# SPA-Net for HHH→6b Resolved and Boosted Jet Assignment

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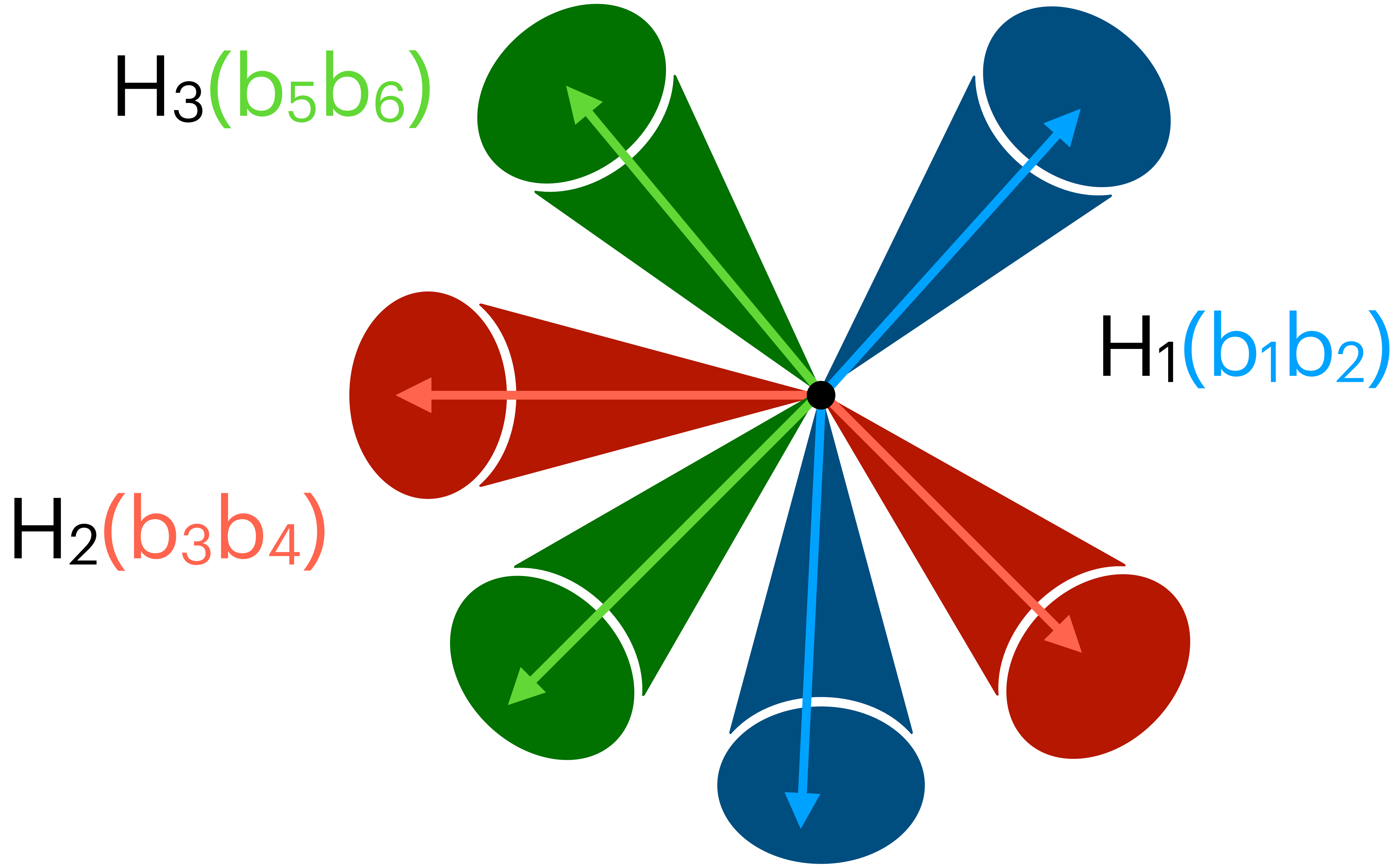
# Introduction & Outline

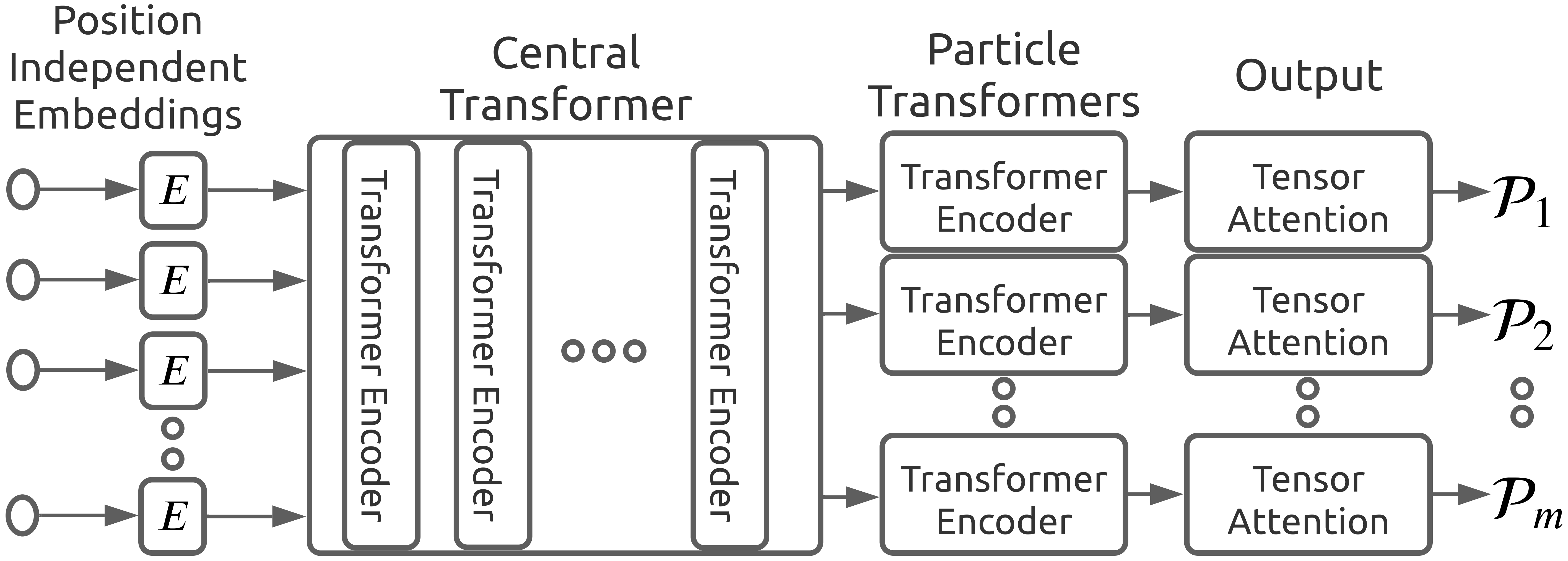
[1] arXiv:2010.09206
[2] arXiv:2106.03898

- $HHH$ has a complex final state with many different types of (partial) reconstruction possible, including overlapping small- and large-radius jets
- Idea: use ML to optimize reconstruction efficiency
  - Existing approach (SPA-Net [1, 2]) may work out of the box for fully-resolved case (6 small-radius b jets), would like to generalize to mixed resolved/boosted cases
  - Fully exploit event topology and kinematic correlations of jets to pair the 3 Higgs correctly
- Outline of rest of talk
  - Overview of SPA-Net
  - Baseline methods for $HHH$
  - Preliminary results with $HHH$
  - Discussion



# Symmetry Preserving Attention Networks (SPA-Net)

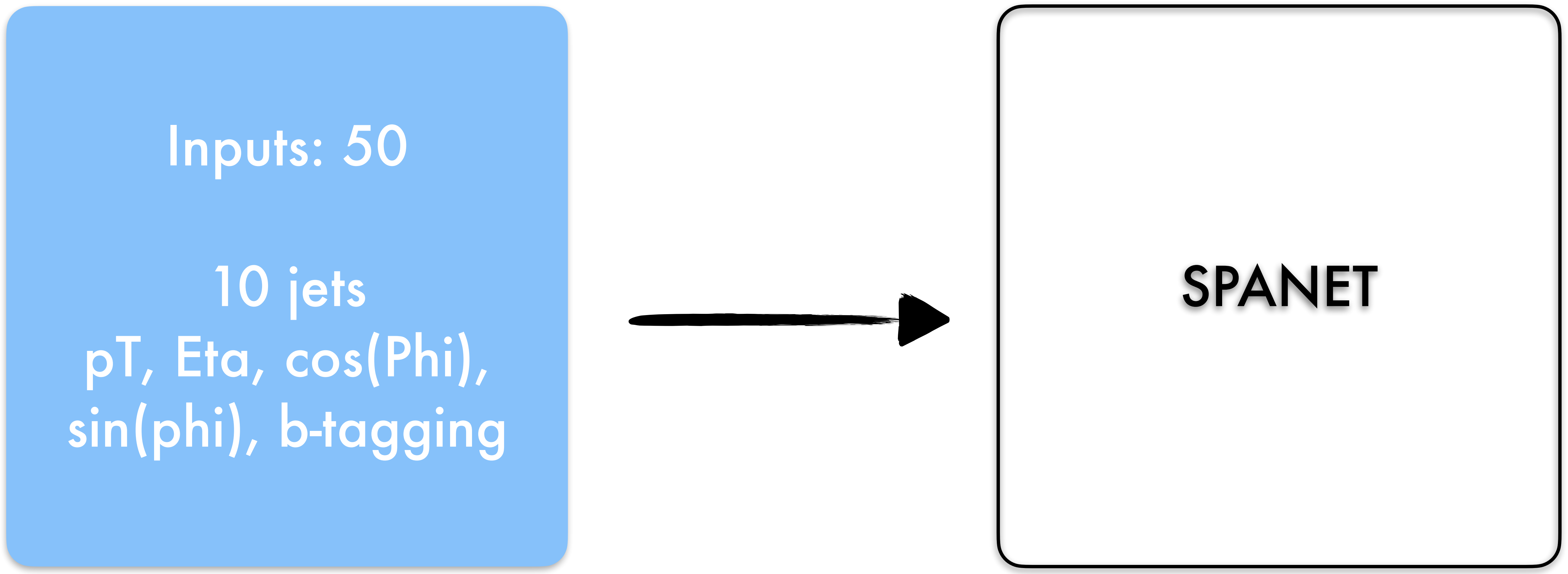
- Consider all valid permutations using *symmetric tensor attention*
  - Resonance particle $p$ (e.g., Higgs) is associated with $k_p$ partons (e.g., 2 b quarks); maximum of $N$ reconstructed jets (e.g., 10)
  - Input: matrix of transformer-encoded jets $X_p \in \mathbb{R}^{N \times D}$
  - Output: rank-$k_p$ tensor $\mathcal{P}_p \in \mathbb{R}^{N \times N \times \cdots \times N}$ the joint distribution over $k_p$-jet assignments
    - $\sum \mathcal{P}_p = 1$
    - Valid solutions => $\mathrm{diag}(\mathcal{P}_p) = 0$

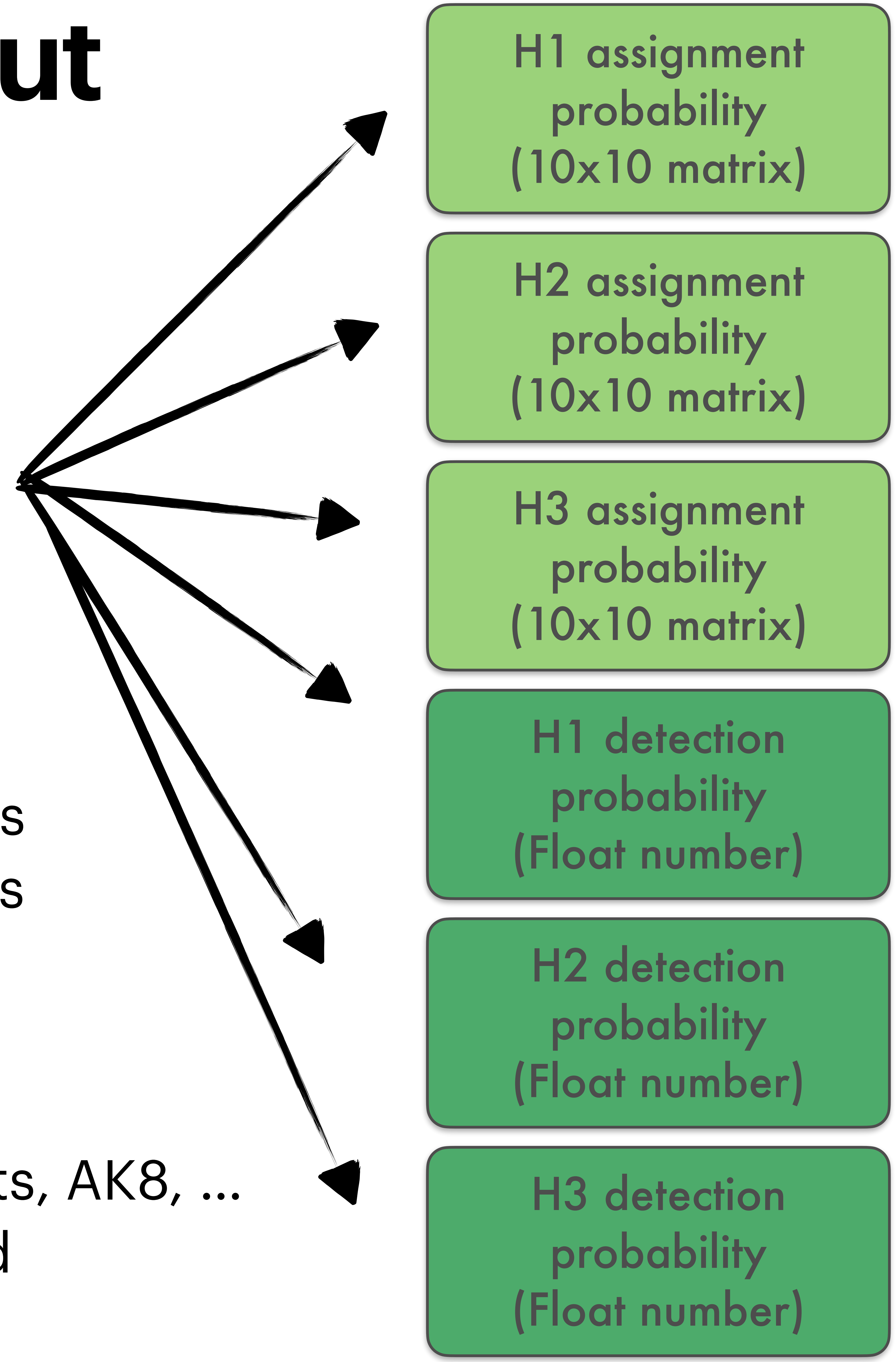


# SPA-Net Output

- [**Detection probability**, **assignment distribution**] x $N_c$ candidates
  - For each particle candidate in $N_c$ candidates
    - 1. If DP is lower than the threshold, SPANet did not find the particle, and the corresponding AD is ignored.
    - 2. The peak of AD indicates which combination SPANet predicts correct:
      - E.g. If $k_p$ = 3 and argmax$(P_{ijk}) = (1,2,3)$, then jet 1, 2, 3 reconstruct this particle candidate. The AD tensor is symmetric by design.
- Optional auxiliary output:
  - Classifications, regressions learned from your customized dataset.
  - MLP classifiers and regressors can take selected or grouped inputs from different depth (event level, particle level, jet level).

# SPA-Net Output



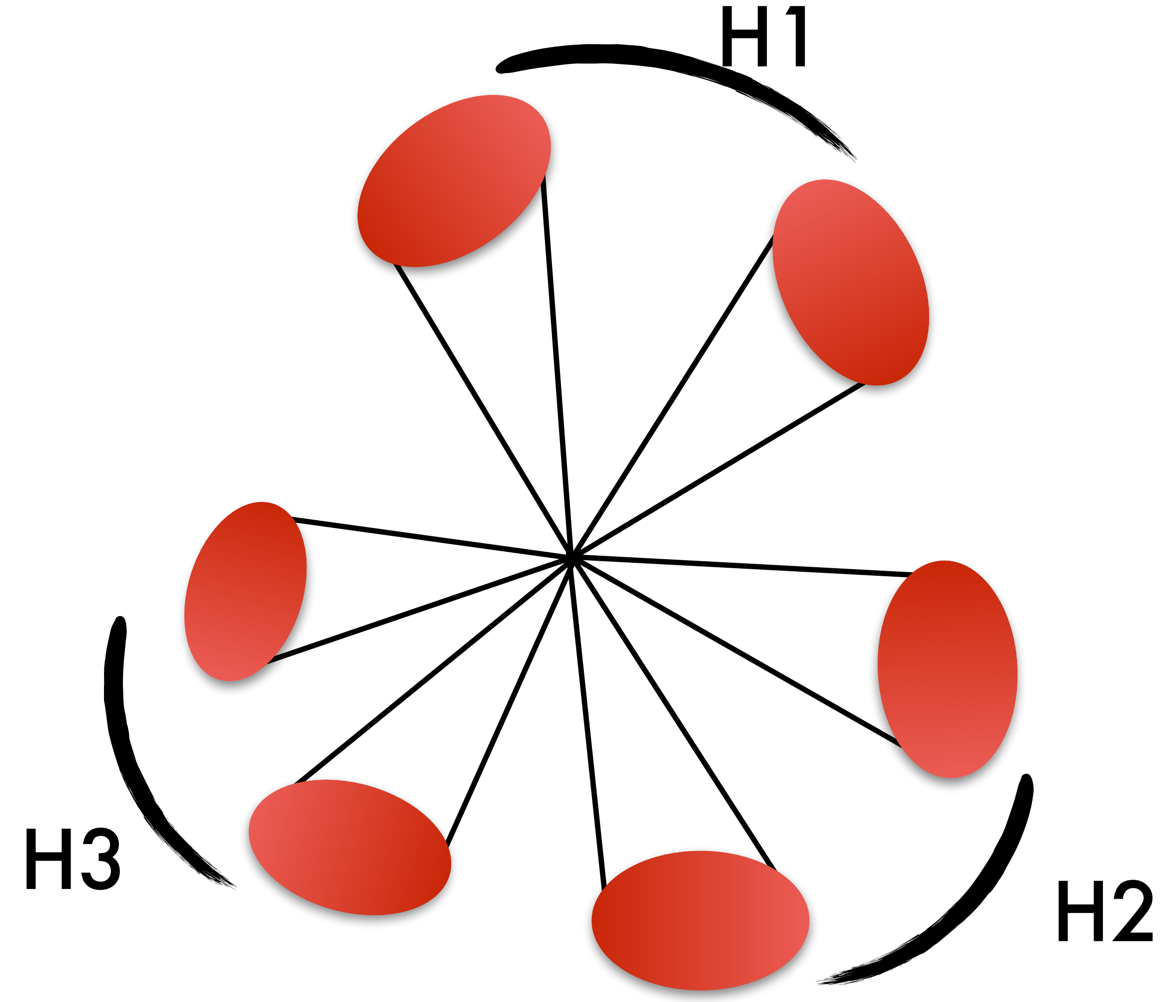

Inputs: start from signal samples HHH used for analysis

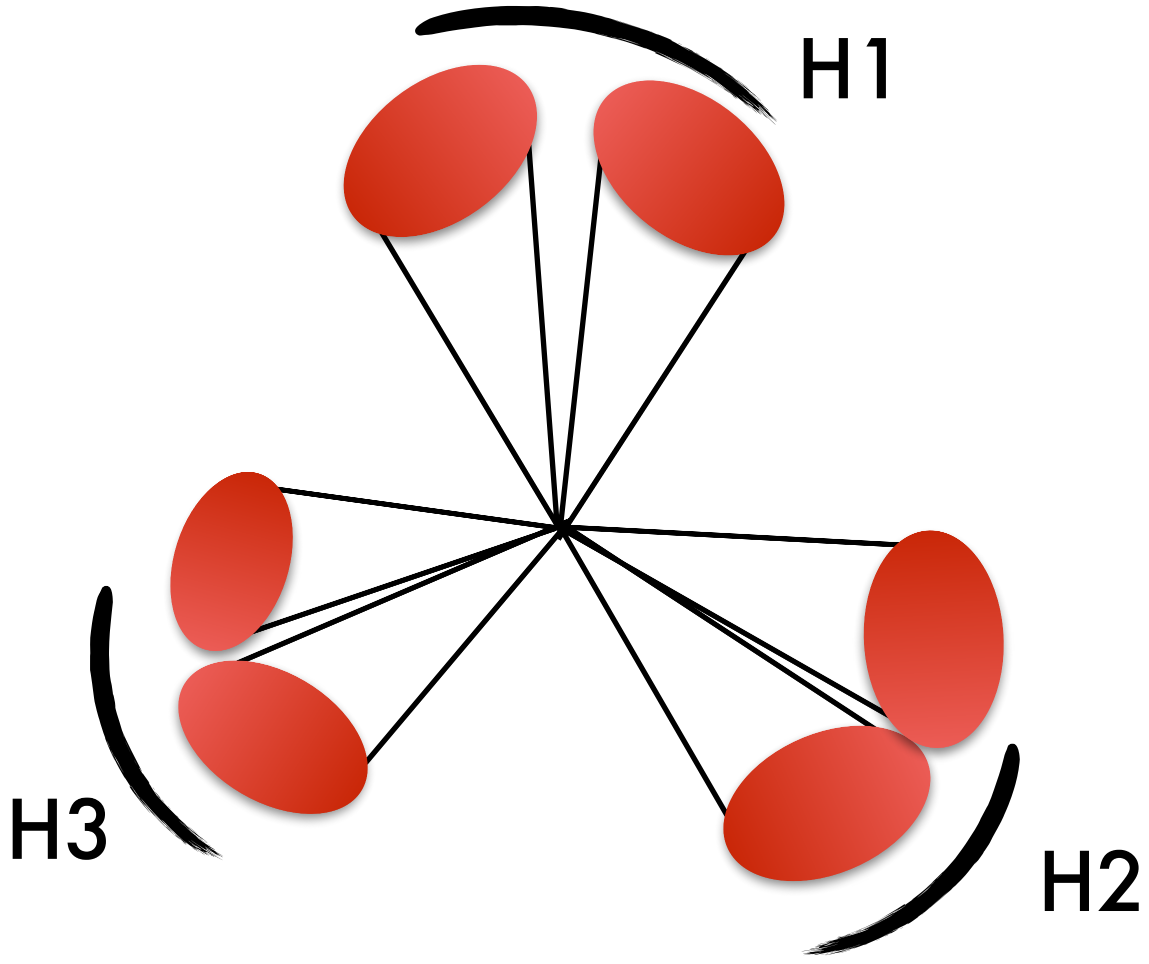
- Inputs: 10 AK5 jets with pT, eta, phi, b-tagging, mass

Advantages of SPANET:

- Exploit kinematic information of all jets (not just 6)
- Dynamical: each event has a different number of jets, AK8, ...
- Interesting interplay between boosted and resolved

# Resolved HHH topologies



# Event Configuration: fully resolved

- To give a flavor of how we use it, we can compare the event configurations for $t\bar{t} \to (bq\bar{q})(\bar{b}q\bar{q})$ (one of the original use cases) and $HHH \to (b\bar{b})(b\bar{b})(b\bar{b})$

```
# ----------------------------------------------------------------------
# REQUIRED - EVENT - Complete list of resonance particles and daughters.
# ----------------------------------------------------------------------
EVENT:
  t1:
    - q1: Jets
    - q2: Jets
    - b:  Jets
  t2:
    - q1: Jets
    - q2: Jets
    - b:  Jets
# ----------------------------------------------------------
# REQUIRED KEY - PERMUTATIONS - List of valid permutations.
# ----------------------------------------------------------
PERMUTATIONS:
    EVENT:
      - [ t1, t2 ]
    t1:
      - [ q1, q2 ]
    t2:
      - [ q1, q2 ]
```

```
# ----------------------------------------------------------------------
# REQUIRED - EVENT - Complete list of resonance particles and daughters.
# ----------------------------------------------------------------------
EVENT:
  h1:
    - b1: Jets
    - b2: Jets
  h2:
    - b1: Jets
    - b2: Jets
  h3:
    - b1: Jets
    - b2: Jets
# ----------------------------------------------------------
# REQUIRED KEY - PERMUTATIONS - List of valid permutations.
# ----------------------------------------------------------
PERMUTATIONS:
    EVENT:
      - [ h1, h2, h3 ]
    h1:
      - [ b1, b2 ]
    h2:
      - [ b1, b2 ]
    h3:
      - [ b1, b2 ]
```

# Dataset & Input Features

Madgraph model provided by
A. Papaefstathiou, T. Robens, G. Tetlalmatzi-Xolocotzi
JHEP 05 (2021) 193

- Using 14 TeV pp-collisions simulated HHH6b Madgraph+Py8+Delphes:
  - ~1M events for training+validation; ~300k events for testing
- Truth matching condition:
  - Gen b-quark from Higgs boson decay is within $\Delta R \leq 0.5$ of AK5 jet
  - Added hadron “b” flavor requirement on AK5 jet
- Higgs boson is “reconstructible” if both b quark daughters match to AK5 jets
- Up to 10 AK5 Jets are considered per event (ranked by pT)
- Input jet features:
  - $p_{\mathrm{T}}$ (log-normalized), $\eta$ (normalized), $\sin\phi$, $\cos\phi$, and boolean b-tag score

# Partial vs Complete events

| Inclusive >= 6 AK5 jets | Event proportion |
| --- | --- |
| 0 H | 10% |
| 1 H | 20% |
| 2 H | 50% |
| 3 H | 20% |

- SPANet allows to reconstruct partial events as well as complete events
- 10% of events have 0 Higgs Boson reconstructible
- 20% have 1 Higgs reconstructible and 50% with 2 Higgs reconstructible
- 20% have 3 Higgs Boson reconstructible from 2 AK5 jets

# HHH — Baseline Method

- Baseline (Higgs mass): $m_H = 125\,\mathrm{GeV}$
    - Note: background mass sculpting

$$\chi^2 = (m_{b_1b_2} - m_H)^2 + (m_{b_3b_4} - m_H)^2 + (m_{b_5b_6} - m_H)^2$$

- Estimate SPA-Net performance improvements with respect to Chi2 baseline

# Baseline Method 1 ($m_H = 125$ GeV) — HHH

Note: using top 6 jets in each event ordered by b-tag and pT

| Event Type | Event Purity | H Purity |
|---|---|---|
| 1-3 H | 22% | 39% |
| 3H | 23% | 43% |

$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

$$\text{H Purity} = \frac{\text{Number of reconstructed HIggs}}{\text{Total number of Higgs}}$$

- For 1-3 reconstructible Higgs, **22%** of events correctly reconstructed; **39%** of Higgs correctly reconstructed
- For 3 reconstructible Higgs, **23%** of events correctly reconstructed; **52%** of Higgs correctly reconstructed

# SPA-Net — HHH

| Event Type | Event Purity | H Purity |
|---|---|---|
| 1-3 H | 34% | 52% |
| 3H | 38% | 58% |

$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

$$\text{H Purity} = \frac{\text{Number of reconstructed HIggs}}{\text{Total number of Higgs}}$$

- For 1-3 reconstructible Higgs, **34% (+54%)** of events correctly reconstructed; **52% (+33%)** of Higgs correctly reconstructed
- For 3 reconstructible Higgs, **38% (+65%)** of events correctly reconstructed; **58% (+34%)** of Higgs correctly reconstructed

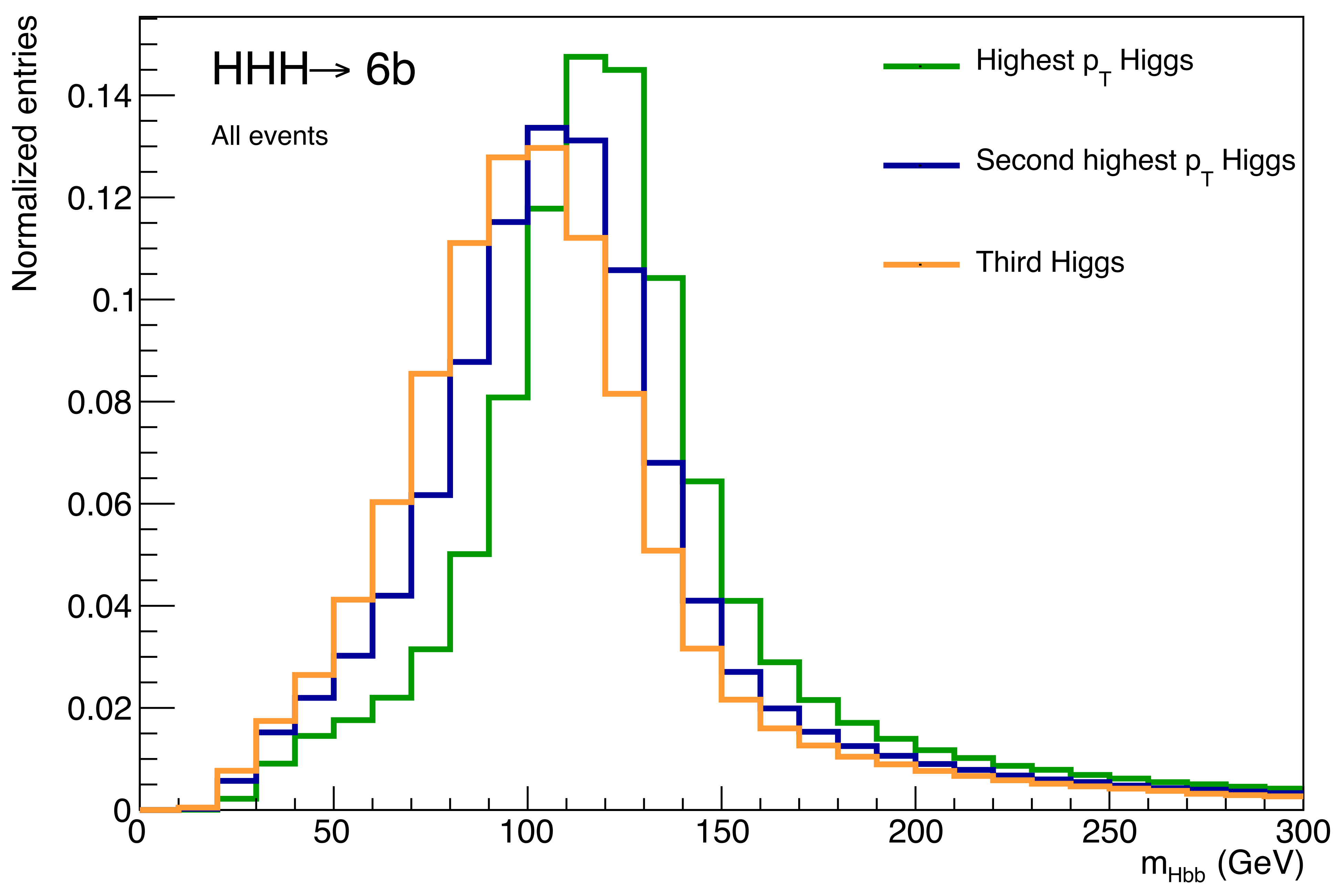
# SPA-Net — HHH

| H purity | 1-3h | | 3h | |
|---|---|---|---|---|
| | Chi2 baseline | SPA-Net | Chi2 baseline | SPA-Net |
| 6 jets | 46.3% | 53.4% (**+15%**) | 64.1% | 67.8% (**+6%**) |
| 7 jets | 40.2% | 53.3% (**+33%**) | 47.0% | 61.9% (**+32%**) |
| >= 8 jets | 33.8% | 49.8% (**+47%**) | 36.4% | 54.2% (**+49%**) |
| Full | 38.6% | 51.7% (**+33%**) | 43.1% | 58.2% (**+35%**) |

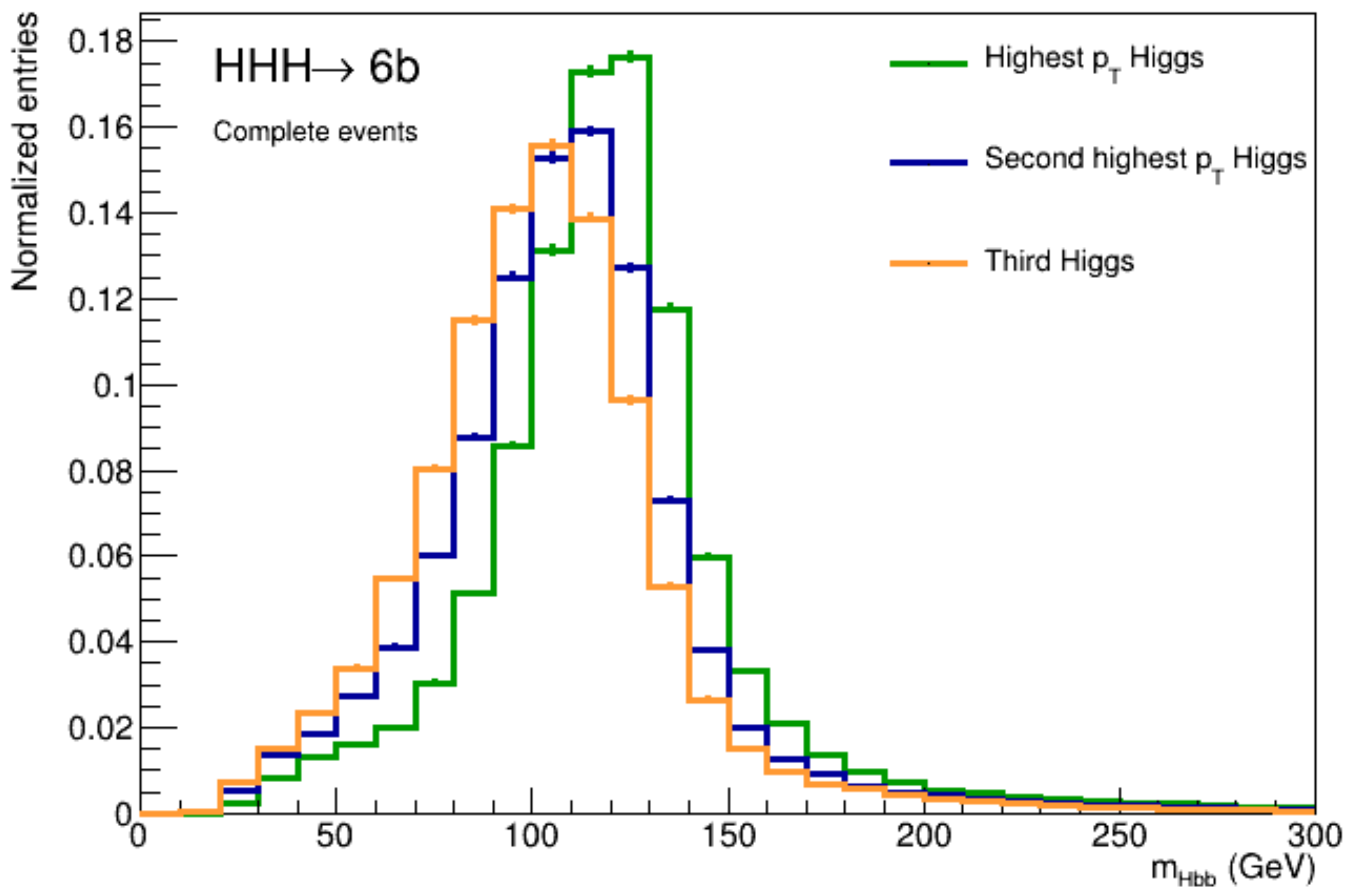
- Performance improvements in H purity:
  - Largest improvements observed with higher number of jets
  - SPA-Net: achieves similar H purity in different jet multiplicity categories

# Mass Reconstruction

1-3h events

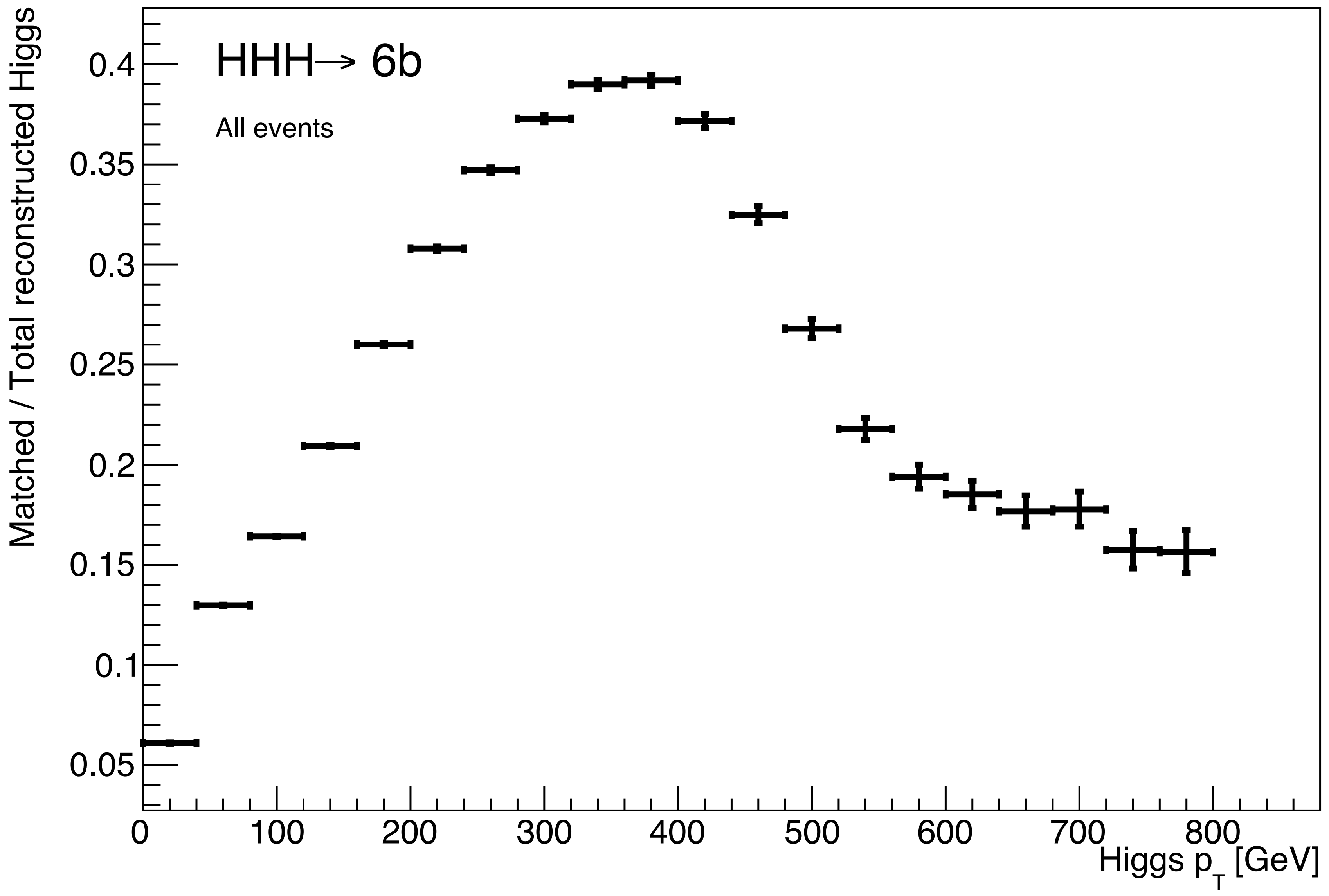



3h events



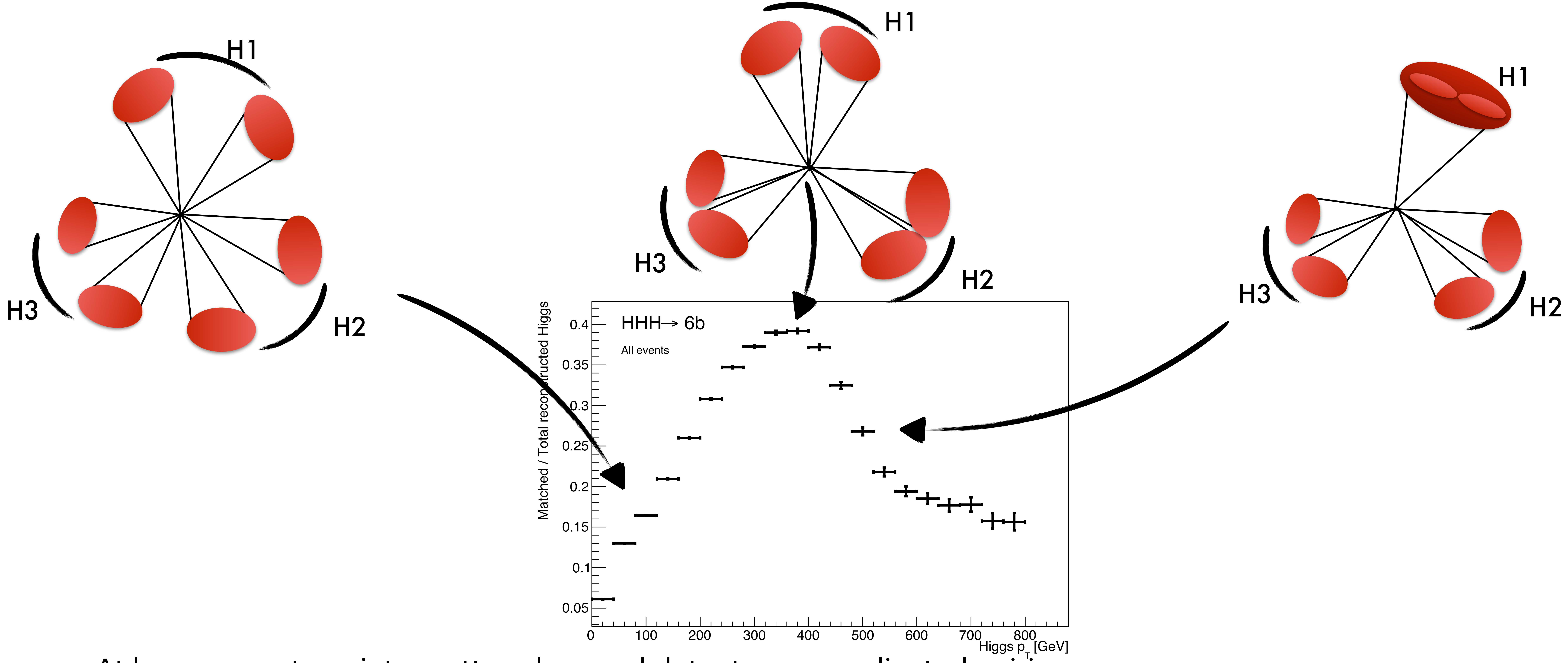

- SPA-Net reconstructs the mass of each Higgs candidate appropriately
- Work in progress: investigating mass sculpting of backgrounds

# Differential matching efficiency



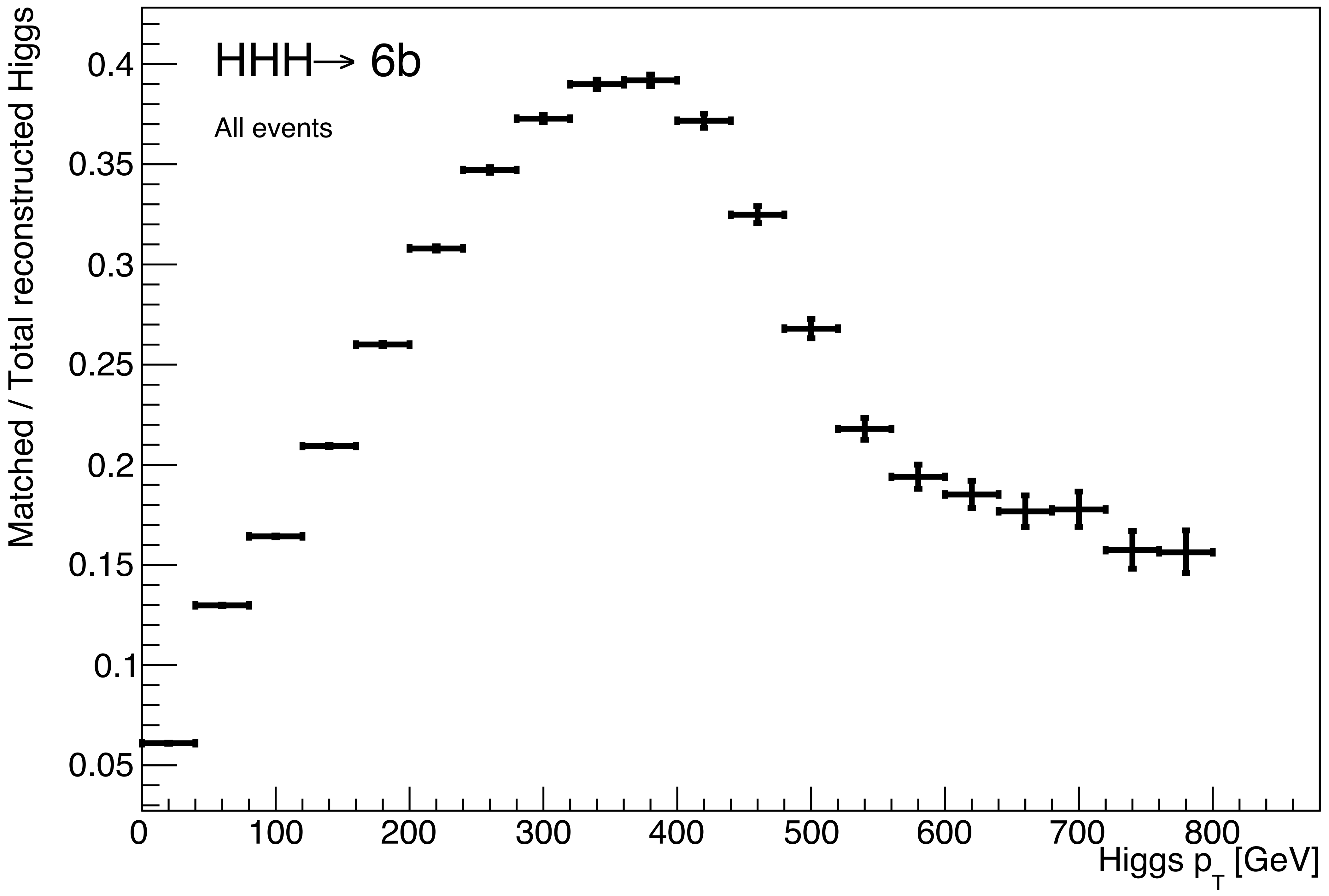

- Matching efficiency: strong dependence on momentum of the Higgs bosons

# Differential matching efficiency

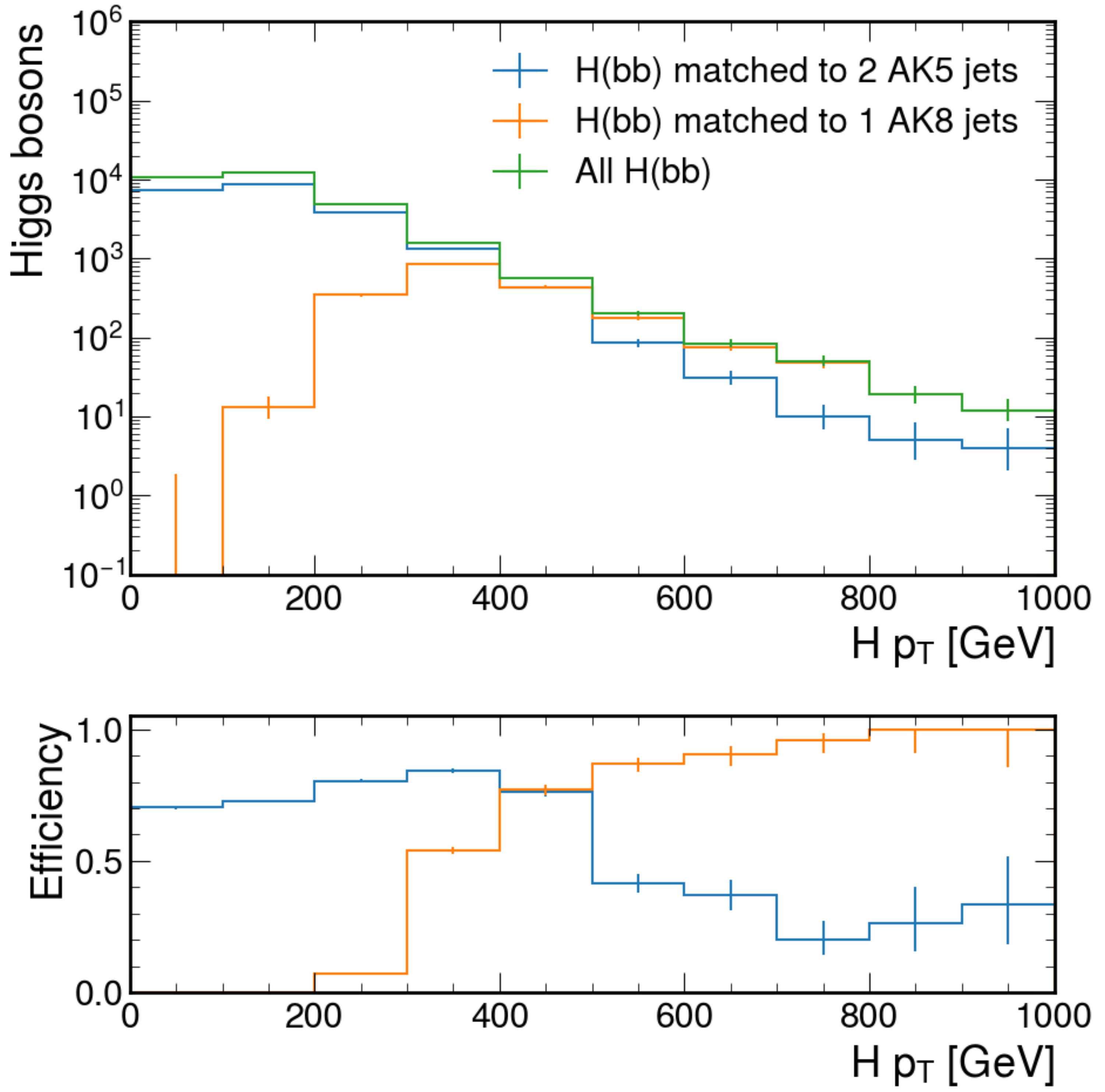



- At low momentum, jets scattered around detector = complicated pairing
- At higher momentum, jets from Higgs boson more and more collimated, clearer correlation
- At very high momentum ( pT > 400 GeV), matching efficiency drops and Higgs reconstructed in AK8 jets

# Differential matching efficiency



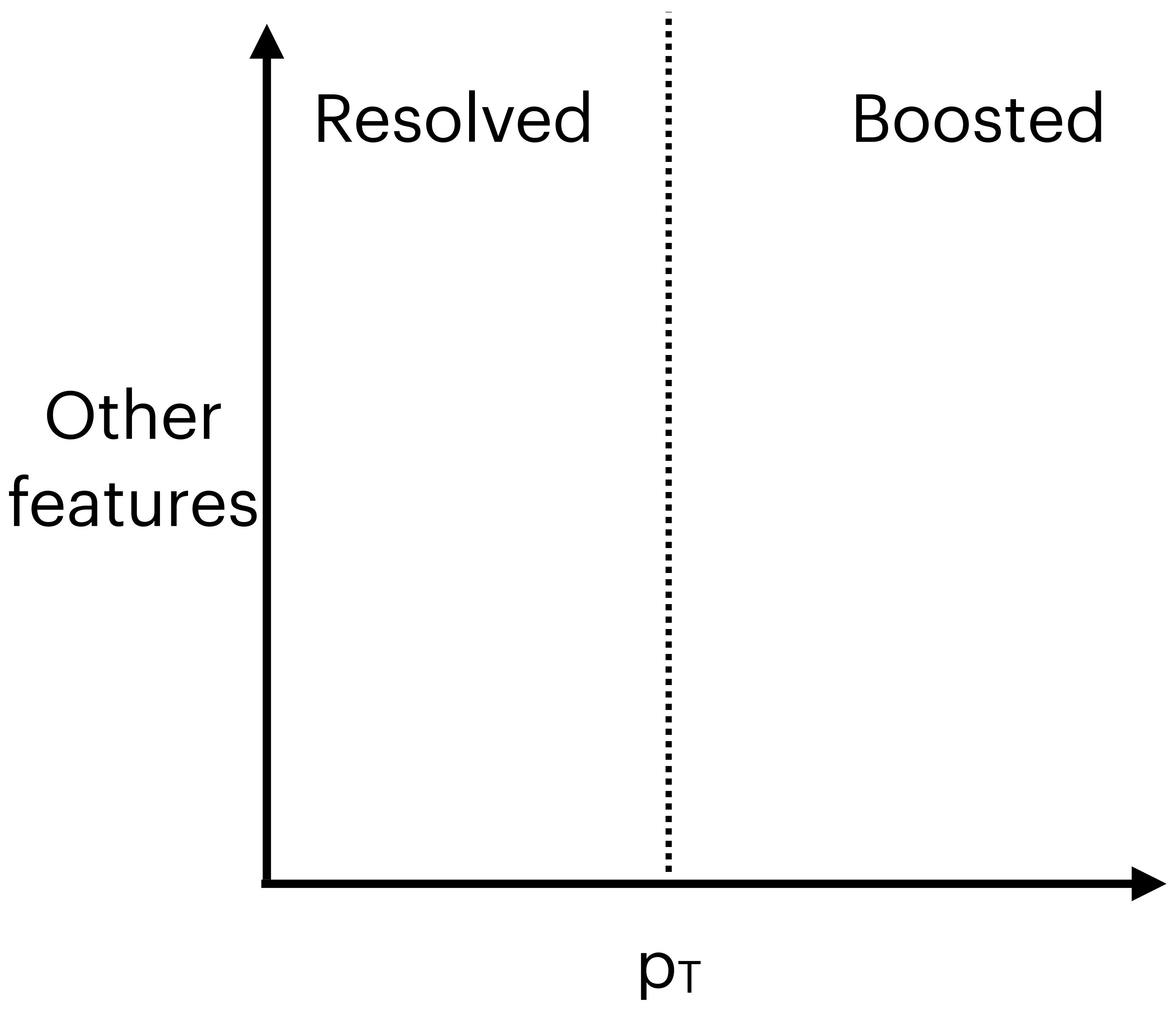

Truth level: Higgs reconstructed in 2 AK5 vs 1 AK8

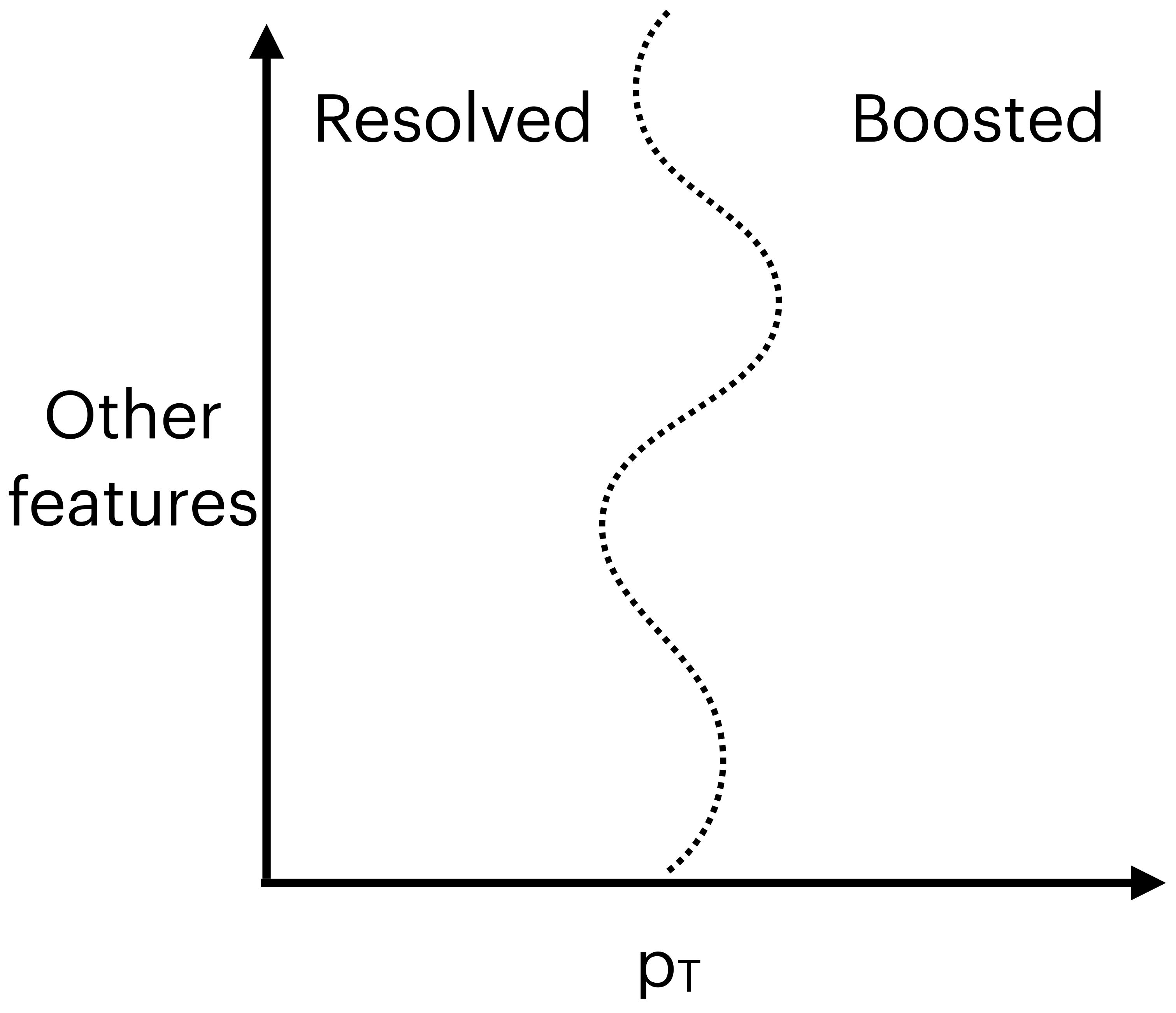



- At 400 GeV, Higgs more likely to be reconstructed in 1 AK8 than 2 AK5
- Optimal performance: generalize approach to both boosted + resolved topologies

# Boosted + Resolved topologies

- Generalizing to **boosted + resolved** topologies
- Idea is to help us divide phase space between boosted and resolved in more intelligent way



# Event Configuration: resolved + boosted

- Comparison boosted+resolved configurations $HHH \to (b\bar{b})(b\bar{b})(b\bar{b})$

```
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# REQUIRED - EVENT - Complete list of resonance particles and daughters.
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    - q1: Jets
    - q2: Jets
    - b:  Jets
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    - q2: Jets
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# ---------------------------------------------------------
# REQUIRED KEY - PERMUTATIONS - List of valid permutations.
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PERMUTATIONS:
    EVENT:
      - [ t1, t2 ]
    t1:
      - [ q1, q2 ]
    t2:
      - [ q1, q2 ]
```
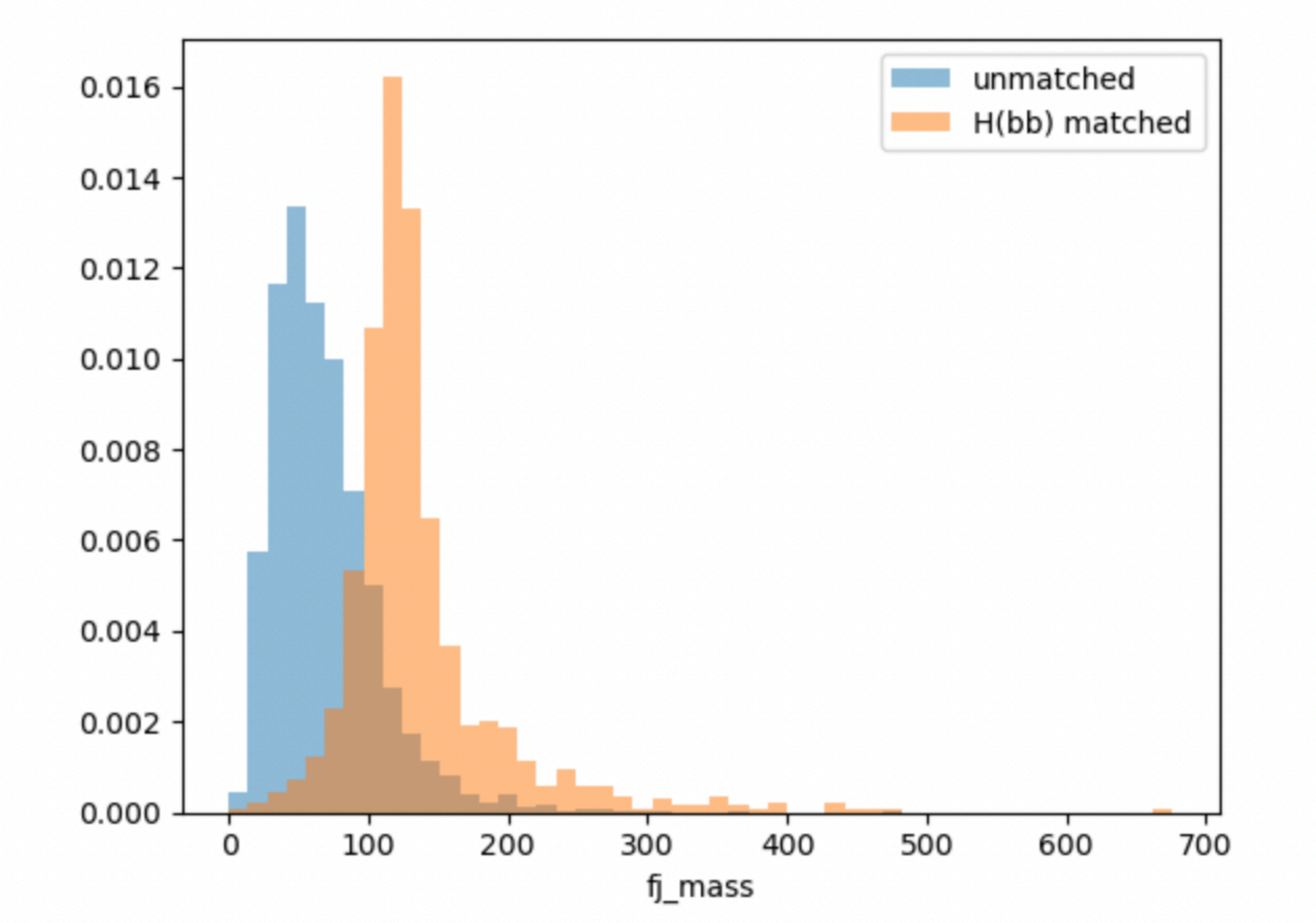

```
# ---------------------------------------------------------------------
# REQUIRED - EVENT - Complete list of resonance particles and daughters.
# ---------------------------------------------------------------------
EVENT:
  h1:
    - b1: Jets
    - b2: Jets
  h2:
    - b1: Jets
    - b2: Jets
  h3:
    - b1: Jets
    - b2: Jets
  bh1:
    - bb: BoostedJets

  bh2:
    - bb: BoostedJets

  bh3:
    - bb: BoostedJets

# ---------------------------------------------------------
# REQUIRED KEY - PERMUTATIONS - List of valid permutations.
# ---------------------------------------------------------
PERMUTATIONS:
    EVENT:
      - [[h1, h2], [bh1,bh2]]

      - [[h1, h3], [bh1,bh3]]

      - [[h2, h3], [bh2,bh3]]

    h1:
      - [ b1, b2 ]
    h2:
      - [ b1, b2 ]
    h3:
      - [ b1, b2 ]
```

**All permutations of AK5 and AK8 jets**

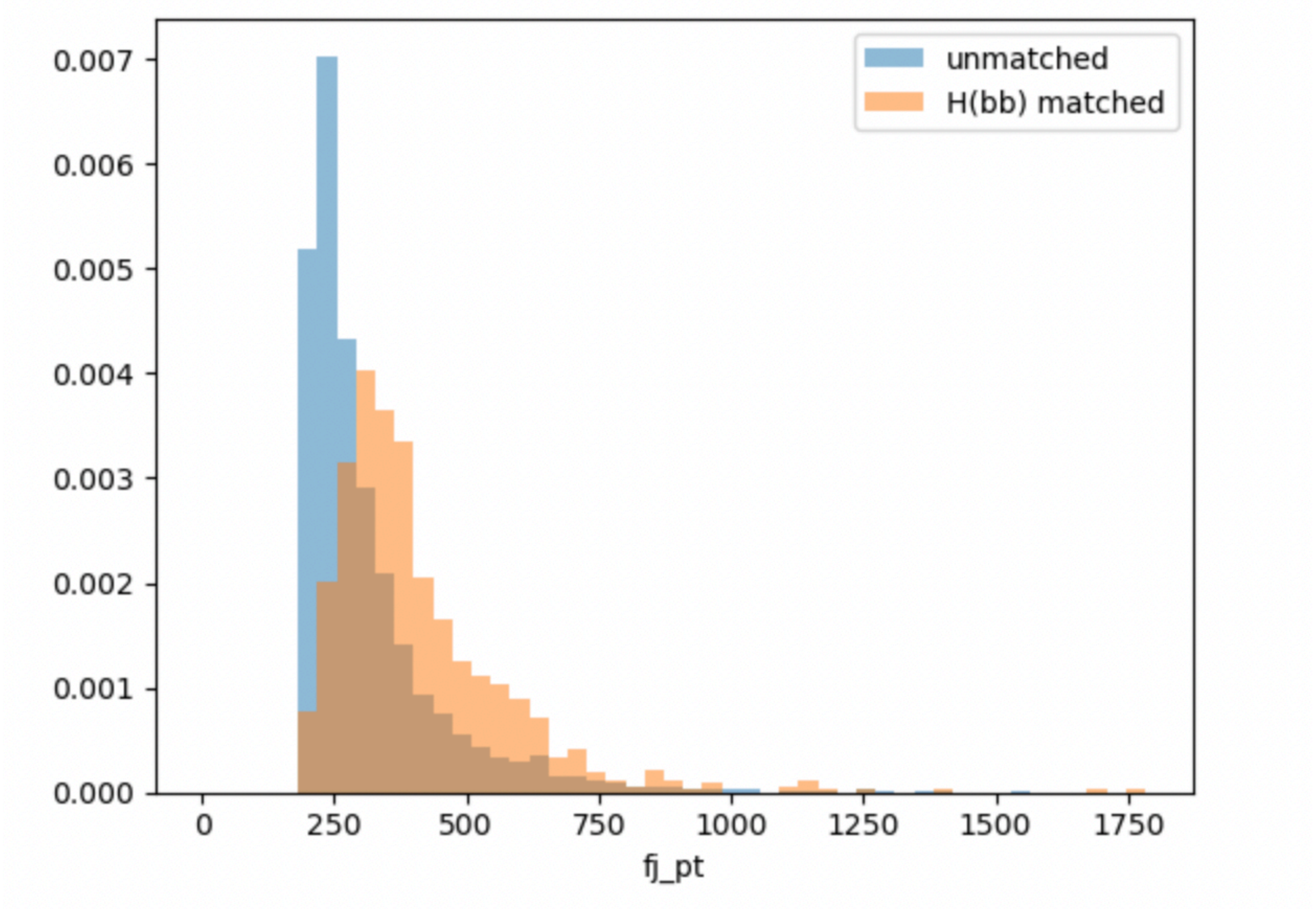
# Boosted variables

- Use boosted jets mass, pT, eta, phi, and subjettiness variables as inputs

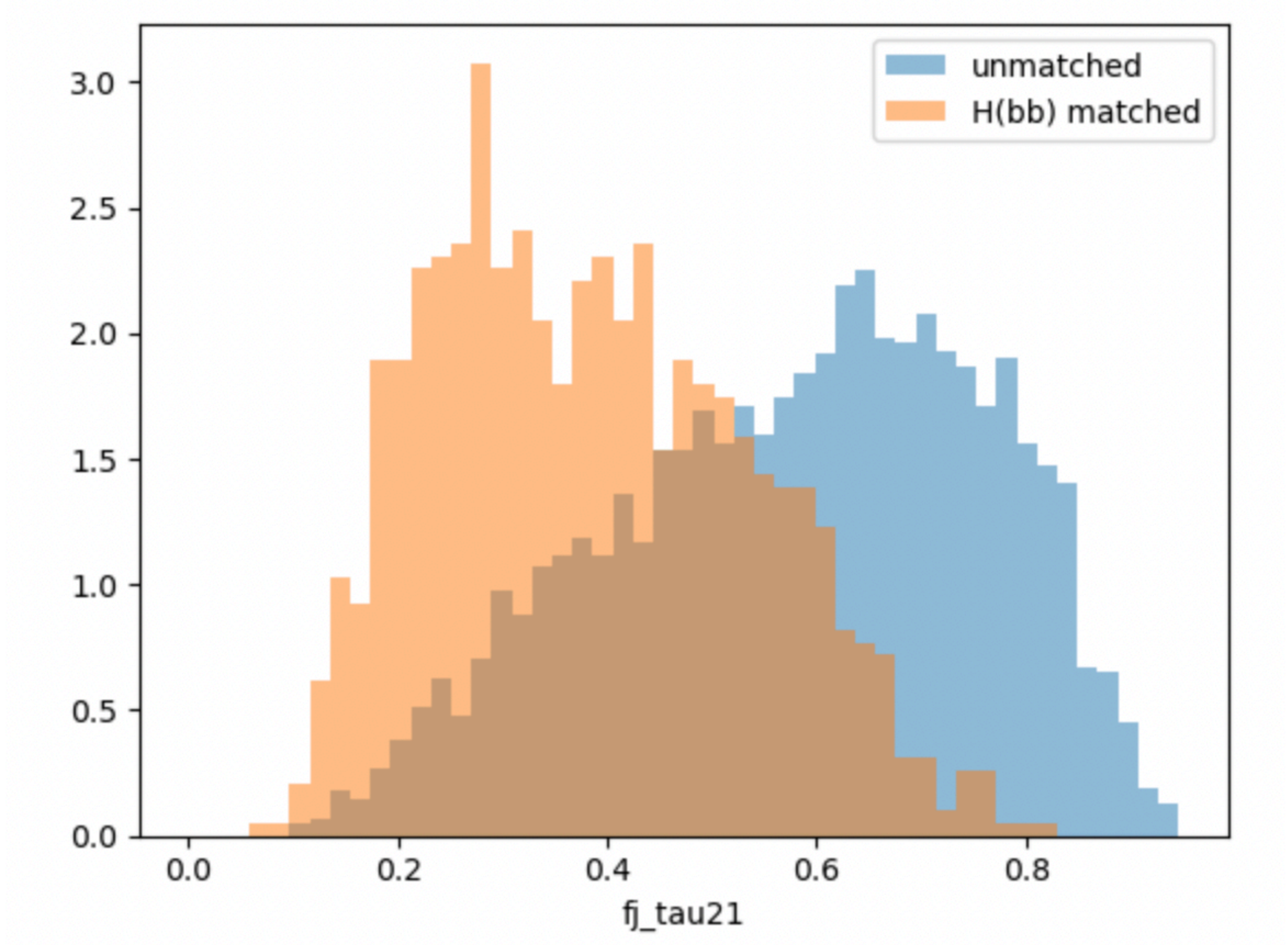
Mass




pT




Subjettiness




- On-going work to include both resolved and boosted jets in one training
    - Currently studying network hyperparameters and metrics to compare to resolved
    - Stay tuned: more results will be published in the paper

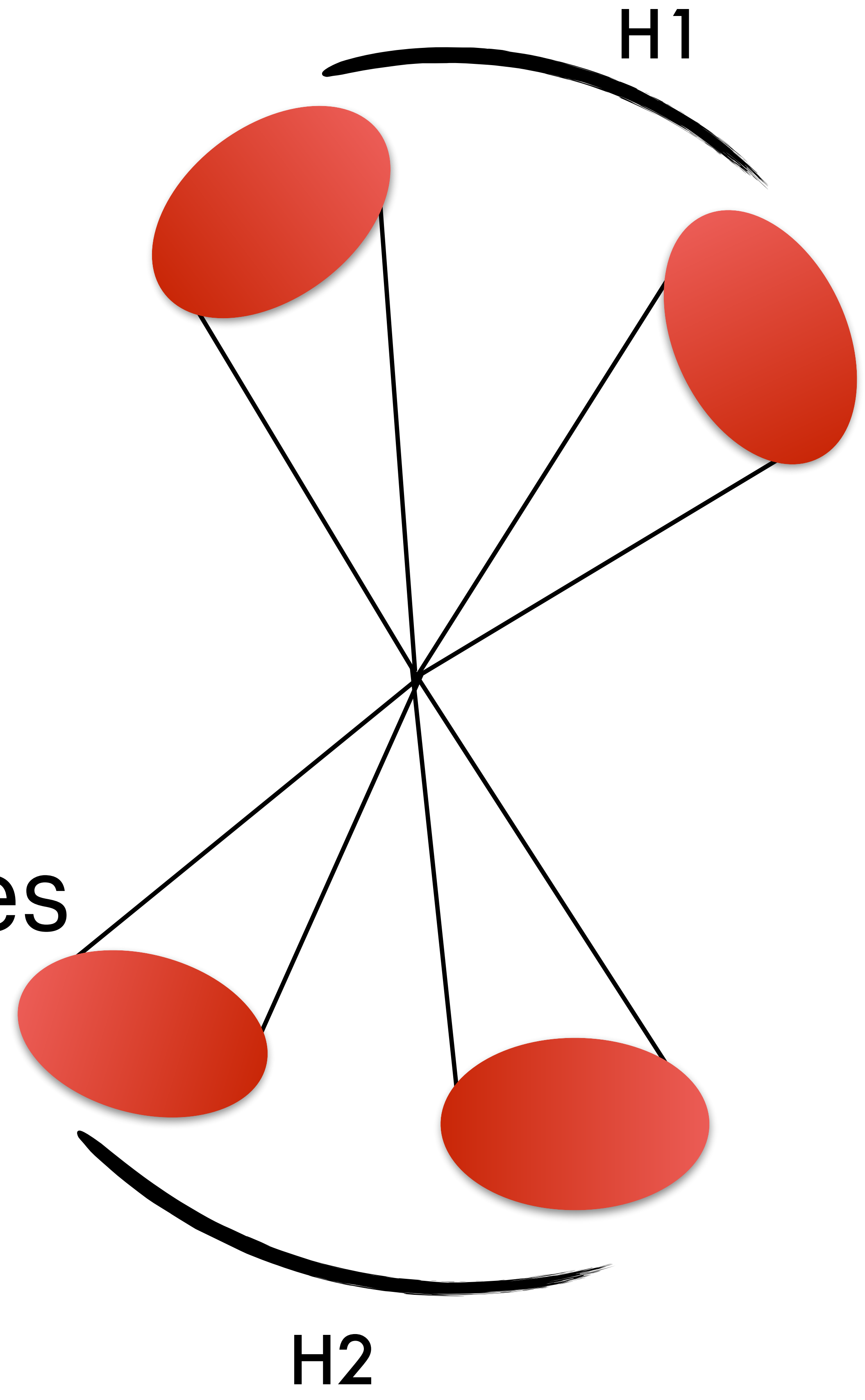
# HH pairing comparisons

- Baseline (Higgs mass): $m_H = 125\,\mathrm{GeV}$
  - Note: background mass sculpting

$$\chi^2 = (m_{b_1b_2} - m_H)^2 + (m_{b_3b_4} - m_H)^2$$

- Mass agnostic distance method:
  - Find pairs based on minimal distance between 2 Higgs masses

$$D = |\,m(b_1, b_2) - k \times m(b_3, b_4)\,| / \sqrt{1 + k^2}, k = 125/120$$



# Baseline Method 1 ($m_H$ = 125 GeV) — HH

Note: using top 4 jets in each event

| Event Type | Event Purity | H Purity |
|---|---|---|
| 1-2 H | 44% | 57% |
| 2H | 21% | 53% |

$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

$$\text{H Purity} = \frac{\text{Number of reconstructed HIggs}}{\text{Total number of Higgs}}$$

- For 1-2 reconstructible Higgs, **44%** of events correctly reconstructed; **57%** of Higgs correctly reconstructed
- For 2 reconstructible Higgs, **21%** of events correctly reconstructed; **53%** of Higgs correctly reconstructed

# SPA-Net — Resolved HH training

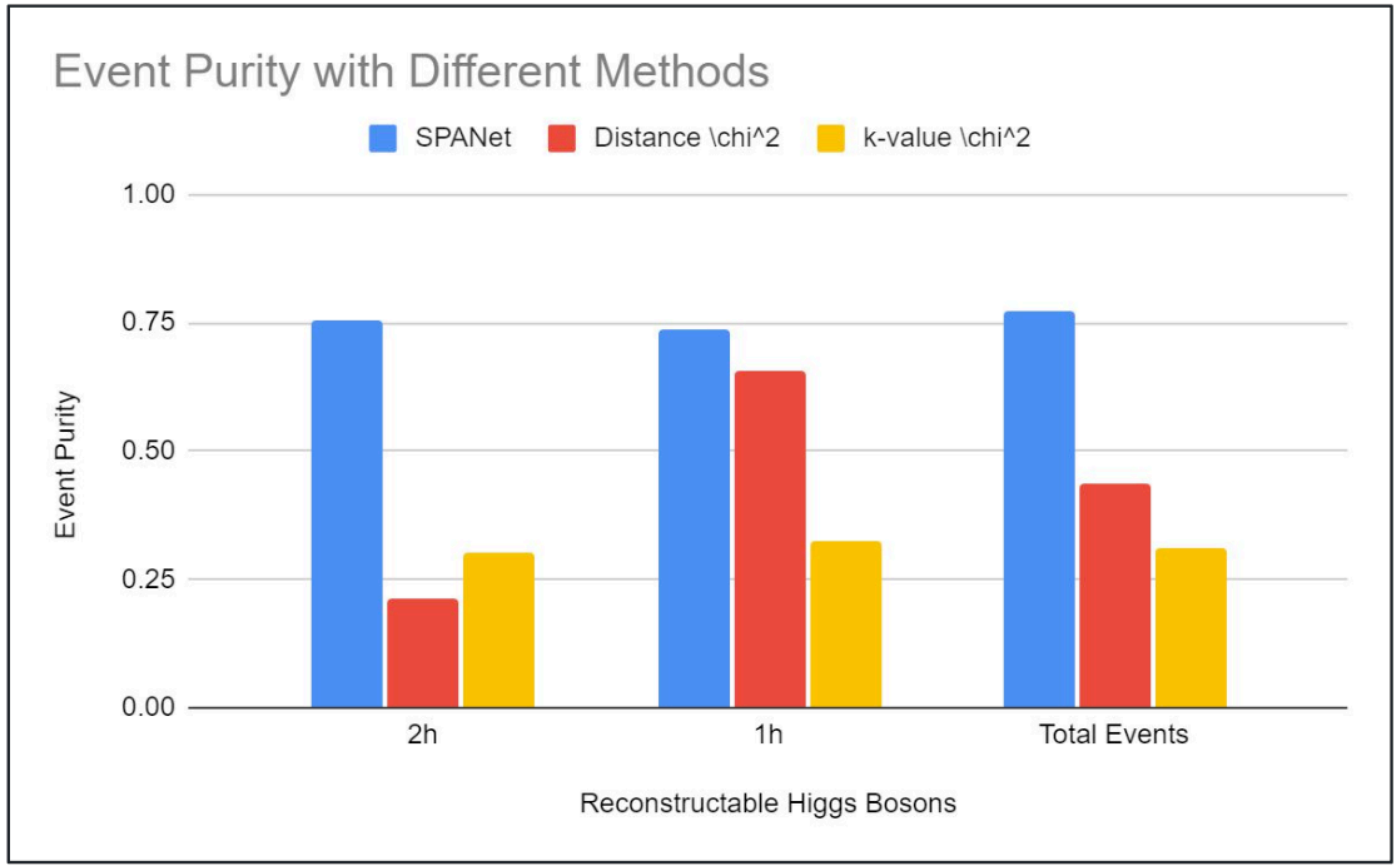
| Event Type | Event Purity | H Purity |
|---|---|---|
| 1-2 H | 76% | 81% |
| 2H | 77% | 84% |

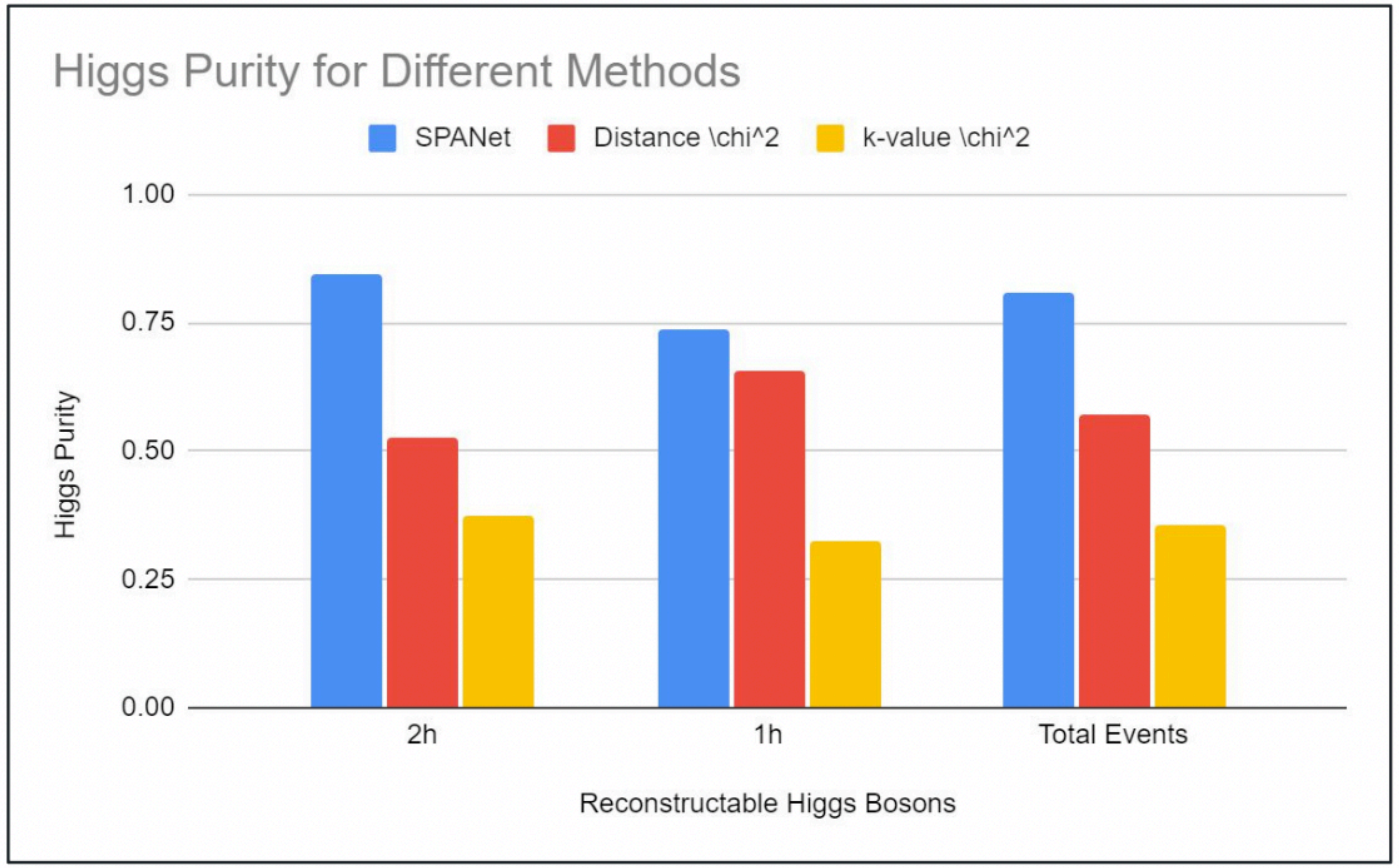
$$\text{Event Purity} = \frac{\text{Number of events that all Higgs are reconstructed}}{\text{Total number of events}}$$

$$\text{H Purity} = \frac{\text{Number of reconstructed HIggs}}{\text{Total number of Higgs}}$$

- For 1-2 reconstructible Higgs, **76% (+72%)** of events correctly reconstructed; **81% (+42%)** of Higgs correctly reconstructed
- For 2 reconstructible Higgs, **77% (+360%)** of events correctly reconstructed; **84% (+58%)** of Higgs correctly reconstructed - result to be understood in context of analysis, limitations of Delphes…

# Method comparison

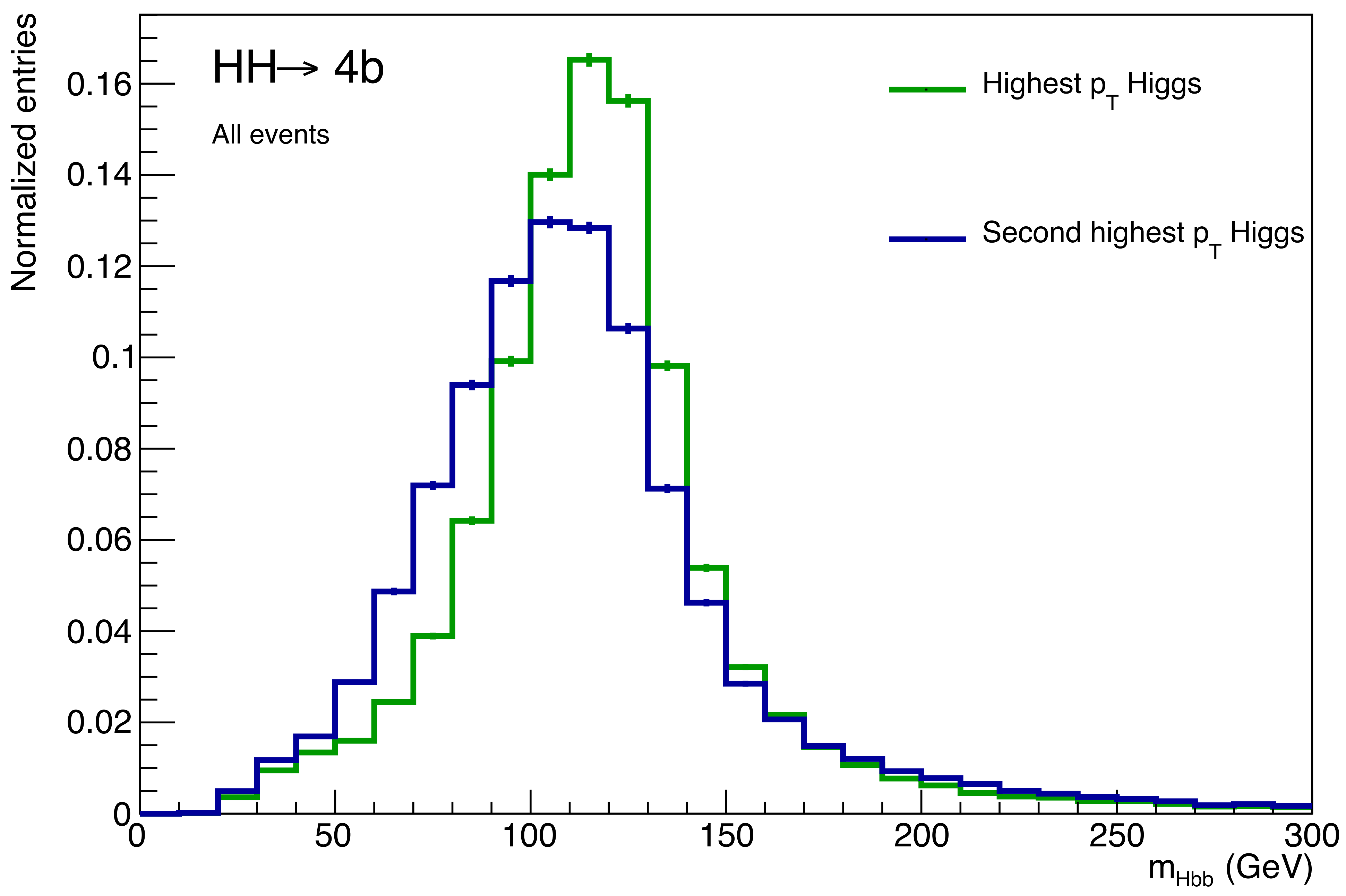


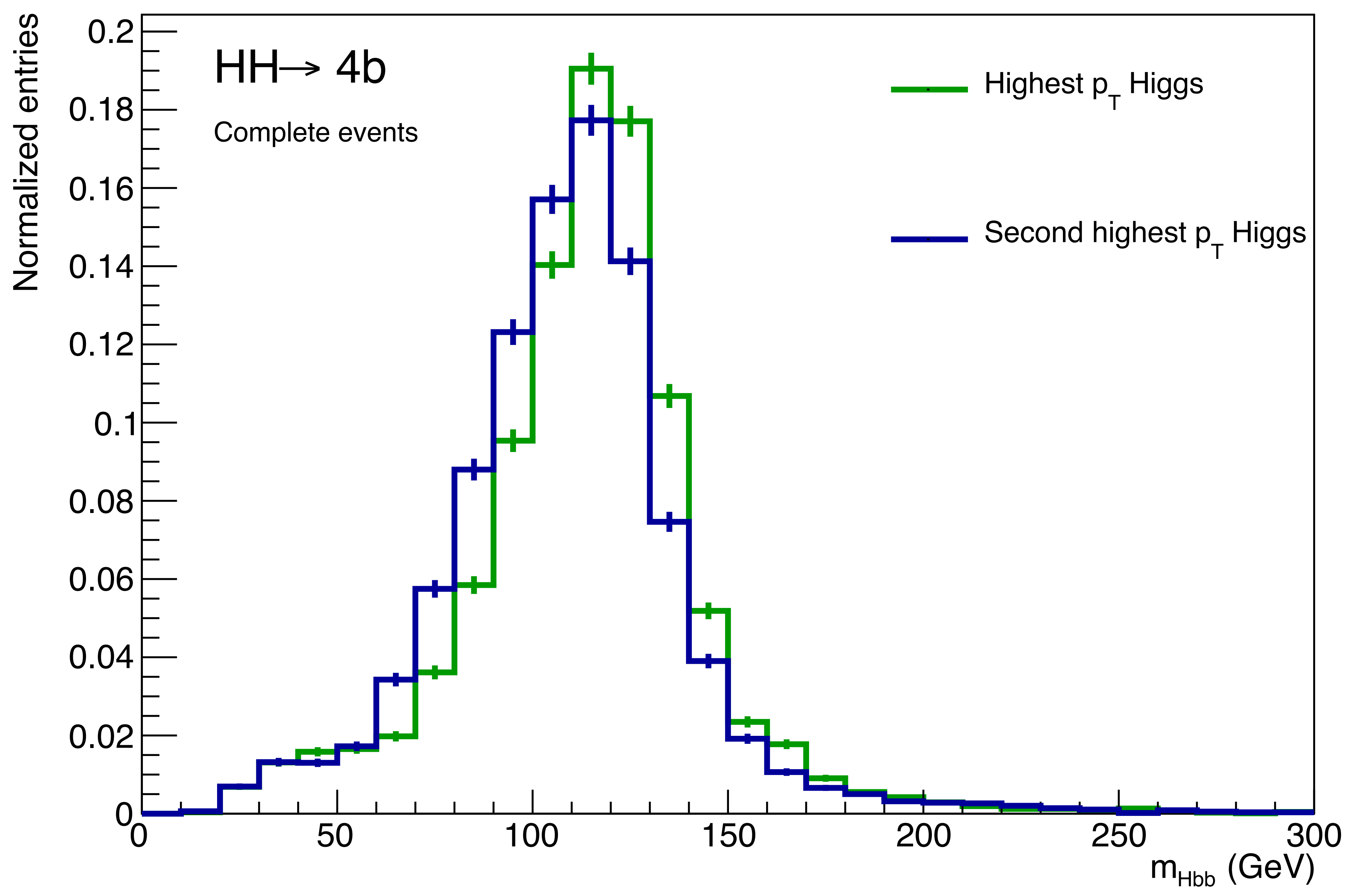

- SPA-Net: clear performance improvements with respect to both baselines
- Short-comings: chi2 methods consider only 4 jets so far, boolean b-tagging
  - Likely lower improvements than in analysis specific reconstruction
  - However, great potential for SPA-Net to improve HH and HHH analyses

# Mass Reconstruction HH

1-2h events



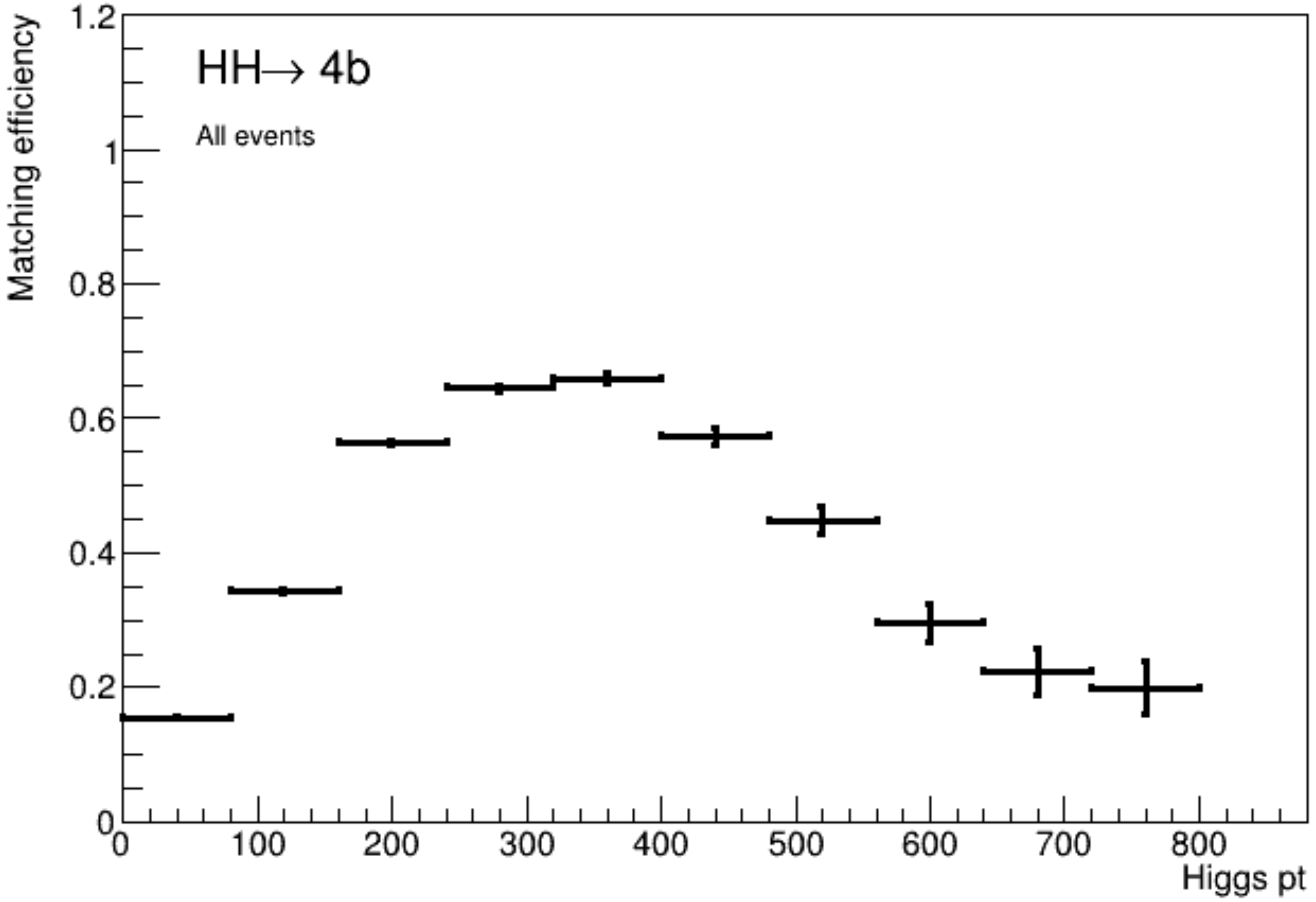

2h events



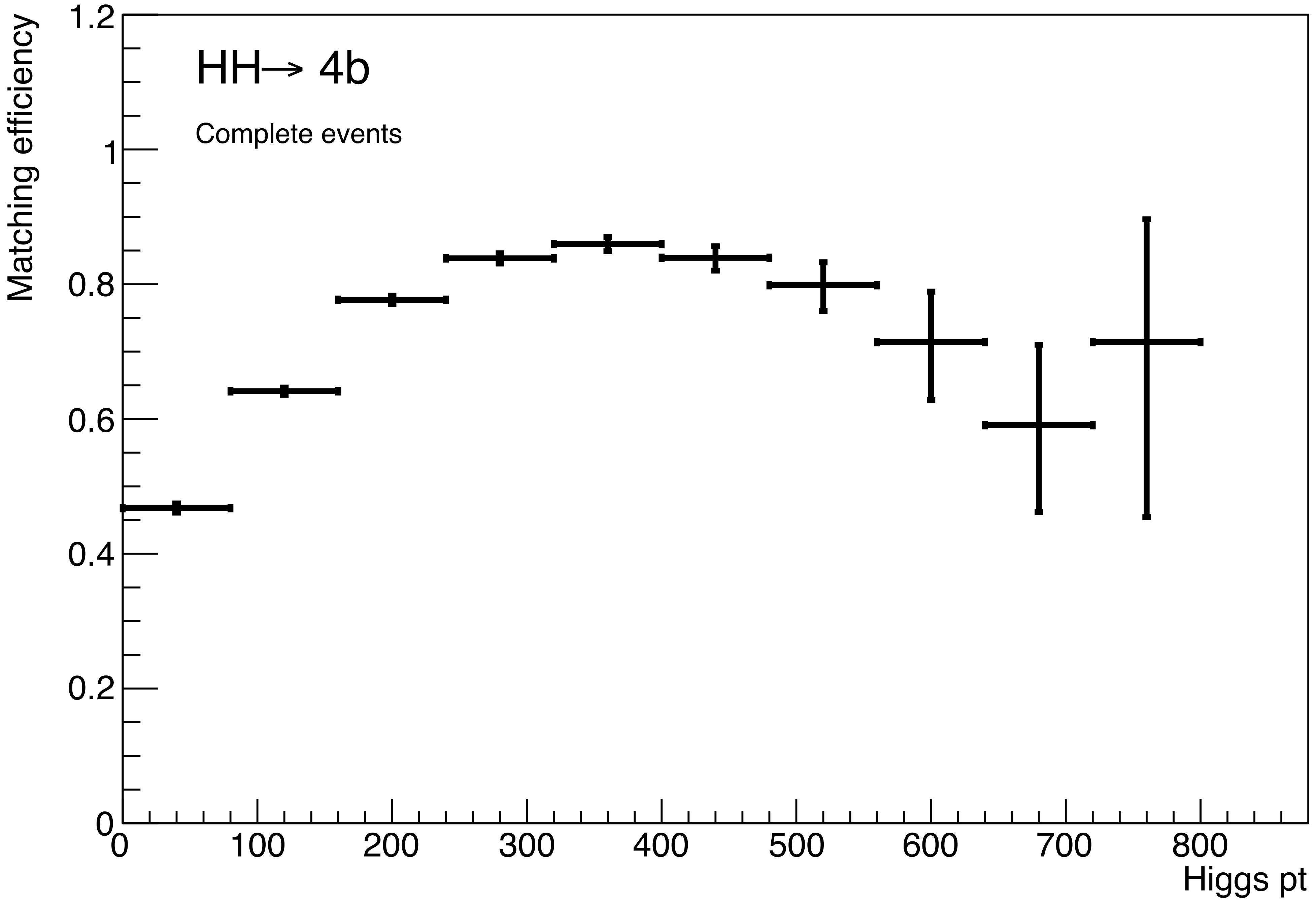

- SPA-Net reconstructs the mass of each Higgs candidate appropriately
- Work in progress: investigating mass sculpting of backgrounds
- Investigating boosted + resolved reconstruction to validate SPA-Net method

# Differential matching efficiency

1-2h events




2h events




- At 400 GeV, Higgs more likely to be reconstructed in 1 AK8 than 2 AK5
- Optimal performance: generalize approach to both boosted + resolved topologies
  - On-going work to define best strategy and compare results with HHH

# Summary

- SPANet: A transformer model for particle reconstruction.
- SPANet shows better performance than chi2 in our preliminary study of HHH6b.
  - Unique algorithm to pair fully resolved, semi-boosted, fully boosted simultaneously
  - Performance improvements validated on HH4b signal too

- Stay tuned: paper being finalised and will be published soon!

# Backup

# Symmetric Tensor Attention

- Note $\mathscr{P}_p$ is an "overparameterization" of the valid jet assignments: many represent the same physical combinations.
  - For example for the $HHH \to 6b$ case, 10 jets maximum
    - Each $\mathscr{P}_p$ has 100 entries
    - But we can swap $(b_1, b_2)$ for each $H$, and can swap $H_1, H_2, H_3$
    - In the end we end up with only 3150 unique physical assignments!

$$\forall \sigma \in G_p \ \left(j_1, j_2, \ldots, j_{k_p}\right) \simeq \left(j_{\sigma(1)}, j_{\sigma(2)}, \ldots, j_{\sigma\left(k_p\right)}\right) \iff \mathscr{P}_p^{j_1 j_2 \cdots j_{k_p}} = \mathscr{P}_p^{j_{\sigma(1)} j_{\sigma(2)} \cdots j_{\sigma\left(k_p\right)}}$$

$$\mathcal{S}^{i_1 i_2 \cdots i_{k_p}} = \sum_{\sigma \in G_p} \Theta^{i_{\sigma(1)} i_{\sigma(2)} \cdots i_{\sigma\left(k_p\right)}},$$

$$\mathcal{O}^{j_1 j_2 \cdots j_{k_p}} = X_{i_1}^{j_1} X_{i_2}^{j_2} \cdots X_{i_{p_k}}^{j_{p_k}} \mathcal{S}^{i_1 i_2 \cdots i_{k_p}},$$

$$\mathscr{P}_p^{j_1 j_2 \cdots j_{k_p}} = \frac{\exp(\mathcal{O}^{j_1 j_2 \cdots j_{k_p}})}{\sum_{j_1, j_2, \ldots, j_{p_k}} \exp(\mathcal{O}^{j_1 j_2 \cdots j_{k_p}})}.$$

# Combined Symmetric Loss

- Symmetric attention layers produce solutions $\{\mathscr{P}_1, \mathscr{P}_2, \ldots, \mathscr{P}_m\}$ for each particle's jet-carton assignment sub-problem
- True assignments are delta-distributions containing one possible valid jet assignment $\{\mathscr{T}_1, \mathscr{T}_2, \ldots, \mathscr{T}_m\}$.
- Loss for each sub-problem is the categorical cross entropy for each particle $p$
- Permutation group $G_E$ induces an equivalence relation over particles: $\forall \sigma \in G_E, \ (\mathscr{T}_1, \mathscr{T}_2, \ldots, \mathscr{T}_m) \simeq (\mathscr{T}_{\sigma(1)}, \mathscr{T}_{\sigma(2)}, \ldots, \mathscr{T}_{\sigma(m)})$
- Incorporate these symmetries by allowing network to fit any equivalent jet assignment (minimize loss over a given equivalence class)

# Partial Event Reconstruction

- Though each parton is usually expected to produce a jet, some particles are impossible to reconstruct
- Mask unreconstrable particles and only include the loss contributed by reconstructable particles
- Also, scale the loss based on the distribution of events present in the training dataset by computing the effective class count for each partial combination

$$\mathscr{L}_{\min}^{\text{masked}} = \min_{\sigma \in G_E} \left( \sum_{i=1}^{m} \frac{\mathscr{M}_{\sigma(i)} CE(\mathscr{P}_i, \mathscr{T}_{\sigma(i)})}{CB\left(\mathscr{M}_{\sigma(1)}, \mathscr{M}_{\sigma(2)}, \ldots, \mathscr{M}_{\sigma(m)}\right)} \right).$$

# Generalization to Boosted & Resolved

- $N_j$ : number of AK4 jets; $N_{fj}$ : number of AK8 jets
- Two sets of reconstruction targets: [h1, h2] and [bh1 (boosted H 1), bh2]
  - Output AK4 jet AD tensor is a $N_j$ x $N_j$ symmetric matrix,
  - Output AK8 jet AD tensor is a vector of size $N_{fj}$.
- `{h_i}` and `{bh_i}` are the same set of particles ordered by pT, so `h_i` and `bh_i` have one-to-one correspondence.
- `bh_i` will be selected if it is detected. If not, `h_i` will be selected if it is detected.

# Model Configuration

- Many hyperparameters to tune!
- We suggested the following:


```yaml
assignment_loss_scale: 1.0
balance_classifications: false
balance_jets: 0
balance_losses: true
balance_particles: 1
batch_size: 4096
classification_loss_scale: 0.0
combinatorial_scale: 0.0
combine_pair_loss: min
dataset_limit: 1.0
dataset_randomization: 0
detection_loss_scale: 0.0
dropout: 0.2
epochs: 250
event_info_file: event_files/hhh_masses.yaml
focal_gamma: 0.0
gradient_clip: 0.0
hidden_dim: 64
initial_embedding_dim: 16
initial_embedding_skip_connections: 1
kl_loss_scale: 0.0
l2_penalty: 0.0002
learning_rate: 0.0015
learning_rate_cycles: 1
learning_rate_warmup_epochs: 1.0
limit_to_num_jets: 0
linear_activation: gelu
linear_block_type: GRU
linear_prelu_activation: true
mask_sequence_vectors: 1
masking: Filling
normalization: LayerNorm
normalize_features: 1
num_attention_heads: 4
num_branch_embedding_layers: 3
num_branch_encoder_layers: 3
num_classification_layers: 3
num_dataloader_workers: 4
num_detector_layers: 2
num_embedding_layers: 10
num_encoder_layers: 4
num_gpu: 1
num_jet_embedding_layers: 0
num_jet_encoder_layers: 2
num_regression_layers: 3
optimizer: AdamW
partial_events: 1
position_embedding_dim: 16
regression_loss_scale: 0.0
skip_connections: 1
split_symmetric_attention: 1
testing_file: ''
train_validation_split: 0.95
training_file: data/hhh_training_masses.h5
transformer_activation: gelu
transformer_dim: 32
transformer_dim_scale: 2.0
transformer_type: Gated
trial_output_dir: ./test_output
trial_time: ''
usable_gpus: ''
validation_file: ''
verbose_output: false
```