Inclusive semileptonic decays

K. Keri Vos

Maastricht University & Nikhef

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established, precise framework
- Extract important CKM parameters V_{cb} and V_{ub}
- Extract power corrections from data
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem → (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \dots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B | O_i^{(k)} | B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \dots | B \rangle$ non-perturbative matrix elements → string of iD
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v (iD_\mu) \dots (iD_{\mu_n}) b_v|B\rangle$$

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to Λ_{QCD}
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B \right\rangle$$

  - $C_i^{(k)}$ perturbative Wilson coefficients
  - $\langle B|\ldots|B\rangle$ non-perturbative matrix elements $\to$ string of iD
  - operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$$

- HQE parameters extracted from lepton energy, hadronic mass and q^2 moments

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD, iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD, iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

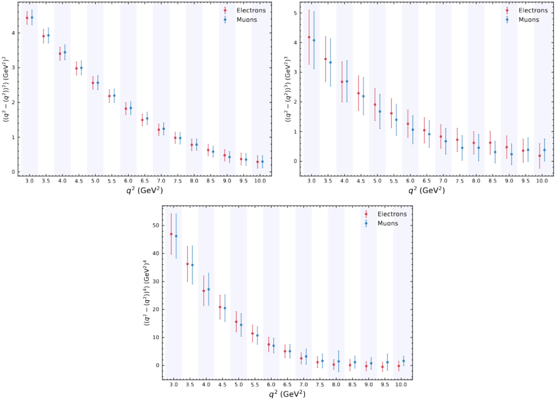
Dilepton momentum

$$\left\langle (q^2) \right\rangle_{\rm cut} = \frac{\int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use some cut on the leptons

q^2 moments

Belle Collaboration [2109.01685, 2105.08001]



Centralized moments as function of q^2_{cut}

Determining V_{cb} and the HQE elements

$$\langle E_\ell^n \rangle, \langle (M_X^2)^n \rangle \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_d^3, r_E, r_G, s_E, s_B, s_{qB}, + \cdots$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb}$$

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)\right.$$
$$\left. + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}(\Gamma^{(D,0)} + \Gamma_0^{(1)}\left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

- Include terms up to $1/m_b^4$* see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

$$|V_{cb}|^{\mathrm{BCG}}_{\mathrm{incl}} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|^{q^2}_{\mathrm{incl}} = (41.69 \pm 0.63) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) \right.$$

$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

Challenges:

- Include higher-order $1/m_b$ and α_s corrections
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

Towards the ultimate precision in inclusive V_{cb}

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)\right.$$
$$\left. + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}(\Gamma^{(D,0)} + \Gamma_0^{(1)}\left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

Challenges:

- Include higher-order $1/m_b$ and α_s corrections
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109
- Information on HQE elements from lattice Talk by Gambino [2005.13730]
- Test $SU(3)$ symmetry for B_s

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\ v_\mu = \delta v_\mu \text{ and } \delta_{RP}\ iD_\mu = -m_b \delta v_\mu$$

- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\ v_\mu = \delta v_\mu \ \text{ and } \ \delta_{RP}\ iD_\mu = -m_b \delta v_\mu$$

  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)

- q^2 moments enable a full extraction up to $1/m_b^4$

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- Agreement with BCG extraction (differs due to branching ratio inputs)

  Bordone,Capdevila, Gambino [2021]

$$|V_{cb}|^{\rm BCG}_{\rm incl} = (42.00 \pm 0.51) \times 10^{-3}$$

- Higher order terms reduce value by 0.25%.

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\mathrm{GeV}^4$$

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\text{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\text{GeV}^4$$

- Inputs for $B \to X_u \ell\nu$ Next, B lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.
- Additional 0.23 uncertainty due to missing higher orders

q^2 moments only analysis

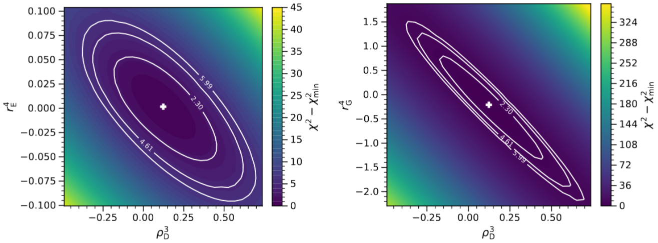
Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\text{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\text{GeV}^4$$

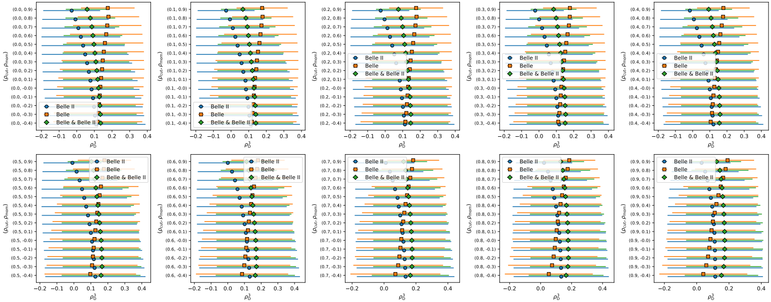
- Inputs for $B \to X_u \ell\nu$ Next, B lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.
- Additional 0.23 uncertainty due to missing higher orders



What about theory correlations?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$
- Included by adding a penalty term to the χ^2
- Scan over large range of values
- V_{cb} constant w.r.t. theory correlations

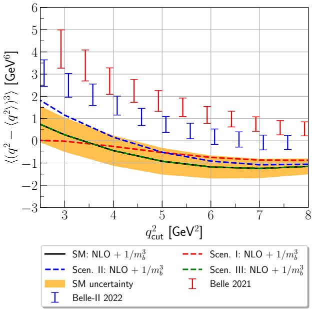


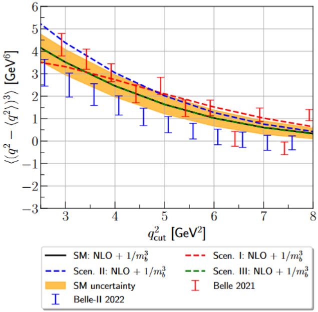
What about ρ_D^3?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Large uncertainties on HQE elements
- Important: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included

Rahimi, Fael, Vos [2208.04282]



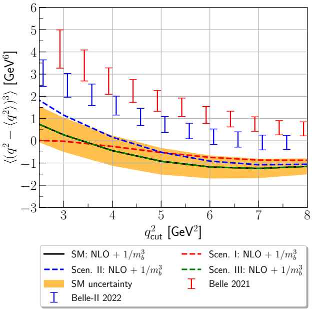


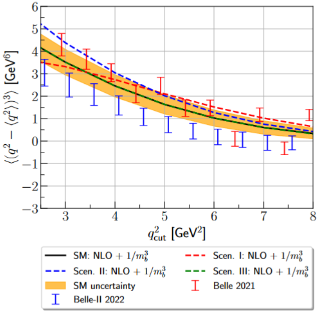
What about ρ_D^3?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Large uncertainties on HQE elements
- Important: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included
- Corrections are negative Steinhauser, Fael, Schoenwald [2205.03410]
- Full analysis including all data is necessary! Bernlochner, Fael, Prim, KKV [in progress]

Rahimi, Fael, Vos [2208.04282]





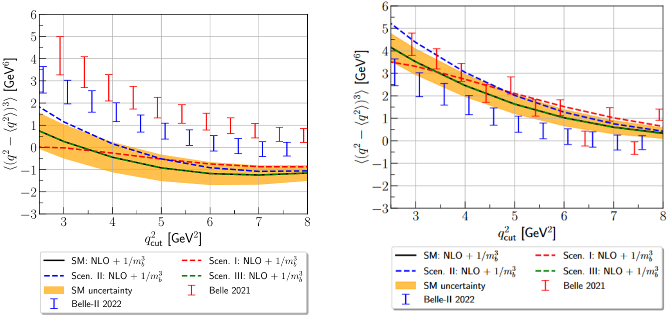
Even higher corrections?

Mannel, Mulatin, KKV [in progress]

- HQE set up with $m_c/m_b \sim \mathcal{O}(1)$
- IR sensitive terms for $m_c \to 0$ Bigi, Mannel, Turczyk, Uraltsev [0911.3322]
  - at dim-6: $1/m_b^3 ln m_c^2$
  - at dim-8: $1/m_b^5 m_b^2/m_c^2 \sim 1/m_b^3 1/m_c^2$
- Numerically: $m_c^2 \sim m_b \Lambda_{\mathrm{QCD}}$
- in progress: Calculation and estimate of these effects

New Physics?

Fael, Rahimi, KKV [2208.04282]



- NP would also influence the moments of the spectrum
- Requires a simultaneous fit of hadronic parameters and NP In progress..

Interplay between exclusive and inclusive: $b \to c$

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ

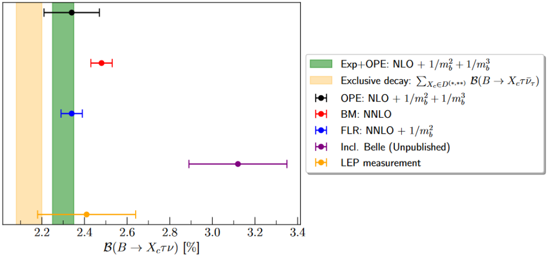
Interplay between exclusive and inclusive: $b \to c$

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ
- Next step ratios with τ!

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$

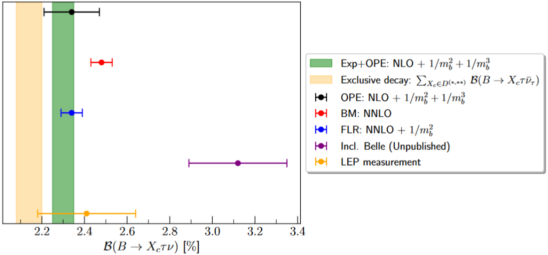


KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ
- Next step ratios with τ! Need new measurements!

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$



The challenge of inclusive $B \to X_u$ decays

The challenge of V_{ub}

Exclusive $B \to \pi\ell\nu$

- Only one form factor
- Combining Lattice QCD [FNAL/MILC, RBC/UKQCD] and QCD sum rules

Recent update:

Leljak, Melic, van Dyk [2102.07233]

$$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$$

The challenge of V_{ub}

Exclusive $B \to \pi\ell\nu$

- Only one form factor
- Combining Lattice QCD [FNAL/MILC, RBC/UKQCD] and QCD sum rules

Recent update:

Leljak, Melic, van Dyk [2102.07233]

$$|V_{ub}|_{\text{excl}} = (3.77 \pm 0.15) \cdot 10^{-3}$$

Inclusive $B \to X_u\ell\nu$

- Experimental cuts necessary to remove charm background
- Local OPE as in $b \to c$ cannot work
- Switch to different set-up using light-cone OPE
- Introduce non-perturbative shape functions ($\sim$ parton DAs in DIS)
- Different frameworks: BLNP, GGOU, DGE, ADFR

Recent update:

Belle [2102.00020]

$$|V_{ub}|_{\text{incl}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))b_v|B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y)-\omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

→ H: Hard scattering kernel at $\mathcal{O}(m_b)$
→ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
→ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

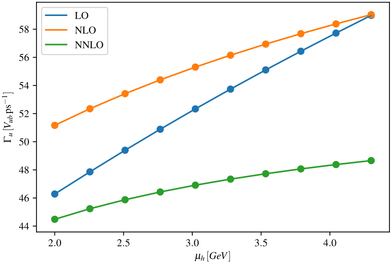
$$d\Gamma = H \otimes J \otimes S$$

→ H: Hard scattering kernel at $\mathcal{O}(m_b)$
→ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
→ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

- In progress: include known α_s^2 corrections

Shape function parametrization

Preliminary! Olschewsky, Lange, Mannel, KKV [2306.xxxx]



- α_s^2 corrections give large corrections [see also Pezcjak 2019]
- Required to make precision predictions

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

  → H: Hard scattering kernel at $\mathcal{O}(m_b)$
  → J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
  → S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- In progress: include known α_s^2 corrections

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

→ H: Hard scattering kernel at $\mathcal{O}(m_b)$
→ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
→ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \to c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$

BLNP update - in progress -

Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

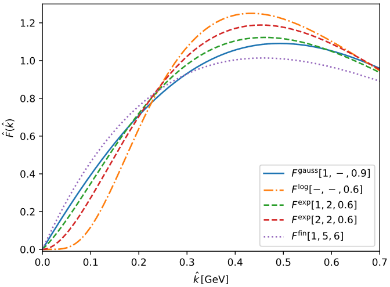
- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

  - $\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
  - $\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
  - $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \to c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization

Shape function parametrization

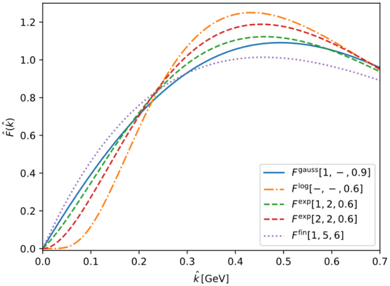
Olschewsky, Lange, Mannel, KKV [2306.xxxx]



- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

Shape function parametrization

Olschewsky, Lange, Mannel, KKV [2306.xxxx]



- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

In progress:

Lange, Mannel, Olschewsky, KKV [in progress]

$$|V_{ub}|_{\text{incl}} = \quad \text{Stay Tuned!}$$

Heavy quark expansion for charm?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

$\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

$\rightarrow$ V_{ub}

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\mathrm{QCD}}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \to s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \to s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Challenges:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?

Challenges:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

In short: how to handle the charm mass?

The HQE for charm

I: $m_Q \sim m_q \gg \Lambda_{\rm QCD}$ OPE for $b \to c\ell\bar{\nu}$

- q is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v(iD^\alpha \cdots iD^\beta)Q_v$
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \ldots\right]\frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

II: $m_Q \gg m_q \gg \Lambda_{\rm QCD}$ start with q dynamical

- four-quark operators $(\bar{Q}_v\Gamma q)(q\bar{\Gamma}Q_v)$

$\to$ removed when matching onto two-quark operators

- RGE running gives $\log(m_q/m_Q)$

III: $m_Q \gg m_q \sim \Lambda_{\rm QCD}$ OPE for $c \to s\ell\bar{\nu}$

- q dynamical degree of freedom
- four-quark operators remain in OPE
- no explicit $\log(m_q/m_Q)$: hidden inside new non-perturbative HQE parameters

IV: $m_Q \gg \Lambda_{\rm QCD} \gg m_q$ for $b \to u$ and $c \to d$ transitions

HQE for Charm revisited

- The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

- Expansion parameters:
  - $1/m_c$
  - α_s
  - m_s/m_c

Fael, Mannel, KKV, hep-ph/1910.05234

HQE for Charm revisited

Fael, Mannel, KKV

- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{4\mathrm{q}}|D\rangle$
- Up to $1/m_c^3$ only <u>one</u> extra HQE param:

$$\tau_0 = 128\pi^2\left(T_1(\mu) - T_2(\mu) - 2\frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c}\right)$$
$$+ \log\left(\frac{\mu^2}{m_c^2}\right)\left[8\tilde{\rho}_D^3 + \frac{1}{m_c}\left(\frac{16}{3}r_G^4 - \frac{16}{3}r_E^4 + \frac{8}{3}s_E^4 - \frac{1}{3}s_{qB}^4 - 12m_s^4\right)\right]$$

- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.

HQE for charm revisited

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV

$$\frac{\Gamma(D \to X_s \ell\nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

HQE for charm revisited

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

Key question: HQE indeed applicable to inclusive charm decays?

HQE for charm revisited

$\rho = m_s^2/m_c^2$

Fael, Mannel, KKV

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- RPI quantities (q^2 moments) depend on reduced set
- Data required to test description
- Comparison of extracted HQE parameters with B decays

Key question: How to handle the charm mass?

How to handle the charm mass?

Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- $\overline{\mathrm{MS}}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\mathrm{kin}} = m_Q^{\mathrm{Pole}} - \left[\overline{\Lambda}\right]_{\mathrm{pert}} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\mathrm{pert}} + \ldots$$

$$[\overline{\Lambda}]_{\mathrm{pert}} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\mathrm{pert}} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.

Short-Distances Masses

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- $\overline{\rm MS}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\rm kin} = m_Q^{\rm Pole} - \left[\overline{\Lambda}\right]_{\rm pert} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\rm pert} + \ldots$$

$$[\overline{\Lambda}]_{\rm pert} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\rm pert} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\boldsymbol{\Lambda_{\rm QCD} < \mu < m_Q}$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??

    $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$

    $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Kinetic Mass

Putting all power corrections to zero!

- $m_c^{\rm kin}(1\text{ GeV}) = 1.16\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 7.7\frac{\alpha_s(m_c)}{\pi} + 69\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

- $m_c^{\rm kin}(0.5\text{ GeV}) = 1.4\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 1.2\frac{\alpha_s(m_c)}{\pi} + 17\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

Kinetic Mass

Putting all power corrections to zero!

- $m_c^{\rm kin}(1\text{ GeV}) = 1.16\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 7.7\frac{\alpha_s(m_c)}{\pi} + 69\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

- $m_c^{\rm kin}(0.5\text{ GeV}) = 1.4\text{ GeV}$ ($m_s \to 0$ limit)

$$\Gamma(c \to s\ell\nu)^{\rm kin} = \Gamma_0 \left[1 + 1.2\frac{\alpha_s(m_c)}{\pi} + 17\left(\frac{\alpha_s(m_c)}{\pi}\right)^2\right]$$

$\mu = 0.5$ GeV touches upon the non-perturbative regime?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Start from vacuum correlator

$$\int d^4x\, e^{-iqx}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2)$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Expand around $q^2 = 0$: ($\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots$)

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right)$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right) = \Pi(0) + \frac{q^2}{12\pi^2}\int\frac{ds}{s}\frac{R(s)}{s-q^2}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right) = \Pi(0) + \frac{q^2}{12\pi^2}\int\frac{ds}{s}\frac{R(s)}{s-q^2}$$

- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s) \qquad (1)$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Expand around $q^2 = 0$: $(\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots)$

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right) = \Pi(0) + \frac{q^2}{12\pi^2}\int\frac{ds}{s}\frac{R(s)}{s-q^2}$$

- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s) \qquad (1)$$

- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344Boushmelev, Mannel, KKV [2301.05607]

$$
\begin{aligned}
\Gamma(c \to s\ell\nu) &= \frac{G_F^2 |V_{cs}|^2}{192\pi^3} \left(\frac{1}{2} \left(\frac{\bar{C}_n}{M_n} \right)^{1/2} \right)^5 \left(1 + \frac{\alpha_s(\mu)}{\pi} a_1 + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 a_2 + \cdots \right) \\
&= \frac{G_F^2 |V_{cs}|^2}{6144\pi^3} \left(\frac{\bar{C}_n^{(0)}}{M_n} \right)^{5/2} \left(1 + \frac{\alpha_s(\mu)}{\pi} \left[a_1 + \frac{5}{2n} \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right] \right. \\
&\quad \left. + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left[a_2 + \frac{5}{2n} a_1 \frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} + \frac{5}{2n} \frac{\bar{C}_n^{(2)}}{\bar{C}_n^{(0)}} + \frac{5}{4n} \left(\frac{5}{4n} - 1 \right) \left(\frac{\bar{C}_n^{(1)}}{\bar{C}_n^{(0)}} \right)^2 \right] + \cdots \right)
\end{aligned}
$$

- Conclusion: pert. series improves a bit
- Scale at which α_s^2 vanishes rather low: 0.7 m_b
- In progress: Similar approach for the charm + power corrections

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision
- Still many things to work on!

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision
- Still many things to work on!
- Stay tuned for new data and updated theory predictions

Outlook - inclusive decays

We are in the High-precision Era in Flavour Physics!

- Reached impressive precision
- Still many things to work on!
- Stay tuned for new data and updated theory predictions

Close collaboration between theory and experiment necessary!

Backup

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV [arXiv: 2105.02163]

Avoid background subtraction by calculating the full inclusive width:

$$\mathrm{d}\Gamma(B \to X\ell) = \mathrm{d}\Gamma(B \to X_c \ell \bar{\nu}) + \mathrm{d}\Gamma(B \to X_u \ell \bar{\nu}) + \mathrm{d}\Gamma(B \to X_c(\tau \to \ell \bar{\nu} \nu)\bar{\nu})$$

- $b \to u \ell \nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \to c(\tau \to \mu \nu \bar{\nu})\bar{\nu}$ contribution: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \to u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \to X_c \ell\nu$ by taking $m_c \to 0$ limit
- For V_{ub} determination
  - large charm background requires experimental cuts
  - reduces the inclusivity and local OPE no longer converges
  - spectrum described by non-local OPE
  - convolution of pert. coefficients with shape function

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2$+$1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \to X_c \ell\nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

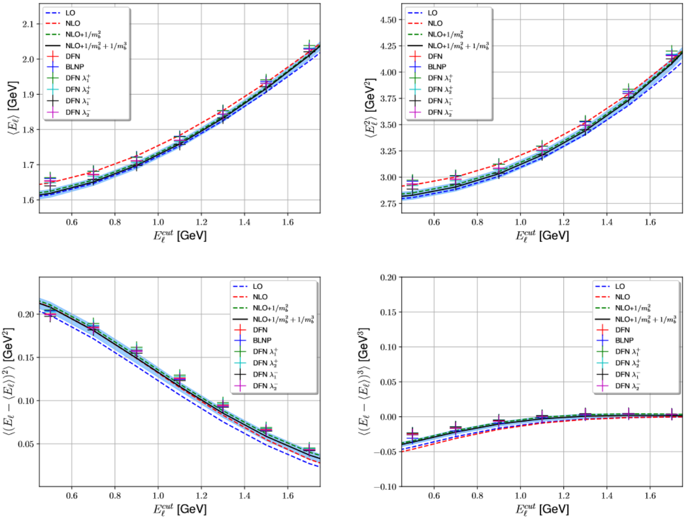
Monte Carlo:

- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
  - Inputs from $B \to X_c \ell \nu$ and $B \to X_s \gamma$ data using KN-scheme Kagan, Neubert 1998
  - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi \ell \bar{\nu}$ and $\bar{B} \to \rho \ell \bar{\nu}$

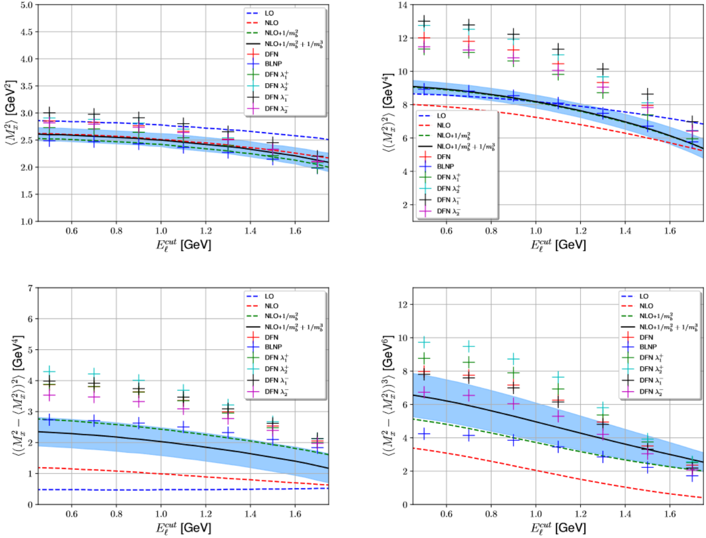
Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

Monte Carlo versus HQE

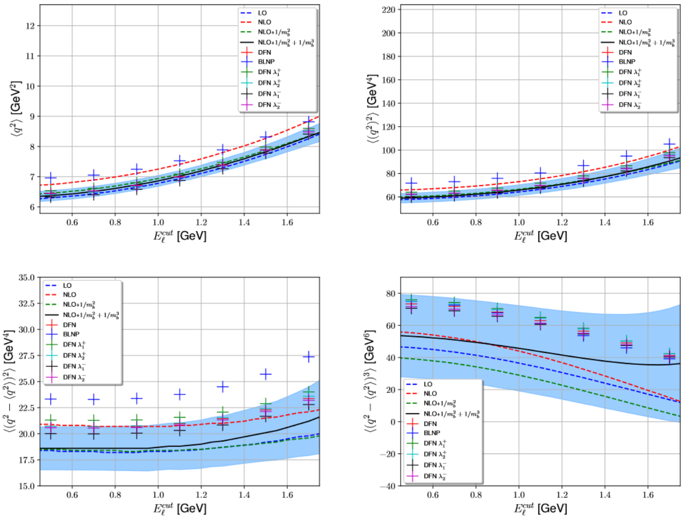
Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Wide spread between MC for higher moments

Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Monte Carlo versus local OPE: remarks

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
  - should include higher moments of the shape-function model?
  - include subleading shape functions?
- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Five-body τ contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\rm tot}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\rm tot}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

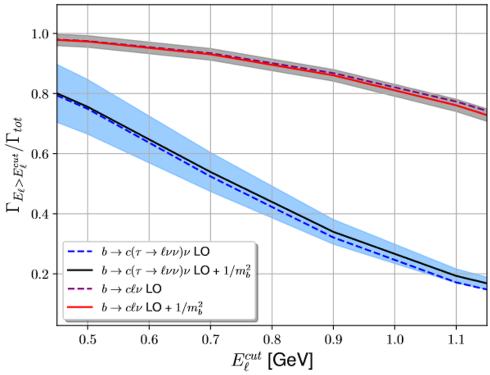
- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

$$\frac{d^8\Gamma}{dq^2\, dq^2_{\nu\bar{\nu}}\, dp^2_{X_c}\, d^2\Omega\, d\Omega^*\, d^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2 - m_\tau^2)(m_\tau^2 - q^2_{\nu\bar{\nu}})\mathcal{B}(\tau \to \mu\nu\nu)}{2^{17}\pi^5 m_\tau^8\, m_b^3 q^2} W_{\mu\nu} L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Rahimi, Mannel, KKV[arXiv: 2105.02163];



No MC data available to test with

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]; Gambino [in progress]
- Full determination up to $1/m_b^4$ from data possible?

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

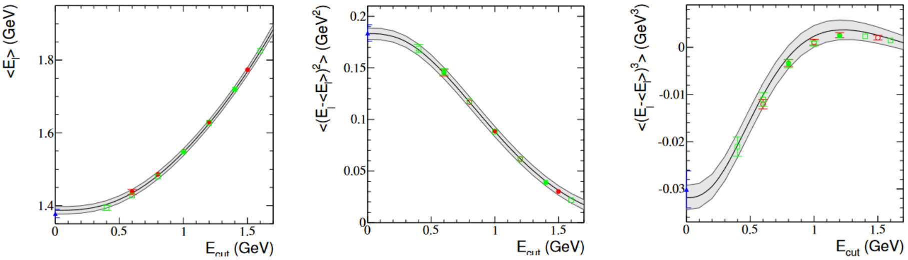
Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

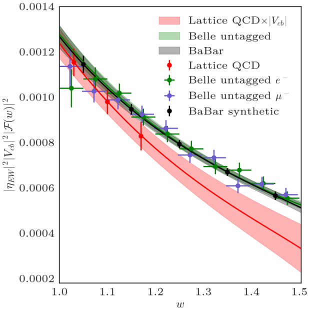
- -0.25% shift due to power corrections

Gambino, Schwanda Phys. Rev. D 89, 014022 (2014)



$B \to D^*$ form factors

Fermilab-MILC [2105.14019]



- Tension between the slope of the lattice and experimental data
- Same form factors determine SM predictions for $R_{D^{(*)}}$
- New experimental and lattice data needed!

The V_{cb} puzzle: Inclusive versus Exclusive decays

Exclusive $B \to D^{(*)}\ell\bar{\nu}$

- Form factor required (only for $B \to D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
  - BGL: model independent based on unitarity and analyticity
  - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

Inclusive $B \to X_c\ell\nu$

- Determined fully data driven including $1/m_b$ power corrections

Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

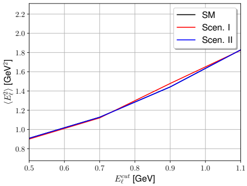
Stay tuned!

Inclusive $b \to c$

Mannel, Rahimi, KKV [in progress]

NP in the τ sector

- Affects also inclusive $B \to X_c \tau \nu$ Rusov, Mannel, Shahriaran [2017]
- Lepton and hadronic moments challenging to measure
- Recently moments of the five-body decay $B \to X_c \tau (\to \mu\nu\nu)\nu$ investigated Mannel, Rahimi, KKV [2105.02163]
- Would also be influenced by NP [in progress]
- Specific NP scenarios from global fit Mandal, Murgui, Penuela, Pich [2004.06726]



Preliminary!

Five-body τ contribution

Rahimi, Mannel, KKV JHEP 09 (2021) 051 [arXiv: 2105.02163];

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\rm tot}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\rm tot}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

$$\frac{d^8\Gamma}{dq^2\, dq^2_{\nu\bar{\nu}}\, dp^2_{X_c}\, d^2\Omega\, d\Omega^*\, d^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2 - m_\tau^2)(m_\tau^2 - q^2_{\nu\bar{\nu}})\mathcal{B}(\tau \to \mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu} L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y) - \omega) f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
$\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

  $\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
  $\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
  $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

- Other approach: OPE with hard-cutoff μ Gambino, Giordano, Ossola, Uraltsev
  - Use pert. theory above cutoff and parametrize the infrared
  - Different definition of the shape functions
- Shape functions have to be parametrized and obtained from data

New Physics explanation?

- Too many to count: exclusive $B \to D^{(*)}$ in combination with

$$R_{D^{(*)}} = \frac{B \to D^{(*)}\tau\nu}{B \to D^{(*)}\mu\nu}$$

- For inclusive $b \to c$ less analyses
  - RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
  - RH-current to moments Feger, Mannel, et. al. [2010]
  - NP for moments KKV, Fael, Rahimi [in progress]