CP violation in D decays to two pseudoscalars: A SM-based calculation

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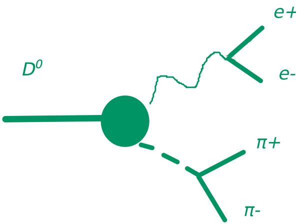
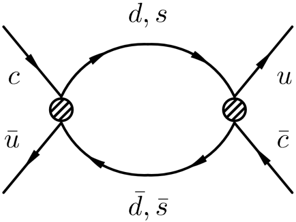
Contents

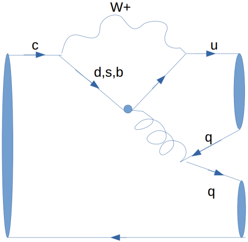
A new Flavour Physics 'anomaly' or an incomplete theory prediction?

- Flavour Physics beyond B-anomalies
- Charm Physics is growing (LHCb, Belle II, BESIII)

Rare decays | Mixing | CP violation in decays







- CPV in hadronic D modes: only discovery of CPV in the charm sector
- Plus new result of *KK* has puzzling implications

CP violation in D decays: just a SM system or gateway to New Physics?

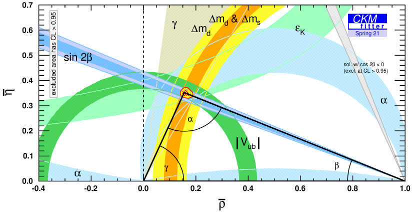
$$\Delta A_{CP}^{exp} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) = [-1.54 \pm 0.29] \cdot 10^{-3}$$

$$\Delta A_{CP}^{dir,exp} = [-1.57 \pm 0.29] \cdot 10^{-3}$$ [LHCb 2019]

NEW!!! $$A_{CP}(D^0 \to K^+K^-) = [6.8 \pm 5.4(\text{stat}) \pm 1.6(\text{syst})] \cdot 10^{-4}$$ [LHCb 2022]

$$A_{CP}^{dir}(D^0 \to \pi^+\pi^-) = [23.2 \pm 6.1] \cdot 10^{-4}$$

- Is the SM theoretical prediction in agreement?
- Weak sector (CKM parameters) already probed by kaons, B mesons



CPV in D: the strong sector

- Does a beyond-naive estimation of hadronic effects matter?

$$\mathscr{A} = |A_1|e^{i\delta_1 + i\phi_1} + |A_2|e^{i\delta_2 + i\phi_2}$$

$$\overline{\mathscr{A}} = |A_1|e^{i\delta_1 - i\phi_1} + |A_2|e^{i\delta_2 - i\phi_2}$$

$$a_{CP}^{dir} \sim |A_1||A_2| \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$$

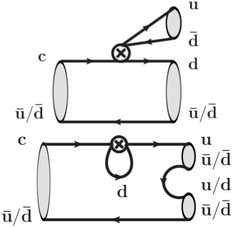
Need different weak phases AND different strong phases

$$\mathscr{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$

$$\mathscr{A}(\overline{D^0} \to f) = A(f) - i r_{CKM} B(f)$$

$$a_{CP}^{dir} \approx 2 r_{CKM} \frac{|B(f)|}{|A(f)|} \cdot \sin \arg \frac{A(f)}{B(f)}$$

($r_{CKM} = Im \frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}}$, rephasing-invariant)



Tree topology

Penguin topology

Non-perturbative QCD methods

- In K decays: Chiral Perturbation Theory
- In B decays: HQET
- $\Lambda_{\chi PT} \approx m_\rho < m_D = 1865$ MeV, $\frac{\Lambda_{QCD}}{m_c} = \mathcal{O}(1)$
  $\rightarrow$ neither approach is strictly valid in charm!
- Approaches in charm use symmetries to combine observables

  [Müller, Nierste, Schacht '15]

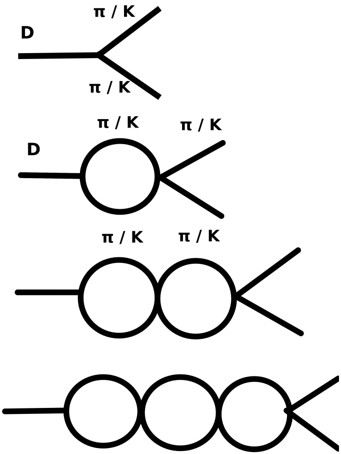
  or set bounds for the strong phases [Khodjamirian, Petrov '17]

Contents

A way to look at the problem: rescattering

- Strong process, blind to the weak phase
- Isospin (u↔d) is a good symmetry of strong interactions
- In I=0, two channels:

$$S_{strong} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to KK \\ KK \to \pi\pi & KK \to KK \end{pmatrix}$$



Rescattering & what we learn about strong phases

- S matrix is **unitary**, as well as strong sub-matrix
- For I=0, S-wave:

$$\begin{pmatrix} A_0^0(D\to\pi\pi) \\ A_0^0(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}}_{S_{strong}} \cdot \begin{pmatrix} A_0^{0*}(D\to\pi\pi) \\ A_0^{0*}(D\to KK) \end{pmatrix}$$

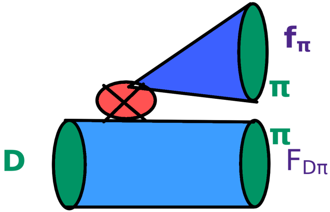
- The phases are related to the rescattering phases **which are known from data/other experiments**
- Watson's theorem (elastic rescattering limit): $argA_0^0(D\to\pi\pi)=\delta_1\equiv argA(\pi\pi\to\pi\pi)mod\pi$
- With inelasticities:

$$argA_0^0(D\to\pi\pi)=\delta_1+arccos\sqrt{\frac{(1+\eta)^2-\left(\frac{|A_0^0(D\to KK)|}{|A_0^0(D\to\pi\pi)|}\right)^2(1-\eta^2)}{4\eta}}$$

depends on the ratio $\lambda_{\pi K}=\frac{|A_0^0(D\to\pi\pi)|}{|A_0^0(D\to KK)|}$

What about magnitudes?

- Rescattering also affects the *magnitudes* of amplitudes, apart from the *phases*
- An estimate for magnitudes: factorisation/large number-of-colors (N_C)



CKM × Wilson coefficient ×factorisation

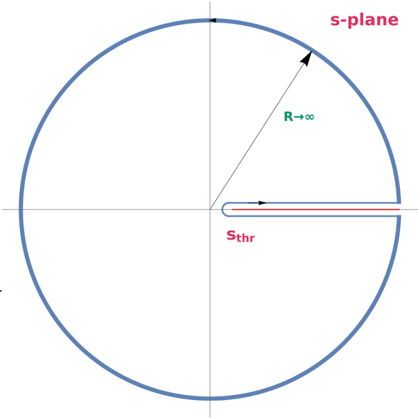
- Does not take rescattering into account
- Decay constant and form factor come from data and/or lattice $< \pi^- | (\overline{d}\gamma_\mu c) | D^0 >= \frac{m_D^2 - m_\pi^2}{m_\pi^2} q_\mu f_0^{D\pi}(m_\pi^2) + (\textit{vanishing contr.})$

Basic property of scattering amplitudes: analyticity

- Fundamental, model-independent property related to **causality**
- Cauchy's theorem: $A(s) = \frac{1}{2\pi i} \oint_C ds' \frac{A(s')}{s'-s}$ leads to

$$ReA(s) = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{ImA(s')}{s'-s}$$

(Dispersion relation)



- Unitarity of S-matrix & dispersion relation:

$$\underbrace{ReA(s)}_{\text{Re at a point}} = \frac{1}{\pi} \underbrace{PV \int_{s_{thr}}^{\infty} ds' \frac{\tan \delta(s')}{s'-s} ReA(s')}_{\text{integral of Re along the physical region}}$$

Analyticity & what we learn about magnitudes

- Integral equation, studied by Muskhelishvili-Omnes
- One subtraction: needs one piece of physical information
- Single channel case (& one subtraction at s_0), **physical** solution:

$$|A_I(s)| = A_I(s_0) \underbrace{exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_I(z)}{(z-s_0)(z-s)}\}}_{\text{Omnes factor } \Omega}$$

We need more than just large N_C!

$|A_I(s=m_D^2)| = (\text{large } N_C \text{ result}) \times (\text{Omnes factor})_I$

- Behaviour at large s: $\Omega(s) \sim \frac{1}{s^n}$, $n = \frac{\delta_I(\infty)}{\pi}$

Dispersion relations for multiple channels

- More channels: Equally more solutions.
- The equivalent of the dispersion relation in the 2-channel case:

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{s'-s} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix} \tag{1}$$

$T = T_0^0 = -i(S_0^0 - I)$

- No analytical solution
- Closed-form equation: $\lambda_{\pi K}(s) \equiv \frac{|A_0^0(D \to \pi\pi)(s)|}{|A_0^0(D \to KK)(s)|} = func(\int \eta(z), \delta_1(z), \delta_2(z), \lambda_{\pi K}(z))$
- Gives an analytical solution only in the case of small phases

Solving 2-channel dispersion relations

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{1}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{s'-s} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix}$$

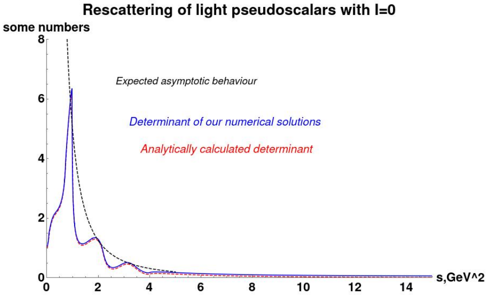
- Two 'fundamental' solutions $\Omega^{(1)}(s) = \begin{pmatrix} \Omega_{\pi 1}(s) \\ \Omega_{K1}(s) \end{pmatrix}$, $\Omega^{(2)}(s) = \begin{pmatrix} \Omega_{\pi 2}(s) \\ \Omega_{K2}(s) \end{pmatrix}$ for which $det\Omega \equiv det\left(\Omega^{(1)}|\Omega^{(2)}\right) \xrightarrow{s\to\infty} \frac{1}{s^n}$, $n = \frac{\delta_1(\infty)+\delta_2(\infty)}{\pi}$
- The $det\Omega(s)$ always has an explicit analytical solution
- In our case $n = 2$ and the fundamental solutions go as $\frac{1}{s}$
- The physical solution is unique:

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \Omega(s) \cdot \begin{pmatrix} P_1(s) \\ P_2(s) \end{pmatrix}$$

Numerical solution of 2-channel case

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{s - s_0}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{(s'-s)(s'-s_0)} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- We discretise following the method from [Moussallam et al. hep-ph/9909292]
- To pick the *fundamental* solutions, we
  - check they behave as expected at infinity
  - make sure the numerical determinant behaves as the (known) analytical determinant

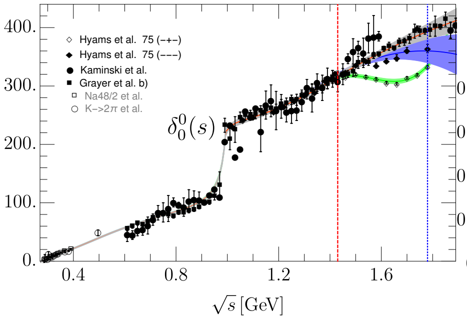


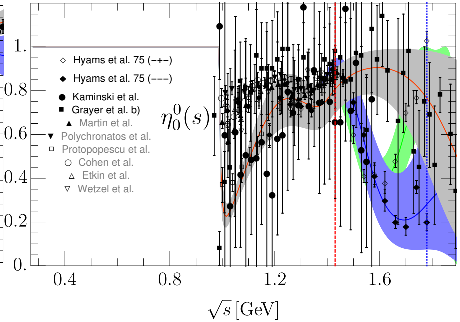
Summary of our method

- Factor out weak phases
- Flavour basis to isospin
- Isospin blocks:
  - I=0 with 2 channels: $\pi\pi$ and KK
  - I=1 with KK elastic rescattering
  - I=2 with $\pi\pi$ elastic rescattering
- Isospin amplitudes treated with dispersion relations calculated numerically
- Physical input: unitarity (for integrand), large N_C limit (for polynomial ambiguity/subtraction point)

Data deployed: phase-shifts & inelasticities of I=0

- Use inelasticity and phase-shift parameterisations [Pelaez et al., 1907.13162],[Pelaez et al., 2010.11222]
- Data: nuclear experiments from the 70'-80's
- Analytical parameterisation in partial waves, encompassing effect of known resonances
- Respect dispersion relations up to some energy, within uncertainties
- Parameterisations available up to energies $\sim m_D$ - extrapolate for higher & vary relevant parameters for uncertainties


Hyams et al. 75 (−+−)
Hyams et al. 75 (−−−)
Kaminski et al.
Grayer et al. b)
Na48/2 et al.
K−>2π et al.
$\delta_0^0(s)$
$\sqrt{s}$ [GeV]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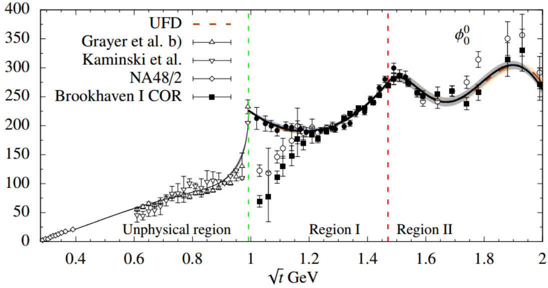
Protopopescu et al.
Cohen et al.
Etkin et al.
Wetzel et al.
$\eta_0^0(s)$
$\sqrt{s}$ [GeV]


UFD
Grayer et al. b)
Kaminski et al.
NA48/2
Brookhaven I COR
ϕ_0^0
Unphysical region
Region I
Region II
$\sqrt{t}$ GeV

The weak part & short-distance contributions

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} z_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu)\right) - \lambda_b \Sigma_{i=3}^{6} v_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu)\right]$$

$\lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b$

$|\lambda_d| \approx |\lambda_s| = \mathscr{O}(\lambda)$, usually $Re\lambda_d = -Re\lambda_s$

$$\begin{aligned} Q_1^d &= (\bar{d}c)_{V-A}(\bar{u}d)_{V-A} \\ Q_2^d &= (\bar{d}_j c_i)_{V-A}(\bar{u}_i d_j)_{V-A} \\ Q_1^s &= (\bar{s}c)_{V-A}(\bar{u}s)_{V-A} \\ Q_2^s &= (\bar{s}_j c_i)_{V-A}(\bar{u}_i s_j)_{V-A} \end{aligned}$$

$$\begin{aligned} Q_3 &= (\bar{u}c)_{V-A}\Sigma_q(\bar{q}q)_{V-A} \\ Q_4 &= (\bar{u}_j c_i)_{V-A}\Sigma_q(\bar{q}_i q_j)_{V-A} \\ Q_5 &= (\bar{u}c)_{V-A}\Sigma_q(\bar{q}q)_{V+A} \\ Q_6 &= (\bar{u}_j c_i)_{V-A}\Sigma_q(\bar{q}_i q_j)_{V+A} \\ Q_{8g} &= -\frac{g_s}{8\pi^2} m_c \bar{u}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu} c \end{aligned}$$

μ	z_1	z_2	v_3	v_4	v_5	v_6
1.3 GeV	1.21	−0.41	0.02	−0.06	0.02	−0.06
2 GeV	1.15	−0.31	0.01	−0.04	0.01	−0.03

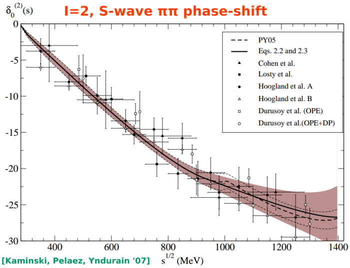
Data deployed: isospins 1 and 2

- For I=1 and 2 we can deploy Br's of $A(D^+ \to \pi^+\pi^0) \sim A_{I=2}$, $A(D^+ \to K^+\overline{K^0}) \sim A_{I=1}$, isospin-pure channels
- Extract Omnes factors' magnitudes from those

Phases: there are available data for I=2 $\pi\pi$, but not well behaved

No data for I=1 KK

Not elastic channels



- It is exact to assume Omnes factors' magnitudes from the charged D channels
- It is *not exact* to extract the phases, so we leave them free

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Omnes factors

For the isospin=0 channels we **calculate** numerically the Omnes matrix at $s = m_D^2$:

$$\Omega_{I=0} = \begin{pmatrix} 0.58e^{1.8i} & 0.64e^{-1.7i} \\ 0.58e^{-1.4i} & 0.61e^{-2.3i} \end{pmatrix}$$

(In data: inelasticity taken mainly from $\pi\pi$ rescattering - solution I from Pelaez et al. '19)

Compare to Watson's theorem prediction: $argA(\pi\pi \to \pi\pi) = 7rad$, $argA(KK \to KK) = -1.7rad$

The **physical solution** is

$$\begin{pmatrix} \mathbf{A}(D \to \pi\pi) \\ \mathbf{A}(D \to KK) \end{pmatrix} = \Omega_{I=0} \cdot \begin{pmatrix} \mathbf{A}_{\text{factorisation}}(D \to \pi\pi) \\ \mathbf{A}_{\text{factorisation}}(D \to KK) \end{pmatrix}$$

(Same for **B** instead of **A**)

$$\text{This way } arg\mathbf{A}(D \to \pi\pi) = 1.6,\ arg\mathbf{A}(D \to KK) = -1.1,$$
$$arg\mathbf{B}(D \to \pi\pi) = -1.3,\ arg\mathbf{B}(D \to KK) = 1.7rad$$

Flavour amplitudes breakdown

- $$\underbrace{\mathscr{A}_f}_{\text{flavour-specific decay amplitudes}} = \underbrace{U_f}_{\text{isospin to flavour matrix}} \underbrace{\mathscr{A}_I}_{\text{isospin-specific decay amplitudes}}$$
- $\mathscr{A}_I = \Omega_I \mathscr{A}_I^{fac}$
- $\mathscr{A}_{\mathscr{I}} = U_f^{-1} \mathscr{A}_f^{fac}$

These give

$$\mathscr{A}(D^0 \to \pi^+\pi^-) \approx \lambda_d \underbrace{f_\pi F_{D\pi}(m_\pi^2)(m_D^2 - m_\pi^2)}_{\text{factorised hadronic matrix element } (D \to \pi\pi)_{fac}} \left(|\Omega_{I=2}| e^{i\delta_{2\pi}} (\frac{1}{3} C_1 - \frac{1}{3} C_2) + \Omega_{11} (\frac{2}{3} C_1 + \frac{1}{3} C_2) \right)$$

$$+ \lambda_s \overbrace{f_K F_{DK}(m_K^2)(m_D^2 - m_K^2)}^{\text{factorised hadronic matrix element } (D \to KK)_{fac}} \tfrac{1}{3} \Omega_{12} C_1 + \underbrace{\#(C_4, C_6)}_{\text{penguin operators}}$$

The contribution of penguin operator insertions to the magnitude of the amplitudes can be ignored

CPV sources

The main term in the CP asymmetry is (for $D^0 \to \pi^+\pi^-$)

$$a_{CP} \sim J * (D \to KK)_{fac}(D \to \pi\pi)_{fac}\{\underbrace{-\overbrace{(2C_1^2 + C_1C_2)}^{\text{curr.-curr. operators}}\ \omega_1}_{I=0 \text{ vs } I=0 \text{ interference}} + \underbrace{\overbrace{(C_1^2 - C_1C_2)}^{\text{curr.-curr. operators}}\ |\Omega_{I=2}|(r_{12}\sin\delta_{\pi\pi}^{I=2} - i_{12}\cos\delta_{\pi\pi}^{I=2})}_{I=2 \text{ vs } I=0 \text{ interference}}\}$$

$$\sim J * (D \to KK)_{fac}(D \to \pi\pi)_{fac}\{-2.4\omega_1 + 2.0|\Omega_{I=2}|(r_{12}\sin\delta_{2\pi} - i_{12}\cos\delta_{2\pi})\}$$

where $\omega_1 = Im(\Omega_{11}\Omega_{12}^*)$(of I=0), $J = Im(\lambda_d\lambda_s^*) \sim$ Jarlskog

Note: in $D \to \pi\pi$ main contribution from $I = 2, I = 0$ interference; in $D \to KK$ from $I = 0, I = 0$ interference

The interference with the short-distance penguins (suppressed by GIM) is

$$J * (D \to KK)_{fac}(D \to \pi\pi)_{fac}\{0.13\omega_1 + 0.25|\Omega_{I=2}|\ldots\} + J * (D \to \pi\pi)_{fac}^2 0.13|\Omega_{I=2}|\ldots$$

much smaller than the tree-tree interference

Comparison to the $K \to \pi\pi$ CPV problem

$$\mathscr{A}(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{2}} A_2 + A_0$$

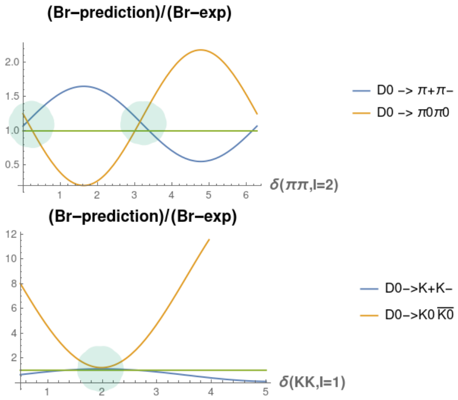
- Follow the same procedure as in the D decays [Gisbert, Pich '17]
  $\pi\pi$ rescattering only elastic
  $\Rightarrow argA(I = 0) = argB(I = 0) = argA(\pi\pi \to \pi\pi)$
  I=2, I=0 different strong phases $\to \frac{\epsilon'}{\epsilon}$
  Interference between I=2-tree and I=0-penguin only:

$$\mathscr{A}(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{2}} \lambda_u |\Omega_2| e^{i\delta_2} T_2 + \lambda_u |\Omega_0| e^{i\delta_0} T_0 + \lambda_t |\Omega_0| e^{i\delta_0} P_0$$

$$a_{CP} \sim Im(\lambda_u^* \lambda_t) \sin(\delta_2 - \delta_0)$$

Branching fraction estimation

We adjust $\delta^{\pi\pi}_{I=2}, \delta^{KK}_{I=1}$



Branching fraction estimation

We find:

$\frac{Br_{theo}}{Br_{exp}}$

Decay channel	**Our method** (preliminary)	Naive factorisation	Watson's theorem, no DRs	Correct phases, no DRs
$D^0 \to \pi^+\pi^-$	1.1	1.7	0.63	1.0
$D^0 \to \pi^0\pi^0$	1.1	0.1	2.1	0.8
$D^0 \to K^+K^-$	1.1	0.9	0.070	0.7
$D^0 \to K^0\overline{K^0}$	1.2	0 ($1/N_C$-suppressed)	12	0.7

$$\left(\frac{Br(D^0 \to K^+K^-)}{Br(D^0 \to \pi^+\pi^-)}\right)_{theo} = \left(\frac{Br(D^0 \to K^+K^-)}{Br(D^0 \to \pi^+\pi^-)}\right)_{exp} \approx 2.8$$

Old $D \to \pi\pi$, KK puzzle seems to be solved!

CP asymmetries

$\Delta A_{CP}^{dir,exp} = (-1.57 \pm 0.29) \cdot 10^{-3}$ [LHCb 2019]

$A_{CP}(D^0 \to K^+K^-) = (6.8 \pm 5.4(\text{stat}) \pm 1.6(\text{syst})) \cdot 10^{-4}$ [LHCb 2022]

We predict $\Delta A_{CP}^{dir,theo} \leq \mathcal{O}(10^{-4})$!! (preliminary)

and $a_{CP}^{dir}(D^0 \to \pi^+\pi^-) \approx 3 \cdot 10^{-4}$, $a_{CP}^{dir}(D^0 \to K^+K^-) \approx -1 \cdot 10^{-4}$ with no prior assumption about U-spin

Also predict $a_{CP}^{dir}(D^0 \to \pi^0\pi^0) = \mathcal{O}(10^{-4})$

$$\text{Recall: } a_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\sim 6 \cdot 10^{-4}} \underbrace{\frac{|B(f)|}{|A(f)|}}_{\text{needs be } \mathcal{O}(1)} \cdot \underbrace{\sin\arg\frac{A(f)}{B(f)}}_{\text{needs be close to 1}}$$

- The short-distance GIM-suppressed diagrams are not the only generator of CP-odd amplitudes
- Yet we **do not** find a sufficient enough enhancement of B's, or very large phase-shift differences between A and B to compensate

Caveats & points to improve

- Something big missing in $I = 2$?
  Less likely: no established particle of I=2 as per PDG
- Third channel in $I = 0$?
  - *Yes:* 4π is known, but its effect on 2π, $2K$ difficult to estimate
  - No available data over energy
  - Future work!
- SM calculation - "strong" statement, needs to be scrutinised
- If everything fails, it's time for NP! *(See talk by T. Höhne)*

Summary

- SM approach deploying
  1. S-matrix unitarity, scattering amplitude analyticity, isospin symmetry and factorisation
  2. as much data as possible (rescattering, form factors and decay constants, Br's of D^+ decays)
- We succeed in calculating the branching fractions in reasonable agreement with experiment, from scratch
- We still estimate the CP asymmetry **an order of magnitude too small** compared to the experimental value!
- The SM discussion is still open, but seems difficult to accommodate the current exp. value in our SM calculation...

Thank you very much!
Stay tuned!

(Preprint coming soon!)

Contents

Isospin-2 and -1 fixing

$$\mathscr{A}(D^+ \to \pi^+\pi^0) = \frac{3}{2\sqrt{2}} A^{\pi}_{I2}$$

$$\mathscr{A}(D^+ \to K^+\overline{K^0}) = A^K_{I1}$$

We fix $|A^{\pi}_{I2}|$, $|A^K_{I1}|$ from the Br's and use them in e.g.

$$\mathscr{A}(D^0 \to \pi^+\pi^-) = -\frac{1}{2\sqrt{3}} A^{\pi}_{I2} + \frac{1}{\sqrt{6}} A^{\pi}_{I0}$$

If I=2 elastic then $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2}$
If inelastic $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2} + \text{(mixing)}$ but we use directly
$A^{\pi}_{I2} = |A^{\pi}_{I2}| exp\{i\delta^{\pi\pi}_{I=2}\}$, phase left free

Comments on 2203.04056

$$
\begin{aligned}
\mathcal{A}_{D^0\to KK} =& \eta\, e^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} \\
& + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}\,, \\
\mathcal{A}_{D^0\to \pi\pi} =& \eta\, e^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} \\
& + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}\,.
\end{aligned} \tag{8}
$$

$\alpha_{\pi\pi}$, α_{KK} referred to as "tree level processes" with no phases BUT extracted from the branching fraction:

from the amplitudes given in Eq. (8). By taking into account that $\sqrt{1-\eta^2} << 1$ at the D^0 mass, we have:

$$\Gamma_{\pi\pi} \approx \eta |V_{cd}^* V_{ud}|^2\, a_{\pi\pi}^2 \text{ and } \Gamma_{KK} \approx \eta |V_{cs}^* V_{us}|^2 a_{KK}^2\,. \tag{11}$$

Eq. (8) in the elastic limit would give:

$$
\begin{aligned}
\mathscr{A}_{D^0\to KK} &= e^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} \\
\mathscr{A}_{D^0\to \pi\pi} &= e^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi}
\end{aligned}
$$

$\Rightarrow \delta(D^0 \to KK) = 2\delta_{KK},\ \delta(D^0 \to \pi\pi) = 2\delta_{\pi\pi}$!
Compare to ours:

$$
\begin{aligned}
\mathscr{A}_{D^0\to KK} &= \Omega_{22}\, V_{cs}^* V_{us}\, a_{KK} + \Omega_{21}\, V_{cd}^* V_{ud}\, a_{\pi\pi} \\
\mathscr{A}_{D^0\to \pi\pi} &= \Omega_{11}\, V_{cd}^* V_{ud}\, a_{\pi\pi}\, \Omega_{12}\, V_{cs}^* V_{us}\, a_{KK}
\end{aligned}
$$

The full $\mathscr{A}$'s coincide with the transition amplitude from the branching fraction

Numerical solution of 2-channel case

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{s-s_0}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{(s'-s)(s'-s_0)} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- We discretise following the method from [Moussallam et al. hep-ph/9909292] into

$$\begin{pmatrix} ReA^{\pi}(s_i) \\ ReA^{K}(s_i) \end{pmatrix} = \frac{s_i-s_0}{\pi} \sum_j \hat{w}_j \frac{(ReT)^{-1}(ImT)(s_j)}{(s_j-s_i)(s_j-s_0)} \begin{pmatrix} ReA^{\pi}(s_j) \\ ReA^{K}(s_j) \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

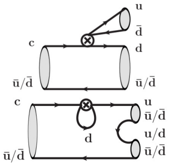
- This creates an **invertible** matrix which gives *a* (discrete) solution
- Subtleties taken care of as in [Moussallam et al. hep-ph/9909292]
- To pick the *fundamental* solutions, we fix the vector at an unphysical point $s < 0$ and
  - check they behave as $\frac{1}{s}$ for large s
  - make sure the numerical determinant behaves as the (known) analytical determinant

CPV in mesons

$$A_{CP}(f) = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D^0} \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D^0} \to \overline{f})}$$

$$\approx \frac{A(D^0 \to f) - A(\overline{D^0} \to \overline{f})}{A(D^0 \to f) + A(\overline{D^0} \to \overline{f})} + \frac{< t_f >}{\tau_{D^0}} a_{CP}^{ind}$$

- $A_\Gamma = -a_{CP}^{ind} = (-2.8 \pm 2.8) \cdot 10^{-4}$
- For the decay $D^0 \to \pi^+\pi^-$: apply unitarity of the CKM matrix
  $A(D^0 \to \pi^+\pi^-) = \lambda_d A_d + \lambda_b A_b$
  $\to a_{CP}^{dir} \sim |\lambda_d||\lambda_b||A_d||A_b| \sin arg \frac{V_{cd}^* V_{ud}}{V_{cb}^* V_{ub}} \cdot \sin arg \frac{A_d}{A_b}$



Isospin decomposition

- $\pi\pi$ states can have isospin=0,2. KK can have isospin=0,1.

$$\begin{pmatrix} A(\pi^+\pi^-) \\ A(\pi^0\pi^0) \\ A(K^+K^-) \\ A(K^0\overline{K}^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} A_\pi^2 \\ A_\pi^0 \\ A_K^1 \\ A_K^0 \end{pmatrix}$$

CPV in I=0

$$\begin{pmatrix} A^\pi \\ A^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Re\lambda_d T^\pi + ... \\ Re\lambda_s T^K + ... \end{pmatrix}$$

$$\begin{pmatrix} B^\pi \\ B^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Im\lambda_d T^\pi + \sum_i Im\lambda_{d_i} P_i^\pi \\ Im\lambda_s T^K + \sum_i Im\lambda_{d_i} P_i^K \end{pmatrix}$$

Can consider either $Im\lambda_d = 0$ or $Im\lambda_s = 0$, not both simultaneously $\Rightarrow$ In a_{CP}^{dir} there always exists a term $\sim T^\pi T^K$, both for $\pi\pi$ and for KK

Large N_C limit & effective operators

- $T_{fac}(D^0 \to \pi^+\pi^-) = \lambda_d C_1 \frac{G_F}{\sqrt{2}} F_0^{D\pi}(m_\pi^2) f_\pi \cdot (m_D^2 - m_\pi^2)$
- $P_{fac}(D^0 \to \pi^+\pi^-) =$
  $\lambda_d(C_4 - 2C_6 \frac{M_\pi^2}{(m_u+m_d)(m_c+m_d)}) \frac{G_F}{\sqrt{2}} F_0^{D\pi}(m_\pi^2) f_\pi \cdot (m_D^2 - m_\pi^2)$
- $Q_1(i) = (\overline{d}_i c)_{V-A}(\overline{u} d_i)_{V-A}$, $Q_2(i) = (\overline{d}_i d_i)_{V-A}(\overline{u} c)_{V-A}$,
  $Q_{5,3} = (\overline{u} c)_{V-A} \sum_q (\overline{q} q)_{V\pm A}$,
  $Q_4 = \sum_q (\overline{u} q)_{V-A}(\overline{q} c)_{V-A}$, $Q_6 = -2\sum_q (\overline{u} q)_{S+P}(\overline{q} c)_{S-P}$
- $C_1 = 1.15$, $C_2 = -0.31$, $C_3 = 0.01$, $C_4 = -0.04$, $C_5 = 0.01$, $C_6 = -0.03$
- $\lambda_d = V_{cd}^* V_{ud} \approx 0.22$
- $\overline{m_c}(2GeV) = 1.045 GeV$
- Compare $m_D = 1865$ MeV to $\Lambda_{\chi PT} \approx m_\rho = 775$ MeV