Non-local Form Factors
Discussion Flavour@TH

Danny van Dyk

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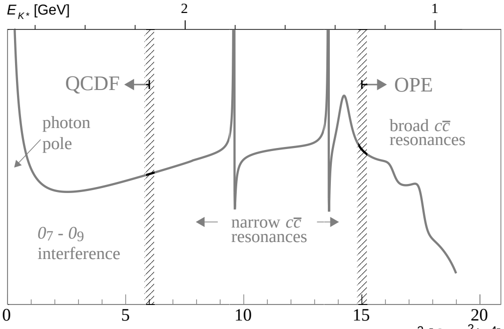
Spectrum

$$\mathcal{H}_\lambda = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\, J^\mu_{\bar{c}c}(x), [C_1 O_1^c + C_2 O_2^c](0)\,\}\, |H_b\rangle$$



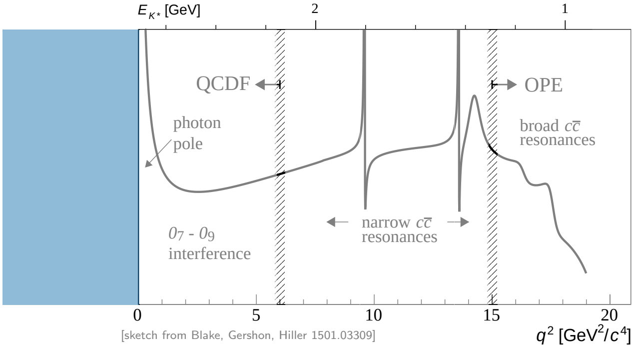
[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$
- $J^\mu_{\bar{c}c} = Q_c \bar{c}\gamma^\mu c$

crucial for decision if $b \to s\mu^+\mu^-$ anomalies are BSM physics!

Spectrum

$$\textcolor{red}{\mathcal{H}_\lambda} = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ \textcolor{green}{J^\mu_{\bar{c}c}(x)}, \textcolor{orange}{[C_1 O^c_1 + C_2 O^c_2](0)} \} |H_b\rangle$$



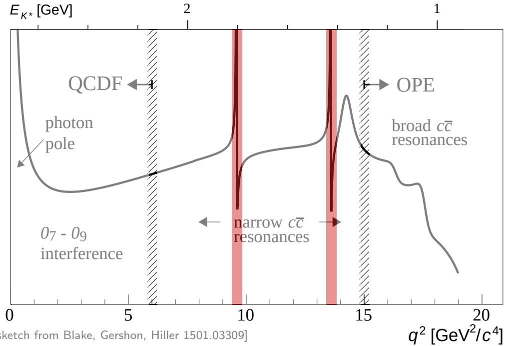
[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\,[\bar{c}\Gamma' c]$
- $J^\mu_{\bar{c}c} = Q_c \bar{c}\gamma^\mu c$

- leading contributions expressed through local form factors $\mathcal{F}_\lambda$
- correction suppressed by $1/(q^2 - 4m_c^2)$ can by systematically obtained

Spectrum

$$\mathcal{H}_\lambda = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\, J^\mu_{\bar{c}c}(x), [C_1 O^c_1 + C_2 O^c_2](0)\,\} |H_b\rangle$$



[sketch from Blake, Gershon, Hiller 1501.03309]

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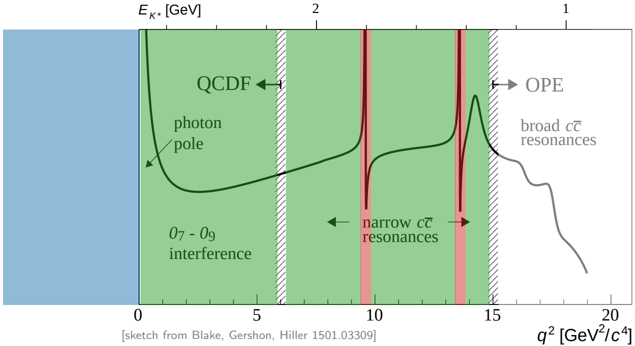
- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by hadronic decays
- experimental measurements provide additional information about $\mathcal{H}_\lambda$

Spectrum

$$\mathcal{H}_\lambda = P(\lambda)_\mu \langle H_s | \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\, J^\mu_{\bar{c}c}(x), [C_1 O_1^c + C_2 O_2^c](0)\,\} | H_b \rangle$$



[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\,[\bar{c}\Gamma' c]$
- $J^\mu_{\bar{c}c} = Q_c \bar{c}\gamma^\mu c$

[Bobeth,Chrzaszcz,DvD,Virto '17]

strategy

- compute $\mathcal{H}_\lambda$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$ using suitable parametrization
- include information from hadronic decays to narrow charmonia J/ψ and $\psi(2S)$

Assumptions

- extrapolation from $q^2 < 0$ to $q^2 \geq 0$ relies on a limited number of assumptions
- crucially: reliance on the analytic structures
  - only singularities due to on-shell intermediate states; no "anomalous" cuts
  - two isolated poles due to charmonia J/ψ & $\psi(2S)$
  - numerically dominant branch cut starts at $q^2 = 4M_D^2$
  - "light hadron" branch cut starting at $q^2 = 0$ can be split off and treated in a model
- consequences:
  - $\mathcal{H}_\lambda$ are complex-valued even at $q^2 < 0$

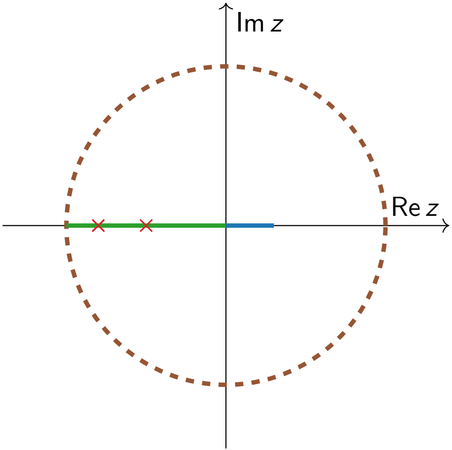
Parametrisation using z mapping

- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]
  - branch cut is mapped onto unit circle in z
- real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
  - data and theory live insides the unit circle
- expand in z

\+ resonances J/ψ, $\psi(2S)$ can be included (poles/Blaschke factors)

\+ easy to use in a fit to theory and data

\+ compatible with analyticity



Open Questions

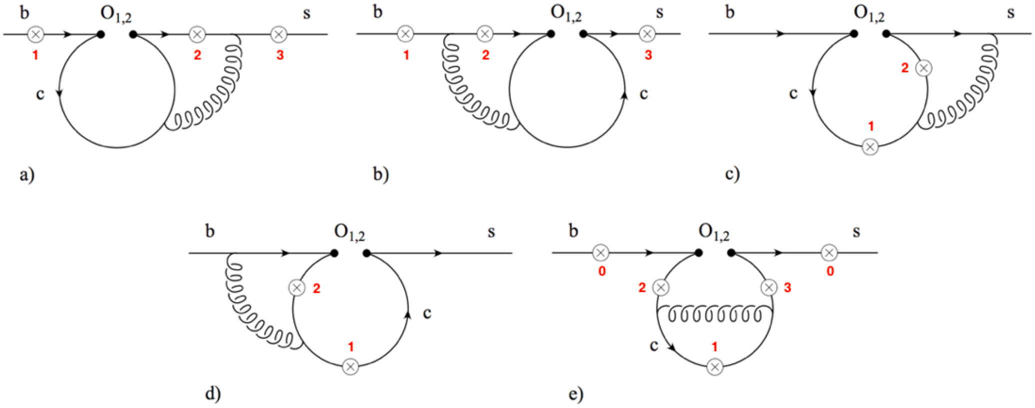
Probably the biggest problem:

- z param approach assumes that all cuts of the form factor are physical cuts
  - supported by two-loop OPE results, which do not show any anomalous cut(s)

  $\Rightarrow$ a-priori no reason to consider such cuts
- Rome group is concerned by “triangle diagrams” involving e.g. $D_s\overline{D}$ intermediate states
- no Lagrangian put forward that governs their calculation
- these diagrams, when taken at face value as Feynman diagrams, do produce anomalous cuts

How do these two approaches relate to each other?

NLO OPE – Diagrams

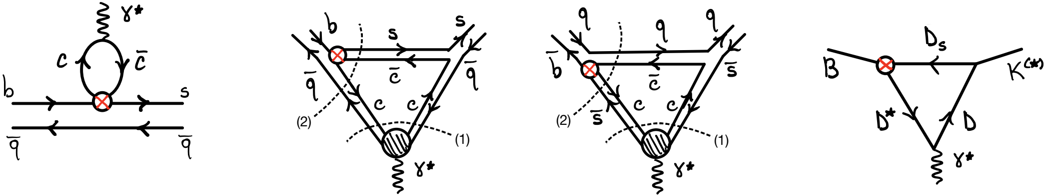


[Asatrian, Greub, Virto 1912.09099]

NLO OPE – Dispersion Relations

- AGV have tested their conclusions on the analytic structure by applying dispersion relations to their results
- no indication that anomalous cuts exist!
- topology of triangle diagrams can be found in OPE Feynman diagrams
  - these topologies produce a cut in p_B^2 as expected
  - lead to fact that OPE result is complex-valued even to the left of all physical cuts in q^2

Triangle Diagrams



(a) (b) (c)

[Ciuchini,Fedele,Franco,Paul,Silvestrini,Valli 2212.10516]

Triangle Diagrams

- diagram (c) can be expressed in terms of Passarino-Veltman function C_0
- in this diagram, an anomalous cut is present, starting from $q^2 = 4M_D^2$ to a point in the lower q^2 half plane.
  - infinite tower of such cuts is produced by all possible other mesonic intermediate states, but relation to the physical cut remain unchanged
  - to be shown: anomalous cuts are or are not singularities of the full amplitude

Different Points of View

Luca:

- We do know that rescattering invalidates QCD factorization results in non-leptonic B decays (QCD factorization without power corrections gives $B \to K\pi$ BR's a factor of two below exp value)
- The singularities of triangle diagrams correspond to long-distance contributions that do not admit an OPE

Danny:

- concern should be taken seriously and investigated
- currently no indication for any effect whatsoever
  - OPE result contains triangle topology
- is mesonic picture of rescattering the correct interpretation?
  - if yes, would invalidate basically all QCD factorization results in non-leptonic B decays