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Luminosity calibration and beam-beam interactions at hadron colliders with Q-Gaussian beams

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Motivation

- ***Luminosity*** ***($\mathcal{L}$)*** – Is the quantity that characterizes the intensity of particles collisions at the interaction point
- When two beams approach each other => strong electromagnetic interactions => beam perturbations by the non-linear beam-beam force (bb) which results in: orbit shift of the whole bunch (coherent effect) and particle redistribution inside the bunch (incoherent effect).

- This bb effect impacts collider luminosity in two ways.[1,2]
  - the number of collisions is changed from expected values;
  - the luminosity calibration via van-der-Meer scan is biased.

- It was observed that the actual bunch profile does not follow the exact Gaussian distribution, but they have a different tail's population than Gaussian [3].

- The current models for bb interaction assume that the colliding beams have Gaussian particle densities, but for high precision luminosity calibration, for example less than 1% for HL-LHC, more precise models of bb interaction that account for the deviation from Gaussianity should be considered.

Basic Concepts

- Luminosity is characteristic of the beam collision, and its definition from the beam parameters is (simplified expression for ultrarelativistic beams, zero crossing angle*:

$$\mathcal{L}(\Delta \boldsymbol{r}_{\perp}) = N_1 N_2 \int \rho_1^{lab,\perp}(\boldsymbol{r}_{\perp} - \Delta \boldsymbol{r}_{\perp})\, \rho_2^{lab,\perp}(\boldsymbol{r}_{\perp}) d\boldsymbol{r}_{\perp},$$

where:

$N_{1,2}$ – intensity of the colliding bunches;

$\rho_2^{lab,\perp}(\boldsymbol{r}_{\perp})$ – transverse normalized particle distribution densities in the colliding bunches in the lab frame;

$\Delta \boldsymbol{r}_{\perp}$– transverse distance between the colliding beams orbits “beam separation”.

- The integral represents the convolution of beam densities, and it is called ***Overlap integral (Ω)***

Ω represents the probability of single collision

- Luminosity $\mathcal{L}$ is related to the overlap integral as [4]:

$$\mathcal{L} = N_1 N_2 \Omega$$

Basic Concepts

- Luminosity is determined based on some well-known process or phenomena, i.e in a practice there is the measured quantity proportional to "true" luminosity:

$$\mu = \sigma_{vis}\mathcal{L}$$

where μ is the reaction rate; and σ_{vis} is the visible cross-section.

- σ_{vis} is a characteristic of the detector-luminometer, and it measures the rates of measured quantity (number of tracks, energy deposited etc)

- The visible cross-section is known as the luminosity calibration constant: by finding the σ_{vis} at specifically optimized beam conditions (usually, head-on collisions, low beam intensity, high beta function)

$$\sigma_{vis} = \frac{1}{N_1 N_2}\frac{\mu_{cond}}{\Omega_{cond}}$$

- the luminosity at any other beam conditions is found as

$$\mathcal{L} = \mu/\sigma_{vis}$$

Basic Concepts

- ***Van-Der-Meer Scan (vdM)*** – it is a beam separation scan method Is used for luminosity calibration where the two beams are separated in the transverse directions by $(\Delta x, \Delta y)$ while the resulting reaction rate $(\mu(\Delta x, 0), (\mu(0, \Delta y))$ is recorded and then fitted using a suitable fit model. The convolved beam Σ_u in the transverse directions is determined as

$$\Sigma_x = \frac{1}{C}\frac{\int \mu(\Delta x, 0) d\Delta x}{\mu(0,0)}, \qquad \Sigma_y = \frac{1}{C}\frac{\int \mu(0, \Delta y) d\Delta y}{\mu(0,0)}, \qquad (1)$$

- as a result, the maximum Ω is found as

$$\Omega(0,0) = \frac{1}{C^2 \ \Sigma_x \Sigma_y};$$

For Gaussian
$C = \sqrt{2\pi}$

- Consequently, the calibration constant σ_{vis} is determined at scan conditions as:

$$\sigma_{vis} = \frac{1}{N_1 N_2}\frac{\mu_{\max}(0,0)}{\Omega(0,0)}. \qquad (2)$$

- The vdM scan is widely used for luminosity calibration at hadron colliders such as in RHIC [5,6] and LHC [7,8] and it is also planned to be used at NICA [9]

Q-Gaussian distribution

It was observed that the Q-Gaussian provides a more realistic model for the bunch profile for LHC and also for the HL-LHC upgrade [10, 11].

$$QG(\mathrm{u}; q, \beta_q) = \frac{\sqrt{\beta_q}}{C_q} e_q(-\beta_q u^2) \qquad (3)$$

q: weight of tails

$$e_q(-\beta_q u^2) = \begin{cases} exp(-\beta_1 u^2), & q = 1 \\ [1-(1-q)\beta_q u^2]_+^{\frac{1}{1-q}}, & q \neq 1 \end{cases}$$

$$\mathrm{C_q} = \begin{cases} \dfrac{2}{(3-q)\sqrt{1-q}} Beta\left(\dfrac{1}{1-q}, \dfrac{1}{2}\right), & q < 1 \\ \sqrt{\pi}, & q = 1 \\ \dfrac{1}{\sqrt{q-1}} \mathrm{Beta}\left(\dfrac{1}{q-1} - \dfrac{1}{2}, \dfrac{1}{2}\right), & 1 < q < 3 \end{cases}$$

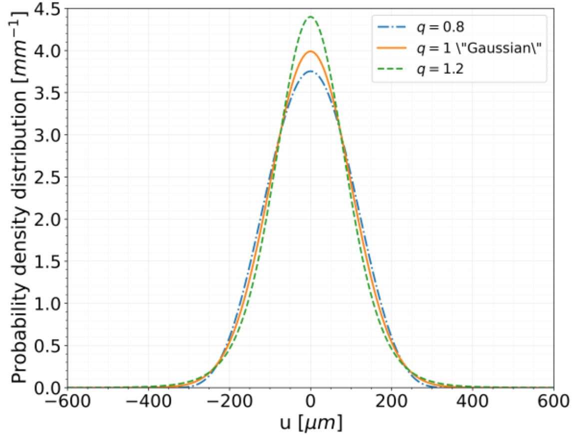

Fig. 2: Q-Gaussian bunch profile with same RMS $\sigma_q = 100\ \mu m$ and different tail density q = 0.8, 1 "Gaussian" and 1.2

Standard deviation

$$\sigma_q = \begin{cases} \sqrt{1/\beta_q(5-3q)}, & q < 5/3 \\ \infty, & 5/3 \leq q < 2 \\ \text{undefined}, & 2 \leq q < 3 \end{cases}$$

Overlap integral of Q-Gaussian beams [12]

For two equal-size Q-Gaussian beams ($\sigma_1 = \sigma_2 = \sigma,\ \beta_1 = \beta_2 = \beta$) with tail density ($q_1 = q_2 = q$), the beam overlap is given by*:

for finite light-tailed distribution "$q < 1$":

$_2F_1$ Gaussian Hypergeometric function

$$\Omega(\Delta; q, \beta_q) = \frac{\sqrt{\beta_q}}{\sqrt{1-q}\,C_q^{\,2}} \left(1-\sqrt{(1-q)\beta_q \frac{\Delta^2}{4}}\right)\left(1-(1-q)\beta_q\frac{\Delta^2}{4}\right)^{\frac{2}{1-q}} Beta\left(\frac{1}{2}, \frac{2-q}{1-q}\right) {}_2F_1\left(\frac{1}{q-1}, \frac{1}{2}; \frac{5-3q}{2-2q}; \frac{\left(1-\sqrt{(1-q)\beta_q\frac{\Delta^2}{4}}\right)^2}{\left(1+\sqrt{(1-q)\beta_q\frac{\Delta^2}{4}}\right)^2}\right) \quad (4)$$

for infinite heavy-tailed distribution "$q > 1$":

$$\Omega(\Delta; q, \beta_q) = \frac{\sqrt{\beta_q}}{\sqrt{q-1}\,C_q^{\,2}} Beta\left(\frac{1}{2}, \frac{5-q}{2q-2}\right) {}_2F_1\left(\frac{1}{q-1}, \frac{5-q}{2q-2}; \frac{q+1}{2q-2}; (1-q)\beta_q\frac{\Delta^2}{4}\right) \quad (5)$$

for q=1 "Gaussian":

$$\Omega(\Delta; \beta_q) = \sqrt{\frac{\beta_q}{\pi}}\, exp(-\beta_q \Delta^2) \quad (6)$$

At the limit of $q \to 1$, The formulas of overlap integral of Q-Gaussian beams tends to the Gaussian case.

Overlap integral of Q-Gaussian beams

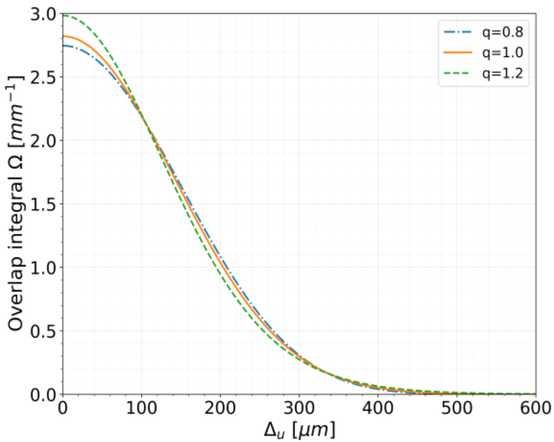

Fig. 2 The overlap integral of Q-Gaussian bunches with RMS $\sigma_q = 100\ \mu m$ and different tail density q = 0.8, 1 "Gaussian" and 1.2 as a function of the distance between beams orbits

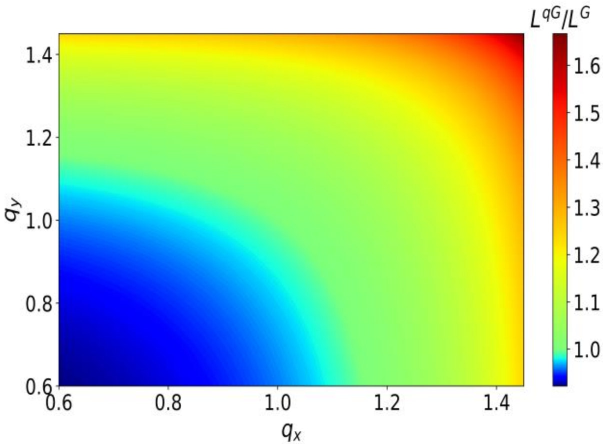

Fig. 3: Luminosity of Q-Gaussian beams w.r.t. Gaussian beams at zero separation, where the tail's weight of transverse bunch profile are q_x and q_y. and the colliding beams have the same RMS size

Total Kick

- When an ultra-relativistic charged particle ($\beta \approx 1$) with charge Q_2 passes through (or near) a charged particle density ρ_1 at a zero crossing-angle, it gains a total transverse momentum kick given by [1,13]:

$$\Delta \boldsymbol{p}_2(x_p, y_p) = \frac{Q_2}{c} \int \mathbf{E}_1^{\perp}(x_p, y_p, z_p) dz_p = \frac{Z_2 e}{c} \boldsymbol{E}_1(x_p, y_p) \quad (7)$$

- The corresponding beam-beam angular deflection $\Delta\theta$

$$\Delta\theta(x_p, y_p) = \frac{\Delta \boldsymbol{p}_2(x_p, y_p)}{p} = \frac{Z_2 e}{pc} \boldsymbol{E}_1(x_p, y_p)$$

where

$\mathrm{Q}_2 = Z_2 e$ is the charge of the kicked particle,

$\mathbf{E}_1^{\perp}(x_p, y_p, z_p)$ is the electric field produced by the kicker bunch ρ_1 perpendicular to the trajectory of the kicked particle,

$\boldsymbol{E}_1(x_p, y_p)$ is the total transverse field from the transverse distribution ρ_1 at a certain point (x_p, y_p)

p is the total momentum of the kicked particle.

Total kick

- The $\boldsymbol{E}(x_p, y_p)$ produced by the bunch $\rho_1(x, y)$ at a certain point (x_p, y_p) is equal to the sum of electric fields produced by its individual particles, $\boldsymbol{E}(x_p, y_p) = \sum_i \boldsymbol{E}_i(x_p, y_p)$ where

$$\boldsymbol{E}_i(x_p, y_p) = \frac{q_i}{2\pi\,\epsilon_o} \frac{(\boldsymbol{x_p} - \boldsymbol{x_i}) + (\boldsymbol{y_p} - \boldsymbol{y_i})}{(x_p - x_i)^2 + (y_p - y_i)^2}$$

$\boldsymbol{E}_i\,(x_p, y_p)$ is the field produced at (x_p, y_p) by i^{th} particle of the transverse density ρ_1 at (x_i, y_i)

- Since the bunch has continuous transverse q-Gaussian density $\rho_1(x, y) = QG(x; q_{\mathrm{x}}, \sigma_x)\; QG(\mathrm{y}; q_{\mathrm{y}}, \sigma_y)$, the total field can be approximated as

$$\boldsymbol{E}_1(x_p, y_p) = \frac{N_1 Z_1 e}{2\pi\epsilon_o} \int\int \frac{(\boldsymbol{x_p} - \boldsymbol{x}) + (\boldsymbol{y_p} - \boldsymbol{y})}{(x_p - x)^2 + (y_p - y)^2} QG(x; q_{\mathrm{x}}, \sigma_x)\; QG(\mathrm{y}; q_{\mathrm{y}}, \sigma_y)\; dx\, dy \qquad (8)$$

N_1 is the number of particles in the kicker bunch and $Z_1 e$ is the charge

Total kick

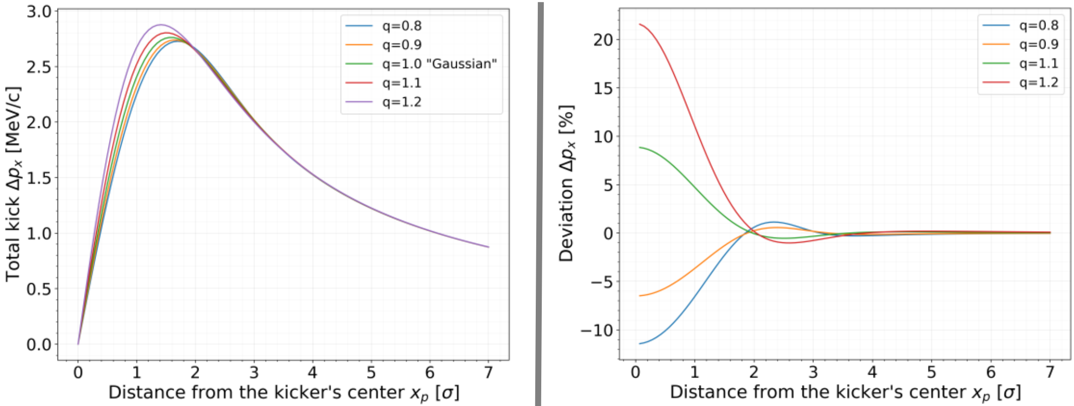

Fig. 4: The horizontal beam-beam kick Δp_x gained by the particle at the distance $(x_p, 0)$ from the center of Q-Gaussian kicker bunch with RMS size $\sigma = 40\ \mu m$ and tail weights q = 0.8, 0.9, 1 ”Gaussian”, 1.1, and 1.2. (right) and its deviation from that of Gaussian (left).

The dependence of the kick on the tail weight is divided into three regions:

- region–1 near the bunch center for $|x| \lesssim 1.8\ \sigma$, the heavier the tails is the stronger kick;
- region–2 at medium range from the bunch center for $1.8\ \sigma \lesssim |x| \lesssim 3.5$, the lighter the tails the stronger the kick;
- region–3 at a further distance from the bunch center for $|x| \gtrsim 3.5\ \sigma$, the of Q-Gaussian tends to that of Gaussian with a deviation up to 0.02 to 0.03%.

Simulation

- The luminosity variations due to the bb effect is estimated as the ratio of beam overlap with and without bb interaction (i.e. ratio of luminosities with and without bb interaction), this ratio is called the beam-beam correction R is

$$R = \Omega_{\mathrm{bb}}/\Omega_{\mathrm{o}}$$

- For Gaussian and Multi-Gaussian beams, R is estimated using the B*B code developed by V. Balagura. We have modified B*B code for q-Gaussian beams.
- The beam-beam bias in the visible cross-section is found as the derivation of the visible cross-section with beam-beam interaction $\sigma_{vis,bb}$ from the visible cross-section without beam-beam interaction $\sigma_{vis,o}$, from eqs. (1, 2), we get

$$\sigma_{vis,o} \propto \frac{1}{N_1 N_2 \Omega_o(0,0)} \int \Omega_o(\Delta x, 0) d\Delta x \times \int \Omega_o(0, \Delta y) d\Delta y \qquad \text{(without bb interaction)}$$

$$\Omega_{\mathrm{bb}}(\Delta x, 0) = \mathrm{R}(\Delta x, 0)\, \Omega_o(\Delta x, 0)$$

$$\sigma_{vis,bb} \propto \frac{1}{N_1 N_2 \Omega_{\mathrm{bb}}(0,0)} \int \Omega_{\mathrm{bb}}(\Delta x, 0) d\Delta x \times \int \Omega_{\mathrm{bb}}(0, \Delta y) d\Delta y \qquad \text{(with bb interaction)}$$

$$\propto \frac{1}{N_1 N_2\, R(0,0)\, \Omega_o(0,0)} \int R(\Delta x, 0) \Omega_o(\Delta x, 0) d\Delta x \times \int R(0, \Delta y) \Omega_o(0, \Delta y) d\Delta y$$

$$\sigma_{vis}\ bias = \frac{\sigma_{vis,bb} - \sigma_{vis,0}}{\sigma_{vis,0}} = \frac{\int R(\Delta x, 0)\, \Omega_o(\Delta x, 0) d\Delta x \times \int R(0, \Delta y)\, \Omega_o(0, \Delta y) d\Delta y}{R(0,0) \left(\int \Omega_o(\Delta x, 0) d\Delta x \times \int \Omega_o(0, \Delta y) d\Delta y \right)} - 1 \qquad (9)$$

Beam parameters for simulations

- Two datasets of beam parameters, ATLAS-2012 [1] and ATLAS/CMS-2018 [14], are considered
- For each dataset, the particle density is assumed to be Q-Gaussian. Five distinct values of tail densities (q) is simulated for $q = 0.8, 0.9, 1$ "Gaussian", 1.1 and 1.2

	ATLAS-2012 *	ATLAS/CMS- 2018**
bunch intensity $N_1 = N_2 = N$ $[10^{10}]$	8.5	8.5
Particle momentum $[GeV/c]$	3500	6499
(Q_x, Q_y)	(0.31, 0.32)	(0.31, 0.32)
$\beta^*[m]$	1.5	19.7
RMS size $[\mu m]$	40	97.1

Results for ATLAS-2012

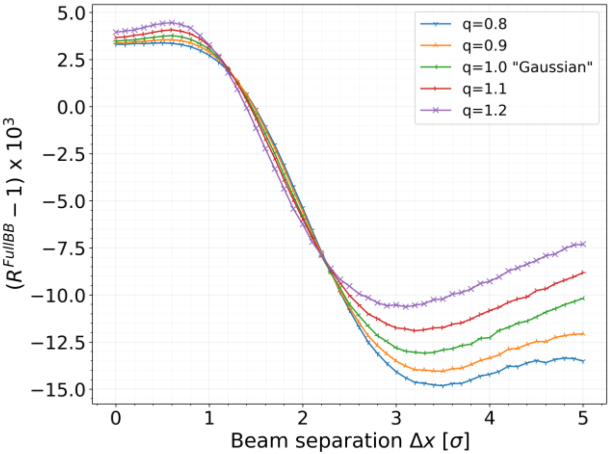

Fig.5: The beam-beam correction R (i.e. Ω_{bb}/Ω_o) during van-der-Meer scan

$\Delta\sigma_{vis}/\sigma_{vis,0}$ [%]					
Tail density "q"	0.8	0.9	1.0 "Gaussian"	1.1	1.2
R(0,0)	1.0033	1.0034	1.0035	1.0037	1.004
$\sigma_{vis} bias$ [%]	-0.215	-0.192	-0.162	-0.124	-0.079

Results for ATLAS/CMS-2018

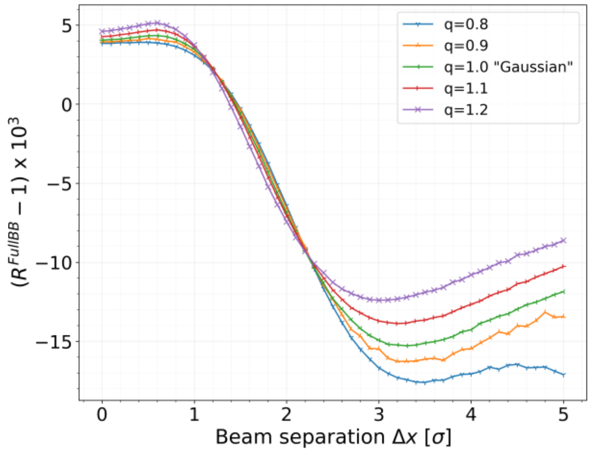

Fig.6: The beam-beam correction R (i.e. Ω_{bb}/Ω_o) during van-der-Meer scan

$\Delta\sigma_{vis}/\sigma_{vis,0}$ [%]					
Tail density "q"	0.8	0.9	1.0 "Gaussian"	1.1	1.2
R(0,0)	1.0039	1.004	1.0041	1.0043	1.0046
$\sigma_{vis} bias$ [%]	-0.268	-0.237	-0.211	-0.158	-0.108

Summary

- The Q-Gaussian distribution is a more natural approach for describing the particle density inside bunches than Gaussian, and Multi-Gaussian distributions, as the Q-Gaussian can describe infinite heavy-tailed distributions "$q > 1$", finite bounded light-tailed distributions "$q < 1$" , and the Gaussian distribution "$q = 1$", through the introduction of single additional parameter 'q'.
- The particle distribution in the colliding bunches has a significant impact on luminosity as well as luminosity calibration at the beam separation scan (Fig. 2 and 3).
- The particle distribution also influences the beam-beam force (Fig. 4), Q-Gaussian bunches with heavier tails exerts a stronger momentum kicks near the bunch centre. At a further distance from the bunch centre for $|x| \gtrsim 3.5\ \sigma$, the momentum kicks of Q-Gaussian tends to that of Gaussian with a deviation up to 0.03%.
- The current model for bb interaction is based on Gaussian beams. However, it is essential to consider that the actual colliding beams deviate from Gaussian which leads to potential misestimate of luminosity as well as its calibration constant. (underestimating the bias for light-tailed beams and overstating the bias for heavy-tailed beams)
- Our Q-Gaussian model shows that for q with a deviation in range of 10% around Gaussian, the bias for Q-Gaussian beams can be approximately 2 times smaller for heavy-tailed beams or approximately 1.25 times larger for light-tailed beams compared to that of Gaussian beams.
- Bearing in mind that the target precision of luminosity measurements is below 1% in HL-LHC, the bb interaction, as well as its influence on beam overlap, should be carefully considered with the appropriate beam distribution.

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