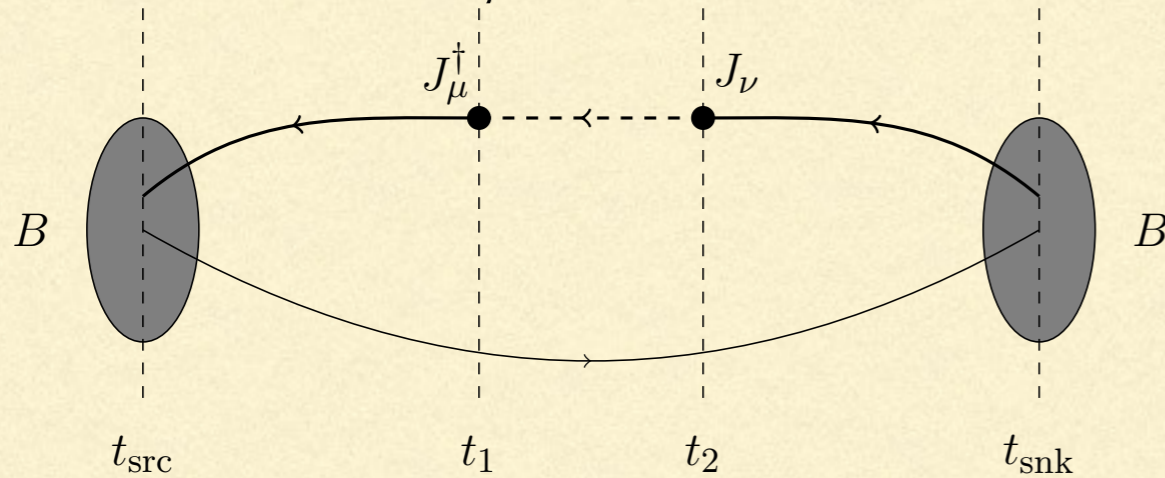


# A NEW APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

$$\sim \langle B | J_\mu^\dagger(\mathbf{x}, t) J_\nu(\mathbf{0}, 0) | B \rangle$$

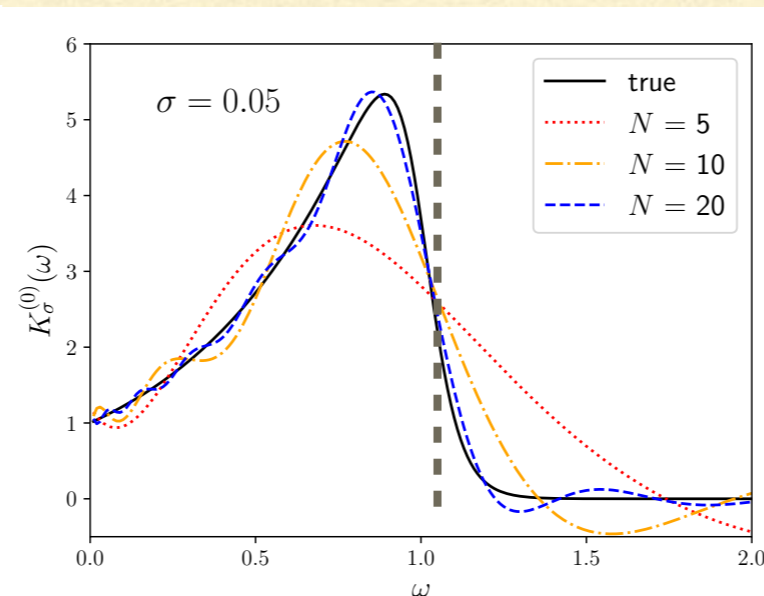
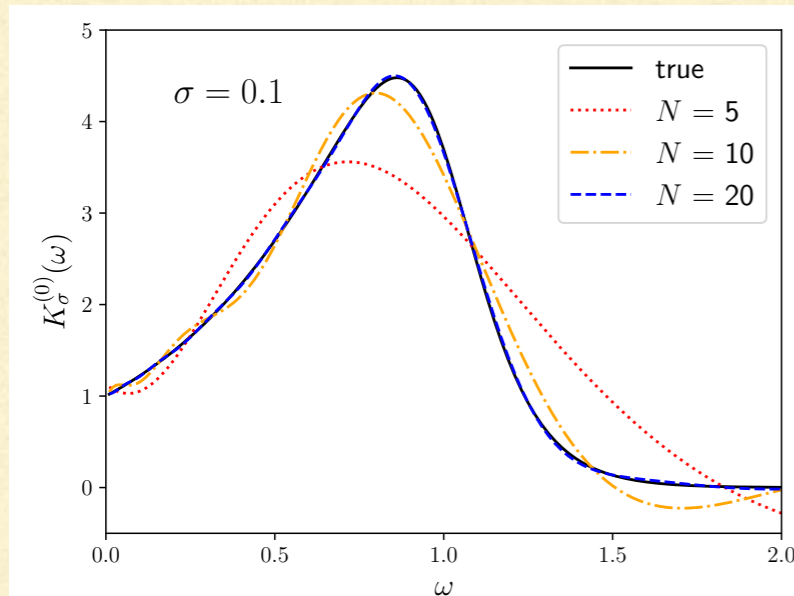


$$d\Gamma \sim L^{\mu\nu} W_{\mu\nu}$$

$$\int d^3x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_B} \langle B | J_\mu^\dagger(\mathbf{x}, t) J_\nu(\mathbf{0}, 0) | B \rangle \sim \int_0^\infty d\omega W_{\mu\nu} e^{-t\omega}$$

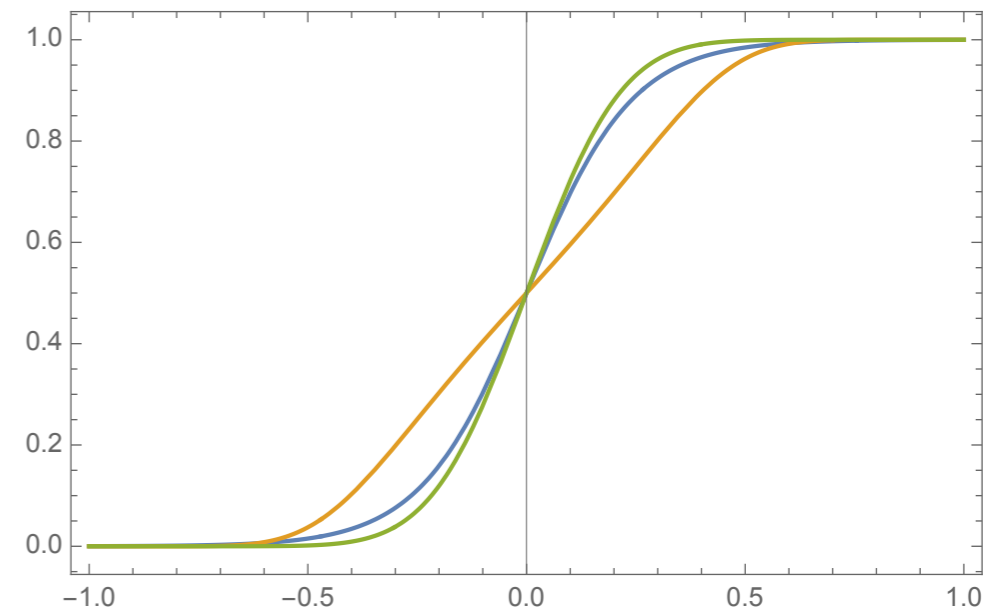
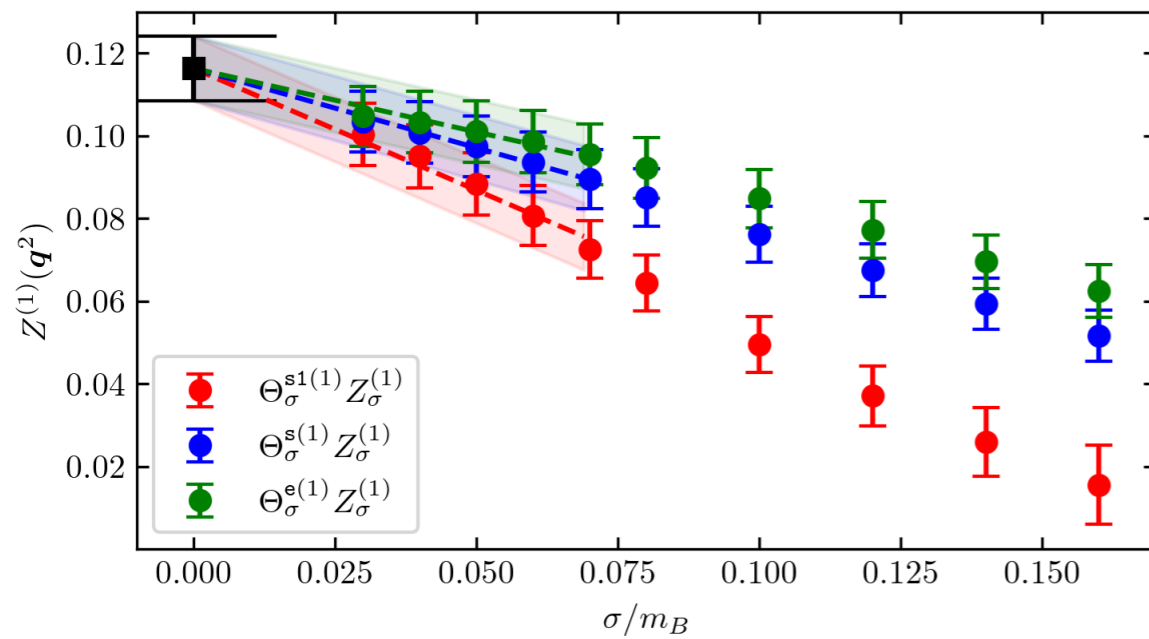
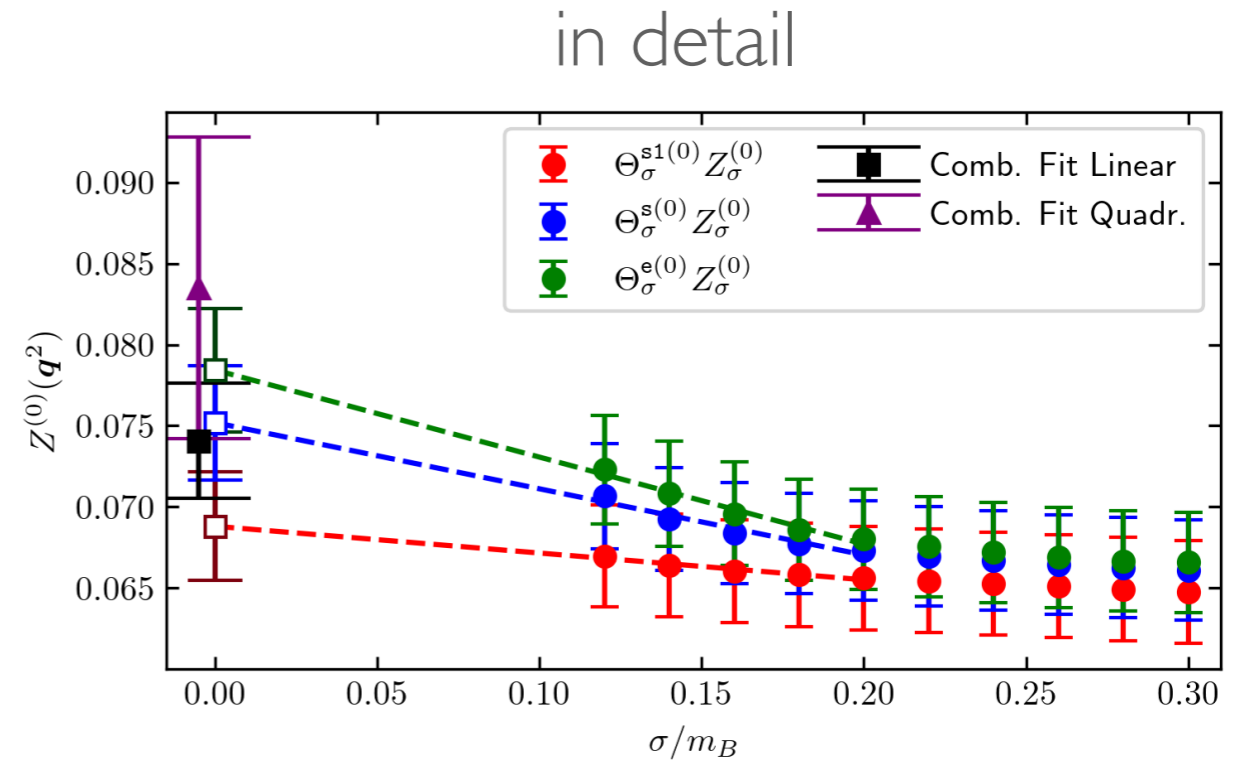
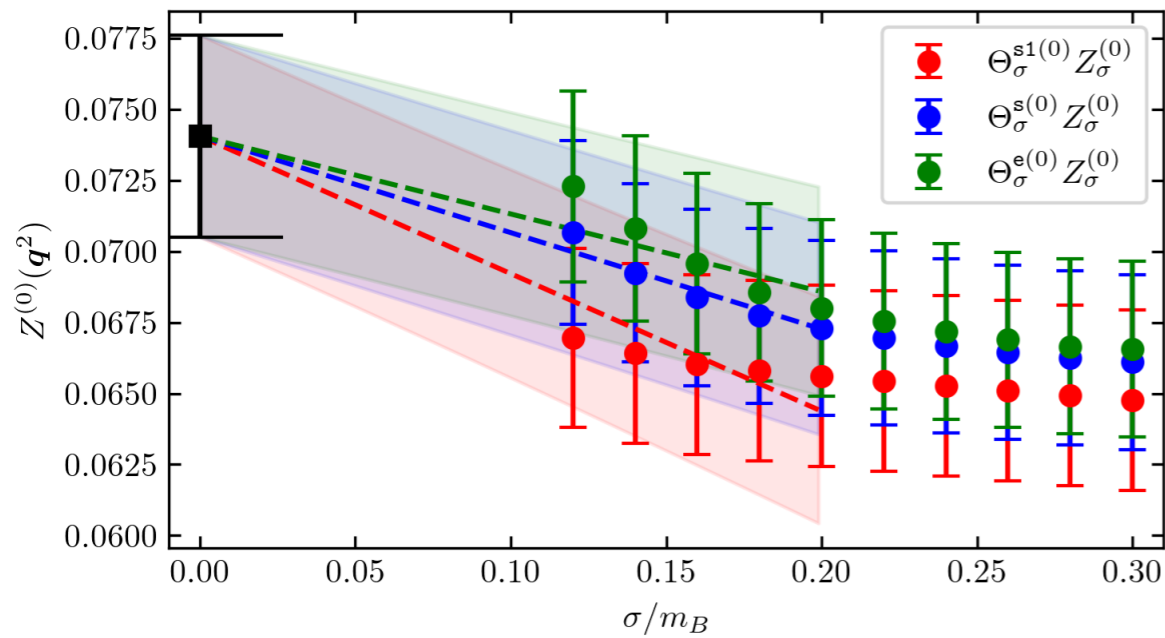
smearing kernel  $f(\omega) = \sum_n a_n e^{-na\omega}$

**The necessary smearing is provided by phase space integration** over the hadronic energy, which is cut by a  $\theta$  with a sharp hedge: sigmoid  $1/(1 + e^{x/\sigma})$  can be used to replace kinematic  $\theta(x)$  for  $\sigma \rightarrow 0$ . Larger number of polynomials needed for small  $\sigma$



Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

$$\lim_{\sigma \rightarrow 0} \lim_{V \rightarrow \infty} \bar{X}_\sigma$$

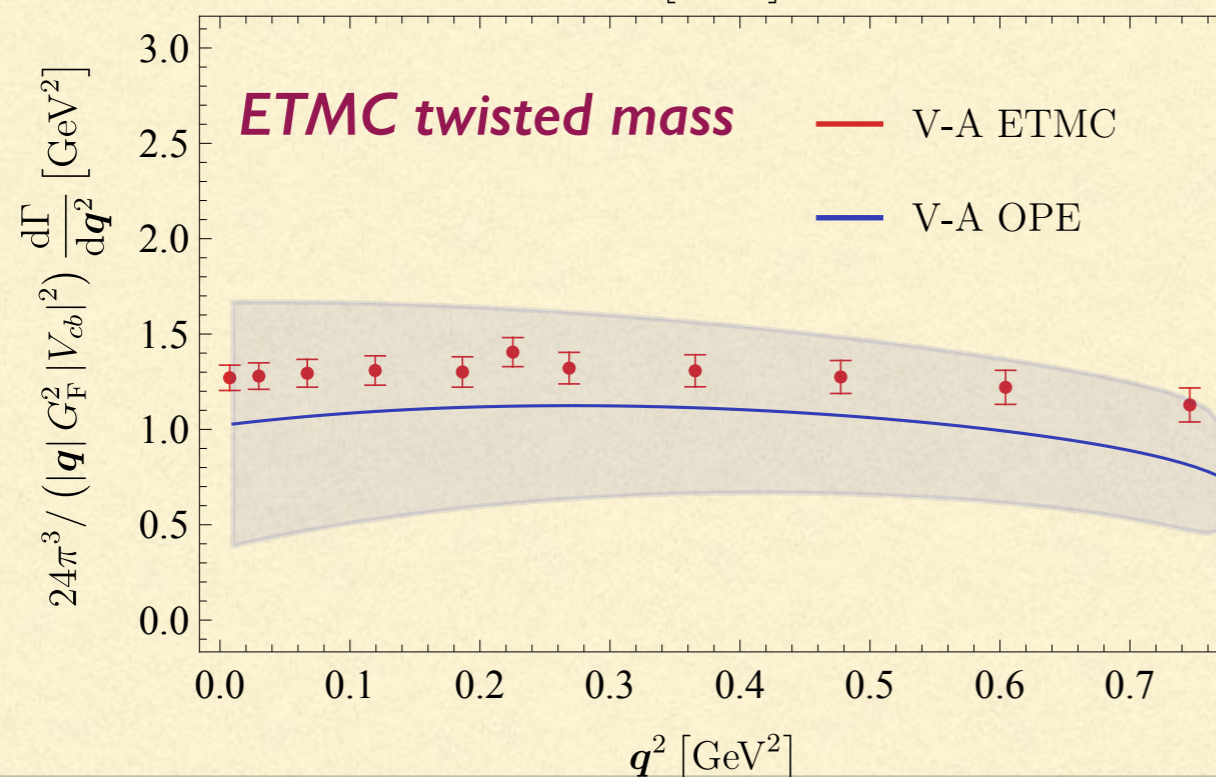
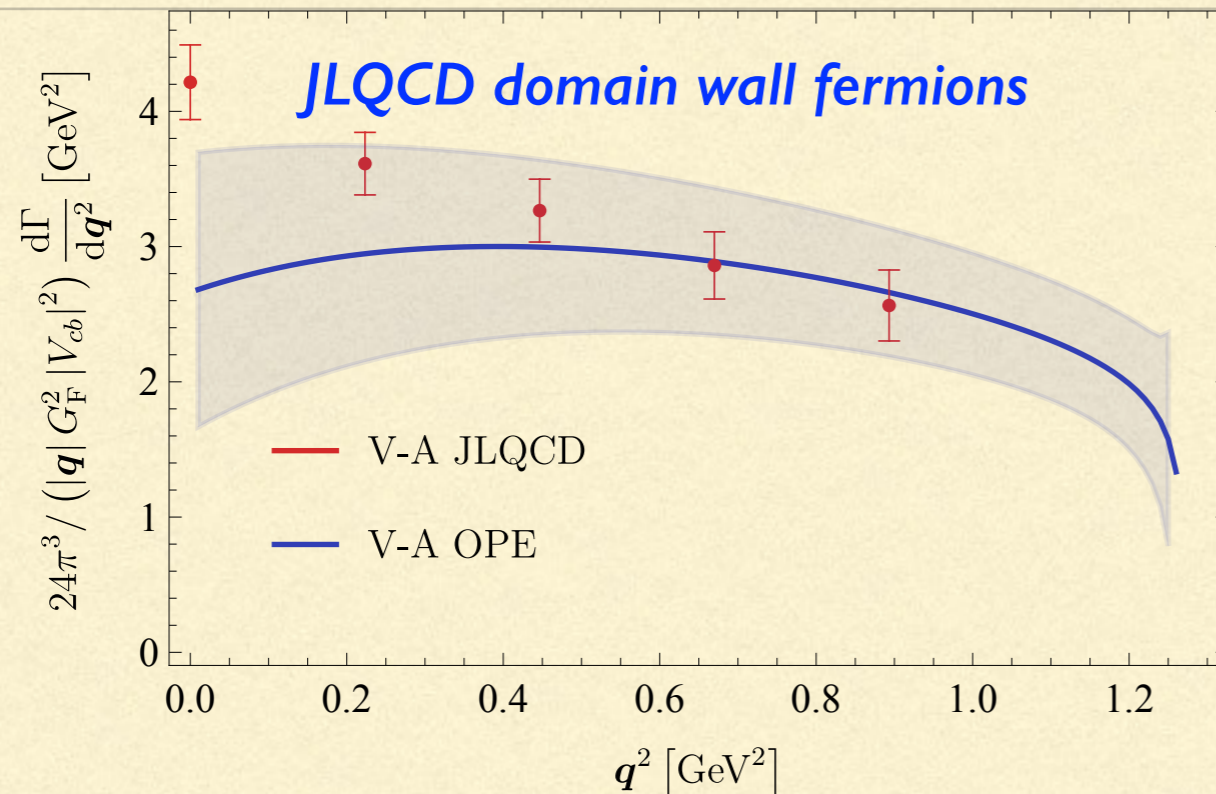


ETMC at  $|\mathbf{q}| = 0.5\text{GeV}$

Using different approx to the kernel  
improves the  $\sigma \rightarrow 0$  extrapolation

Interplay with continuum and infinite volume limits

# LATTICE VS OPE



$m_b^{kin}$ (JLQCD)	$2.70 \pm 0.04$
$\bar{m}_c(2 \text{ GeV})$ (JLQCD)	$1.10 \pm 0.02$
$m_b^{kin}$ (ETMC)	$2.39 \pm 0.08$
$\bar{m}_c(2 \text{ GeV})$ (ETMC)	$1.19 \pm 0.04$
$\mu_\pi^2$	$0.57 \pm 0.15$
$\rho_D^3$	$0.22 \pm 0.06$
$\mu_G^2(m_b)$	$0.37 \pm 0.10$
$\rho_{LS}^3$	$-0.13 \pm 0.10$
$\alpha_s^{(4)}(2 \text{ GeV})$	$0.301 \pm 0.006$

OPE inputs from fits to exp data (physical  $m_b$ ), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include  $O(1/m_b^3)$  and  $O(\alpha_s)$  terms

Hard scale  $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1 - 1.5 \text{ GeV}$

We do not expect OPE to work at high  $|\mathbf{q}|$

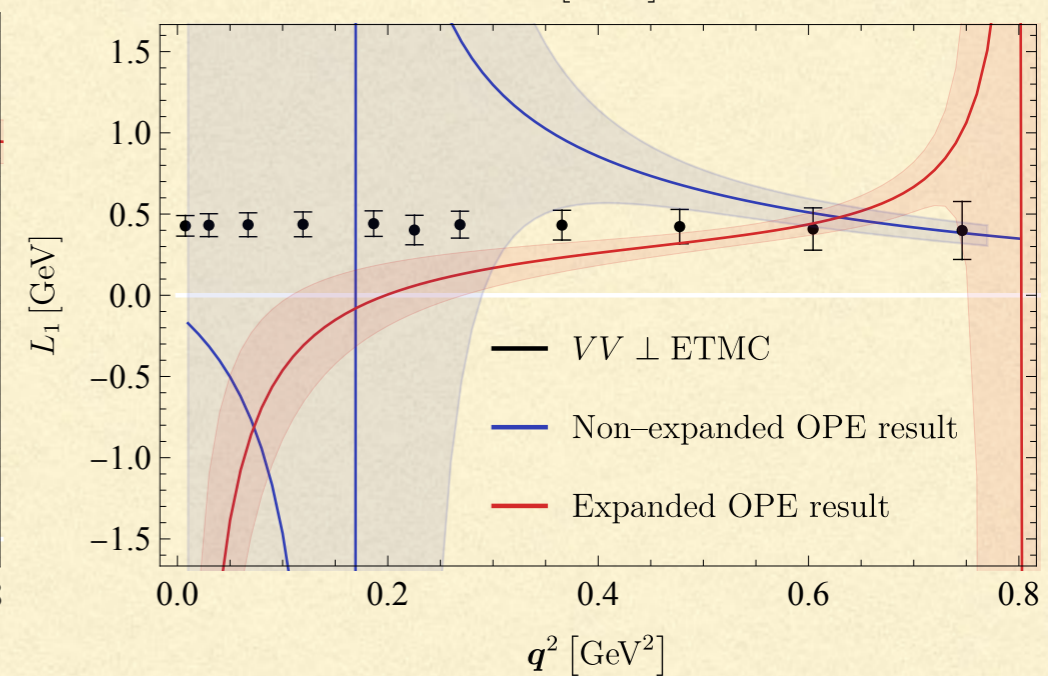
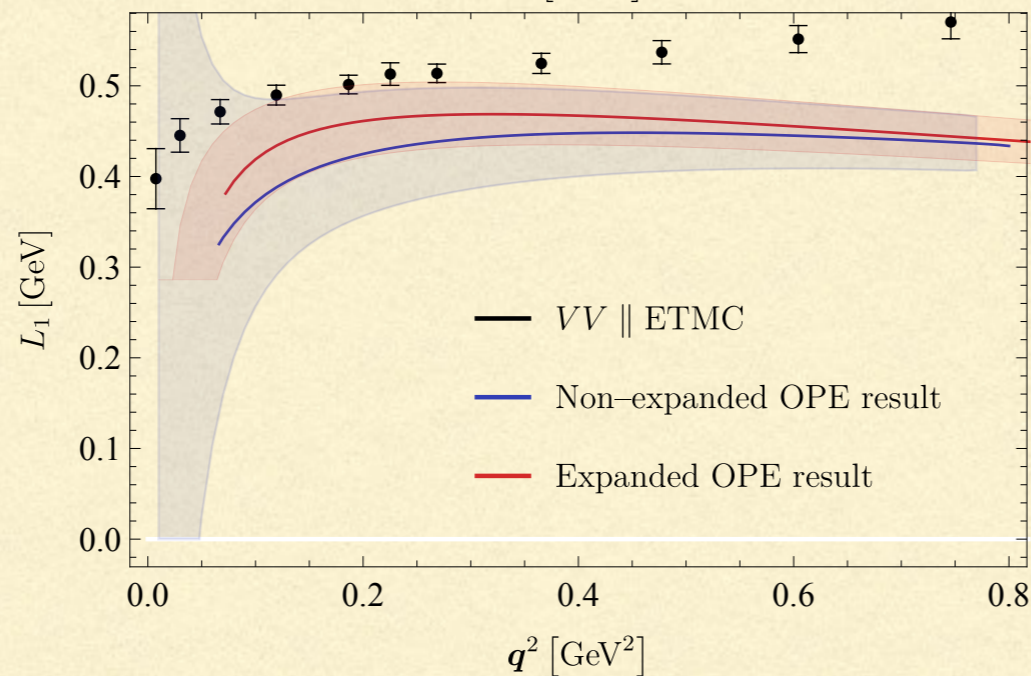
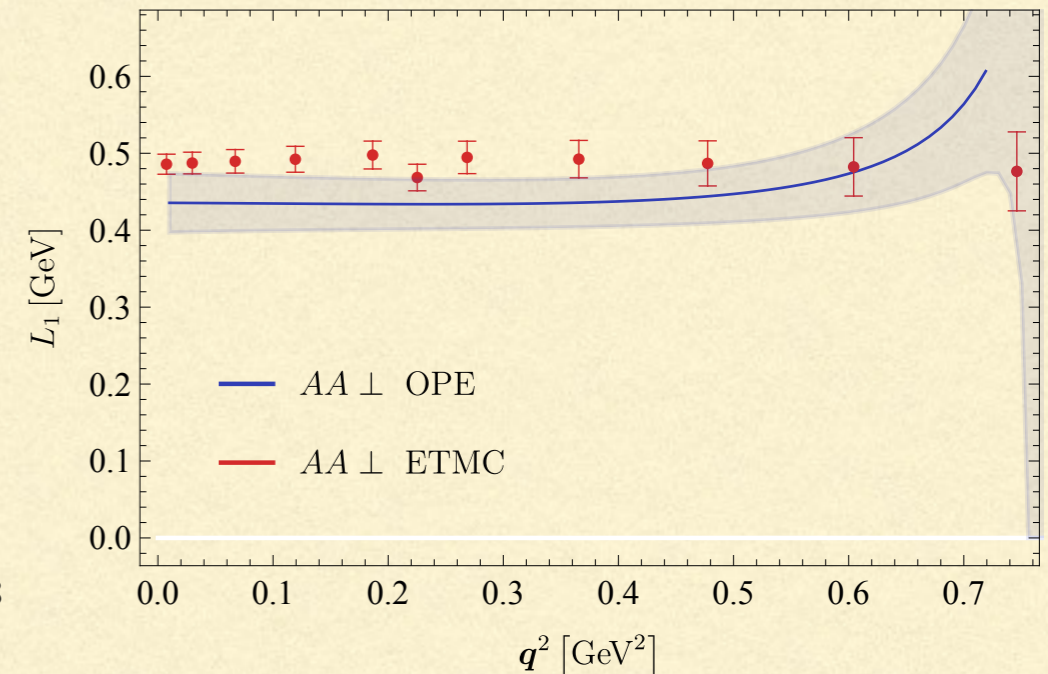
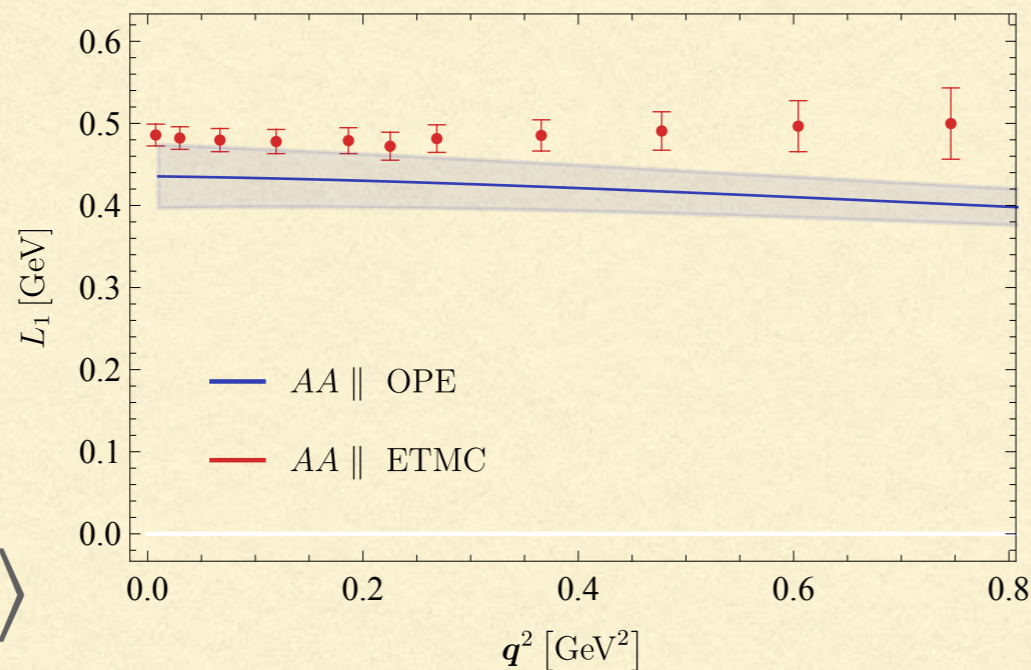
Twisted boundary conditions allow for any value of  $\vec{q}^2$

Smaller statistical uncertainties

# MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalò, 2203.11762

$$L_1 = \langle E_\ell(\mathbf{q}^2) \rangle$$



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

# First results at the physical $b$ mass

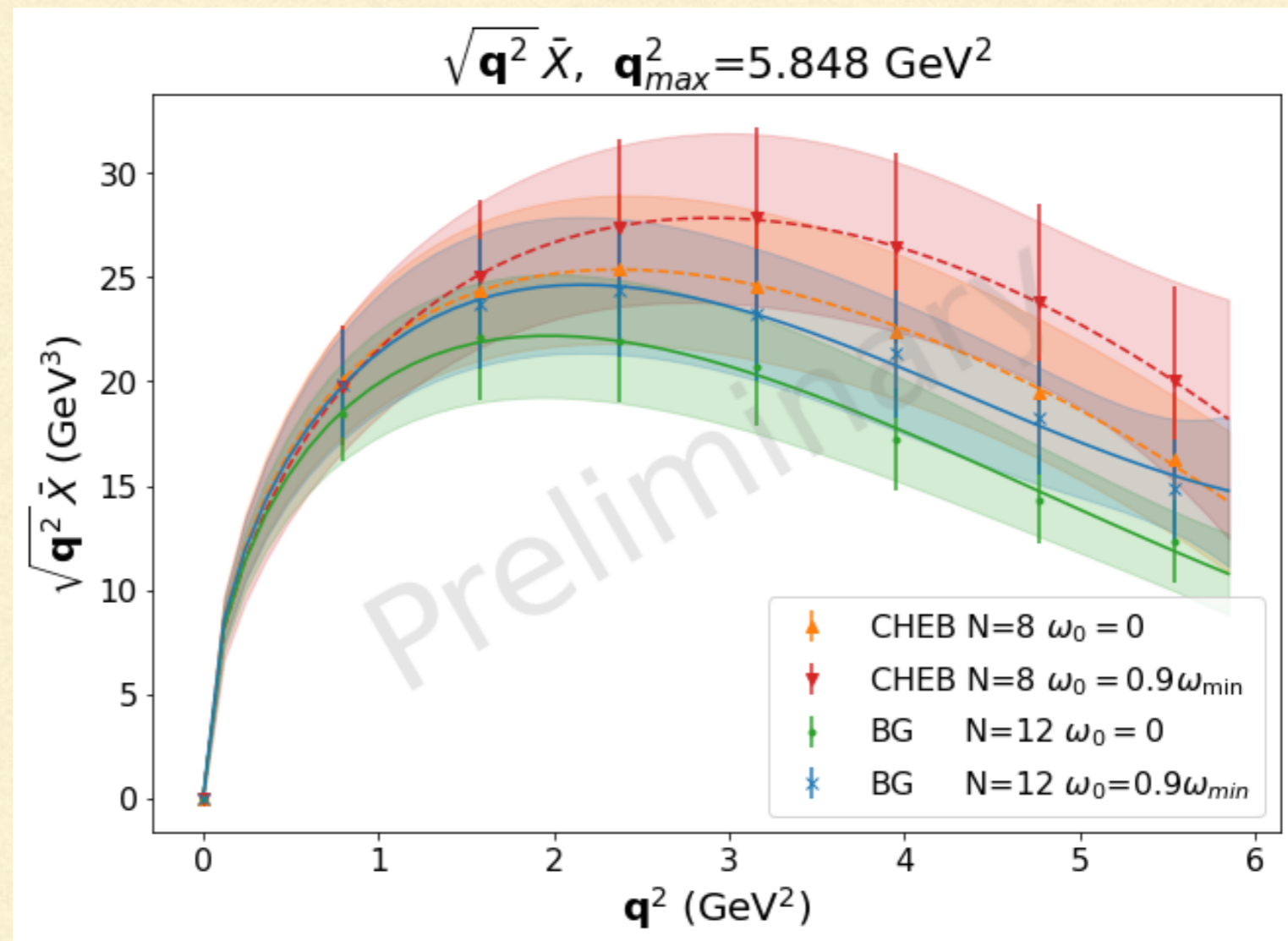
Relativistic heavy quark  
effective action for  $b$   
 $B_s$  decays

domain wall fermions

improved Backus-Gilbert

$\sim 10\%$  determination of  
total width

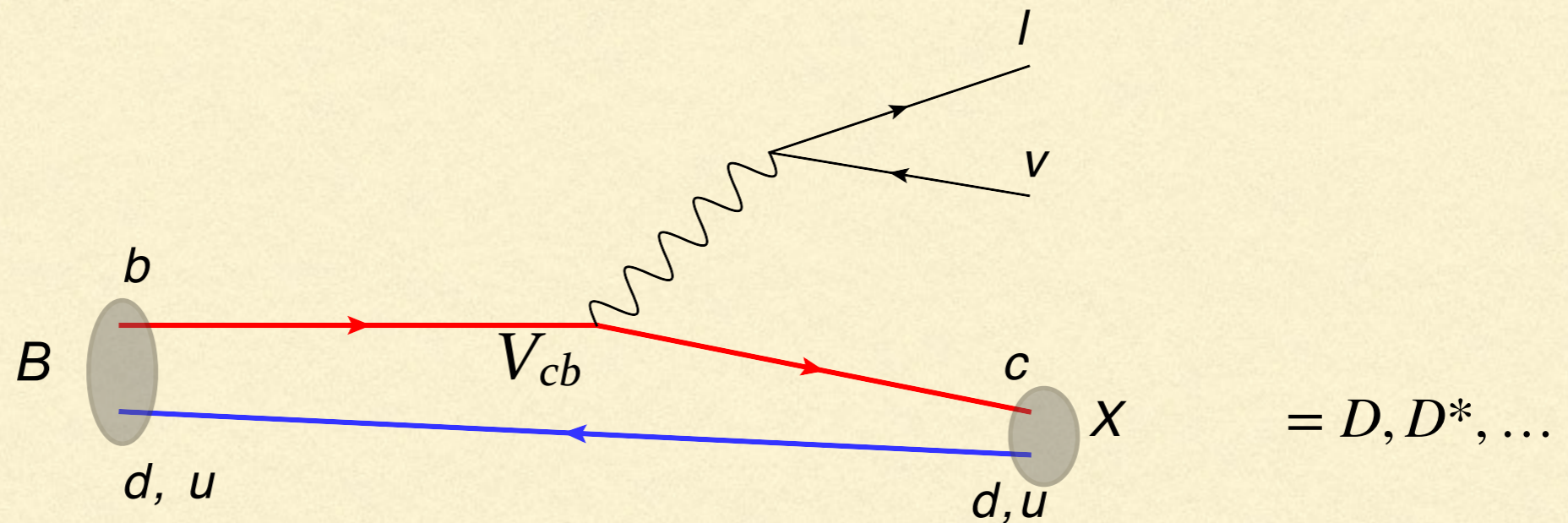
possibly compare with  
partial width at low  $q^2$



Barone, Hashimoto, Juttner, Kaneko, Kellermann, 2211.15623

Ongoing work on **semileptonic  $D_s$  decays** by two collaborations

# EXCLUSIVE DECAYS



There are 1(2) and 3(4) FFs for  $D$  and  $D^*$  for light (heavy) leptons, for instance

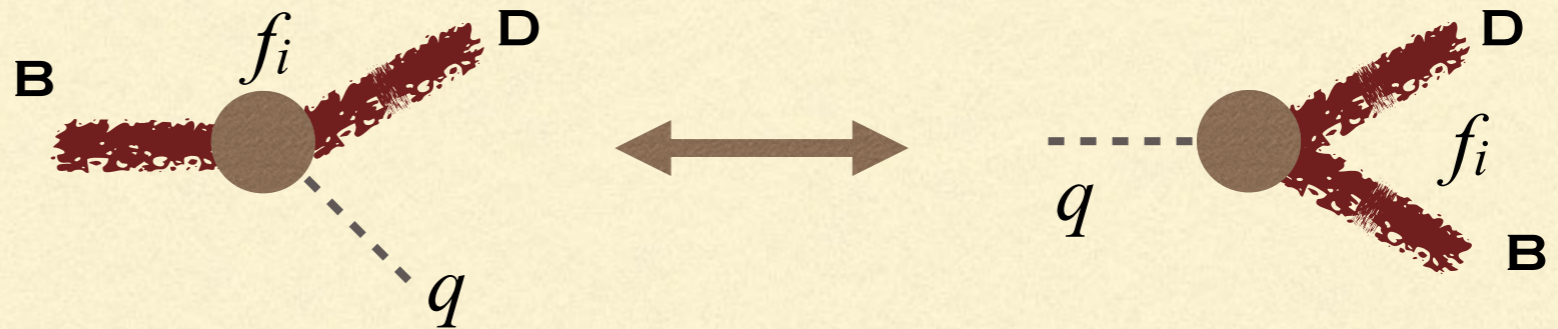
$$\langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = \left[ (p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] f_+^{B \rightarrow D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0^{B \rightarrow D}(q^2)$$

Information on FFs from LQCD (at high  $q^2$ ), LCSR (at low  $q^2$ ), HQE, exp, extrapolation, unitarity constraints, ...

A **model independent parametrization** is necessary

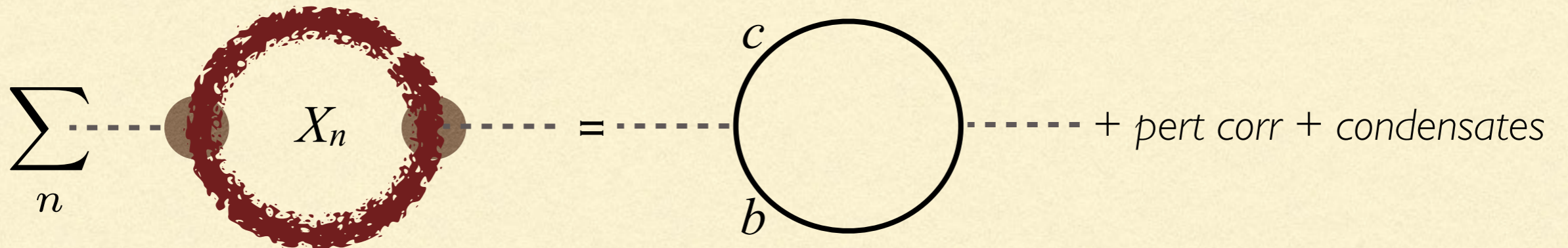
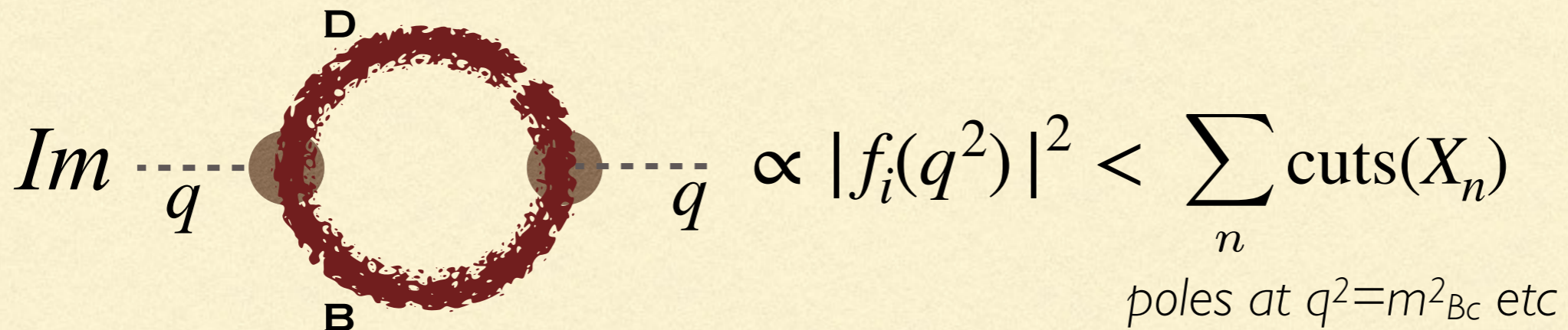
# MODEL INDEPENDENT FF PARAMETRIZATION

crossing +  
analyticity



physical semileptonic region  
 $m_\ell^2 \leq q^2 \leq (m_B - m_D)^2$

2-point correlator cuts  
 $q^2 \geq (m_B + m_D)^2$



using quark-hadron duality (OPE) + dispersion relations

# PARAMETRIZATIONS

- **Boyd-Grinstein-Lebed (BGL)** (1995) based on crossing & analyticity, unitarity constraints

based on OPE

$$F(q^2) = \bar{F}(q^2) \sum_{k=0}^{\infty} a_k z(q^2)^k \quad \text{with} \quad \sum_k a_k^2 \leq 1,$$

$0 < z < \sim 0.06$  in the physical region. Series must be truncated in a controlled way.

- HQET for  $B^{(*)} \rightarrow D^{(*)}$  form factors:

$$F_i(w) = \xi(w) \left[ 1 + c_\alpha^i \frac{\alpha_s}{\pi} + c_b^i \frac{\bar{\Lambda}}{2m_b} + c_c^i \frac{\bar{\Lambda}}{2m_c} + \dots \right]$$

- $c_{b,c}^i$  can be computed using subleading IW functions from QCD sumrules Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

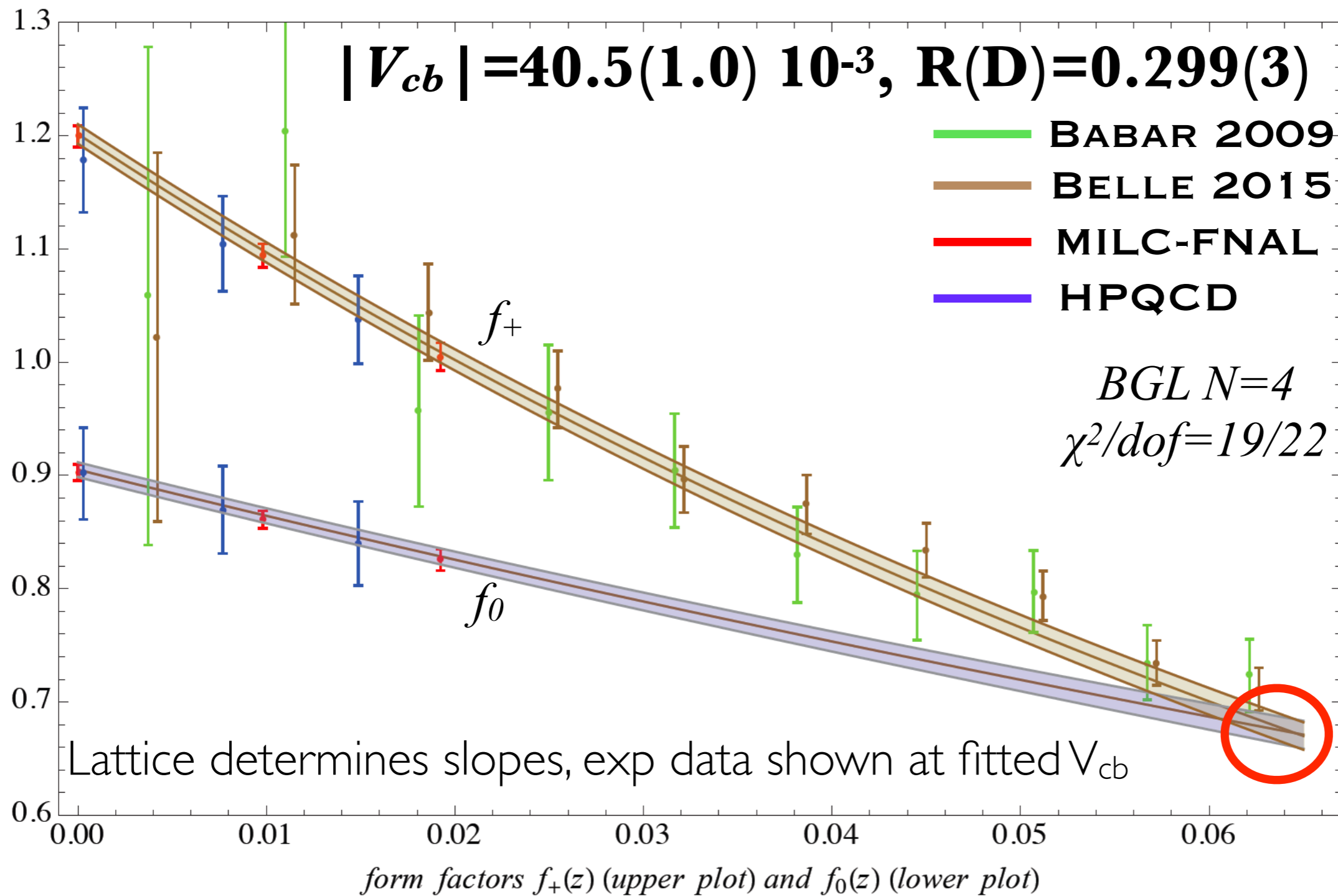
- Ratios free of Isgur-Wise function: can use to get **strong unitarity bounds** but  $1/m_c^2$  corrections can be significant as shown by lattice calculations

- **Caprini-Lellouch-Neubert (CLN)** (1998) parametrization is simpler with fewer parameters, but relies on NLO HQET. All exp analyses before 2017 were based on CLN, did not include uncertainty.



# LATTICE + EXP BGL FIT for $B \rightarrow D/v$

Bigi, PG 1606.08030



$R(D) = 0.299(3)$

$1.9\sigma$  from exp

FLAG has very similar results

CLN cannot fit both ff

kinematic constraint at  $q^2=0$

# D'AGOSTINI BIAS

Standard  $\chi^2$  fits sometimes lead to paradoxical results

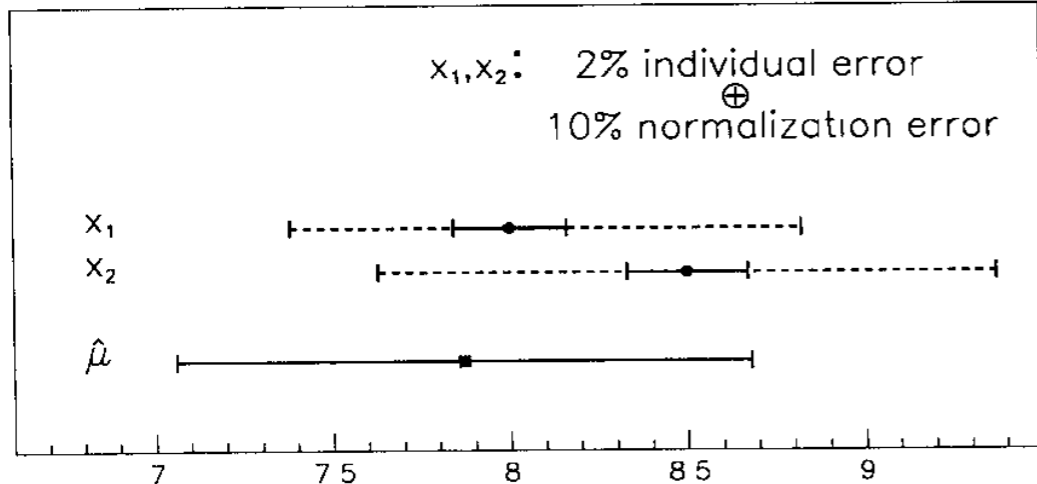


Fig. 1. Best estimate of the true value from two correlated data points, using in the  $\chi^2$  the empirical covariance matrix of the measurements. The error bars show individual and total errors.

$$\hat{k} = \frac{x_1\sigma_2^2 + x_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (x_1 - x_2)^2\sigma_f^2},$$

Many exp systematics are highly correlated. Bias is stronger with more bins

On the use of the covariance matrix to fit correlated data

G. D'Agostini

*Dipartimento di Fisica, Università "La Sapienza" and INFN, Roma, Italy*

(Received 10 December 1993; revised form received 18 February 1994)

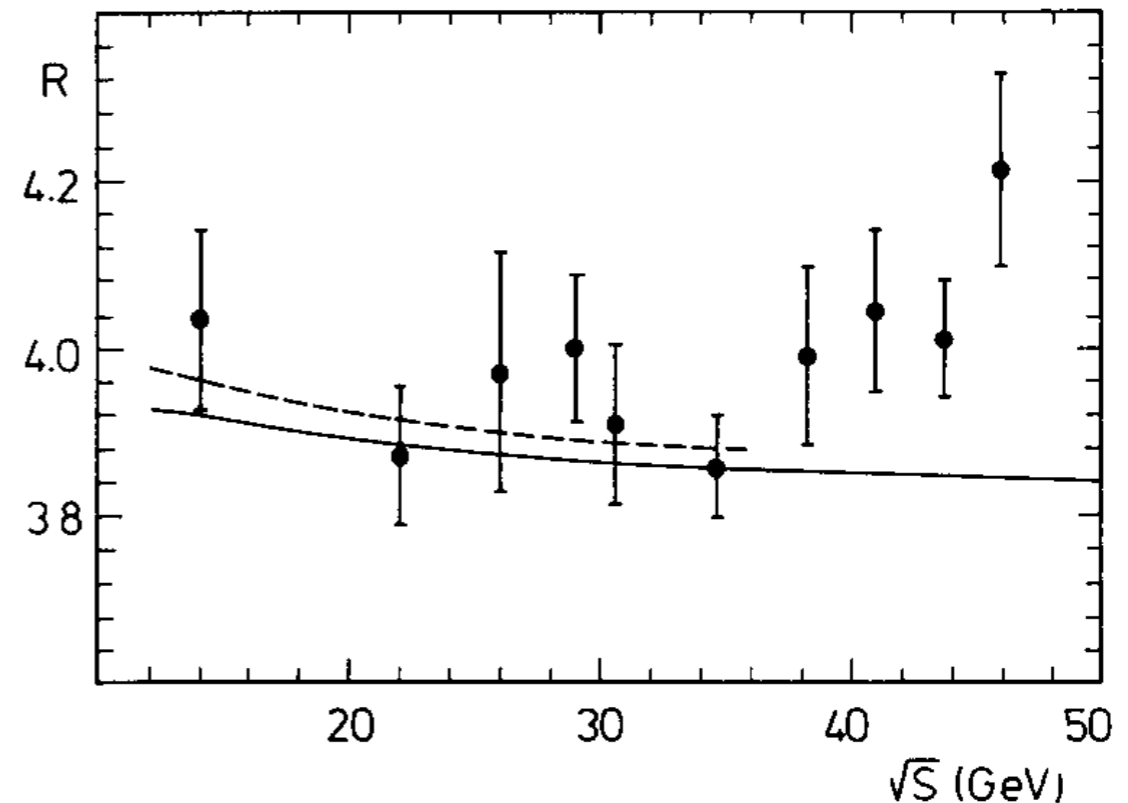
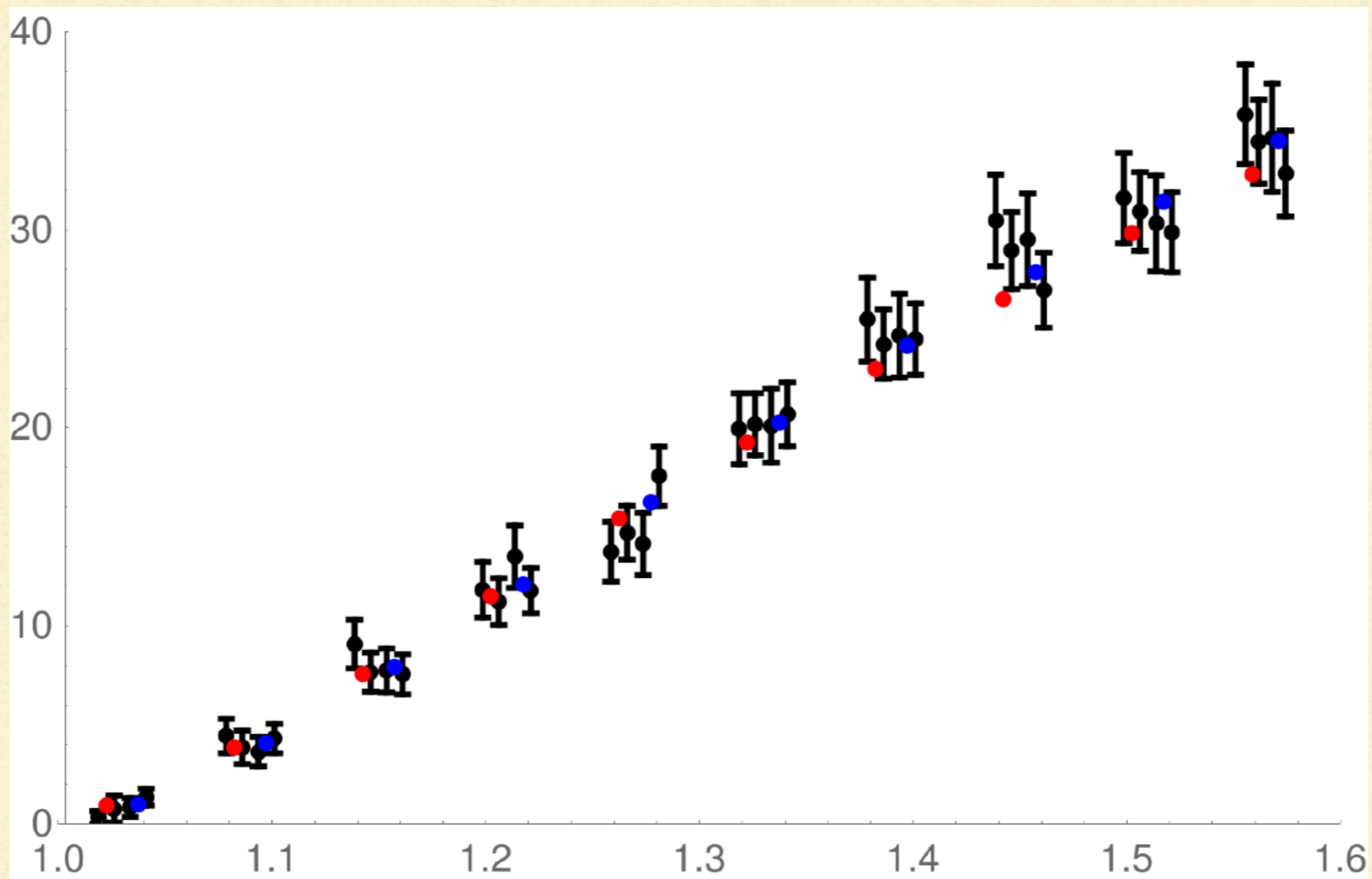


Fig. 2. R measurements from PETRA and PEP experiments with the best fits of QED + QCD to all the data (full line) and only below 36 GeV (dashed line). All data points are correlated (see text).

# $w$ DISTRIBUTION for $B \rightarrow D\ell\nu$



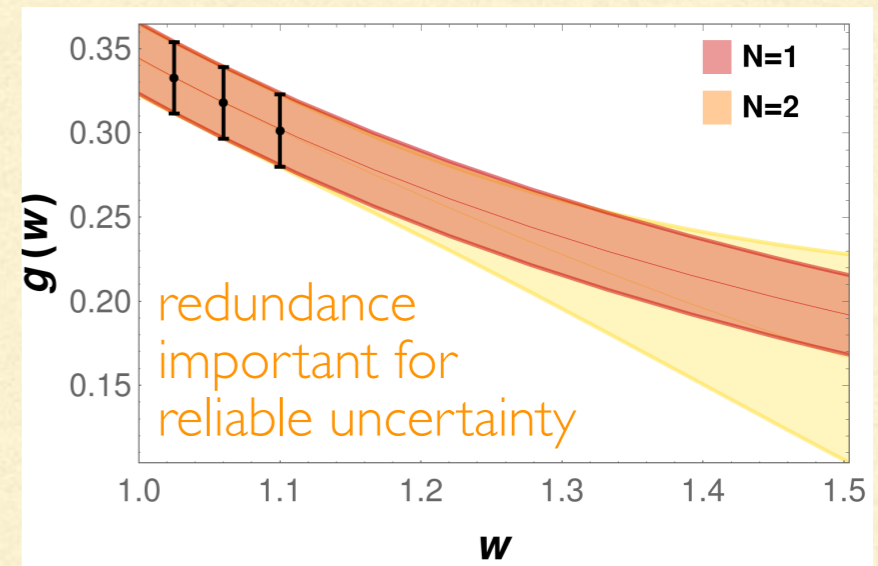
Belle 2015 consider 4 channels ( $B^{0,+}$ ,  $e$ ,  $\mu$ ) for each bin.  
Average (red points) usually lower than all central values. Bias?  
Blue points are average of normalised bins.

**Standard fit** to Belle15+FNAL+HPQCD:  $|V_{cb}| = 40.9(1.2) 10^{-3}$

**Fit to normalised bins** Belle15+FNAL+HPQCD:  $|V_{cb}| = 41.9(1.2) 10^{-3}$  Jung, PG

# TRUNCATION AND UNCERTAINTY

Fits with BGL parametrisation: **model independence vs overfitting**. Where do we truncate the series? How can we include unitarity constraints? These questions are related.



Different options with various pro/cons:

1. Frequentist fits with strong  $\chi^2$  **penalty** outside unitarity; increase BGL order till  $\chi_{min}^2$  is stable. Uncertainties from  $\Delta\chi_{min}^2 = 1$  do not have probabilistic interpretation. Bigi, PG, I 606.08030, Jung, Schacht, PG I 905.08209
2. Frequentist fit with **Nested Hypothesis Test** to determine optimal truncation order: go to order  $N + 1$  if  $\Delta\chi^2 = \chi_{min,N}^2 - \chi_{min,N+1}^2 \geq 1$   
Check unitarity a posteriori Bernlochner et al, I 902.09553
3. **Bayesian inference** using unitarity constraints as prior with BGL Flynn, Jüttner, Tsang 2303.11285 or in the **Dispersive Matrix approach**, Martinelli, Simula, Vittorio et al. 2105.02497

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# $|V_{cb}|$ from $B \rightarrow D^* l \nu$

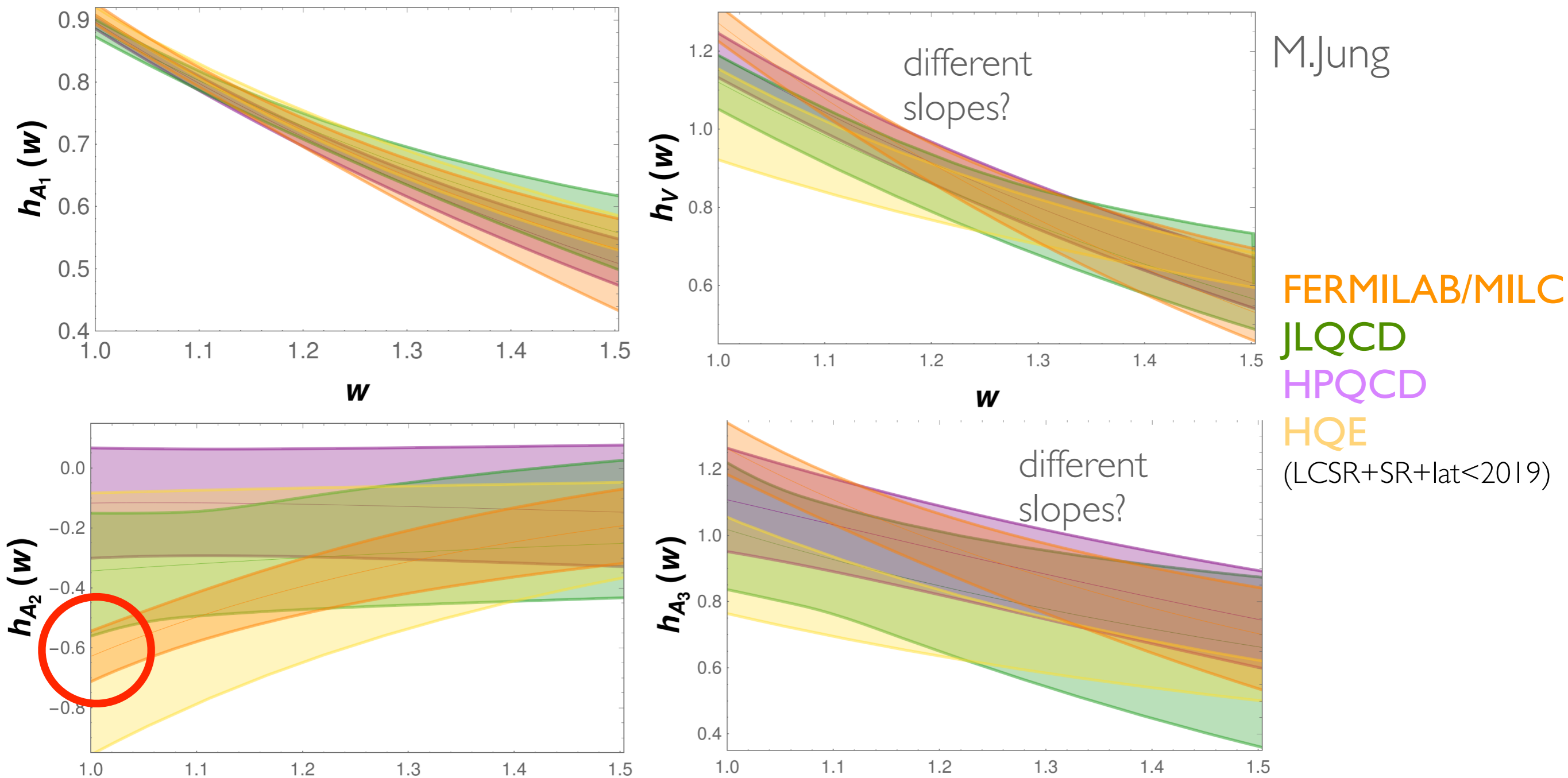
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More complicated: 4 FFs, angular spectra,  $D^*$  unstable. **Present status unclear.**

- 1. Parametrisations matter and the related uncertainties require careful consideration.** Belle 2017 dataset analysed with BGL or CLN leads to 6-8% difference in  $|V_{cb}|$ . Bigi, PG, Schacht, Grinstein, Kobach  
Discard old exp results obtained with CLN and provide data in a parametrisation independent way.
  - 2.** Despite recent progress, **lattice calculations** are indecisive. Tension between **Fermilab/MILC** 2021 and HPQCD 2023 results at non-zero recoil and **Belle** untagged 2018 data, while **JLQCD** preliminary results give a consistent picture.
  - 3. Problems** in Belle 2018 analysis (D'Agostini bias,  $\mu/e$   $4\sigma$  tension in the FB asymmetry) PG, Jung, Schacht & Bobeth, Bordone, van Dyk, Gubernari, Jung  
**other experimental analyses make conflicting claims** but data not yet available for independent fits
-

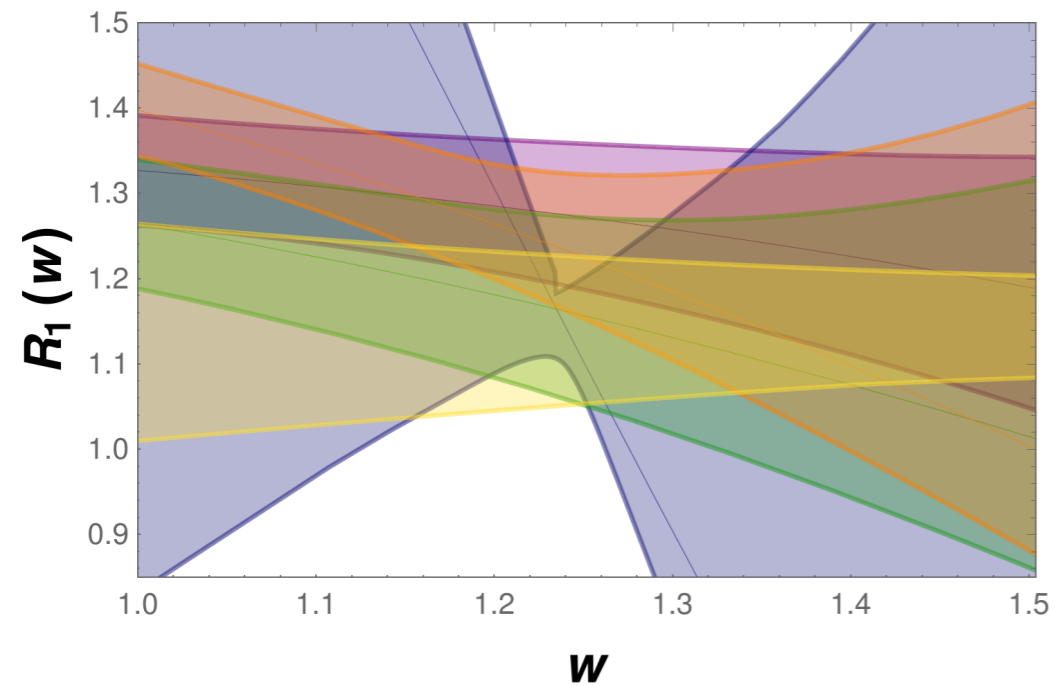
# LATTICE FORM FACTORS AT NONZERO RECOIL

2105.14019, 2112.13775, 2304.03137



BGL fits with weak unitarity. General good agreement, but a few exceptions

# RATIOS OF FORM FACTORS



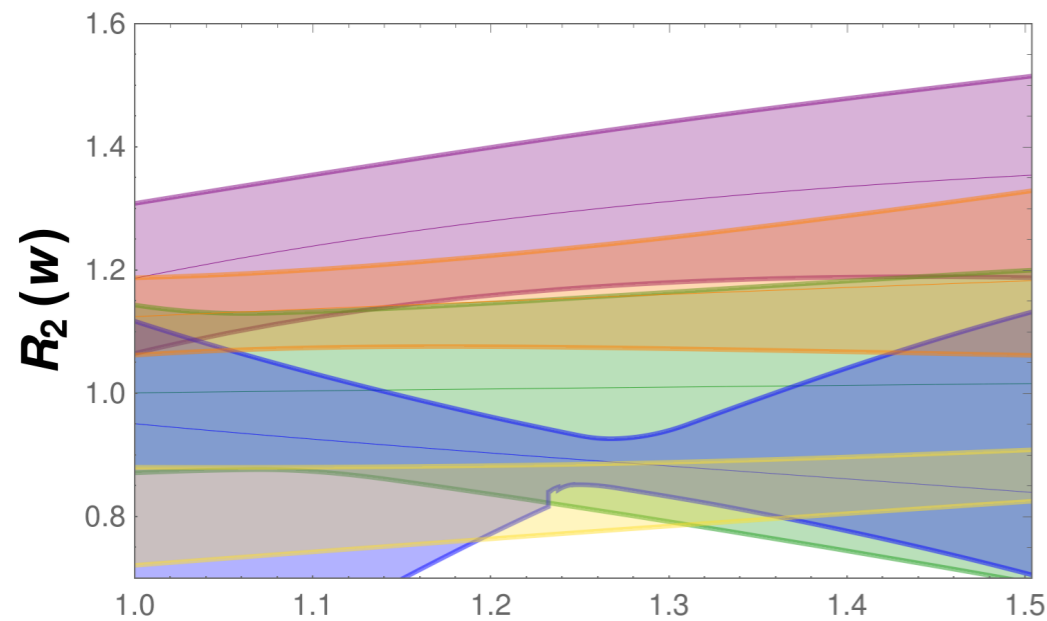
FERMILAB/MILC

JLQCD

HPQCD

HQE (LCSR+SR+lat<2019)

EXP (Belle 2018)

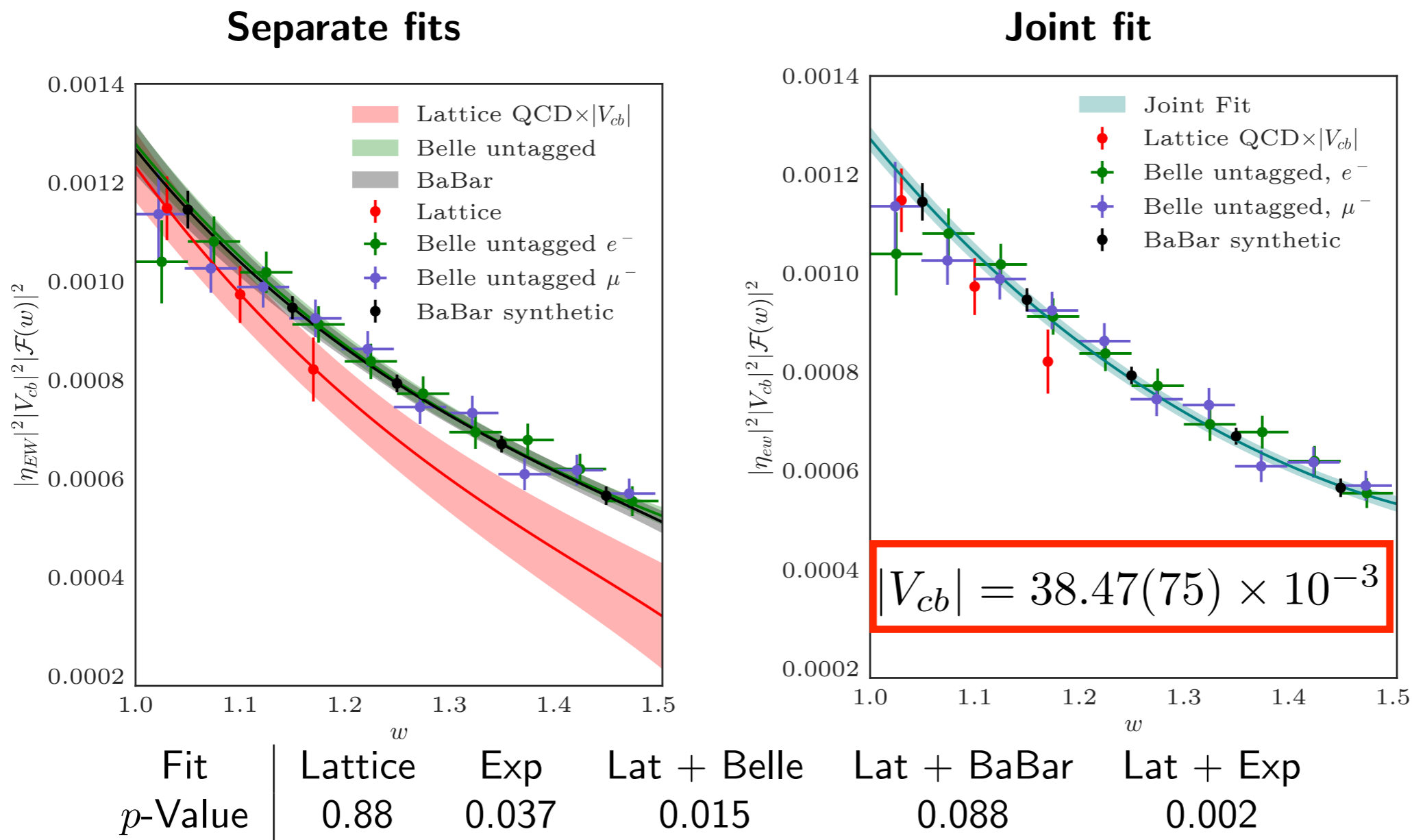


Form factor ratios more sensitive to differences. Stark disagreement between FERMILAB & HPQCD and HQE & EXP in  $R_2$

M.Jung

# FERMILAB/MILC CALCULATION

2105.14019



*First lattice calculation beyond zero recoil for this mode*

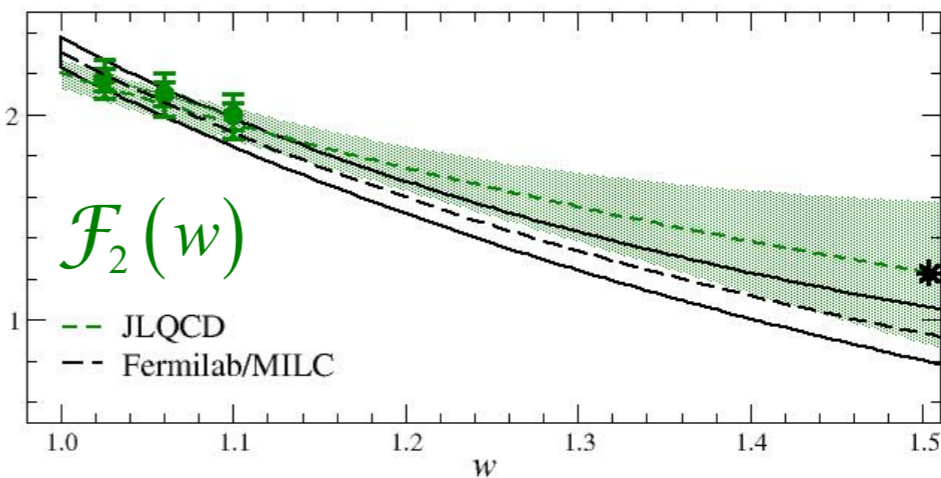
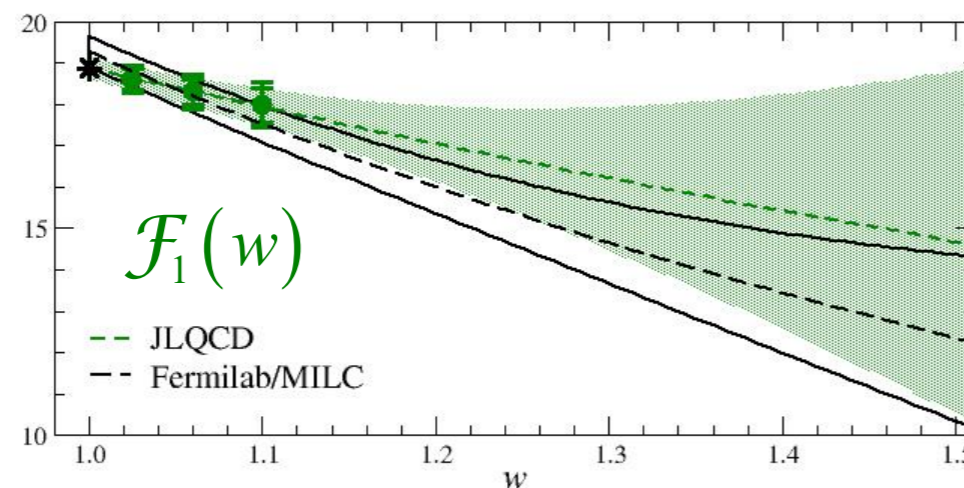
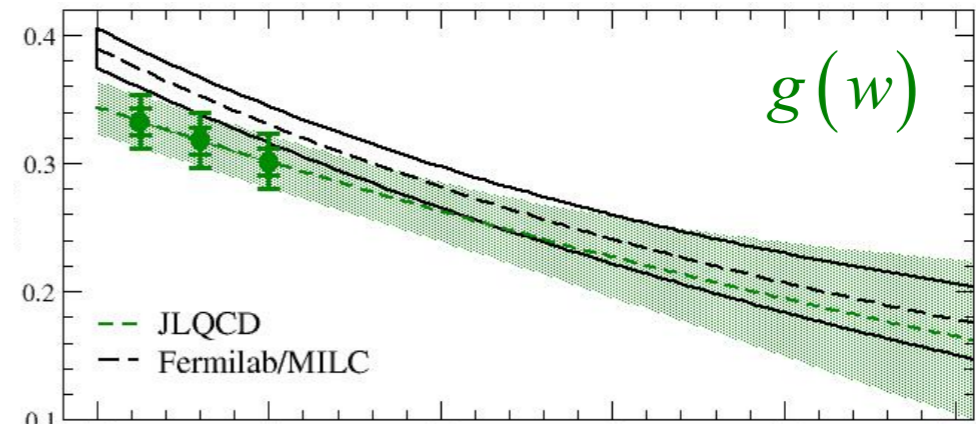
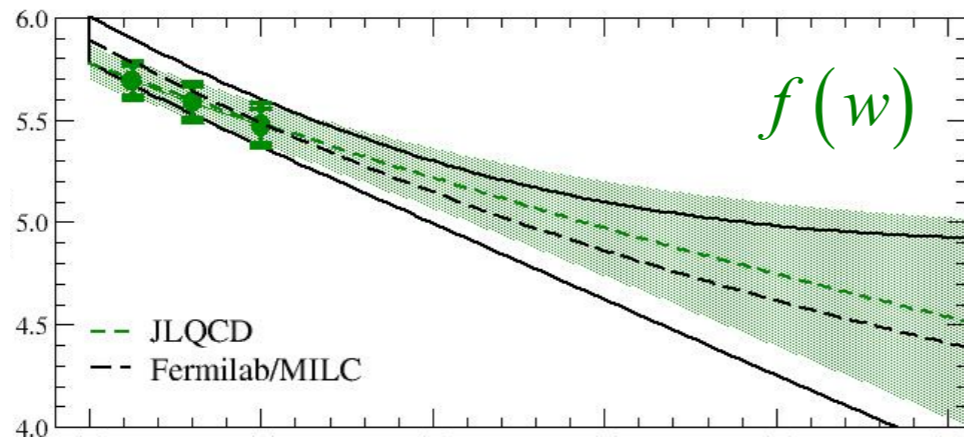
*Our analysis of same exp+lattice data(Jung, PG):*

$$|V_{cb}| = 39.4(9) \cdot 10^{-3} (\chi^2_{min} = 50) \text{ using only total rate } |V_{cb}| = 42.2^{+2.8}_{-1.7} \cdot 10^{-3}$$



# JLQCD PRELIMINARY RESULTS

## JLQCD vs Fermilab/MILC



- reasonably consistent

$\Leftrightarrow g @ w \sim 1$

T. Kaneko @ Barolo workshop 4/2022

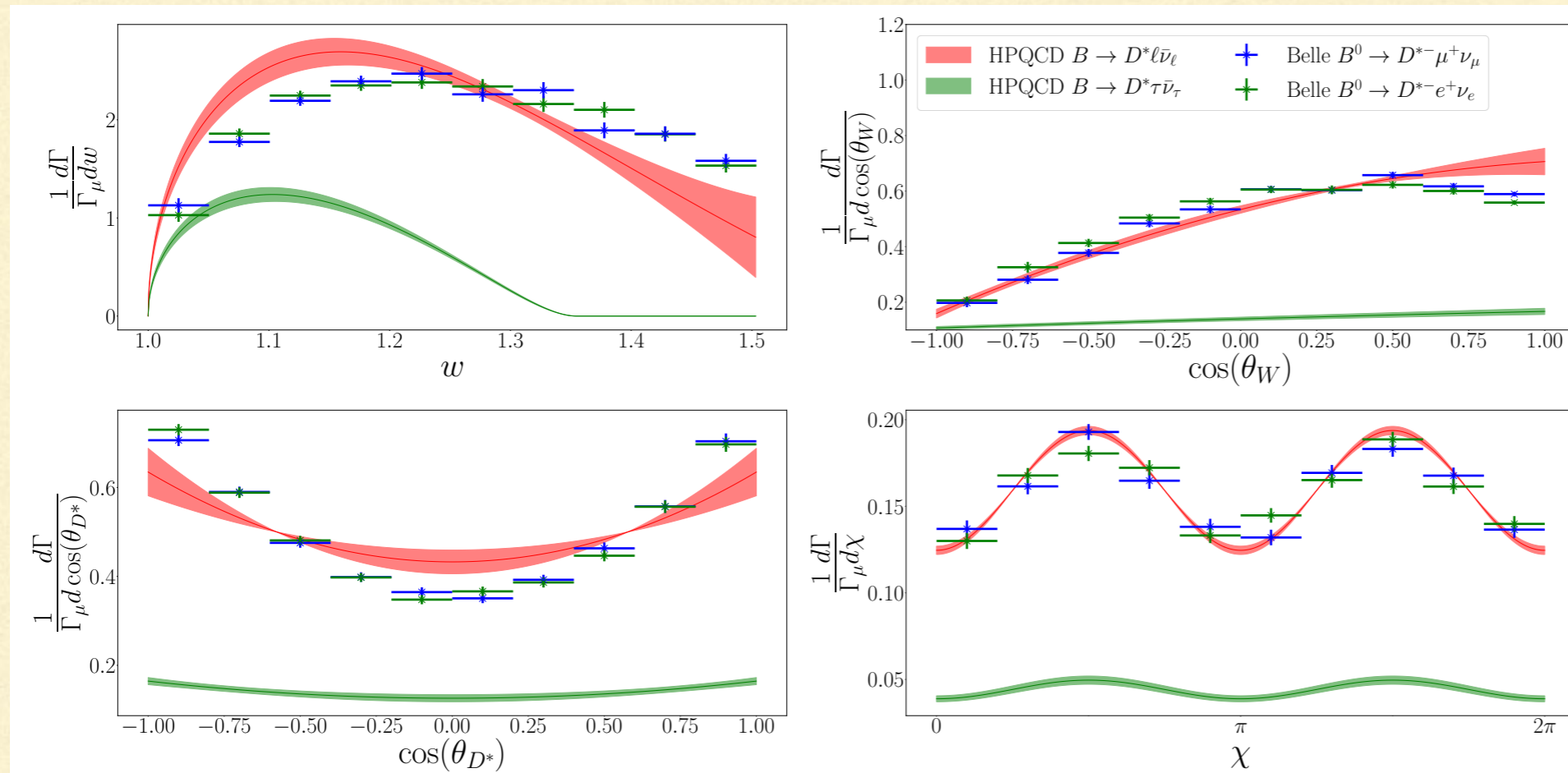
Kaneko et al 2112.13775

Our analysis of same exp (Belle I 8) + JLQCD data (Jung, PG):

$$|V_{cb}| = 40.7(9) \cdot 10^{-3} \quad (\chi_{\min}^2 = 33) \quad \text{using only total rate} \quad |V_{cb}| = 40.8_{-2.3}^{+1.8} \cdot 10^{-3}$$

# NEW HPQCD FFS CALCULATION

2304.03137



Tension with Belle 2018 data similar to FNAL

## Belle I 8+HPQCD

BGL exp	$\chi^2$	$ V_{cb} $
0001	78	41.0(8)
0101	68	41.2(8)
0111	57	40.8(8)
1111	57	40.8(8)
1121	54	40.6(8)
1222	52	40.6(8)
2222	50	40.4(8)
2232	50	40.4(8)
3333	50	40.4(8)

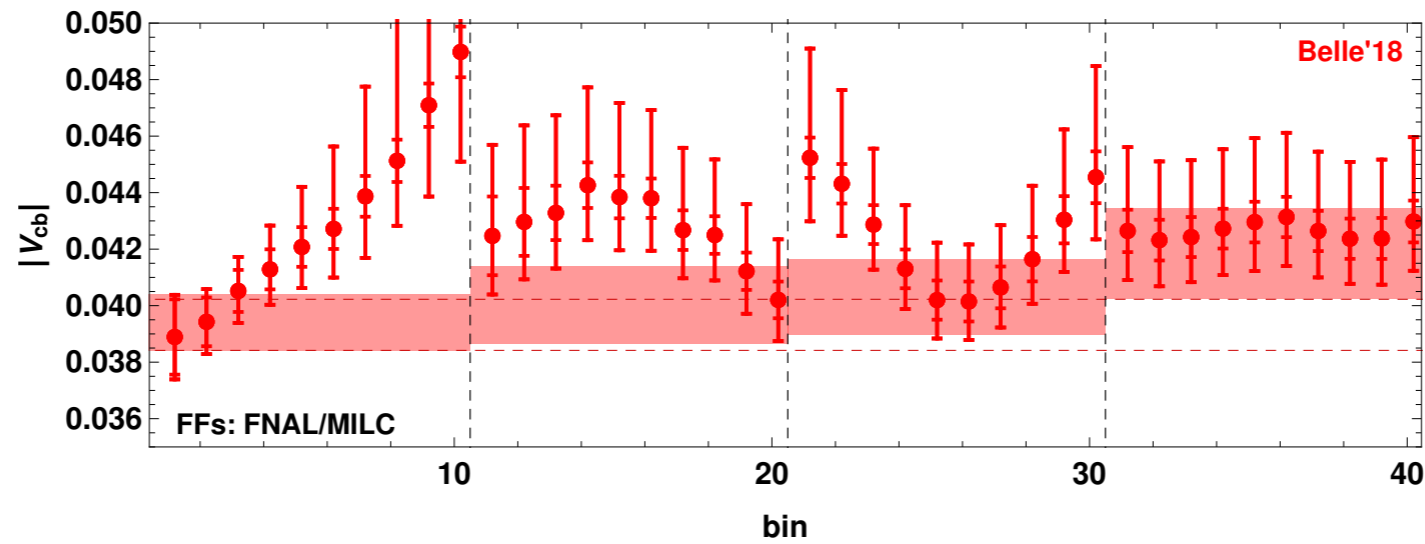
Extrapolation in  $m_h$ , data cover the whole  $w$  region

Our analysis of same exp (Belle I 8) + HPQCD data (Jung, PG):  
 $|V_{cb}| = 40.4(8) \cdot 10^{-3}$  using only total rate  $|V_{cb}| = 44.4 \pm 1.6 \cdot 10^{-3}$

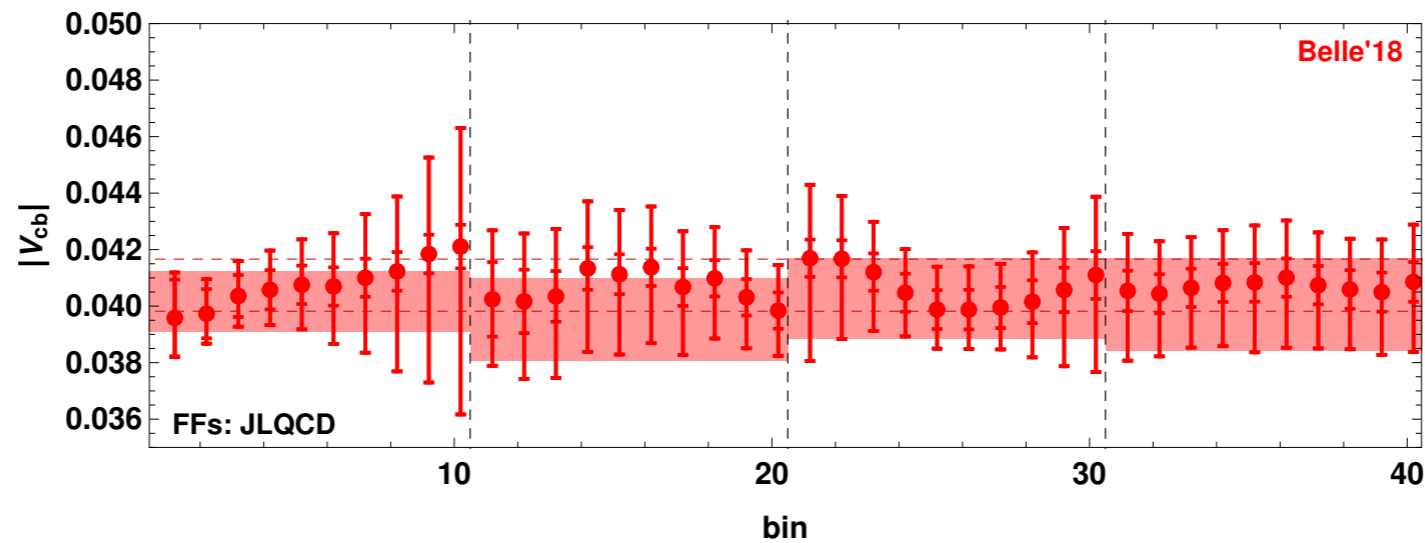
HPQCD and FNAL are not well compatible: adding 16 FNAL points increases  $\chi^2$  by 35

# Binned $V_{cb}$ from Belle'18 data: FNAL/MILC vs JLQCD

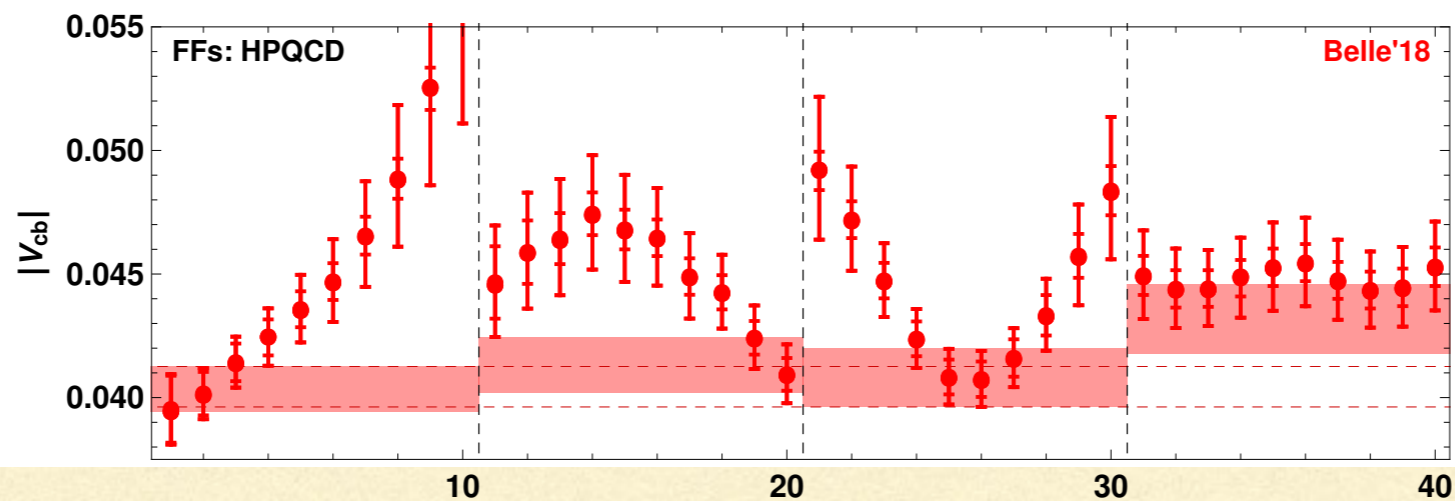
FNAL/MILC



JLQCD



HPQCD



Binned analysis proposed by Martinelli, Simula, Vittorio in DM approach  
 $2105.08674$   
 $2109.15248$

Extracting  $V_{cb}$  from each bin, FFs only determined by lattice QCD

M. Jung

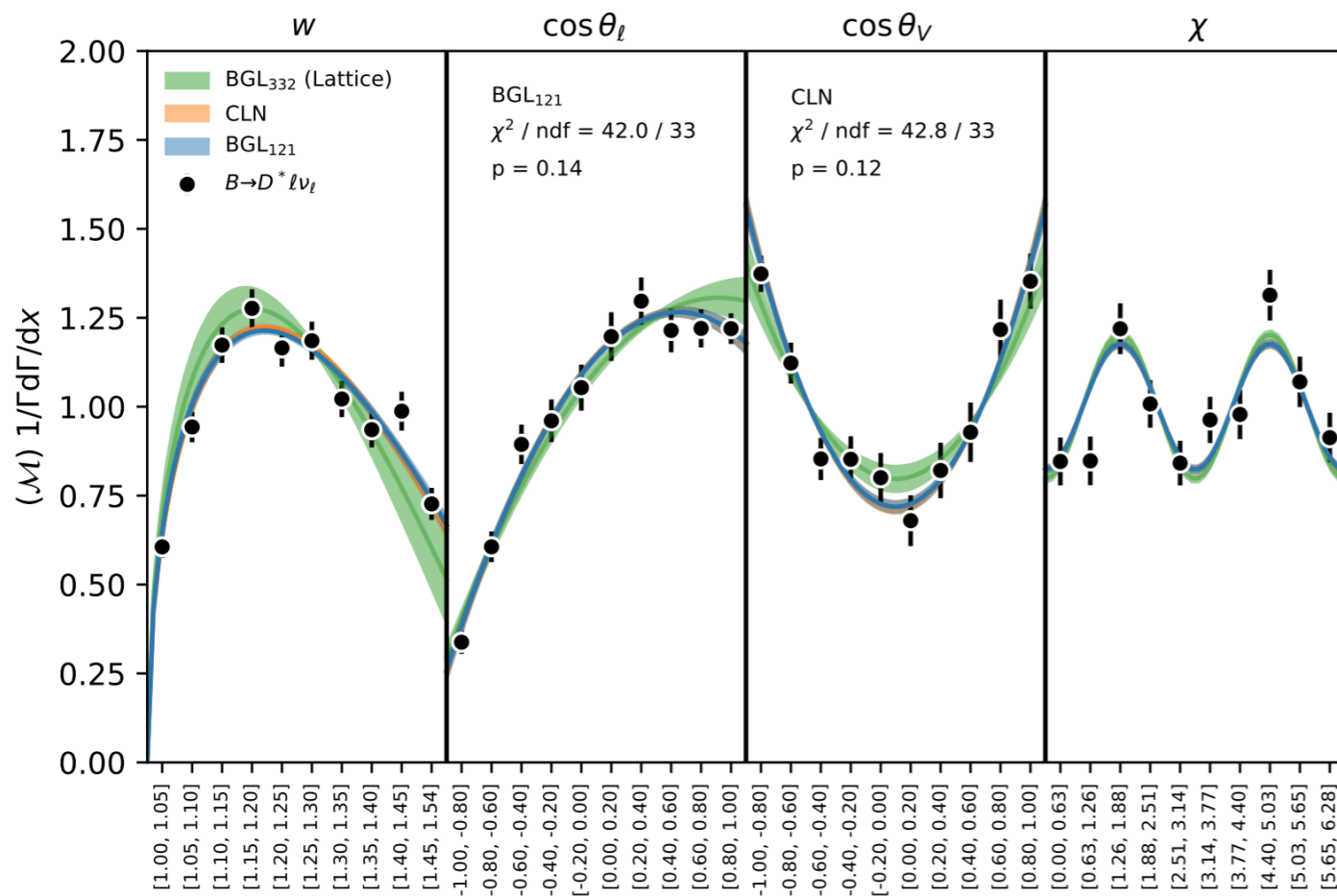
Global BGL fit to Belle I 8+FNAL+JLQCD+HPQCD data:

$$|V_{cb}| = 40.3(7) \cdot 10^{-3} (\chi^2_{min} = 91.2) \text{ using only total rate } |V_{cb}| = 42.4(1.0) \cdot 10^{-3}$$

# Measurement of Differential Distributions of $B \rightarrow D^* \ell \nu_\ell$ and Determination of $|V_{cb}|$



New  
[Preliminary]



Measured Shapes + External Branching Ratio Input

BGL(121)	Value	Correlation				
$a_0 \times 10^3$	$24.93 \pm 1.41$	1.00	0.25	-0.21	0.26	-0.30
$b_0 \times 10^3$	$13.11 \pm 0.18$	0.25	1.00	-0.01	-0.01	-0.62
$b_1 \times 10^3$	$-11.93 \pm 12.72$	-0.21	-0.01	1.00	0.25	-0.48
$c_1 \times 10^3$	$-6.87 \pm 0.97$	0.26	-0.01	0.25	1.00	-0.49
$ V_{cb}  \times 10^3$	$40.77 \pm 0.92$	-0.30	-0.62	-0.48	-0.49	1.00

CLN	Value	Correlation			
$\rho^2$	$1.25 \pm 0.09$	1.00	0.56	-0.89	0.38
$R_1(1)$	$1.32 \pm 0.08$	0.56	1.00	-0.63	-0.03
$R_2(1)$	$0.85 \pm 0.07$	-0.89	-0.63	1.00	-0.15
$ V_{cb}  \times 10^3$	$40.30 \pm 0.86$	0.38	-0.03	-0.15	1.00

Based on the lattice input at zero-recoil:

$$h_{A_1}(1) = 0.906 \pm 0.013$$

2301.07529

# $B^0 \rightarrow D^{*-} \ell^+ \nu$ untagged (189/fb) preliminary [to be submitted to Phys. Rev. D] Belle II

LQCD used only for normalisation at zero recoil ( $w = 1$ )

## BGL fit result

BGL truncation order determined by Nested Hypothesis Test [Phys. Rev. D100, 013005]

	Values	Correlations				$\chi^2/\text{ndf}$
$\tilde{a}_0 \times 10^3$	$0.89 \pm 0.05$	1.00	0.26	-0.27	0.07	40/31
$\tilde{b}_0 \times 10^3$	$0.54 \pm 0.01$	0.26	1.00	-0.41	-0.46	
$\tilde{b}_1 \times 10^3$	$-0.44 \pm 0.34$	-0.27	-0.41	1.00	0.56	
$\tilde{c}_1 \times 10^3$	$-0.05 \pm 0.03$	0.07	-0.46	0.56	1.00	

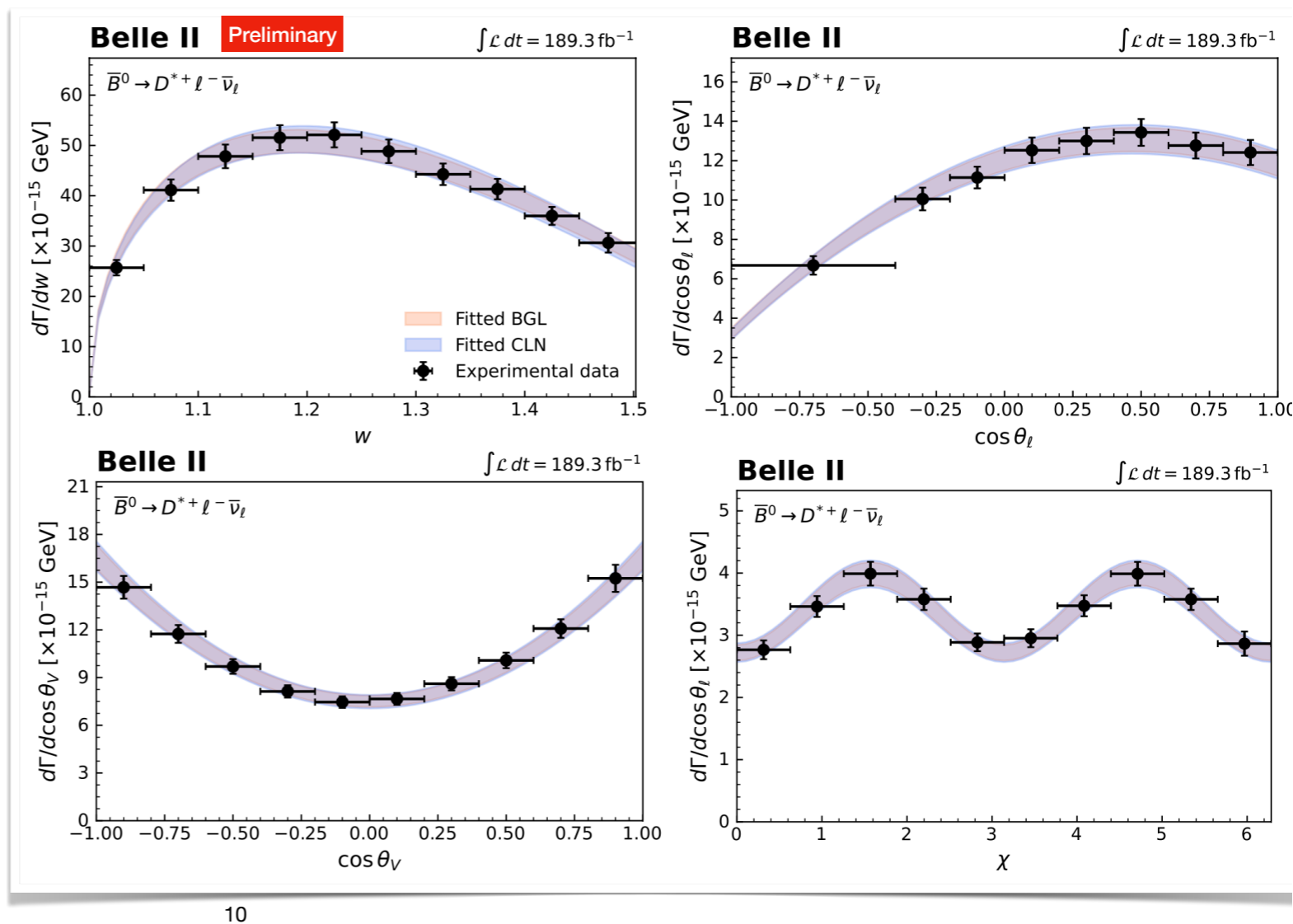
Relative uncertainty (%) Preliminary

	$\tilde{a}_0$	$\tilde{b}_0$	$\tilde{b}_1$	$\tilde{c}_1$
Statistical	3.3	0.7	44.8	35.4
Finite MC samples	3.0	0.7	39.4	33.0
Signal modelling	3.0	0.4	40.0	30.8
Background subtraction	1.2	0.4	24.8	18.1
Lepton ID efficiency	1.5	0.3	3.1	2.5
Slow pion efficiency	1.5	1.5	18.4	22.0
Tracking of $K, \pi, \ell$	0.5	0.5	0.6	0.5
$N_{B\bar{B}}$	0.8	0.8	1.1	0.8
$f_{+-}/f_{00}$	1.3	1.3	1.7	1.3
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+)$	0.4	0.4	0.5	0.4
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	0.4	0.4	0.5	0.4
$B^0$ lifetime	0.1	0.1	0.2	0.1
Total	6.1	2.5	78.3	64.1

Preliminary

$$|V_{cb}| \eta_{\text{EW}} \mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}} \left( \frac{|\tilde{b}_0|}{P_f(0) \phi_f(0)} \right) \quad \mathcal{F}(1) = 0.906 \pm 0.013$$

$$|V_{cb}|_{\text{BGL}} = (40.9 \pm 0.3_{\text{stat}} \pm 1.0_{\text{syst}} \pm 0.6_{\text{theo}}) \times 10^{-3}$$



C. Schwanda, Moriond '23

$$|V_{cb}|_{\text{CLN}} = (40.4 \pm 0.3_{\text{stat}} \pm 1.0_{\text{syst}} \pm 0.6_{\text{theo}}) \times 10^{-3}$$

# RESULTS BY BABAR AND LHCb

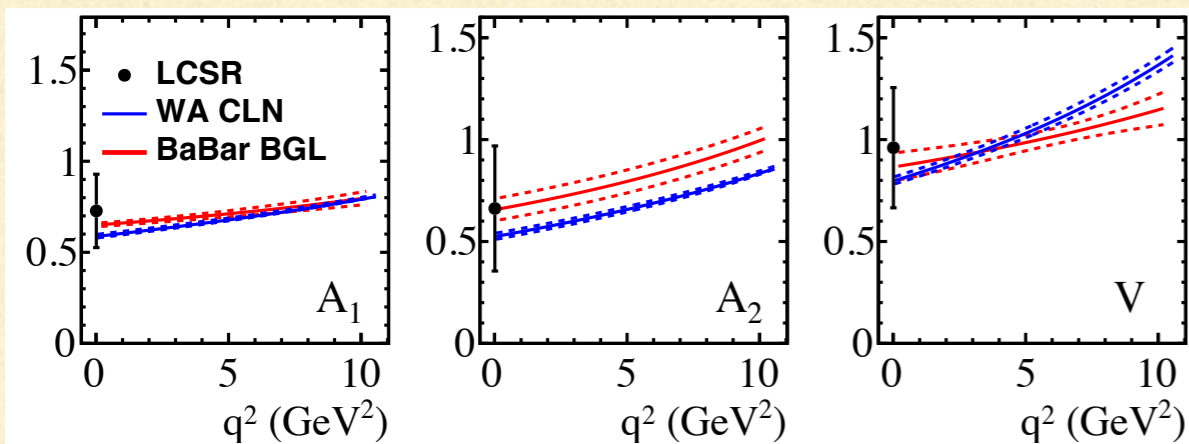
1903.10002, 2001.03225

Reanalysis of tagged  $B^0$  and  $B^+$  data, unbinned 4 dimensional fit with simplified BGL and CLN  
About 6000 events  
No data provided yet



Measurement of  $|V_{cb}|$  with  $B_s^0 \rightarrow D_s^{(*)-} \mu^+ \nu_\mu$  decays

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)},$$
$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)}$$



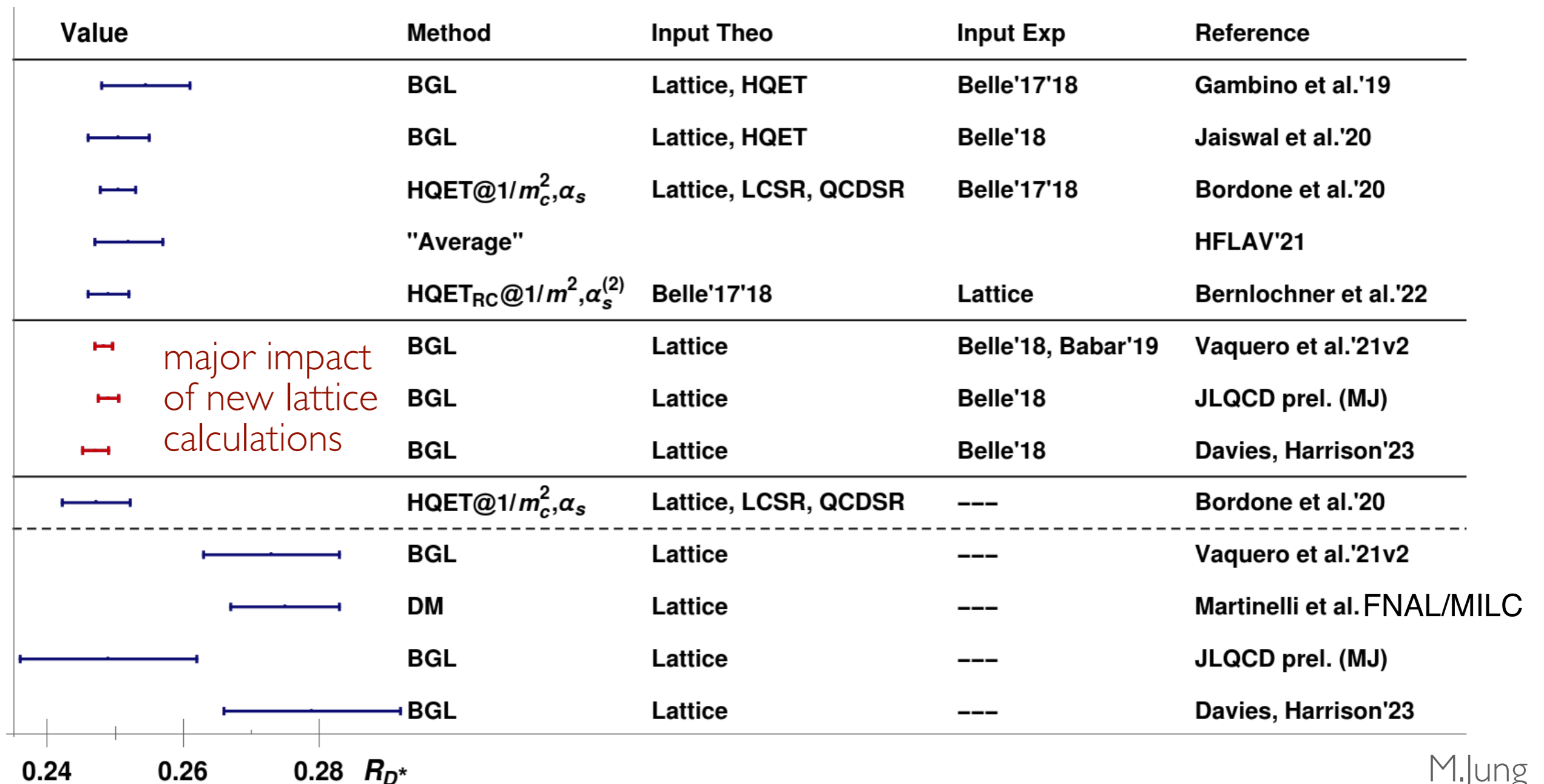
No clear BGL(111)/CLN difference but disagreement with HFLAV CLN ffs

$$\mathbf{V_{cb}=0.0384(9)}$$

$$\mathbf{V_{cb}=0.0414(16) \quad CLN}$$
$$\mathbf{V_{cb}=0.0423(17) \quad BGL^{(222)}}$$

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL<sup>(222)</sup>

# Overview over predictions for $R(D^*)$



M.Jung

Predictions based only on Fermilab & HPQCD lead to larger  $R(D^*)$ , in better agreement with exp, mostly because of the suppression at high  $w$  of the denominator.  
***I see no reason not to use experimental data for a SM test***, especially in presence of tensions in lattice data.

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# SUMMARY

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- *Despite many new theoretical and exp results, the  $V_{cb}$  puzzle persists, but there are reasons for optimism. Great and lasting progress has been achieved.*
  - **Inclusive  $b \rightarrow c$** : new 3loop calculations show pert effects under control, 1.2% accuracy on  $|V_{cb}|$ , work on QED effects,  $q^2$  moments
  - First calculations of **inclusive semileptonic meson decays on the lattice**. Exploratory calculations for  $m_b \sim 2.5\text{GeV}$  in good agreement with OPE, others ongoing. Promising to complement/validate the OPE, still a long way to go.
  - **Exclusive  $b \rightarrow c$** : uncertainties have been underestimated in the past; three lattice groups have computed the  $B \rightarrow D^*$  FFs at non-zero recoil and new exp analyses are under way. The **situation is still unclear**. FNAL & HPQCD in tension with exp spectra, JLQCD gives a more consistent picture with reduced tension with inclusive  $\sim 1.2\sigma$ . New Belle and Belle II analyses may confirm the trend.
-