



Dark Matter production from the inflaton

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34th Rencontres de Blois on Particle Physics and Cosmology

Based on

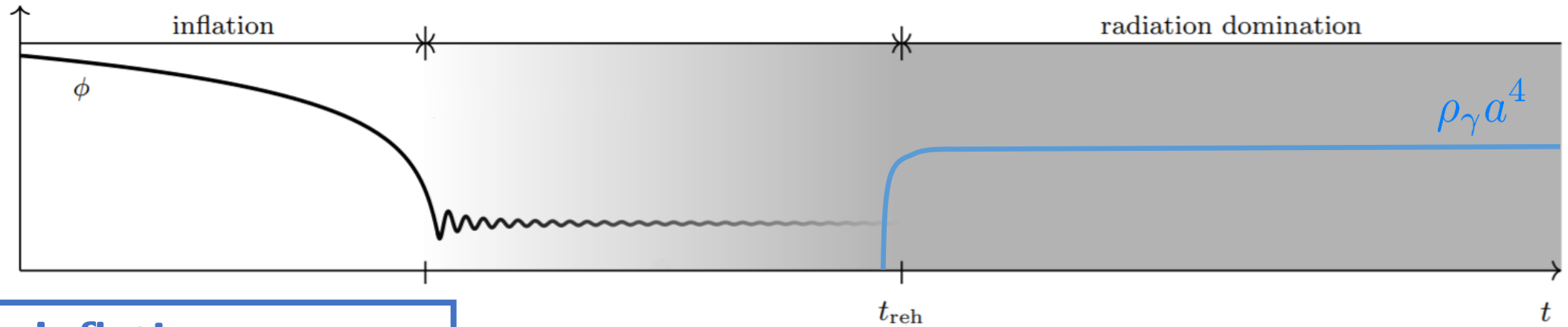
[arXiv:2303.07359 - PRD] & [arXiv:2206.08940 - PRD]

with **S. Verner & M. A. G. Garcia**

Could dark matter be produced from the inflaton?

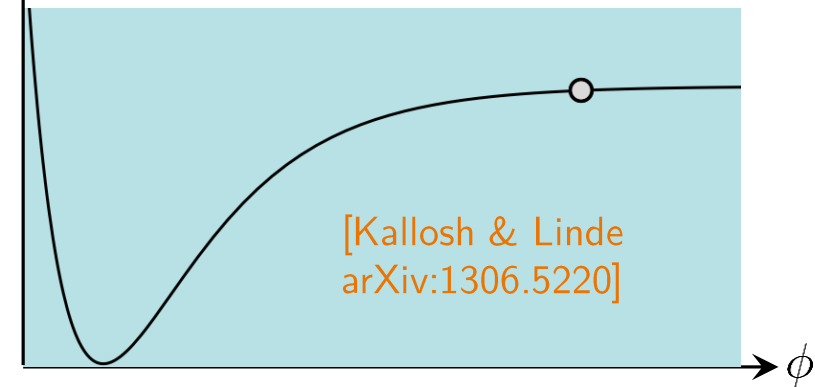
What are the associated signatures/constraints?

The early universe



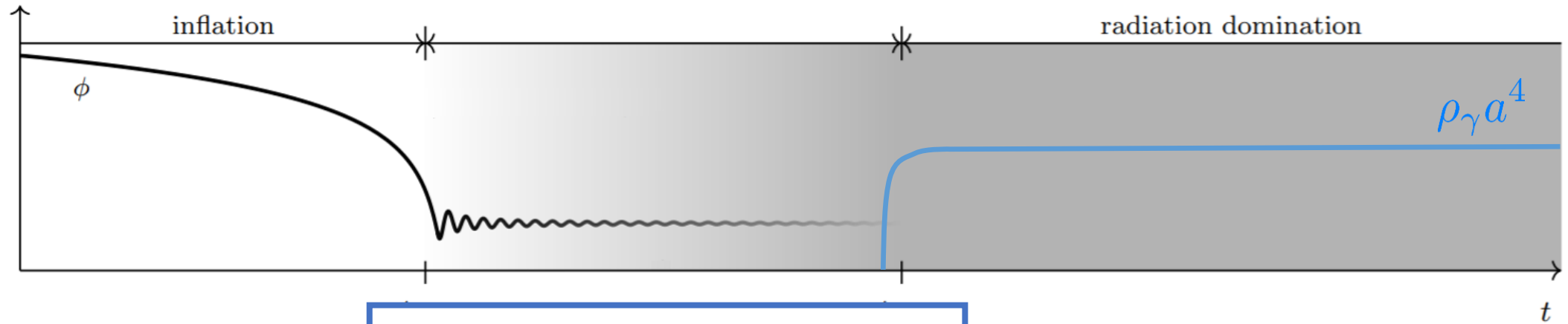
Inflation

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6} \tanh \left(\frac{\phi}{\sqrt{6} M_P} \right) \right]^2$$



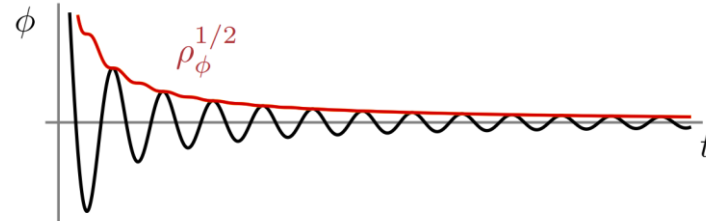
$$\lambda \simeq \frac{18\pi^2 A_s(k_*)}{6N_*^2} \sim 10^{-11}$$

The early universe



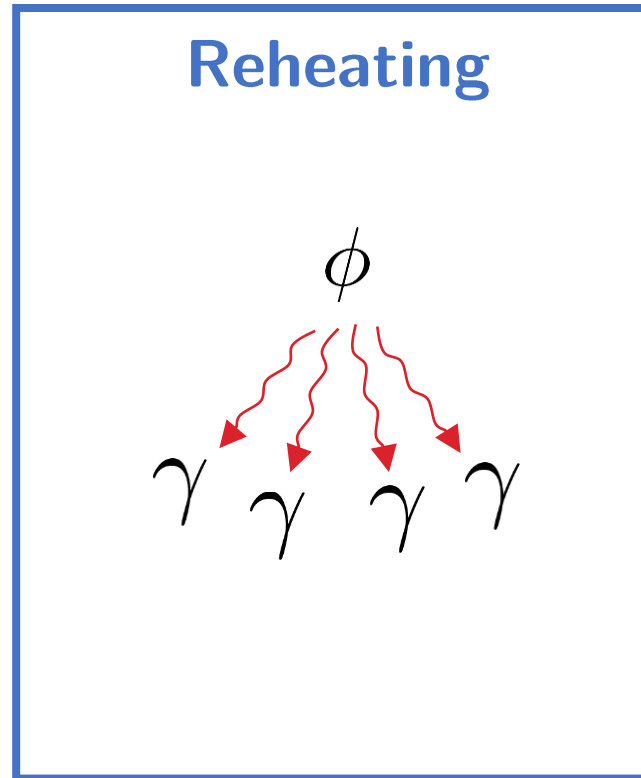
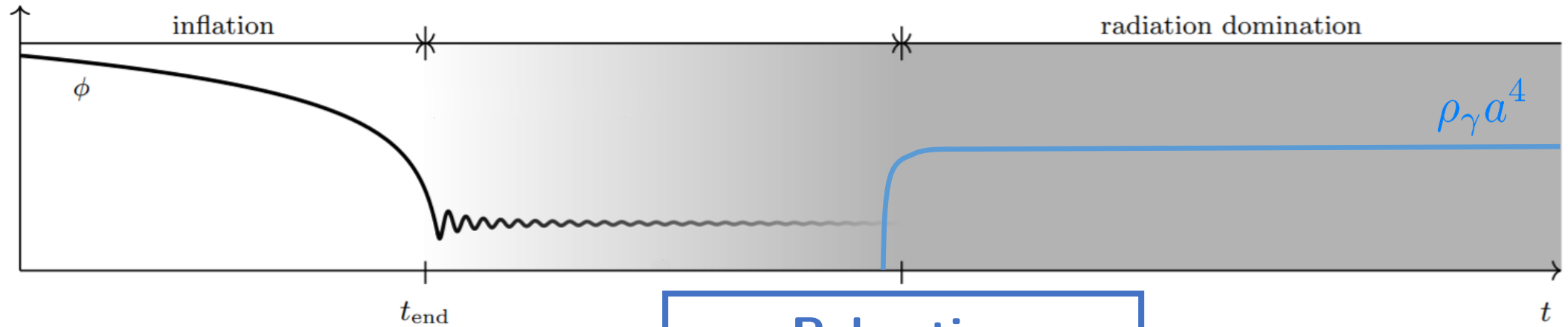
After end of inflation

$$V(\phi) \simeq \frac{1}{2} m_\phi^2 \phi^2 = \lambda \phi^2 M_P^2 \quad (\phi \ll M_P)$$

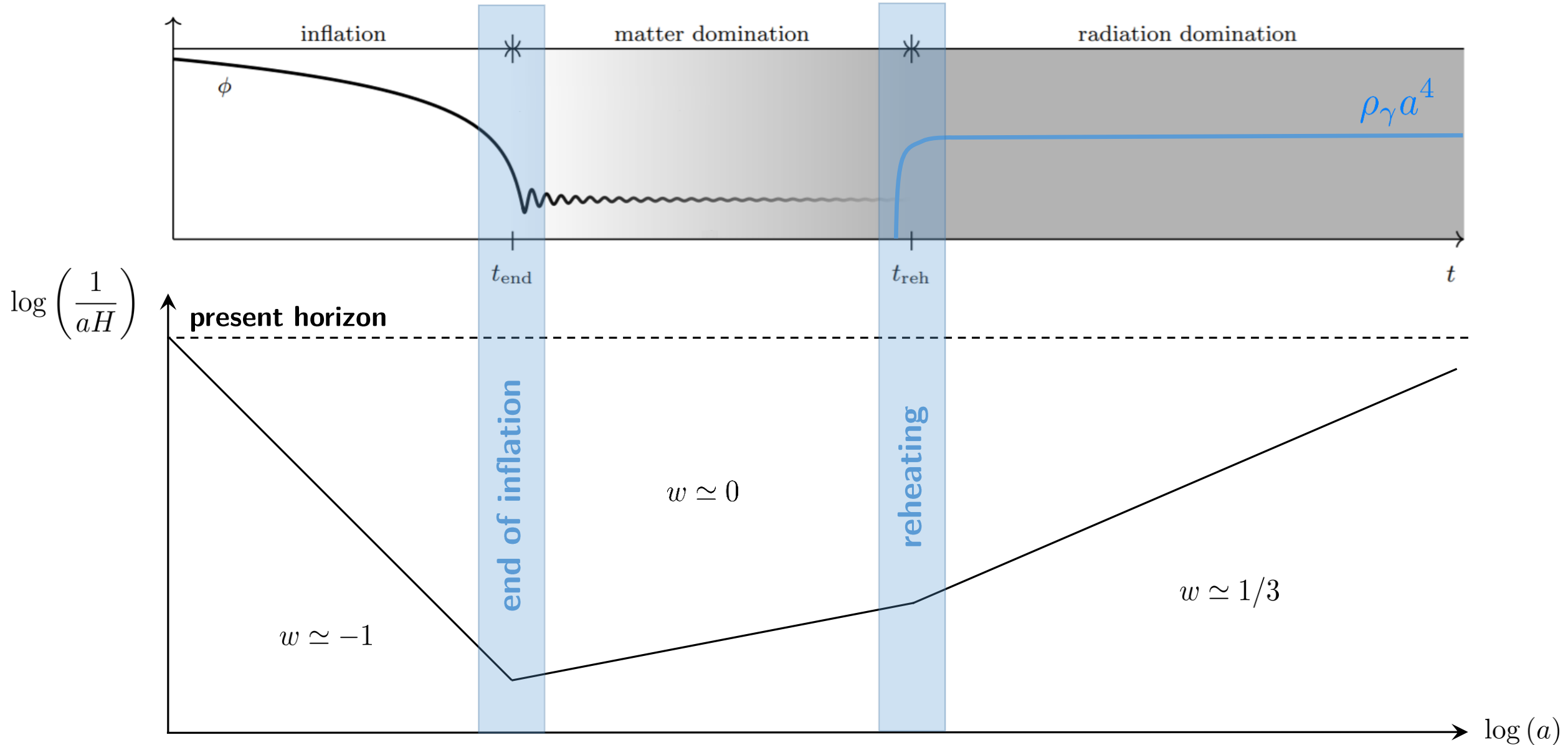


$$\langle w_\phi \rangle \simeq 0$$

The early universe



The early universe



Dark matter production

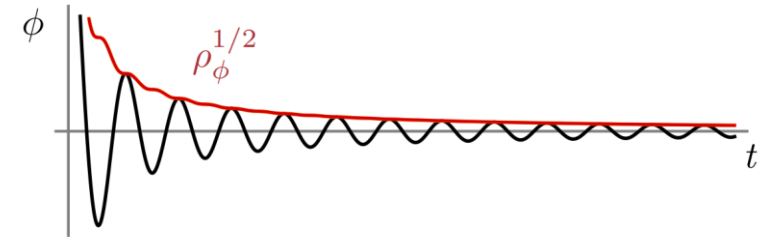
Particle production from the inflaton

- Consider **coupling to scalar dark matter**

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\textcircled{R} + \frac{1}{2}(\partial_\mu\phi)^2 - \lambda\phi^2 M_P^2 \right) \text{ inflaton}$$

minimal coupling to gravity

$$+ \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}\sigma\phi^2\chi^2 - \frac{1}{2}m_\chi^2\chi^2 \text{ scalar}$$



$$\rightarrow m_{\chi,\text{eff}}^2(t) = m_\chi^2 + \sigma\phi^2(t)$$

Goal: Estimate DM production for all regimes of σ/λ

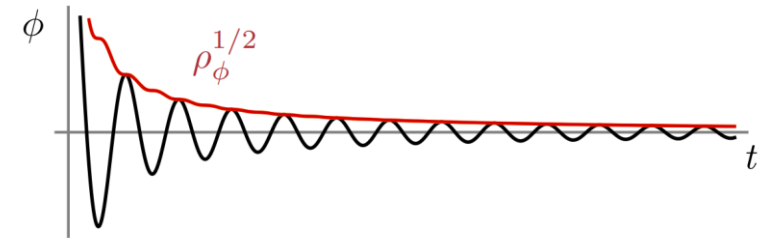
Particle production from the inflaton

- Consider **coupling to scalar dark matter**

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\underbrace{R}_{\text{minimal coupling to gravity}} + \frac{1}{2}(\partial_\mu \phi)^2 - \lambda \phi^2 M_P^2 \right) \text{ inflaton}$$

minimal coupling to gravity

$$+ \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2} \sigma \phi^2 \chi^2 - \frac{1}{2} m_\chi^2 \chi^2 \text{ scalar}$$



$$\rightarrow m_{\chi, \text{eff}}^2(t) = m_\chi^2 + \sigma \phi^2(t)$$

- Estimate **dark matter density produced from inflaton**

First approach: fluid picture – neglect fast oscillations

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3 \mathbf{P} f_\chi(P_0, t)$$

→ **Get phase space distribution**

$$\frac{\partial f_\chi}{\partial t} - H |\mathbf{P}| \frac{\partial f_\chi}{\partial |\mathbf{P}|} = \frac{\pi^2}{\beta^2 m_\phi^3} \rho_\phi \Gamma_{\phi\phi \rightarrow \chi\chi} \delta(|\mathbf{P}| - m_\phi \beta(t)) \boxed{(1 + 2f_\chi(|\mathbf{P}|)) \text{ Bose enhancement}}$$

$$\beta(t) \equiv \sqrt{1 - \frac{m_{\text{eff}}^2(t)}{m_\phi^2}} : \text{kinematic blocking}$$

Scalar production: the field picture

- Treat dark matter as quantum field in curved space-time

Equation of motion:
$$\left(\frac{d^2}{dt^2} - \frac{\nabla^2}{a^2} + 3H \frac{d}{dt} + m_\chi^2 + \sigma\phi^2 \right) \chi = 0$$

- Quantize the (rescaled) field
$$X(\tau, \mathbf{x}) \equiv a\chi = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} e^{-i\mathbf{p}\cdot\mathbf{x}} \left[X_p(\tau)\hat{a}_{\mathbf{p}} + X_p^*(\tau)\hat{a}_{-\mathbf{p}}^\dagger \right]$$

- Harmonic oscillator with time-dependent frequency

$$X_p'' + \omega_p^2 X_p = 0 \quad \omega_p^2(t) = p^2 + a^2(t)\hat{m}_{\text{eff}}^2(t) \quad \hat{m}_{\text{eff}}^2(t) = m_\chi^2 + \sigma\phi^2 + \frac{R}{6}$$

Gravity!

- Distribution function from occupation number

$$n_p = \frac{1}{2\omega_p} \left| \omega_p X_p - iX_p' \right|^2 + \text{Bunch-Davies initial conditions} \quad \rightarrow \quad f_\chi(P, t) = n_{aP}(t)$$

$$\begin{aligned} ' &\equiv \frac{d}{d\tau} \\ dt &= a d\tau \end{aligned}$$

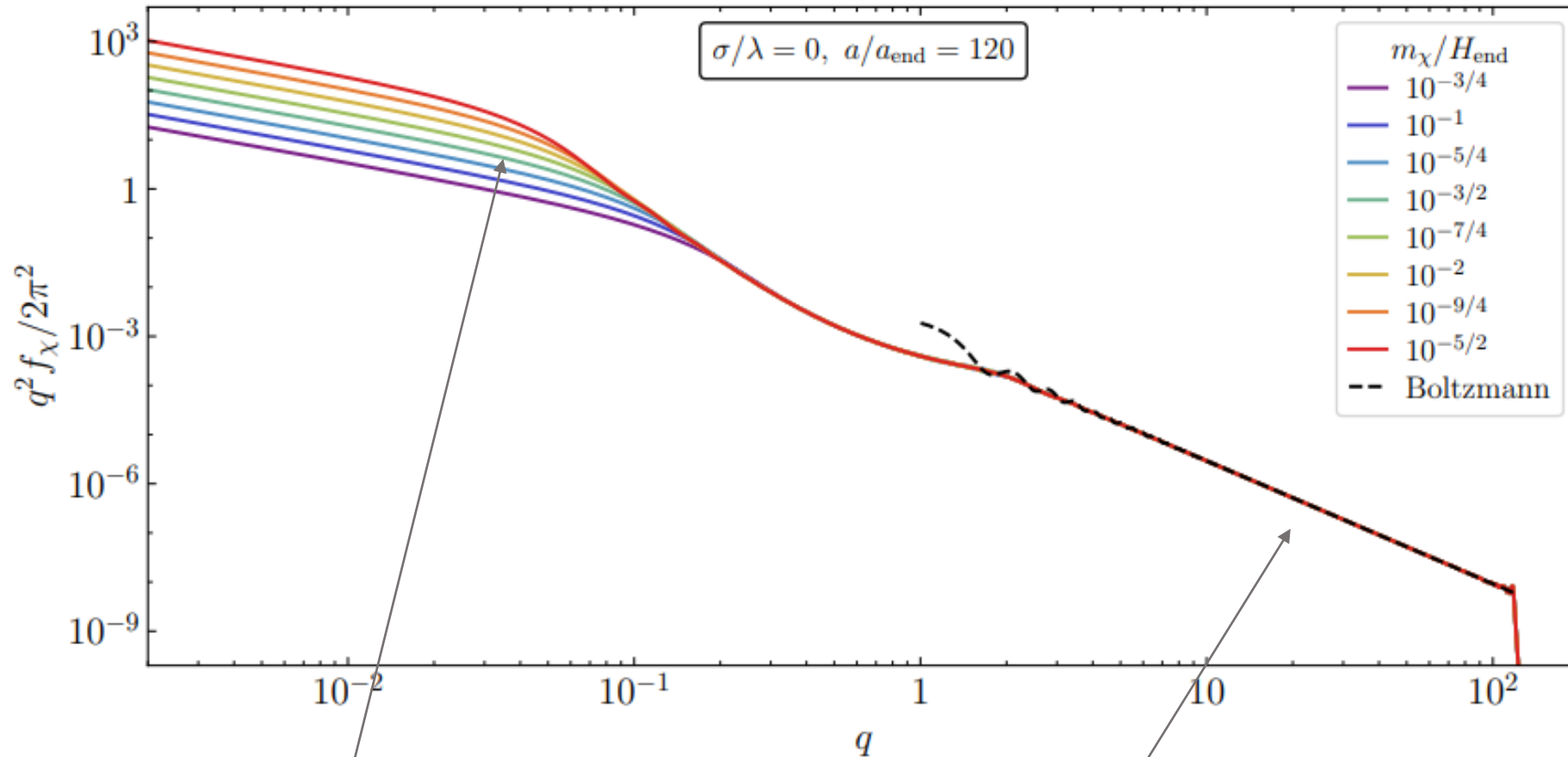
[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452 – arXiv:9405187]

[M. A. G. Garcia, K. Kaneta, K. Olive, Y. Mambrini, S. Verner - arXiv: 2109.13280]

Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]

[S. Ling & A. J. Long - arXiv:2101.11621]

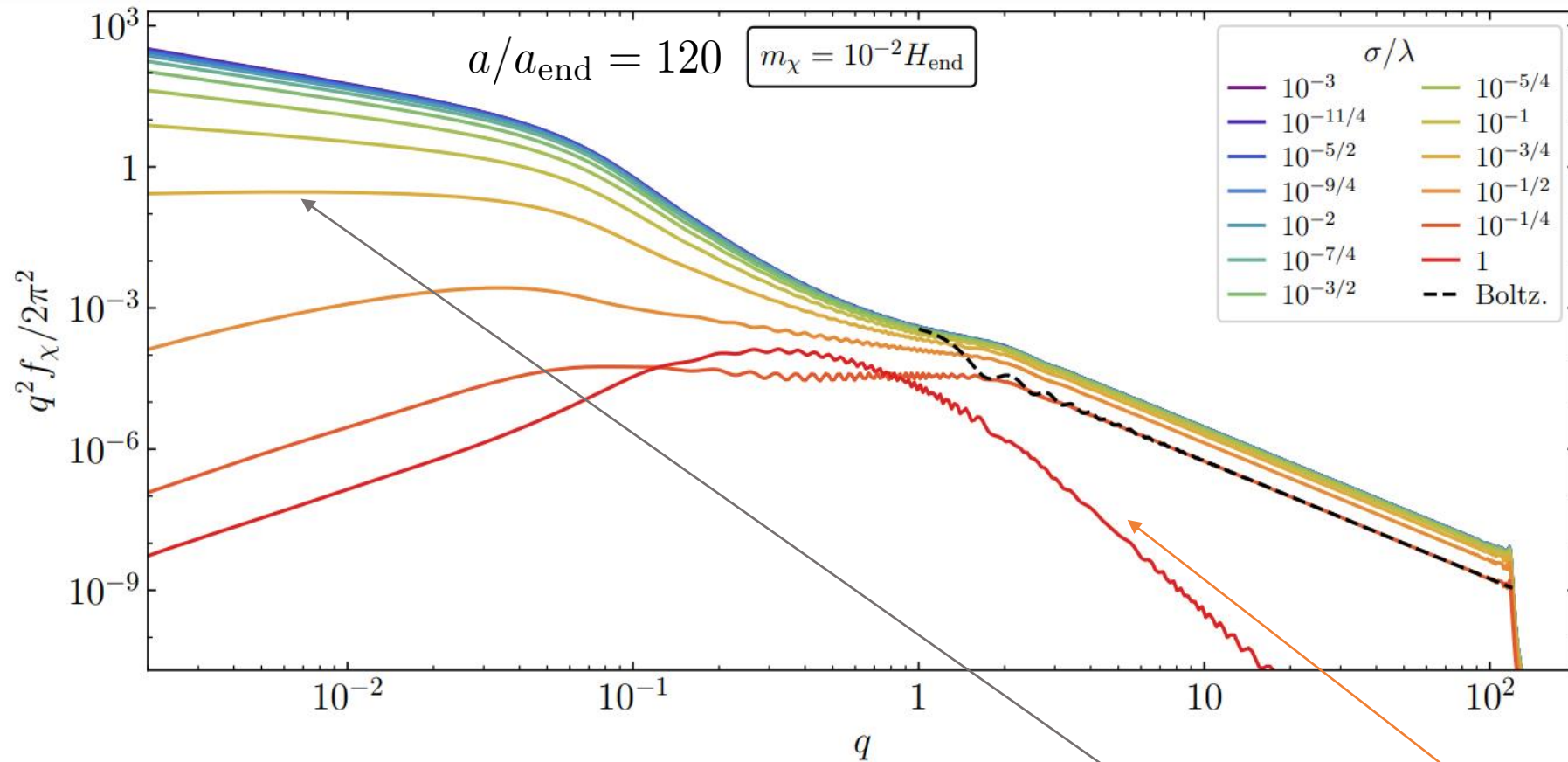


$$q \equiv \frac{P}{T_\star} \left(\frac{a}{a_0} \right)$$

$$T_\star \equiv m_\phi \left(\frac{a_{\text{end}}}{a_0} \right)$$

- ➔ **Excitation of light field in quasi De-Sitter space:** increases as mass decreases
- ➔ Recover fluid (**Boltzmann**) regime at large q $f_\chi \sim q^{-9/2}$

Gravitational interferences $0 < \sigma/\lambda < 1$

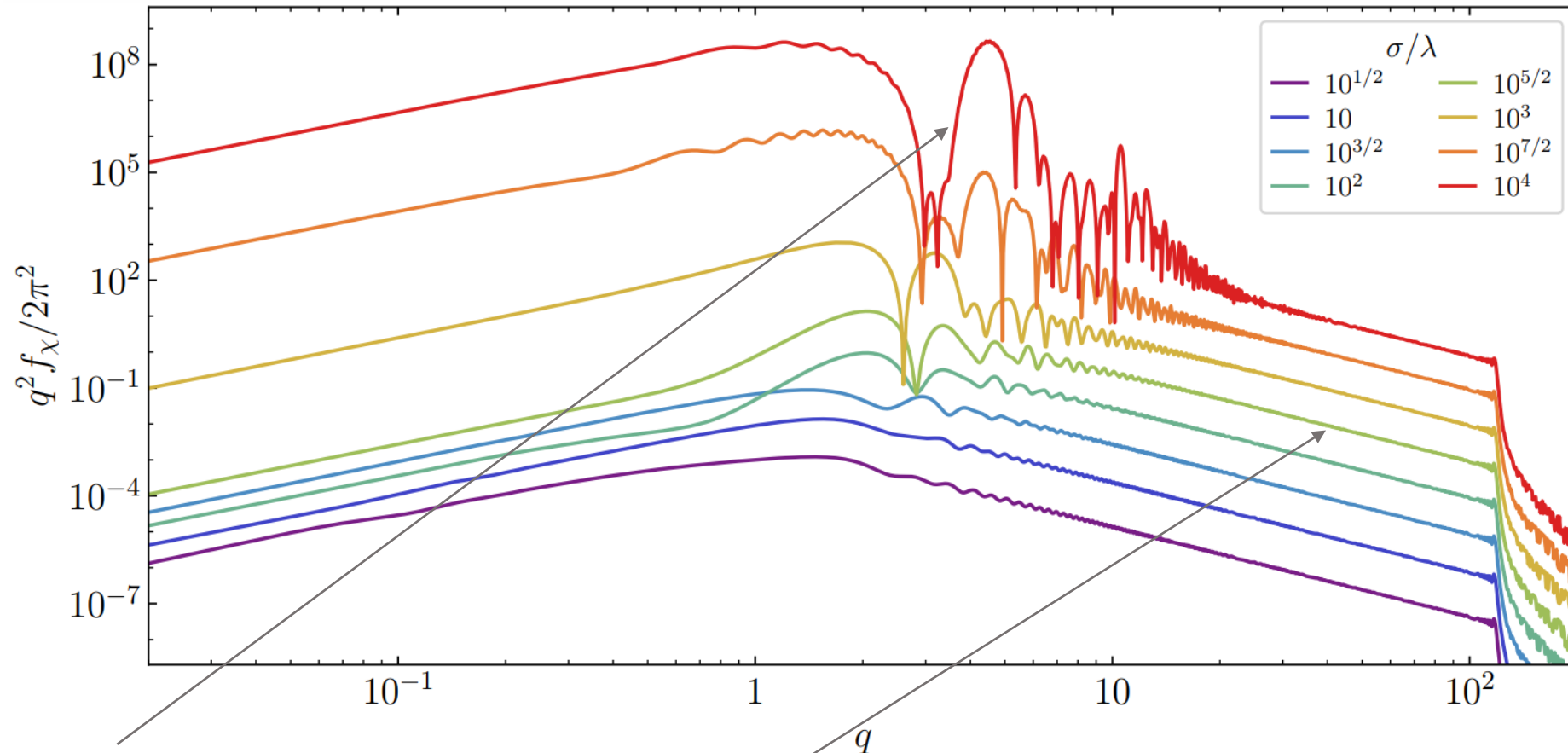


➔ **Effective mass behaves a IR modes regulator and suppresses density production!**

➔
$$\Gamma_{\phi\phi \rightarrow \chi\chi} \sim \left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left\langle \begin{array}{c} \chi \\ \chi \end{array} \right| + \begin{array}{c} \phi \\ \phi \end{array} \left\langle \begin{array}{c} \chi \\ \chi \end{array} \right| \end{array} \right|^2 = \left(\left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left\langle \begin{array}{c} \chi \\ \chi \end{array} \right| \right| \ominus \left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left\langle \begin{array}{c} \chi \\ \chi \end{array} \right| \right|^2 \sim \frac{1}{32\pi} \frac{\rho_\phi^2(t)}{m_\phi^3} \left[\sigma \ominus \lambda \left(1 + \frac{m_\chi^2}{2m_\phi^2} \right) \right]^2$$
 Interferences!

Small couplings $1 < \sigma/\lambda < 10^4$

$$a/a_{\text{end}} = 120$$



➔ **Broad resonances at large coupling from instabilities! Mode equation \iff Mathieu equation**

➔ Recover **Boltzmann regime at large q** $f_\chi \sim q^{-9/2}$

[L. Kofman, A. Linde, A. Starobinsky
arXiv:9704452 – arXiv:9405187]

Large couplings $\sigma/\lambda > 10^4$

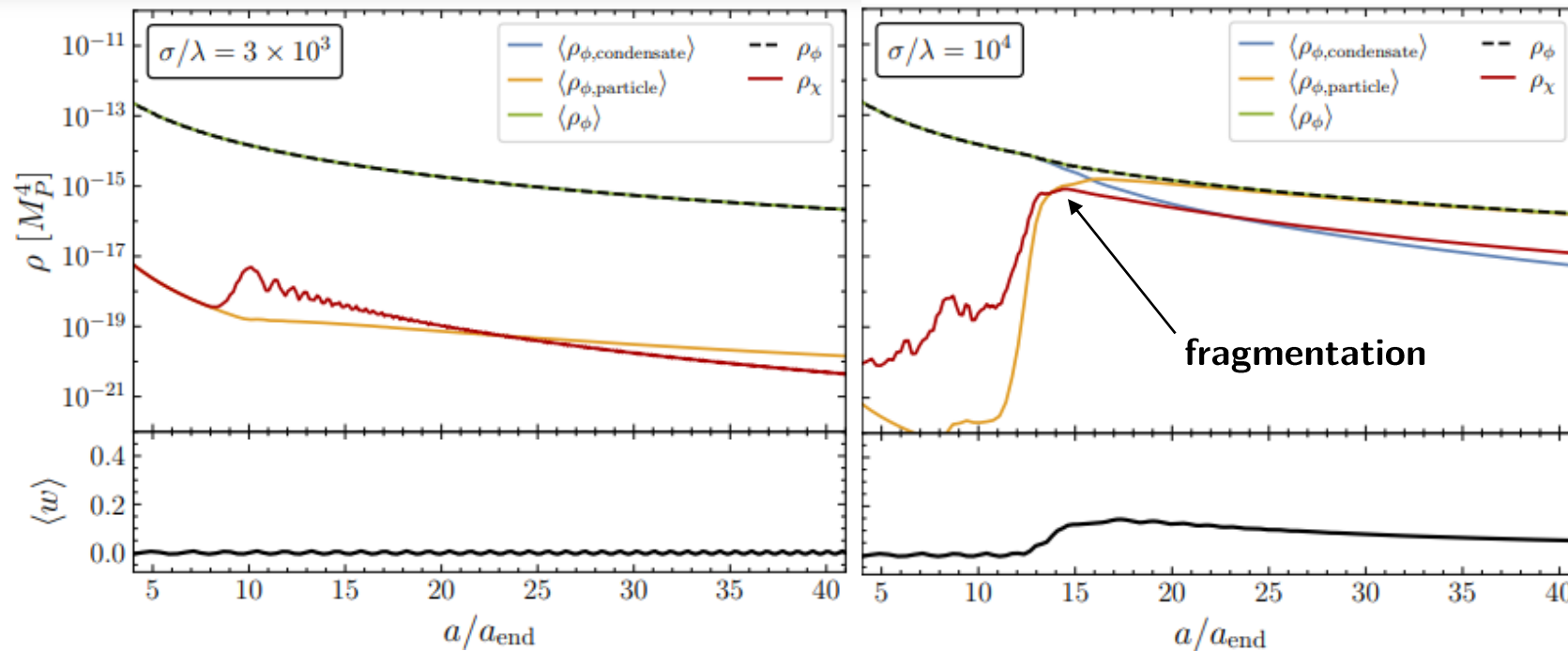
Copiously produced **dark matter disrupts inflaton condensate**

→ Hartree approximation $\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\phi^2\langle\chi^2\rangle = 0$

→ Real space lattice simulations

CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

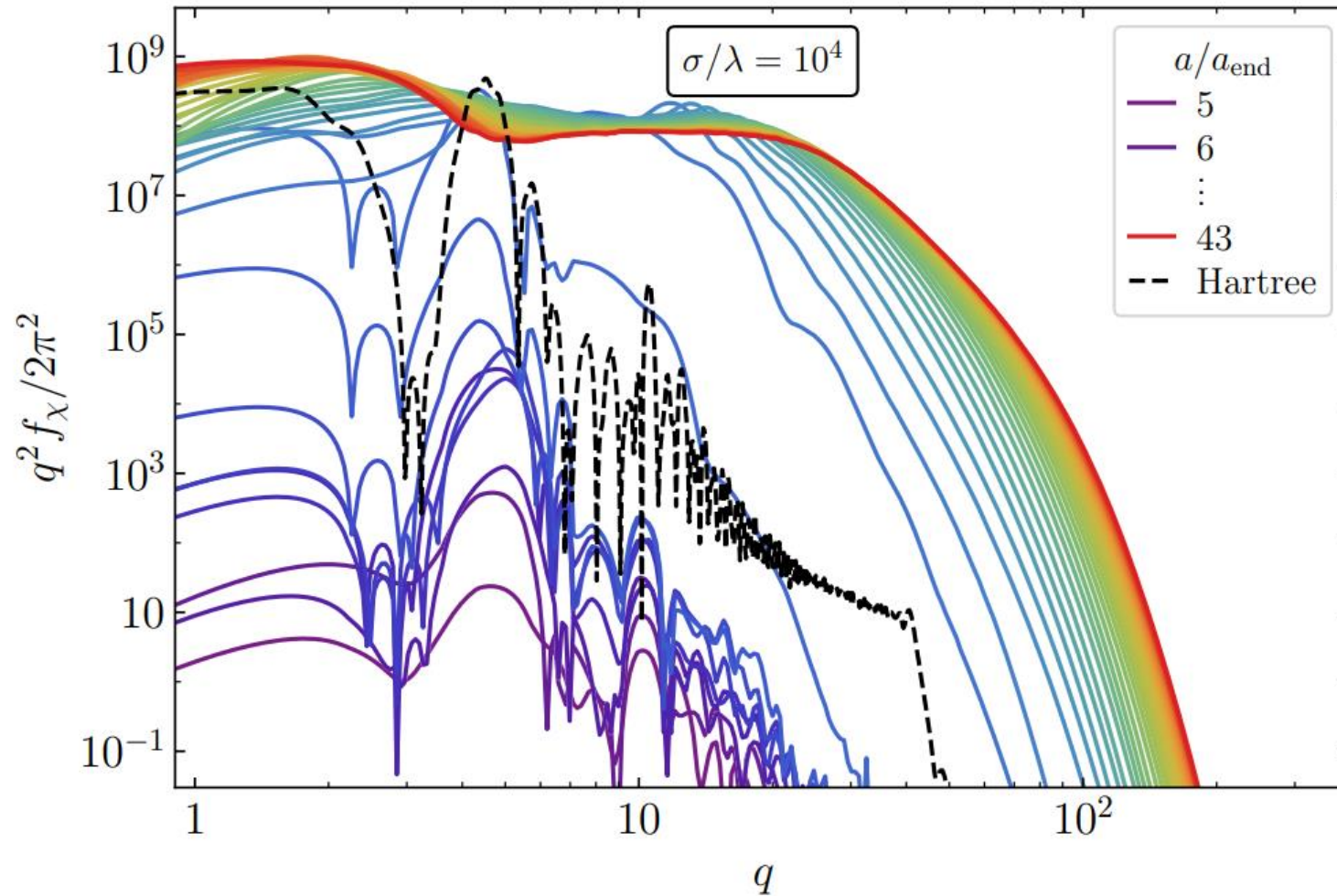
[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]



$$\rho_{\phi,\text{condensate}} \equiv \frac{1}{2}\bar{\phi}^2 + V(\bar{\phi})$$

$$\rho_{\phi,\text{particle}} \equiv \rho_\phi - \rho_{\phi,\text{condensate}}$$

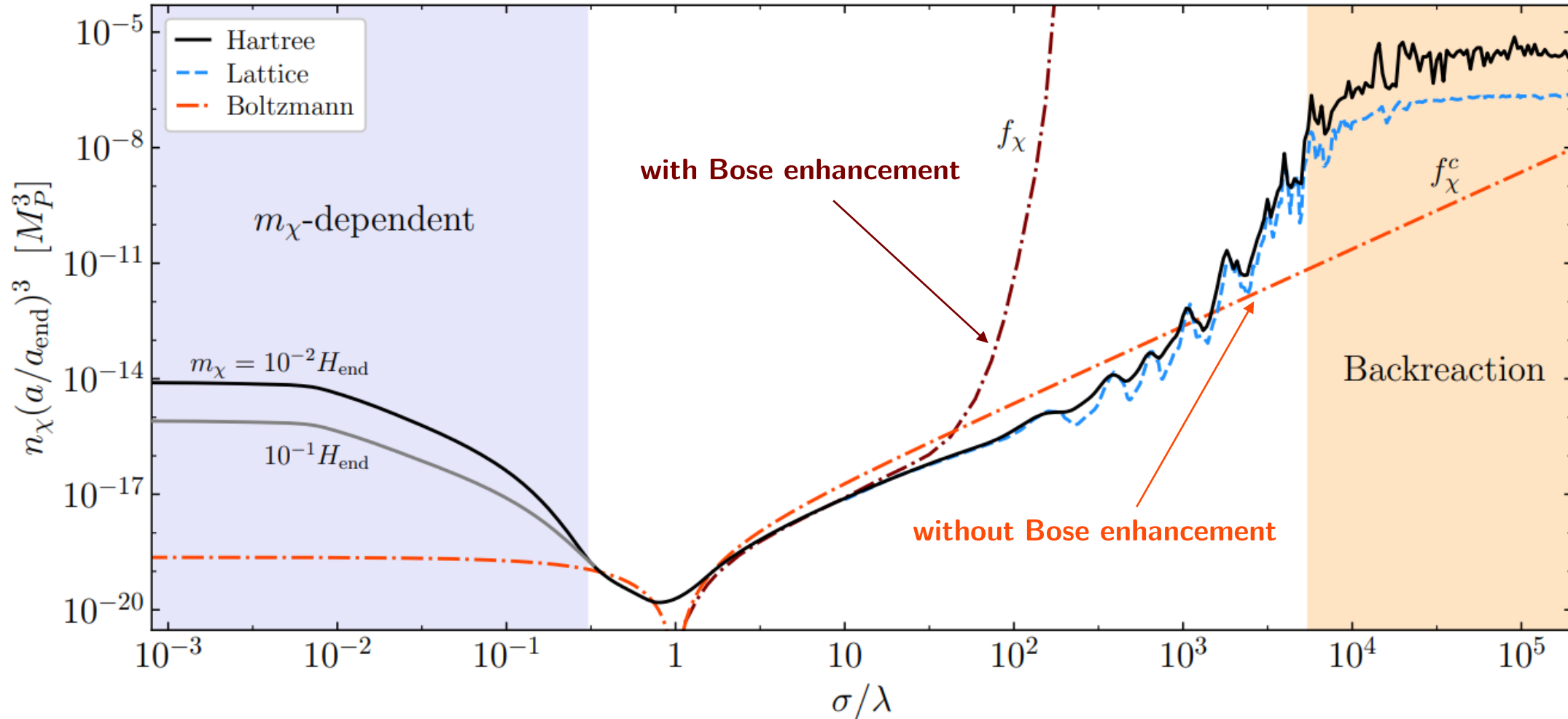
Large couplings $\sigma/\lambda > 10^4$



→ **Quasi-thermal** distribution from lattice

→ **Hartree fails** if backreaction too large

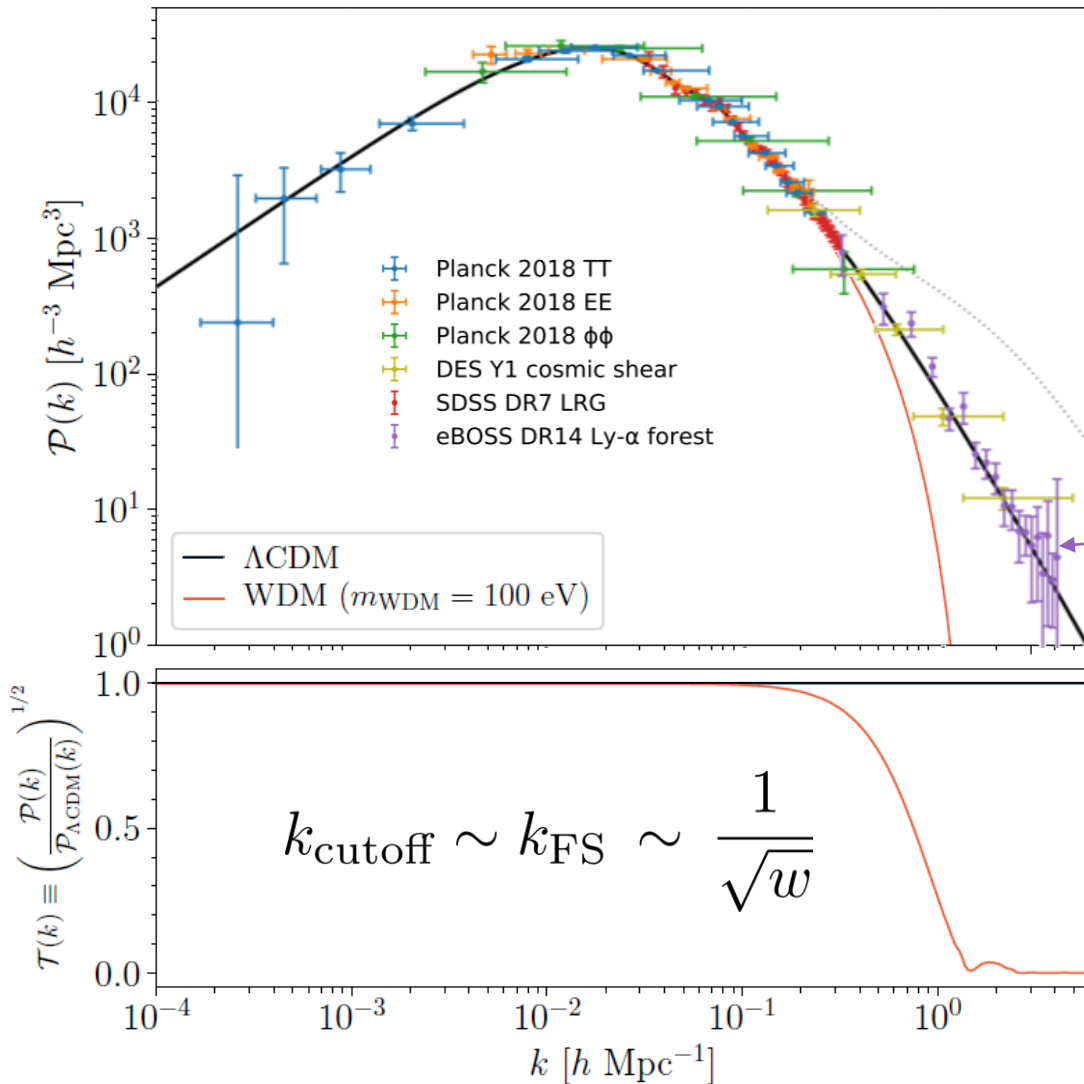
Scalar dark matter production phases



➔ Applicable to **generic light scalar**, not just **dark matter**

Cosmological constraints

Lyman- α constraints on non-cold dark matter



- **Cutoff** determined by **equation-of-state** parameter

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_{\star}^2}{3m_{\chi}^2} \frac{\langle q^2 \rangle}{a^2}$$

- **Find mass** that reproduces **cutoff** constrained by **Lyman- α**

[G. Ballesteros, M A. G. Garcia, **MP**, JCAP 03 (2021) 101]

$$m_{\chi} > 7.5 \text{ keV} \left(\frac{m_{\text{WDM}}^{\text{Ly}-\alpha}}{3 \text{ keV}} \right)^{4/3} \left(\frac{T_{\star}}{T_{\gamma}^0} \right) \sqrt{\langle q^2 \rangle}$$

w - matching

$$\langle q^2 \rangle \equiv \frac{\int dq q^4 f(q)}{\int dq q^2 f(q)}$$

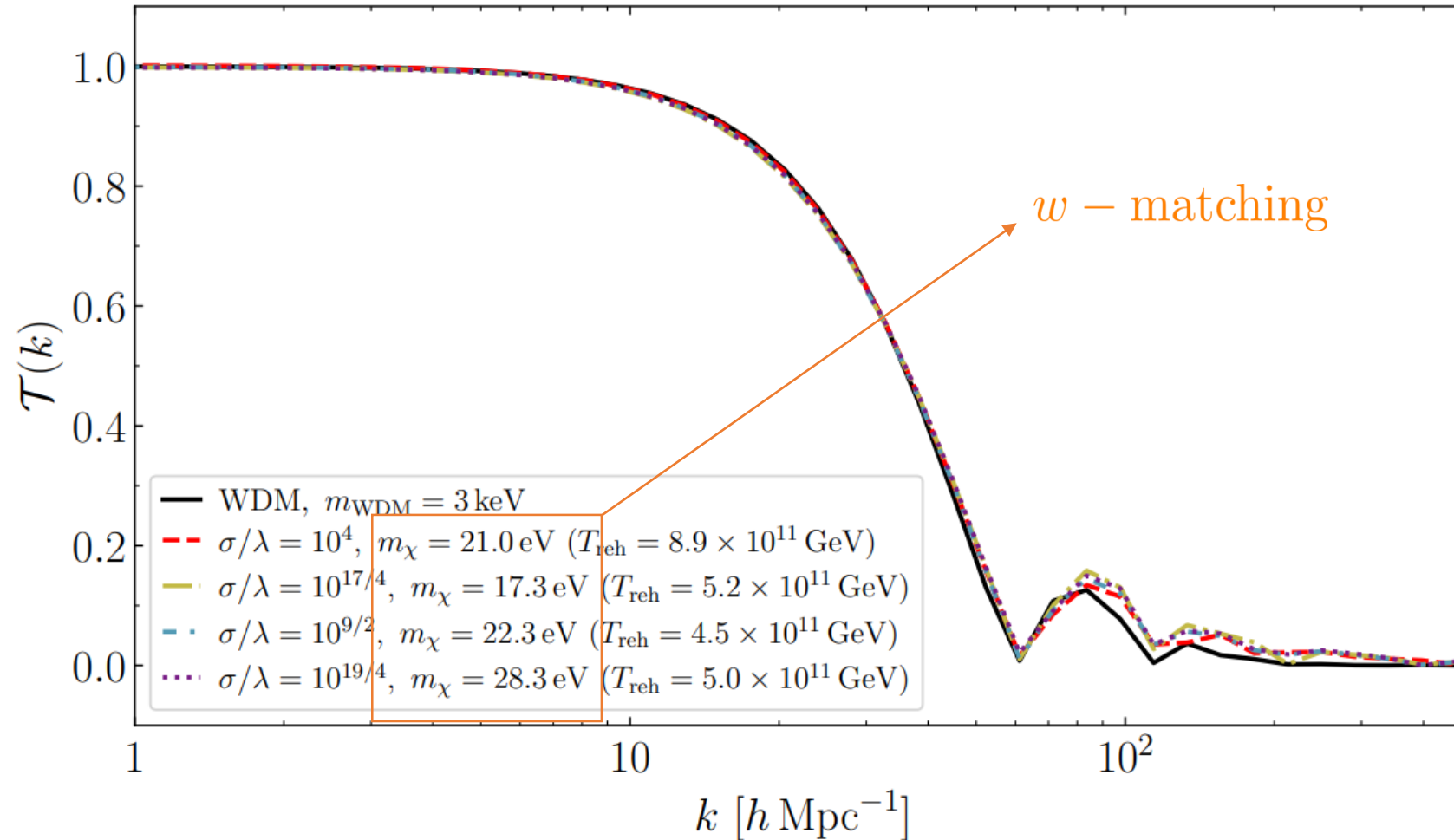
Photon temperature now

[S. Chabanier, M. Millea, N. Palanque-Desabrouille, MNRAS 489 (2019) 2, 2247-2253]

Constraints on preheating production $10^4 < \sigma/\lambda < 10^5$

- Power spectrum computed numerically with CLASS

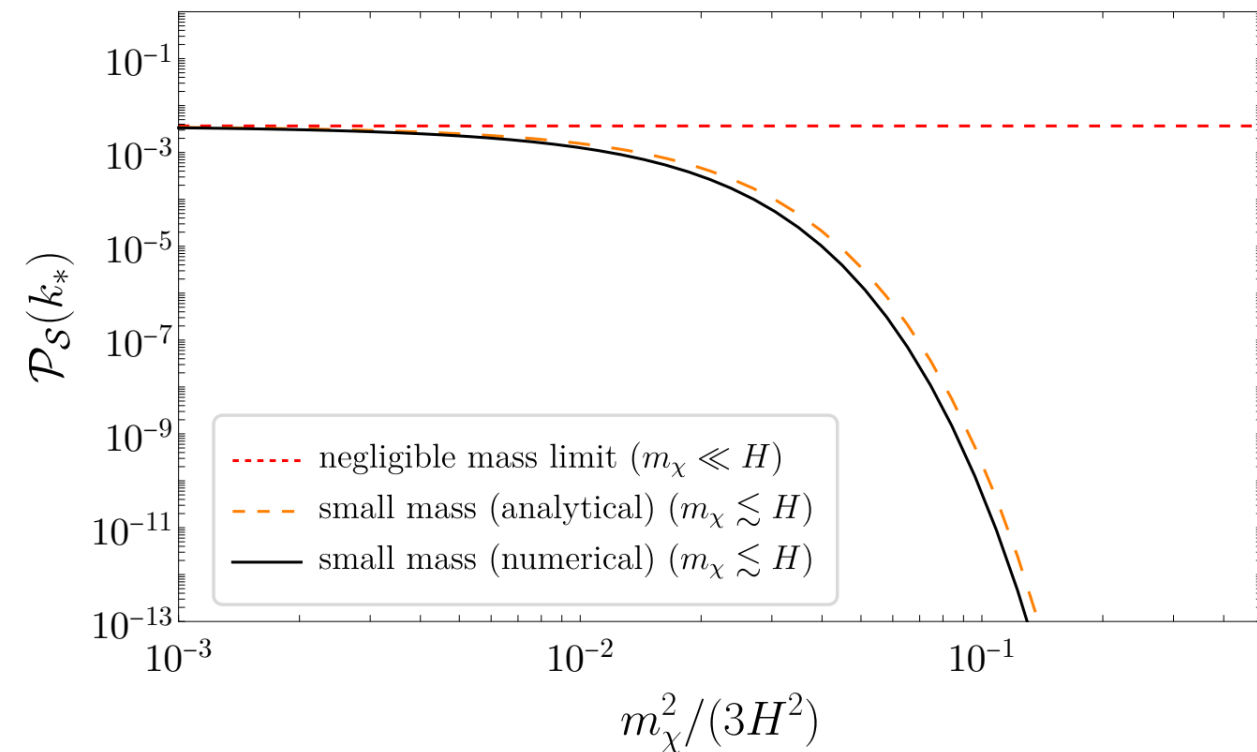
[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



- Excellent agreement with w – matching for **all distributions!**

Isocurvature constraints

- **Single field inflation** predicts *adiabatic* perturbations $\frac{\delta\rho_i}{\dot{\rho}_i} = \frac{\delta\rho_j}{\dot{\rho}_j}$
- Significant **DM production** during inflation departs from “**single clock**” inflation: DM *isocurvature* perturbations constrained by CMB

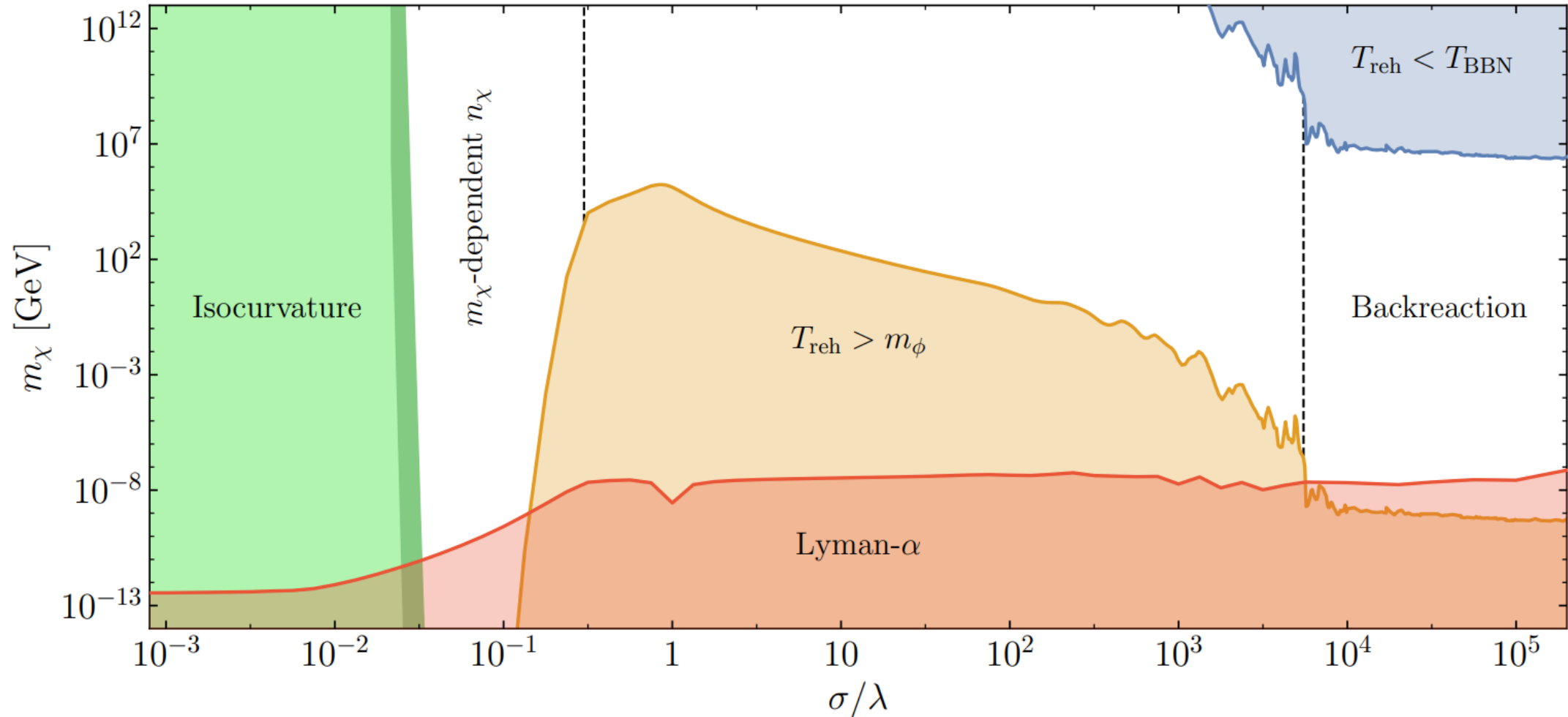


$$\beta_{\text{iso}} \simeq \mathcal{P}_S(k_*)/\mathcal{P}_R(k_*) < \mathcal{O}(1\%)$$

$$\mathcal{P}_S(k) = \frac{k^3}{(2\pi)^5 \rho_\chi^2 a^8} \int d^3\mathbf{p} P_X(p, |\mathbf{p} - \mathbf{k}|)$$

$$P_X(p, q) = |X'_p|^2 |X'_q|^2 + a^4 m_\chi^4 |X_p|^2 |X_q|^2 + a^2 m_\chi^2 [(X_p X_p'^*)(X_q X_q'^*) + \text{h.c.}]$$

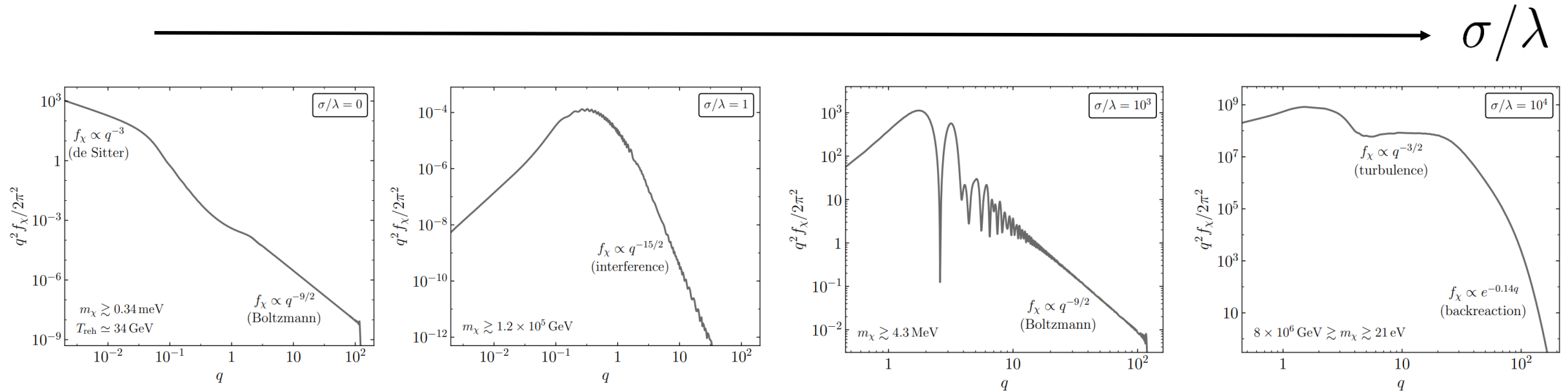
Summary of constraints



→ **Isocurvature:** $\sigma/\lambda > \mathcal{O}(10^{-2})$

→ **Lyman- α :** $m_\chi > \mathcal{O}(10)$ eV

Summary and conclusion



- Characterization of **scalar preheating** over **large spectrum** of **couplings**
- Check **validity** of **several approaches**
- **Dark matter production** constrained by **Lyman- α** and **isocurvature**

Thank you for your attention

Back-up slides

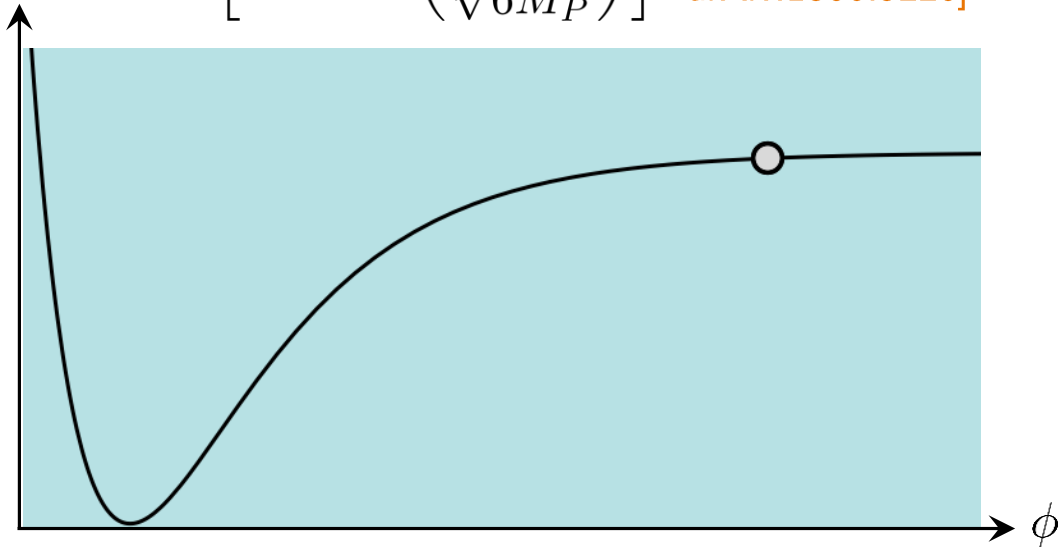
End of inflation

For concreteness, **consider**

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0$$

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6}M_P}\right) \right]^2$$

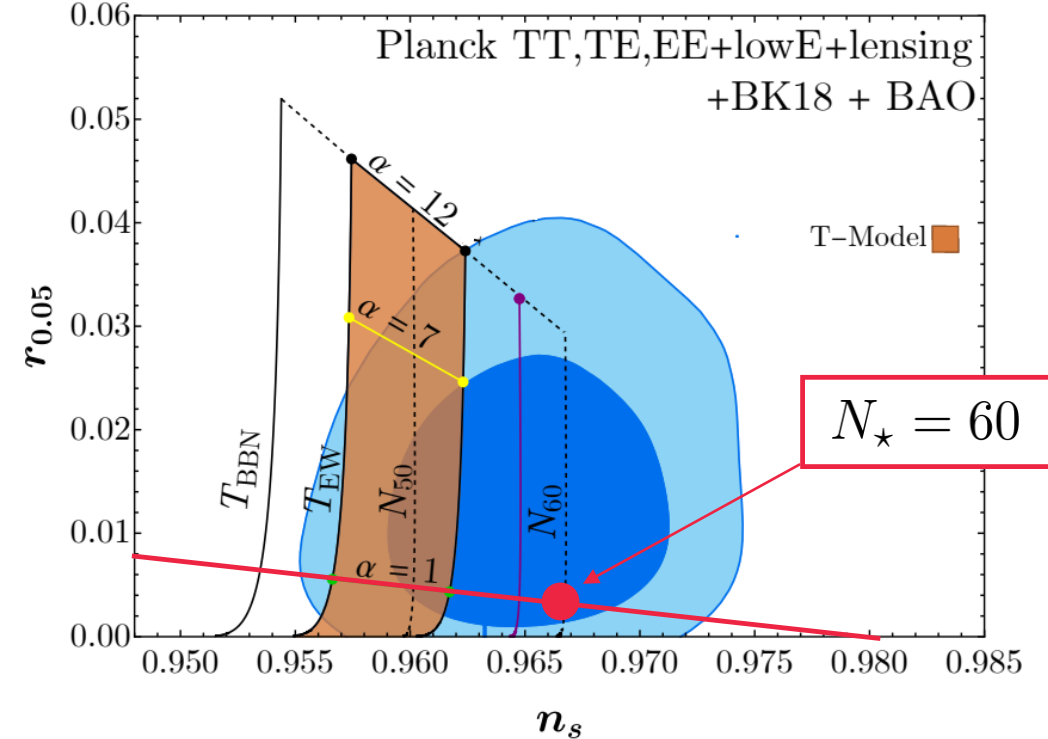
[Kallosh & Linde arXiv:1306.5220]



$$\lambda \simeq \frac{18\pi^2 A_s(k_*)}{6N_*^2} \sim 10^{-11}$$

[J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, K. A. Olive, and S. Verner - arXiv:2112.04466]

$$k_* = 0.05 \text{ Mpc}^{-1}$$



$N_* = 60$

$$r \simeq 16\epsilon_* \simeq \frac{12}{N_*^2} \quad n_s \simeq 1 - 6\epsilon_* + 2\eta_* \simeq 1 - \frac{2}{N_*}$$

$$A_s(k_*) \simeq 2.1 \times 10^{-9} \text{ [Planck 18']}$$

ϕ inflaton

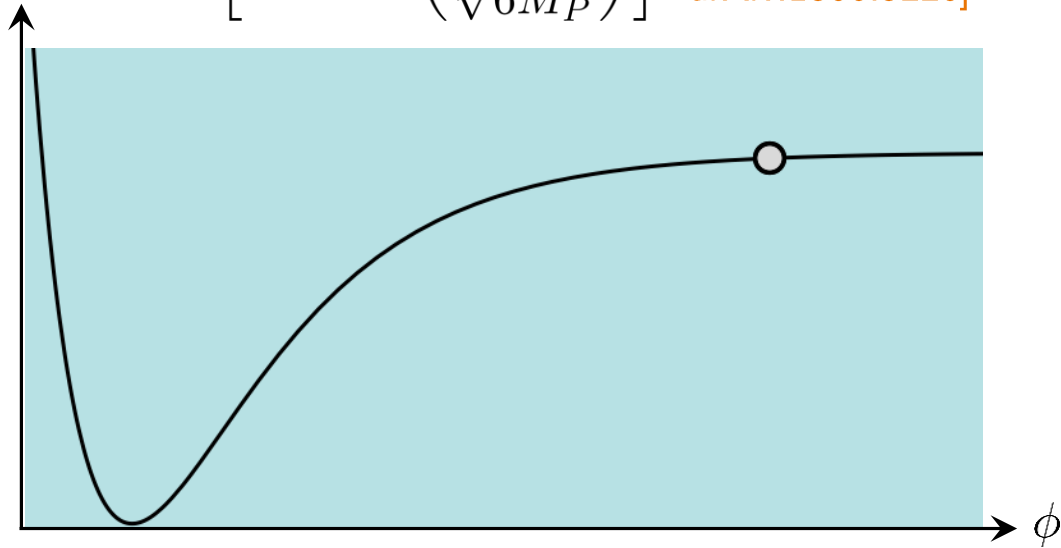
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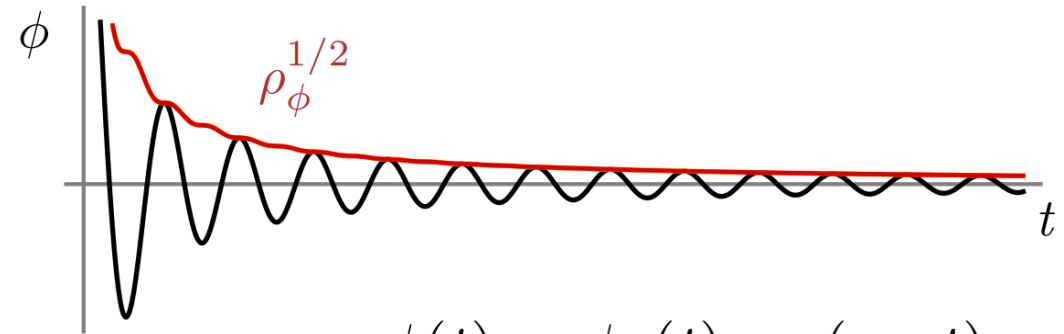
[Kallosh & Linde arXiv:1306.5220]



$$\lambda \simeq \frac{18\pi^2 A_s(k_*)}{6N_*^2} \sim 10^{-11}$$

Close to the minimum

$$V(\phi) \simeq \frac{1}{2} m_{\phi}^2 \phi^2 = \lambda \phi^2 M_P^2 \quad (\phi \ll M_P)$$



$$\rightarrow \phi(t) \simeq \phi_0(t) \cos(m_{\phi} t)$$

$$\langle P_{\phi} \rangle = \frac{1}{2} \langle \dot{\phi}^2 \rangle - \langle V(\phi) \rangle \simeq 0$$

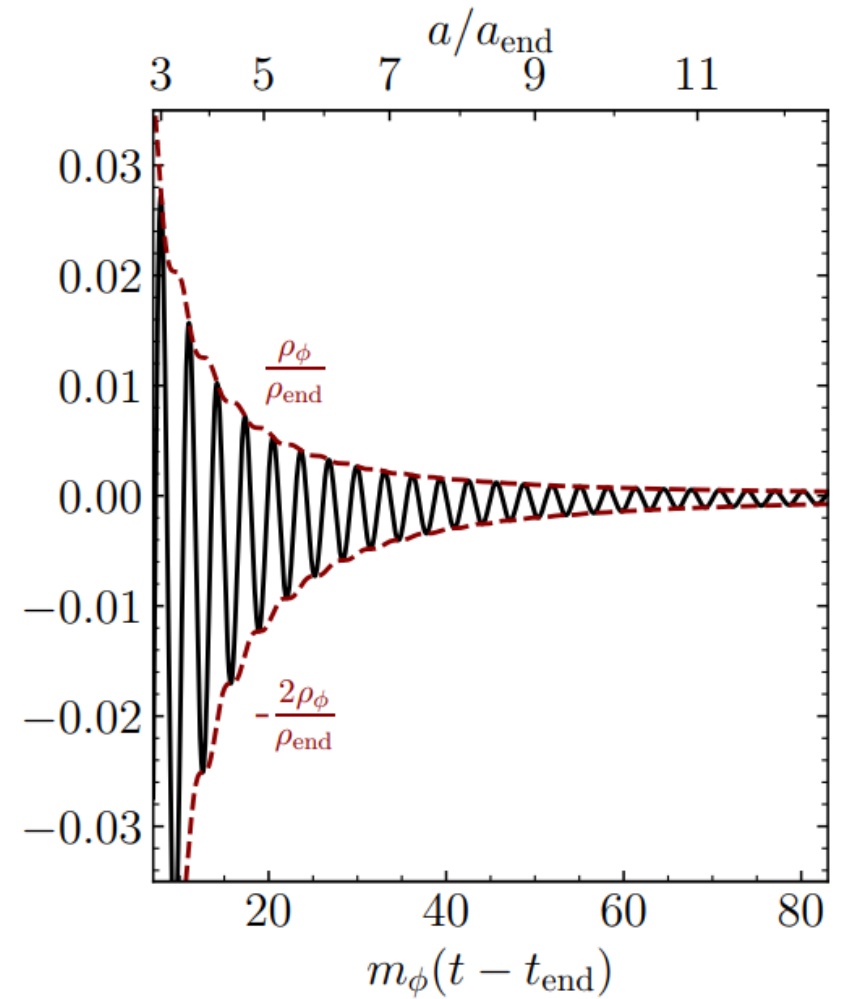
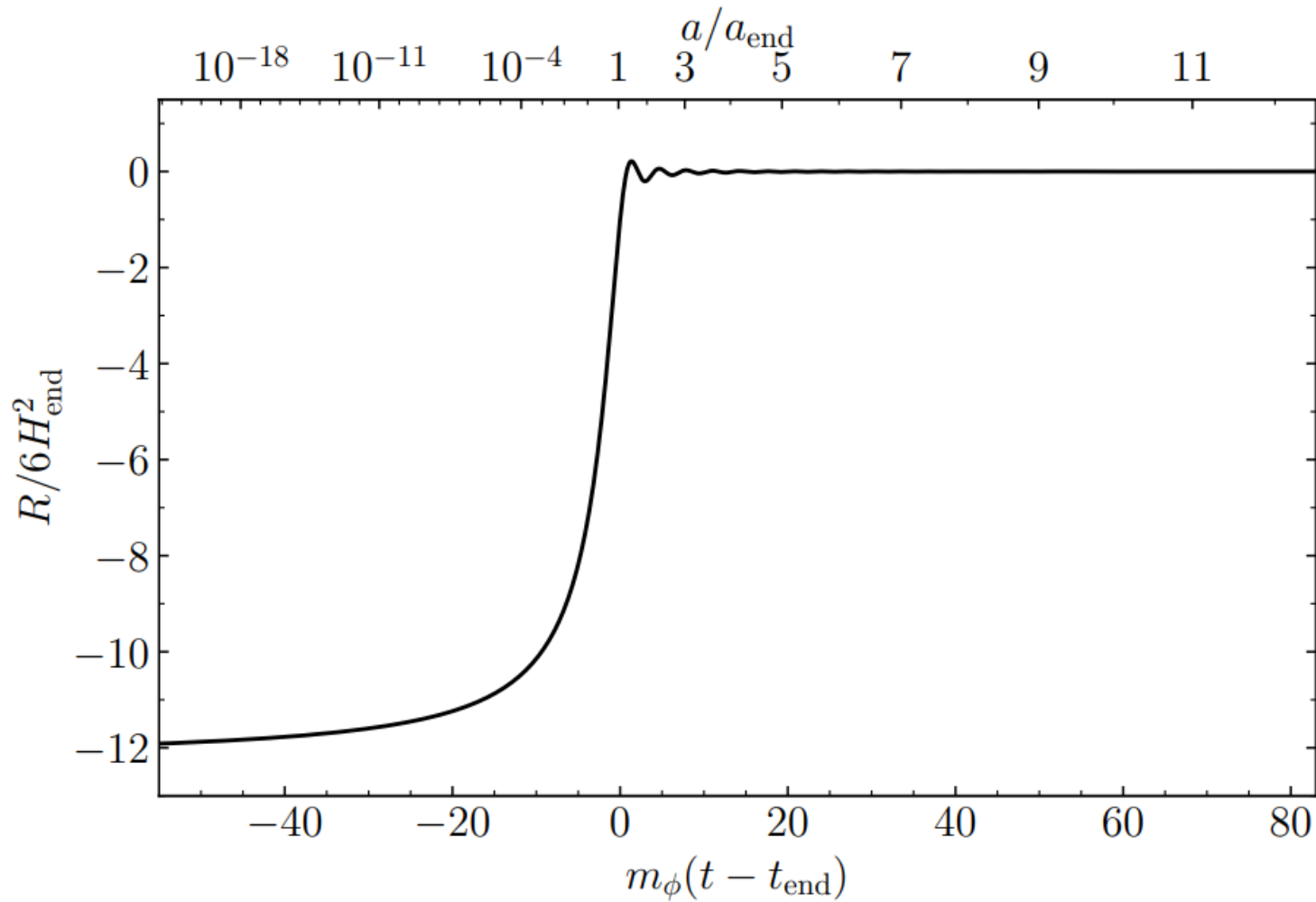
$$\langle w_{\phi} \rangle \simeq 0$$

$$\langle \rho_{\phi} \rangle = \frac{1}{2} \langle \dot{\phi}^2 \rangle + \langle V(\phi) \rangle \simeq V(\phi_0)$$

Perturbative reheating $\rightarrow \rho_{\phi}(t) \simeq \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-3} e^{-\Gamma_{\phi}(t-t_{\text{end}})} \quad \Gamma_{\phi} \ll m_{\phi}$

ϕ : inflaton

Gravitational contribution to the effective mass



Reheating

- In **fluid picture**: transition to radiation era via **dissipation** term $\equiv \Gamma_\phi \rho_\phi (1 + w_\phi)$

$$T_{\text{tot}}^{\mu\nu} = T_\phi^{\mu\nu} + T_\gamma^{\mu\nu}$$

$$\nabla_\mu T_{\text{tot}}^{\mu\nu} = 0$$

$$\nabla_\mu T_\phi^{\mu\nu} = -\nabla_\mu T_\gamma^{\mu\nu}$$

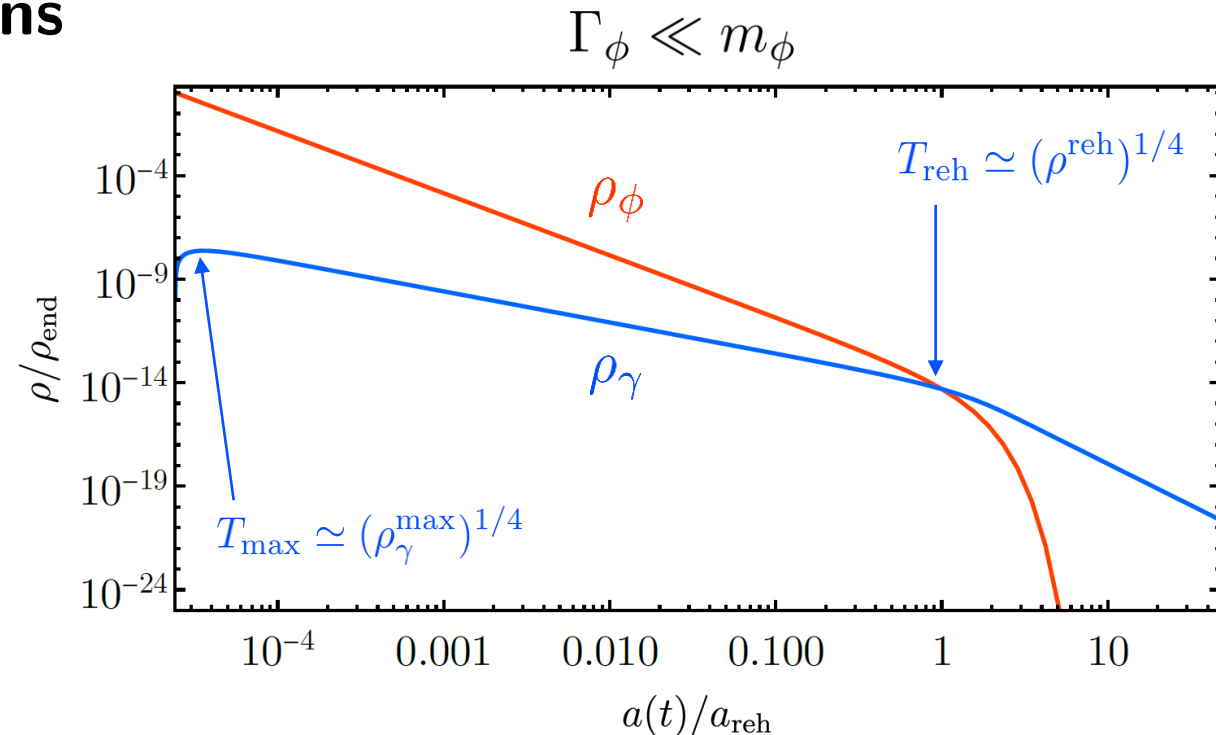
- System of **Friedmann-Boltzmann equations**

$$\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = -\Gamma_\phi \rho_\phi (1 + w_\phi)$$

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma_\phi \rho_\phi (1 + w_\phi)$$

$$H^2 = \frac{1}{3M_{\text{Pl}}^2}(\rho_\phi + \rho_\gamma)$$

$$\rightarrow \rho_\phi(t) \simeq \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-3} e^{-\Gamma_\phi(t-t_{\text{end}})}$$



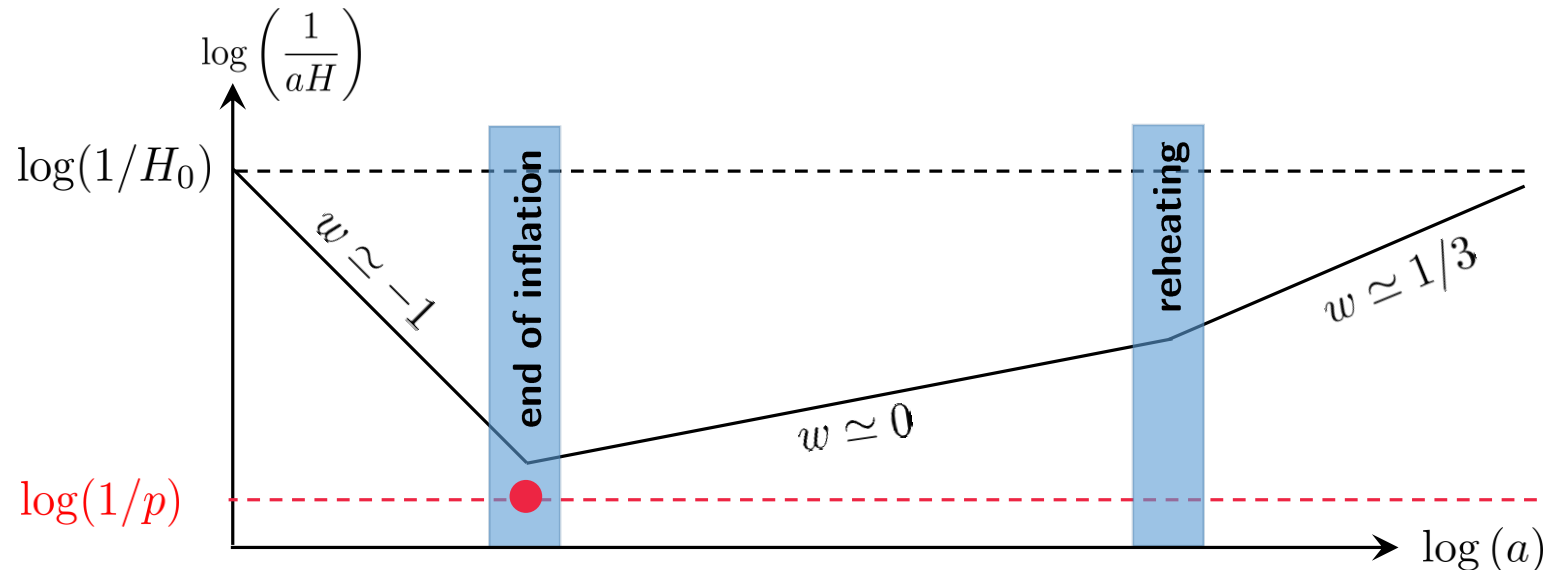
Scalar preheating: the field picture

- Set **initial condition** for **mode functions** in **Bunch-Davies** vacuum

$$X_p(\tau_0) = \frac{1}{\sqrt{2\omega_p}} \quad X'_p(\tau_0) = -\frac{i\omega_p}{\sqrt{2\omega_p}}$$

$$\omega_p^2(t_{\text{end}}) = p^2 + a_{\text{end}}^2 (m_\chi^2 + \sigma\phi^2 - H_{\text{end}}^2)$$

- For small physical scales **modes** always inside horizon $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\text{end}}$



Scalar preheating: the field picture

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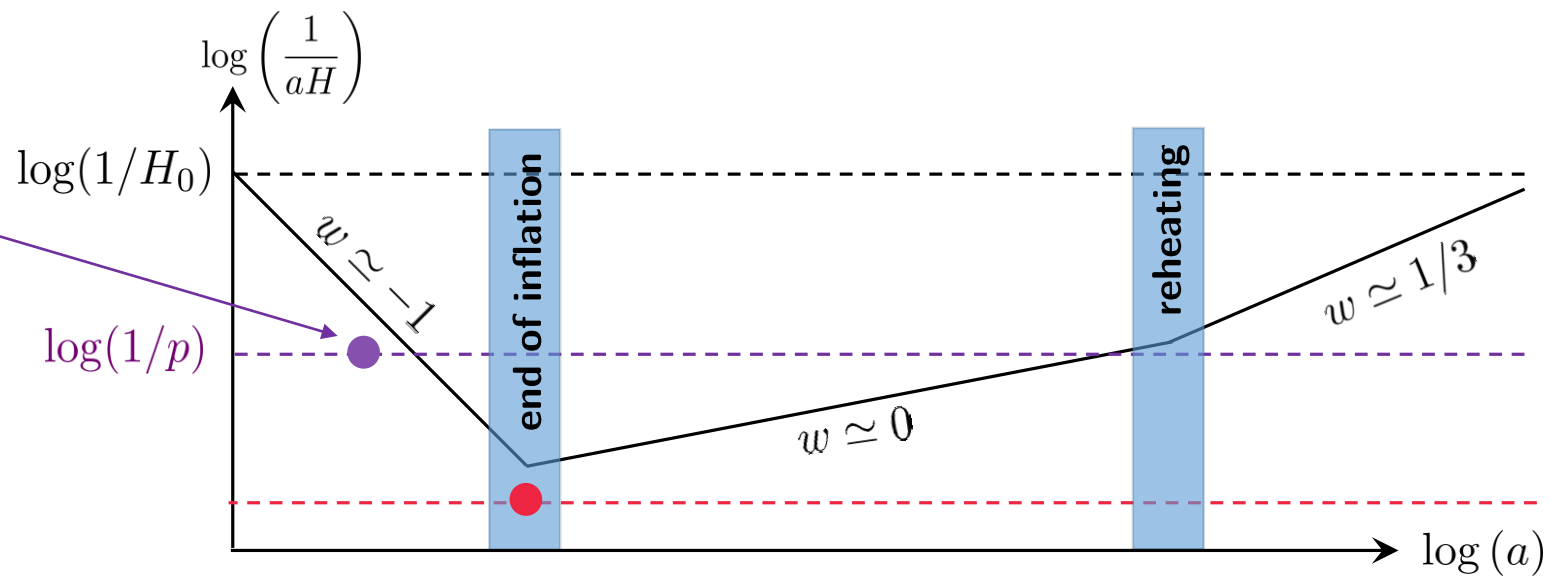
$$\omega_p^2(t_{\text{end}}) = p^2 + a_{\text{end}}^2 (m_\chi^2 + \sigma\phi^2 - H_{\text{end}}^2)$$

- For small physical scales **modes** always inside horizon $\omega_p^2 > 0$ ● $\tau_0 = \tau_{\text{end}}$
- For $m_\chi^2 + \sigma\phi^2 < H_{\text{end}}^2$ **modes** with $p^2 < a_{\text{end}}^2 (H_{\text{end}}^2 - m_\chi^2 - \sigma\phi^2)$ $\omega_p^2(t_{\text{end}}) < 0$

- $\tau_0 < \tau_{\text{end}}$

$$p^2 \gg a(\tau_0)^2 H(\tau_0)^2$$

➔ **Particle production!**
Red-tilt of the spectrum!



Scalar preheating: the oscillating condensate

- Treat inflaton as coherent oscillating condensate

$$\phi(t) \simeq \phi_0(t) \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega_\phi t} \quad E_n = n\omega_\phi \quad \rightarrow \quad \Gamma_{\phi\phi \rightarrow \chi\chi} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^{\infty} E_n \beta_n |\mathcal{M}_n|^2$$

$$|\mathcal{M}|_{\phi\phi \rightarrow \chi\chi}^2 = \left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left. \begin{array}{c} \text{---} h_{\mu\nu} \text{---} \\ \text{---} \sigma \text{---} \end{array} \right| + \left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left. \begin{array}{c} \text{---} \sigma \text{---} \\ \text{---} h_{\mu\nu} \text{---} \end{array} \right| \Bigg|^2 = \left(\left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left. \begin{array}{c} \text{---} h_{\mu\nu} \text{---} \\ \text{---} \sigma \text{---} \end{array} \right| \ominus \left| \begin{array}{c} \phi \\ \phi \end{array} \right\rangle \begin{array}{c} \chi \\ \chi \end{array} \left. \begin{array}{c} \text{---} \sigma \text{---} \\ \text{---} h_{\mu\nu} \text{---} \end{array} \right| \right)^2 \quad \rightarrow \quad \Gamma_{\phi\phi \rightarrow \chi\chi} = \frac{1}{32\pi} \frac{\rho_\phi^2(t)}{m_\phi^3} \left[\sigma \ominus \lambda \left(1 + \frac{m_\chi^2}{2m_\phi^2} \right) \right]^2 \beta_2$$

$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2 \frac{h_{\mu\nu}}{M_P}$

- Approximate solution for $\beta \simeq 1$, $t_{\text{end}} < t < t_{\text{reh}}$

$$f_\chi(q, t) \sim q^{-9/2} \theta(q-1) \theta\left(\frac{a}{a_{\text{end}}} - q\right)$$

$$q \equiv \frac{P}{T_\star} \left(\frac{a}{a_0} \right)$$

$$T_\star \equiv m_\phi \left(\frac{a_{\text{end}}}{a_0} \right)$$

- $n_\chi \left(\frac{a}{a_{\text{end}}} \right)^3 = \frac{m_\phi^3}{2\pi^2} \int dq q^2 f_\chi(q, t)$: time independent when DM production stops

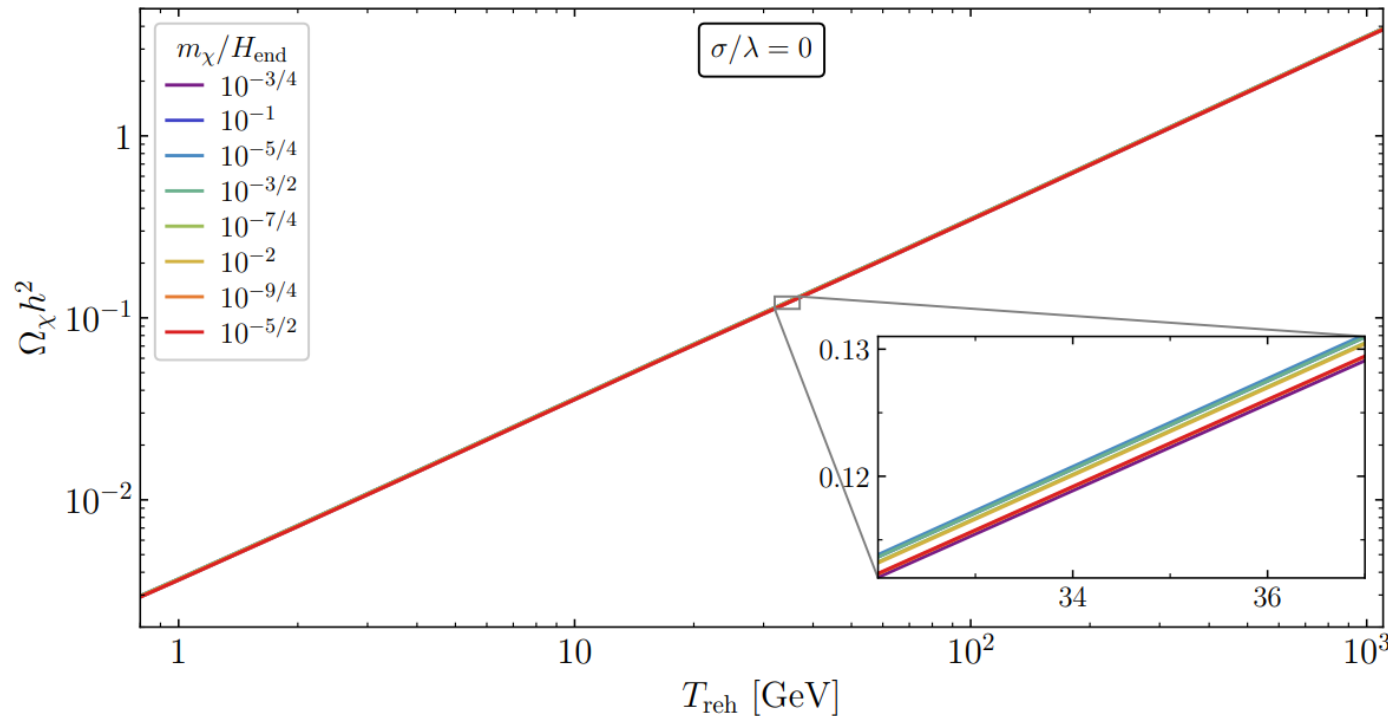
Gravitational production $\sigma/\lambda \ll 1$

[N. Herring, D. Boyanovsky & A. R. Zentner - arXiv:1912.10859]

[S. Ling and A. J. Long - arXiv:2101.11621]

→ Take IR cutoff as present horizon

$$q_{\text{IR}} = q_0 = \left(\frac{90}{\pi^2}\right)^{1/4} \left(\frac{11}{43}\right)^{1/3} g_{\text{reh}}^{1/12} \left(\frac{H_{\text{end}} M_P}{m_\phi^2}\right)^{1/2} \frac{H_0}{T_0} \left(\frac{a_{\text{reh}}}{a_{\text{end}}}\right)^{1/4} \Leftrightarrow p_0 = a_0 H_0$$



**For small DM mass
For $T_{\text{reh}} > 30$ GeV
Overclose the universe!**

Backreaction

Backreaction important at **large coupling** $\sigma/\lambda > 10^4$

- **Hartree approximation** $\ddot{\phi} + 3H\dot{\phi} + V_{\phi} + \sigma\phi^2\langle\chi^2\rangle = 0$
 - ➔ To simulate **energy transferred** back to inflaton
 - ➔ Ok but neglects **rescattering** and **disruption** of the **condensate**
- **Lattice simulations** in real space
 - ➔ Occupation numbers are **large**, **classical approach** justified
 - ➔ Cannot be **used** for **smaller couplings**
 - ➔ Does not **account** for **metric perturbations**

CosmoLattice

*A modern code for lattice simulations of scalar
and gauge field dynamics in an expanding universe*

[D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]

Scalar production: resonance effects

[M. A. G. Garcia, K. Kaneta, Y. Mambrini, K. A. Olive, S. Verner, arXiv:2109.13280]

- **Changing variables**

$$x_p \equiv a^{1/2} X_p \quad z \equiv m_\phi t + \frac{\pi}{2}$$

$$A_p = \frac{p^2}{m_\phi^2 a^2} + 2\hat{q}$$

("energy²")

$$\hat{q} = \frac{\sigma \phi_0^2}{4m_\phi^2}$$

("mass²")

- **Equation for mode functions** \iff **Mathieu equation**

$$\ddot{x}_p + (A_p - 2\hat{q} \cos(2z))x_p = 0$$

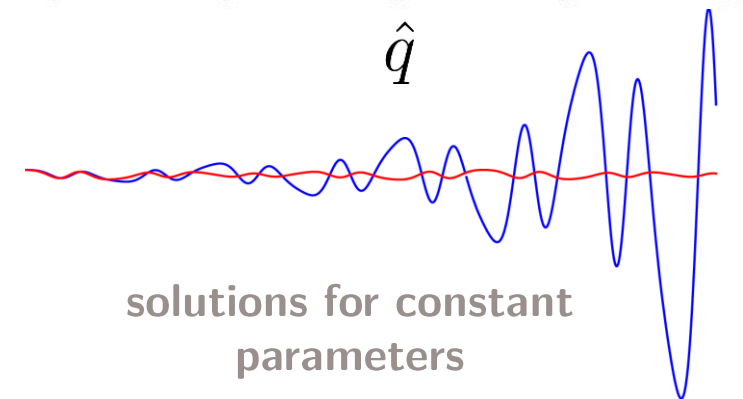
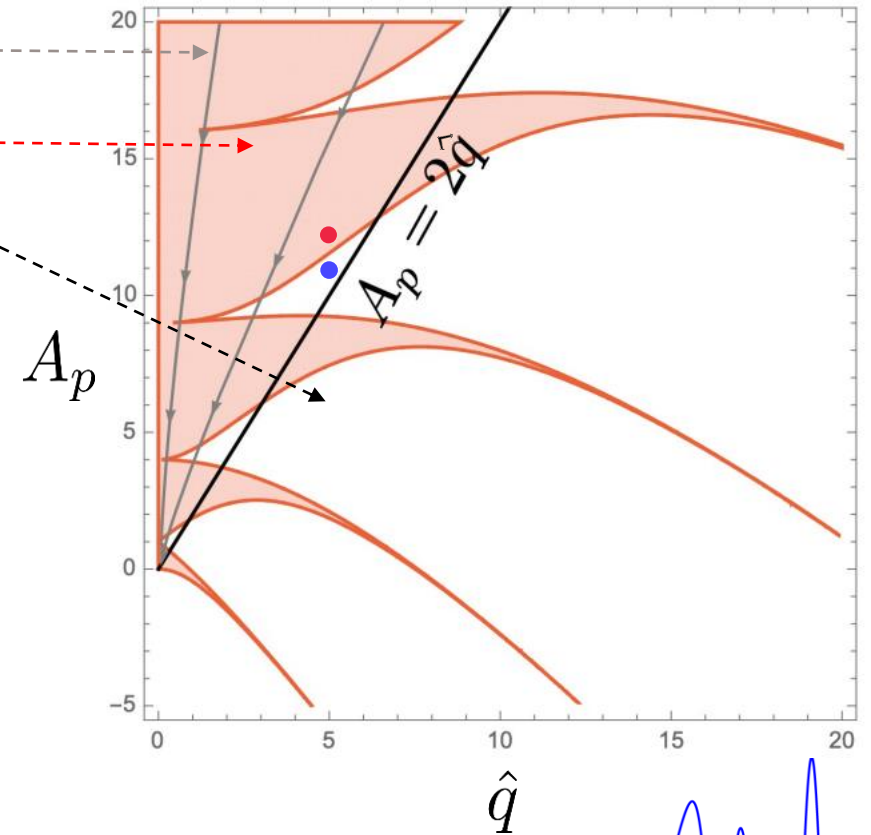
- For **large** momentum, i.e. $A_p \gg \hat{q}$ **narrow resonance**
 $A_p \sim 1 \pm \hat{q} \rightarrow$ **stable**

- Close to $A_p \sim 2\hat{q}$, **efficient** particle production
as **crossing** several **instability bands over** $\Delta t \sim \hat{q}/H$

mode evolution

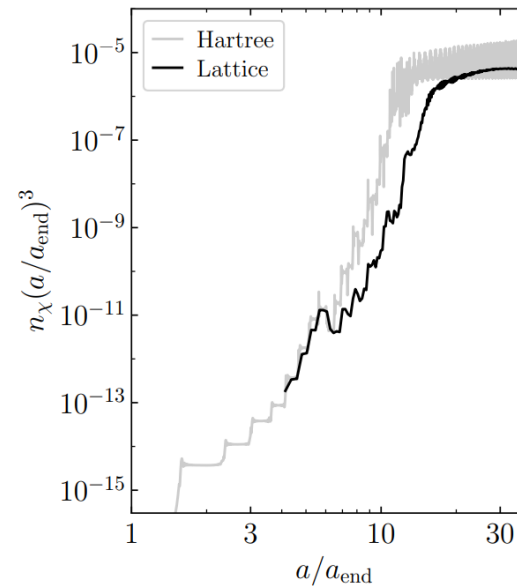
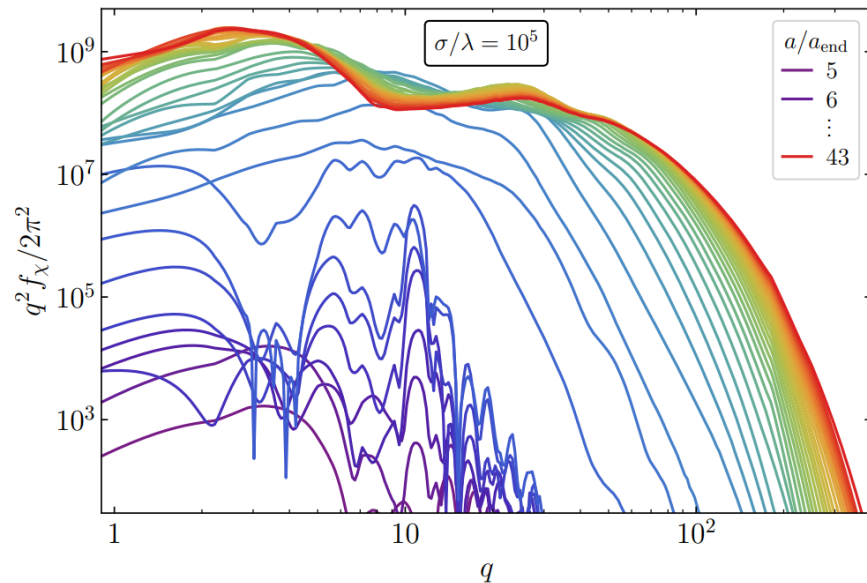
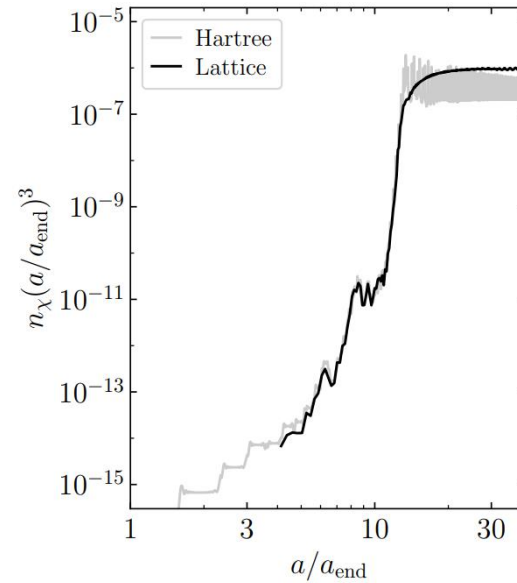
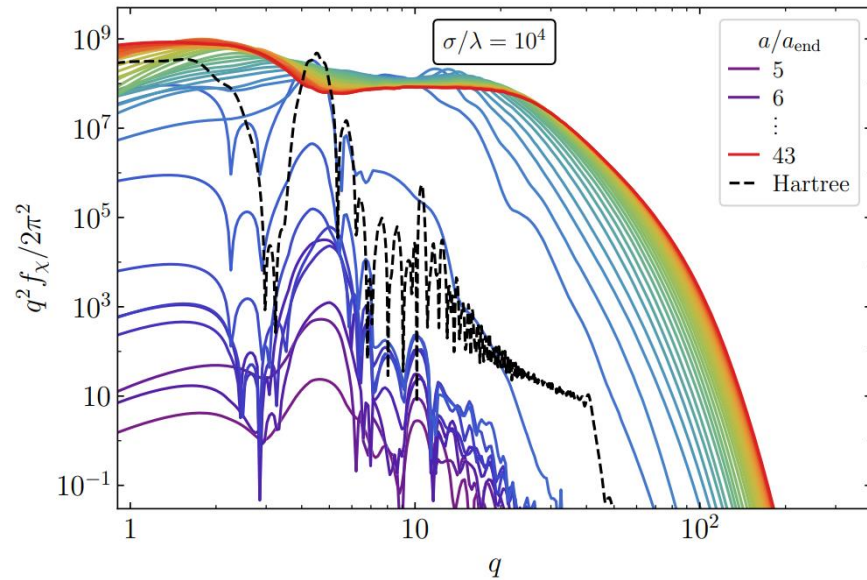
stable

unstable



[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452]

Lattice simulations: phase space distribution

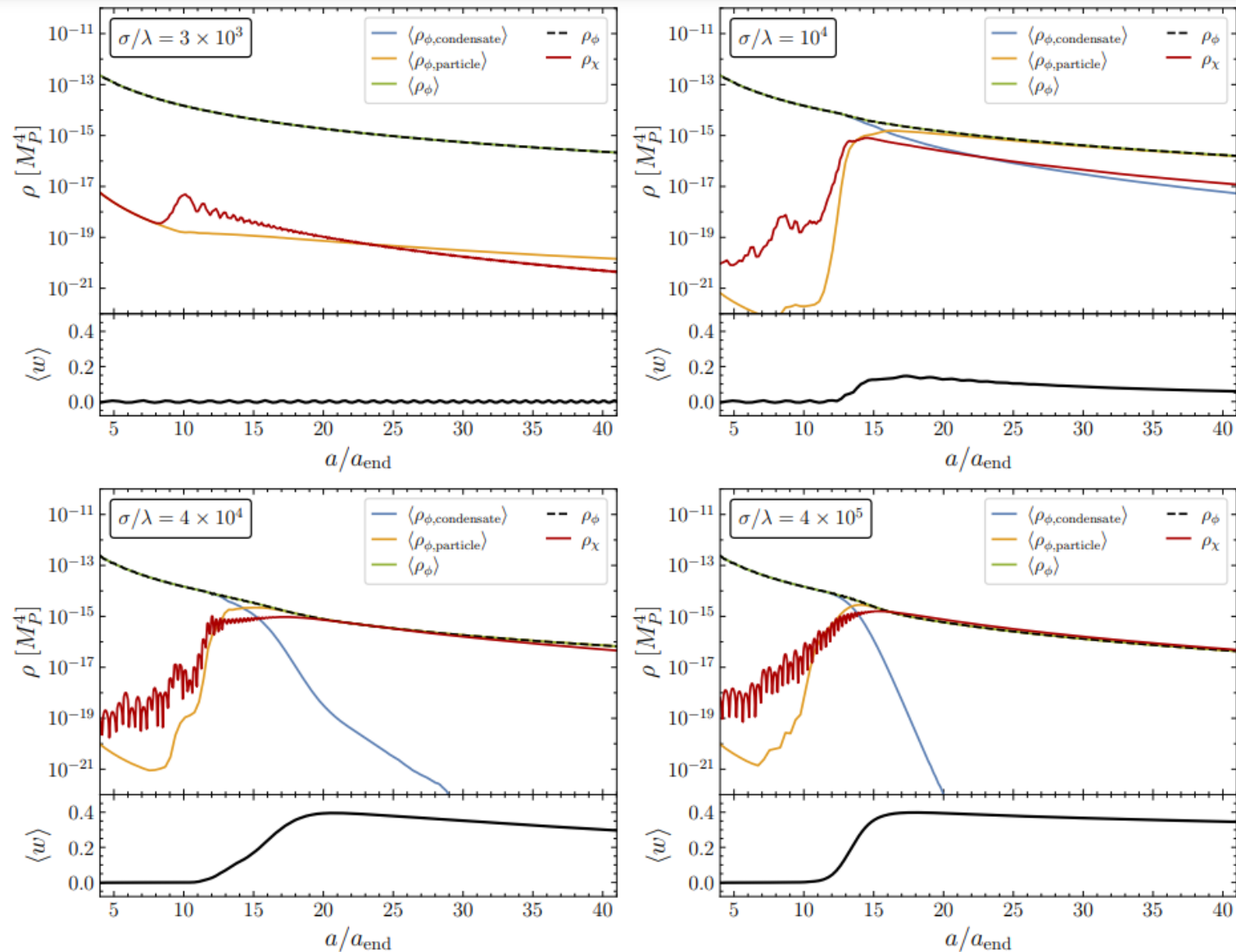


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Lattice simulations: energy density and fragmentation



$$\rho_{\phi, \text{condensate}} \equiv \frac{1}{2} \dot{\bar{\phi}}^2 + V(\bar{\phi})$$

$$\rho_{\phi, \text{particle}} \equiv \rho_{\phi} - \rho_{\phi, \text{condensate}}$$

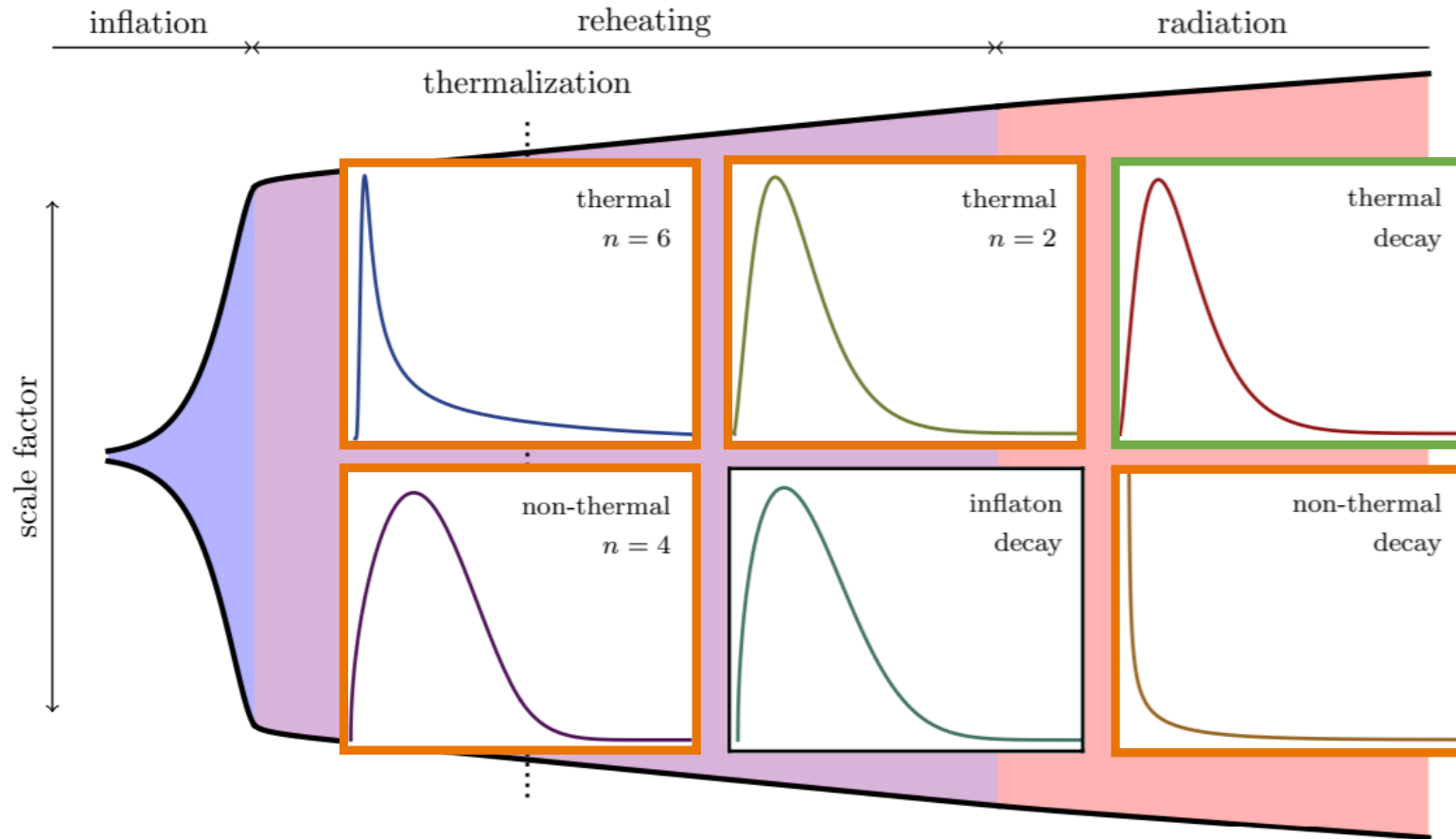
$$\langle w(a) \rangle \equiv \frac{1}{\Delta a} \int_a^{a+\Delta a} w(\tilde{a}) d\tilde{a}$$

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DM phase space distribution from freeze-in scenarios



[G. Ballesteros, M A. G. Garcia,
MP, JCAP 03 (2021) 101]

Freeze-in via
scattering

Freeze-in via
decay

➔ Previous analysis for cases with “**well-behaved**” distributions

$$f(q) \propto q^\alpha \exp(-\beta q^\gamma)$$