B-meson lifetimes vs. hadronic B-meson decays

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Inclusive B-meson decays

Motivation

- The lifetime $\tau = \Gamma^{-1}$ is a fundamental property of particles
- For heavy hadrons H_Q, systematic framework to compute Γ

  $m_Q \gg \Lambda_{QCD}$
- Focus on the B-system $m_b \sim 4.5\,\text{GeV} \gg 0.5\,\text{GeV} \sim \Lambda_{QCD}$
  - Experimental precision very high $\mathcal{O}(‰)$ [HFLAV, PDG]
  - Aim at competitive theoretical precision to both
    - Test the SM and the framework used
    - Perform indirect NP searches

The total decay width of a B-meson

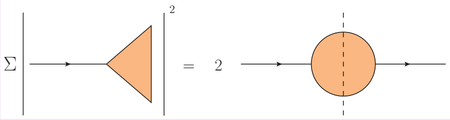
- Start from the definition

$$\Gamma(B) = \frac{1}{2m_B} \sum_n \int_{\text{PS}} (2\pi)^4 \delta^{(4)}(p_n - p_B) |\langle n|\mathcal{H}_{eff}|B\rangle|^2$$

- Use optical theorem to rewrite [Shifman, Voloshin '85]

$$\Gamma(B) = \frac{1}{2m_B} \text{Im} \langle B| i \int d^4x \, \text{T}\Big\{\mathcal{H}_{eff}(x), \mathcal{H}_{eff}(0)\Big\}|B\rangle$$

- $\mathcal{H}_{eff}$ - weak effective Hamiltonian describing b-quark decays



The heavy quark expansion (HQE)

- The b-quark carries most of the hadron momentum $p_B^\mu = m_B v^\mu$

- Convenient parametrisation

$$p_b^\mu = m_b v^\mu + k^\mu \qquad k \sim \Lambda_{QCD} \ll m_b$$

- Introduce rescaled b-quark field

$$b(x) = e^{-i m_b v \cdot x}\, b_v(x)$$

- Action of the covariant derivative becomes

$$i D_\mu b(x) = e^{-i m_b v \cdot x} \big(m_b v_\mu + i D_\mu \big) b_v(x)$$

$D_\mu = \partial_\mu - i A_\mu^a(x) t^a$

The HQE

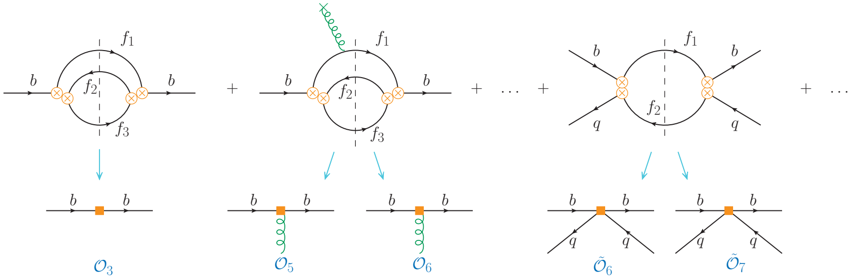
- Obtain systematic expansion

$$\Gamma(B) = \underbrace{\Gamma_3}_{\Gamma(b)} + \underbrace{\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]}_{\delta\Gamma(B)}$$

  - $\Gamma_d, \tilde{\Gamma}_d$ - short distance coefficients
  - $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local operators bilinear in the heavy quark field
  - $\Gamma(b)$ - total decay width of free b quark
  - $\delta\Gamma(B)$ - effects due to interaction with soft gluons and quarks

The HQE

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$



Very advanced framework thanks to huge effort of big community

Status of the HQE: perturbative side

$$\Gamma_d = \Gamma_d^{(0)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)\Gamma_d^{(1)} + \left(\frac{\alpha_s(m_b)}{4\pi}\right)^2 \Gamma_d^{(2)} + \ldots$$

Semileptonic modes (SL)	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 Czakon, Czarnecki, Dowling '21
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi '13 Mannel, Pivovarov, Rosenthal '15
$\Gamma_6^{(1)}$	Mannel, Moreno, Pivovarov '19
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10
$\tilde{\Gamma}_6^{(1)}$	Lenz, Rauh '13

Non-leptonic modes (NL)	
$\Gamma_3^{(2)}$	Czarnecki, Slusarczyk, Tkachov '05 *
$\Gamma_3^{(1)}$	Ho-Kim, Pham '83; Altarelli, Petrarca '91 Bagan et al. '94; Krinner, Lenz, Rauh '13 Lenz, Nierste, Ostermaier '97
$\Gamma_5^{(1)}$	Mannel, Moreno, Pivovarov '23 **
$\Gamma_6^{(0)}$	Lenz, MLP, Rusov '20 Mannel, Moreno, Pivovarov '20
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste '02 Franco, Lubicz, Mescia, Tarantino '02
$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03

* Only partial result

** Only massless final states

Status of the HQE: non-perturbative side

	B_d, B^+	B_s
$\langle \mathcal{O}_5 \rangle$	Fits to SL data $^{\diamond}$ Lattice QCD $^{+}$ HQET sum rules *	Spectroscopy relations **
$\langle \mathcal{O}_6 \rangle$	Fits to SL data $^{\diamond}$ EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$	Sum rules estimates ** EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET sum rules ‡	HQET sum rules ‡
$\langle \tilde{\mathcal{O}}_7 \rangle$	Vacuum insertion approximation	

$^{\diamond}$[Bordone, Capdevila, Gambino '21; Bernlochner, Fael, Olschewsky, Persson, van Tonder, Vos, Welsch '22]

$^{+}$[Gambino, Melis, Simula '17; Bazavov et al. '18] *[Ball, Braun '94; Neubert '96]

**[Bigi, Mannel, Uraltsev '11] ‡[Kirk, Lenz, Rauh '18; King, Lenz, Rauh '20]

The dim-6 two-quark operator contributions

- Sizeable contribution to $\Gamma(B)$ due to Darwin operator

[Lenz, MLP, Rusov '20; Mannel, Moreno, Pivovarov '20]

$$\Gamma(B) = \Gamma_0 \left[5.53 - 0.14 \frac{\mu_\pi^2(B)}{\text{GeV}^2} - 0.24 \frac{\mu_G^2(B)}{\text{GeV}^2} - 1.35 \frac{\rho_D^3(B)}{\text{GeV}^3} + \ldots \right]$$

where

$$\rho_D^3(B) = \frac{\langle B|\bar{b}_v (iD_\mu)(iv \cdot D)(iD^\mu) b_v |B\rangle}{2m_B}$$

- Potential large effect, particularly in $\tau(B_s)/\tau(B_d)$

What is the value of ρ_D^3?

- Tension between different extractions of ρ_D^3 from fits see talk by K. Vos

  [Bordone et al, 21; Bernlochner et al. '22]

- Alternatively, use EOM for gluon field strength tensor

  e.g. [Bigi, Mannel, Uraltsev '11]

$$\mathcal{O}_{\rho_D} = \frac{1}{4m_B}\bar{b}_v[iD_\mu,[iD^\rho,iD^\mu]]v_\rho b_v = -\frac{g_s^2}{4m_B}(\bar{b}_v\gamma^\mu t^a b_v)\sum_q(\bar{q}\gamma_\mu t^a q) + \mathcal{O}\left(\frac{1}{m_b}\right)$$

  - Determine ρ_D^3 from dim-6 four-quark matrix elements
  - However obtain large $\mathrm{SU}(3)_F$ breaking effects ~ 50%!

$$\frac{\rho_D^3(B_s)}{\rho_D^3(B_d)} = \frac{f_{B_s}^2\, m_{B_s}}{f_B^2\, m_B} \approx 1.5$$

The observables

- Compute total widths

$$\Gamma(B) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

- And lifetime ratios

$$\tau(B^+_{(s)})/\tau(B_d) = 1 + \left[\delta\Gamma(B_d)^{\mathrm{HQE}} - \delta\Gamma(B^+_{(s)})^{\mathrm{HQE}}\right] \tau(B^+_{(s)})^{\mathrm{exp}}$$

- No two-quark contributions for $\tau(B^+)/\tau(B_d)$ in isospin limit

- Crucial role of $\mathrm{SU(3)_F}$ breaking effects for $\tau(B_s)/\tau(B_d)$

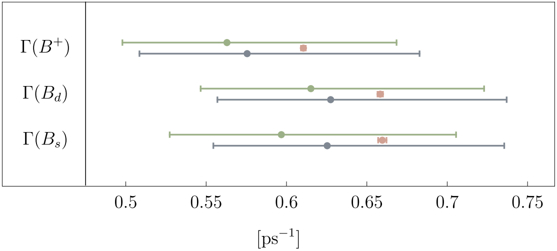
Results

Scenario A

- ◇ Larger inputs for B_d

[Bordone et al. '21]

- ◇ Larger $\text{SU}(3)_\text{F}$ breaking

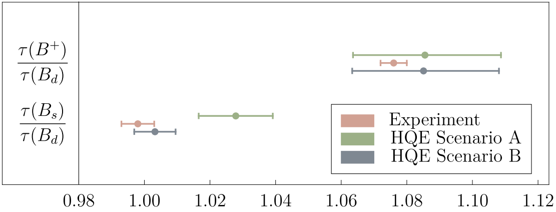


Scenario B

- ◇ Smaller inputs for B_d

[Bernlochner et al. '22]

- ◇ Smaller $\text{SU}(3)_\text{F}$ breaking



[Lenz, MLP, Rusov '22]

Results

- Overall good agreement of HQE and data for B-system
- For the total decay widths
  - Large uncertainties, dominated by scale variation in Γ_3

    Only NLO-QCD corrections included so far
  - Crucial the computation of α_s^2-corrections to NL b-decays

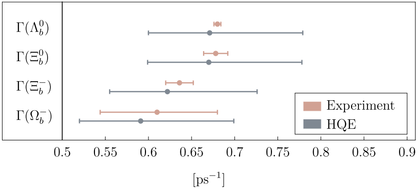
    [Egner, Fael, Schönwald, Steinhauser (in progress)]
- For the ratio $\tau(B^+)/\tau(B_d)$
  - Dominant uncertainties due to four-quark matrix elements

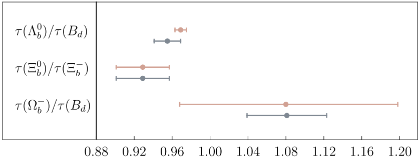
    Lattice determination of bag parameters highly desirable
- For the ratio $\tau(B_s)/\tau(B_d)$
  - Dominant uncertainties due to two-quark matrix elements
  - Tension with data in one scenario

    Need better control over size of non-pert inputs and $SU(3)_F$ break.

What about other heavy hadrons?

HQE for b-baryons



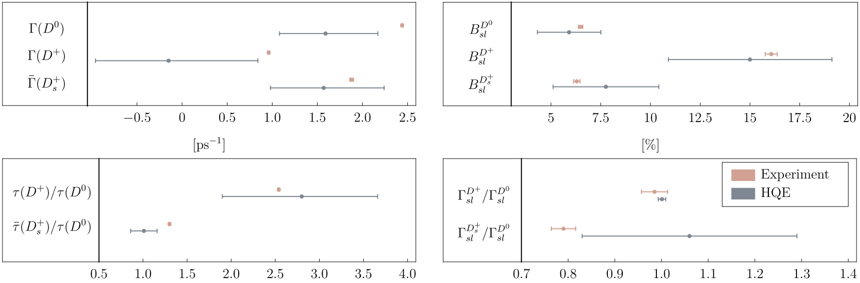


[Gratrex, Lenz, Melić, Nišandžić, MLP, Rusov ’23]

- Very good agreement of HQE predictions with data
- Main sources of uncertainties
  - For total widths - scale variation in leading term Γ_3
  - For lifetime ratios - dim-6 four-quark matrix-elements

No first principle determinations for all baryons, rely on simplified models of QCD

Test the HQE in the charm sector



[King, Lenz, MLP, Rauh, Rusov, Vlahos '21]

- HQE able to explain observed pattern
- But very large uncertainties, mainly due to
  - Charm quark mass
  - Poorly known non-perturbative inputs

see also [Gratrex, Melić, Nišandžić '22; Dulibič, Gratrex, Melić, Nišandžić '23]

Back to the B-system

B-lifetime ratios

- Lifetime ratios are theoretically more clean
- In presence of NP effects

$$\underbrace{\frac{\tau(B^+)}{\tau(B_d)}}_{\text{exp.}} = 1 - \underbrace{\tau(B^+)\left[\delta\Gamma(B^+) - \delta\Gamma(B_d)\right]^{\text{HQE}}}_{\text{theory}} - \underbrace{\tau(B^+)\left[\delta\Gamma(B^+) - \delta\Gamma(B_d)\right]^{\text{NP}}}_{\text{indirectly constrained}}$$

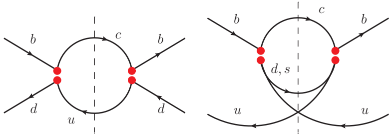
- Potential to constrain specific BSM operators
- Mainly limited by theory uncertainties
- Until further insights on $\tau(B_s)/\tau(B_d)$, use only $\tau(B^+)/\tau(B_d)$

However larger uncertainties!

BSM effects in $\tau(B^+)/\tau(B_d)$ and mixing

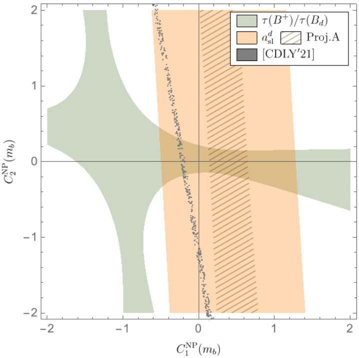
- How large is space for NP in $b \to c\bar{u}d(s)$ decays ?
- Repeat computation with 20 additional NP operators, also for a_{sl}^d

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}^{\rm SM} + \mathcal{H}_{eff}^{\rm NP}$$



Compare with study of decays like $\bar{B}_s \to D_s^+ \pi^-$

[Cai, Deng, Li, Yang '21]



[Lenz, Müller, MLP, Rusov '22]

Conclusions

- Up-to-date analysis of B-meson lifetimes (ratios) within HQE
- Overall, good agreement with data but larger uncertainties
- Plenty of room for improvement
  - Higher order QCD corrections e.g. $\Gamma_3^{(2)}, \tilde{\Gamma}_6^{(2)}$

    Planned by U. Nierste, M. Steinhauser et al. in Karlsruhe
  - Determination of $\langle \tilde{O}_6 \rangle$ by lattice QCD

    Planned by O. Witzel, M. Black in Siegen
  - Better control on two-quark non-perturbative inputs

    Crucial impact on $\tau(B_s)/\tau(B_d)$
- With higher precision, potential to constrain some NP operators

Study of the hadronic decays

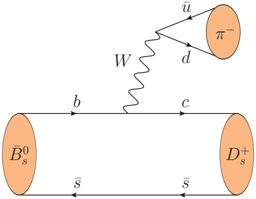
$$\bar{B}^0_{(s)} \to D^+_{(s)} K^-(\pi^-)$$

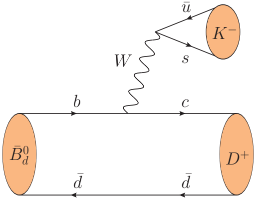
within LCSRs

[Balitsky, Braun, Kolesnischenko '89]

Work in progress in collaboration with A. Rusov

The non-leptonic decays $\bar{B}^0_{(s)} \to D^+_{(s)} K^-(\pi^-)$





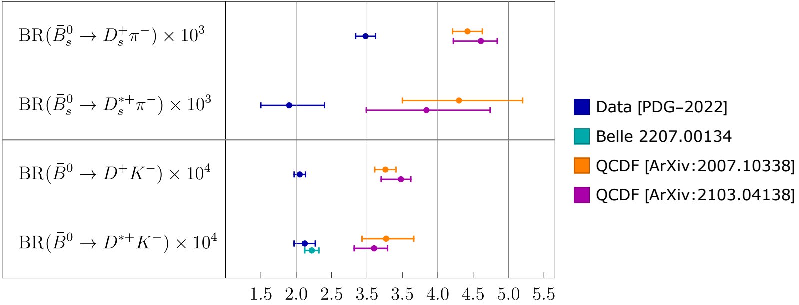
- Tree-level decays induced by $b \to c\bar{u}q$ transitions with $q = d, s$
- Theoretically "clean" channels

  no pollution due to penguin and annihilation topologies
- Golden modes for QCD-factorisation (QCDF) framework

  [Beneke, Buchalla, Neubert, Sachrajda '99 -'01]

◇ Tension between QCDF predictions and data ranging $(2-7)\,\sigma$

Credit to A. Rusov



■ [Bordone, Gubernari, Huber, Jung, van Dyk '20] ■ [Cai, Deng, Li, Yang '21]

Status of power corrections

- Systematic study of power corrections is challenging in QCDF
- First estimates of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections

[Bordone, Gubernari, Huber, Jung, van Dyk '20]

  - Computed non-factorisable soft-gluon exchange within LCSRs
  - Found very small effect

$$\frac{\mathcal{A}(\bar{B}^0_{(s)} \to D^+_{(s)} L^-)_{\rm NLP}}{\mathcal{A}(\bar{B}^0_{(s)} \to D^+_{(s)} L^-)_{\rm LP}} \simeq -[0.06, 0.6]\%$$

- Can we obtain an alternative estimate?

The decay amplitude

- Use the weak effective Hamiltonian

$$\mathcal{A}(\bar{B}_s^0 \to D_s^+ \pi^-) = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \Big[C_1 \langle O_1 \rangle + C_2 \langle O_2 \rangle \Big]$$

$$O_1 = \big(\bar{c}\gamma_\mu (1-\gamma_5) b\big)\big(\bar{d}\gamma^\mu (1-\gamma_5) u\big) \qquad O_2 = \big(\bar{c}\gamma_\mu (1-\gamma_5) t^a b\big)\big(\bar{d}\gamma^\mu (1-\gamma_5) t^a u\big)$$

- In naive QCDF

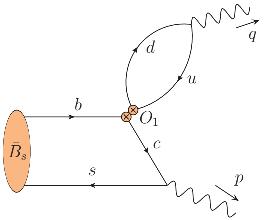
$$\langle O_1 \rangle \overset{\text{NQCDF}}{=} i f_\pi (m_{B_s}^2 - m_{D_s}^2) f_0^{B_s D_s}(m_\pi^2) \qquad \langle O_2 \rangle \overset{\text{NQCDF}}{=} 0$$

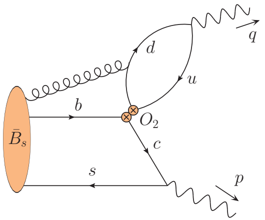
- First estimate of $\langle O_2 \rangle$ beyond NQCDF using two-point sum rule

[Blok, Shifman '93]

$$C_2 \langle O_2 \rangle / C_1 \langle O_1 \rangle \sim 13\%$$

New estimate of decay amplitude within LCSRs





- Start from three-point correlation function see e.g. [Khodjamirian '00]

$$\mathcal{F}_\mu^{O_i}(p,q) = i^2 \int d^4x \int d^4y \; e^{ip\cdot x} e^{iq\cdot y} \langle 0 | T\{ j_5^D(x),\, O_i(0), j_\mu^\pi(y)\} | \bar{B}(p+q)\rangle$$

$$j_5^D(x) = i m_c (\bar{s}\gamma_5 c)(x) \qquad j_\mu^\pi(y) = (\bar{u}\gamma_\mu\gamma_5 d)(y)$$

Light-cone OPE for the correlation functions

- Consider the kinematical region of large and negative p^2 and q^2

$$P^2 \equiv -p^2 \qquad Q^2 \equiv -q^2 \qquad P^2 \sim Q^2 \sim m_B \Lambda$$

- Dominant contribution to the correlator comes from

$$x^2 \sim 0 \qquad y^2 \sim 0 \qquad (x-y)^2 \nsim 0$$

x and y are aligned along different light-cone directions!

- Double LC expansion of the correlator $\mathcal{F}_\mu^{O_2}$ not feasible

$$\langle 0|\bar{q}(z_1 n) G_{\mu\nu}(z_2 \bar{n}) h_v(0)|\bar{B}(v)\rangle = ?$$

$v^\mu = (n^\mu + \bar{n}^\mu)/2 \qquad n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$

- Perform instead LC-local expansion around $x^2 \sim 0$ but $y^\mu \sim 0$

Light-cone OPE for the correlation functions

- For light-quark loop use local expansion of propagator up to $G_{\mu\nu}$

e.g. [Balitsky, Braun '89]

$$S^{(q)}_{ij}(x,y) = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \left[\frac{\delta_{ij}\, \not{k}}{k^2 + i\varepsilon} - \frac{G^a_{\alpha\beta} t^a_{ij}}{4} \frac{(\not{k}\,\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{k})}{(k^2+i\varepsilon)^2} \right] + \dots$$

- Use two- and three-particle B-meson LCDAs up to twist-six

[Braun, Ji, Manashov '17]

$$\langle 0|\bar{q}(x) G_{\mu\nu}(0) h_v(0)|\bar{B}(v)\rangle \sim \int_0^\infty d\omega_1\, e^{-i\omega_1 v\cdot x} f_{\mu\nu}\big(\{\phi_3, \phi_4, \dots, \phi_6\}(\omega_1)\big)$$

$$\langle 0|\bar{q}(x) h_v(0)|\bar{B}(v)\rangle \sim \int_0^\infty d\omega\, e^{-i\omega v\cdot x} f\big(\{\phi_+, \phi_-, g_+, g_-\}(\omega)\big)$$

The OPE results

- The correlators take the following form

$$\mathcal{F}_\mu^{O_i} = \left(q_\mu (p\cdot q) - p_\mu q^2\right) \mathcal{F}^{O_i}(p^2, q^2)$$

  - Result is transversal with respect to q^μ

- Arrive at OPE for the invariant amplitudes

$$[\mathcal{F}_q^{O_2}(p^2,q^2)]_{\text{OPE}} \sim \int_0^\infty d\omega_1 \sum_{\psi=\phi_3,\ldots} \psi(\omega_1) \sum_{n=1}^{3} \frac{c_n^\psi(\omega_1,q^2)}{\left[\tilde{s}(\omega_1,q^2) - p^2 - i\varepsilon\right]^n}$$

  - Similarly for $\mathcal{F}_q^{O_1}$ - including both 2- and 3-particle contributions

Link OPE to hadronic matrix elements

- Derive double dispersion relations in p^2- and q^2-channels
- Obtain sum-rule for hadronic matrix elements

$$i\langle O_2\rangle = \underbrace{\frac{(m_\pi^2 - q^2)}{f_\pi f_D m_D^2}\int_{m_c^2}^{s_0^D} ds \int_0^\infty d\omega_1 \left[\sum_\psi \psi(\omega_1)\sum_{n=1}^{3}\frac{c_n^\psi(\omega_1,q^2)}{(n-1)!}\, e^{(m_D^2 - s)/M^2}\,\delta_s^{(n-1)}\big(\tilde{s}(\omega_1,q^2) - s\big)\right]}_{\text{contribution due to } \pi \text{ pole}}$$

$$- \underbrace{\frac{(m_\pi^2 - q^2)}{f_\pi}\int_{s_h'^{(\pi)}}^\infty ds' \,\frac{\rho_h(s')}{(s' - q^2)}}_{\text{excited states and continuum}}$$

- Analogously for $\langle O_1\rangle$, however use also QHD in q^2-channel

Inputs and preliminary results

- Use exponential model for LCDAs
  - For $\phi_+, \phi_-, g_+, \phi_3, \phi_4, \tilde{\psi}_4, \psi_4$ use models from [Braun, Ji, Manashov '17]
  - For $g_-, \tilde{\phi}_5, \psi_5, \tilde{\psi}_5, \phi_6$ use models from [Lü, Shen, Wang, Wei '18]
  - Inclusion of $\tilde{\phi}_5, \ldots \psi_6$ necessary to preserve local limit of 3p ME

Also lift of some cancellations between LCDAs!

- Main limitations due to poorly known input parameters
  - Dominant uncertainty coming from λ_H^2, λ_B
- From preliminary analysis
  - Non-factorisable corrections large and positive $\sim (1 - 10)\%$

But with very large uncertainties!

Conclusions

- Study of $\bar{B}_{(s)} \to D_{(s)} K^-(\pi^-)$ decays within LCSRs
- Estimate fact. and non-fact. contributions with same framework

  Alternative to QCDF, however larger uncertainties

- Non-factorisable contributions might be large (but positive)
- Many non-perturbative inputs still poorly known
- Much more to be understood and clarified

Thanks for the attention