An Improved Method for Laser Speckle MTF Measurements

Dan Weatherill, Dan Wood, Ian Shipsey, Daniela Bortoletto, Wei Wei Liu, Sebastian Banfield

Oxford Physics Microstructure Detector Laboratory

Outline

- Motivation for new MTF measurement methods (astronomy, atom interferometry)
- Background on MTF measurement
- Past attempts at laser speckle MTF
- Our adjustment of the method
- Preliminary results on a 2.75um CMOS APS sensor

Motivation 1 – Rubin Observatory

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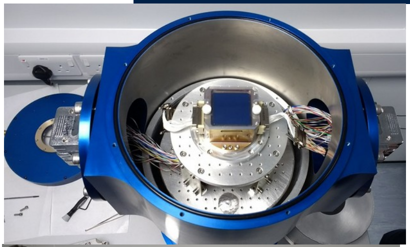
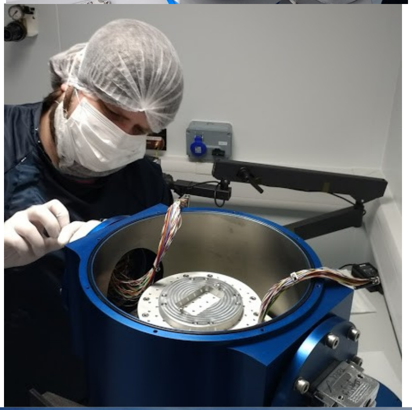
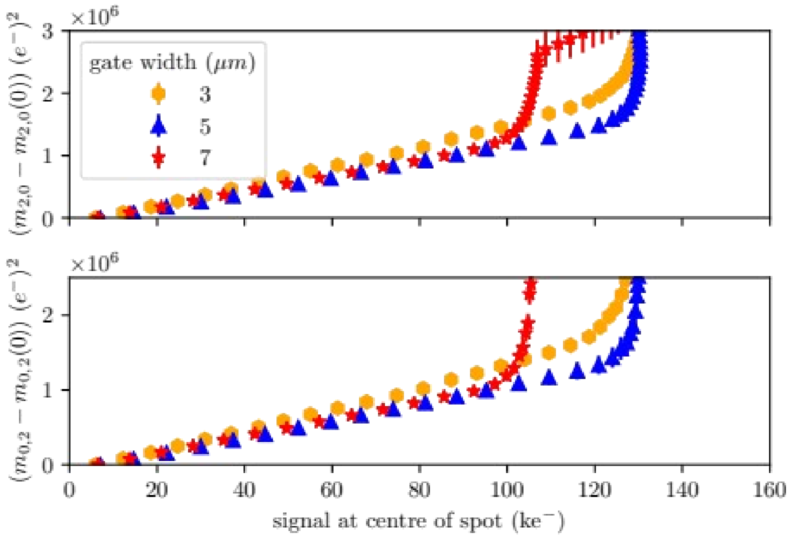
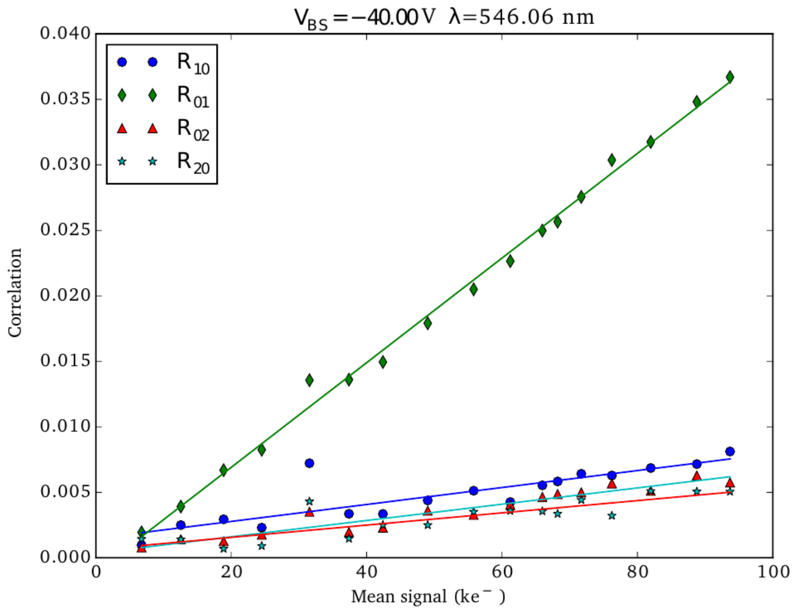
- One of our tasks within the LSST:UK consortium is to develop new methods to probe the detector PSF of the LSSTCam camera.
- Laser speckle MTF methods have several interesting possibilities in this area:
  - In principle can probe MTF in different areas of detector with high statistical power
  - Laser speckle measurement can probe MTF / PSF in different contrast regimes, which may be a powerful direct probe for the brighter-fatter effect – currently accessible only by low contrast (flat field correlations) or very high contrast (spot broadening) measurements.





However: this is future work!





Motivation2 – Atom Interferometry

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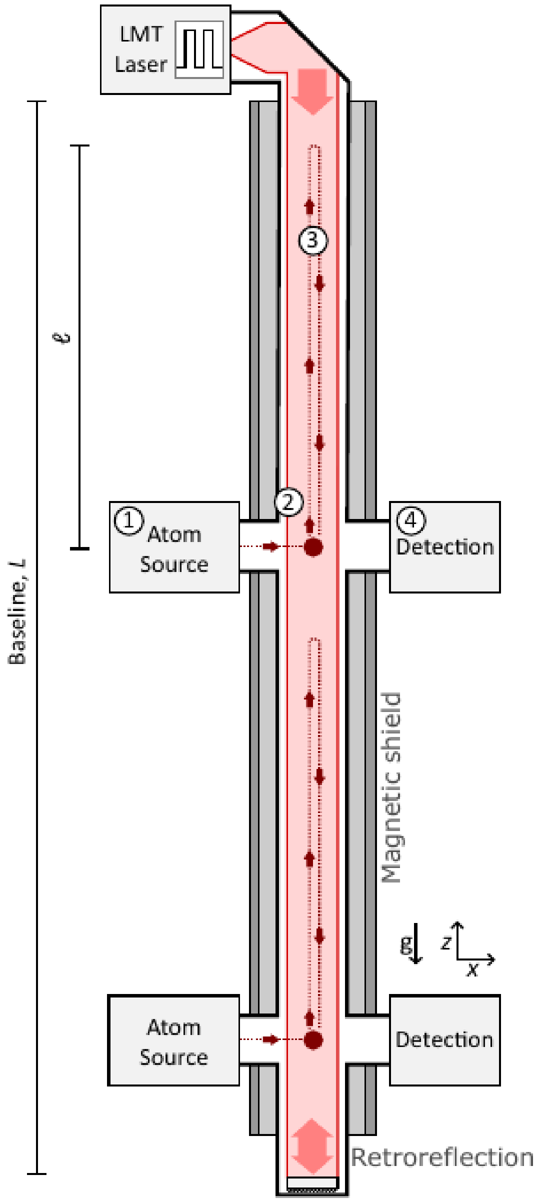
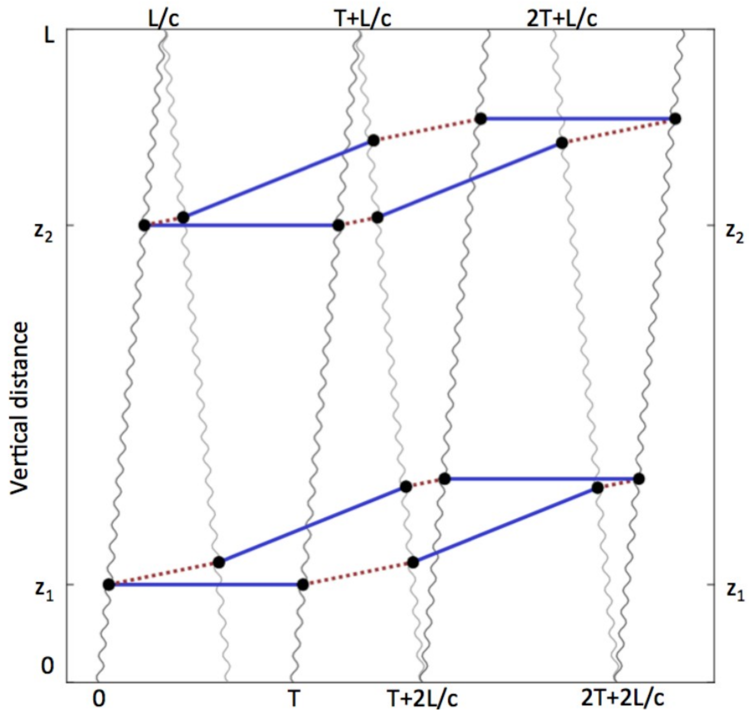
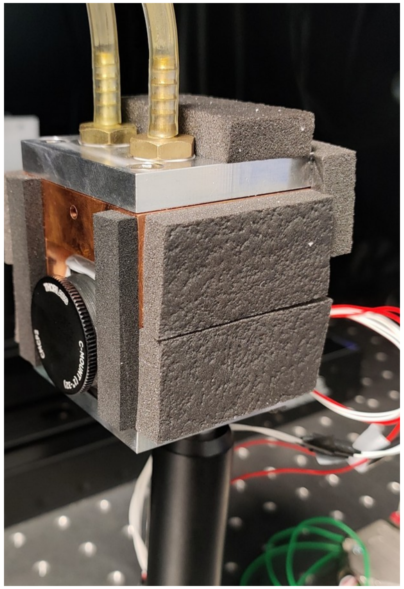




Images from Badurina et al (2020)

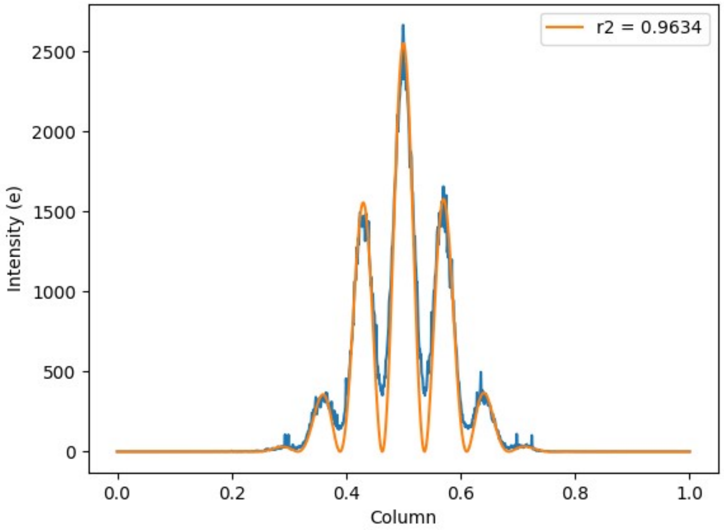
- Vertical atom interferometer experiments (MAGIS-100 & AION-10) use laser pulses to prepare clouds of free-falling cold strontium atoms in quantum superposition states, and fluoresce these atoms to observe the interference patterns
- In principle sensitive to ultra-light dark matter (via its effects on the atom transition frequencies) and gravitational waves (by their modulation of the light travel time in the detector)

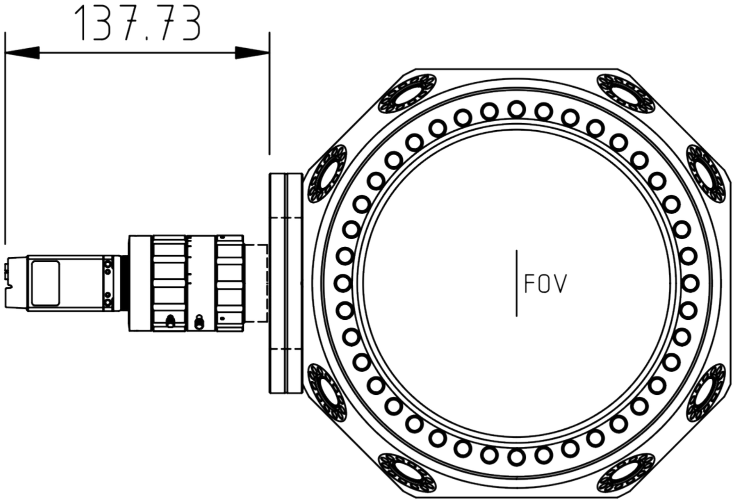
MAGIS-100 /AION-10 imagers



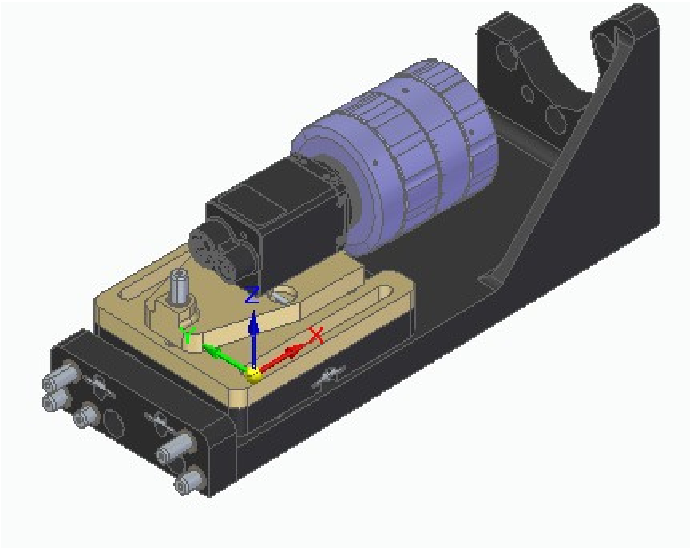
Oxford OPMD group is responsible for designing, procuring and testing the main science imagers for the MAGIS-100 instrument in the first instance.

Conventional high-quality CMOS-APS imagers. Low noise (~2e-), **small pixels (2.75um)**, QE 71%@450nm, imaging the fluorescence transition at 461nm. Feature scale of interest in object space is ~100um.





Understanding of system transfer function is important for science image correction – the detector oversamples the optics but **only just**. Therefore we need to know detector MTF component



Background - MTF

PSF (Point Spread Function) is a **real valued** object. **OTF** (Optical Transfer Function), therefore (being its Fourier transform) is a complex (but hermitian) object.

MTF (Modulation transfer function) is just the **magnitude** of the **incoherent OTF**

MTFs combine multiplicatively in the frequency domain (equivalent to convolving PSFs in spatial domain)

$$\mathrm{MTF}_{\mathrm{sys}} = \mathrm{MTF}_{\mathrm{optics}} \times \mathrm{MTF}_{\mathrm{sensor}}$$

Important – a pixellated detector is NOT LTI system! We actually generally measure
“pseudo-MTF”, which averages over all possible relative positions of input pattern.

Various physical mechanisms contribute to sensor MTF, - sampling, micro-lenses, diffusion.

Therefore MTF is temperature and wavelength dependent.

Other MTF Measurement Methods

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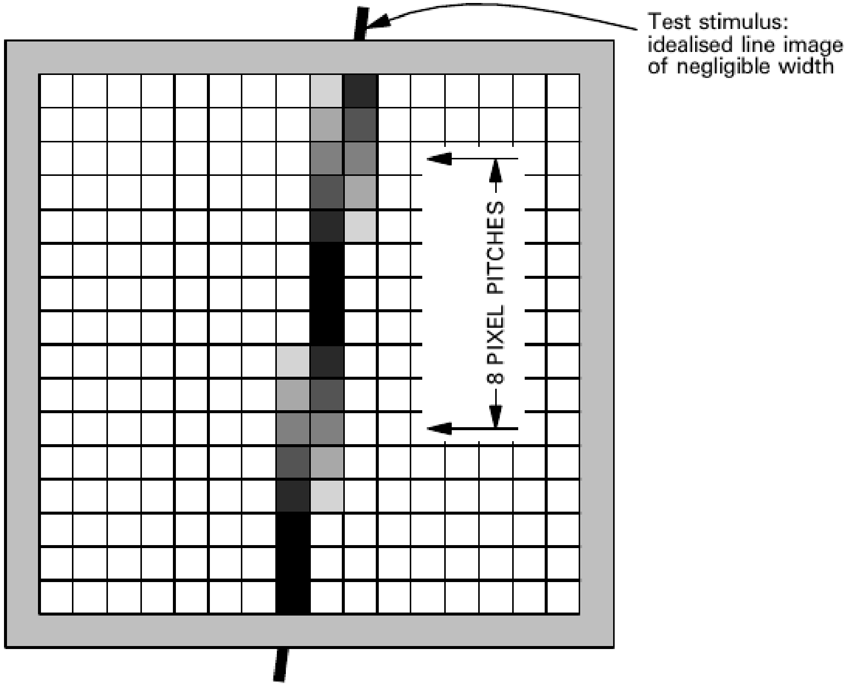


Notes

[1] In this example a 16 x 16 pixel region is designated on the sensor, e.g. enclosed by a one pixel width border suitably highlighted on the image display. The test image is focused as shown with a slope of 1 in 8 with respect to the pixel columns or rows.

[2] An almost ideal sensor will yield a response as illustrated above. The shaded pixels indicate partial responses because the line energy is shared between adjacent pixels.

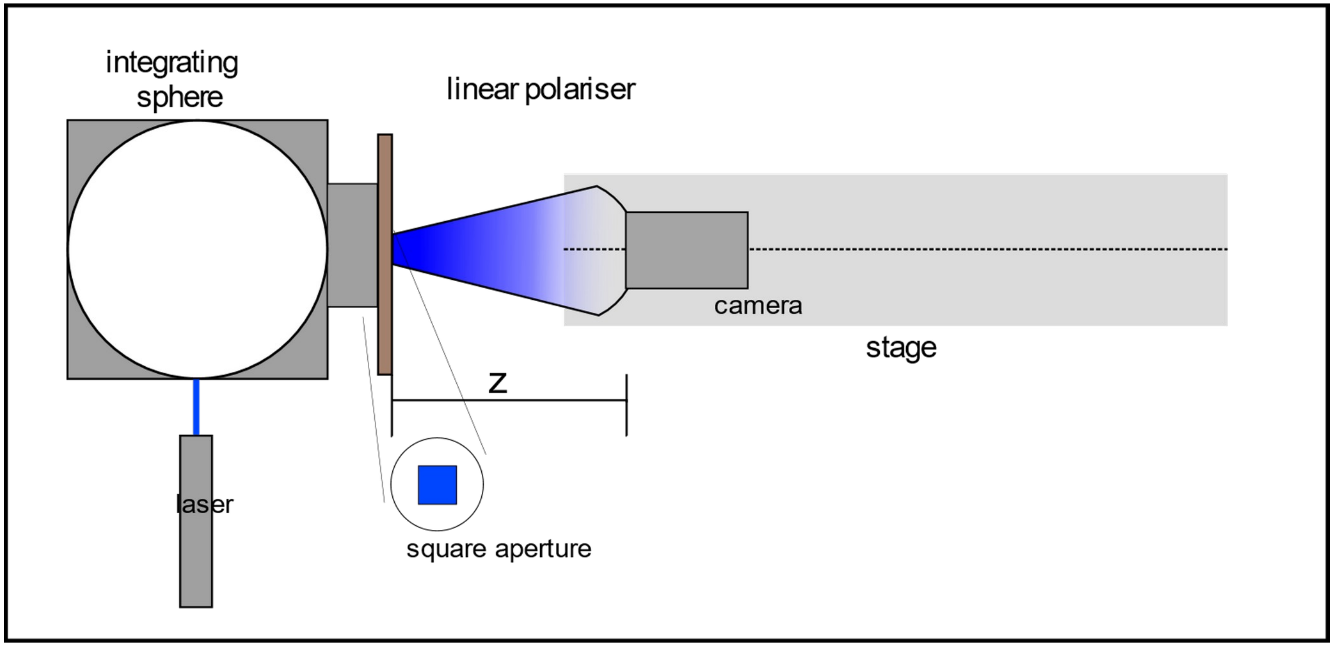
(image left, from te2v tech note "modulation transfer function of charge coupled devices")

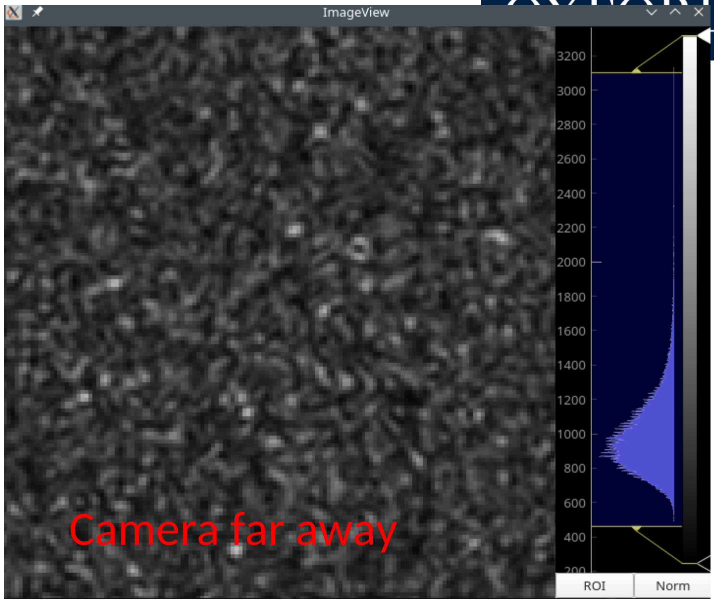
Traditionally, to measure sensor MTF, we project some pattern (typically a line) onto the detector, at a slight angle to the pixel grid. This gives us a vernier method to work out the "line spread function" which is just the integral of the point spread function.

Problem: this is OK for 10um CCD pixels…. But at 2.75um CMOS APS pixels, **how do we do that projection?** The NA required (particularly at short wavelengths), is prohibitive.

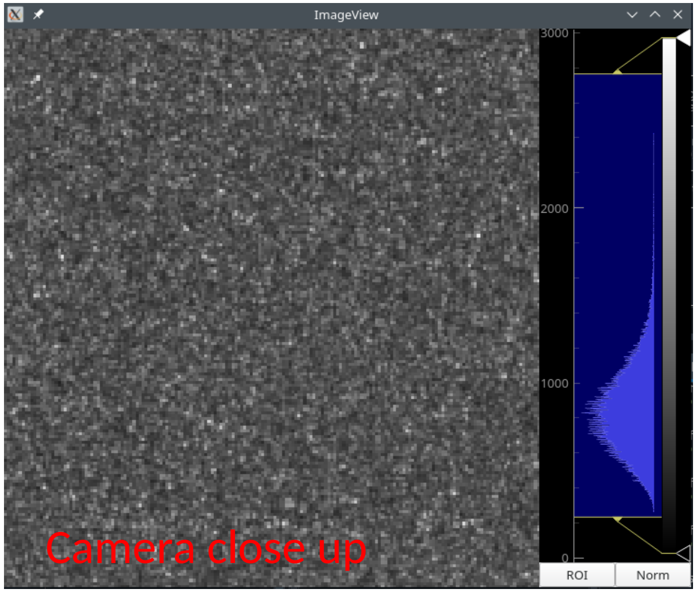
If we know (very accurately) the OTF of the projection optics, this is still fine. But that itself is quite a big ask.

Laser Speckle MTF Principle





- Laser light passed through an integrating sphere (rough internal surface) forms a "speckle pattern" – an interference pattern caused multiple scattering from the rough surface
- Speckle pattern passed through a square aperture has **theoretically known** spatial power spectral density, depending only on distance, wavelength and aperture size. This PSD is (**crucially) band limited**
- Directly projected onto sensor (no optics!) it is thus convolved with only the sensor (+ coverglass) MTF component



Crucially: NO REFRACTIVE OPTICS INVOLVED!!

Past Work

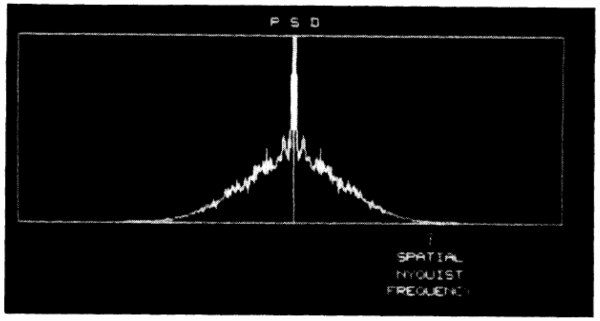
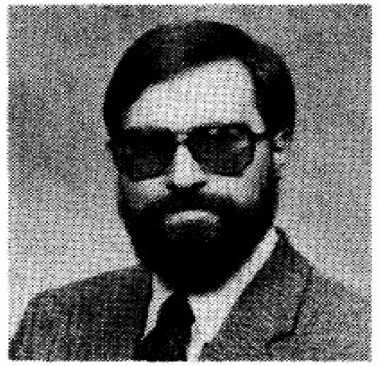


Fig. 6. PSD measured for speckle inside the shadow region. The arrow indicates the spatial Nyquist frequency of 21.5 cycles/mm.



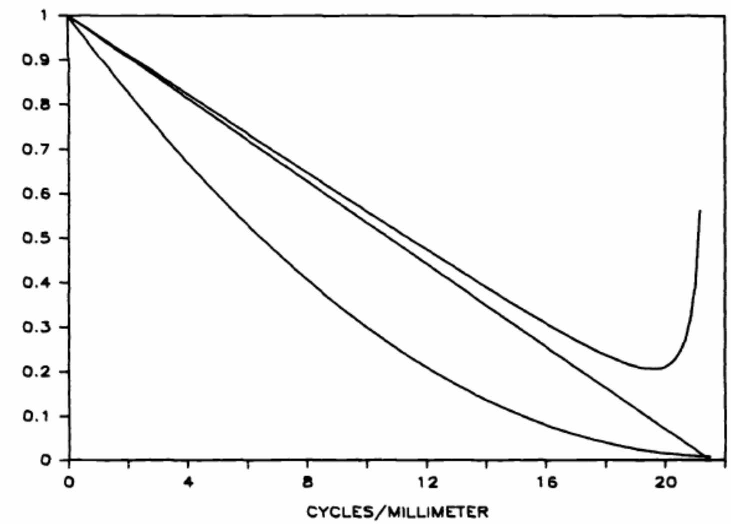


Fig. 8. Upper curve is the calculated MTF of the detector array. The middle curve is the input PSD of the speckle. The bottom curve is the polynomial fit to the normalized output PSD of the speckle from Fig. 7.

Boreman et al, SPIE Op Eng 29, 1990 first demonstrated attempted MTF measurement via laser speckle and taking the power spectral density

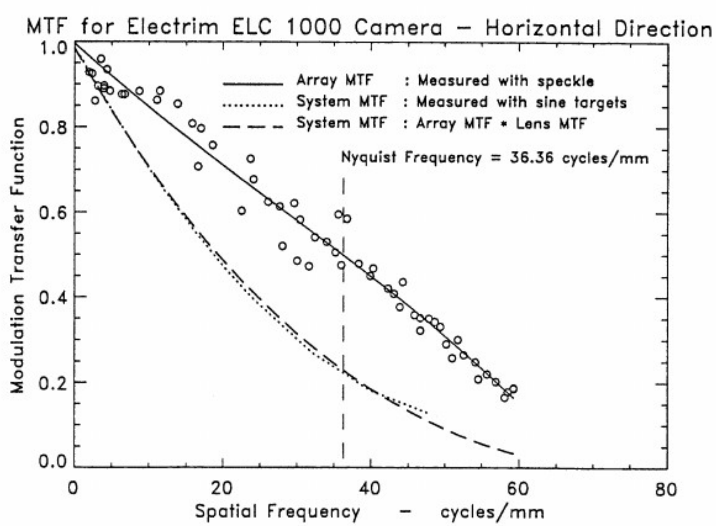


Figure 6. MTF of Electrim camera array using laser speckle and system MTF with sine targets vs system MTF as an array MTF times lens MTF.

Later, Sensiper, Boreman et al (1992) developed this method Further using spaced slit aperture.

This has a known cutoff frequency, by moving the aperture along the optic axis, MTF can be reconstructed
Similar to our new method but with less statistical power

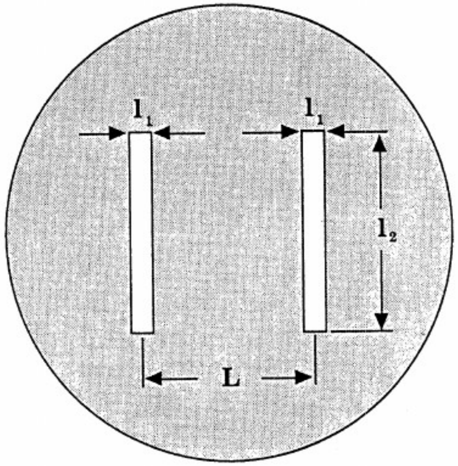
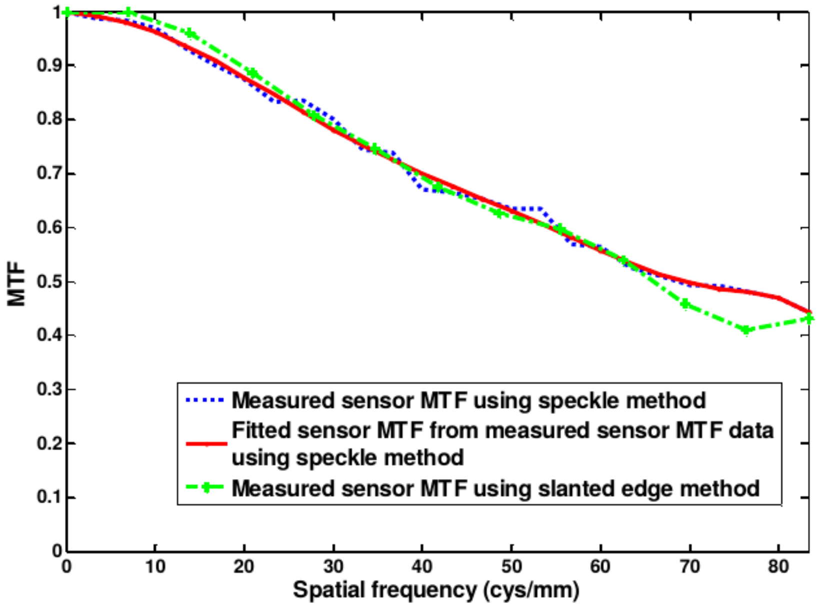


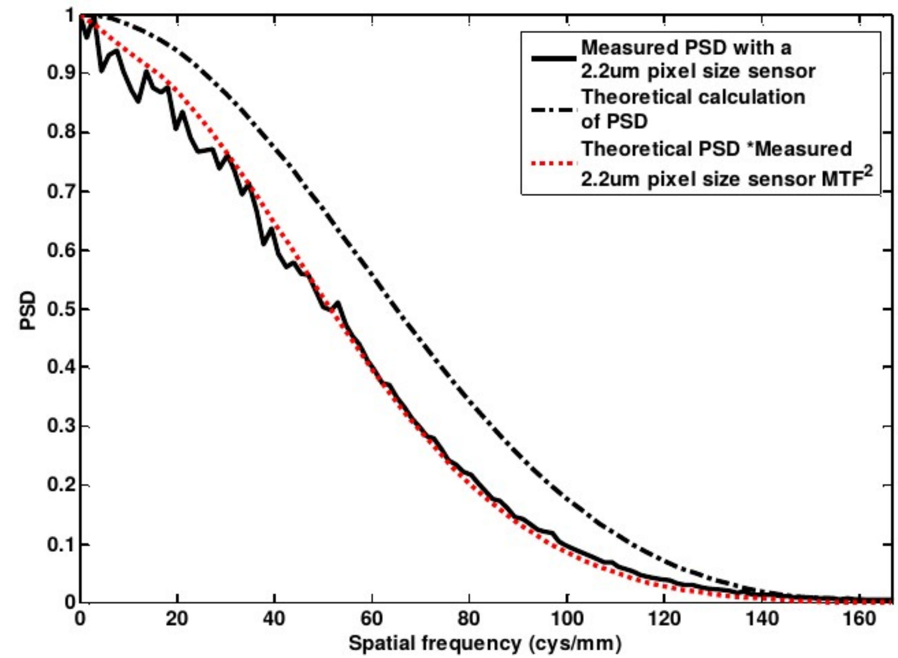
Figure 2. One-dimensional aperture.

Past Work (2)



Chen et al (2008) did a **very** impressive measurement of speckle MTF and compared with slanted edge projection on a 2.2um pitch sensor.

However, to get this measurement beyond Nyquist they had to oversample the speckle by moving the detector laterally by half a pixel!!



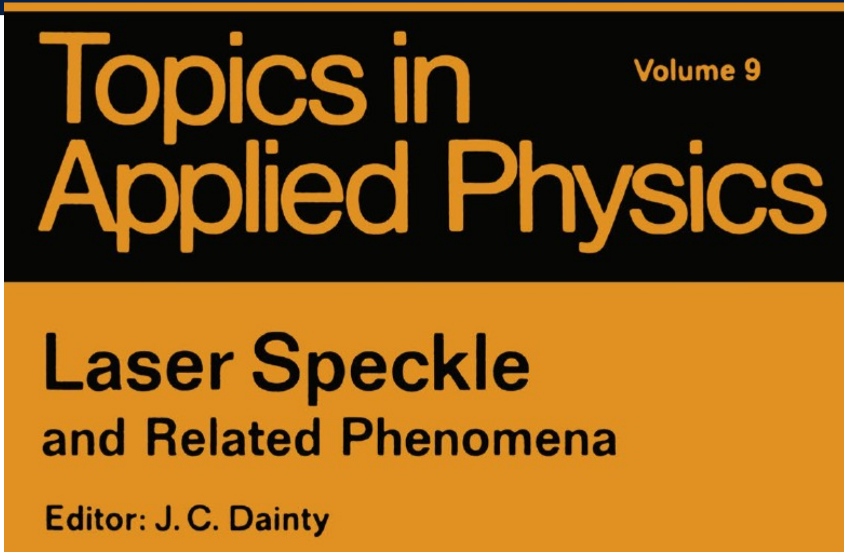
Chen et al method is highly sensitive to the accuracy of these lateral stage movements.

Also it requires an incredibly vibration stable speckle pattern (the exact same pattern must be imaged 4 times to get the oversampling)

It has the major advantage that only one projection distance is needed to reconstruct the full MTF

All above methods require pixel size to be known beforehand!

(some) Theory



$$R_I(\Delta x, \Delta y) = \langle I \rangle^2 \left[1 + \mathrm{sinc}^2 \frac{L\Delta x}{\lambda z} \mathrm{sinc}^2 \frac{L\Delta y}{\lambda z} \right],$$

^ Autocorrelation function!

Wiener-Khintchin Theorem

$$\Lambda(x) = 1 - |x| \quad \text{for} \quad |x| \le 1,$$

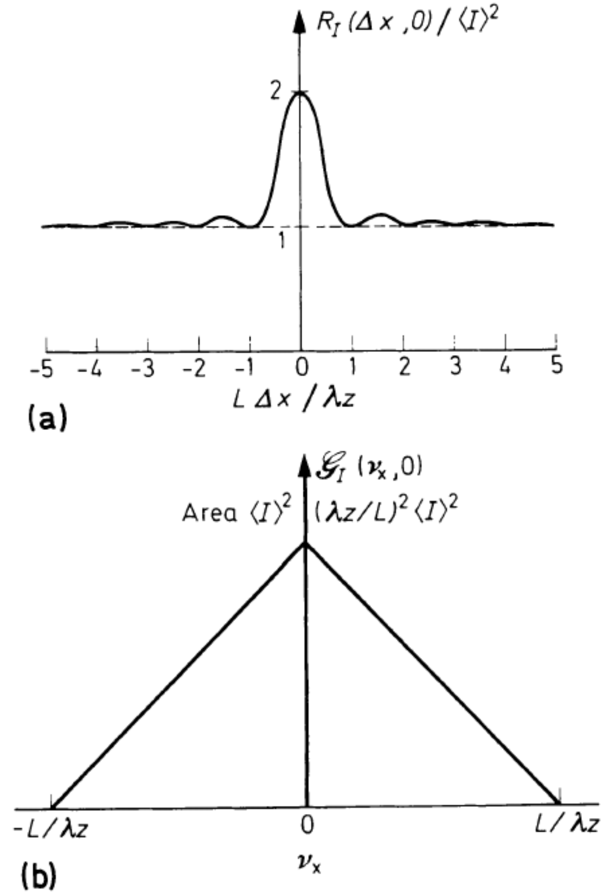


Fig. 2.15a and b. Form of the (a) autocorrelation function and (b) power spectral density of speckle produced by a square scattering spot

$$\mathscr{G}_I(\nu_X, \nu_Y) = \langle I \rangle^2 \left[\delta(\nu_X, \nu_Y) + \left(\frac{\lambda z}{L}\right)^2 \Lambda\left(\frac{\lambda z}{L} \nu_X\right) \Lambda\left(\frac{\lambda z}{L} \nu_Y\right) \right]$$

Power Spectral Density (PSD)

More Theory (just a bit)

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$$\mathcal{G}_I(\nu_X,\nu_Y) = \langle I \rangle^2 \left[\delta(\nu_X,\nu_Y) + \left(\frac{\lambda z}{L}\right)^2 \Lambda\left(\frac{\lambda z}{L}\nu_X\right) \Lambda\left(\frac{\lambda z}{L}\nu_Y\right) \right]$$

THIS is a problem – need proper Normalisation!

(We solved this by using Parseval's theorem)
- **this is probably the most important key To the whole method**

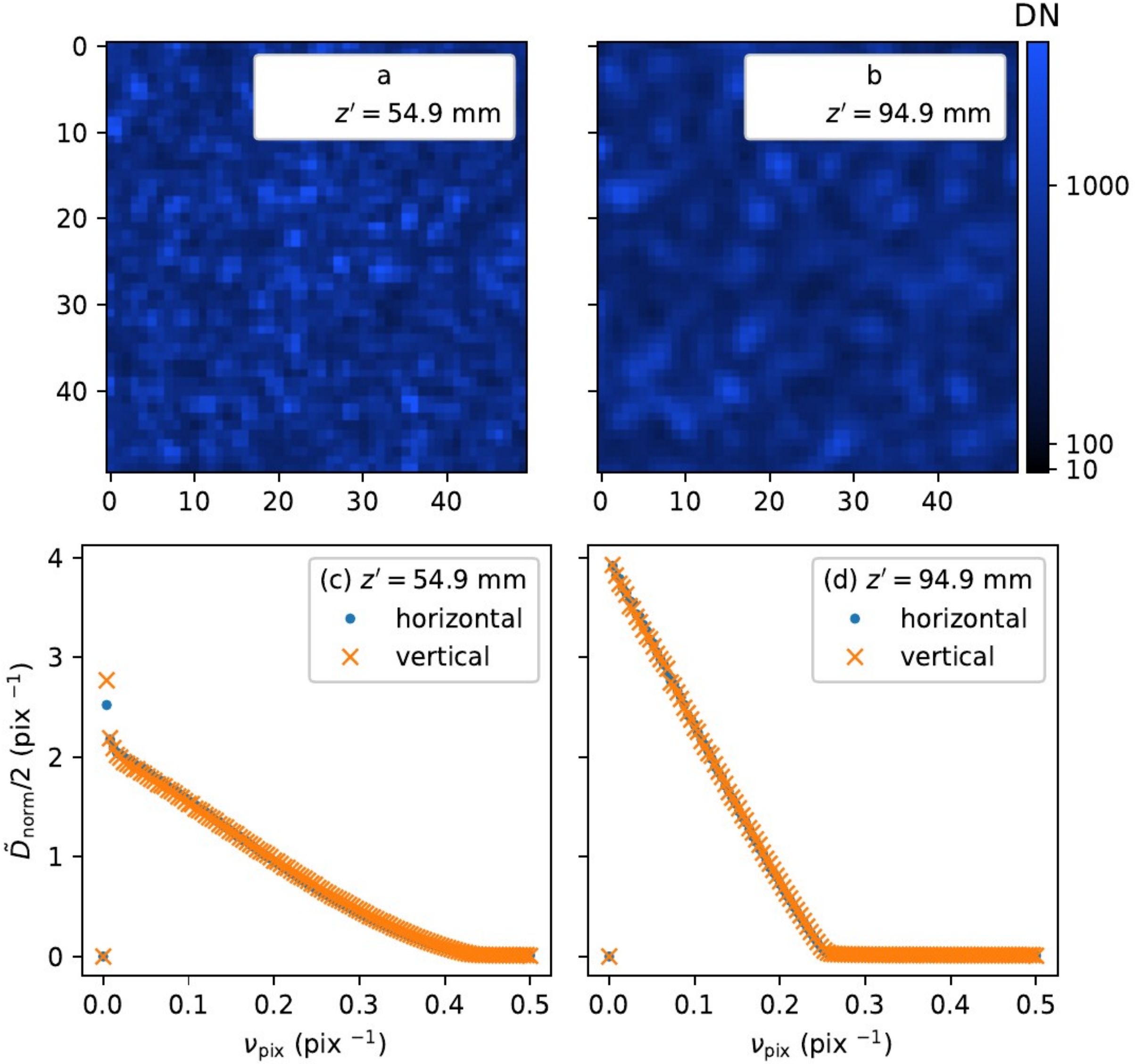
This is NOT a problem – Welch's Method removes this component

L – aperture size
Z – distance to aperture

Then, we work out MTF from our measured PSD using the below equation

$$PSD_{\text{meas}}(\nu) = MTF(\nu)^2 \times PSD_{\text{speckle}}(\nu)$$

PSD (!!) Measurement



- Square aperture allows us to leverage Welch's method for **power spectral density (PSD)** information to give high statistical power, at the cost of losing xtalk information (which we expect to be not a problem e.g. in CMOS APS).
- At far distances, output looks almost exactly like the theoretical prediction (a downward sloping straight line)
- At close distances, cutoff frequency increases, until eventually we see aliasing effects.
- Normalising this PSD is a challenge, and had not previously been done properly in the literature. Key insight: Parseval's theorem implies total power in PSD is equal to image variance

PSD Fitting

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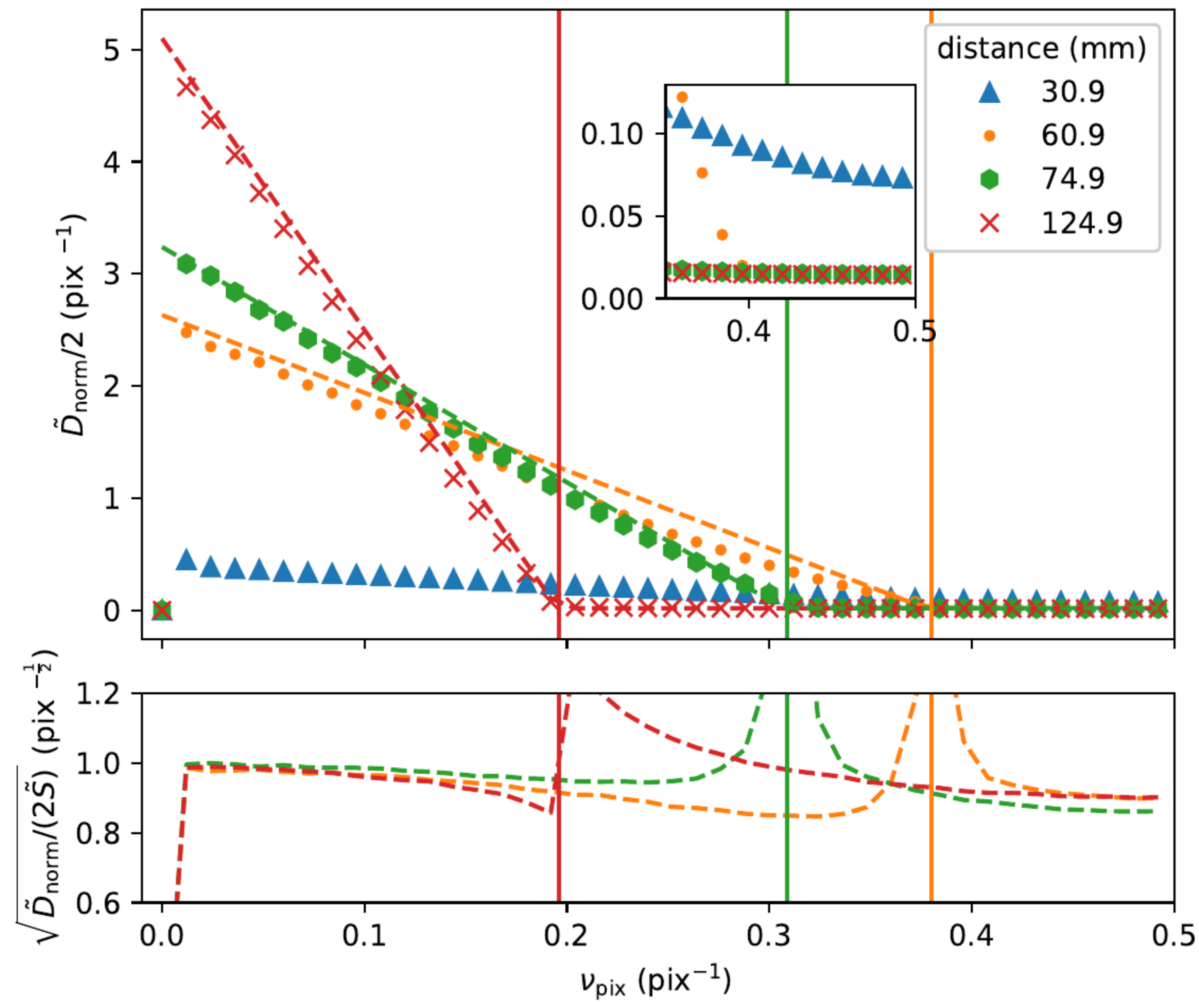
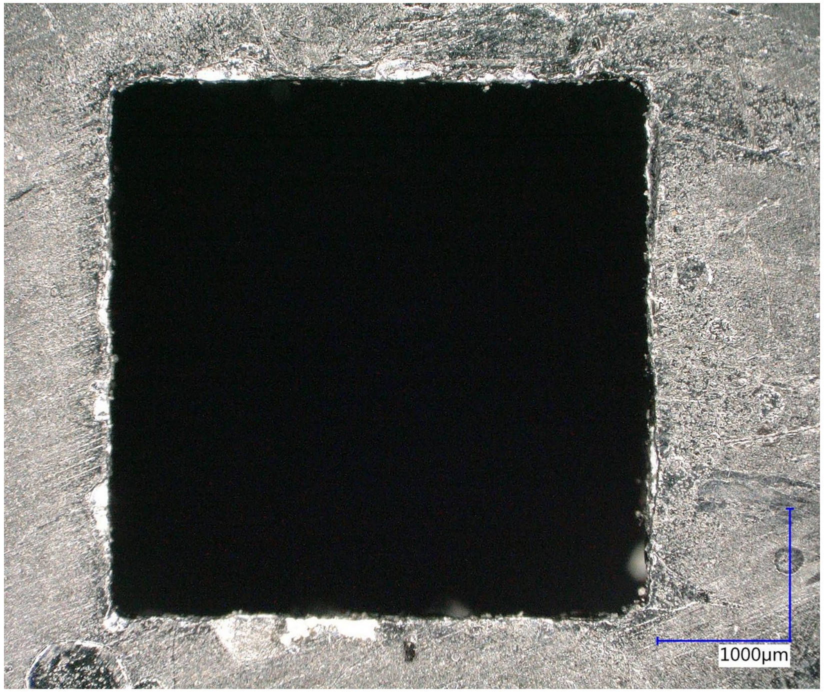
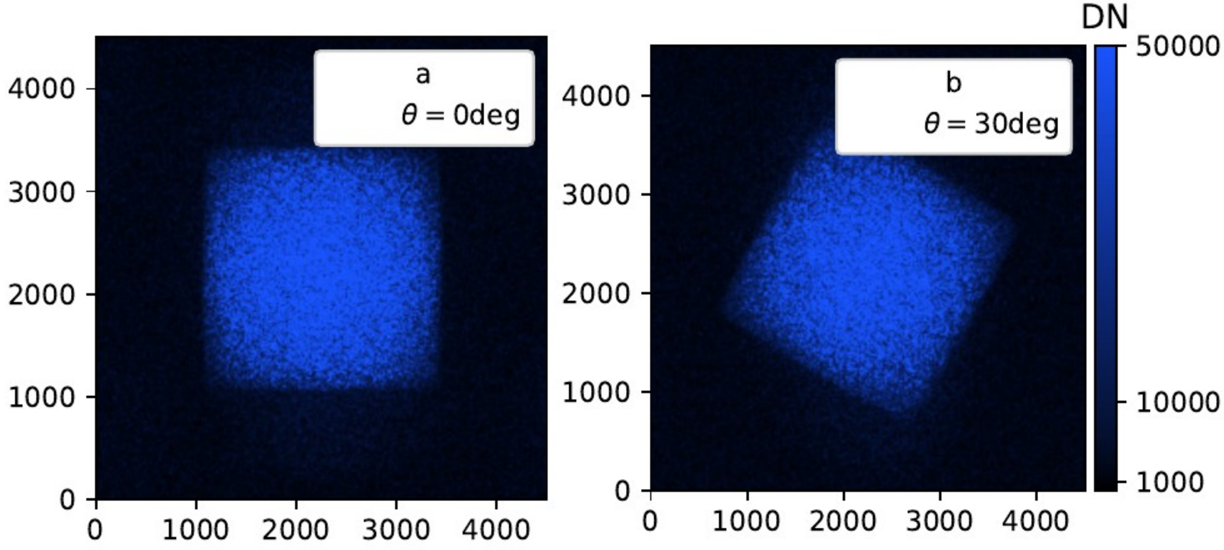
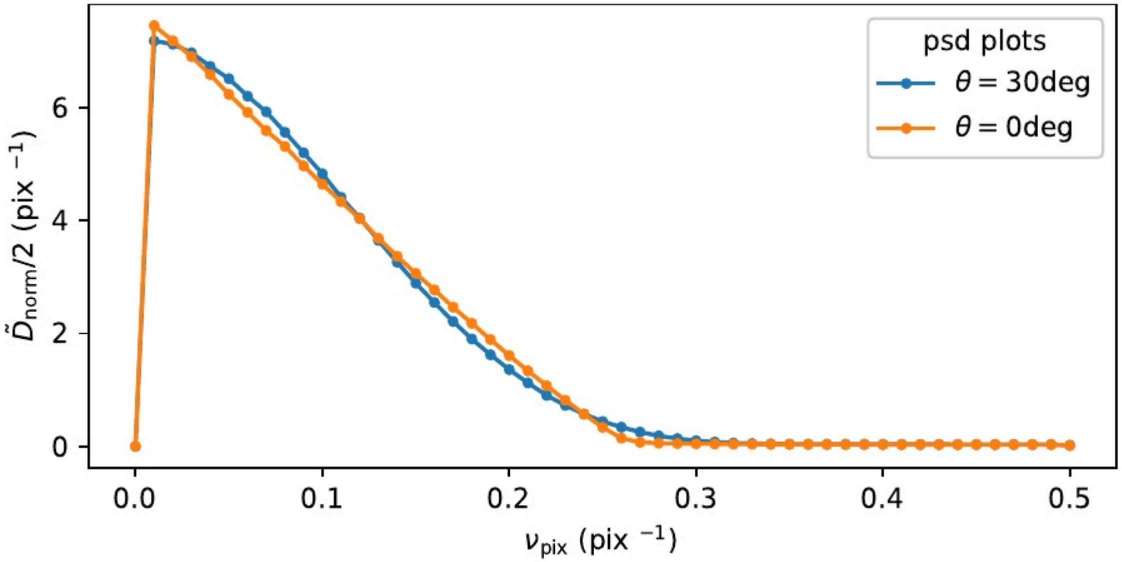
- We fit the closest theoretical PSD for the speckle to the data to extract the cutoff frequency.
- These are **not just straight line fits!** They are further constrained because the theoretical PSD has known gradient and area for a given cutoff frequency. Proper normalisation allows us to powerfully constrain the fits beyond simple straight lines.
- At this point it is **in principle** possible to extract MTF by simply dividing the data by the theoretical PSD (lower panel). However it is subject to the problem of 0 / 0 : the input speckle pattern is by design a low pass filter, so de-convolving it naively leads to infinities for a system with some noise. We have a way round this (in a couple of slides)



Data Verification (angle)







Accuracy of the method depends pretty critically on knowing the size of the aperture we are using, it being actually square, and it being in the correct orientation.
This can be checked by looking at the 2D fourier transforms of images (right)

NB the "real space" images look identical to the eye!

Fitting Pixel Size & true distance

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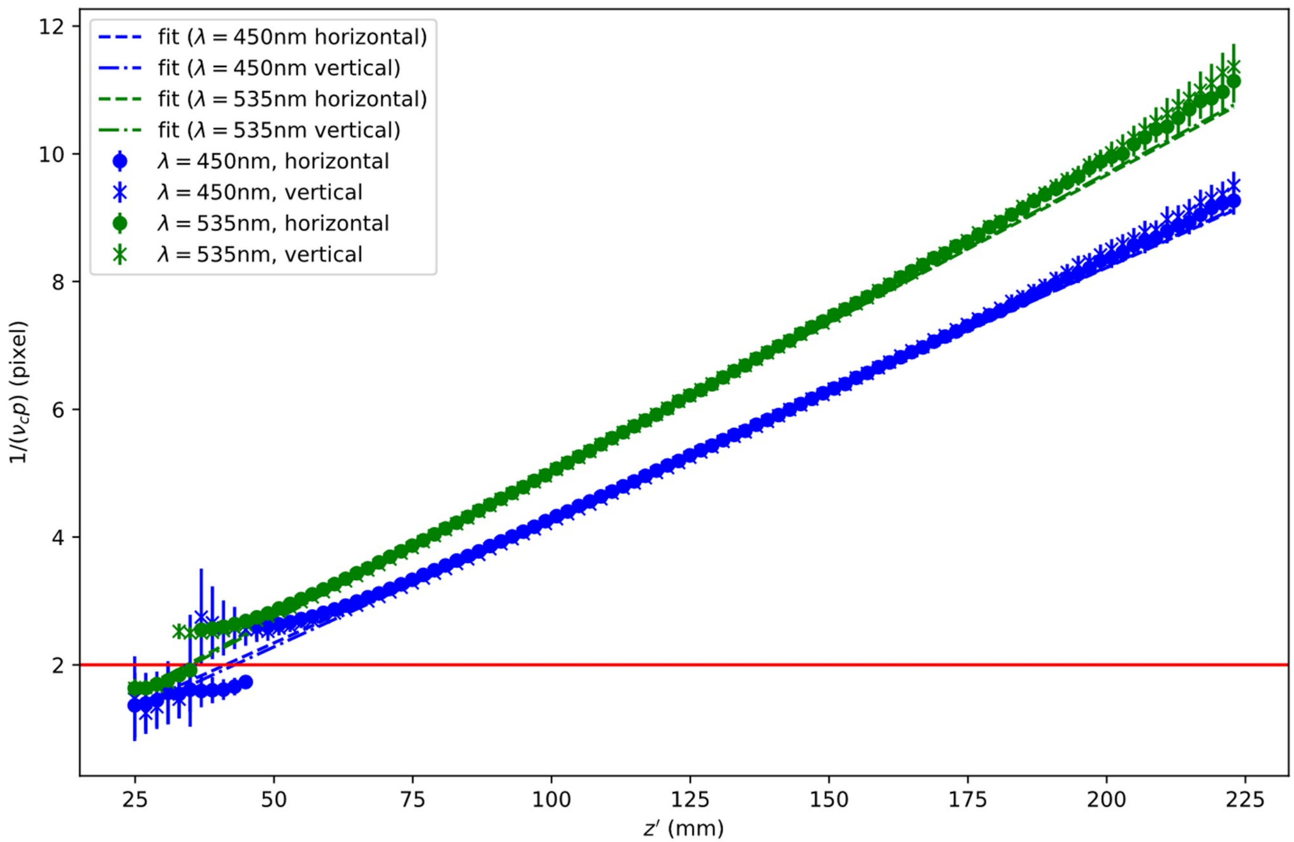

direction	λ (nm)	fitted gradient (m^{-1})	z_{offset} (mm)	p (μm)
horizontal	450	(39.2 ± 0.1)	(9.9 ± 0.4)	(2.80 ± 0.01)
vertical	450	(39.7 ± 0.1)	(7.5 ± 0.3)	(2.76 ± 0.02)
horizontal	535	(46.3 ± 0.2)	(9.0 ± 0.4)	(2.82 ± 0.01)
vertical	535	(46.7 ± 0.2)	(7.7 ± 0.3)	(2.79 ± 0.02)

- The theory says that the cutoff frequency depends reciprocally on distance, left shows fit to 1/cutoff freq vs distance.
- There are some experimental effects due to stray light and exposure time shot noise we need to take better account of (only basic implementation so far), but the fit looks pretty good
- As a by product, we get an estimate of the pixel size from this (conversion factor from mm to pixels!
  (note nominal pixel size is 2.75um)
  This calibration gives us "true" distance from our motor distance

MTF Extraction

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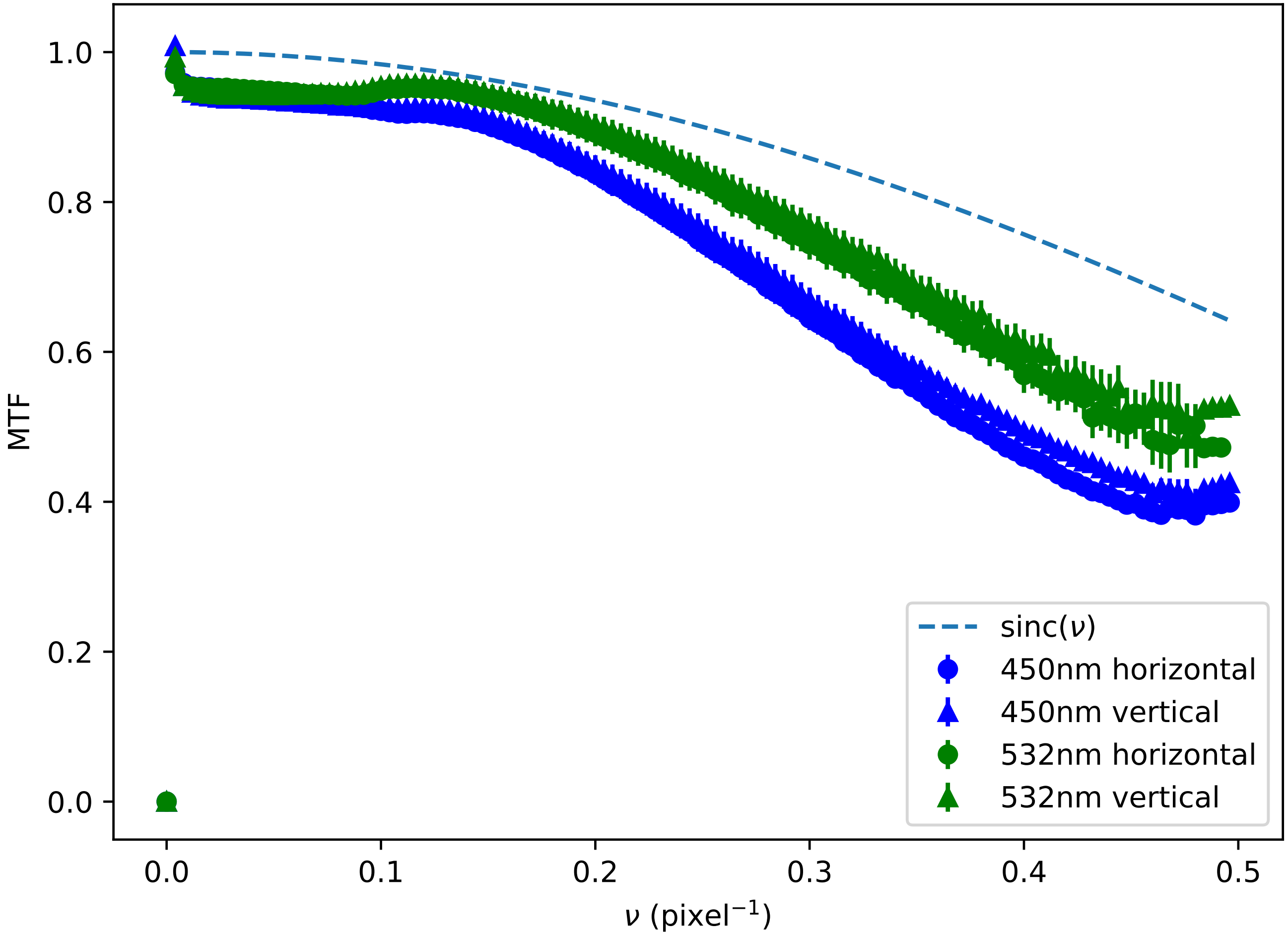
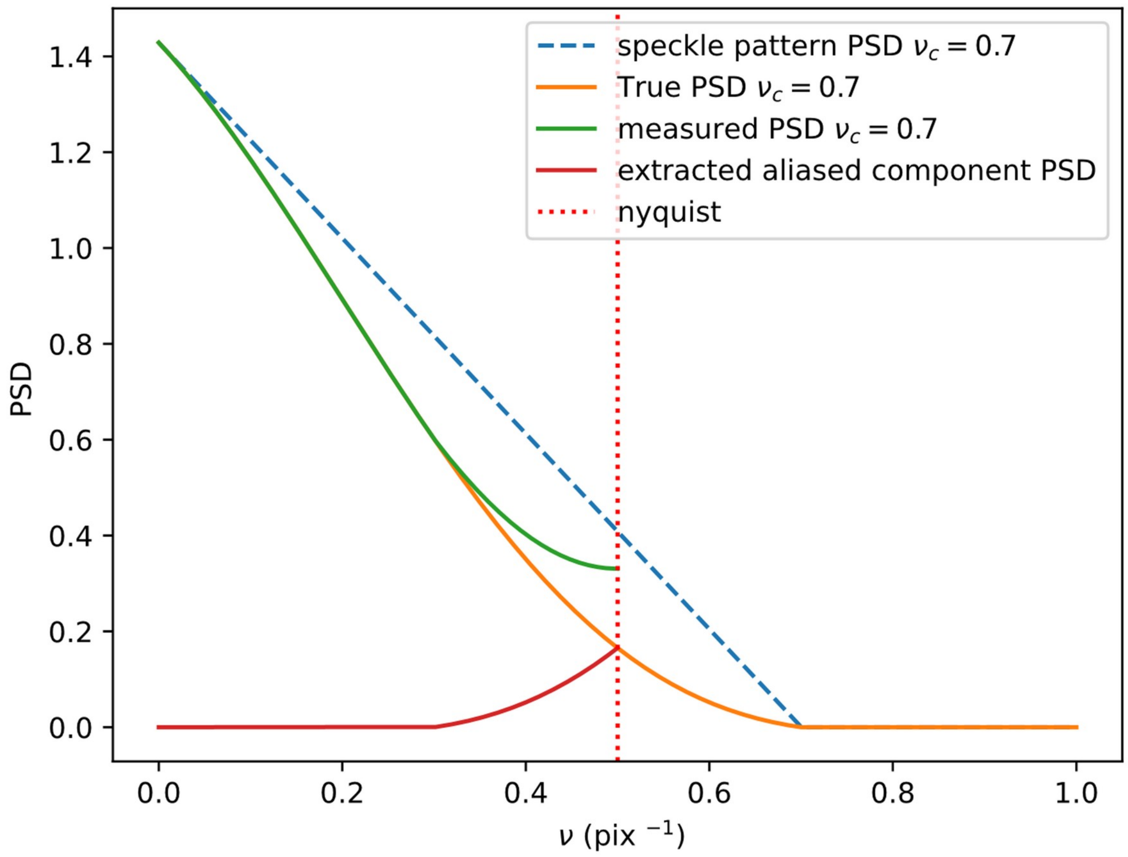


- Once we know the "true" distances and cutoff frequencies, we can then fit PSD vs distance at each frequency bin, rather than fitting the cutoff frequency for each distance measured PSD.

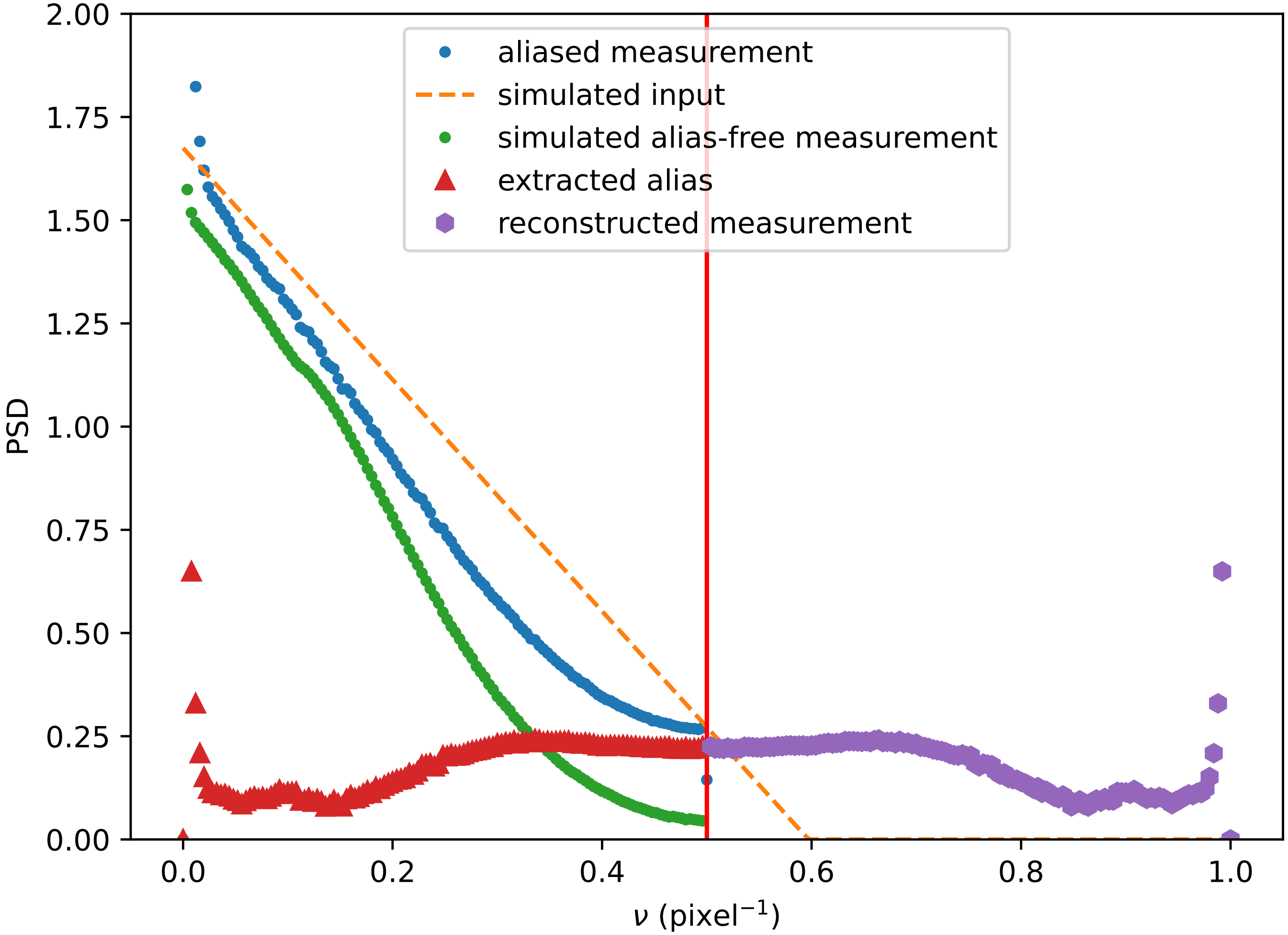
- The result yields a single MTF point for each fit.
- Care must be taken for several confounding effects (only some of which we've so far fully dealt with – **see proceeding**)
- With that caveat, an example sensor MTF measurement is shown here (with sinc function for reference)
- We stress again, **no functional form or polynomial fitting has been used to obtain this!**

Scheme for super-Nyquist



1) We already measured MTF upto (just below) Nyquist.
2) We already measured “true distance” and thus can **simulate** input speckle PSDs which have an aliased component
3) Deliberately drive the system into aliasing, but with cutoff between Nyquist & sampling freq.
4) Simulate projected measurement (using measured MTF and known input) **without aliasing**
5) Measure difference between this simulation and actual measurement
6) That is the component that was aliased back into the passband!
7) Flip it around the Nyquist frequency
8) … profit?

Preliminary Results



- So far, the basic concept is working but there is a problem with normalisation in aliased measurements
- **See Proceeding** and watch this space, we are still working on it!

Conclusions / Future Work

- Laser speckle MTF measurement has some attractive properties that recommend it to being used for small pixel sensors
- We have developed an extension of the usual laser speckle MTF method which does not need high precision positioning stages, and does not require knowledge of pixel size a-priori
- A scheme for extracting above-Nyquist MTF measurements using our new technique has been demonstrated, though improvements and some more control over systematics are needed, which are ongoing

Future Work

- Comparison with direct knife edge measurements on larger pixel sensors
- Include the crosstalk terms, use as tool to probe brighter-fatter effect

Thanks

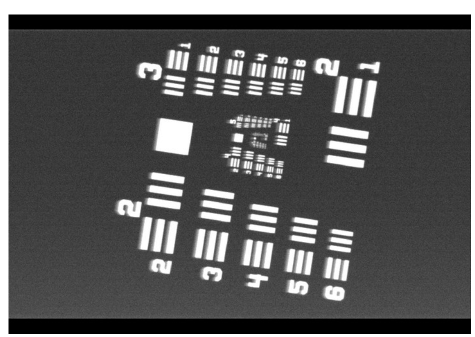
All questions, comments & suggestions very enthusiastically received

Daniel.weatherill@physics.ox.ac.uk

The rest of the OPMD AION/MAGIS team:
Daniela Bortoletto, Ian Shipsey, Dan Wood

The rest of the OPMD Group

Our brilliant AION/MAGIS undergraduate project students:
Simai Jia (2022)
Wei Wei Liu (2022/23)
Harry King (2022/23)
Sebastian Banfield (2023)



Background – OTF, PSF

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“OTF for dummies”

The PSF is the thing you convolve an input illuminance with to get the measured image.

The OTF is its Fourier Transform.

It’s different for coherent vs incoherent illumination

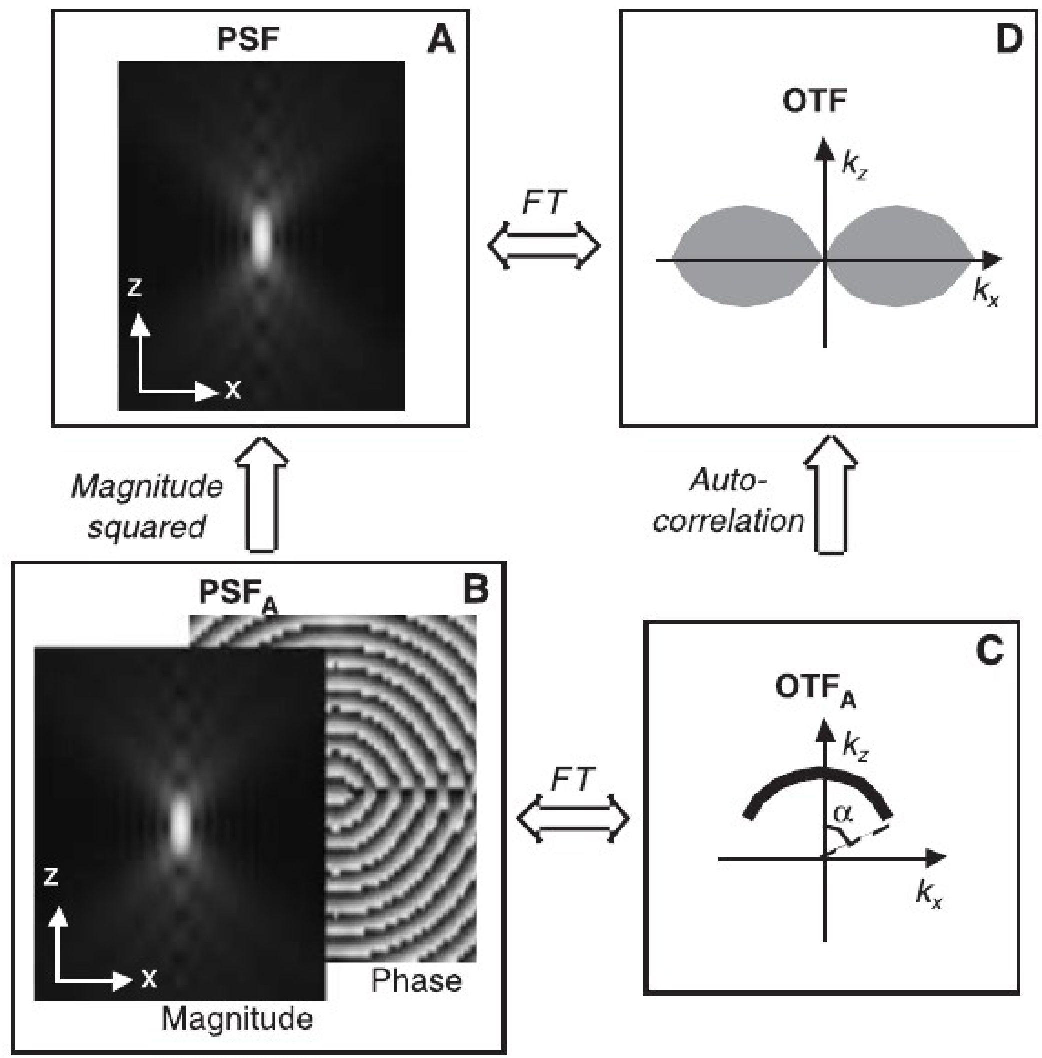
If you know the former, you can get the latter (the other way requires phase retrieval)

Applies only to LTI systems (e.g. lenses).

In atom interferometry and astronomy, we are looking at incoherent light

But nice to measure coherent OTF because contains more information

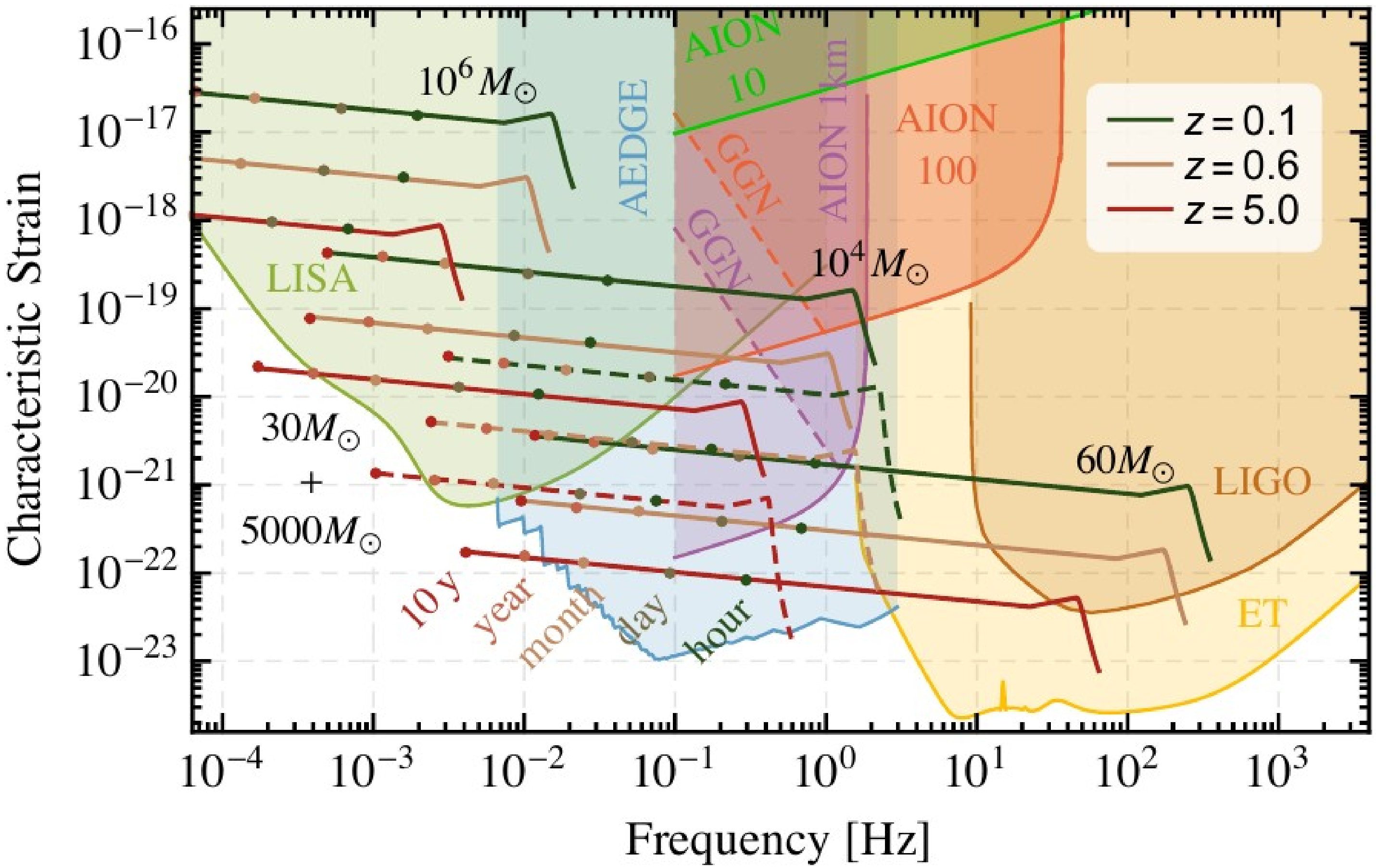
Incoherent Light



Coherent Light

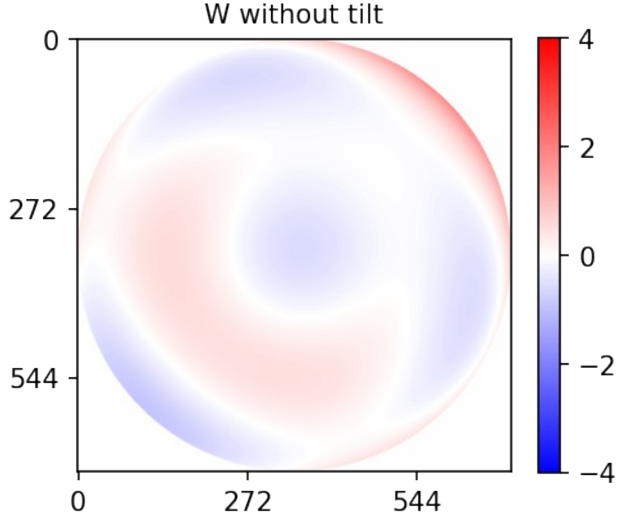
(image from Hanser et al, 2004)

Backup



Badurina et al (2020)

Backup


W without tilt
0
272
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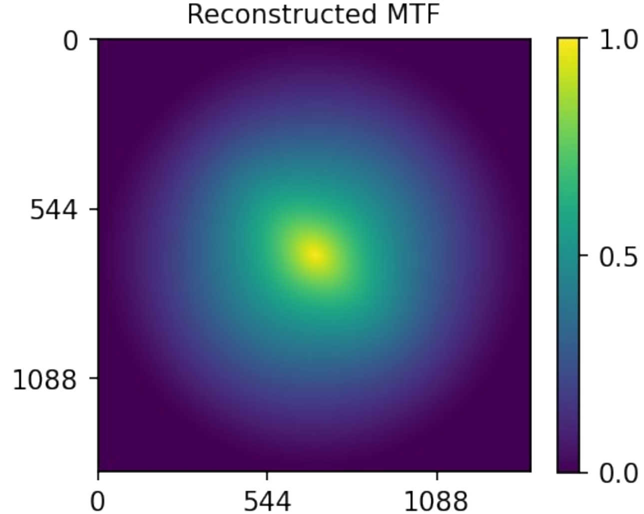
Reconstructed MTF
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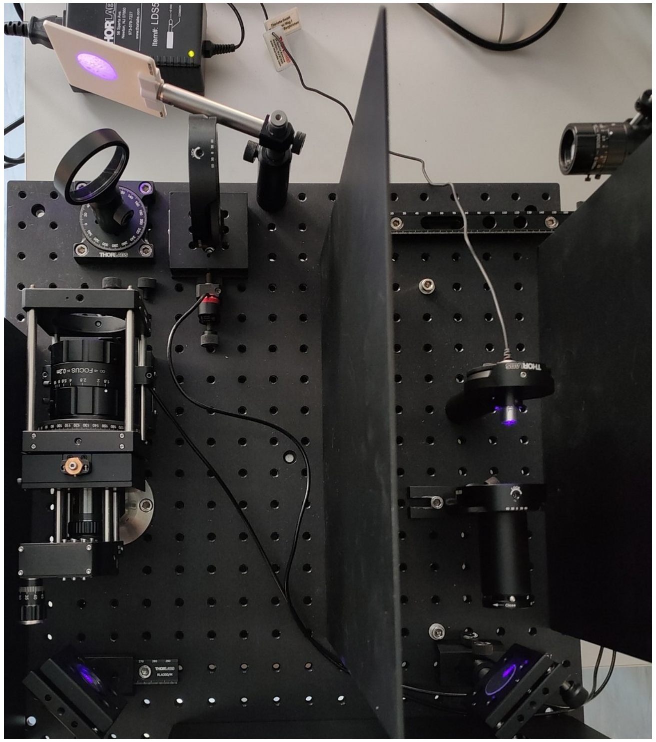

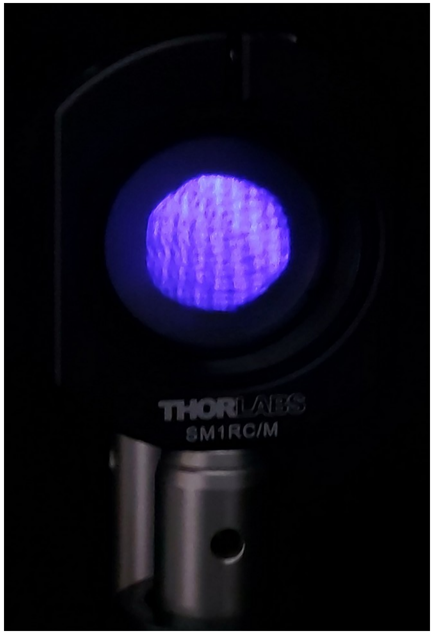
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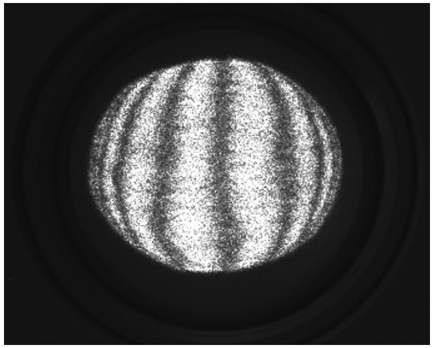

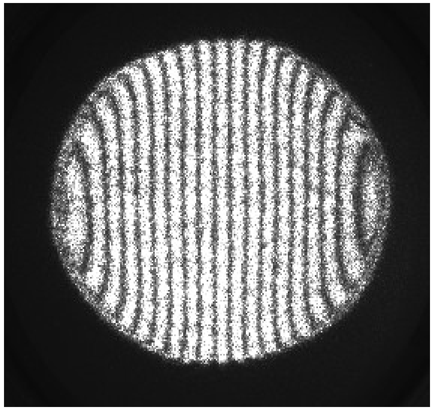

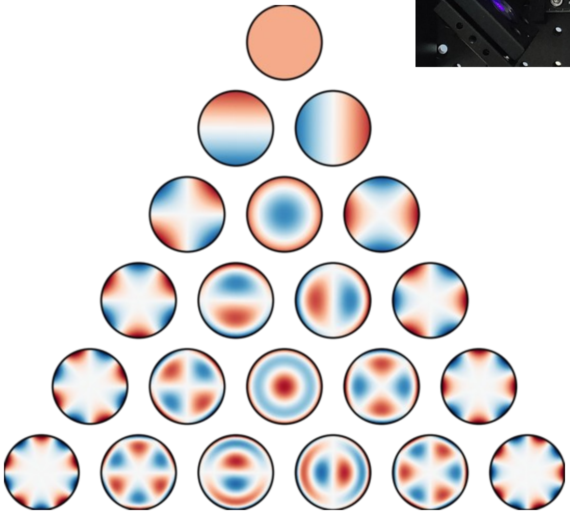

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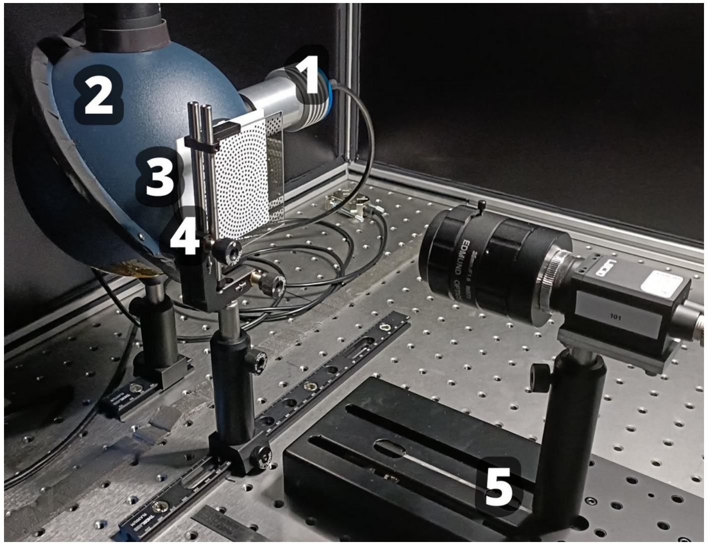
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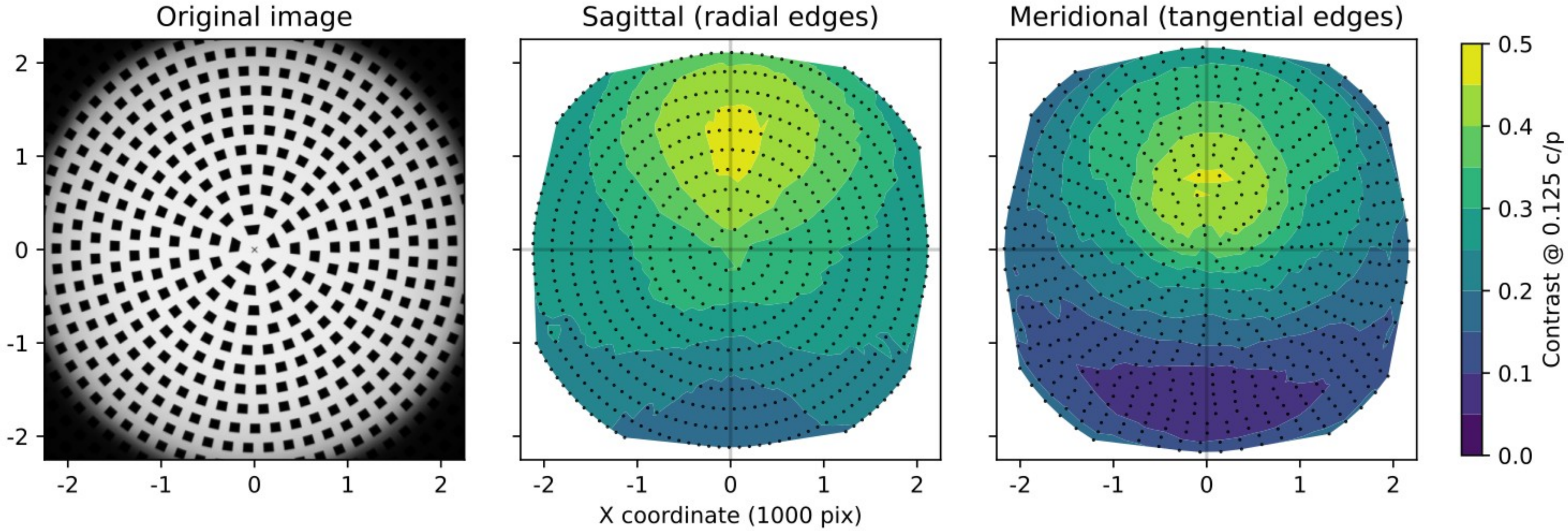
4
5


Original image
Sagittal (radial edges)
Meridional (tangential edges)
X coordinate (1000 pix)
Contrast @ 0.125 c/p
0.5
0.4
0.3
0.2
0.1
0.0
-2
-1
0
1
2

Backup – Charge Diffusion MTF

$$MTF(k_x, k_y) = \frac{M_e(k_x, k_y) + M_{zce}(k_x, k_y)}{M_e(0,0) + M_{zce}(0,0)}.$$

$M_e(k_x, k_y)$ and $M_{zce}(k_x, k_y)$ are given by the following expressions:

$$M_e(k_x, k_y) = qD_e\left(-\frac{A_e}{L_{ke}} \cdot e^{-\frac{z_d}{L_{ke}}} + \frac{B_e}{L_{ke}} \cdot e^{\frac{z_d}{L_{ke}}} - \gamma_e \alpha e^{-\alpha \cdot z_d}\right) \cdot \mathrm{sinc}(\pi k_x \cdot p) \cdot \mathrm{sinc}(\pi k_y \cdot p) \quad (48)$$

$$M_{zce}(k_x, k_y) = \phi_0(e^{-\alpha z_d} - e^{-\alpha z_{epi}})\mathrm{sinc}(\pi k_x \cdot p) \cdot \mathrm{sinc}(\pi k_y \cdot p) \quad (49)$$

where p is the pixel pitch.

Djite, Estribeau et al, 2012