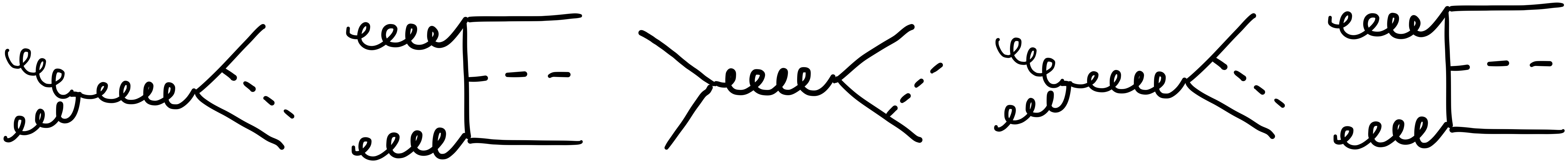


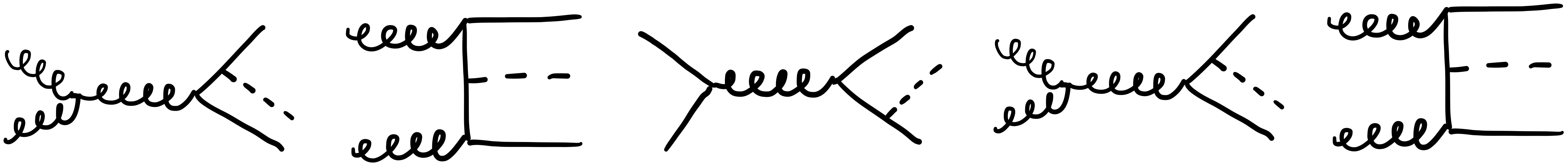
$t\bar{t}H$ PRODUCTION IN NNLO QCD

34th Rencontres de Blois

UNIVERSITÀ
DEGLI STUDI
DI MILANO

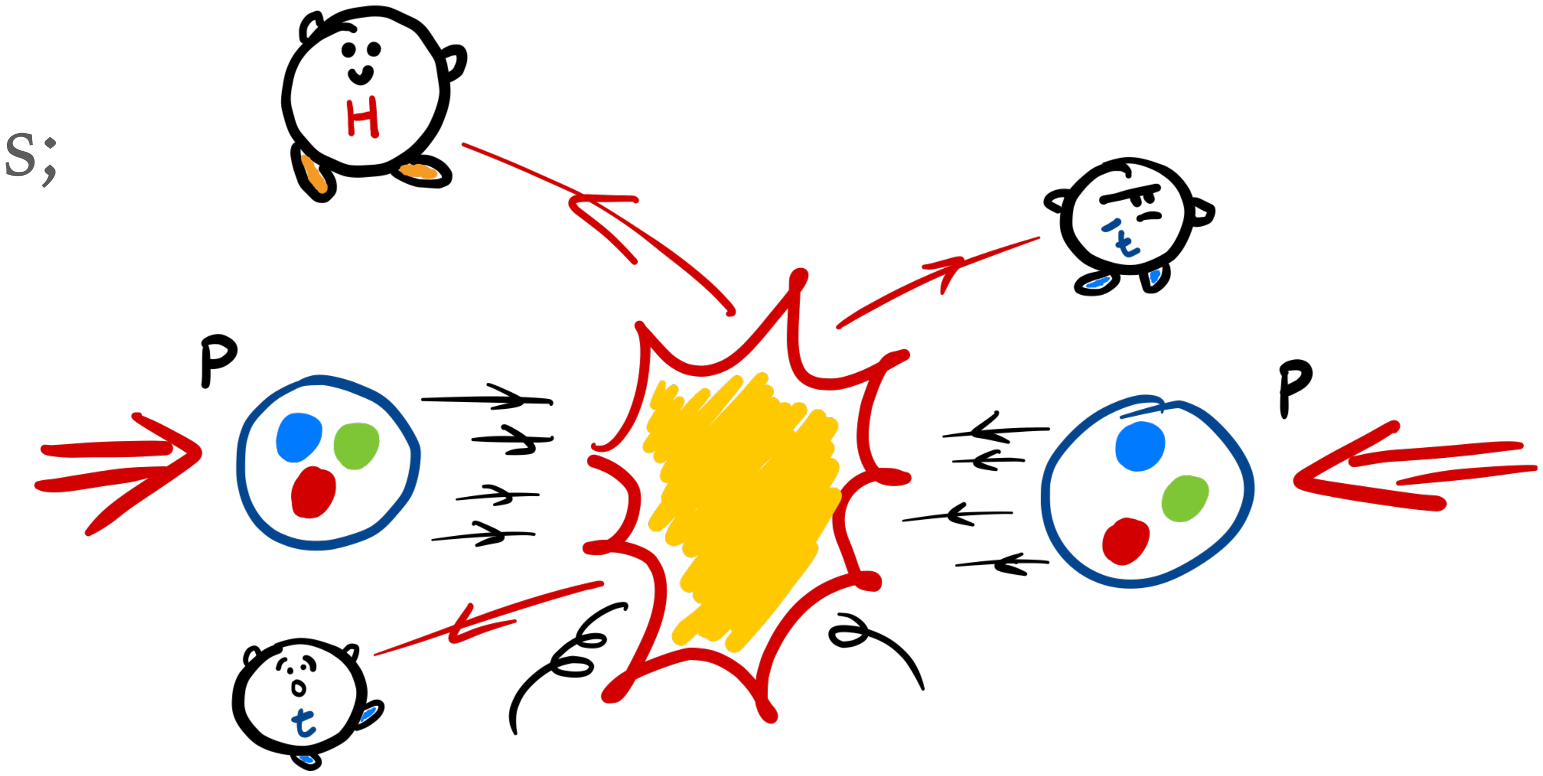
Simone Devoto

In collaboration with:
S. Catani, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini



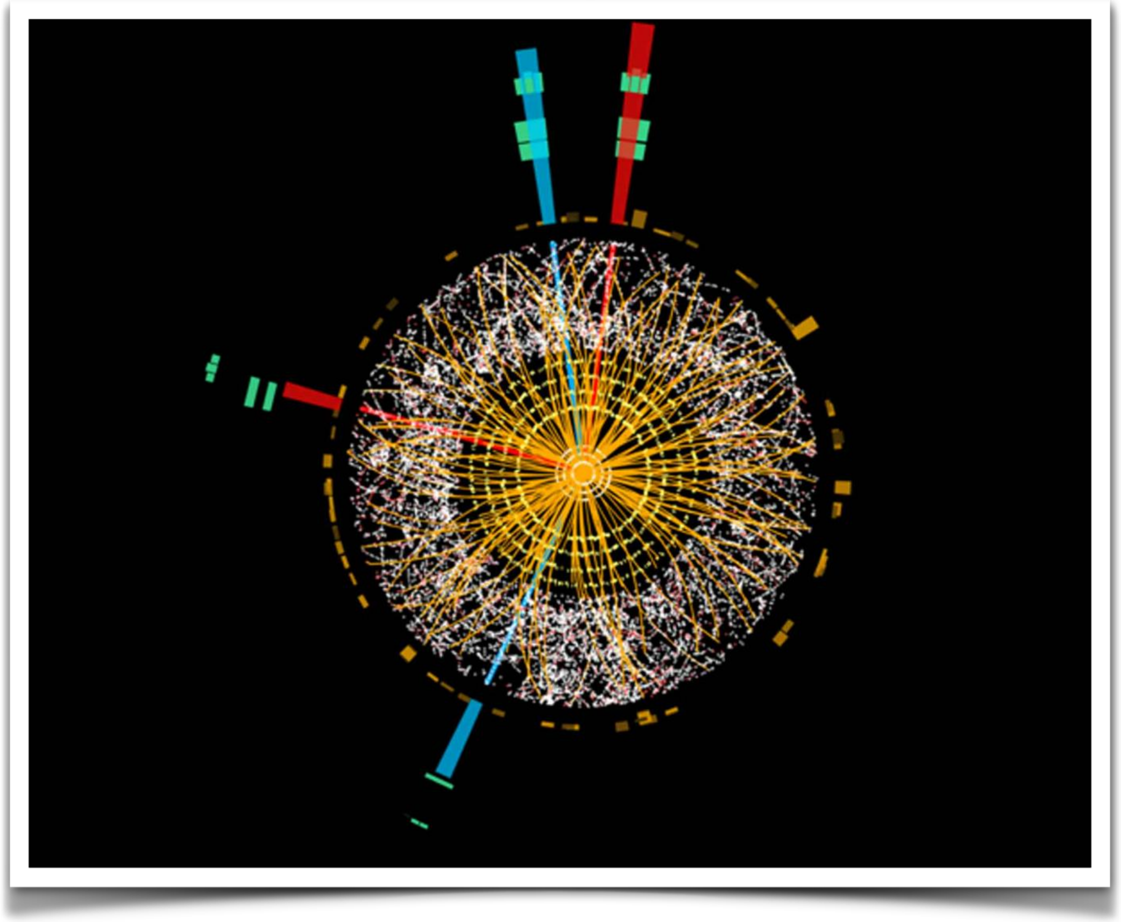
CONTENTS

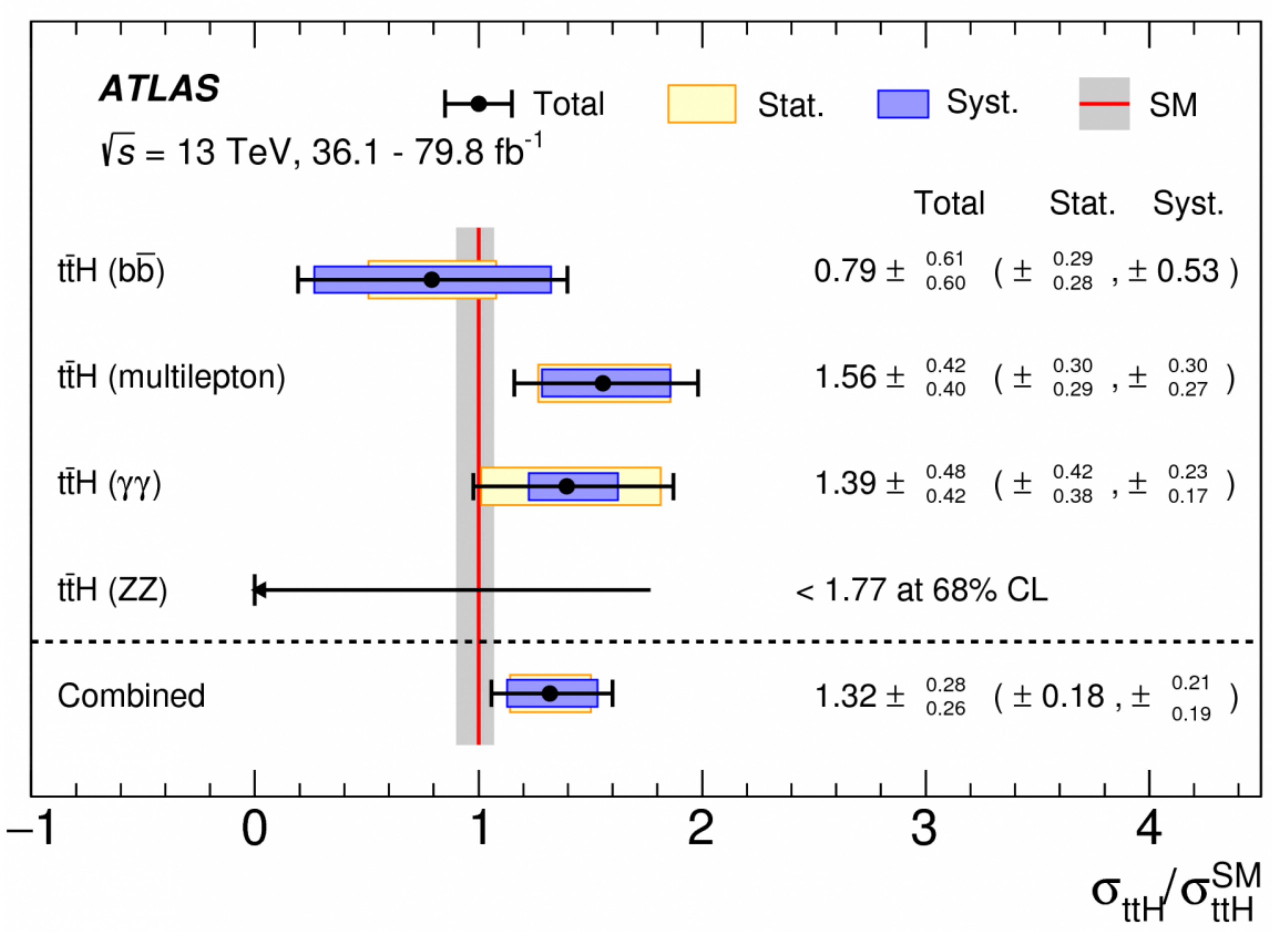
- **Introduction**;
- $t\bar{t}H$ @ NNLO, **theory bottlenecks**:
  - subtraction;
  - two-loop amplitudes;
- **Results**.



INTRODUCTION

- The **discovery of the Higgs boson** in 2012 confirmed one of the most glaring predictions of the Standard Model.
- The **study of the Higgs boson** proprieties is one of the priorities of LHC.
- Special role played by the **top quark**: strong coupling because of the top mass!
- $t\bar{t}H$ production allows direct measurement of the **top-quark Yukawa coupling**! (possible window on new physics scenarios…)





[M.Cepeda et al.: arXiv 1902.00134]

STATUS OVERVIEW

THEORY

- NLO QCD:

  [W. Beenakker, S. Dittmaier, M. Krämer, B. Plumper, M. Spira, and P. Zerwas; 0107081, 0211352], [L. Reina and S. Dawson; 0107101], [L. Reina, S. Dawson, and D. Wackeroth; 0109066], [S. Dawson, L. Orr, L. Reina, and D. Wackeroth; 0211438], [S. Dawson, C. Jackson, L. Orr, L. Reina, and D. Wackeroth; 0305087], [A. Denner and R. Feger, 1506.07448];

- NLO EW:

  [S. Frixione, V. Hirschi, D. Pagani, H. Shao, and M. Zaro; 1407.0823, 1504.03446], [Y. Zhang, W.-G. Ma, R.-Y. Zhang, C. Chen, and L. Guo; 1407.1110];

- NLO QCD + EW:

  [A. Denner, JN. Lang, M. Pellen, and S. Uccirati; 1612.07138];

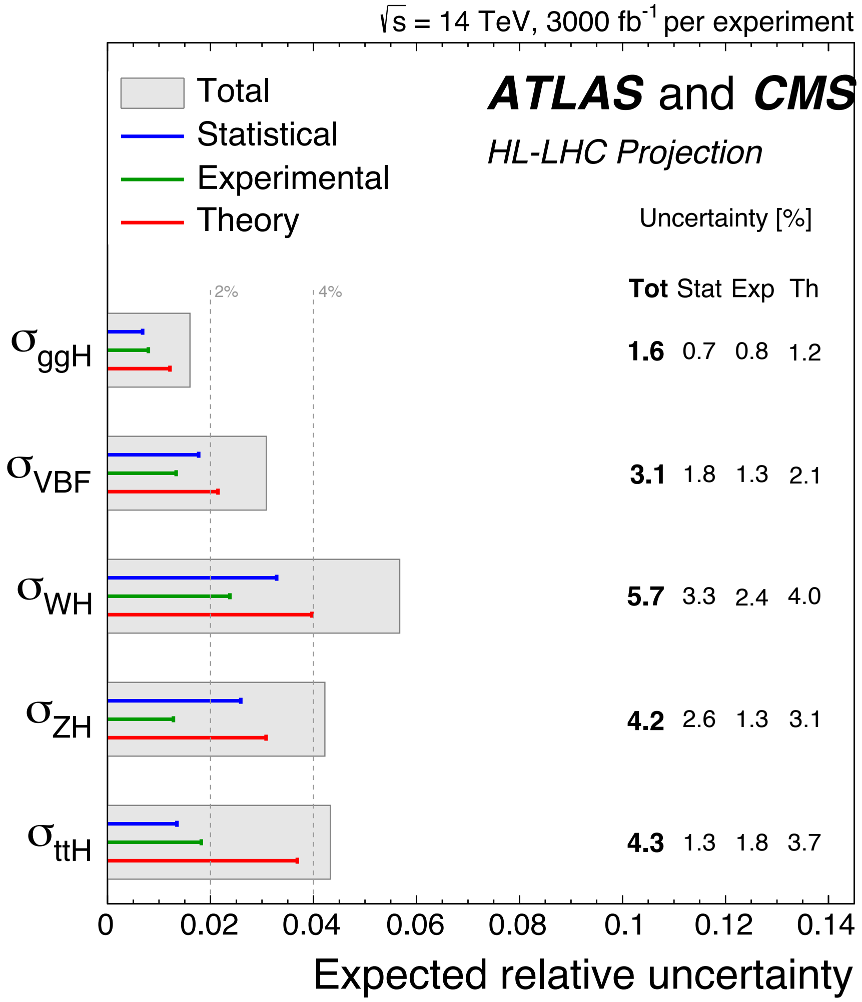
- Resummation of soft gluons:

  [A. Kulesza, L. Motyka, T. Stebel, and V. Theeuwes; 1509.02780, 1704.03363], [A. Broggio, A. Ferroglia, B. D. Pecjak, A. Signer, and L. L. Yang; 1510.01914], [A. Broggio, A. Ferroglia, B. D. Pecjak, and L. L. Yang; arXiv:1611.00049], [A. Broggio, A. Ferroglia, R. Frederix, D. Pagani, B. D. Pecjak, and I. Tsinikos; 1907.04343], [W.-L. Ju and L. L. Yang; 1904.08744], [A. Kulesza, L. Motyka, D. Schwartländer, T. Stebel, and V. Theeuwes; 2001.03031]

- Current theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

- ATLAS collaboration: *[1806.00425];*
- CMS collaboration: *[1804.02610].*

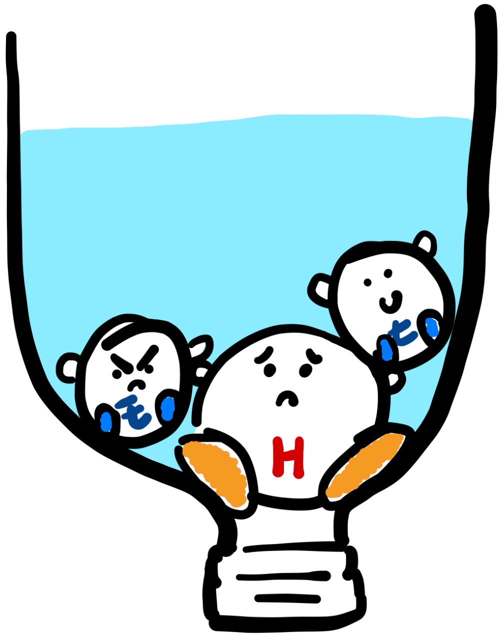


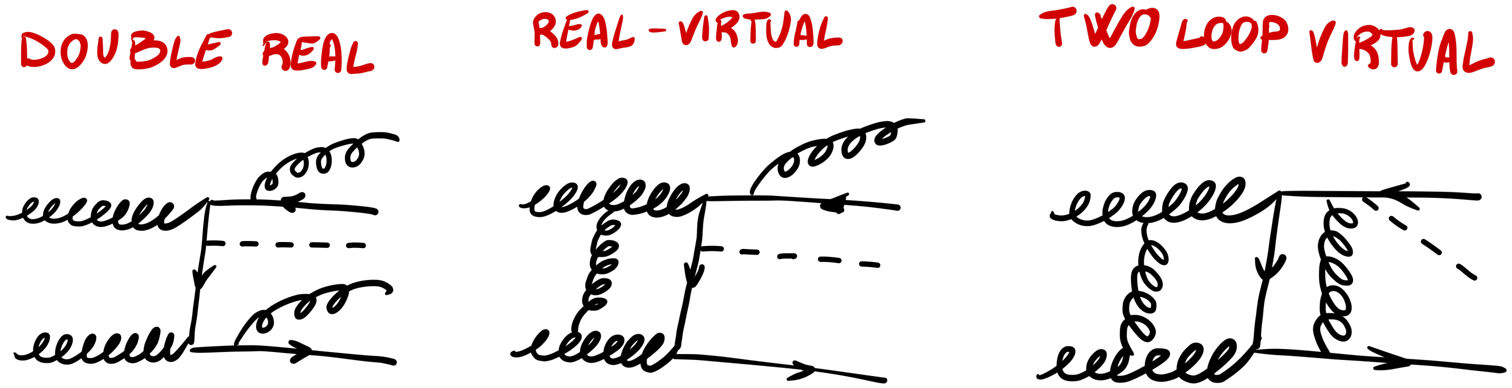
[M.Cepeda et al.: arXiv 1902.00134]

- Current experimental uncertainties $\mathcal{O}(20\%)$
- Expected at the end of HL-LHC $\mathcal{O}(2\%)$

THEORY BOTTLENECKS

- First steps: computation of NNLO QCD off-diagonal channels *[S. Catani, I. Fabre, M. Grazzini, S. Kallweit : arXiv:2102.03256]*
- Full NNLO corrections: *[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]* THIS TALK!





What makes NNLO challenging?

Subtraction procedure

- We use **q_T-subtraction**;
- We **generalised** the method to this class of processes.

Two loop amplitudes

- Not known: current frontier!
- We develop a **soft Higgs approximation**.

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini Phys.Rev.Lett. 98 (2007)]

$$d\sigma^F_{NNLO} = d\sigma^F_{NNLO}\Big|_{q_T=0} + d\sigma^F_{NNLO}\Big|_{q_T\neq 0}$$

$$d\sigma^{F+jets}_{NLO}$$

$$\boxed{d\sigma^F_{NNLO} = \mathscr{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{NLO} - d\sigma^{CT}_{NLO}\right]}$$

HARD COLLINEAR COEFFICIENT

Contains information on virtual corrections to the process.

$$\mathscr{H}^F_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}^{(2)}$$

Contains the genuine 2-loop contribution:

$$H^{(2)} = \frac{2\,Re\left(\mathscr{M}^{(2)}(\mu_{IR},\mu_R)\mathscr{M}^{(0)}\right)}{|\,\mathscr{M}^{(0)}\,|^2}$$

- APPROXIMATED -

Includes:

- one-loop squared contribution;
- soft parton contribution.

- EXACT -

SOFT PARTON CONTRIBUTION

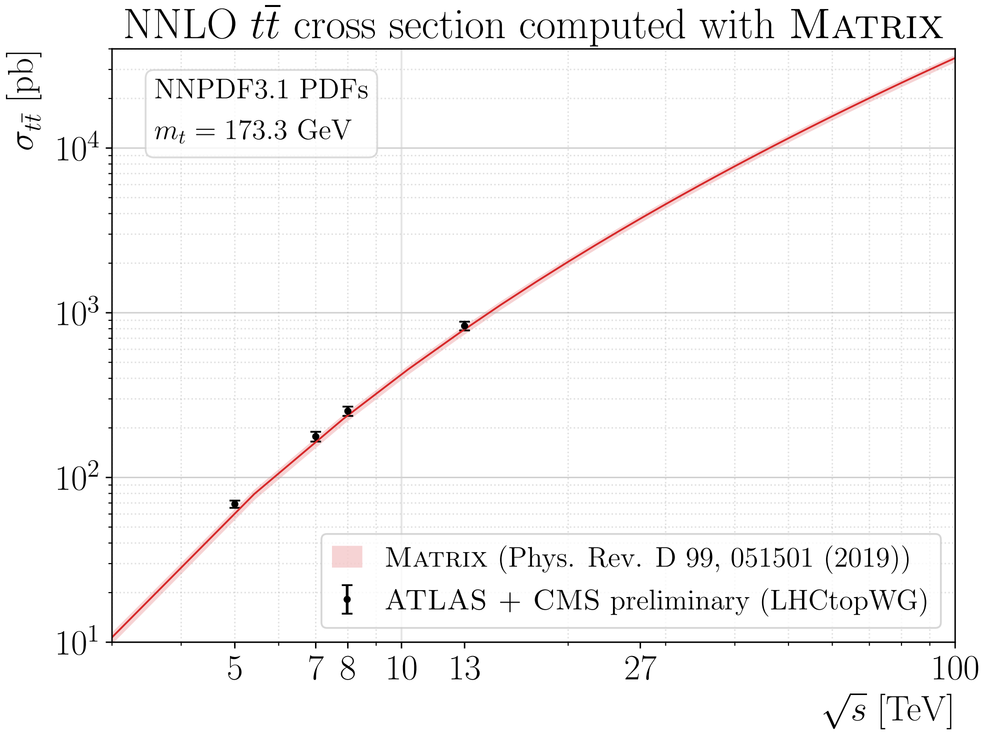
[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786
SD, J. Mazzitelli, In preparation]

The soft contribution from a massive final state was a key ingredient to extend q_T subtraction to a massive coloured final state.

[S. Catani, SD, M. Grazzini , J. Mazzitelli: 2301.11786]

Soft contributions to heavy-quark (Q) production

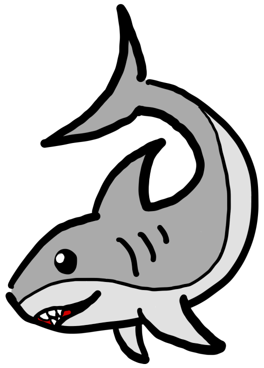
- Applied to top pair and bottom pair production: *[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan: 2019, 2020]*;
- Mostly **analytic** expressions;
- Assumption of $Q\bar{Q}$ **back-to-back** at LO.



NEW: generalisation to $Q\bar{Q}F$ kinematics

[SD, J. Mazzitelli, IN PREPARATION]

- removed the back-to-back assumption;
- Extra contribution computed **numerically**;
- On-the-fly numerical integration implemented in a **library**: **SHARK**
  Soft function for **H**eavy quark production in **AR**bitrary **K**inematics



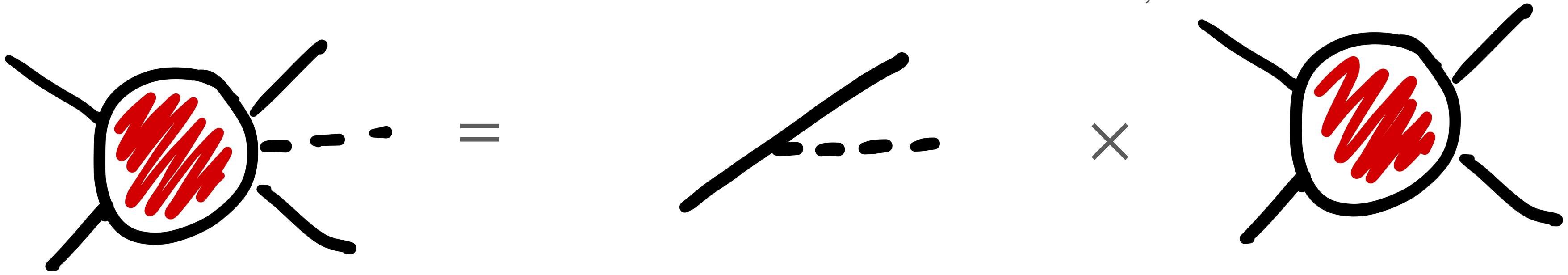
2-LOOP CONTRIBUTION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

Process: $c(p_1) + \bar{c}(p_2) \to t(p_3) + \bar{t}(p_4) + H(k)$

Soft Higgs approximation: $k \to 0, \quad m_H \ll m_t$

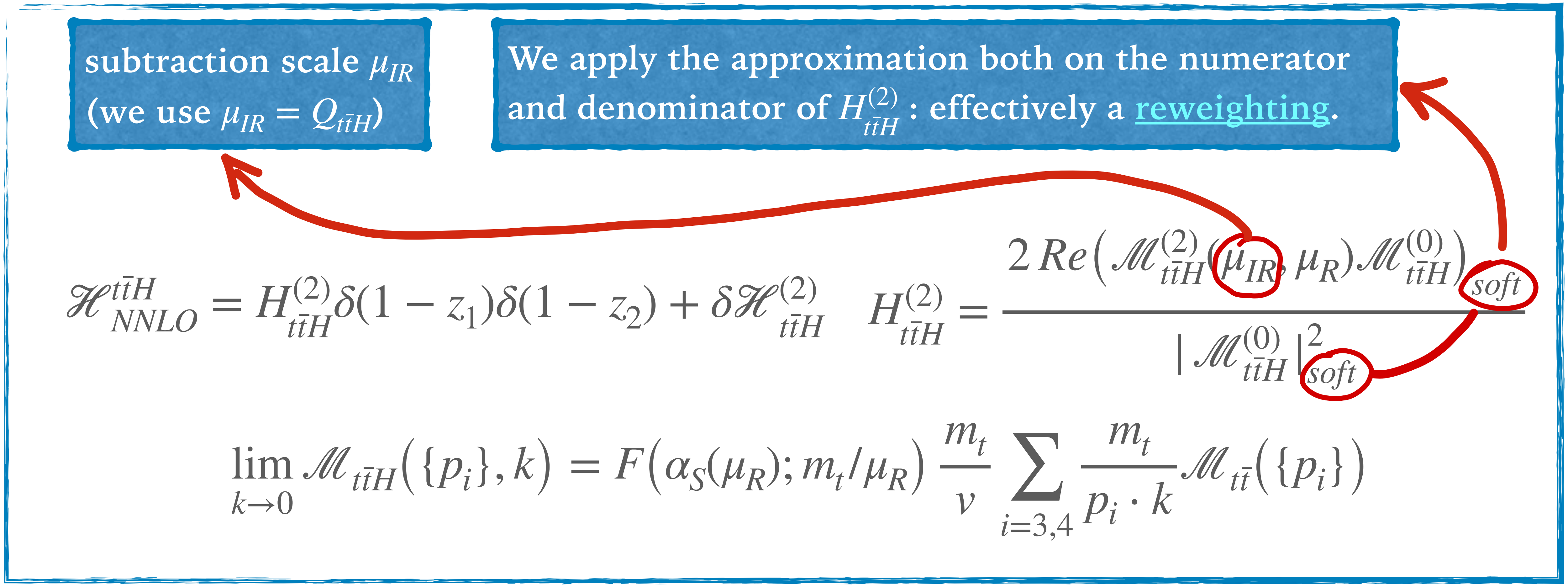
$$\lim_{k\to 0} \mathscr{M}_{t\bar{t}H}\big(\{p_i\},k\big) = F\big(\alpha_S(\mu_R); m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathscr{M}_{t\bar{t}}\big(\{p_i\}\big)$$



- The formula captures the leading behaviour in the soft limit $k \to 0$: the emission from highly off-shell top propagators is not captured.
- The formula can be obtained both from the eikonal approximation and the low energy theorems;
- The perturbative function $F\big(\alpha_S(\mu_R); m_t/\mu_R\big)$ is an effective coupling which also takes into account the renormalisation of the mass and of the wave function.

2-LOOP CONTRIBUTION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]



subtraction scale μ_{IR} (we use $\mu_{IR} = Q_{t\bar{t}H}$)

We apply the approximation both on the numerator and denominator of $H^{(2)}_{t\bar{t}H}$: effectively a reweighting.

$$\mathscr{H}^{t\bar{t}H}_{NNLO} = H^{(2)}_{t\bar{t}H}\,\delta(1-z_1)\,\delta(1-z_2) + \delta\mathscr{H}^{(2)}_{t\bar{t}H} \qquad H^{(2)}_{t\bar{t}H} = \frac{2\,Re\left(\mathscr{M}^{(2)}_{t\bar{t}H}(\mu_{IR},\mu_R)\,\mathscr{M}^{(0)}_{t\bar{t}H}\right)_{soft}}{|\,\mathscr{M}^{(0)}_{t\bar{t}H}\,|^2_{soft}}$$

$$\lim_{k\to 0}\mathscr{M}_{t\bar{t}H}\left(\{p_i\},k\right) = F\left(\alpha_S(\mu_R); m_t/\mu_R\right)\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\,\mathscr{M}_{t\bar{t}}\left(\{p_i\}\right)$$

- To use the approximation, we need a **recoil prescription** to map the $t\bar{t}H$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}H} \to Q_{t\bar{t}}$);
- We use the **q_T recoil prescription**:
  - We reabsorb the Higgs momentum equally in the initial-state parton momenta;
  - We leave unchanged the top and anti-top momenta.

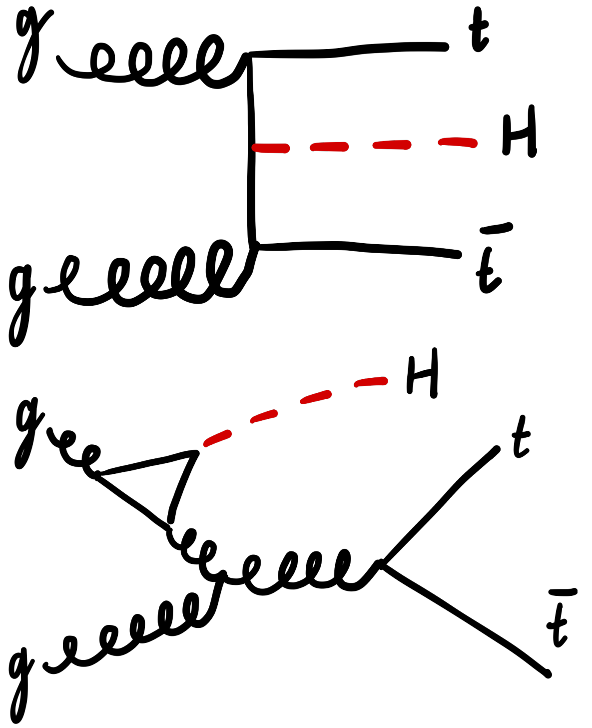
TESTING THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]

To **validate** our procedure: test the approximation at NLO!

$\Delta\sigma_{\mathrm{NLO,H}}$[fb]	13 TeV		100 TeV	
	gg	$q\bar{q}$	gg	$q\bar{q}$
Exact	88.62	7.826	8205	217.0
Soft Approximation	61.92	7.413	5612	206.0
Difference	**30.1%**	**5.27%**	**31.6%**	**5.06 %**

- Deviation w.r.t. exact computation is about **30%** for the gg **channel** and **5%** for the $q\bar{q}$ **channel**;
- Deviation **independent** of kinematic variables;
- **Better agreement** for $q\bar{q}$ channel can be explained by the presence, both at LO and NLO, of diagrams where a **Higgs boson is radiated from a virtual top** only present in the gg channel.



UNCERTAINTIES ESTIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: 2210.07846]

How to estimate the NNLO uncertainties?

- We use the **deviation from the exact results at NLO** as a **lower bound** on the NNLO uncertainty;
- We multiply by a **tolerance factor** of **3**;
- We combined **linearly** the uncertainty for the gg and $q\bar{q}$ channel;

How to test the NNLO uncertainties?

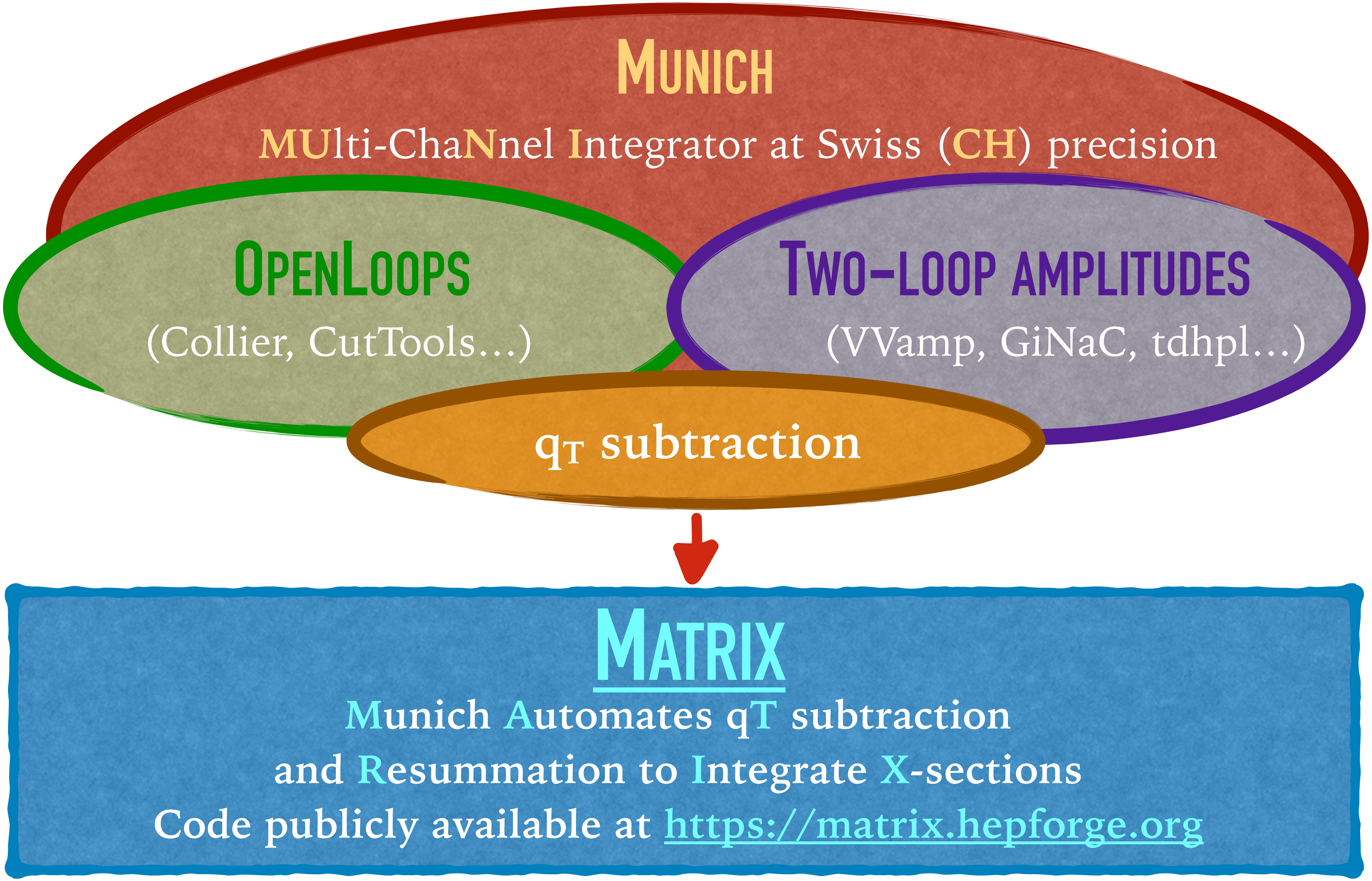
- Check the effect of using **different recoil prescription**;
- Check the effect of using a **different subtraction scales** $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$.

Final uncertainty: • $\pm 15\,\%$ **on** $\Delta\sigma_{\mathrm{NNLO}}$ • $\pm 0.6\,\%$ **on** σ_{NNLO}

Effect on the total cross section modulated by the (small) contribution of the hard factor: about ***1%*** *of the LO cross section in the* gg *and* ***2-3%*** *in the* $q\bar{q}$ *channel.*

THE MATRIX PROJECT

[M.Grazzini, S. Kallweit, M. Wiesemann: 1711.06631]



MUNICH

MUlti-Cha**N**nel **I**ntegrator at Swiss (**CH**) precision

OPENLOOPS

(Collier, CutTools…)

TWO-LOOP AMPLITUDES

(VVamp, GiNaC, tdhpl…)

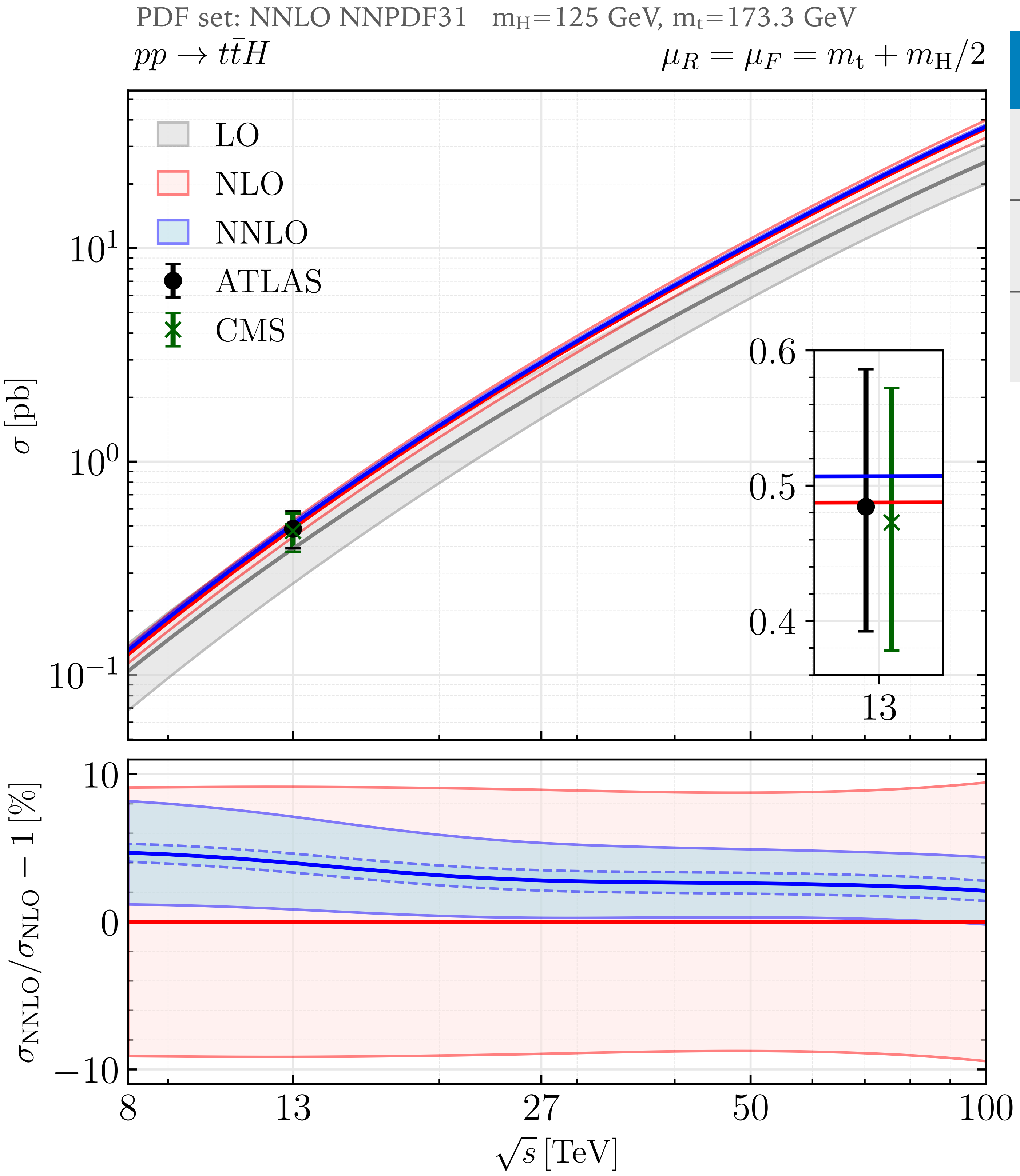
q_T subtraction

MATRIX

Munich **A**utomates q**T** subtraction
and **R**esummation to **I**ntegrate **X**-sections
Code publicly available at https://matrix.hepforge.org

RESULTS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: 2210.07846]



σ [pb]	13 TeV	100 TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

Statistical + soft Higgs uncertainties

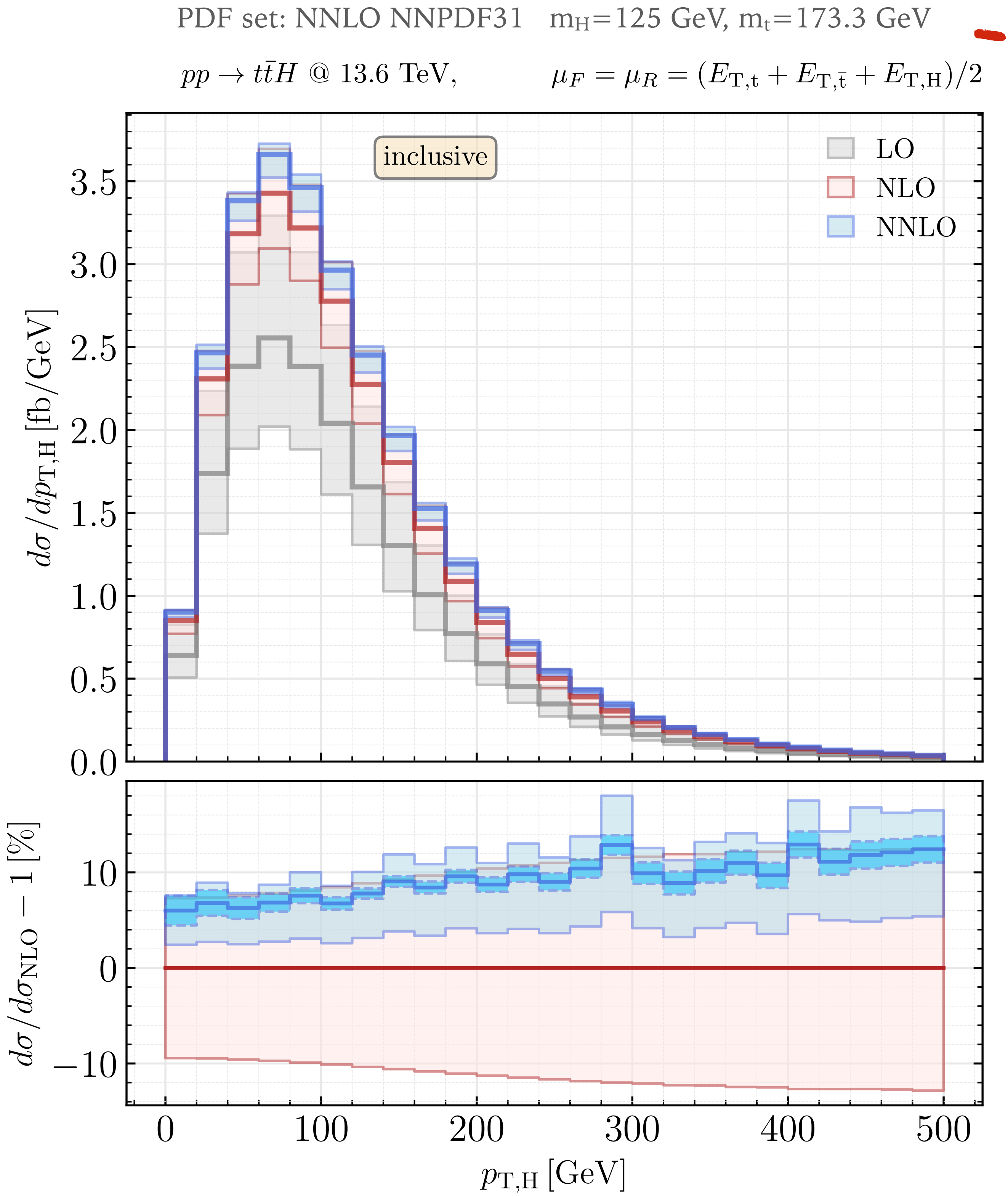
Scale uncertainties

- **NNLO corrections**: **+4%** (13 TeV), **+2%** (100 TeV);
- Reduction of **scale uncertainties**;
- Soft approximation uncertainty significantly **smaller** than remaining perturbative uncertainties.

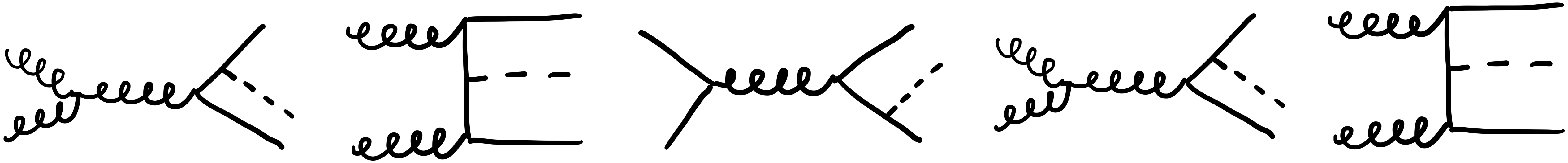
RESULTS

[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: IN PREPARATION]

-PRELIMINARY RESULTS-

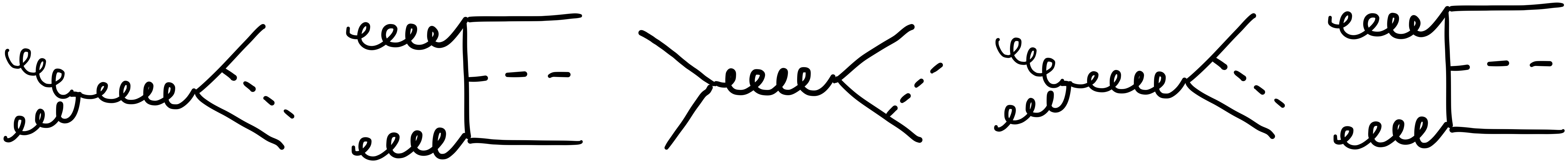


- First results for **differential distributions**;
- Soft approximation uncertainty computed on a **bin-by-bin basis**;
- NLO and NNLO uncertainty bands **overlap**;
- Soft approximation uncertainty **of the same order** over all the spectrum.



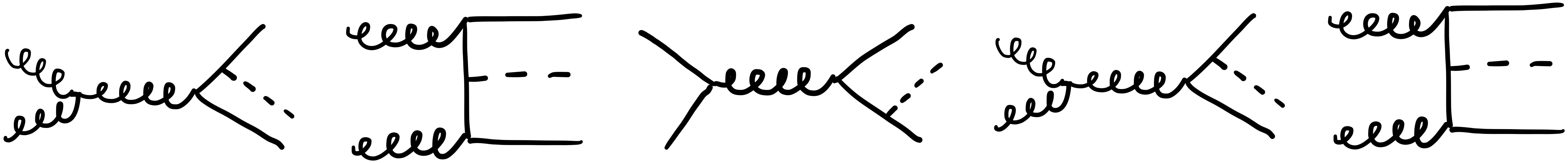
SUMMARY & OUTLOOK

- We computed within q_T subtraction formalism the **NNLO QCD corrections** to $t\bar{t}H$ production;
- The **missing ingredients** we needed for the computation are:
  - **NNLO soft contribution** in arbitrary kinematics;
  - **two-loop amplitudes** (**soft Higgs boson approximation**);
- **First** (almost) exact computation at NNLO QCD for a **2 → 3 process** with massive coloured particles.



SUMMARY & OUTLOOK

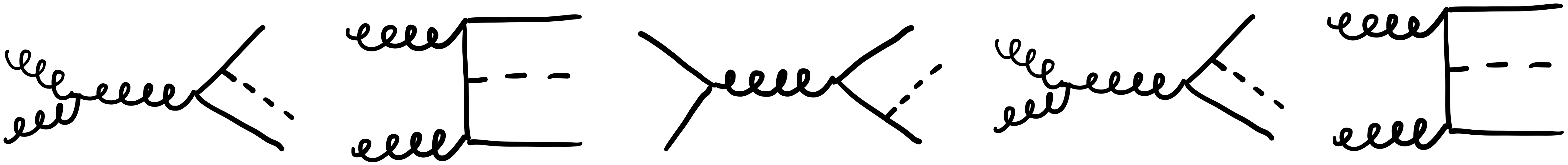
- Phenomenological results with **differential distributions**;
- Combination with **EW corrections**;
- The framework is ready to be applied to **different processes** in the same class.



SUMMARY & OUTLOOK

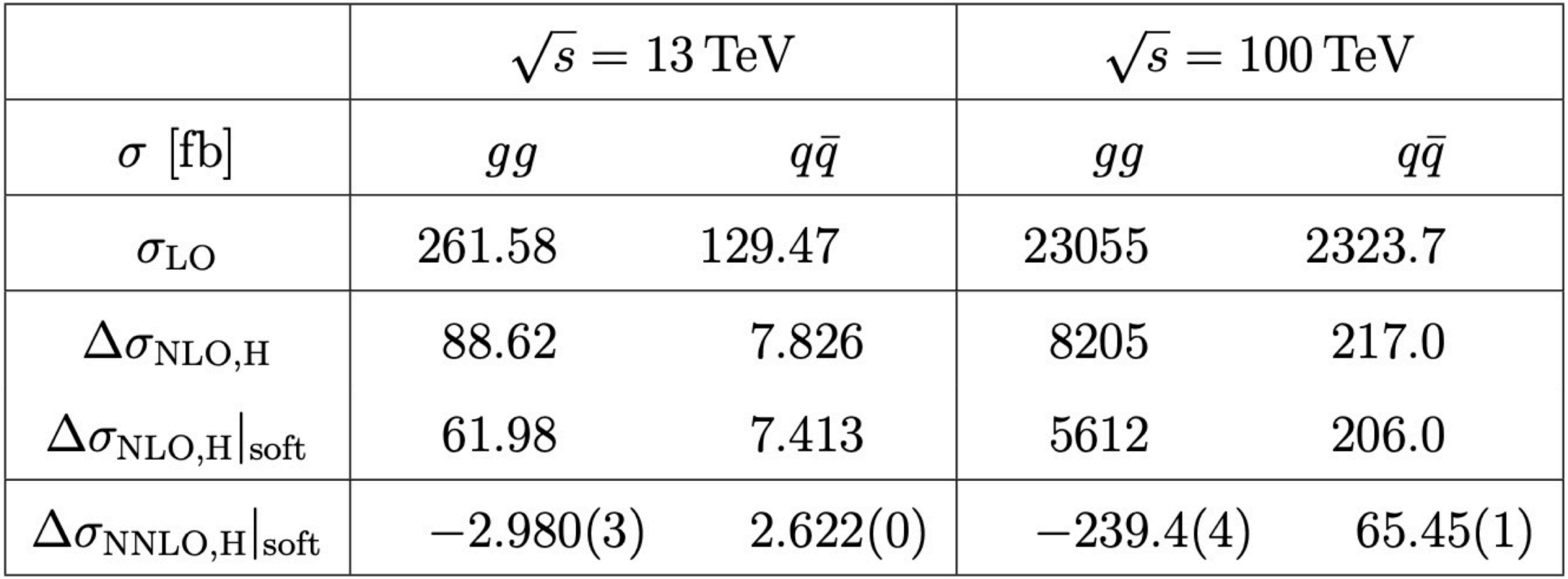
- Phenomenological results with **differential distributions**;
- Combination with **EW corrections**;
- The framework is ready to be applied to **different processes** in the same class.

THANKS!



BACKUP SLIDES

TOTAL CROSS SECTION



	$\sqrt{s} = 13\,\text{TeV}$		$\sqrt{s} = 100\,\text{TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}}\vert_{\text{soft}}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

- Soft Higgs approximation at LO:
  - gg channel: factor 2.3 ($\sqrt{s} = 13$ TeV)/factor 2.0 ($\sqrt{s} = 100$ TeV)
  - $q\bar{q}$ channel: factor 1.11 ($\sqrt{s} = 13$ TeV)/factor 1.06 ($\sqrt{s} = 100$ TeV)
- At LO there is no reweighting!

CHANGING THE SUBTRACTION SCALE

$$H^{(2)}_{t\bar{t}H} = \frac{2\,Re\left(\mathcal{M}^{(2)}_{t\bar{t}H}(\mu_{IR},\mu_R)\mathcal{M}^{(0)}_{t\bar{t}H}\right)_{soft}}{|\,\mathcal{M}^{(0)}_{t\bar{t}H}|^2_{soft}}$$

- The subtraction scale μ_{IR} is the scale at which the IR poles are subtracted (equivalently, at which the soft approximation is applied);
- Effect of using a different subtraction scales $\mu_{IR} \to 2\,\mu_{IR}$, $\mu_{IR} \to 1/2\,\mu_{IR}$.
  - gg channel +164%/-25% ($\sqrt{s} = 13$ TeV)
    +142%/-20% ($\sqrt{s} = 100$ TeV)
  - $q\bar{q}$ channel +4%/-0% ($\sqrt{s} = 13$ TeV)
    +3%/-0% ($\sqrt{s} = 100$ TeV)

SOFT HIGGS APPROXIMATION

Eikonal approximation

$$\lim_{k\to 0}\mathscr{M}_{t\bar{t}H}\big(\{p_i\},k\big) = F\big(\alpha_S(\mu_R);m_t/\mu_R\big)\,\frac{m_t}{v}\sum_{i=3,4}\frac{m_t}{p_i\cdot k}\mathscr{M}_{t\bar{t}}\big(\{p_i\}\big)$$

Low Energy Theorem

$$\lim_{q\to 0}\mathscr{M}^{\text{bare}}(p\to p+q) = \frac{1}{v}m_0\frac{\partial}{\partial m_0}\mathscr{M}^{\text{bare}}(p\to p)\bigg|_{p^2=m^2}$$

$$\begin{aligned}F(\alpha_S(\mu_R);m_t/\mu_R) = {}& 1+\frac{\alpha_S(\mu_R)}{2\pi}\big(-3\,C_F\big)\\ &+\left(\frac{\alpha_S(\mu_R)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2-\frac{185}{12}C_FC_A+\frac{13}{6}C_F(n_L+1)-6\,C_F\beta_0\ln\frac{\mu_R^2}{m_t^2}\right)+\mathcal{O}(\alpha_S^3)\end{aligned}$$

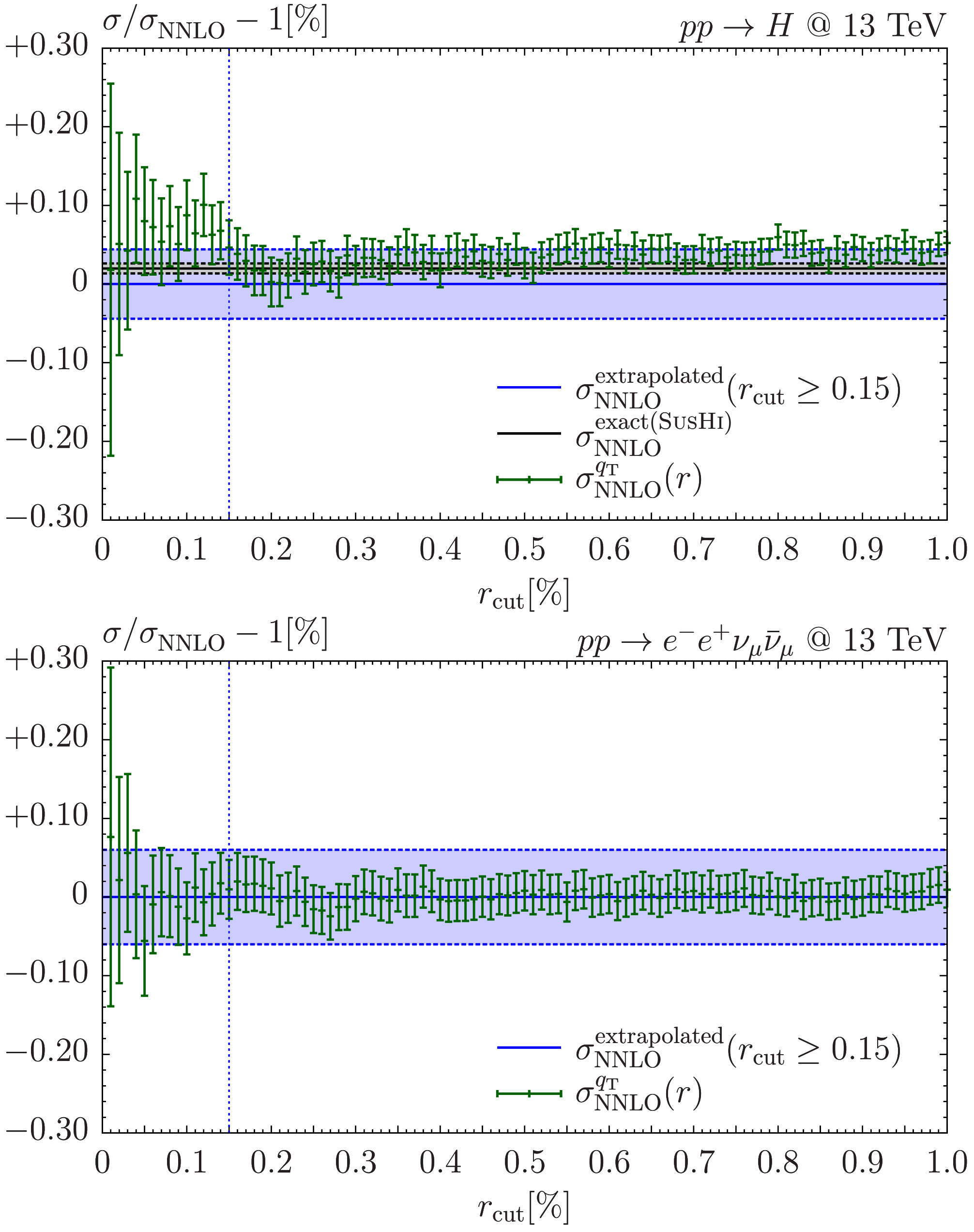
THE SLICING

$$d\sigma^{F}_{(N)NLO} = \mathscr{H}^{F}_{(N)NLO} \otimes d\sigma^{F}_{LO} + \left[d\sigma^{F+jets}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right]$$

$d\sigma^{F+jets}_{(N)LO}$ and $d\sigma^{CT}_{(N)LO}$ are separately divergent. In practice, q$_T$ subtraction is implemented as a slicing method:

- introducing a cutoff $r_{cut} = Q/M$;
- performing the limit $r_{cut} \to 0$.

Quality of the $q_T \to 0$ extrapolation can be understood looking at the r_{cut} dependence



[M.Grazzini, S. Kallweit, M. Wiesemann: arXiv 1711.06631]

r_{CUT} DEPENDENCE