

# Planck stars, White Holes Remnants, and Planck-mass quasi-particles

## The quantum gravity phase in black holes' evolution and its manifestations

DRAFT

Carlo Rovelli

A review recent developments in the exploration of quantum gravity aspects of black hole physics.

### CONTENTS

I. Introduction	1
A. A sketch of the scenario	2
B. The domain of validity of classical gravity	3
II. Non dissipative aspects of the transition	4
A. Planck stars	4
B. Black to white transition	5
C. The exterior metric	5
D. The Boundary Region	7
E. The LQG transition amplitude and the Christodoulou-D'Ambrosio result	8
F. White holes	9
III. Dissipative aspects of the transition	10
A. Black hole lifetime	10
B. How big is the black hole interior?	11
C. Instability	11
D. Planckian Remnants	12
E. The lifetime of the white hole	13
F. There is no information paradox	14
IV. Phenomenology	17
A. Dark Matter	17
B. Direct detection	17
C. Cosmological considerations	19
D. Primordial holes and erebons	19
E. Modeling remnants emission	20
References	23

### I. INTRODUCTION

Quantum gravity is a theory with a mass scale:  $m_P = \sqrt{\hbar c/G}$ , a fraction of microgram. This is very small in astrophysics and very large in high-energy physics. It is reasonable to study the possibility that the spectrum of the theory could include a stable or semi-stable non-perturbative object at this scale: a Planck-mass quasi-particle. Recent developments in classical general relativity and in loop quantum gravity bring credence to this possibility.

These developments regard the dynamics of black holes. We expect black holes to evolve into spacetime regions dominated by strong quantum gravity effects. These regions have not been much explored in the traditional literature on quantum effects on black holes, often

focused on what happens before the hole reaches these regions, for instance at Page time. But a number of recent lines of research have addressed these regions revealing a plausible physical scenario, which we detail in the next section, for the full evolution of a black hole [1].

Several ingredients have contributed to this scenario. These include a new solution of the Einstein equations [2] showing that a trapping horizon can evolve into an anti-trapping one, a better understanding of the interior of white holes and black holes, and numerous applications of a variety of Loop Quantum Gravity techniques – canonical, covariant, and numerical– to describe the genuine non-perturbative regions.

Three aspects of this scenario are particularly appealing. It provides a candidate for dark matter that does not require any new physical hypothesis (such as new fields, particles, or modifications of the field equations): just general relativity and its possible quantum properties [3]. (On the idea that primordial black holes could play a key role for dark matter, see also [4, 5].) It offers a natural solution to the black hole information ‘paradox’. It is in principle, and perhaps even in practice, directly testable: Planck-mass quasi-particle may be [6].

The scenario includes distinct quantum phenomena happening in different spacetime regions. It includes dissipative as well as non-dissipative aspects. Its analysis employs different approximations and truncations for treating these different phenomena. Because of this complexity, it can only be addressed ‘à la Fermi’, estimating the relevance and the import of the various physical effects, rather than within a single mathematical-physics idealization. This complexity motivates the present review paper, which brings together the various ingredients of this scenario, scattered in the literature.

We start with a quick sketch of the scenario (Section IA) and an analysis of the regions where classical GR is unreliable (Section IB). Then we break the presentation into two parts: a first part (Section II) where we discuss the non dissipative aspects of the global dynamics of a black hole, and a second part (Section III) where we take the dissipative (time oriented) effects into account.

In the first, we discuss the possible bounce of the collapsing star (Section IIA), namely the “Planck Star” phenomenon, and the external metric of the global black hole dynamics (Section IIB). A side section discusses the counter intuitive classical geometry of white holes, because this play a role in global picture (Section IIF). This part ends presenting the current state of the calculations of the Black to White transition probability [7].

In the second part, based on these calculations, we discuss the relevant temporal regimes, the black hole information paradox (Section III F), and the structure of and stability of the remnants (Section III D). The last part (Section IV A) focuses on the possibility of observations supporting the scenario described.

Unless indicated, we use Planck units  $c = G = \hbar = 1$ .

### A. A sketch of the scenario

Perhaps the most interesting discovery of the last decades is the abundance and variety of the astrophysical object we now call black holes. Current direct and indirect observations of these objects are all well accounted for by classical GR. But classical GR is insufficient to account for the phenomena that happen in their high-curvature regions, where genuine quantum gravity effects are most likely non-negligible. Therefore GR does not provide a reliable *global* picture of the dynamics of these objects.

In a black hole produced by a collapsing star there are three distinct spatiotemporal regions where we expect quantum gravity effects to dominate: (i) the moment when the star reaches Planck density; (ii) the moments, outside the star but inside the horizon, when Planck curvature is reached; (iii) the moment *outside* the horizon, where the Planck curvature is reached because of the shrinking of the horizon due to the Hawking radiation. These events are spacelike with respect to one another. Each of them needs to be treated separately and understood on its own terms. But all of them can be described using Loop Quantum Gravity (LQG) as a guide, utilizing tools and results from various branches of LQG, in particular the spinfoam amplitudes of covariant LQG [8, 9], Quantum Cosmology [10] and numerical calculations [11].

The intuition underpinning the scenario that emerges [12] is that quantum-gravitational pressure can stop gravitational collapse and cause a bounce [13], permitting the entire content of the black hole to eventually dissipate [14]. Indeed, quantum cosmology (which we review below) indicates that the dominant quantum effect at high density is a quantum pressure sufficient to counterbalance weight and reverse collapse.

This quantum gravitational pressure stops gravitational collapse when the energy density becomes Planckian, yielding a new phase in the life of gravitationally collapsed object. At the bounce itself, the star has maximal density and is called a ‘‘Planck star’’ [13]. After this, which the collapsing object bounces out. In the meanwhile, the geometry outside the star, including the horizon also bounces, tunnelling [15] from a black to a white hole geometry. This process can be short in proper time, as well as, due to the huge gravitational time dilation, very long for an external observer.

This process may at first seem incompatible with the Birkhoff’s theorem (in the spherically symmetric case), but it is not so, because the Birkhoff’s theorem is lo-

cal but not global. In fact, it was realized in [2] that there is an exact solution of the Einstein equations, locally –but not globally– isometric to the Kruskal spacetime, which can surround a black hole that tunnels into a white hole, containing the quantum violation Einstein’s equation only within the small high curvature spatiotemporal region. Thus, a local quantum violation of classical GR in a compact region, namely a conventional tunnelling effect, permits the black holes to evolve into an anti-trapped region, that is, a white hole. This scenario has been explored in [12, 16–21].

If the transition happens at the end of the evaporation, when quantum gravity becomes relevant also outside the horizon, and where quantum gravitational transition amplitudes indicate its probability to approach unity, most of the energy of the black hole has already been radiated away into Hawking radiation, which is a dissipative phenomenon that breaks time-reversal invariance. Therefore, even if the transition itself can be described as an elastic non-dissipative phenomenon, the overall life of a black hole is very far from being time-reversal symmetric, and we may expect the white hole emerging from the transition to be a remnant with mass at the Planck scale.

The internal geometry of these objects is a remarkably precise realization, fully consistent with classical GR, of the ‘cornucopia’ intuition considered in the 1980’s and later abandoned on the basis of arguments that (as we shall discuss below) are not anymore cogent [22, 23].

There is a well known instability affecting eternal and macroscopic white holes: they can easily fall into a black hole as in the process described by the classical Kruskal geometry. On the the other hand, there are arguments indicating that a finite live Planck-mass white hole can be stabilized by quantum gravity [24], by shifting it into a quantum superposition of black and white geometry. The key of this stabilization is the Loop Quantum Gravity area gap, because of which there is likely no black (or white) hole with a mass smaller than the Planck mass.

Therefore the scenario predicts the existence of quasi-stable remnants of a known mass. A large number of these, of primordial or pre-big-bang origin, would behave precisely as dark matter. The possibility of direct detection of these object has been explored in [6].

Once quantum gravity phenomena are taken into account, the notion itself of horizon is necessarily different than the one relevant for the classical theory. *Event* horizons loses their relevance, because they are likely not to form at all. The Hawking radiation drags the dynamical evolution of the trapping (apparent, dynamical) horizon into regions where the Einstein equations do not hold anymore. Accordingly, the discussion on the so called black-hole information paradox needs to be reconsidered.

Even more dramatically, the traditional *definition* of a black hole as an object characterized by an *event* horizon becomes misleading: in the light of quantum gravity, the horizon of the realistic astrophysical black holes is likely *not* an *event* horizon, because while the Kerr geometry and its perturbations used in modelling astrophysical ob-

jects may be appropriate, its future time evolution (which determine whether the horizon is an event horizon) is not the one determined by the Einstein equations.

While the scenario illustrated takes fully into account quantum gravitational phenomena in the high curvature region, it assumes that classical GR and quantum field theory on curved spacetimes are good approximations before these regions. Contrary to what sometimes claimed, GR and quantum field theory taken together, yield no inconsistencies in the entire region where quantum gravity can be neglected. As argued in detail below, the common claim that inconsistencies appear already at the Page time derives from certain strong versions of the holographic hypothesis [25], or from confusions between ADM mass and Bondi mass, between thermodynamic and von-Neumann entropies, or between event and dynamical horizons. Thus, the scenario considered here is only based on conventional GR and conventional quantum ideas.

Yet, it implies a number of remarkable new phenomena. Due to the huge time dilation in a black hole, the process can last micro-seconds in local proper time, but billions of light-years observed from the outside; the internal volume of the hole can remain huge even as the size of the horizon shrinks; the star's bounce volume is much larger than Planckian, because the onset of quantum-gravity effects is governed by density, not size; the interior of an evaporating hole can keep memory of the initial state, without information loss. Information can follow a different path from energy, and be slowly released by the remnants, while most of the energy was previously lost into the Hawking radiation. This shows that unpalatable phenomena like 'firewalls' [26] are not to be expected on the basis of GR and quantum theory alone, without arbitrary holographic assumptions that are only string-motivated.

In the rest of the paper, we describe all this in detail.

## B. The domain of validity of classical gravity

The Carter-Penrose diagram of a (classical) Schwarzschild black hole created from a gravitational collapse is depicted on Figure 1. The dark grey region is where the classical theory becomes unreliable, due to quantum gravitational effects. We expect this to happen when the curvature becomes Planckian, for instance when the Kretschmann scalar

$$K^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48 \frac{M^2}{r^6} \quad (1)$$

becomes of order 1 in natural units. Here  $M$  is the black hole mass and  $r$  is the Schwarzschild radius. This happens before the  $r = 0$  singularity inside the black hole. Importantly the surface where this happens is space-like.

Just outside the horizon,  $K \sim \frac{1}{M^2}$ . The Hawking evaporation steadily decreases the mass  $M$  of an isolated black hole, bringing it down to Planckian values, hence

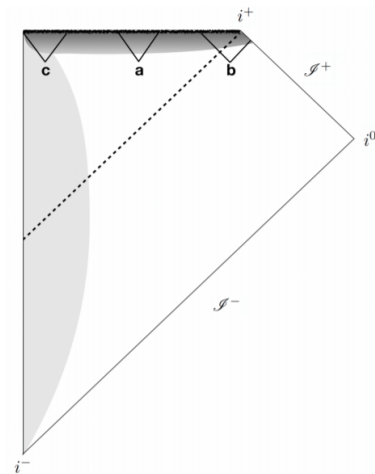


Figure 1. The Carter-Penrose diagram of a Schwarzschild black hole until the onset of quantum gravity. The light grey region is the collapsing star. The dark grey region is where quantum gravity becomes relevant.

the quantum region extends outside the horizon. As we will see later on, results in [7] indicate that the transition probability  $P$  from black holes to white holes is proportional to

$$P \sim e^{-M^2}. \quad (2)$$

Thus becoming dominant at the end of the evaporation, where  $M \sim 1$ . It is not however excluded that quantum effects could appear even earlier [2, 27], at a (retarded) time of order  $M^2$  after the collapse.

For the moment, we are not concerned with these estimates. The important point is that sooner or later the quantum region extends outside the horizon. This region can therefore be organized into three sub regions [28] (see Figure 1):

- Region **a** : The region which is neither directly causally connected to the horizon nor to the collapsing star.
- Region **b** : The region in the vicinity of the horizon.
- Region **c** : The collapsing star region.

The phenomena in these three regions can be considered causally disconnected, as they are separated by space-like distance. For a black hole of (initial) mass  $M$ , this distance is of the order [28]

$$L \simeq M^{\frac{10}{3}}, \quad (3)$$

which is huge for a macroscopic black hole.

The locus of the onset of quantum gravity is not 'at a point', but rather 'at a time'. It is not 'somewhere' but rather, 'at some time'. What happens inside a black hole cannot be understood in terms of a static or stationary model.

The fact that the various onsets of quantum gravity are spatially related implies by locality that what happens in a region of this locus is causally disconnected, hence independent from what happens in other regions. To understand the physics of the end of a black hole, we have to understand the quantum evolution of each of these three regions independently.

## II. NON DISSIPATIVE ASPECTS OF THE TRANSITION

### A. Planck stars

Let us start by studying what happens in the interior of a collapsing star, after it enters its horizon, when it reaches Planckian density. The simplest modelizations of a collapsing star is provided by the Oppenheimer-Snyder model [29]: a spherically symmetric pressure-less homogenous star free-falling under its own weight. Assuming the star to start at rest at past infinity, the metric inside a such a star can be written in co-mouving and proper time coordinates  $(T, R)$  as

$$ds^2 = -dT^2 + a^2(T)(dR^2 + R^2 d\Omega^2), \quad (4)$$

where the  $T = \text{constant}$  slices define the homogeneity foliation,  $d\Omega^2$  is the metric of the unit 2-sphere,  $R \in [0, R_{\text{boundary}}]$  and  $a(T)$  is known as the scale factor. The radial comoving coordinate of the boundary of the star can be chosen to be  $R_{\text{boundary}} = 1$  without loss of generality. The uniform density of the star is then  $\rho = m/\frac{4}{3}\pi a^3$ , where  $m$  is the total mass. Inserting this metric in the Einstein field equations gives the Friedmann equation for  $a(t)$ :

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho, \quad (5)$$

where the overdot means differentiation with respect to  $T$ . Eq. (5) can be solved:

$$a(T) = \left( \frac{9m(T - T_0)^2}{2R_{\text{star}}^3} \right)^{1/3}. \quad (6)$$

Without loss of generality we can take the time at which the star collapses to zero physical radius to be  $T = 0$ .

How does quantum gravity affects this dynamics? A major result in loop quantum cosmology [30–33] is that the Friedmann equation for the scale factor, Eq. (5), is modified by quantum gravity effect into

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\rho \left( 1 - \frac{\rho}{\rho_c} \right), \quad (7)$$

where the critical density  $\rho_c = \sqrt{3}c^2/(32\pi^2\gamma^3\hbar G^2) \sim c^2/\hbar G^2$ ,  $\gamma$  being the Barbero-Immirzi parameter, is a constant with the dimension of a density and Planckian value. This equation can be integrated to give

$$a(T) = \left( \frac{9mT^2 + Am}{2R_{\text{star}}^3} \right)^{1/3}, \quad (8)$$

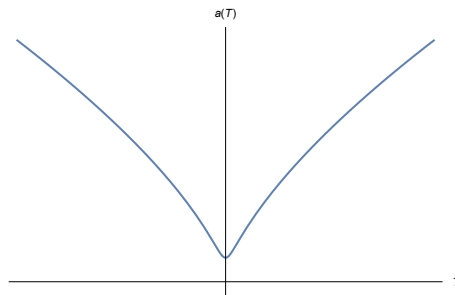


Figure 2. The scale factor  $a(T)$  in eq. (9) that gives the standard LQC bounce.

where  $A = 3/(2\pi\rho_c)$  is a parameter of Planckian value. In units where  $G = c = 1$ , the constant is  $A \sim \hbar \sim m_{Pl}^2$ . The last equation can be integrated, giving

$$a(T) = \left( \frac{9mT^2 + Am}{2} \right)^{1/3}. \quad (9)$$

Notice that  $a(T)$  is positive for the whole range  $T \in [-\infty, \infty]$ : it decreases for  $T < 0$ , reaches a minimum  $a_0 = \sqrt[3]{Am/2}$  for  $T = 0$  and then increases for  $T > 0$ . See Fig. 2. This is the characteristic bounce of loop quantum cosmology. This result shows that the line element in eq. (4) is well defined everywhere, without singular collapse: the star reaches a maximal density, where it is called a ‘Planck star’, and then bounces and expands.

Of course the time-reversal symmetric bounce is an approximation, because dissipative effects break this symmetry. The Hawking radiation generates an ingoing flux of negative energy that is likely to reduce the energy of the star, in the interior of the hole. We will address these phenomena later on, when discussing the dissipative aspects of the dynamics of the black hole.

To get a feeling of the physics of the bounce, we can rewrite (7) in the form

$$\dot{a}^2 = \frac{2m}{a} - \frac{Am^2}{a^4}. \quad (10)$$

The coordinate  $T$  is the proper time along the comoving worldlines, hence it is also the proper time on the boundary of the star. This means that Eq. (10) gives also the evolution of the physical radius  $r_b(T) = a(T)$  of the star in its own proper time, hence

$$\dot{r}_b^2 = \frac{2m}{r_b} - \frac{Am^2}{r_b^4}. \quad (11)$$

This shows that a mass element on the boundary of the star falling in its own proper time feels a potential that is the Newtonian one, precisely as in classical GR, but corrected by a repulsive quantum term proportional to the inverse of the 4th power of the radius. This is the short scale repulsive force due to the quantum pressure.

Is this dynamics compatible with the dynamics of the surrounding geometry?

## B. Black to white transition

In the classical Oppenheimer-Snyder model, the geometry of the collapsing star is compatible with a surrounding Schwarzschild geometry. That is, matching conditions between the two geometries are satisfied on the star boundary. To confirm this, consider the surface of the star as a free falling spherically symmetric shell in a Schwarzschild metric

$$ds^2 = -F(r)dt^2 + dr^2/F(r) + r^2 d\Omega^2, \quad (12)$$

where  $F(r) = 1 - 2m/r$ . The shell follows a radial time-like geodesic and therefore satisfies

$$-1 = -F(r)\dot{t}^2 + \dot{r}^2/F(r) \quad (13)$$

The conservation law associated to the time-like Killing symmetry gives  $F(r)\dot{t} = E$  where  $E$  is a constant that can be taken equal to unit if the shell starts at rest at infinite radius. Hence  $-F(r) = -E^2 + \dot{r}^2$ , that is

$$\dot{r}^2 = \frac{2m}{r}. \quad (14)$$

A comparison with (11) shows that this is exactly the change of the Schwarzschild radius in proper time of the boundary of the star, in the classical ( $A=0$ ) case.

But this suggest immediately the form of a metric which is compatible with the bouncing star, in the quantum case: it is again (12) but with

$$F(r) = 1 - \frac{2m}{r} + \frac{Am}{r^4}. \quad (15)$$

This is an interesting metric. (For earlier attempts to write the black hole metric in the quantum region see for instance [34–46]) It was suggested already in [13] and has been derived by various quantum gravity research groups, using different methods [21, 45, 47–51], as a credible candidate for the effective metric in the high curvature of a spherically symmetric black hole. For instance, it is uniquely determined by requiring it to satisfy matching conditions with the collapsing star and keep the Killing symmetry of the Schwarzschild geometry (which in the interior of the horizon is space-like, not time-like as in the exterior.)

This metric has inner and outer Killing horizons. See [52] for full details. For  $m \gg m_{Pl}$ , that is  $m^2 \gg A$ , the outer one is at

$$r_+ = 2m + O(A/m) \sim r_{Schwarzschild} \quad (16)$$

in the classical region. If  $m$  is large, this is a negligible modification of the usual Schwarzschild horizon. While the inner one is at

$$r_- = \sqrt[3]{Am/2} + O(A^{2/3}/m^{1/3}) \sim \sqrt[3]{m/m_{Pl}} l_{Pl} \quad (17)$$

deep inside the quantum region, where the spacetime curvature has Planckian size. These are all also apparent

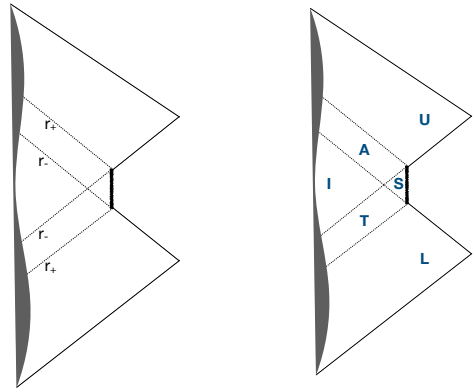


Figure 3. Conformal diagram of the maximal extension of the spacetime representing the star and the exterior region defined by eqs. (12) and (15).

horizons, that is, they separate trapped, non-trapped and anti-trapped regions [52].

The Carter-Penrose diagram of the maximal extension of this metric is depicted in Figure 3, which shows the relative location of these horizons.

And additional physical reasons that make this geometry interesting. It has the global structure (including the inner horizons) very similar to the Kerr and the Reissner-Nordström black holes, and the general Kerr-Newman black holes. The full bounce in the Reissner-Nordström case has been described in [53], while the Kerr case has not been constructed yet.

Notice that it has (i) two asymptotic regions and (ii) a time-like singularity in a high curvature region. These two features make it an unlikely candidate for describing the actual physics of realistic black holes. But a surprising result on spherically symmetric solutions of the Einstein's field equations, solve both these difficulties. This is illustrated in the next section.

## C. The exterior metric

The Birkhoff's theorem is often presented as stating that the only spherically symmetric spacetime compatible with the Einstein's field equations is the Kruskal geometry. At a sufficient distance from a black hole, we expect quantum effects to be negligible and therefore the theorem to hold. Hence we might expected that whatever happens away from the quantum region, namely where quantum effects are negligible, must form a subset of a Kruskal geometry. In the Kruskal metric, there is an anti-trapped region, but it is before, not after the trapped region. Hence there seem no way to join the metric described above with a single external asymptotic region.

But the above is wrong. The reason is that Birkhoff's theorem is local, not global: it states that a spherically

symmetric solution of the Einstein field equations is locally, not necessarily globally, isometric to the Kruskal metric. This seems a subtlety, but has momentous consequences for understanding the dynamics of quantum black holes.

In fact, as surprisingly realized in [2], there is an *exact* solution of the Einstein's field equations, with a single asymptotic region, that can surround a quantum transition from a black to a white hole. The Carter-Penrose diagram of this solution is depicted in the left hand side of Figure 44, while the right hand side shows how this can be *locally* isometrically mapped onto the Kruskal spacetime. Notice that the map is not injective.

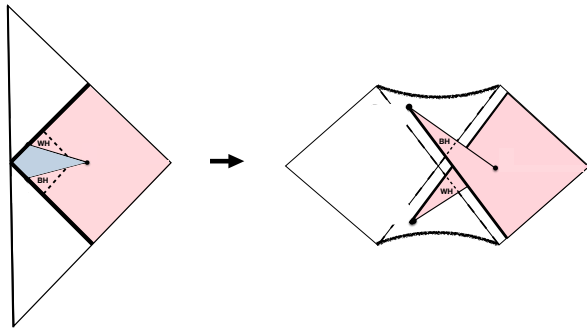


Figure 4. The spacetime discovered in [2], describing the collapse of a null shell into a black hole and its bounce out a white hole (left); and the way it can be locally isometrically mapped onto the Kruskal spacetime (right). The pink region is an exact solution of the classical Einstein equations.

This solution describes a null shell collapsing into a black hole and bouncing out from a white hole. The key point is that the black hole (the trapped region) is in the past of the white hole (the anti-trapped region) while in the Kruskal spacetime it is its future. This shows that an exact solution of the Einstein equations is compatible with a quantum process happening in a compact high curvature region tunnelling a black into a white hole.

The same idea solves the two difficulties mentioned at the end of last section, providing a spacetime with a single asymptotic region and no singularities for the bouncing star. This can be constructed by cutting and gluing the maximal extension of the metric defined by eqs. (12) and (15), as follows.

First, pick a point  $\alpha$  in the interior region, on the surface invariant under time reversal and a point  $\beta_L$  outside the horizons, in the first asymptotic region, as in the left panel of Figure 5. This choice depends on three parameters: the advanced times  $v(\alpha)$  and  $v(\beta_L)$  and the Schwarzschild radius of  $\beta_L$ . Next, consider the blue line of the figure. This has a null portion joining  $\alpha$  and  $\beta_L$  with their last common past event and a spacelike region joining  $\beta_L$  with spacelike infinity along a constant Schwarzschild time. Next draw the line symmetric to this under time reversal, as in the right panel of Figure 5.

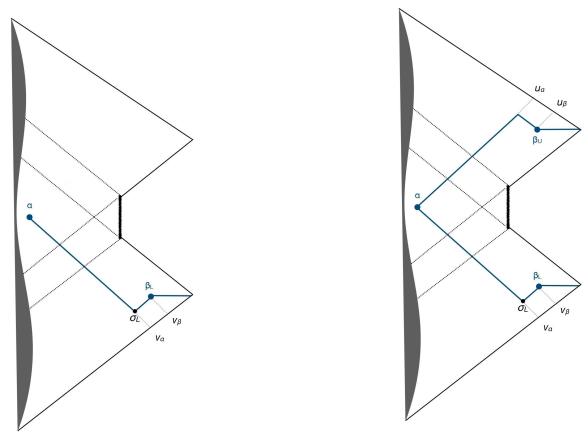


Figure 5. The construction of the single asymptotic region spacetime, I: choosing the cut and glue surface

Next, delete the entire spacetime region enclosed in the blue line and glue its two space-like portions as in Figure 6. This is clearly possible since they are both constant Schwarzschild time surfaces.

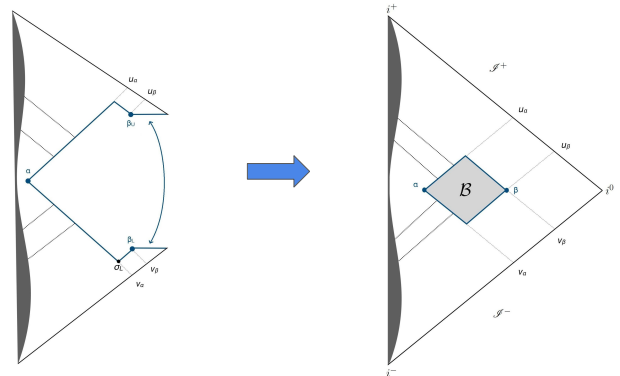


Figure 6. The construction of the single asymptotic region spacetime, II: deleting the singular region and gluing the two equal time exterior surfaces.

The resulting spacetime has a single asymptotic region, is everywhere locally isomorphic to the maximal extension of the metric (12-15), and has a hole, depicted as the  $B$  diamond of Figure 6. In [52] it is shown that the  $B$  region admits a regular Lorentzian metric compatible with rest of the spacetime geometry. The  $B$  region and its physics will be discussed in detail later on.

The popular idea that at the end of the evaporation the black hole disappears is not supported by any theory and contradicts unitarity. The above construction offers a far more plausible alternative. See Figure 7.

The relation of the resulting geometry with the various quantum gravity phenomena is depicted in Figure 8. (See also Figure 8 in [54].)

We have constructed the above metric introducing three parameters ( $v(\alpha)mv(\beta_L)$ ) and the Schwarzschild ra-

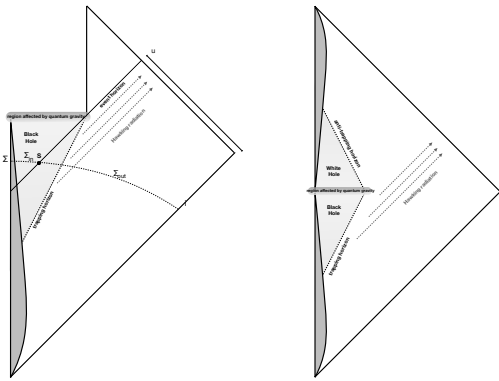


Figure 7. Left: a popular idea of what happens at the end of the evaporation. This is not supported by any theory and contradicts unitarity. On the right, the plausible scenario.

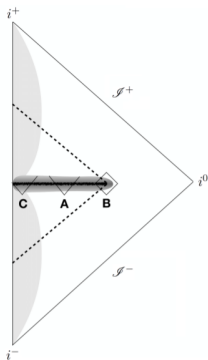


Figure 8. The Carter-Penrose diagram of the black to white transition. The dark grey region is the quantum gravity region. The black hole (trapped region) is below the quantum gravity region while the white hole is above. The trapping horizons are the dashes lines.

dius of  $\beta_L$ .) The resulting geometry (a part from the choice of the metric inside  $B$ ) depends then on these parameters, plus the mass of the star. Of the three parameters, two simply locate the  $B$  region: they determine the minimal and maximal radius of the diamond defining it. The last one, on the other hand determines the global geometry of the spacetime, in the same sense in which the radius of a cylinder determines the global geometry of a locally flat cylinder. Its geometrical and physical interpretation can be seen as follows: it determines the time  $T$  at which an observer at large from the hole sees the duration of the process.

This is illustrated in Figure 9, where the red line represents an observer at a fixed radial distance  $R$ . As shown in [52], this time is

$$T = 2R + 4m \ln r - 2m - 4m \ln \delta. \quad (18)$$

The first term on the right hand side can be interpreted as the back and forth travel time for the light from the observer to the hole. The second term is the standard relativistic correction. The last term is independent from

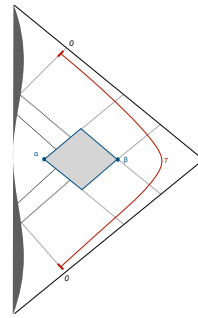


Figure 9. The definition of the duration of the process. The red line represents an observer at fixed radius.

the radius and represents a property of the transition itself. The quantity  $\delta$  entering it depends on the choice of  $\beta_L$  as follows:  $r = 2m + \delta$  is the radius at which the constant Schwarzschild-time surface passing by  $\beta_L$  intersect the surface of the falling star. This is clearly very close to the collapse (the point where the surface enters entirely in its horizon), if the lifetime of the hole is long. Thus

$$\tau = -4m \ln \delta \quad (19)$$

can be taken as a definition if the duration of the quantum process, as observed from infinity. In our construction, it is determined by how the geometry has been constructed cutting and gluing. Physically, it is determined by the quantum theory of the geometry of the  $B$  region, to which we now turn.

#### D. The Boundary Region

In the previous sections, we have constructed a space-time geometry that could describe the evolution of a black hole past the region where quantum gravitational effects dominate. How well supported by known physics is this picture?

The bounce of the star is supported by the LQG modelling of the quantum dynamics of symmetric spacetimes. The physical plausibility of the effective metric outside the star but inside the outer horizons is also supported by various LQG modelling of the quantum dynamics of symmetric spacetimes. The structure of the  $B$  region, on the other hand, has only been guessed as a plausible solution to the requirement of a global dynamics compatible with the external geometry dictated by the classical Einstein equations. Does it follow from quantum gravity? Is it allowed by it?

To address this question we need to step back and think more precisely what we are doing. First, let us remind once more that for the moment we are disregarding the explicit taking into account of the dissipative effects that break time reversal invariance. These will be studied in the next section. Then, consider the fact that we are

describing a quantum process. There are different ways of describing quantum processes. In some regimes, these can be described as corrections to the classical equations of motion. This is probably not viable for the quantum transition of the horizon, which is a non-perturbative process. Another possibility is to describe the evolution of a quantum state. Quantum states that approximate classical configurations, such as coherent states, spread. We could therefore consider the full quantum state of the geometry after the actual quantum process as a quantum state, namely a quantum superposition of geometries. This is analogous to describing nuclear radioactivity in terms of a wave function of the emitted particle widely spread in space and time. Correct, but incomplete. What we observe in nuclear radioactivity, indeed, is not a wave function of the emitted particle widely spread in space and time. Rather, it is a specific space and time where this particle is detected, say by a Geiger counter. In Copenhagen terms, we observe the result of a measurement. In Many World terms, we observe the position of the particle in one branch, after decoherence. In Relational Quantum Mechanics, we observe the metric relative to us, again after decoherence. The theory, of course, does not predict the specific outcome of the observation, but only the probability of the possible alternatives. To compute these, we need to compute transition amplitudes. This is how we can address the problem of describing the  $B$  region in loop quantum gravity.

That is, we can compute the transition amplitude for the geometry to make a quantum transition from its value in the past boundary of  $B$  to its future boundary. Notice that the geometry (both intrinsic and extrinsic) of these two boundaries is known from the construction of the above section. We have simply to compute the LQG transition amplitudes between quantum states approximating these geometries.

The covariant formulation of LQG is precisely formulated to do this calculation, using approximate truncations of the degrees of freedom. The calculation has been performed, under some drastic simplifications by Christodoulou and D'Ambrosio using analytical techniques in [18, 28, 55]. The result had been confirmed numerically in [11]. Numerical calculations are in course to go beyond these approximations. The way this is performed is summarized in the next section.

### E. The LQG transition amplitude and the Christodoulou-D'Ambrosio result

The covariant formulation of Loop Quantum Gravity defines transition amplitudes between quantum states of the geometry, in suitable truncations of the numbers of degrees of freedom. The number of degrees of freedom can be truncated by approximating the 3d geometry of the boundary by means of a cellular decomposition. The

two skeleton of the dual of the decomposition form a graph and the truncated variables can be taken to  $SU(2)$  group elements  $U_l$  on the links  $l$  of this graph. The quantum states of the geometry can then be taken to functions of the  $U_l$ 's in the space  $L_2[SU(2)^L/SU(2)^N]$ , where  $L$  and  $N$  are respectively the numbers of links and nodes of  $\Gamma$ , as in lattice Yang Mills theory. The algebra formed by the group elements  $U_l$  themselves and the Left invariant operators is the observable algebra, which has a natural interpretation in terms of functions of the discretized geometry. A large literature has developed a theory of coherent states for this algebra, representing semiclassical geometrical states.

Transition amplitudes can be intuitively understood as Feynman sums over geometries. Concretely, they are defined in a truncation. A truncation is here given by a two-complex bounded by the boundary graph. The amplitude is defined by the product of the vertex amplitude

$$A_v(\psi) = P_{SL(2,C)} Y_\gamma \psi(\mathbb{1}) \quad (20)$$

where  $P_{SL(2,C)}$  is the projector on the  $SL(2,C)$  invariant part and  $Y_\gamma$  is the 'simplicity map' from  $SU(2)$  to  $SL(2,C)$  representations defined, in the canonical basis, by

$$|m; j\rangle \mapsto |\gamma j, j; j, m\rangle. \quad (21)$$

See [8, 9] for the full details. This amplitude has been shown to give the Einstein dynamics in suitable limits [56] and can be taken as a definition of the covariant LQG theory.

To apply this theory to the  $B$  region we need to find a suitably simple cellular decomposition of this region. A step in this direction has been taken in [57], but a full analytical calculation has been completed only in a simpler setting, where the entire quantum region

The complexity of the calculation on the  $B$  region alone is given by the topology of  $B$ , which is the product of a two-sphere and a disk. The disk is the product of a finite time interval and a finite radial interval and it is delimited by an exterior two-sphere  $S_+$  that surrounds the horizon and by an interior two-sphere  $S_-$  surrounded by the horizon (sitting on the bounce radius of the transition of the internal geometry of the black hole). The two two-spheres  $S_+$  and  $S_-$  split  $\Sigma$  into a past component  $\Sigma^p$  and a future component  $\Sigma^f$ .

Considering instead the entire quantum region simplifies the topology. The boundary of the quantum region can be taken to be as depicted in blue Figure 10. The tip, at the largest radius, is a two sphere, chosen outside the Planckian curvature region. The lowest (past) part of the boundary and the upper (future) one are both 3d balls.

Each of this can be triangulated with 4 tetrahedra and the interior can be triangulated with two four-simplices sharing an internal tetrahedron, as depicted in Figure 11.

We refer to [18, 28, 55] for analytical calculation. Numerical calculations have been performed in [58] and in



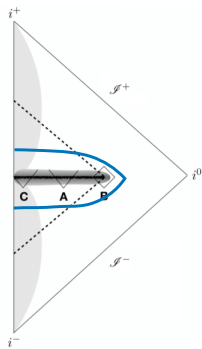


Figure 10. The boundary of the quantum region for the simplified calculation of the transition amplitude.

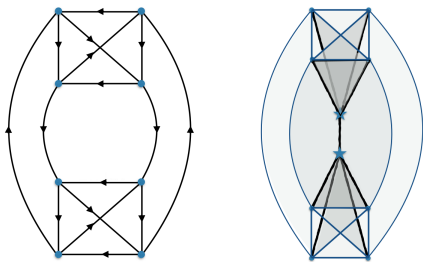


Figure 11. The graph of the spin network and the two complex of the spinfoam for the calculation of the transition.

[59]. The first utilizes the `sl2cfoam-next` code for computing spinfoam transition amplitudes. A good introduction to this is in [60]. The code permits to explore the low-spin (small-mass) regime of the amplitude. The second utilizes the complex saddle point method for computing spinfoam amplitudes, and permits to explore the high-spin (large-mass) regime. A key result of these investigations (that contradicts previous expectations) is that the transition amplitude gives a transition probability proportional to

$$P \sim e^{-\frac{Gm^2}{ch}} \sim e^{-\left(\frac{m}{m_P}\right)^2}. \quad (22)$$

Notice that this is non-analytic in the Planck constant. Therefore it is a genuine non perturbative result that cannot be obtained in conventional perturbation theory expanding around a classical solution. The exponential is characteristic of quantum tunnelling phenomena. The transition, in fact, is forbidden classically (classically the geometry evolves into a singularity) and is characteristic quantum tunnelling effect.

Another way of understanding this result is to connect to the asymptotic behaviour of the amplitudes

$$W \sim e^{iS_{Regge}} \sim e^{i\sum_f j_f \Theta_f(j)}, \quad (23)$$

where  $S_{Regge}$  is the Regge action: a sum over the faces of the face spin times the dihedral angle of the face, which is a function of all the spins, determined by the interpo-

lating flat geometry. Here there is no interpolating geometry and the angles turn out to be imaginary, giving

$$W \sim e^{-c\sum_f j_f} \sim e^{-c\sum_f A_f}, \sim e^{-cm^2}. \quad (24)$$

The simplest way to interpret this result is in analogy to nuclear radioactivity: as a transition probability per unit of (here Planck-) time. The immediate consequence of this result is that the transition probability is exponentially suppressed if the mass of the hole is larger than the Planck mass.

Anticipating the discussion about dissipative effects, we can see that toward the end of the Hawking radiation,  $m$  decreases and approaches  $m_P$ . At this stage the transition becomes increasingly probable, going to probability unit when the mass becomes actually Planckian.

In other words, the calculation indicates that the transition can happen and it is likely to happen at the end of the Hawking evaporation, and give birth to a white hole with near Planckian mass.

## F. White holes

Before addressing the dissipative aspects of the life of a hole, let us pause to clarify a few properties of the white holes. White holes, like black holes, are exact solutions of the Einstein's equation. Like black holes, they have long been considered playing no role in our universe by a majority of physicists. The situation has changed for blackholes in the last decades. It has not yet equally changed for white holes, but it might soon.

A white hole spacetime is simply the time reversal of a black hole spacetime. For instance, a classical black hole formed by a collapsing star and its time reversal, a white hole from which star emerges, are depicted in Figure 12.

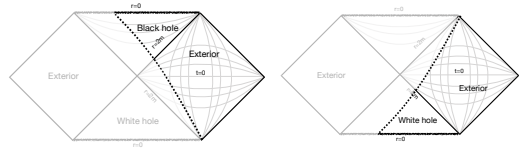


Figure 12. The spacetimes of a black and a white hole (outside the respective stars, as subsets of the Kruskal geometry).

The reason black holes have been traditionally taken more seriously as candidates for real objects than white holes is that we know a well understood classical scenario for how a black hole can form, but not so for a white hole. However, since the two are the time reversal of one another, this means that we know a well understood classical scenario for how a white hole can end, but not so for a black hole. The end of a black hole is definitely a quantum phenomenon, because it involved high curvature regions, where quantum theory cannot be neglected. The same must be true for the birth of a white

hole. All this points to a simple possibility: the end of a black hole is the birth of a white hole, via a quantum transition process. This is precisely the scenario we are studying here. A white hole can be originated by a dying black hole.

The difference between a black hole and a white hole is not very pronounced. In fact, observed from the outside (say from the exterior of a sphere of radius  $r = 2m + \epsilon > 2m$ , where  $m$  is the mass of the hole) and for a finite amount of time, a white hole cannot be distinguished from a black hole.

This is clear from the usual Schwarzschild line element, which is symmetric under time reversal, and therefore describes equally well the exterior of a black hole and the exterior of a white hole. Equivalently, zone II of the maximal extension of the Schwarzschild solution is equally the outside of a black hole and the outside of a white hole (see Fig. 1, Left). Analogous considerations hold for the Kerr solution. The continuation of the external metric of a stationary Kerr or Schwarzschild spacetime inside the radius  $r = 2m + \epsilon$  contains both a trapped region (a black hole) and an anti-trapped region (a white hole).

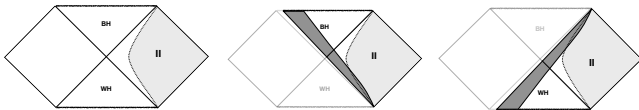


Figure 13. *Left: in the extended Schwarzschild spacetime, which is stationary, the (light grey) region outside  $r = 2m + \epsilon$  (dotted line) is equally the outside of a black and a white hole. Center: A collapsing star (dark grey) replaces the white hole region (“WH”) in the non-stationary collapse metric. Right: The time reversed process. The difference between the last two can only be detected looking at the past, or the future.*

What distinguishes then the objects we call ‘black holes’ from ‘white holes’? The objects in the sky we call ‘black holes’ are described by a stationary metric only approximately and for a limited time. We expect that (at least) in the past their metric was definitely non-stationary, and they were produced by gravitational collapse. The energy contained inside  $r = 2m + \epsilon$  was less than  $m$  in the past and the continuation of the metric inside this radius contains the trapped region, but not the anti-trapped region, which is instead replaced by the region describing the collapsing star. Therefore seen from the outside a ‘black hole’ (as opposite to a ‘white hole’) is only characterised by the fact that in the past it does not have an anti-trapped region (see Fig. 1, Center). Viceversa, a white hole is an object that from the exterior and for a finite time is undistinguishable from a black hole, but in the future ceases to be stationary, the amount of energy inside  $r = 2m + \epsilon$  decreases, and there is no trapped region in the future (see Fig.1, Right).

### III. DISSIPATIVE ASPECTS OF THE TRANSITION

#### A. Black hole lifetime

The most important dissipative phenomenon in the life of a black hole is the Hawking radiation. This is a markedly irreversible process that dissipates energy of the collapsed star into heat of the emitted thermal radiation.

The lifetime  $\tau_{BH}$  of a black hole is known by Hawking radiation theory. It can be estimated as follows. The Hawking radiation is thermal, namely it has Planck spectrum. The peak of the spectrum is on a wavelength  $\lambda$  that is determined by the only length parameter in the problem: the Schwarzschild radius  $2Gm/c^2$ . Therefore the radiation is mostly composed by quanta of frequency

$$\nu = c/\lambda \sim \frac{c^3}{Gm} \quad (25)$$

These have an energy  $E = h\nu$  and therefore (being thermal) a temperature  $T$  given by

$$kT = E \sim \frac{c^3 \hbar}{Gm} \quad (26)$$

This is (up to a numerical factor) Hawking’s temperature. The emission of the horizon can be modelled as the emission of a sphere at this temperature with the area of the horizon. This gives an emitted power (going back to natural units)

$$P = \frac{dm}{dt} \sim AT \sim m^2 m^{-4} = m^{-2}. \quad (27)$$

This is a differential equation for  $m(t)$  that can be immediately integrated giving  $m^3 \sim t$ . Therefore we expect that the finite lifetime of a black hole due to the Hawking evaporation is of the order

$$\tau_{BH} \sim m_o^3 \quad (28)$$

where we have indicated as  $m_o$  the *initial* mass of the hole.<sup>1</sup>

The evaporation shrinks the horizon, bringing it close to Planckian size, where the black to white transition has high probability to happen. The white hole generated by the process has then an horizon of Planckian size. (This lifetime can be shorter if quantum gravity fluctuations trigger an earlier tunnelling. This could be as early as  $\tau_{BH} \sim m_o^2$  as suggested in [2, 27] (see also [61–65]). This possibility will be considered elsewhere. Here we focus on the possibility that the transition happens at the end of the evaporation.)

<sup>1</sup> The effect of the back reaction on the metric has been studied in [132]. See references there.

However, this does not mean that the white hole produced is “small” in all sense of the world. In fact, its interior is vast. To understand this point, a detour on the size of the interior of black holes is important, as this is a frequently misunderstood issue.

### B. How big is the black hole interior?

In Minkowski spacetime, we say that a space-like sphere of radius  $r$  encloses a space-like ball of volume  $4\pi r^3/3$ . As a 3d surface in Minkowski space, ball is characterized by two properties: it is linear, and it is the surfaces that maximizes the volume among those bounded by the sphere. Any deformation in fact, is time-like and decreases the volume. In a curved Lorentzian manifold linearity is meaningless, but we can still talk about the volume enclosed in a spacelike (topological) 2-sphere by defining it as the volume of the 3d surface bounded by the sphere that maximizes the volume. This definition, introduced in [66], allows us to talk of the interior volume of a black hole and provides a simple intuition of the internal geometry of a black hole, by defining a natural foliation of this geometry.

Consider therefore the spherical symmetric spacetime of a collapsing star and choose a retarded time  $v$ . The intersection of the retarded time with the horizon defines a sphere on the horizon. Let  $V(v)$  be the volume of the maximal-volume spacelike ball bounded by this sphere. This is the volume of the interior of the black hole at the retarded time  $v$  on its horizon. The dependence of this volume on the retarded time has been computed both for an eternal horizon and for an evaporating one [17, 66–70] and in both cases it is linear in  $v$ .

This result is important. The interior of a horizon is not stationary: it is dynamical. In the natural foliation considered above, its volume increases steadily in time. More specifically, the interior is like a tube whose radial dimensions shrink in time while its longitudinal dimension increases, in such a way that the total volume increases.

This means that that old evaporated black hole has a small horizon but a huge internal volume. Ignoring this fact has nourished a wrong intuition about the geometry of evaporated black holes.

At the moment of the quantum transition, therefore a macroscopic black hole is a narrow and extremely long geometrical entity. It is radially shrinking and growing in length. The transition is a bounce that reverses the process: the white hole is expanding radially and shortening.

### C. Instability

A classic macroscopic white hole is an unstable solution of the Einstein equations (see Chapter 15 in [71] and references therein). This means that there are solutions

with initial data that are arbitrarily close to the white hole initial data, but have a qualitatively different future.

The qualitative different future is the formation of a black hole in the future of the white hole. That is, a white hole is unstable toward becoming a black hole. The transformation of a white hole into a black hole is the process described by the vacuum Kruskal spacetime.

To see why there is this instability it is easier to address the time reversed scenario, because we have a better intuition about it: to show that solutions with *final* data arbitrarily close to those of a *black* hole, must have a qualitatively different past from the black hole. Or in other words, that the data on future null infinity on a black hole spacetime cannot admit certain arbitrary small variations.

To see this, consider a black hole of mass  $m$ , formed by a collapsed star. In the unperturbed solution, no energy reaches future null infinity (here we are considering classical physics: no Hawking radiation). Now, consider a perturbation of this spacetime in which there is a small pulse of null radiation with total energy  $\epsilon$  arriving on null infinity. Here  $\epsilon$  can be arbitrarily small. This radiation must be emitted by the surface of the star before this enters the horizon. Say it was emitted when the radius of the star was  $2m + \delta$ . This must be outside the Schwarzschild radius of the star plus the energy to be emitted, otherwise it could not reach null infinity. Hence necessarily  $2m + \delta > 2m + \epsilon$  that is, we must have  $\delta > \epsilon$ . Now let's  $u_\delta$  the retarded time of the emission point. Necessarily, the energy pulse of energy  $\epsilon$  must reach null infinity *before*  $u_\delta$ . Therefore, no perturbation with energy  $\epsilon$  can reach null infinity *after*  $u_\delta$ .

Can a perturbation with energy  $\epsilon$  reach null infinity *after*  $u_\delta$ , with some different past? Yes it does! A white hole of mass  $2m + \epsilon$  can emit the pulse and then become a black hole of mass  $m$ ! In this case, the pulse is never at a radius where it can trigger an increase of the size of the blackhole.

Just time reversing this scenario shows immediately why a classical eternal white hole of mass  $m$  is unstable: an arbitrary small pulse originating from past null infinity sufficiently early in time will bring enough energy to be inside the Schwarzschild radius before reaching the star emerging from the white hole, thus triggering the formation of a black hole. As we shall see, quantum theory may alter this picture dramatically. For this, however, we have to start considering the dissipative aspects of the life of a black hole.

In other words, the spacetime depicted in the Center panel of Figure 1 does not change much under a small arbitrary modification of its initial conditions on past null infinity; but it is drastically modified if we modify its final conditions on future null infinity. This is intuitively simple to grasp: if we sit on future null infinity and look back towards the hole, we see a black disk. This is the final condition. A slightly perturbed final condition includes the possibility of seeing radiation arriving from this disk. This is impossible in the spacetime of the Center panel of

Figure 13, because of the huge red shift of the radiation moving next to the horizon, but it is possible in the Left spacetime, because the radiation may have crossed over from the other asymptotic region.

The same is true for a white hole, reversing the time direction. In the spacetime depicted in the Right panel, with some radiation, there is necessarily a dark spot in the *incoming* radiation from past null infinity. If we perturb this configuration, and add some incoming radiation in this dark spot, the evolution generically gives the spacetime of the Left panel.

Physically, what happens is that this radiation moves along the horizon, is blue shifted, can meet radiation coming out of the white hole and this is more mass than  $m$  at a radius  $r \sim 2m$ : it is mass inside its Schwarzschild radius. At this point the region is trapped, and a black hole forms. Consequently the evolution of the perturbed initial conditions yields the spacetime of the Left, not the one of the Right: the white hole is unstable and decays into a black hole. This is the standard “instability of white holes”.

#### D. Planckian Remnants

The arguments given above suggest that at the end of the evaporation a black hole undergoes a quantum transition to a white hole with a Planckian-size horizon and a vast interior. The possibility of remnants with such a structure was considered in the 1990’s [72]. What was not realized at that time is that classical GR does in fact predict the existence of objects with the similar properties: white holes; and quantum theory could account for their formation at the end of the evaporation and for their stability.

Let us indeed discuss how the instability discussed in the last paragraph can affect the remnants formed at the end of a black hole evaporation. There are reasons to believe it does not. First, as observed in [12], the wavelength of the perturbation needed to trigger the instability must be smaller than the size of the hole. To make a Planck size black hole unstable, we need trans-Planckian radiation, and this is likely not allowed by quantum gravity.

Independently from this, if a Planck-scale white hole is unstable, its decay mode is into a Planck-scale black hole. Let us introduce some notation to describe this. We denote as  $|m_o, m\rangle_B$  the state of a black hole evaporated from an initial mass  $m_o$  to a final mass  $m$  and as  $|m_o, m\rangle_W$  a white hole that can emerge from the quantum tunnelling of such black hole. Namely from the process

$$|m_o, m\rangle_B \xrightarrow{\text{tunnelling}} |m_o, m\rangle_W. \quad (29)$$

Now, the instability of a white hole means the possibility of the process

$$|m_o, m\rangle_W \xrightarrow{\text{instability}} |m_o, m\rangle_B. \quad (30)$$

A similar possibility of oscillations between these two states has been also studied in [73–75]. Quantum mechanically the joint possibility of the two transitions imply that if the system is free to dissipate energy into radiation it will settle on its lowest energy state, which will be a superposition of the two states, of the form

$$|m_o, m\rangle = \alpha|m_o, m\rangle_W + \beta|m_o, m\rangle_B. \quad (31)$$

Since the black and white hole are indistinguishable from the exterior, this superposition has no effect on the exterior. The minimal energy means of these objects corresponds to the minimal area of their horizon. In loop quantum gravity, there is a minimal non vanishing area, which is

$$A_{min} = 4 \frac{\sqrt{3}}{\pi} \gamma \hbar G / c^3 \quad (32)$$

where  $\gamma$  is a constant of order unit. This corresponds to an approximate Planckian mass  $m_P$ .

Once a remnant has attained this minimal size, it can still radiate away all its energy, and dissipate into flat space. However, this last transition

$$|m_o, m_P\rangle \rightarrow |0\rangle \quad (33)$$

is likely to be strongly suppressed for several reasons.

Recall that the a hole like  $|m_o, m\rangle_B$  is not uniquely by its mass. Rather, holes of the same mass can have different interiors, depending on the initial mass  $m_o$ . In other words,  $m_o$  represents quantum numbers in addition to the mass.

In flat space physics, a small volume with a small energy can contain only a finite number of different states. This is what prevents the blackbody UV catastrophe. Intuitively, to have many states we need many particles and to have many particles with small total energy we need to have them with long wavelength, but this is not available in the volume contained within a small sphere. small volume. In general relativistic physics, however, these constraints do not hold, because on a curved geometry an arbitrarily large volume can be enclosed into an arbitrary small sphere. Hence an arbitrarily small horizon can enclose any number of different states, limited only by the maximal size  $m_o$  of the parent black hole and by the finiteness of the time during which the black hole interior have had the opportunity to grow.

The possibility of this oscillation has been considered in [73–75]. For macroscopic black holes, the existence of these oscillations depend on the tunneling probability when  $m \gg m_{Pl}$ . This was estimated to be large by [73–75], on the basis of a black-hole lifetime computation giving  $\tau_{BH} \sim m_o$ . In [18] this same timescale has been reinterpreted as the duration of the actual tunnelling transition, while the transition probability has been estimated to be suppressed by a factor

$$P \sim e^{-\left(\frac{m}{m_{Pl}}\right)^2} \quad (34)$$

which ceases rendering the transition highly suppressed only towards the end of the evaporation when  $m \sim m_{P\ell}$ . In all cases, remnants are stable.

If the evolution does allow both processes (30) and (29), from a quantum mechanical perspective it may be reasonable to consider a ground state of a hole with large  $m_o$  as formed by a quantum superposition of the two states:

$$|m_o, m_{P\ell}\rangle = \frac{1}{\sqrt{2}} \left( |m_o, m_{P\ell}\rangle_W + |m_o, m_{P\ell}\rangle_B \right) \quad (35)$$

as the quantum state of the remnant. In any case, white hole instability does not affect the fact that remnants remain Planck size objects with Planck mass for a very long time.

Possible astrophysical implications have been explored in [3, 76–86], and its relevance for are currently under intense investigation.

### E. The lifetime of the white hole

What is then the lifetime of the subsequent white hole? A number of arguments [12] suggest that this should be long. The main reason has to do with the information trapped inside the hole. The Hawking radiation is genuinely thermal: it is describe by a quantum state which is not pure: it is a density matrix. This is because the Hawking radiation that escapes to infinity forms together with negative radiation that falls inside the hole. The two are entangled, therefore the part escaping to infinity has von Newman entropy. The total entropy of the Hawking radiation is of the order of the Bekenstein Hawking entropy

$$S_{BG} = \frac{A}{4}. \quad (36)$$

where  $A \sim m_o^2$  is the *initial* area of the black hole, before the beginning of the evaporation. Some scientists expect that the total entropy of the Hawking radiation start decreasing at Page time because late Hawking quanta are correlated with early ones. We believe that this expectation is wrong as we will explain in detail later on.

This implies that an amount of information of the order  $S_{BH}$  is trapped inside the hole. This must later be emitted in the form or radiation by the white hole. If the white hole has a total available energy of the order of the Planck mass, namely unit in natural units, then it must be able to emit a large amount of information with a very small energy. To carry enough information, we need

a number of particles of order  $N$  where  $2^N \sim e^{S_{BH}} \sim e^{m_o^2}$  and therefore a temperature  $T \sim m_o^2$ . Now, consider the radial directions along which these particles are emitted. We can model them as a one dimensional thermal bath of particles. This has an entropy  $S \sim LT$ , where  $L$  is the length of the region occupied by the bath. From this we get  $L \sim m_o^4$ . Assuming massless particles, we need a time

$$\tau_{WH} \sim m_o^4 \quad (37)$$

to fill it. (See also [87].) This is the expected order of magnitude of the white hole remnant. This is only a lower bound (not an upper bound, as considered in some phenomenology investigations).

Notice also that  $m_o$  is the mass of the hole before the evaporation. While the classical evolution of the *exterior* of a (stationary, spherically symmetric) black hole is uniquely determined by its current horizon area, its *internal* properties are not. The interior of a recently collapsed black hole with mass  $m$  is different from the interior of a black hole that has evaporated down to mass  $m$  from a larger earlier initial mass  $m_o$ . As we have seen, the internal volume of a stationary black hole, indeed, keeps increasing with time. In the classical theory, the interior has no effect on the exterior, but in the quantum theory it does. In particular, the internal volume is conserved in the transition from black to white hole, and can affect the lifetime of the child white hole.

Therefore the state of the black hole at some given time, which determines its full future evolution, is not specified by its sole current mass  $m$ , but also by the internal geometry, which in turn is determined by the initial mass of the black hole.

We can account for this by writing the quantum state of the black hole at some given time in the form  $|m_o, m\rangle_B$ , where the first quantum number is the initial mass, which determines the size of the interior, while the second is the current mass, which determines the area  $A = 16\pi m^2$  of the horizon on the given time slice and decreases in the evaporation. We do not need to keep track of other (possible) quantum numbers here.

At formation, the hole is in the state  $|m_o, m_o\rangle_B$ , then  $m$  decreases by Hawking evaporation until the state  $|m_o, m_{P\ell}\rangle_B$ . This state tunnels to a white hole state with the same quantum numbers, which we denote  $|m_o, m_{P\ell}\rangle_W$ . The tunnelling process itself from black to white is short and takes a time of the order of the current mass [18, 74]. Here is therefore the full life cycle of a gravitationally collapsed object

$$\xrightarrow{\text{collapse}} |m_o, m_o\rangle_B \xrightarrow[\text{black hole}]{\tau_{WH} \sim m_o^3} |m_o, m_{P\ell}\rangle_B \xrightarrow[\text{tunnelling}]{\tau_T \sim m_{P\ell}} |m_o, m_{P\ell}\rangle_W \xrightarrow[\text{white hole}]{\tau_{WH} \sim m_o^4} |m_{P\ell}, m_{P\ell}\rangle_W \xrightarrow{\text{end}} . \quad (38)$$

These are time scales for a distant observer. The process is very long if measured from a distance. But it is

extremely short (order  $m$ , which is the time light takes to cross the radius of the star) if measured on the bouncing

star itself. The huge difference is due to the extreme gravitational time dilation [2]. Time slows down near high density mass. An observer (capable of resisting the tidal forces) landing on a Planck star will find herself nearly immediately in the distant future, at the time where the black hole ends its evaporation. The *proper* lifetime of a Planck star is short: from its own perspective, the star is essentially a bounce. A black hole is a shortcut to the distant future.

### F. There is no information paradox

This Section follows closely [88] and [89]. The black hole information problem, as raised by Don Page in 1993 [90], regards the physics of a spacetime region *before* the quantum gravity region. Figure 14 is the Carter-Penrose diagram of a spherically symmetric spacetime geometry around a collapsing star, on which there is an evolving quantum field  $\phi$ . The geometry takes into account the back-reaction of the Hawking radiation of the field. The collapsed star generates a trapped region: the black hole. The boundary of this region is the (trapping) horizon. In the limit in which we disregard the back-reaction of the Hawking radiation, the horizon is null, but taking back-reaction into account, it is time-like. The notion of *event* horizon is not defined, because no assumption is made about the distant future, which depends on quantum gravity and is not relevant for the firewall theorem. Considerations on this region are sufficient for Page’s argument for the information problem.

Here is Page’s key observation. Consider a sphere  $S$  on the horizon at retarded time  $u$  (see Figure 14). The region of future null infinity preceding the time  $u$  receives the Hawking radiation emitted until the horizon has reached  $S$ . Consider the case where the black hole is “old” at  $S$ , namely the area  $A$  of  $S$  is much smaller than the the initial horizon area  $A_0$  at  $S_0$ . The Hawking radiation arriving at future infinity around  $u$  is in a mixed state. If the initial state of the field was pure and evolution is unitary, this radiation must be correlated with something else: with what?

There are two reasonable possibilities:

- (a) it is correlated with degrees of freedom inside the horizon;
- (b) it is correlated with degrees of freedom outside the horizon. In particular, late Hawking quanta may be correlated with early Hawking quanta.

A part of the theoretical community has got convinced that the correct answer must be (b) because (a) is ruled out by the following argument by Don Page. Assume that:

*Assumption A: The number  $N$  of states of a black hole with which external degrees of freedom can be entangled at some given time is bounded by  $N \sim e^{A/4}$  in Planck units ( $\hbar = G = c = k = 1$ ), where  $A$  is the area of the horizon at that time.*

There are several arguments supporting the idea that

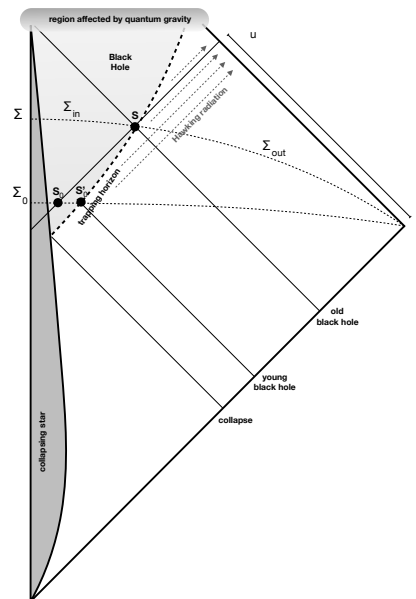


Figure 14. *The portion of spacetime relevant for the information ‘paradox’.*

the thermodynamical interaction between a black hole and its surroundings is well described by treating the black hole as a system with  $N_{BH} = e^{A/4}$  (orthogonal) states, where  $A$  is the horizon area. These arguments are convincing. However, it has then become fashionable to deduce from this fact that the black hole itself cannot have more than  $N_{BH}$  states (see for instance the discussion in [91] and references therein). This further step is not convincing, because it relies on a hypothesis that is only realized in specific approaches to quantum gravity. The actual number  $N$  of independent states of a black hole of area  $A$  can be larger than  $N_{BH}$ .

The possibility of a distinction between  $N$  and  $N_{BH}$  is opened by the fact that according to classical general relativity the interaction between a black hole and its surroundings is entirely determined by what happens in the vicinity of the horizon. The number  $N_{BH}$  counts only states that can be distinguishable from the exterior, which may be called “surface” states. On the other hand,  $N$  counts also states that can be distinguished by local observables *inside* the horizon. These do not contribute to the thermodynamical entropy  $S = A/4$ , but can contribute to the von Newman entropy, which can therefore remain high even when  $A$  shrinks.

The fact that a blackhole can have more states than  $N_{BH}$  follows from elementary considerations of causality.

To show this, consider a gravitationally collapsed object and let  $\Sigma_1$  be a Cauchy surface that crosses the horizon but does not hit the singularity, see Figure 15. Let  $\Sigma_2$  be a later similar Cauchy surface and  $i = 1, 2$ . Let  $A_i$  be the area of the intersection of  $\Sigma_i$  with the horizon. Assume that no positive energy falls into the horizon during the interval between the two surfaces. Let quantum fields

live on this geometry, back-reacting on it [92]. Finally, let  $\Sigma_i^{in}$  be the (open) portions of  $\Sigma_i$  inside the horizon.

Care is required in specifying what is meant here by ‘horizon’, since there are several such notions (event horizon, trapping horizon, apparent horizon, dynamical horizon...) which in this context may give tiny (exponentially small in the mass) differences in location. For precision, by ‘horizon’ I mean here the event horizon, if this exist. If it doesn’t, I mean the boundary of the past of a late-time spacelike region lying outside the black hole (say outside the trapping region). With this definition, the horizon is light-like.

Because of the back-reaction of the Hawking radiation, the area of the horizon shrinks and therefore

$$A_2 < A_1. \quad (39)$$

Now consider the evolution of the quantum fields from  $\Sigma_1$  to  $\Sigma_2$ . We are in a region far away from the singularity and therefore (assuming the black hole is large) from high curvature. Therefore we expect conventional quantum field theory to hold here, without strange quantum gravity effects, at least up to high energy scales. Since the horizon is light-like,  $\Sigma_1^{in}$  is in the causal past of  $\Sigma_2^{in}$ . This implies that any local observable on  $\Sigma_1^{in}$  is fully determined by observables on  $\Sigma_2^{in}$ . That is, if  $\mathcal{A}_i$  is the local algebra of observables on  $\Sigma_i^{in}$  then  $\mathcal{A}_1$  is a subalgebra of  $\mathcal{A}_2$ :

$$\mathcal{A}_1 \subset \mathcal{A}_2. \quad (40)$$

Therefore any state on  $\mathcal{A}_2$  is also a state on  $\mathcal{A}_1$  and if two such states can be distinguished by observables in  $\mathcal{A}_1$  they certainly can be distinguished by observables in  $\mathcal{A}_2$  as the first are included in the latest. Therefore the states that can be distinguished by  $\mathcal{A}_1$  —which is to say: on  $\Sigma_1^{in}$ — can also be distinguished by  $\mathcal{A}_2$  —which is to say: on  $\Sigma_2^{in}$ . Therefore the distinguishable states on  $\Sigma_1^{in}$  are a subset of those in  $\Sigma_2^{in}$ . How many are them? Either there is an infinite number of them, or a finite

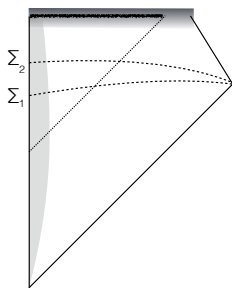


Figure 15. The (lowest part) of the conformal diagram of a gravitational collapse. The clear grey region is the object, the dotted line is the horizon, the thick upper line is the singularity, the dark upper region is where quantum gravity effect may become relevant (this region play no role in this paper.) The two Cauchy surfaces used in the paper are the dashed lines.

number due to some high-energy (say Planckian) cut-off. If there is an infinite number of them, then immediately the number of states distinguishable from inside the black hole is larger than  $N_{NB}$ , which is finite. If there is a finite number of them, then the number  $N_2$  of distinguishable states on  $\Sigma_2^{in}$  must be equal or larger than the number  $N_1$  of states distinguishable on  $\Sigma_1^{in}$ , because the second is a subset of the first. That is

$$N_2 \geq N_1. \quad (41)$$

Comparing equations (15) and (41) shows immediately that it is impossible that  $N_i = e^{A_i/4}$ , as the exponential is a monotonic function.

The conclusion is that the number of states distinguishable from the interior of the black hole must be different from the number  $N_{BH} = e^{A/4}$  of the states contributing to the Bekenstein-Hawking entropy. Since the second is shrinking to zero with the evaporation, the first must overcome the second at some point. Therefore in the interior of a black hole there are more possible states than  $e^{A/4}$ .

The physical interpretation of the conclusion is simple: the thermal behaviour of the black hole described by the Bekenstein-Hawking entropy  $S = A/4$  is determined by the physics of the vicinity of the horizon.

In classical general relativity, the effect of a black hole on its surroundings is independent from the black hole interior. A vivid expression of this fact is in the numerical simulations of black hole merging and radiation emission by oscillating black holes: in writing the numerical code, it is routine to cut away a region inside the (trapping) horizon: it is irrelevant for whatever happens outside! This is true in classical general relativity, and there is no compelling reason to suppose it to fail if quantum fields are around. Therefore a natural interpretation of  $S_{BH}$  is to count states of near-surface degrees of freedom, not interior ones. This is of course not a new idea: it has a long history [93–99] and see in particular [100], [101] in support of this idea from two different research camps, loops and strings. The argument presented here strongly support this idea, by making clear that there are interior states that do not affect the Bekenstein-Hawking entropy.

This conclusion is not in contrast with the various arguments leading to identify Bekenstein-Hawking entropy with a counting of states. To the opposite, evidence from it comes from the membrane paradigm [102] and from Loop Quantum Gravity [100, 103–105], which both show explicitly that the relevant states are surface states, but also from the string theory counting [106, 107], because the counting is in a context where the relevant state space is identified with the scattering state space, which could be blind to interior observables. For a discussion on different viewpoints about these alternatives, see [108]

If there are more states available in a black hole than  $e^{A/4}$ , then Page argument for the information loss paradox fails.

Recall indeed that the interior of an old black hole can have large volume even if its horizon has small area.

It was in fact shown in [66] that at a time  $v$  after the collapse, a black hole with mass  $m$  has interior volume

$$V \sim 3\sqrt{3}\pi m^2 v \quad (42)$$

for  $v \ll m$ . See also [70]. This volume may store large number of states. When the evaporation ends this information can leak out, slowly, if much of it is in long wavelength modes. Therefore information *can* emerge from the hole, before total dissipation, and is not lost.

These observations go against diffused prejudices regarding holography, but the result presented here does not invalidate holographic ideas: it sharpens them by pointing out that what is bound by the area of the boundary of a region is not the number of possible states in the region, but only the number of states distinguishable from observations outside the region.

If information remains trapped inside, the final stage of the black hole at the end of the evaporation must store an large amount of information in spite of its expected planckian size [35, 72, 90]. As we are seeing in these notes, this is perfectly possible.

Another consequence of the unsupported assumption that the number of states of a black hole is bound by the area is the firewall theorem [26], which is said to prove the existence of a diverging energy-momentum tensor, on the horizon. In popular accounts (for a recent one, see for instance [109]), three assumptions are (erroneously, as we shall see) said to be proven incompatible: (i) unitarity of the quantum evolution, (ii) equivalence principle (absence of firewalls), and (iii) quantum field theory on curved spacetimes. There is a more subtle assumption in the above arguments, which is the one very much likely to be unphysical: the assumption that the number of internal states of black hole must shrink when the hole shrinks.

Notice that there is a general argument relating the entropy in a region and the area of the region, but it does not apply to evaporating black holes. A celebrated observation by Raphael Bousso is that the entropy bounds cannot be applied naively to volumes (here the hypersurface  $\Sigma_{in}$  in Figure 1); it can be shown to fail in this form. Rather, it must be applied covariantly to null surfaces [110]. To apply Busso's covariant version of the entropy bound to a null surface, this must be a Light-Sheet, in the terminology of [110], namely its expansion must be negative moving away from the surfaces and the null surface must close. The problem is that *there is no Light-Sheet for surface  $S$* . The four null surfaces emanating from a sphere  $S$  on horizon of an old black hole are depicted in Figure 1. Three of these are expanding and the fourth does not close in the region where quantum gravity can be neglected. The crucial one is the internal past light surface, namely the lower left one in the Figure: naively, one may think that this has negative expansion when moving away from  $S$ , but that's not true. This is because the area of the sphere  $S_0$ , situated in the internal past light cone of  $S$  is exponentially close to the area of the sphere  $S'_0$ , situated on the trap-

ping horizon, which is much bigger than the area of  $S$ , precisely because the black hole is evaporating. Therefore: *the entropy bound does not apply after the black hole has shrunk*. (Nor is the quantum version of the entropy bound, as mentioned in [111].) This implies that all the standard arguments showing the maximal entropy falling into a region is bound by the area start failing as soon as the horizon surface shrinks.

To unravel the widespread confusion about black hole entropy is then necessary to distinguish the thermodynamical entropy from the von Neumann entropy.

To illustrate this point, consider a physical system confined inside a finite box. Suppose this system has two kinds of degrees of freedom. A set of degrees of freedom  $q$  that can interact with the exterior of the box and a set of degrees of freedom  $x$  that for a finite but long time interval  $T$  are isolated for all practical purposes. Let  $\mathcal{H}^{(q)}$  be the Hilbert space of the first,  $\mathcal{H}^{(x)}$  the Hilbert space of the second, and assume that both are finite dimensional. Suppose we are outside the box and interact with the box during the time interval  $T$ . What is the relevant maximal thermodynamical entropy  $S^{\text{therm}}$  describing the thermal behaviour of the box?

The answer is  $S^{\text{therm}} = \dim \mathcal{H}^{(q)}$ , because entropy governs the heat and energy exchanges of a system with the exterior; if the  $x$  degrees of freedom are decoupled, their value cannot change in the interaction, therefore they do not participate in these exchanges. Energy never thermalises to them. Therefore they are irrelevant for the thermal behaviour of the box. Therefore they do not contribute to  $S^{\text{therm}}$

Now let us ask a different question. If the entire box and its exterior are in a pure state, what is the maximum entanglement entropy  $S^{\text{ent}}$  between the exterior of the box and the interior? Now there is no reason not to include the  $x$  degrees of freedom in the counting, because the full state may be in a quantum superposition including different values of these variables, for instance established before the interval  $T$  during which these degrees of freedom are effectively decoupled. Hence

$$S^{\text{ent}} = \dim(\mathcal{H}_q \otimes \mathcal{H}_x) > \dim(\mathcal{H}_q) = S^{\text{therm}}. \quad (43)$$

Therefore: *entanglement entropy can be larger than thermodynamical entropy*.

Let us see how these considerations play out in the black hole case. There is evidence that the thermal interactions of a black hole with horizon area  $A$  are governed by a thermodynamical entropy  $S^{\text{therm}} = A/4$ . I assume here, as commonly done, that this is the case. Is the entanglement entropy of a black hole bounded by its thermodynamical entropy?

For a young black hole, the Bousso bound gives us a positive answer, because before any evaporation the interior past light cone of a sphere on the horizon has a past Light-sheet and the bound applies. Since the entropy that may have crossed the Light-Sheet is bounded, so is the maximum number of possible interior states that



may have had the chance of getting entangled with the exterior.

But after the back-reaction leading to the evaporation starts, the Bousso entropy bound cannot be invoked anymore. The Area decreases: does the maximal entanglement decrease as well?

As shown in the previous section, this is impossible.

A common popular hypothesis in the literature is that at the end of the Hawking evaporation a black hole simply disappears from the universe, popping into nothing. This scenario is represented by the popular left space-time diagram in Figure 8. It is important to emphasize that nothing in current physics really implies that this is the geometry of a fully evaporating black hole: when the area of the black hole becomes very small, we are deeply into the quantum gravitational regime. The idea that the black hole may disappear “just because it is small” is ungrounded and superficial, especially because its internal volume (defined above) remains big [70].<sup>2</sup>

#### IV. PHENOMENOLOGY

Measuring quantum gravity effects is notoriously difficult [113]. Still, two very interesting options are opened by the scenario described above. The first is the possibility that remnants contribute to dark matter. The second is the possibility of direct detection of these remnants.

##### A. Dark Matter

The possibility that remnants of evaporated black holes of primordial origin could form a component of dark matter was suggested by MacGibbon [114] thirty years ago and has been explored by many authors [115–121]. Since there are no strong observational constraints on this potential contribution to dark matter [122], the weak point of the scenario has been, until, the question of the physical nature of the remnants.

The scenario discussed in these notes has changed the picture: conventional physics provides a candidate for remnants: small-mass white holes with large interiors, these can be stable if they are sufficiently light, produced at the end of the evaporation and stabilized by quantum gravity. A preliminary phenomenological analysis of the possibility of detection of these signals has been carried out in [123] (where however the white hole’s lifetime is unnecessarily assumed to be an upper bound).

---

<sup>2</sup> A different possibility is that the information is taken by correlations with fundamental pre-geometric structures [112].

##### B. Direct detection

Particles of this kind (with masses at the Planck scale—a fraction of a microgram—and Planck-length effective diameter) might arise in a quantum theory of gravity [134] and have been discussed in the context of loop quantum gravity [135]. If their sole interaction is gravitational, they could account for the required present density of dark matter leftover from a hot big bang [136, 137]. Planck mass particles that interact only or almost only gravitationally are intriguing dark matter candidates for several reasons. The Planck scale is the fundamental scale in quantum gravity and it is plausible to expect stable or quasi-stable objects at this scale as part of the spectrum. This is a dark matter candidate that does not require exotic assumptions of new forces, or particles or corrections to the Einstein equations, or physics beyond the standard model. It only requires general relativity and quantum theory to hold together. Furthermore, the strength of the interaction of such particles, combined with the assumption of a sufficiently hot big bang, leads to a density of these objects at decoupling whose order of magnitude is compatible with the present dark matter density [136, 137].

Direct detection of this form of dark matter using classical sensing is challenging [139], due to the extreme weakness of the gravitational interaction. But there may be a quantum technology that could open a window to do the detection. In fact, recent developments in the area of table-top experiments involving gravity and quantum phenomena (we follow especially [140], see for instance [141] for up to date references) open the theoretical possibility of direct detection of purely-gravitationally-interacting dark matter particles. Here we first illustrate an idealized detector where the center of detector mass is set in a superposition of locations. Then discuss a more concrete tentative protocol, which employs Josephson junctions.

Consider a quantum particle of mass  $m$  (the “detector”, or D particle) split into a superposition of two positions and then recombined. For concreteness, imagine it is a particle with spin  $1/2$ , prepared in the  $|+\rangle_z$  eigenstate of the spin in the  $z$  direction, and split according to the eigenstates  $|\pm\rangle_y$  of the spin in the  $y$  direction. Upon recombination, the particle will still be in the  $|+\rangle_z$  state. But say a (classical) particle with mass  $M$  (the “dark matter”, or DM particle) flies rapidly next to one of the two positions, during the time the state was split. The DM particle transfers different amounts of momentum to the two branches of the D particle, altering their relative phase. Upon recombination, the phase shift can give rise to a non-vanishing probability of measuring the  $|-\rangle_z$  eigenstate. Figure 16 illustrates the setting.

Let us estimate the magnitude of the effect. We take the D particle as the source of an external potential for the DM particle. The displacement of the D particle due

to the passage of the DM particle is negligible:

$$\Delta d \approx \frac{c^2}{v^2} \frac{M}{m_p} \ell_p, \quad (44)$$

where  $d$  and  $v$  are defined in Figure 1 and  $\ell_p$ ,  $m_p$  and  $c$  are the Planck length, the Planck mass, and the speed of light, respectively. Indeed for  $M \approx m_p$  and  $v \approx 10^{-3}c$  (the mean velocity of DM particles in the galactic halo [144])  $\Delta d$  is of the order of  $10^{-6}\ell_p$ . Thus, detection using the classical response would be nearly impossible. We may thus assume the interaction not to change positions of the D particle significantly. We can then estimate the relative quantum phase between the two superimposed configurations as

$$\Delta S = \int dt \left( \frac{GmM}{\sqrt{d^2 + (vt)^2}} - \frac{GmM}{\sqrt{(d+\epsilon)^2 + (vt)^2}} \right), \quad (45)$$

which only involves the difference of the integrated Newtonian potential in the two superimposed configurations of the D particle separated by the distance  $\epsilon$ .  $G$  is the Newton constant. The integration of each term is logarithmically divergent, but the integration of the difference is finite. A direct evaluation gives

$$\Delta S = 2 \frac{GmM}{v} \log(1 + \epsilon/d) \approx 2 \frac{GmM}{v} \frac{\epsilon}{d} \quad (46)$$

An improved calculation that takes into account the modification of the trajectory of the DM particle is given in the Appendix. It changes the factor 2 in Eq.(46) into a 3.

The difference in the action gives a phase difference in the evolution of the two branches of the overall quantum state

$$\Delta\phi = \frac{\Delta S}{\hbar} = 3 \frac{mM}{m_p^2} \frac{c}{v} \frac{\epsilon}{d}. \quad (47)$$

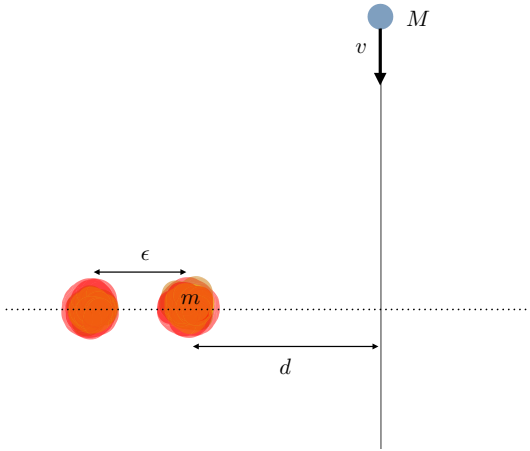


Figure 16. A particle of mass  $m$  in a superposition state with separation  $\epsilon$ . The DM particle passes by with velocity  $v$  and a closest approach distance  $d$ .

where  $m_p$  is the Planck mass (a fraction of a microgram).

There is another way of looking at this result. The difference of the action between the two branches is equal to the change of the Hamilton function for the motion of the DM particle in the field of the D particle. In turn, this is precisely the change in momentum, by the general relation  $\partial S/\partial x = -p$ . Hence the above calculation can be seen as an evaluation of the difference in momentum transfer between the two branches [142].

A non-negligible phase shift can give rise to a non-vanishing probability  $P$  of measuring the recombined D particle in the state  $|-\rangle_z$ , as

$$P = \frac{1 - \cos \Delta\phi}{2}. \quad (48)$$

If the dark matter particles have Planckian mass [135–137],  $M \sim m_p$ , then

$$\Delta\phi \sim \frac{\epsilon}{d} \frac{c}{v} \frac{m}{m_p}. \quad (49)$$

It is useful to estimate the size of the effect using some numbers. It is possible to put a mass of the order  $m \approx 10^{-17}\text{Kg} = 2 \times 10^{-8}m_p$  into quantum superposition [143]. The speed of cold DM particles in the galactic halo leads to an expected mean velocity on earth of  $v \approx 10^{-3}c$  [144]. This gives

$$\Delta\phi \approx 10^{-5} \frac{\epsilon}{d}. \quad (50)$$

Due to the amplifying nature of the factor  $c/v$  in eq. (49), pushing technology to masses  $m \sim 10^{-3}m_p$  is required, in order for the prefactor of  $\epsilon/d$  to become order unity<sup>3</sup>. An intriguing possibility is to consider the effect of the phase shift (47) on a large number of particles in a coherent state. A device that allows to exploit this possibility is a superconducting Josephson junction (JJ)<sup>4</sup>. This realization of the detector has the advantage that the collective state of the electrons translates the probabilistic response of (48) into a directly measurable signal, circumventing the need of a statistical reconstruction of the phase. It is easy to see that the phase shift due to the interaction of the electrons with the DM particle gives rise to the current across the junction

$$I = I_c \sin(\Delta\phi_e), \quad (51)$$

where  $\Delta\phi_e \approx 10^{-19}\epsilon/d$  is given by (47) with  $m_e \approx 10^{-22}m_p$  (the electron mass),  $\epsilon$  is the insulator width

<sup>3</sup> Notice that in the theoretical derivation of the effect we have taken the gravitational field of the detector particle, and hence spacetime geometry, to be itself in quantum superposition, as pointed out in [140].

<sup>4</sup> Josephson junctions in SQUID's have been suggested as supersensitive gravitational detectors [145, 146] as well as for cold dark matter search [147].

of the JJ, and  $I_c$  the critical current. For small  $\epsilon$  one has that  $I_c \approx e\hbar n_s a / (m_e \epsilon)$  where  $a$  is the area of the JJ and (at low temperatures) the density of superconducting electrons  $n_s$  approaches the Fermi density  $n_s \approx n_f = (3\pi^2)^{-1} (2m_e \epsilon_f / \hbar^2)^{3/2}$  with  $\epsilon_f$  is the Fermi energy of the material [150]. Present technology allows for the integration of transistors at close to the nanometer scales [148] and it seems possible to produce JJ with  $\sqrt{a} \sim 50$  nm in the future. Using this, and the value  $\epsilon_f \sim 7$  eV (for copper) we estimate  $I \sim (\epsilon/d) s^{-1}$  (electrons per second). In an idealized aligned configuration of about  $10^8$  junctions connected in parallel along one meter (as in Fig ??) the DM particle would induce a current of the order of  $10^7 \epsilon / ds^{-1}$  (electrons per second) with a single DM event ( $I \approx 10^{-11} (\epsilon/d) A$ ).

To exceed the thermal noise current  $I_T \approx ekT/\hbar \approx 10^{-7} T / (1K) A$  would require  $T < 10^{-1} mK$  in the setting of this letter. Much lower temperatures have been attained in the lab in small controlled environments [149], but achieving this at the space and time scales necessary for a realistic detector configuration is likely to be the key challenge. Interestingly, WIMP-cryogenic detectors already operate at the mK regime [153]. Among the issues of a concrete detection are also the fact that if DM particles have Planckian mass the flux on earth is expected to be of the order of one particle per meter square per year [139]. However, given that the JJ protocol is a one-shot detection, covering an area of several square meters with such detectors could give a significant rate of signal.

Challenges are significant, but it is remarkable that quantum mechanics can amplify effects—which classically reduce to undetectable Planck scale displacements (44)—to macroscopic observable levels. Rapidly evolving quantum computing technologies combined with the growing interest in experiments testing the interface of gravity and quantum mechanics can be used to address crucial questions in astrophysics, and possibly provide direct validation of certain implications of quantum gravity.

### C. Cosmological considerations

Let us now come to the hypothesis that white-hole remnants are a constituent of dark matter. To give an idea of the density of these objects, a local dark matter density of the order of  $0.01 M_\odot / pc^3$  corresponds to approximately one Planck-scale white hole per each  $10.000 K m^3$ . On the other hand, these objects could be moving fast with respect to our local frame, since we are rotating with the galaxy at hundreds of Km per second, while dark matter probably isn't.

Say lifetime of the remnants is of the order or lower to  $m_o^4$  (it might be longer). Indeed, for these objects to be still present now we need that their lifetime be larger or equal than the Hubble time  $T_H$ , that is

$$m_o^4 \geq T_H. \quad (52)$$

On the other hand, we expect these to be produced by evaporated black holes, therefore the lifetime of the black hole must be shorter than the Hubble time. Therefore

$$m_o^3 < T_H. \quad (53)$$

This gives an estimate on the possible value of  $m_o$ :

$$10^{10} gr \leq m_o < 10^{15} gr. \quad (54)$$

These are the masses of primordial black holes that could have given origin to dark matter present today in the form of remnants. Their Schwarzschild radius is in the range

$$10^{-18} cm \leq R_o < 10^{-13} cm. \quad (55)$$

According to primordial black hole formation theory, black holes of a given mass could have formed when their Schwarzschild radius was of the order of the horizon. Remarkably, the horizon was presumably in this range at the end of inflation, during or just after reheating. This concordance supports the plausibility of the scenario.

### D. Primordial holes and erebons

An alternative possibility for the generation of remnants is that they were formed in a contracting phase before the current expanding one, in a big bounce scenario (for a review of classical and quantum bouncing cosmologies, see [124, 125] and references therein).

The possibility that black holes could live across the big bounce and represent a component of dark matter has been considered in [126]. Recently Roger Penrose has coined the name *erebons*, from the Greek god of darkness Erebus, to refer to matter crossing over from one eon to the successive one [127] in his cyclic cosmology [128]. Large black holes evaporated before the bounce could have given rise to a population of Planck-size white holes remnants that has crossed the bounce and formed what we see today as dark matter. For this to happen, their density should have been sufficient to balance the huge dilution in an eventual inflationary phase.

An surprising aspect of this scenario is that it might address the apparent low-entropy of the initial cosmological state. The current arrow of time can be entirely traced back to the homogeneity of the gravitational field at the beginning of the expansion: it is this homogeneity the vast reserve of low-entropy that drives all current irreversible phenomena. (The reason is that generic states of the gravitational field towards which evolution is driven are crumpled, not homogeneous. The initial homogeneity begins to crumple gravitationally into galaxies and stars, with a constant increase of entropy. Crumpling generates entropy, so do the nuclear reactions in the Sun, and the biosphere phenomena nourished by the low entropy of the Sun, and so on.) If the big bounce was actually finely dotted by white holes with large interiors, the metric was

not homogeneous at that point. Rather, its low entropy is a consequence of the fact that we do not have access to the vast interiors of these holes. In other words, it is determined by the peculiar subset of observable to which we happen to have access.

This can be consistent and can represent a concrete realisation of the perspectival interpretation of entropy suggested in [129]. The idea that entropy is perspectival is the following. Entropy depends on a microstate of a system *and a choice of macroscopic observables*, or coarse graining. It is a fact that for any microstate there is some choice of macroscopic observable such that the entropy of the corresponding macrostate is low. This observation opens the possibility that the early universe was not in a very low entropy configuration because its microstate was peculiar, but because the coarse graining under which we access it is peculiar. In other words, past low entropy, and its consequent irreversible evolution can be a real, but perspectival phenomenon, like the apparent rotation of the sky around us.

If the cosmos at the big bounce was finely dotted by white holes with large interiors, then the gravitational field was not in the very improbable low entropy homogeneous or nearly homogeneous configuration. It was in a high entropy crumpled configuration. But being *outside* all white holes, we are in a special place, and from the special perspective of this place we see the universe under a coarse graining which defines an entropy that was low in the past.

### E. Modeling remnants emission

Remnants must emit, in order to release the information that was trapped with the in-falling Hawking radiation. If a black hole ends up tunneling into a white hole, its horizon is not an event horizon, and information can exit. The interior of the hole is causally connected with future null infinity, the von Neumann entanglement entropy across it can remain high when the Bekenstein-Hawking entropy decreases with the evaporation. The smallness of the white hole horizon's area and energy implies that this can only happen slowly [87]. Bringing out a large amount of information involving only little energy is what gives rise to the low energy radiation that we model here.

The hypothesis of the model we study are the following. A black hole of initial mass  $m$  evaporates via Hawking evaporation, leaving a remnant of Planckian mass which contains an amount of information sufficient to purify its Hawking radiation, namely of order

$$S \sim \frac{A}{4} = 4\pi m^2 \quad (56)$$

in natural units  $\hbar = G = c = k = 1$ . Here  $A$  is the area of the horizon *at the formation*. This information can be emitted in the form of radiation. Since the radiation is emitted radially, we model it as a uniform one-

dimensional gas of photons in thermal equilibrium, emitted by the surface of the remnant during the lifetime  $\tau$ , following [91]. Assuming for simplicity a steady emission, at the end of the remnant lifetime the radiation covers a length  $L = \tau$ . The energy  $E$  available for this gas is only that of the mass of the remnant, which is of the order of the Planck mass, namely unit in natural units.

$$E \sim 1, \quad (57)$$

while its total entropy, needed to purify the Hawking radiation is (56), uniformly distributed over a length  $L$ .

A standard derivation, which for completeness we report in the appendix, shows that the entropy  $S$  and energy  $E$  of a one dimensional photon gas of temperature  $T$  in a space of length  $L$  are [154, 155]

$$S = \frac{2\pi}{3}LT, \quad E = \frac{1}{6}LT^2. \quad (58)$$

Rearranging these two relations we find

$$L = \frac{3S^2}{8\pi^2E} = 6m^4, \quad T = \frac{4\pi E}{S} = \frac{1}{m^2} \quad (59)$$

The life-time of a white hole would be equal to time required for the photons to travel a distant  $L$ . We therefore have

$$\tau_W \sim 6m^4 \quad (60)$$

which matches previous estimates of the time needed to release the information in the remnant. The estimate of the temperature shows that the temperature of the white hole remnant to be much lower than that of the initial Hawking temperature of the parent black hole, which is  $\sim 1/m$ .

We can compute the total number of photons emitted from the white hole remnant. To this end we assume the system under study resembles a black body radiation where the frequency of the emitted photons follows a Planckian distribution. The peak frequency of a Planckian distribution of photons is at

$$\nu = \alpha T = \frac{\alpha}{m^2} \quad (61)$$

where  $\alpha \sim 2.82$ . We have reported the steps of deriving  $\alpha$  in the appendix for completeness. In natural units the relation between the energy  $\epsilon$  of a single photon and its frequency  $\nu$  is of course  $\epsilon = \nu$ , hence we can derive the total number of photons emitted by the remnant of a black hole of initial mass  $m$  to be

$$N_\gamma = \frac{E}{\epsilon} = \frac{m^2}{\alpha}. \quad (62)$$

The energy emitted by a single remnant and the number of photons emitted are not uniformly distributed in space, but those emitted by a uniform gas of remnant of the same mass and age are. Therefore we can estimate

the average energy density and photon number by dividing the total values by the volume of the region covered by the emission, which is  $V \sim L^3$ . We obtain the average energy density per unit remnant

$$\rho_o = \frac{E}{\frac{4}{3}\pi L^3} = \frac{1}{288\pi} m^{-12} \quad (63)$$

and the average photon density per remnant

$$n_\gamma = \frac{N_\gamma}{\frac{4}{3}\pi L^3} = \frac{1}{288\pi c} m^{-10}, \quad (64)$$

at the end of the process. Let's consider a uniform distribution of remnants, and let  $\Omega$  be their number density. The total energy emitted at the end of the process must be equal to their total initial mass. Since they have unit mass (in natural units), this is equal to their total number. Hence, the energy density of the radiation  $\rho_{tot}$  at the end of the process is equal to the initial number density of remnants.

$$\rho_{tot} = \Omega \quad (65)$$

and the total photons density is

$$n = \Omega n_\gamma = \frac{\Omega}{288\pi\alpha} m^{-10}, \quad (66)$$

Consider a population of black holes formed at a time  $t = 0$ , with mass  $m$  and uniformly distributed in space. Assume that they all evaporate around time  $\tau_B \sim m^3$  as predicted by Hawking radiation theory, and survive as white hole remnants for a time  $\tau_W$  as in (60). Between times  $\tau_B$  and  $\tau_B + \tau_W$ , they emit a steady radiation as described above. Assuming  $m \gg 1$ , we approximate  $\tau_B + \tau_W \sim \tau_W$ . What is the radiation observed by an observer at time  $t$ ? For  $t < \tau_B$  there is none. For  $\tau_B < t < \tau_B + \tau_W$  the observer will receive only the radiation emitted by the remnants within a distance  $r < (t - \tau_B)$  because radiation emitted by more distant remnants has not had the time to reach the observer. Radiation emitted at a distance  $r$  is diluted by distance by a factor  $1/r^2$  but the number of emitters at this distance is proportional to  $r^2$ , hence the radiation received is proportional to  $r < (t - \tau_B)$ . For the same reason, if  $t > \tau_B + \tau_W$  the radiation received remains constant in time. That is, the radiation density changes in time as

$$\rho(t) \begin{cases} = 0 & \text{for } t < m^3, \\ = \left(\frac{t - \tau_B}{\tau_W - \tau_B}\right)\Omega & \text{for } m^3 < t < 6m^4, \\ = \Omega & \text{for } t > 6m^4. \end{cases} \quad (67)$$

In other words, the process is a steady (linear in time) transformation of dust into radiation, on a  $m^4$  timescale.

Since the energy in a white hole is related to the area of its horizon, a continuous energy emission as the one described above implies a continuously decreases of the white hole horizon area, below the Planck area. According to LQG, however, any physical area is quantized, with

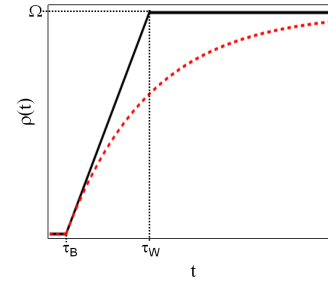
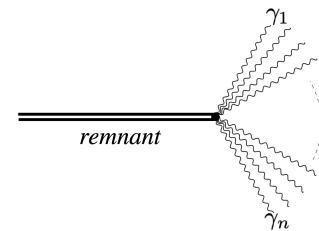


Figure 17. Background white hole radiation as a function of time. The solid black line represents a classical linear emission while the dashed red line represents a quantum emission.

the minimum non-zero eigenvalue (the "area gap") of the order of the Planck area  $A_{Pl}$  [130, 131]. We are thus led to consider a more refined description of the process, in which a remnant with near-Planckian mass and area can make a single quantum leap into radiation, in analogy with conventional nuclear radioactivity, where a steady emission of a macroscopic bulk of material is realised by individual quantized emissions governed by a probability distribution [156].

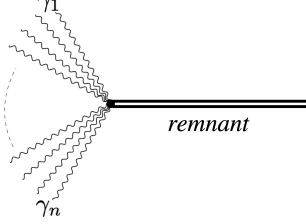
More precisely, an area gap of the order of the Planck area implies that the energy of the lowest non-vanishing energy states of the remnants is Planckian. Therefore in first order perturbation theory the only allowed transition with the emission of radiation (which necessarily has energy) is emission of the entire Planck energy of the remnant.

Let us see what could the correspond vertex describing the transition be, in the language of quantum field theory. The essential point is that (black and) white holes have many internal degrees of freedom that reflect their internal structure. A white hole parented by an old black hole evaporated from an initial mass  $m$  has an interior capable of holding information compatible with (56) even if the area of its horizon is small. A vertex coupling such remnant to a single or a few photons is therefore forbidden by conservation of information (unitarity), because a few photons do not have enough degrees of freedom to match the large number of quantum numbers describing the white hole interior. Few photons cannot carry the entire information that can be stored in the remnant. Hence the only possible transition is a transition  $remnant \rightarrow \gamma_1 \dots \gamma_n$  to a large number of low energy photons:



This conclusion is interesting in view of an old objection

to the remnant scenario, because of which this scenario was abandoned in the Nineties. The objection is that the large number of remnant internal states would make them too easy to produce in particle physics experiments. Here we see clearly why that conclusion was too quick. The effective vertex responsible for a remnant production would be



in order to create a long living Planck size remnant. The number of photons emitted by a single remnant is given in (62). If the number of photons is small, these can be high energy, but the remnant produced correspond to a remnant whose parent is a black hole of Planckian size, which is short lived. The process would not be distinguishable by the standard possibility of collapse predicted by conventional quantum gravity. To produce an actual long living remnant, on the other hand, we need  $m$  to be large, and hence we would need to focus a *large number of low energy photons*.

For instance to produce a remnant similar to the one left over from a primordial black hole formed at reheating (see below) the number of low energy photons to focus would be staggering:

$$5 \cdot 10^{38} < N_\gamma < 5 \cdot 10^{48}. \quad (68)$$

Creating such remnant in the lab is clearly unlikely due to the huge number of photons required for the process to happen. Therefore not being able to create remnants by the present experimental settings is not a reason to reject the theory of black holes turning into Planck size white hole remnants at the end of their life time cycle.

If the probability of transition is constant in time, the total energy density  $\rho_{rem}$  of a population of remnants will decay exponentially, starting at  $t = \tau_B$  as

$$\rho_{rem}(t) = \Omega e^{-\lambda(t-\tau_B)}. \quad (69)$$

If the lifetime of the white hole is of order  $\tau_W$ , we expect the decay constant to be

$$\lambda \sim (\tau_W - \tau_B)^{-1}, \quad (70)$$

The changes in the radiation density as a function of time is then

$$\rho(t) \begin{cases} = 0 & \text{for } t < m^3, \\ = \left(1 - e^{-\frac{t-\tau_B}{\tau_W-\tau_B}}\right) \Omega & \text{for } t > m^3. \end{cases} \quad (71)$$

In Fig. 1 we have plotted the energy density change of the linear emission (67) as a black solid line and the quantum

emission (71) as a dashed red line. The two converge in the two limits  $\tau_B < t \ll \tau_W$  and  $t \gg \tau_W$ .

The energy density  $\rho_{rad}$  of the radiation emitted by a population of remnants with current energy density  $\rho_{rem}$  generated by parents black holes with mass  $m$  formed at a time  $t$  in the past is then easily obtained, using also (71), as

$$\rho_{rad} = \sinh\left(\frac{1 - tm^{-3}}{1 - 6m}\right) \rho_{rem} \quad (72)$$

Using (61) we can write the mass in terms of the frequency of the radiation, and give the energy density in radiation as

$$\rho_{rad} = \sinh\left(\frac{1 - t(\nu/\alpha)^{3/2}}{1 - 6\sqrt{\alpha/\nu}}\right) \rho_{rem}. \quad (73)$$

In the cosmological standard model, primordial black holes may have formed at reheating. To get a sense of the characteristic of the diffuse radiation remnants may emit, we estimate its parameter in this simplest case. We use here a rough model, that neglects the effect of expansion.

For these black holes, we can approximate  $t$  in the above formulas with the Hubble time  $t = t_H$ . Notice that this allows us to deduce the density of an otherwise dark population of remnants just from the observation of the emitted radiation. (In other cosmological scenarios, in particular in bouncing models [157–159],  $t$  can be larger.)

Restoring physical units, and denoting the Plank mass, energy, frequency and time as  $m_{Pl}$ ,  $E_{Pl}$ ,  $\nu_{Pl}$ ,  $t_{Pl}$  we have that a population of primordial black holes of mass  $m$  and number density per unit co-moving volume  $\Omega$  gives rise to remnants producing a radiation with density

$$\rho = \Omega E_{Pl} \left(1 - e^{-\frac{t_H/t_{Pl} (m/m_{Pl})^{-3}}{1 - 6m/m_{Pl}}}\right) \quad (74)$$

and frequency given by (61), namely

$$\nu = \alpha \left(\frac{m}{m_{Pl}}\right)^{-2} \nu_{Pl}. \quad (75)$$

If we are in the era where this radiation forms, we must have  $\tau_B < t_H < \tau_W$ . This gives

$$(m/m_{Pl})^3 < t_H/T_{Pl} < 6(m/m_{Pl})^4. \quad (76)$$

Since  $\tau_H \sim 10^{61} t_{Pl}$  this gives the approximate mass range  $10^{15} m_{Pl} < m < 10^{20} m_{Pl}$ . The model is thus entirely determined by a single parameter or order of unity, that can be taken to be

$$x = \log_{10}(m/m_{Pl}) \in [15, 20]. \quad (77)$$

And the relevant quantities are

$$m = 10^{x-5} \text{gr}, \quad (78)$$

$$\nu = 10^{-2x+32} \text{Hz} \quad (79)$$

$$\rho_{rad} = \sinh\left(\frac{10^{61} - 10^{3x}}{10^{4x} - 10^{3x}}\right) \rho_{rem} \quad (80)$$

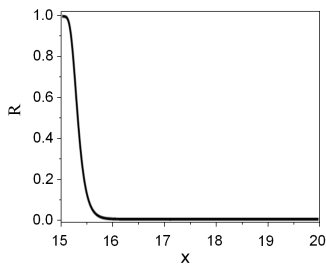


Figure 18. Ratio of radiation to total mass as function of the single parameter of the model  $x$ .

This is a mass range

$$10^{10}gr < m < 10^{15}gr, \quad (81)$$

and a frequency range

$$10^{14}Hz > \nu > 10^4Hz. \quad (82)$$

The ratio of the radiation density to the total density or remnants and radiation, as a function of  $x$ , is shown in Figure 2. Notice that remnants originating from parent black holes in the mass range of  $10^{10}gr < m < 2 \cdot 10^{10}gr$  have emitted most of their photons while remnants originating from more massive black holes,  $4 \cdot 10^{10} < m < 10^{15}$  have emitted close to zero. This is because of the long

lifetime of white hole remnants which is in the order of  $\tau_W = 6m^4$ .

In conclusion, a diffused radiation at frequency  $\nu$  and density  $\rho_{rad}$  can witness a cold dark component formed by white hole remnants, descending from primordial black holes of mass

$$m = 10^x m_{Pl} \quad (83)$$

and with density

$$\rho_{rem} = \sinh^{-1} \left( \frac{1 - 10^{61} \cdot 10^{-3x}}{1 - 6 \cdot 10^{-x}} \right) \rho_{rad} \quad (84)$$

where  $x$  can be measured directly from the frequency of the diffused radiation:

$$x = -\frac{1}{2} \log_{10} \frac{\nu}{\alpha \nu_{Pl}} \quad (85)$$

Strong constraints on the fraction of dark matter formed after the big bang have been studied in [160]. In other cosmological scenarios such as big bounce or matter bounce scenarios white hole remnants might account for an important portion of dark matter. In this scenario, the energy density of dark matter can be written as a function of two parameters: the time  $t$  since the black hole formation and their mass  $m$  (which can still be deduced from the frequency), using equation (72).

- 
- [1] C. Rovelli, “Black Hole Evolution Traced Out with Loop Quantum Gravity,” *Physics* **11** (jan, 2018) 127, [1901.04732](#).
  - [2] H. M. Haggard and C. Rovelli, “Quantum-gravity effects outside the horizon spark black to white hole tunneling,” *Physical Review D* **92** (2015), no. 10, 104020, [1407.0989](#).
  - [3] C. Rovelli and F. Vidotto, “Small black/white hole stability and dark matter,” *Universe* **4** (nov, 2018) 127, [1805.03872](#).
  - [4] T. Banks and W. Fischler, “Entropy and Black Holes in the Very Early Universe,” [2109.05571](#).
  - [5] T. Banks and W. Fischler, “Primordial Black Holes as Dark Matter,” *AIP Conference Proceedings* **1053** (aug, 2020) 129–136, [2008.00327](#).
  - [6] A. Perez, C. Rovelli, and M. Christodoulou, “Detecting Gravitationally Interacting Dark Matter with Quantum Interference,” [2309.08238](#).
  - [7] M. Christodoulou and F. D’Ambrosio, “Characteristic time scales for the geometry transition of a black hole to a white hole from spinfoams,” *arXiv preprint arXiv:1801.03027* (2018).
  - [8] C. Rovelli, “Zakopane lectures on loop gravity,” *PoS QGGS2011* (2011) 3, [1102.3660](#).
  - [9] C. Rovelli and F. Vidotto, *Covariant loop quantum gravity: An elementary introduction to quantum gravity and spinfoam theory*. Cambridge Univeristity Press, 2015.
  - [10] A. Ashtekar, T. Pawlowski, P. Singh, and K. Vandersloot, “Loop quantum cosmology of k=1 FRW models,” *Phys. Rev.* **D75** (2007) 24035, [0612104](#).
  - [11] P. Frisoni, “Numerical approach to the black-to-white hole transition,” *Phys.Rev.D* **107** (jun, 2023).
  - [12] E. Bianchi, M. Christodoulou, F. D’Ambrosio, H. M. Haggard, and C. Rovelli, “White holes as remnants: A surprising scenario for the end of a black hole,” *Classical and Quantum Gravity* **35** (nov, 2018) 225003, [1802.04264](#).
  - [13] C. Rovelli and F. Vidotto, “Planck stars,” *International Journal of Modern Physics D* **23** (oct, 2014) 1442026, [1401.6562](#).
  - [14] A. Ashtekar and M. Bojowald, “Black hole evaporation: A paradigm,” *Class. Quant. Grav.* **22** (2005) 3349–3362, [0504029](#).
  - [15] P. Dona, H. M. Haggard, C. Rovelli, and F. Vidotto, “Tunneling of quantum geometries in spinfoams,” [2402.09038](#).
  - [16] M. Christodoulou, C. Rovelli, S. Speziale, and I. Vilensky, “Planck star tunneling time: An astrophysically relevant observable from background-free quantum gravity,” *Physical Review D* **94** (2016) 084035, [1605.05268](#).
  - [17] T. De Lorenzo and A. Perez, “Improved black hole fireworks: Asymmetric black-hole-to-white-hole tunneling scenario,” *Physical Review D* **93** (2016) 124018, [1512.04566](#).

- [18] M. Christodoulou and F. D’Ambrosio, “Characteristic Time Scales for the Geometry Transition of a Black Hole to a White Hole from Spinfoams,” [1801.03027](#).
- [19] F. D’Ambrosio and C. Rovelli, “How information crosses Schwarzschild’s central singularity,” *Classical and Quantum Gravity* **35** (2018), no. 21, [1803.05015](#).
- [20] P. Martin-Dussaud and C. Rovelli, “Interior metric and ray-tracing map in the firework black-to-white hole transition,” *Classical and Quantum Gravity* **35** (2018), no. 14, [1803.06330](#).
- [21] F. Fazzini, C. Rovelli, and F. Soltani, “Painlevé-Gullstrand coordinates discontinuity in the quantum Oppenheimer-Snyder model,” *Physical Review D* **108** (jul, 2023) [2307.07797v2](#).
- [22] T. Banks, A. Dabholkar, M. R. Douglas, and M. O’Loughlin, “Are horned particles the climax of Hawking evaporation?,” *Phys.Rev.D* **45** (1992), no. 10, 3607–3616.
- [23] T. Banks and M. O’Loughlin, “Classical and quantum production of cornucopions at energies below 1018 GeV,” *Physical Review D* **47** (1993) 540–553, [9206055](#).
- [24] C. Rovelli and F. Vidotto, “Small black/white hole stability and dark matter,” *Universe* **4** (2018), no. 11, [1805.03872](#).
- [25] C. Rovelli, “Black holes have more states than those giving the Bekenstein-Hawking entropy: a simple argument,” [1710.00218](#).
- [26] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, “Black holes: Complementarity or firewalls?,” *Journal of High Energy Physics* **2013** (2013) 1–19, [1207.3123](#).
- [27] H. M. Haggard and C. Rovelli, “Quantum gravity effects around Sagittarius A\*,” *International Journal of Modern Physics D* **25** (2016), no. 12, [1607.00364](#).
- [28] F. D’Ambrosio, M. Christodoulou, P. Martin-Dussaud, C. Rovelli, and F. Soltani, “The end of a black hole’s evaporation – Part I,” [2009.05016](#).
- [29] J. R. Oppenheimer and H. Snyder, “On Continued Gravitational Contraction,” *Physical Review* **56** (1939), no. 5, 455–459.
- [30] A. Ashtekar, T. Pawłowski, and P. Singh, “Quantum nature of the big bang,” *Phys. Rev. Lett.* **96** (2006) 141301, [0602086](#).
- [31] A. Ashtekar, T. Pawłowski, and P. Singh, “Quantum nature of the big bang: An analytical and numerical investigation. I,” *Phys. Rev. D* **73** (2006) 124038, [0604013](#).
- [32] I. Agullo and A. Corichi, “Loop Quantum Cosmology,” [1302.3833](#).
- [33] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, “Black Hole Collapse and Bounce in Effective Loop Quantum Gravity,” *Classical and Quantum Gravity* **38** (dec, 2020) 04LT01, [2006.09325](#).
- [34] L. Modesto, “Disappearance of the black hole singularity in loop quantum gravity,” *Physical Review D - Particles, Fields, Gravitation and Cosmology* **70** (2004), no. 12, 5, [0407097v2](#).
- [35] S. Hossenfelder and L. Smolin, “Conservative solutions to the black hole information problem,” *Physical Review D - Particles, Fields, Gravitation and Cosmology* **81** (jan, 2010) [0901.3156](#).
- [36] R. Gambini and J. Pullin, “A scenario for black hole evaporation on a quantum Geometry,” *Proceedings of Science* **15-18-July** (2014) [1408.3050](#).
- [37] J. Olmedo, S. Saini, and P. Singh, “From black holes to white holes: a quantum gravitational, symmetric bounce,” [1707.07333](#).
- [38] D. Malafarina, “Classical collapse to black holes and quantum bounces: A review,” mar, 2017.
- [39] A. Ashtekar, J. Olmedo, and P. Singh, “Quantum extension of the Kruskal spacetime,” *Physical Review D* **98** (jun, 2018) [1806.02406](#).
- [40] A. Ashtekar, J. Olmedo, and P. Singh, “Quantum Transfiguration of Kruskal Black Holes,” *Physical Review Letters* **121** (jun, 2018) [1806.00648](#).
- [41] K. E. A. Clements, “Black Hole to White Hole Quantum Tunnelling,”.
- [42] N. Bodendorfer, F. M. Mele, and J. Münch, “Effective quantum extended spacetime of polymer Schwarzschild black hole,” *Classical and Quantum Gravity* **36** (feb, 2019) [1902.04542](#).
- [43] G. E. Volovik, “From black hole to white hole via the intermediate static state,” [2103.10954](#).
- [44] J. Ben Achour, S. Brahm, S. Mukohyama, and J. P. Uzan, “Consistent black-to-white hole bounces from matter collapse,” apr, 2020.
- [45] J. Münch, “Effective Quantum Dust Collapse via Surface Matching,” [arXiv:2010.13480 \[gr-qc\]](#).
- [46] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, “Black hole collapse and bounce in effective loop quantum gravity,” *Classical and Quantum Gravity* **38** (jun, 2021) [2006.09325](#).
- [47] J. G. Kelly, R. Santacruz, and E. Wilson-Ewing, “Effective Loop Quantum Gravity Framework for Vacuum Spherically Symmetric Space-Times,” *Physical Review D* **102** (nov, 2020) 106024, [2006.09302](#).
- [48] K. Giesel, M. Han, B.-F. Li, H. Liu, and P. Singh, “Spherical symmetric gravitational collapse of a dust cloud: polymerized dynamics in reduced phase space,” [2212.01930](#).
- [49] J. B. Achour, S. Brahma, S. Mukohyama, and J. P. Uzan, “Towards consistent black-to-white hole bounces from matter collapse,” *Journal of Cosmology and Astroparticle Physics* **2020** (apr, 2020) [2004.12977](#).
- [50] J. Lewandowski, Y. Ma, J. Yang, and C. Zhang, “Quantum Oppenheimer-Snyder and Swiss Cheese models,” [2210.02253](#).
- [51] M. Bobula and T. Pawłowski, “Rainbow Oppenheimer-Snyder collapse and the entanglement entropy production,” *Physical Review D* **108** (jul, 2023).
- [52] M. Han, C. Rovelli, and F. Soltani, “Geometry of the Black-to-White Hole Transition within a Single Asymptotic Region,” *Phys. Rev. D* **107** (mar, 2023) 64011, [2302.03872](#).
- [53] A. Rignon-Bret and C. Rovelli, “Black to white transition of a charged black hole,” *Physical Review D* **105** (aug, 2022) [2108.12823](#).
- [54] R. Carballo-Rubio, F. D. Filippo, S. Liberati, and M. Visser, “Geodesically complete black holes,” 2020.
- [55] M. Christodoulou, F. D’Ambrosio, and C. Theofilis, “Geometry Transition in Spinfoams,” [2302.12622](#).
- [56] J. W. Barrett, R. J. Dowdall, W. J. Fairbairn, F. Hellmann, and R. Pereira, “Lorentzian spin foam amplitudes: graphical calculus and asymptotics,” *Class. Quant. Grav.* **27** (2010) 165009, [0907.2440](#).
- [57] F. Soltani, C. Rovelli, and P. Martin-Dussaud, “End of a black hole’s evaporation. II.,” *Physical Review D* **104** (may, 2021) 106014, [2105.06876](#).



- [58] P. Frisoni, “Numerical approach to the black-to-white hole transition,” *Physical Review D* **107** (apr, 2023) [2304.02691v2](#).
- [59] M. Han, D. Qu, and C. Zhang, “Spin foam amplitude of the black-to-white hole transition,” [2404.02796](#).
- [60] P. Donà and P. Frisoni, “How-to Compute EPRL Spin Foam Amplitudes,” *Universe* **8** (mar, 2022) 208.
- [61] R. Gregory and R. Laflamme, “Black Strings and p-Branes are Unstable,” [9301052](#).
- [62] R. Casadio and B. Harms, “Black hole evaporation and large extra dimensions,” *Physics Letters B* **487** (aug, 2000) 209–214.
- [63] R. Casadio and B. Harms, “Black hole evaporation and compact extra dimensions,” *Physical Review D* **64** (2001), no. 2, 024016.
- [64] R. Emparan, J. García-Bellido, and N. Kaloper, “Black Hole Astrophysics in AdS Braneworlds,” *Journal of High Energy Physics* **2003** (2003), no. 01, 079–079.
- [65] B. Kol and E. Sorkin, “Black-brane instability in an arbitrary dimension,” *Classical and Quantum Gravity* **21** (2004), no. 21, 4793–4804.
- [66] M. Christodoulou and C. Rovelli, “How big is a black hole?,” *Physical Review D* **91** (2015) 064046, [1411.2854](#).
- [67] I. Bengtsson and E. Jakobsson, “Black holes: Their large interiors,” *Mod. Phys. Lett. A* **30** (2015) 1550103, [arXiv:1502.0190](#).
- [68] Y. C. Ong, “The persistence of the large volumes in black holes,” *General Relativity and Gravitation* **47** (mar, 2015) [1503.08245v4](#).
- [69] S.-J. Wang, X.-X. Guo, and T. Wang, “Maximal volume behind horizons without curvature singularity,” [1702.05246](#).
- [70] M. Christodoulou and T. De Lorenzo, “Volume inside old black holes,” *Physical Review D* **94** (2016) 104002, [1604.07222](#).
- [71] V. Frolov and I. Novikov, *Black Hole Physics: Basic Concepts and New Developments*. Springer, 2012.
- [72] S. B. Giddings, “Black holes and massive remnants,” *Physical Review D* **46** (mar, 1992) 1347–1352, [9203059](#).
- [73] C. Barceló, R. Carballo-Rubio, L. J. Garay, and G. Jannes, “The lifetime problem of evaporating black holes: mutiny or resignation,” *Classical and Quantum Gravity* **32** (2015) 35012, [1409.1501](#).
- [74] C. Barceló, R. Carballo-Rubio, and L. J. Garay, “Black holes turn white fast, otherwise stay black: no half measures,” *Journal of High Energy Physics* **2016** (2016) 1–21, [1511.00633](#).
- [75] L. J. Garay, C. Barceló, R. Carballo-Rubio, and G. Jannes, “Do stars die too long?,” in *The Fourteenth Marcel Grossmann Meeting*, vol. 2, pp. 1718–1723. World Scientific Press, dec, 2017.
- [76] A. Barrau, C. Rovelli, and F. Vidotto, “Fast radio bursts and white hole signals,” *Physical Review D* **90** (2014), no. 12, 127503, [1409.4031](#).
- [77] A. Barrau and C. Rovelli, “Planck star phenomenology,” *Physics Letters B* **739** (2014) 405–409, [1404.5821](#).
- [78] F. Vidotto, A. Barrau, B. Bolliet, M. Shutten, and C. Weimer, “Quantum-gravity phenomenology with primordial black holes,” [1609.02159](#).
- [79] C. Rovelli, “Planck stars: new sources in radio and gamma astronomy?,” *Nature Astronomy* (2017), no. 1, 0065, [1708.01789](#).
- [80] A. Barrau, B. Bolliet, M. Shutten, and F. Vidotto, “Bouncing black holes in quantum gravity and the Fermi gamma-ray excess,” *Physics Letters B* **772** (2017) 58–62, [1606.08031](#).
- [81] A. Raccanelli, F. Vidotto, and L. Verde, “Effects of primordial black holes quantum gravity decay on galaxy clustering,” *Journal of Cosmology and Astroparticle Physics* **2018** (aug, 2018) [1708.02588](#).
- [82] F. Vidotto, “Quantum insights on Primordial Black Holes as Dark Matter,” in *Proceedings of Science*, vol. 335. Sissa Medialab Srl, nov, 2018. [1811.08007](#).
- [83] C. Rovelli and F. Vidotto, “Pre-big-bang black-hole remnants and past low entropy,” *Universe* **4** (2018), no. 11, [1805.03224](#).
- [84] F. Vidotto, “Measuring the Last Burst of Non-singular Black Holes,” *Foundations of Physics* **48** (mar, 2018) 1380–1392, [1803.02755](#).
- [85] E. Barausse et.al., “Prospects for fundamental physics with LISA,” *General Relativity and Gravitation* **52** (jan, 2020) [2001.09793](#).
- [86] A. Barrau, L. Ferdinand, K. Martineau, and C. Renevey, “Closer look at white hole remnants,” *Physical Review D* **103** (jan, 2021) [2101.01949](#).
- [87] J. Preskill, “Do Black Holes Destroy Information?,” in *An international symposium on Black Holes, Membranes, Wormholes and Superstrings*, S. Kalara and D. Nanopoulos, eds., p. 22. World Scientific, Singapore, 1993.
- [88] C. Rovelli, “Black holes have more states than those giving the Bekenstein-Hawking entropy: a simple argument,” [1710.00218](#).
- [89] C. Rovelli, “The subtle unphysical hypothesis of the firewall theorem,” *Entropy* **21** (2019), no. 9, [1902.03631](#).
- [90] D. Page, “Information in black hole radiation,” *Physical Review Letters* **71** (dec, 1993) 3743–3746, [9306083](#).
- [91] D. Marolf, “The Black Hole information problem: past, present, and future,” *Reports on Progress in Physics* **80** (2017) 092001, [1703.02143](#).
- [92] R. M. Wald, *Quantum field theory in curved space-time and black hole thermodynamics*. Univ. Pr., Chicago, USA, 1994.
- [93] J. W. York Jr, “Dynamical origin of black-hole radiance,” *Phys. Rev. D* **28** (1983), no. 12, 2929–2945.
- [94] W. H. Zurek and K. Thorne, “Statistical Mechanical Origin of the Entropy of a Rotating, Charged Black Hole,” *Phys. Rev. Lett.* **54** (1885) 2171.
- [95] A. Wheeler, *A Journey into Gravity and Spacetime*. Freeman, New York, 1990.
- [96] G. ’tHooft, “The black hole interpretation of string theory,” *Nucl. Phys. B* **335** (1990) 138.
- [97] L. Susskind, L. Thorlacius, and R. Uglum, “The Stretched Horizon and Black Hole Complementarity,” *Phys Rev D* **54** (1985) 2171.
- [98] V. Frolov and I. Novikov, “Dynamical Origin of the Entropy of a Black Hole,” *Phys Rev D* **48** (1993) 4545.
- [99] F. Larsen and F. Wilczek, “Internal structure of black holes,” *Phys. Lett.* **375** (1996) 37.
- [100] C. Rovelli, “Loop Quantum Gravity and Black Hole Physics,” *Helv. Phys. Acta* **69** (1996) 482, [9608032](#).
- [101] A. Strominger, “Black hole entropy from near horizon microstates,” *JHEP* **9802** (1998) 9, [hep-th/9712251](#).
- [102] K. Thorne, R. Price, and D. Macdonald, *Black Holes:*

- the Membrane Paradigm*. Yale University Press, New Haven, 1986.
- [103] C. Rovelli, “Black Hole Entropy from Loop Quantum Gravity,” *Physical Review Letters* **77** (1996), no. 16, 3288–3291, [9603063](#).
- [104] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, “Quantum geometry and black hole entropy,” *Phys. Rev. Lett.* **80** (1998) 904–907, [9710007](#).
- [105] A. Perez, “Black holes in loop quantum gravity,” mar, 2017.
- [106] A. Strominger and C. Vafa, “Microscopic origin of the Bekenstein-Hawking entropy,” *Phys. Lett.* **B379** (1996) 99–104, [hep-th/9601029](#).
- [107] G. T. Horowitz and A. Strominger, “Counting States of Near-Extremal Black Holes,” *Phys. Rev. Lett.* **77** (1996) 2368–2371.
- [108] T. Jacobson, D. Marolf, and C. Rovelli, “Black hole entropy: inside or out?,” *arXiv.org hep-th* (2005).
- [109] C. Wuthrich, “Quantum gravity from general relativity,” *Routledge Companion to the Philosophy of Physics*. (2019).
- [110] R. Bousso, “The holographic principle,” *Reviews of Modern Physics* **74** (2002), no. 3, 825–874, [0203101](#).
- [111] R. Bousso, Z. Fisher, S. Leichenauer, and A. C. Wall, “Quantum focusing conjecture,” *Physical Review D* **93** (2016) 064044, [1506.02669](#).
- [112] A. Perez, “No firewalls in quantum gravity: The role of discreteness of quantum geometry in resolving the information loss paradox,” *Classical and Quantum Gravity* **32** (2015), no. 8, [1410.7062](#).
- [113] S. Liberati and L. Maccione, “Quantum Gravity phenomenology: achievements and challenges,” [1105.6234](#).
- [114] J. H. MacGibbon, “Can Planck-mass relics of evaporating black holes close the Universe?,” *Nature* **329** (1987), no. 24, 308.
- [115] J. D. Barrow, E. J. Copeland, and A. R. Liddle, “The cosmology of black hole relics,” *Physical Review D* **46** (jul, 1992) 645–657.
- [116] B. J. Carr, J. H. Gilbert, and J. E. Lidsey, “Black hole relics and inflation: Limits on blue perturbation spectra,” *Phys Rev D* **50** (1994) 4853.
- [117] A. Liddle and A. Green, “Primordial black holes and early cosmology,”.
- [118] A. Barrau, G. Boudoul, and L. Derome, “An improved gamma-ray limit on the density of primordial black holes,” in *International Cosmic Ray Conference 28 ICRC 2003*, pp. 1697–1699, Universal Academy Press. 2003.
- [119] P. Chen and R. J. Adler, “Black hole remnants and dark matter,” *Nuclear Physics B - Proceedings Supplements* **124** (2003) 103–106.
- [120] P. Chen, “Inflation Induced Planck-Size Black Hole Remnants As Dark Matter,” [0406514](#).
- [121] K. Nozari and S. H. Mehdipour, “Gravitational Uncertainty and Black Hole Remnants,” *Modern Physics Letters A* **20** (2008), no. 38, 2937–2948, [0809.3144](#).
- [122] B. Carr, F. Kuhnel, and M. Sandstad, “Primordial Black Holes as Dark Matter,” [1607.06077](#).
- [123] A. Barrau and C. Rovelli, “Planck star phenomenology,” [1404.5821](#).
- [124] R. Brandenberger and P. Peter, “Bouncing Cosmologies: Progress and Problems,” [1603.05834](#).
- [125] I. Agullo and P. Singh, “Loop Quantum Cosmology: A brief review,” 2016. [1612.01236](#).
- [126] B. Carr, T. Clifton, and A. Coley, “Black holes as echoes of previous cosmic cycles,” [1704.02919](#).
- [127] R. Penrose, “Correlated “noise” in LIGO gravitational wave signals: an implication of Conformal Cyclic Cosmology,” *arXiv e-prints* (2017) [1707.04169](#).
- [128] R. Penrose, “The basic ideas of conformal cyclic cosmology,” in *AIP Conference Proceedings*, vol. 1446, pp. 233–243. 2012.
- [129] C. Rovelli, “Is Time’s Arrow Perspectival?,” in *The Philosophy of Cosmology*, K. Chamcham, J. Silk, J. Barrow, and S. Saunders, eds., pp. 285–296. Cambridge University Press, 2016. [1505.01125](#).
- [130] C. Rovelli, “A generally covariant quantum field theory and a prediction on quantum measurements of geometry,” *Nuclear Physics, Section B* **405** (sep, 1993) 797–815.
- [131] C. Rovelli and L. Smolin, “Discreteness of area and volume in quantum gravity,” *Nucl. Phys.* **B442** (1995) 593–622, [9411005](#).
- [132] P. Martin-Dussaud and C. Rovelli, “Evaporating black-to-white hole,” *Classical and Quantum Gravity* **36** (may, 2019) [1905.07251](#).
- [133] N. Aghanim et al. (Planck Collaboration), *Astron. Astrophys* **641**, A6 (2020).
- [134] V. Kuzmin and I. Tkachev, *Phys. Rev. D* **59**, 123006 (1999). E. W. Kolb and A. J. Long, *Phys. Rev. D* **96**, 103540 (2017). D. J. H. Chung, E. W. Kolb, and A. Riotto, *Phys. Rev. D* **59**, 023501 (1998). K. Harigaya, T. Lin, and H. K. Lou, *J. High Energy Phys.* **2016**, 14 (2016). E. Babichev, D. Gorbunov, and S. Ramazanov, *Phys. Lett. B* **794** (2019). D. Hooper, G. Krnjaic, and S. D. McDermott, *J. High Energy Phys.*, **2019**, 1 (2019). H. Kim and E. Kuflik, *Phys. Rev. Lett.* **123**, 191801 (2019).
- [135] Carlo Rovelli, Francesca Vidotto “Small black-white hole stability and dark matter”, *Universe* **2018**, *4*(11), 127, [arXiv:1805.03872](#).
- [136] A. Barrau, K. Martineau, F. Moulin and J. F. Ngono, *Phys. Rev. D* **100**, no.12, 123505 (2019) [doi:10.1103/PhysRevD.100.123505](#) [arXiv:1906.09930](#).
- [137] L. Amadei and A. Perez, “Planckian discreteness as seeds for cosmic structure,” *Phys. Rev. D* **106**, no.6, 063528 (2022) [doi:10.1103/PhysRevD.106.063528](#) [arXiv:2104.08881](#).
- [138] S.W. Hawking, “Black hole explosions?” *Nature*, **248** (1974), 30-31.
- [139] D. Carney, S Ghosh, G Krnjaic, JM Taylor “Proposal for gravitational direct detection of dark matter”, *Physical Review D*, 2020, [arXiv:1903.00492](#).
- [140] Marios Christodoulou, Carlo Rovelli. “On the possibility of laboratory evidence for quantum superposition of geometries”, *Physics Letters B*, **792** (2019) 64-68. [arXiv:1808.05842](#).
- [141] M. Christodoulou, A. Di Biagio, M. Aspelmeyer, Č. Brukner, C. Rovelli and R. Howl, “Locally Mediated Entanglement in Linearized Quantum Gravity,” *Phys. Rev. Lett.* **130**, no.10, 100202 (2023) [arXiv:2202.03368](#).
- [142] J. Pedernales and M.B. Plenio. [arXiv:2311.04745](#).
- [143] Fein, Y.Y., Geyer, P., Zwick, P. et al. “Quantum superposition of molecules beyond 25 kDa”, *Nat. Phys.* **15**, 1242-1245 (2019).

- <https://doi.org/10.1038/s41567-019-0663-9>
- [144] K. Freese, M. Lisanti and C. Savage, *Rev. Mod. Phys.* **85**, 1561-1581 (2013)  
doi:10.1103/RevModPhys.85.1561 arXiv:1209.3339.
- [145] M. Cerdonio, M., Falferi, P., Prodi, G. A., C. Rovelli, and S. Vitale, In *Proceedings of the 7th Italian Conf. on General Relativity and Gravitational Physics*, Bruzzo, V., Cianci, R., and Massa, E., eds. (World Scientific, Singapore 1986) p. 497–501.
- [146] M. Cerdonio, G.A. Prodi, S. Vitale. Dragging of inertial frames by the rotating Earth: Proposal and feasibility for a ground-based detection. *Gen Relat Gravit* **20**, 83–87 (1988).  
<https://doi.org/10.1007/BF00759259>.
- [147] S. J. Asztalos, G. Carosi, C. Hagmann, D. Kinion, K. van Bibber, M. Hotz, L. J Rosenberg, G. Rybka, J. Hoskins, J. Hwang, P. Sikivie, D. B. Tanner, R. Bradley, J. Clarke, SQUID-Based Microwave Cavity Search for Dark-Matter Axions, *Physical Review Letters* **104**, (2010), doi: 10.1103/physrevlett.104.041301.
- [148] Soloviev, I. I. and Bakurskiy, S. V. and Ruzhickiy, V. I. and Klenov, N. V. and Kupriyanov, M. Yu. and Golubov, A. A. and Skryabina, O. V. and Stolyarov, V. S. *Physical Review Applied* **16**, no.4, 044060 (2021) doi:10.1103/PhysRevApplied.16.044060 arXiv:044060.
- [149] C. Deppner, W. Herr, M. Cornelius, P. Stromberger, T. Sternke, C. Grzeschik, A. Grote, J. Rudolph, S. Herrmann and M. Krutzik, *et al.* *Phys. Rev. Lett.* **127**, no.10, 100401 (2021) doi:10.1103/PhysRevLett.127.100401
- [150] L.D. Landau, E.M. Lifshitz, and L.P. Pitaevskii. *Statistical Physics: Theory of the Condensed State*. Course of theoretical physics. Elsevier Science, 1980.
- [151] M. Aspelmeyer, “When Zeh Meets Feynman: How to Avoid the Appearance of a Classical World in Gravity Experiments,” *Fundam. Theor. Phys.* **204** (2022), 85-95 doi:10.1007/978-3-030-88781-0\_5 arXiv:2203.05587.
- [152] S. Rijavec, M. Carlesso, A. Bassi, V. Vedral and C. Marletto, “Decoherence effects in non-classicality tests of gravity,” *New J. Phys.* **23** (2021) no.4, 043040 doi:10.1088/1367-2630/abf3eb arXiv:2012.06230.
- [153] M. Schumann, *J. Phys. G* **46**, no.10, 103003 (2019) doi:10.1088/1361-6471/ab2ea5 [arXiv:1903.03026 [astro-ph.CO]].
- [154] F. Schwabl, *Statistical Mechanics*. Springer Science & Business Media, June, 2006. Google-Books-ID: 7VnKAW284PgC.
- [155] V. V. Skobelev, “Entropy and energy of a photon gas in an n-dimensional space,” *Russian Physics Journal* **55** (Feb., 2013) 999–1004.
- [156] B. R. Martin and G. Shaw, *Nuclear and Particle Physics: An Introduction*. John Wiley & Sons, Apr., 2019. Google-Books-ID: pWmEDwAAQBAJ.
- [157] A. Ashtekar and A. Barrau, “Loop quantum cosmology: From pre-inflationary dynamics to observations,” *Class. Quant. Grav.* **32** (2015), no. 23, 234001. eprint: 1504.07559.
- [158] R. Brandenberger and P. Peter, “Bouncing Cosmologies: Progress and Problems,” *Found. Phys.* **47** (2017), no. 6, 797–850. eprint: 1603.05834.
- [159] A. Ijjas and P. J. Steinhardt, “Bouncing Cosmology made simple,” *Classical and Quantum Gravity* **35** (July, 2018) 135004. arXiv:1803.01961 [astro-ph, physics:gr-qc].
- [160] A. Barrau, L. Ferdinand, K. Martineau, and C. Renevey, “Closer look at white hole remnants,” *Physical Review D* **103** (Feb., 2021) 043532.