Neutral meson mixings on the lattice

Felix Erben

Flavour@TH CERN

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OVERVIEW

1) theory: $B_{(s)} - \bar{B}_{(s)}$ mixing on the lattice
2) available lattice results
3) ongoing project by RBC/UKQCD and JLQCD
4) comparison to $K - \bar{K}$, $D - \bar{D}$ mixing on the lattice

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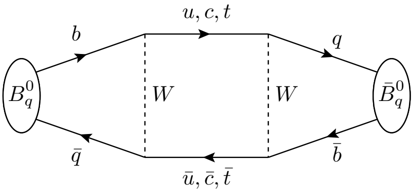
B_q MESON MIXING

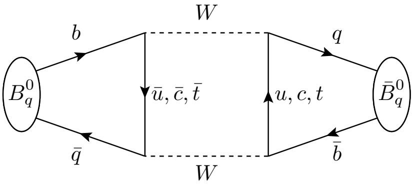
B-mesons B_d, B_s have mass eigenstates

$$|B^0_{qL}\rangle = p_q|B^0_q\rangle + q_q|\bar{B}^0_q\rangle$$
$$|B^0_{qH}\rangle = p_q|B^0_q\rangle - q_q|\bar{B}^0_q\rangle$$



with mass m_{qL} and total decay width Γ_{qL} for the lighter eigenstate. Splittings:

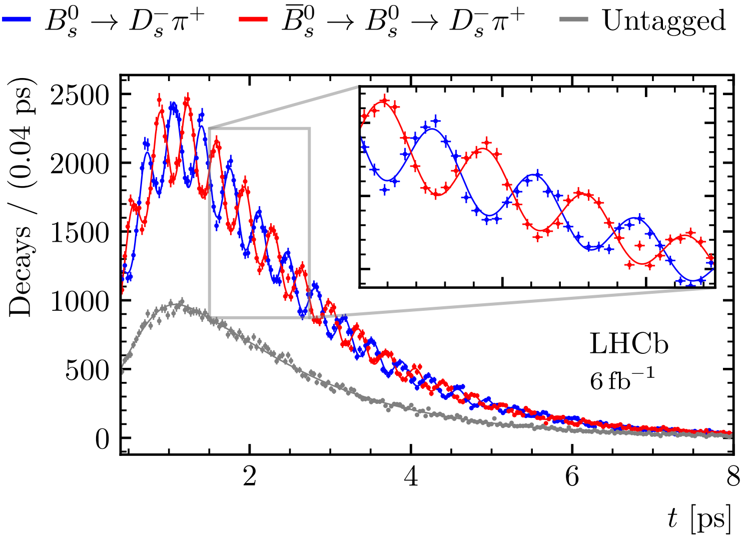
$$\Delta m_q = m_{qH} - m_{qL}$$
$$\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$$



Experimentally, time dependent probabilities give access to the splittings, e.g.

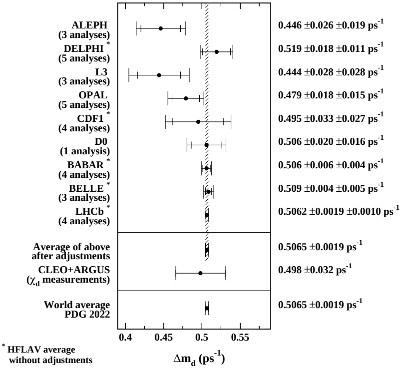
$$\mathcal{P}(B^0_q \to \bar{B}^0_q) = \frac{1}{2} e^{-\Gamma_q t}[\cosh(\frac{1}{2}\Delta\Gamma_q t) - \cos(\Delta m_q t)]|q_q/p_q|^2$$

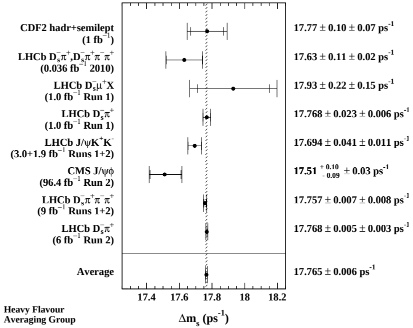
LHCb 2021 measurement [Nature Phys. 18 (2022) 1, 1-5]



B_q MESON MIXING

Experimental results, HFLAV 2021 [Phys.Rev.D 107 (2023) 5]





$$\Delta m_d = 0.5065(19) ps^{-1}$$

$$\Delta m_s = 17.765(6) ps^{-1}$$

B_q MESON MIXING

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle = \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} + \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{LD}$$
$$= \langle B_q^0|\mathcal{H}_W^{\Delta B=2}|\bar{B}_q^0\rangle + \sum_n \frac{\langle B_q^0|\mathcal{H}_W^{\Delta B=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta B=1}|\bar{B}_q^0\rangle}{M_{B_q} - E_n}$$

short-distance contribution:

- t-loop enhancement
- additional CKM hierachy enhancement

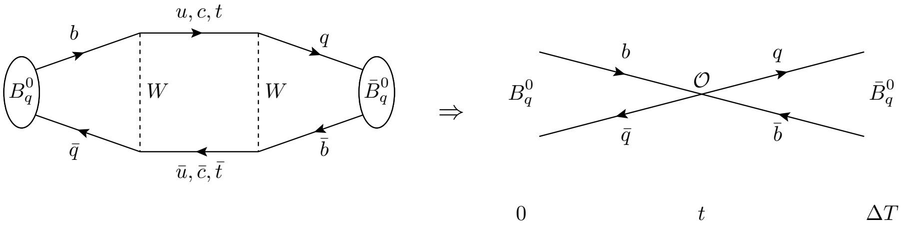
$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} \sim \left(\sum_{q'=u,c,t} V_{q'q}^* V_{q'b} S_0(m_{q'}^2/M_W^2) \right)^2$$

long-distance contribution:

- CKM-suppressed

B_q-mixing dominated by short-distance contribution

THEORY



- $\Delta B = 2$ process
- enhanced by top quark $\Rightarrow$ short-distance dominated
- OPE shrinks box diagram to local four-quark operator

$$\langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle \sim \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle$$

- 5 parity-even, dimension 6, $\Delta B = 2$ operators $\mathcal{O}_i$

- bag parameters $\mathcal{B}$ give access to mass splittings Δm

$$\mathcal{B}_{B_q}^{[i]} = \frac{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle}{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle_{\mathrm{VSA}}}$$

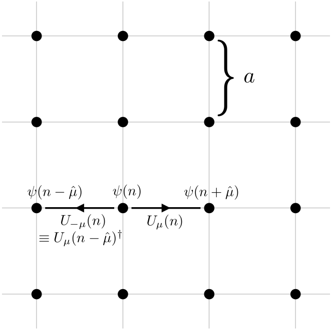
$$\Delta m_q = |V_{td} V_{tq}^*|^2 \mathcal{K} M_{B_q} f_{B_q}^2 \mathcal{B}_{B_q}^{[1]}$$

- $\mathcal{K}$ known (perturbative)
- M_{B_q}, f_{B_q}, $\mathcal{B}_{B_q}^{[i]}$ non-perturbatively from lattice QCD
- Δm_q as input $\Rightarrow |V_{tq}|$ (or other way round)
- Additional $\mathcal{B}_{B_q}^{[i]}$ give access to $\Delta\Gamma_q$ and constrain various BSM models

LATTICE QCD

- Discrete, finite Euclidean space-time grid
  - quark fields ψ on sites n
  - gluons U_μ as gauge links
  - finite lattice spacing a (UV regulator)
  - finite volume L, T (IR regulator)
- Path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dU d\psi d\bar{\psi}\, \mathcal{O}[U, \psi, \bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$$



- even relatively small grids have size $\Lambda = (L/a) \times (T/a) = 24^3 \times 48$
  - exact evaluation prohibitively expensive
  - $\Rightarrow$ stochastic sampling of ensembles

LATTICE QCD

Fermionic part of the path integral can be solved explicitly

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dU\, e^{-S_G[U]} d\psi d\bar{\psi}\, e^{-S_F[U,\psi,\bar{\psi}]} \mathcal{O}[U,\psi,\bar{\psi}]$$
$$= \frac{1}{Z} \int dU\, e^{-S_G[U]} \Big(\prod_f \det[D_f(U)] \Big) \mathcal{O}[U,\psi,\bar{\psi}]$$

Monte-Carlo simulation: interpret

$$Z^{-1} e^{-S_G[U]} \Big(\prod_f \det[D_f(U)] \Big)$$

as probability weight.

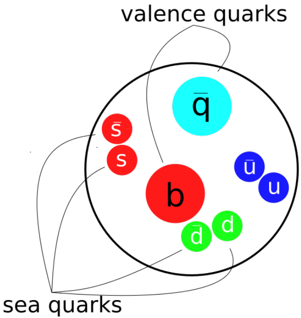
- D_f is matrix with $12 \times \Lambda$ rows and columns

$\Rightarrow$ brute-force inversion prohibitively expensive

$\Rightarrow$ intricate algorithms needed to account for $\det[D_f]$, beyond the scope of this talk

LATTICE QCD

- Fermion determinants describe “sea quarks”
  - quark pairs in vacuum
  - gives rise to definition of ”valence quarks” giving quantum numbers to hadrons
- ”quenching”: set $\det[D_f] = 1$
  - neglects sea quark contribution
- e.g. ”$N_f = 2 + 1$”
  - $\det[D_u] = \det[D_d] \neq 1$
  - $\det[D_s] \neq 1$
- sea-quark masses m_l, m_s are inputs
  - often $M_\pi > M_\pi^{\mathrm{phys}}$
  - many modern lattice calculations at $M_\pi = M_\pi^{\mathrm{phys}}$



Fermion actions

Various choices for lattice fermion action, with various strengths and weaknesses.
No action exists that simultaneously: (Nielsen–Ninomiya)

- recovers the correct continuum limit $a \to 0$ for Dirac operator D
- is local
- does not introduce unphysical fermions ("doublers")
- respects chiral symmetry $\{\gamma_5, D\} = 0$

The first 3 conditions can be met though with a lattice chiral symmetry (Ginsparg-Wilson)

- $\{\gamma_5, D\} = aD\gamma_5 D$

CONTINUUM LIMIT

We need to control on each ensemble

- light-quark discretisation effects $\Rightarrow M_\pi L \gtrapprox 4$
- heavy-quark discretisation effects am_h

Two approaches for heavy quark:

effective theories

- allow expansion in $1/am_b$
- truncation at some order
- "cheap"
- not easily improvable beyond certain precision

method:

- Relativistic action (HQET, RHQ, Fermilab method)
- Nonrelativistic QCD (NRQCD)

fully relativistic

- $am_h \ll 1$ needed

$\Rightarrow$ fine lattice spacing for am_b^{phys}

- "expensive"
- improvable with finer, larger boxes

method:

- extrapolation $am_h \to am_b$ for multiple $am_h < am_b$
- today impossible to reach am_l^{phys}, am_b^{phys} simultaneously

Full recipe

2pt-functions

$$\langle B_q(t) B_q^\dagger(0) \rangle_{L,a,m_l,m_h} \Rightarrow M_{B_q}(L, a, m_l, m_h), f_{B_q}(L, a, m_l, m_h)$$

3pt-functions

$$\langle B_q(\Delta T) \mathcal{O}_i(t) B_q^\dagger(0) \rangle_{L,a,m_l,m_h} \Rightarrow \mathcal{B}_{B_q}^{[i]}(L, a, m_l, m_h)$$

Leading to

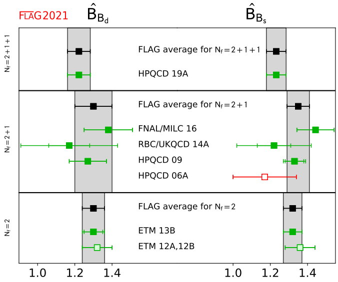
$$\Delta m_q = |V_{td} V_{tq}^*|^2 \mathcal{K} \lim_{a\to 0} \lim_{L\to\infty} \lim_{m_l\to m_l^{\mathrm{p}}} \lim_{m_h\to m_h^{\mathrm{p}}} (M_{B_q} f_{B_q}^2 \mathcal{B}_{B_q}^{[1]})(L, a, m_l, m_h)$$

or more precise results for

$$\xi^2 = \frac{f_{B_s}^2 \mathcal{B}_{B_s}^{[1]}}{f_{B_d}^2 \mathcal{B}_{B_d}^{[1]}} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{M_{B_d}}{M_{B_s}} \frac{\Delta m_s}{\Delta m_d}$$

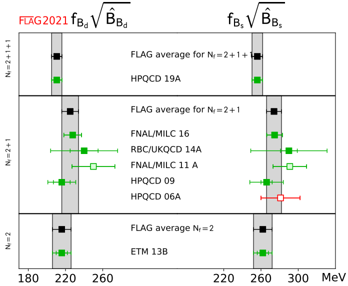
Different groups repeat the exercise for different setups and actions

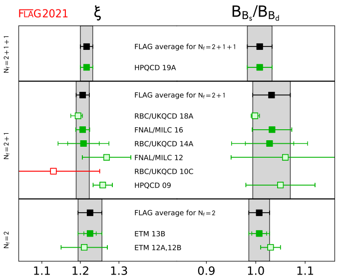
COMMUNITY AVERAGE



FLAG 2021 [Eur.Phys.J.C 82 (2022) 10, 869]

- compilation of lattice results, including averages
- 3 sections for different sea quark configurations
- all viable actions must restore the correct limits (universality)





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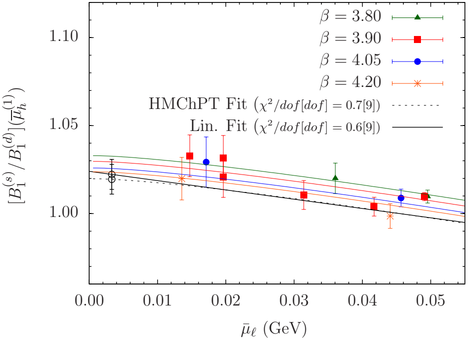
ETMC 2014

[JHEP 03 (2014) 016]

- twisted-mass action: automatically $O(a)$ improved
- $N_f = 2$
- 4 lattice spacings $0.054\mathrm{fm} \leqslant a \leqslant 0.098\mathrm{fm}$
- pion masses $180\mathrm{MeV} \leqslant M_\pi \leqslant 500\mathrm{MeV}$

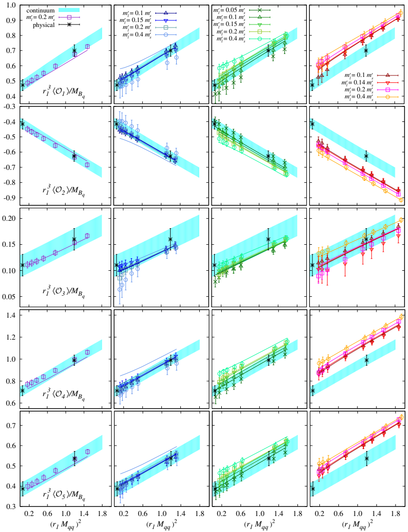


- statistical error dominates (except ξ)
- effective theory for b-quark: perturbtaive matching to HQET, then interpolation to physical am_b
- Non-perturbative renormalization (RI'/MOM)

FNAL / MILC 2016

[Phys.Rev.D 93 (2016) 11]

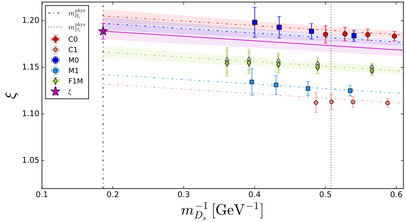
- staggered action (asqtad): $O(a)$, tadpole improved
- effective theory for b-quark via Fermilab action
- $N_f = 2 + 1$
- 4 lattice spacings $0.045\mathrm{fm} \leqslant a \leqslant 0.12\mathrm{fm}$
- pion masses $175\mathrm{MeV} \leqslant M_\pi \leqslant 555\mathrm{MeV}$
- B_d, B_s bag parameters for all 5 operators
- error mostly statistical, largest systematic contributions from chiral-continuum fit, matching and am_h discretisation effects



RBC/UKQCD 2018

[arxiv:1812.08791]

- domain-wall fermion action:
  - chiral symmetry
  - 5D formulation (expensive)
- $N_f = 2 + 1$

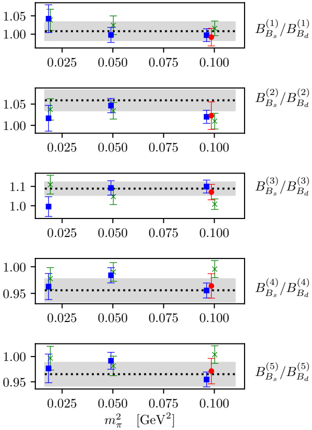


- 3 lattice spacings $0.073\mathrm{fm} \leqslant a \leqslant 0.11\mathrm{fm}$
- pion masses $M_\pi^{phys} = 139\mathrm{MeV} \leqslant M_\pi \leqslant 430\mathrm{MeV}$
- fully-relativistic heavy-quark action
- reach from physical am_c to about $0.5am_b$
- missing renormalisation factors - only ratios $\mathcal{B}_{B_s}/\mathcal{B}_{B_d}$ and ξ
- error mostly statistical, largest systematic contributions from chiral-continuum fit and m_H^{-1} extrapolation

HPQCD 2019

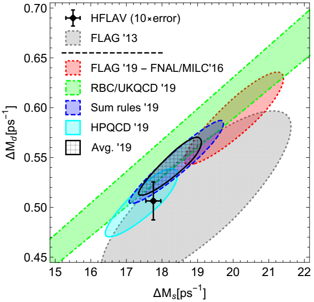
[Phys. Rev. D 100, 094508]

- HISQ staggered action: reduced systematic effects compared to asqtad staggered
- $N_f = 2 + 1 + 1$
- 3 lattice spacings $0.09\mathrm{fm} \leqslant a \leqslant 0.15\mathrm{fm}$
- includes pion mass $M_\pi^{phys} = 130\mathrm{MeV}$ ensemble
- effective theory for b-quark: Nonrelativistic QCD
- $\mathcal{B}_{B_d}, \mathcal{B}_{B_s}$ bag parameters for all 5 operators
- error dominated by matching in $O(\alpha_s \Lambda_{QCD}/M)$, $O(\alpha_s^2)$
- follow-up publication for the width difference $\Delta\Gamma_q$ [Phys.Rev.Lett. 124 (2020) 8]



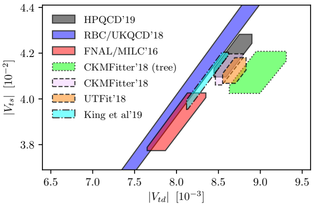
$B_q - \bar{B}_q$ MIXING ON THE LATTICE

- current tension between Δm_d, Δm_s lattice determinations
    - FNAL/MILC '16 is in tension with experiment
    - HPQCD '19 is compatible with experiment
    - RBC/UKQCD '18 result still missing renormalization factors
    - theory uncertainty dominates experimental one



[Di Luzio et al., JHEP 12 (2019) 009]

- similar picture in $|V_{td}|, |V_{ts}|$
    - lattice results in slight tension, but all compatible with sum-rules (King et al. '19)
    - unitarity-triangle fits favour HPQCD '19 result



[HPQCD 19, Phys. Rev. D 100, 094508]
[King et al. 19, JHEP 05 (2019) 034]

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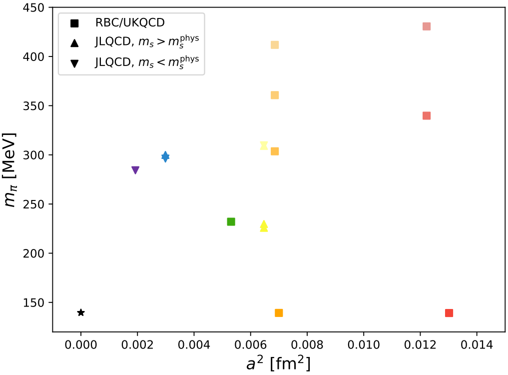
RBC/UKQCD '18 project:

- only ratios $\mathcal{B}_{B_s}/\mathcal{B}_{B_d}$ and ξ
- error dominated by m_H^{-1} extrapolation

Next step: Joint effort with JLQCD collaboration

- 3 additional lattice spacing, down to $a = 0.044$fm
- improved reach to almost am_b^{phys}



[PoS LATTICE2021 (2022) 224]

BSM $B - \bar{B}$ mixing on JLQCD and RBC/UKQCD $N_f = 2+1$ DWF ensembles

Peter Boyle,[a,b] Luigi Del Debbio,[a] Felix Erben,[a,*] Andreas Jüttner,[c,d] Takashi Kaneko,[e,f] Michael Marshall,[a] Antonin Portelli,[a] J Tobias Tsang[g] and Oliver Witzel[h]

& **Jonathan Flynn,[c] Rajnandini Mukherjee[c]**

[a]*Higgs Centre ... y, The University of Edinburgh, Edinburgh EH9 3FD, United Kingdom*

[PoS LATTICE2021 (2022) 224]

- fully-relativistic heavy-quark actions
- We simulate at multiple heavy-quark masses $am_h \sim am_c \rightarrow am_h \lesssim am_b$
  - $\Rightarrow$ Extrapolation in $am_h \rightarrow am_b$ as part of global fit
- Non-perturbative renormalization to obtain $\mathcal{B}_{B_s}, \mathcal{B}_{B_d}$ for all 5 dimension-6 operators
- 6 lattice spacings $a = 0.044\text{fm}$ - $a = 0.11\text{fm}$
- 2 ensembles at M_π^{phys}

Domain-Wall Fermions

- we use "Domain-Wall Fermions"
  - automatic $O(a)$ improvement in absence of odd powers in a

  $\Rightarrow$ reduced discretisation effects
  - chirally symmetric formulation

  $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}$$

$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{2/2} & \mathcal{O}_{2/3} \\ \mathcal{O}_{3/2} & \mathcal{O}_{3/3} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{4/4} & \mathcal{O}_{4/5} \\ \mathcal{O}_{5/4} & \mathcal{O}_{5/5} \end{pmatrix} \end{pmatrix}$$

Block-structure:

- $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix
- linearly independent from each other and from $\mathcal{O}_1$
- more complicated mixing pattern for other lattice fermions

[Boyle et al., JHEP 10 (2017) 054]

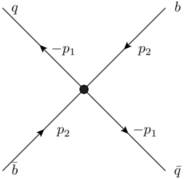
Non-perturbative renormalisation

$$\langle O \rangle_i^{\mathrm{S}}(\mu) = \lim_{a^2 \to 0} \sum_{j=1}^{5} [Z_O^{\mathrm{S}}(a, \mu)]_{ij} \langle O \rangle_j^{\mathrm{bare}}(a)$$

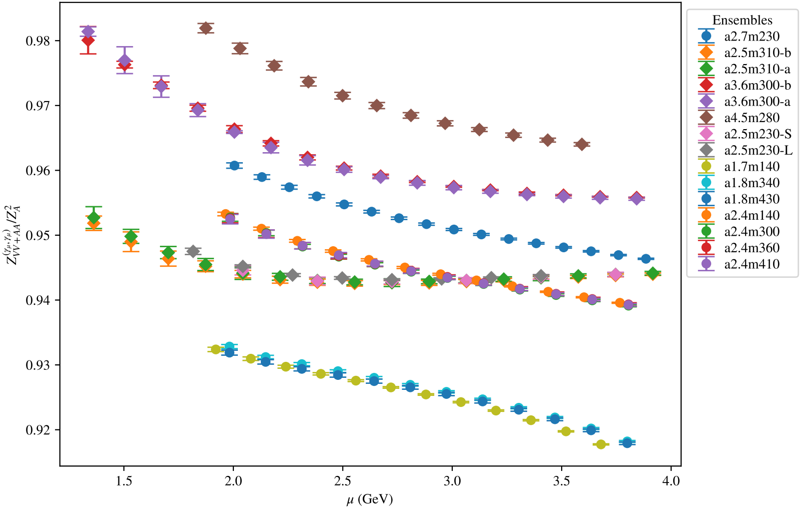
for some regularisation independent scheme S at mass scale μ.
Continuum perturbation theory can then match

$$\langle O \rangle_i^{\overline{\mathrm{MS}}}(\mu) = R^{\overline{\mathrm{MS}} \leftarrow \mathrm{S}} \langle O \rangle_i^{\mathrm{S}}(\mu)$$

We use the "RI-SMOM" scheme. Requires computation of four-quark vertices for $(\bar{b}q) \to (\bar{q}b)$. [Boyle et al., JHEP 10 (2017) 054]

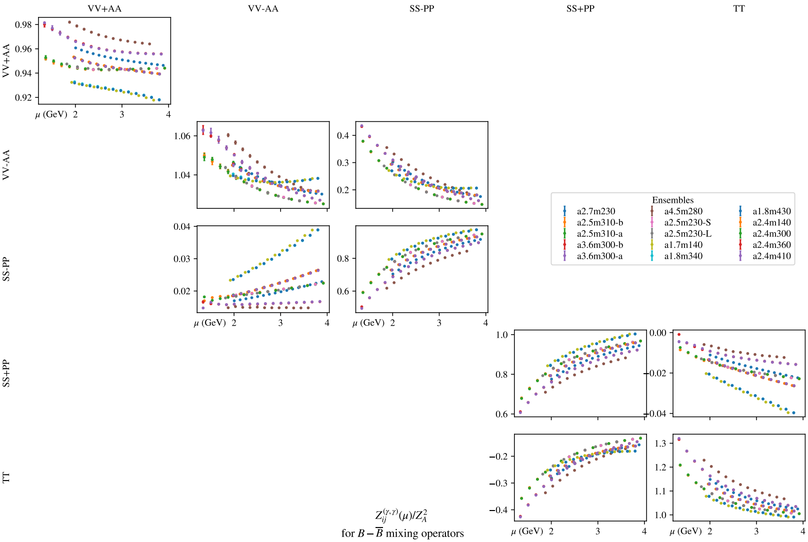


NON-PERTURBATIVE RENORMALISATION - $VV + AA$



plot and work by Rajnandini Mukherjee (University of Southampton)

Non-perturbative renormalisation - full matrix



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FITS TO LATTICE CORRELATION FUNCTIONS

On each ensemble (i.e. at fixed L, a, m_l, m_h) we measure

- 2pt functions

$$\langle B_q(t) B_q^\dagger(0) \rangle \rightsquigarrow M_{B_q}, f_{B_q}$$

- 3pt functions

$$\langle B_q(\Delta T) \mathcal{O}_i(t) B_q^\dagger(0) \rangle \rightsquigarrow M_{B_q}, f_{B_q}, \mathcal{B}_{B_q}^{[i]}$$

- in practice, we define ratios of 3pt and 2pt functions

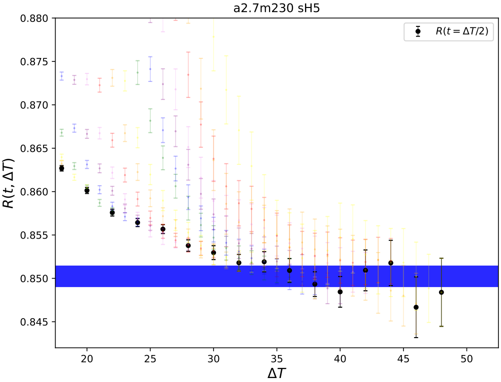
$$R_{B_q}^{[i]}(\Delta T, t) \xrightarrow{t, \Delta T, (\Delta T - t) \to \infty} \mathcal{B}_{B_q}^{[i]}$$

Fits to lattice correlation functions

- 15 ensembles
- 5 operators
- 4-6 heavy-quark masses per ensemble
- heavy-light and heavy-strange sector

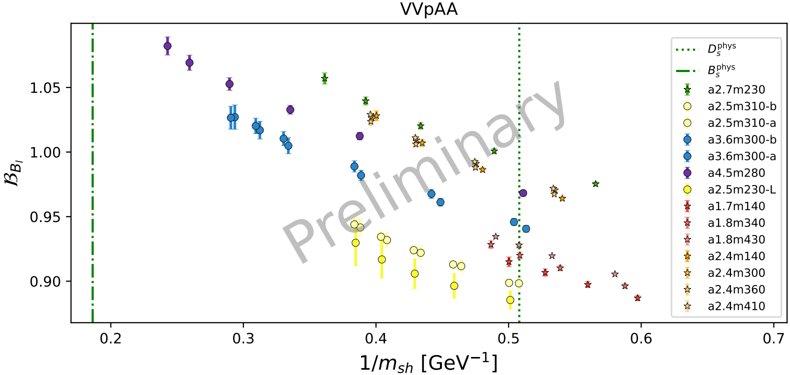
$\Rightarrow$ over 700 combined fits

- multiple values for ΔT to control fits better



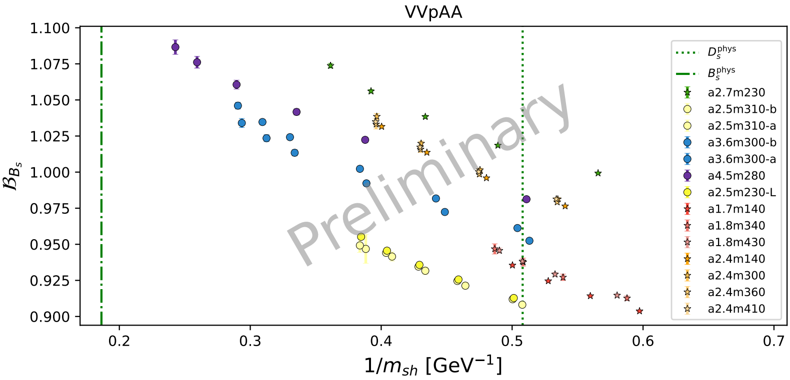
- Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble

BAG PARAMETER $\mathcal{B}_{B_l}$ - $VV + AA$



- each data point from one fit like on previous slide
- very fine JLQCD ensembles reach far towards am_h^{phys} (a4.5m280)
- two ensembles at M_π^{phys} control chiral limit (a1.7m140, a2.4m140)
- 3 lattice spacings each for the RBC/UKQCD and JLQCD sets of ensembles

BAG PARAMETER $\mathcal{B}_{B_s}$ - $VV + AA$



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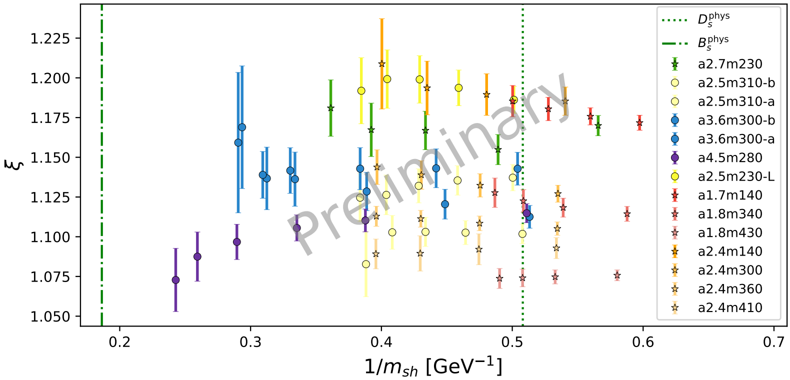

- this $SU(3)$-breaking ratio is close to 1
- only a mild dependence on heavy-quark mass
- small discretisation effects in a

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Other neutral meson mixings

For other neutral mesons $M^0 \in \{K, D, B_q\}$

$$\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle = \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} + \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD}$$
$$= \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle + \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}$$

short-distance contribution:

- t enhancement for K, $B_{(s)}$
- additional CKM hierachy enhancement for $B_{(s)}$
- sub-dominant for D, but ok to describe CP-violating contributions

long-distance contribution:

- relevant but smaller than short-distance for K
- dominant for D
- CKM-suppressed for $B_{(s)}$

SHORT-DISTANCE CONTRIBUTIONS

$$\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} = \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle$$

- very similar theory for K,D,$B_{(s)}$ mixings
- fine lattice spacings to control am_H effects mostly a problem for $B_{(s)}$
  - $\Rightarrow$ we can currently simulate at M_K and M_D, but not at $M_{B_{(s)}}$
- cost dominated by inversions, more expensive for lighter quarks
  - $\Rightarrow$ on a given ensemble, single correlation function for K more expensive than for D than for $B_{(s)}$
  - ... but more correlation functions needed for m_h-extrapolation in $B_{(s)}$

LONG-DISTANCE CONTRIBUTIONS

$$\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD} = \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}$$

- for $K-\bar{K}$ mixing, concepts have been derived in [Christ et al., Phys.Rev.D 91 (2015)]
  - the Euclidean lattice finite-volume estimator has growing exponentials for $E_n(L) < M_M$
  - careful treatment requires knowledge of the FV spectrum $E_n(L)$
- formalism can be extended to more general cases, including $D-\bar{D}$, e.g. [Briceño et al., Phys. Rev. D 101 (2020)], [FE et al., JHEP 04 (2023) 108)], [Jackura et al., PoS LATTICE2022 (2023) 062]
- problem for $D-\bar{D}$ mixing: **finite-volume effects** are not yet controllable above 4-particle thresholds like $D^0 \to K\pi\pi\pi$
- first steps towards accounting for these effects in a programme towards hadronic $D \to K\pi$ decays: [Joswig, Hansen, FE et al., PoS LATTICE2022 (2023) 063]

	short-distance	long-distance
$K-\bar{K}$	"easy"	"difficult"
$D-\bar{D}$	"easy"	"very difficult"
$B_{(s)}-\bar{B}_{(s)}$	"difficult"	negligible

Conclusions

- Multiple lattice groups have computed B_q mixing parameters $\Delta m_d, \Delta m_s$
- also access to $\Delta\Gamma_d, \Delta\Gamma_s$ and BSM models via 5-operator basis
- broad agreement between different lattice actions and approaches
- theory uncertainty vastly dominates experimental uncertainty
- current programme by RBC/UKQCD and JLQCD aims to lower the uncertainty via
  - simple renormalisation for chiral Domain-Wall Fermions
  - fully relativistic treatment of heavy-quark
  - very fine lattice spacings
  - large variety of ensembles to control relevant limits
- Long-distance contribution relevant for $K - \bar{K}$ and in particular $D - \bar{D}$ mixing
- formalism to compute them conceptually clear but very challenging

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Backup

LATTICE SETUP

- RBC-UKQCD's 2+1 flavour domain wall fermions [Blum et al. Phys.Rev.D 93 (2016) 7]
  - pion masses from $M_\pi = 139$ MeV to $M_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5m_b$, using a stout-smeared action ($\rho = 0.1$, $N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ [Boyle et al. arxiv:1812.08791]
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at M_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions [Kaneko et al. EPJ Web Conf. 175 (2018) 13007]
  - pion masses from $M_\pi = 226$ MeV to $M_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

LATTICE SETUP

	L/a	T/a	a^{-1} [GeV]	M_π [MeV]	$M_\pi L$	hits $\times N_{\rm conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48×90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32×100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32×101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64×82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32×83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32×76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32×81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24×100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16×100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16×100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16×100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48×72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24×50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24×50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32×50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last column shows names used by RBC/UKQCD ("R/U") and JLQCD ("J").