# Weak boson production at hadron colliders: combined QCD-QED transverse-momentum resummation formalism

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# Introductions & Motivations

View of the city of Blois

- Drell-Yan (DY) mechanism [1] is a paramount at hadron colliders
  - SM and BSM physics
  - detector calibration
  - SM parameters extraction ($m_W$, $\alpha_S$, ...)
  - ...
- Nowadays an astonishing experimental precision (per-thousand level) is achieved

$\Rightarrow$ Need of computation of **higher-order perturbative corrections**

- In the framework of QCD $q_T$ resummation, predictions are known at percent, or even higher, level of precision $\Rightarrow$ EW corrections must be taken into account: $\alpha \sim \alpha_S^2$

[1] S.D. Drell and T.-M. Yan, 1970

# State-of-the-art of $q_T$ resummation: N3LL accuracy

Disclaim: a short review, far-from being exhaustive

- The procedure to implement resummation of $q_T$ logarithmic-enhanced terms is known since long time ([Parisi,Petronzio('79)],[Kodaira,Trentadue('82)], [Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01) ]
- Nowadays, different procedures to perform resummation have been developed in the aim of N3LL and beyond accuracy
- Resummation in direct $q_T$ space → DY predictions up to N3LL+N3LO ( [W. Bizo, X. Chen, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, P. Torrielli, D. M. Walker ('18, '19, '21, '22)])
- Resummation in the framework of Effective Theories → some relevant kynematic distributions known up to N3LL ([M. A. Ebert, J.K.L. Michel, I. W. Stewart, F. J. Tackmann ('21)]), N3LL+NNLO ([T. Becher, T, Neumann ('20)]), $\text{N4LL}_P$ + N3LO ([T. Neumann, J. Campbell ('22)])
- Studies within Transverse-Momentum dependent (TMD) factorization and TMD parton densities → TMD parton distributions up to N3LL ([A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, M. Radici ('20)] )

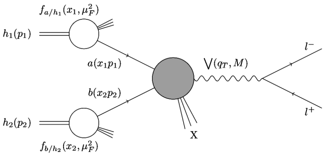
- In this paper we apply to Drell-Yan transverse-momentum distribution the resummation formalism developed by ([Catani, de Florian, Grazzini ('01)]) and firstly applied for the case of Higgs boson production ([Bozzi, Catani, de Florian, Grazzini ('03, '06, '08)])
- State of the art $q_T$ distributions can be computed at N3LL+N3LO ([Camarda et al. ('20)], [S. Camarda, L.Cieri, G.Ferrera ('21)] ) and even at **N4LL** [S. Camarda, L.Cieri, G.Ferrera ('23)] ) [hep-ph:2303.12781 ]
- QED corrections at NLL+NLO were computed only for on-shell Z production (L. Cieri, G. Ferrera, G.F.R. Sborlini, 2018) $\rightarrow$ we extend the formalism to reach $\text{NLL}_{QED} + \text{NLO}_{EW}$ accuracy for both charged and neutral current on-shell Drell-Yan processes

## Drell-Yan $q_T$ distribution

$$\frac{d\sigma_V}{dq_T^2}(q_T, M, s) \overset{\text{factorization theorem}}{=}$$

$$= \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2)$$

$$\times \frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2}\left(q_T, M, \hat{s}, \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2\right)$$




- In the region $q_T \gtrsim M_V$ the perturbative fixed-order expansion is reliable:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{d\hat{\sigma}^{(0)}_{V_{ab}}}{dq_T^2} + \frac{\alpha_S}{\pi}\frac{d\hat{\sigma}^{(1)}_{V_{ab}}}{dq_T^2} + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{d\hat{\sigma}^{(2)}_{V_{ab}}}{dq_T^2} + \mathcal{O}\left[\left(\frac{\alpha_S}{\pi}\right)^3\right]$$

- In the region $q_T << M_V$ (bulk of the events) large logarithmic corrections of the type $\alpha_S^n \ln^m(M_V^2/q_T^2)$, due to soft and/or collinear parton radiations, spoil the convergence

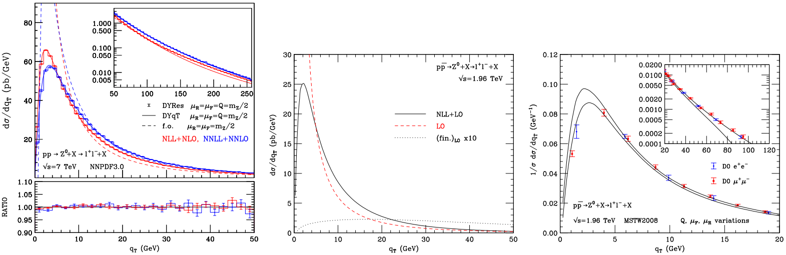
$$\Downarrow$$

Resummation at all perturbative orders is mandatory:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{\hat{\sigma}^{(0)}_V}{q_T^2} \sum_{n=1}^{+\infty} \sum_{m=0}^{2n-1} A^V_{n,m} ln^m\left(\frac{M^2}{q_T^2}\right) \alpha_S^n(M^2), \quad \alpha_S^n \ln^m(M_V^2/q_T^2) >> 1$$

# Analytic Resummation formalism in $q_T$

**G. Bozzi, S. Catani, D. de Florian, M. Grazzini** hep-ph/0508068




G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini [1007.2351[hep-ph]], [1507.06937[hep-ph]]

- Partonic cross section is explicitly splitted as:

$$\frac{d\hat{\sigma}_{a_1 a_2 \to V}}{dq_T^2} = \frac{d\hat{\sigma}^{res}_{a_1 a_2 \to V}}{dq_T^2} + \frac{d\hat{\sigma}^{fin}_{a_1 a_2 \to V}}{dq_T^2}, \quad \text{with} \quad \lim_{Q_T \to 0} \int_0^{Q_T} dq_T^2 \frac{d\hat{\sigma}^{fin.}_{a_1 a_2 \to V}}{dq_T^2} = 0$$

- Resummation is performed in impact parameter (**b**) space

$$\frac{d\hat{\sigma}^{res.}_{a_1 a_2 \to V}}{dq_T^2}(q_T; M, \hat{s}; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}^V_{a_1 a_2}(b; M, \hat{s}, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2).$$

$\mathcal{W}^V$ can be expressed in an exponential and factorized form in the Mellin space $\rightarrow$ $z = M_V^2/\hat{s}$, $f_N = \int_0^1 dz\, z^{N-1} f(z)$: [2]

$$\mathcal{W}_N^V = \mathcal{H}_N^V(M, \alpha_S(\mu_R^2)) \times exp\, \mathcal{G}_N\left(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2\right),$$

$$L = \ln\left(\frac{Q^2 b^2}{b_0^2} + 1\right),\ b_0 = 2exp(-\gamma_E), \gamma_E = 0.5772\ldots$$

$$\mathcal{H}_N^V\left(M, \alpha_S, \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right) = \hat{\sigma}_0^V(M)\left[1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_N^{V(n)}\left(\frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right)\right],$$

$$\mathcal{G}_N\left(\alpha_S(\mu_R^2), L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2})\right) = -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left(A(\alpha_S(q^2))\log\left(\frac{M^2}{q^2}\right) + \tilde{B}_N(\alpha_S(q^2))\right) =$$

$$= L g_N^{(1)}(\alpha_S(\mu_R^2)\, L) + g_N^{(2)}\left(\alpha_S(\mu_R^2)\, L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-2} g_N^{(n)}\left(\alpha_S(\mu_R^2)\, L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right)$$

- $\mathcal{H}_N^V$, $A(\alpha_S)$ and $B(\alpha_S)$ have a customary $\alpha_S$-expansion:

$$\Downarrow$$

**Perturbative structure of the resummed component**:
LL accuracy ($\sim \alpha_S^n L^{n+1}$): $g_N^{(1)}$; NLL accuracy ($\sim \alpha_S^N L^n$): $g_N^{(2)}$ , $\mathcal{H}_N^{(1)}$ ; NNLL accuracy: ($\sim \alpha_S^N L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$ ; N3LL accuracy ($\sim \alpha_S^N L^{n-2}$): $g_N^{(4)}$ , $\mathcal{H}_N^{(3)}$

[2] Flavour indices are understood

## On-shell Z boson production

- QED corrections at NLL+NLO known
  - [L. Cieri, G. Ferrera, G. F. R. Sborlini 1805.11948[hep-ph]]
  - Direct abelianization of QCD resummation formalism for a colourless final state
- We incorporated also weak corrections at one loop within the hard factor $\mathcal{H}$
  - We consider one-loop renormalized form factor
  - Modifications only in the hard factor $\mathcal{H}$ (massive loop corrections)
  - Accuracy at $\mathrm{NLL}_{\mathrm{QED}} + \mathrm{NLO}_{\mathrm{EW}}$

## On-shell W boson production **(NEW)**

- Charged final state $\rightarrow$ a "naive abelianization" of the QCD formulation for DY process is not suitable
- We use the formalism of $t\bar{t}$ production **S. Catani, M. Grazzini, A. Torre 1408.4564[hep-ph]**)

1. Replacement: $t\bar{t} \rightarrow W$ (colour charged $\rightarrow$ electrically charged)
2. Abelianization of QCD result
   - absence of colour correlations involving initial and finale state (abelian limit)

- We start from the QCD resummation program generalized to a colourful final state:

$$W_N^V(b,M) = \sum_{c a_1 a_2} \sigma^{(0)}_{c\bar{c},V}(\alpha_S(M^2)) f_{a_1/h_1,N}(b_0^2/b^2) f_{a_2/h_2,N}(b_0^2/b^2) \times S_c(M,b) \times [(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]_{c\bar{c},a_1,a_2;N}(M^2, b_0^2/b^2)]$$

  - $[(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]$ hard factor; $S_c$ Sudakov form factor
  - $\mathbf{\Delta}$ related to soft (non-collinear) wide-angle radiation from final state and from initial-final state interferences ($\mathbf{\Delta} = 1$ for neutral final states)

- Applying the abelianization procedure to Eqs. (15-18) of Ref [1408.4564], we obtain:

$$\Delta(\alpha; Q, b) = \exp\left\{ - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

$$D'(\alpha) = \frac{\alpha}{\pi} D'^{(1)} + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n D'^{(n)}$$

- Resummation of additional logarithmic-enhaced contributions of the type: $\alpha^n \log(Qb)^k$
  - $D'(\alpha)$ starts to contribute at NLL accuracy (as $B'_N$)

## Sudakov form factor

- The coefficient $D$ can be absorbed in the colourless Sudakov form factor
  - Exponentiation of single-logarithmic enhanced terms due to a charged final state
- In combined QCD-QED resummation formalism, we finally obtain the following generalization:

$$\mathcal{G}'_N(\alpha, L) = -\int_{b_0^2}^{b^2} \frac{dq^2}{q^2} \left( A'(\alpha(q^2)) \log\left(\frac{M^2}{Q^2}\right) + \tilde{B}'_N(\alpha(q^2)) + D'(\alpha(q^2)) \right) =$$

$$= Lg'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g_N'^{(n)}(\alpha L)$$

- $A'(\alpha)$ and $\tilde{B}'_N(\alpha)$ are related to QED initial-state radiation $\rightarrow$ direct abelianization of QCD analogous through the replacement:

$$2\, C_F \rightarrow (e_q^2 + e_{\bar{q}'}^2)$$

- $D'(\alpha)$ is instead an additional term, due to a charged final state, characteristic of final-state massive radiation
  - it can be obtained by a suitable abelianization of the soft anomalous dimension matrix of Eqs. (15)-(17) of Ref. ([1408.4564[hep-ph]])

$$D^{(1)} = \frac{-e_W^2}{2}$$

- Observation: the additional resummed contribution **implies** the replacement $B_1 \rightarrow B_1 + D_1$ in all the ingredients of the original formalism ($\Sigma$, $\mathcal{H}$, $\tilde{S}$)

## Hard collinear coefficient function

- We started from $t\bar{t}$ subtraction operator of **1408.4564[hep-ph]**, transforming it properly
- The one-loop virtual renormalized form factor was included in $\mathcal{H}$

**Combined QCD-QED resummation formalism**

- As widely discussed we consistently included QED effects in the well-known QCD resummation formalism
- The generalized expressions are thus double perturbative expansion in $\alpha$ and $\alpha_S$:

$$\mathcal{G}'_N(\alpha_S,\alpha,L) = \mathcal{G}_N(\alpha_S L) + L g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{(n-2)} g_N'^{(n)}(\alpha L) +$$

$$+ g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{n,m=1;n+m\neq 2}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L,\alpha L)$$

and:

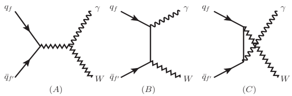
$$\mathcal{H}_N'^{V}(\alpha_S,\alpha) = \mathcal{H}_N^{V}(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}_N'^{V(1)} + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n}\mathcal{H}_N'^{V(n)} + \sum_{n,m=1}^{+\infty}\mathcal{H}_N'^{V(n,m)}$$

- We also considered the mixed QCD-QED contributions at LL, by including $g'^{(1,1)}(\alpha_S L,\alpha L)$ (1805.11948)
- For the sake of completeness, $f_{\gamma/h}(x,\mu_F^2)$ and QED effects in PDF evolution were included in the factorization formula

$$\Downarrow$$

$$\boxed{(\text{NNLL+NNLO})_{\text{QCD}} + (\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}})}$$

# Small $q_T$ expansion of real cross sections at NLO: photon emission




- This calculation reproduces the logarithmic structure of the fixed-order expansion of the Sudakov form factor
  - Cross-check of our formulas
  - Confirms the validity of the replacement $t\bar{t} \to W$ and abelianization procedures

$$\text{Hadronic cross section : } \sigma = \sum_{ab} \tau \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \frac{1}{z} \int dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}$$

with: $\tau = Q^2/S$

$$\text{Partonic inclusive cross section : } \hat{\sigma}_{ab}(z) = \int_{(q_T^{cut})^2}^{(q_T^{max})^2} dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2},$$

Inclusive hadronic cross section, introducing $f(a) = 2\sqrt{a}(\sqrt{1+a} - \sqrt{a})$, $a \equiv \dfrac{(q_T^{cut})^2}{Q^2}$ :

$$\sigma^{>(1)}_{q_f \bar{q}_{f'}} = \tau \int_0^{1-f(a)} \frac{dz}{z} \mathcal{L}_{q_f \bar{q}_{f'}}\left(\frac{\tau}{z}\right) \frac{1}{z} \hat{\sigma}^{(1)}_{q_f \bar{q}_{f'}}(z) = \tau \int_\tau^1 \frac{dz}{z} \mathcal{L}_{q_f \bar{q}_{f'}}\left(\frac{\tau}{z}\right) \hat{\sigma}^{(0)} \hat{G}^{(1)}_{q_f \bar{q}_{f'}}(z), \text{ with :}$$

$$\hat{G}_{q_f \bar{q}_{f'}} = \sum_{m,r} \log^m(a) a^{\frac{r}{2}} \hat{G}^{(1,m,r)}_{q_f \bar{q}_{f'}}(z), \text{ power series in the cutoff}$$

- Final expression obtained:

$$\hat{G}^1_{q_f \bar{q}_{f'}} = \log(a) \left( \frac{3}{2}\delta(1-z)\frac{\left(e^2_{q_f} + e^2_{q_{f'}}\right)}{2} - \frac{1}{2}\left( \left(P^{\mathrm{QED}}\right)_{q_f q_f} + e^2_W \delta(1-z) - \left(P^{\mathrm{QED}}\right)_{\bar{q}_{f'} \bar{q}_{f'}} \right) \right) +$$

$$+\frac{1}{2}\log^2(a)\,\delta(1-z)\frac{\left(e^2_{q_f} + e^2_{q_{f'}}\right)}{2} + \sqrt{a}\,\frac{1}{2}e^2_W\left(2\pi\delta'(1-z) - 3\pi\delta(1-z)\right)$$

$$+\textit{finite terms} + \textit{higher order terms}$$

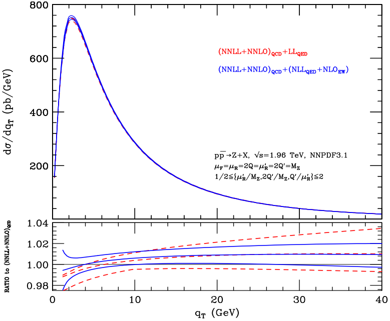
- $P^{QED}_{q_f \bar{q}_f}$ AP splitting functions in QED (D. de Florian, G. Rodrigo, G. F. R. Sborlini: 1611.04785[hep-ph], 1512.00612[hep-ph], 1606.02887[hep-ph])
- We reproduce the known $A$ and $B$ perturbative coefficient of the QCD resummation formalism, modulo $C_F \to \frac{\left(e^2_{q_f} + e^2_{q_{f'}}\right)}{2}$
- Additional logarithmic divergence from the charged final state $\sim D'_1 \log(a)$, $D'_1 = -\frac{e^2_W}{2}$
- A linear power correction in the cutoff ($\sqrt{a}$) and proportional to the charged final state is present
  - Accordingly with L. Buonocore, M. Grazzini, F. Tramontano: 1911.10166 [hep-ph] (massive leg emission $\to$ linear power correction)

# Numerical Results at hadron colliders

- On-shell $W$ and $Z$ boson production
- Resummation formalism together with a consistent matching procedure is implemented in the FORTRAN program $\mathrm{DYQT}$ **G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, [1007.2351 [hep-ph]], [0812.2862 [hep-ph]]**
- We reached the accuracy: $(\mathrm{NNLL}+\mathrm{NNLO})_{\mathrm{QCD}} + (\mathrm{NLL}_{\mathrm{QED}} + \mathrm{NLO}_{\mathrm{EW}})$
- Input parameters $\alpha(m_Z^2)$, $m_W$, $m_Z$, $|V_{CKM}|$ (PARTICLE DATA GROUP collaboration, 2022)
- To compute EW corrections at NLO we took as input values also $m_H$ and $m_t$ (PARTICLE DATA GROUP collaboration, 2022)
- PDF: NNPDF3.1.luxQED at NNLO in QCD (NNPDF Collaboration, 1706.00428[hep-ph], 1712.07053[hep-ph] ), with inclusion of $f_\gamma$ and LO QED effects in the evolution
- Perturbative uncertainty: scale variation method
  - QCD scales: $\mu_F = \mu_R = 2Q = m_V$
  - QED scales: simultaneous variations of $Q'$ and $\mu'_R$, according to:

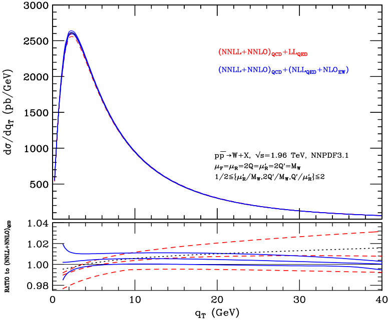
$$m_V/2 \leq \{\mu'_R, 2Q'\} \leq 2m_V,\ 1/2 \leq \{\mu'_R/Q'\} \leq 2,\ \mu'_F = m_V$$

## Z Boson production at Tevatron, $\sqrt{s} = 1.96$TeV



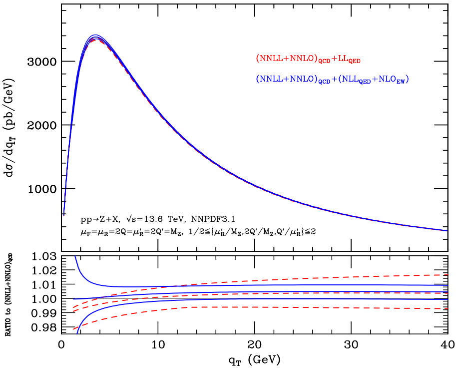

- $LL_{QED}$:
  - Spectrum slightly harder (effects of $\mathcal{O}(1\%)$)
  - Scale variation band: $\mathcal{O}(2-4\%)$
- $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$:
  - Effects of $\mathcal{O}(0.5\%)$
  - Scale variation band: reduction by roughly a factor 2
- Analogy with ([L. Cieri, G. Ferrera, G. F. R. Sborlini, 1805.11948[hep-ph] ], QED effects up to $(\text{NLL+NLO})_{\text{QED}}$

## W Boson production at Tevatron, $\sqrt{s} = 1.96$TeV



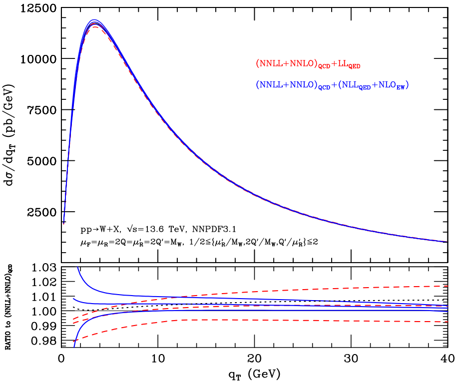

- $\text{LL}_{\text{QED}}$:
  - Spectrum slighly harder (effects bit less than $\mathcal{O}(1\%)$)
  - Scale variation band: $\mathcal{O}(2-3\%)$
- $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$:
  - Effects: $+(-)\ \mathcal{O}(1\%)$
  - Soft wide angle radiation makes the spectrum softer ($D'_1$ is negative as $B'_1$) (analogy with S. Catani, M. Grazzini, H. Sargsyan 1806.01601[hep-ph], QCD resummation for $t\bar{t}$)
  - Scale variation band: reduction of a factor 1.5-2 (up to 3)

- $Z$ **boson production at LHC,** $\sqrt{s} = 13.6$**TeV**



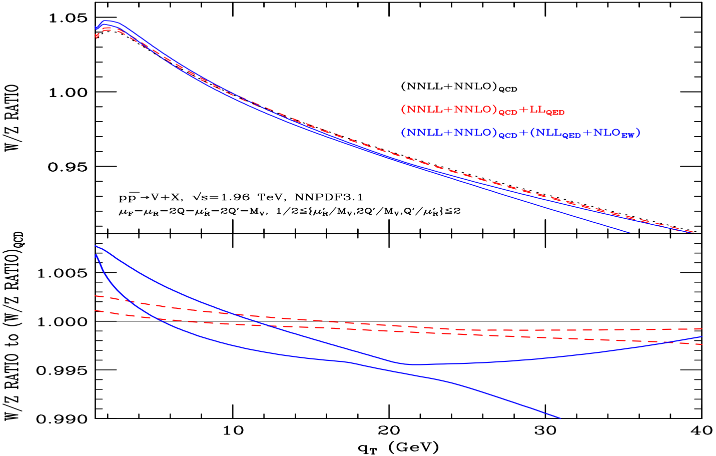

- Impact of QED corrections less than Tevatron case (enhancement of gluon PDF)
- Similar qualitative behaviour of QED effects
- $\mathrm{LL_{QED}}$: $-\mathcal{O}(1\%)$ $(+\mathcal{O}(0.5\%))$ (harder spectrum); scale variation band: $\mathcal{O}(2\%)$
- $\mathrm{NLL_{QED}} + \mathrm{NLO_{EW}}$: effects $+\mathcal{O}(0.5\%)$; scale variation band: reduction by roughly a factor 1.5-2
- Analogy with ( [L. Cieri, G. Ferrera, G. F. R. Sborlini, 1805.11948[hep-ph]])

- $W$ **boson production at LHC,** $\sqrt{s} = 13.6$**TeV**



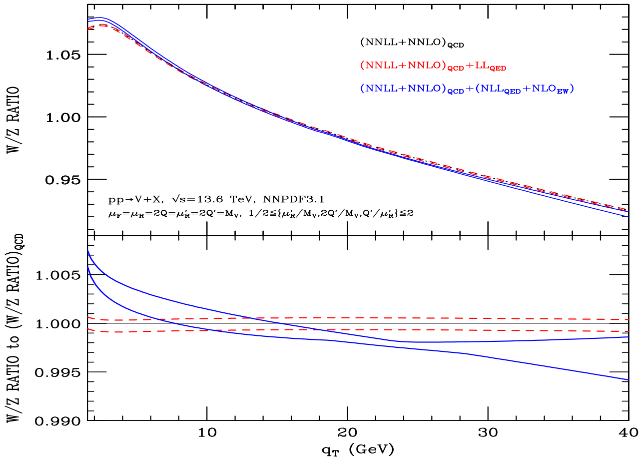

- Impact of QED radiation less than Tevatron case (enhancement of gluon PDF)
- Similar qualitative behaviour of QED effects
- $\mathrm{LL_{QED}}$: spectrum harder; scale variation band $\mathcal{O}(2\%)$
- $\mathrm{NLL_{QED}} + \mathrm{NLO_{EW}}$: Soft wide angle radiation makes the spectrum softer; scale variation band: reduction of factor $1.5 - 2$ (up to 4)

- $R(q_T) = \frac{\frac{1}{\sigma_W}\frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z}\frac{d\sigma_Z}{dq_T}}$, **Tevatron** $\sqrt{s} = 1.96$**TeV**




- Relevant for $m_W$ extraction $\boxed{q_T(Z)^{meas.} + R(q_T)^{theo.}} \rightarrow \boxed{q_T(W)}$
- $LL_{QED}$: slightly softer spectrum; per-thousand level effects and scale variation band
- $NLL_{QED} + NLO_{EW}$ : QED contributions not suppressed; softer spectrum ($\mathcal{O}(0.5 - 1\%)$); scale variation band: $\underline{\mathcal{O}(0.1\%) - \mathcal{O}(1\%)}$

$$R(q_T) = \frac{\frac{1}{\sigma_W}\frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z}\frac{d\sigma_Z}{dq_T}}$$, **LHC** $\sqrt{s} = 13.6$**TeV**




- Less impact of QED radiation (enhancement of gluon luminosity)
- $LL_{QED}$: less than per-thousand level effects and scale variation band
- $NLL_{QED} + NLO_{EW}$ : QED contributions not suppressed; softer spectrum ($\mathcal{O}(0.5\%)$); scale variation band: $\mathcal{O}(0.1\%) - \mathcal{O}(0.5\%)$

an overlap of the bands is not observed (Tevatron, LHC)

# Summary & Outlook

- We considered **QED corrections** to $q_T$ **resummation formalism in QCD** for weak boson production (focusing especially on charged-current process )
- Final state radiation is fully included by direct abelianization the resummation formalism for a coloured final state ( $t\bar{t}$ )
- Expansion at small $q_T$ of the real inclusive cross section has confirmed the validity of the **abelianization procedure**
- Through the use of the numerical code DYqT we presented numerical predictions at $(\text{NNLL+NNLO})_{\text{QCD}} + (\text{NLL})_{\text{QED}} + (\text{NLO})_{\text{EW}}$, finding QED effects from **per thousand** to **percent** level
- We considered also the ratio distribution $q_T(W)/q_T(Z)$, in the direction of $m_W$ extraction $\rightarrow$ a sizeable **reduction of scale-variation band** is observed at **LL**, while the predictions at **NLL+NLO do not** benefit from the cancellation of common uncertainties

**Prospects**

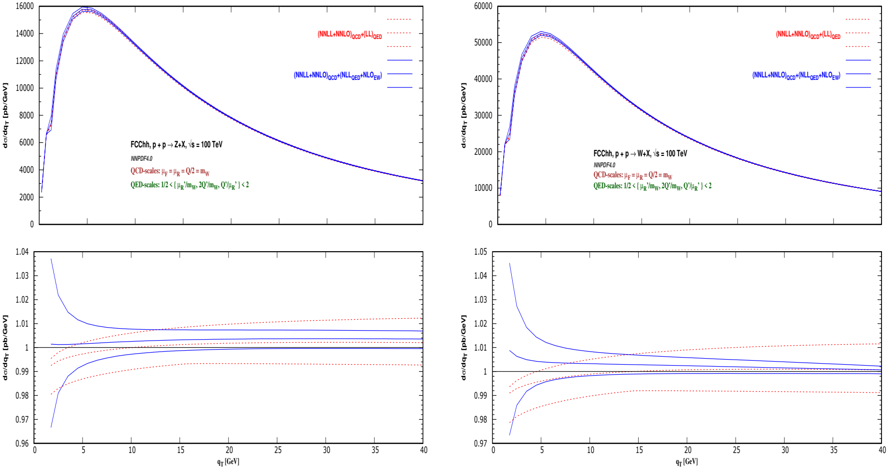
- Inclusion of the decay of weak boson and the radiation from leptonic final state
- **On-going work**: Effects added in DYTURBO
  (fast and precise numerical predictions)
  - Code optimisation ( starting from DYqT, DYRES, DYNNLO)
  - Factorization into production and decay variables
  - Numerical integration based on interpolating functions
  - ...
- Correction to production (along the line of DYqT code ) and decay (NEW)

...

**Thank you for the attention!!**

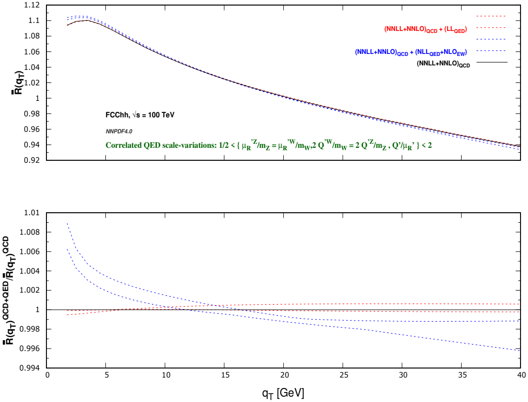
# BACKUP SLIDE

# Predictions at FCChh, $\sqrt{s} = 100\text{TeV}$, $q_T$-spectra




- Qualitative features of QED effects analogous to Tevatron and LHC cases ( LL: spectrum harder; NLL+NLO: different functional behaviour between Z and W; reduction of bandwidths at NLL+NLO ...
- Strong suppression of quark-induced reactions )
- Slightly worse overlapping of the band with respect to Tevatron and LHC

# Predictions at FCChh, $\sqrt{s} = 100\text{TeV}$, $q_T(W)/q_T(Z)$-spectra




- $LL_{QED}$: less than per-thousand effects and scale-variation band
- $NLL_{QED} + NLO_{EW}$: QED contributions and scale variation band at per-thousand level
- Non-overlapping of the bands

**At FCC-hh PDF extrapolation is challenging (out of experimental accessible range)**

## Matching procedure

- For intemerdiate $q_T$ values, the resummed component should be properly combined with fixed order expansion $\rightarrow$ matching procedure:
  - recover fixed-order series for $q_T \lesssim M_V$ where $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{res.} \rightarrow 0$
  - avoid double counting of logarithmic terms $\rightarrow$ counterterm $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{asym}$:
- The finite part is $\left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{fin.} = \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\sigma_{ab}^{res}}{dq_T^2}\right]_{f.o.} = \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{f.o.} - \left[\frac{d\sigma_{ab}}{dq_T^2}\right]_{asym}$
- The counterterm, in impact parameter and Mellin space, is given by the expansion:

$$\left[\mathcal{H}^V_{a_1 a_2, N} \times \exp\left(\mathcal{G}_{a_1 a_2, N}\right)\right] \approx \sigma^{(0)}_{c\bar{c},V}(\alpha_S, M)\left[\delta_{ca_1}\delta_{ca_2}\delta(1-z)\right.$$

$$+ \sum_k \left(\frac{\alpha_S}{\pi}\right)^k \Sigma^{V,(k)}_{c\bar{c}\leftarrow a_1 a_2}(z, L; M^2/\mu_R^2, M^2/\mu_F^2 m M^2/Q^2)$$

$$\left. + \sum_k \left(\frac{\alpha_S}{\pi}\right)^k \mathcal{H}_{c\bar{c}\leftarrow a_1 a_2}(z; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2)\right]$$

**Coloured (charged) final state** S. Catani, M. Grazzini, A. Torre, [1408.4564 [hep-ph]]

- Drell-Yan final state is coulourless
- In case of charged final state (e.g. $t\bar{t}$) production the hadronic resummed contribution is generalized according to:

$$W_N^V(b,M) = \sum_{ca_1a_2} \sigma^{(0)}_{c\bar{c},V}(\alpha_S(M^2)) f_{a_1/h_1,N}(b_0^2/b^2) f_{a_2/h_2,N}(b_0^2/b^2)$$

$$\times S_c(M,b) \times [(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]_{c\bar{c},a_1,a_2;N}(M^2, b_0^2/b^2)]$$

  - $[(\mathbf{H}^V \mathbf{\Delta} C_1 C_2)]$ hard factor; $S_c$ Sudakov form factor
  - $\mathbf{\Delta}$ related to soft wide-angle radiation from final state ( $\mathbf{\Delta} = 1$ for colourless final states)
- We extend this formalism for electrically charged emission in the proceeding of the talk

# Higgs production at the LHC using $q_T$ subtraction formalism at $N^3LO$ QCD

- **L. Cieri (a,b) , X. Chen (b) , T. Gehrmann (b) , E.W.N. Glover (c) and A. Huss 1807.11501[hep-ph]**
- the central prediction at $N^3LO$ almost coincides with the upper edge of the band

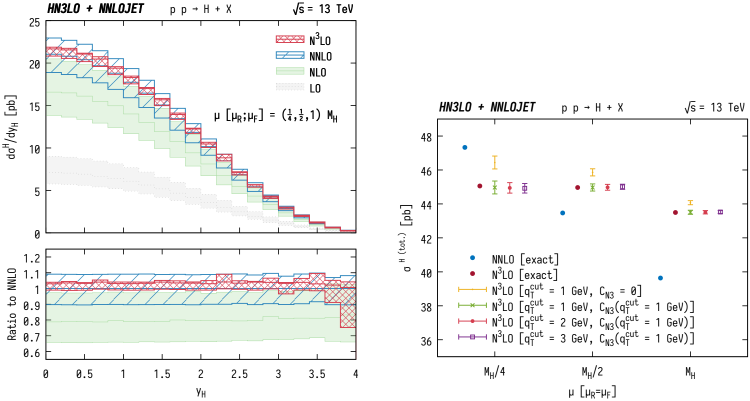



Figure: Rapidity distribution of the Higgs boson computed using the $q_T$ subtraction formalism up to $N^3LO$ (left panel) and the total cross section of the same process.