Pushing the boundaries of threshold resummation

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CERN Theory Colloquium

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Pushing the boundaries of threshold resummation

Part A
Extension to a higher number of particles that carry colour → 4top production

Part B
Understanding subleading-power contributions

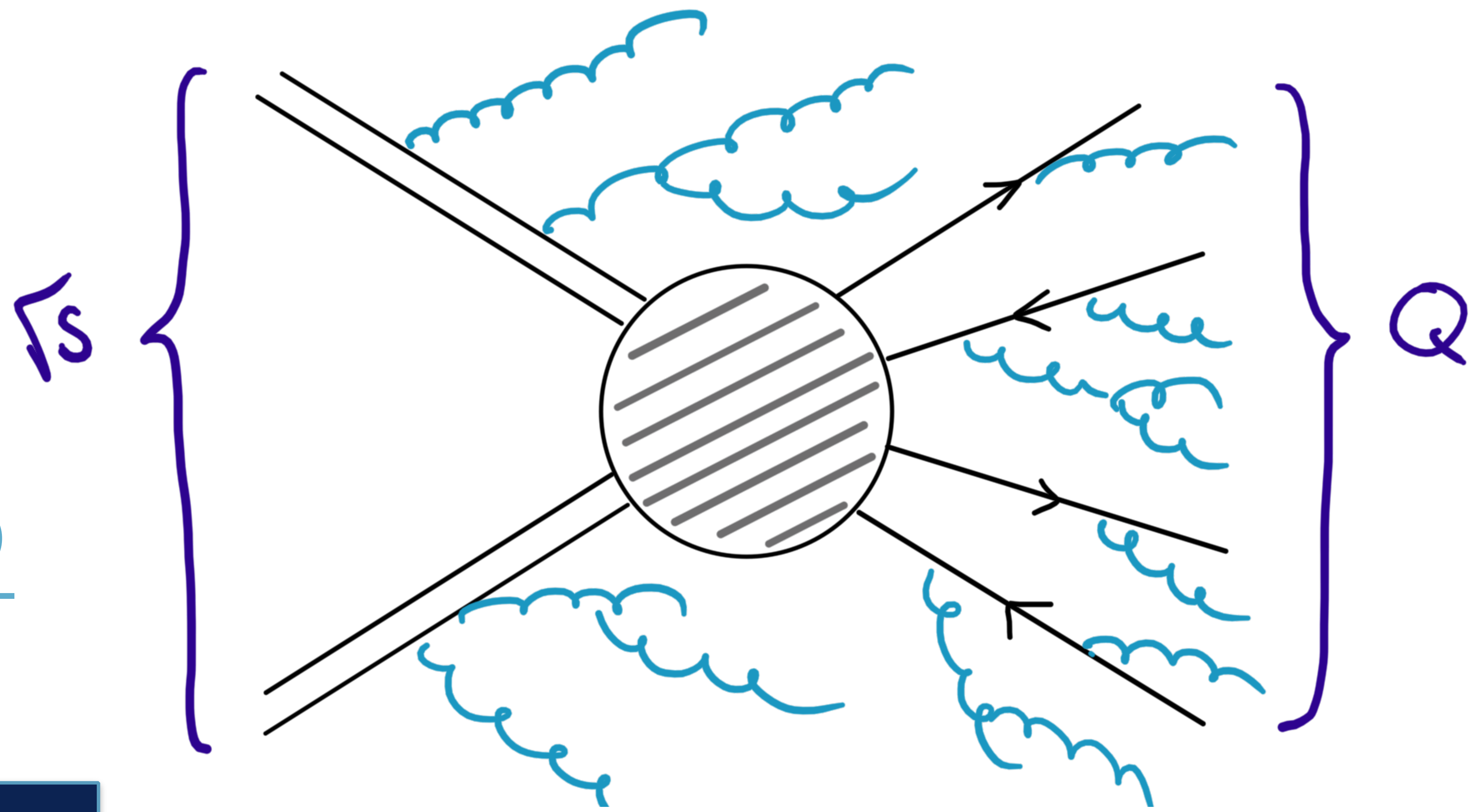
Improving perturbation theory

Fixed-order description of a differential cross section

$$\mathrm{d}\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1\alpha_s + c_2\alpha_s^2 + \ldots$$

$$c_n = \sum_{k=0}^{k_{\max,n}} d_{nk} L^k + f_n,$$

L depends on observable, e.g. $L^k = \dfrac{\ln^{2k-1}(1 - Q^2/s)}{1 - Q^2/s}$



Corrections can be predicted accurately in the limit that $L^k \to \infty$ for $Q^2 \simeq s$ (soft/collinear gluon emissions)

Resummation: A new series

Based on the seminal papers by Parisi (1980); Curci and Greco (1980); Sterman (1987); Catani and Trentadue (1989, 1991)

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	…
N^{n}LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	…

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N^nLO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

Leading-Log (LL)

Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N^{n}LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

Next-to-Leading-Log (NLL)

Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
N^{n}LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	...

$$\sigma_{\text{resum}} = h(\alpha_s) e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

obeys perturbative expansion;
including $\mathcal{O}(\alpha_s)$ leads to NLL' accuracy

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Threshold resummation for a coloured final state

- Colour can be factorised when up to three hard partons are present

  Can be proven using conservation of colour charge $\sum \mathbf{T}_i = 0$

- Colour interferences are created with four or more partons

- Resummation has been understood for $2 \to 2$ processes…

  See e.g. Contopanagos, Laenen, Sterman [1996]; Kidonakis, Sterman [1997]

- … $2 \to 2 + X$ with $X = h, W, Z, \gamma$

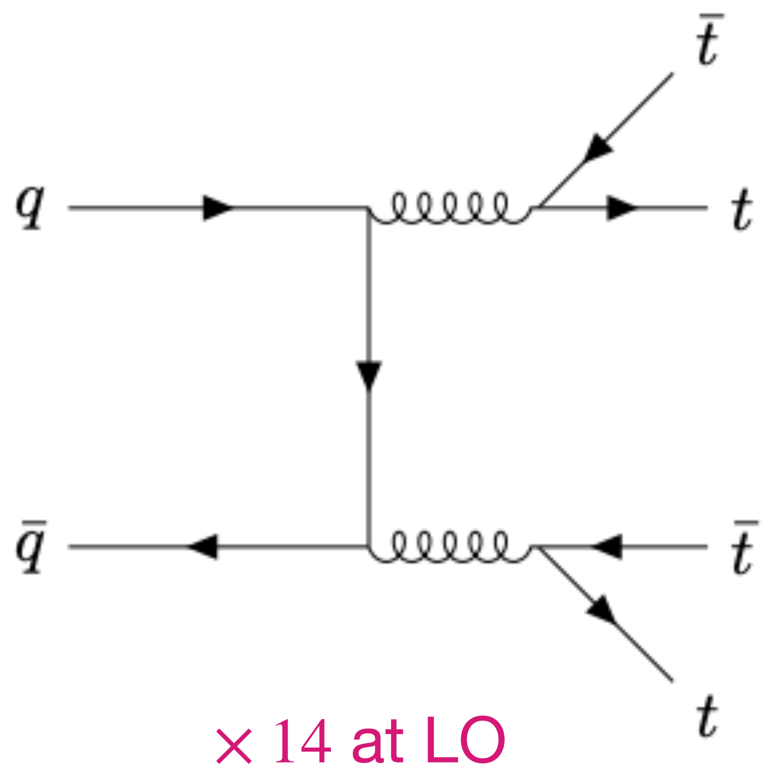
  See e.g. [1409.1460], [1509.02780], [1510.01914], [1901.04243], [2001.03031], [2012.09170], [2212.00096]

- But what about $2 \to 4$, such as **4top production**?

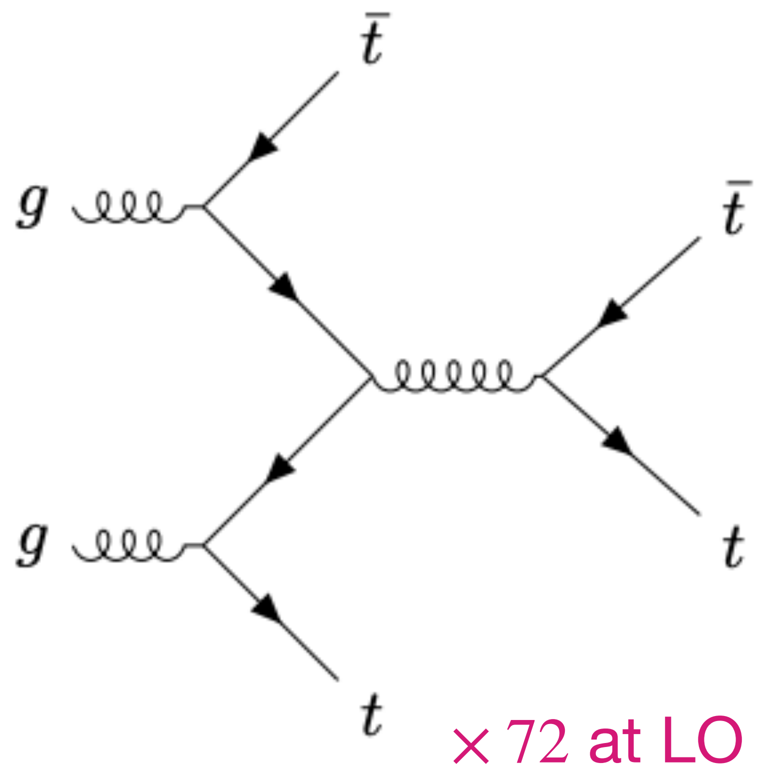
Part A
Threshold resummation for the 4top production process

[2212.03259]
with Anna Kulezsa and Laura Moreno Valero

Status of 4top - theory



$\times$ 14 at LO

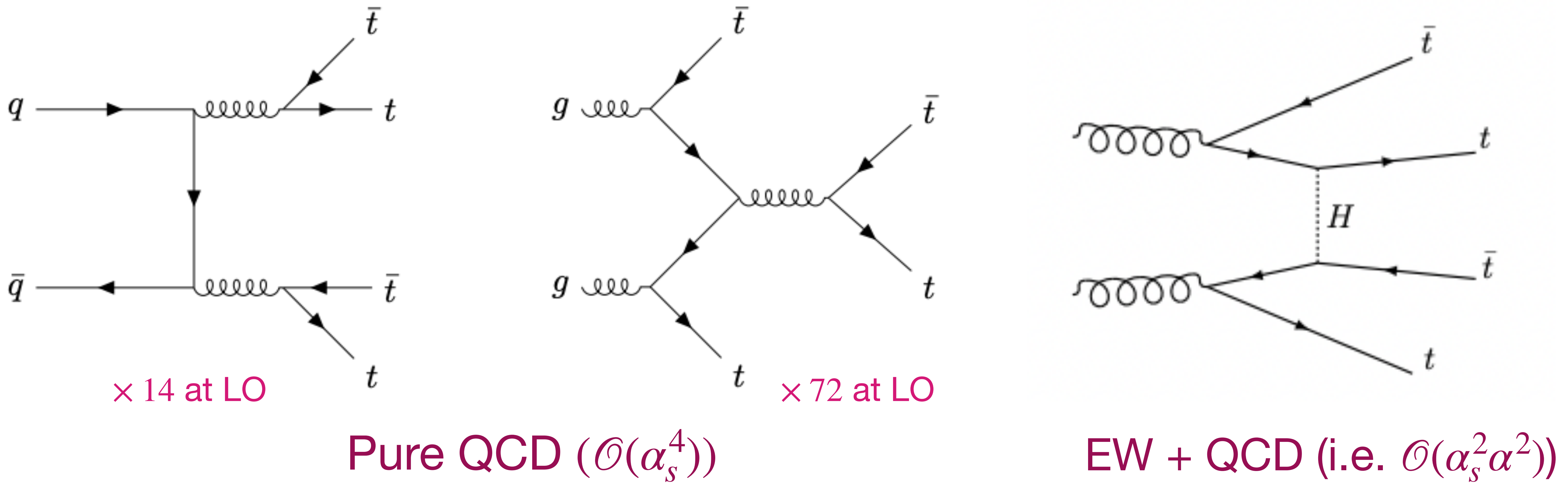


$\times$ 72 at LO

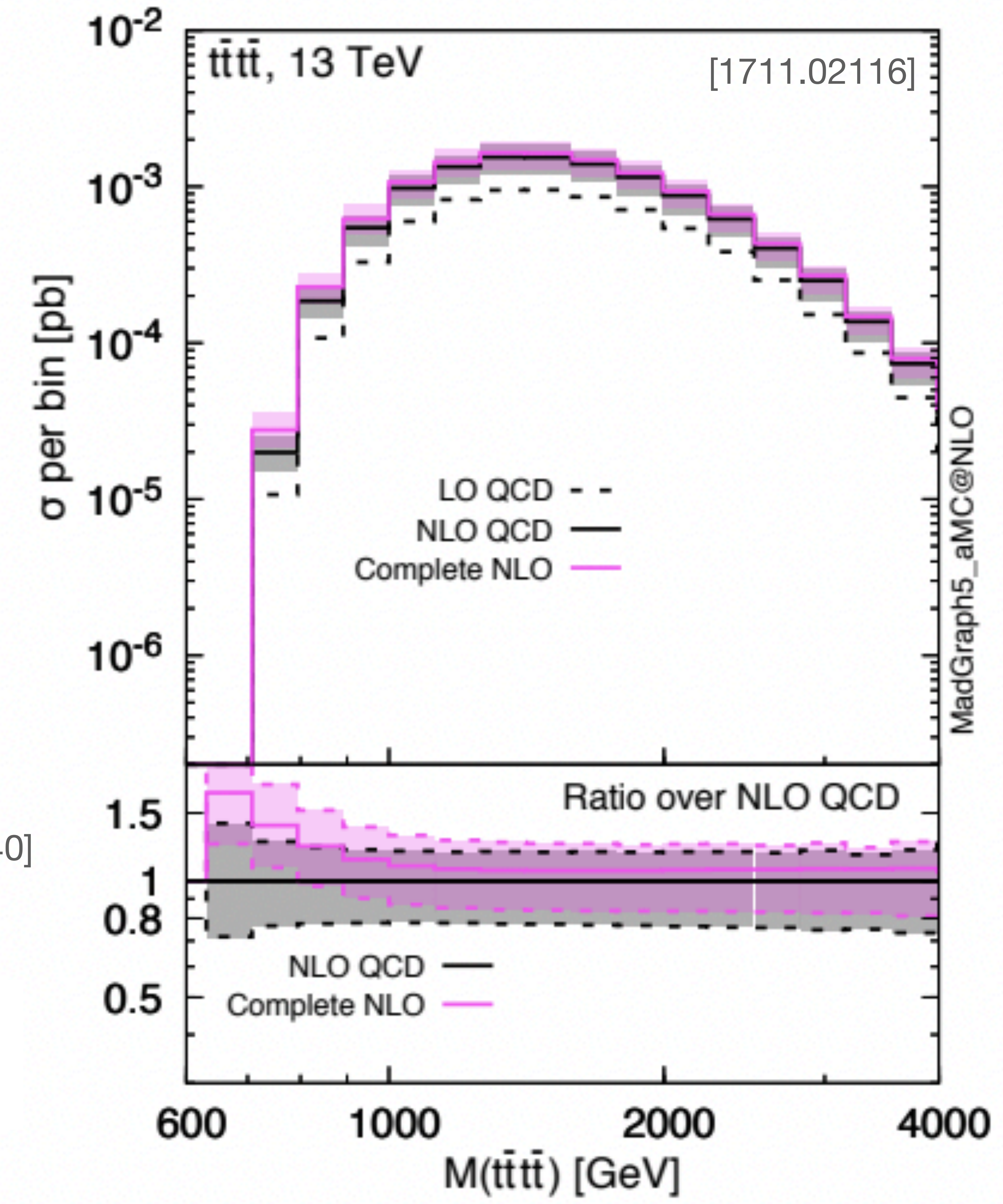
Pure QCD ($\mathcal{O}(\alpha_s^4)$)

- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]

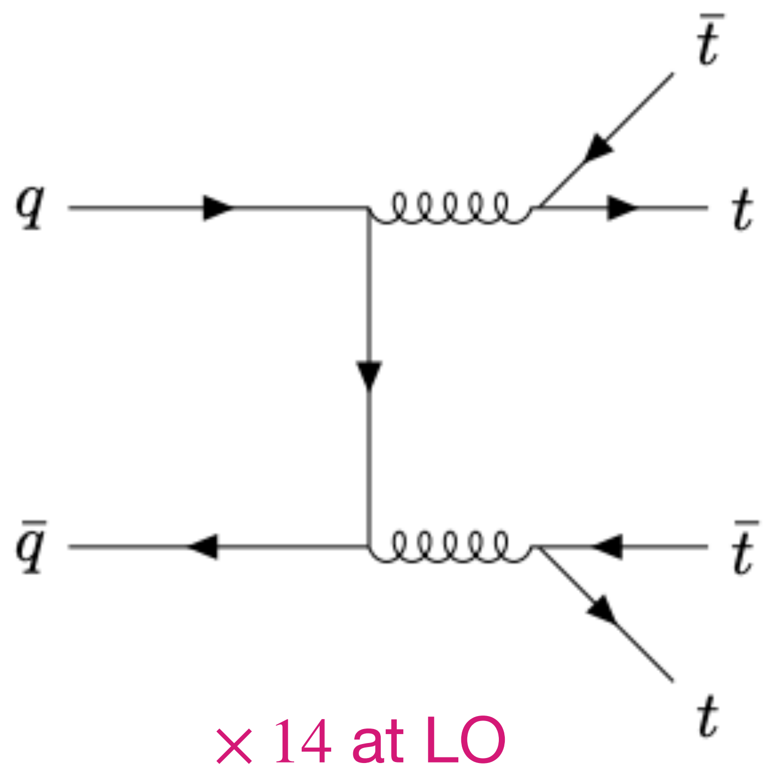
Status of 4top - theory



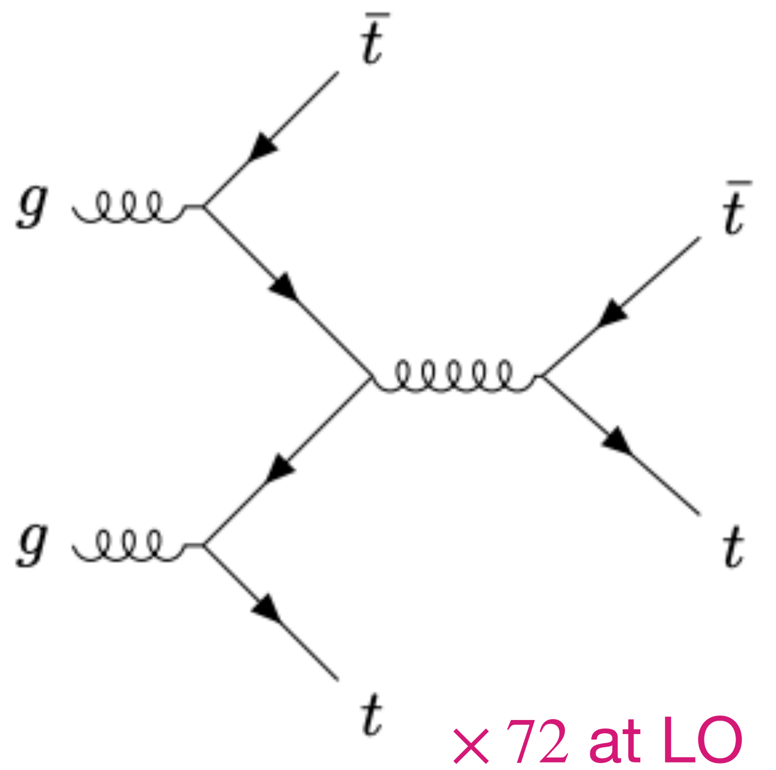
- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]
- Including EW corrections [1711.02116]



Status of 4top - theory

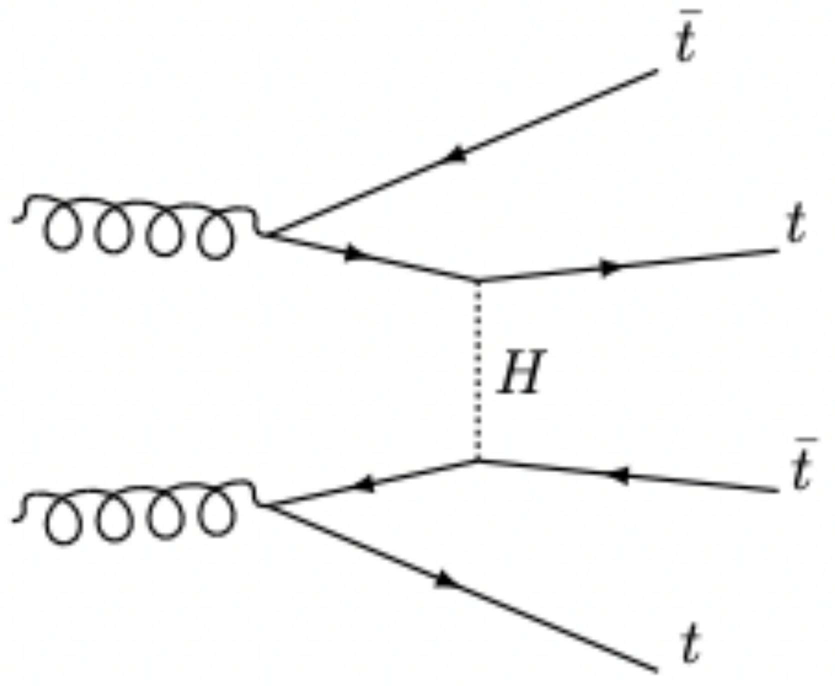


$\times$ 14 at LO



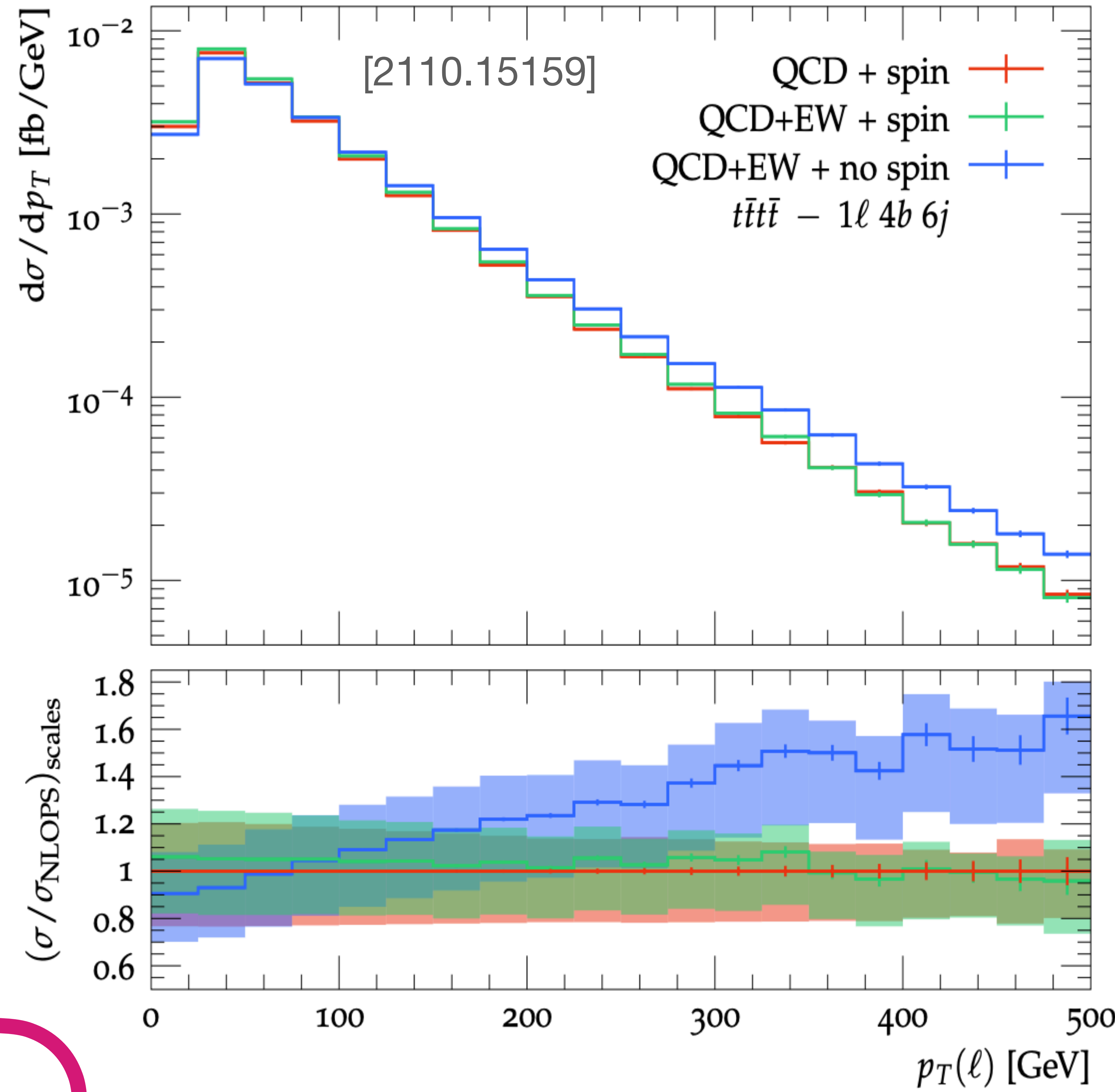
$\times$ 72 at LO

Pure QCD ($\mathcal{O}(\alpha_s^4)$)



EW + QCD (i.e. $\mathcal{O}(\alpha_s^2\alpha^2)$)

- NLO calculated some time ago [1206.3064]
- MadGraph@NLO matched with parton showers [1405.0301, 1507.05640]
- Including EW corrections [1711.02116]
- POWHEG-BOX implementation with spin correlations [2110.15159]



[2110.15159]

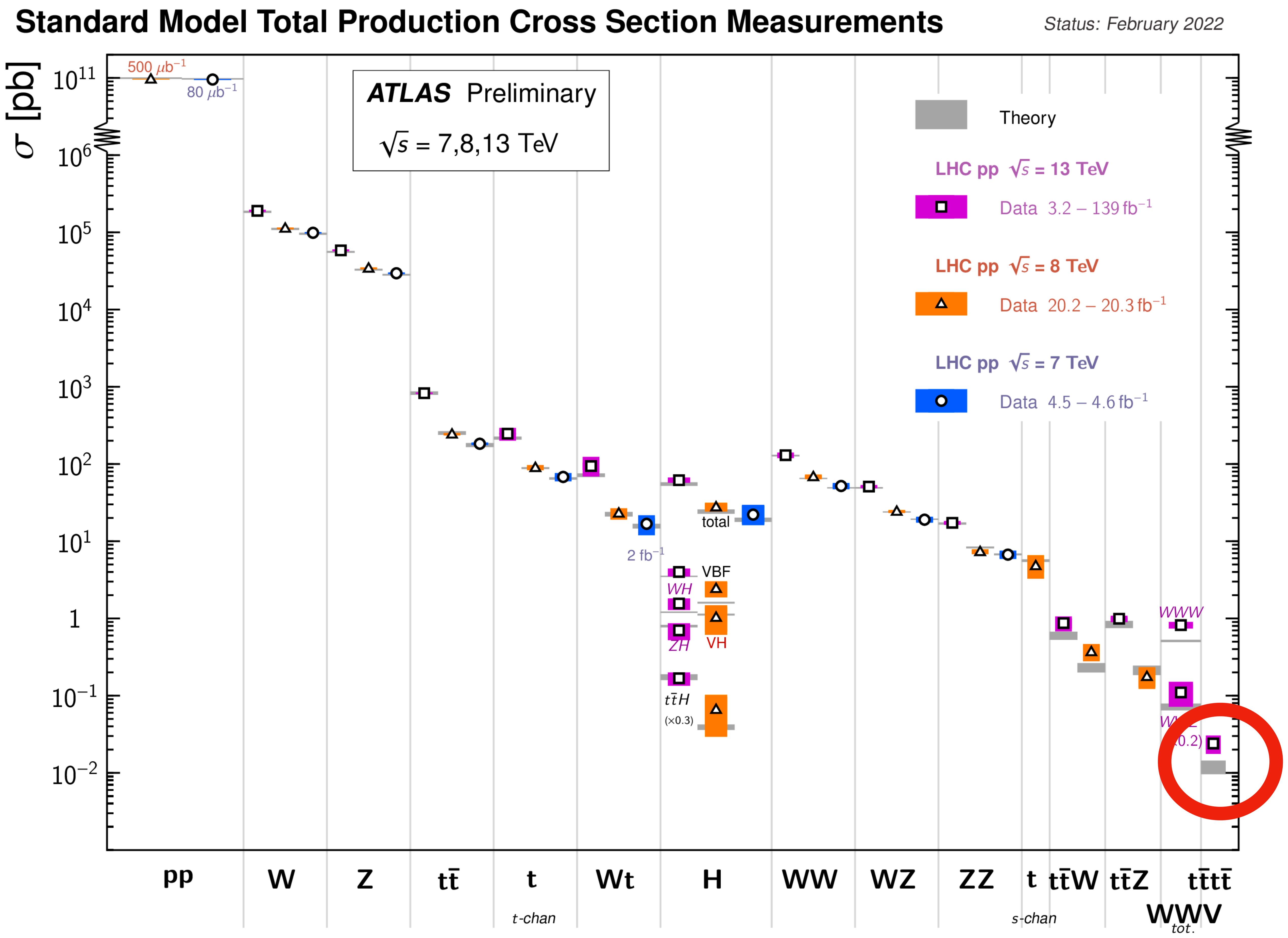
PDF	$\sigma^{\rm NLO}$ [fb]	$\delta_{\rm scale}$	$\delta_{\rm PDF}$	$\mathcal{K} = \frac{\rm NLO}{\rm LO}$
NNPDF3.1	11.65	$+1.98$ (17%) -2.57 (22%)	$+0.28$ (2%) -0.28 (2%)	1.33
MMHT	11.62	$+1.95$ (17%) -2.54 (22%)	$+0.63$ (5%) -0.53 (5%)	1.04
CT18	11.74	$+1.97$ (17%) -2.56 (22%)	$+0.46$ (4%) -0.36 (3%)	1.06

$\mu_0 = H_T/4$

$m_t = 172.5$ GeV

Status of 4top - experiment

First hints for its existence found by ATLAS and CMS!



In one/two opposite-sign lepton final-state channel

+ 10/8 jets…

$$\sigma^{\mathrm{ATLAS}}_{t\bar{t}t\bar{t}} = 26 \pm 8(\mathrm{stat.})^{+15}_{-13}(\mathrm{syst.})\ \mathrm{fb}$$ [2106.11683]

$$\sigma^{\mathrm{CMS}}_{t\bar{t}t\bar{t}} = 38^{+13}_{-11}\ \mathrm{fb}$$ [CMS-PAS-TOP-21-005]

In two same-sign / > 3 lepton final-state channel

$$\sigma^{\mathrm{ATLAS}}_{t\bar{t}t\bar{t}} = 24 \pm 5(\mathrm{stat.})^{+5}_{-4}(\mathrm{syst.})\ \mathrm{fb}$$ [2007.14858]

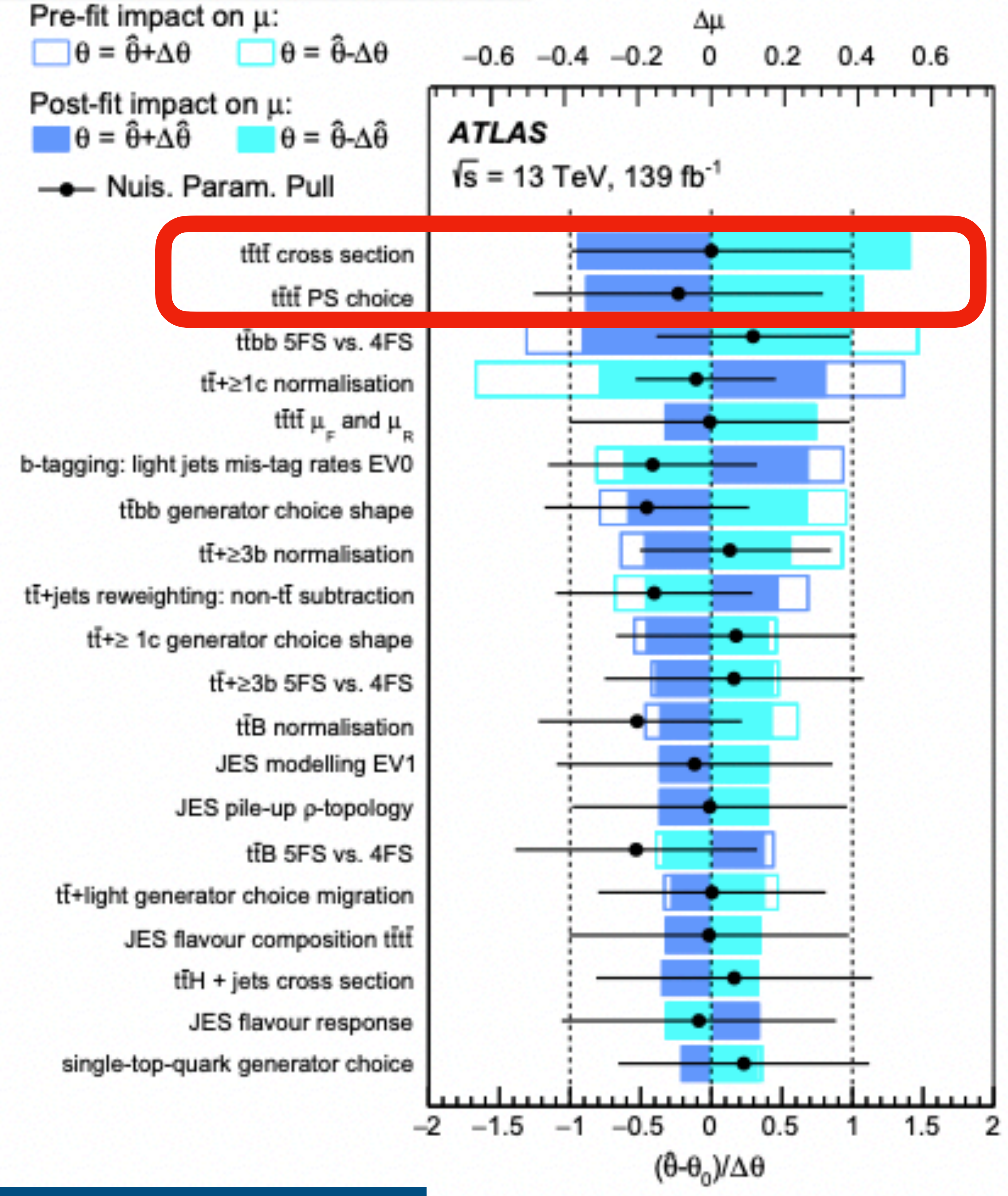
$$\sigma^{\mathrm{CMS}}_{t\bar{t}t\bar{t}} = 12.6^{+5.8}_{-5.2}\ \mathrm{fb}$$ [1908.06463]

Uncertainties 4top

Uncertainty dominated by systematics!

Uncertainty source	$\Delta\mu$	
Signal modelling		
$t\bar{t}t\bar{t}$ cross section	+0.56	−0.31
$t\bar{t}t\bar{t}$ modelling	+0.15	−0.09
Background modelling		
$t\bar{t}W$+jets modelling	+0.26	−0.27
$t\bar{t}t$ modelling	+0.10	−0.07
Non-prompt leptons modelling	+0.05	−0.04
$t\bar{t}H$+jets modelling	+0.04	−0.01
$t\bar{t}Z$+jets modelling	+0.02	−0.04
Other background modelling	+0.03	−0.02
Charge misassignment	+0.01	−0.02
Instrumental		
Jet uncertainties	+0.12	−0.08
Jet flavour tagging (light-flavour jets)	+0.11	−0.06
Simulation sample size	+0.06	−0.06
Luminosity	+0.05	−0.03
Jet flavour tagging (b-jets)	+0.04	−0.02
Jet flavour tagging (c-jets)	+0.03	−0.01
Other experimental uncertainties	+0.03	−0.01
Total systematic uncertainty	+0.70	−0.44
Statistical	+0.42	−0.39
Non-prompt leptons normalisation (HF, Mat. Conv., Low m_{γ^*})	+0.05	−0.04
$t\bar{t}W$ normalisation	+0.04	−0.04
Total uncertainty	+0.83	−0.60

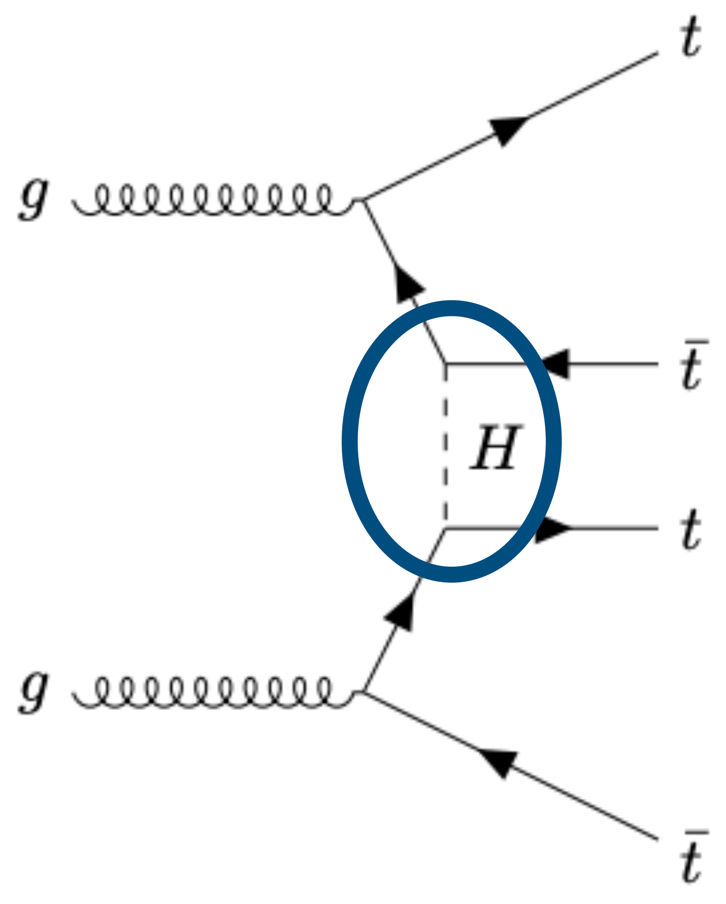
ATLAS [2106.11683]

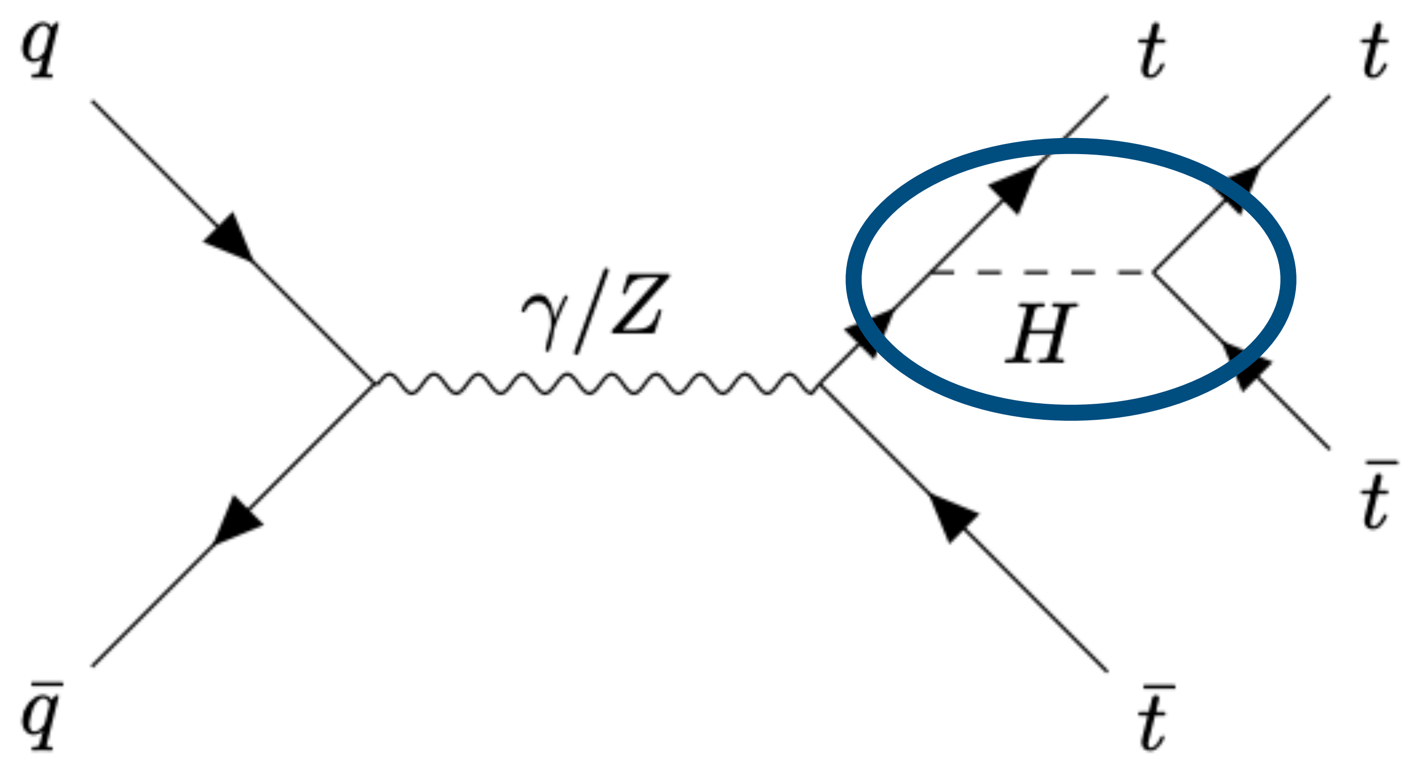


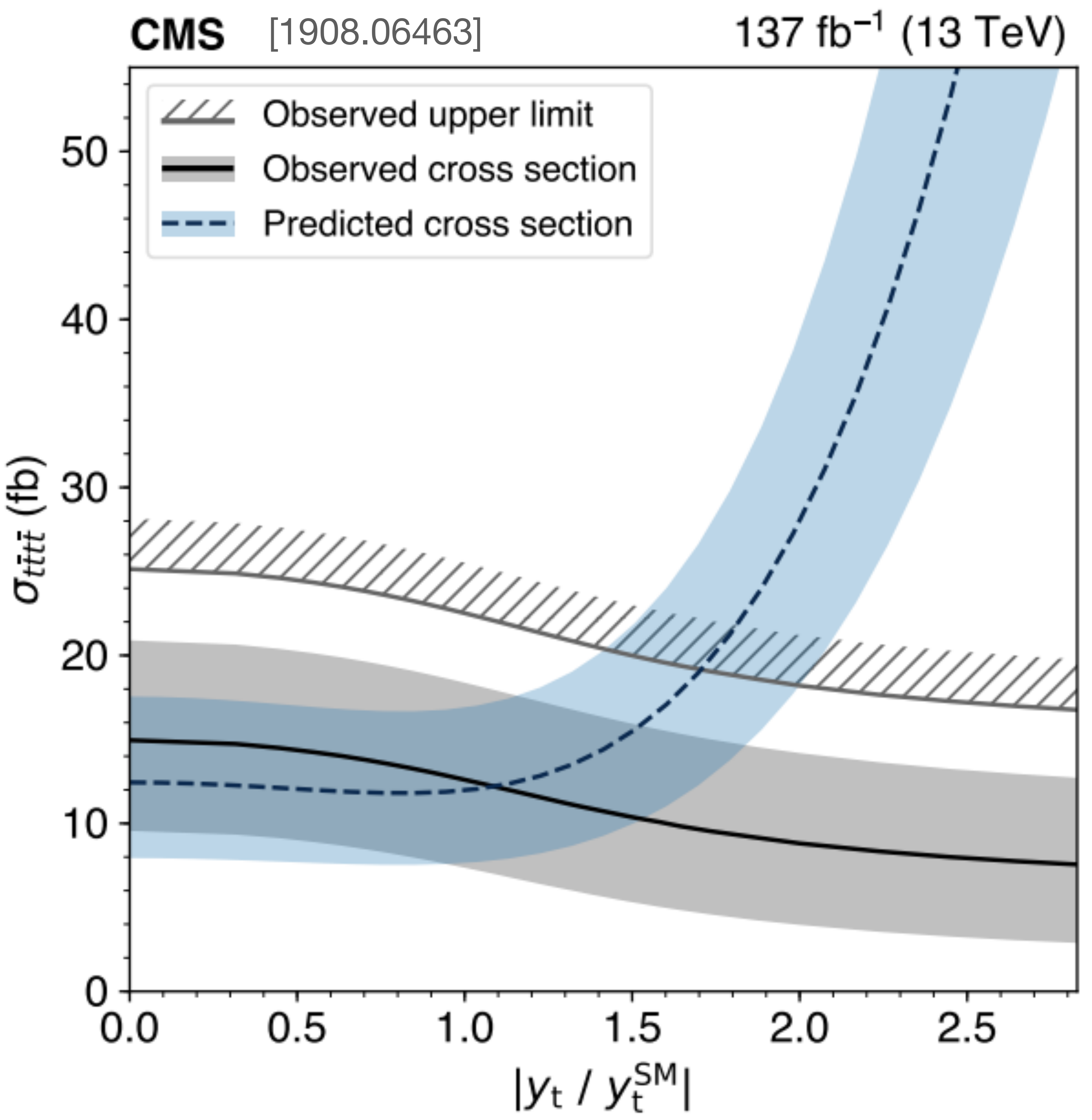
Besides $t\bar{t}t\bar{t}$, uncertainties in $t\bar{t}W$, $t\bar{t}b\bar{b}$ background modeling and in jet energy scale / resolution contribute substantially

Why 4top?

- Sensitive to the Yukawa coupling



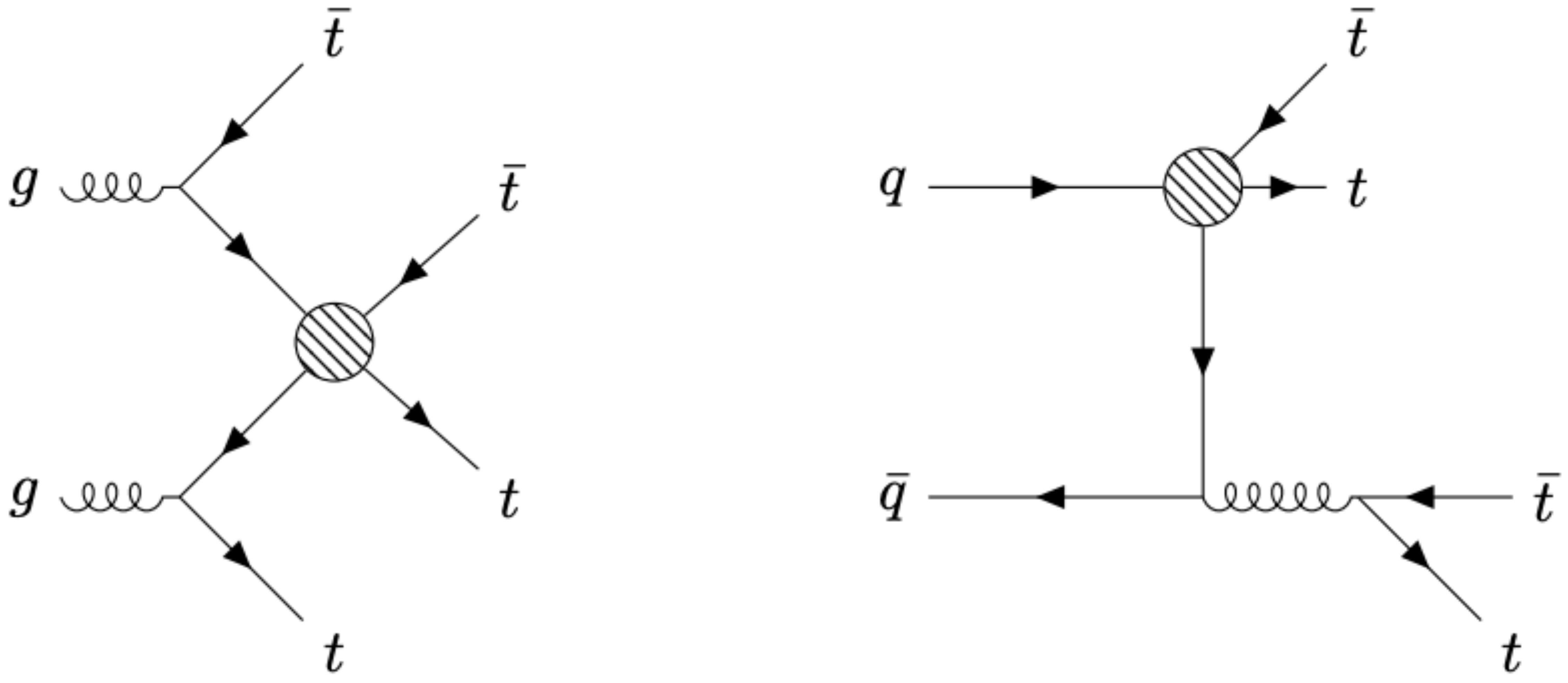




Why 4top?

- Sensitive to the Yukawa coupling

- Constrain EFT coefficients see e.g. [2208.04962]
  - Effective four-heavy-quark operators ($QQQQ$) and two-heavy-two-light four-quark operators ($QQqq$)
  - Dimension-6 operator $\hat{H}$ that modifies Higgs propagator



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Why 4top?

- Sensitive to the Yukawa coupling

- Constrain EFT coefficients
  - Effective four-heavy-quark operators ($QQQQ$) and two-heavy-two-light four-quark operators ($QQqq$)
  - Dimension-6 operator $\hat{H}$ that modifies Higgs propagator

- Probe the presence of new particles
  - Simplified DM models
  - Type II two Higgs doublet models
  - SUSY (both minimal and non-minimal models)

Threshold resummation for 4top

Logarithmic corrections grow large when $\sqrt{s} \to M \equiv 4m_t$ *(the absolute-threshold limit)*

$$\sigma_{t\bar{t}t\bar{t}}(N) = \int_0^1 \mathrm{d}\tau\, \tau^{N-1} \sigma_{t\bar{t}t\bar{t}}(\tau) \qquad \tau = \frac{M^2}{S} = x_1\, x_2\, \rho$$

Transformation to Mellin space helps to obtain closed forms for the phase-space integrals

Usual factorisation for the hadronic total production cross section:

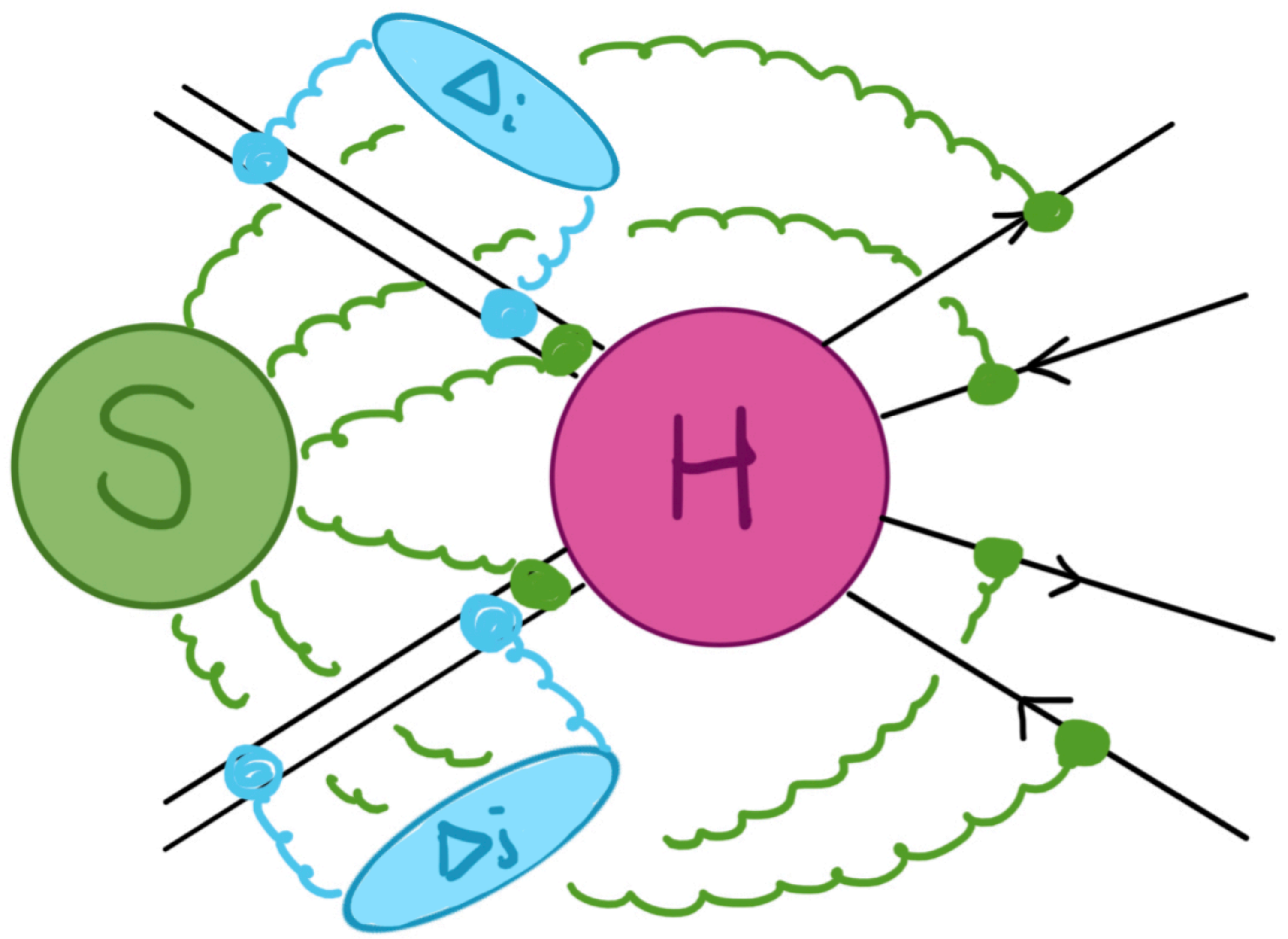
$$\sigma_{t\bar{t}t\bar{t}}(\tau) = \int_0^1 \mathrm{d}x_1 f_i(x_1, \mu_F^2) \int_0^1 \mathrm{d}x_2 f_j(x_2, \mu_F^2) \int_0^1 \mathrm{d}\rho\, \delta\left(\rho - \frac{\tau}{x_1 x_2}\right) \hat{\sigma}_{ij \to t\bar{t}t\bar{t}}(\rho)$$

partonic cross section obeys refactorisation when all radiation is soft and/or collinear

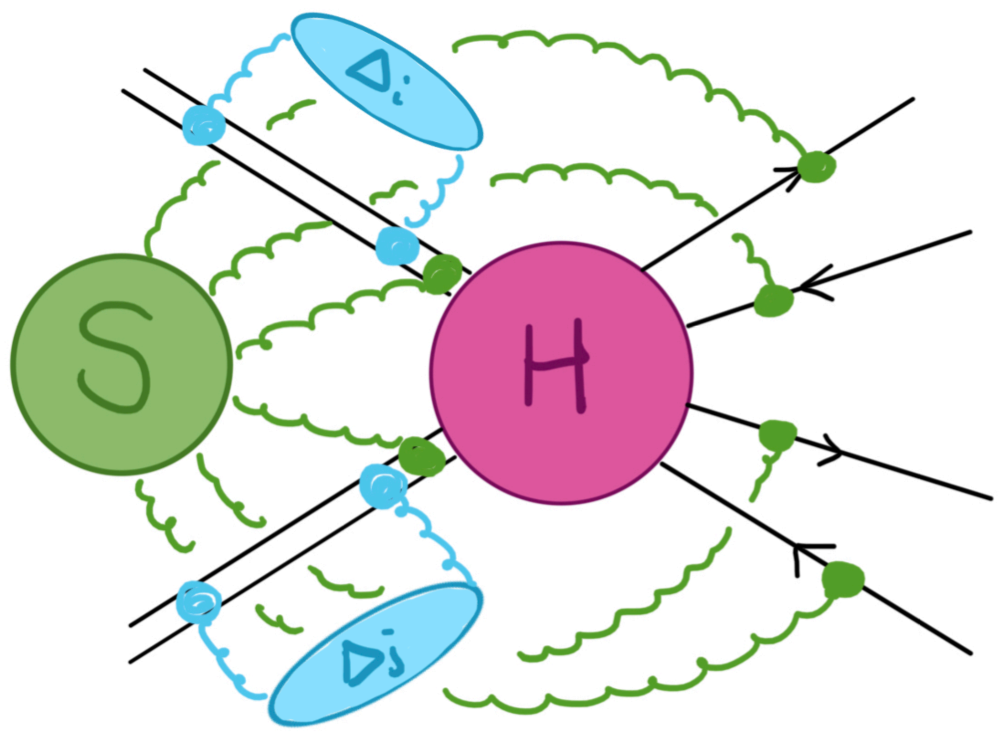
Resummation for 4top

Mellin-space resummed cross section

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N) = \text{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\, \tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right] \Delta_i\, \Delta_j$$



Resummation for 4top



Mellin-space resummed cross section

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N) = \text{Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}} \;\bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\,\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\,\mathbf{U}_{ij\to t\bar{t}t\bar{t}} \,\right] \Delta_i \Delta_j$$

Incoming jet functions

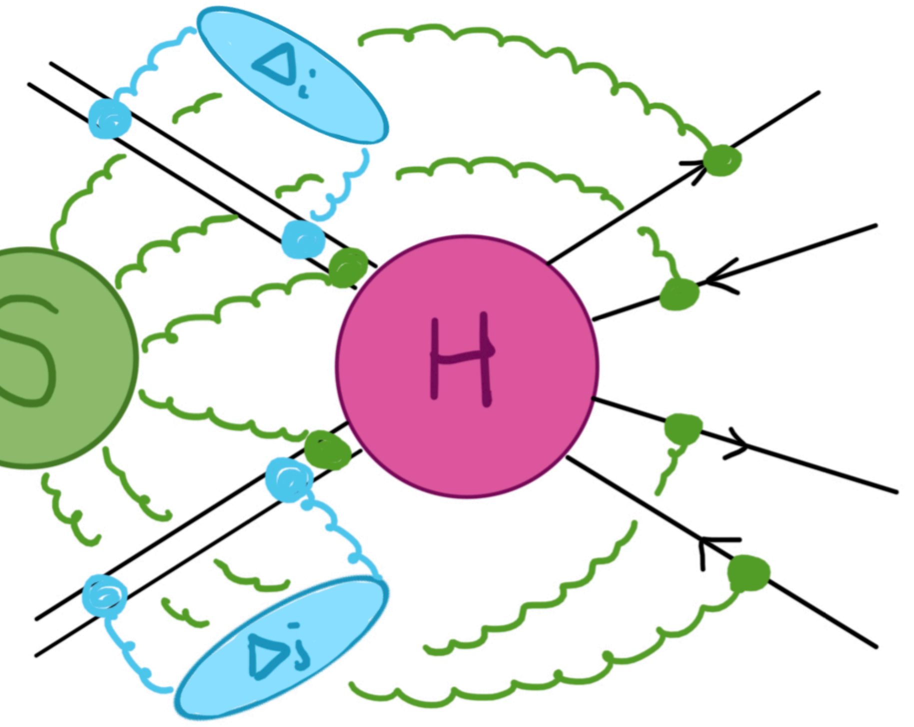
capture soft-collinear enhancements

$$\Delta_i = \exp\left[\frac{1}{\alpha_s} g_1(\lambda) + g_2(\lambda, \mu_R/M, \mu_F/M)\right]$$

Needed at LL Needed at NLL

$$\lambda = \alpha_s \ln\left(\bar{N}\right) = \alpha_s\left(\ln N + \gamma_E\right)$$

Resummation for 4top



Mellin-space resummed cross section

Soft function
captures wide-angle-soft enhancements

$$\hat{\sigma}^{\rm res}_{ij\to t\bar{t}t\bar{t}}(N) = {\rm Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\, \tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \right] \Delta_i\, \Delta_j$$

Incoming jet functions
capture soft-collinear enhancements

Result of RGE equation with evolution matrix

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathscr{P}\exp\left[\int_{\mu_R}^{M/\bar{N}} \frac{{\rm d}q}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

And boundary condition at $\bar{N} = M/\mu_R$

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\tilde{\mathbf{S}}^{(1)} + \dots$$

Needed at NLL

Needed at NLL'

Colour structure 4top

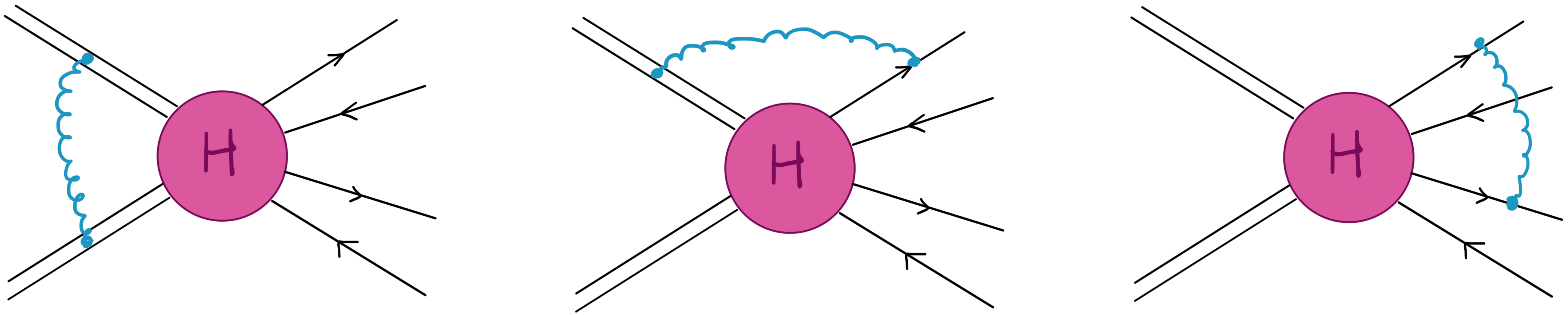
$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathscr{P}\exp\left[\int_{\mu_R}^{M/\bar{N}} \frac{\mathrm{d}q}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

Soft-anomalous dimension (SAD) matrix

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \ldots$$

Needed at NLL

Needed at NNLL



Colour structure 4top

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathscr{P}\exp\left[\int_{\mu_R}^{M/\bar{N}} \frac{\mathrm{d}q}{q}\, \mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

$$\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}(\alpha_s) = \left(\frac{\alpha_s}{\pi}\right)\mathbf{\Gamma}^{(1)}_{ij\to t\bar{t}t\bar{t}} + \left(\frac{\alpha_s}{\pi}\right)^2\mathbf{\Gamma}^{(2)}_{ij\to t\bar{t}t\bar{t}} + \dots$$

Needed at NLL — Needed at NNLL

$q\bar{q} \to t\bar{t}t\bar{t}$

$$\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

leads to a 6-dimensional SAD

$gg \to t\bar{t}t\bar{t}$

$$\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

leads to a 14-dimensional SAD

SAD for $q\bar{q}$ (Full kinematics)

In colour basis of [1207.0609]

$$\Gamma 11=\frac{(L34+L56+2)\left(Nc^2-1\right)}{2\,Nc}$$

$$\Gamma 12=\frac{(-L35+L36+L45-L46)\sqrt{Nc^2-1}}{2\,Nc}$$

$$\Gamma 13=\frac{\sqrt{Nc^2-1}\,(-T15+T16+T25-T26)}{2\,Nc}$$

$$\Gamma 14=\frac{\sqrt{Nc^2-1}\,(-T13+T14+T23-T24)}{2\,Nc}$$

$$\Gamma 15=0$$

$$\Gamma 16=0$$

$$\Gamma 21=\frac{(-L35+L36+L45-L46)\sqrt{Nc^2-1}}{2\,Nc}$$

$$\Gamma 22=-\frac{-L36\,Nc^2-L45\,Nc^2-2\,Nc^2+L34-2\,L35+2\,L36+2\,L45-2\,L46+L56+2}{2\,Nc}$$

$$\Gamma 23=\frac{-T13+T14+T23-T24}{2\,Nc}$$

$$\Gamma 24=\frac{-T15+T16+T25-T26}{2\,Nc}$$

$$\Gamma 25=\frac{\sqrt{Nc^2-4}\,(-T13+T14-T15+T16+T23-T24+T25-T26)}{2\sqrt{2}\,Nc}$$

$$\Gamma 26=\frac{T13+T14-T15-T16-T23-T24+T25+T26}{2\sqrt{2}}$$

$$\Gamma 31=\frac{\sqrt{Nc^2-1}\,(-T15+T16+T25-T26)}{2\,Nc}$$

$$\Gamma 32=\frac{-T13+T14+T23-T24}{2\,Nc}$$

$$\Gamma 33=-\frac{-L34\,Nc^2+T15\,Nc^2+T26\,Nc^2-Nc^2+L34+L56-2\,T15+2\,T16+2\,T25-2\,T26+2}{2\,Nc}$$

$$\Gamma 34=\frac{-L35+L36+L45-L46}{2\,Nc}$$

$$\Gamma 35=\frac{\sqrt{Nc^2-4}\,(-L35+L36+L45-L46-T13+T14+T23-T24)}{2\sqrt{2}\,Nc}$$

$$\Gamma 36=\frac{L35+L36-L45-L46+T13-T14+T23-T24}{2\sqrt{2}}$$

$$\Gamma 41=\frac{\sqrt{Nc^2-1}\,(-T13+T14+T23-T24)}{2\,Nc}$$

$$\Gamma 42=\frac{-T15+T16+T25-T26}{2\,Nc}$$

$$\Gamma 43=\frac{-L35+L36+L45-L46}{2\,Nc}$$

$$\Gamma 44=-\frac{-L56\,Nc^2+T13\,Nc^2+T24\,Nc^2-Nc^2+L34+L56-2\,T13+2\,T14+2\,T23-2\,T24+2}{2\,Nc}$$

$$\Gamma 45=\frac{\sqrt{Nc^2-4}\,(-L35+L36+L45-L46-T15+T16+T25-T26)}{2\sqrt{2}\,Nc}$$

$$\Gamma 46=\frac{-L35+L36-L45+L46-T15+T16-T25+T26}{2\sqrt{2}}$$

$$\Gamma 51=0$$

$$\Gamma 52=\frac{\sqrt{Nc^2-4}\,(-T13+T14-T15+T16+T23-T24+T25-T26)}{2\sqrt{2}\,Nc}$$

$$\Gamma 53=\frac{\sqrt{Nc^2-4}\,(-L35+L36+L45-L46-T13+T14+T23-T24)}{2\sqrt{2}\,Nc}$$

$$\Gamma 54=\frac{\sqrt{Nc^2-4}\,(-L35+L36+L45-L46-T15+T16+T25-T26)}{2\sqrt{2}\,Nc}$$

$$\Gamma 55=-\frac{-L36\,Nc^2-L45\,Nc^2+T13\,Nc^2+T15\,Nc^2+T24\,Nc^2+T26\,Nc^2-2\,Nc^2+2\,L34-6\,L35+6\,L36+6\,L45-6\,L46+2\,L56-6\,T13+6\,T14-6\,T15+6\,T16+6\,T23-6\,T24+6\,T25-6\,T26+4}{4\,Nc}$$

$$\Gamma 56=-\frac{1}{4}\sqrt{Nc^2-4}\,(L36-L45+T13-T15-T24+T26)$$

$$\Gamma 61=0$$

$$\Gamma 62=\frac{T13+T14-T15-T16-T23-T24+T25+T26}{2\sqrt{2}}$$

$$\Gamma 63=\frac{L35+L36-L45-L46+T13-T14+T23-T24}{2\sqrt{2}}$$

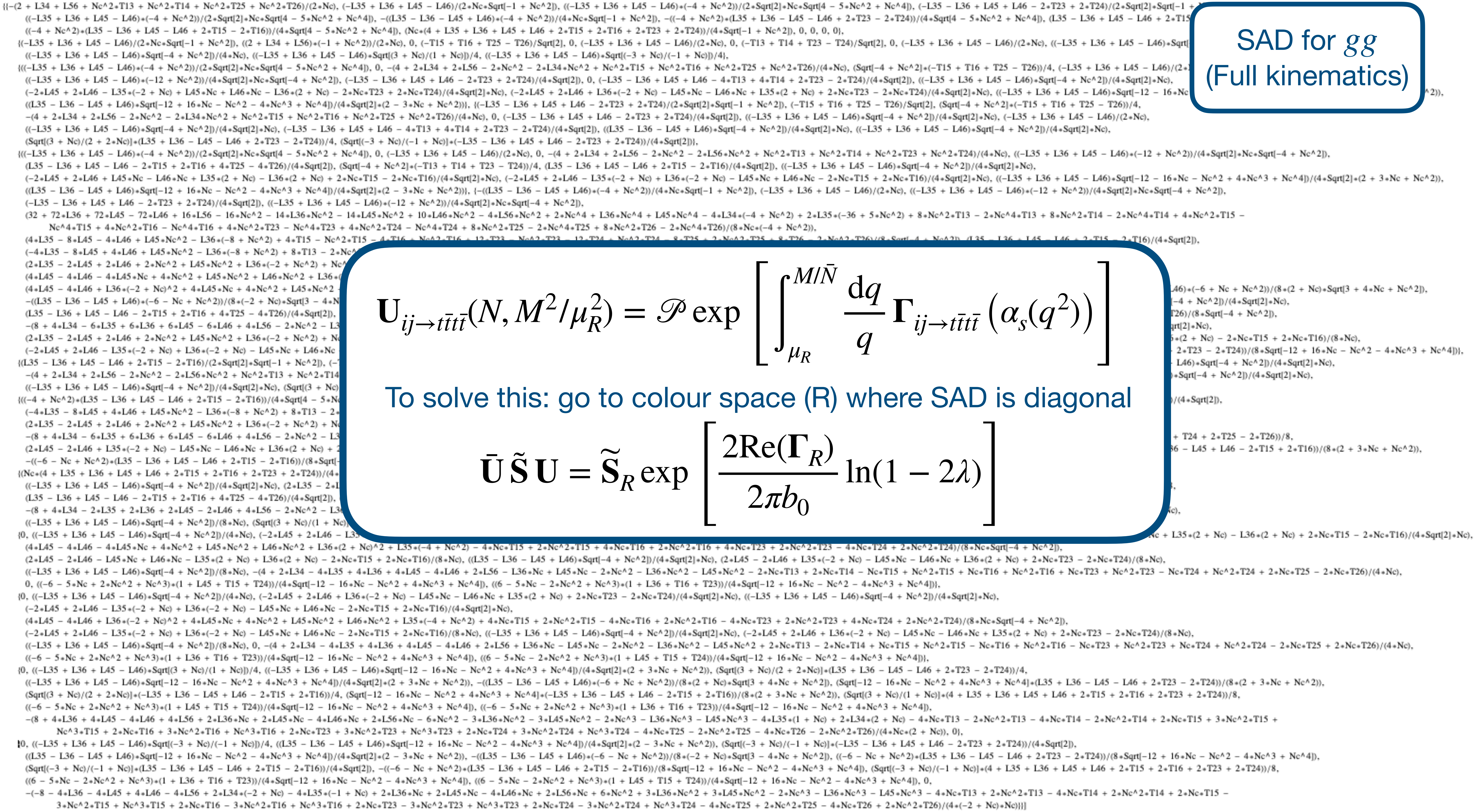
$$\Gamma 64=\frac{-L35+L36-L45+L46-T15+T16-T25+T26}{2\sqrt{2}}$$

$$\Gamma 65=-\frac{1}{4}\sqrt{Nc^2-4}\,(L36-L45+T13-T15-T24+T26)$$

$$\Gamma 66=-\frac{-L36\,Nc^2-L45\,Nc^2+T13\,Nc^2+T15\,Nc^2+T24\,Nc^2+T26\,Nc^2-2\,Nc^2+2\,L34-2\,L35+2\,L36+2\,L45-2\,L46+2\,L56-2\,T13+2\,T14-2\,T15+2\,T16+2\,T23-2\,T24+2\,T25-2\,T26+4}{4\,Nc}$$

SAD for gg
(Full kinematics)

```
{{-(2 + L34 + L56 + Nc^2*T13 + Nc^2*T14 + Nc^2*T25 + Nc^2*T26)/(2*Nc), (-L35 + L36 + L45 - L46)/(2*Nc*Sqrt[-1 + Nc^2]), ((-L35 + L36 + L45 - L46)*(-4 + Nc^2))/(2*Sqrt[2]*Nc*Sqrt[4 - 5*Nc^2 + Nc^4]), (-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(2*Sqrt[2]*Sqrt[-1 +
  ((-L35 + L36 + L45 - L46)*(-4 + Nc^2))/(2*Sqrt[2]*Nc*Sqrt[4 - 5*Nc^2 + Nc^4]), -((L35 - L36 - L45 + L46)*(-4 + Nc^2))/(4*Nc*Sqrt[-1 + Nc^2]), -((-4 + Nc^2)*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(4*Sqrt[4 - 5*Nc^2 + Nc^4]), (L35 - L36 + L45 - L46 + 2*T15
  ((-4 + Nc^2)*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(4*Sqrt[4 - 5*Nc^2 + Nc^4]), (Nc*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/(4*Sqrt[-1 + Nc^2]), 0, 0, 0, 0},
 {(-L35 + L36 + L45 - L46)/(2*Nc*Sqrt[-1 + Nc^2]), ((2 + L34 + L56)*(-1 + Nc^2))/(2*Nc), 0, (-T15 + T16 + T25 - T26)/Sqrt[2], 0, (-L35 + L36 + L45 - L46)/(2*Nc), 0, (-T13 + T14 + T23 - T24)/Sqrt[2], 0, (-L35 + L36 + L45 - L46)/(2*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[(3 + Nc)/(1 + Nc)])/4, ((-L35 + L36 + L45 - L46)*Sqrt[(-3 + Nc)/(-1 + Nc)])/4},
 {((-L35 + L36 + L45 - L46)*(-4 + Nc^2))/(2*Sqrt[2]*Nc*Sqrt[4 - 5*Nc^2 + Nc^4]), 0, -(4 + 2*L34 + 2*L56 - 2*Nc^2 - 2*L34*Nc^2 + Nc^2*T15 + Nc^2*T16 + Nc^2*T25 + Nc^2*T26)/(4*Nc), (Sqrt[-4 + Nc^2]*(-T15 + T16 + T25 - T26))/4, (-L35 + L36 + L45 - L46)/(2*
  ((-L35 + L36 + L45 - L46)*(-12 + Nc^2))/(4*Sqrt[2]*Nc*Sqrt[-4 + Nc^2]), (-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(4*Sqrt[2]), 0, (-L35 - L36 + L45 + L46 - 4*T13 + 4*T14 + 2*T23 - 2*T24)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (-2*L45 + 2*L46 - L35*(-2 + Nc) + L45*Nc + L46*Nc - L36*(2 + Nc) - 2*Nc*T23 + 2*Nc*T24)/(4*Sqrt[2]*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45*Nc - L46*Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(4*Sqrt[2]*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc ^2)),
  ((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2))}, {(-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(2*Sqrt[2]*Sqrt[-1 + Nc^2]), (-T15 + T16 + T25 - T26)/Sqrt[2], (Sqrt[-4 + Nc^2]*(-T15 + T16 + T25 - T26))/4,
  -(4 + 2*L34 + 2*L56 - 2*Nc^2 - 2*L34*Nc^2 + Nc^2*T15 + Nc^2*T16 + Nc^2*T25 + Nc^2*T26)/(4*Nc), 0, (-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-L35 + L36 + L45 - L46)/(2*Nc),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-L35 - L36 + L45 + L46 - 4*T13 + 4*T14 + 2*T23 - 2*T24)/(4*Sqrt[2]), ((L35 - L36 - L45 + L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (Sqrt[(3 + Nc)/(2 + 2*Nc)]*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/4, (Sqrt[(-3 + Nc)/(-1 + Nc)]*(-L35 - L36 + L45 + L46 - 2*T23 + 2*T24))/(4*Sqrt[2])},
 {((-L35 + L36 + L45 - L46)*(-4 + Nc^2))/(2*Sqrt[2]*Nc*Sqrt[4 - 5*Nc^2 + Nc^4]), 0, (-L35 + L36 + L45 - L46)/(2*Nc), 0, -(4 + 2*L34 + 2*L56 - 2*Nc^2 - 2*L56*Nc^2 + Nc^2*T13 + Nc^2*T14 + Nc^2*T23 + Nc^2*T24)/(4*Nc), ((-L35 + L36 + L45 - L46)*(-12 + Nc^2))/(4*Sqrt[2]*Nc*Sqrt[-4 + Nc^2]),
  (L35 - L36 + L45 - L46 - 2*T15 + 2*T16 + 4*T25 - 4*T26)/(4*Sqrt[2]), (Sqrt[-4 + Nc^2]*(-T13 + T14 + T23 - T24))/4, (L35 - L36 + L45 - L46 + 2*T15 - 2*T16)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (-2*L45 + 2*L46 + L45*Nc - L46*Nc + L35*(2 + Nc) - L36*(2 + Nc) + 2*Nc*T15 - 2*Nc*T16)/(4*Sqrt[2]*Nc), (-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45*Nc + L46*Nc - 2*Nc*T15 + 2*Nc*T16)/(4*Sqrt[2]*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 + 3*Nc + Nc^2)),
  ((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2))}, {-((L35 - L36 - L45 + L46)*(-4 + Nc^2))/(4*Nc*Sqrt[-1 + Nc^2]), (-L35 + L36 + L45 - L46)/(2*Nc), ((-L35 + L36 + L45 - L46)*(-12 + Nc^2))/(4*Sqrt[2]*Nc*Sqrt[-4 + Nc^2]),
  (-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*(-12 + Nc^2))/(4*Sqrt[2]*Nc*Sqrt[-4 + Nc^2]),
  (32 + 72*L36 + 72*L45 - 72*L46 + 16*L56 - 16*Nc^2 - 14*L36*Nc^2 - 14*L45*Nc^2 + 10*L46*Nc^2 - 4*L56*Nc^2 + 2*Nc^4 + L36*Nc^4 + L45*Nc^4 - 4*L34*(-4 + Nc^2) + 2*L35*(-36 + 5*Nc^2) + 8*Nc^2*T13 - 2*Nc^4*T13 + 8*Nc^2*T14 - 2*Nc^4*T14 + 4*Nc^2*T15 -
     Nc^4*T15 + 4*Nc^2*T16 - Nc^4*T16 + 4*Nc^2*T23 - Nc^4*T23 + 4*Nc^2*T24 - Nc^4*T24 + 8*Nc^2*T25 - 2*Nc^4*T25 + 8*Nc^2*T26 - 2*Nc^4*T26)/(8*Nc*(-4 + Nc^2)),
  (4*L35 - 8*L45 - 4*L46 + L45*Nc^2 - L36*(-8 + Nc^2) + 4*T15 - Nc^2*T15 - 4*T16 + Nc^2*T16 + 12*T23 - Nc^2*T23 - 12*T24 + Nc^2*T24 - 8*T25 + 2*Nc^2*T25 + 8*T26 - 2*Nc^2*T26)/(8*Sqrt[-4 + Nc^2]), (L35 - L36 + L45 - L46 + 2*T15 - 2*T16)/(4*Sqrt[2]),
  (-4*L35 - 8*L45 + 4*L46 + L45*Nc^2 - L36*(-8 + Nc^2) + 8*T13 - 2*Nc^2*T13 - 8*T14 + 2*Nc^2*T14 - 12*T15 + Nc^2*T15 + 12*T16 - Nc^2*T16 - 4*T23 + Nc^2*T23 + 4*T24 - Nc^2*T24)/(8*Sqrt[-4 + Nc^2]),
  (2*L35 - 2*L45 + 2*L46 + 2*Nc^2 + L45*Nc^2 + L36*(-2 + Nc^2) + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24)/(8*Nc),
  (4*L45 - 4*L46 - 4*L45*Nc + 4*Nc^2 + L45*Nc^2 + L46*Nc^2 + L36*(2 + Nc)^2 + L35*(-4 + Nc^2) - 4*Nc*T15 + 2*Nc^2*T15 + 4*Nc*T16 + 2*Nc^2*T16 + 4*Nc*T23 + 2*Nc^2*T23 - 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]),
  (4*L45 - 4*L46 + L36*(-2 + Nc)^2 + 4*L45*Nc + 4*Nc^2 + L45*Nc^2 + L46*Nc^2 + L35*(-4 + Nc^2) + 4*Nc*T15 + 2*Nc^2*T15 - 4*Nc*T16 + 2*Nc^2*T16 - 4*Nc*T23 + 2*Nc^2*T23 + 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]), -((L35 - L36 - L45 + L46)*(-6 + Nc + Nc^2))/(8*(2 + Nc)*Sqrt[3 + 4*Nc + Nc^2]),
  -((L35 - L36 - L45 + L46)*(-6 - Nc + Nc^2))/(8*(-2 + Nc)*Sqrt[3 - 4*Nc + Nc^2])}, {-((-4 + Nc^2)*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(4*Sqrt[4 - 5*Nc^2 + Nc^4]), 0, (-L35 - L36 + L45 + L46 - 2*T23 + 2*T24)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (L35 - L36 + L45 - L46 - 2*T15 + 2*T16 + 4*T25 - 4*T26)/(4*Sqrt[2]), (4*L35 - 8*L45 - 4*L46 + L45*Nc^2 - L36*(-8 + Nc^2) + 4*T15 - Nc^2*T15 - 4*T16 + Nc^2*T16 + 12*T23 - Nc^2*T23 - 12*T24 + Nc^2*T24 - 8*T25 + 2*Nc^2*T25 + 8*T26 - 2*Nc^2*T26)/(8*Sqrt[-4 + Nc^2]),
  -(8 + 4*L34 - 6*L35 + 6*L36 + 6*L45 - 6*L46 + 4*L56 - 2*Nc^2 - L36*Nc^2 - L45*Nc^2 + 2*Nc^2*T13 + 2*Nc^2*T14 + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24 + 2*Nc^2*T25 + 2*Nc^2*T26)/(8*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (2*L35 - 2*L45 + 2*L46 + 2*Nc^2 + L45*Nc^2 + L36*(-2 + Nc^2) + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24)/(8*Nc), (Sqrt[-4 + Nc^2]*(-L36 + L45 - 2*T13 + 2*T14 + T15 - T16 + T23 - T24))/8, (2*L45 - 2*L46 - L45*Nc + L46*Nc - L35*(2 + Nc) + L36*(2 + Nc) - 2*Nc*T15 + 2*Nc*T16)/(8*Nc),
  (-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45*Nc + L46*Nc - 2*Nc*T15 + 2*Nc*T16)/(8*Nc), (Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*(2 + 3*Nc + Nc^2)), ((-6 - Nc + Nc^2)*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])},
 {(L35 - L36 + L45 - L46 + 2*T15 - 2*T16)/(2*Sqrt[2]*Sqrt[-1 + Nc^2]), (-T13 + T14 + T23 - T24)/Sqrt[2], 0, (-L35 + L36 + L45 - L46)/(2*Nc), (Sqrt[-4 + Nc^2]*(-T13 + T14 + T23 - T24))/4, (L35 - L36 + L45 - L46 + 2*T15 - 2*T16)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  -(4 + 2*L34 + 2*L56 - 2*Nc^2 - 2*L56*Nc^2 + Nc^2*T13 + Nc^2*T14 + Nc^2*T23 + Nc^2*T24)/(4*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (L35 - L36 + L45 - L46 - 2*T15 + 2*T16 + 4*T25 - 4*T26)/(4*Sqrt[2]), ((L35 - L36 - L45 + L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (Sqrt[(3 + Nc)/(2 + 2*Nc)]*(-L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/4, (Sqrt[(-3 + Nc)/(-1 + Nc)]*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(4*Sqrt[2])},
 {((-4 + Nc^2)*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(4*Sqrt[4 - 5*Nc^2 + Nc^4]), 0, (-L35 - L36 + L45 + L46 - 4*T13 + 4*T14 + 2*T23 - 2*T24)/(4*Sqrt[2]), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (L35 - L36 + L45 - L46 + 2*T15 - 2*T16)/(4*Sqrt[2]),
  (-4*L35 - 8*L45 + 4*L46 + L45*Nc^2 - L36*(-8 + Nc^2) + 8*T13 - 2*Nc^2*T13 - 8*T14 + 2*Nc^2*T14 - 12*T15 + Nc^2*T15 + 12*T16 - Nc^2*T16 - 4*T23 + Nc^2*T23 + 4*T24 - Nc^2*T24)/(8*Sqrt[-4 + Nc^2]),
  (2*L35 - 2*L45 + 2*L46 + 2*Nc^2 + L45*Nc^2 + L36*(-2 + Nc^2) + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24)/(8*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  -(8 + 4*L34 - 6*L35 + 6*L36 + 6*L45 - 6*L46 + 4*L56 - 2*Nc^2 - L36*Nc^2 - L45*Nc^2 + 2*Nc^2*T13 + 2*Nc^2*T14 + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24 + 2*Nc^2*T25 + 2*Nc^2*T26)/(8*Nc), (Sqrt[-4 + Nc^2]*(-L36 + L45 - T15 + T16 - T23 + T24 + 2*T25 - 2*T26))/8,
  (2*L45 - 2*L46 + L35*(-2 + Nc) - L45*Nc - L46*Nc + L36*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45*Nc - L46*Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc), (Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(-L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/(8*(2 + 3*Nc + Nc^2)),
  -((-6 - Nc + Nc^2)*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])},
 {(Nc*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/(4*Sqrt[-1 + Nc^2]), (-L35 + L36 + L45 - L46)/(2*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-L35 - L36 + L45 + L46 - 4*T13 + 4*T14 + 2*T23 - 2*T24)/(4*Sqrt[2]),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (2*L35 - 2*L45 + 2*L46 + 2*Nc^2 + L45*Nc^2 + L36*(-2 + Nc^2) + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24)/(8*Nc), (Sqrt[-4 + Nc^2]*(-L36 + L45 - 2*T13 + 2*T14 + T15 - T16 + T23 - T24))/8,
  (L35 - L36 + L45 - L46 - 2*T15 + 2*T16 + 4*T25 - 4*T26)/(4*Sqrt[2]), (Sqrt[-4 + Nc^2]*(-L36 + L45 - T15 + T16 - T23 + T24 + 2*T25 - 2*T26))/8,
  -(8 + 4*L34 - 2*L35 + 2*L36 + 2*L45 - 2*L46 + 4*L56 - 2*Nc^2 - L36*Nc^2 - L45*Nc^2 + 2*Nc^2*T13 + 2*Nc^2*T14 + Nc^2*T15 + Nc^2*T16 + Nc^2*T23 + Nc^2*T24 + 2*Nc^2*T25 + 2*Nc^2*T26)/(8*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc), (Sqrt[(3 + Nc)/(1 + Nc)]*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8, (Sqrt[(-3 + Nc)/(-1 + Nc)]*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8},
 {0, ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Nc), (-2*L45 + 2*L46 - L35*(-2 + Nc) + L45*Nc + L46*Nc - L36*(2 + Nc) - 2*Nc*T23 + 2*Nc*T24)/(4*Sqrt[2]*Nc), ((L35 - L36 - L45 + L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-2*L45 + 2*L46 + L45*Nc - L46*Nc + L35*(2 + Nc) - L36*(2 + Nc) + 2*Nc*T15 - 2*Nc*T16)/(4*Sqrt[2]*Nc),
  (4*L45 - 4*L46 - 4*L45*Nc + 4*Nc^2 + L45*Nc^2 + L46*Nc^2 + L36*(2 + Nc)^2 + L35*(-4 + Nc^2) - 4*Nc*T15 + 2*Nc^2*T15 + 4*Nc*T16 + 2*Nc^2*T16 + 4*Nc*T23 + 2*Nc^2*T23 - 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]),
  (2*L45 - 2*L46 - L45*Nc + L46*Nc - L35*(2 + Nc) + L36*(2 + Nc) - 2*Nc*T15 + 2*Nc*T16)/(8*Nc), ((L35 - L36 - L45 + L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (2*L45 - 2*L46 + L35*(-2 + Nc) - L45*Nc - L46*Nc + L36*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc), -(4 + 2*L34 - 4*L35 + 4*L36 + 4*L45 - 4*L46 + 2*L56 - L36*Nc + L45*Nc - 2*Nc^2 - L36*Nc^2 - L45*Nc^2 - 2*Nc*T13 + 2*Nc*T14 - Nc*T15 + Nc^2*T15 + Nc*T16 + Nc^2*T16 + Nc*T23 + Nc^2*T23 - Nc*T24 + Nc^2*T24 + 2*Nc*T25 - 2*Nc*T26)/(4*Nc),
  0, ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])},
 {0, ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45*Nc - L46*Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(4*Sqrt[2]*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc),
  (-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45*Nc + L46*Nc - 2*Nc*T15 + 2*Nc*T16)/(4*Sqrt[2]*Nc),
  (4*L45 - 4*L46 + L36*(-2 + Nc)^2 + 4*L45*Nc + 4*Nc^2 + L45*Nc^2 + L46*Nc^2 + L35*(-4 + Nc^2) + 4*Nc*T15 + 2*Nc^2*T15 - 4*Nc*T16 + 2*Nc^2*T16 - 4*Nc*T23 + 2*Nc^2*T23 + 4*Nc*T24 + 2*Nc^2*T24)/(8*Nc*Sqrt[-4 + Nc^2]),
  (-2*L45 + 2*L46 - L35*(-2 + Nc) + L36*(-2 + Nc) - L45*Nc + L46*Nc - 2*Nc*T15 + 2*Nc*T16)/(8*Nc), ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(4*Sqrt[2]*Nc), (-2*L45 + 2*L46 + L36*(-2 + Nc) - L45*Nc - L46*Nc + L35*(2 + Nc) + 2*Nc*T23 - 2*Nc*T24)/(8*Nc),
  ((-L35 + L36 + L45 - L46)*Sqrt[-4 + Nc^2])/(8*Nc), 0, -(4 + 2*L34 - 4*L35 + 4*L36 + 4*L45 - 4*L46 + 2*L56 + L36*Nc - L45*Nc - 2*Nc^2 - L36*Nc^2 - L45*Nc^2 + 2*Nc*T13 - 2*Nc*T14 + Nc*T15 + Nc^2*T15 - Nc*T16 + Nc^2*T16 - Nc*T23 + Nc^2*T23 + Nc*T24 + Nc^2*T24 - 2*Nc*T25 + 2*Nc*T26)/(4*Nc),
  ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])},
 {0, ((-L35 + L36 + L45 - L46)*Sqrt[(3 + Nc)/(1 + Nc)])/4, ((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 + 3*Nc + Nc^2)), (Sqrt[(3 + Nc)/(2 + 2*Nc)]*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/4,
  ((-L35 + L36 + L45 - L46)*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 + 3*Nc + Nc^2)), -((L35 - L36 - L45 + L46)*(-6 + Nc + Nc^2))/(8*(2 + Nc)*Sqrt[3 + 4*Nc + Nc^2]), (Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*(2 + 3*Nc + Nc^2)),
  (Sqrt[(3 + Nc)/(2 + 2*Nc)]*(-L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/4, (Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]*(-L35 + L36 - L45 + L46 - 2*T15 + 2*T16))/(8*(2 + 3*Nc + Nc^2)), (Sqrt[(3 + Nc)/(1 + Nc)]*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8,
  ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]), ((-6 - 5*Nc + 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 - 16*Nc - Nc^2 + 4*Nc^3 + Nc^4]),
  -(8 + 4*L36 + 4*L45 - 4*L46 + 4*L56 + 2*L36*Nc + 2*L45*Nc - 4*L46*Nc + 2*L56*Nc - 6*Nc^2 - 3*L36*Nc^2 - 3*L45*Nc^2 - 2*Nc^3 - L36*Nc^3 - L45*Nc^3 - 4*L35*(1 + Nc) + 2*L34*(2 + Nc) - 4*Nc*T13 - 2*Nc^2*T13 - 4*Nc*T14 - 2*Nc^2*T14 + 2*Nc*T15 + 3*Nc^2*T15 +
     Nc^3*T15 + 2*Nc*T16 + 3*Nc^2*T16 + Nc^3*T16 + 2*Nc*T23 + 3*Nc^2*T23 + Nc^3*T23 + 2*Nc*T24 + 3*Nc^2*T24 + Nc^3*T24 - 4*Nc*T25 - 2*Nc^2*T25 - 4*Nc*T26 - 2*Nc^2*T26)/(4*Nc*(2 + Nc)), 0},
 {0, ((-L35 + L36 + L45 - L46)*Sqrt[(-3 + Nc)/(-1 + Nc)])/4, ((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2)), (Sqrt[(-3 + Nc)/(-1 + Nc)]*(-L35 - L36 + L45 + L46 - 2*T23 + 2*T24))/(4*Sqrt[2]),
  ((L35 - L36 - L45 + L46)*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4])/(4*Sqrt[2]*(2 - 3*Nc + Nc^2)), -((L35 - L36 - L45 + L46)*(-6 - Nc + Nc^2))/(8*(-2 + Nc)*Sqrt[3 - 4*Nc + Nc^2]), ((-6 - Nc + Nc^2)*(L35 + L36 - L45 - L46 + 2*T23 - 2*T24))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]),
  (Sqrt[(-3 + Nc)/(-1 + Nc)]*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(4*Sqrt[2]), -((-6 - Nc + Nc^2)*(L35 - L36 + L45 - L46 + 2*T15 - 2*T16))/(8*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), (Sqrt[(-3 + Nc)/(-1 + Nc)]*(4 + L35 + L36 + L45 + L46 + 2*T15 + 2*T16 + 2*T23 + 2*T24))/8,
  ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L36 + T16 + T23))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), ((6 - 5*Nc - 2*Nc^2 + Nc^3)*(1 + L45 + T15 + T24))/(4*Sqrt[-12 + 16*Nc - Nc^2 - 4*Nc^3 + Nc^4]), 0,
  -(-8 - 4*L36 - 4*L45 + 4*L46 - 4*L56 + 2*L34*(-2 + Nc) - 4*L35*(-1 + Nc) + 2*L36*Nc + 2*L45*Nc - 4*L46*Nc + 2*L56*Nc + 6*Nc^2 + 3*L36*Nc^2 + 3*L45*Nc^2 - 2*Nc^3 - L36*Nc^3 - L45*Nc^3 - 4*Nc*T13 + 2*Nc^2*T13 - 4*Nc*T14 + 2*Nc^2*T14 + 2*Nc*T15 -
     3*Nc^2*T15 + Nc^3*T15 + 2*Nc*T16 - 3*Nc^2*T16 + Nc^3*T16 + 2*Nc*T23 - 3*Nc^2*T23 + Nc^3*T23 + 2*Nc*T24 - 3*Nc^2*T24 + Nc^3*T24 - 4*Nc*T25 + 2*Nc^2*T25 - 4*Nc*T26 + 2*Nc^2*T26)/(4*(-2 + Nc)*Nc)}}
```



SAD for gg (Full kinematics)

$$\mathbf{U}_{ij\to t\bar{t}t\bar{t}}(N, M^2/\mu_R^2) = \mathscr{P}\exp\left[\int_{\mu_R}^{M/\bar{N}} \frac{\mathrm{d}q}{q}\,\mathbf{\Gamma}_{ij\to t\bar{t}t\bar{t}}\left(\alpha_s(q^2)\right)\right]$$

To solve this: go to colour space (R) where SAD is diagonal

$$\bar{\mathbf{U}}\,\tilde{\mathbf{S}}\,\mathbf{U} = \widetilde{\mathbf{S}}_R \exp\left[\frac{2\mathrm{Re}(\mathbf{\Gamma}_R)}{2\pi b_0}\ln(1-2\lambda)\right]$$

Colour structure 4top

$$\bar{\mathbf{U}}\,\tilde{\mathbf{S}}\,\mathbf{U} = \widetilde{\mathbf{S}}_R \exp\left[\frac{2\mathrm{Re}(\boldsymbol{\Gamma}_R)}{2\pi b_0}\ln(1-2\lambda)\right]$$

$C_2(\mathbf{1}) = 0$

$C_2(\mathbf{8}_{(\mathbf{A/S})}) = N_c = 3$

$C_2(\mathbf{10}, \bar{\mathbf{10}}) = 2N_c = 6$

$C_2(\mathbf{27}) = 2(N_c+1) = 8$

$C_2(N_c^2(N_c-3)(N_c+1)/4 = \mathbf{0}) = 2(N_c-1) = 4$

$q\bar{q} \to t\bar{t}t\bar{t}$

$$\mathbf{3}\otimes\bar{\mathbf{3}} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{1}\oplus\mathbf{8} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\bar{\mathbf{10}}\oplus\mathbf{27}$$

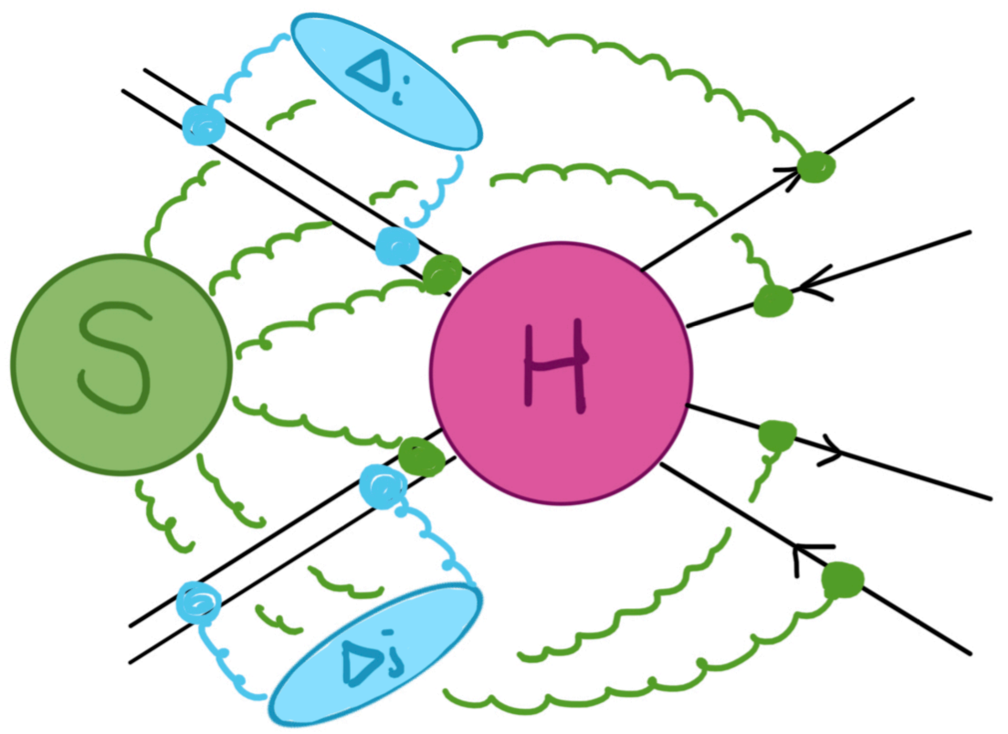
Absolute threshold limit: $2\mathrm{Re}[\boldsymbol{\Gamma}^{(1)}_{R,q\bar{q}}] = \mathrm{diag}\left(0, 0, -3, -3, -3, -3\right)$

$gg \to t\bar{t}t\bar{t}$

$$\mathbf{8}\otimes\mathbf{8} = \mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}} \longrightarrow \mathbf{0}\oplus\mathbf{1}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\bar{\mathbf{10}}\oplus\mathbf{27} = \mathbf{0}\oplus(2\times\mathbf{1})\oplus(2\times\mathbf{8})\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{10}\oplus\bar{\mathbf{10}}\oplus\mathbf{27}$$

Absolute threshold limit: $2\,\mathrm{Re}[\boldsymbol{\Gamma}^{(1)}_{R,gg}] = \mathrm{diag}\left(-8, -6, -6, -4, -3, -3, -3, -3, -3, -3, -3, -3, 0, 0\right)$

Resummation for 4top



Mellin-space resummed cross section

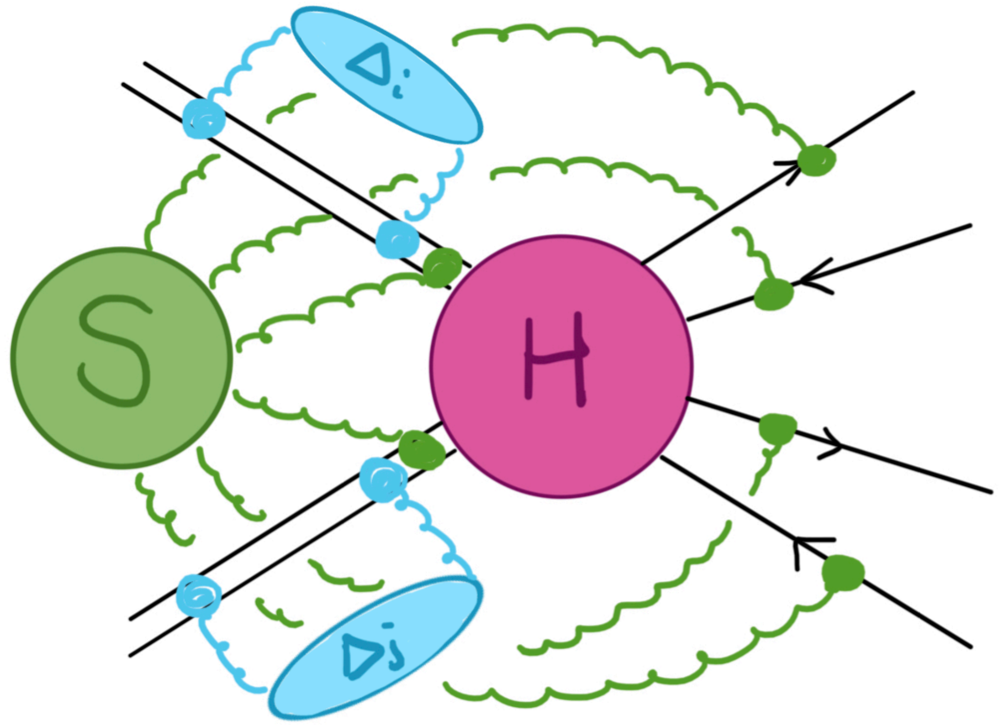
Soft function
captures wide-angle-soft enhancements

$$\hat{\sigma}^{\rm res}_{ij\to t\bar{t}t\bar{t}}(N) = {\rm Tr}\left[\, \mathbf{H}_{ij\to t\bar{t}t\bar{t}}\; \bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\, \tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\, \mathbf{U}_{ij\to t\bar{t}t\bar{t}} \right] \Delta_i\, \Delta_j$$

Hard function
captures constant contributions as $N \to \infty$

Incoming jet functions
capture soft-collinear enhancements

Resummation for 4top



Mellin-space resummed cross section

Soft function
captures wide-angle-soft enhancements

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N) = \text{Tr}\left[\mathbf{H}_{ij\to t\bar{t}t\bar{t}}\,\bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\,\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\,\mathbf{U}_{ij\to t\bar{t}t\bar{t}}\right]\Delta_i\,\Delta_j$$

Hard function
captures constant contributions as $N \to \infty$

Incoming jet functions
capture soft-collinear enhancements

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots,$$

NLL

Projection on colour space R

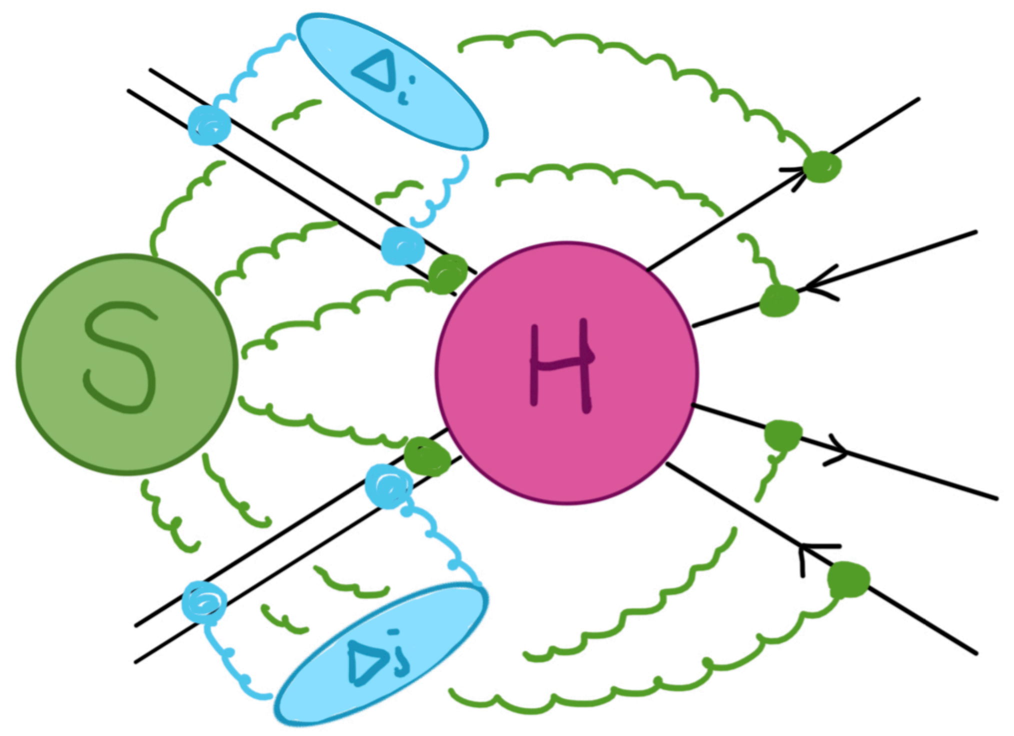
$$H^{(0)}_{IJ} = \frac{1}{2s}\int_0^1 \mathrm{d}\rho\,\rho^{N-1}\int \mathrm{d}\Phi^B \sum_{\text{colour(K,L),spin}} \mathscr{A}^{(0)}_K \mathscr{A}^{\dagger(0)}_L \langle c_L | c_J\rangle\langle c_I | c_K\rangle$$

Born phase-space integration

Colour-stripped amplitude

Melissa van Beekveld

Resummation for 4top



Mellin-space resummed cross section

Soft function
captures wide-angle-soft enhancements

$$\hat{\sigma}^{\text{res}}_{ij\to t\bar{t}t\bar{t}}(N) = \text{Tr}\left[\,\mathbf{H}_{ij\to t\bar{t}t\bar{t}}\;\bar{\mathbf{U}}_{ij\to t\bar{t}t\bar{t}}\,\tilde{\mathbf{S}}_{ij\to t\bar{t}t\bar{t}}\,\mathbf{U}_{ij\to t\bar{t}t\bar{t}}\right]\Delta_i\,\Delta_j$$

Hard function
captures constant contributions as $N \to \infty$

Incoming jet functions
capture soft-collinear enhancements

$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s(\mu_R)}{\pi}\mathbf{H}^{(1)} + \ldots,$$

NLL'

$$\mathbf{H}^{(1)} = \mathbf{V}^{(1)} + \mathbf{C}^{(1)}$$

$\mathbf{V}^{(1)}$: Virtual one-loop contributions

$\mathbf{C}^{(1)}$: Collinear end-point contributions

(MG5@NLO to extract $\mathbf{H}^{(0)}$ and $\mathbf{V}^{(1)}$)

Matching to fixed order

Go back to physical space by an inverse Mellin transform

$$\sigma^{\mathrm{NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \int_{\mathscr{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N} f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)$$

Match to fixed-order predictions through

$$\sigma^{\mathrm{NLO+NLL}(')}_{t\bar{t}t\bar{t}}(\tau) = \sigma^{\mathrm{NLO}}_{t\bar{t}t\bar{t}}(\tau) + \int_{\mathscr{C}} \frac{\mathrm{d}N}{2\pi i}\, \tau^{-N} f_i(N+1,\mu_F^2)\, f_j(N+1,\mu_F^2) \times \left[\hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N) - \hat{\sigma}^{\mathrm{res}}_{ij\to t\bar{t}t\bar{t}}(N)\big|_{\mathrm{NLO}}\right]$$

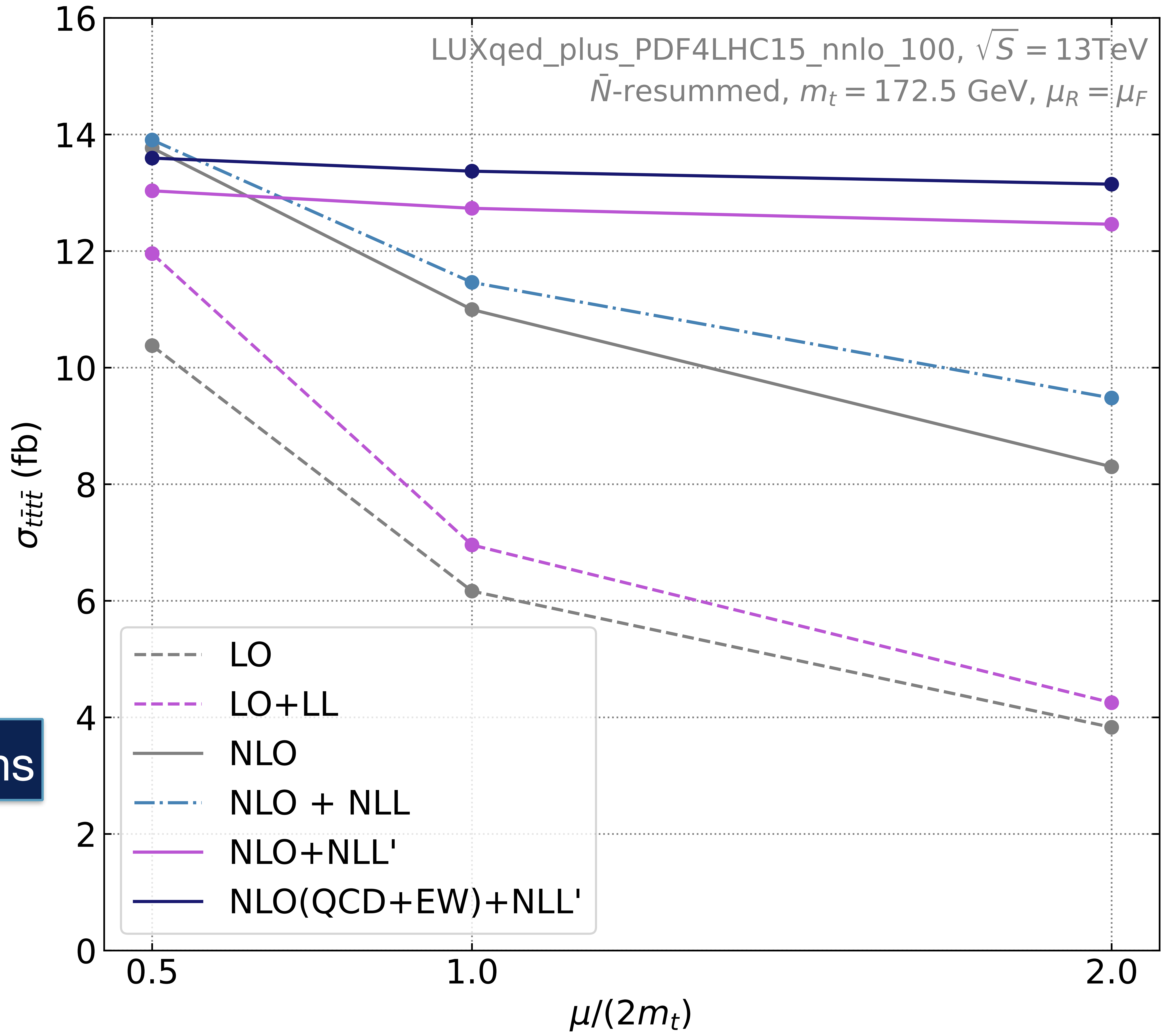
Consider both QCD-only NLO corrections
and QCD + EW (LO only)

Results

- LHC at 13 TeV
- $\mu_R = \mu_F = 2m_t$
- $m_t = 172.5$ GeV
- PDF4LHC with LUXqed PDF set

(includes photon PDFs)

Result is extremely stable under μ variations



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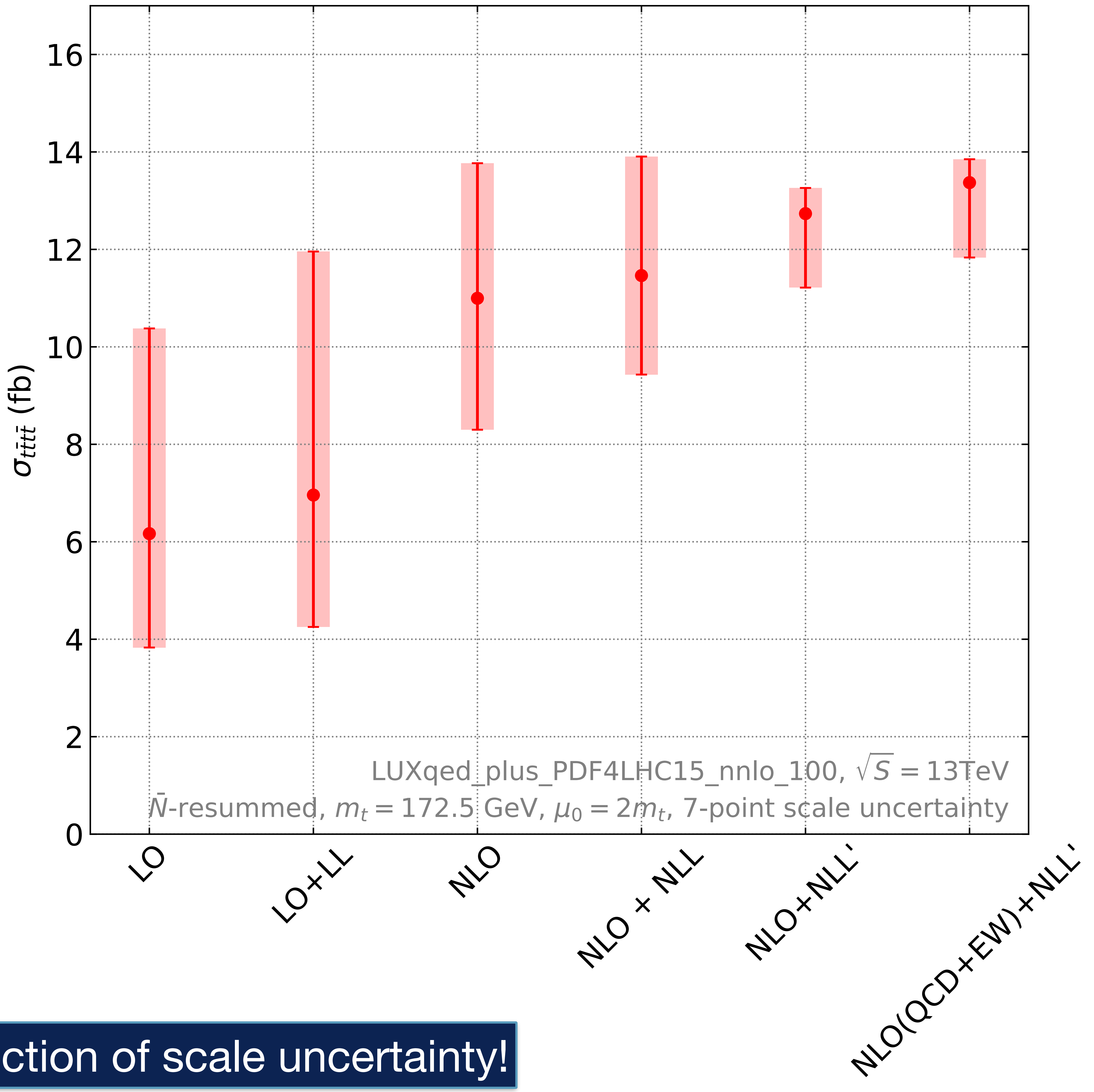
[2212.03259]

Results

	$\sigma_{t\bar{t}t\bar{t}}$ (fb)	K-factor
NLO	$11.00(2)^{+25.2\%}_{-24.5\%}$	
NLO+NLL	$11.46(2)^{+21.3\%}_{-17.7\%}$	1.04
NLO+NLL'	$12.73(2)^{+4.1\%}_{-11.8\%}$	1.16
NLO(QCD+EW)	$11.64(2)^{+23.2\%}_{-22.8\%}$	
NLO(QCD+EW)+NLL	$12.10(2)^{+19.5\%}_{-16.3\%}$	1.04
NLO(QCD+EW)+NLL'	$13.37(2)^{+3.6\%}_{-11.4\%}$	1.15

PDF error is 6.9 %

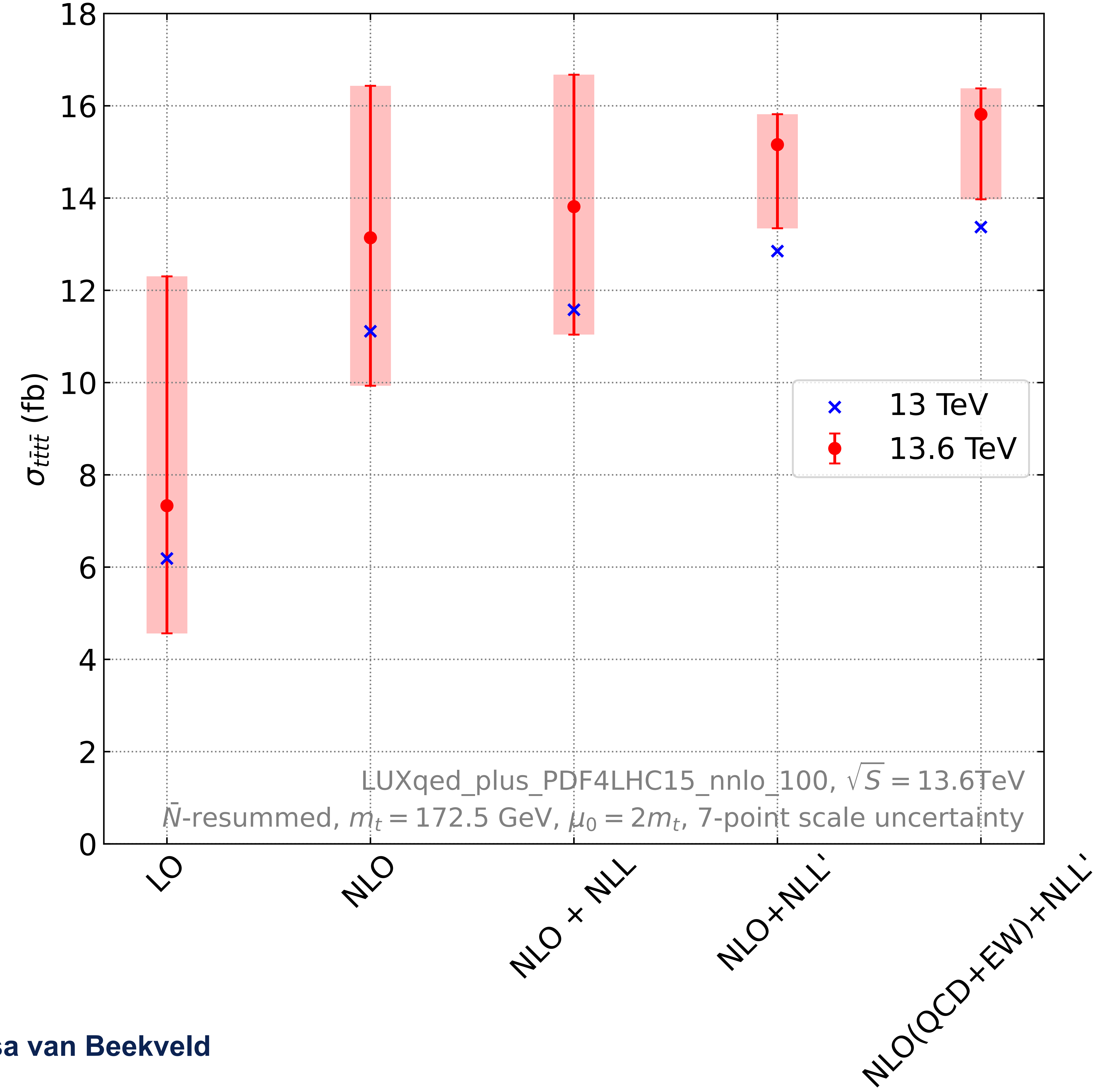


We find a huge reduction of scale uncertainty!

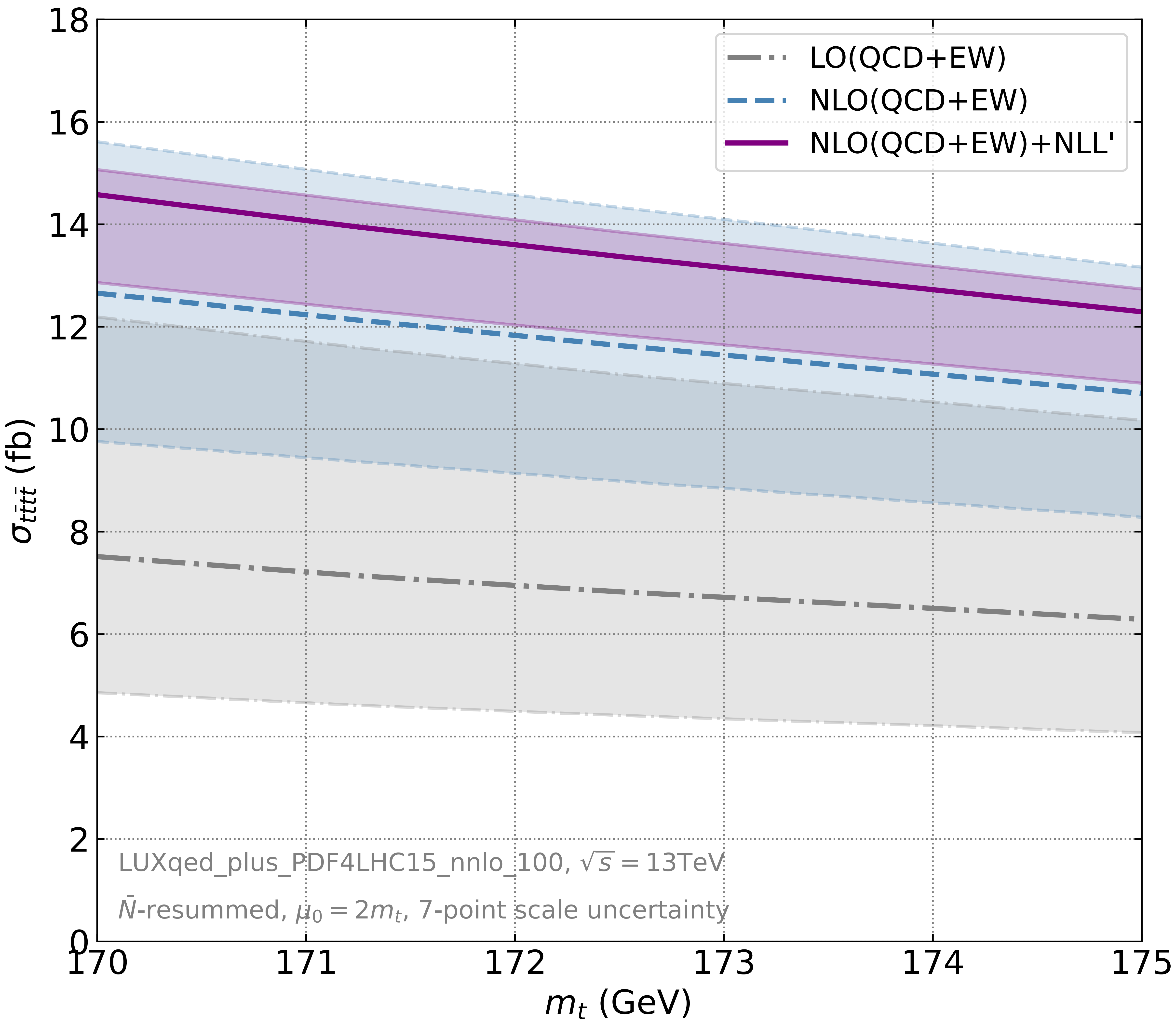
Results for 13.6 TeV

	$\sigma_{t\bar{t}t\bar{t}}$ (fb)	K-factor
NLO	$13.14(2)^{+25.1\%}_{-24.4\%}$	
NLO+NLL	$13.81(2)^{+20.7\%}_{-20.1\%}$	1.05
NLO+NLL'	$15.16(2)^{+4.3\%}_{-11.9\%}$	1.15
NLO(QCD+EW)	$13.80(2)^{+22.9\%}_{-22.6\%}$	
NLO(QCD+EW)+NLL	$14.47(2)^{+18.4\%}_{-18.5\%}$	1.05
NLO(QCD+EW)+NLL'	$15.81(2)^{+3.6\%}_{-11.6\%}$	1.14

PDF error is $6.7\ \%$



Results for different top masses



Part B
Next-to-leading power corrections

[2101.07270], [2109.09752]

with Eric Laenen, Jort Sinninghe Damste, Leonardo Vernazza, Chris White

Threshold expansion of cross sections

Consider hadronic observable of colour-singlet production $\sigma_{pp\to H+X}$

$$\mathrm{d}\sigma_{pp\to H+X} = \sigma_0^H \sum_{i,j} \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \Delta_{H,ij}(z) \qquad \tau = \frac{m_H^2}{S} = x_1 x_2 z$$

Threshold expansion of partonic coefficient function up to leading power (LP):

$$\Delta_{H,ij}(z) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(ij),\mathrm{LP}} \left(\frac{\ln^m(1-z)}{1-z}\right)_+ + c_n^\delta \delta(1-z) + \ldots$$

Threshold expansion of cross sections

Consider hadronic observable of colour-singlet production $\sigma_{pp\to H+X}$

$$\mathrm{d}\sigma_{pp\to H+X} = \sigma_0^H \sum_{i,j} \int_\tau^1 \frac{\mathrm{d}z}{z} \mathscr{L}_{ij}\left(\frac{\tau}{z}\right) \Delta_{H,ij}(z) \qquad \tau = \frac{m_H^2}{S} = x_1 x_2 z$$

Threshold expansion of partonic coefficient function up to next-to-leading power (NLP):

$$\Delta_{H,ij}(z) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(ij),\mathrm{LP}} \left(\frac{\ln^m(1-z)}{1-z}\right)_+ + c_n^\delta \delta(1-z) + \sum_{m=0}^{2n-1} c_{nm}^{(ij),\mathrm{NLP}} \ln^m(1-z) + \ldots$$

NLP contributions - not generally understood, but lots of recent progress!

Diagrammatic - [0807.4412], [1905.08741], [1905.11771], [1905.13710], [2101.07270], [2109.09752] …
SCET - [1809.10631], [1910.12685], [1910.14038], [1912.01585], [2008.04943], [2107.07353] …

[1905.13710]

NLP LL resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic invariant-mass distribution at NLP:

$$d\sigma = \frac{1}{2s}\left[\int d\Phi_{\mathrm{LP}}\,|\mathscr{M}|^2_{\mathrm{LP+NLP}} + \int d\Phi_{\mathrm{NLP}}\,|\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

NLP LL resummation for colour-singlet processes

Consider $p_1 + p_2 \rightarrow Q + k_1 + \ldots + k_n$

Partonic invariant-mass distribution at NLP:

$$\mathrm{d}\sigma = \frac{1}{2s}\left[\int \mathrm{d}\Phi_{\mathrm{LP}} |\mathscr{M}|^2_{\mathrm{LP+NLP}} + \int \mathrm{d}\Phi_{\mathrm{NLP}} |\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

Factorises into hard phase space and n 1-body soft-gluon phase spaces

NLP LL resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$
Partonic invariant-mass distribution at NLP:

$$d\sigma = \frac{1}{2s}\left[\int d\Phi_{\rm LP}\,|\mathcal{M}|^2_{\rm LP+NLP} + \int d\Phi_{\rm NLP}\,|\mathcal{M}|^2_{\rm LP} + \ldots\right]$$

NLP NLL only!

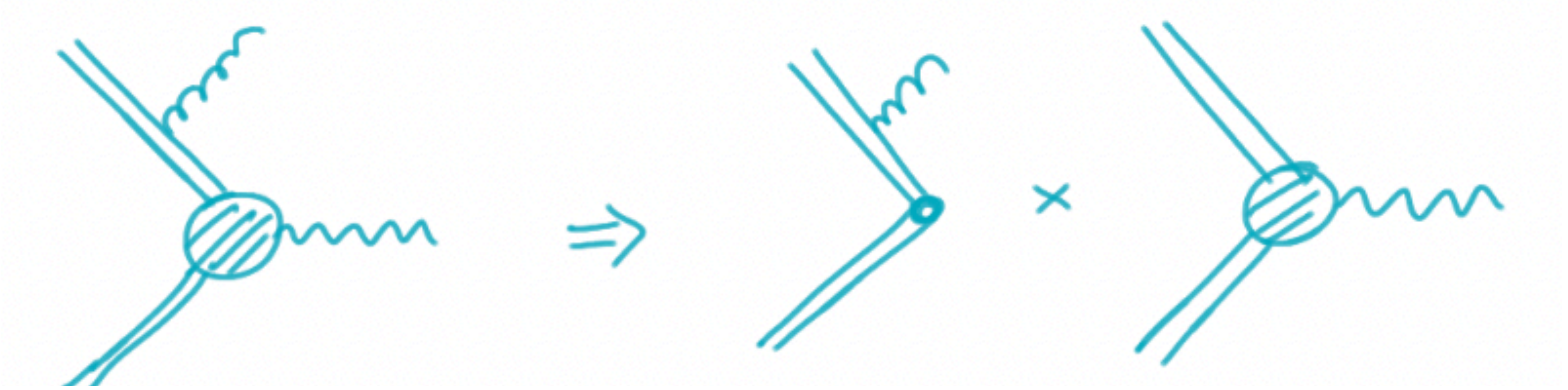
NLP LL resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic invariant-mass distribution at NLP:

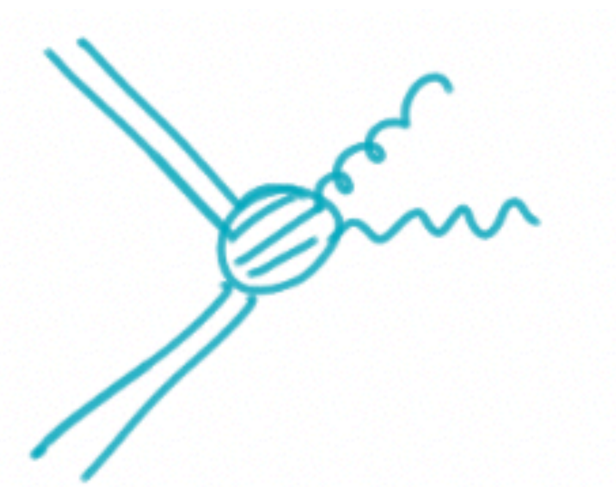
$$d\sigma = \frac{1}{2s}\left[\int d\Phi_{\mathrm{LP}} \, |\mathscr{M}|^2_{\mathrm{LP+NLP}} + \int d\Phi_{\mathrm{NLP}} \, |\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

Contains only next-to-soft corrections at LL [1410.6406, 1807.09246]



External next-to-soft-gluon emissions exponentiate

[0811.2067,1010.1860]



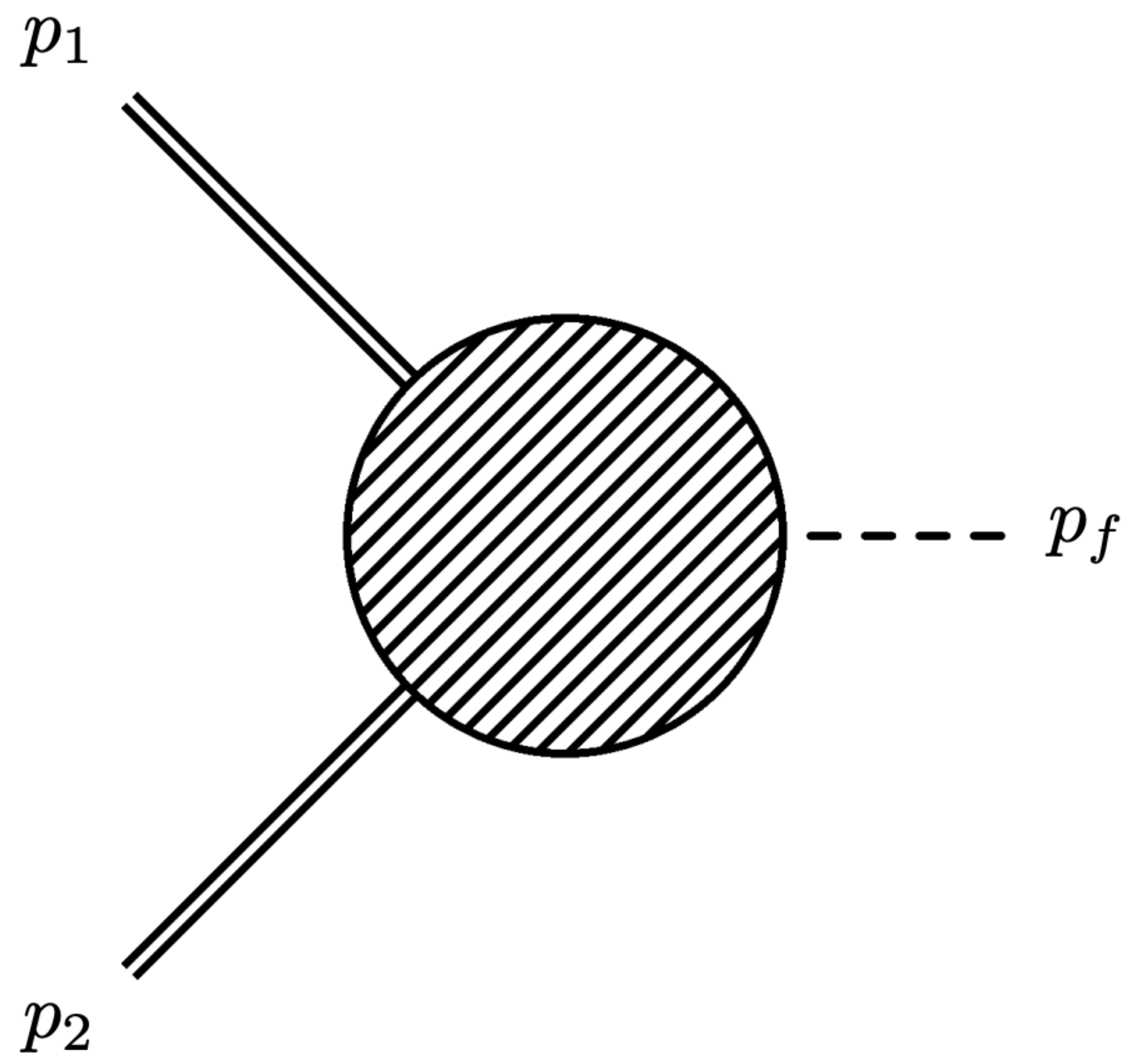
Non-factorisable (internal) emissions are linked to external ones by kinematic shift of soft function

[1706.04018,1905.08741]

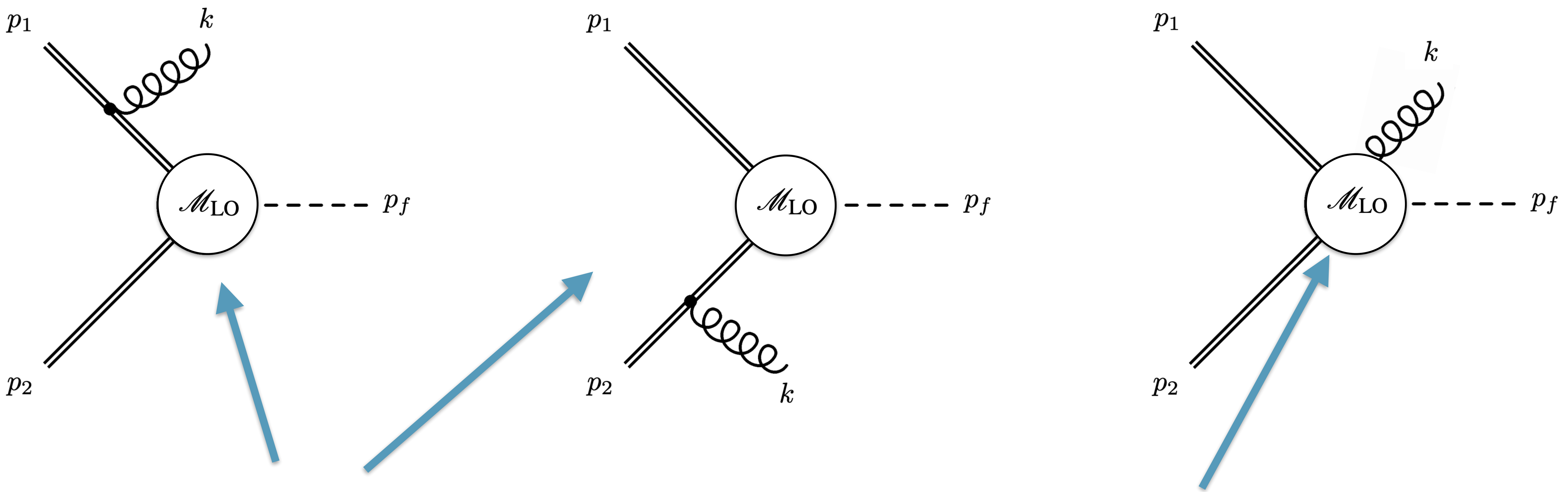
$$d\sigma^{\mathrm{res,NLP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 dz^{N-1} z \, S_{\mathrm{LP}}(z)\right]$$

Universality of NLP logs

Consider the production of a colourless final state



NLO gluon-emission amplitude at NLP

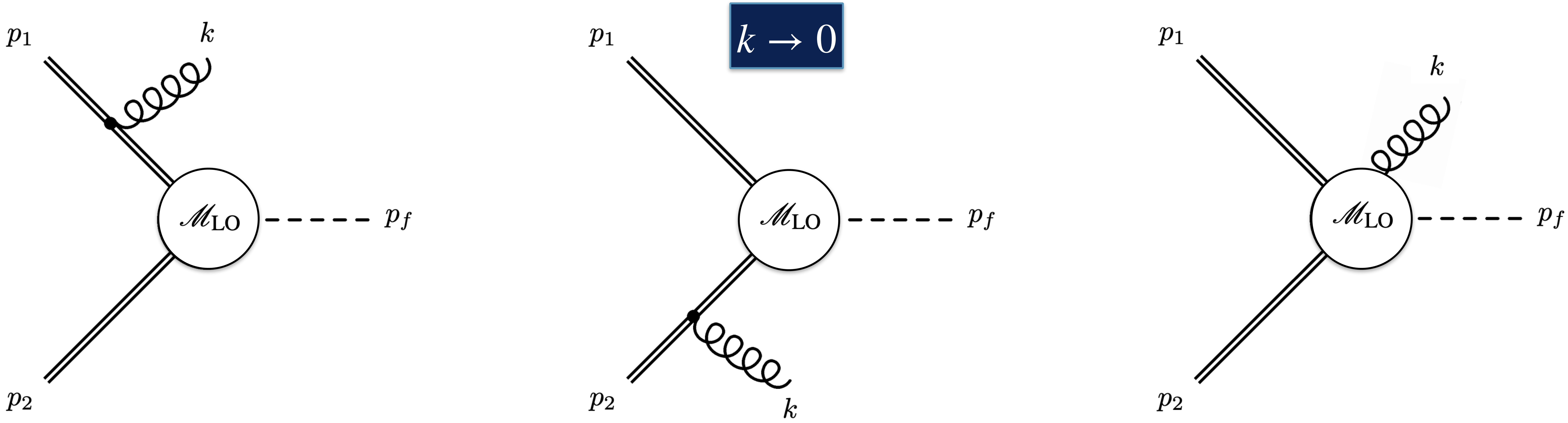


LP diagrams

At NLP, also one ‘internal’ emission contributes

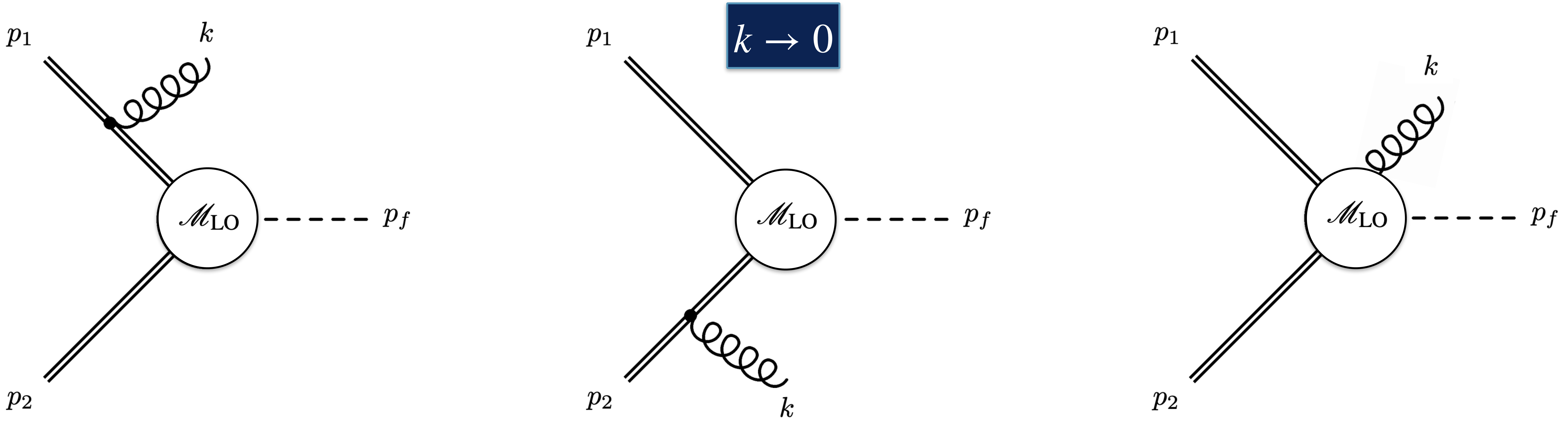
Low (1958); Burnett and Kroll (1968)

NLO gluon-emission amplitude at NLP



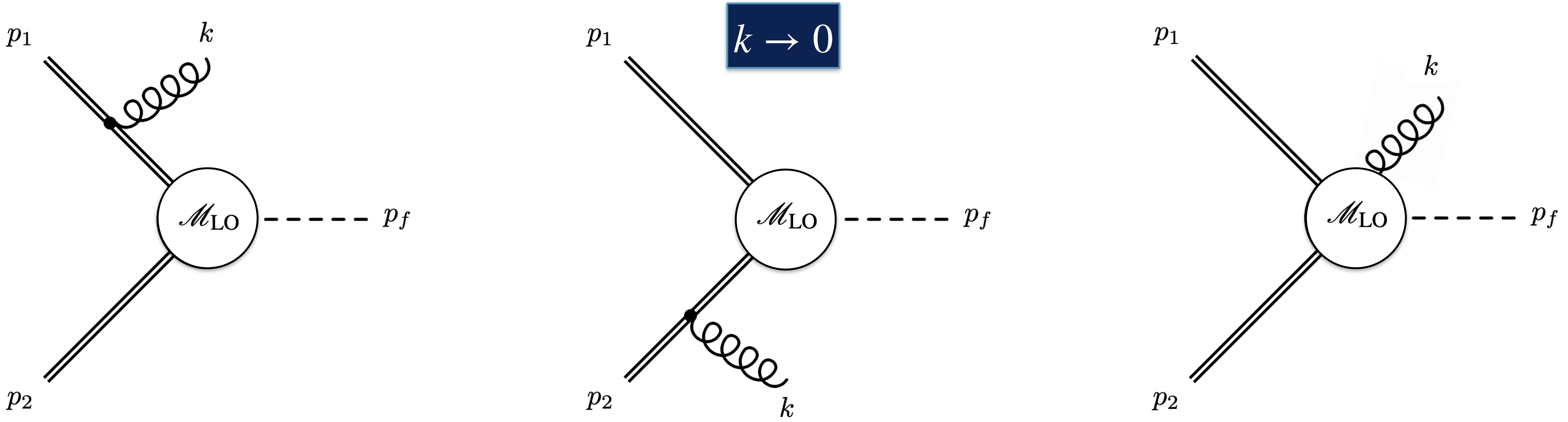
$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

NLO gluon-emission amplitude at NLP



$$\mathcal{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

NLO gluon-emission amplitude at NLP



$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

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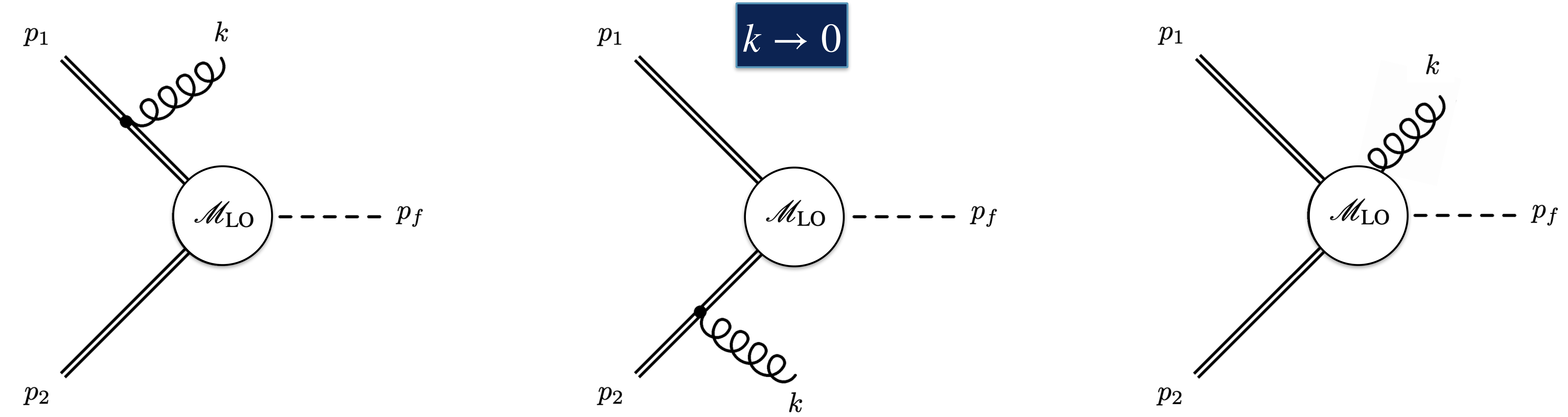

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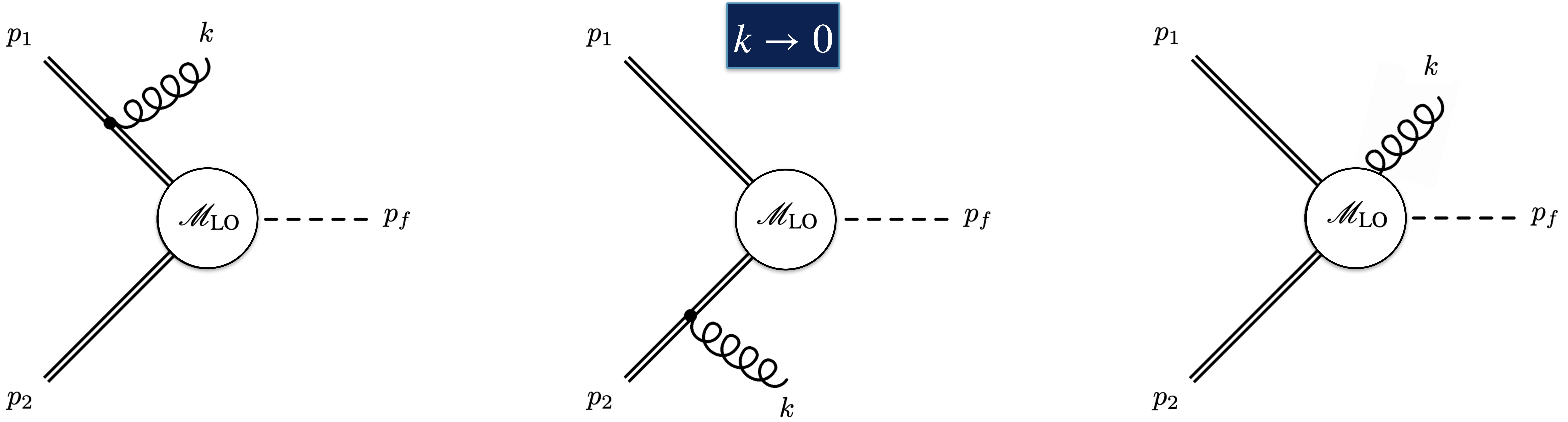
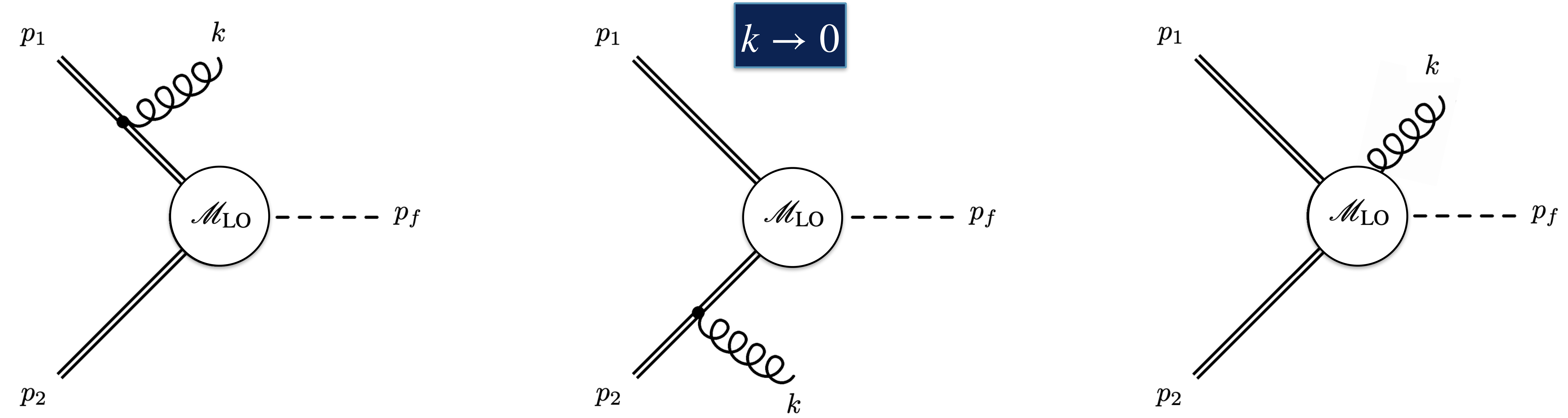


$$\mathcal{A}_{\text{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\text{LO}}\, \varepsilon_\sigma^*(k)$$

Eikonal

NLO gluon-emission amplitude at NLP



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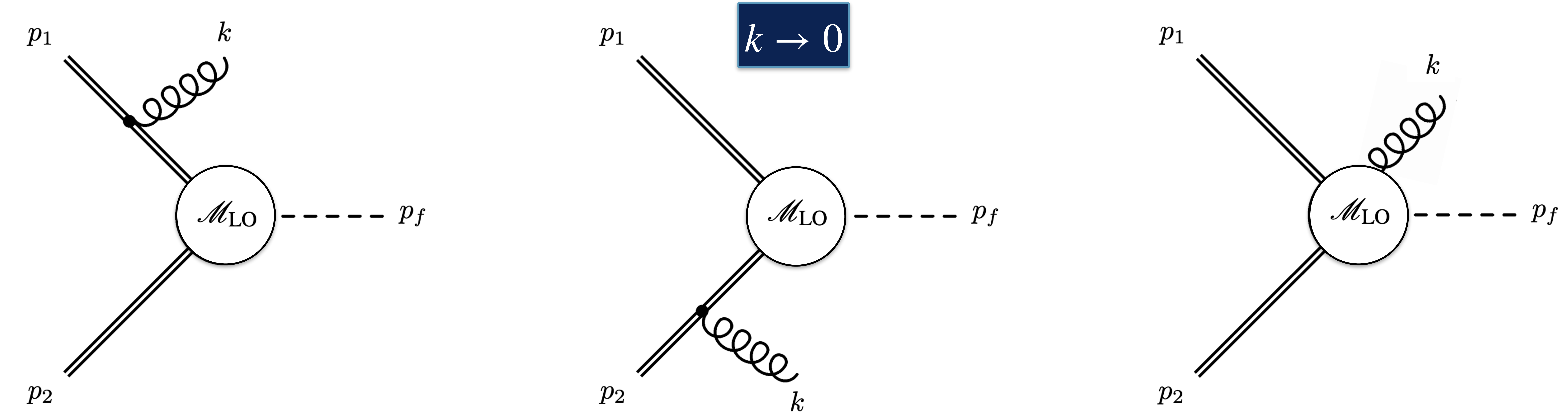
Spin

$$\begin{aligned} \Sigma^{\sigma\alpha} &= \frac{i}{4}[\gamma^\sigma, \gamma^\alpha] \equiv S^{\sigma\alpha} \\ &= i(g^{\rho\sigma}g^{\alpha\nu} - g^{\sigma\nu}g^{\alpha\rho}) \equiv M^{\sigma\alpha,\rho\nu} \end{aligned}$$

NLO gluon-emission amplitude at NLP



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Orbital

$$L^{\sigma\alpha} = -i \left(p_i^\sigma \frac{\partial}{\partial p_{i\alpha}} - p_i^\alpha \frac{\partial}{\partial p_{i\sigma}} \right)$$

Towards the NLP cross section

$$|\mathscr{A}_{\mathrm{NLP}}|^2 = \sum_{\mathrm{colors}} |\mathscr{A}_{\mathrm{scal}}|^2 + 2\mathrm{Re}\left[(\mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}})^\dagger \mathscr{A}_{\mathrm{scal}}\right]$$

$$= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)|^2$$

Towards the NLP cross section

$$|\mathscr{A}_{\text{NLP}}|^2 = \sum_{\text{colors}} |\mathscr{A}_{\text{scal}}|^2 + 2\text{Re}\left[(\mathscr{A}_{\text{spin}} + \mathscr{A}_{\text{orb}})^\dagger \mathscr{A}_{\text{scal}}\right]$$

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Eikonal factor

Towards the NLP cross section

$$|\mathscr{A}_{\text{NLP}}|^2 = \sum_{\text{colors}} |\mathscr{A}_{\text{scal}}|^2 + 2\text{Re}\left[(\mathscr{A}_{\text{spin}} + \mathscr{A}_{\text{orb}})^\dagger \mathscr{A}_{\text{scal}}\right]$$

$$= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\text{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)|^2$$

Shift in Born matrix element

$$\delta p^\alpha_{i;j} \equiv -\frac{1}{2}\left(k^\alpha + \frac{p_j \cdot k}{p_i \cdot p_j} p^\alpha_i - \frac{p_i \cdot k}{p_i \cdot p_j} p^\alpha_j\right)$$

Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

[1706.04018]

Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

$$K_{\mathrm{NLP}} = \frac{\alpha_s}{\pi}\left(\frac{\bar{\mu}^2}{s}\right)^{\epsilon}\left[-\frac{2}{\epsilon}\left(\left(\frac{1}{1-z}\right)_+ - 1\right) + 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 4\ln(1-z) + \frac{1}{\varepsilon^2}\delta(1-z) + \ldots\right]$$

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NLP log with the same coefficient as the LP log!

[1706.04018]

Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

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$$= zK_{\mathrm{LP}}$$

The gluonic NLP colour-singlet cross section is directly obtainable from the LP one!

$$\mathrm{d}\sigma^{\mathrm{res,NLP}} = \sigma_{\mathrm{hard}}\exp\left[\int_0^1 \mathrm{d}z^{N-1} z\, S_{\mathrm{LP}}(z)\right]$$

NLP resummation for colour-singlet processes

$$\sigma^{\text{res,NLP}} = \sigma_{\text{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} z\, S_{\text{LP}}(z)\right]$$

Wide-angle coefficient contributes at NNLL (unchanged for NLP LL)

$$= \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

$$P_{ii}^{\text{NLP}} = \frac{\alpha_s}{2\pi} C_i \left[\left(\frac{1}{1-z}\right)_+ - 1 + \cdots\right] + \mathcal{O}(\alpha_s^2)$$

Key is that the LL LP and NLP contributions come from a pole in ϵ that needs to be absorbed in parton distribution functions

→ the NLP expansion of the splitting function generates this information

Melissa van Beekveld

NLP resummation for colour-singlet processes

$$\sigma^{\text{res,NLP LL}} = \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1}\int_{\mu_F^2}^{Q^2(1-z)^2}\frac{\mathrm{d}q^2}{q^2}P_{ii}^{\text{NLP}}(z,\alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1}\frac{1}{1-z}D(\alpha_s((1-z)^2Q^2))\right]$$

$$= \sigma_{\text{hard}} \exp\left[\frac{2}{\alpha_s}g_a^{(1)}(\lambda) + \ldots + 2h_a^{(1)}(\lambda,N)\right]$$

[2101.07270]

NLP resummation for colour-singlet processes

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$$= \sigma_{\text{hard}} \exp \left[\frac{2}{\alpha_s} g_a^{(1)}(\lambda) + \ldots + 2h_a^{(1)}(\lambda, N) \right]$$

[2101.07270]

Can be obtained from the LP with a derivative:

$$h_a^{(1)}(\lambda, N) = -\frac{A_a^{(1)}}{2\pi b_0} \frac{\ln(1-2\lambda)}{N} = \frac{1}{2\alpha_s} \frac{\partial}{\partial N} g_a^{(1)}(\lambda)$$

NLP resummation for colour-singlet processes

$$\sigma^{\text{res,NLP LL}} = \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1}\int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1}\frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

Note that this only works at NLP LL for 'LP-induced' colour-singlet processes:

★ Beyond LL the phase space needs to be modified (leading to $Q^2(1-z)^2 \to Q^2(1-z)^2/z$)
★ Need to identify (and find a resummation pattern for) sources of logarithms beyond the NLP LL

See [2205.04479] for progress to achieve beyond-NLP-LL resummation in SCET

★ The qg-induced channels are not considered (yet)

★ The kinematic shift for channels with more than two coloured legs is not factorisable

[2101.07270]

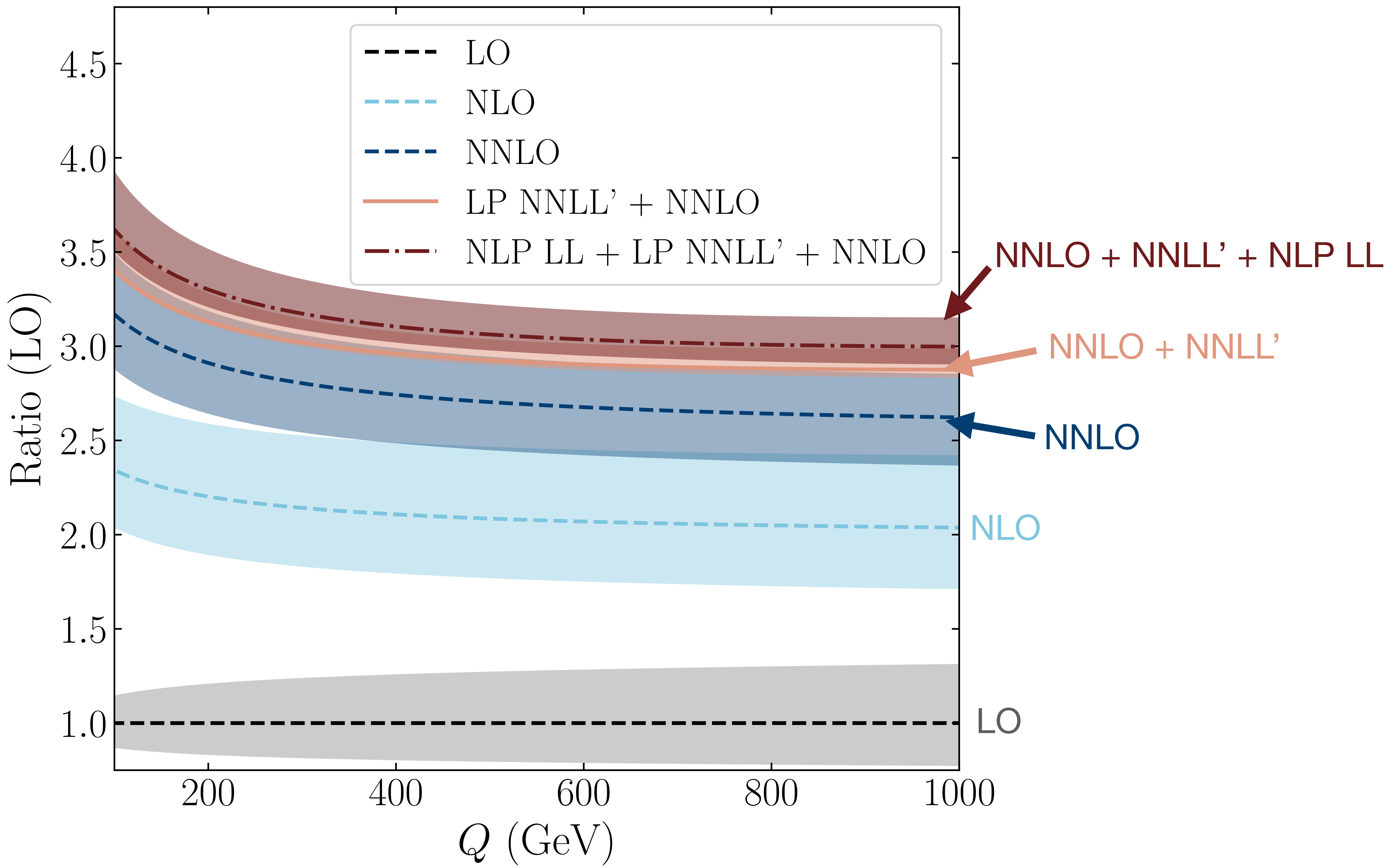
What about the numerical size?

Consider single Higgs and DY production

Take both processes at LP NNLL’ + NLP LL and match to NNLO

Use PDF4LHC NNLO PDF set with $\mu_R = \mu_F$

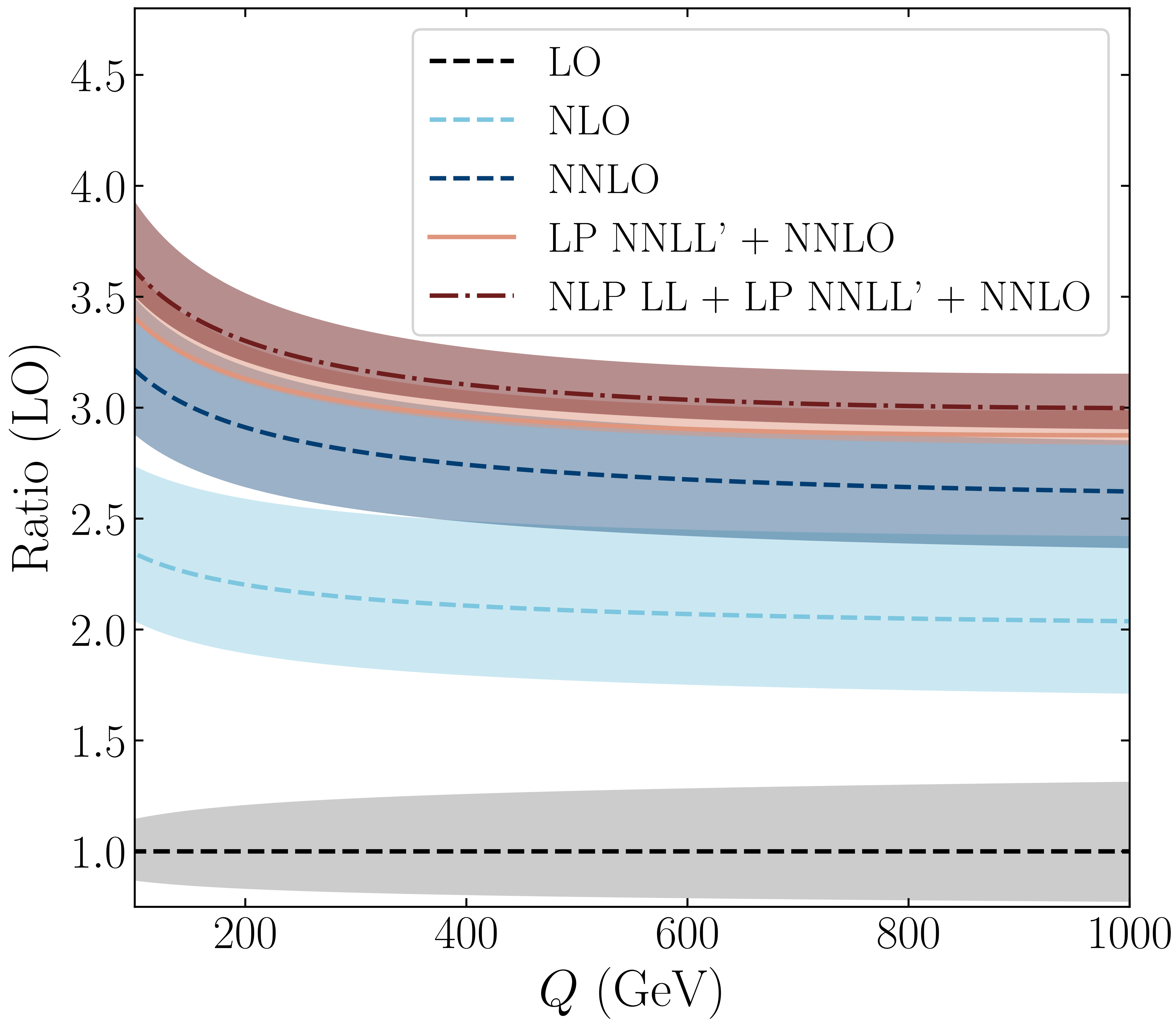
Consider single Higgs and DY

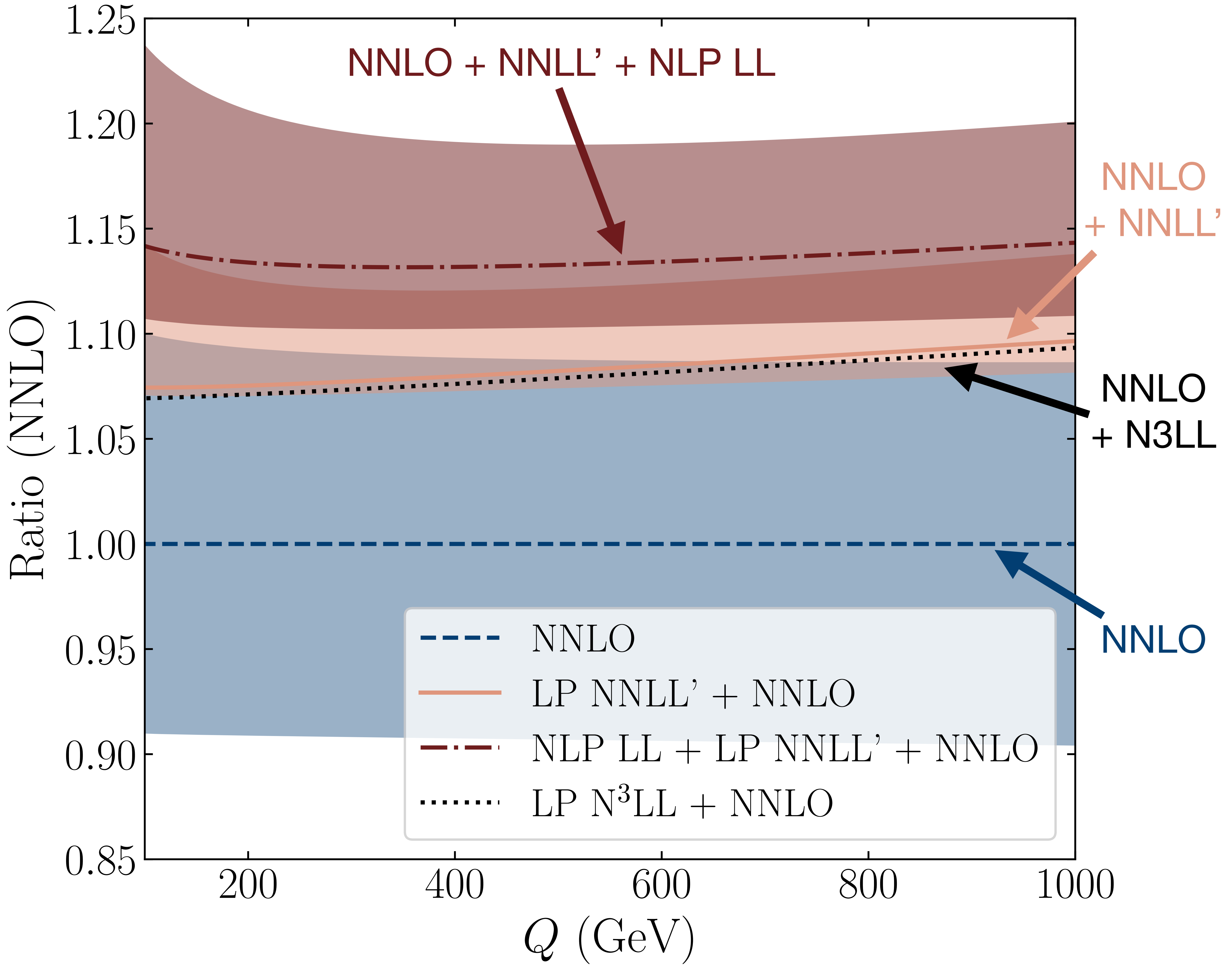


We vary $Q = m_h$

It seems that the NLP contribution is indeed subleading…

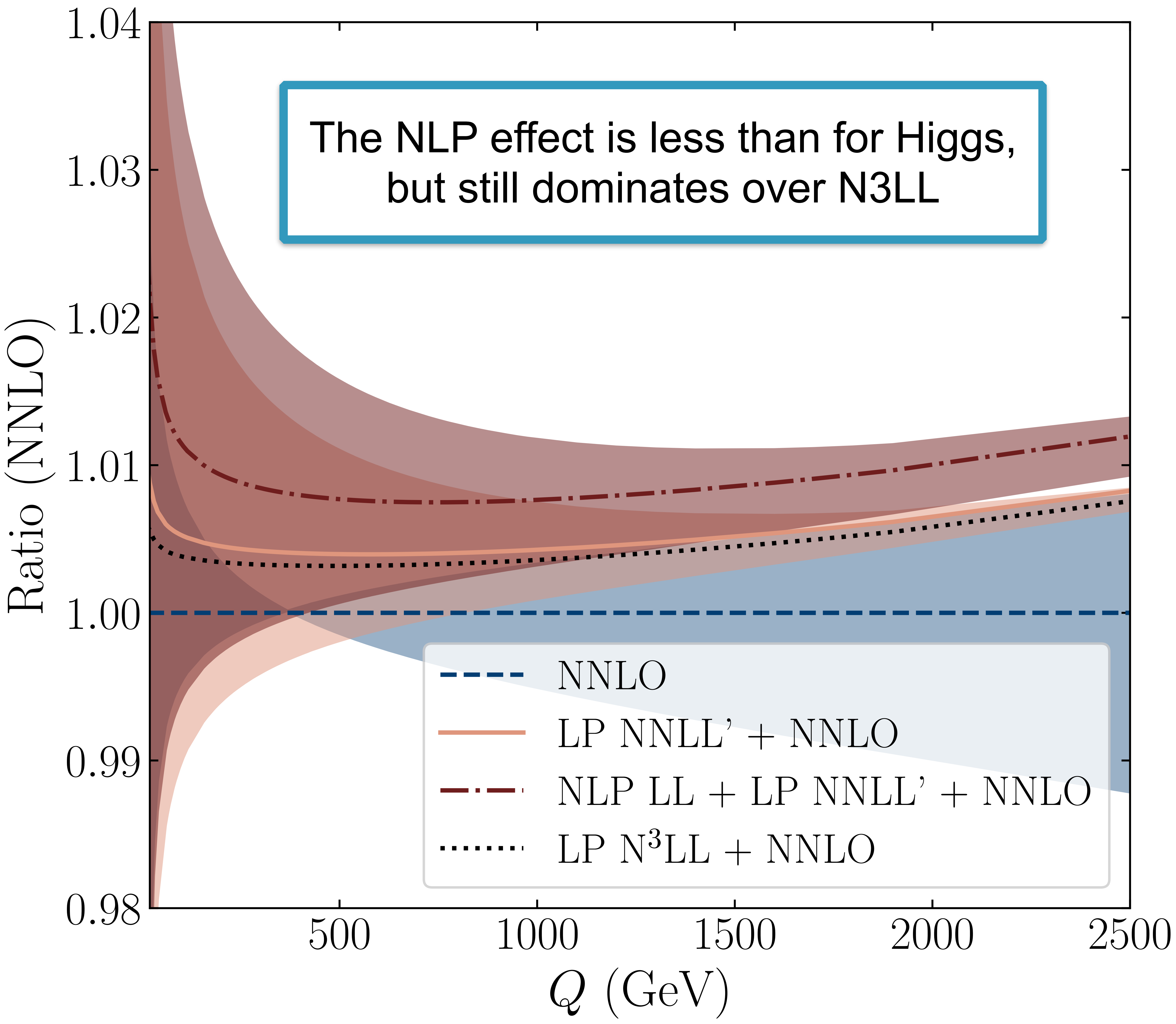
Consider single Higgs and DY





NLP has an $5-7\,\%$ enhancement of NLP on top of LP NNLL

Consider single Higgs and DY



SCET vs dQCD at NLP

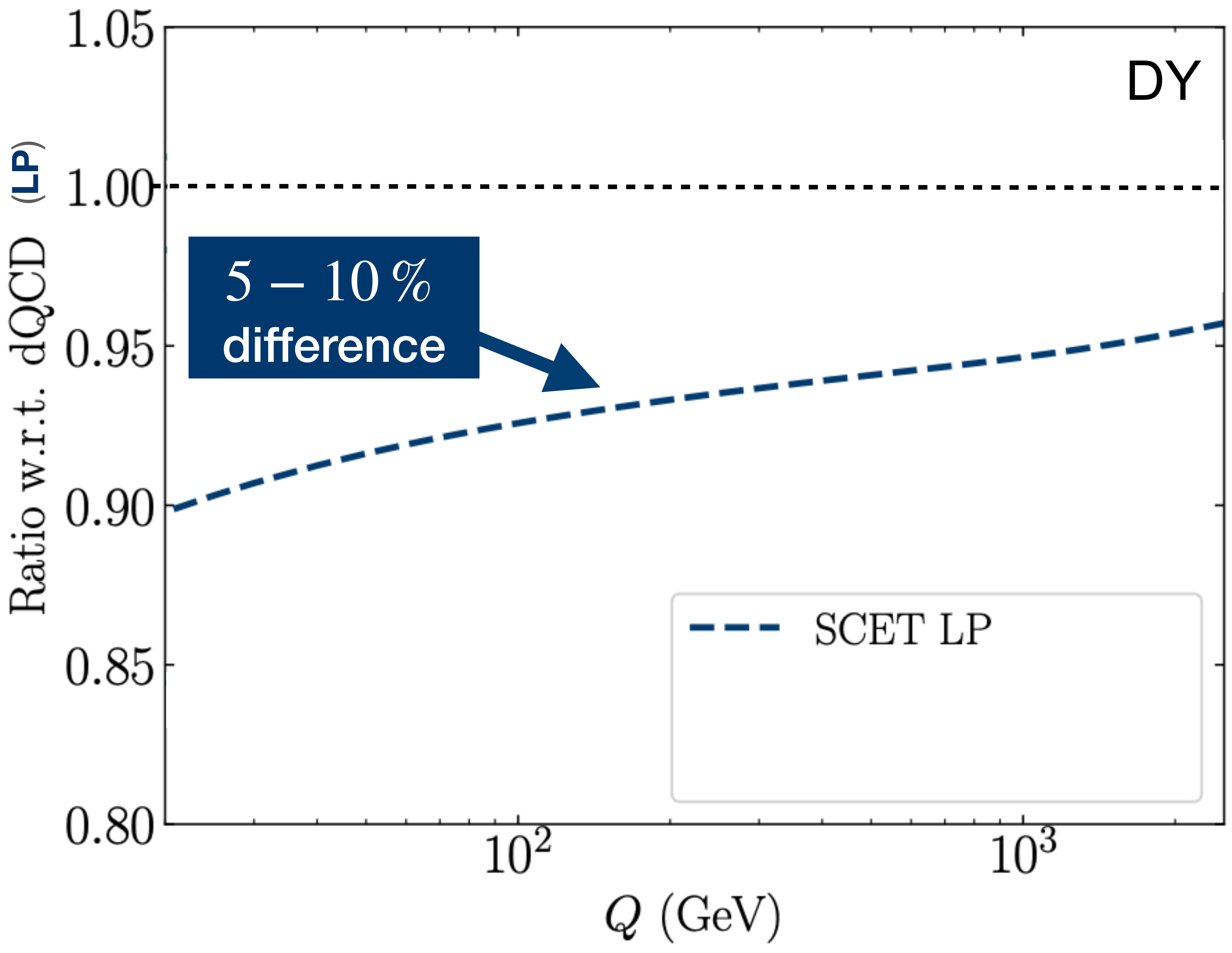
– DY and Higgs resummed in SCET at NLP LL as well [1809.10631,1910.12685]

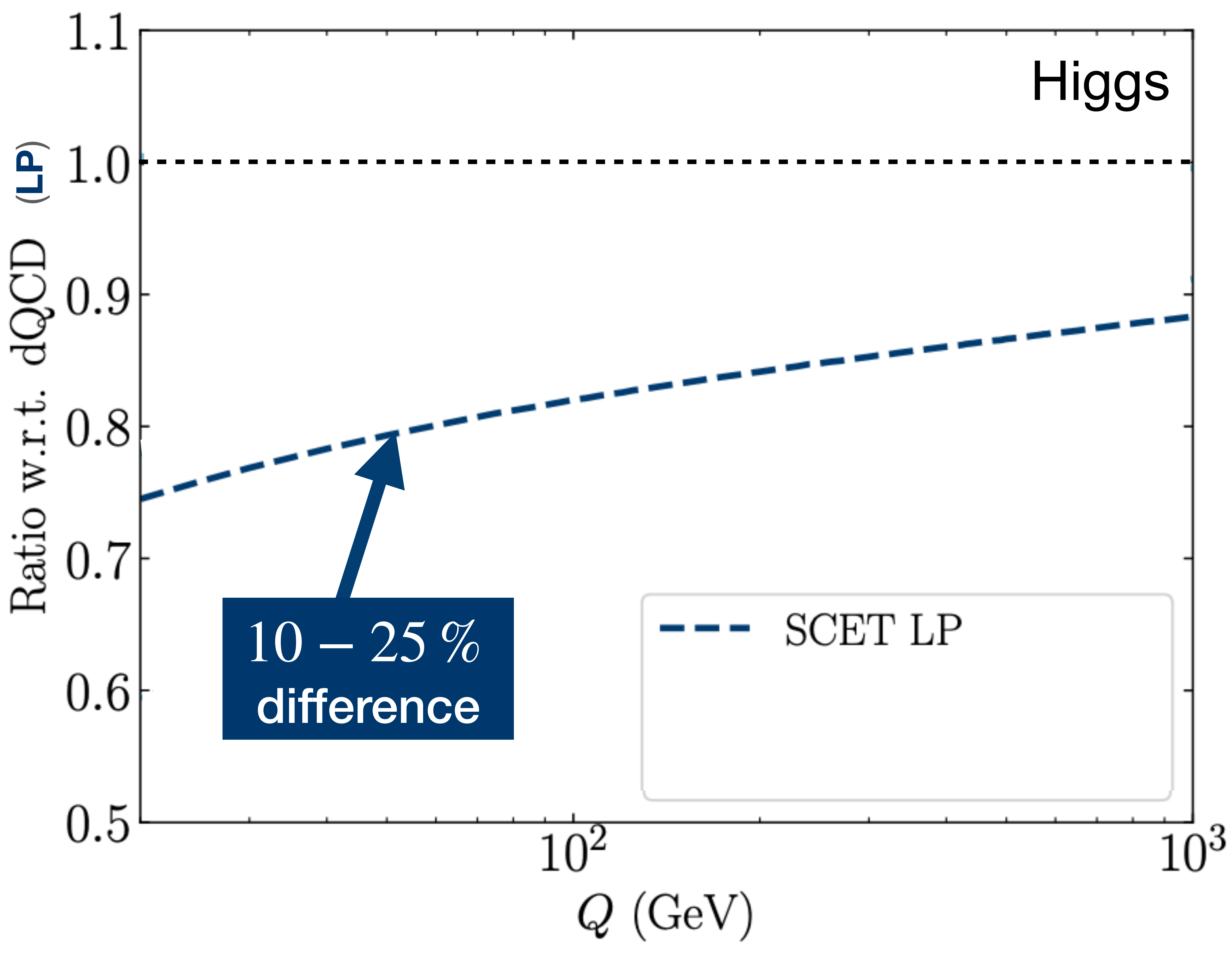
– Numerical differences can become sizable between two approaches at LP [0601048, 0809.4283, 1201.6364, 1301.4502, 1409.0864]

– Shown that these differences originate from power-suppressed contributions

Can we obtain analytical and numerical agreement at NLP LL?

SCET vs dQCD at NLP





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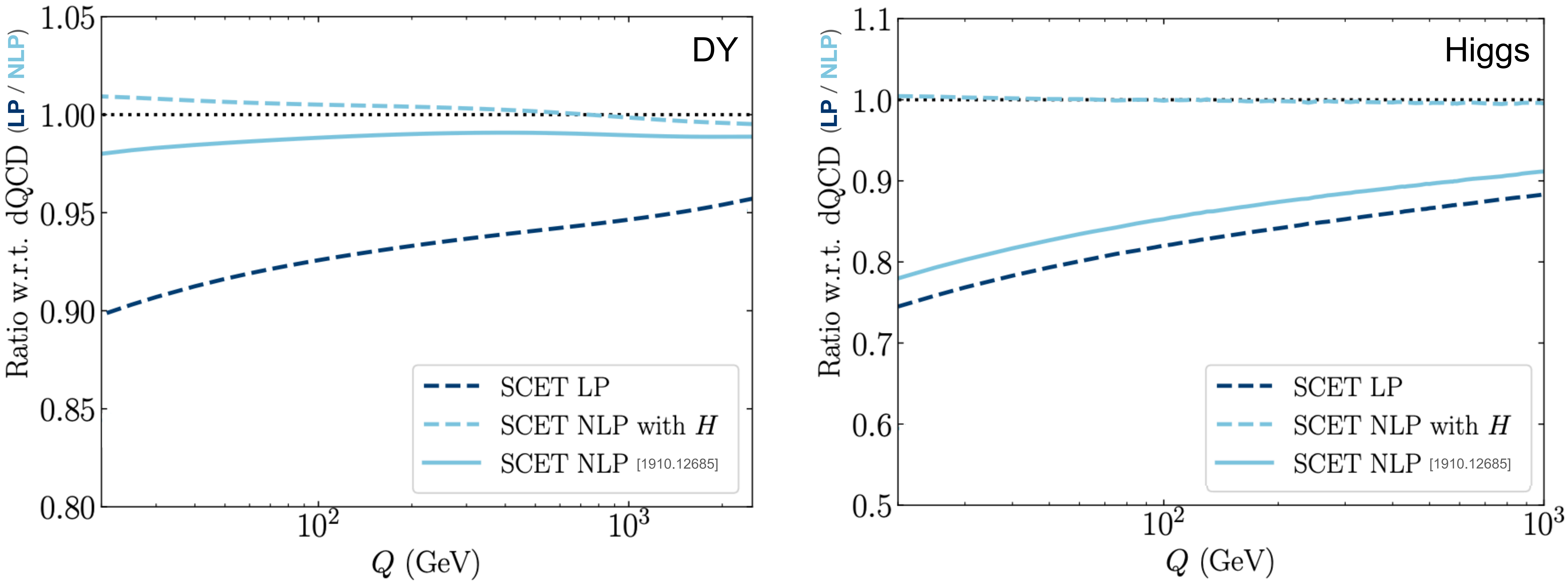
SCET vs dQCD at NLP



Remaining differences originate from:

- Truncations of higher-logarithmic terms
- Truncations of higher-power terms

NLP resummation for colour-singlet processes

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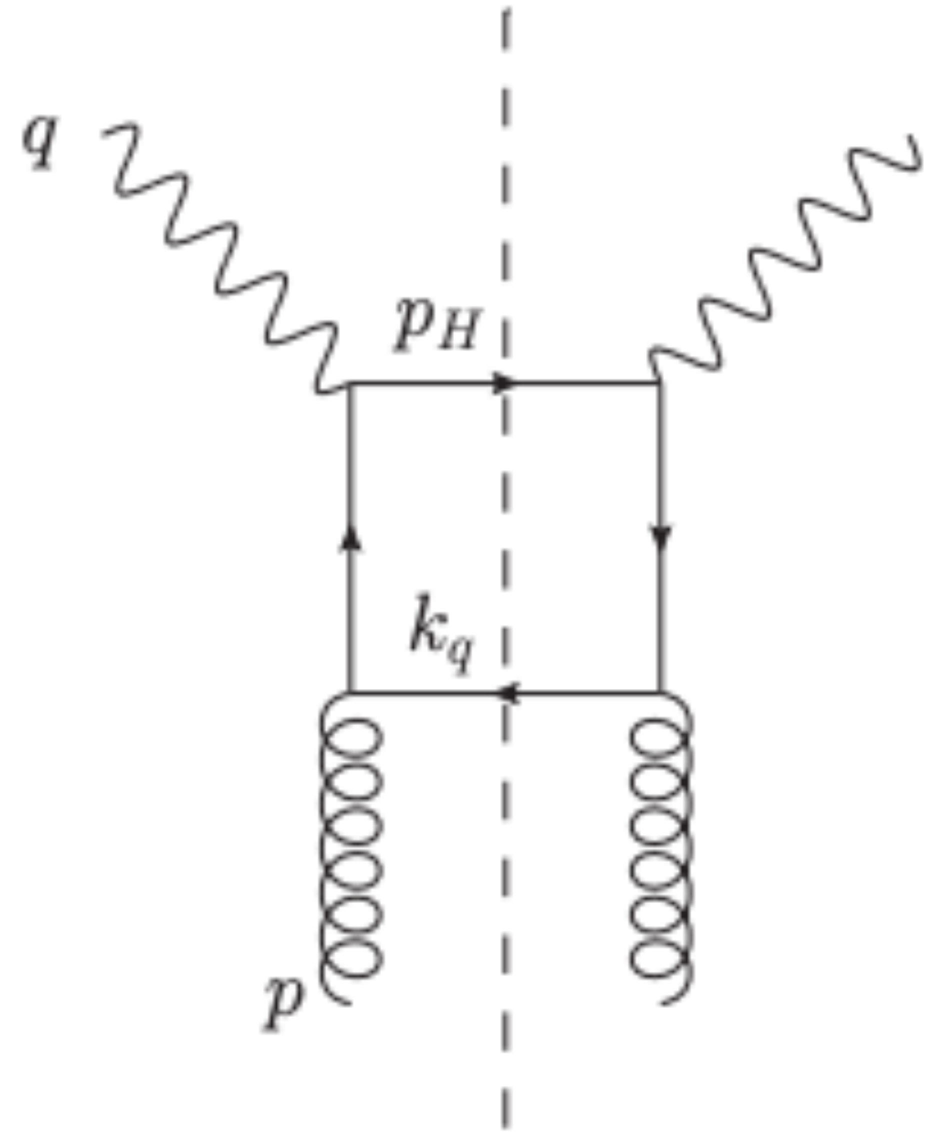
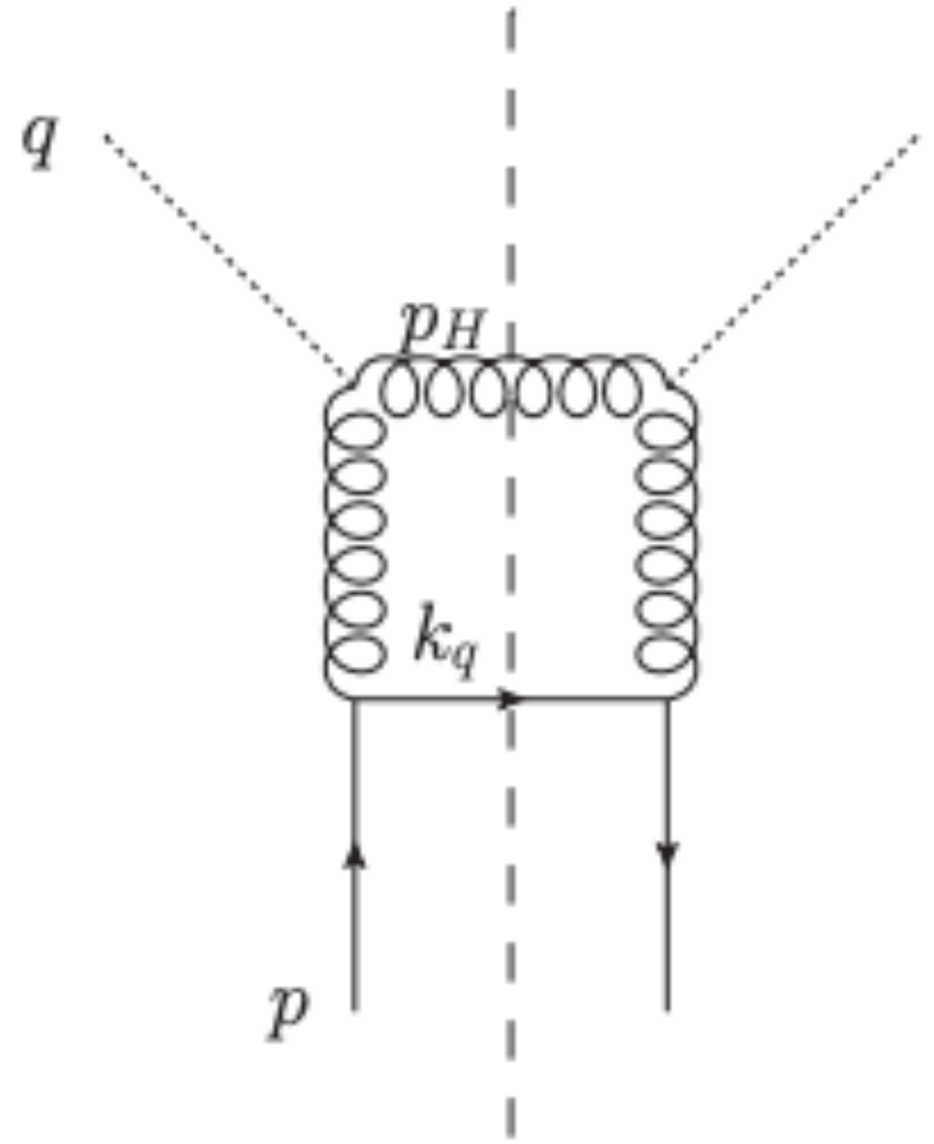
Can we resum soft-quark emissions in DY and Higgs production?

What about quark emissions?

Consider DIS-like configurations with one soft quark emission



$g(p) + \gamma^*(q) \to q(p_H) + \bar{q}(k_q)$



$q(p) + h(q) \to g(p_H) + q(k_q)$

These are power-suppressed due to spinor sum

Multiple quark emissions would put it beyond NLP!

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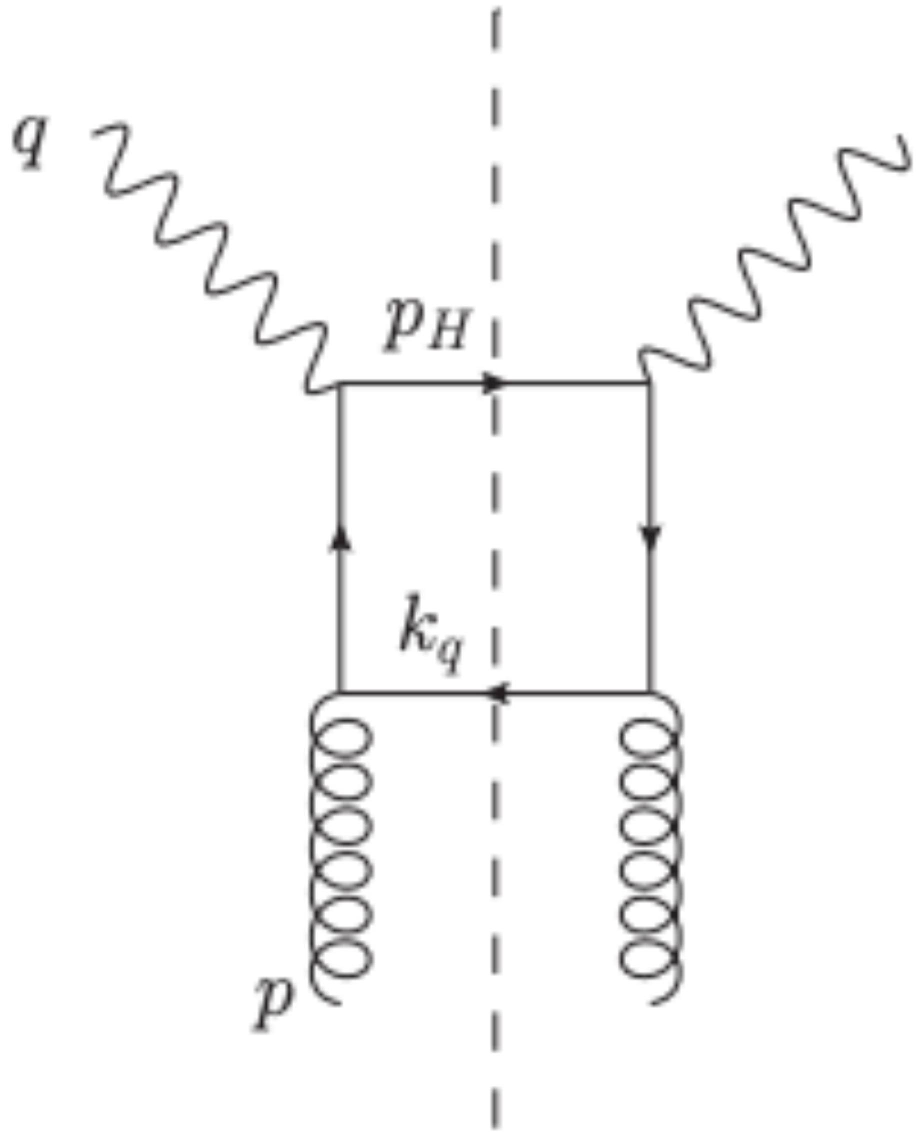

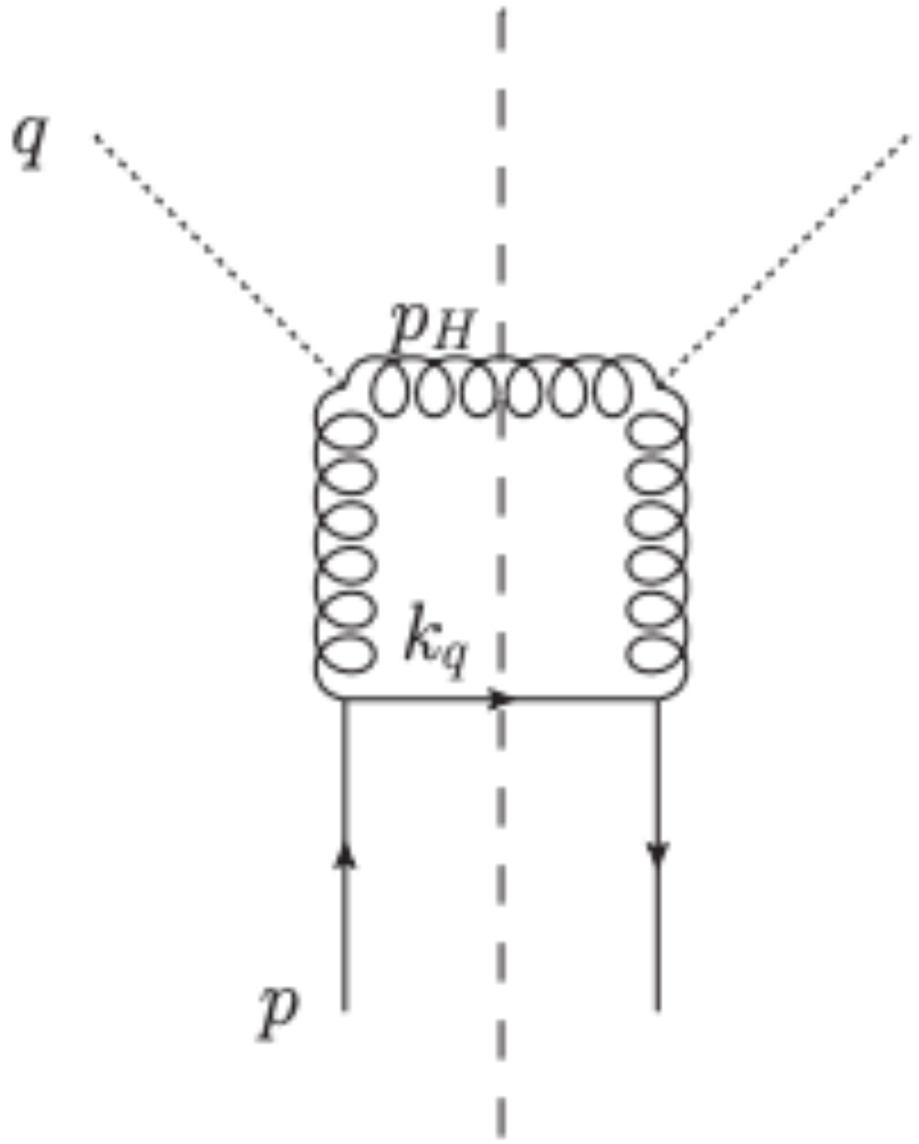

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Multiple quark emissions would put it beyond NLP!

$g(p) + \gamma^*(q) \to q(p_H) + \bar{q}(k_q) + \sum g(k_i)$

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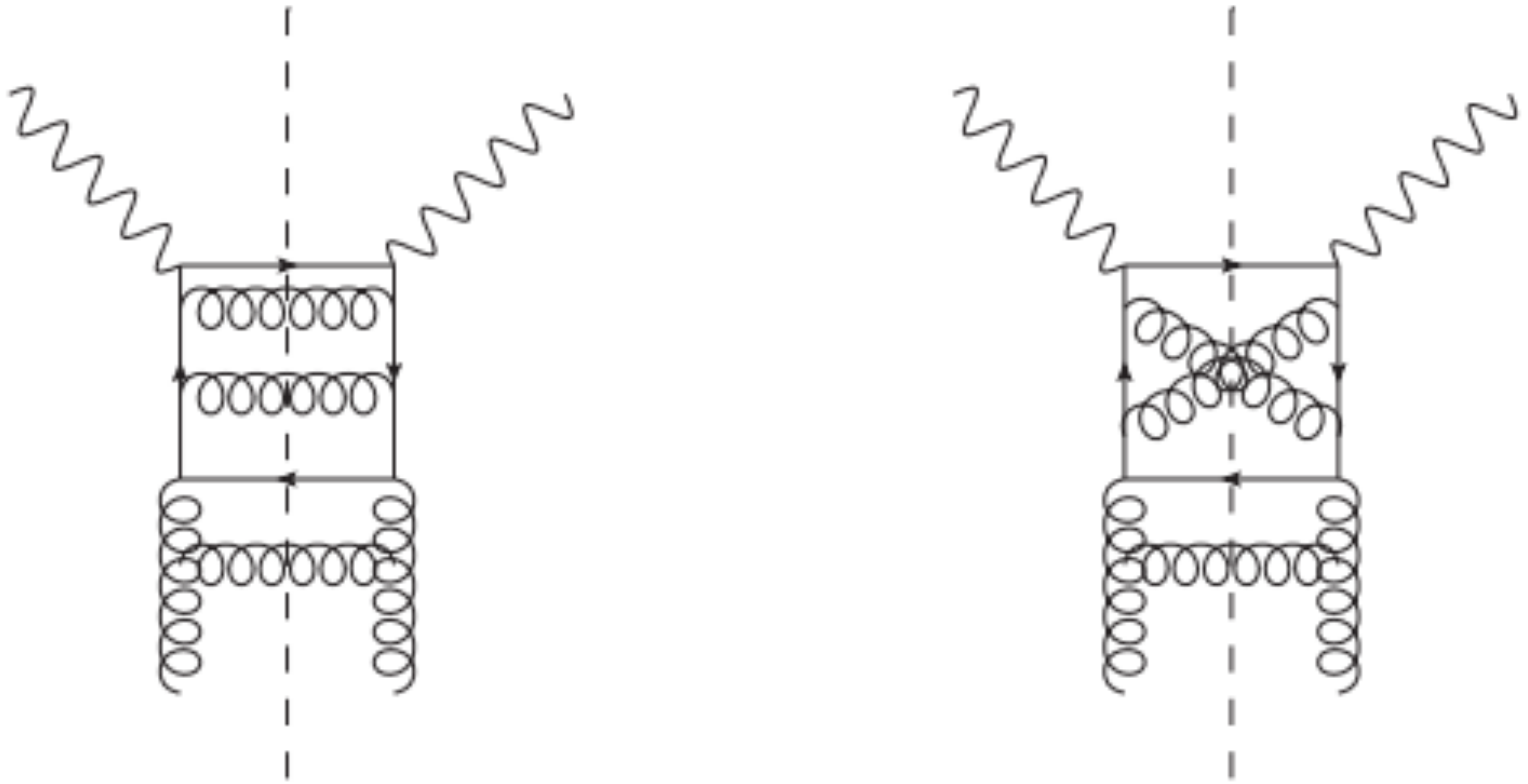

1. Phase-space can be treated in LP approximation
2. LLs originate from strongly-ordered emissions
3. By picking a suitable gauge crossed ladder diagrams may be neglected

As may be found in many textbooks, e.g. Dokshitzer, Diakonov, Troian (1980)

What about quark emissions?

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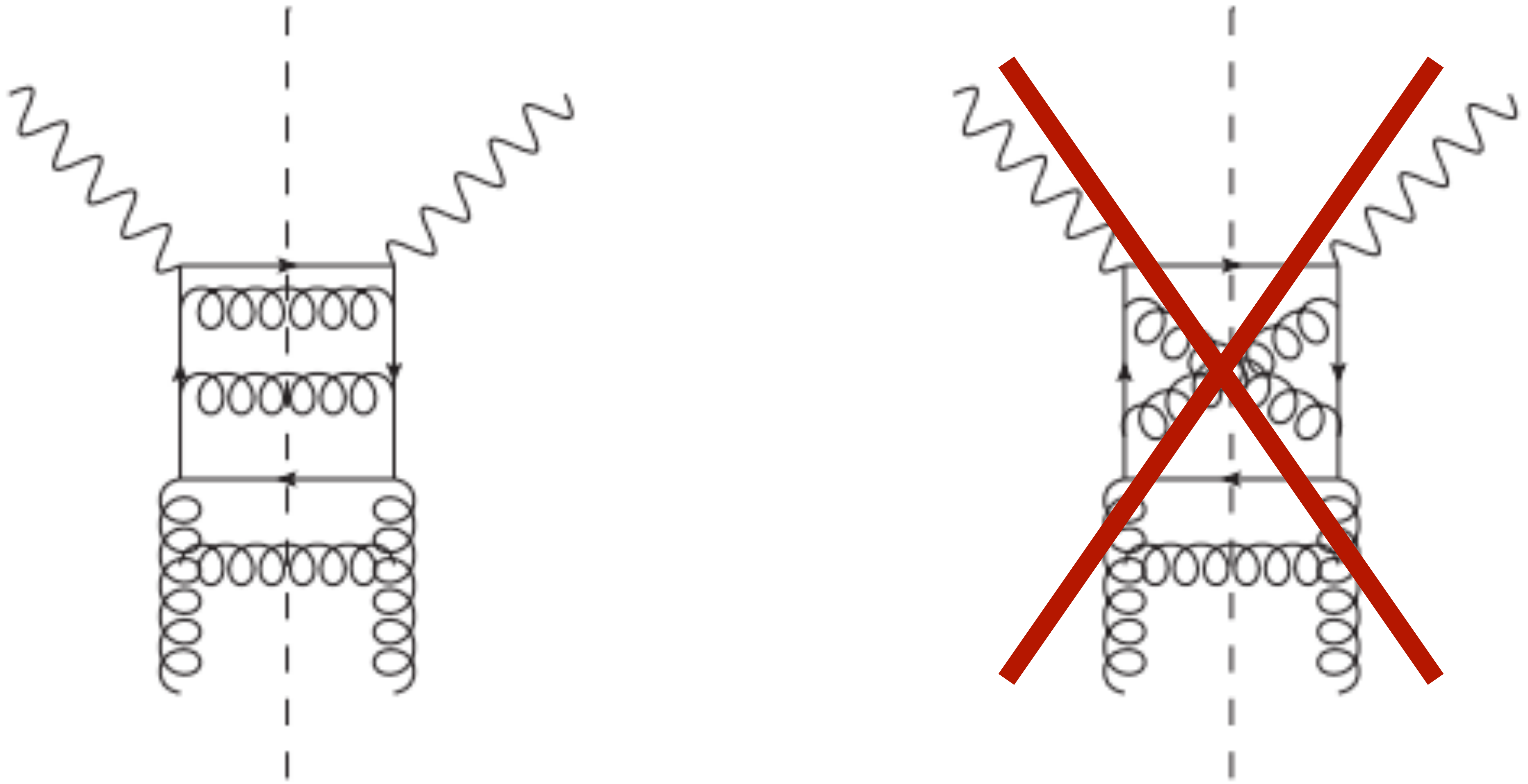

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$$\sum_{\text{pol}} \epsilon^{\dagger}_{\mu}(k)\epsilon_{\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n\cdot k} \qquad n = q + xp \qquad x = \frac{-q^2}{2q\cdot p}$$

DIS structure function at NLP LL

$$g(p) + \gamma^*(q) \to q(p_H) + \bar{q}(k_q) + \sum g(k_i)$$

1. Phase-space can be treated in LP approximation
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3. By picking a suitable gauge crossed ladder diagrams may be neglected

$$W^{\mathrm{real}}_{\gamma^* g}(N) = \frac{1}{\sigma_0}\int_0^1 \mathrm{d}x\, x^{N-1} \sum_{i=1}^{\infty} \int \mathrm{d}\Phi_i \,|\,\mathscr{M}_{g\gamma^* \to q\bar{q}g_1\ldots g_i}|^2$$

$$= -\frac{2a_s n_f}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_A - C_F}\left(\frac{4a_s N^\epsilon}{\epsilon^2}\right)^{-1}\left[\exp\left(\frac{4a_s C_A N^\epsilon}{\epsilon^2}\right) - \exp\left(\frac{4a_s C_F N^\epsilon}{\epsilon^2}\right)\right]$$

Notice pattern of colour charges!

Also seen for the off-diagonal splitting function [1005.1606], and gluon thrust [1910.14038], [2205.04479]

DIS structure function at NLP LL

$$g(p) + \gamma^*(q) \to q(p_H) + \bar{q}(k_q) + \sum g(k_i)$$

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Virtuals can be added implicitly through requiring soft pole cancellation

$$W_{\gamma^* g}\Big|_{\text{LL}} = -\frac{2a_s n_f}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_A - C_F}\left(\frac{4a_s (N^\epsilon - 1)}{\epsilon^2}\right)^{-1}\left[\exp\left(\frac{4a_s C_A (N^\epsilon - 1)}{\epsilon^2}\right) - \exp\left(\frac{4a_s C_F (N^\epsilon - 1)}{\epsilon^2}\right)\right]$$

DIS structure function at NLP LL

Coefficient function extracted after mass factorisation: $W_{\gamma^* g} = \underset{\text{NLP}}{C_{\gamma^* g}} \times \overset{\text{LP}}{Z_{gg}} + \overset{\text{LP}}{C_{\gamma^* q}} \times \underset{\text{NLP}}{Z_{qg}}$

$$C_{\gamma^* g}\Big|_{\text{LL}} = \frac{1}{2\ln N}\frac{n_F}{C_A - C_F}\left[\mathscr{B}_0(4a_s(C_A - C_F)\ln^2 N)\exp\left(2a_s C_F \ln^2 N\right) - \exp\left(2a_s C_A \ln^2 N\right)\right]$$

- Higgs-quark process may be obtained by setting $n_f \to C_F$ and $C_F \leftrightarrow C_A$ elsewhere
- Result proves conjectures of [1005.1606], [1407.1553]
- Result is consistent with SCET results of [2008.04943]

Extending to DY/single Higgs

Approach may straightforwardly be extended to DY/single Higgs production

$$W^{\text{real}}_{\text{DY},g\bar{q}}\Big|_{\text{LL}} = -\frac{2a_s T_R N^{2\epsilon}}{\epsilon}\frac{1}{N}\left(\frac{4a_s N^{2\epsilon}}{\epsilon^2}\right)^{-1}\frac{1}{C_F - C_A}\left[\exp\left(\frac{4a_s C_F N^{2\epsilon}}{\epsilon^2}\right) - \exp\left(\frac{4a_s C_A N^{2\epsilon}}{\epsilon^2}\right)\right]$$

Notice the factor of 2 (stems directly from the phase space)

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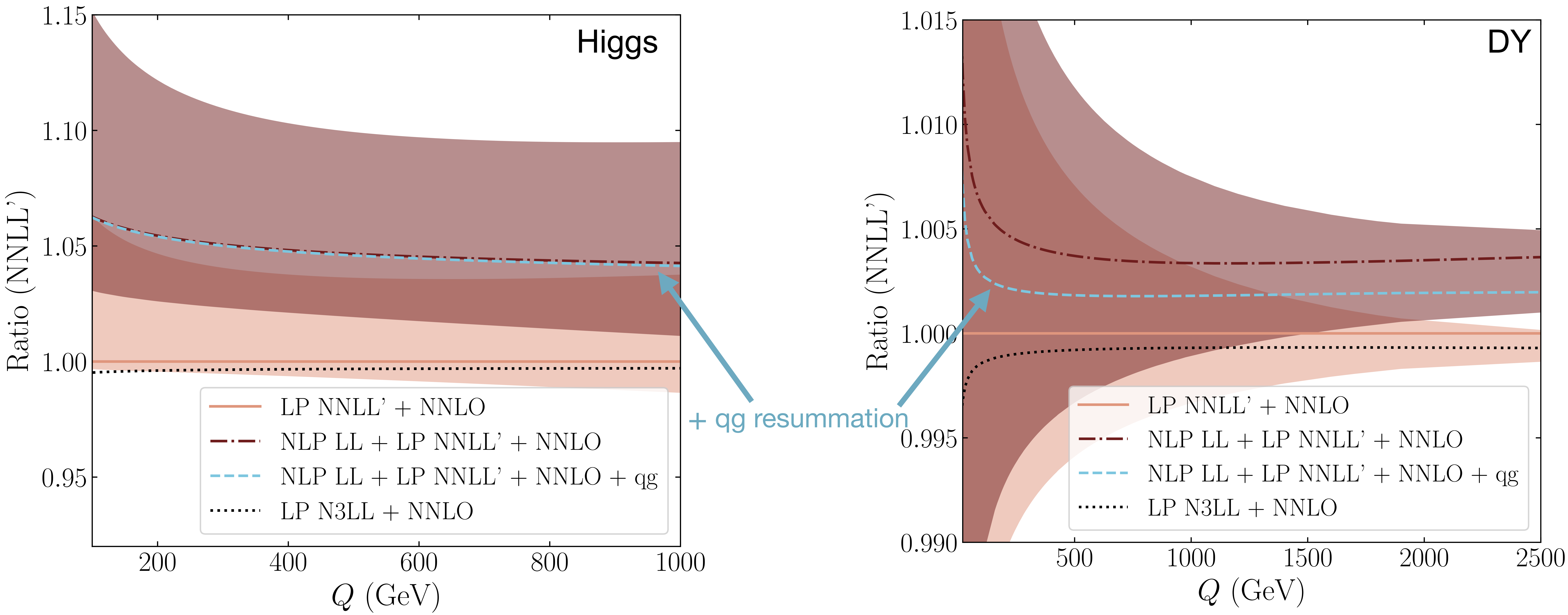
Notice the factor of 2 (stems directly from the phase space)

Requiring soft-pole cancellation + a finite coefficient function after mass factorisation:

$$C_{{\rm DY},g\bar{q}}\Big|_{\rm LL} = \frac{T_R}{C_A - C_F}\frac{1}{2N\ln N}\left[\mathscr{B}_0(4a_s(C_A - C_F)\ln^2 N)\exp\left(8a_s C_F \ln^2 N\right) - \exp\left(2a_s(C_F + 3C_A)\ln^2 N\right)\right]$$

Adding to the NLP soft-gluon resummation



Conclusions part A

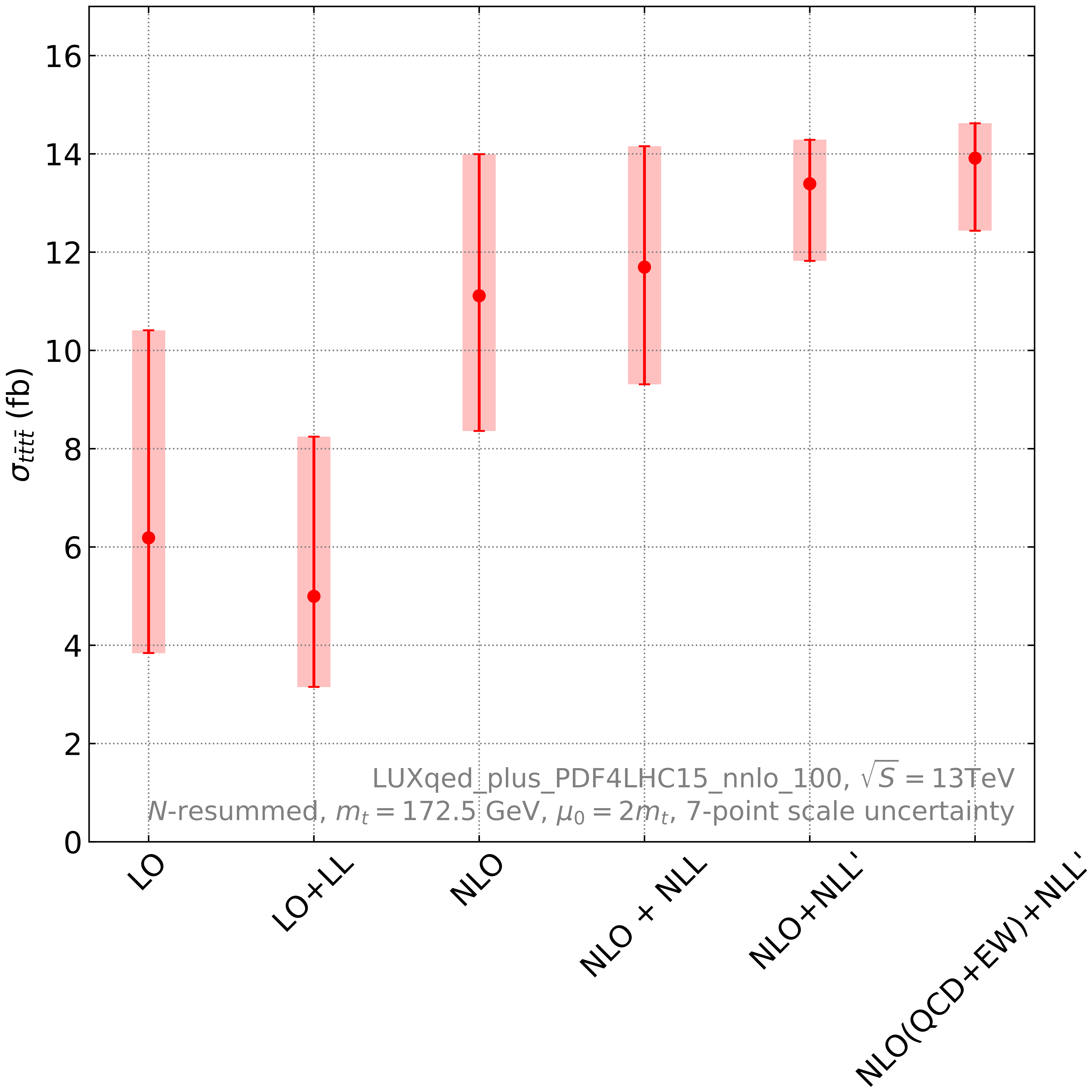
- Performed the first resummation of a process with 6 coloured particles at Born level
- Find a significant reduction of the total scale uncertainty
- Next: $t\bar{t}b\bar{b}$, invariant-mass resummation, extension to NNLL

Conclusions part B

- NLP LL soft-gluon contributions for colour-singlet processes are linked to the LP LL ones, this allows their resummation
- Quark channels may be resummed like LP ones, but with more complicated combinatorics
- Numerical contribution of LL NLP terms varies for different processes, but in general it is not ‘negligible’

Back-up

N resummed result



Colour basis $q\bar{q}$

$$c_1^{q\bar{q}} = \frac{1}{\sqrt{N_c^3}} \delta_{c_1 c_3} \delta_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_2^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} \delta_{c_1 c_3} t^{a_1}_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_3^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} t^{a_1}_{c_1 c_3} \delta_{c_2 c_4} t^{a_1}_{c_6 c_8}$$

$$c_4^{q\bar{q}} = \frac{1}{T_R \sqrt{N_c(N_c^2-1)}} t^{a_1}_{c_1 c_3} t^{a_1}_{c_2 c_4} \delta_{c_6 c_8}$$

$$c_5^{q\bar{q}} = \frac{\sqrt{N_c}}{T_R^2 \sqrt{2(N_c^4 - 5N_c^2 + 4)}} t^{a_1}_{c_1 c_3} d^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8}$$

$$c_6^{q\bar{q}} = \frac{1}{T_R^2 \sqrt{2N_c(N_c^2-1)}} t^{a_1}_{c_1 c_3} i f^{a_1 a_2 b_3} t^{a_2}_{c_2 c_4} t^{b_3}_{c_6 c_8}$$

Colour basis gg

$$\bar{c}_1^{gg} = \frac{3\sqrt{3}}{8}c_1^{gg} + \frac{3}{10}\sqrt{\frac{3}{2}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{3}{2}}c_{10}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{11}^{gg} - \frac{1}{4}\sqrt{\frac{3}{10}}c_{12}^{gg} + \frac{7}{40}c_{13}^{gg}\,,$$

$$\bar{c}_2^{gg} = -\frac{\sqrt{5}}{4}c_1^{gg} + \sqrt{\frac{2}{5}}c_6^{gg} - \frac{1}{2\sqrt{2}}c_{11}^{gg} - \frac{1}{2\sqrt{2}}c_{12}^{gg} + \frac{1}{4}\sqrt{\frac{3}{5}}c_{13}^{gg}\,,$$

$$\bar{c}_3^{gg} = -\frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_5^{gg} = -\frac{1}{2\sqrt{2}}c_1^{gg} - \frac{1}{2}c_6^{gg} - \frac{1}{2}c_{10}^{gg} + \frac{1}{2}\sqrt{\frac{3}{2}}c_{13}^{gg}\,,$$

$$\bar{c}_6^{gg} = -\frac{1}{2}\sqrt{\frac{5}{14}}c_1^{gg} + \frac{3}{2\sqrt{35}}c_6^{gg} - \frac{1}{2}\sqrt{\frac{5}{7}}c_{10}^{gg} + \frac{2}{\sqrt{7}}c_{12}^{gg} - \frac{3}{2}\sqrt{\frac{3}{70}}c_{13}^{gg}\,,$$

$$\bar{c}_7^{gg} = -\frac{1}{2\sqrt{7}}c_1^{gg} + \frac{3}{5\sqrt{14}}c_6^{gg} - \frac{1}{\sqrt{14}}c_{10}^{gg} + \sqrt{\frac{7}{10}}c_{11}^{gg} - \frac{3}{\sqrt{70}}c_{12}^{gg} - \frac{3}{10}\sqrt{\frac{3}{7}}c_{13}^{gg}\,,$$

$$\bar{c}_8^{gg} = \frac{1}{\sqrt{2}}c_7^{gg} + \frac{1}{\sqrt{2}}c_9^{gg}\,,$$

$$\bar{c}_{13}^{gg} = \frac{1}{8}c_1^{gg} + \frac{1}{2\sqrt{2}}c_6^{gg} + \frac{1}{2\sqrt{2}}c_{10}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{11}^{gg} + \frac{1}{4}\sqrt{\frac{5}{2}}c_{12}^{gg} + \frac{3\sqrt{3}}{8}c_{13}^{gg}\,,$$

$$\bar{c}_4^{gg} = c_{14}^{gg}\,, \quad \bar{c}_9^{gg} = c_8^{gg}\,, \quad \bar{c}_{10}^{gg} = c_5^{gg}\,, \quad \bar{c}_{11}^{gg} = c_4^{gg}\,, \quad \bar{c}_{12}^{gg} = c_3^{gg}\,, \quad \bar{c}_{14}^{gg} = c_2^{gg}\,.$$

Colour basis gg

With

$$c_1^{gg} = \frac{1}{T_R}\frac{1}{N_c^2-1} t^{a_1}_{c_2c_4} t^{a_2}_{c_6c_8}\,,$$

$$c_2^{gg} = \frac{1}{Nc\sqrt{N_c^2-1}}\delta_{a_1a_2}\delta_{c_2c_4}\delta_{c_6c_8}\,,$$

$$c_3^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}}\delta_{c_2c_4} d_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_4^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}}\delta_{c_2c_4} i f_{a_1a_2\,b_1} t^{b_1}_{c_6c_8}\,,$$

$$c_5^{gg} = \frac{1}{T_R}\frac{1}{\sqrt{2(N_c^4-5N_c^2+4)}} t^{b_1}_{c_2c_4} d_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_6^{gg} = \frac{1}{T_R^2}\frac{N_c}{2(N_c^2-4)\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}$$

$$c_7^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} d_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_8^{gg} = \frac{1}{T_R}\frac{1}{N_c\sqrt{2(N_c^2-1)}} t^{b_1}_{c_2c_4} i f_{b_1a_1a_2}\delta_{c_6c_8}\,,$$

$$c_9^{gg} = \frac{1}{T_R^2}\frac{1}{2\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} d_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{10}^{gg} = \frac{1}{T_R^2}\frac{1}{2N_c\sqrt{N_c^2-1}} t^{b_1}_{c_2c_4} i f_{b_1a_1b_2} i f_{b_2a_2b_3} t^{b_3}_{c_6c_8}\,,$$

$$c_{11}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{10}{}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{12}^{gg} = \frac{1}{T_R}\frac{2}{\sqrt{N_c^4-5N_c^2+4}} t^{b_1}_{c_2c_4} \mathbf{P}^{\overline{10}}{}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{13}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2+2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{27}{}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,,$$

$$c_{14}^{gg} = \frac{1}{T_R}\frac{2}{N_c\sqrt{N_c^2-2N_c-3}} t^{b_1}_{c_2c_4} \mathbf{P}^{0}{}_{a_1b_1a_2b_2} t^{b_2}_{c_6c_8}\,.$$

Colour basis gg

With

$$\mathbf{P}^{10(\overline{10})}_{a_1b_1a_2b_2} = \frac{1}{4}\left(\delta_{a_1b_3}\delta_{b_1b_4} - \delta_{a_1b_4}\delta_{b_1b_3}\right)\left(\delta_{b_3a_2}\delta_{b_4b_2} \pm \frac{1}{T_R^2}\mathrm{Tr}\left[t^{b_3}t^{b_2}t^{b_4}t^{a_2}\right]\right) - \frac{1}{4N_cT_R}f_{a_1b_1b_5}f_{b_5a_2b_2}\,,$$

$$\mathbf{P}^{27}_{a_1b_1a_2b_2} = \frac{1}{4}\left(\delta_{a_1b_3}\delta_{b_1b_4} + \delta_{a_1b_4}\delta_{b_1b_3}\right)\left(\delta_{b_3a_2}\delta_{b_4b_2} + \frac{1}{T_R^2}\mathrm{Tr}\left[t^{b_3}t^{b_2}t^{b_4}t^{a_2}\right]\right) - \frac{1}{4T_R(2+N_c)}d_{a_1b_1b_3}d_{b_3a_2b_2} - \frac{1}{2N_c(1+N_c)}\delta_{a_1b_1}\delta_{a_2b_2}\,,$$

$$\mathbf{P}^{0}_{a_1b_1a_2b_2} = \frac{1}{4}\left(\delta_{a_1b_3}\delta_{b_1b_4} + \delta_{a_1b_4}\delta_{b_1b_3}\right)\left(\delta_{b_3a_2}\delta_{b_4b_2} - \frac{1}{T_R^2}\mathrm{Tr}\left[t^{b_3}t^{b_2}t^{b_4}t^{a_2}\right]\right) + \frac{1}{4T_R(2-N_c)}d_{a_1b_1b_3}d_{b_3a_2b_2} + \frac{1}{2N_c(1-N_c)}\delta_{a_1b_1}\delta_{a_2b_2}$$

SCET vs dQCD at NLP

Resummation at LP: [0710.0680, 0809.4283]

$$\Delta^{\text{SCET,LP}} = H(Q,\mu)U(Q,\mu_s,\mu)\tilde{s}\left(\ln\frac{Q^2}{\mu_s^2} + \frac{\partial}{\partial\eta},\mu_s\right)\frac{z^{-\eta}}{(1-z)^{1-2\eta}}\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}$$

Resummation at NLP: $\Delta^{\text{SCET,LP+NLP}} = \Delta^{\text{SCET,LP}} + \Delta^{\text{SCET,NLP}}$ [1809.10631,1910.12685]

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$$\Delta^{\text{SCET,NLP}} = -\beta(\alpha_s(\mu_s^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{\text{LL}}(Q,\mu_s)$$ [2101.07270]

As in the dQCD case, NLP contributions can be obtained directly from the LP ones with a derivative. In N-space, these forms are identical!

SCET vs dQCD at NLP

Important to include for analytical agreement at NLP

Resummation at LP:

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SCET vs dQCD at NLP

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Resummation at LP:

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Resummation at NLP: $\Delta^{\mathrm{SCET,LP+NLP}} = \Delta^{\mathrm{SCET,LP}} + \Delta^{\mathrm{SCET,NLP}}$

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Important for numerical agreement

Explicit forms - dQCD

$$g_a^{(1)}(\lambda, N) = \frac{A_a^{(1)}}{2\pi b_0^2}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h_a^{(1)}(\lambda, N) = -\frac{A_a^{(1)}}{2\pi b_0}\frac{\ln(1-2\lambda)}{N}$$

$$\frac{1}{\alpha_s}g_a^{(1)}(\lambda, N) + h_a^{(1)}(\lambda, N) = \frac{1}{\alpha_s}\left(1 + \frac{1}{2}\frac{\partial}{\partial N}\right)g_a^{(1)}(\lambda)$$

Explicit forms - SCET

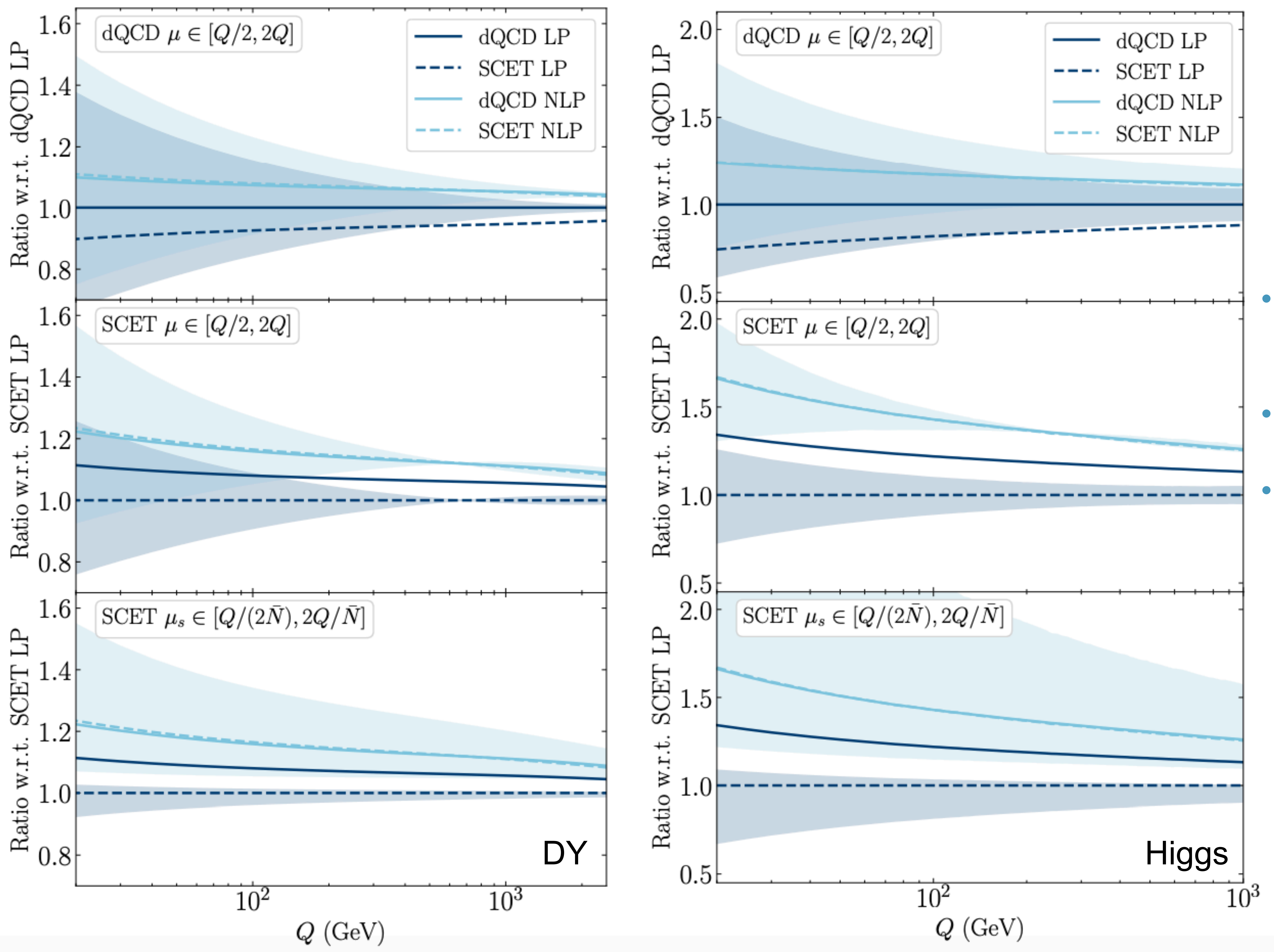
$$U_{aa}(Q,\mu_h,\mu,\mu_s) = \exp\left[S_a(\mu_h^2,\mu_s^2) - a_{\gamma_{V/S}}(\mu_h^2,\mu_s^2) + 2a_{\gamma_a}(\mu_s^2,\mu^2) - a_{\Gamma_{\mathrm{cusp,a}}}(\mu_h^2,\mu_s^2)\ln\frac{Q^2}{\mu_h^2}\right]$$

$$\Delta^{\mathrm{SCET,NLP}}(z,Q,\mu,\mu_s) = -\frac{2A_a^{(1)}}{\pi b_0}\ln\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_s^2)}\exp\left[S_{a,\mathrm{LL}}(Q^2,\mu^2) - S_{a,\mathrm{LL}}(\mu_s^2,\mu^2)\right]$$

$$S_{a,\mathrm{LL}}(\mu^2,\nu^2) = \frac{A_a^{(1)}}{b_0^2\pi}\left[\frac{1}{\alpha_s(\mu^2)} - \frac{1}{\alpha_s(\nu^2)} - \frac{1}{\alpha_s(\mu^2)}\ln\frac{\alpha_s(\nu^2)}{\alpha_s(\mu^2)}\right]$$

$$\Delta^{\mathrm{SCET,NLP}}(z,Q,\mu,\mu_s) = -\beta(\alpha_s(\mu^2))\frac{\partial}{\partial\alpha_s(\mu_s^2)}U_{aa,\mathrm{LL}}(Q,\mu_h=Q,\mu,\mu_s)$$

$$= \frac{\beta(\alpha_s(\mu^2))}{\alpha_s^2 b_0^2}\frac{A_a^{(1)}}{\pi}\ln\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_s^2)}U_{aa,\mathrm{LL}}(Q,\mu_h=Q,\mu,\mu_s)$$



- dQCD uncertainty from varying μ is larger than that of SCET
- LP SCET does not contain central dQCD result
- Explicit soft scale variations in SCET induce a large uncertainty at NLP

Example of a non-colour-singlet production

Prompt photon production

Here we do not know the NLP resummation, but can we use what we have learned from the DY and Higgs cases to estimate the class of NLP contributions that arise due to next-to-soft collinear momentum configurations?

Option 1: use diagonal splitting functions at NLP
Option 2b: use the DGLAP equations with off-diagonal dependence up to LL NLP
Option 2c: use the DGLAP equations with off-diagonal dependence without approximating

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But remember: no interference effects are taken into account in this way!

What about prompt photon?

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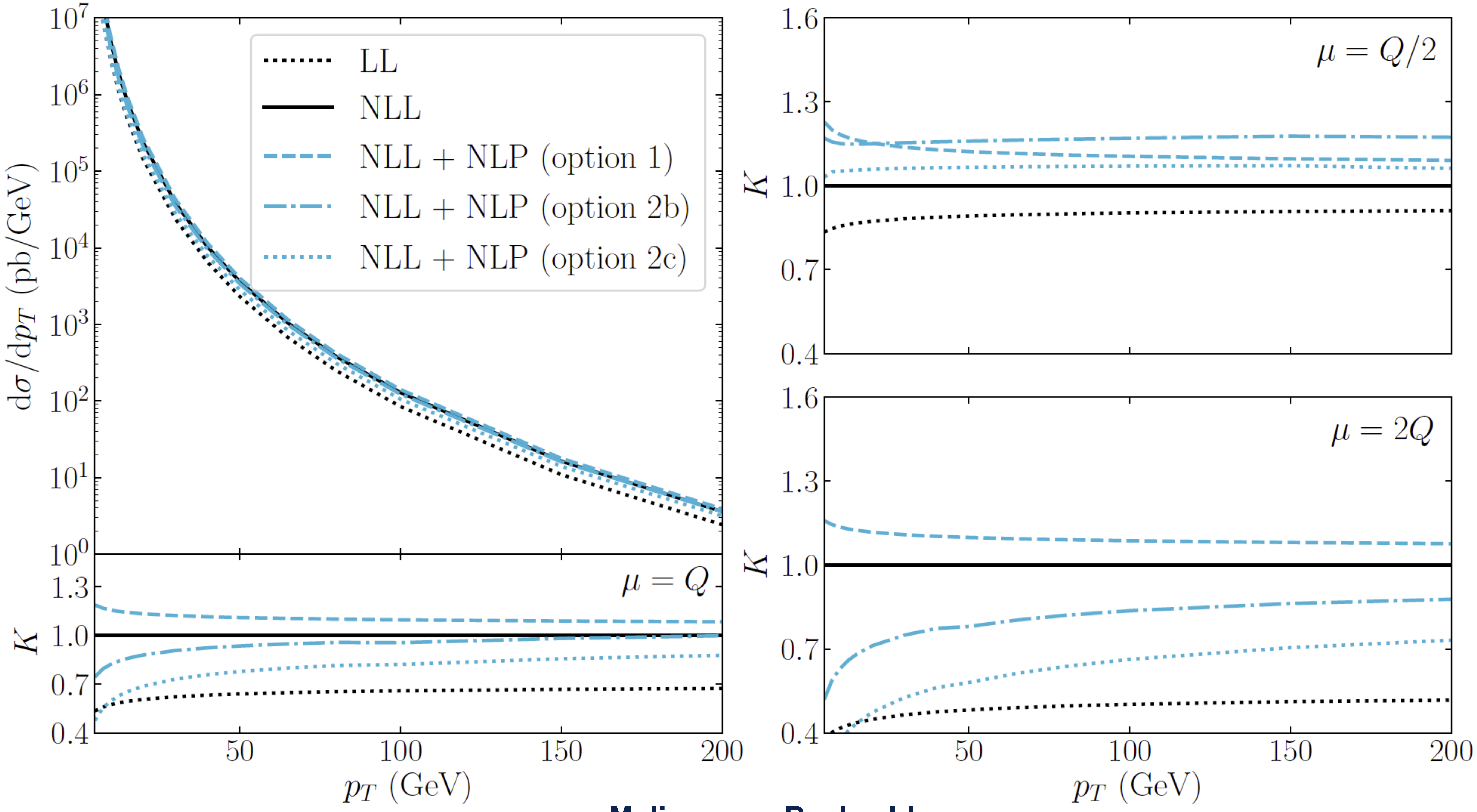

Melissa van Beekveld

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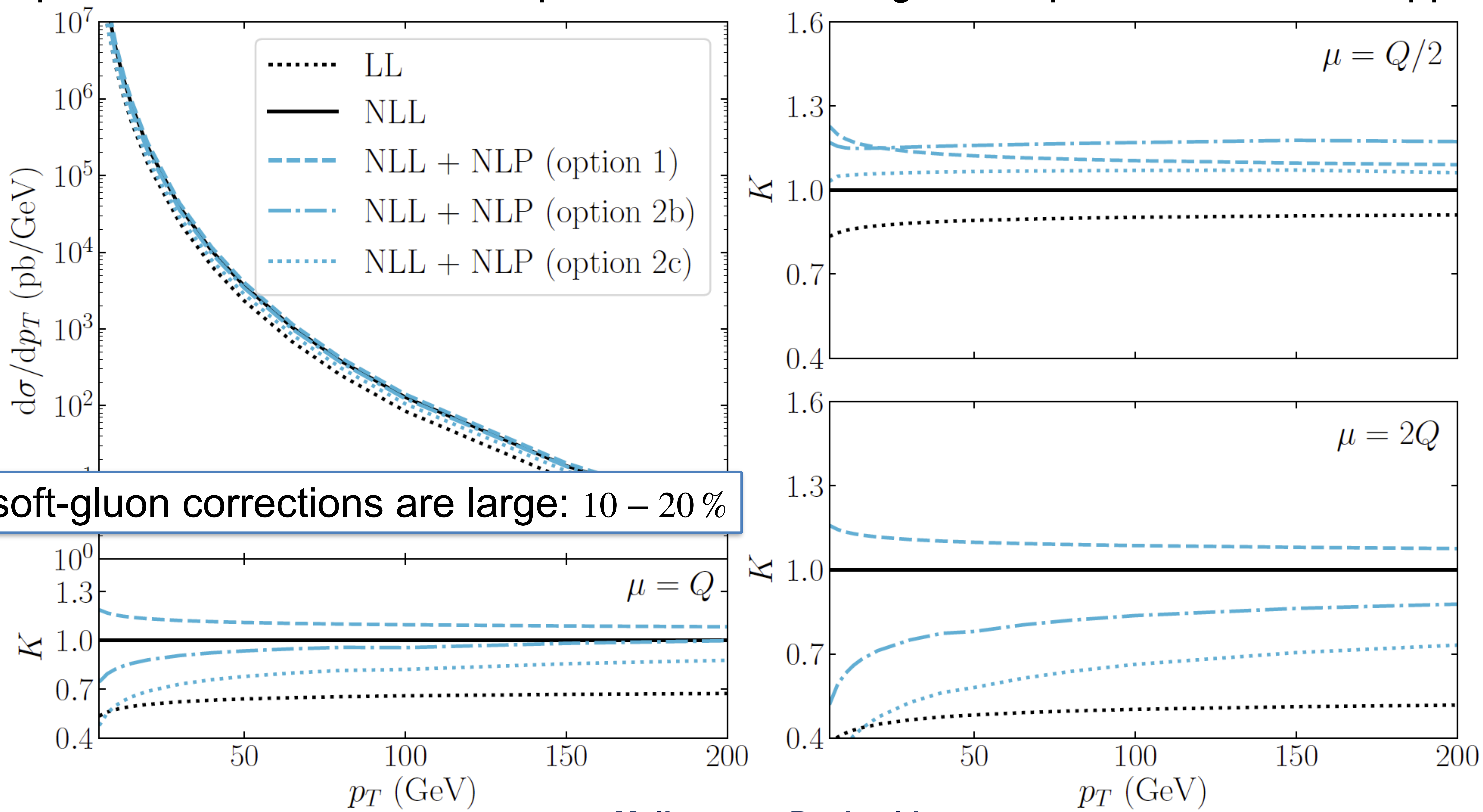

NLP LL soft-gluon corrections are large: $10-20\,\%$

What about prompt photon?

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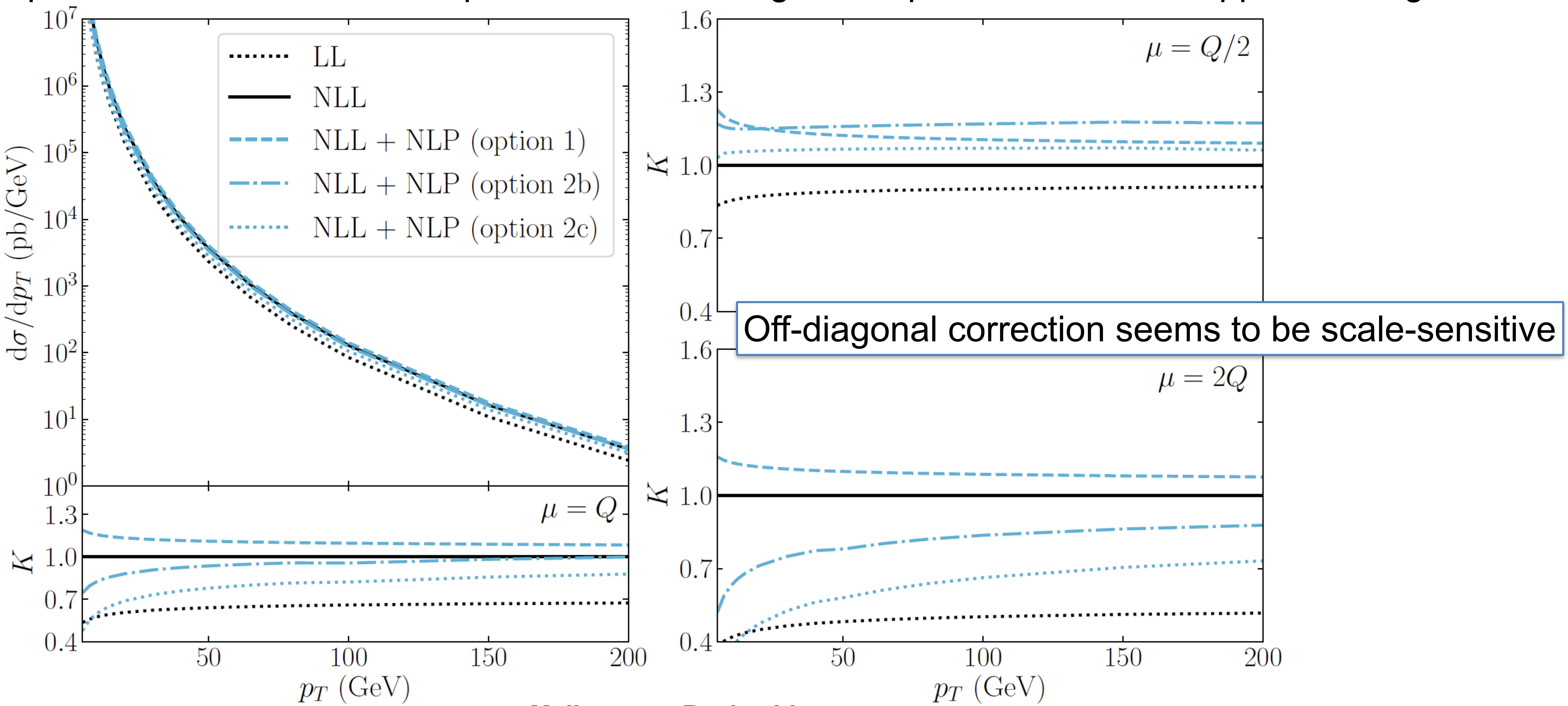

[1905.08741], [1905.11771]

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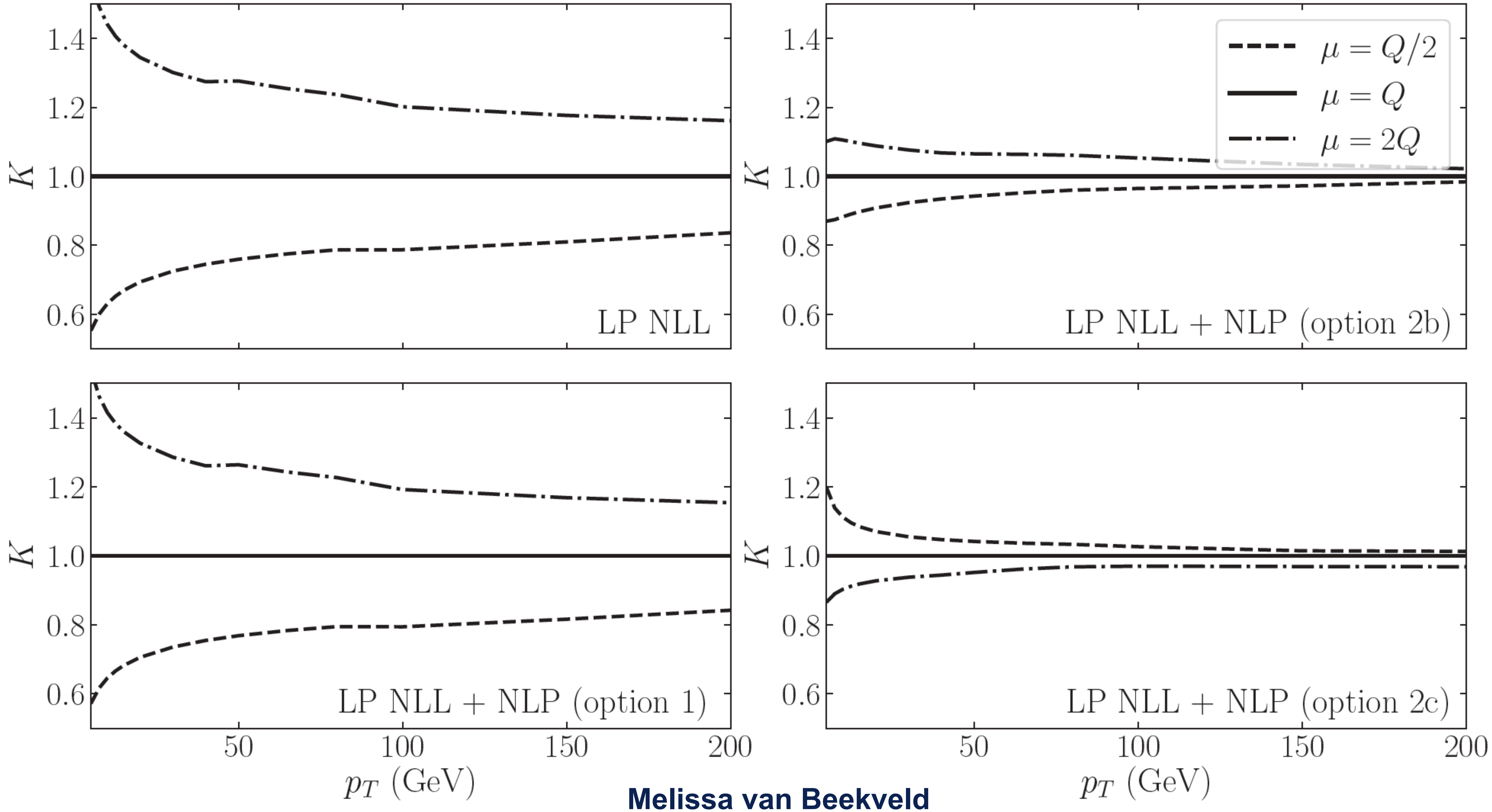

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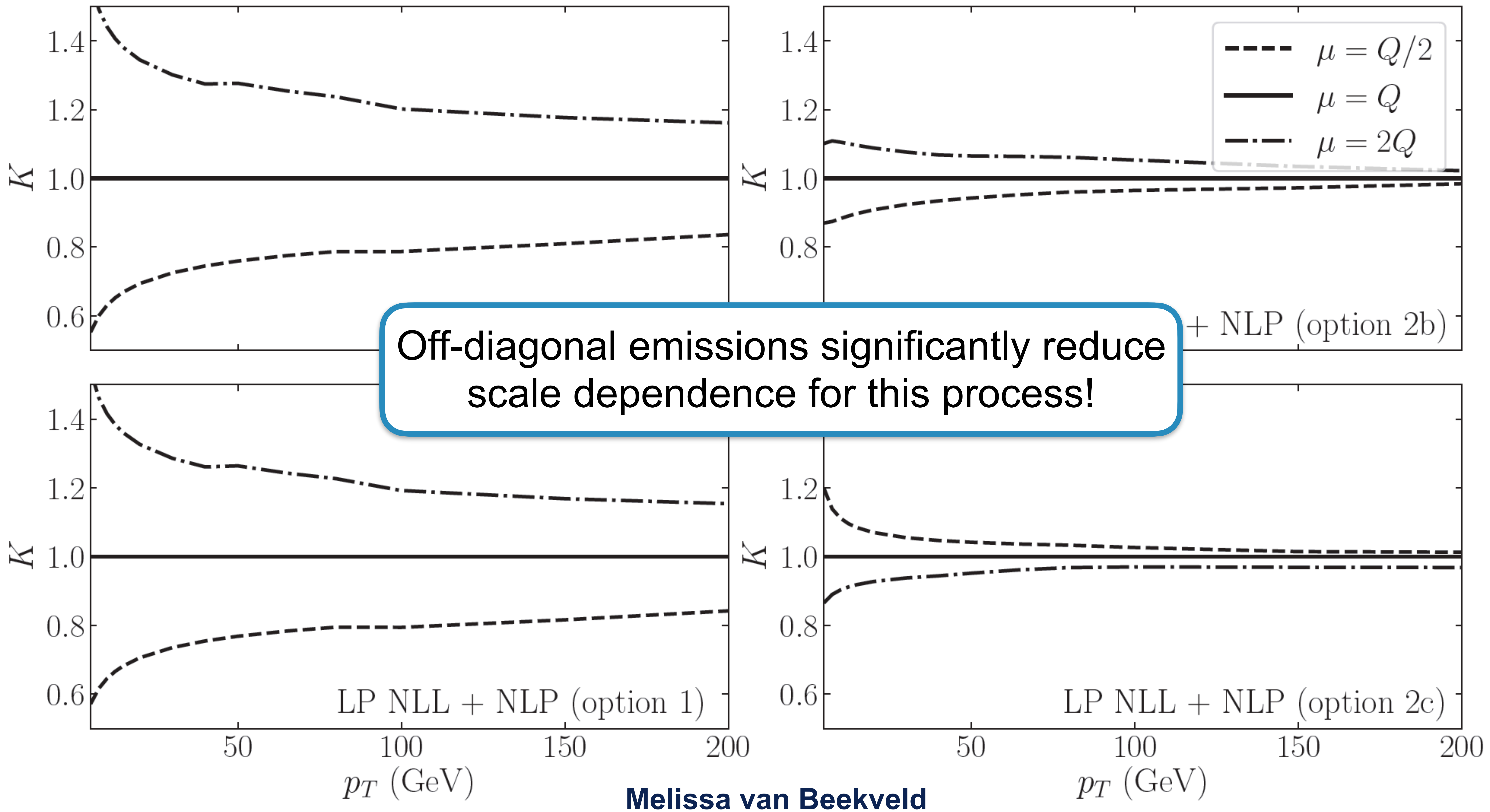

Off-diagonal emissions significantly reduce scale dependence for this process!