Constraining Higgs-Higgs-Z couplings in the 3HDM

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WG2- CP-violation and WG3 - Extended scalars, CERN, Jan 11, 2023

2HDM vs 3HDM gauge couplings

focus on

vector-vector-scalar, VVS

vector-scalar-scalar, VSS

(also quartic couplings VVSS)

Notation - Higgs basis

Consider first 3HDM, later “simplify” to 2HDM

Field notation:

$$\phi_i = e^{i\theta_i} \begin{pmatrix} \phi_i^+ \\ (v_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, 3$$

Higgs basis:

$$H_1 = \begin{pmatrix} G^+ \\ (v + \eta_1^{\mathrm{HB}} + iG_0)/\sqrt{2} \end{pmatrix}, \quad H_i = \begin{pmatrix} \varphi_i^{\mathrm{HB}\,+} \\ (\eta_i^{\mathrm{HB}} + i\chi_i^{\mathrm{HB}})/\sqrt{2} \end{pmatrix}, \quad i = 2, 3$$

Mass eigenstates:

$$h_i = R_{ij}\varphi_j^{\mathrm{HB}}$$

$$\varphi_i^{\mathrm{HB}} = \{\eta_1^{\mathrm{HB}}, \quad \eta_2^{\mathrm{HB}}, \quad \eta_3^{\mathrm{HB}}, \quad \chi_2^{\mathrm{HB}}, \quad \chi_3^{\mathrm{HB}}\}, \quad i = 1, \ldots 5.$$

Kinetic terms:

$$\mathcal{L}_{\mathrm{kin}} = \sum_{i=1,2,3} (D_\mu \phi_i)^\dagger (D^\mu \phi_i)$$

3HDM Gauge couplings

Cubic gauge-gauge-scalar part:

VVS

$$\mathcal{L}_{VVh} = \left(g m_W W^+_\mu W^{\mu -} + \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu \right) \sum_{i=1}^{5} \frac{e_i}{v} h_i$$

$$m_W = \tfrac{1}{2} g v$$

For the cubic gauge-scalar-scalar terms, we find

VSS   VSS+

$$\mathcal{L}_{Vhh} = -\frac{g}{2\cos\theta_W} \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{\lambda_{ij}}{v} (h_i \overleftrightarrow{\partial}_\mu h_j) Z^\mu + \frac{g}{2} \sum_{i=1}^{5} \sum_{k=1}^{2} \left[\frac{f_{ki}}{v} (h_k^+ \overleftrightarrow{\partial}_\mu h_i) W^{\mu -} + \text{h.c.} \right]$$

$$+ \left(ie A^\mu + \frac{ig\cos 2\theta_W}{2\cos\theta_W} Z^\mu \right) \sum_{k=1}^{2} (h_k^+ \overleftrightarrow{\partial}_\mu h_k^-)$$

2HDM Gauge couplings

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$$+ \left(ieA^\mu + \frac{ig\cos 2\theta_W}{2\cos\theta_W} Z^\mu \right) (h^+ \overleftrightarrow{\partial}_\mu h^-)$$

VVS & VSS Gauge couplings

Higgs basis:

$$\text{3HDM, VVS:} \quad e_i = vR_{i1}$$

$$\text{3HDM, VSS:} \quad \lambda_{ij} = v(R_{i2}R_{j4} + R_{i3}R_{j5}) - (i \leftrightarrow j)$$

$$\text{2HDM, VVS:} \quad e_i = vR_{i1}$$

$$\text{2HDM, VSS:} \quad \lambda_{ij} = v(R_{i2}R_{j3}) - (i \leftrightarrow j) = v\epsilon_{ijk}R_{k1} = \epsilon_{ijk}e_k$$

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3HDM, VVS: $e_i = vR_{i1}$

3HDM, VSS: $\lambda_{ij} = v(R_{i2}R_{j4} + R_{i3}R_{j5}) - (i \leftrightarrow j)$

2HDM, VVS: $e_i = vR_{i1}$

2HDM, VSS: $\lambda_{ij} = v(R_{i2}R_{j3}) - (i \leftrightarrow j) = v\epsilon_{ijk}R_{k1} = \epsilon_{ijk}e_k$

Note that this is given in terms of the VVS coupling!
Not true for the 3HDM!

2HDM Gauge couplings

For a 3×3 orthogonal matrix;

$$R_{i2}R_{j3} - R_{j2}R_{i3} = \epsilon_{ijk}R_{k1} = \epsilon_{ijk}e_k/v$$

Example decomposition:

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

Pick $i = 1$, $j = 2$:

$$\begin{aligned} R_{12}R_{23} - R_{22}R_{13} &= (s_1c_2)(c_2s_3) - (c_1c_3 - s_1s_2s_3)s_2 \\ &= s_1c_2^2s_3 - c_1s_2c_3 + s_1s_2^2s_3 = s_1s_3 - c_1s_2c_3 = R_{31} \end{aligned}$$

$$i.e., \quad \lambda_{ij} = \epsilon_{ijk}e_k$$

3HDM Rotation matrix

$h_i VV$ vertex $e_i = vR_{i1}$

R can be constructed from 10 independent angles, two constructions:

$$1: \quad R = \mathcal{O}^{(12)}\mathcal{O}^{(13)}\mathcal{O}^{(14)}\mathcal{O}^{(15)}\mathcal{O}^{(23)}\mathcal{O}^{(24)}\mathcal{O}^{(25)}\mathcal{O}^{(34)}\mathcal{O}^{(35)}\mathcal{O}^{(45)}$$

$$2: \quad R = \mathcal{O}^{(45)}\mathcal{O}^{(35)}\mathcal{O}^{(34)}\mathcal{O}^{(25)}\mathcal{O}^{(24)}\mathcal{O}^{(23)}\mathcal{O}^{(15)}\mathcal{O}^{(14)}\mathcal{O}^{(13)}\mathcal{O}^{(12)}$$

with non-zero elements:

$$\mathcal{O}^{(ij)}_{ii} = \cos\theta_{ij}, \qquad \mathcal{O}^{(ij)}_{ij} = \sin\theta_{ij}$$
$$\mathcal{O}^{(ij)}_{ji} = -\sin\theta_{ij}, \qquad \mathcal{O}^{(ij)}_{ii} = \cos\theta_{ij}$$
$$\mathcal{O}^{(ij)}_{kk} = 1 \text{ for } k \neq i, j$$

By construction “1” above:

$$R_{11} = \cos\theta_{12}\cos\theta_{13}\cos\theta_{14}\cos\theta_{15}$$
$$R_{21} = -\sin\theta_{12}\cos\theta_{13}\cos\theta_{14}\cos\theta_{15}$$
$$R_{31} = -\sin\theta_{13}\cos\theta_{14}\cos\theta_{15}$$
$$R_{41} = -\sin\theta_{14}\cos\theta_{15}$$
$$R_{51} = -\sin\theta_{15}$$

Only four angles are needed for e_1, e_2, e_3, e_4, e_5, independent of other six angles

3HDM Rotation matrix

Modulo the factor v, the e_i are "embedded" in R as follows:

$$R = \begin{pmatrix} e_1 & x & x & x & x \\ e_2 & x & x & x & x \\ e_3 & x & x & x & x \\ e_4 & x & x & x & x \\ e_5 & x & x & x & x \end{pmatrix}$$

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whereas λ_{ij} involves the remaining elements of the rows i and j:

$$\lambda_{ij} = (R_{i2}R_{j4} + R_{i3}R_{j5}) - (i \leftrightarrow j)$$

$$R = \begin{pmatrix} e_1 & \boxed{x \quad x \quad x \quad x} \\ e_2 & x \quad x \quad x \quad x \\ e_3 & x \quad x \quad x \quad x \\ e_4 & x \quad x \quad x \quad x \\ e_5 & x \quad x \quad x \quad x \end{pmatrix}$$

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whereas λ_{ij} involves the remaining elements of the rows i and j:

illustrate for $i = 1$, $j = 2$:

$$R = \begin{pmatrix} e_1 & R_{12} & R_{13} & R_{14} & R_{15} \\ e_2 & R_{22} & R_{23} & R_{24} & R_{25} \\ e_3 & x & x & x & x \\ e_4 & x & x & x & x \\ e_5 & x & x & x & x \end{pmatrix}$$

$\eta_1^{\rm HB}, \quad \eta_2^{\rm HB}, \quad \eta_3^{\rm HB}, \quad \chi_2^{\rm HB}, \quad \chi_3^{\rm HB}$

$$\lambda_{12} = (R_{12}R_{24} - R_{22}R_{14})$$

"cross product"

blue × red

3HDM Rotation matrix

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$$R = \begin{pmatrix} e_1 & x & x & x & x \\ e_2 & x & x & x & x \\ e_3 & x & x & x & x \\ e_4 & x & x & x & x \\ e_5 & x & x & x & x \end{pmatrix}$$

whereas λ_{ij} involves the remaining elements of the rows i and j:

illustrate for $i = 1$, $j = 2$:

$$\lambda_{12} = (R_{12}R_{24} - R_{22}R_{14}) + (R_{13}R_{25} - R_{23}R_{15})$$

"cross product"
blue × red

$$R = \begin{pmatrix} e_1 & R_{12} & R_{13} & R_{14} & R_{15} \\ e_2 & R_{22} & R_{23} & R_{24} & R_{25} \\ e_3 & x & x & x & x \\ e_4 & x & x & x & x \\ e_5 & x & x & x & x \end{pmatrix}$$

$\eta_1^{\mathrm{HB}}, \quad \eta_2^{\mathrm{HB}}, \quad \eta_3^{\mathrm{HB}}, \quad \chi_2^{\mathrm{HB}}, \quad \chi_3^{\mathrm{HB}}$

numerology

- 10 angles
- 5 e_i [four angles + v]
- 10 non-zero λ_{ij} $[(5 \times 4)/2 - 5]$

The e_i and λ_{ij} are not all independent!

Constraints:

- $\sum_i e_i^2 = v^2$
- $\sum_j \lambda_{ij}^2 = v^2 - e_i^2$
- $\sum_{i,j} \lambda_{ij}^2 = v^2(5-1) = 4v^2$

non-trivial relations

Relation 1
$$e_i e_j = -\sum_k \lambda_{ik}\lambda_{jk}$$

Relation 2
$$e_i = -\frac{1}{8v}\sum_{j,k,l,m} \epsilon_{ijklm}\lambda_{jk}\lambda_{lm}$$

Relation 3
$$\lambda_{ij} = -\frac{1}{2v}\sum_{k,l,m} \epsilon_{ijklm} e_k \lambda_{lm}$$

May write seven λ_{ij} in terms of e's and three other λ's. Choose

$$\lambda_{12} = a\lambda_{34} + b\lambda_{35} + c\lambda_{45},$$

and similarly for λ_{13}, λ_{14}, λ_{15}, λ_{23}, λ_{24}, λ_{35}

non-trivial relations

Adding the squares (subject to the normalization), we arrive at

$$
\begin{aligned}
&(v^2 - e_5^2)\lambda_{34}^2 + (v^2 - e_4^2)\lambda_{35}^2 + (v^2 - e_3^2)\lambda_{45}^2 \\
&+ 2e_4e_5\lambda_{34}\lambda_{35} - 2e_3e_5\lambda_{34}\lambda_{45} + 2e_3e_4\lambda_{35}\lambda_{45} - (e_1^2 + e_2^2)v^2 = 0
\end{aligned}
$$

$\lambda_{34},\ \lambda_{35},\ \lambda_{45}$ located on ellipsoid

May solve quadratic equation for one of them

non-trivial relations

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$$(v^2 - e_5^2)\lambda_{34}^2 + (v^2 - e_4^2)\lambda_{35}^2 + (v^2 - e_3^2)\lambda_{45}^2$$
$$+ 2e_4e_5\lambda_{34}\lambda_{35} - 2e_3e_5\lambda_{34}\lambda_{45} + 2e_3e_4\lambda_{35}\lambda_{45} - (e_1^2 + e_2^2)v^2 = 0$$

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Bottom line: In addition to five e_i need only TWO λ_{ij} to describe all λ_{ij}

non-trivial relations

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$$(v^2 - e_5^2)\lambda_{34}^2 + (v^2 - e_4^2)\lambda_{35}^2 + (v^2 - e_3^2)\lambda_{45}^2$$
$$+ 2e_4 e_5 \lambda_{34}\lambda_{35} - 2e_3 e_5 \lambda_{34}\lambda_{45} + 2e_3 e_4 \lambda_{35}\lambda_{45} - (e_1^2 + e_2^2)v^2 = 0$$

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Bottom line: In addition to five e_i need only TWO λ_{ij} to describe all λ_{ij}

4 angles (five e_i) 2 functions of 10 angles (TWO λ_{ij})

non-trivial relations

Consider CP conservation in the 3HDM
5 neutral states: 3 even, 2 odd.

From the "non-trivial" relations, it follows that

$$\boxed{e_i = e_j = \lambda_{ij} = 0}$$

is a sufficient condition for no CP violation
in couplings between gauge bosons and neutral scalars

Alignment

Alignment:

$$e_1 = v \quad e_2 = e_3 = e_4 = e_5 = 0$$

Then

$$\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = 0$$

The remaining ones are pairwise equal:

$$\lambda_{23} = -\lambda_{45}, \quad \lambda_{24} = \lambda_{35}, \quad \lambda_{25} = -\lambda_{34}$$

We also have

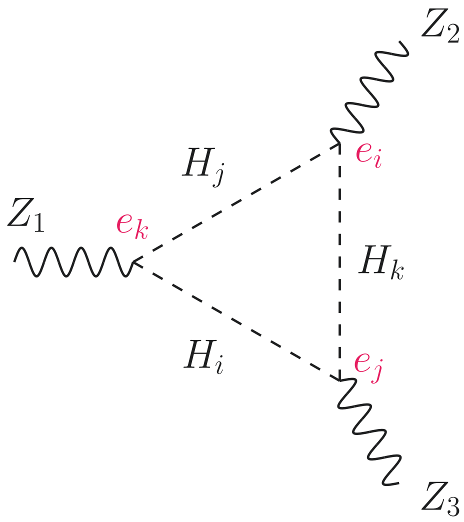
$$\lambda_{34}^2 + \lambda_{35}^2 + \lambda_{45}^2 = v^2$$

The moduli are located on a sphere

Solving for one, we are left with two free parameters

CP violation and alignment

In a CP-violating 2HDM, all pairs of neutral scalars couple to the Z, allowing the triangle diagram



The existence of these couplings induces a CP-violating amplitude,

CP violation and alignment

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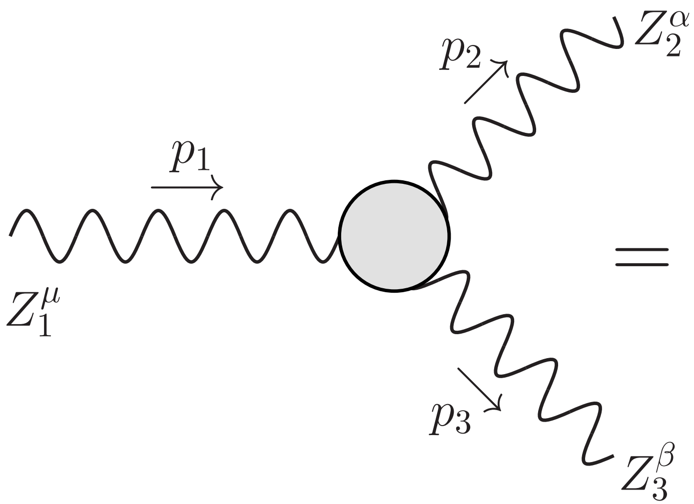


$$= \quad ie\Gamma^{\mu\alpha\beta}$$

$$e\Gamma^{\alpha\beta\mu}_{ZZZ} = ie\frac{p_1^2 - M_Z^2}{M_Z^2}\left[f_4^Z(p_1^\alpha g^{\mu\beta} + p_1^\beta g^{\mu\alpha}) + f_5^Z\epsilon^{\mu\alpha\beta\rho}\ell_\rho\right]$$

violates CP

CP violation and alignment

Also other diagrams, but importantly

$$f_4^Z \text{ is proportional to the invariant } \mathrm{Im}\, J_2 \propto e_1 e_2 e_3$$

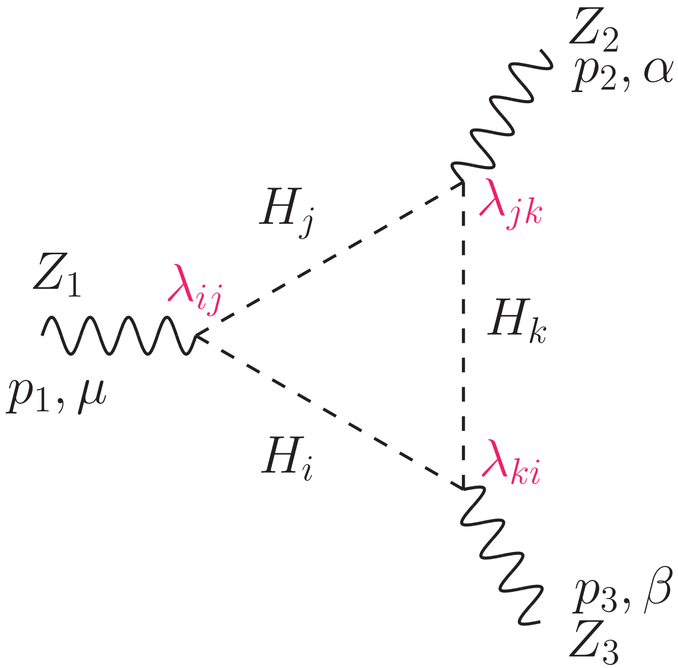
CP-violating

In the alignment limit, two of the e_i vanish, the ZZZ amplitude vanishes

$$e_i \to v \quad \Rightarrow \quad e_j, e_k \to 0 \qquad \mathrm{Im}\, J_2 \to 0$$

CP violation and alignment

In a 3HDM



contributions proportional to $\lambda_{ij}\lambda_{jk}\lambda_{ki}$

This does not vanish in the alignment limit!

CP violation and alignment

Alignment limit

Let h_i be aligned, $e_i \to v$, $e_j \to 0$ for $j \neq i$
Consequence: $\lambda_{ij} \to 0$, but λ_{jk} unconstrained, for $j, k \neq i$

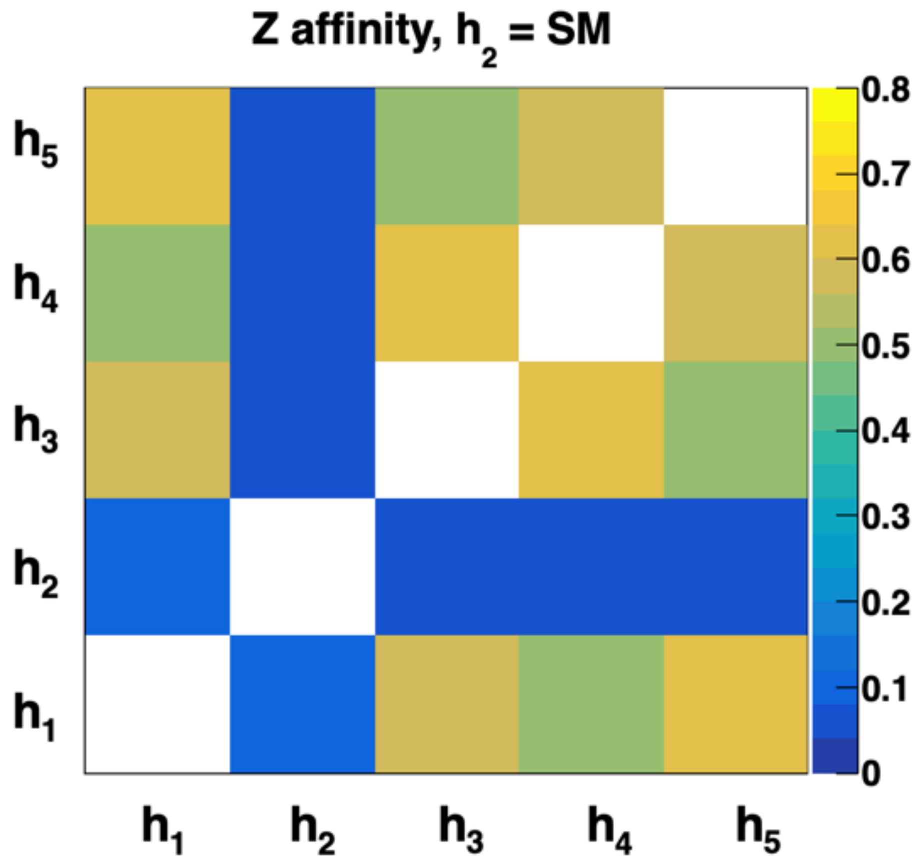
Example, Weinberg potential $\quad Z_2 \times Z_2$ symmetric:

$$V = V_2 + V_4$$

$$\begin{aligned}
V_2 &= -[m_{11}(\phi_1^\dagger \phi_1) + m_{22}(\phi_2^\dagger \phi_2) + m_{33}(\phi_3^\dagger \phi_3)] \\
V_4 &= V_0 + V_{\rm ph}, \\
V_0 &= \lambda_{11}(\phi_1^\dagger \phi_1)^2 + \lambda_{12}(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_{13}(\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3) + \lambda_{22}(\phi_2^\dagger \phi_2)^2 \\
&\quad + \lambda_{23}(\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + \lambda_{33}(\phi_3^\dagger \phi_3)^2 \\
&\quad + \lambda'_{12}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \lambda'_{13}(\phi_1^\dagger \phi_3)(\phi_3^\dagger \phi_1) + \lambda'_{23}(\phi_2^\dagger \phi_3)(\phi_3^\dagger \phi_2) \\
V_{\rm ph} &= \lambda_1(\phi_2^\dagger \phi_3)^2 + \lambda_2(\phi_3^\dagger \phi_1)^2 + \lambda_3(\phi_1^\dagger \phi_2)^2 + \text{ h.c.}
\end{aligned}$$

CP violation and alignment

λ_{ij}/v



h_2 aligned

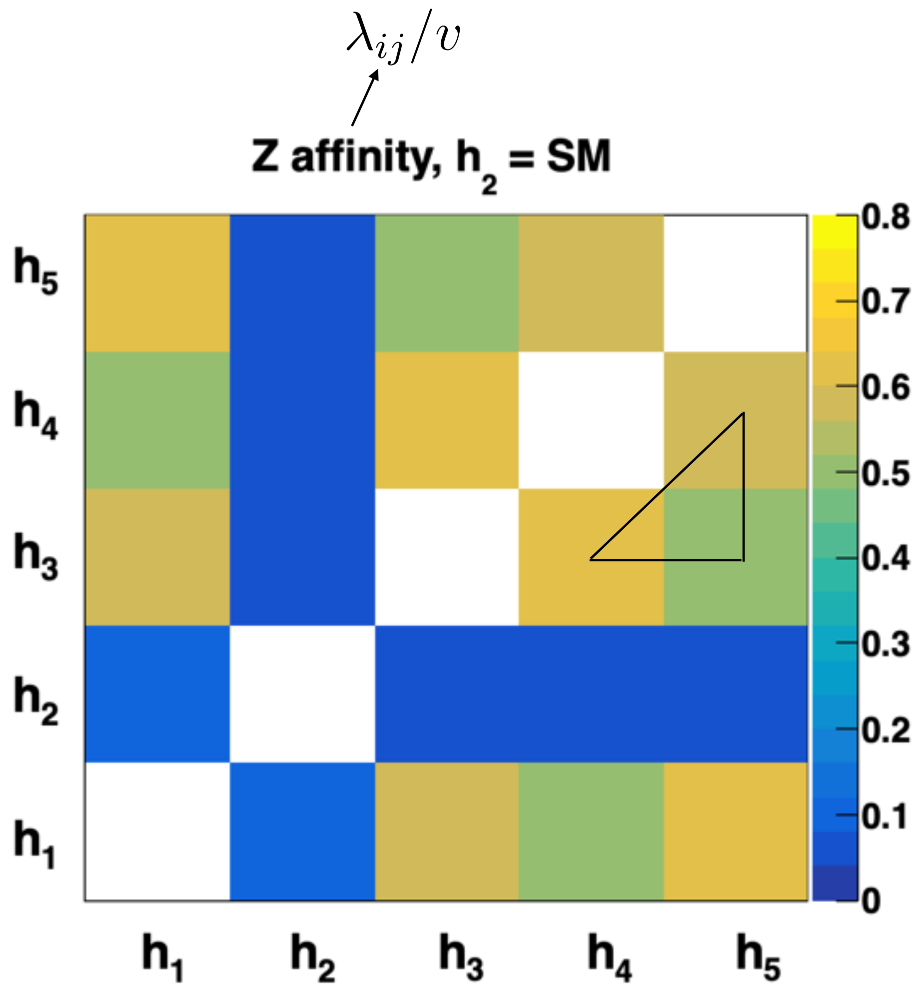
Figure from parameter scan in 2209.06499

CP violation and alignment

h_2 aligned

for example:

$$\lambda_{34}\lambda_{45}\lambda_{53} \neq 0$$



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Summary

- vector (gauge)-scalar couplings:
  - VVS (WWh_i, ZZh_i): e_i
  - VSS (h_ih_jZ): λ_{ij}
  - VSS ($h_ih_k^+W^-$): d_{ki}
- determined by potential, relations among them
- 2HDM: $\lambda_{ij} = \epsilon_{ijk}e_k \quad (*)$
- 2HDM: ZZZ amplitude CP conserving in Alignment limit
- 3HDM: Relation $(*)$ not valid
- 3HDM: ZZZ amplitude CP violating also in Alignment limit

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work in progress