STATE OF THE ART IN PERTURBATIVE QCD FOR LHC

Alexander Huss

CERN

34th **Rencontres de Blois** on *"Particle Physics and Cosmology"* — May 16th 2023

STATE OF THE ART IN PERTURBATIVE QCD FOR LHC*

* focus on *recent* results that are *representative* for on-going *progress* and *relevant* for the LHC *(personal selection)*

Alexander Huss

CERN

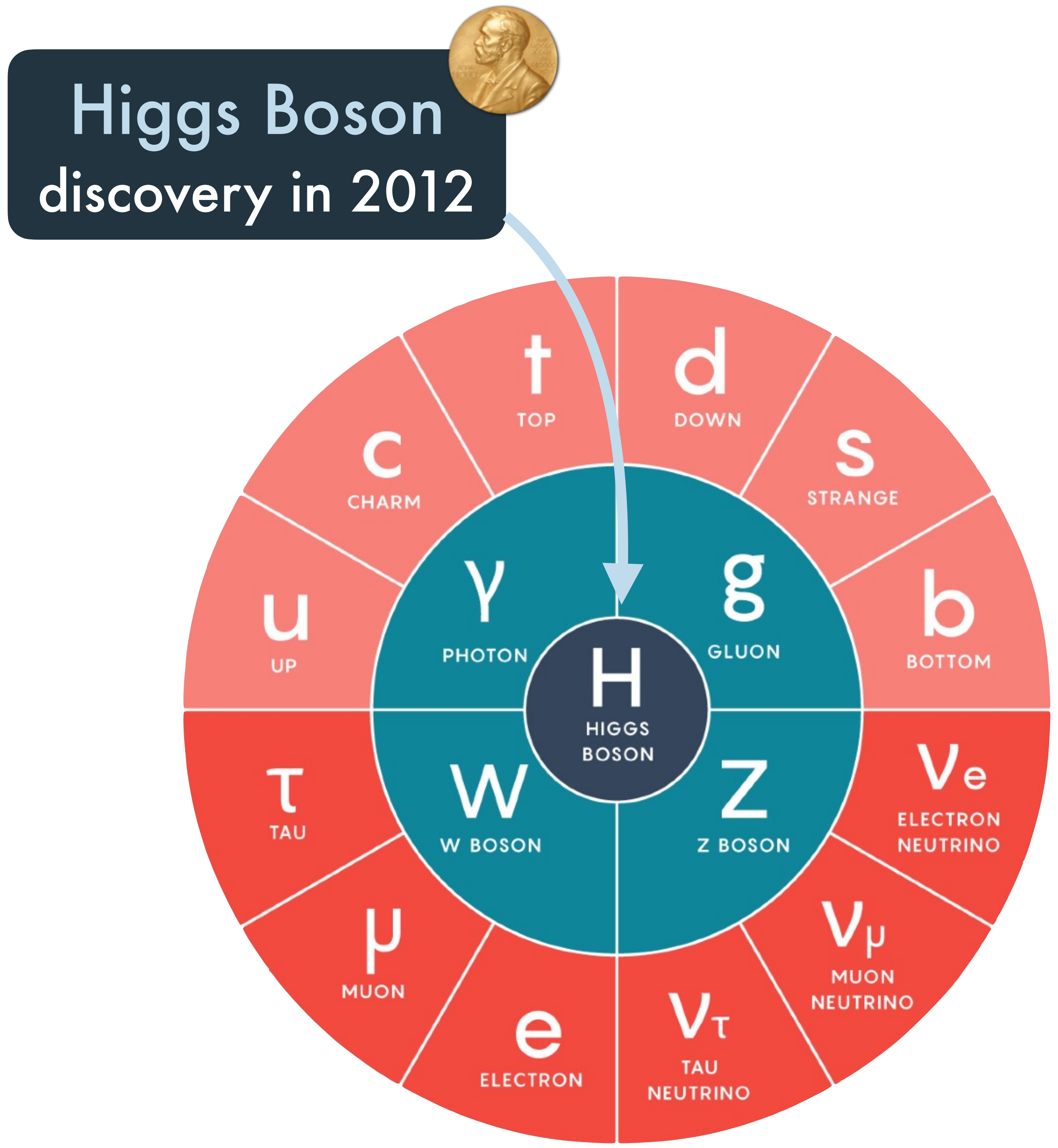
34th **Rencontres de Blois** on *"Particle Physics and Cosmology"* — May 16th 2023

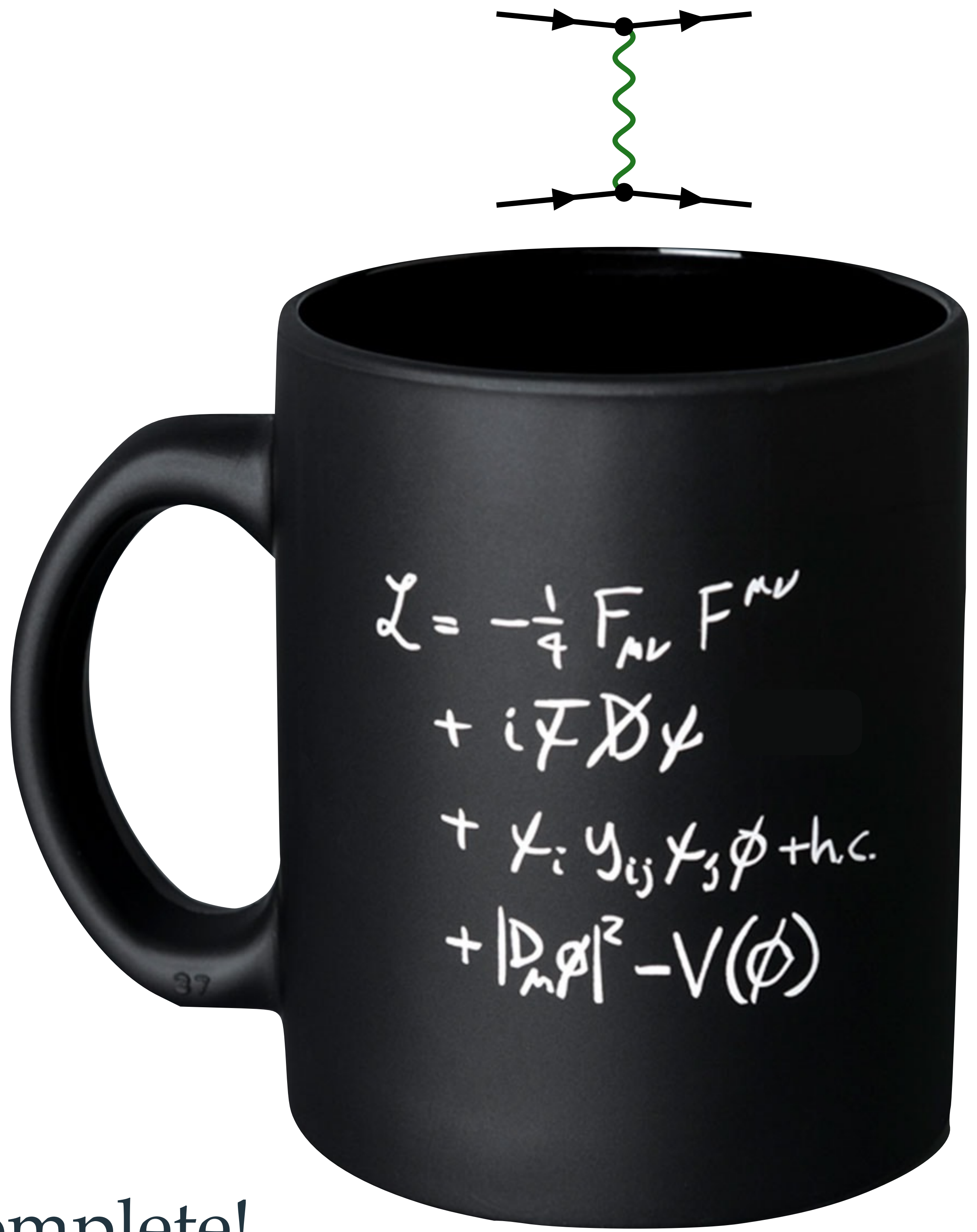
The Standard Model of Particle Physics

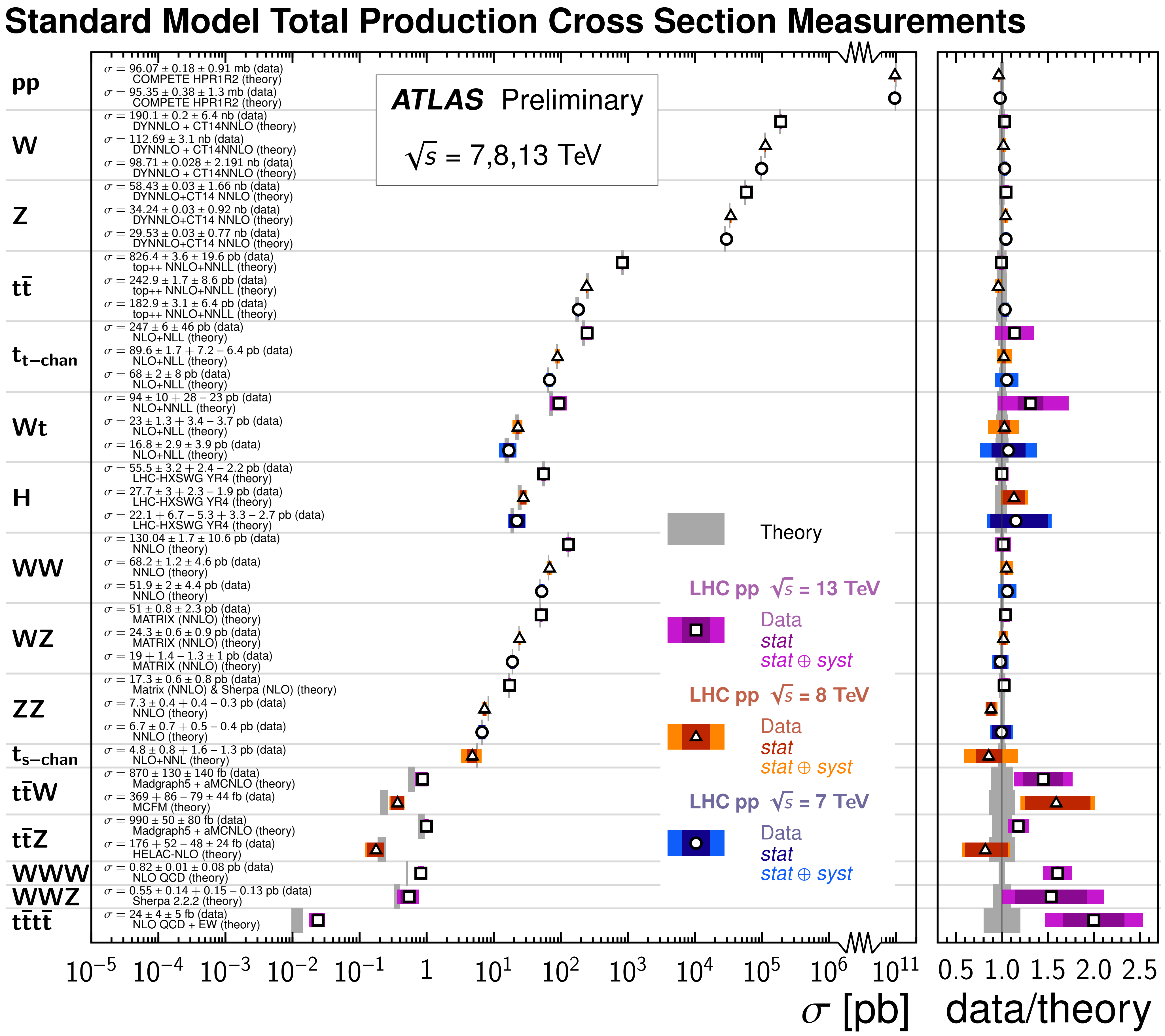
Particles

Interactions

A Remarkable Success Story...





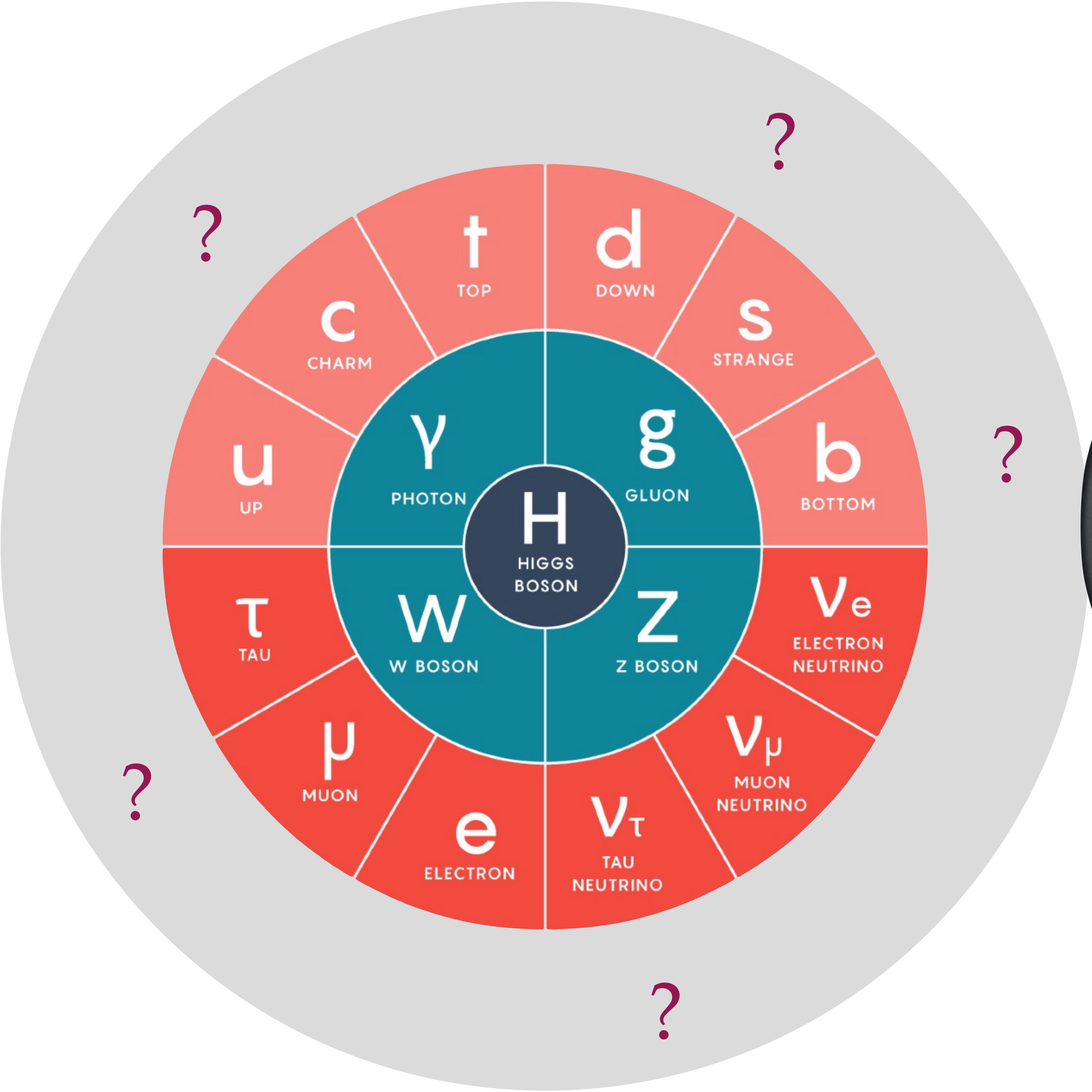


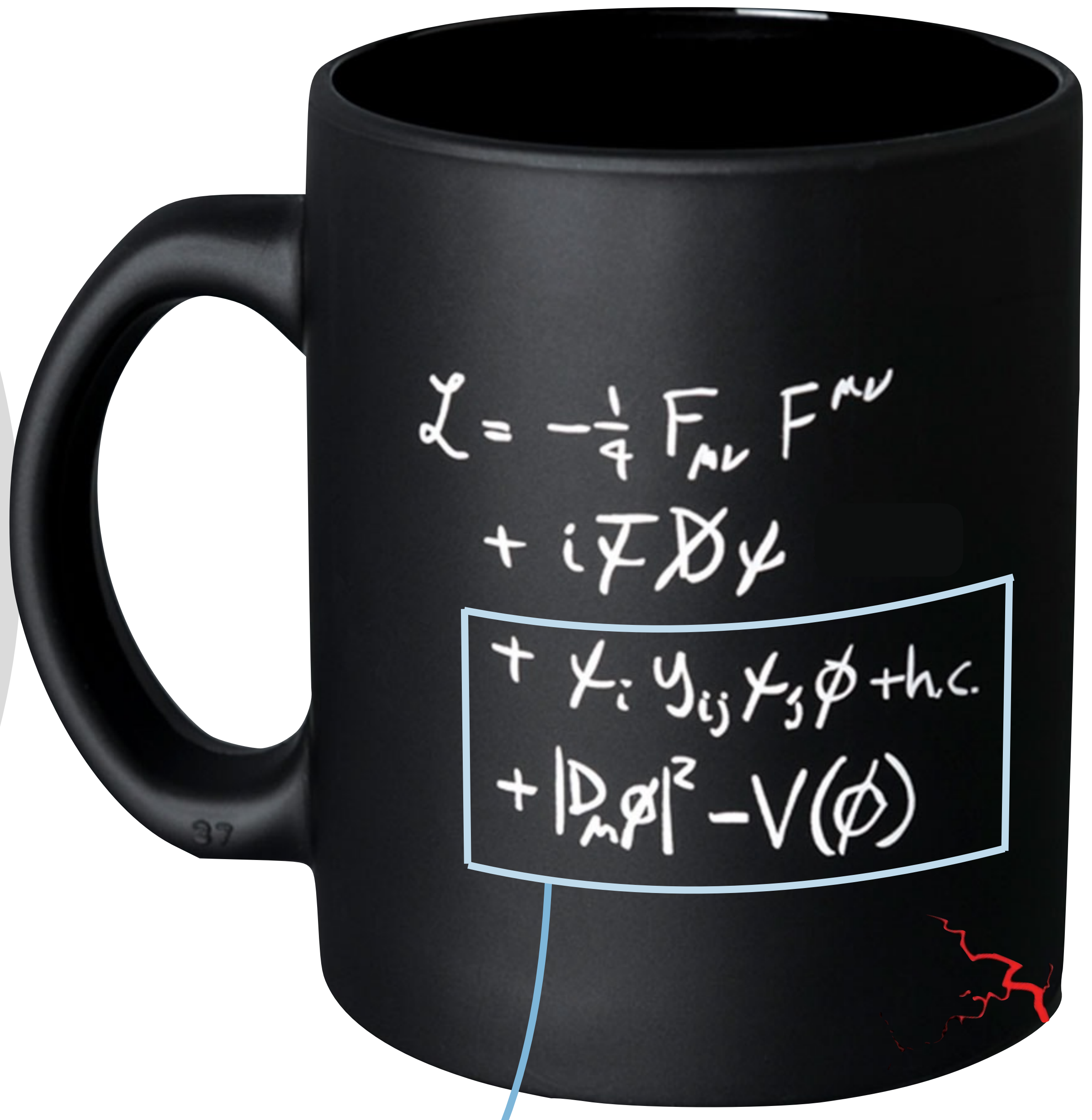
SM is now complete!

- no free parameters (M_{H} last)
- fully predictive theory

THE STANDARD MODEL IS NOT ENOUGH

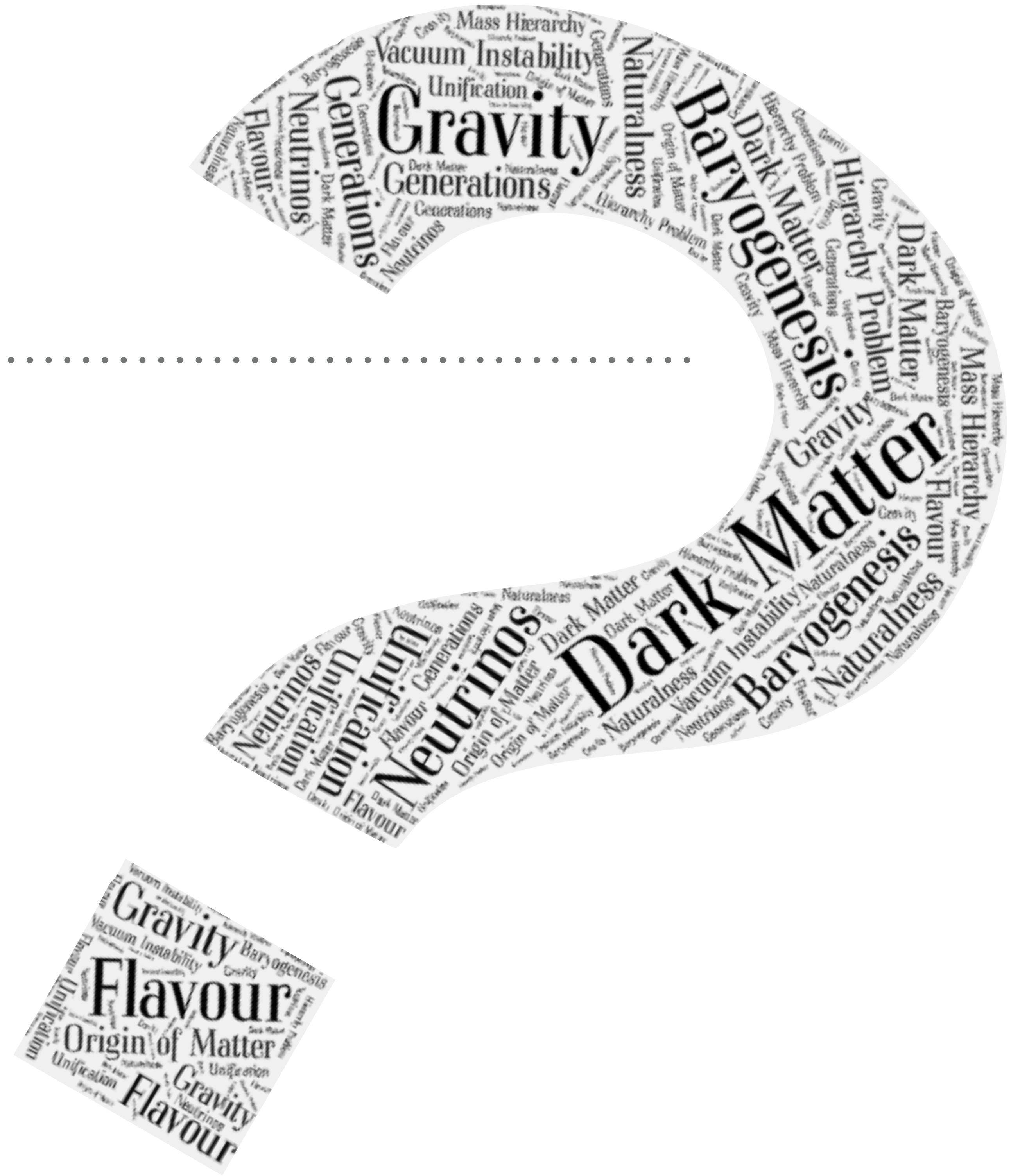
Particles ⇝ Interactions





Only 10 years old!

✶ some tensions
(B decays, $g_\mu - 2$, …)



precision phenomenology:

- *"standard candles"*
  - ↪ measured precisely
    & predicted reliably
- scrutinise the *Higgs sector*
  - ↪ first & only elementary scalar
    ⇝ sensitive to New Physics

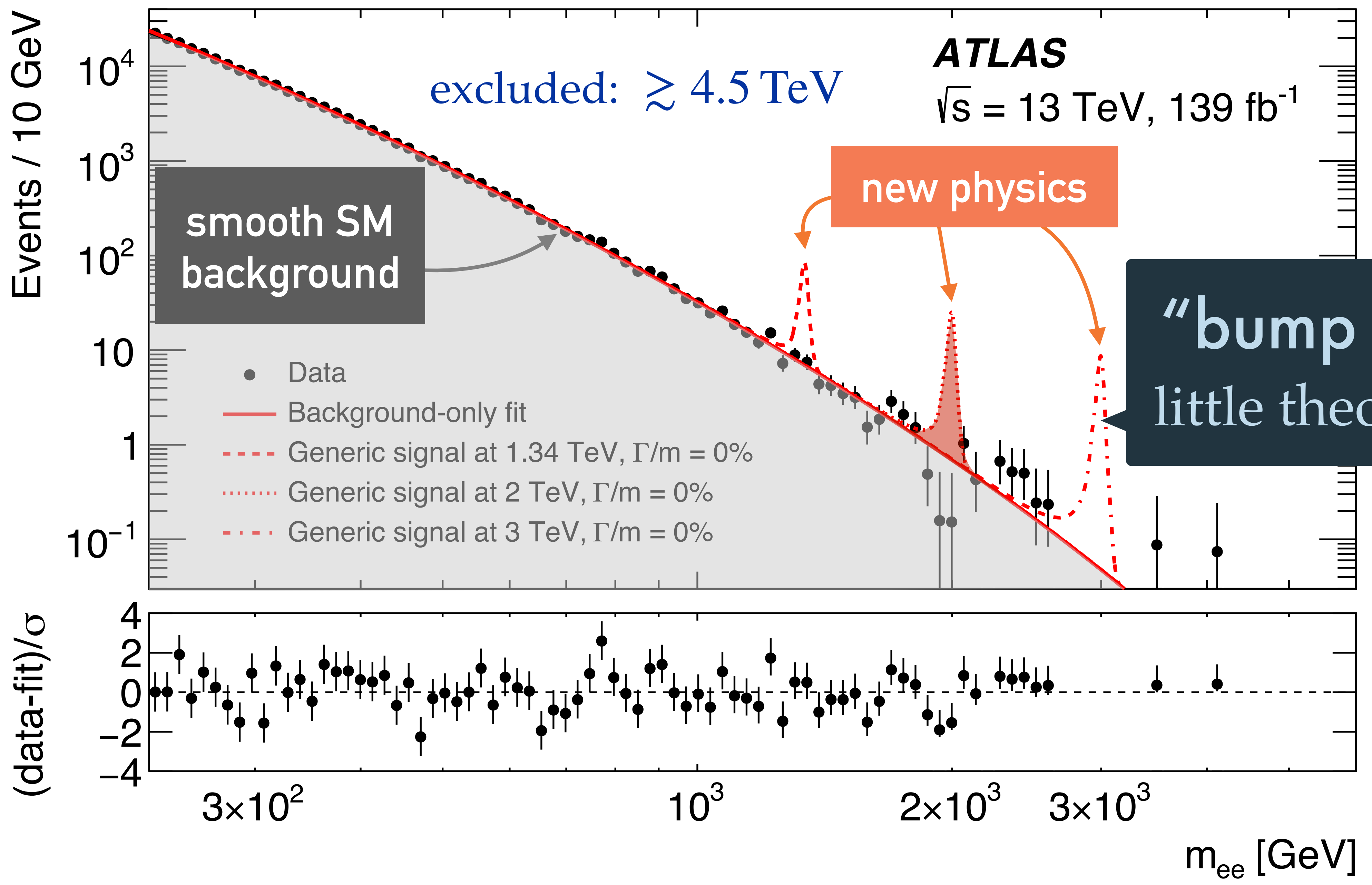
talks by S. Chang & A. Nogamova

New Physics Searches

no striking signals so far

Direct

$Z' \to \ell^+\ell^-$



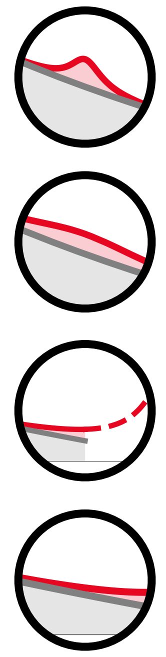
smooth SM background

new physics

"bump hunting"
little theory needed

Hiding in small & subtle effects?

- interaction weak
- wide resonance
- too heavy
- shape distortion
- …

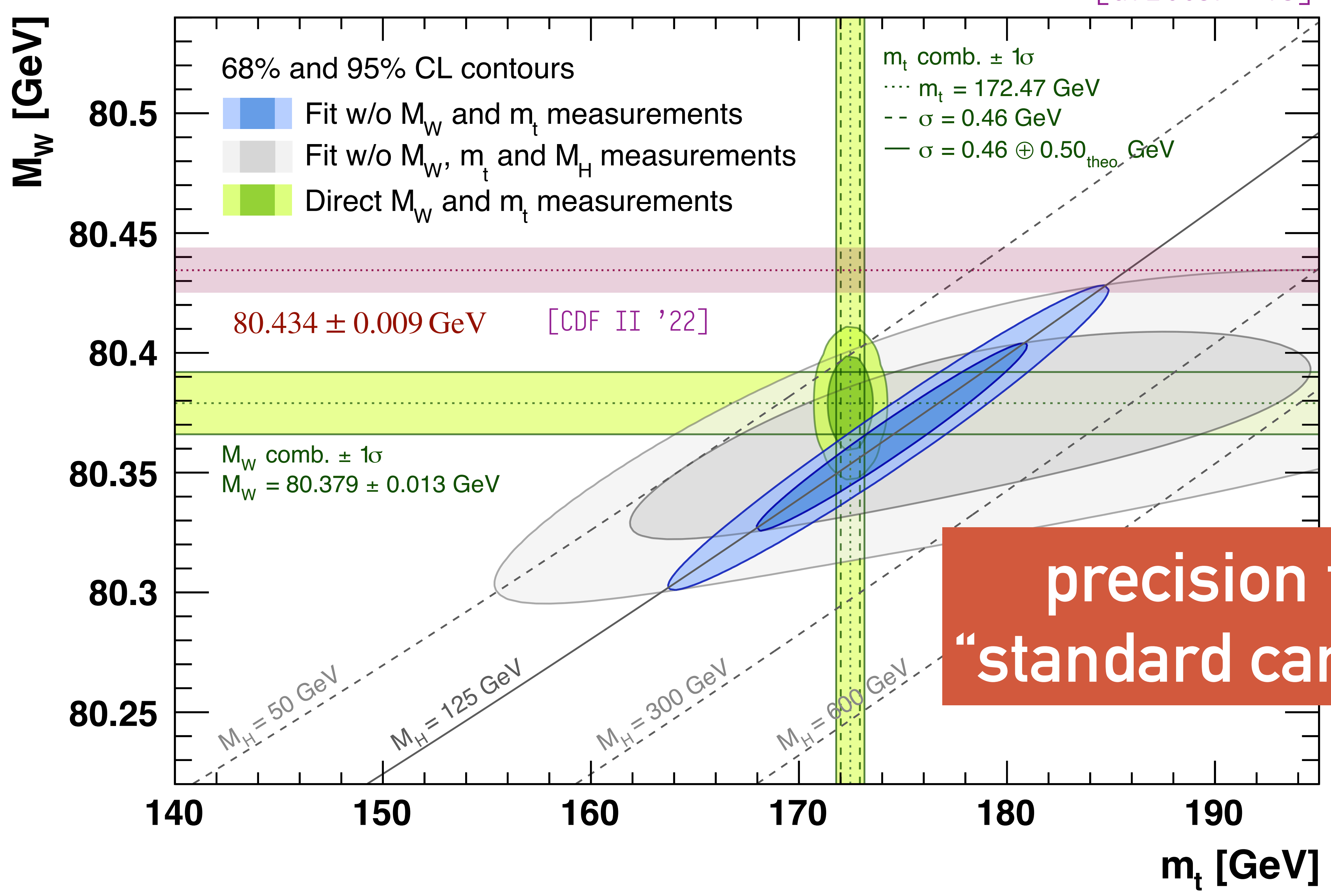


control of SM backgrounds

Indirect

- constrained system ⇝ self consistent?
- sensitivity $\delta\mathcal{O} \sim Q^2/\Lambda_{NP}^2$
  ⇝ per-cent @ EW scale ⇒ probe $\Lambda_{NP} \sim$ TeV

[Gfitter '18]

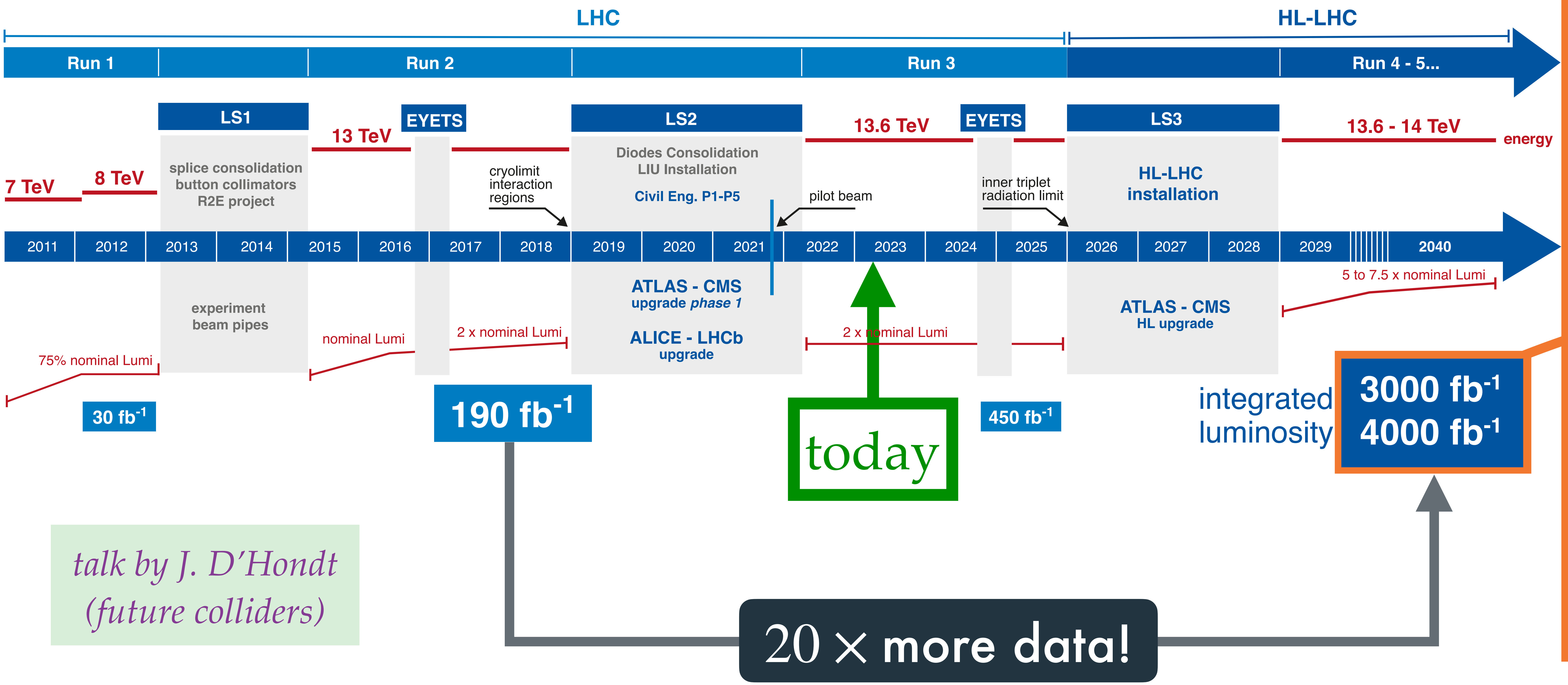


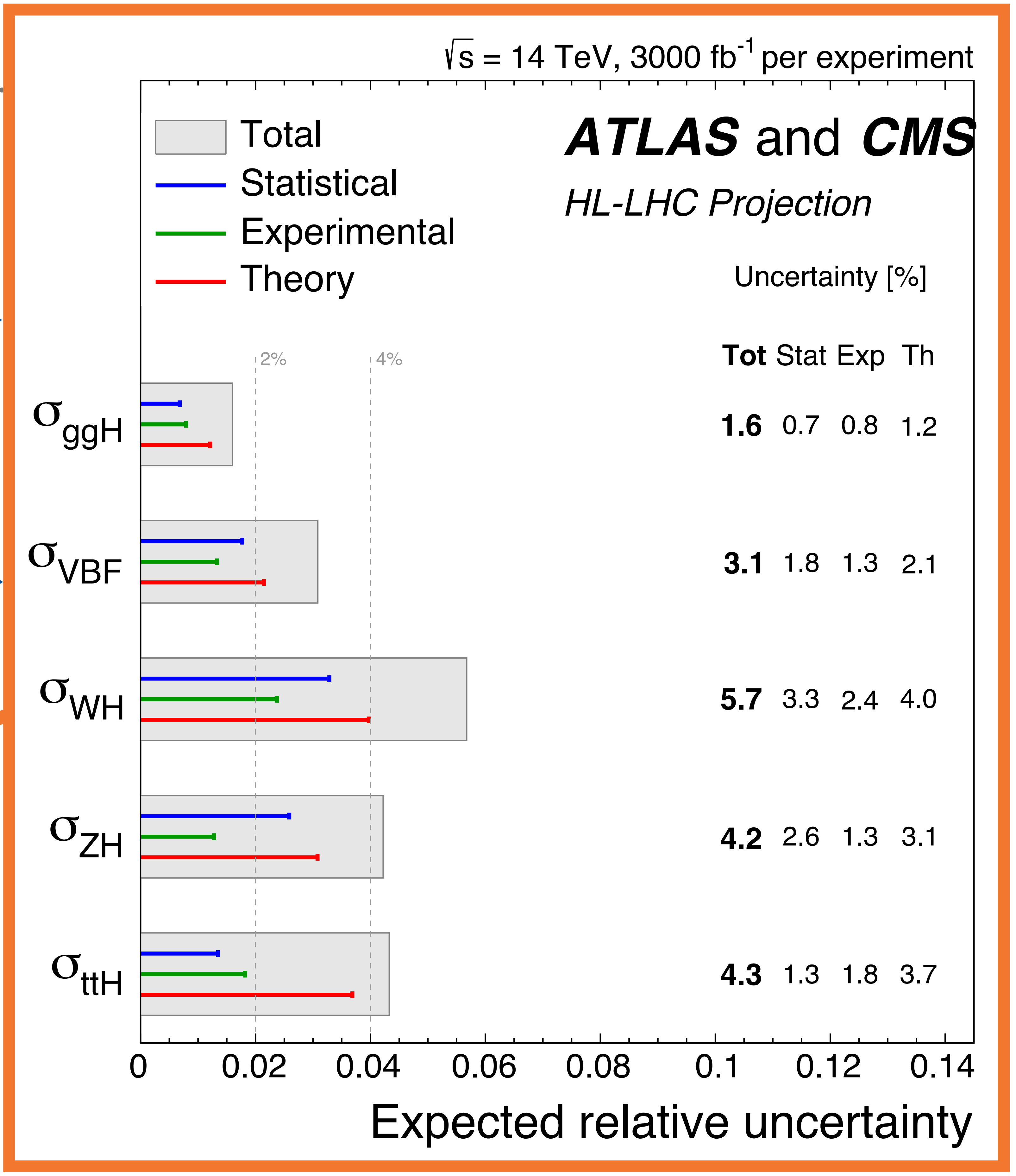
precision for "standard candles"

m/GeV	measured	fit value
m_t	172.47 ± 0.68	176.4 ± 2.1
M_H	125.1 ± 0.2	90^{+21}_{-18}
M_W	80.379 ± 0.013	80.354 ± 0.007

How much Precision? ⇝ $\mathcal{O}(1\%)$

[CERN Yellow Report '19]



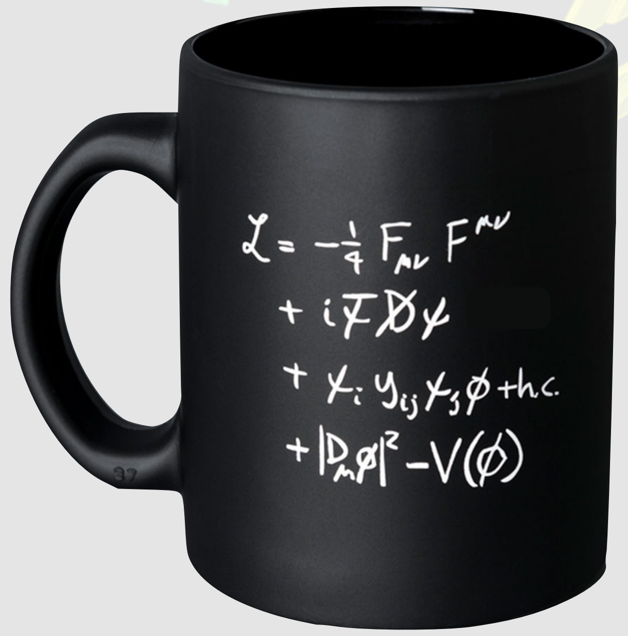


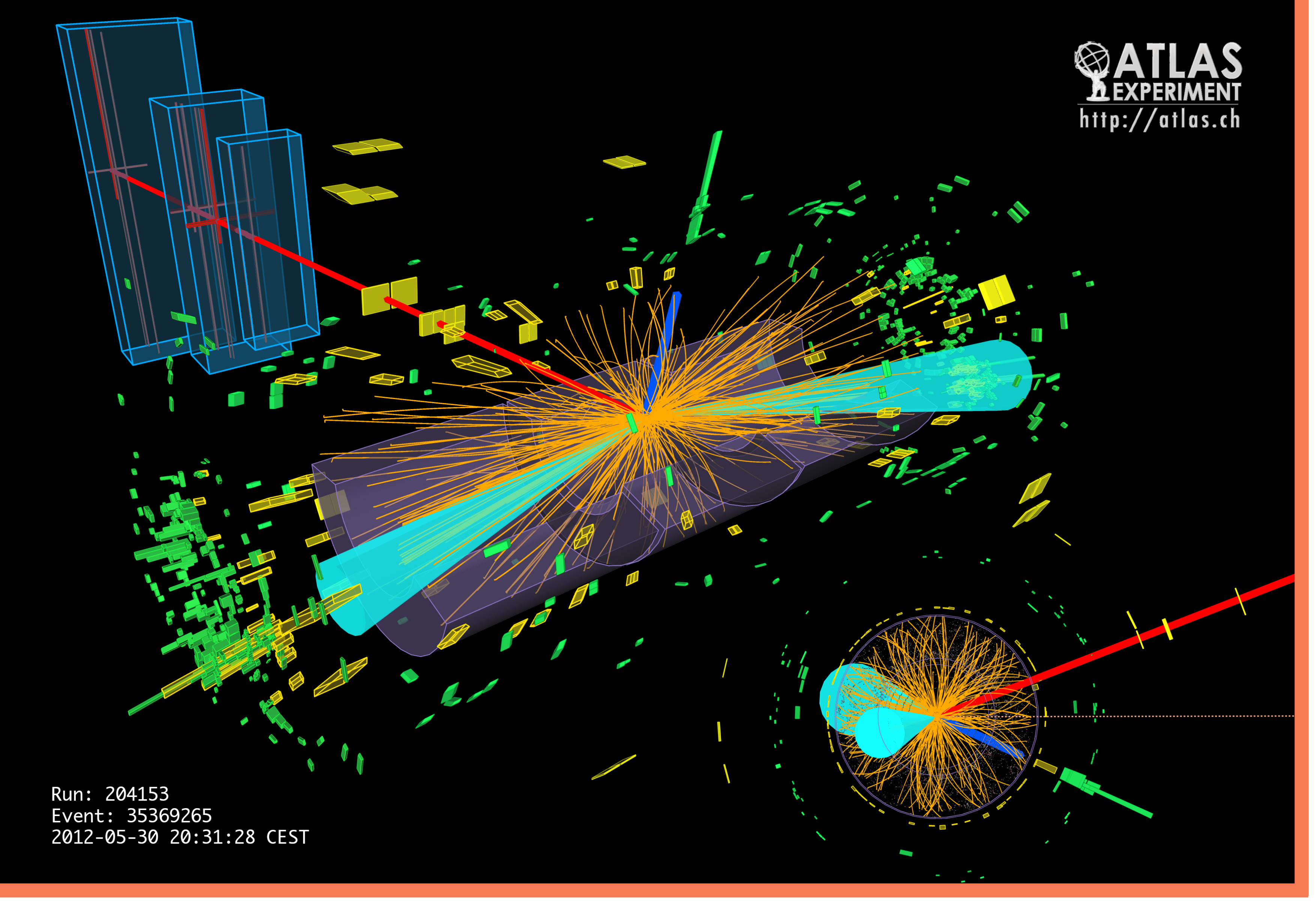
talk by J. D'Hondt (future colliders)

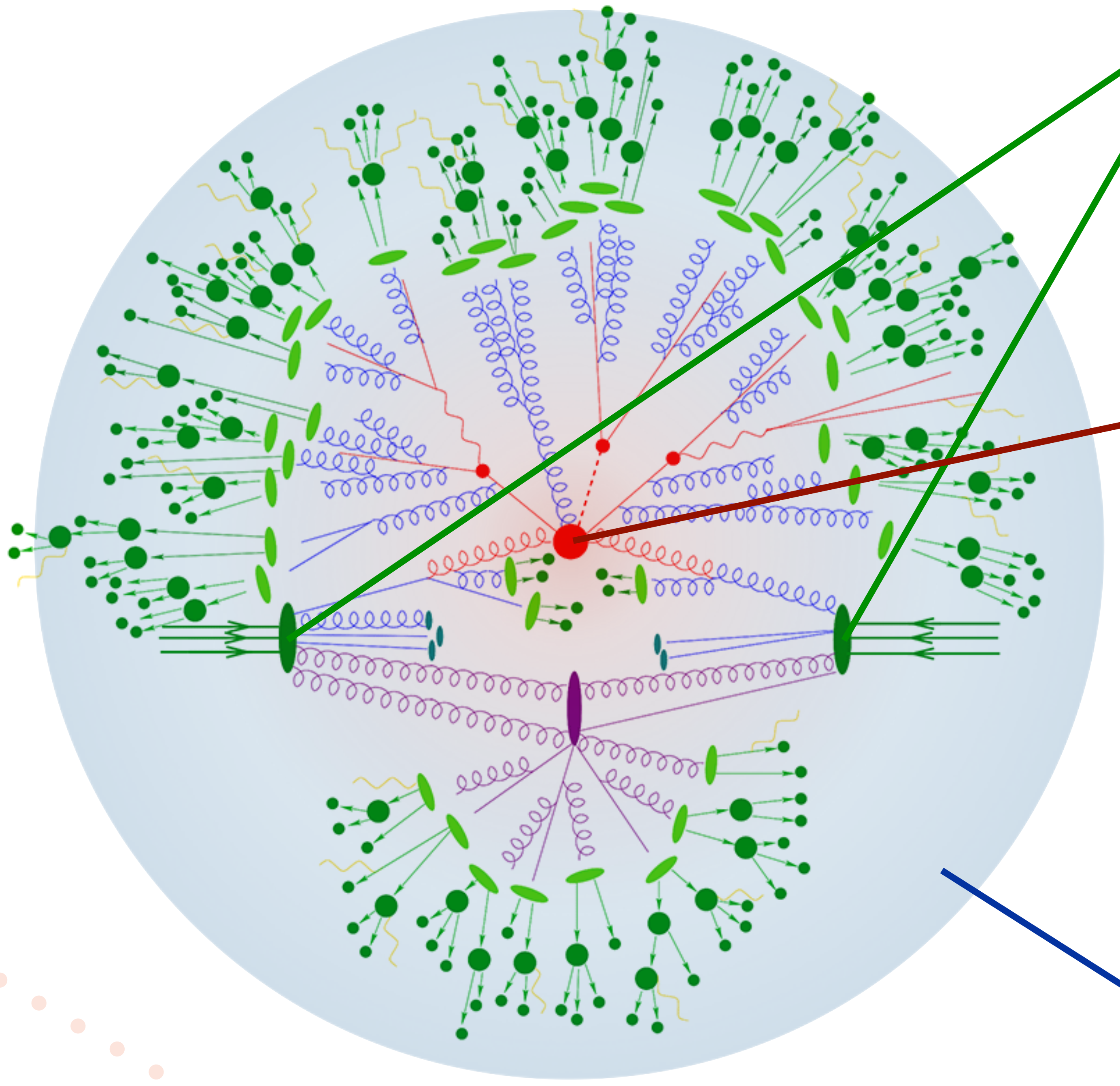
- ⇝ access to rare & complex processes
- ⇝ very high precision measurements

theory uncertainties scaled down by factor 2

How do we predict this from Theory?







incoming protons

- quarks & gluons

short distance “hard”

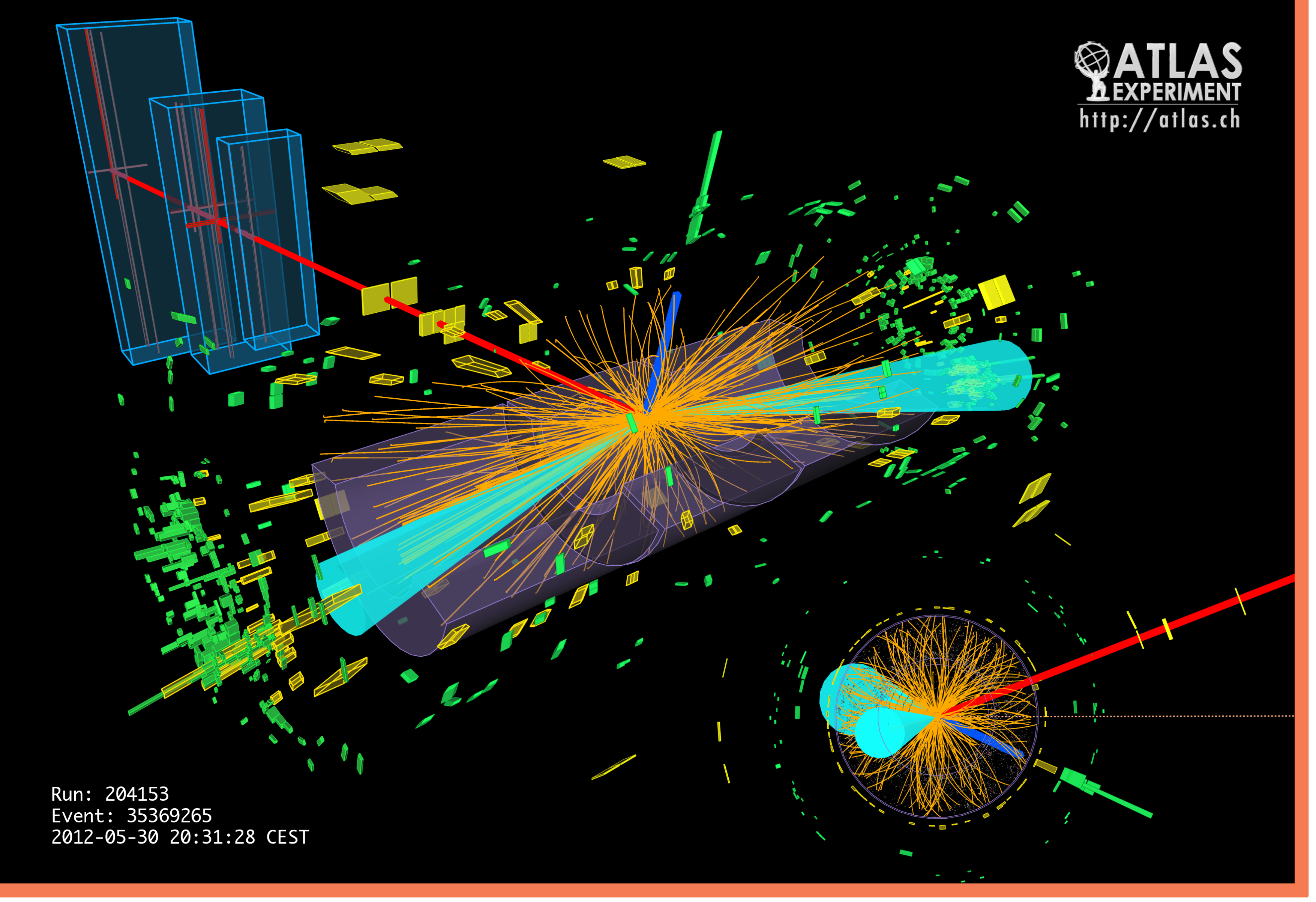
- high scales: 10^2–10^3 GeV

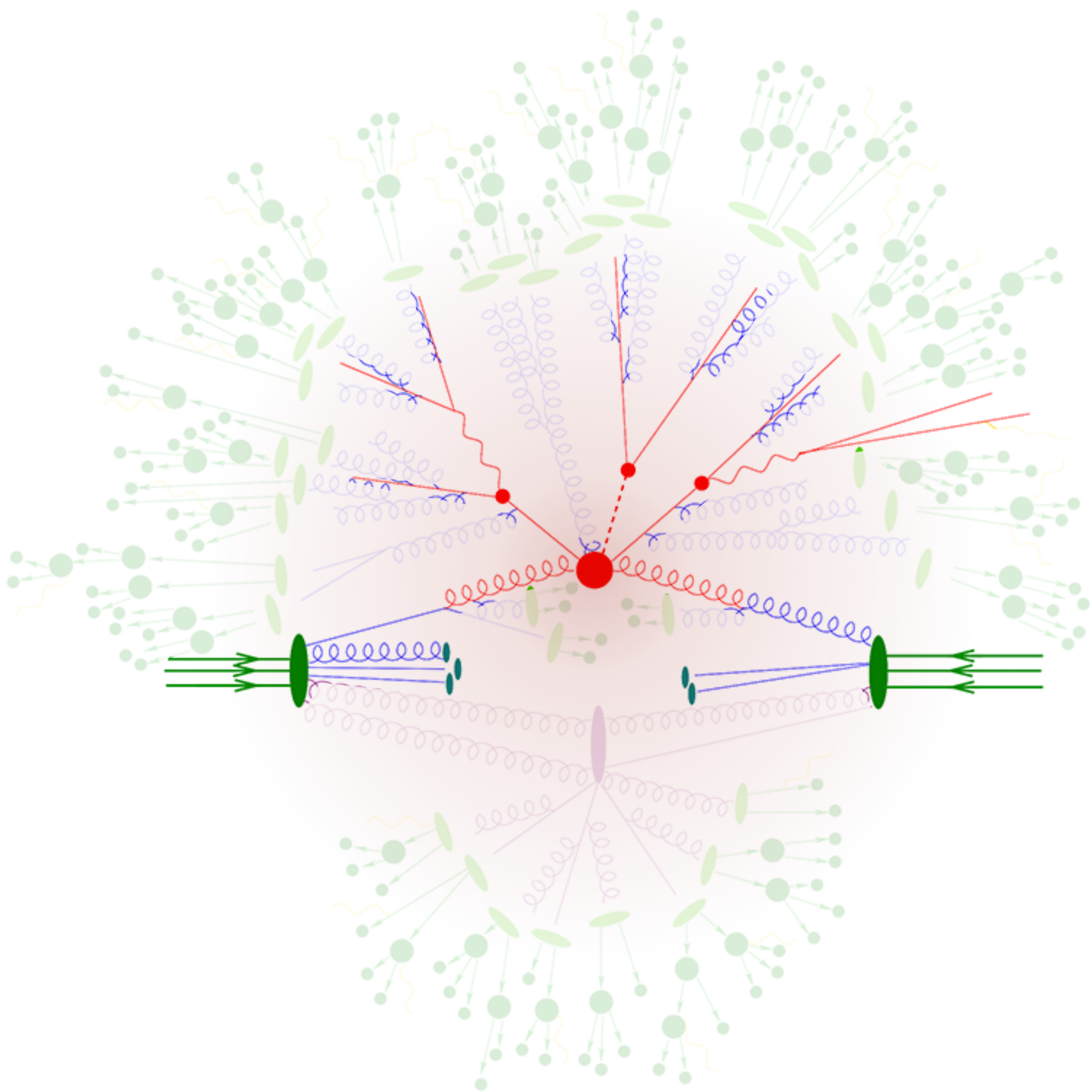
evolution towards a physical observable state

long distance “soft”

- low scales: $\mathcal{O}$(few GeV)

> “Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always.”
>
> Frank Wilczek





- **Focus:**

  high momentum transfer

  & clean signatures

- perturbation theory:

  $\sigma = \sigma_0 \times \left(1 + \alpha_x + \alpha_x^2 + \alpha_x^3 + \ldots\right)$

  fixed order: LO NLO NNLO N³LO ...

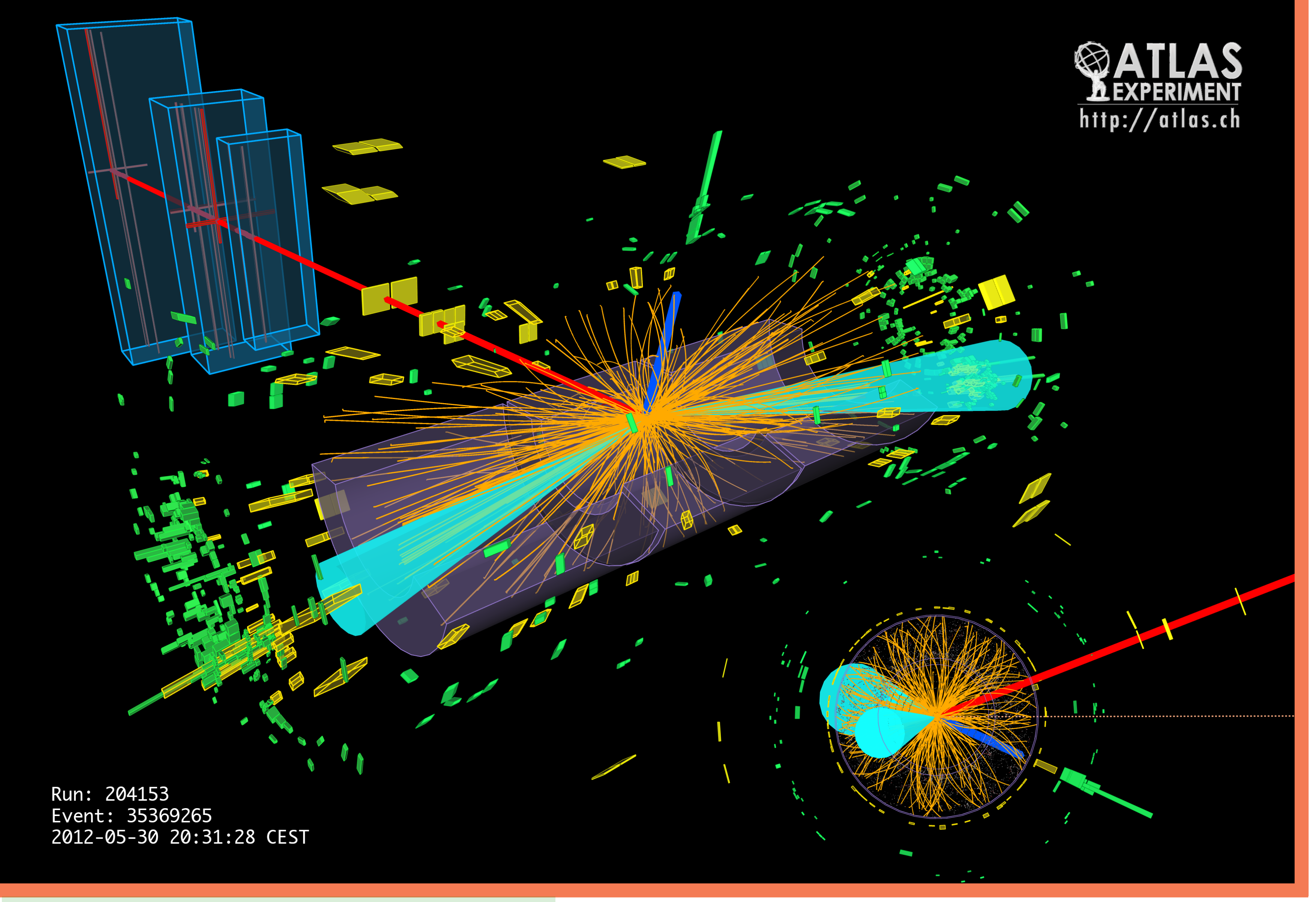
- $\alpha_s \sim 0.1$ & $\alpha_{ew} \sim 0.01$

  1% target $\leftrightarrow$ $\mathcal{O}(\alpha_s{}^2, \alpha_{ew})$

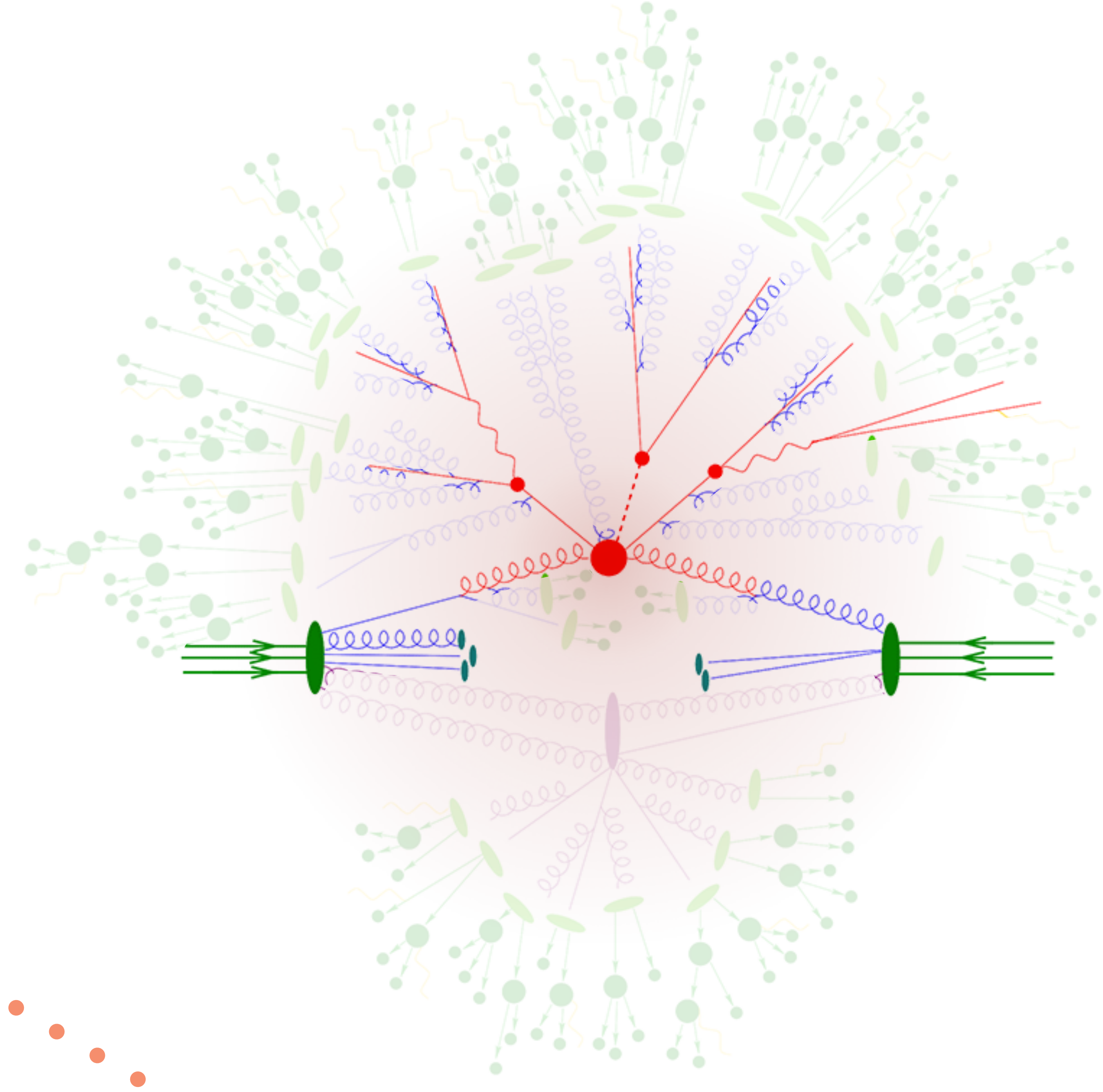
  $\rightsquigarrow$ $\mathcal{O}(\alpha_s{}^3, \alpha_s\,\alpha_{ew})$

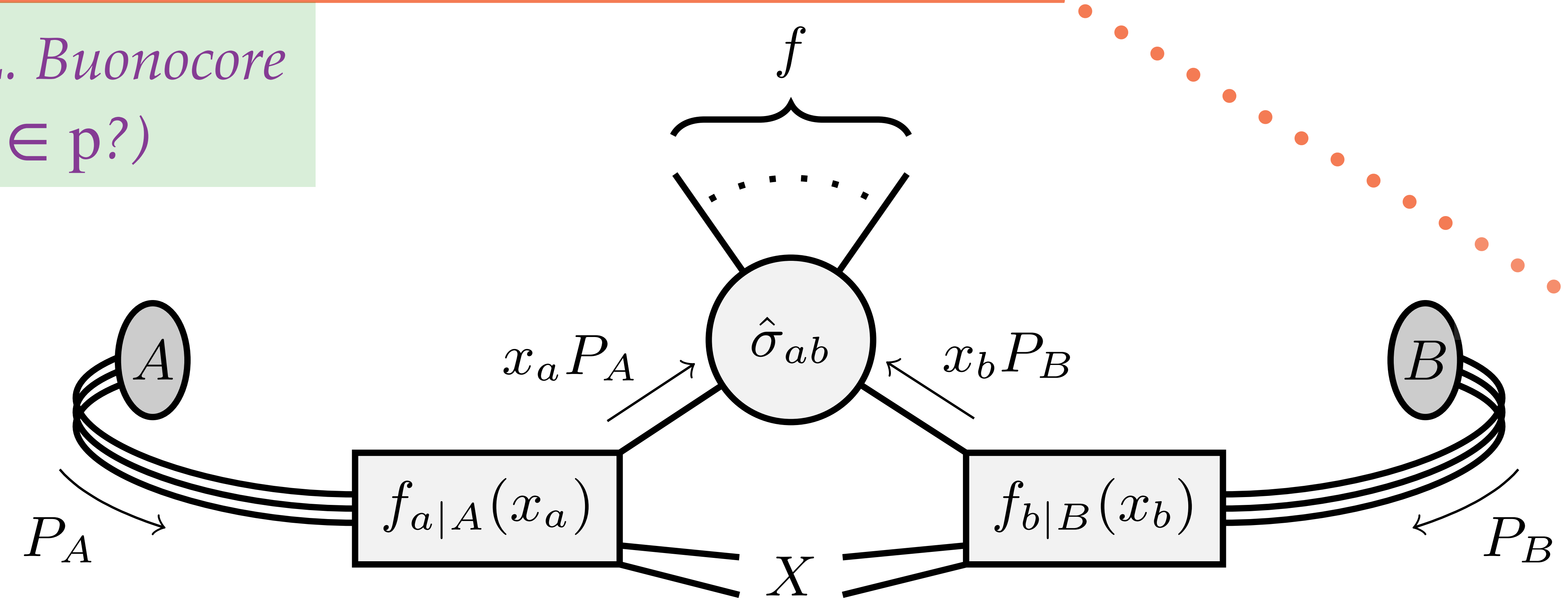
> "Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always."
>
> Frank Wilczek

1. asymptotic freedom
2. factorization



talk by L. Buonocore
($\ell \in$ p?)





$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \; f_{a|A}(x_a)\; f_{b|B}(x_b)\; \hat{\sigma}_{ab}(x_a, x_b)\; (1 + \mathcal{O}(\Lambda_{\rm QCD}^p/Q^p))$$

parton distribution functions (PDFs)
(non-perturbative, universal)

hard scattering
(perturbation theory)

non-perturbative effects
(power suppressed)

- **Focus:**

  high momentum transfer
  & clean signatures

- perturbation theory:

  $\sigma = \sigma_0 \times \left(1 + \alpha_x + \alpha_x^2 + \alpha_x^3 + \ldots\right)$

  fixed order: LO NLO NNLO N³LO ...

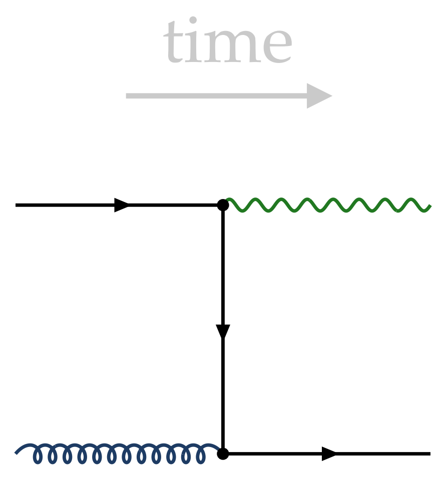
- $\alpha_s \sim 0.1$ & $\alpha_{ew} \sim 0.01$

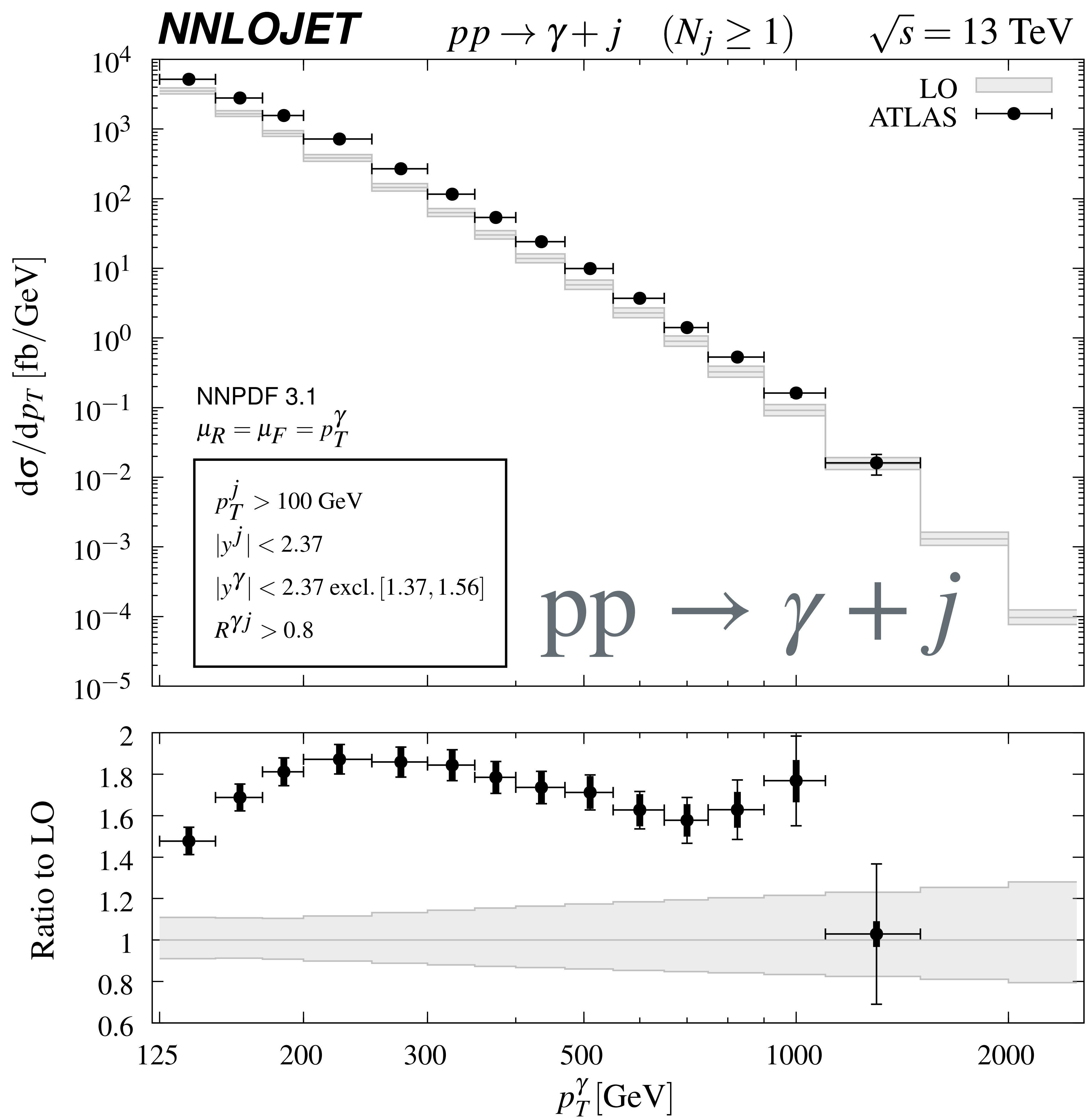
  1% target $\leftrightarrow$ $\mathcal{O}(\alpha_s^2, \alpha_{ew})$

  $\rightsquigarrow$ $\mathcal{O}(\alpha_s^3, \alpha_s\, \alpha_{ew})$

Perturbation Theory @ Leading Order

LO



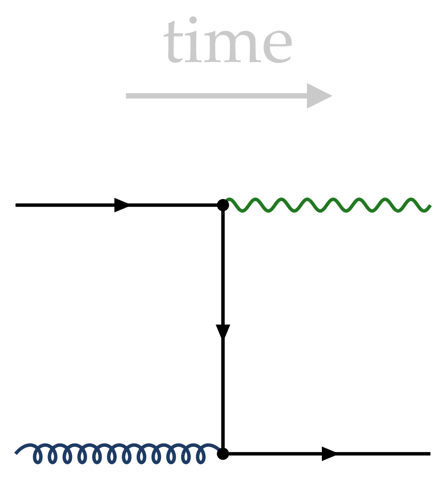


[Chen, Gehrmann, Glover, Höfer, **A.Huss** '19]

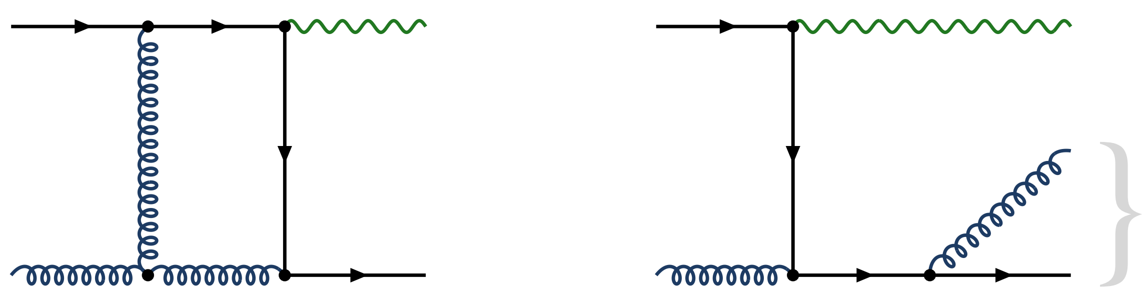
Only captures gross features & unreliable uncertainty estimates

Perturbation Theory @ Next-to-Leading Order

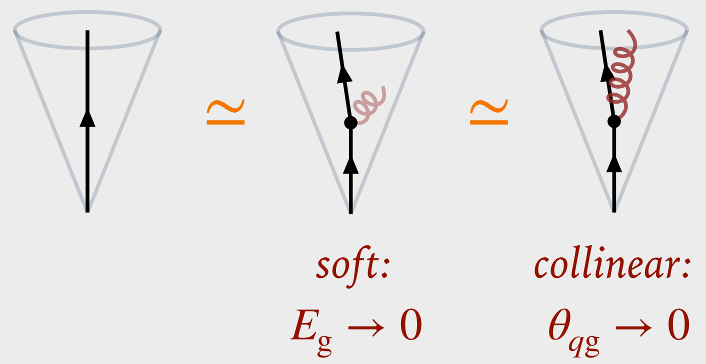
LO



NLO

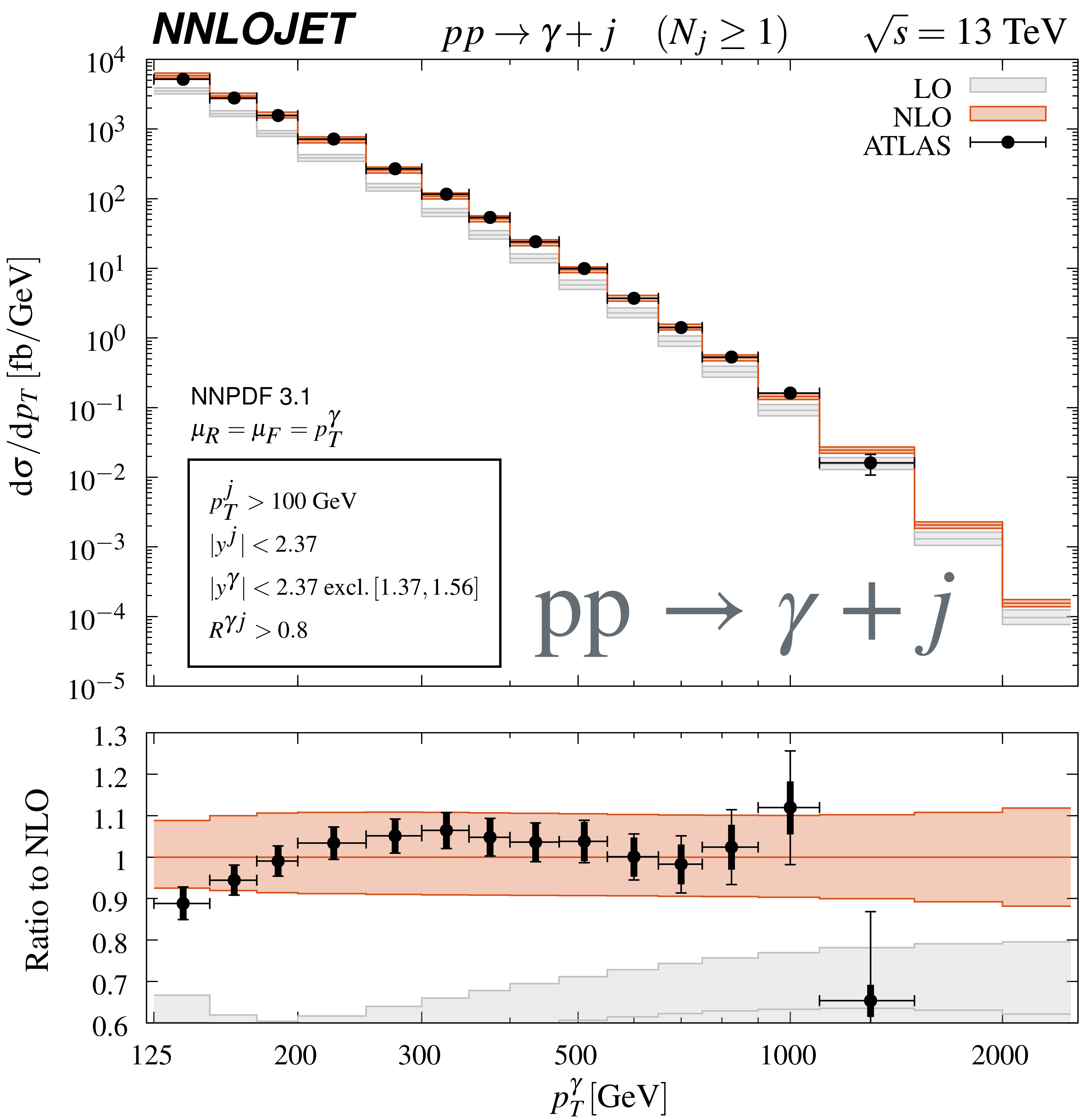


can become unresolved
↭ "infrared" (IR)



higher order: more loops & legs

typically $\mathcal{O}(10-30\%)$ precision
here: limits the *interpretation* of data!



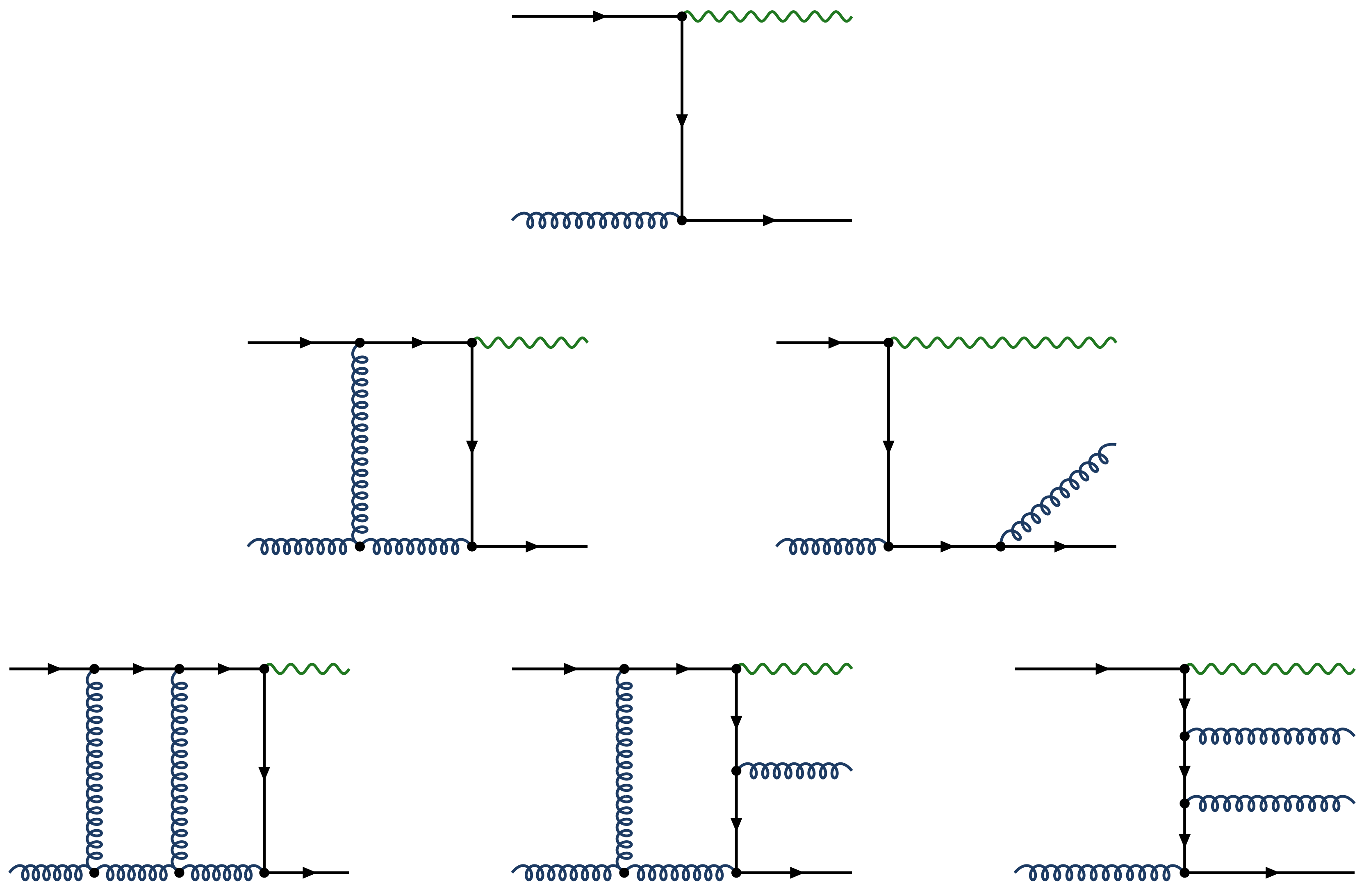
[Chen, Gehrmann, Glover, Höfer, **A.Huss** '19]

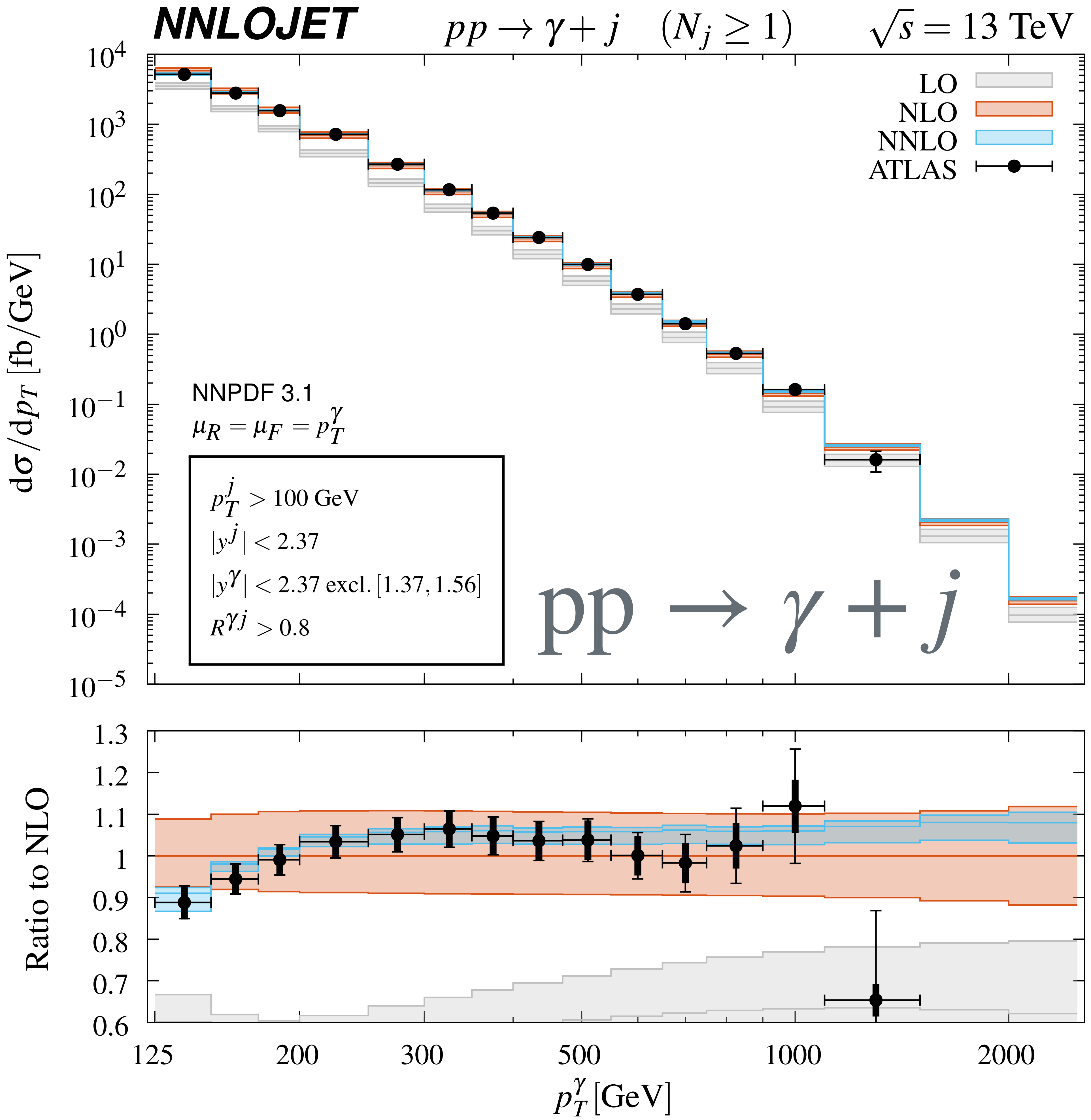
Perturbation Theory @ Next-to-Next-to-Leading Order

LO

NLO

NNLO





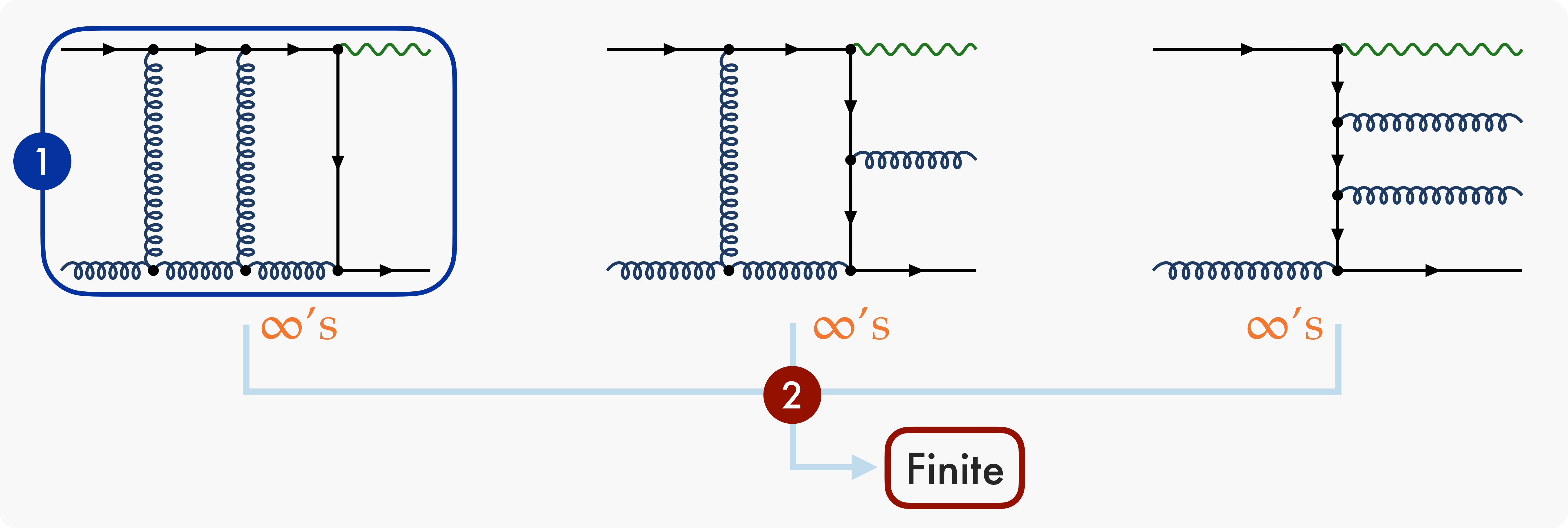
[Chen, Gehrmann, Glover, Höfer, **A.Huss** '19]

mandatory to achieve *single digit* of relative precision

Main Challenges @ NNLO

$$\sigma \sim \int \mathrm{d}\Phi \, \left|\mathcal{M}\right|^2$$

cross section ~ phase space & scattering amplitude



complexity often quantified by the multiplicity ("#legs"): $2 \to n$

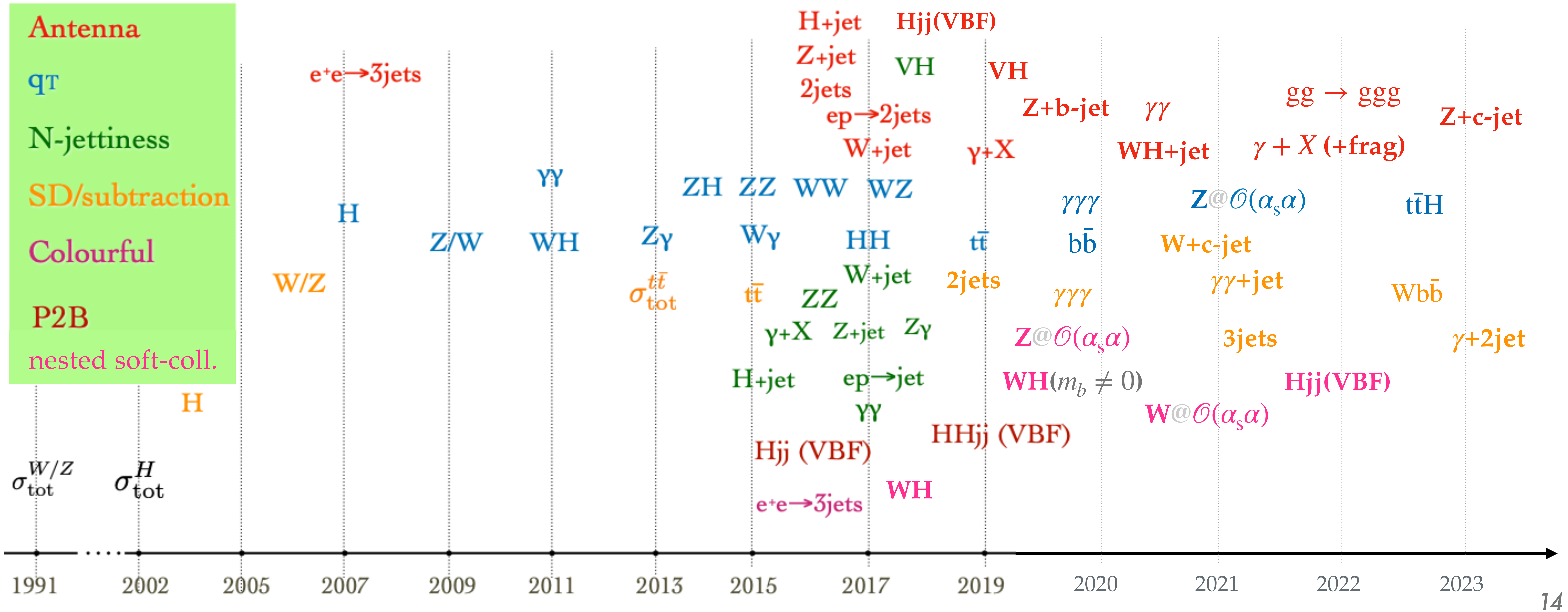
1. **amplitudes** & multi-loop integrals
   - rapid growth in complexity with number of scales
     ↭ kinematic invariants & particle masses (int./ext.)
2. **infrared subtractions** ↔ realistic setup (arbitrary cuts, observables, …)
   - extract IR singularities in $\mathrm{d}\Phi$ without performing the integration
     ↭ more difficult with more coloured legs (simpler if massive)

What can we do Today? — The NNLO Timeline

[adapted from slide by M. Grazzini]

Tremendous progress in the past ~ 10 years!

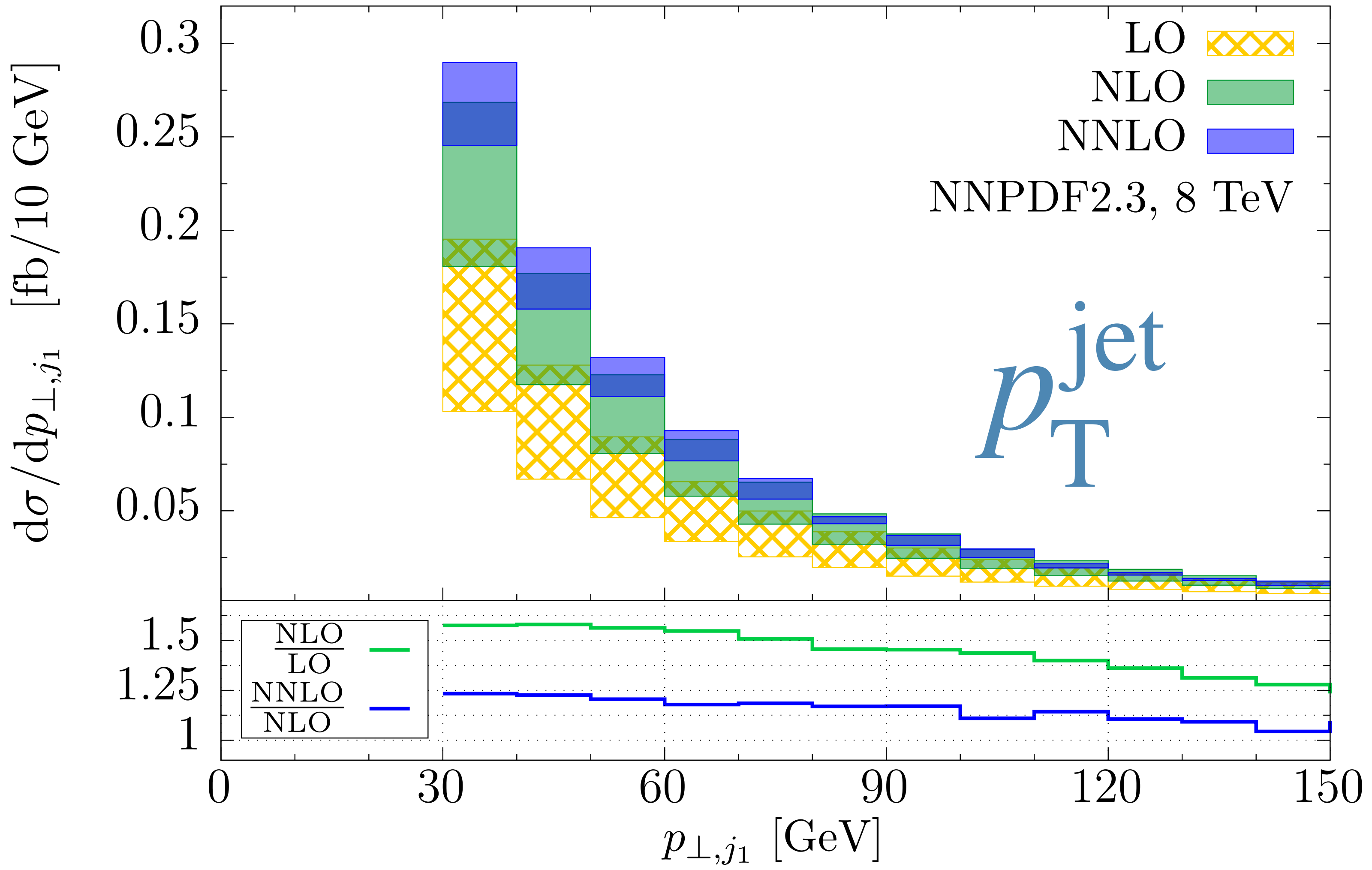
↪ $2 \to 2$ under good control; $2 \to 3$ steady progress



NNLO Reaching Maturity

"Standard" $2 \to 2$ well established ⇝ independent calculations *(validation!)*

H+jet



residue subtraction

[Caola, Melnikov, Schulze '15]

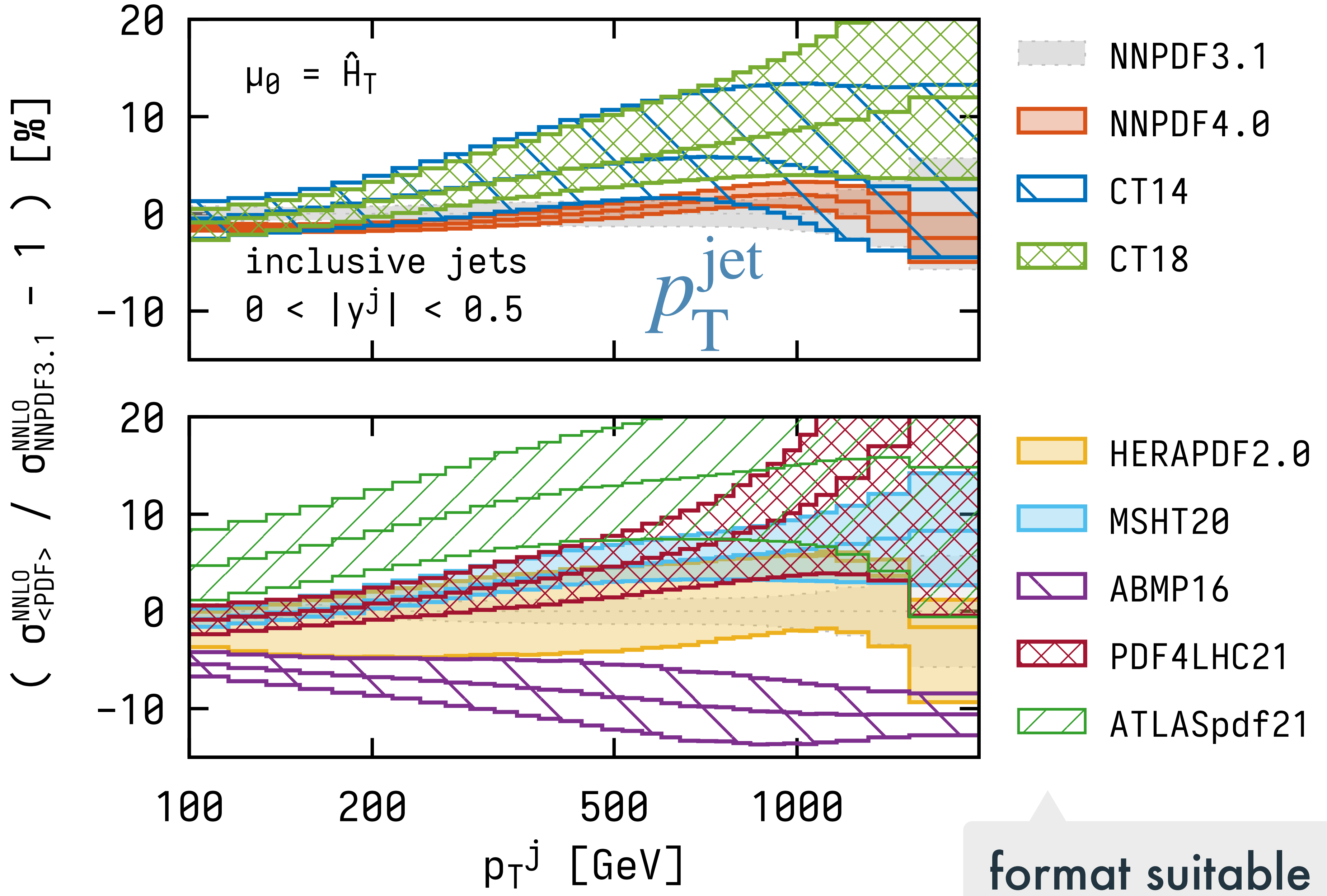
τ_1 jettiness subtraction

[Boughezal, Focke, Giele, Liu, Petriello '15]
[Campbell, Ellis, Seth '19]

antenna subtraction

[Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '16]

jets



format suitable for PDF fits

[NNLOJET+APPLfast '22]

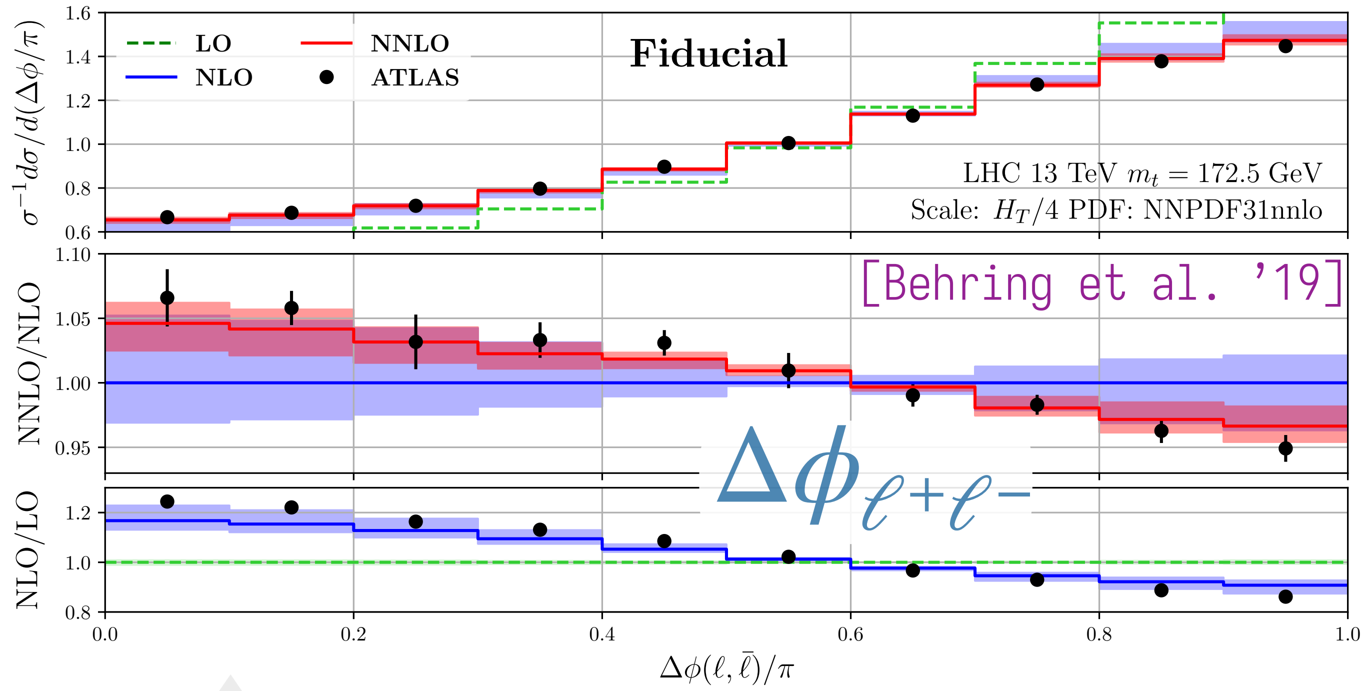
antenna subtraction

[Currie, Glover, Pires '16] (LC)
[Chen, Gehrmann, Glover, AH, Mo '22]

Stripper

[Czakon, van Hameren, Mitov, Poncelet '19]

Top Pairs — $t\bar{t}$



comparison in fiducial volume essential for agreement

Stripper

[Czakon, Heymes, Mitov '15]

q_T subtraction

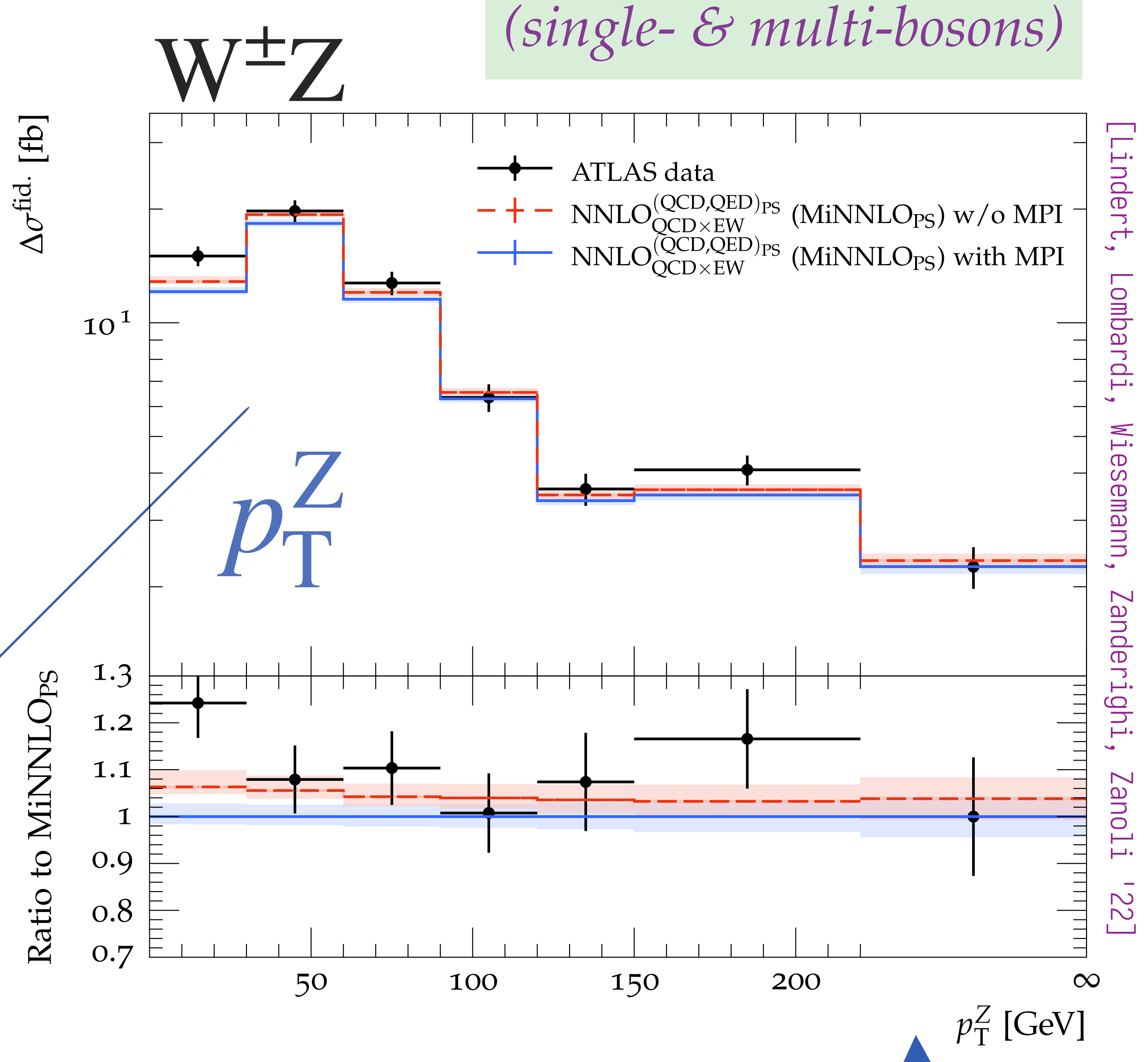
[Catani, Devoto, Grazzini, Kallweit, Mazzitelli '19]

talk by J. Mazzitelli (top-quark physics)

Beyond "Standard" $2 \to 2$ Calculations

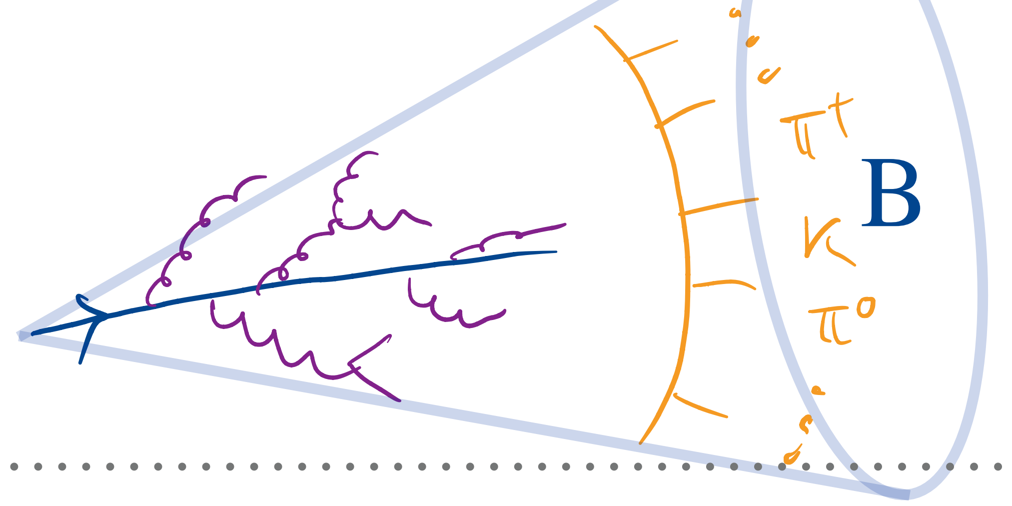
talk by K. Krizka (single- & multi-bosons)

- **adding flavour** (also: $Wb\bar{b}$)
  - Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
  - W+c-jet [Czakon, Mitov, Pellen, Poncelet '20,'23]
  - Z+c-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Garcia, Stagnitto '23]
- **adding masses**
  - $pp \to WH\ (H \to b\bar{b})$ [Behring, Bizoń, Caola, Melnikov, Röntsch '20]
  - $pp \to b\bar{b}$ [Catani, Devoto, Grazzini, Kallweit, Mazzitelli '21]
- **identified particles / fragmentation**
  - hadron fragmentation [Czakon, Generet, Mitov, Poncelet '21,'22]
  - isolated photons [Gehrmann, Schürmann '22; + Chen, Glover, Höfer, AH '22]
- **beyond approximations**
  - non-factorizable corrections { VBF [Liu, Melnikov, Penin '19]; [Dreyer, Karlberg, Tancredi '20]; single-t [Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22]
  - Higgs beyond HTL ($m_t \to \infty$) [Czakon, Harlander, Klappert, Niggetiedt '20]
- **NNLO ♡ PS** ⇝ H, V, HV, VV, $t\bar{t}$ [Geneva, MiNNLOPS, UN²LOPS]



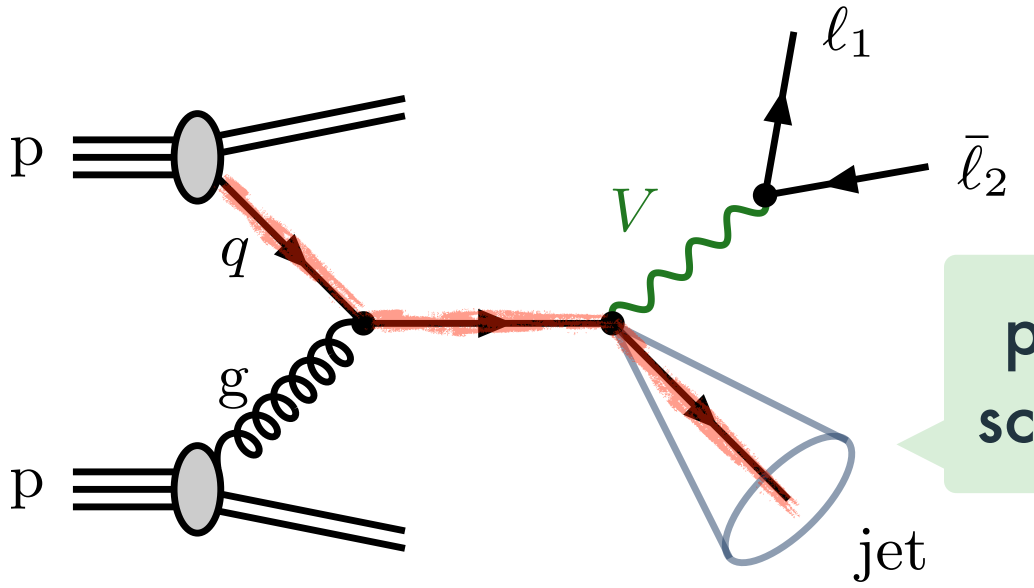
(NNLO$_{\mathrm{QCD}}$+NLO$_{\mathrm{EW}}$)

Jet Flavour ⇝ Proxy for q_f



Flavoured jets are everywhere:

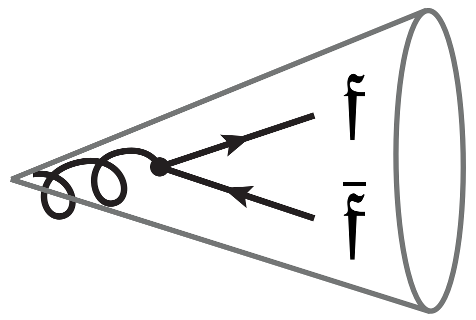
- Higgs physics ⇝ couplings
  (60% $\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}$)
- top physics ⇝ PDFs, α_s, BSM
  ($|V_{tb}| \sim 1$)
- f-jet + E_T^{miss} ⇝ BSM
- $V + f$-jet ⇝ PDFs, α_s, BG



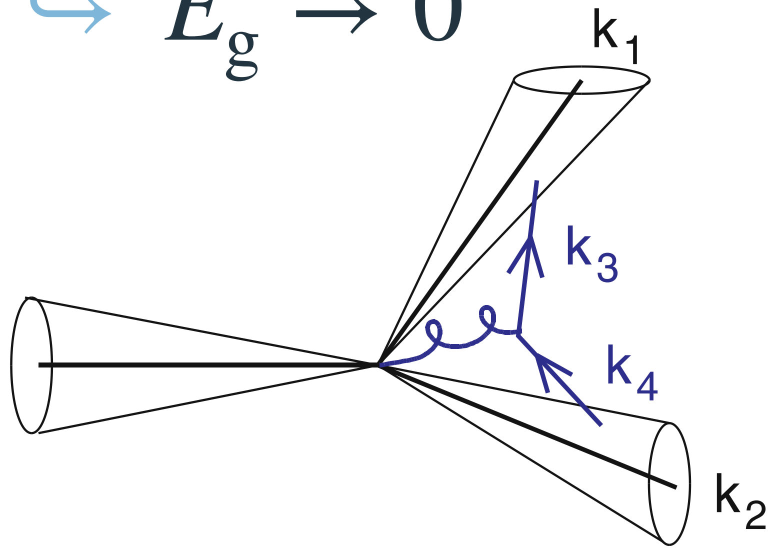
pinpoint specific scattering process

TH: ill-defined/divergent ($m_q \equiv 0$)

- collinear (NLO)
  ↪ $\mathrm{g} \leftrightarrow (\mathrm{f} \parallel \bar{\mathrm{f}})$



- soft (NNLO)
  ↪ $E_g \to 0$



not IR safe!

※ LHC experiments: naive anti-k_T

- *original solution:* flavour-k_T [Banfi, Salam, Zanderighi '06]
  ↪ (un-)folding, e.g. Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, AH, Majer '20]
- *new ideas* in the past year (⇝ compatibility with anti-k_T)
  - flavoured anti-k_T [Czakon, Mitov, Poncelet '22]
  - SoftDrop grooming [Caletti, Larkoski, Marzani, Reichelt '22]
  - Winner-Take-All axis [Caletti, Larkoski, Marzani, Reichelt '22]
  - flavour dressing [Gauld, AH, Stagnitto '22]
  - … [1507.00508; 1512.05265; 1702.02947; 2104.06920; 2202.05082]

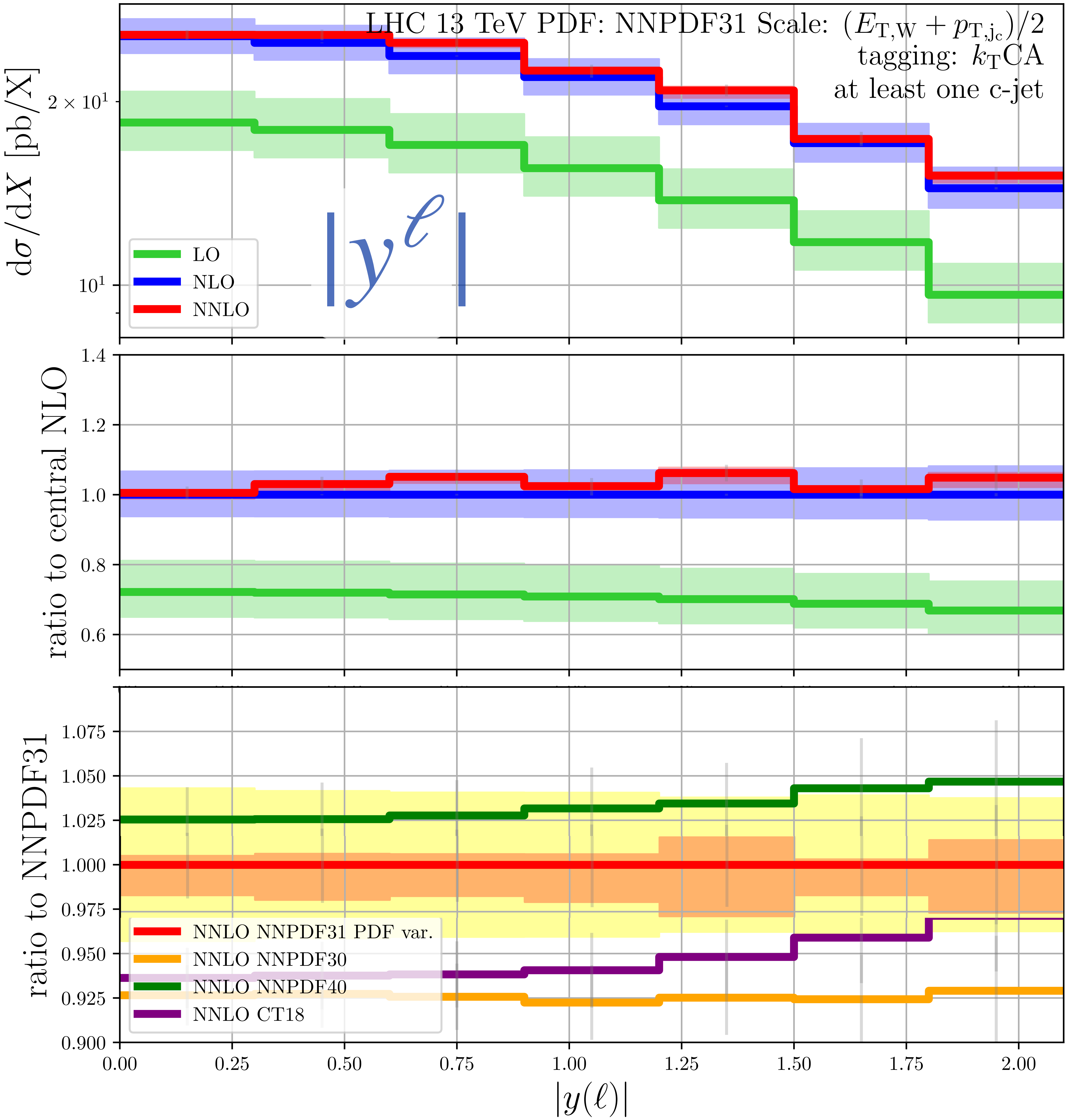
W+c-Jet

[Czakon, Mitov, Pellen, Poncelet '23]

- probe *strange* content of proton
  - ↪ e.g. from 3-loops: [Catani et al. '04]

    $$f_s(x) \neq f_{\bar{s}}(x)$$

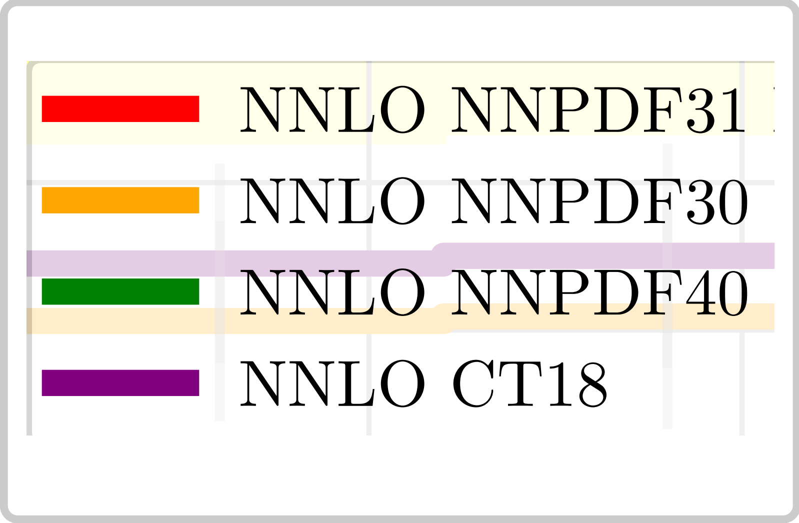
- flavour anti-k_T algorithm



NNLO stabilizes perturbative series

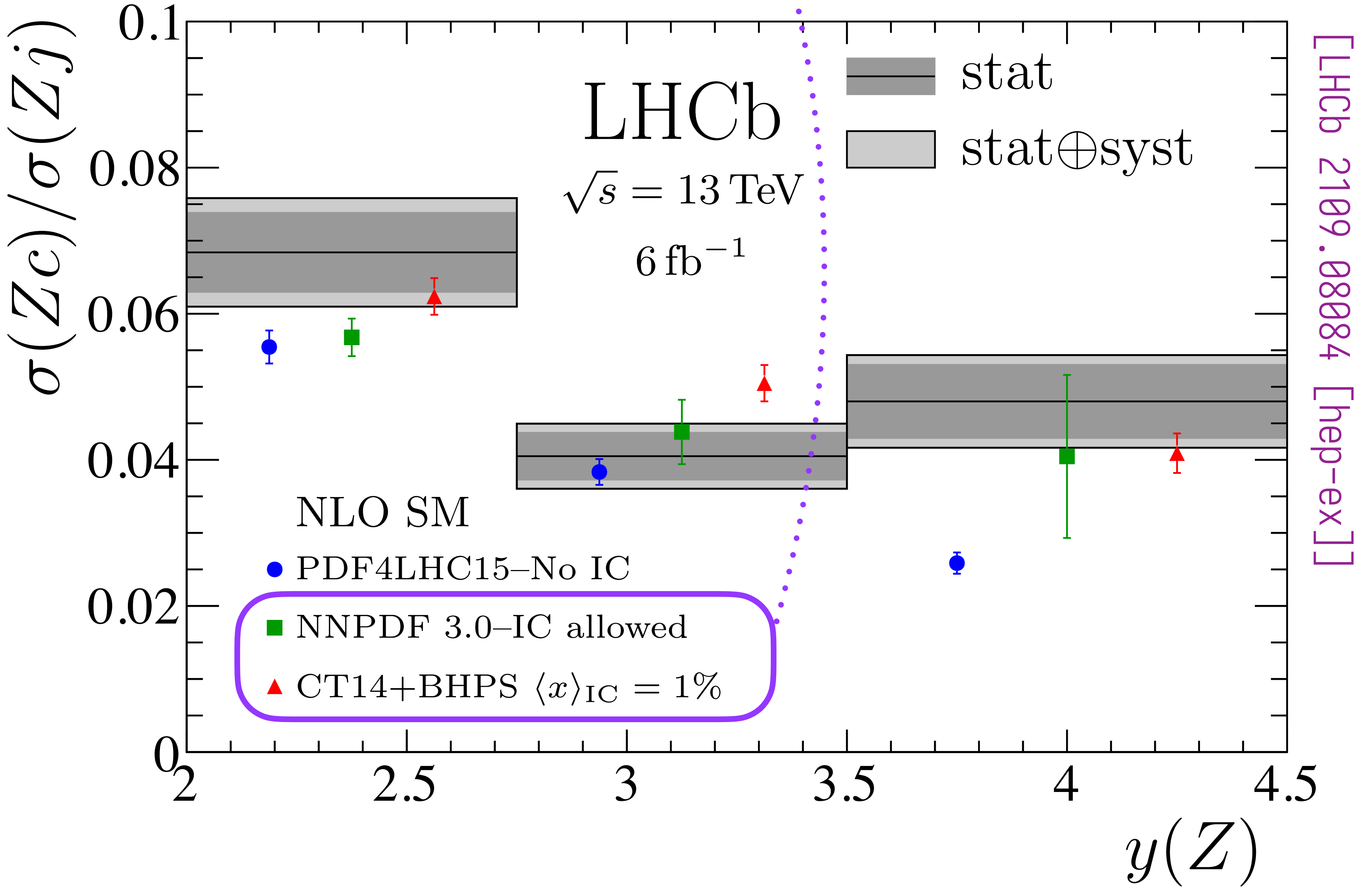
$\Delta^{\mathrm{NNLO}}_{\mathrm{scl}} < \Delta_{\mathrm{PDF}}$

sensitivity to constrain PDFs



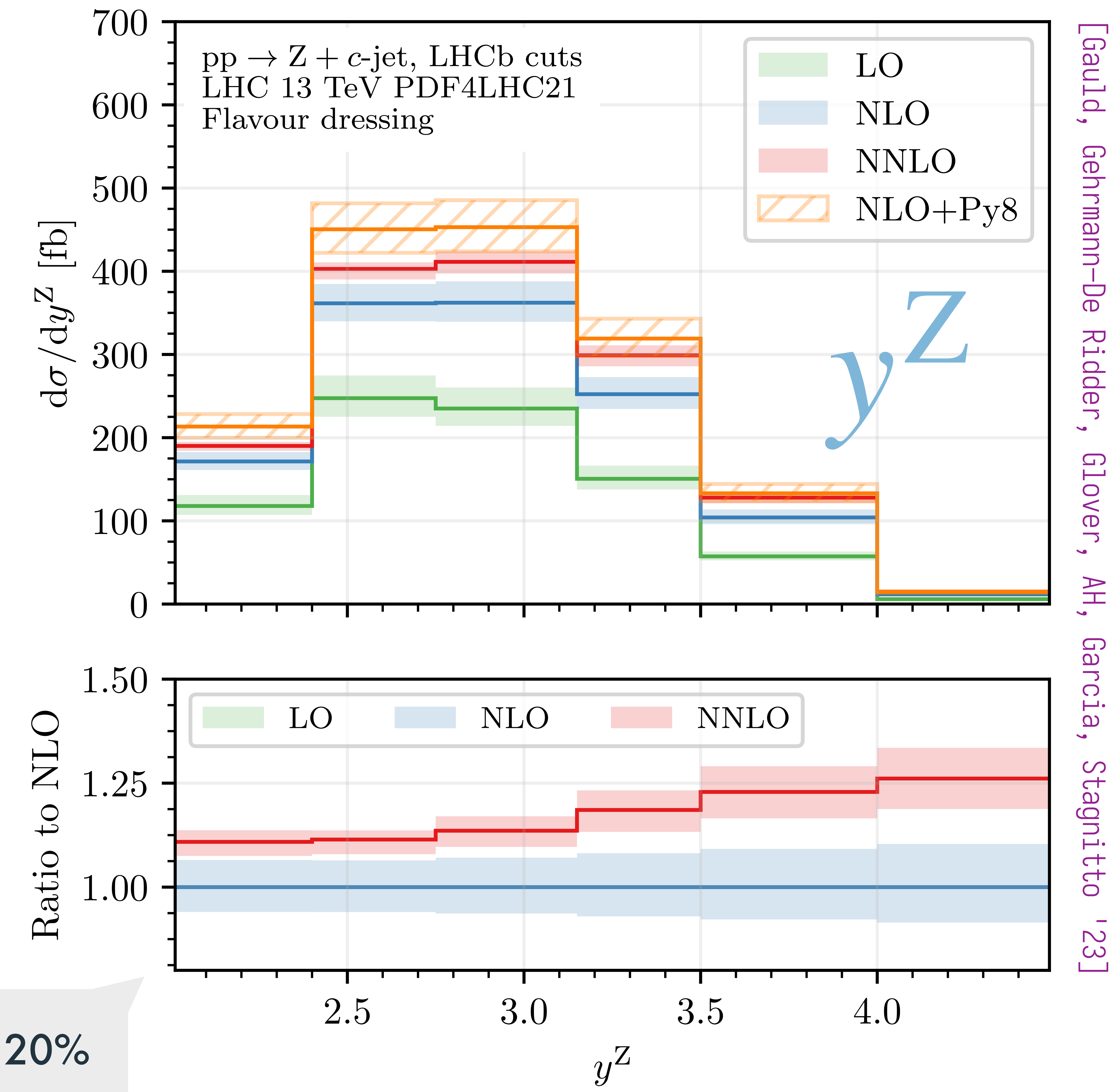
Z+c-Jet

- *"is there an intrinsic charm (IC) component in the proton?"*
  ↪ evidence (3σ) [NNPDF Nature 608 (2022)]
- LHCb kinematics (very forward)
  ⇝ sensitivity to IC



- NNLO 10–20%
- outside of NLO
- affects *shape*

(flavour-dressing)

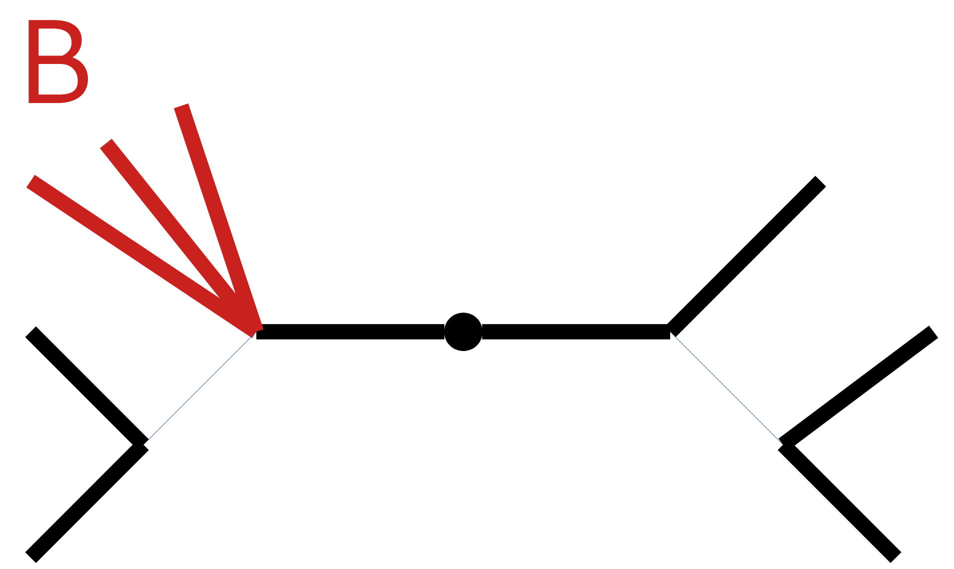


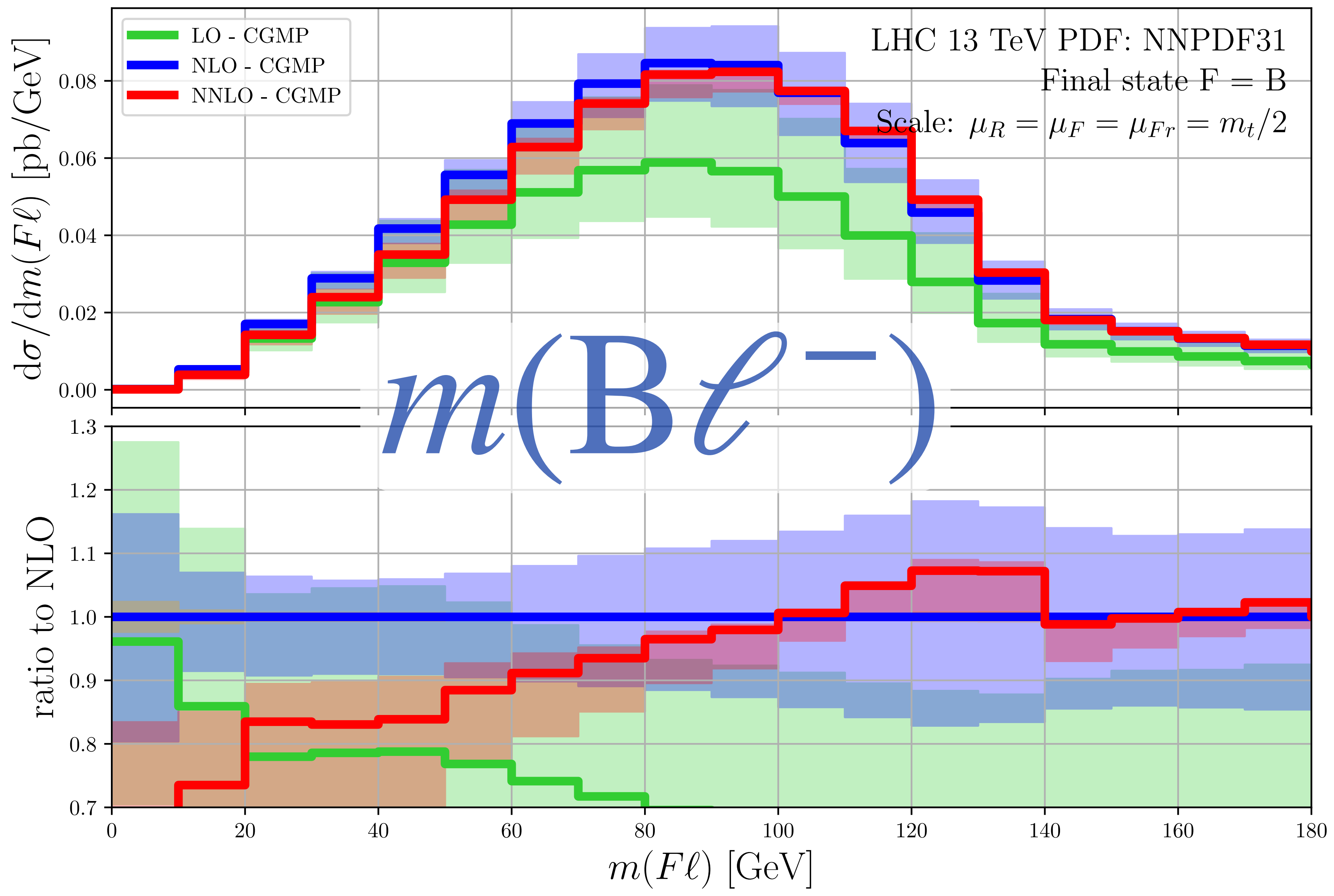
Attention: different c-jet definitions!
additional IR safety issues in EXP:
- tag if *at least* one c-hadron ($g \not\to c\bar{c}$)
- p_T(c-hadron) > 5 GeV ($c \not\to cg$)

B-Hadron in $t\bar{t}$

talk by J. Mazzitelli (top-quark physics)

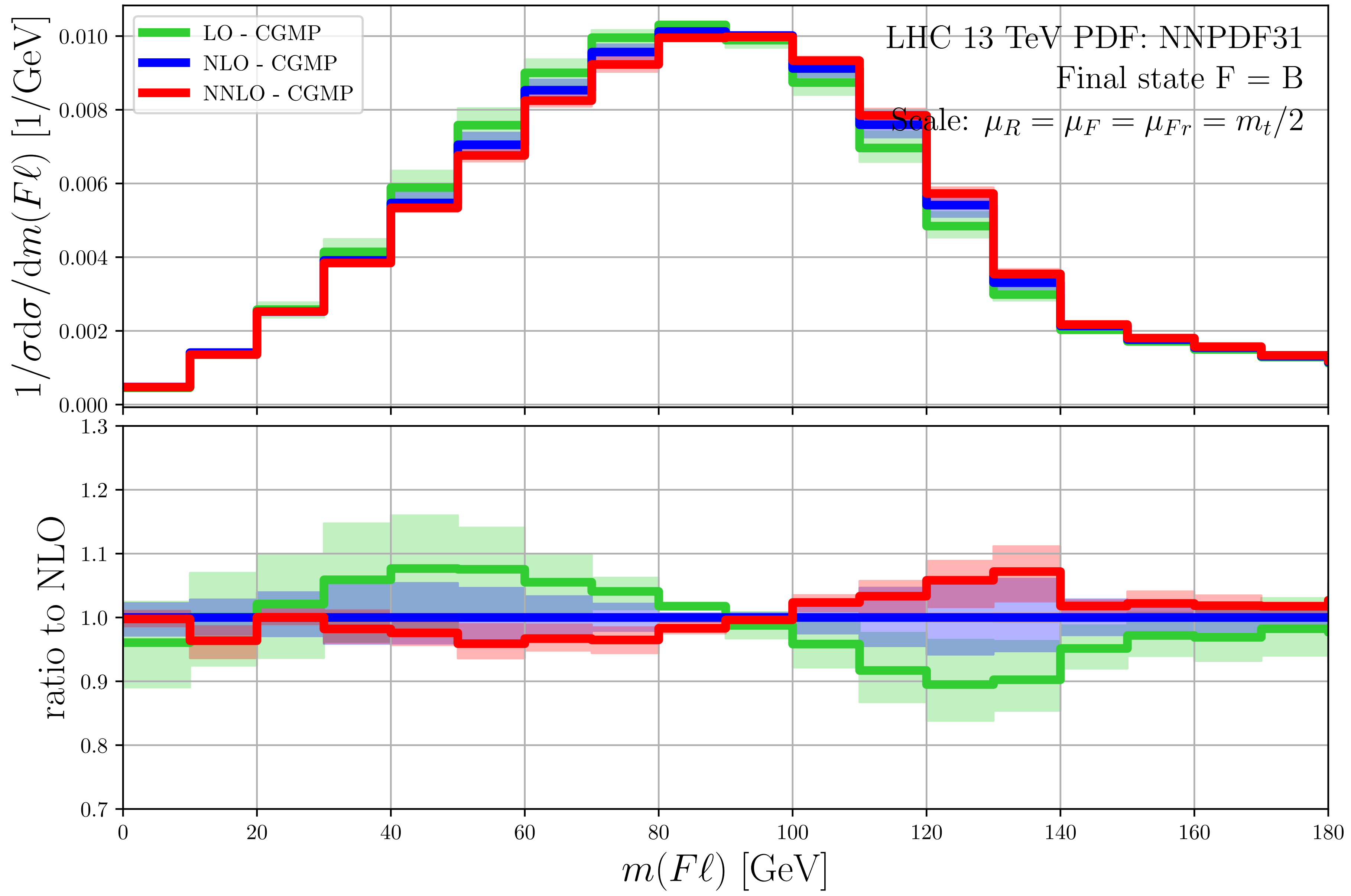
[Czakon, Generet, Mitov, Poncelet '21,'22]





non-overlap $m(\mathrm{B}\ell) \lesssim 50\,\mathrm{GeV}$

normalized
⊖ jet cuts



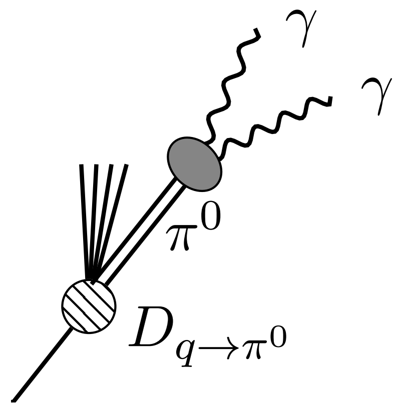
- $t\bar{t}$ ⇝ high purity & statistics
- B-hadrons measured precisely
  ↪ precise m_t extraction?
- $m_t \gg m_b$
  ↪ small power corrections
- extract $D_{i\to\mathrm{B}}$ from e^+e^- data

shape sensitive to m_t

δ_{NNLO} shape distortion ⇝ $\Delta m_t \sim 1\,\mathrm{GeV}$

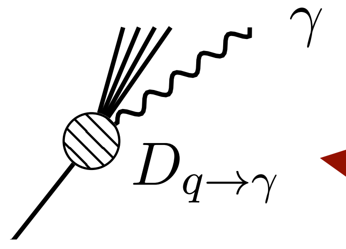
ISOLATED PHOTONS $\gamma + \text{jet}$

EXP: require *photon isolation* to eliminate

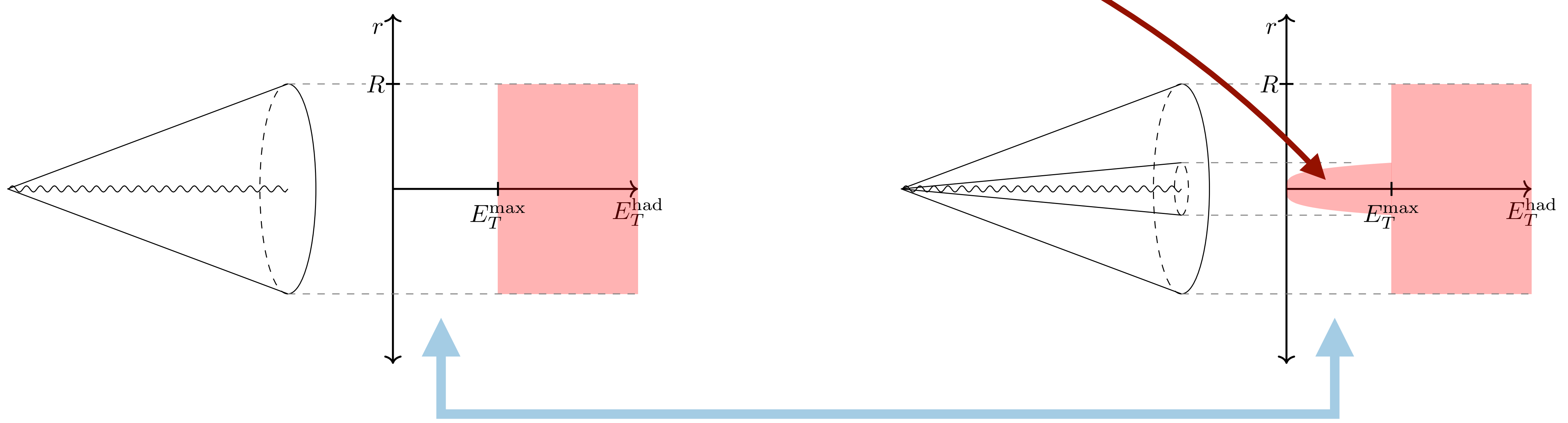


overwhelming background
from hadronization

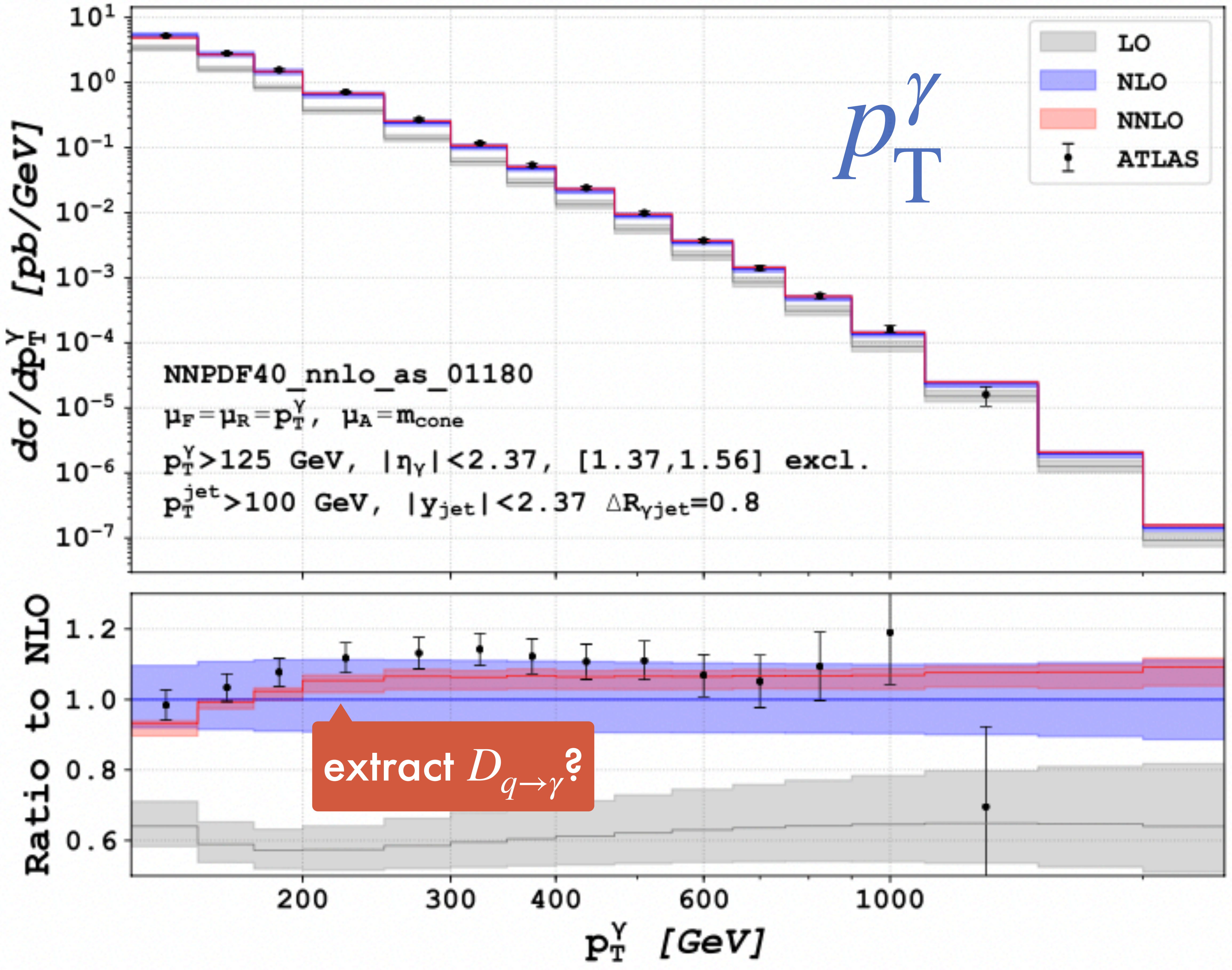
TH: so far relied on *idealized isolations*
[Frixione '98]



remove complex
fragmentation component

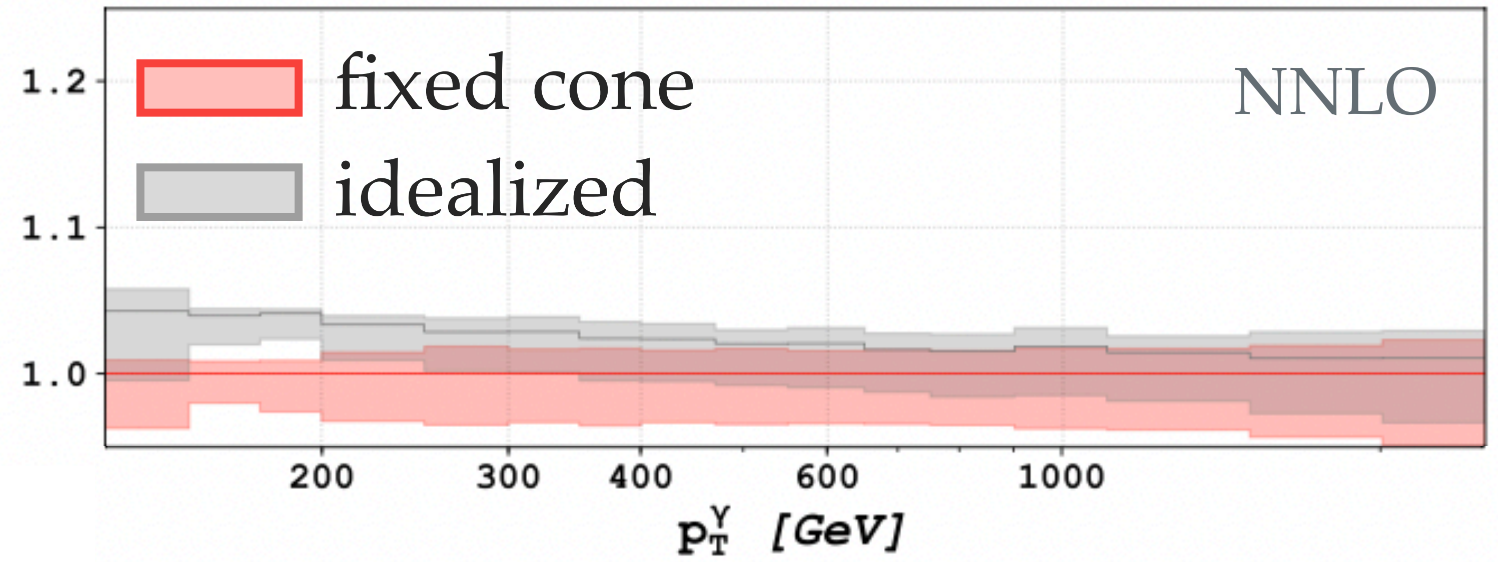


mismatch: few–10% [LH '13 '15]
$\sim \mathcal{O}(\Delta_{\text{exp}}, \Delta_{\text{scl}}^{\text{NNLO}})$



[Chen, Gehrmann, Glover, Höfer, AH, Schürmann '22]

matters
at NNLO!

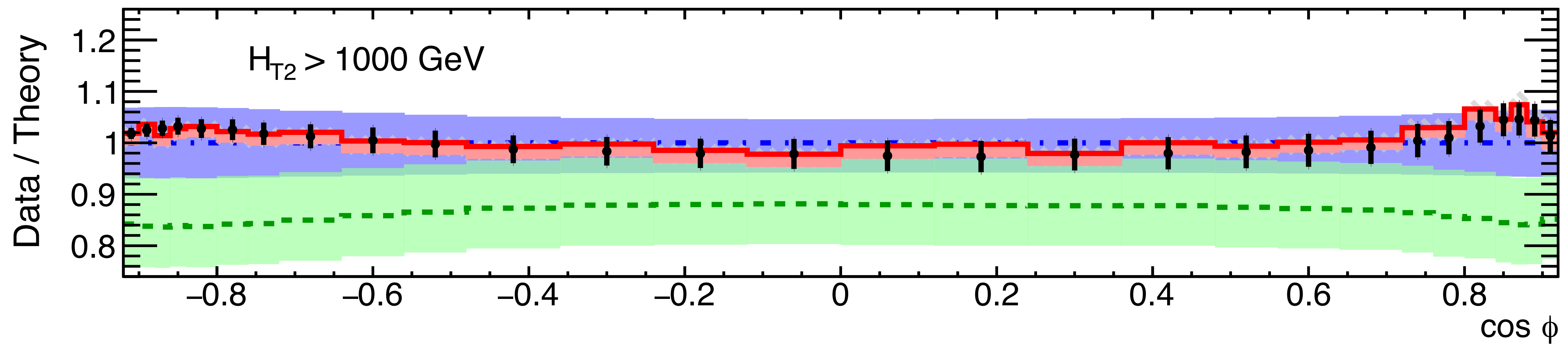


THE 2 → 3 FRONTIER:

among the most complex NNLO calculations

- $pp \to \gamma\gamma\gamma$
  [Chawdhry, Czakon, Mitov, Poncelet '19]
  [Kallweit, Sotnikov, Wiesemann '20]
- $pp \to \gamma\gamma + j$
  [Chawdhry, Czakon, Mitov, Poncelet '21]
  (gluon-fusion @ NLO ≃ N³LO)
  ↪ [Badger, Gehrmann, Marcoli, Moodie '21]
- $pp \to jjj$
  [Czakon, Mitov, Poncelet '21]
  ($gg \to ggg$; antenna automation)
  ↪ [Chen, Gehrmann, Glover, Huss, Marcoli '22]
- $pp \to Wb\bar{b}$
  [Hartanto, Poncelet, Popescu, Zoia '22]
  [Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22] (approx. m_b)
- $pp \to t\bar{t}H$
  [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]
- $pp \to \gamma + jj$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

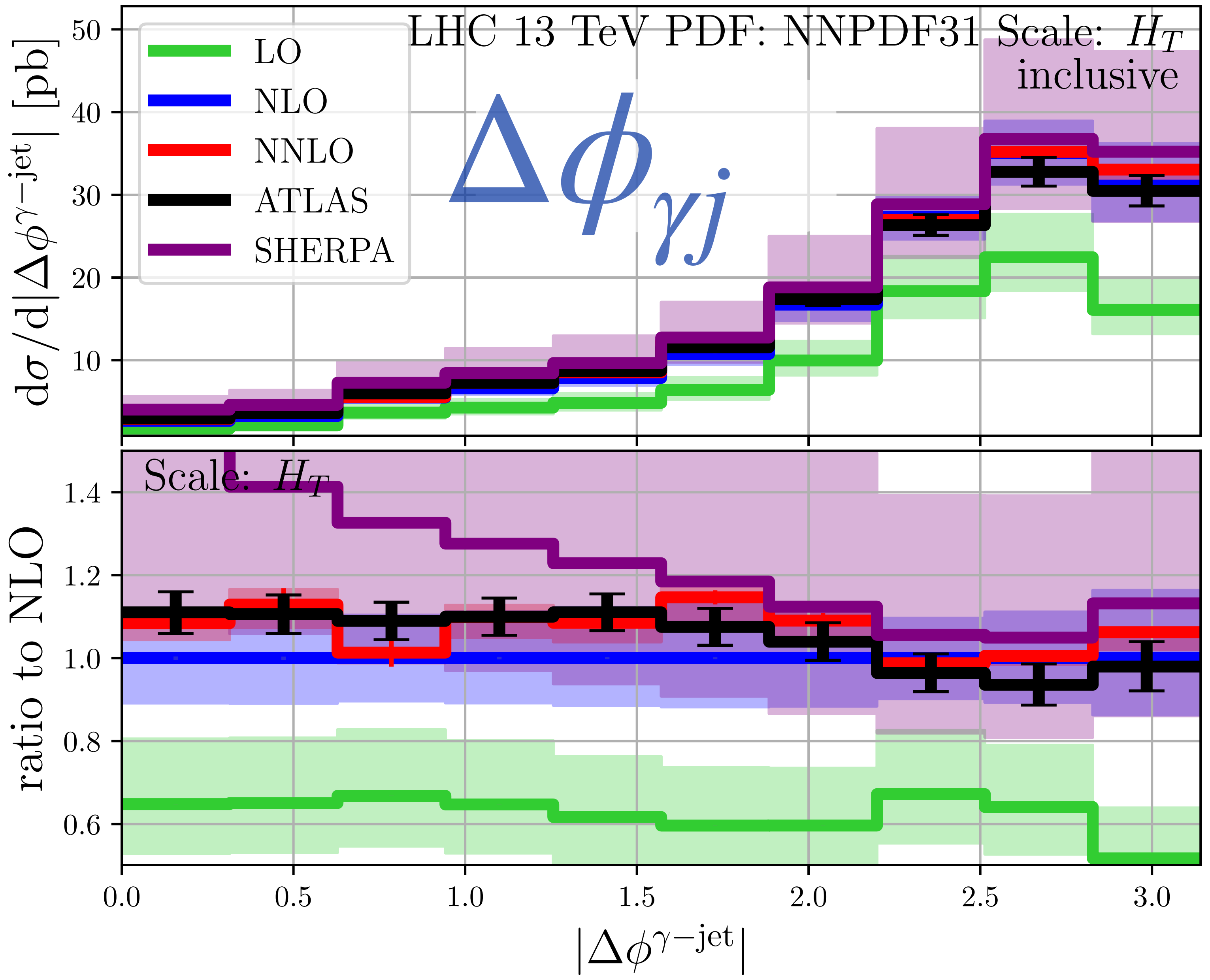
$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{\sigma}\int \sum_{i,j} d\sigma \frac{k_{T,i}\, k_{T,j}}{\sum_k k_{T,k}} \delta(\cos\phi - \cos\phi_{ij})$$

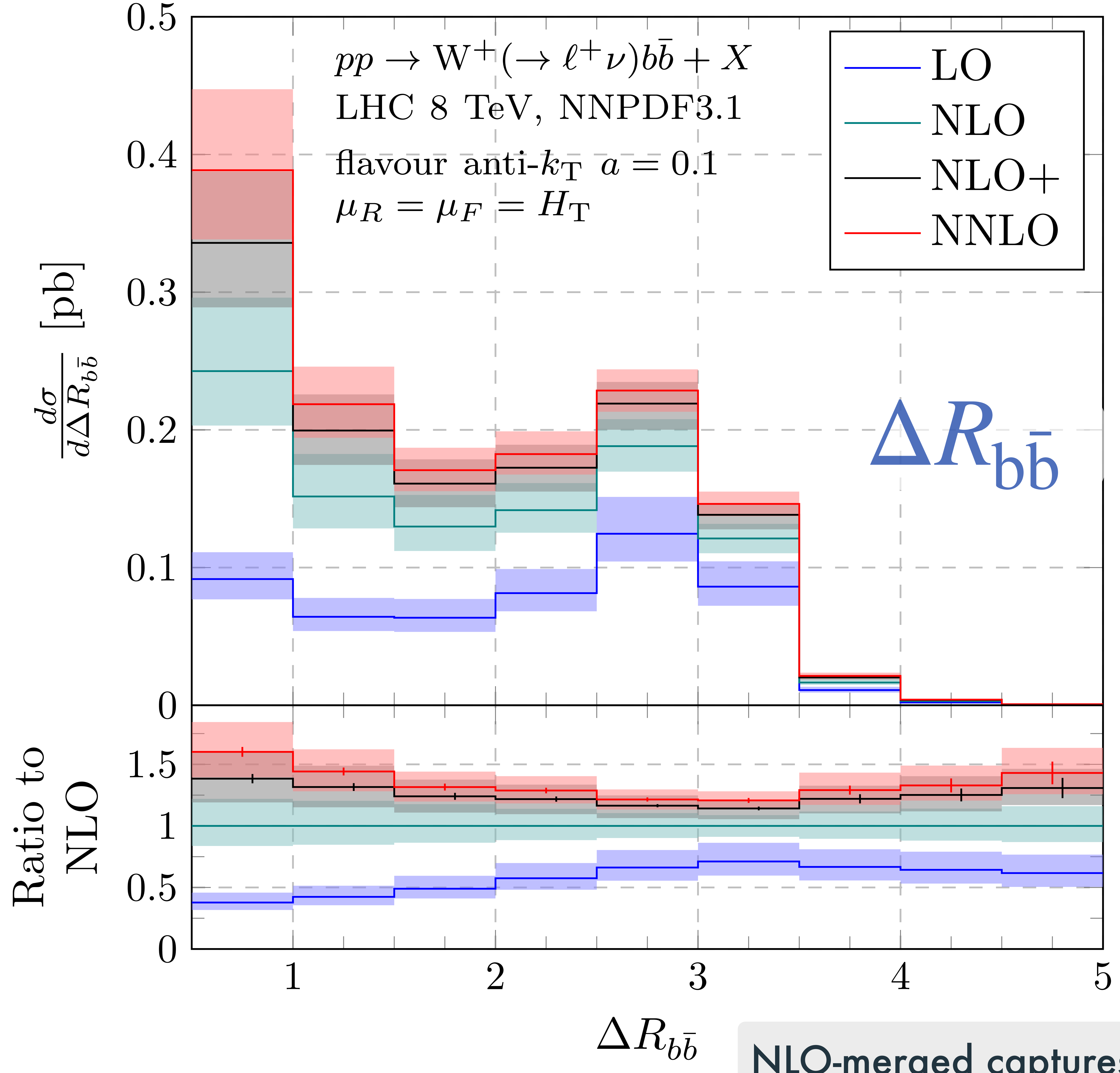


⇝ $\alpha_s(m_Z) = 0.1175 \pm 0.0006\,(\text{exp.})^{+0.0034}_{-0.0017}\,(\text{theo.})$

[ATLAS arXiv:2301.09351]

LO → NLO (× 1.6)
NNLO ⇝ pert. stabilisation & good description of shape
Sherpa (≃ NLO)
⇝ large uncertainties (?!) overshoots at low $\Delta\phi_{\gamma j}$





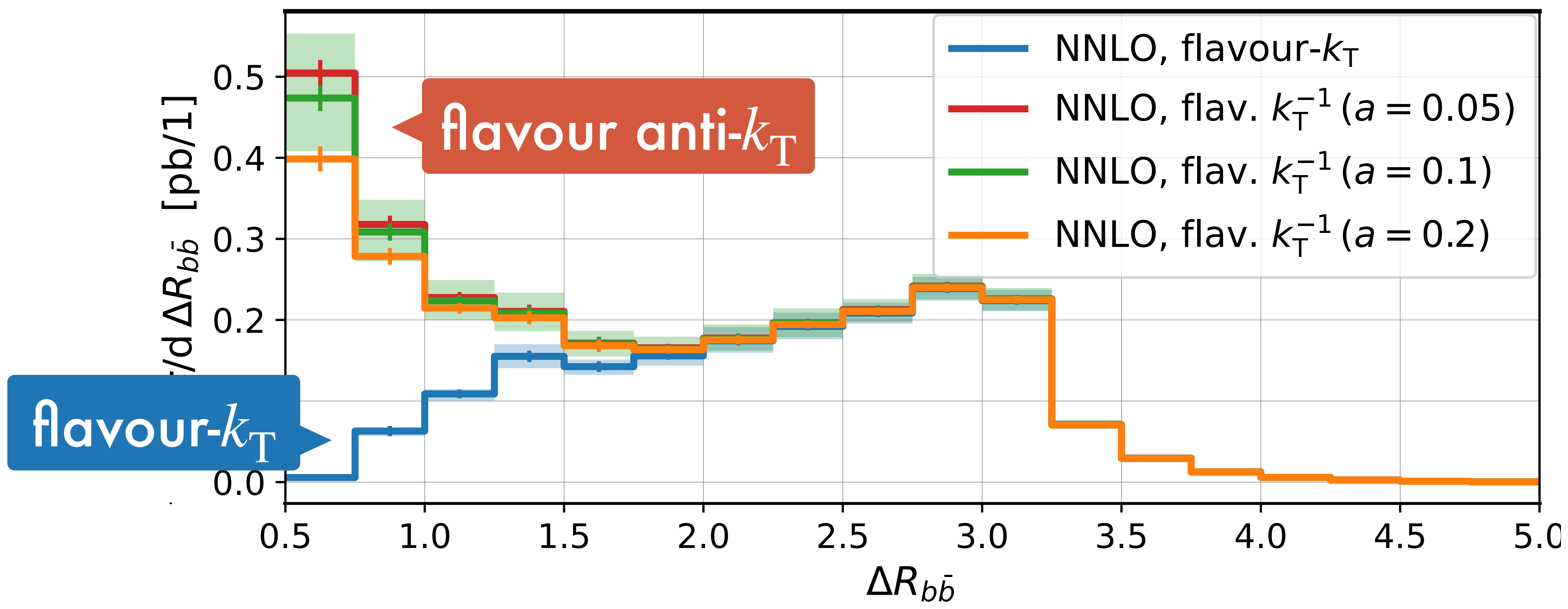
$$\sigma^{\text{NLO+}}_{\text{Wb}\bar{\text{b}},\,\text{inc.}} \equiv \sigma^{\text{NLO}}_{\text{Wb}\bar{\text{b}},\,\text{exc.}} + \sigma^{\text{NLO}}_{\text{Wb}\bar{\text{b}}j,\,\text{inc.}}$$

NLO-merged captures significant part of NNLO corrections

W+b$\bar{\text{b}}$

[Hartanto, Poncelet, Popescu, Zoia ’22]

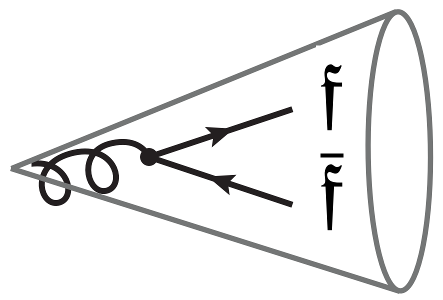
- first NNLO $2 \to 3$ w/ external mass
  - ⇝ *challenge:* 2-loop amplitude [Abreu et al. ’22]
- irreducible background to
  - ↪ *VH*, single top, BSM searches
- test perturbative QCD
  - ↪ large NLO corrections, 4FS vs. 5FS, modelling of *flavoured jets*



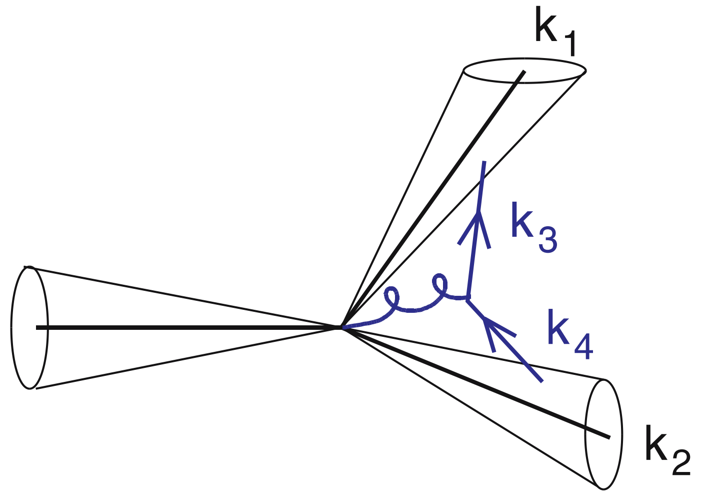
W+b$\bar{\text{b}}$ — Masses as Regulators

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22]

- use massive bottom quarks (4FS)



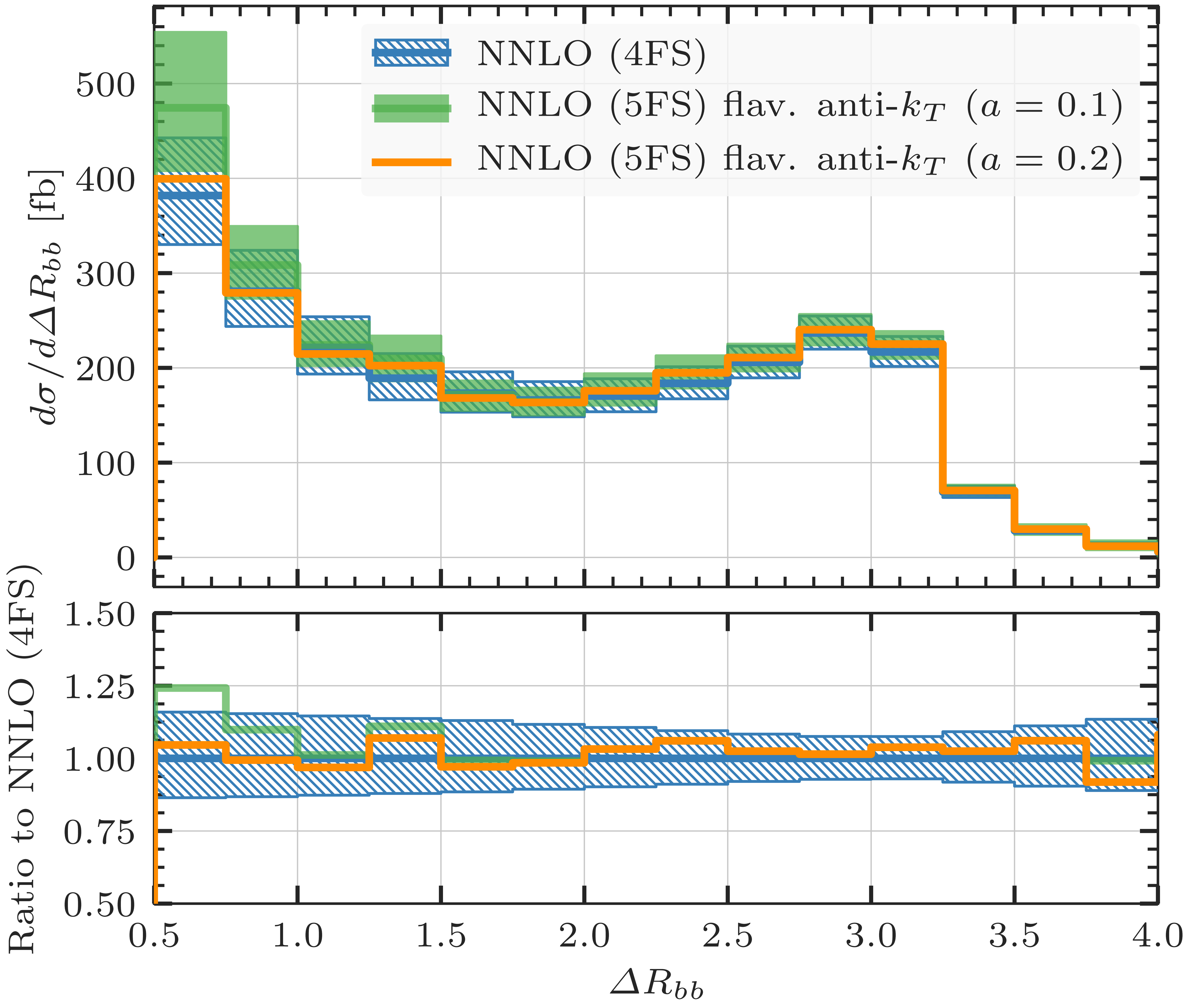
$\rightsquigarrow$ $\alpha_s \log(Q^2/m_f^2)$



$\rightsquigarrow$ $\alpha_s^2 \log(Q^2/m_f^2)$

- 2-loop amplitude
  unknown: 2 → 3 (w/ 2 masses)
  $\rightsquigarrow$ "massifivation" [Mitov, Moch '07]
  ⇒ approximate up to $\mathcal{O}(m_b^2/Q^2)$

- overall good agreement between
  4FS (blue) & 5FS (green, orange)



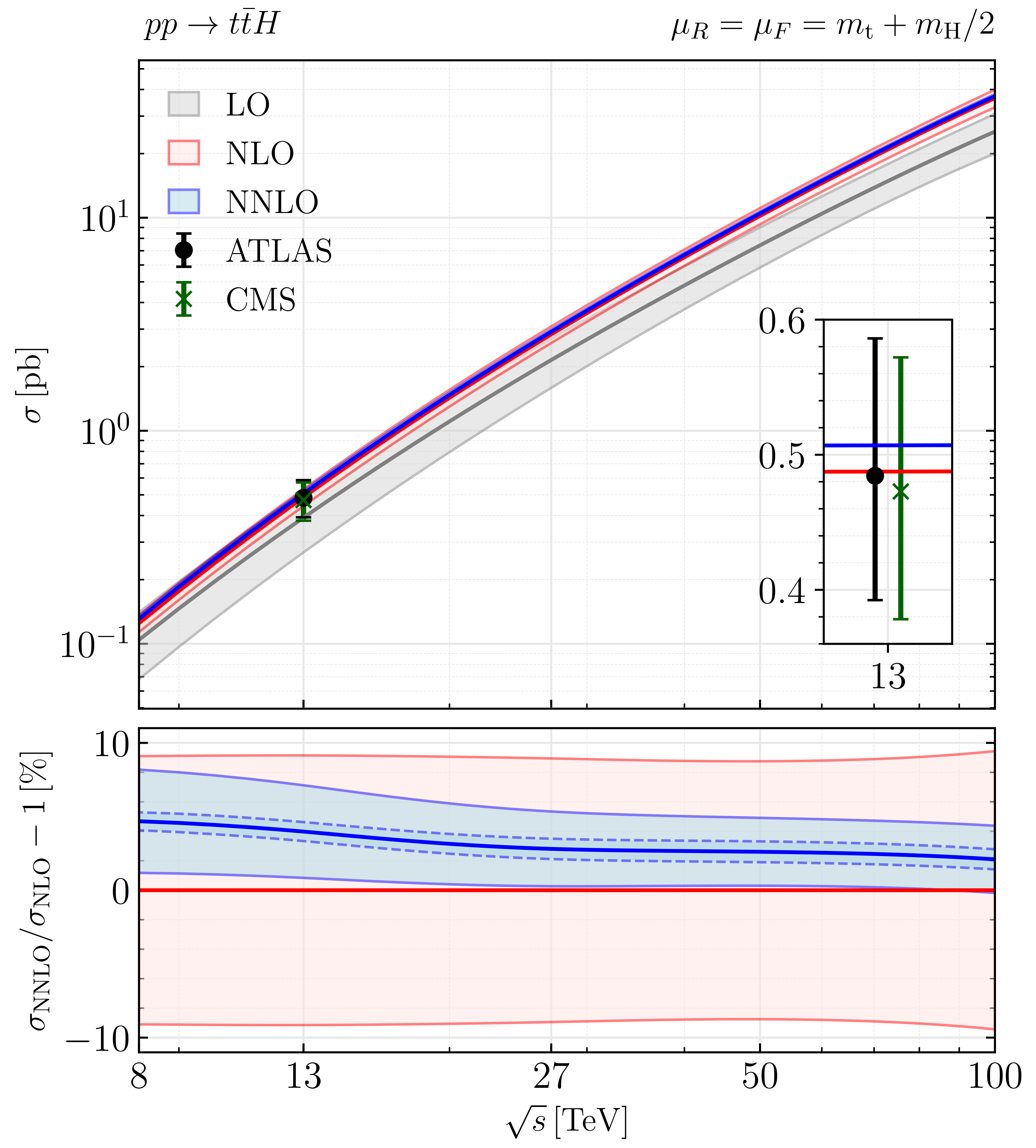
$t\bar{t}H$ — An Eikonal Higgs?

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini ’22]

- a direct probe of the top Yukawa
  ↪ HL-LHC projection (exp): $\mathcal{O}(2\%)$
- missing ingredient: 2-loop amplitude
  ⇝ $2 \to 3$ (+ 2 masses): current frontier
- *apply:* soft Higgs approximation

$$\mathcal{M}^{t\bar{t}H}(p_t, p_{\bar{t}}, p_H) \simeq F(\alpha_s; m_t/\mu_R)\, J(p_H)\, \mathcal{M}^{t\bar{t}}(p_t, p_{\bar{t}})$$

- $\Delta^{\rm NLO}_{\rm scl} \simeq \pm 9\,\%$ $\gg$ $\Delta^{\rm NNLO}_{\rm scl} \simeq \pm 3\,\%$
- error estimate for approximation
  ⇝ $\pm 0.6\,\%$ (▭) on NNLO

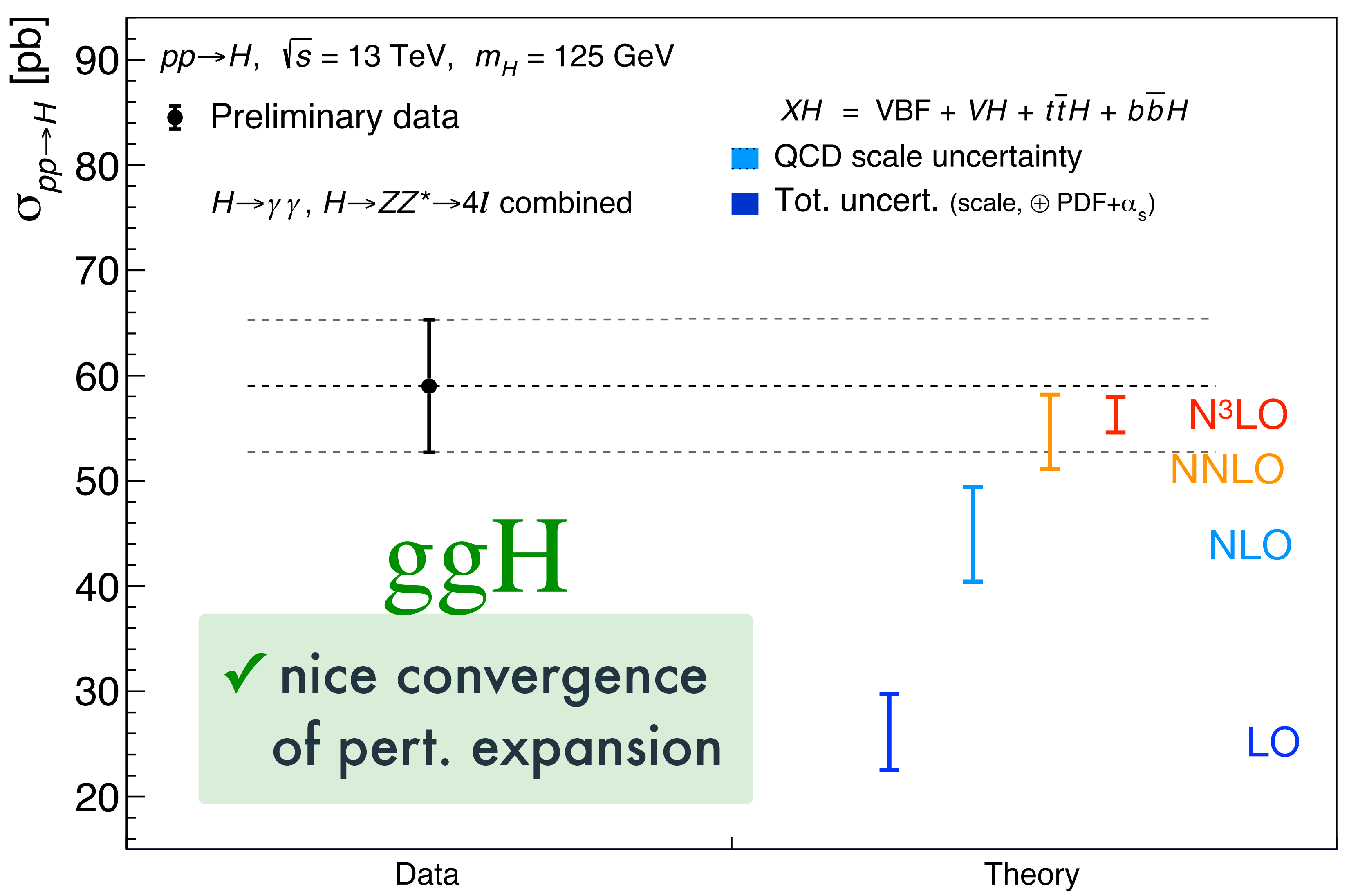


future: valid approximation also for $t\bar{t}Z$ & $t\bar{t}W^{\pm}$?

Going Beyond ⇝ N³LO

Some processes require us to even push to the next order:

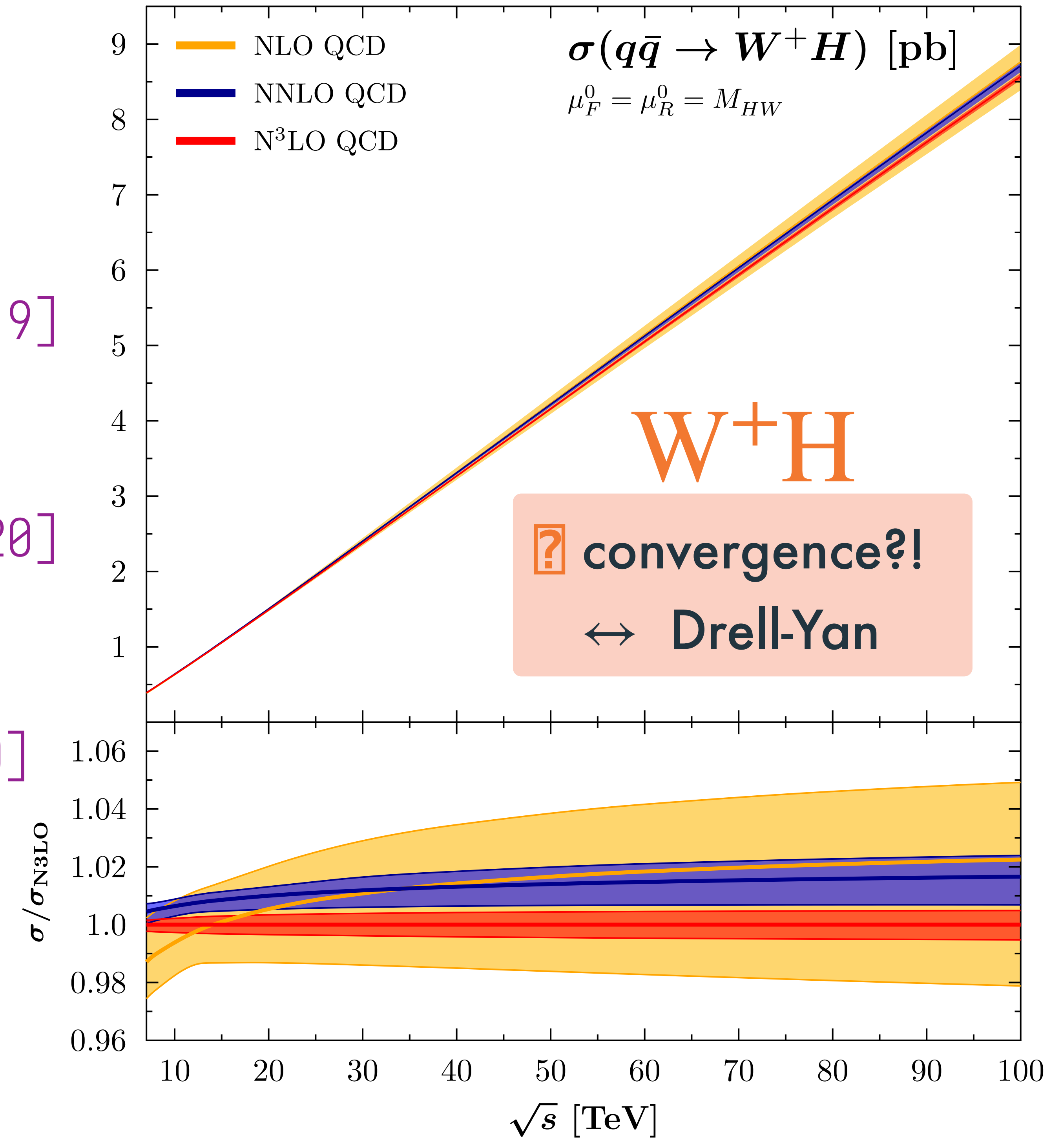
- "Standard candles"
  - ↪ very precisely measured
- slow perturbative convergence
  - ↪ pp → $\gamma\gamma$
  - ↪ Higgs production (gg → H)



[Anastasiou et al. '15]

Fully Inclusive calculations ⇝ σ_{tot}

- gg → H ✓
  [C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger '15]
- VBF-H ✓, VBF-HH ✓
  [F. Dreyer, A. Karlberg '16, '18]
- $b\bar{b}$ → H ✓
  [C. Duhr, F. Dulat, B. Mistlberger '19]
- pp → γ^* ?, pp → $W^\pm$?
  [C. Duhr, F. Dulat, B. Mistlberger '20]
- gg → HH ✓
  [L. Chen, H. Li, H. Shao, J. Wang '20]
- pp → γ^*/Z ?
  [C. Duhr, B. Mistlberger '21]
- pp → VH ?
  [J. Baglio, C. Duhr, B. Mistlberger, R. Szafron '22]



Fully Differential ggH @ N³LO

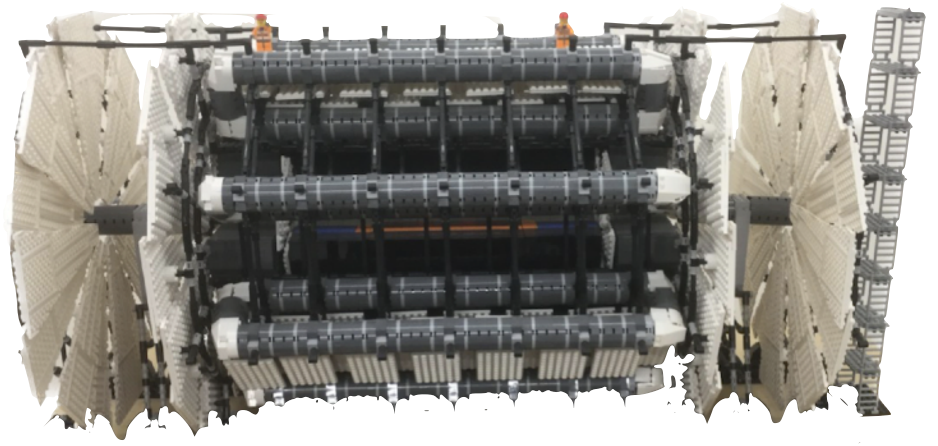
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

Fully Inclusive

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}$(sec)

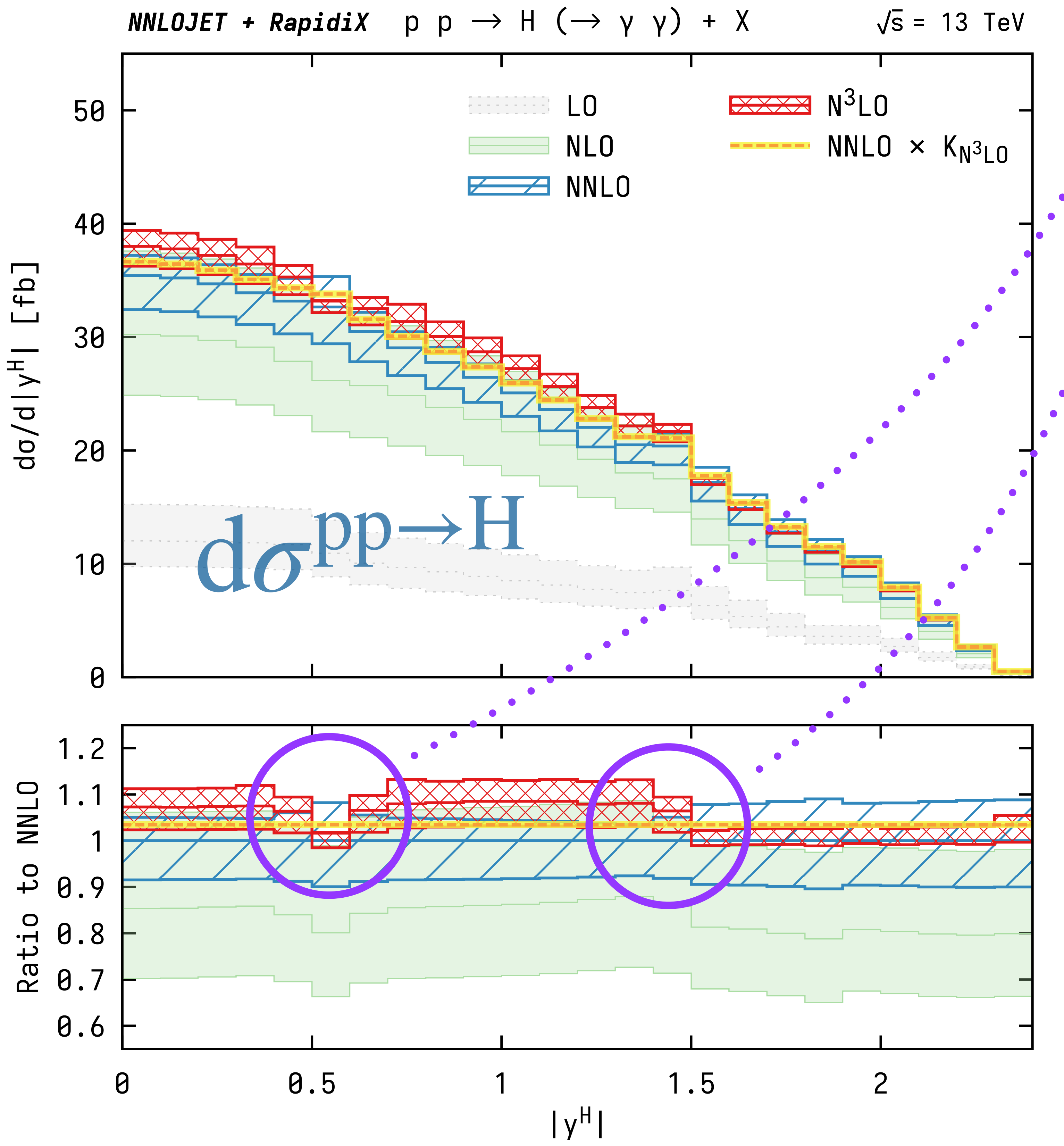
Fully Differential



✓ $d\sigma$ ⇝ fiducial cuts, arbitrary distributions, …

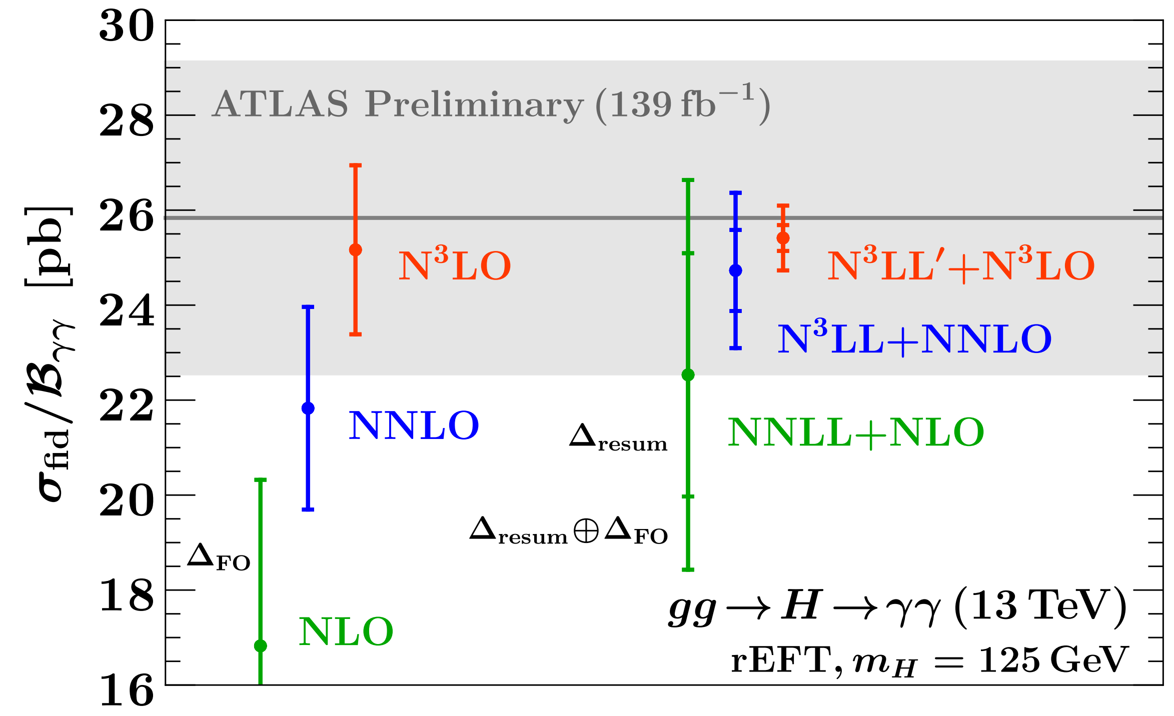
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

ATLAS Cuts



linear fiducial power corrections ⇝ *instabilities*

[Billis, Dehnadi, Ebert, Michel, Tackmann '21]



⊕ can be cured by resummation

⊖ hard $\sigma^{\text{fid.}}$ should not need resummation

Fully Differential ggH @ N³LO

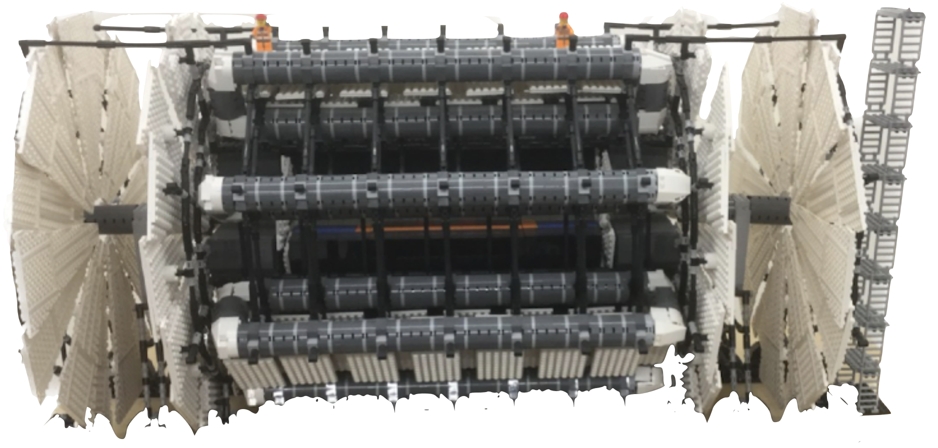
[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

Fully Inclusive

✗ limited to σ^{tot}

✓ very efficient $\mathcal{O}(\text{sec})$

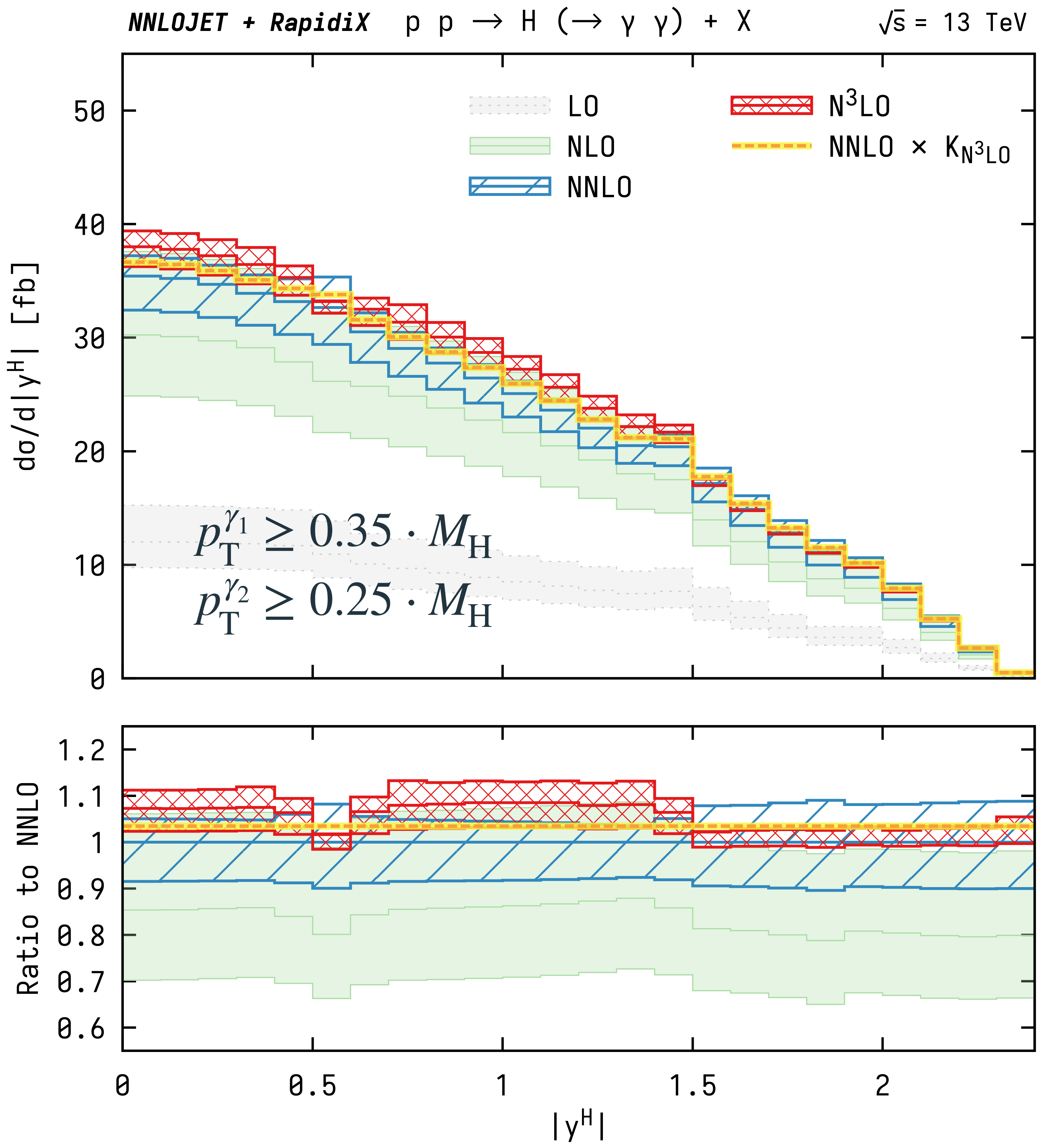
Fully Differential



✓ dσ ⇝ fiducial cuts, arbitrary distributions, …

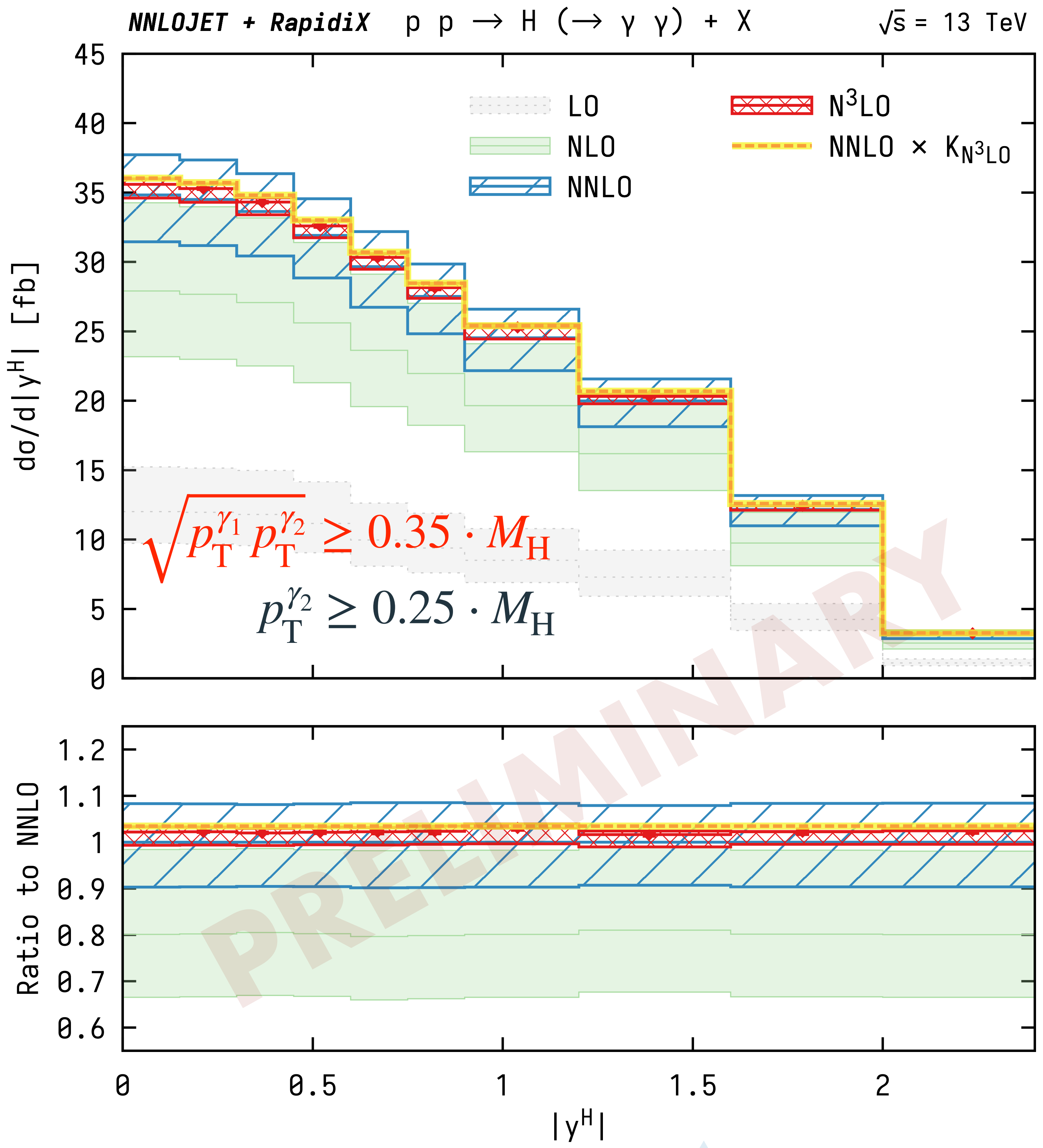
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

ATLAS Cuts



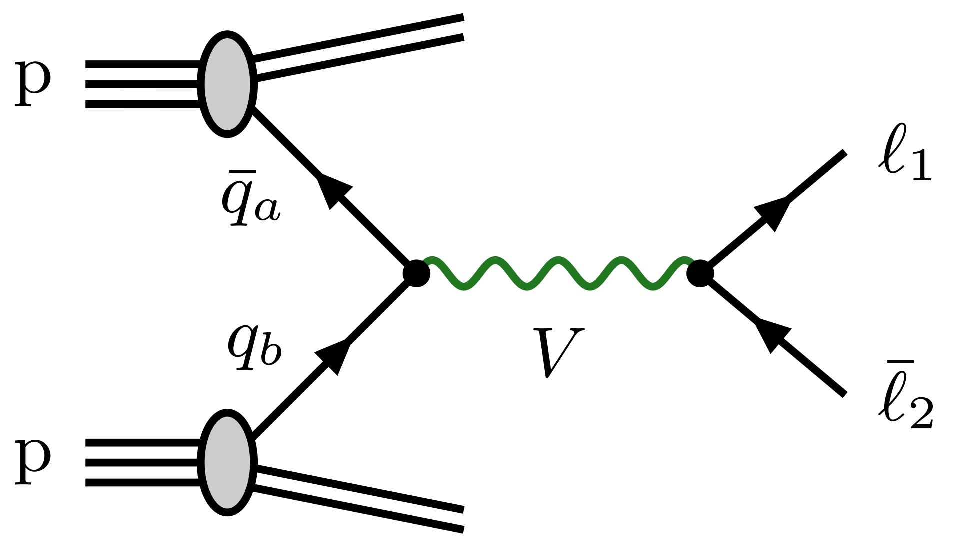
Product Cuts

~~lin. fid. power corr.~~
[Salam, Slade '21]



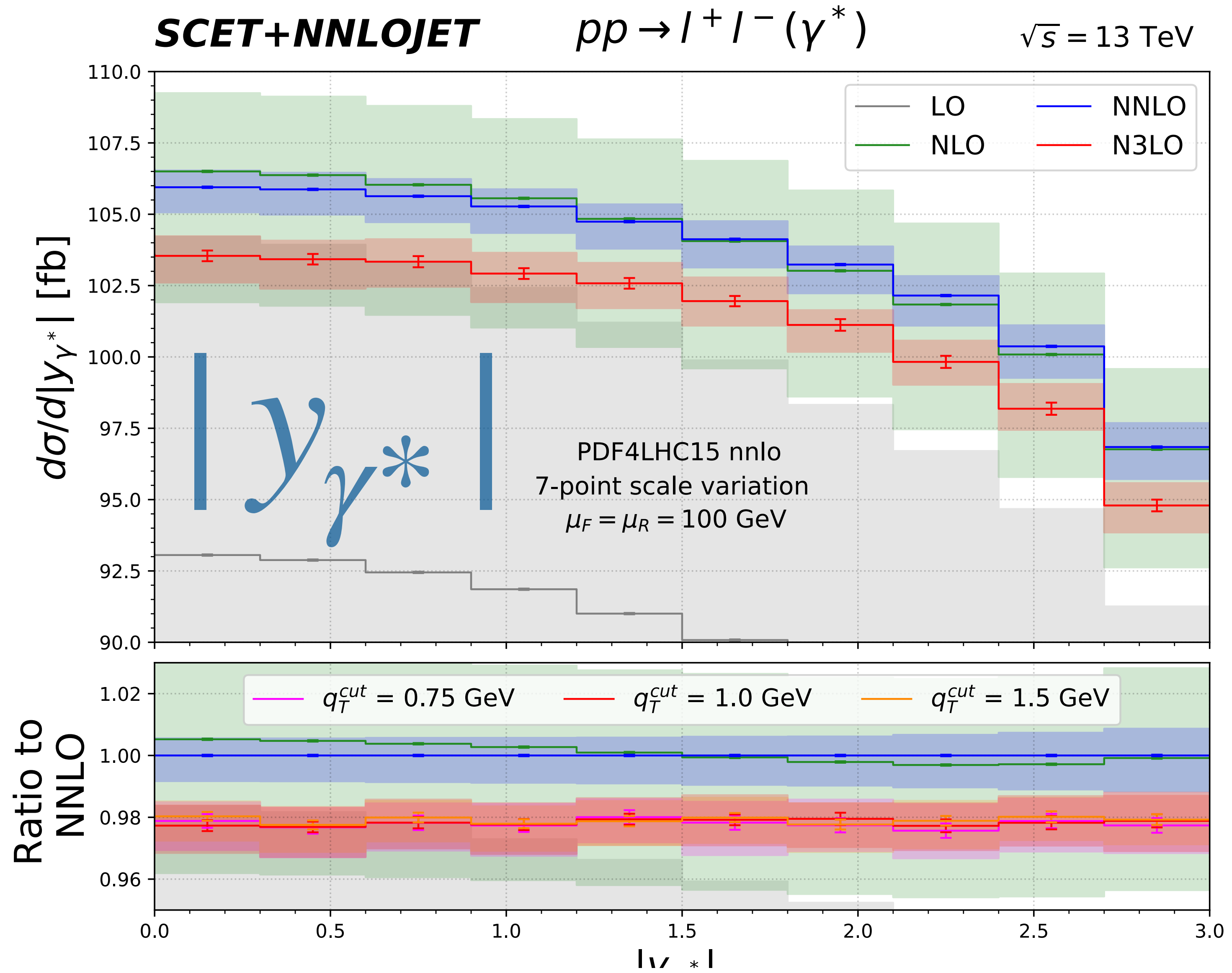
no instabilities & flat K-factor: N³LO $\simeq$ NNLO $\times$ $K_{\text{N}^3\text{LO}}$

Drell Yan — a Standard Candle

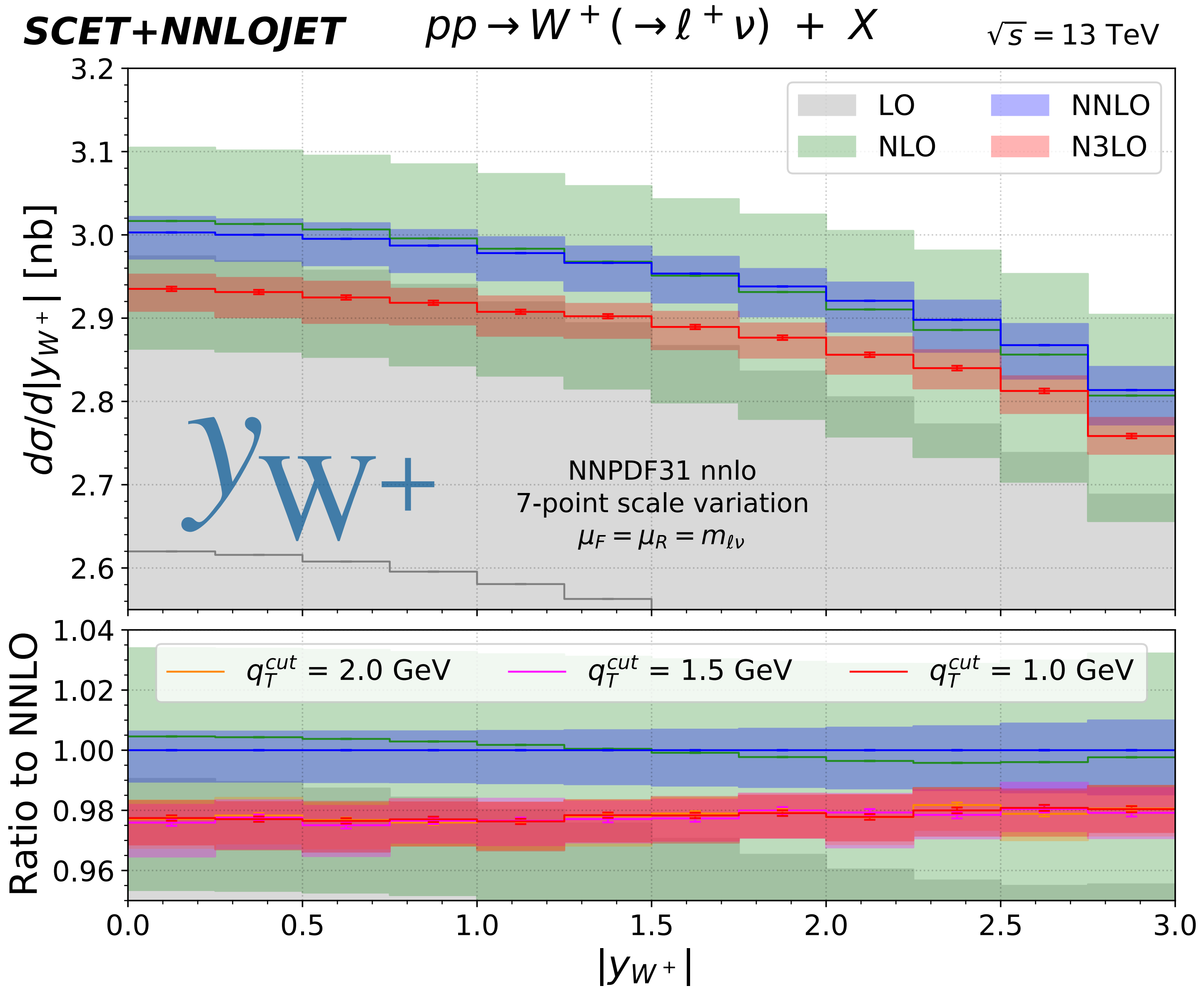


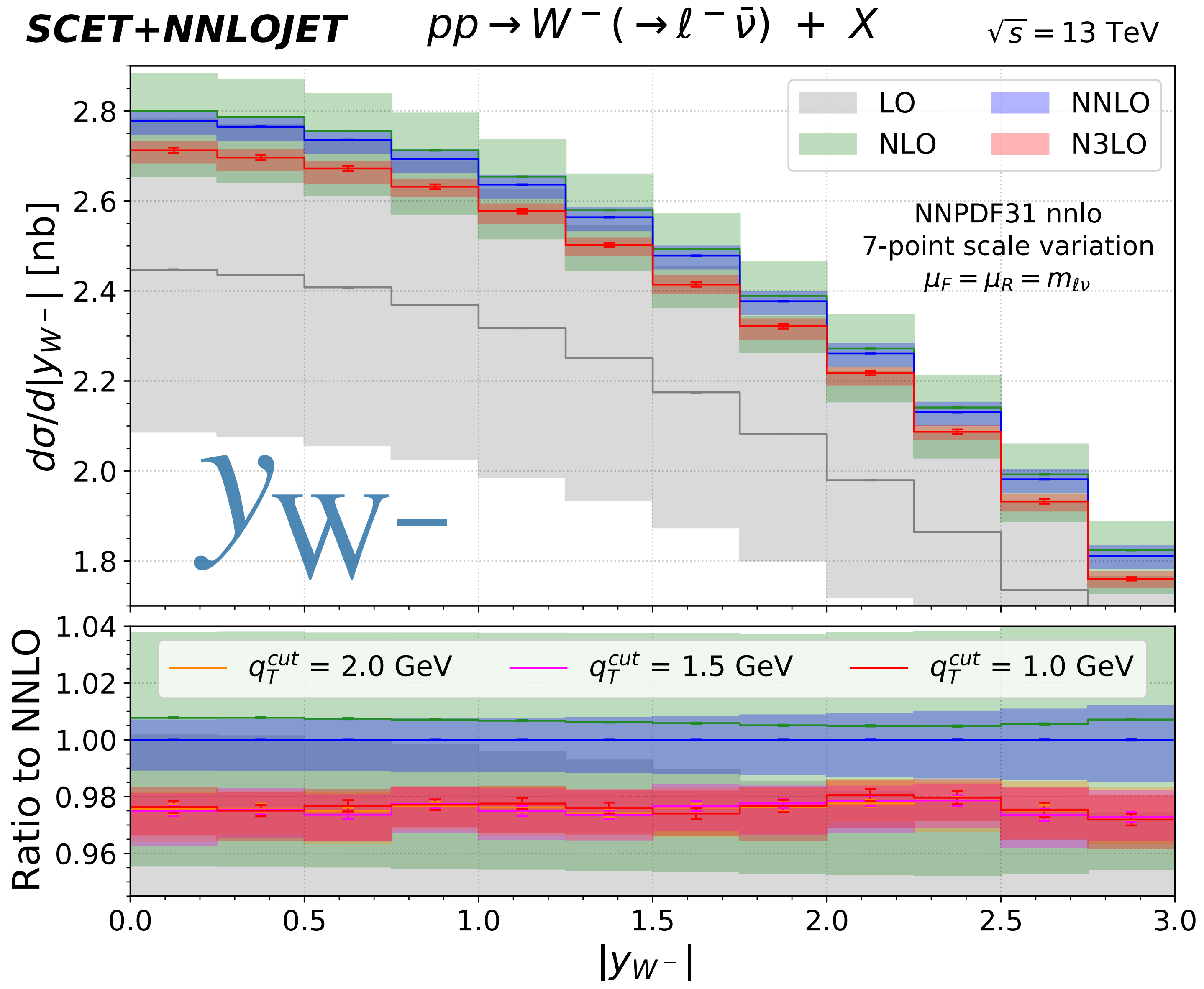
- clean signature $(\ell^\pm, E_T^{\text{miss}})$
  & large cross section:
  $(\sim 1000\ Z\ \&\ \sim 4000\ W^\pm)$ / sec *
- detector calibration, BSM searches, luminosity monitor, PDFs, …
- precision measurements:
  ↪ $\sin^2(\theta_w),\ M_W$

[Chen, Gehrmann, Glover, AH, Yang, Zhu '21,'22]



- almost universal NNLO → N³LO corrections!
- NC & CC± probe different parton content



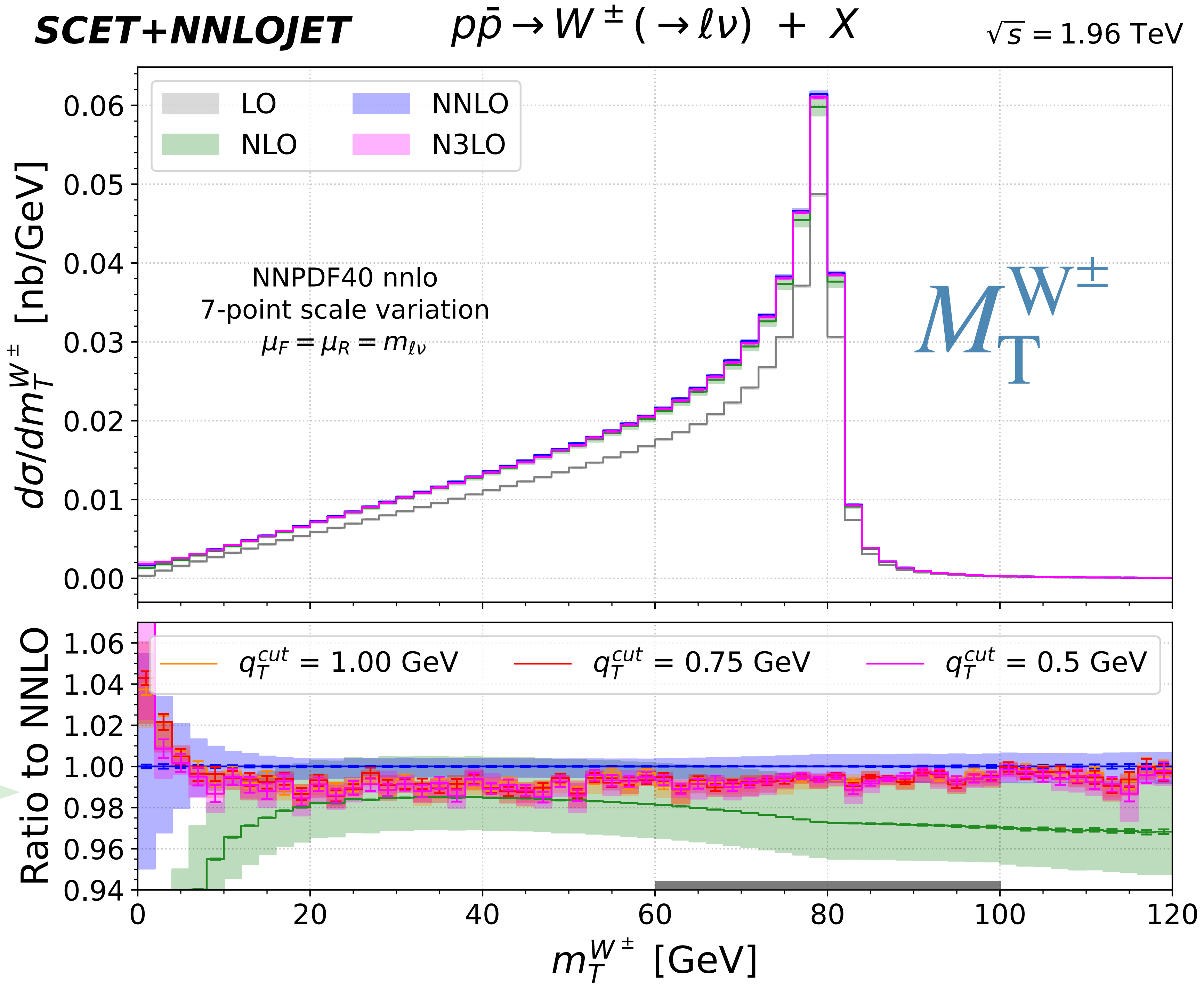


$^*\ \mathscr{L} = 2 \times 10^{34}\ \text{cm}^{-2}\ \text{s}^{-1}$

W Production — Absolute Spectrum

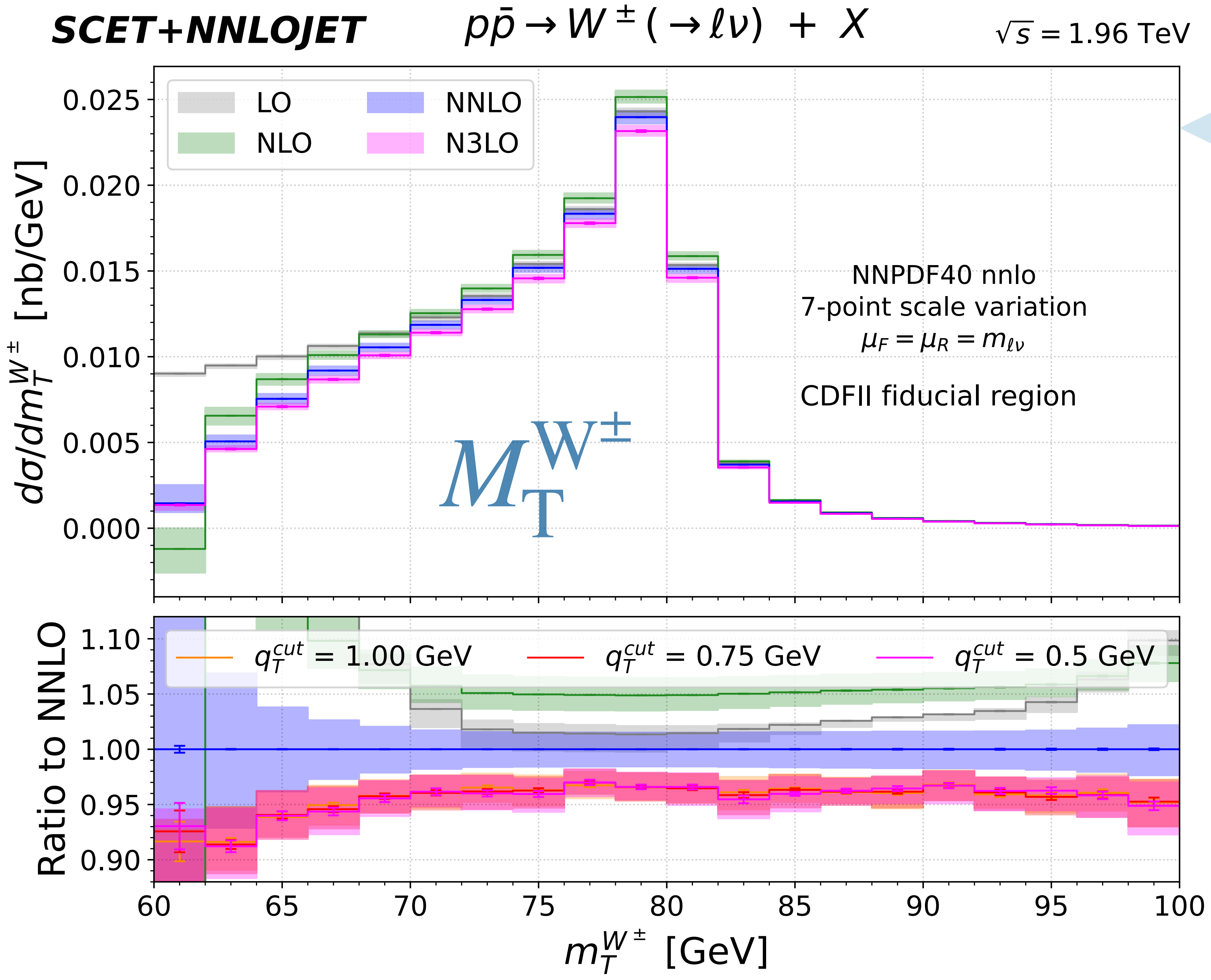
$$M_T^W \equiv \sqrt{E_T^\ell \, E_T^\nu \, (1 - \cos \Delta\varphi_{\ell\nu})}$$

Inclusive



1.96TeV – better convergence as seen in σ^{tot}

Fiducial (CDF II)



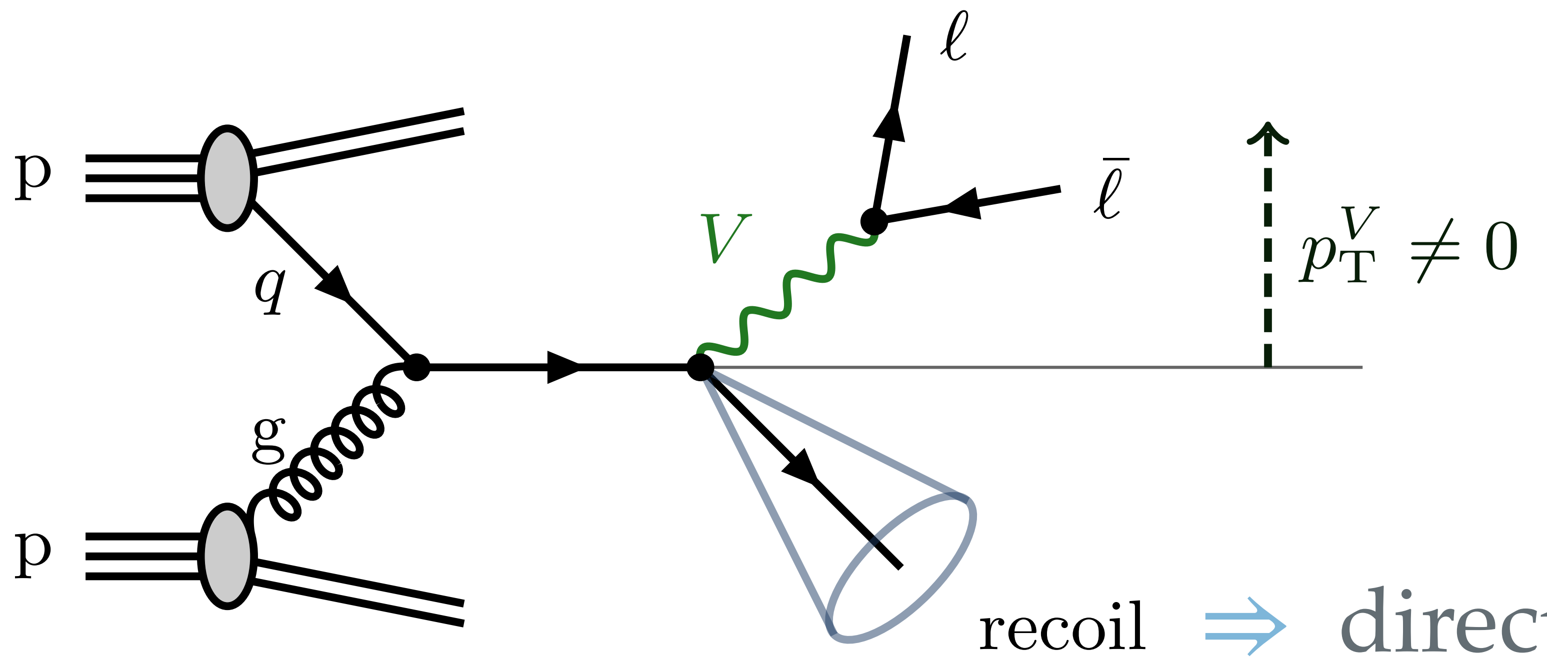
$p_T^\ell,\, E_T^{miss} \in [30, 55]$ GeV
$|\eta^\ell| < 1$
$p_T^W < 15$ GeV

LO
NLO
NNLO
N3LO

- remain largely flat around peak; larger corrections at low M_T^W
- fiducial cuts impact pattern of radiative corrections
- larger N^3LO corrections ($-1\,\%$ [inc.] vs. $-4\,\%$ [fid.])

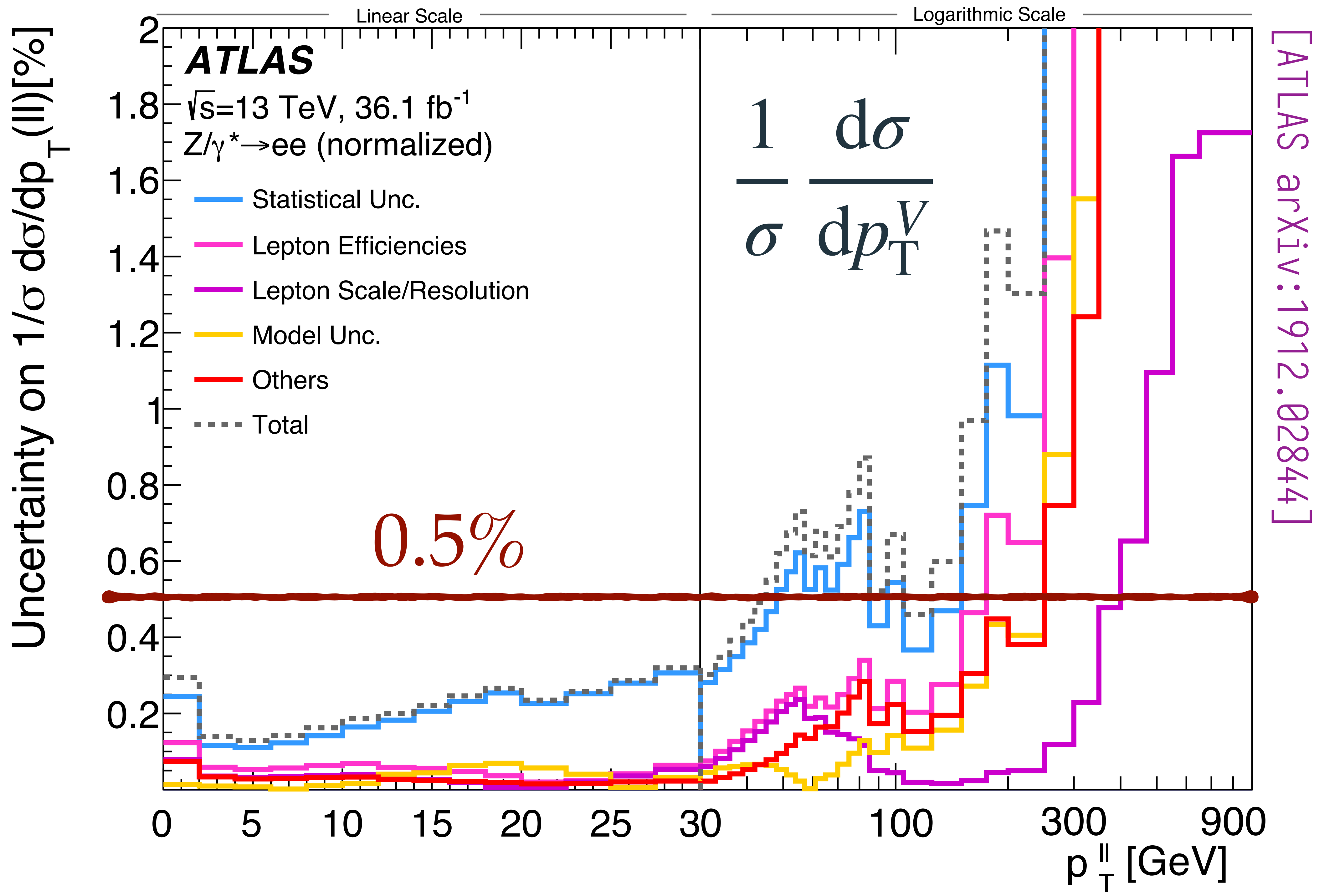
fiducial power corrections?

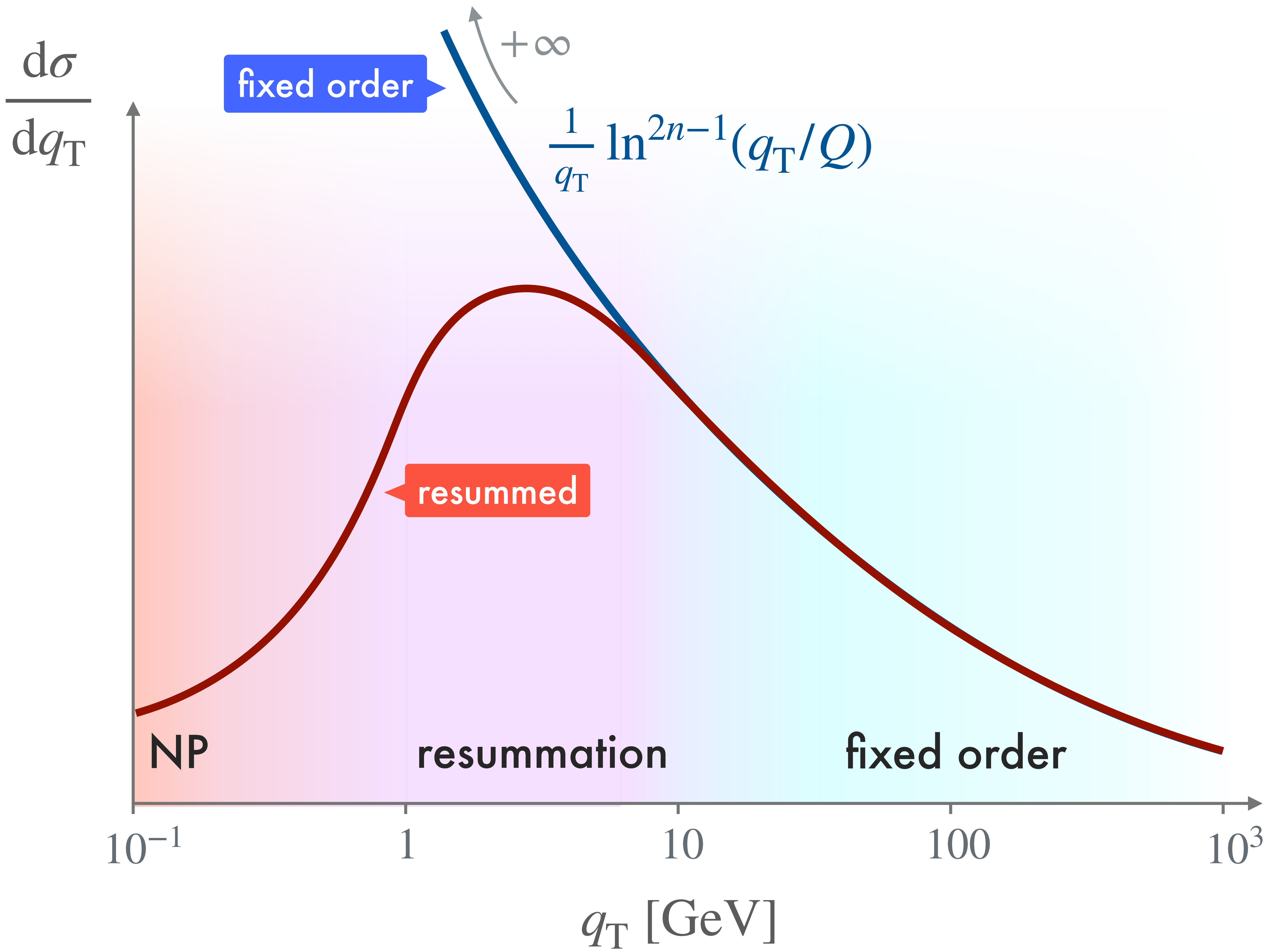
The Transverse Momentum



⇒ direct sensitivity to α_s

& PDFs (high-x gluon)





precision TH tests

↪ non-perturbative QCD ⟩ quark masses ⟩ resummation ⟩ fixed-order ⟩ EW Sudakovs ⟩ …

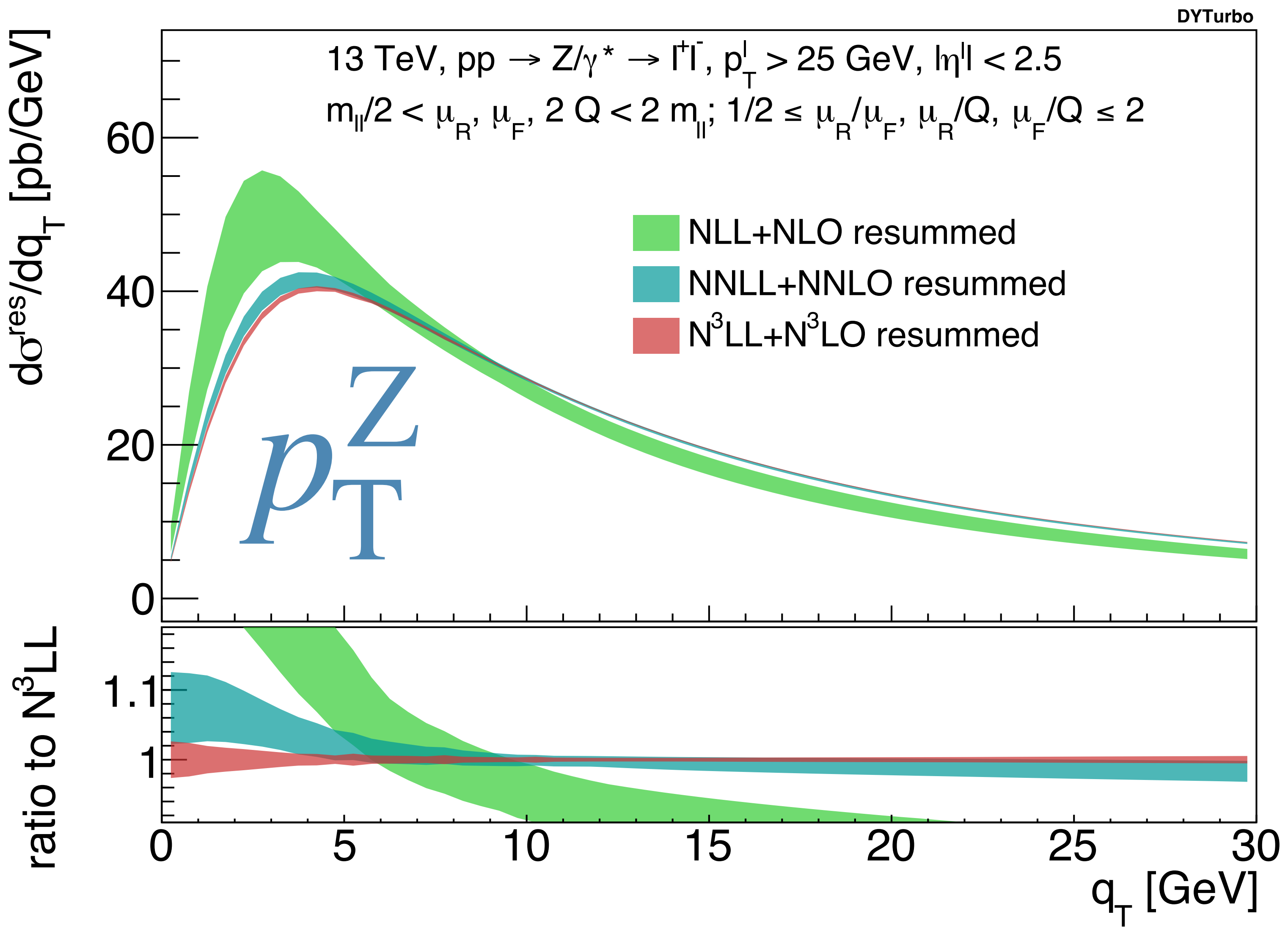
↪ crucial ingredient in many precision measurements

N3LO + Resummation

improved convergence ⇝ uncertainties: *few %*

DYTurbo [Camarda, Cieri, Ferrera '22]

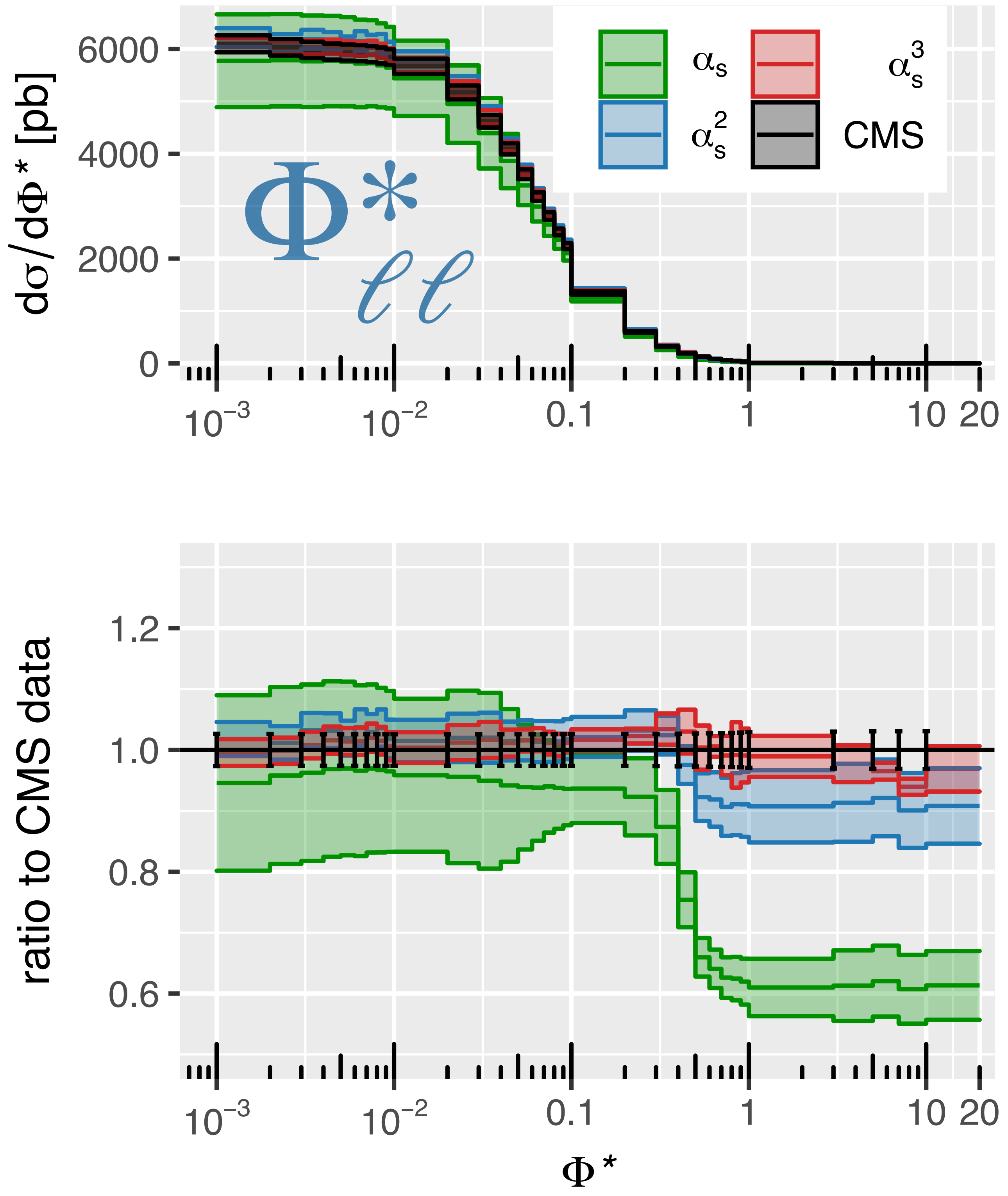
- more robust & reduced uncertainties



talk by A. Autieri (QCD×QED)

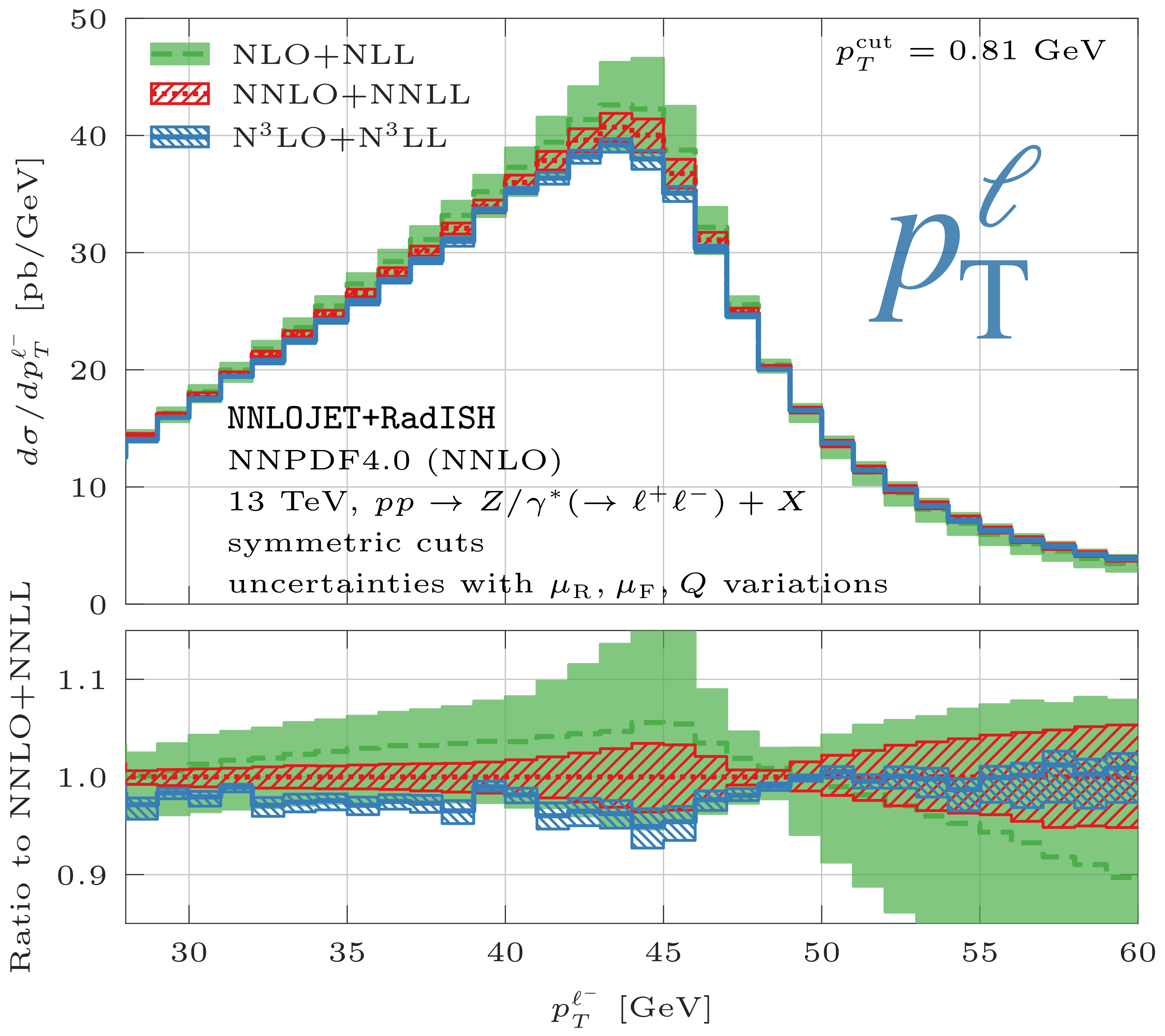
CuTe-MCFM [Neumann, Campbell '22]

- good agreement with data



NNLOjet+RadISH

[Chen, Gehrmann, Glover, AH, Monni, Rottoli, Re, Torrielli '22]



- some shape distortion ⇝ impact on M_W for CC$^\pm$?

CONCLUSIONS & OUTLOOK

- perturbative calculations *crucial* to scrutinise the Standard Model
  ⇝ exploit the full potential of the LHC / future colliders & uncover subtle hints for New Physics
- NNLO in good shape (reduced uncertainties & improved TH-data comparison)
  - $2 \to 2$ largely done, steady progress for $2 \to 3$ ⇝ methods reaching maturity
  - *loop amplitudes* becoming a bottleneck again ⇝ approximations in the interim
  - identified objects ⇝ photon isolation, flavour tagging, hadron fragmentation, …
- N^{3}LO computation of *inclusive* $2 \to 1$ processes mature
  - differential predictions for $\mathrm{pp} \to$ "colour neutral" appearing
    ⇝ $\mathrm{pp} \to \gamma\gamma$, $\mathrm{pp} \to VH$ within reach
- percent-level phenomenology: *everything becomes relevant*
  ⇝ PDFs, parametric, QCD×EW, non-perturbative, …

Conclusions & Outlook

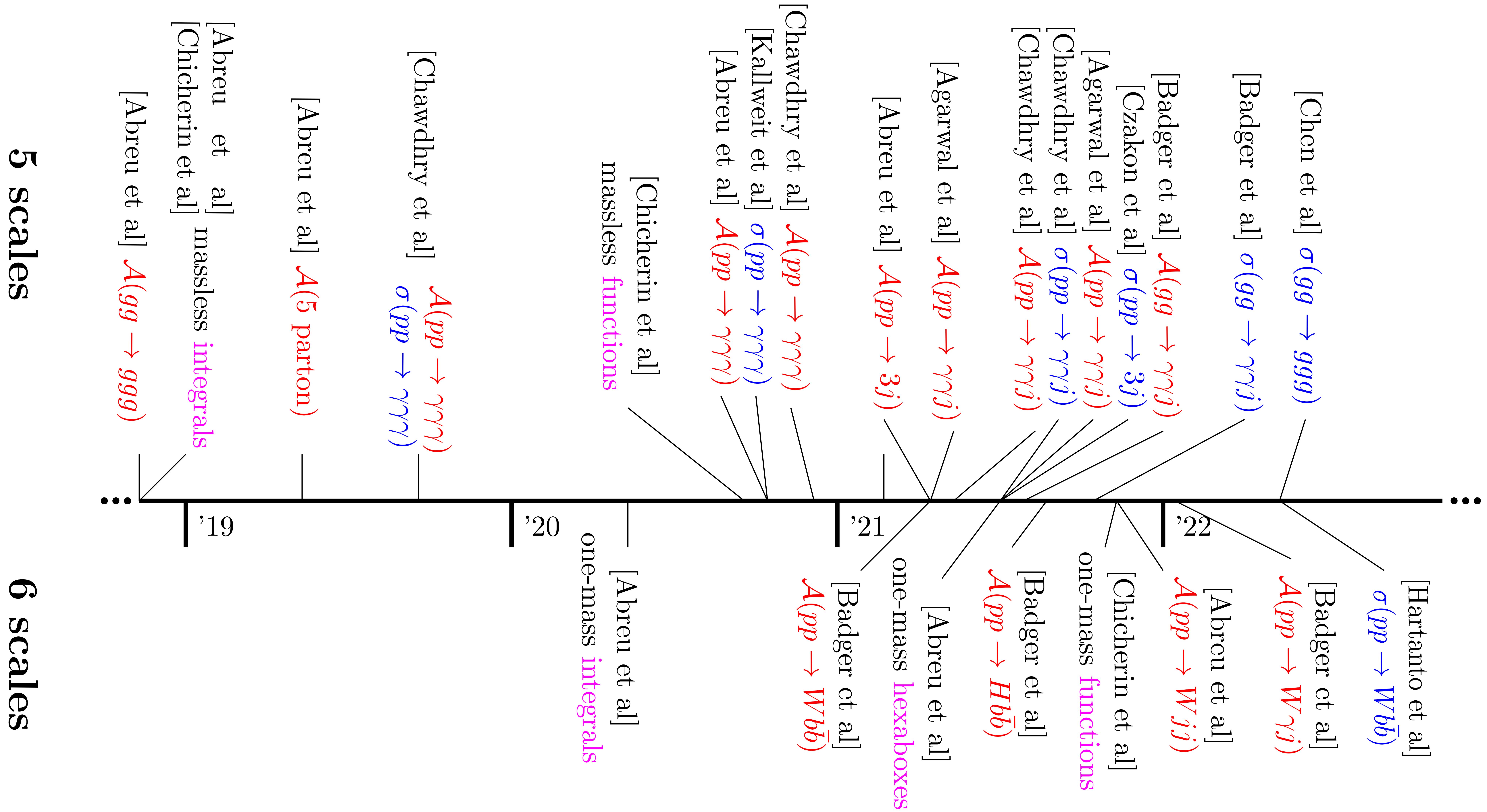
- perturbative calculations *crucial* to scrutinise the Standard Model
  ⇝ exploit the full potential of the LHC / future colliders & uncover subtle hints for New Physics
- NNLO in good shape (reduced uncertainties & improved TH-data comparison)
  - $2 \to 2$ largely done, steady progress for $2 \to 3$ ⇝ methods reaching maturity
  - *loop amplitudes* becoming a bottleneck again ⇝ approximations in the interim
  - identified objects ⇝ photon isolation, flavour tagging, hadron fragmentation, …
- N^3LO computation of *inclusive* $2 \to 1$ processes mature
  - differential predictions for pp → "colour neutral" appearing
    ⇝ $pp \to \gamma\gamma$, $pp \to VH$ within reach
- percent-level phenomenology: *everything becomes relevant*
  ⇝ PDFs, parametric, QCD×EW, non-perturbative, …

Thank you!

BACKUP

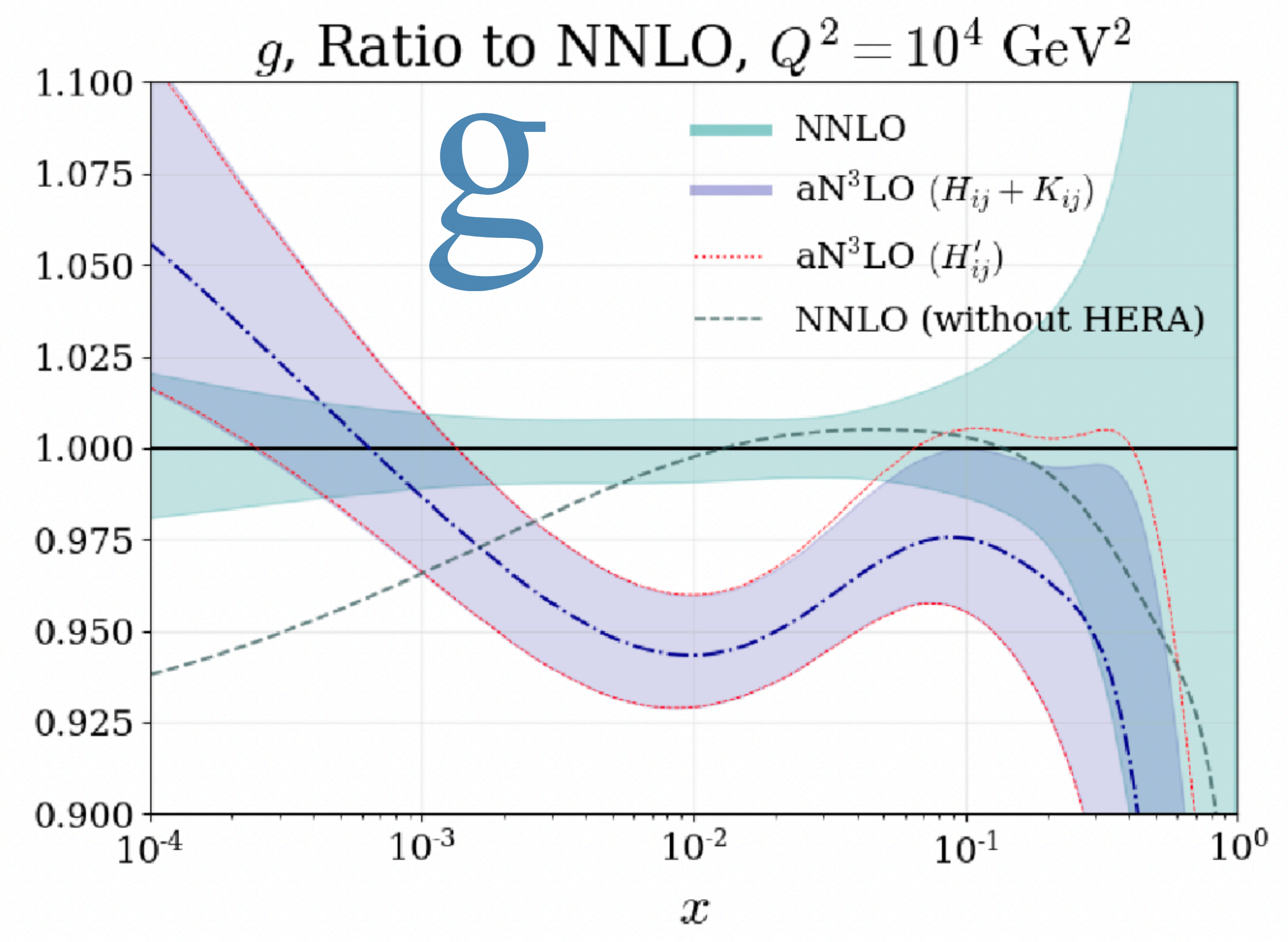
Two-Loop $2 \to 3$ Timeline

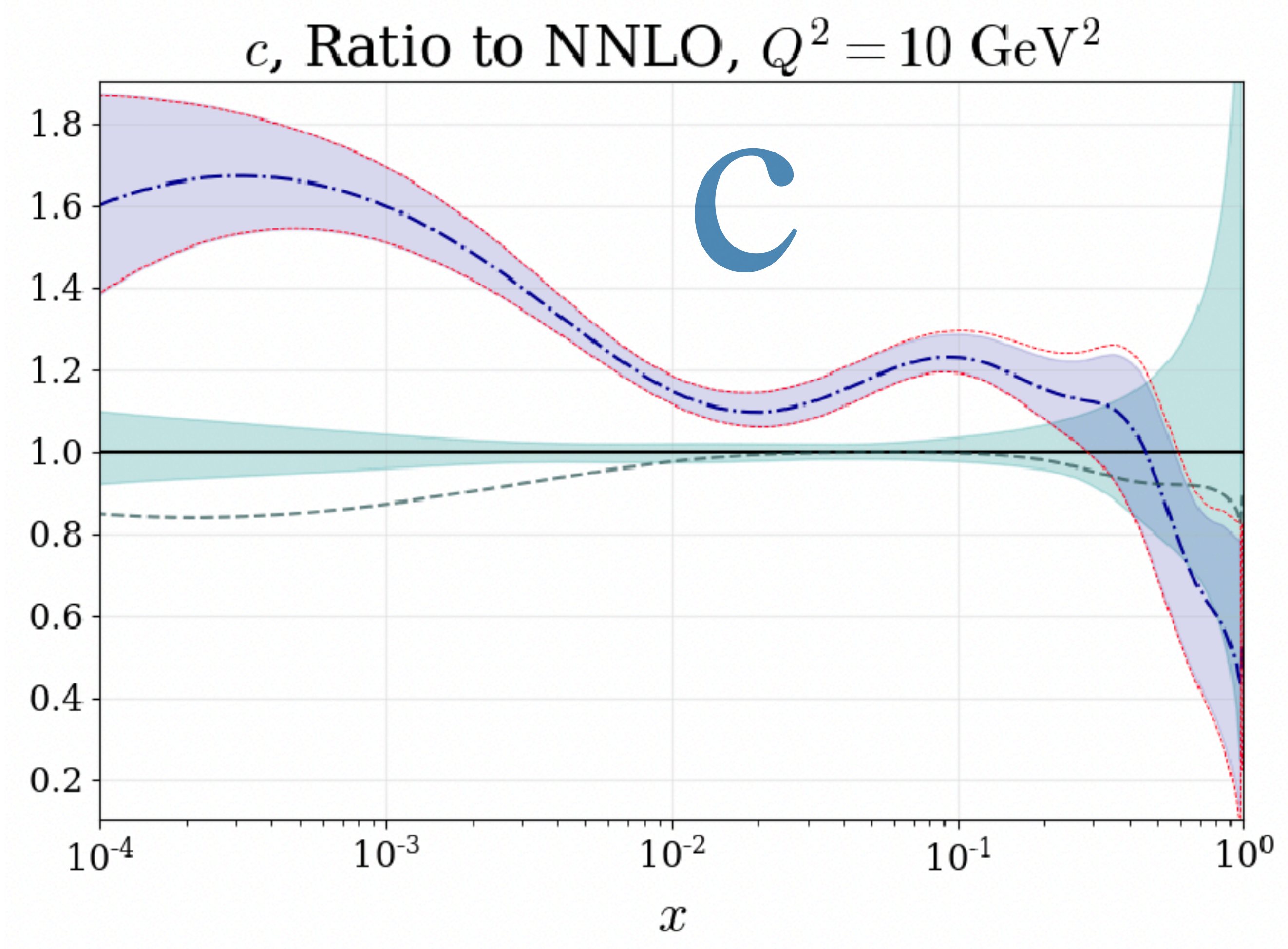
[slide by B. Page — HP2 '22]



N^3LO Parton Distribution Functions

- N3LO evolution
  - ⇝ 4-loop splitting functions
    [Moch, Ruijl, Ueda, Vermaseren, Vogt '17,'18,'22, in progress]
- aN3LO PDFs (MSHT)
  [McGowan, Cridge, Harland-Lang, Thorne '22]

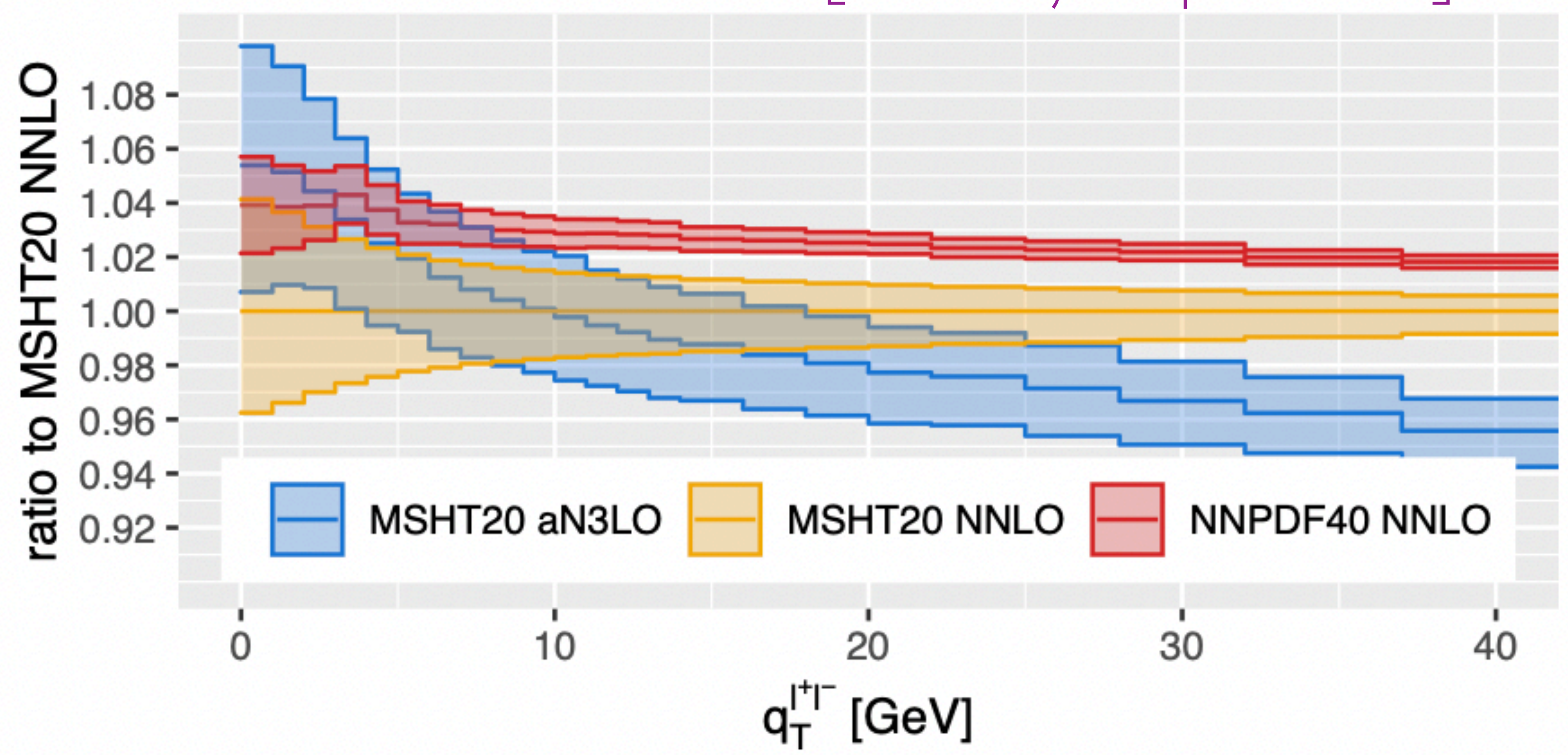




ggH: $\delta\sigma^{N^3LO}$ ↘ VBF: $\delta\sigma^{N^3LO}$ ↗

- purely resummed p_T^Z spectrum
  - ⇝ PDF uncertainties

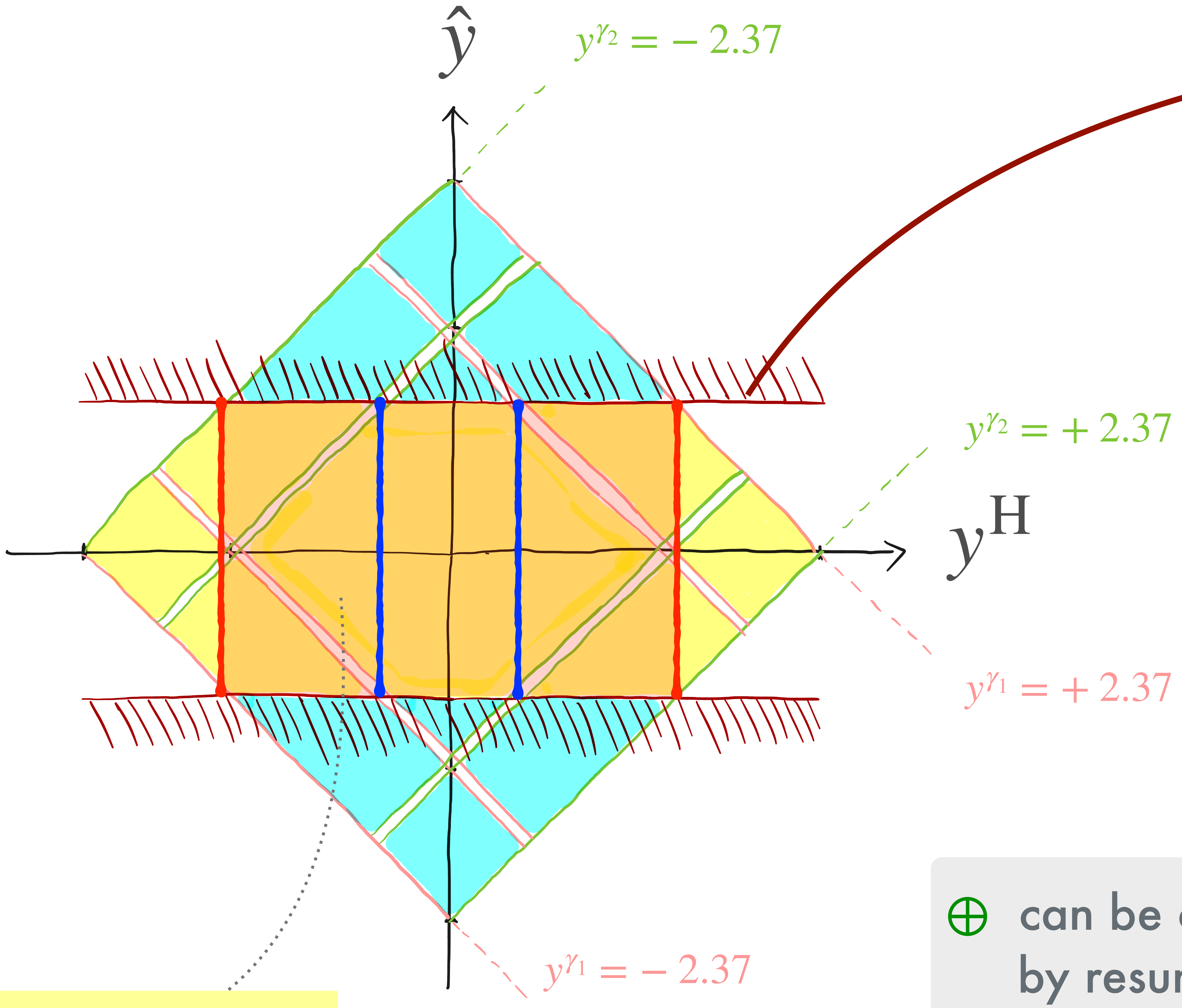
[Neumann, Campbell '22]



syst. differences between PDFs

PDF (NNLO → N³LO) $\delta\sigma^{N^3LO}$ ↗ (?)

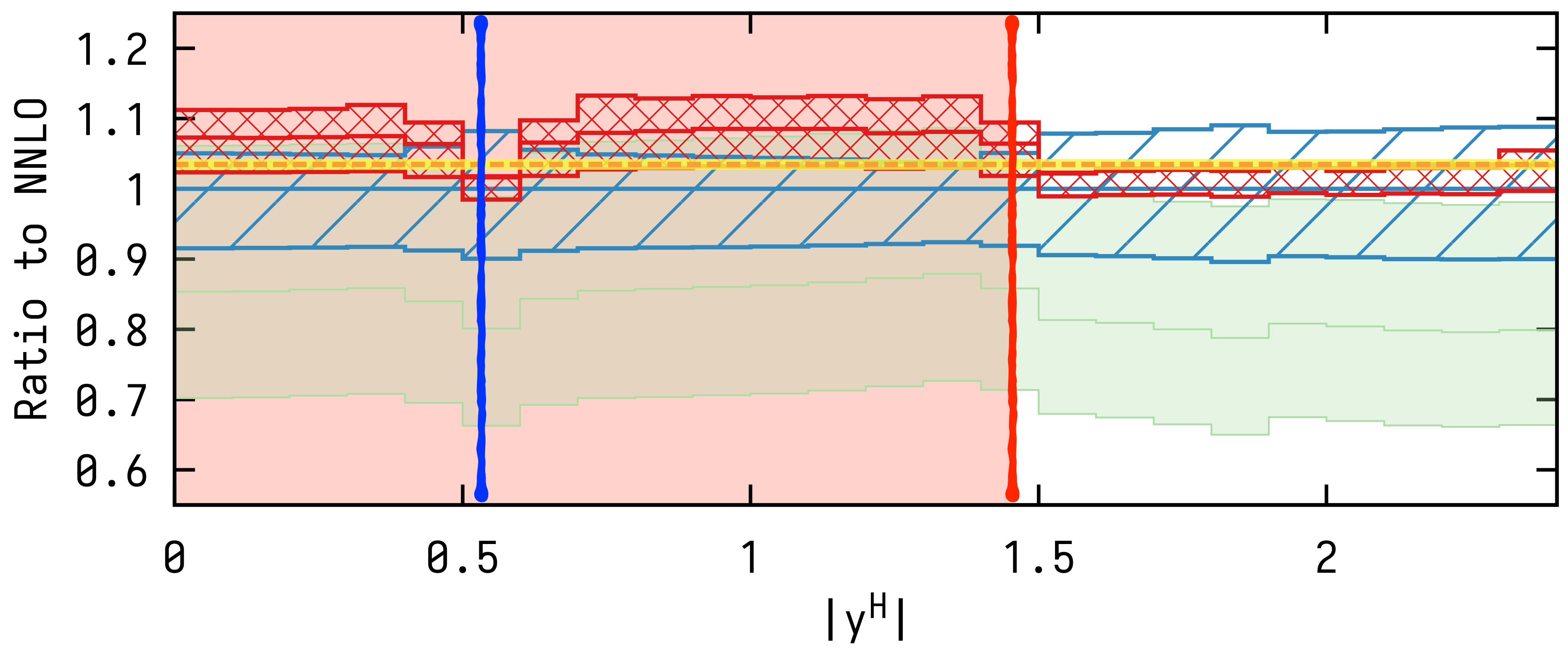
Fiducial Acceptances & y_H



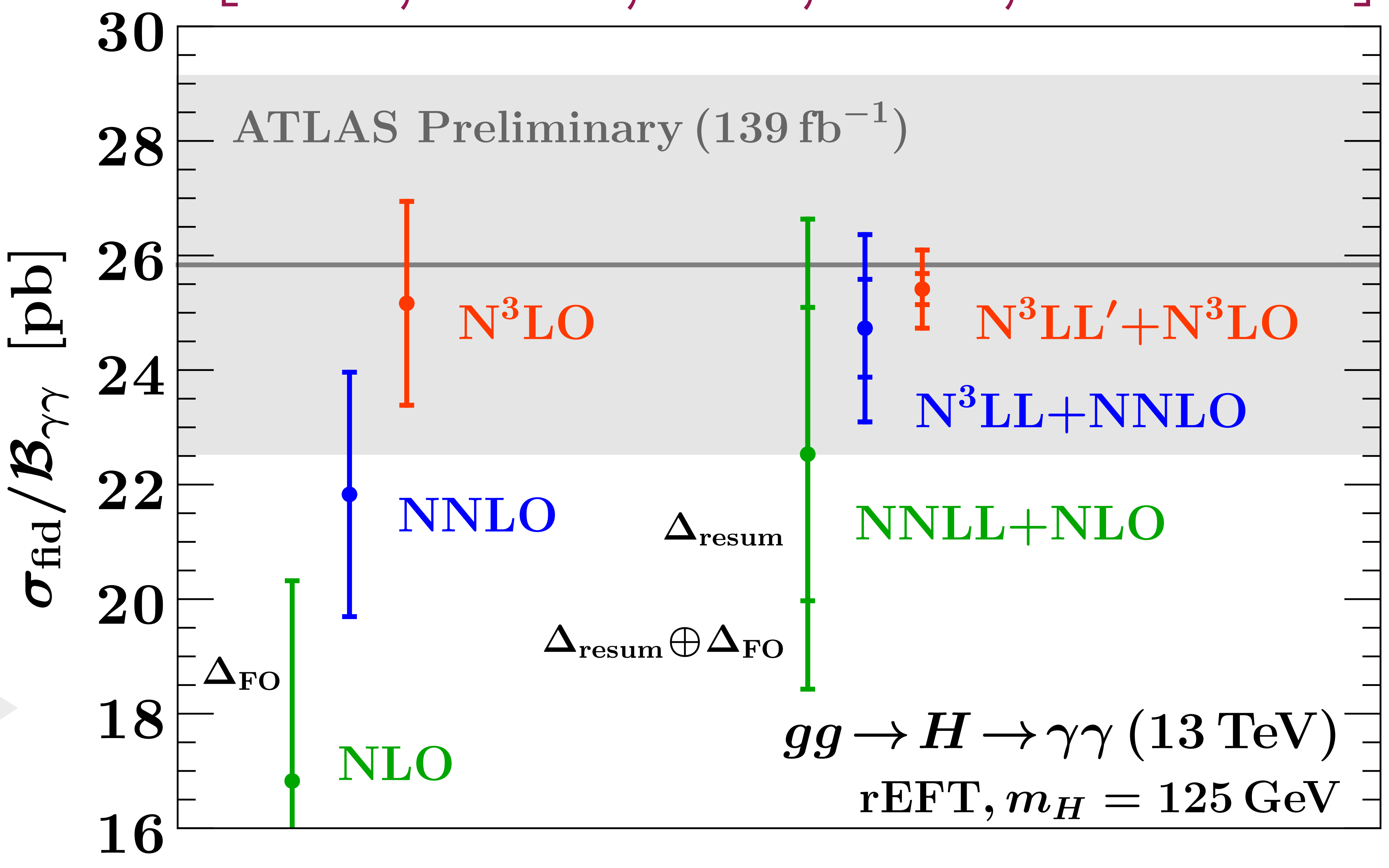
Born acceptance

$$\hat{y} \sim \frac{1}{2}\,|\,\Delta y(\gamma_1,\gamma_2)\,| \le \cosh^{-1}\left(\frac{M_H}{2p_T^{\min}}\right) \approx 0.9$$

- linear fiducial power corrections!



[Billis, Dehnadi, Ebert, Michel, Tackmann '21]

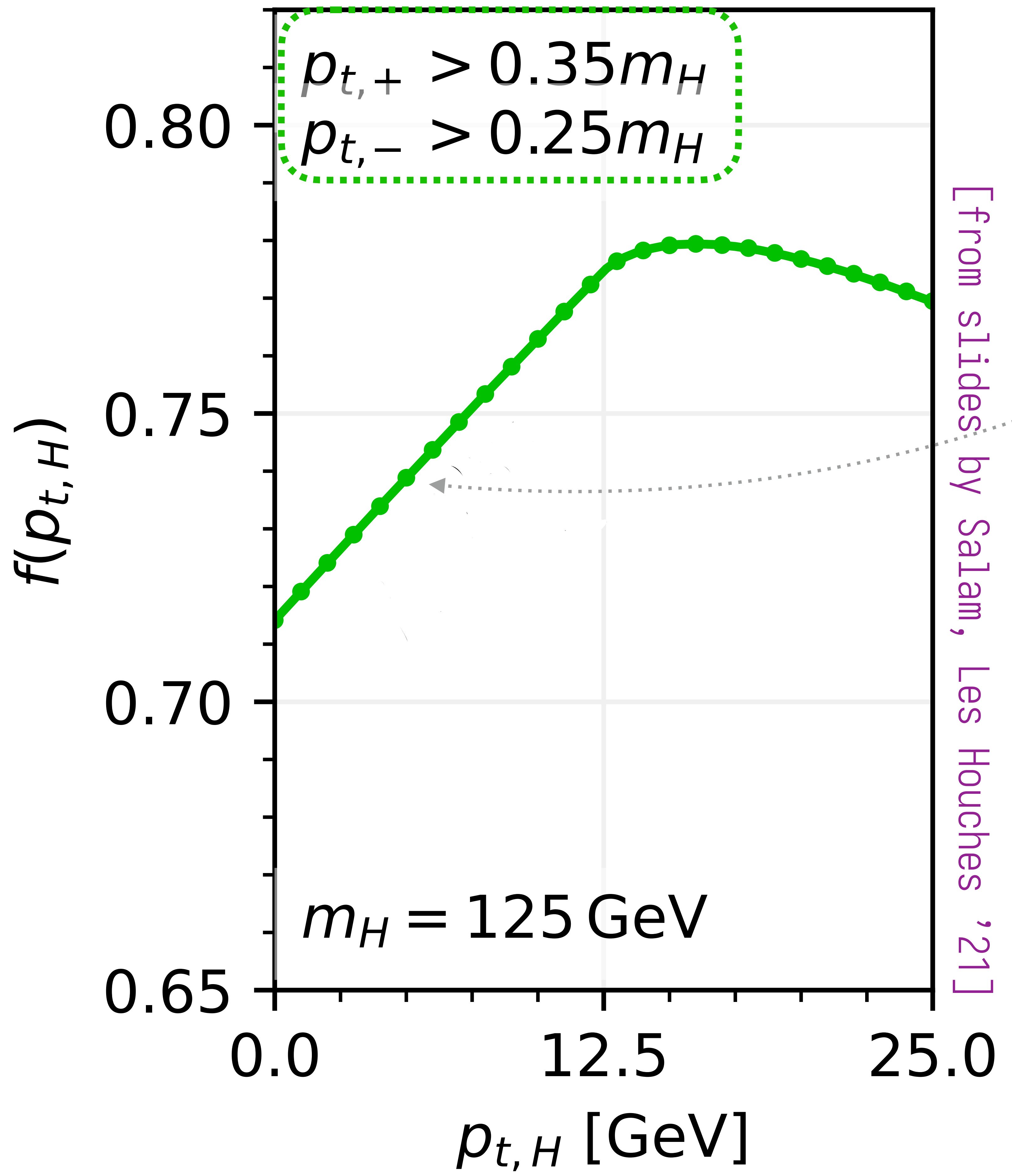


⊕ can be cured by resummation

⊖ hard $\sigma^{\text{fid.}}$ should not need resummation

Acceptance $f(p_T^H)$

“with cuts” / “no cuts”



$$f(p_T^H) = f_0 + f_1 \cdot p_T^H + \mathcal{O}\left((p_T^H)^2\right)$$

[Frixione, Ridolfi ’97; Ebert, Tackmann ’19 + Michel, Stewart ‘21; Alekhin et al. ’21]

- Linear p_T^H dependence
  - **factorial growth** for fixed-order
  - ***sensitivity*** to very low p_T^H

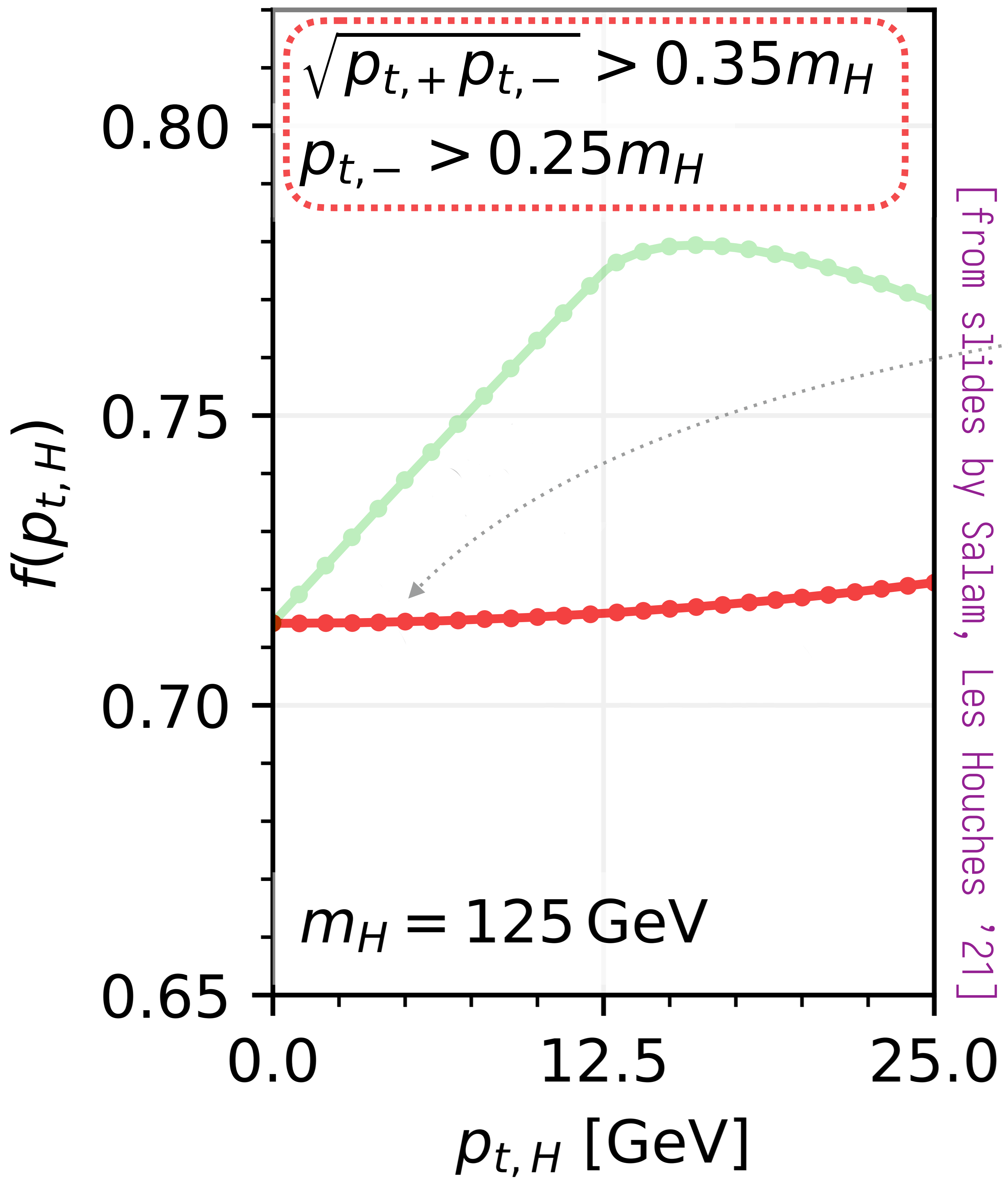
$$\frac{\sigma_{\text{asym}} - f_0\,\sigma_{\text{inc.}}}{\sigma_0\, f_0} \simeq 0.18_{\alpha_s} - 0.15_{\alpha_s^2} + 0.31_{\alpha_s^3} + \ldots$$

$$\simeq 0.12 \text{ @ N}^3\text{LL}$$

[Salam, Slade '21]

Acceptance $f(p_T^H)$

"with cuts" / "no cuts"



$$f(p_T^H) = f_0 + \cancel{f_1 \cdot p_T^H} + f_2 \cdot (p_T^H)^2 + \mathcal{O}\left((p_T^H)^3\right)$$

- Quadratic p_T^H dependence
  - *suppress* factorial growth
  - fixed order $\simeq$ resummation ✓

$$\frac{\sigma_{\text{prod}} - f_0\,\sigma_{\text{inc.}}}{\sigma_0\,f_0} \simeq 0.005_{\alpha_s} + 0.002_{\alpha_s^2} - 0.001_{\alpha_s^3} + \ldots$$

$$\simeq 0.006 \text{ @ N}^3\text{LL}$$

[Salam, Slade '21]

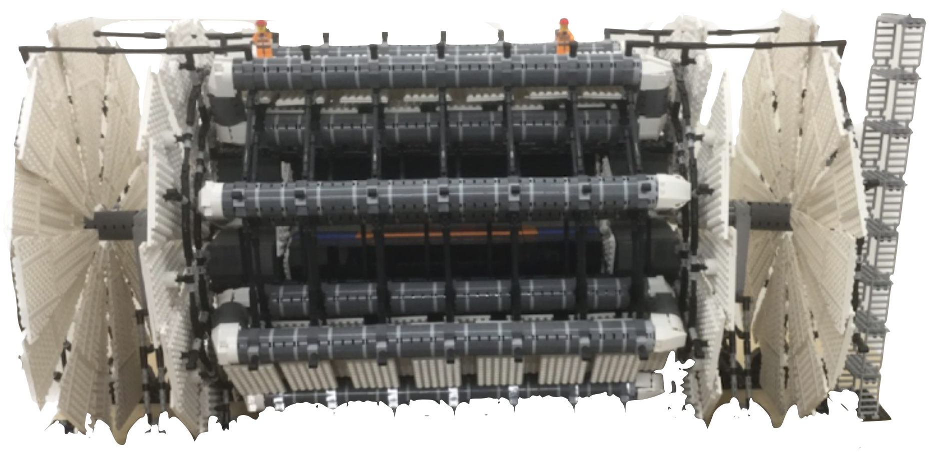
Going Differential @ N³LO — q_T Subtraction

Fully Inclusive

✗ limited to $\sigma^{\rm tot}$

✓ very efficient $\mathcal{O}(\text{sec})$

Fully Differential

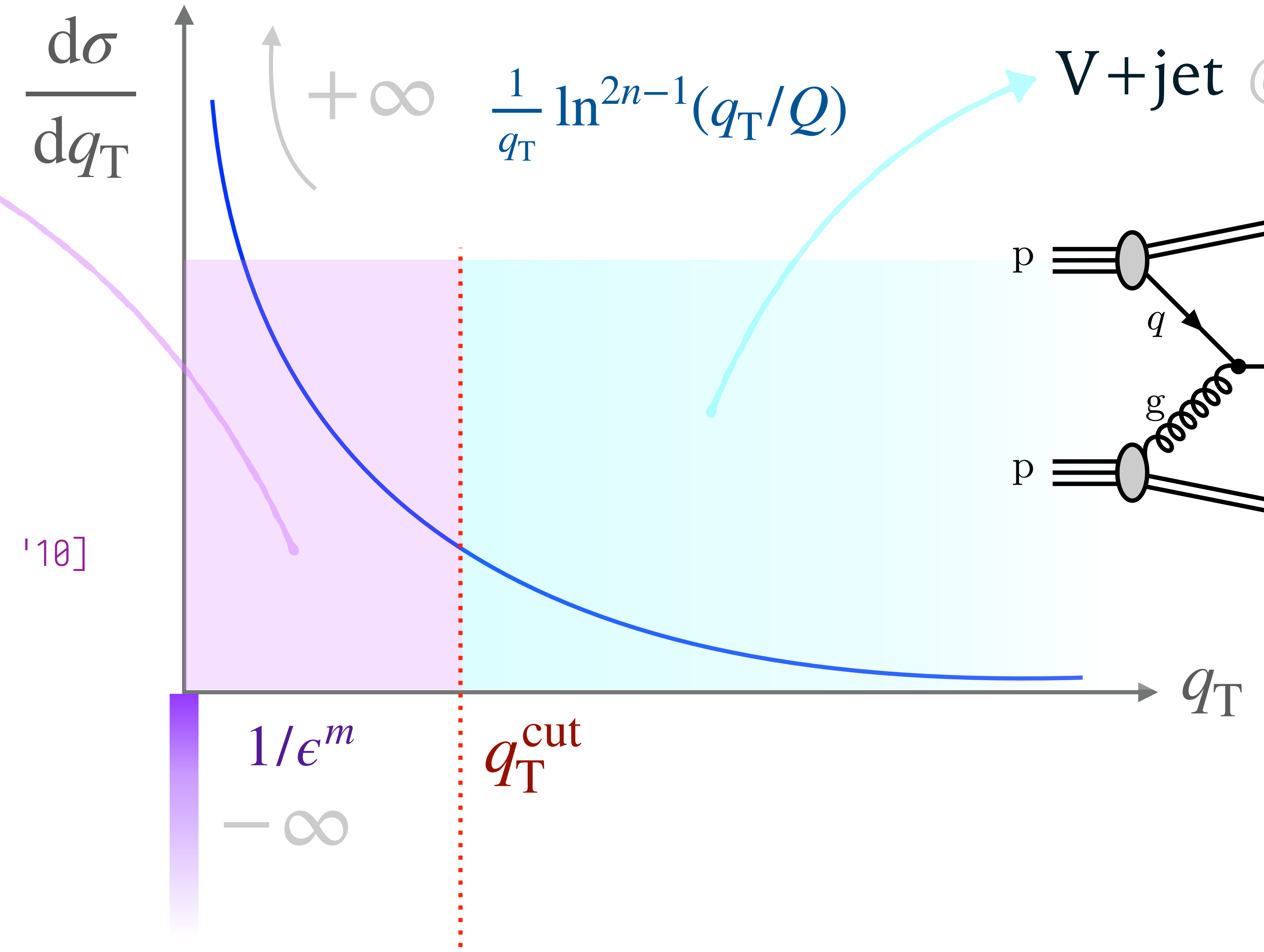


✓ $\mathrm{d}\sigma$ ⇝ fiducial cuts, arbitrary distributions, …

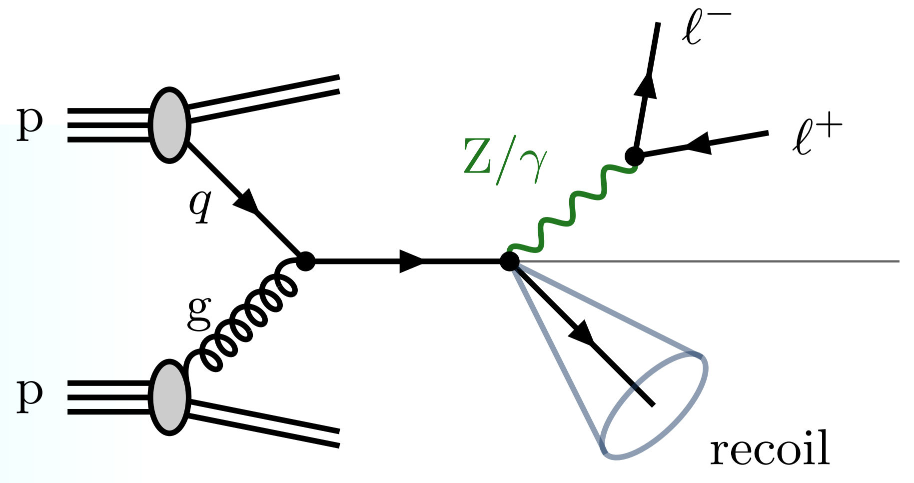
✗ computationally expensive $\mathcal{O}(10^5\text{-}10^6)$ h

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$ [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$ [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$ [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$\mathrm{d}\sigma^V_{\rm N^3LO} = \mathrm{d}\sigma^V_{\rm N^3LO}\Big|_{q_T<q_T^{\rm cut}} + \mathrm{d}\sigma^V_{\rm N^3LO}\Big|_{q_T>q_T^{\rm cut}}$$

$$= \mathcal{H}^V_{\rm N^3LO}\otimes \mathrm{d}\sigma^V_{\rm LO} + \Big[\mathrm{d}\sigma^{V+\rm jet}_{\rm NNLO} - \mathrm{d}\sigma^{V,\rm CT}_{\rm N^3LO}\Big]_{q_T>q_T^{\rm cut}} + \mathcal{O}\left((q_T^{\rm cut}/Q)^n\right)$$

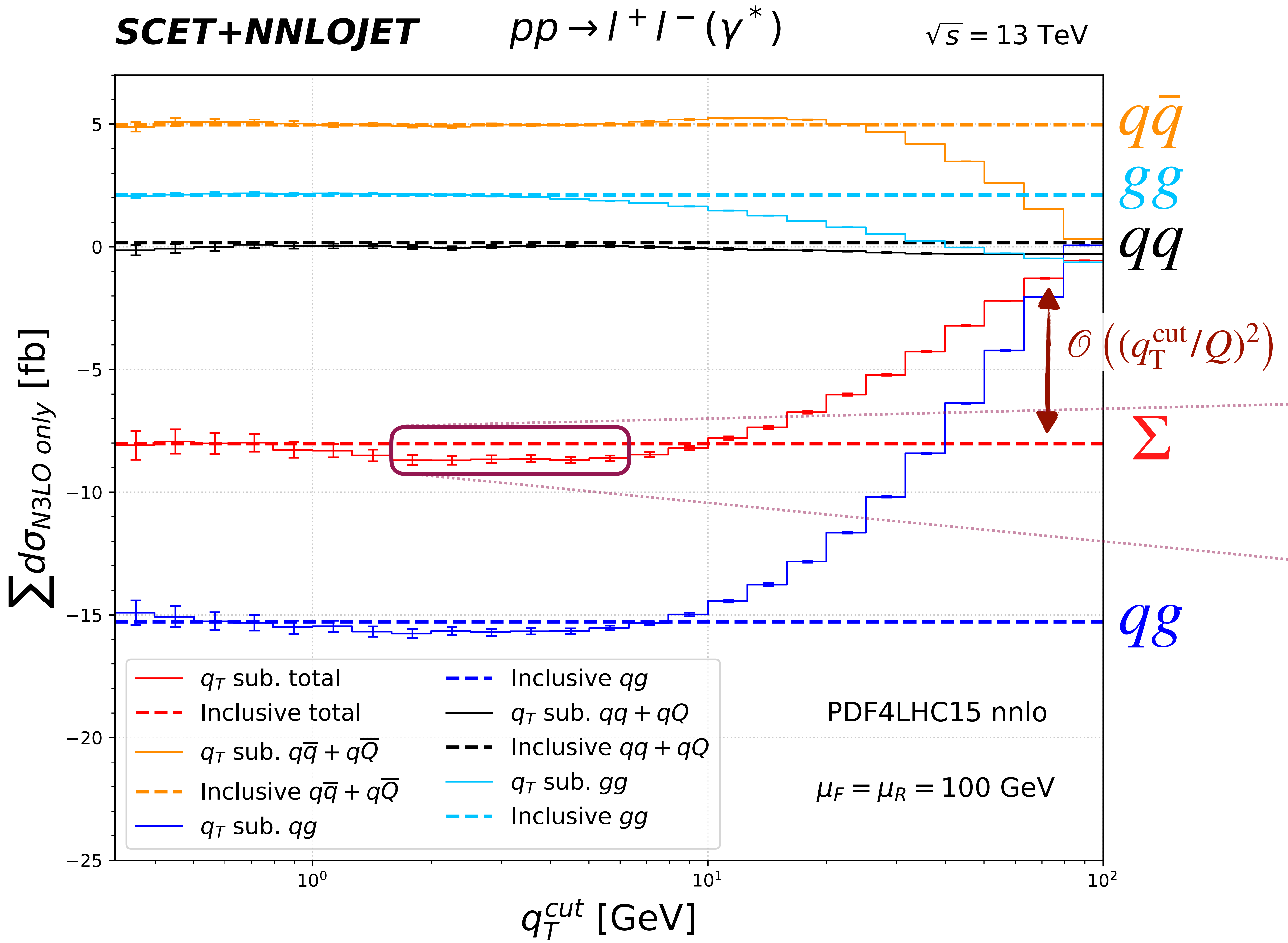
slicing error

$q_T^{\rm cut}$ as small as possible ⇜⇝ $q_T^{\rm cut}$ as large as possible

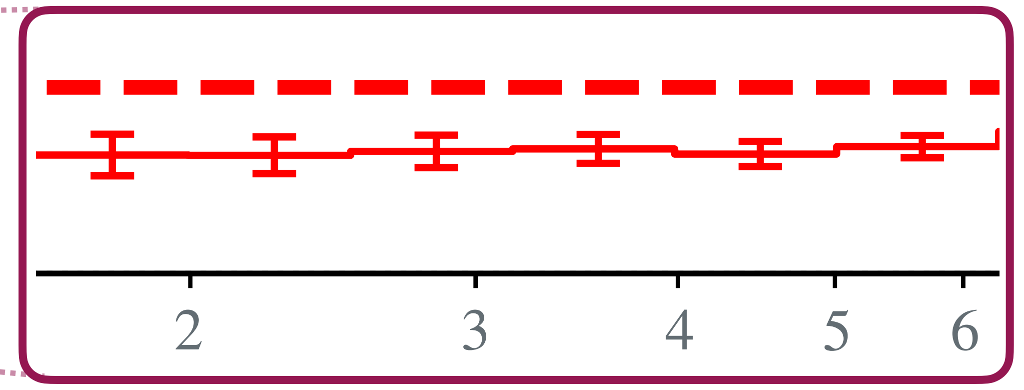
↪ suppress power corrections — ↪ numerical stability & efficiency

Validation

[Chen, Gehrmann, Glover, AH, Yang Zhu ‘21, ‘22]



- *fully independent* calculation of the inclusive cross section
- - - - - ⇝ analytic result [Duhr, Dulat, Mistlberger ’20]
- “fake” plateau: $q_T^{cut} \in [2, 5]$ GeV
  ↪ 12% error on δN³LO!



- converges to correct result for $q_T^{cut} \lesssim 1$ GeV
- fit & extrapolate?
  ⇝ marginal gains for potentially uncontrolled systematics

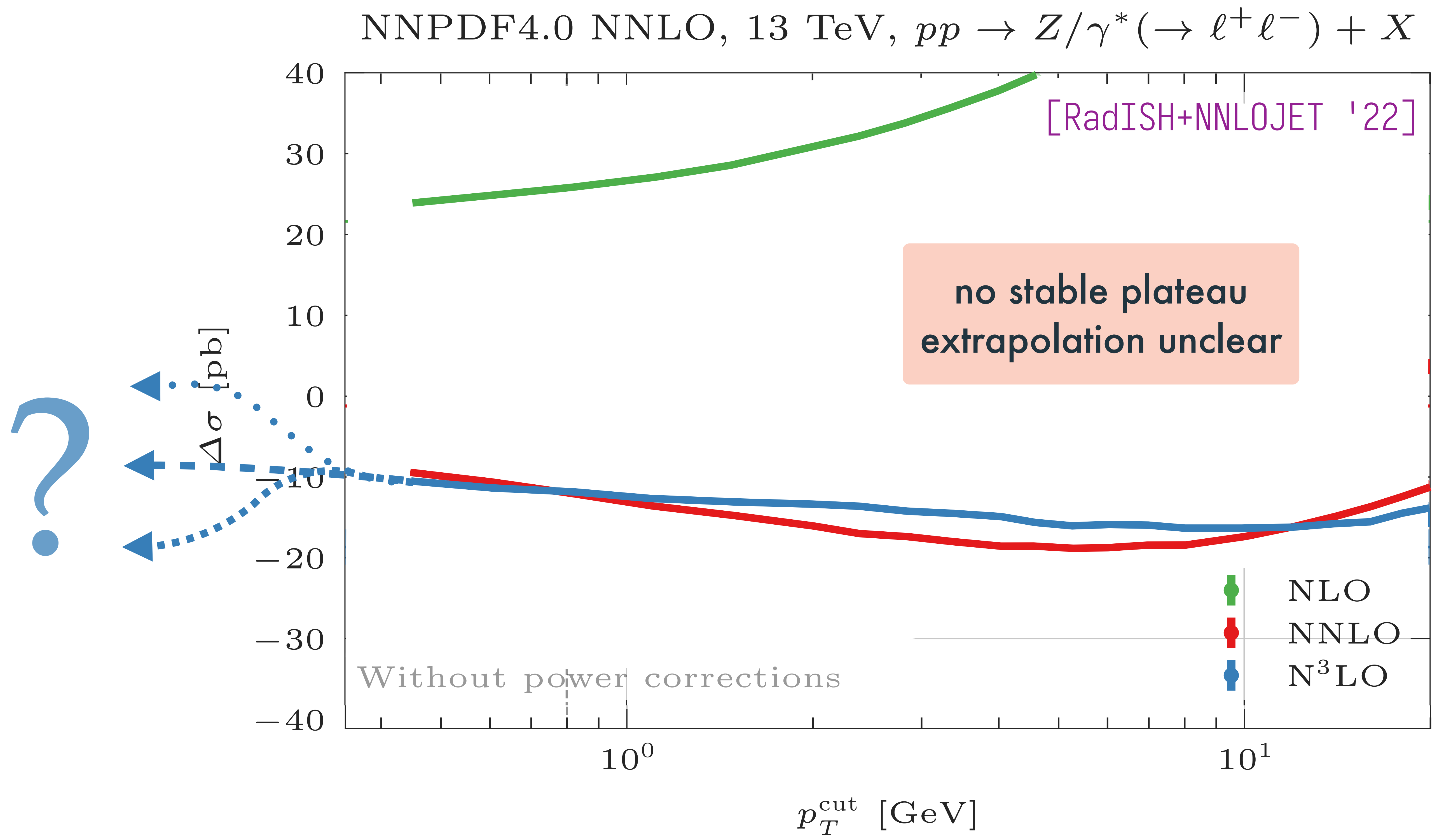
FIDUCIAL CUTS AND LINEAR POWER CORRECTIONS — N^{3}LO SLICING

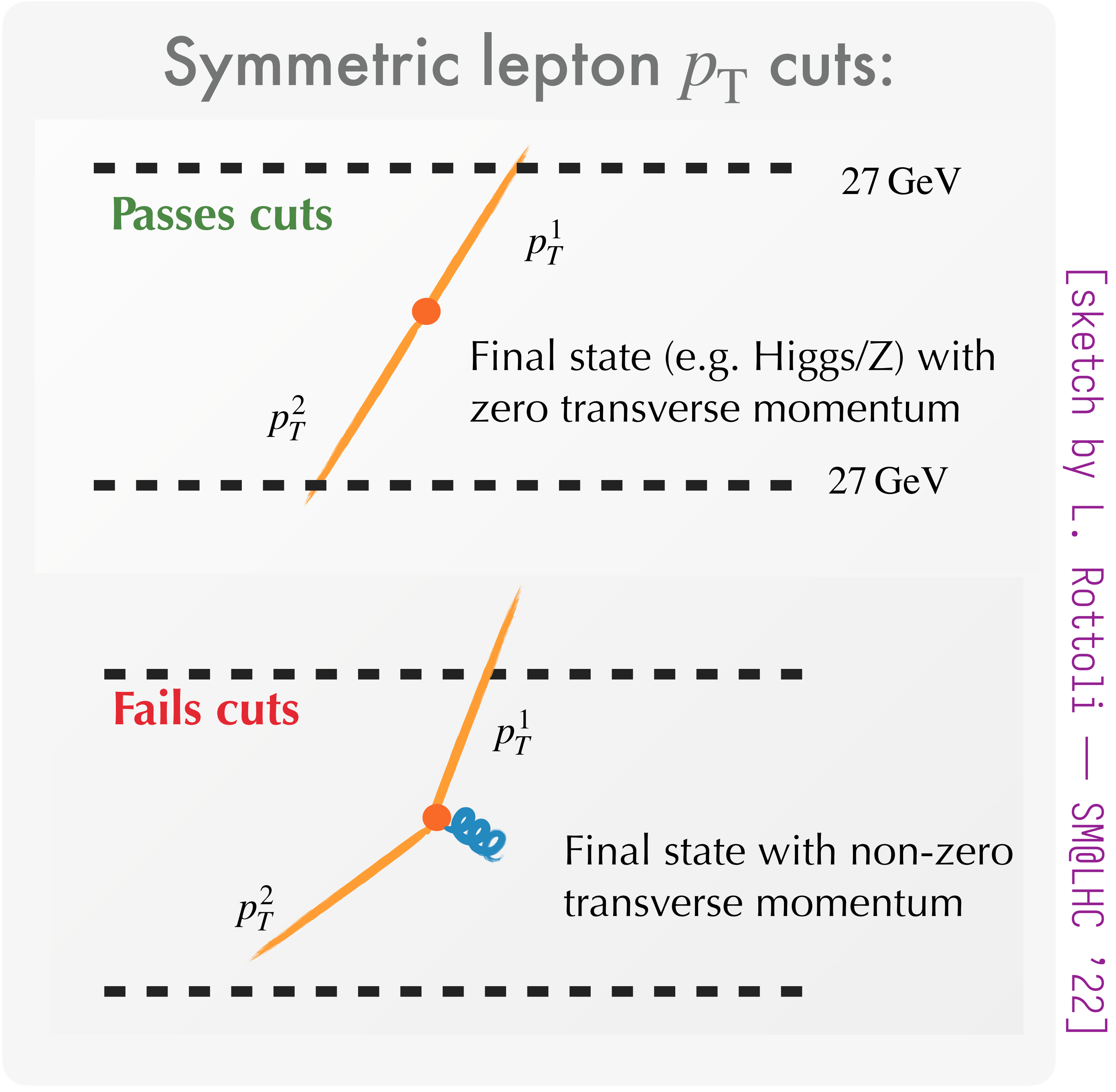
- fiducial cuts ⇝ can induce linear power corrections

[Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]

  - can jeopardise q_T slicing $\mathcal{O}\left((q_T^{cut}/Q)^2\right)$ ⇝ $\mathcal{O}\left(q_T^{cut}/Q\right)$

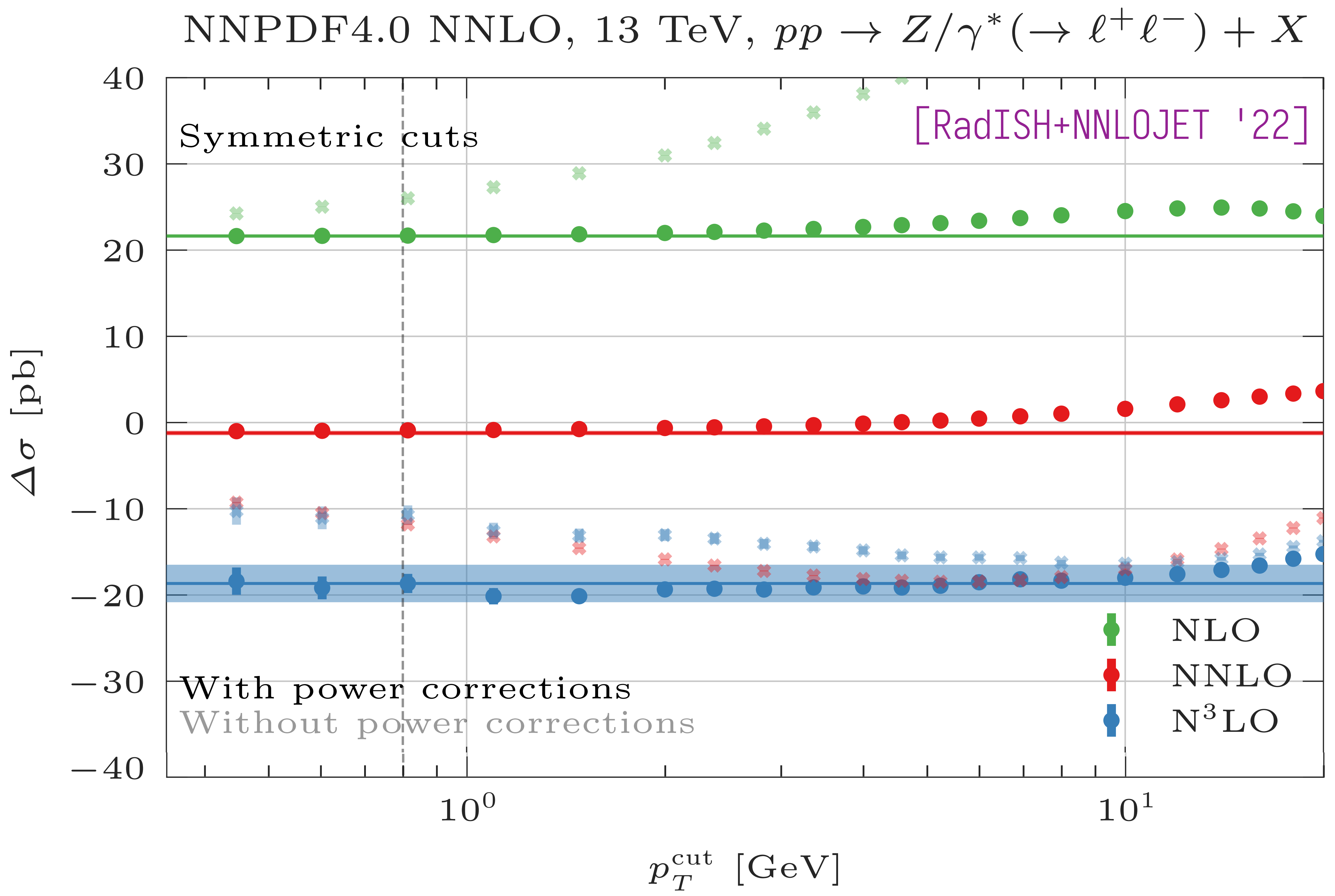
    $[q_T^{cut} \lesssim 1\,\text{GeV}]$ $[q_T^{cut} \lesssim 10^{-2}\,\text{GeV}\ ?!]$

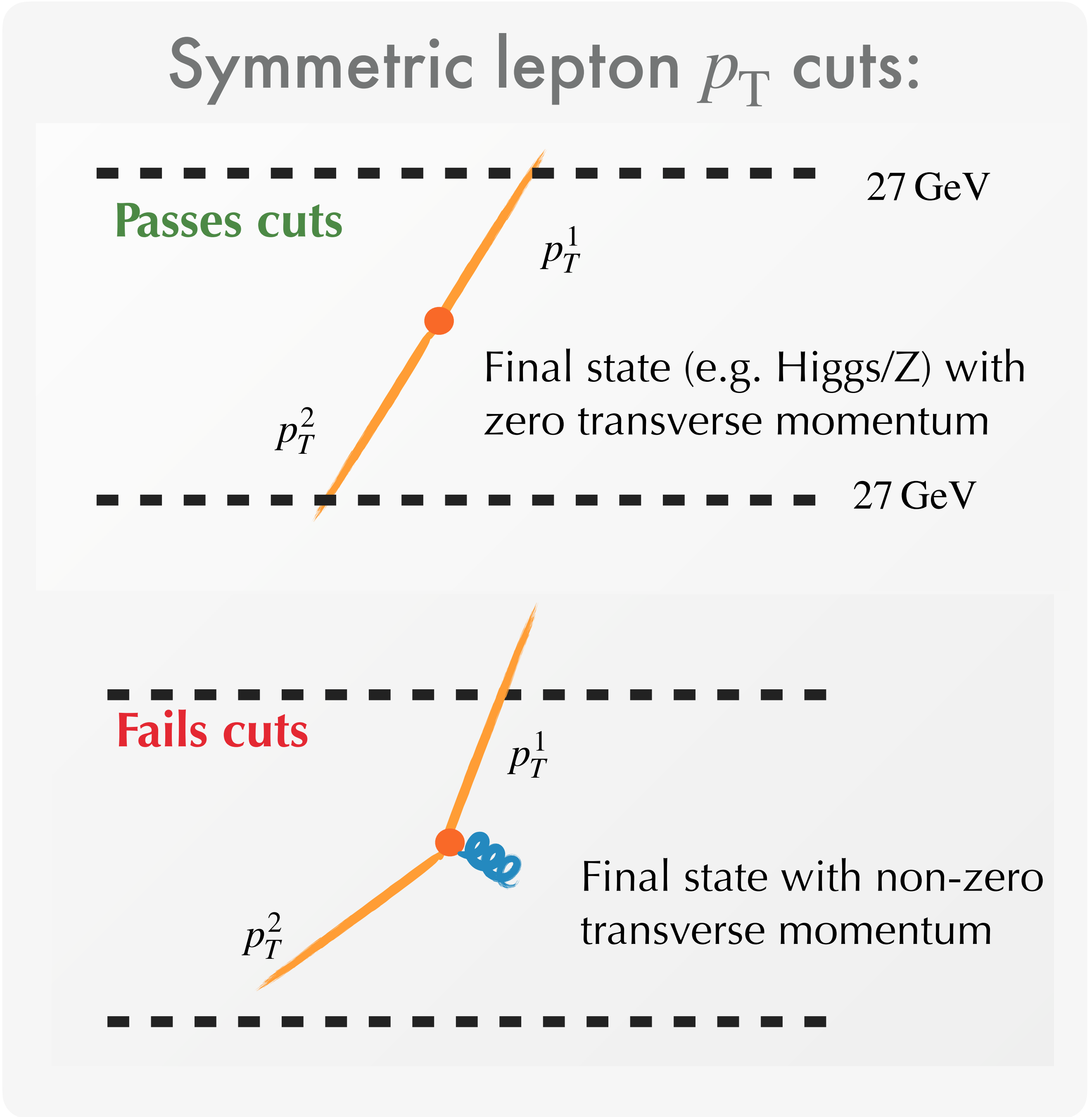




Fiducial Cuts and Linear Power Corrections — N^{3}LO Slicing

- fiducial cuts ⇝ can induce linear power corrections
  [Tackmann, Ebert '19][Alekhin, Kardos, Moch, Trócsányi '21][Salam, Slade '21]
  - can jeopardise q_T slicing $\mathcal{O}\left((q_T^{cut}/Q)^2\right)$ ⇝ $\mathcal{O}\left(q_T^{cut}/Q\right)$
    $[q_T^{cut} \lesssim 1\,\text{GeV}]$ $[q_T^{cut} \lesssim 10^{-2}\,\text{GeV}\ ?!]$





[sketch by L. Rottoli — SM@LHC '22]

can *compute* & *subtract* the linear term:

↪ simple boost of $V \to \ell\bar{\ell}$ system

(pure kinematics & acceptance effect)

[Catani, de Florian, Ferrera, Grazzini '15]
[Ebert, Michel, Stewart, Tackmann '21]