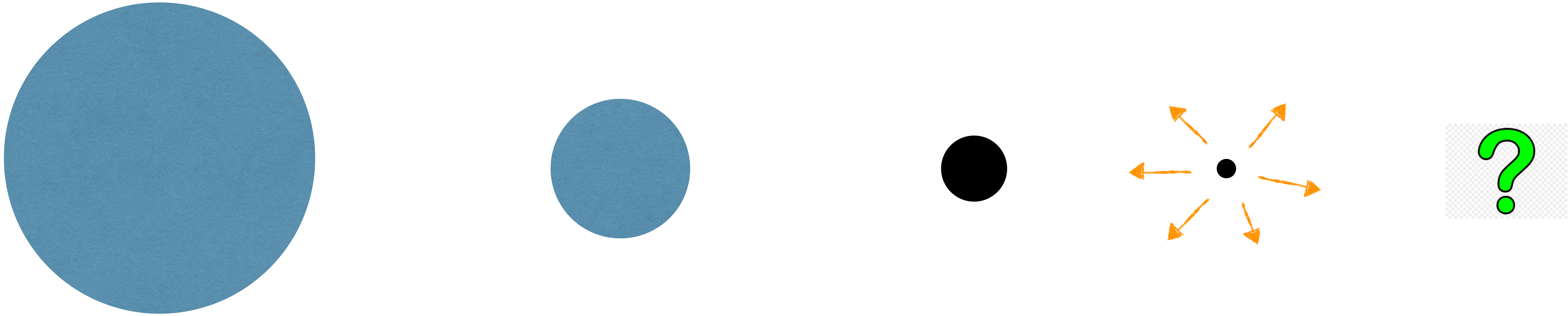
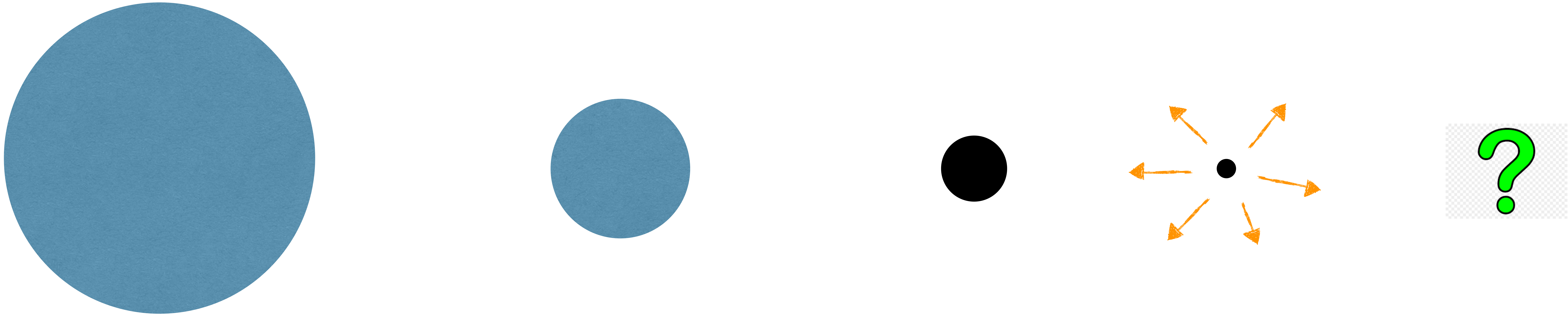
Study what happens in the distant future to a large cloud of matter

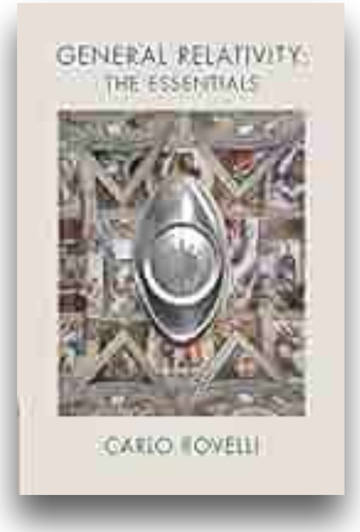


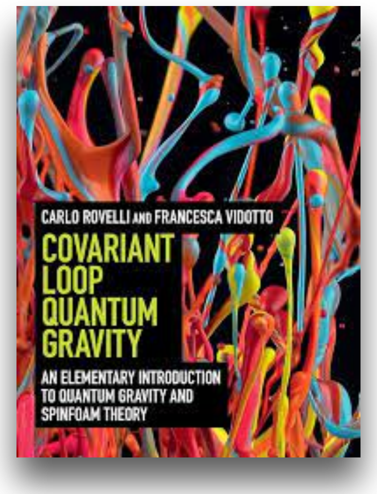
Study what happens in the distant future to a large cloud of matter



Using: $R_{ab} - \frac{1}{2} R\, g_{ab} = 8\pi G T_{ab}$ $\quad \ddot{x}^a - \Gamma^a_{bc}\dot{x}^b\dot{x}^c = 0$

and: $\mathcal{H}_\Gamma = L_2[SU2^L/SU2^N]$ $\quad W(\psi) = (P_{SL2C} Y_\gamma \psi)\,(\mathbb{1})$





Using:

$$R_{ab} - \frac{1}{2} R\, g_{ab} = 8\pi G T_{ab}$$

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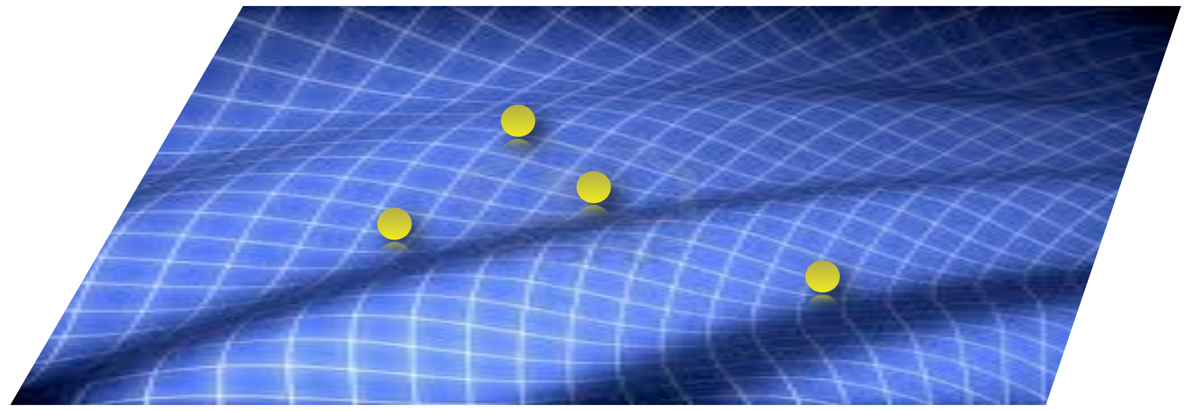
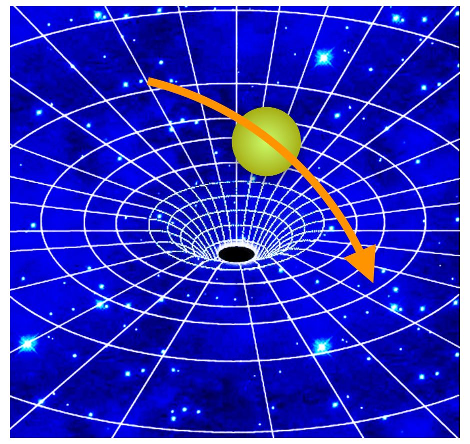


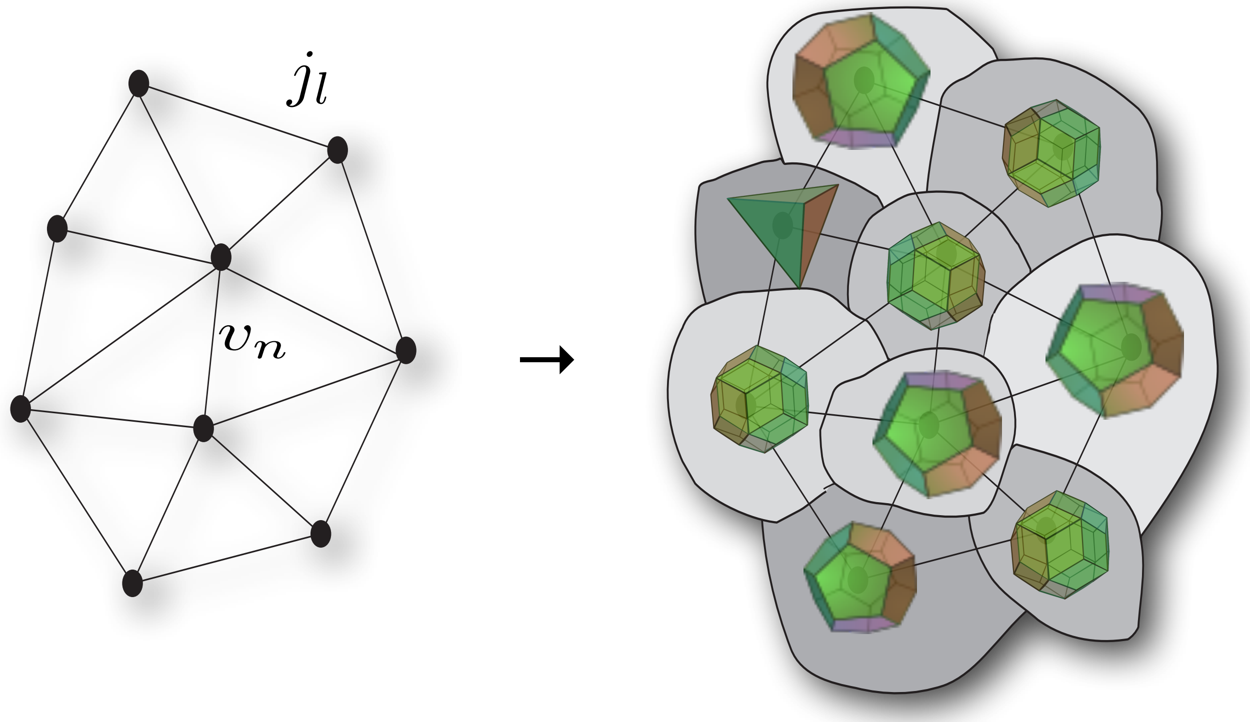


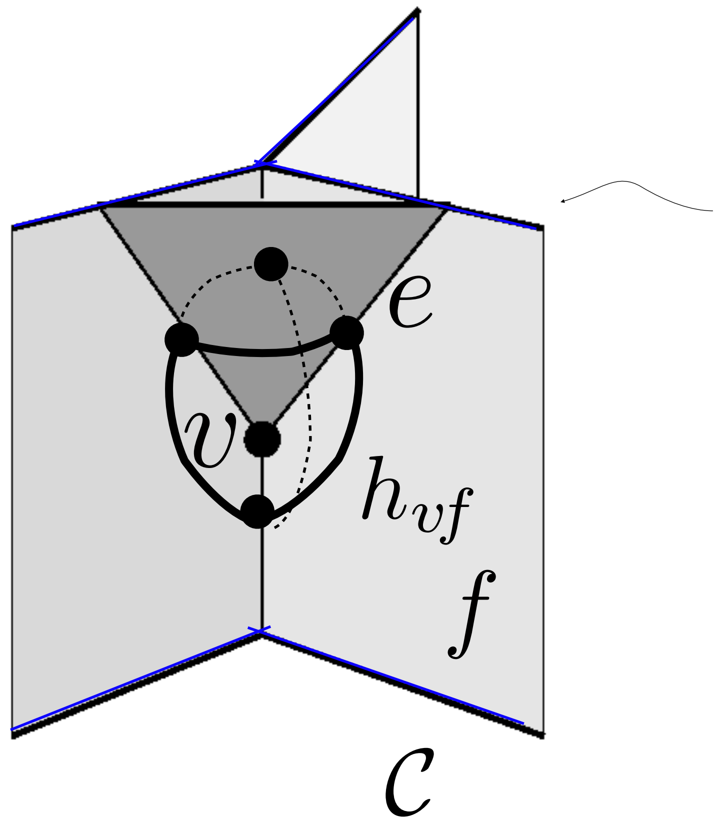
GENERAL RELATIVITY
THE ESSENTIALS
CARLO ROVELLI

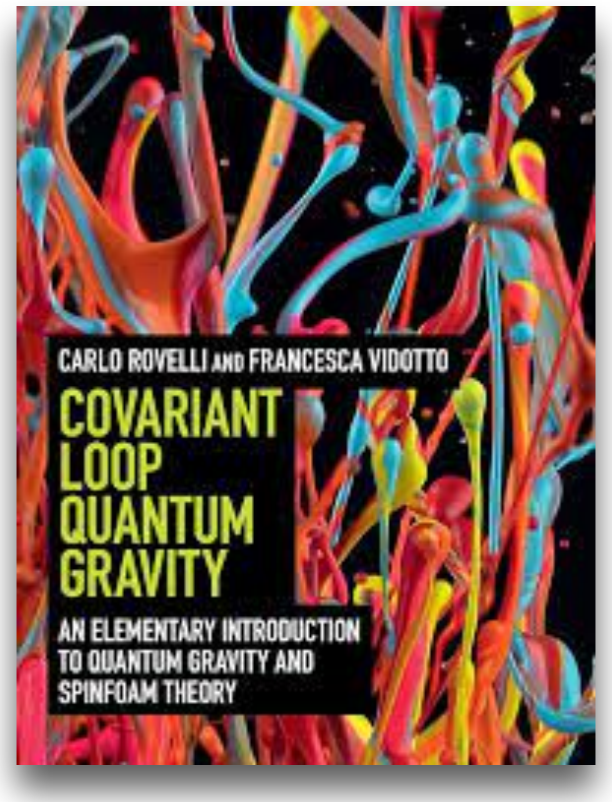
and:

$$H_n = L_2[SU2^{n(n-1)/2}/SU2^n]$$

$$W(\psi) = (P_{SL2C} Y_\gamma \psi)\,(\mathbb{1})$$



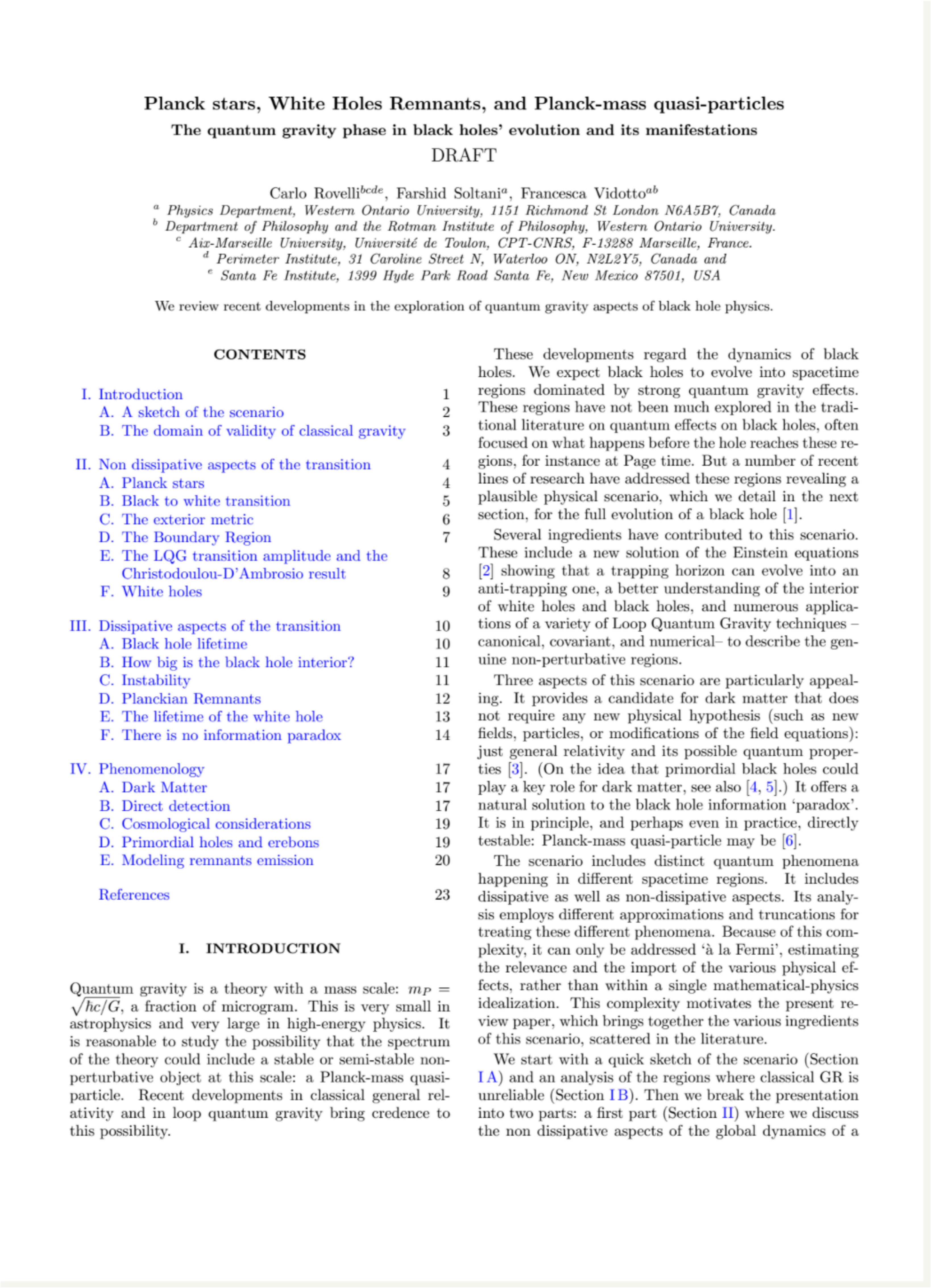




CARLO ROVELLI AND FRANCESCA VIDOTTO
COVARIANT LOOP QUANTUM GRAVITY
AN ELEMENTARY INTRODUCTION TO QUANTUM GRAVITY AND SPINFOAM THEORY

Lecture notes (review paper) in dropbox

https://rb.gy/q23joa



Planck stars, White Holes Remnants, and Planck-mass quasi-particles

The quantum gravity phase in black holes' evolution and its manifestations

DRAFT

Carlo Rovelli[bcde], Farshid Soltani[a], Francesca Vidotto[ab]
[a] *Physics Department, Western Ontario University, 1151 Richmond St London N6A5B7, Canada*
[b] *Department of Philosophy and the Rotman Institute of Philosophy, Western Ontario University.*
[c] *Aix-Marseille University, Université de Toulon, CPT-CNRS, F-13288 Marseille, France.*
[d] *Perimeter Institute, 31 Caroline Street N, Waterloo ON, N2L2Y5, Canada and*
[e] *Santa Fe Institute, 1399 Hyde Park Road Santa Fe, New Mexico 87501, USA*

We review recent developments in the exploration of quantum gravity aspects of black hole physics.

CONTENTS

I. INTRODUCTION

Quantum gravity is a theory with a mass scale: $m_P = \sqrt{\hbar c/G}$, a fraction of microgram. This is very small in astrophysics and very large in high-energy physics. It is reasonable to study the possibility that the spectrum of the theory could include a stable or semi-stable non-perturbative object at this scale: a Planck-mass quasi-particle. Recent developments in classical general relativity and in loop quantum gravity bring credence to this possibility.

These developments regard the dynamics of black holes. We expect black holes to evolve into spacetime regions dominated by strong quantum gravity effects. These regions have not been much explored in the traditional literature on quantum effects on black holes, often focused on what happens before the hole reaches these regions, for instance at Page time. But a number of recent lines of research have addressed these regions revealing a plausible physical scenario, which we detail in the next section, for the full evolution of a black hole [1].

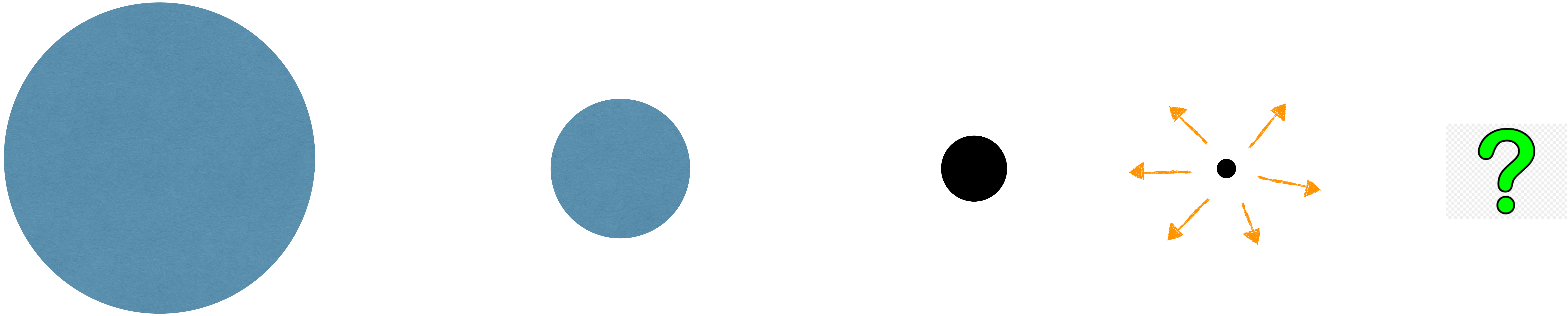
Several ingredients have contributed to this scenario. These include a new solution of the Einstein equations [2] showing that a trapping horizon can evolve into an anti-trapping one, a better understanding of the interior of white holes and black holes, and numerous applications of a variety of Loop Quantum Gravity techniques – canonical, covariant, and numerical– to describe the genuine non-perturbative regions.

Three aspects of this scenario are particularly appealing. It provides a candidate for dark matter that does not require any new physical hypothesis (such as new fields, particles, or modifications of the field equations): just general relativity and its possible quantum properties [3]. (On the idea that primordial black holes could play a key role for dark matter, see also [4, 5].) It offers a natural solution to the black hole information 'paradox'. It is in principle, and perhaps even in practice, directly testable: Planck-mass quasi-particle may be [6].

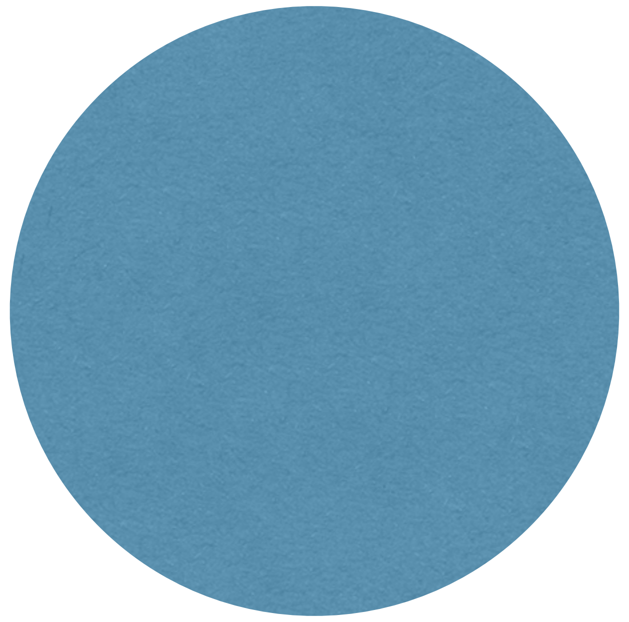
The scenario includes distinct quantum phenomena happening in different spacetime regions. It includes dissipative as well as non-dissipative aspects. Its analysis employs different approximations and truncations for treating these different phenomena. Because of this complexity, it can only be addressed 'à la Fermi', estimating the relevance and the import of the various physical effects, rather than within a single mathematical-physics idealization. This complexity motivates the present review paper, which brings together the various ingredients of this scenario, scattered in the literature.

We start with a quick sketch of the scenario (Section I A) and an analysis of the regions where classical GR is unreliable (Section I B). Then we break the presentation into two parts: a first part (Section II) where we discuss the non dissipative aspects of the global dynamics of a

Study what happens in the distant future to a large cloud of matter



I. At first, ignore dissipative effects



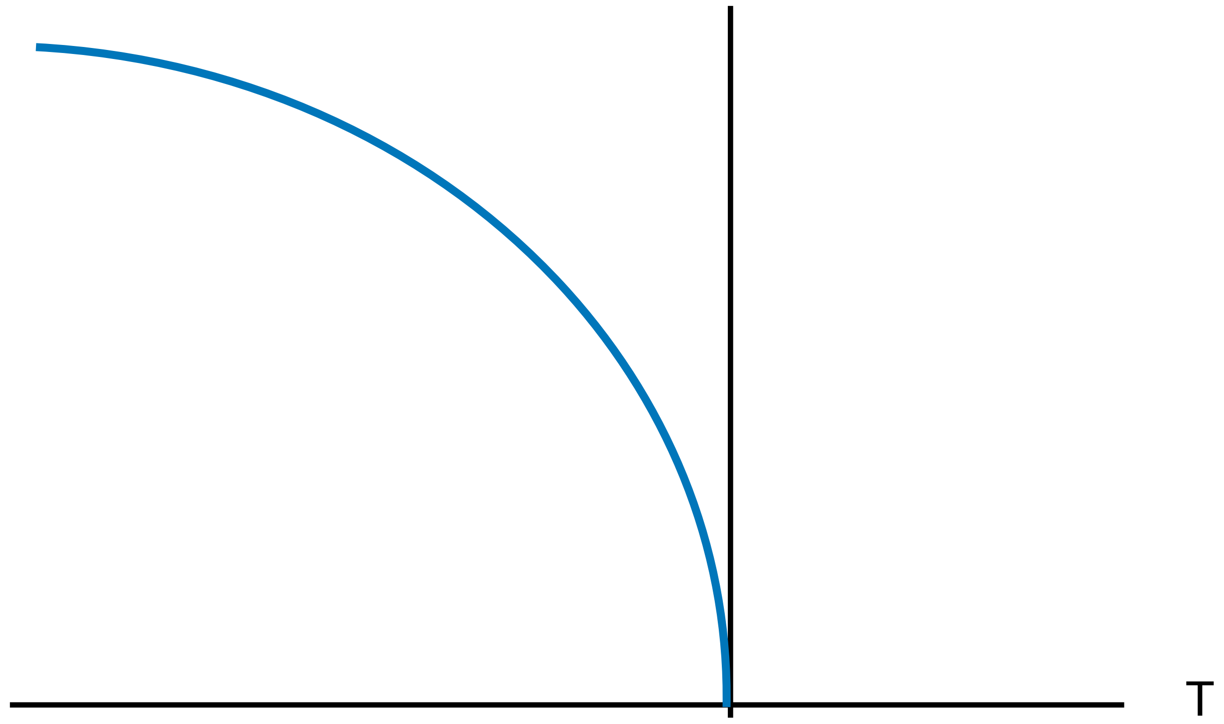
Homogeneous, isotropic, pressure-less.

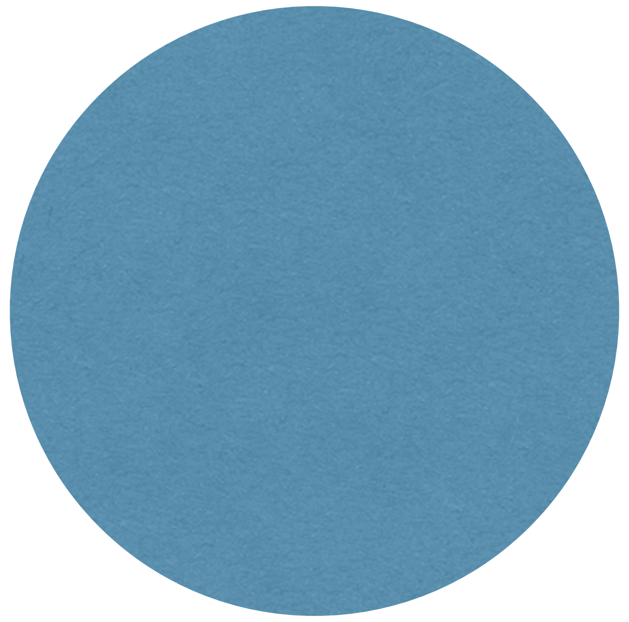
$$ds^2 = -dT^2 + a^2(T)(dR^2 + R^2 d\Omega^2)$$

$$R_{boundary} = 1$$
$$r_b = a(T)R_{boundary}$$

$$\rho = \frac{m}{\frac{4}{3}\pi a^3} \qquad \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho$$

$$a(T) = \left(\frac{9}{2}mT^2\right)^{1/3}$$





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$$a(T) = \left(\frac{9}{2}\, m T^2\right)^{1/3}$$

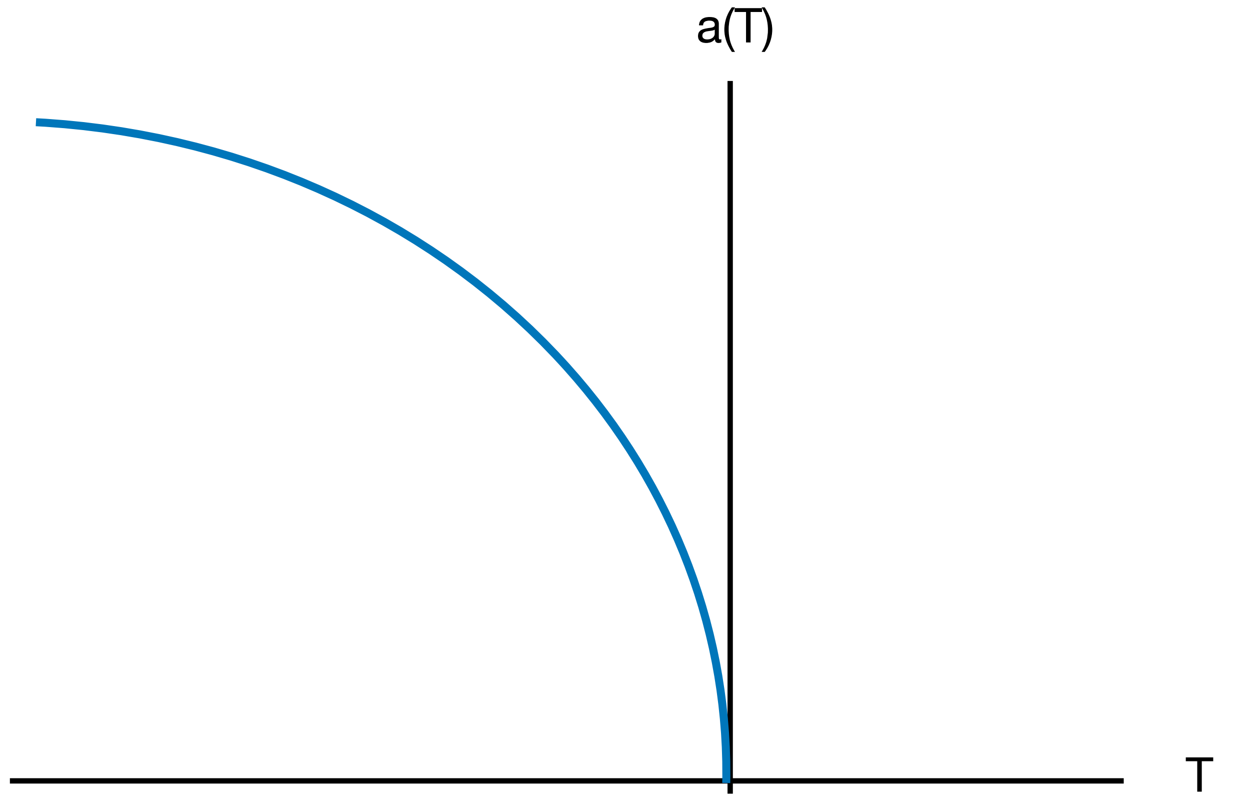
$\hbar \neq 0$

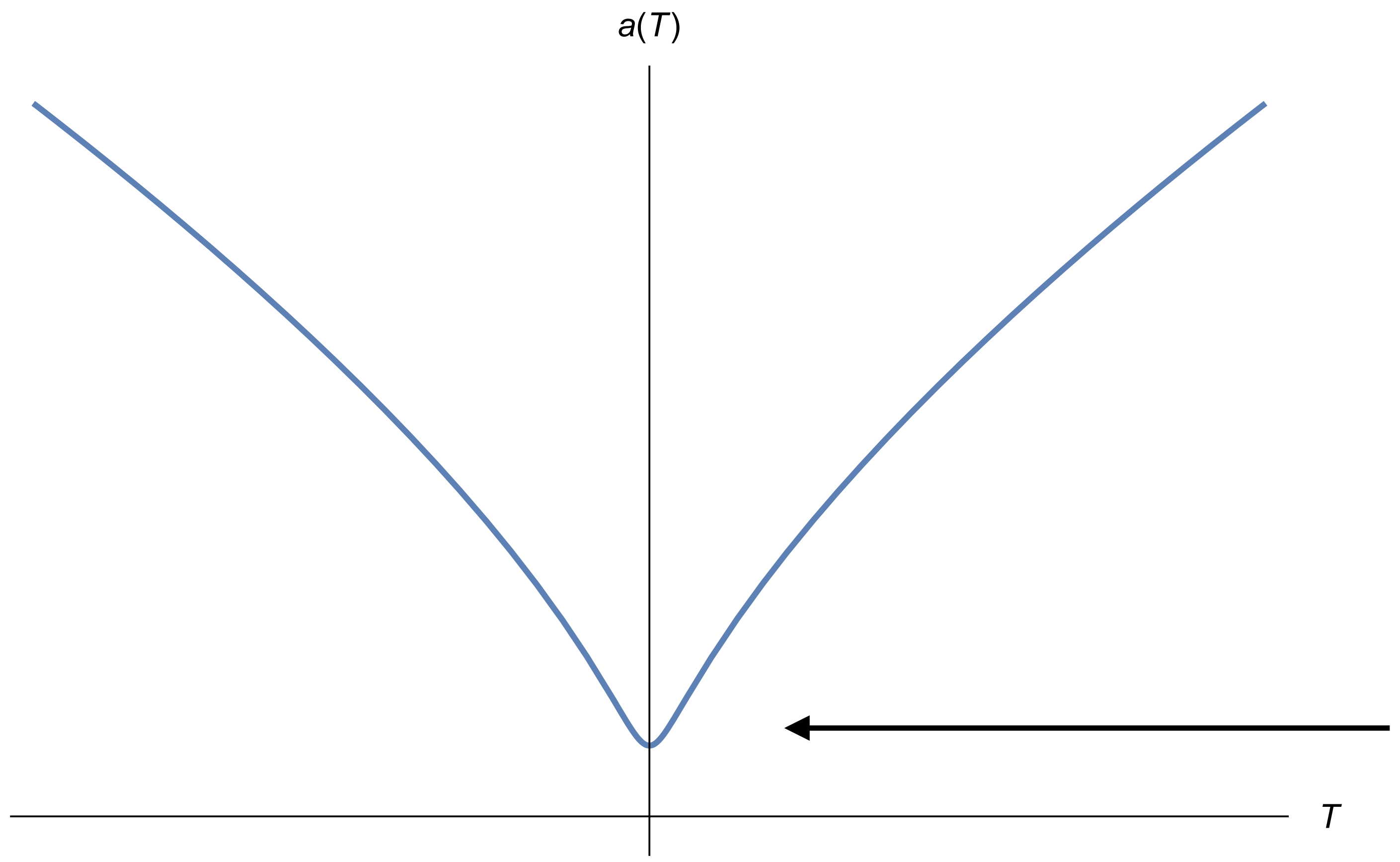
$$\frac{\dot a^2}{a^2} = \frac{8\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$$a(T) = \left(\frac{9}{2}\, m T^2 + Am\right)^{1/3}$$

[Francesca lecture]
[Param Singh lectures]

$$A = 3/(2\pi\rho_c) \sim \hbar \sim m_{Pl}^2$$





Planck star

Vidotto, CR, *Planck Stars*, IJMP 2014. arXiv:1401.6562.

What about *outside* the star?

Birkhoff's theorem (classical GR): that any spherically symmetric solution of the vacuum field equations must be given by the Schwarzschild metric.

$$ds^2 = -F(r)dt^2 + dr^2/F(r) + r^2 d\Omega^2 \qquad F(r) = 1 - 2m/r$$

How does the boundary of the star fall?

From inside:

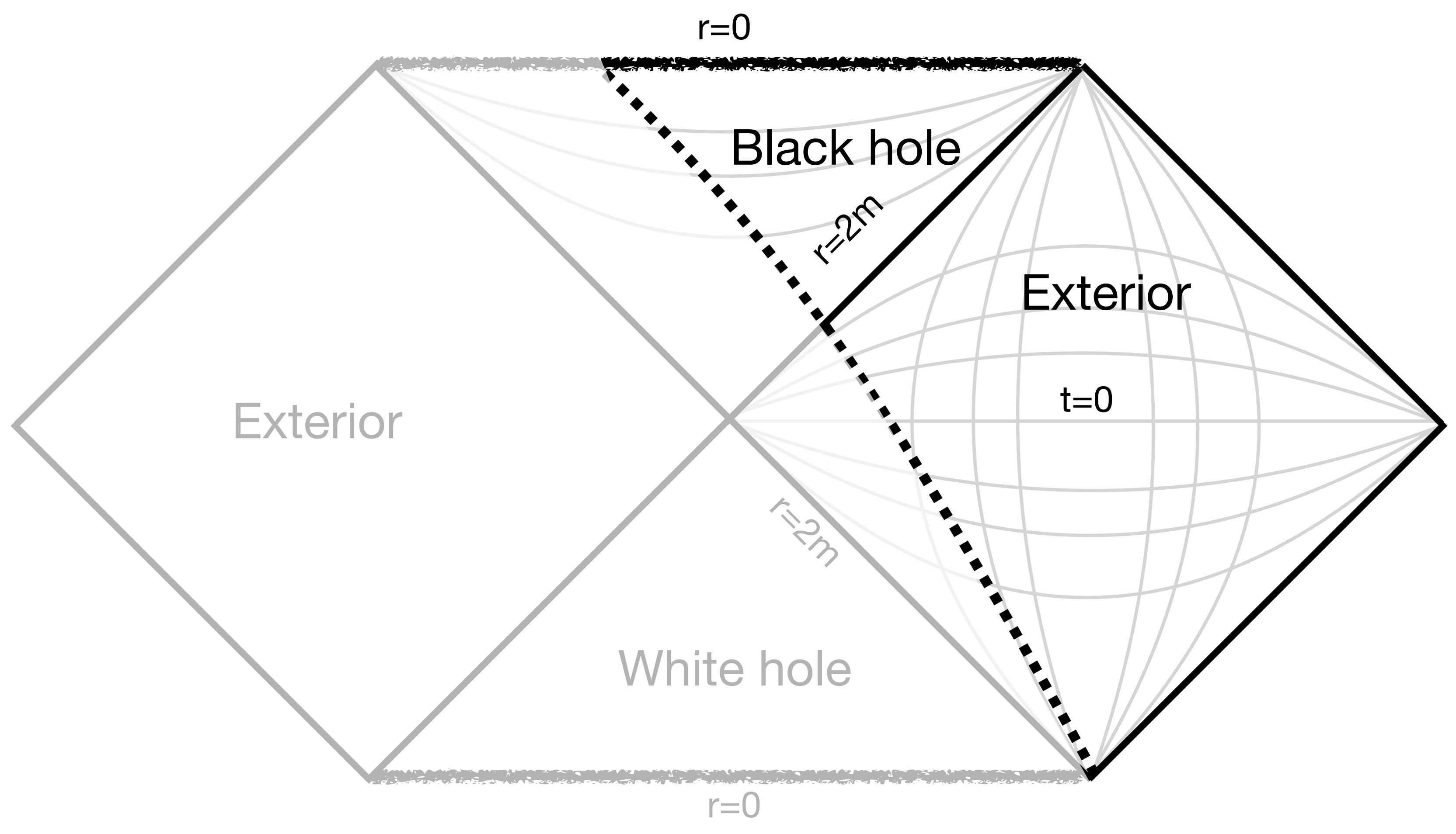
$$\dot{a}^2 = \frac{2m}{a} \qquad \frac{\dot{r}_b^2}{2} - \frac{m}{r_b} = 0$$

Exactly as in Newton!

From outside:

$$-1 = -F(r)\dot{t}^2 + \dot{r}^2/F(r) \qquad F(r)\dot{t} = E$$

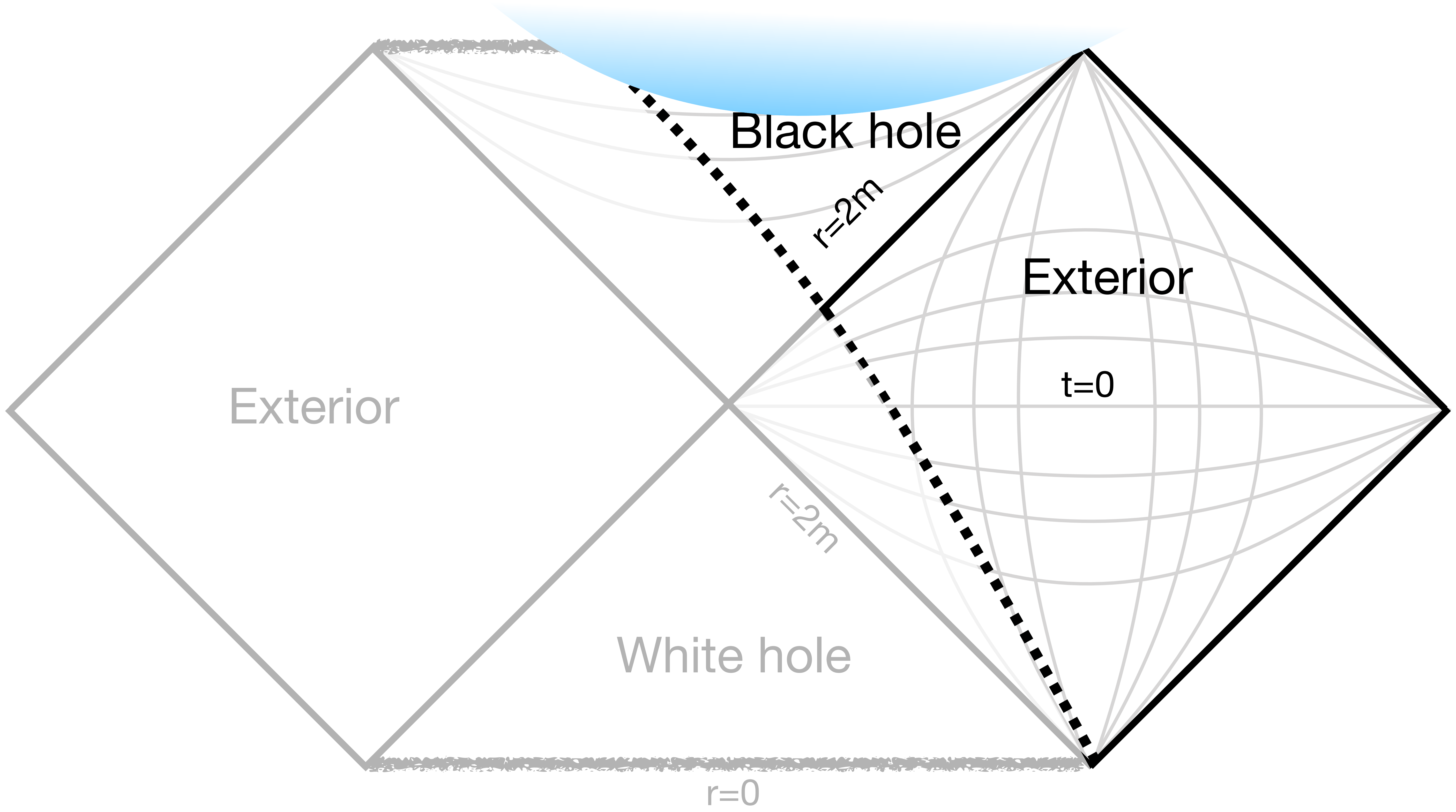
$$\frac{\dot{r}_b^2}{2} - \frac{m}{r_b} = 0$$



Oppenheimer Snyder 1939

$\hbar \neq 0$

$$\mathcal{R} \sim \frac{m}{r^3}$$



$$ds^2 = -F(r)dt^2 + dr^2/F(r) + r^2 d\Omega^2 \qquad F(r) = 1 - 2m/r$$

From inside: $$\dot{a}^2 = \frac{2m}{a} \xrightarrow[\hbar \neq 0]{} \frac{1}{2}\dot{a}^2 = \frac{m}{a} - \frac{Am^2}{2a^4}$$

From outside: $$F(r) = 1 - 2m/r \xrightarrow[\hbar \neq 0]{} F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

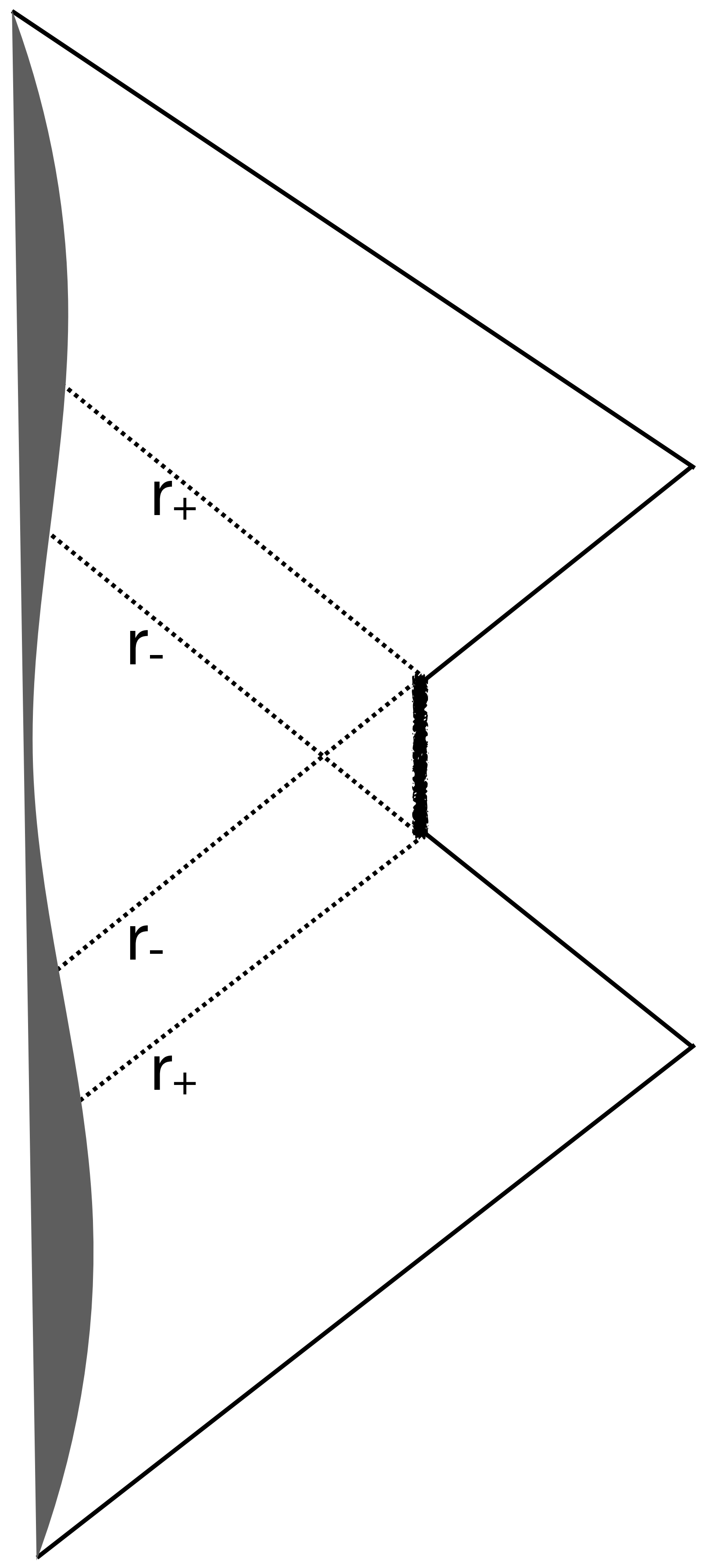
- Killing symmetry respected,
- Matching conditions with the star respected,
- Same metric as the obtained from modifying quantum dynamics!

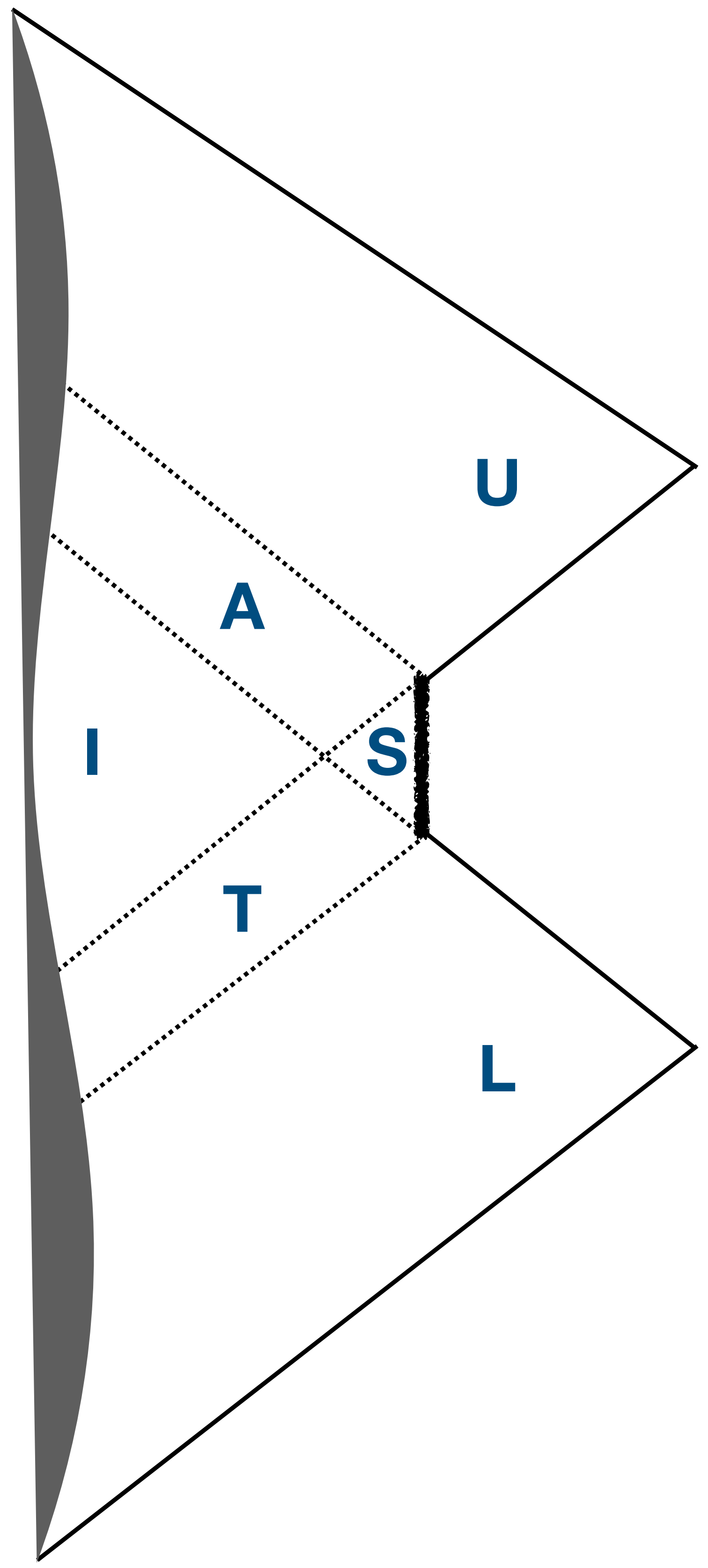
Lewandowski, Ma, Yang, Zhang, PRL 2023

Husain, Kelly, Santacruz, Wilson-Ewing, PRD 2022.

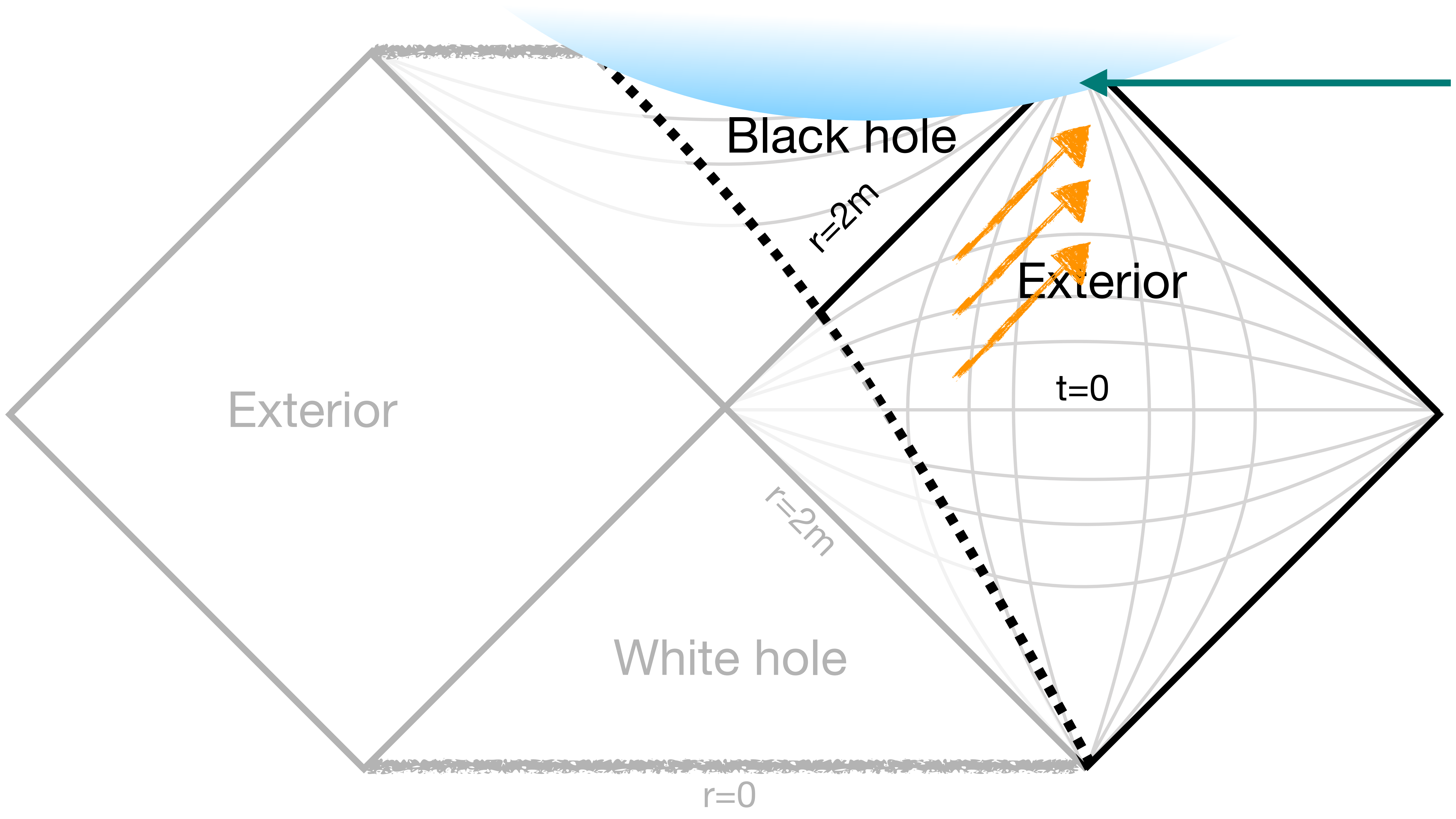
$$ds^2 = -F(r)dt^2 + dr^2/F(r) + r^2 d\Omega^2$$

$$F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$



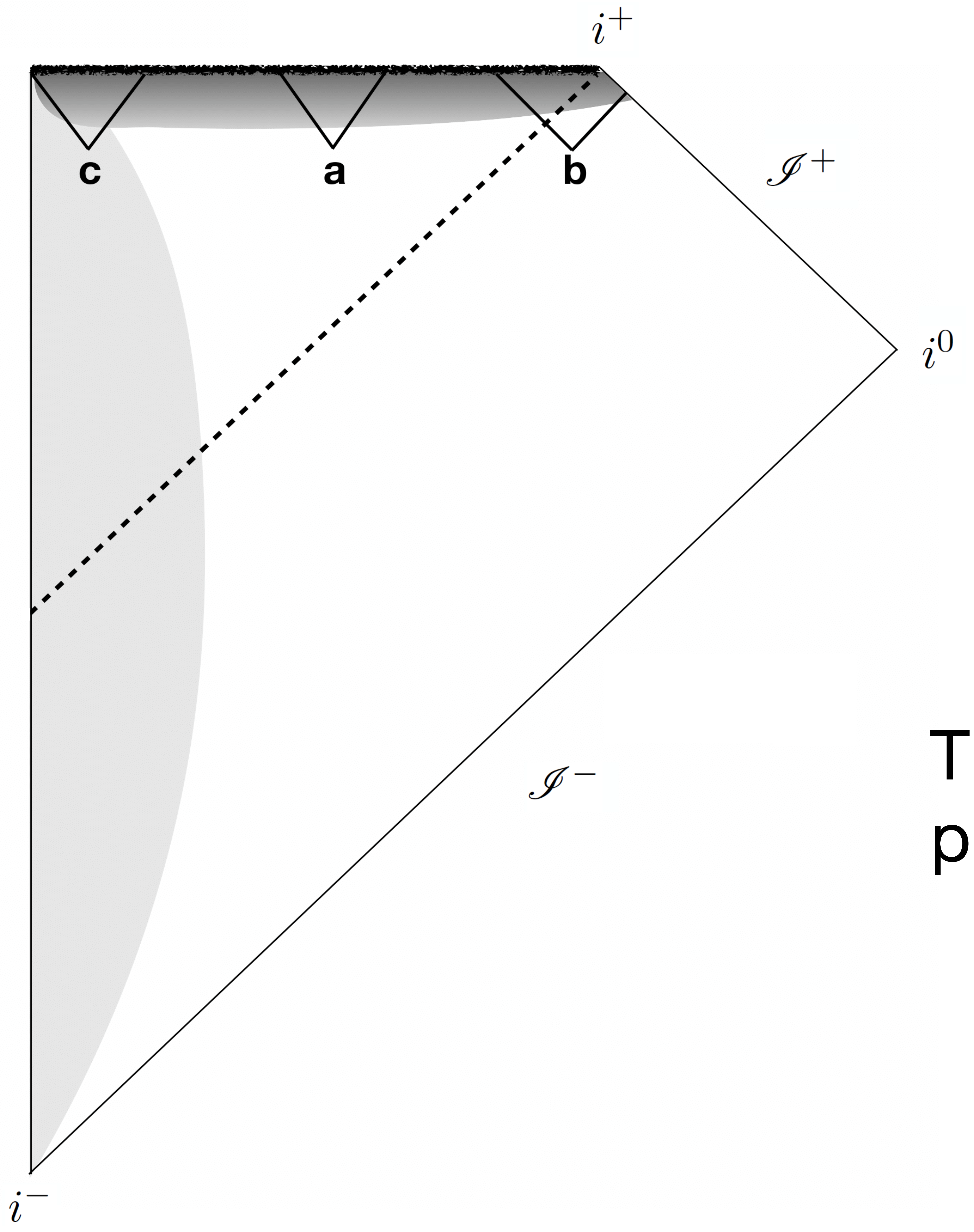


Hawking radiation



$$\mathcal{R} \sim \frac{m}{r^3} \sim \frac{1}{r^2}$$

Outside
the trapping horizon,
the curvature becomes
Planckian !



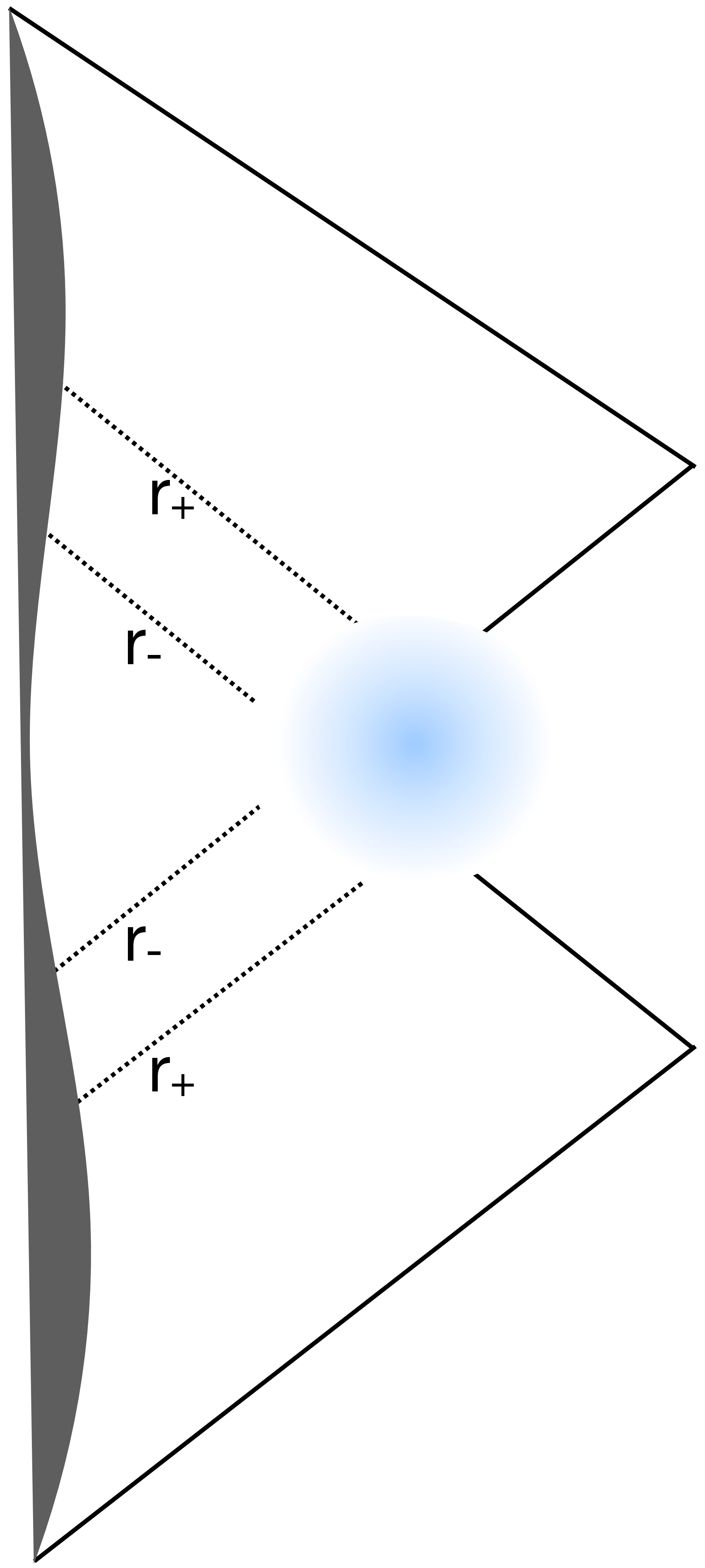
There are three independent physical phenomena happening

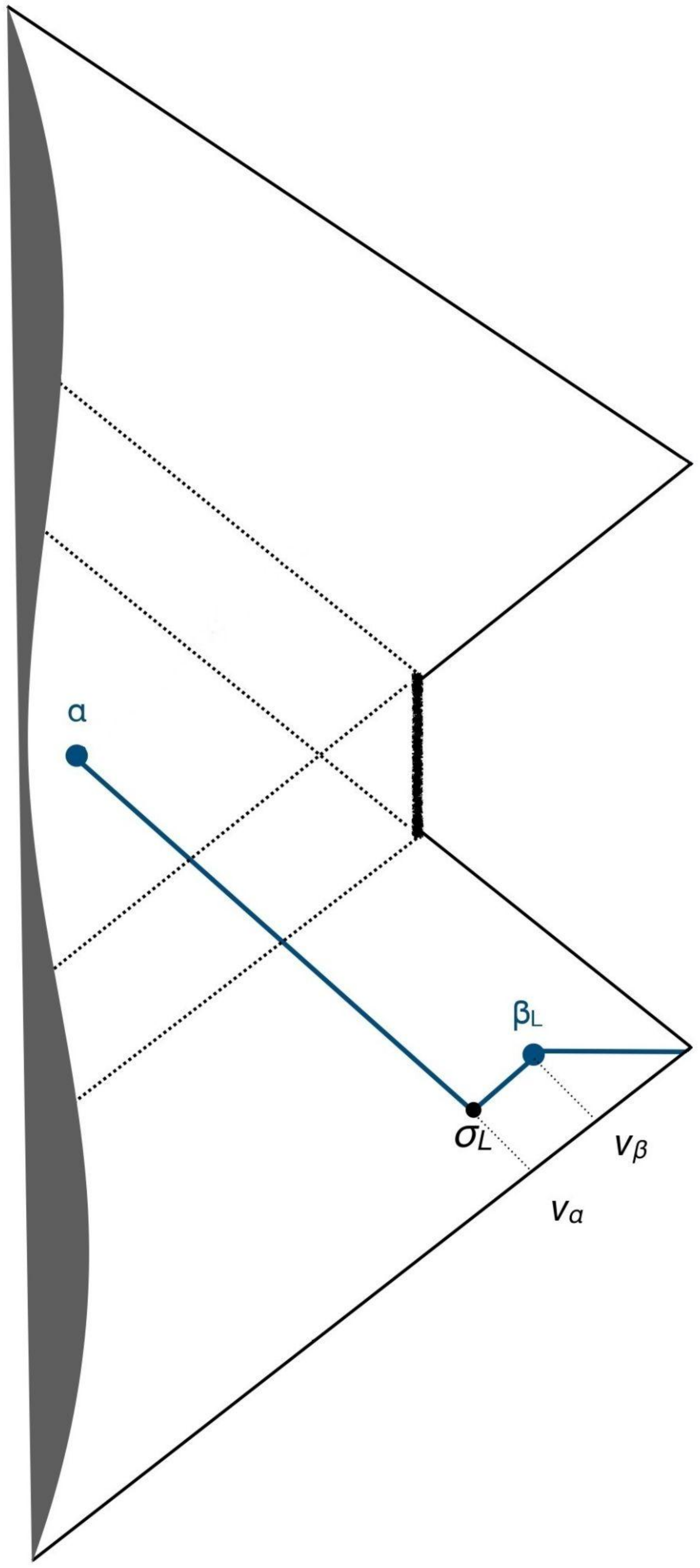
$$ds^2 = -F(r)dt^2 + dr^2/F(r) + r^2 d\Omega^2$$

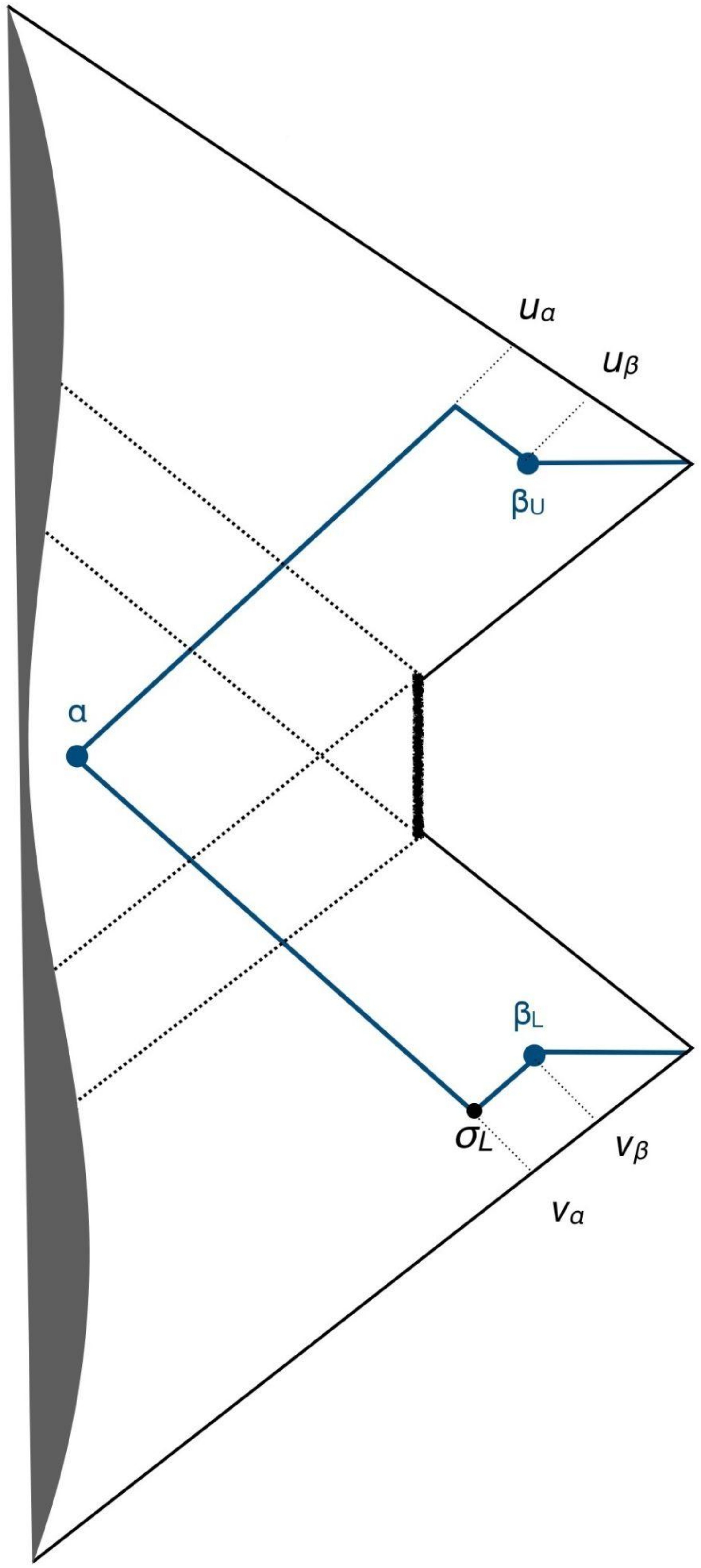
$$F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

A Rignon-Bret, CR, PRD 2022, arXiv:2108.12823.

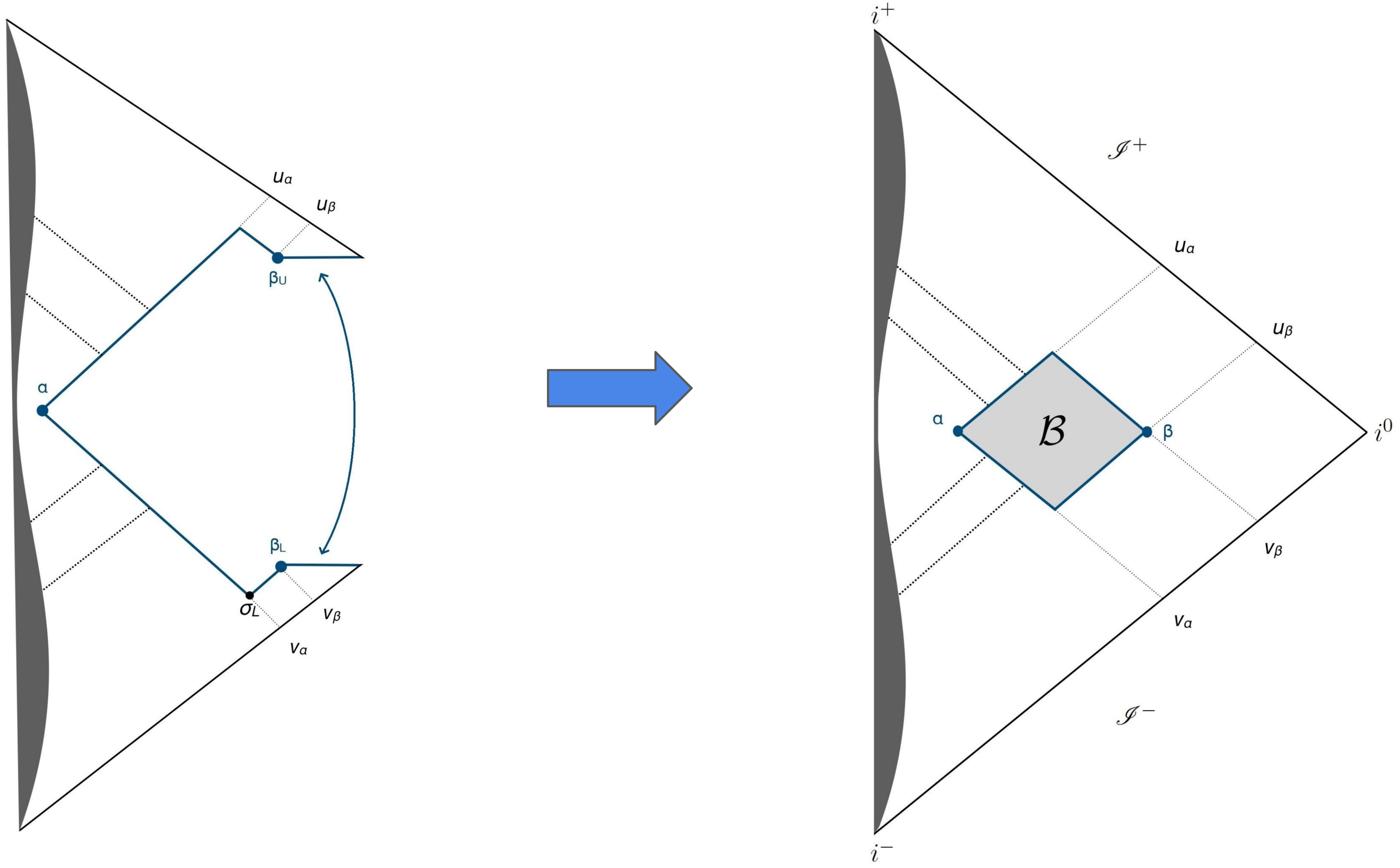
M Han, CR, F. Soltani, PRD (2023), arXiv:2302.03872.







M Han, CR, F. Soltani, PRD (2023), arXiv:2302.03872.


u_α
u_β
β_U
α
β_L
σ_L
v_β
v_α
i^+
$\mathscr{I}^+$
u_α
u_β
α
$\mathcal{B}$
β
i^0
v_β
v_α
$\mathscr{I}^-$
i^-

Global coordinates (diverge on horizons)

$$ds^2 = F(r(u,v))dudv + r^2(u,v)d\Omega^2.$$

$$F(r) = 1 - \frac{2m}{r} + \frac{Am^2}{r^4}$$

$$2r_*(r) = v + u \qquad dr_* = \frac{dr}{F(r)}$$

Good coordinates for L and I

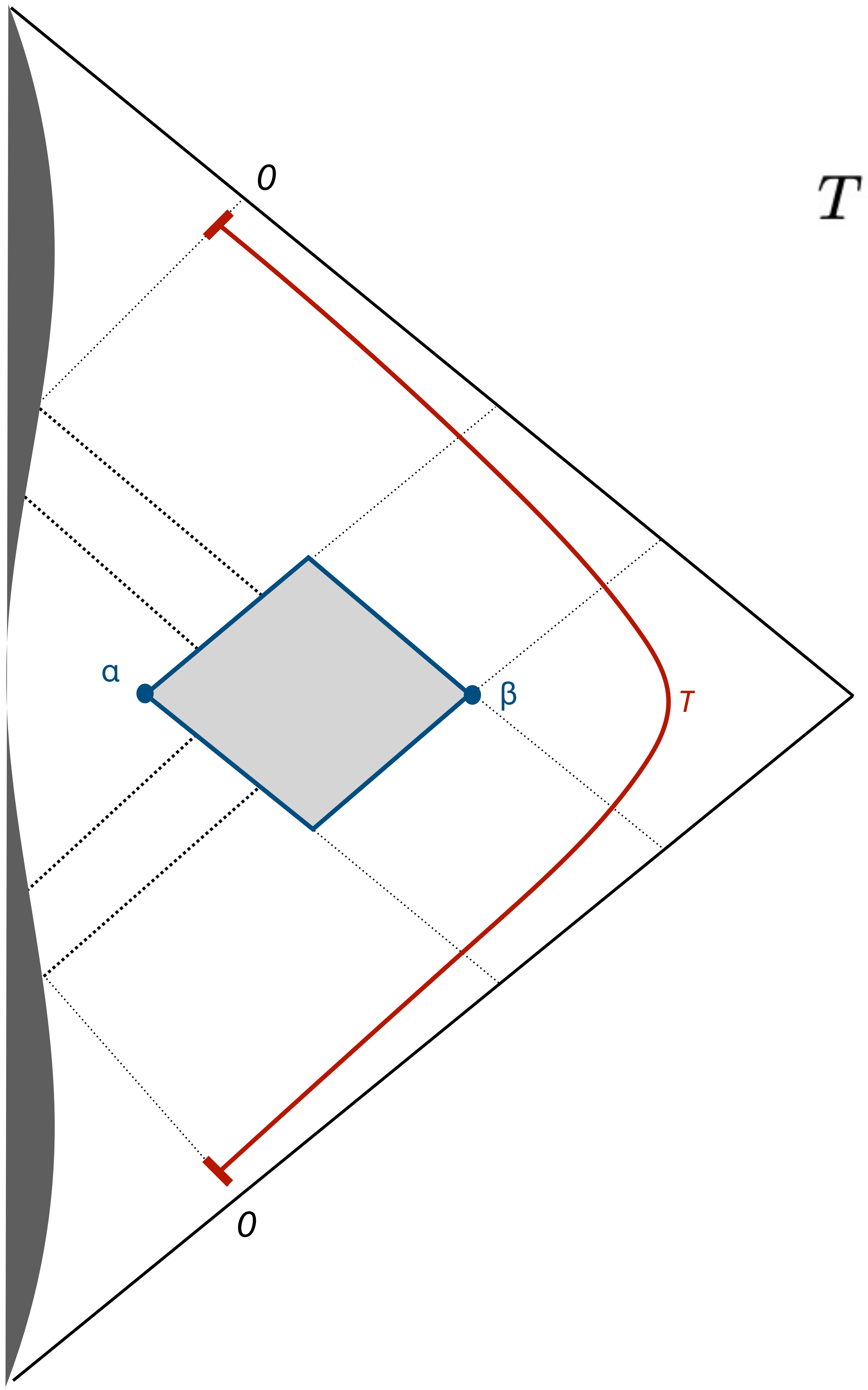
$$ds^2 = -F(r)dv^2 + 2dvdr + r^2d\Omega^2$$

Good coordinates for T and I

$$ds^2 = -F(r)du^2 - 2dudr + r^2d\Omega^2$$

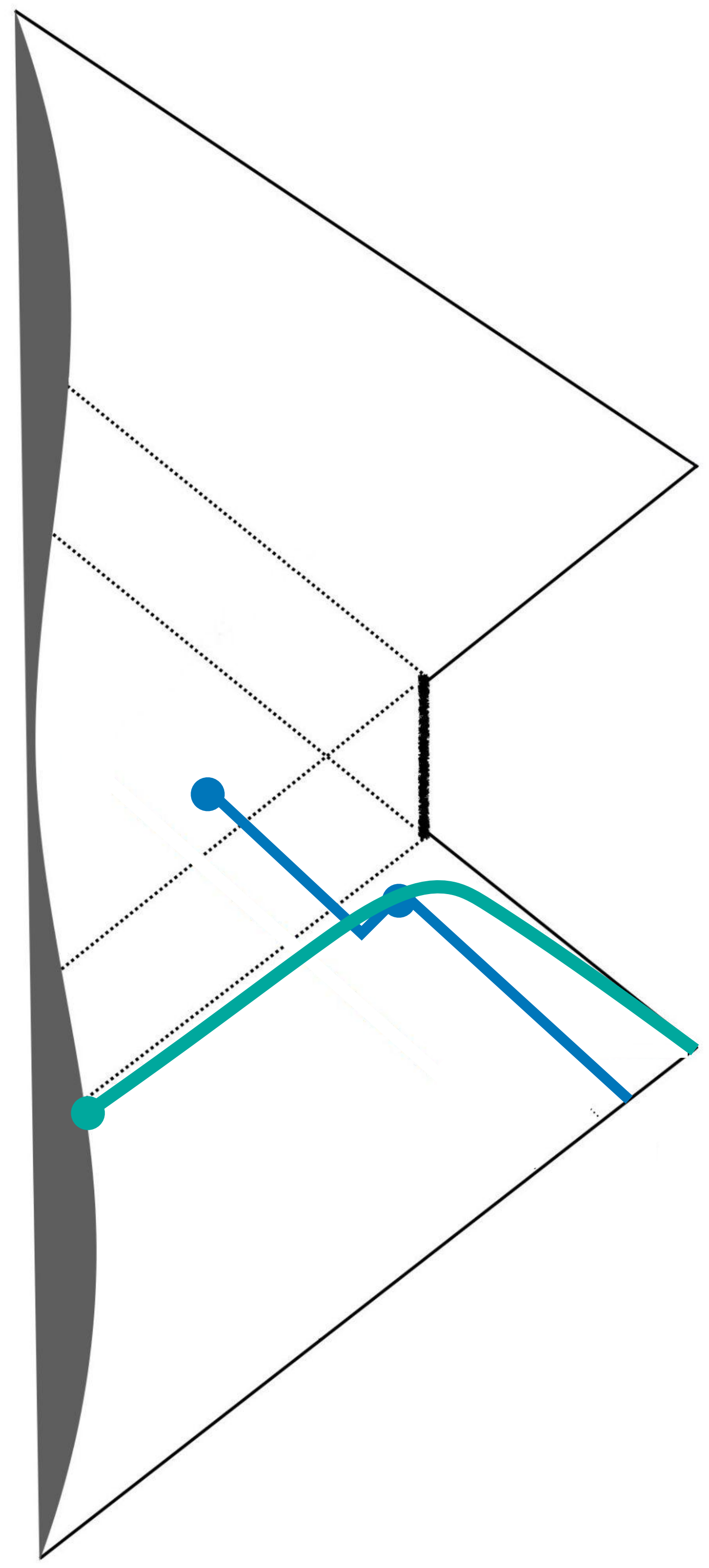
On overlap (I)

$$u = 2r_*(r) - v$$



$$T = 2R + 4m\ln(R-2m) - 4m\ln\delta$$

$$\tau = -4m\ln\delta$$



$$r = 2m(1+\delta)$$

H. Haggard, CR, PRD 2015, arXiv:1407.0989

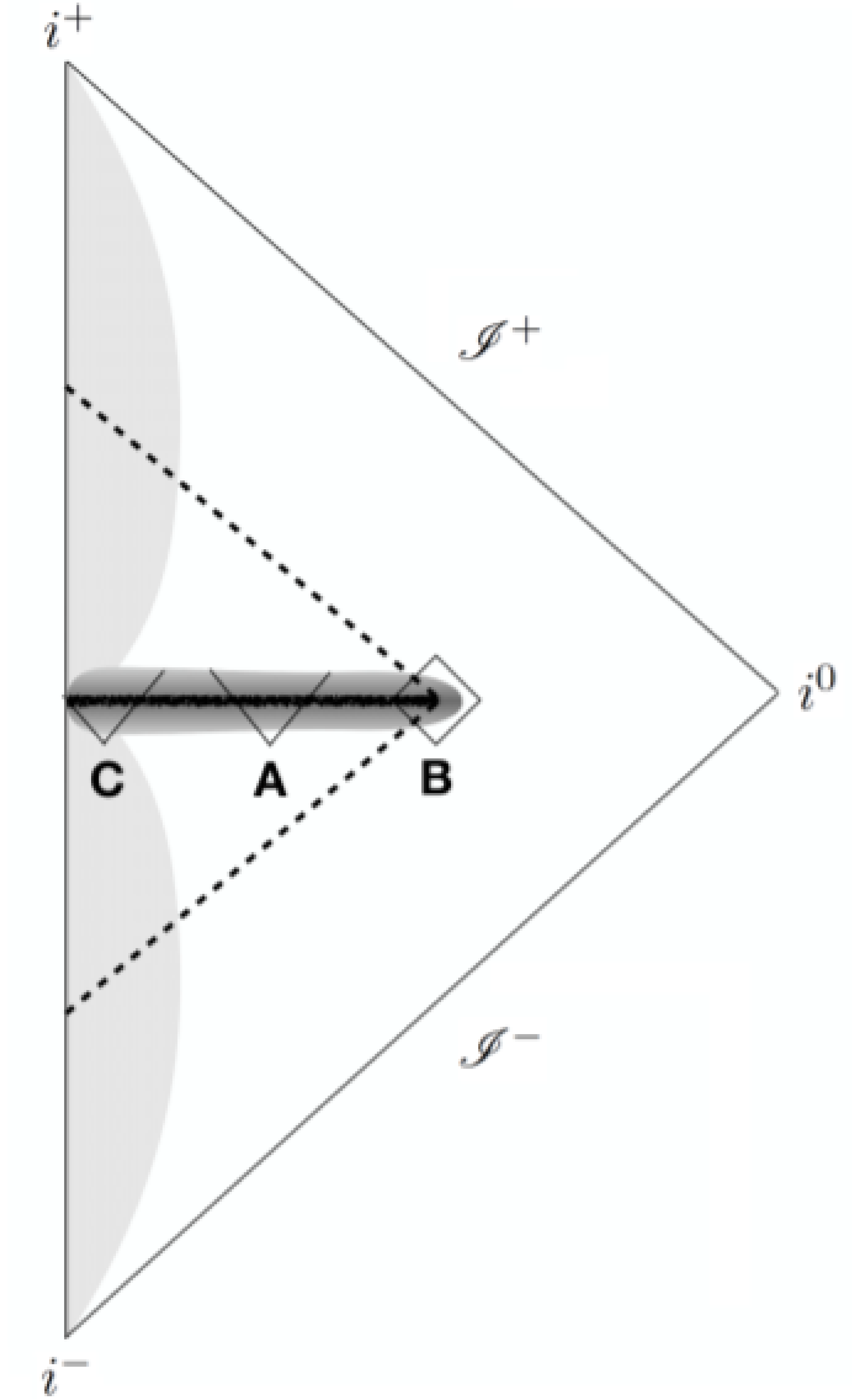
An event horizon is the boundary of the past of future infinity.

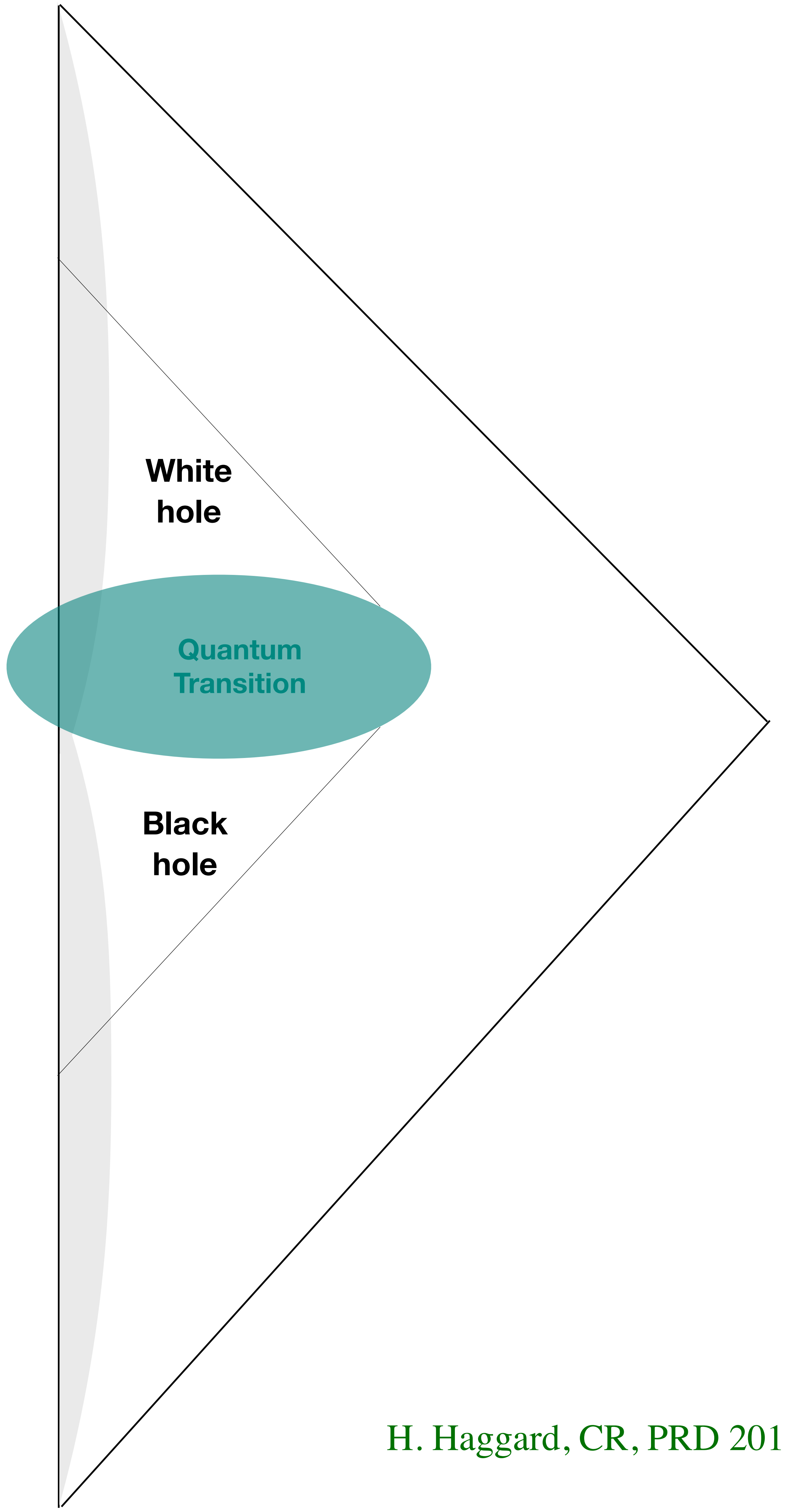
(No light escapes from an event horizon ever.)

A trapping (dynamical) horizon is the boundary of the region where the area of outgoing null surfaces decreases.

(No light escapes from a trapping horizon for a while.)

Real black holes have no event horizon !!

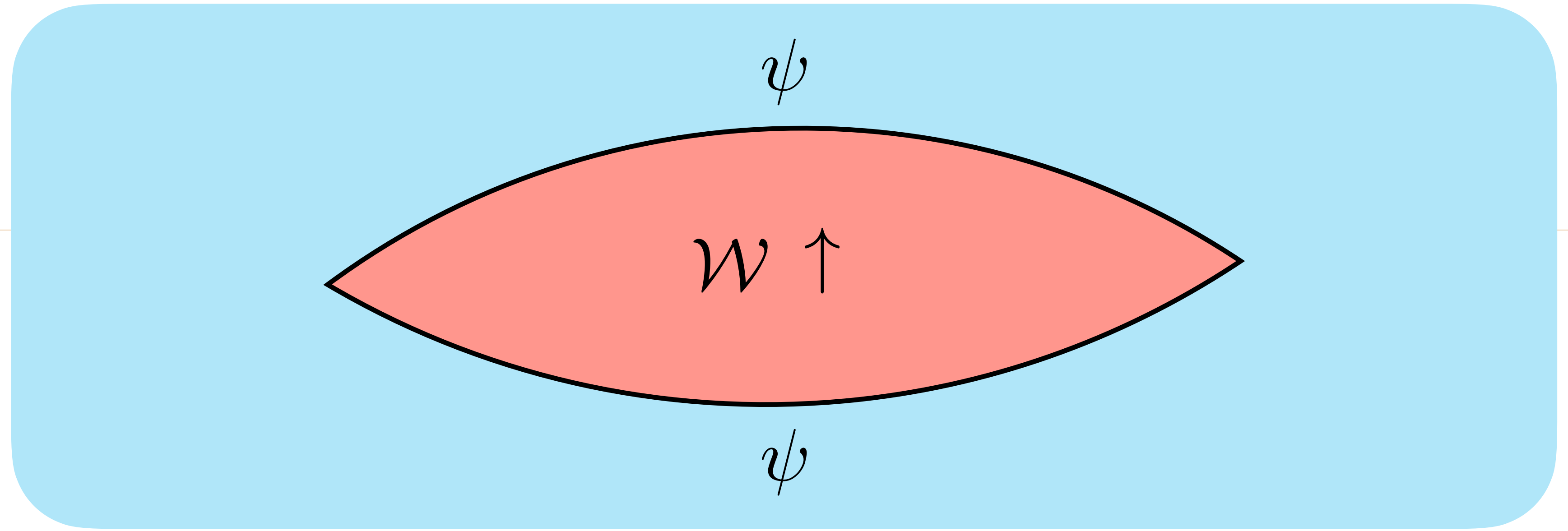




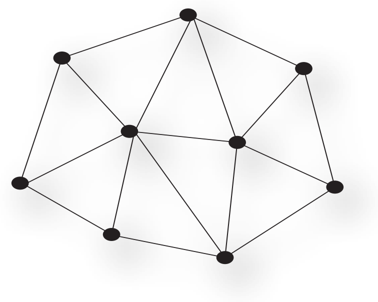
H. Haggard, CR, PRD 2015, arXiv:1407.0989

E. Bianchi, M. Christodoulou, F. D’Ambrosio, H. M. Haggard, CR,
“White holes as remnants: A surprising scenario for the end of a black hole,”
CQG 2018, arXives: 1802.04264.

Structure of the theory

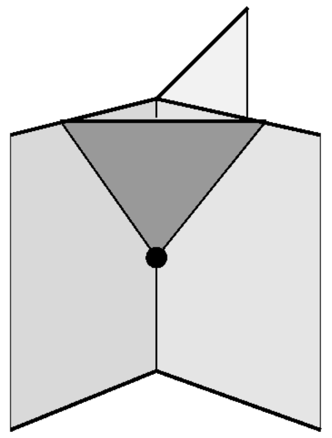


		Truncation
State space	$\mathcal{H} \ni \psi$	$\mathcal{H}_\Gamma$
Operators:	$\vec{L}_l$	$\vec{L}_l$
Transition amplitudes:	$\mathcal{W}$	$\mathcal{W}_\mathcal{C}$



Spin networks

Quantum states of geometry



Spin foams

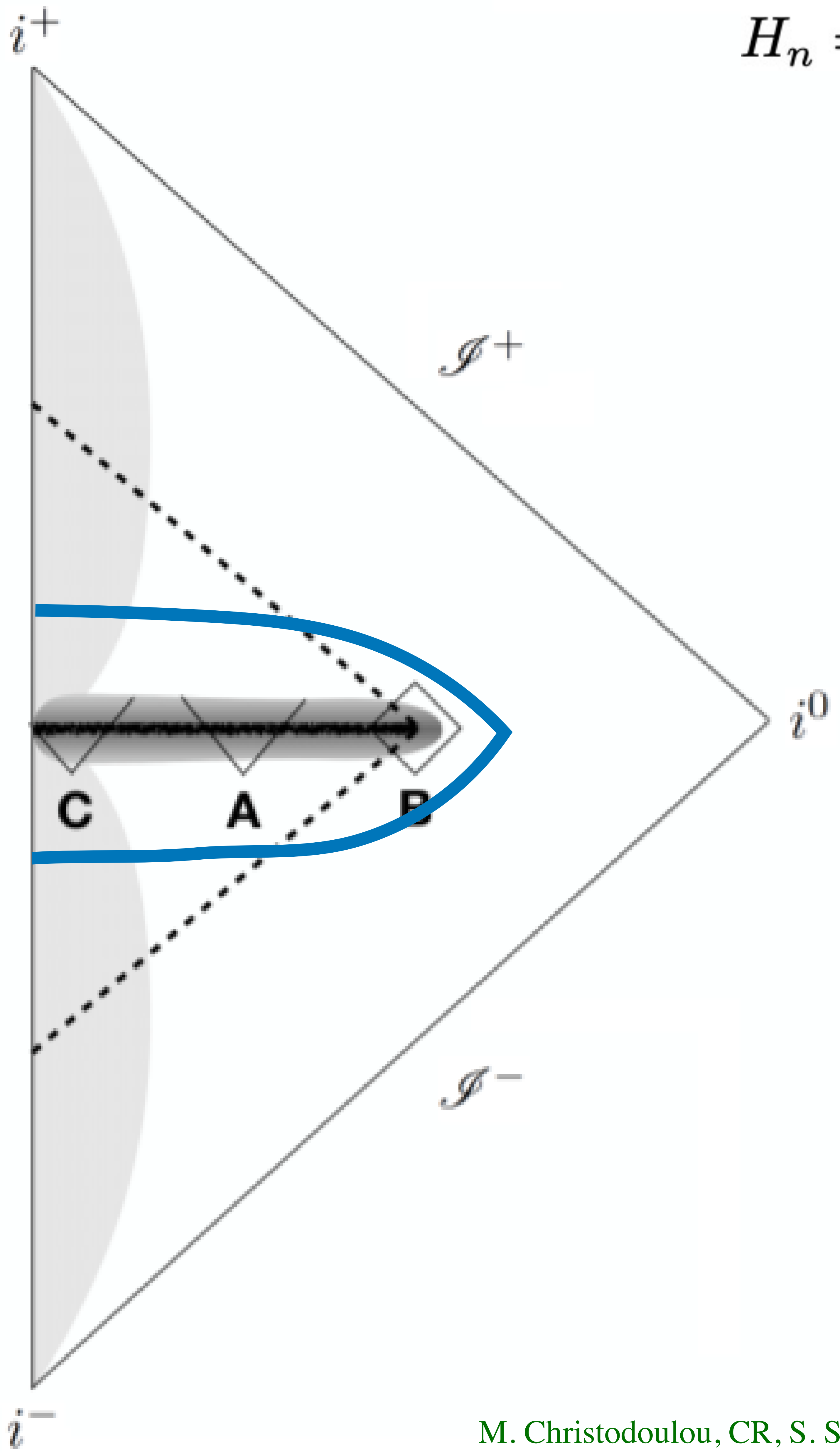
i. Quantum histories of geometries
ii. Discretized spacetime

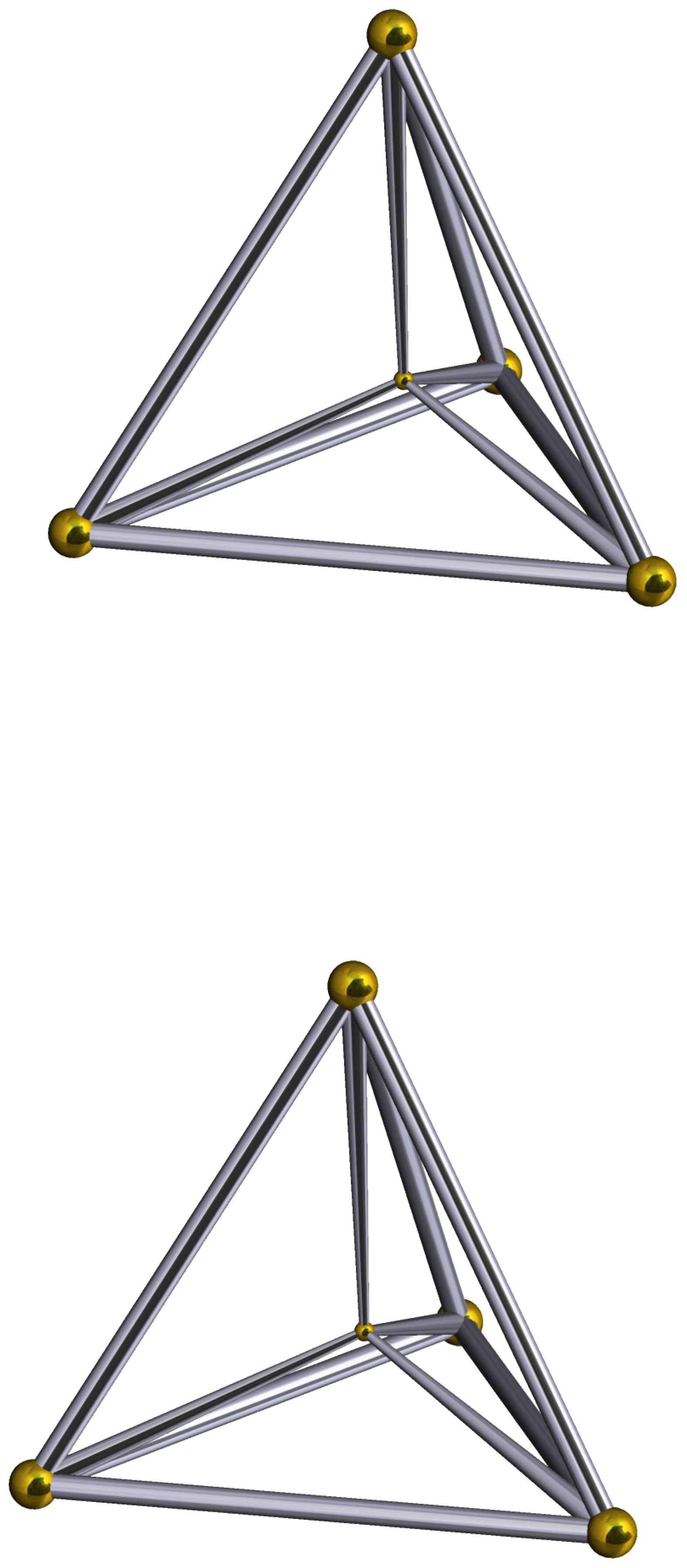
Cfr:

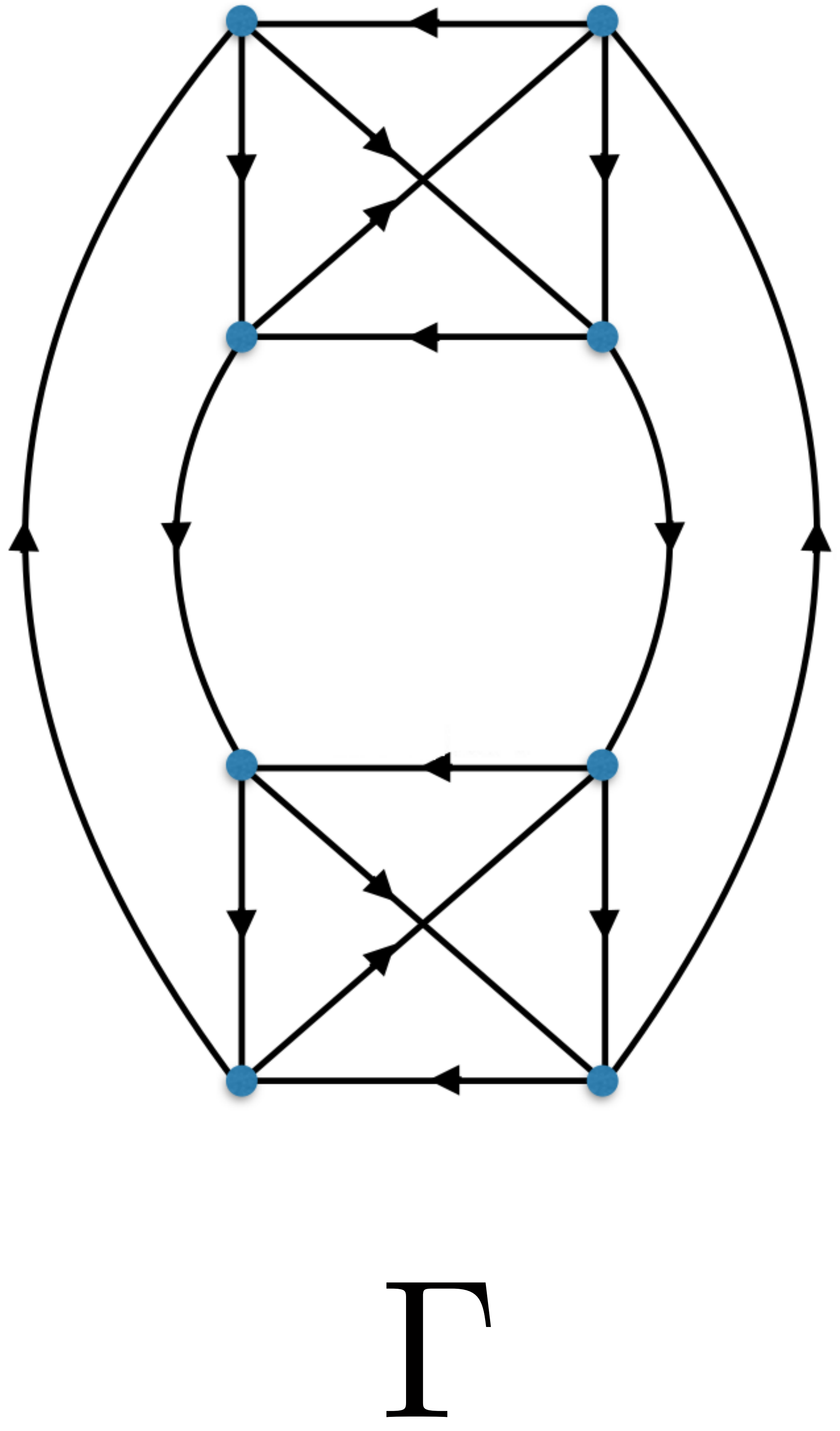
i. Lattice gauge theory
ii. Feynman graph expansion

[Francesca lecture]
[Hongguan lecture]

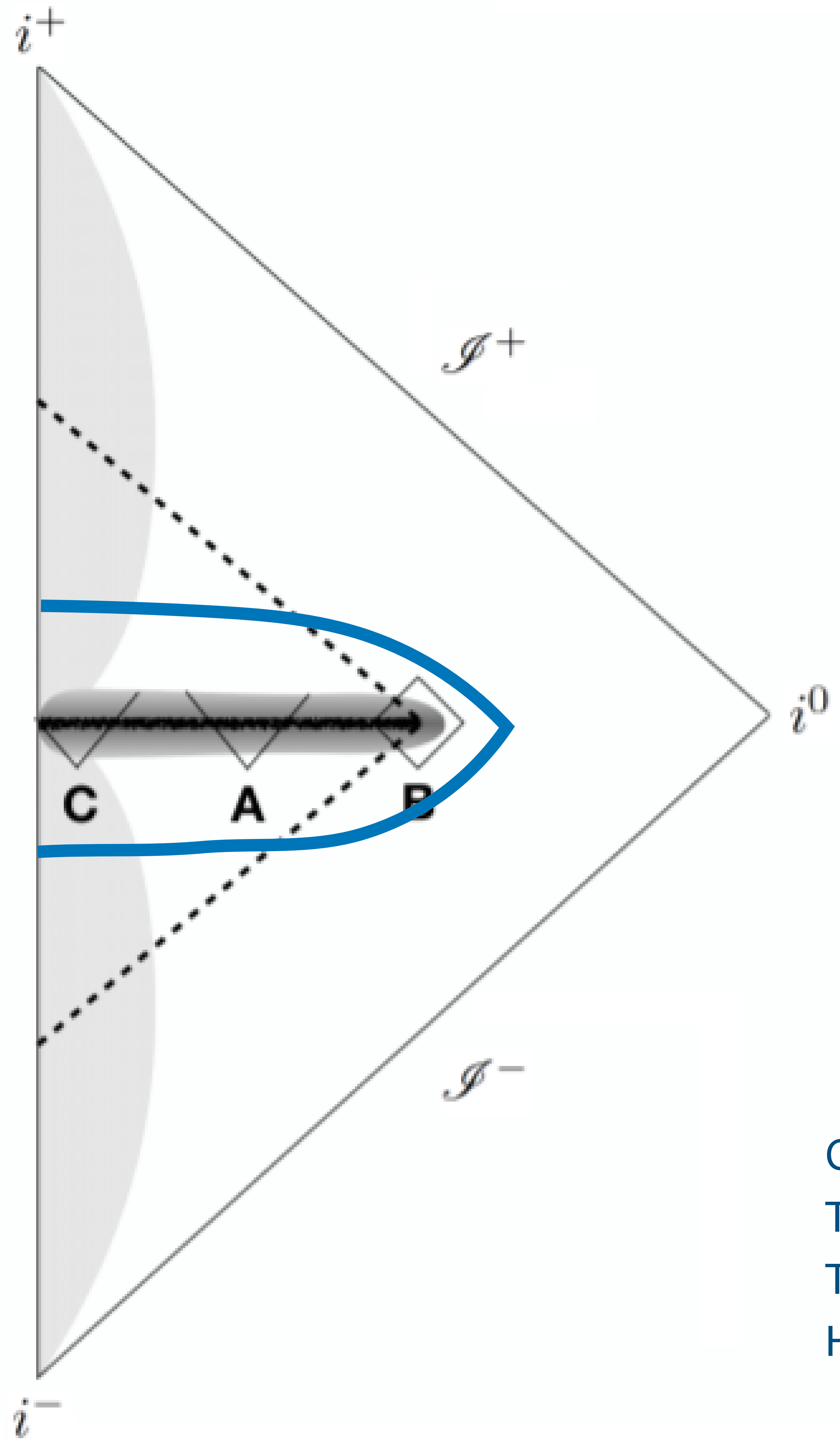
$$H_n = L_2[SU2^{n(n-1)/2}/SU2^n]$$

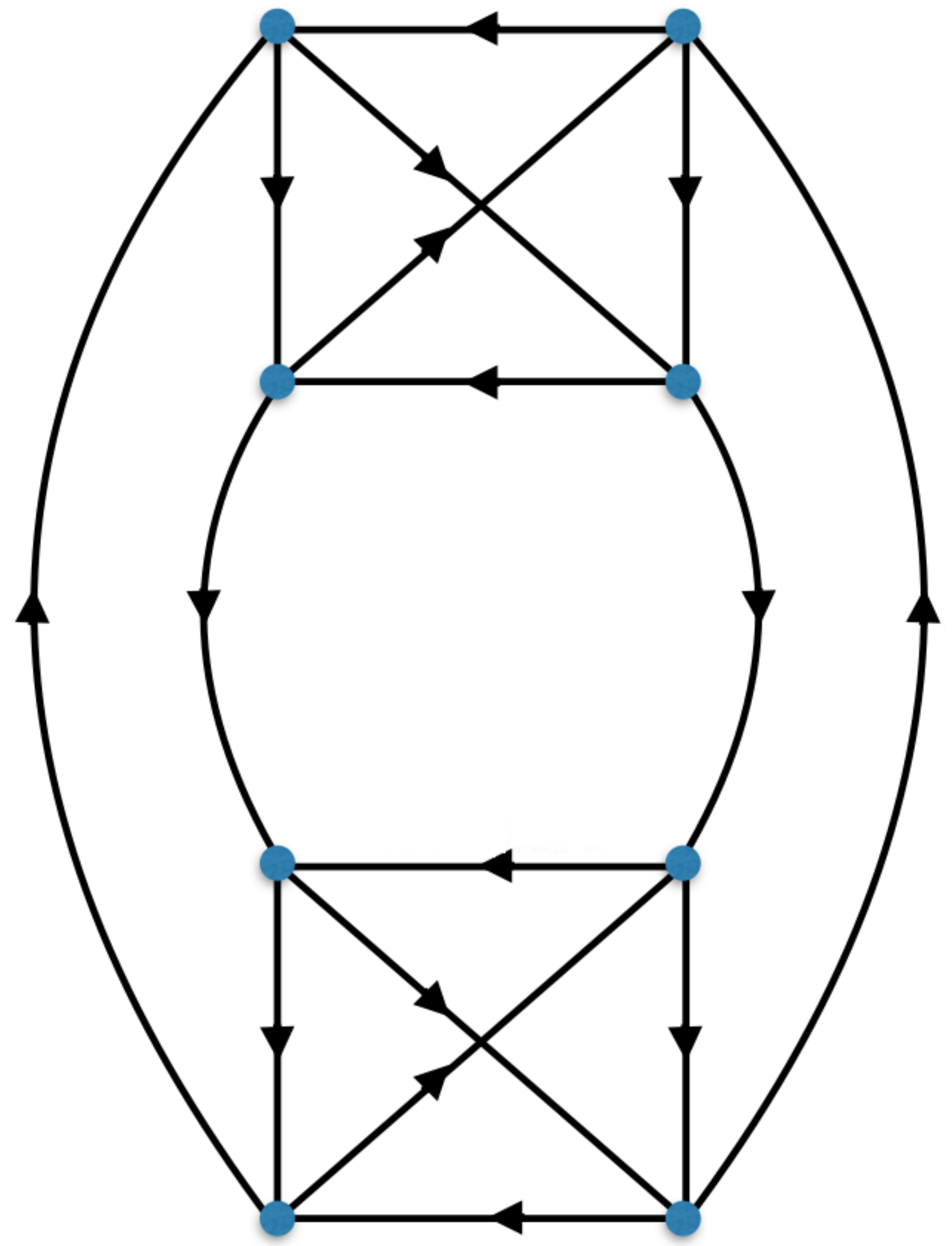






M. Christodoulou, CR, S. Speziale and I. Vilensky, PRD (2016). arXive: 1605.05268.





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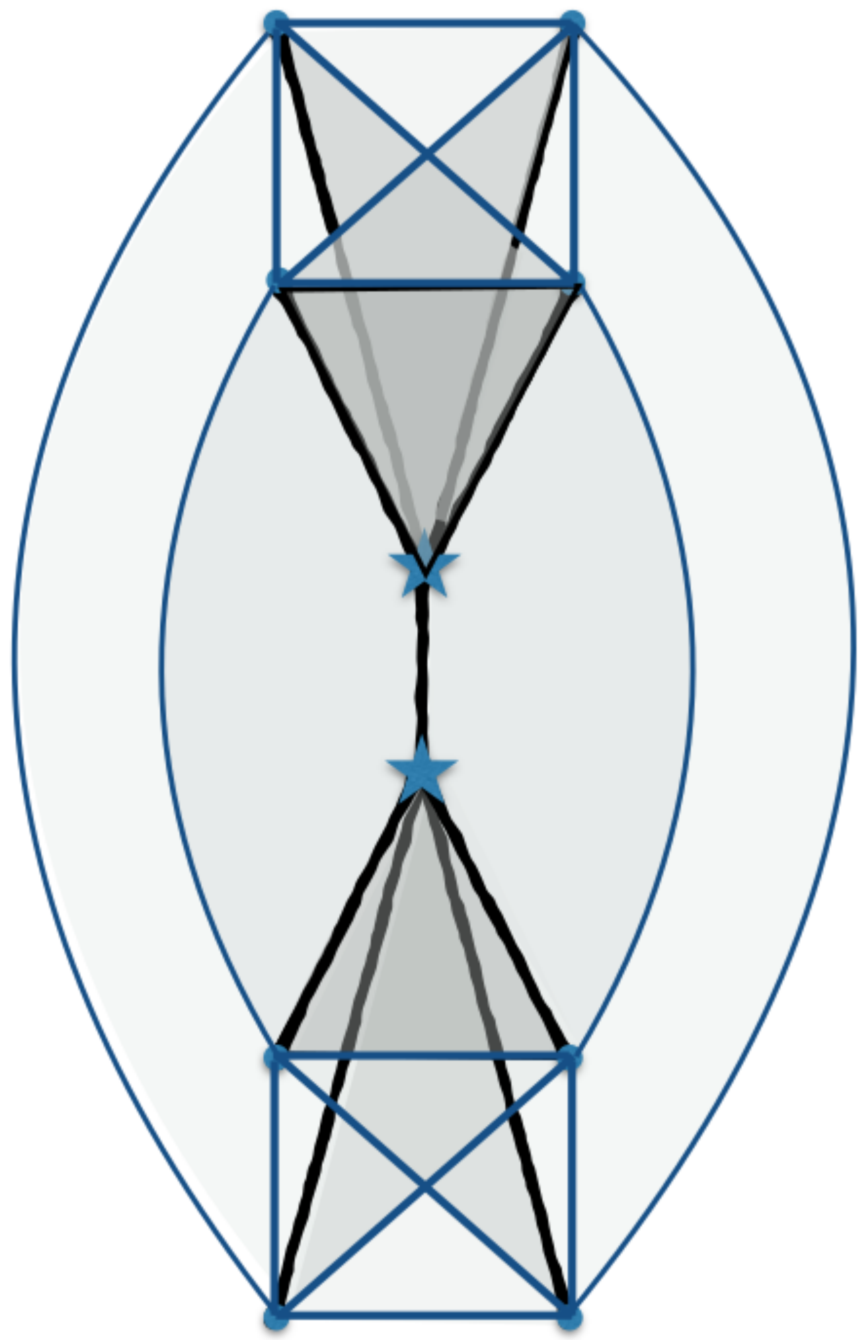
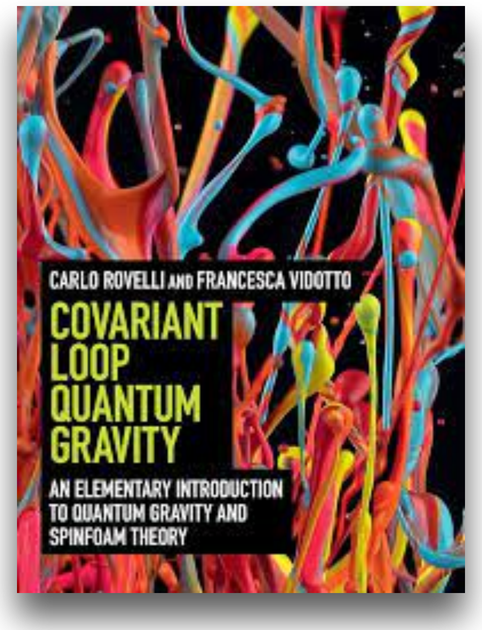


$\mathcal{C}$

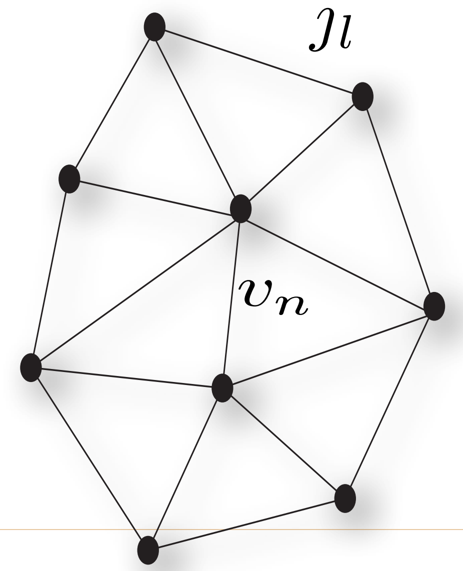
Coherent states depend on intrinsic and extrinsic geometry.
The intrinsic geometry is the same in the past and future surfaces.
The extrinsic geometry has opposite sign.
Hence the transition is a flip in the sign of the extrinsic geometry.

Covariant loop gravity

[Francesca's lectures]



C. Rovelli and F. Vidotto,
Covariant Loop Quantum
Gravity, 2015.

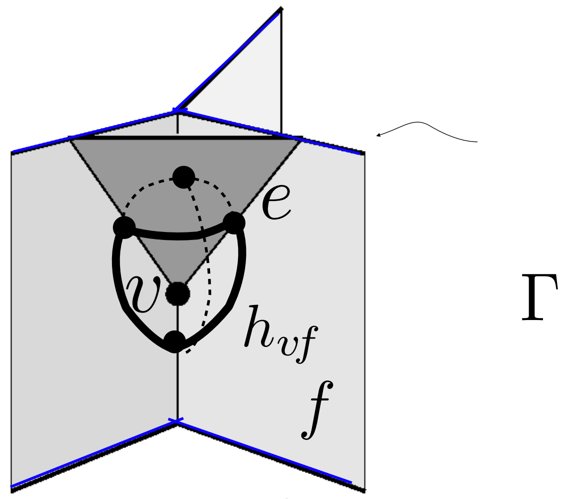


State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N] \quad \psi(h_l) = \psi(\Lambda_n h_l \Lambda_{n'}^{-1})$

Operator: $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

Transition amplitudes $$W_\mathcal{C}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf}) \qquad h_f = \prod_v h_{vf}$$

Vertex amplitude $$A(h_f) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f \, (2j_f+1)\; Tr_j[h_f Y_\gamma^\dagger g_e g_{e'}^{-1} Y_\gamma]$$



2-complex $\mathcal{C}$
(vertices, edges, faces)

Simplicity map $$Y_\gamma \;:\; \mathcal{H}_j \quad \rightarrow \quad \mathcal{H}_{j,\gamma j}$$

$$|j;m\rangle \;\mapsto\; |j,\gamma(j+1);j,m\rangle$$

J. Engle, R. Pereira, CR, PRL 2007, arXiv:0705.2388.
J. Engle, E. Livine, R. Pereira, CR, Nucl. Phys. 2008, arXiv:0711.0146.

1 Coherent states on the boundary spin network

2 Compute the amplitude

F. D'Ambrosio, M. Christodoulou, P. Martin-Dussaud, CR, and F. Soltani, PRD 2021, arXiv:2009.05016.

F. Soltani, CR and P. Martin-Dussaud, PRD 104, 066015 (2021), arXiv:2105.06876..

Analytical calculations:

F. D'Ambrosio, M. Christodoulou,

F. D'Ambrosio, M. Christodoulou, Theophilis, arXiv:2302.12622

Numerical calculations

Low spin regine

P. Frisoni, PRD 107, 2023

F. Gozzini, "A high-performance code for EPRL spin foam amplitudes," CQG 2021

P. Donà, P. Frisoni, "How-to Compute EPRL Spin Foam Amplitudes," Universe 202.
https://doi.org/10.3390%2Funiverse8040208.

[Hongguan lecture]

[Pietro Donà at Loop24]

High spin regine

Muxin Han, Dongxue Qu, Cong Zhang, 2404.02796

https://github.com/czhangUW/BH2WHTranstionInSF

Transition probability $A \sim e^{-\frac{Gm^2}{c\hbar}}$

How to think about this:

- A quantum tunnelling effect [Hal Haggard at Loop24]

P Donà , H Haggard, CR, F Vidotto, arXives: 2402.09038

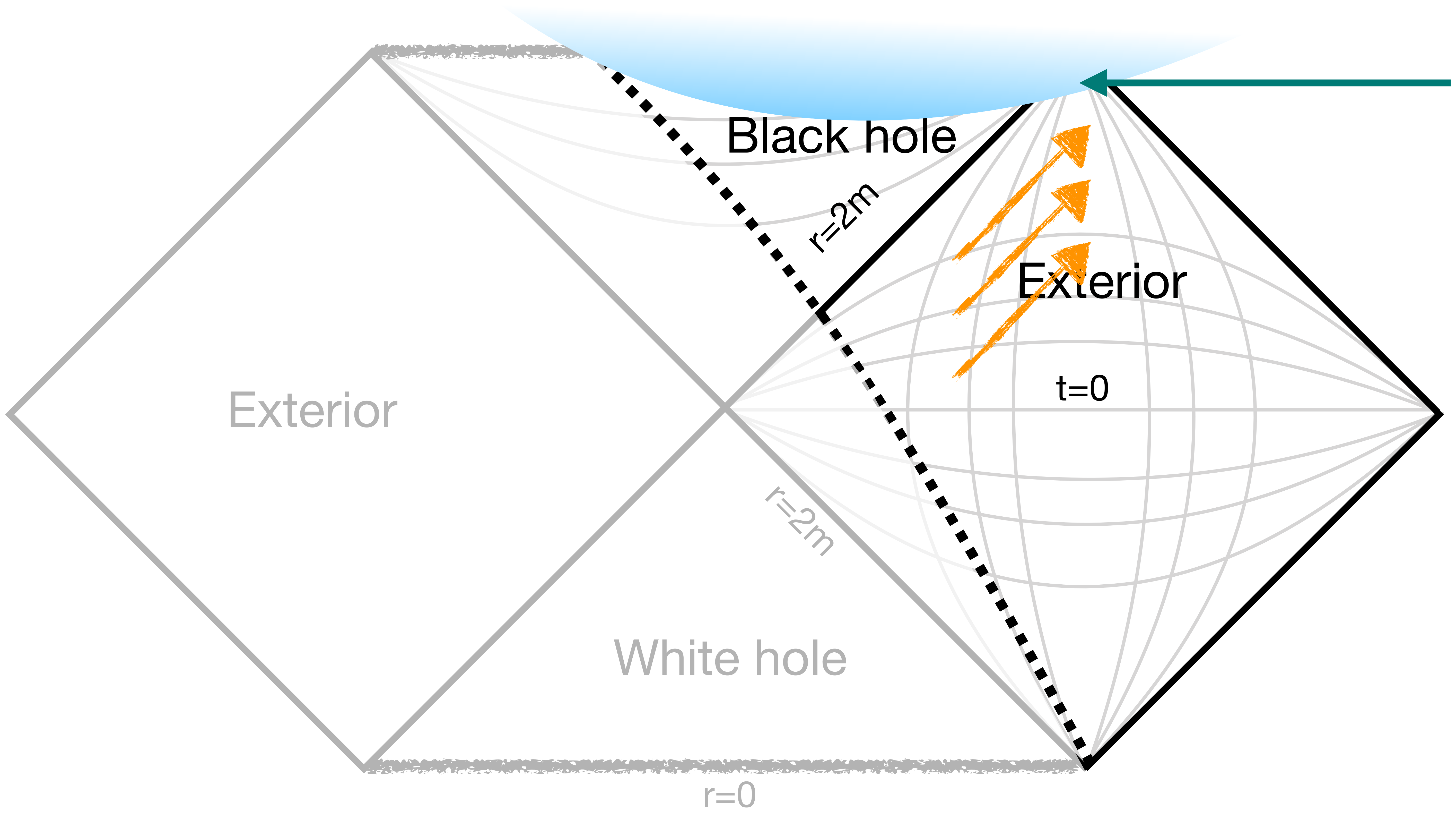
- The amplitude is approximated in the semiclassical regime by

$$A \sim e^{i\, S_{Regge}} \sim e^{i\, \sum_f j_f \theta(j_j)} \sim e^{-\sum_f Area_f}$$

The transition is suppressed for large BH !

II. Dissipative effects

Hawking radiation



$$\mathcal{R} \sim \frac{m}{r^3} \sim \frac{1}{r^2}$$

Outside
the trapping horizon, the
curvature becomes
Planckian !

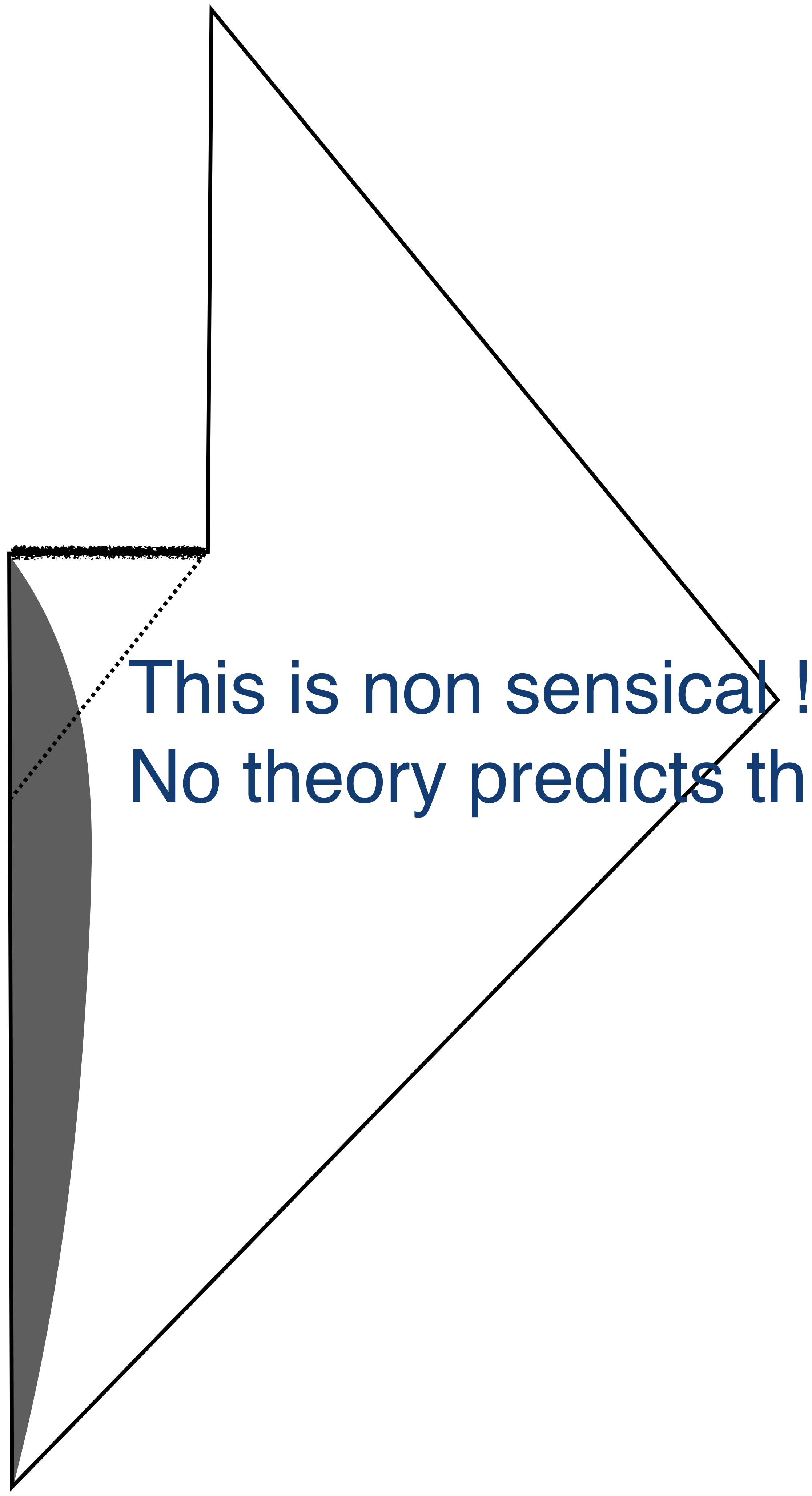
Hawkings radiation: wavelength $\lambda \sim r \sim m$

Temperature: Planck spectrum with max at wavelength: $kT \sim E = \hbar\nu \sim \hbar/\lambda$ $T \sim 1/\lambda \sim 1/m$

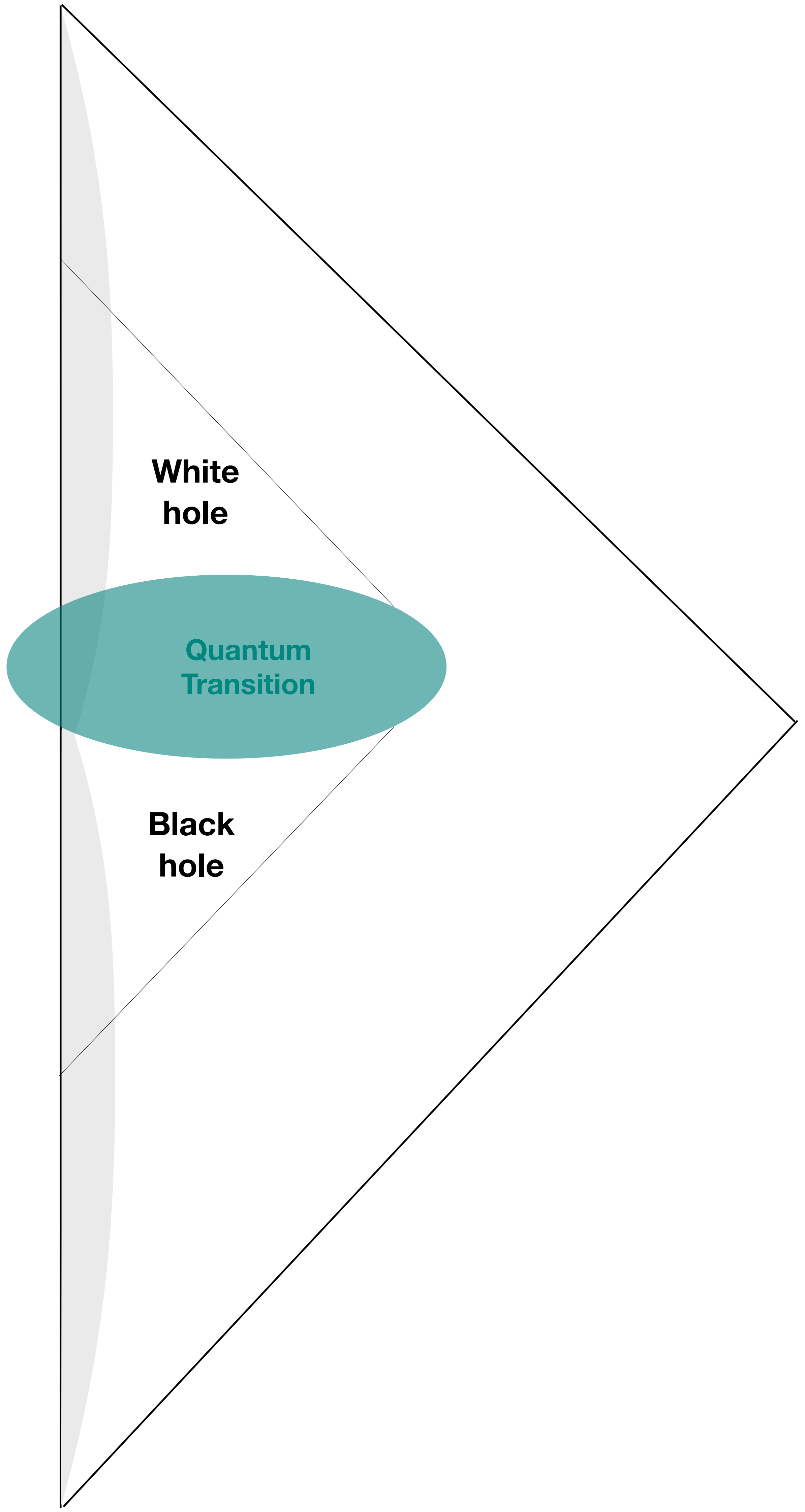
Emitted power: $P = \frac{dm}{dt} = -m^2T^4 \sim -\frac{1}{m^2}$ $m^3 \sim t$

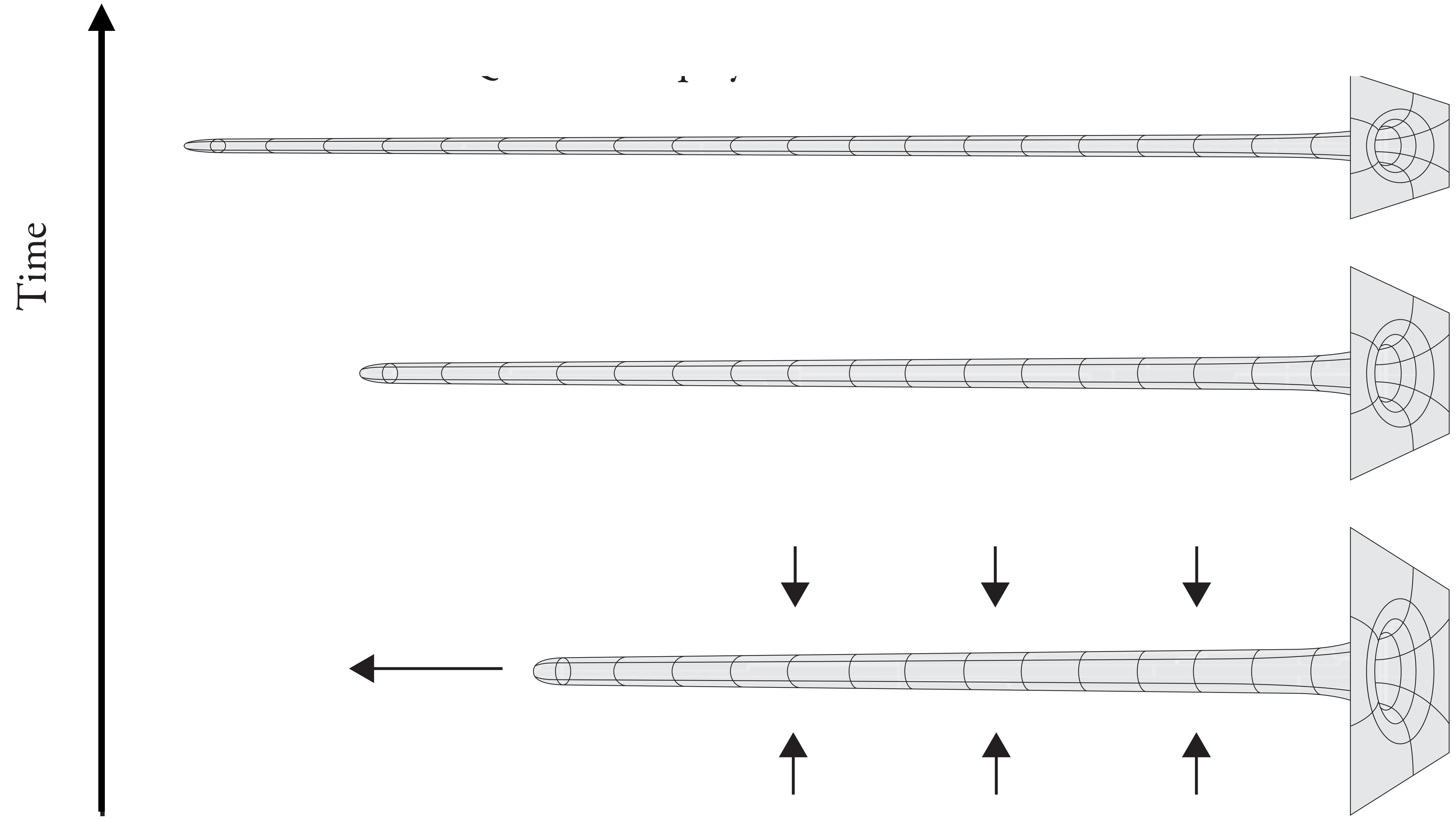
Lifetime of the black hole $\tau_{BH} \sim m_o^3$

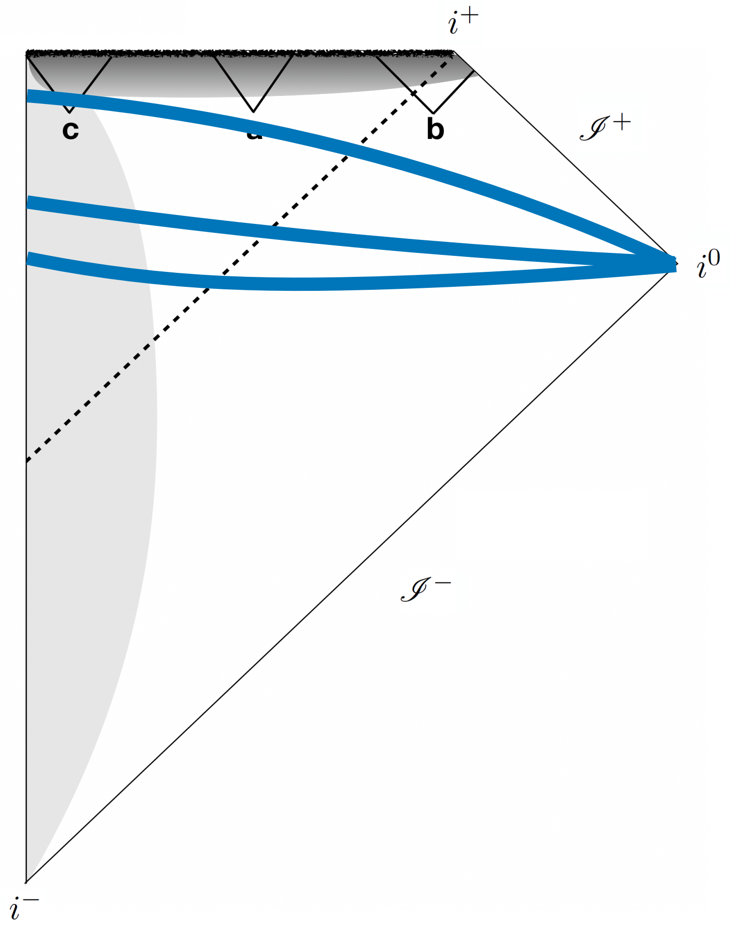
After this lifetime the black hole reaches the size where the transition becomes increasingly probable !



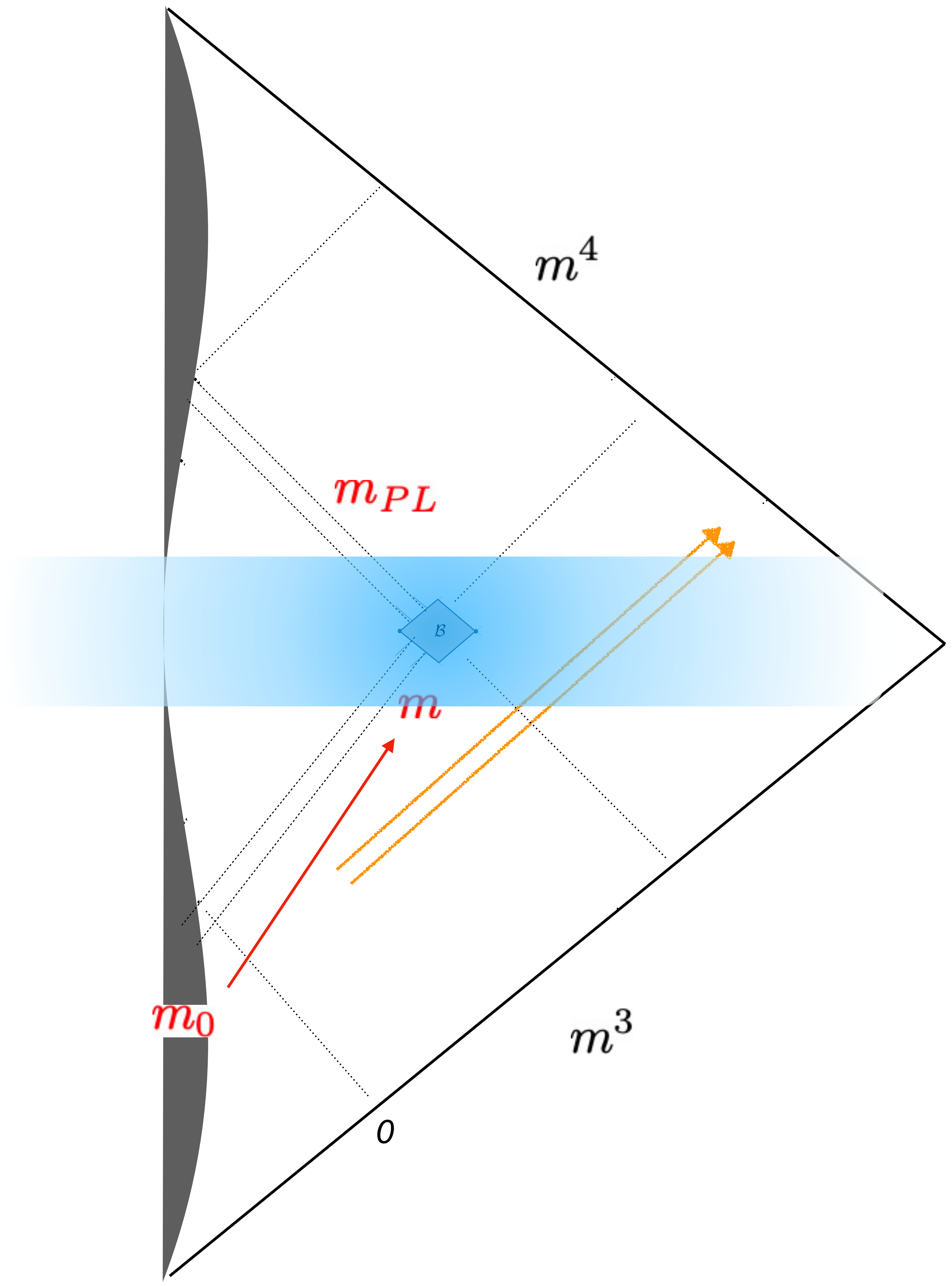
This is non sensical !
No theory predicts this !

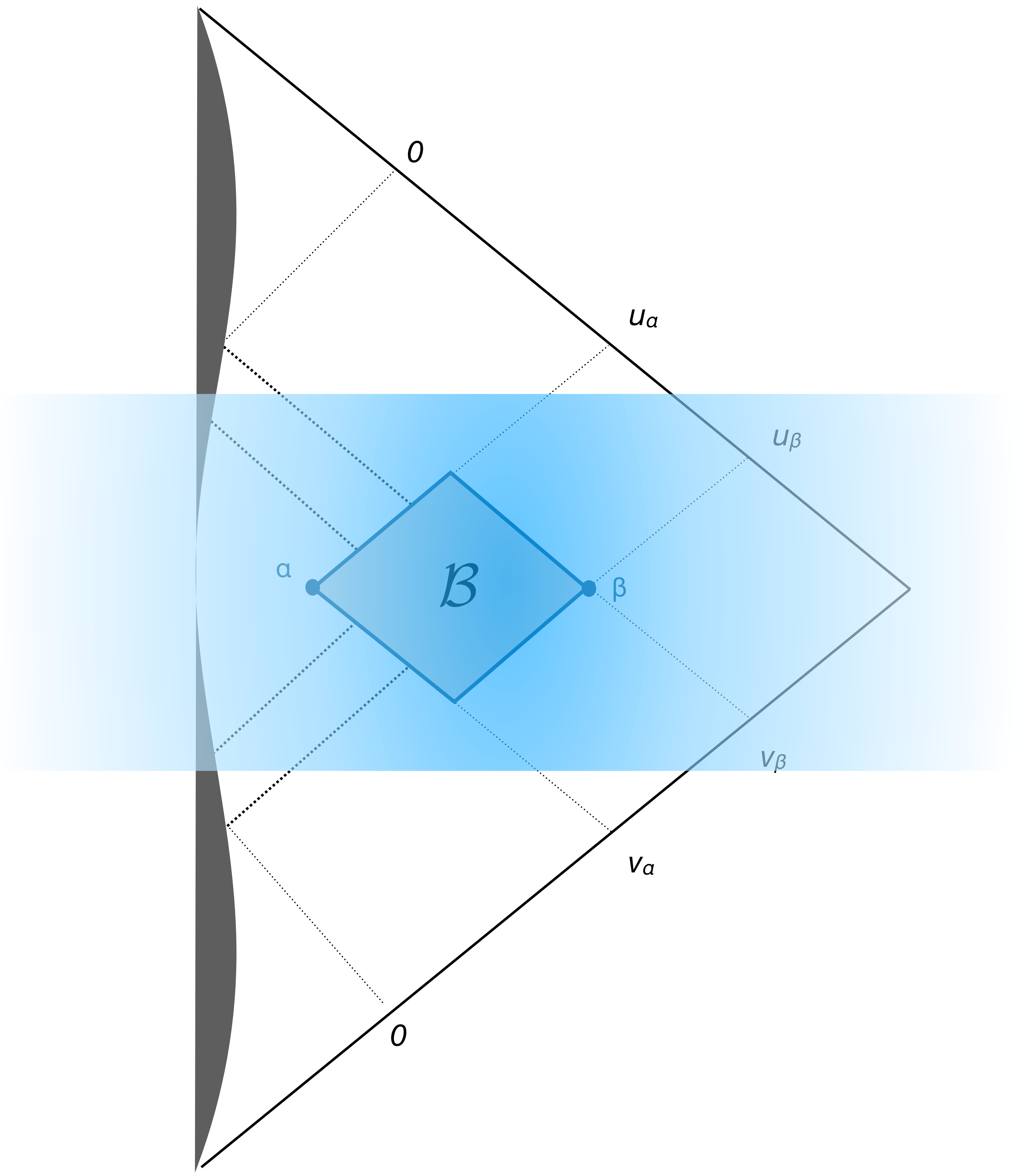


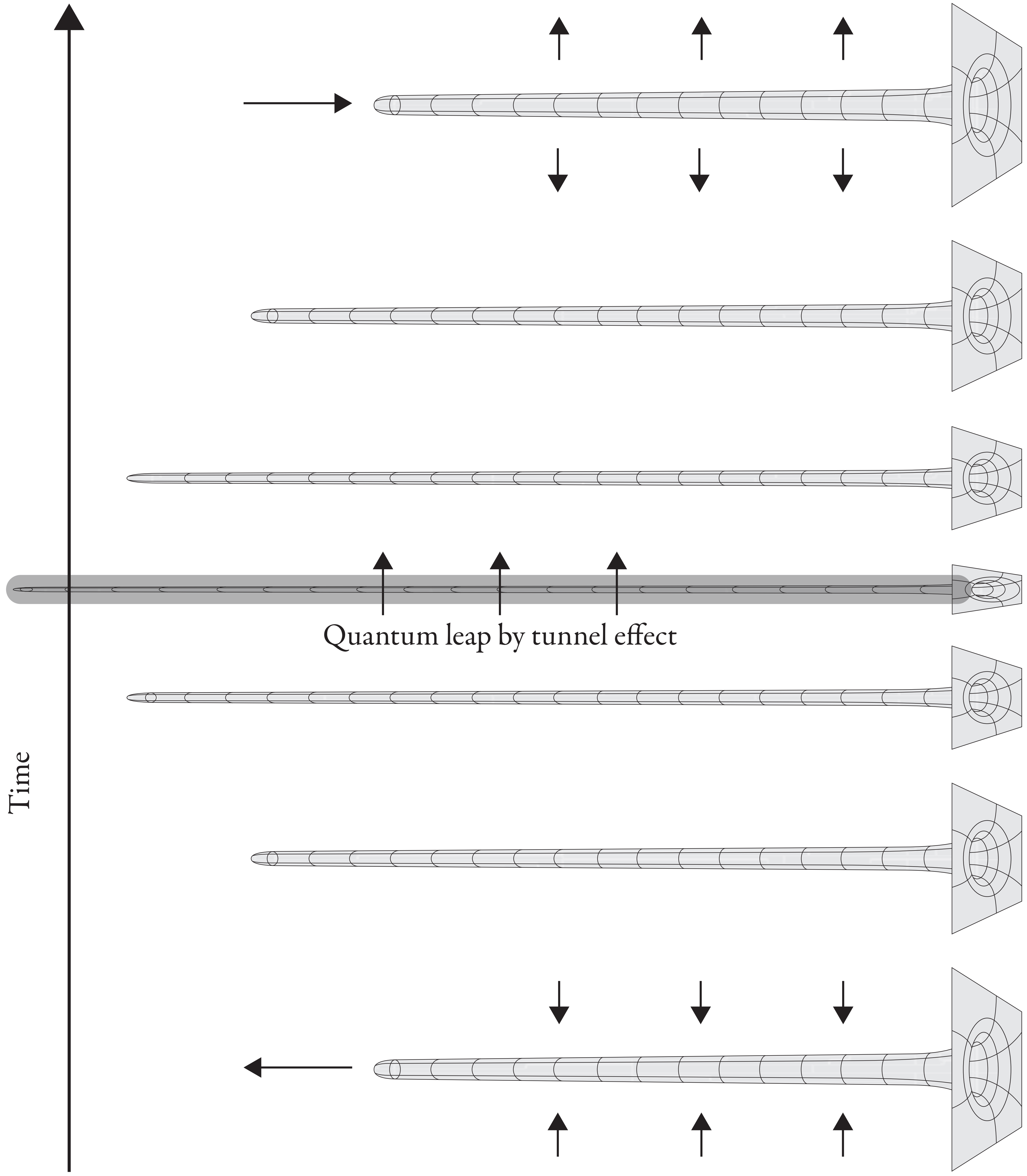




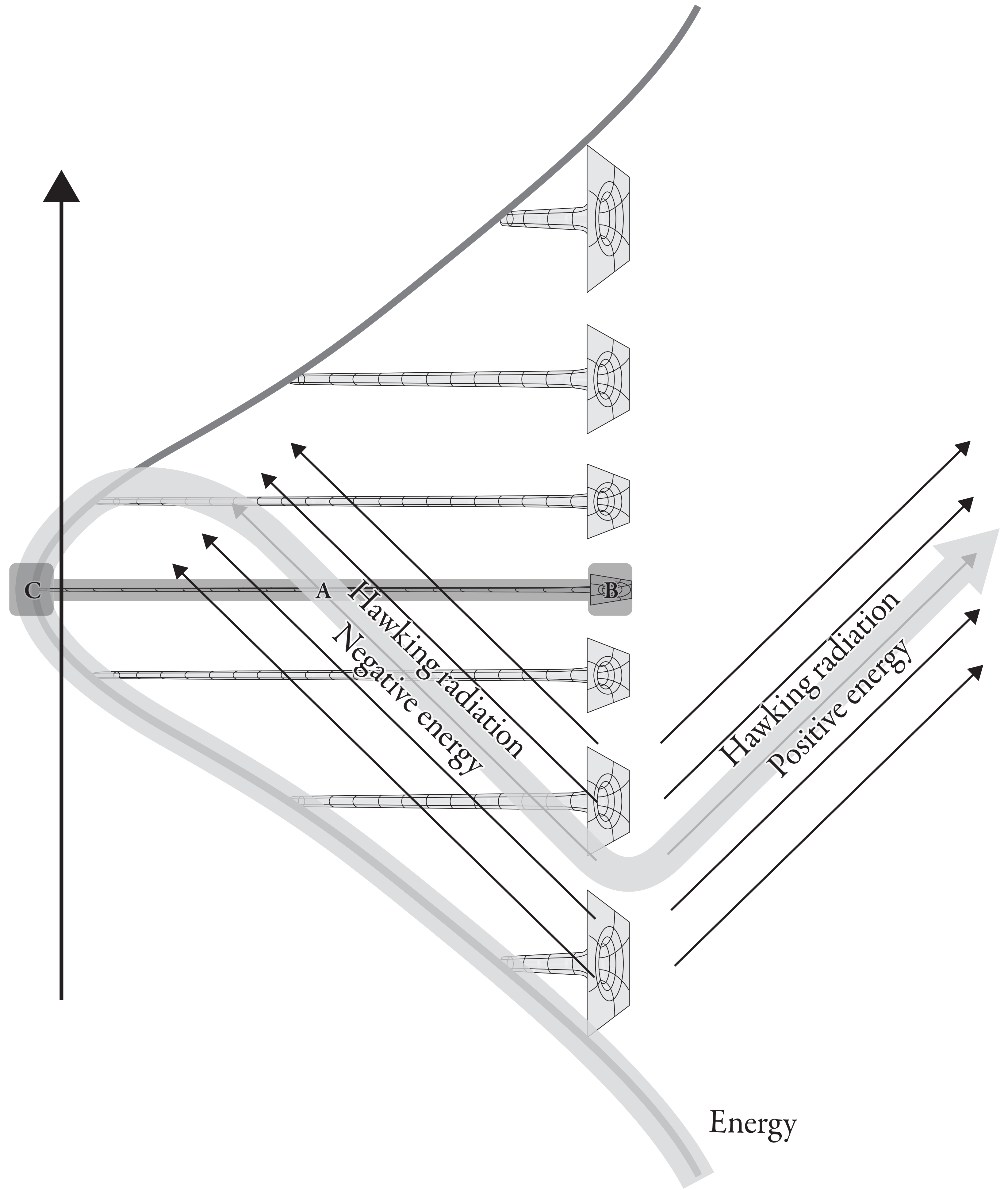
M Christodoulou, CR, How big is a black hole? PRD 2015.

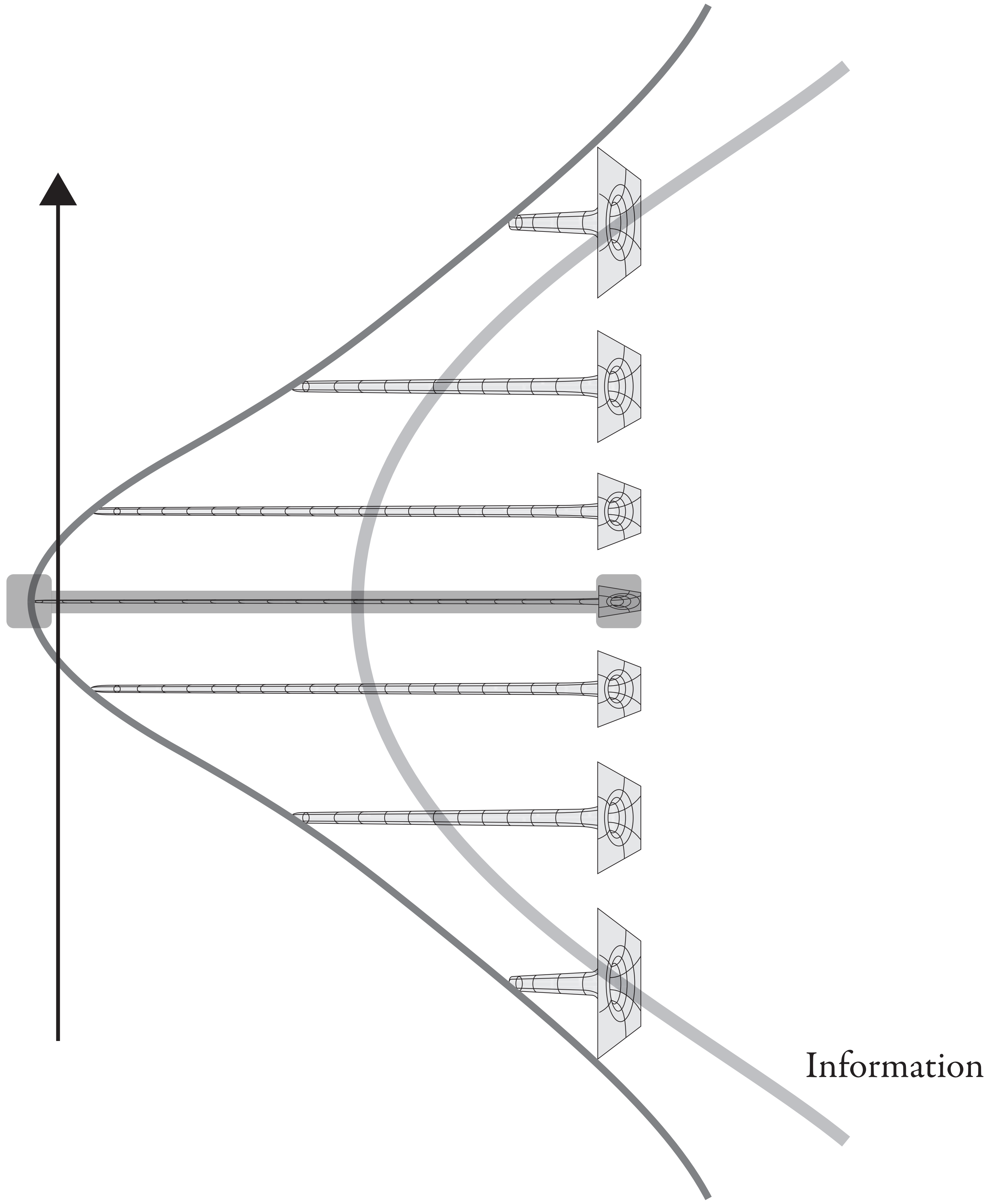

m^4
m_{PL}
$\mathcal{B}$
m
m_0
m^3
0

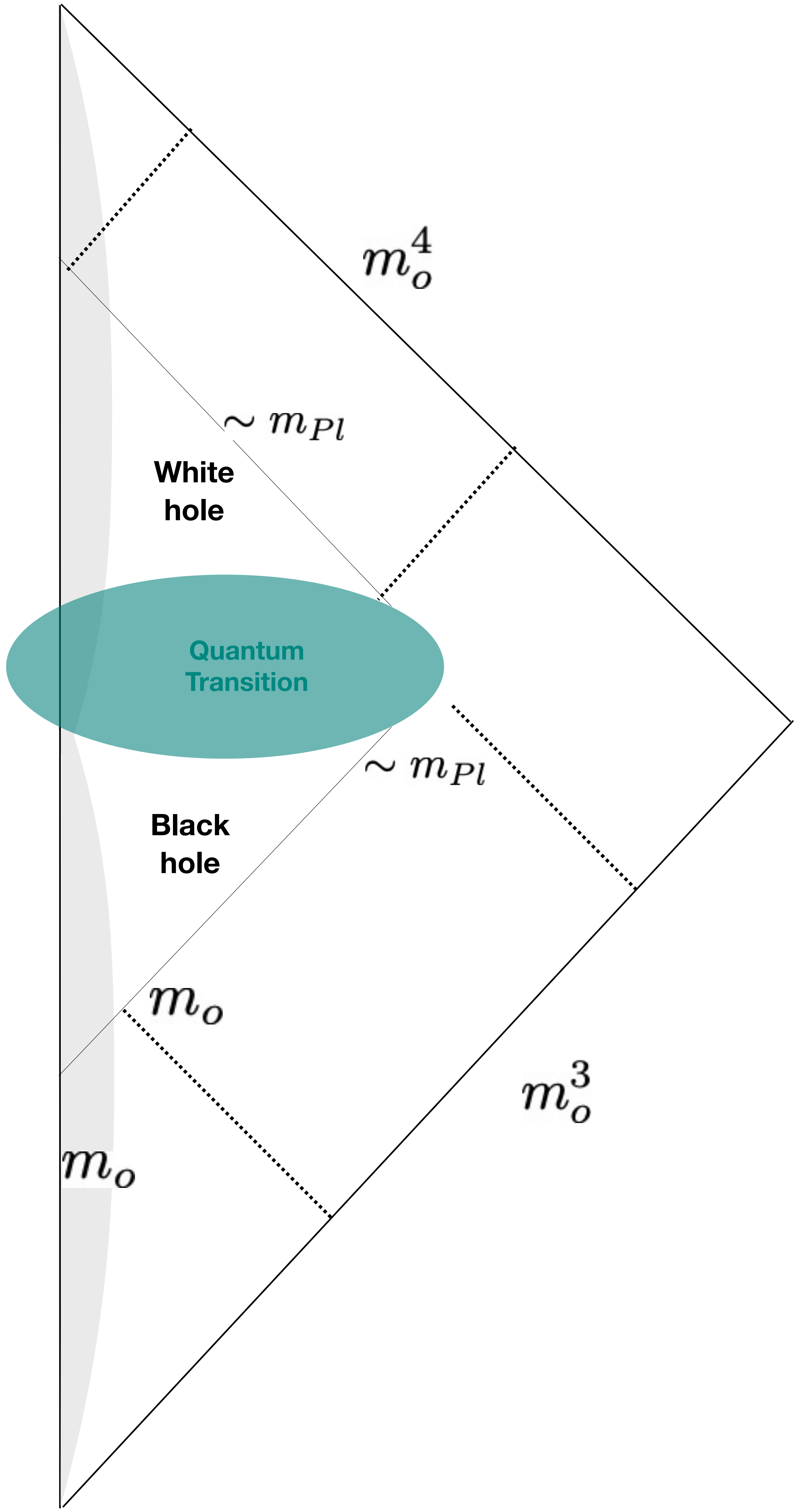

0
u_α
u_β
α
$\mathcal{B}$
β
v_β
v_α
0



M Christodoulou, CR, How big is a black hole? PRD 2015.


C
A
B
Hawking radiation
Negative energy
Hawking radiation
Positive energy
Energy


Information



E. Bianchi, M. Christodoulou, F. D'Ambrosio, H. M. Haggard, CR,
"White holes as remnants: A surprising scenario for the end of a black hole," CQG 2018, arXives: 1802.04264.

$$S \sim \frac{A}{4} = 4\pi m^2$$

$$S = \frac{2\pi}{3} LT, \qquad E = \frac{1}{6} LT^2.$$

$$L = \frac{3\mathrm{S}^2}{8\pi^2\mathrm{E}} = 6m^4, \qquad T = \frac{4\pi\mathrm{E}}{\mathrm{S}} = \frac{1}{m^2}$$

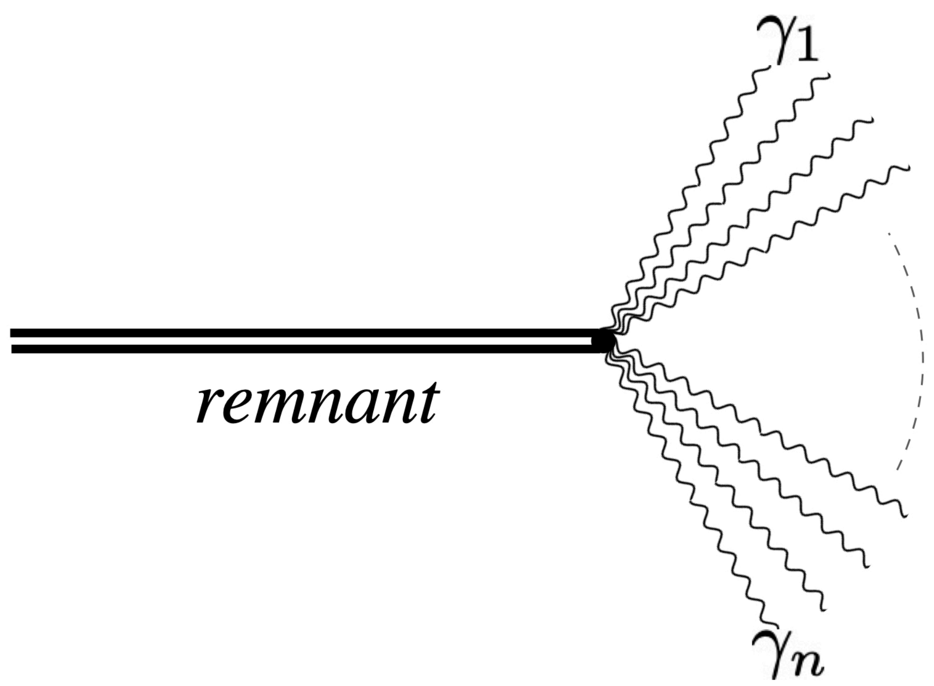
$$\tau_W \sim 6m^4$$

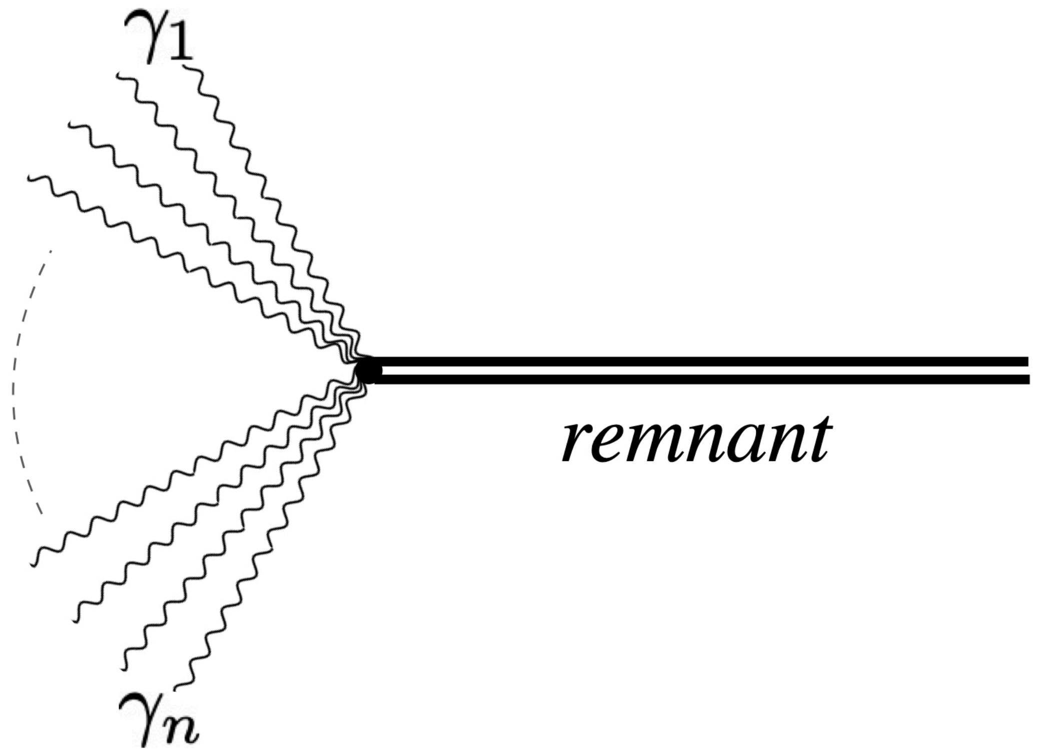
S. Kazemian, M Pascual, F Vidotto, 2022, arXiv:2207.06978.

$$\xrightarrow[\text{collapse}]{} |m_o, m_o\rangle_B \xrightarrow[\text{black hole}]{\tau_{WH}\sim m_o^3} |m_o, m_{P\ell}\rangle_B \xrightarrow[\text{tunnelling}]{\tau_T\sim m_{P\ell}} |m_o, m_{P\ell}\rangle_W \xrightarrow[\text{white hole}]{\tau_{WH}\sim m_o^4} |m_{P\ell}, m_{P\ell}\rangle_W \xrightarrow[\text{end}]{} .$$

$$|m_o, m_P\rangle \rightarrow |0\rangle$$

suppressed!





This also solve the old problem:
Why WH are not easily produced?

The non existence of the information paradox

The thermodynamical entropy $$\Delta S_T = \int \frac{dQ}{T}$$

measures the number of states $$S_T = k \log W \qquad S_T = k \log \dim \mathcal{H} = k \log N$$

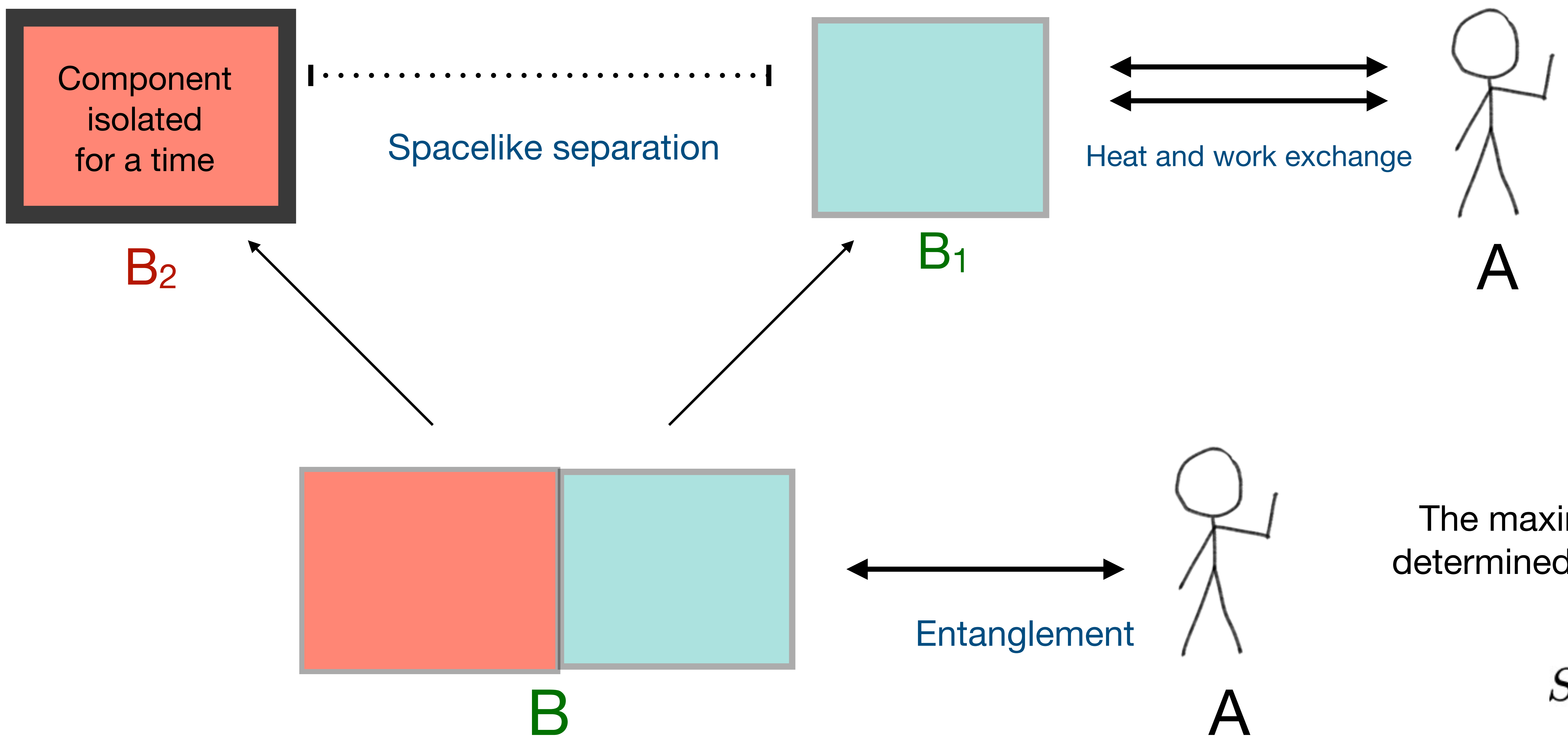
The von Neumann entropy measures entanglement $$\rho_A = Tr_B \Big[|\Psi_{AB}\rangle \langle \Psi_{AB}| \Big]$$

This is only true under (severe) conditions !

$$S_{vN} = -k \; Tr[\rho_A \log \rho_A]$$

It is maximized by $\rho_a = \frac{1}{N}\mathbb{1} \quad S_{vN} < k \log N$

$$\boxed{S_{vN} \leq S_T}$$



Component isolated for a time

Spacelike separation

Heat and work exchange

B$_2$

B$_1$

A

B

Entanglement

A

The thermodynamical entropy is determined by the number of states of B$_1$

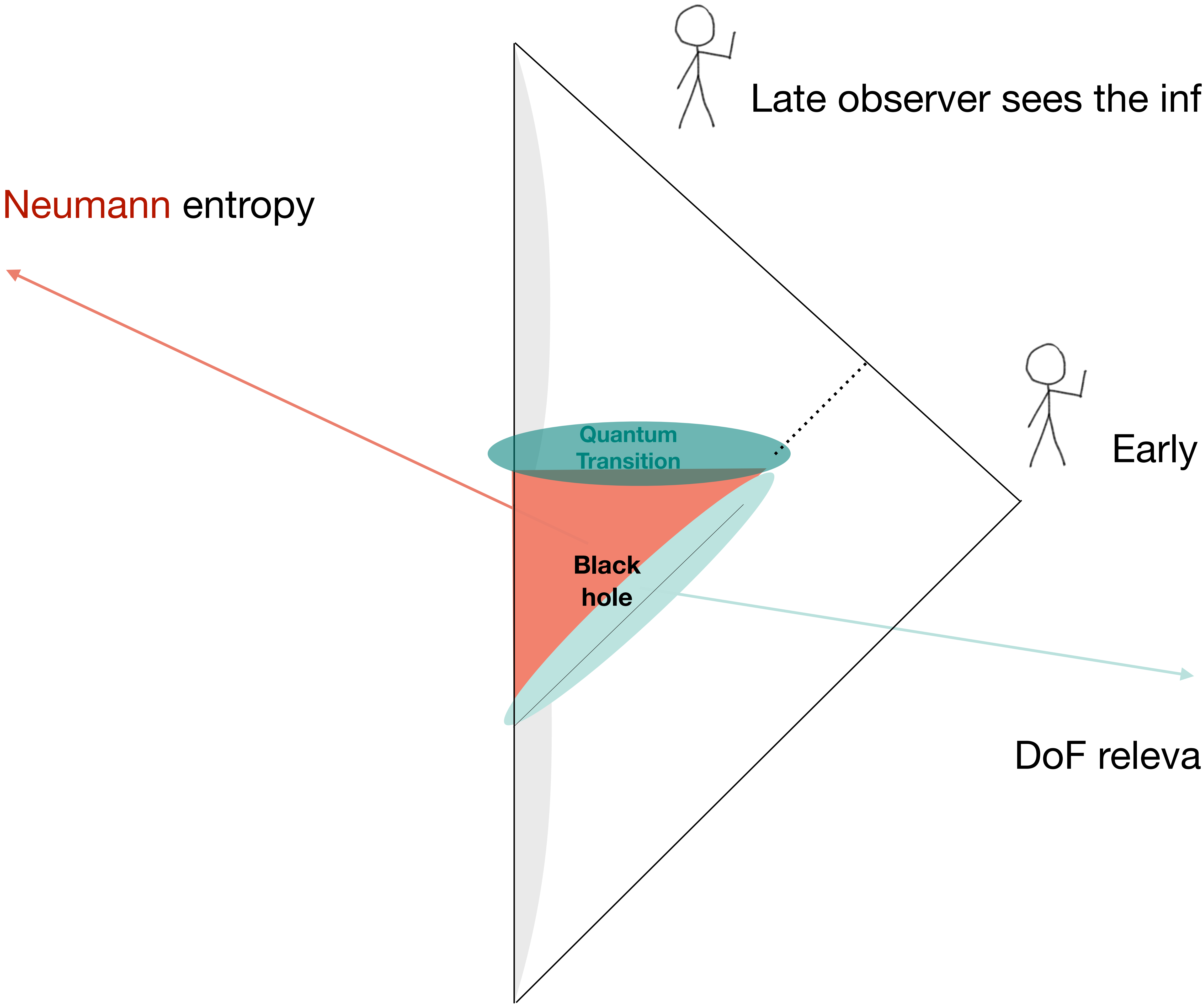
$$S_T = k \log \dim \mathcal{H}_{B_\infty} = k \log N_{B_1}$$

The maximal von Neumann entropy is determined by the **total** number of states of B

$$S_{vN} < -k \log N_B$$

$S_{vN} \leq S_T$ Does not hold anymore!

The von Neumann can be higher that the thermodynamical entropy.



Late observer sees the information coming out

DoF relevant for the von Neumann entropy

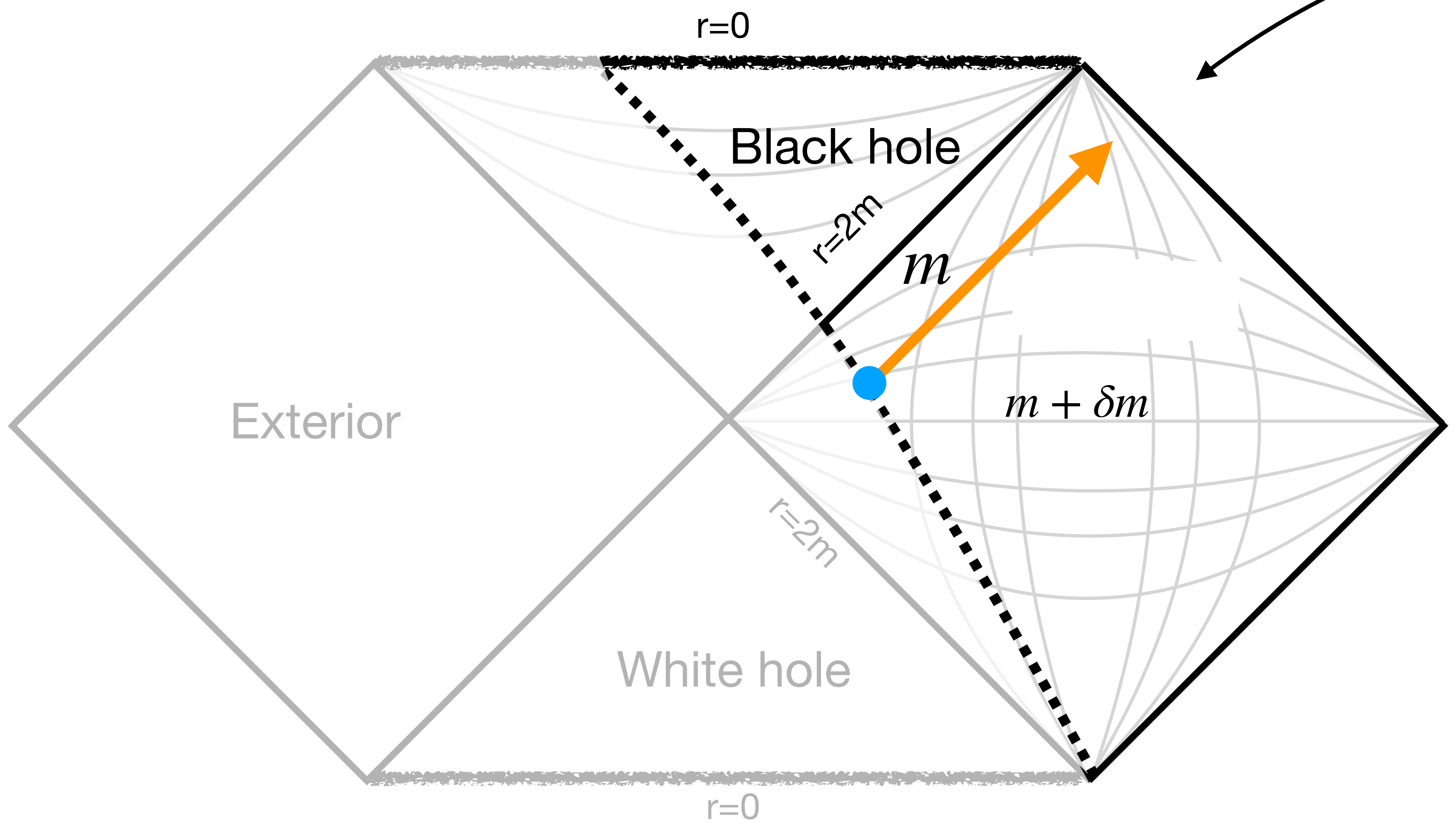
Early observer sees the hole near stationary

DoF relevant for the thermodynamical entropy

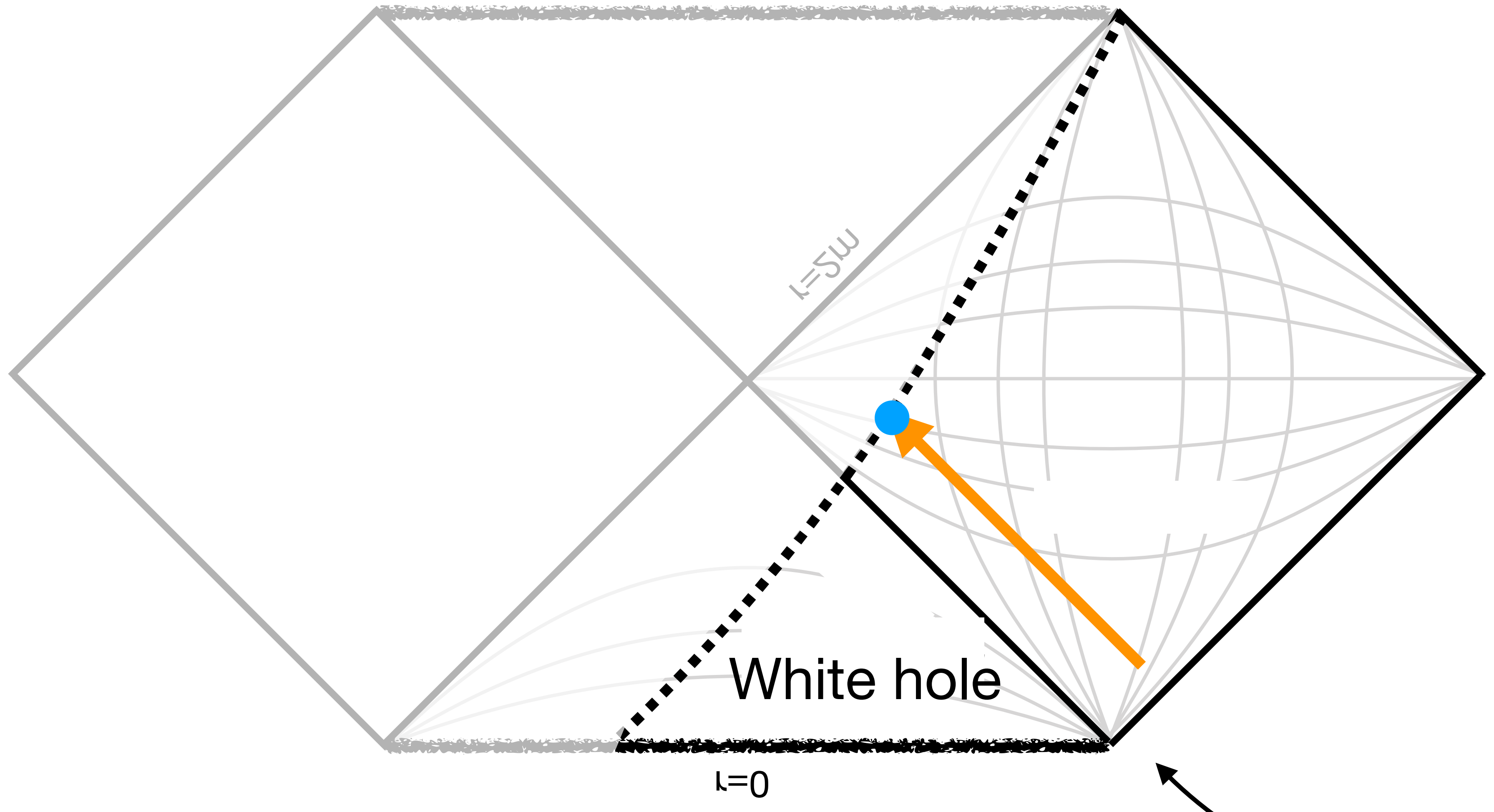
CR, The subtle unphysical hypothesis of the firewall theorem, Entropy 2019.

CR, Black holes have more states than those giving the Bekenstein-Hawking entropy: a simple argument, CQG 2018, arXives:1710.00218

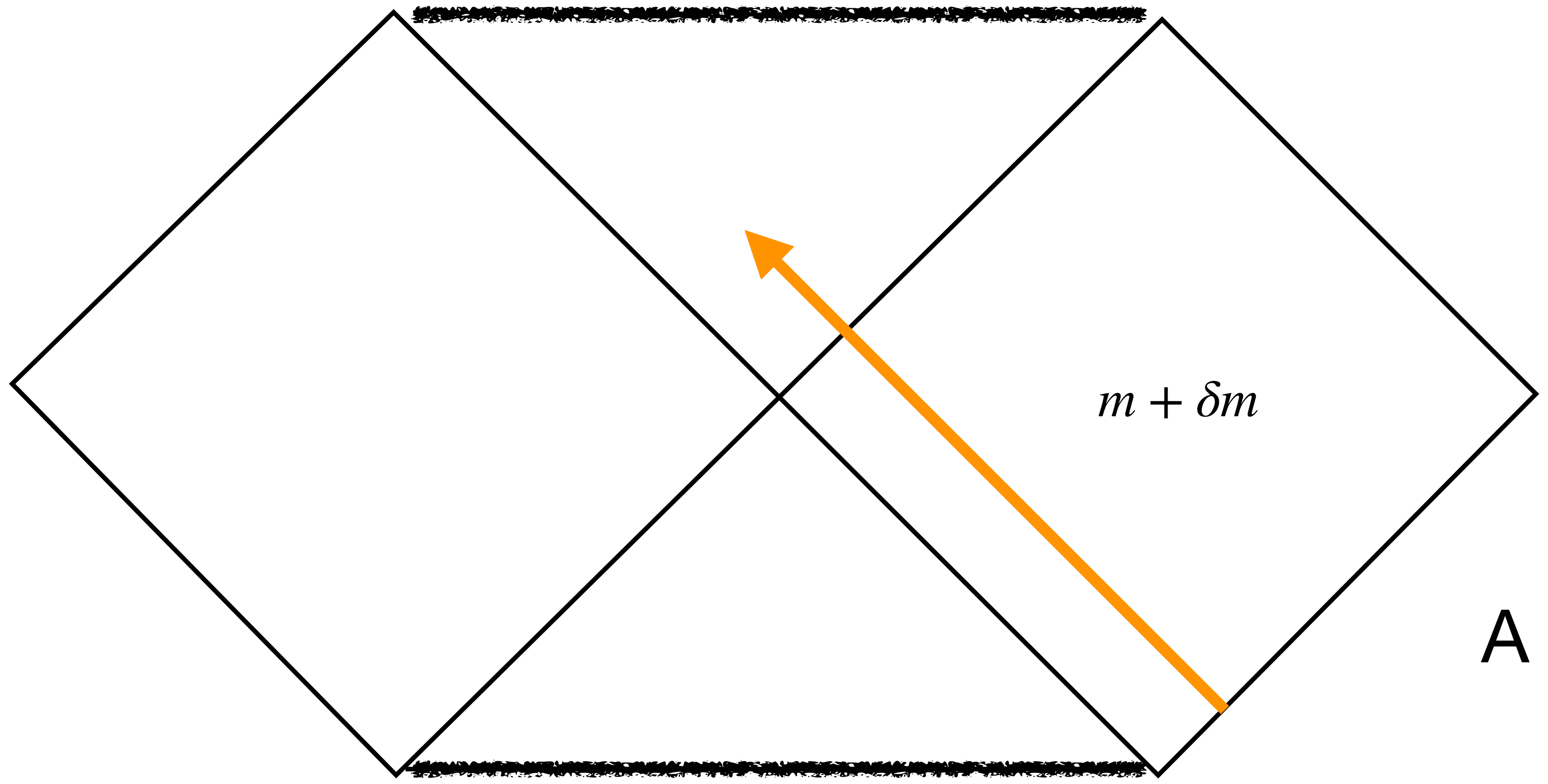
White holes are unstable



The energy pulse cannot be too much in the future



The energy pulse cannot be too much in the past

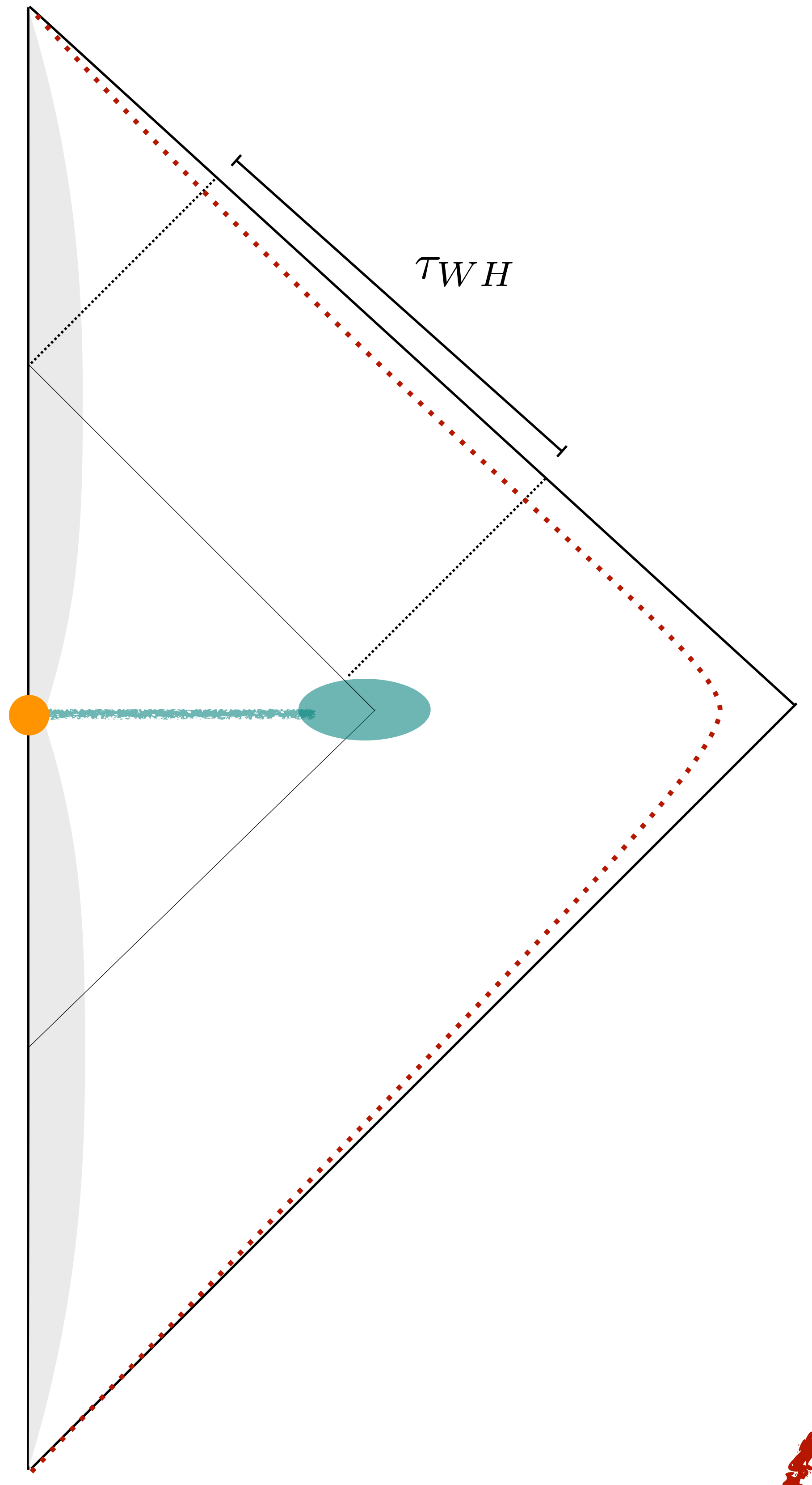


A white hole is unstable toward becoming a black hole

Are remnants stable?
They are stabilized by quantum gravity

Area gap = minimum non vanishing mass

$$A_{min} = 4\frac{\sqrt{3}}{\pi}\gamma\hbar G/c^3$$



$$|m_o, m\rangle_B \xrightarrow[\text{tunnelling}]{} |m_o, m\rangle_W$$

$$|m_o, m\rangle_W \xrightarrow[\text{instability}]{} |m_o, m\rangle_B$$

$$H = \begin{pmatrix} m + 3\sqrt{3}\ i\pi m_o^2\, \frac{\partial}{\partial v} - i\, \frac{\hbar^2}{m^2}\frac{\partial}{\partial m} & b\frac{\hbar}{m} \\ c\frac{\hbar}{m}e^{-m^2/\hbar} & m - 3\sqrt{3}\ i\pi m_o^2\, \frac{\partial}{\partial v} \end{pmatrix}$$

$$|\psi\rangle = \begin{pmatrix} B(m,v) \\ W(m,v) \end{pmatrix}$$

$$|R\rangle = \frac{\sqrt{\frac{a}{b}}|B,\mu\rangle - |W,\mu\rangle}{\sqrt{1+\frac{a}{b}}}$$

$$|m_o, m\rangle = \alpha|m_o, m\rangle_W + \beta|m_o, m\rangle_B$$

Vidotto, CR 2018.

Quasi stable
remnants of
Mass ~ 1μg

Dark matter?

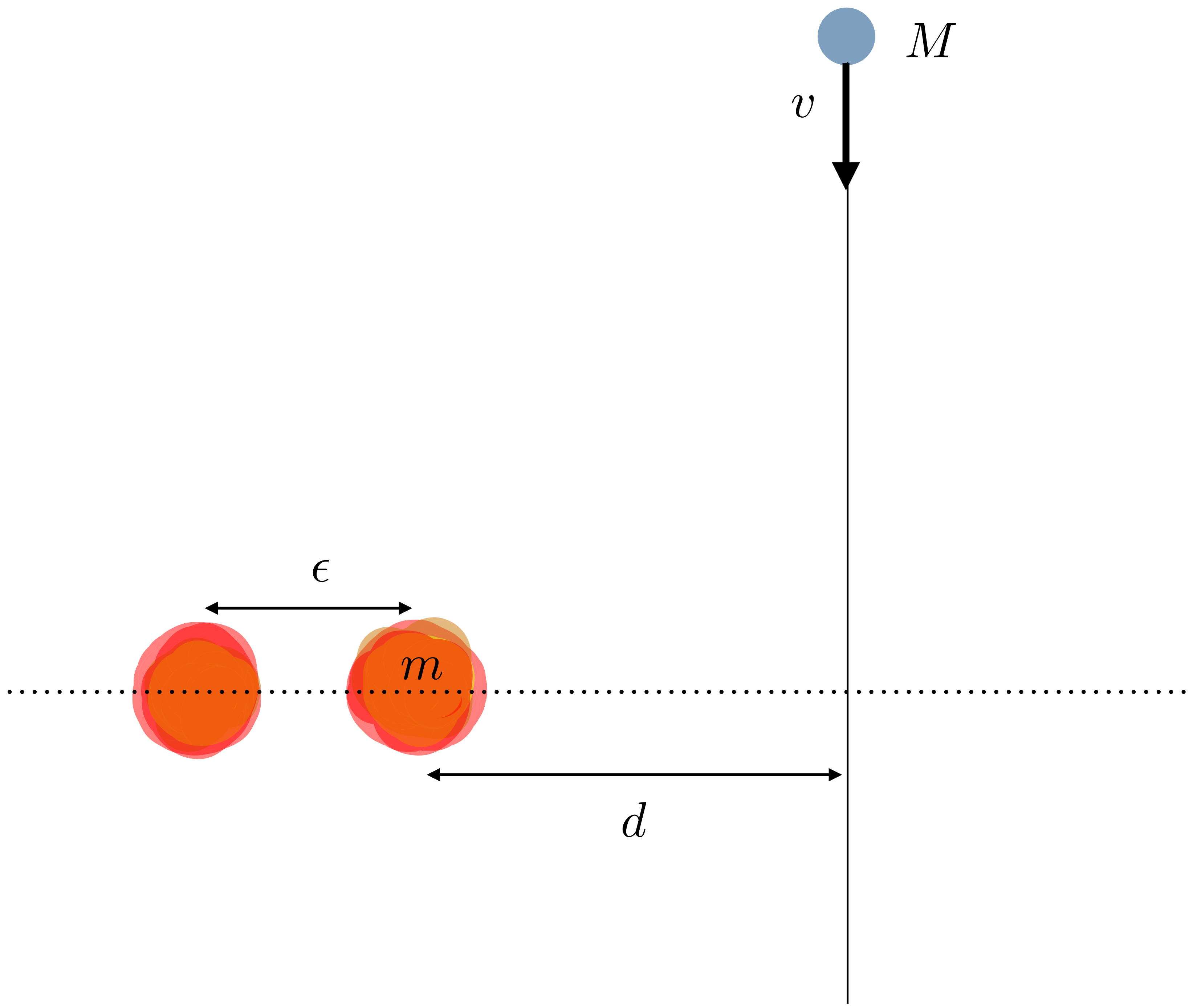
Emission

$$m = 10^{x-5} gr, \quad \nu = 10^{-2x+32} Hz, \quad \rho_{rad} = \sinh\left(\frac{10^{61} - 10^{3x}}{10^{4x} - 10^{3x}}\right)\rho_{rem}$$

$$x = \log_{10}(m/m_{Pl}) \in [15, 20]$$

S. Kazemian, M. Pascual, CR, F. Vidotto, “Diffuse emission from black hole remnants,” CQG 2023.

Direct detection?



A Perez, M Christodoulou, CR,
Detecting Gravitationally Interacting Dark Matter with Quantum Interference, 2024,

Plenty of things still to do !

I trust in you do go ahead !