

Jet physics with Lund plane(s)

Gregory Soyez,

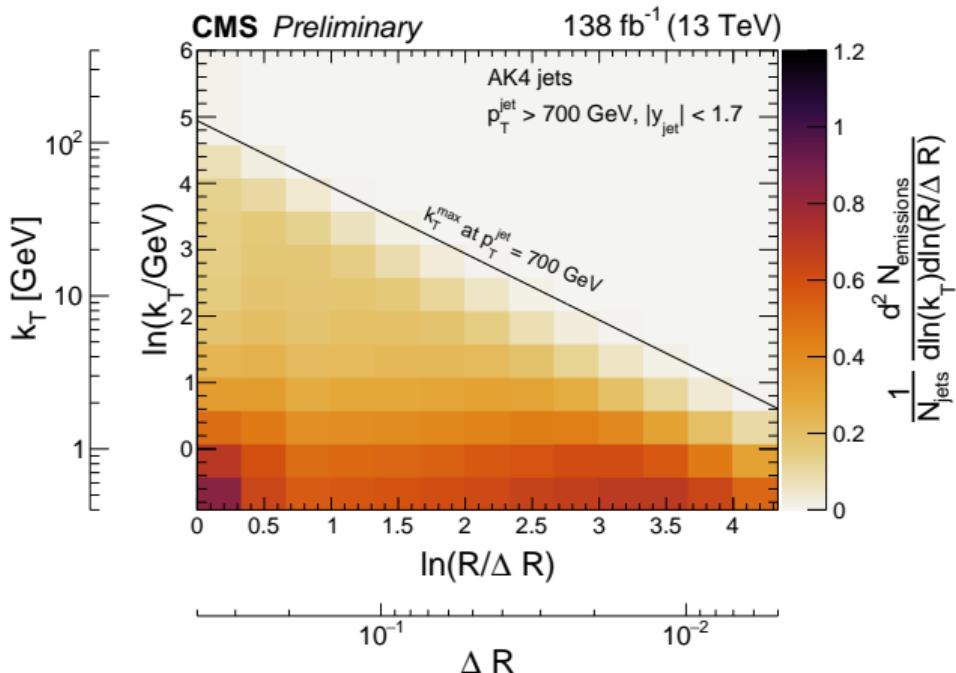
with Frederic Dreyer, Andrew Lifson, Gavin Salam, Adam Takacs, and the PanScales collaboration

IPhT, CNRS, CEA Saclay, CERN

CMS JetMet meeting, Brussels, 15-17 May 2023

Motivation

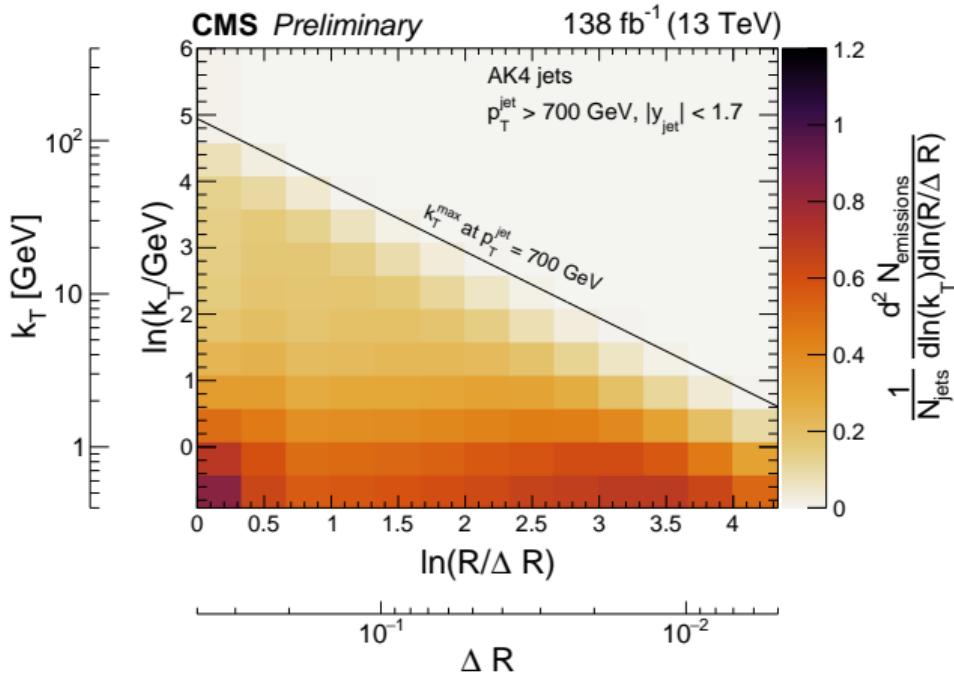
Recent (preliminary) CMS measurement
of the primary Lund plane density



[CMS, PAS-SMP-22-007]

Motivation

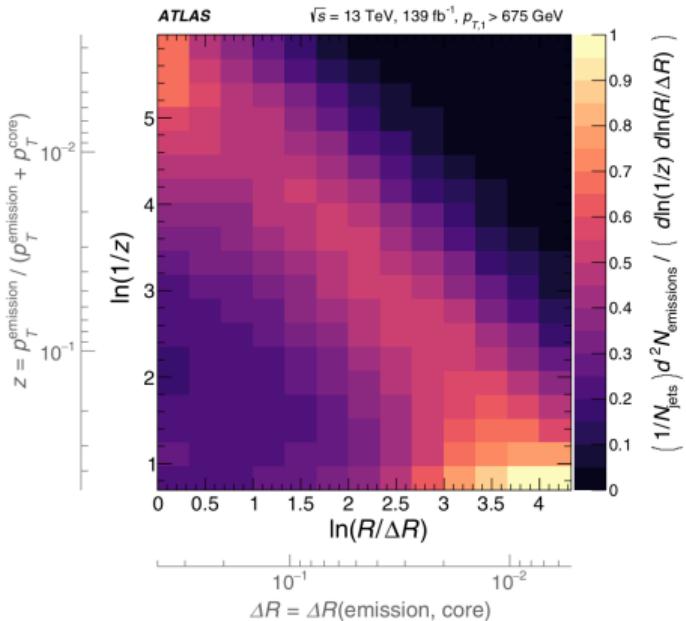
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Jet physics with Lund plane(s)

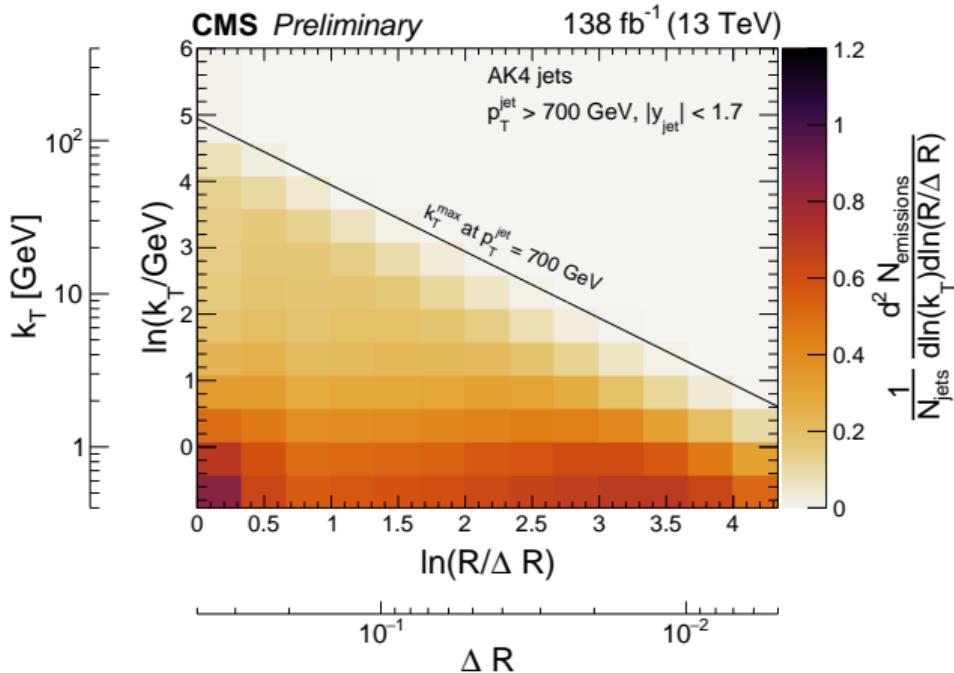


similar measurement in ATLAS

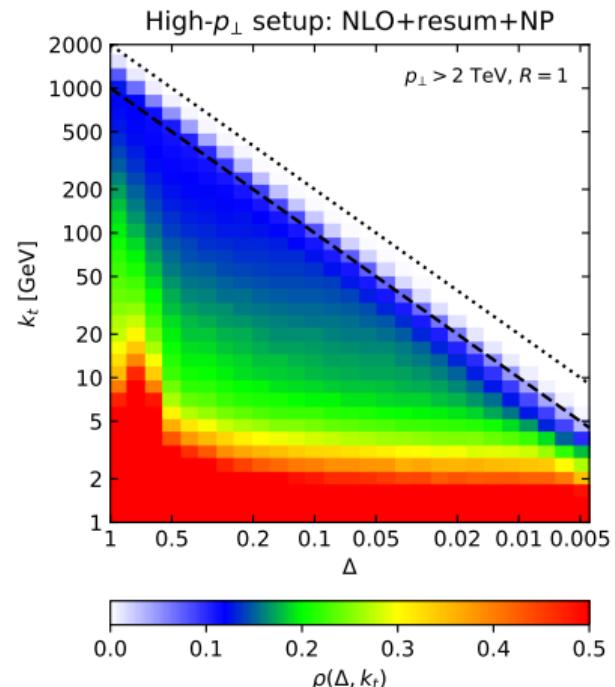
[ATLAS, 2004.03540]

Motivation

Recent (preliminary) CMS measurement
of the primary Lund plane density



[CMS, PAS-SMP-22-007]



with theory calculations available

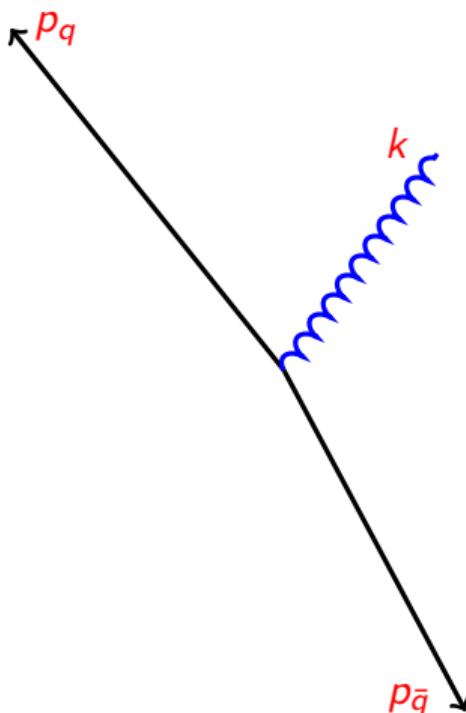
- ➊ Introduction to make sure we are on the same page
 - Lund diagrams: a (historical) conceptual tool for parton showers and resummations
 - promoting to a practical tool for jet physics
- ➋ Overview of a wide range of applications
 - theory background for the primary Lund plane density
 - Lund multiplicity
 - different constructions to see different additional effects
- ➌ Discussions of further interesting observables

Warmup: Lund diagrams

A useful representation of radiation in a jet

Basic features of QCD radiations

Take a gluon emission from a ($q\bar{q}$) dipole



Emission:

$$k^\mu \equiv z_q p_q^\mu + z_{\bar{q}} p_{\bar{q}}^\mu + k_\perp^\mu$$

3 degrees of freedom:

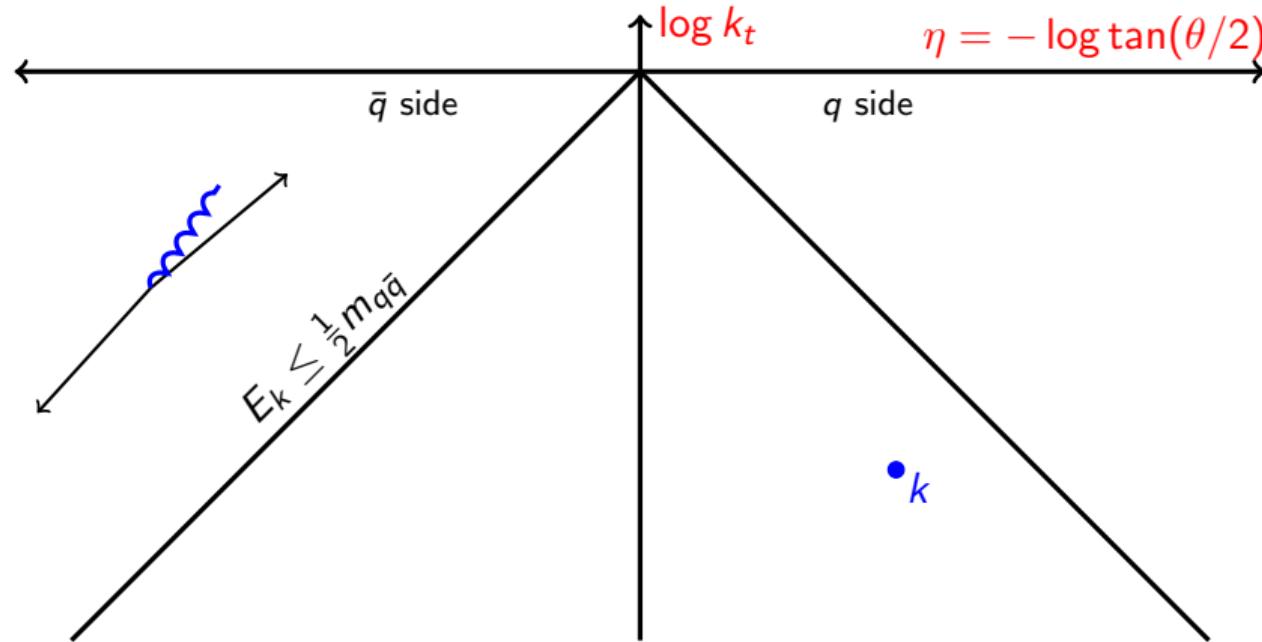
- Rapidity: $\eta = \frac{1}{2} \log \frac{z_q}{z_{\bar{q}}}$
- Transverse momentum: k_\perp
- Azimuth ϕ

In the soft-collinear approximation

$$d\mathcal{P} = \frac{\alpha_s(k_\perp) C_F}{\pi^2} d\eta \frac{dk_\perp}{k_\perp} d\phi$$

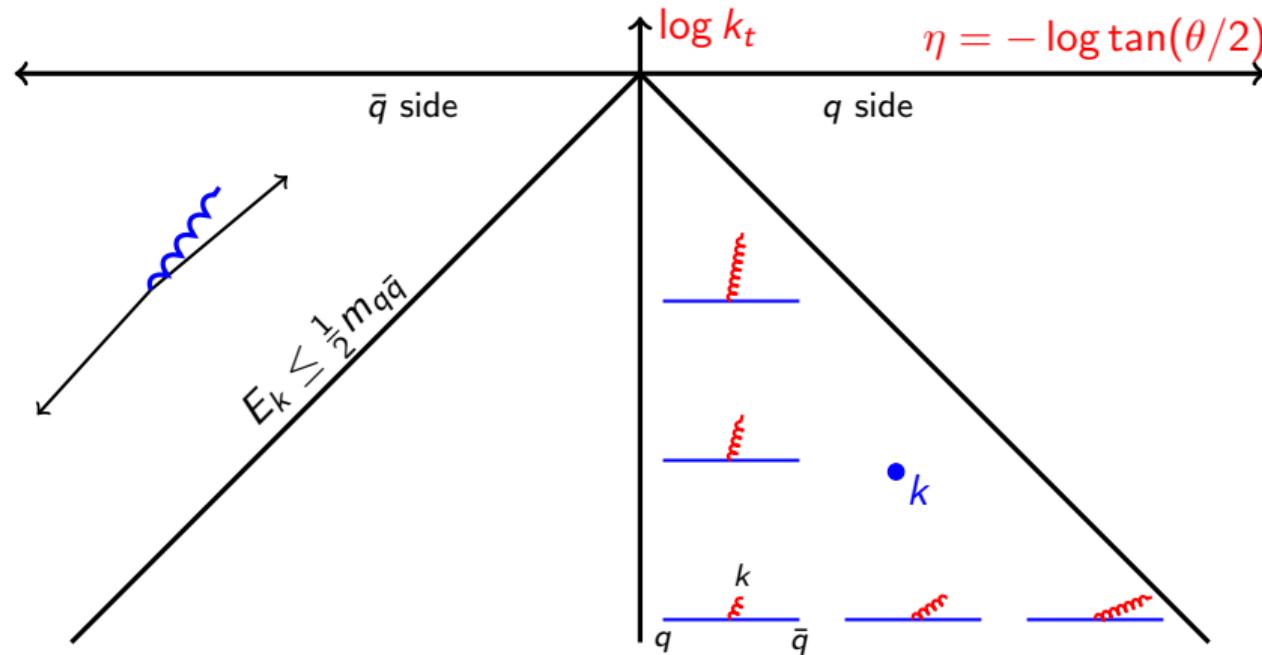
Basic features of QCD radiations: the Lund plane

Lund plane: natural representation uses the 2 “log” variables η and $\log k_\perp$



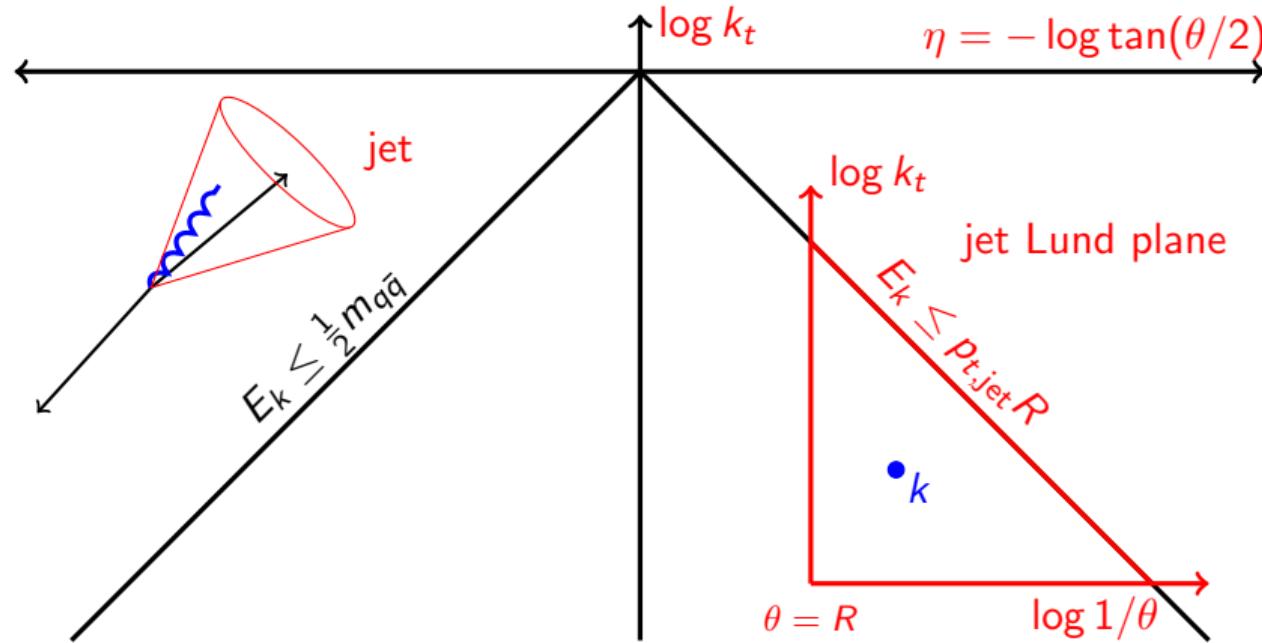
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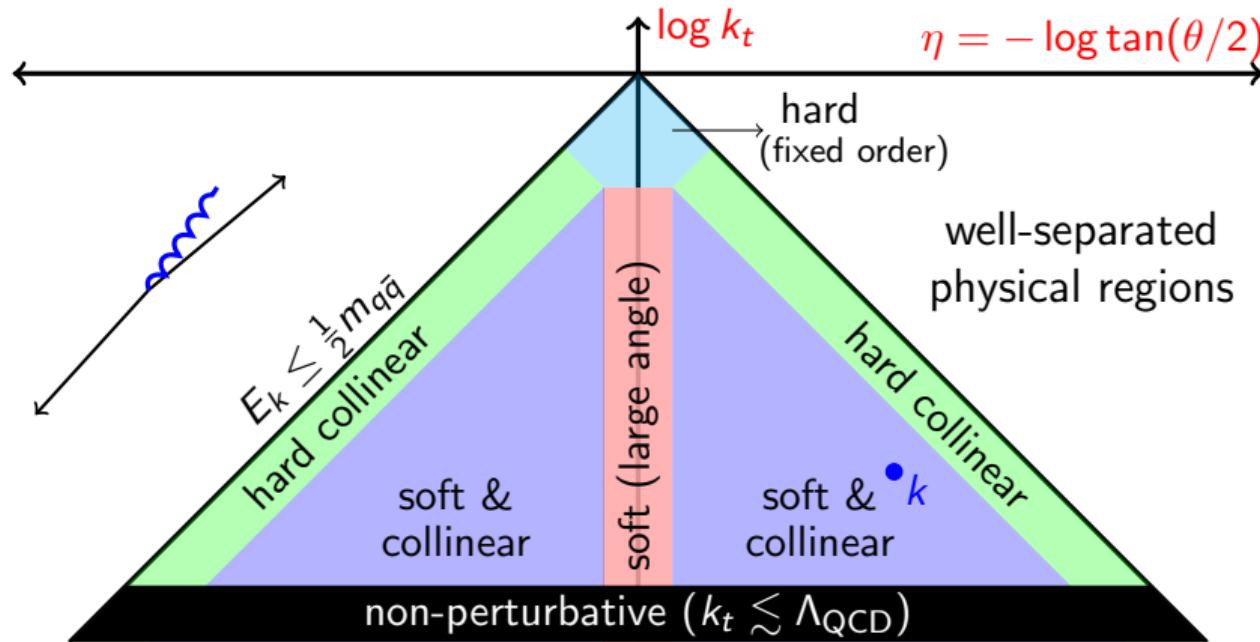
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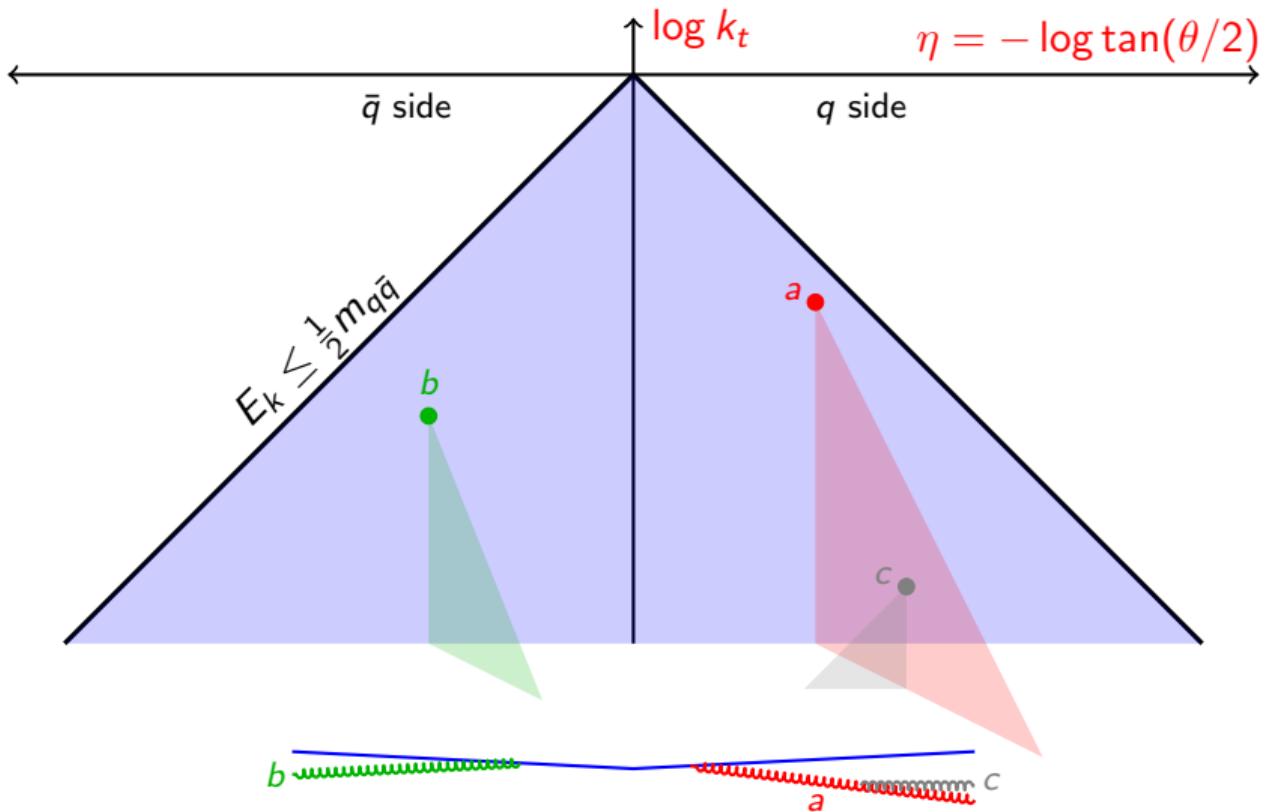


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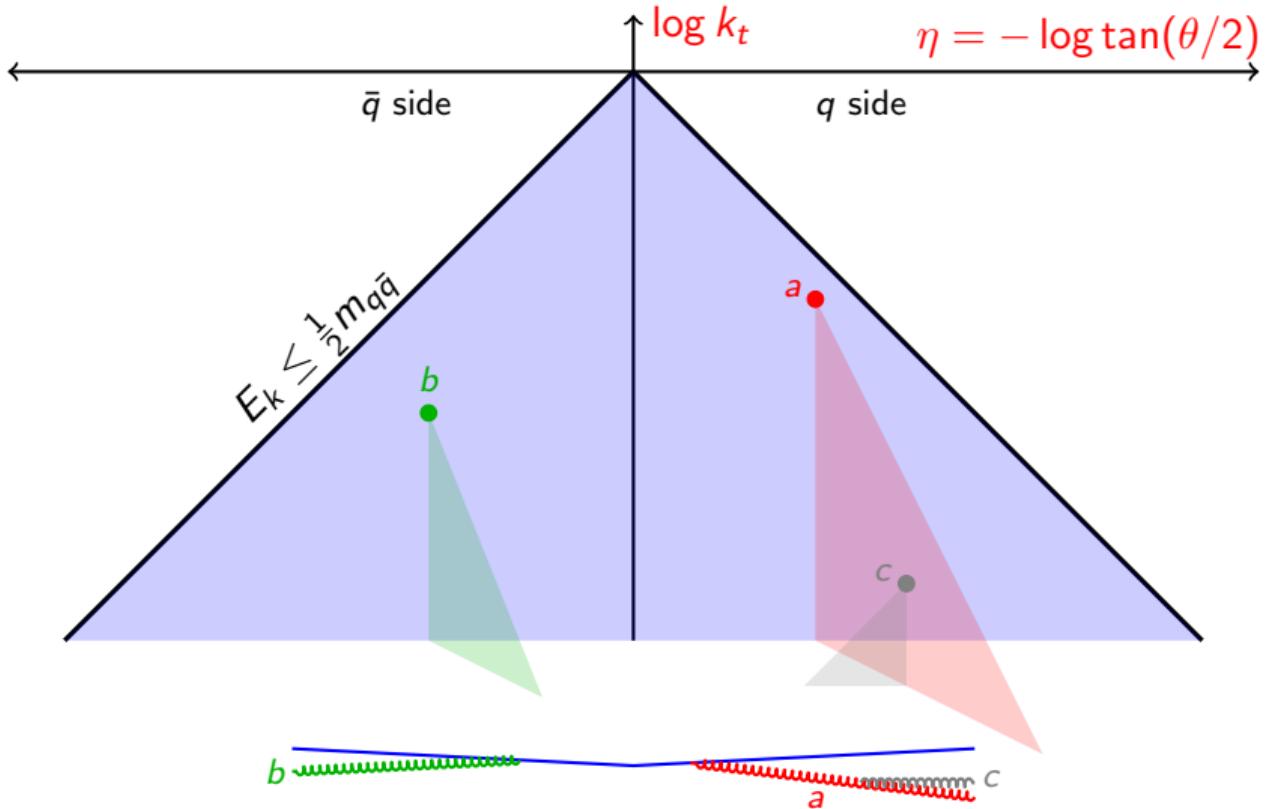


Multiple emissions in the Lund plane



Each emission spawns
its own plane
 a, b primary
 c secondary
...

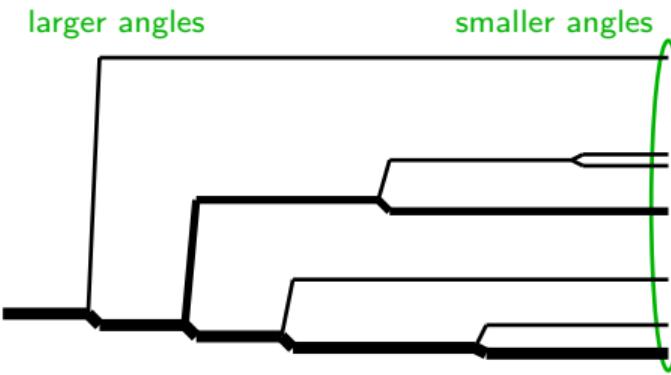
Multiple emissions in the Lund plane



Each emission spawns
its own plane
 a, b primary
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...

Respects angular
ordering
 $(\theta_c < \theta_a)$

The Lund plane(s) representation



For a given jet

- recluster (the constituents) with the Cambridge/Aachen algorithm

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

i.e. cluster from small to large angular distance

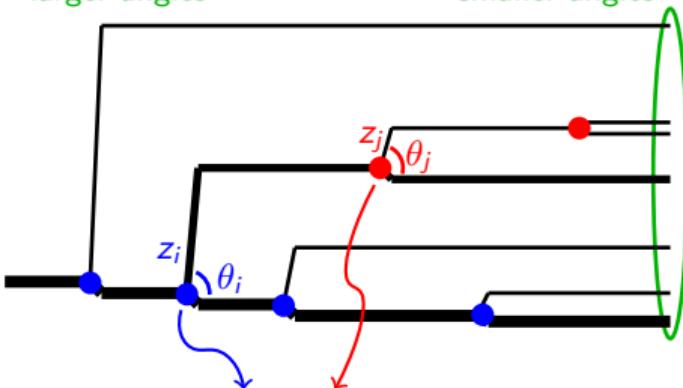
- gives a tree structure on the jet

[F.Dreyer,G.Salam,GS,arXiv:1807.04758]

The Lund plane(s) representation

larger angles

smaller angles



$$\mathcal{T}_i \equiv \{\theta_i, k_{t,i}, z_i, \psi_i, m_i, \dots\}$$

Lund coordinates at each vertex

For a given jet

- recluster (the constituents) with the Cambridge/Aachen algorithm

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

i.e. cluster from small to large angular distance

- gives a tree structure on the jet

For jets in pp : (similar for ee)

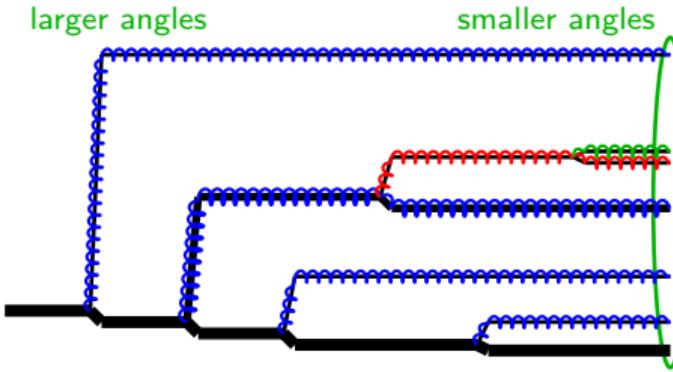
$$\eta = -\ln \Delta R$$

$$k_t = p_{t,\text{soft}} \Delta R, \text{ or } z = \frac{p_{t,\text{soft}}}{p_{t,\text{parent}}}$$

ψ \equiv azimuthal angle

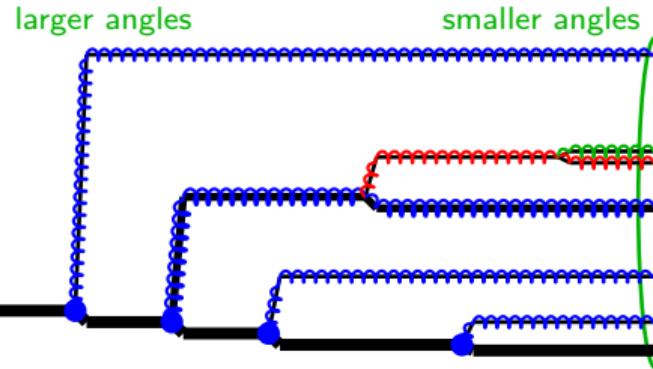
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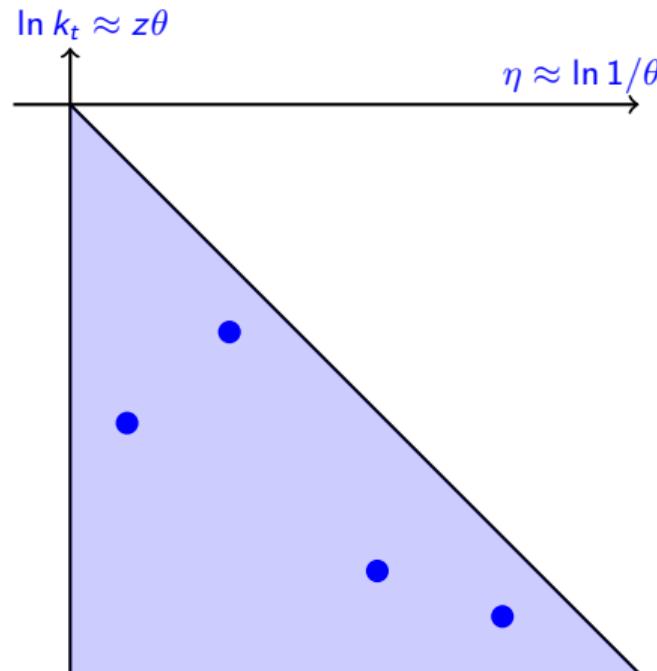


- closely follows **angular ordering**
i.e. mimics partonic cascade

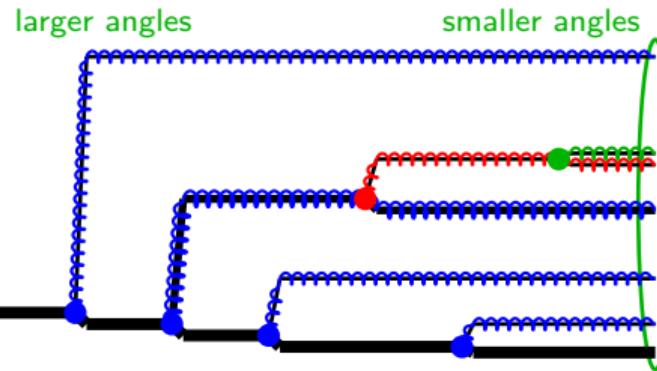
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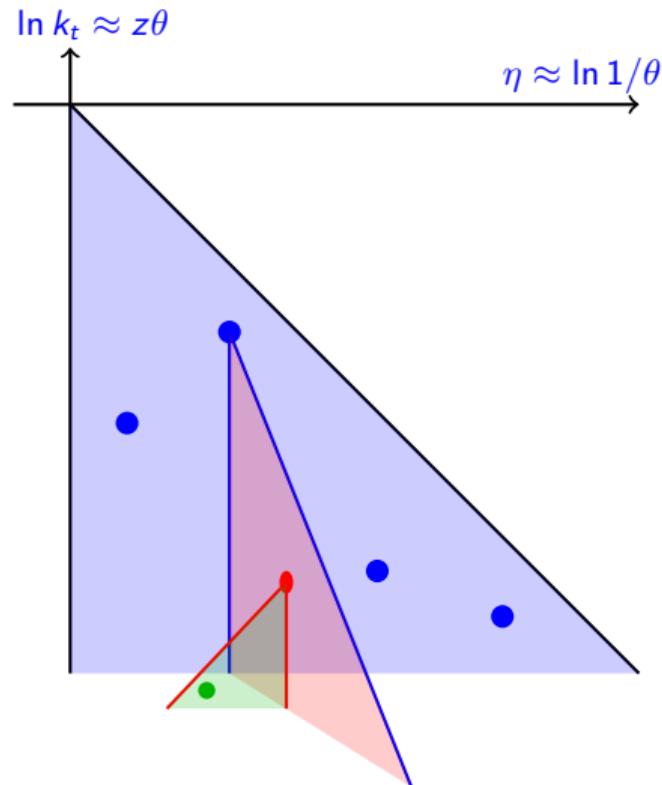
- closely follows angular ordering
i.e. mimics partonic cascade
- can be organised in Lund planes
 - primary



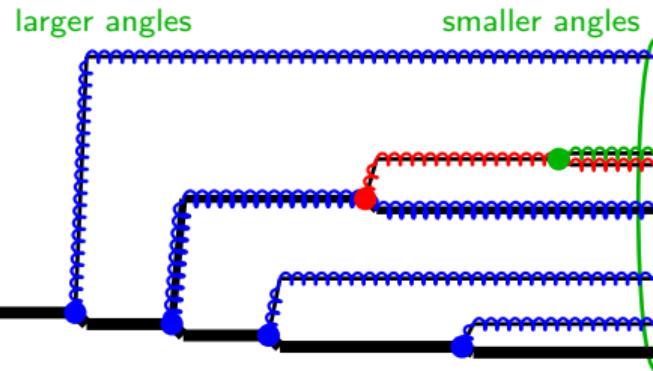
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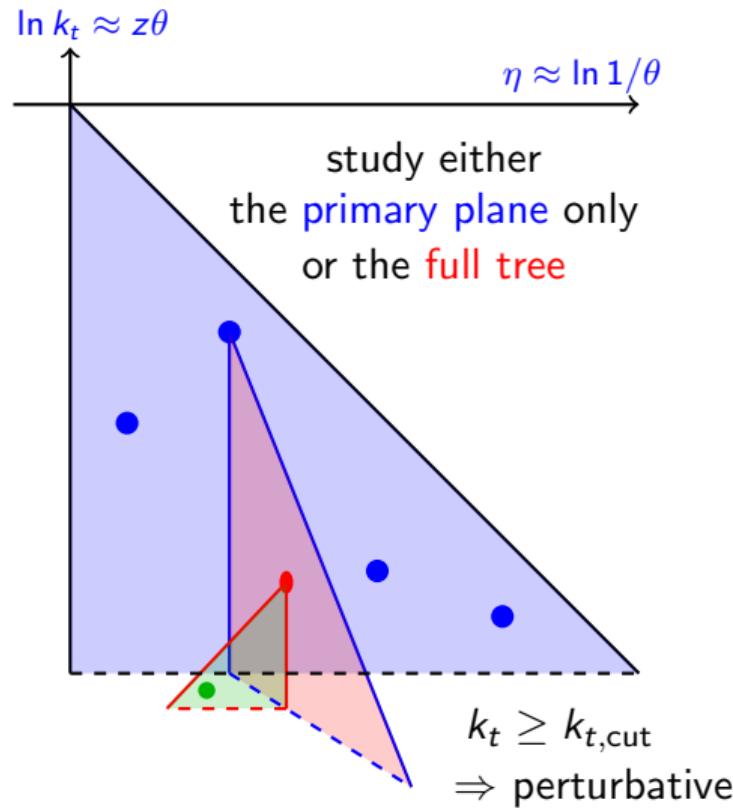
- closely follows **angular ordering**
i.e. mimics partonic cascade
- can be organised in **Lund planes**
 - primary
 - secondary
 - ...



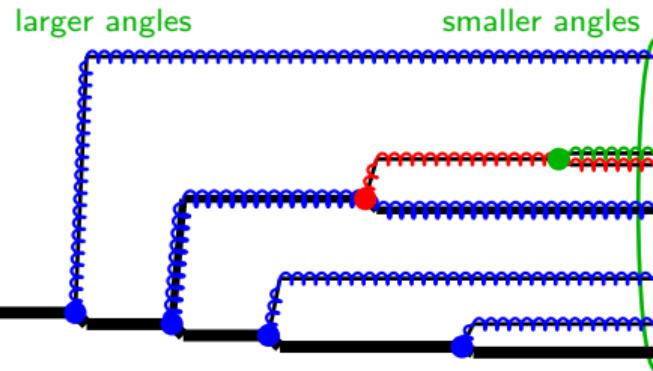
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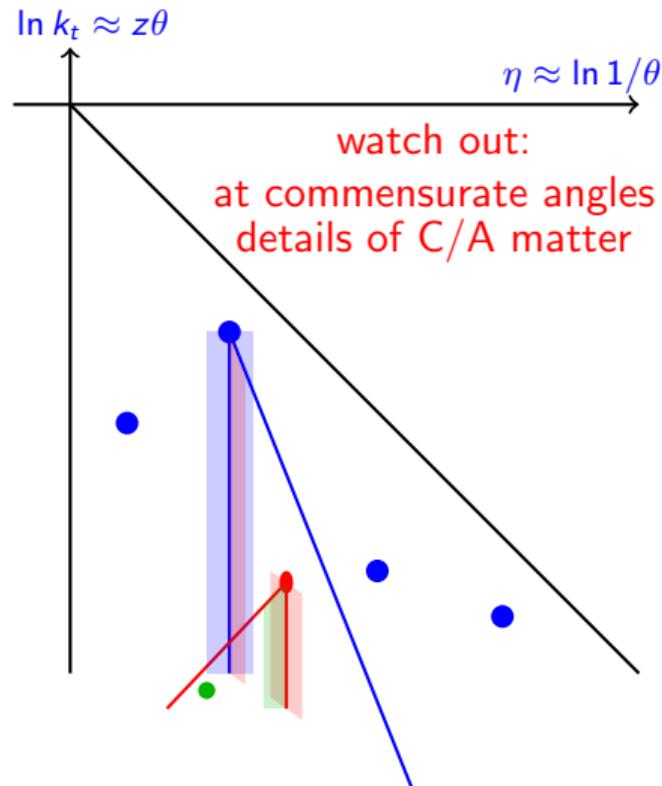
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The Lund plane(s) representation



- closely follows angular ordering
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 - primary
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Message #1

Lund diagrams represent (multiple) radiation across scales

- natural for thinking about resummations and parton showers
- different physical regions (soft, collinear, hard, non-perturbative) well separated
- organised in planes respecting angular ordering

Message #2

For a jet (or an ee event) one can **construct** a Lund-plane(s) structure capturing the properties of Lund diagrams

Imposing a k_t cut allows one to stay perturbative

Application series #1: QCD calculations

Primary Lund plane multiplicity

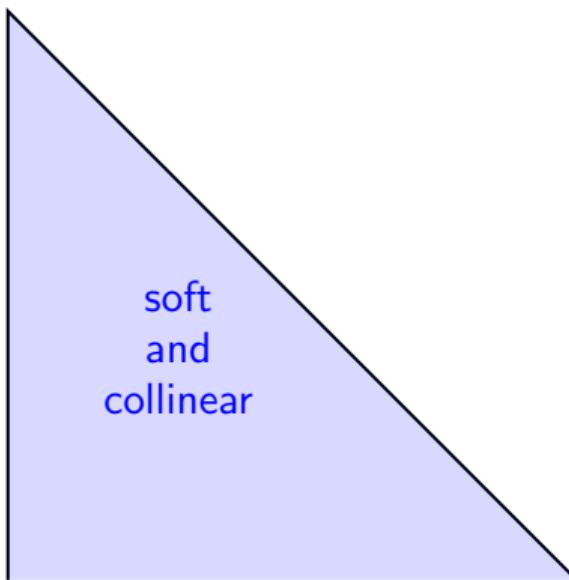
Average number of emission at given k_t , ΔR :

[A. Lifson, G. Salam, GS, arXiv:2007.06578]

$$\rho = \frac{1}{N_{\text{jets}}} \frac{d^2 N}{d \ln \Delta R d \ln k_t}$$

- Double-logarithmic behaviour:

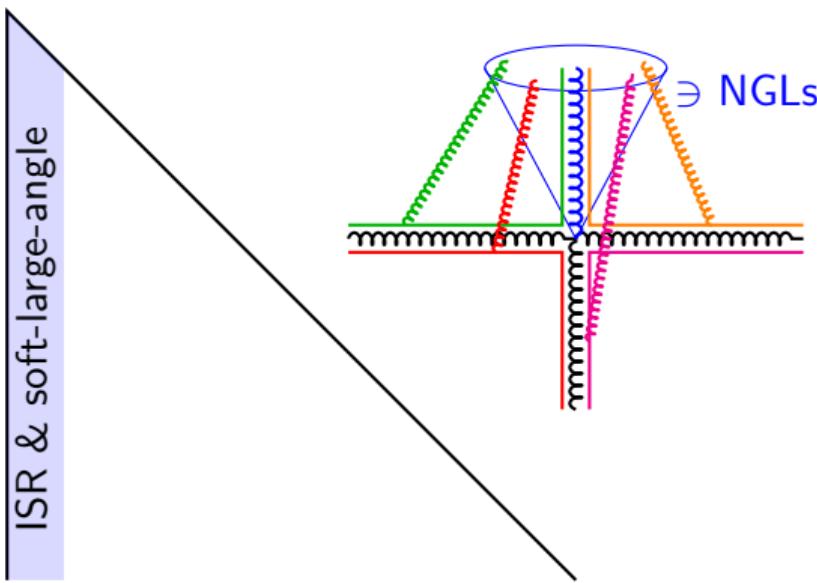
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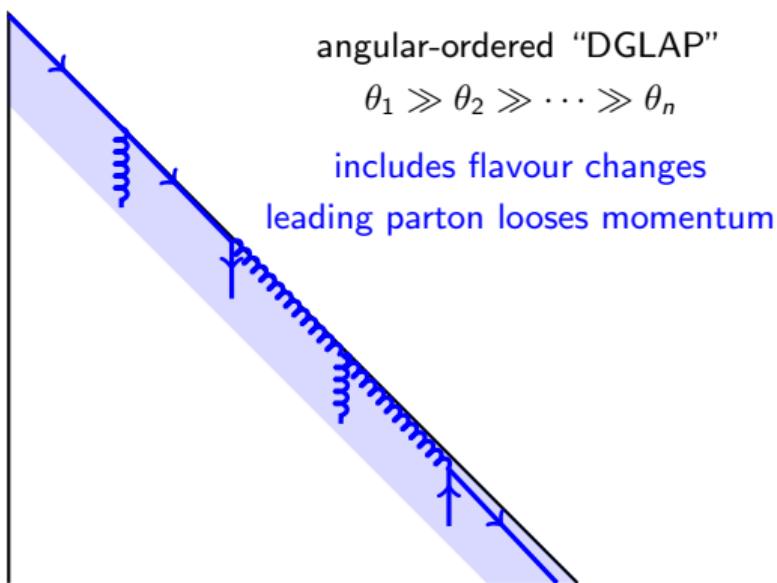
$$\rho = \frac{2\alpha_s(k_t) C_R}{\pi}$$

- Single-log calculation including
 - ✓ Running-coupling (trivial)
 - ✓ ISR+large angle

Primary Lund plane multiplicity

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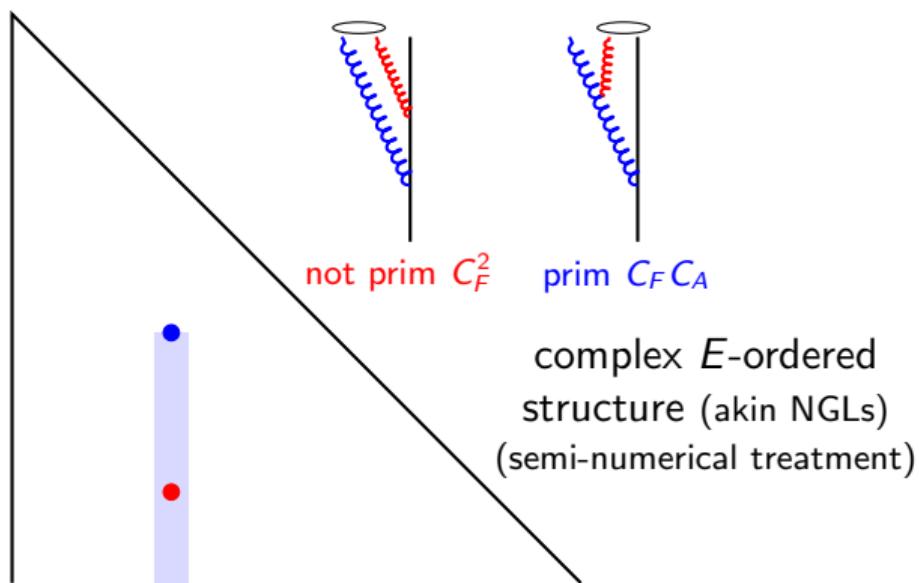
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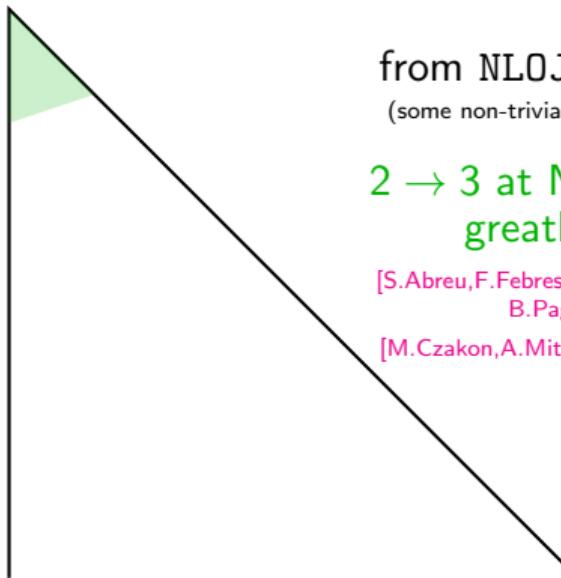
- Single-log calculation including

- ✓ Running-coupling (trivial)
- ✓ ISR+large angle
- ✓ Hard-collinear branchings
- ✓ Clustering effects
($\rightarrow k_t$ dependence beyond $\alpha_s(k_t)$)

Primary Lund plane multiplicity

Average number of emission at given k_t , ΔR :

$$\rho = \frac{1}{N_{\text{jets}}} \frac{d^2 N}{d \ln \Delta R d \ln k_t}$$



from NLO Jet++

(some non-trivial details)

2 → 3 at NNLO would
greatly help!

[S.Abreu,F.Febres Cordero,H.Ita,
B.Page,V.Sotnikov,2102.13609]

[M.Czakon,A.Mitov,R.Poncelet,2106.05331]

[A. Lifson, G. Salam, GS, arXiv:2007.06578]

- Double-logarithmic behaviour:

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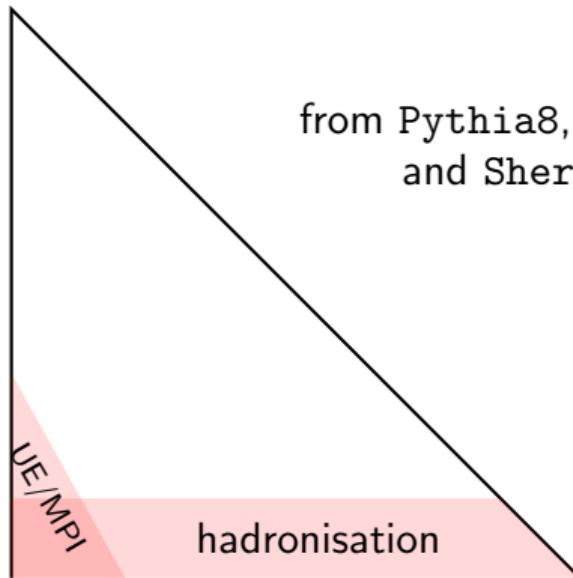
- + Matching to NLO (\sim top)

Primary Lund plane multiplicity

Average number of emission at given k_t , ΔR :

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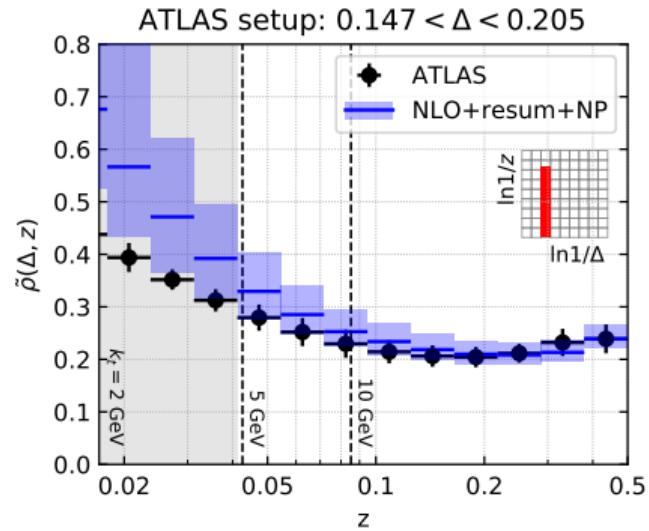


- Double-logarithmic behaviour:

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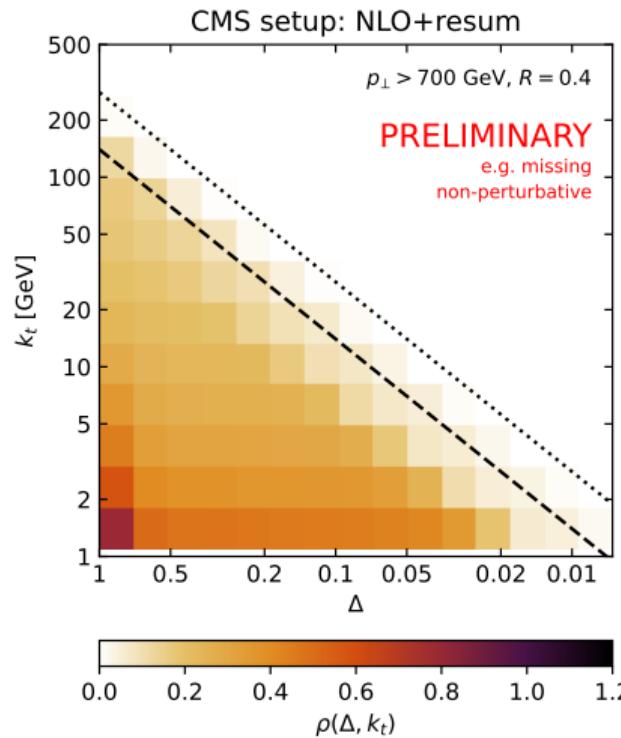
- Single-log calculation including
 - ✓ Running-coupling (trivial)
 - ✓ ISR+large angle
 - ✓ Hard-collinear branchings
 - ✓ Clustering effects
- + Matching to NLO (\sim top)
- + NP corrections (\sim bottom)

Data v. theory

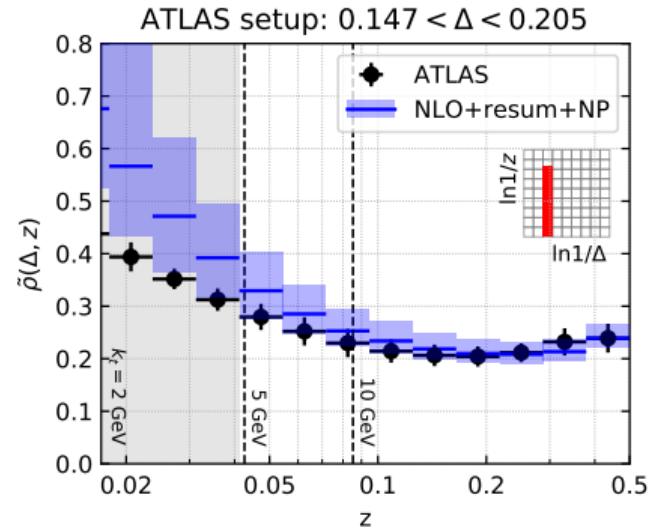


- good agreement (particularly for $k_t \gtrsim 5 \text{ GeV}$)
- commensurate exp.&th. uncert.

Data v. theory



more (hopefully) coming soon!



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- commensurate exp.&th. uncert.

Lund multiplicity (1/2)

Lund multiplicity

count the (average) number of Lund declusterings (full tree)
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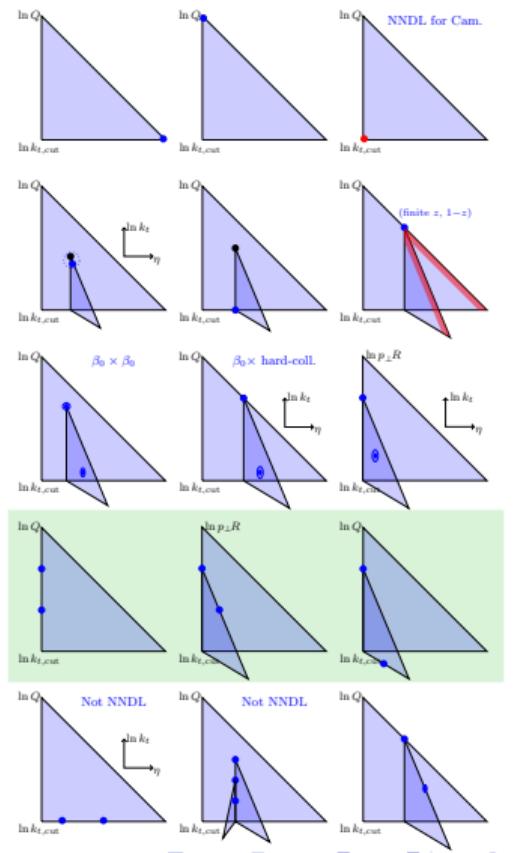
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$$\langle N^{\text{LP}}(L, \alpha_s) \rangle = \underbrace{h_1(\alpha_s L^2) + \sqrt{\alpha_s} h_2(\alpha_s L^2)}_{\text{Since 1992}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{New NNDL!!}} + \dots$$

[R. Medves, A. Soto, GS, 2205.02861]

$$\begin{aligned} 2\pi h_3^{(q)} &= D_{\text{end}}^{q \rightarrow qg} + \left(D_{\text{end}}^{g \rightarrow gg} + D_{\text{end}}^{g \rightarrow q\bar{q}} \right) \frac{C_F}{C_A} (\cosh \nu - 1) + D_{\text{hme}}^{qg} \cosh \nu + \frac{C_F}{C_A} \left[(1 - c_\delta) D_{\text{pair}}^{q\bar{q}} (\cosh \nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}} \right) \frac{\nu}{2} \sinh \nu \right] \\ &+ C_F \left[\left(\cosh \nu - 1 - \frac{1 - c_\delta}{4} \nu^2 \right) D_{\text{clust}}^{(\text{prim})} + (\cosh \nu - 1) D_{\text{clust}}^{(\text{sec})} \right] + \frac{C_F}{C_A} \left[D_{\text{e-loss}}^g \frac{\nu}{2} \sinh \nu + (D_{\text{e-loss}}^q - D_{\text{e-loss}}^g) (\cosh \nu - 1) \right] \\ &+ \frac{C_F \pi^2 \beta_0^2}{C_A 8C_A} \left[3\nu(2\nu^2 - 1) \sinh \nu + (\nu^4 + 3\nu^2) \cosh \nu \right] + \frac{C_F}{2} \left\{ (B_{gg} + c_\delta B_{gq})^2 \nu^2 \cosh \nu + 8 \left[2c_\delta B_{gg} - 2c_\delta B_q - (1 - 3c_\delta^2) B_{gq} \right] B_{gq} \cosh \nu \right. \\ &+ [4B_q(B_{gg} + (2c_\delta + 1)B_{gq}) - (B_{gg} + c_\delta B_{gq})(B_{gg} + 9c_\delta B_{gq})] \nu \sinh \nu + 4(1 - c_\delta^2) B_{gq}^2 \nu^2 + 8 \left[2c_\delta B_q - 2c_\delta B_{gg} + (1 - 3c_\delta^2) B_{gq} \right] B_{gq} \Big\} \\ &\left. + \frac{C_F \pi \beta_0}{C_A 2} \left\{ (B_{gg} + c_\delta B_{gq}) \nu^3 \sinh \nu + [2B_q - 2B_{gg} + (6 - 8c_\delta) B_{gq}] \nu \sinh \nu + 2(B_q + B_{gg} + B_{gq}) \nu^2 \cosh \nu - 4(1 - c_\delta) B_{gq} (2 \cosh \nu - 2 + \nu^2) \right\} \right\} \end{aligned}$$

Only contributes for jets (LHC) [R. Medves, A. Soto, GS, 2212.05076]



Lund multiplicity (1/2)

Lund multiplicity

count the (average) number of Lund declusterings (full tree)
with $k_t \geq k_{t,\text{cut}}$ (full ee event, or inside a pp jet)

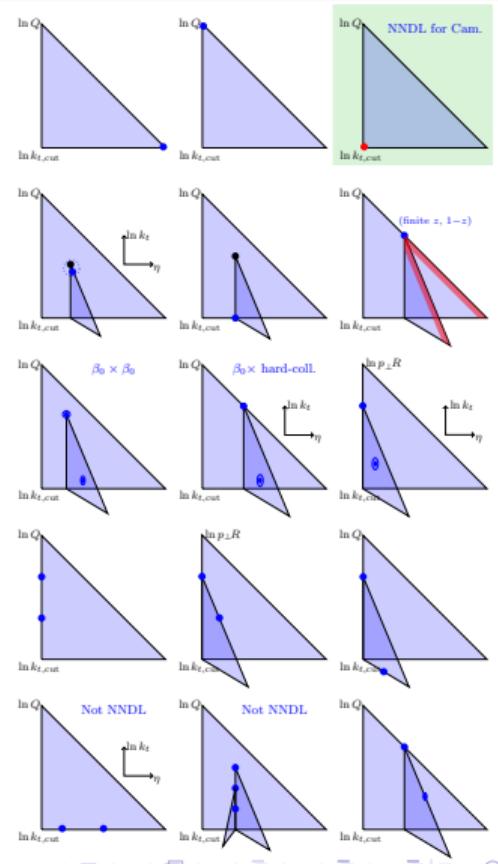
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Side product: NNDL Cambridge multiplicity for $y_{\text{cut}} = k_{t,\text{cut}}^2$



Lund multiplicity (1/2)

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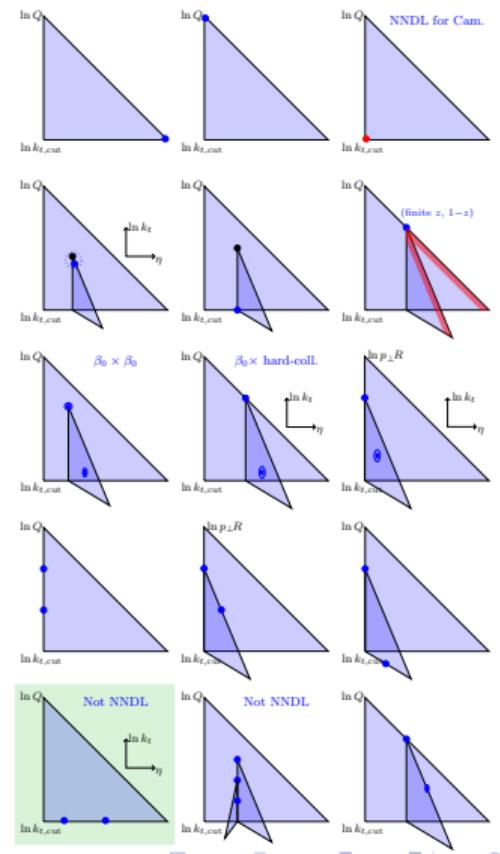
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[R. Medves, A. Soto, GS, 2205.02861]

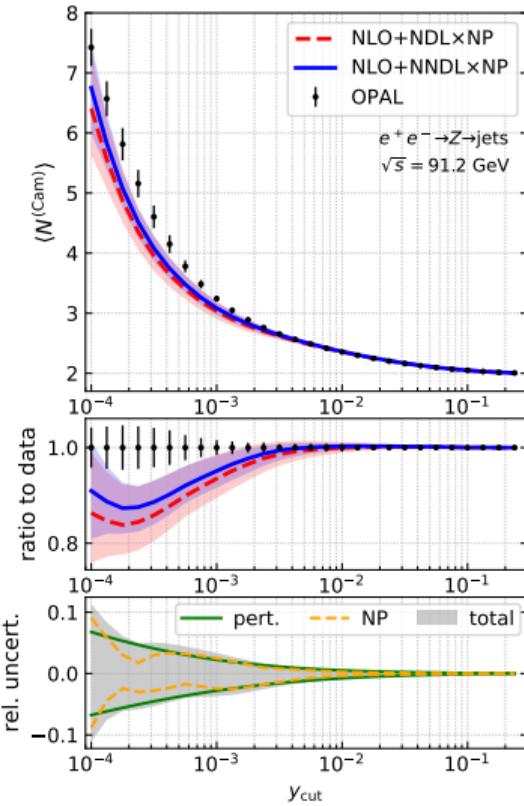
$$\begin{aligned} 2\pi h_3^{(q)} &= D_{\text{end}}^{q \rightarrow qg} + \left(D_{\text{end}}^{g \rightarrow gg} + D_{\text{end}}^{g \rightarrow q\bar{q}} \right) \frac{C_F}{C_A} (\cosh \nu - 1) + D_{\text{hme}}^{qg} \cosh \nu + \frac{C_F}{C_A} [(1 - c_\delta) D_{\text{pair}}^{q\bar{q}} (\cosh \nu - 1) + (K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}}) \frac{\nu}{2} \sinh \nu] \\ &+ C_F \left[\left(\cosh \nu - 1 - \frac{1 - c_\delta}{4} \nu^2 \right) D_{\text{clust}}^{(\text{prim})} + (\cosh \nu - 1) D_{\text{clust}}^{(\text{sec})} \right] + \frac{C_F}{C_A} \left[D_{\text{e-loss}}^g \frac{\nu}{2} \sinh \nu + (D_{\text{e-loss}}^q - D_{\text{e-loss}}^g) (\cosh \nu - 1) \right] \\ &+ \frac{C_F \pi^2 \beta_0^2}{C_A 8C_A} [3\nu(2\nu^2 - 1) \sinh \nu + (\nu^4 + 3\nu^2) \cosh \nu] + \frac{C_F}{2} \left\{ (B_{gg} + c_\delta B_{gq})^2 \nu^2 \cosh \nu + 8 [2c_\delta B_{gg} - 2c_\delta B_q - (1 - 3c_\delta^2) B_{gq}] B_{gq} \cosh \nu \right. \\ &+ [4B_q(B_{gg} + (2c_\delta + 1)B_{gq}) - (B_{gg} + c_\delta B_{gq})(B_{gg} + 9c_\delta B_{gq})] \nu \sinh \nu + 4(1 - c_\delta^2) B_{gq}^2 \nu^2 + 8 [2c_\delta B_q - 2c_\delta B_{gg} + (1 - 3c_\delta^2) B_{gq}] B_{gq} \Big\} \\ &\left. + \frac{C_F \pi \beta_0}{C_A 2} \left\{ (B_{gg} + c_\delta B_{gq}) \nu^3 \sinh \nu + [2B_q - 2B_{gg} + (6 - 8c_\delta) B_{gq}] \nu \sinh \nu + 2(B_q + B_{gg} + B_{gq}) \nu^2 \cosh \nu - 4(1 - c_\delta) B_{gq} (2 \cosh \nu - 2 + \nu^2) \right\} \right\} \end{aligned}$$

No “long-distance effect” \Rightarrow simpler than k_t

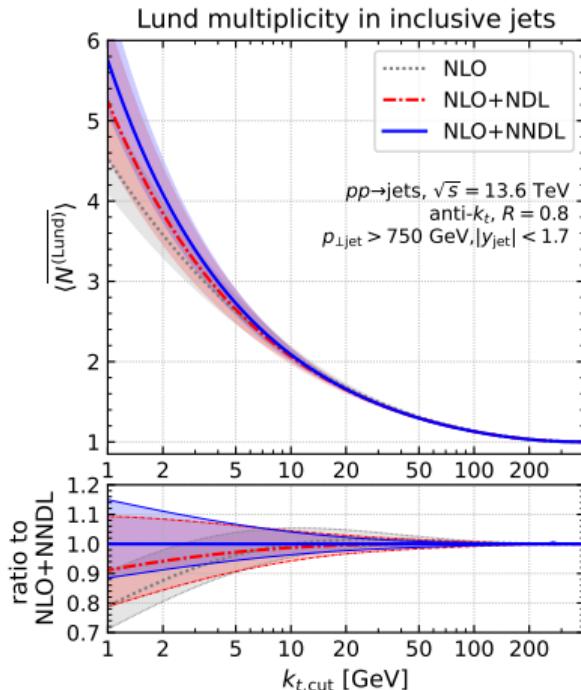


Lund multiplicity (2/2)

OPAL Cambridge multiplicity



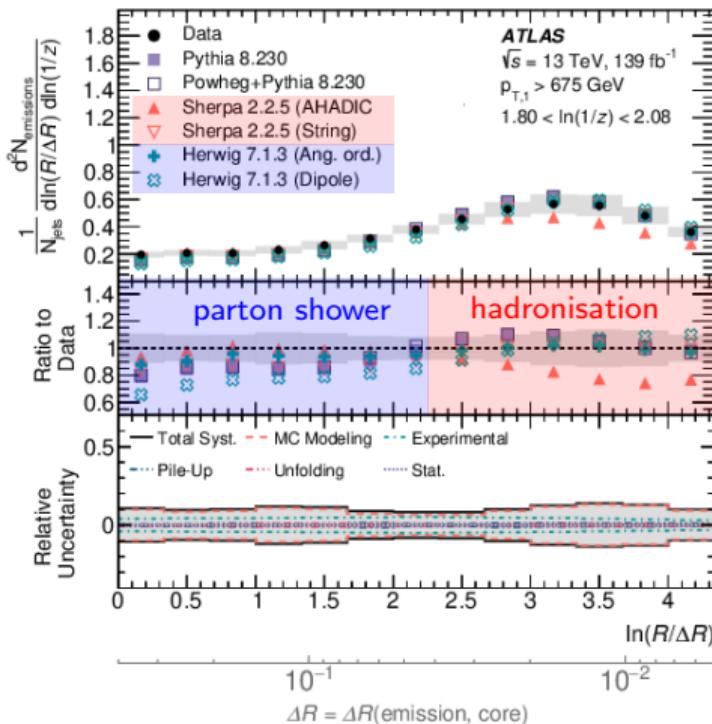
- Clear effect of the resummation
- Clear effect compared to NDL (incl. uncert)
- Can it lead to an α_s measurement?
- NNLO? N³DL?
- Could be done for primary plane multiplicity



Could be measured at LHC

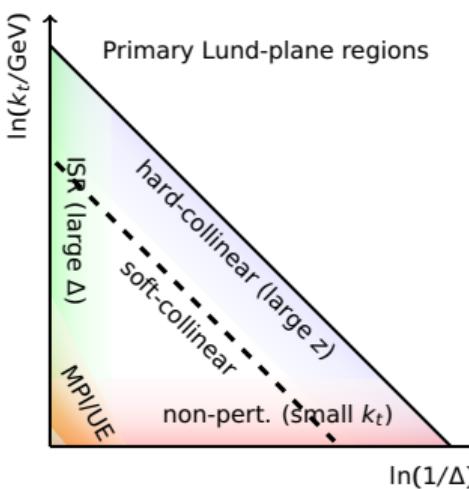
Application series #2: MC development

Obvious comparisons MC vs. data (1/2)

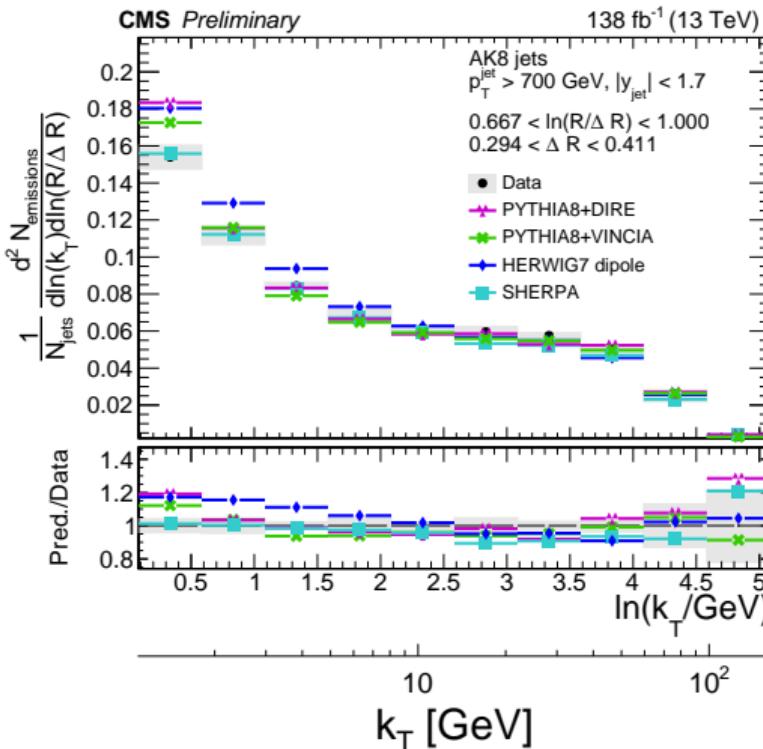


"standard" data vs. Monte Carlo comparison

Recall that different Lund regions are sensitive to different physics:



Obvious comparisons MC vs. data (2/2)

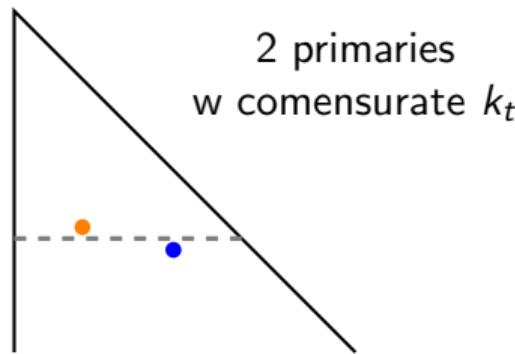
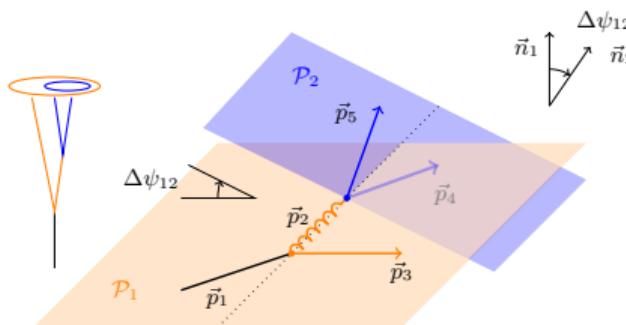


Large spread between Monte Carlo generators also observed by CMS

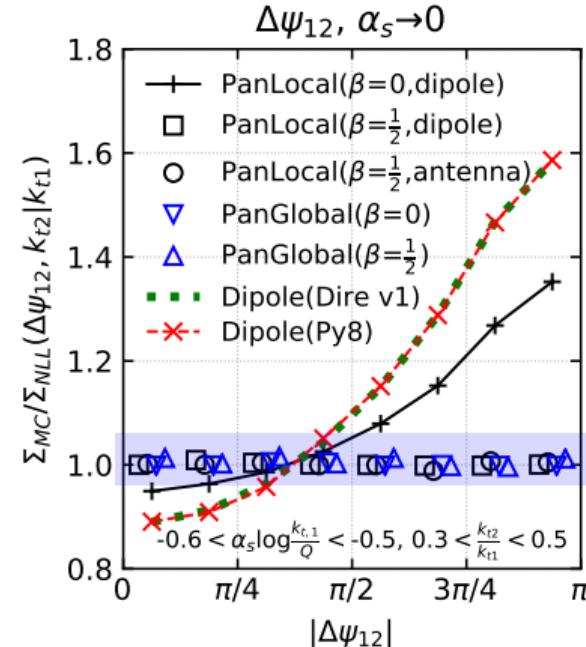
see CMS-PAS-SMP-22-007 for additional comparisons
(scales, tunes, ...)

Crafted observables: example $\Delta\Psi_{12}$

Azimuth between 1st and 2nd prim. declust.



[M.Dasgupta,F.Dreyer,K.Hamilton,P.Monni,G.Salam,GS,2002.11114]

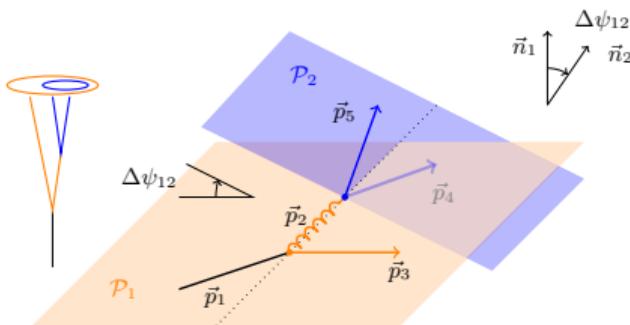


Expected ratio of 1 at NLL

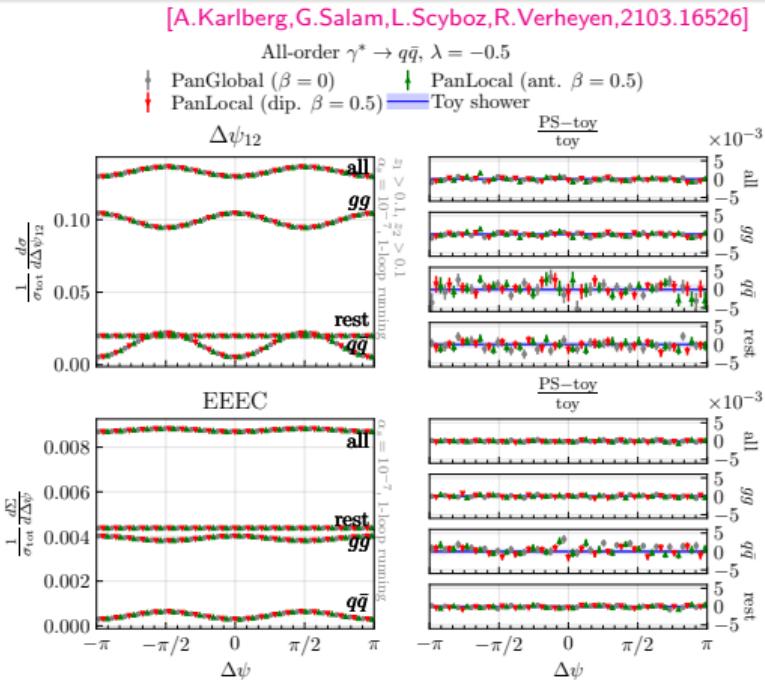
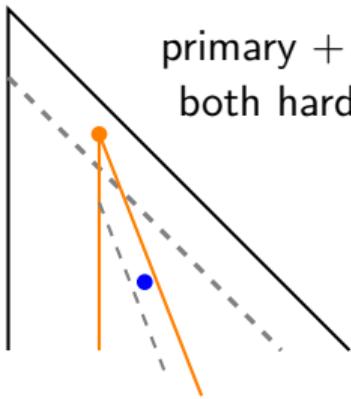
NLL failures for “standard” showers
“New” PanScales shower OK at NLL

Crafted observables: example $\Delta\Psi_{12}$

Azimuth between 1st and 2nd prim. declust.



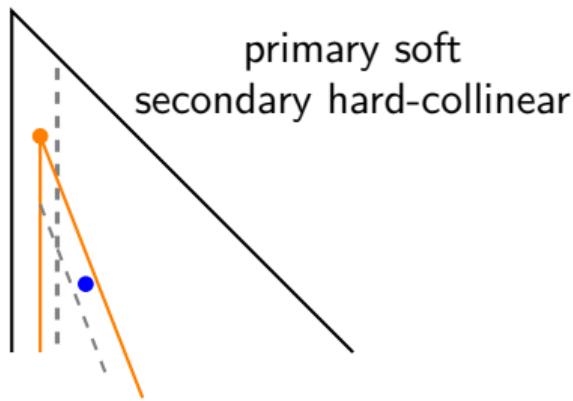
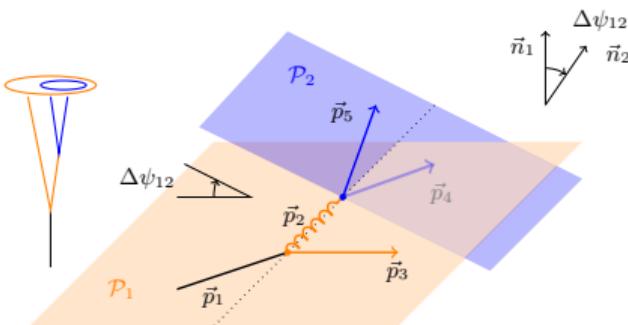
primary + secondary
both hard-collinear



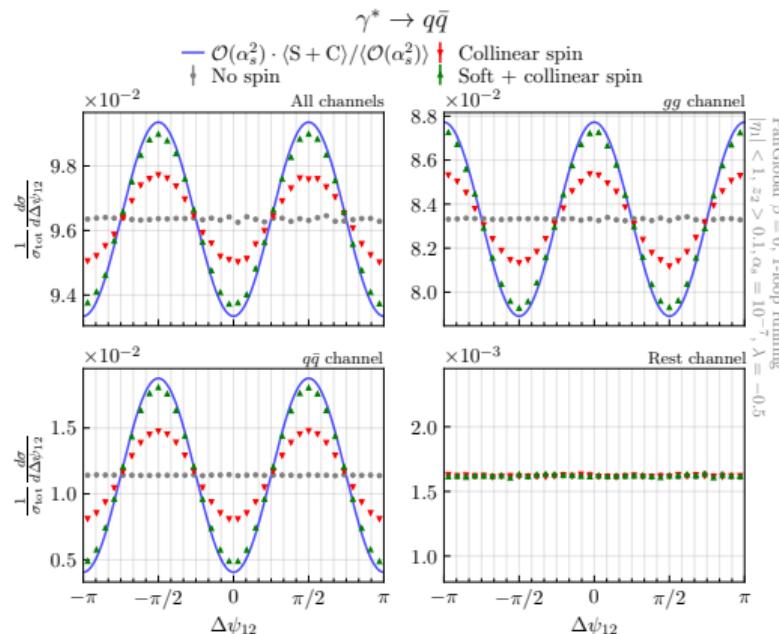
Sensitive to (collinear) spin
“New” PanScales shower have spin at NLL
agrees w EEEC from 2011.02492 (EEEC less sensitive)

Crafted observables: example $\Delta\Psi_{12}$

Azimuth between 1st and 2nd prim. declus.



[K.Hamilton,A.Karlberg,G.Salam,L.Scyboz,R.Verheyen,2111.01161]



Sensitive to (soft) spin
“New” PanScales shower have spin at NLL
first all-order result

Revisiting “standard” substructure observables [skip if needed]

- Equivalent to angularities/EECs:

$$S_\beta = \sum_{i \in \mathcal{L}} E_i e^{-\beta \eta_i}$$

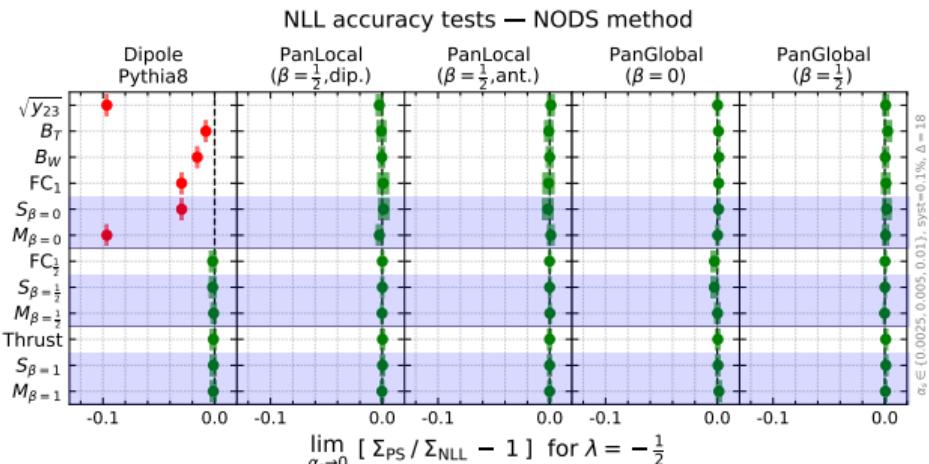
$$M_\beta = \max_{i \in \mathcal{L}} E_i e^{-\beta \eta_i}$$

- ✓ subjets allows for the use of “max”
- ✓ sum \neq max at NLL
- ✓ can be defined in pp

- N -subjettiness-like: sum excluding the N largest

$$\tau_N^{\beta, \text{Lund}} = \sum_{i \in A_N} E_i e^{-\beta \eta_i} \quad \text{with} \quad A_N = \operatorname{argmin}_{X \subset \mathcal{L}, |\mathcal{L} \setminus X| = N-1}$$

- ✓ Could replace sum by max (likely gaining a simpler resummation structure)
- ✓ Could be defined on the primary plane only



[M.Dasgupta,F.Dreyer,K.Hamilton,P.Monni,G.Salam,GS,2002.11114]

[K.Hamilton,R.Medves,G.Salam,L.Scyboz,GS,2011.10054]

Interesting Lund-plane(s)-based observables

Many Lund-based observables potentially interesting/measurable at the LHC

Lund densities

- already proven useful
- potential extensions (e.g. multiplicities)
- heavy quarks (e.g. b jets)
dead cone is a relatively small phase-space, but $b \sim$ light over large region
- other processes? $Z + j?$
top quarks?

$\Delta\Psi_{12}$

Sensitivity to log accuracy and spin correlations

More generally: probes correlations between 2 emissions

expect subleading effects
(compared to above asymptotic studies)

Others?

- Large flexibility to
- (re-)interpret existing tools
(grooming, angularities, N -subjettiness, ...)
 - design taylored observables
(measurements, MC constraints, heavy ions, ...)

- ① Lund diagrams have helped thinking about resummation and MCs
Now they can be reconstructed in practice

- They provide a view of a jet/event which mimics angular ordering
- They provide a separation between different physical effects

In particular, a k_t cut reduces non-perturbative (modelling) effects.

Might be worth considering for tagging e.g. via LundNet (performance v. modelling uncertainty)

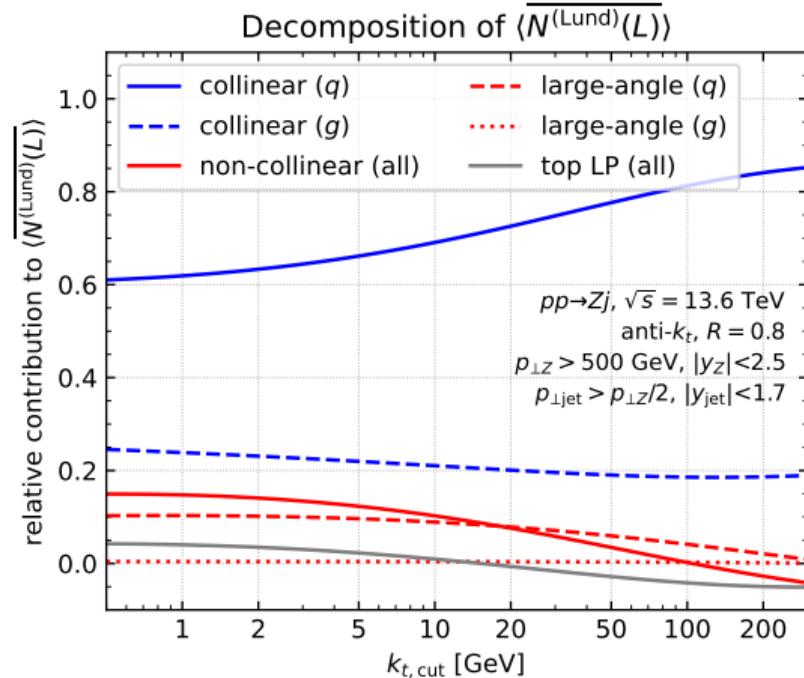
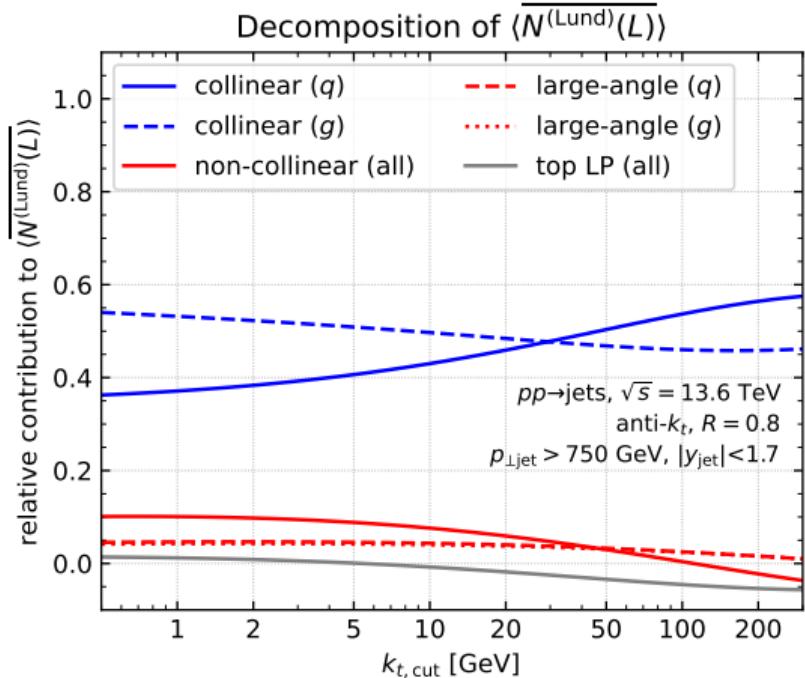
[F.Dreyer,H.Qu,arXiv:2012.08526]

- ② Broad spectrum of applications:

- Wide range of possible (p)QCD calculations
Main limitation: (non-global) clustering logs; can we apply grooming-like techniques?
- Large scope for crafting new observables ((p)QCD calculations, MC devel/validation)
- More connections to deep learning, heavy-ion collisions, ...

Backup

Quark-gluon contributions to multiplicity



Construct the Lund tree in practice: use the Cambridge(/Aachen) algorithm

Main idea: Cambridge(/Aachen) preserves angular ordering

e^+e^- collisions

- ① Cluster with Cambridge ($d_{ij} = 2(1 - \cos \theta_{ij})$)
- ② For each (de)-clustering $j \leftarrow j_1 j_2$:

$$\eta = -\ln \theta_{12}/2$$

$$k_t = \min(E_1, E_2) \sin \theta_{12}$$

$$z = \frac{\min(E_1, E_2)}{E_1 + E_2}$$

$\psi \equiv$ some azimuth,...

Jet in pp

- ① Cluster with Cambridge/Aachen ($d_{ij} = \Delta R_{ij}$)
- ② For each (de)-clustering $j \leftarrow j_1 j_2$:

$$\eta = -\ln \Delta R_{12}$$

$$k_t = \min(p_{t1}, p_{t2}) \Delta R_{12}$$

$$z = \frac{\min(p_{t1}, p_{t2})}{p_{t1} + p_{t2}}$$

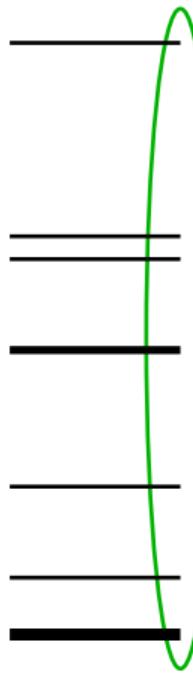
$\psi \equiv$ some azimuth,...

Primary Lund plane

Starting from the jet, de-cluster following the “hard branch” (largest E or p_t)

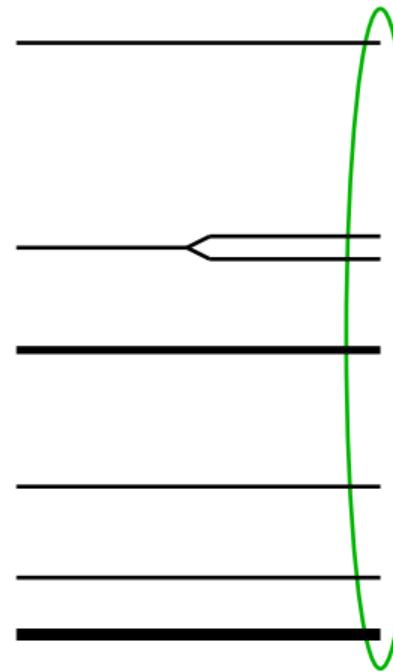
The Lund plane(s) representation: C/A (de)-clustering

use Cambridge/Aachen to iteratively recombine the closest pair



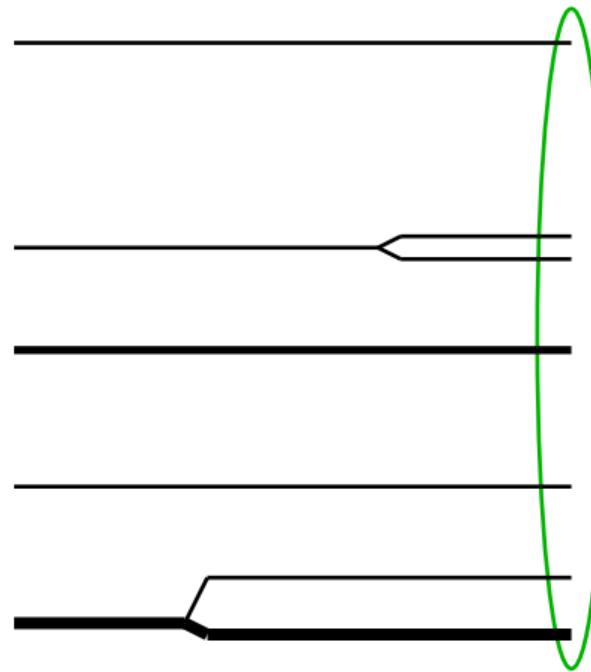
The Lund plane(s) representation: C/A (de)-clustering

use Cambridge/Aachen to iteratively recombine the closest pair



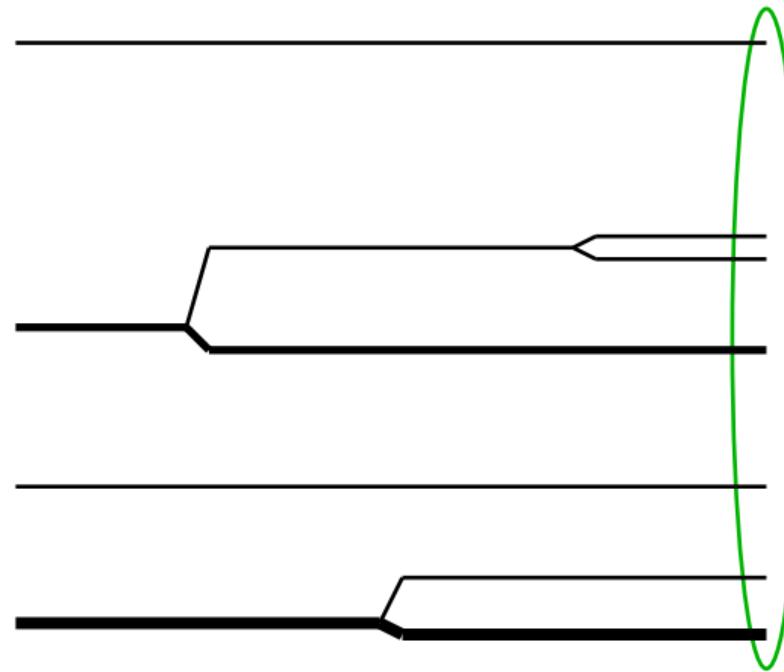
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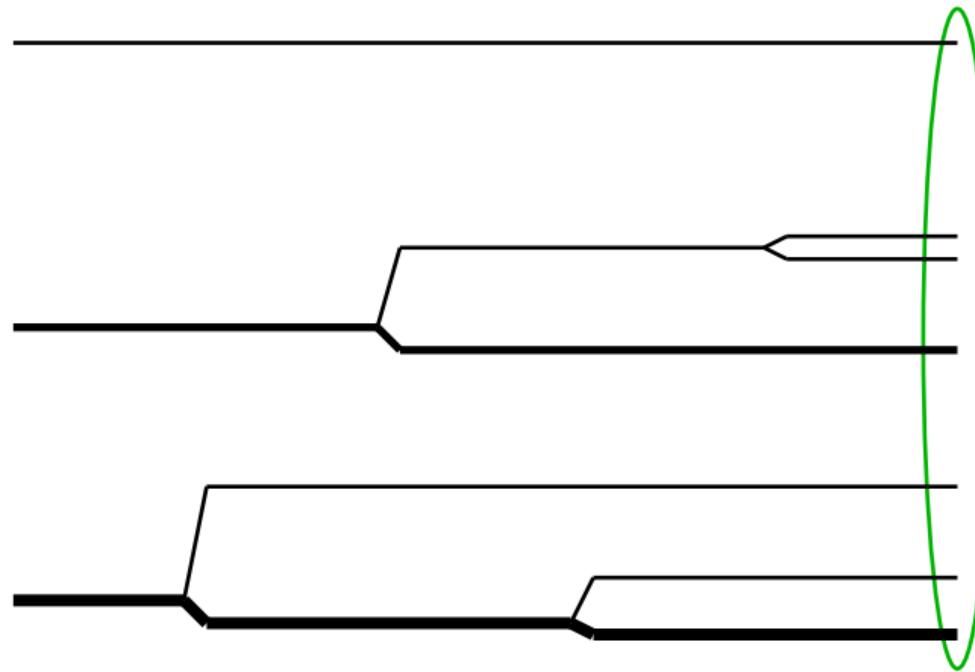
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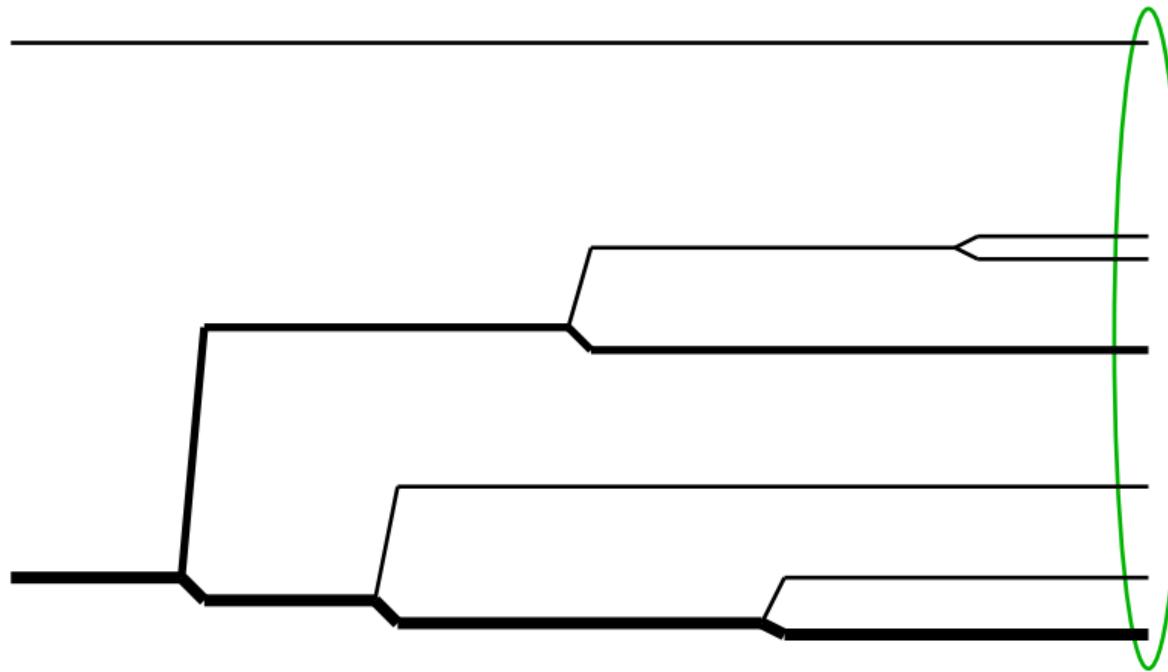
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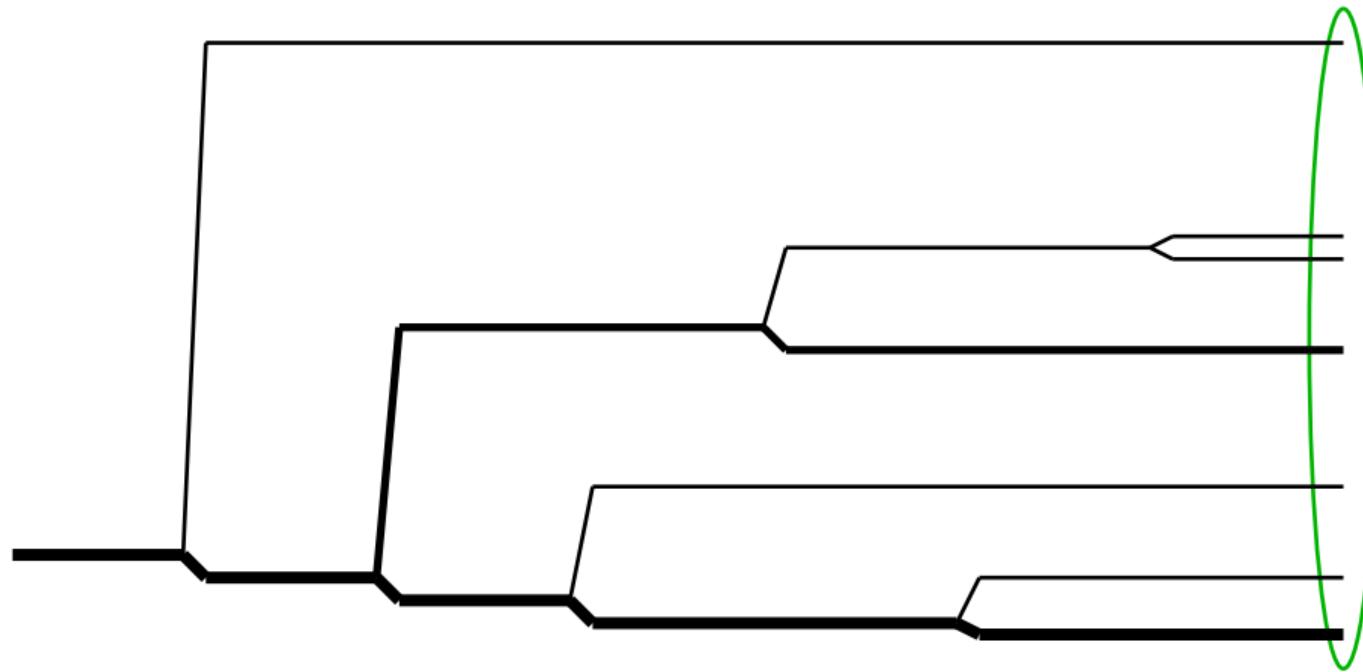
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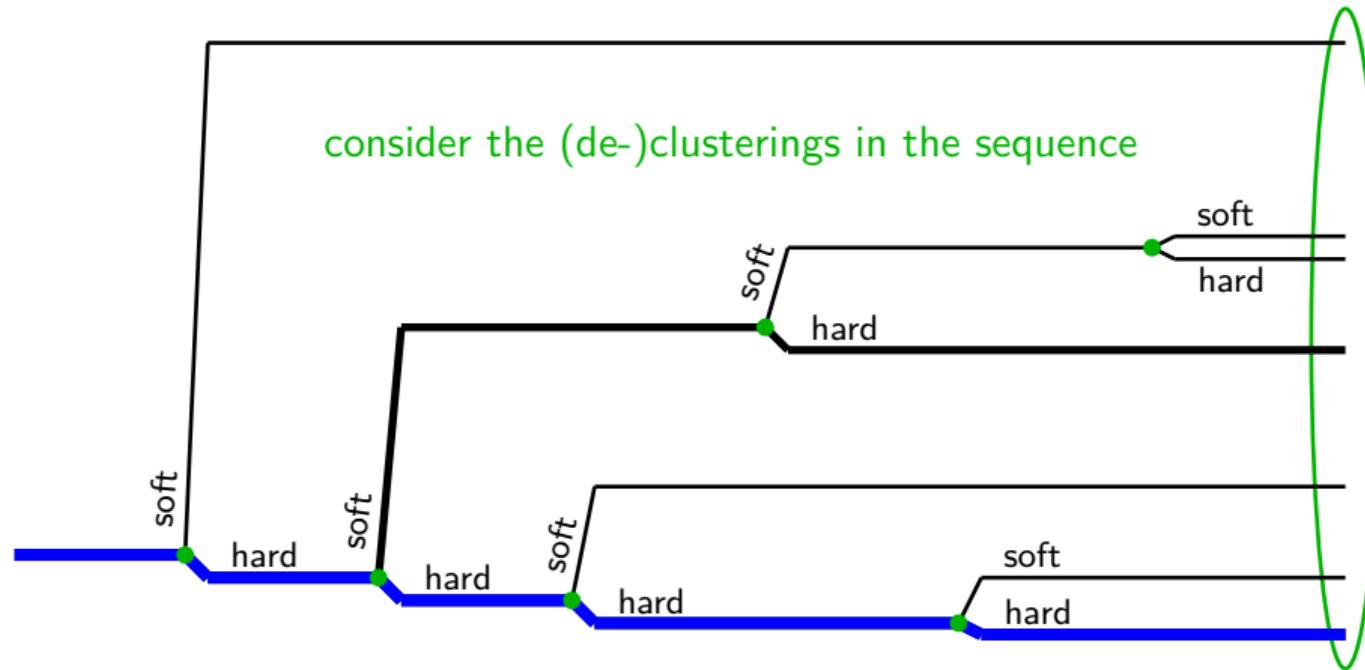
The Lund plane(s) representation: C/A (de)-clustering

use Cambridge/Aachen to iteratively recombine the closest pair



The Lund plane(s) representation: C/A (de)-clustering

use Cambridge/Aachen to iteratively recombine the closest pair



Note: conceptually the largest-energy (p_t or z) branch \equiv emissions from the “leading parton”