Phenomenology of flavoured 3HDMs

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Is the SM Higgs sector overly minimalistic?

Asking to accomplish three different tasks simultaneously:

- W and Z bosons through the kinetic term $|D_\mu H|^2$;
- down-type quarks and leptons through the Yukawa terms $\bar{Q}_L H d_R$;
- up-type quarks through the Yukawa terms $\bar{Q}_L \tilde{H} d_R$ (with $\tilde{H} \equiv i\sigma_2 H^*$)

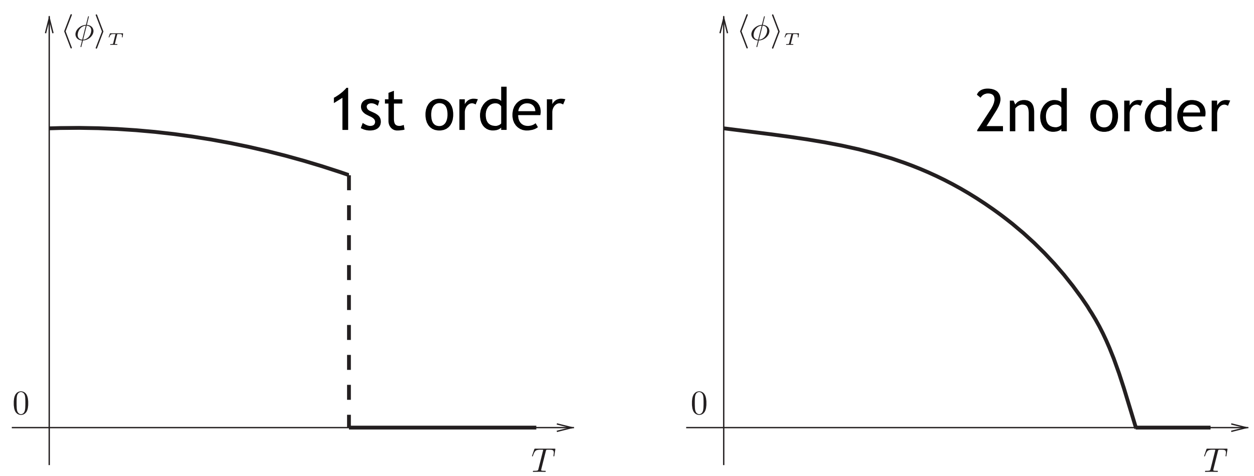
While it is remarkable that the measurements are consistent with one-doublet Higgs sector, the gauge and fermion structure of the SM does not require it to be minimalistic!

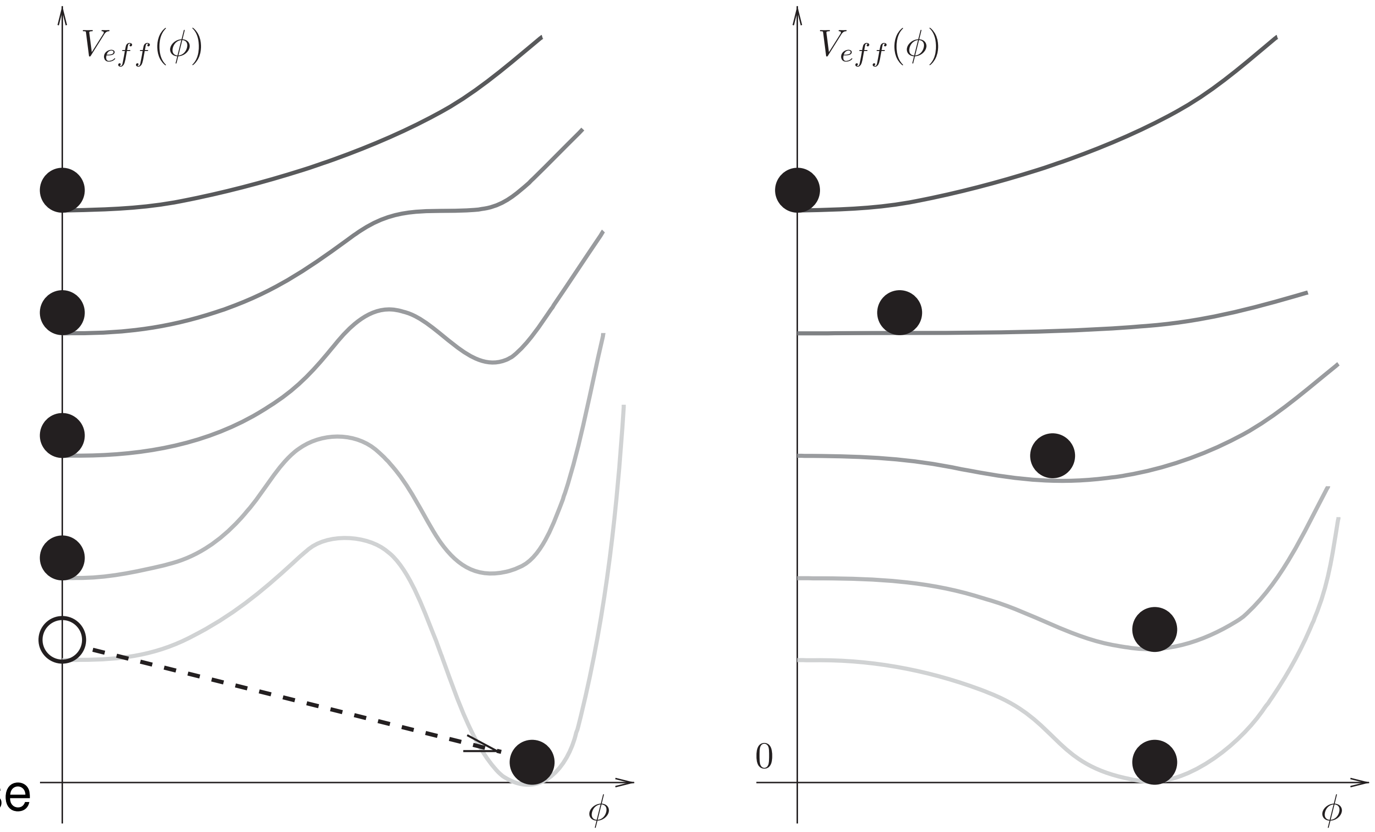
In fact, the SM Higgs sector is totally "exhausted", i.e. cannot do other tasks what it is expected to do, in general:

- does not explain the hierarchical flavour patterns (masses and mixing);
- no FCNCs generated by the Higgs boson exchange (too "boring" flavour properties);
- CP-violation can only be inserted by hands;
- the absence of cosmological EWPT, hence, no sizeable baryon asymmetry.

The Higgs sector can be richer and implement the concept of multiple generations

Cosmological motivation: EW phase transition





Strong cosmological phase transitions (PTs) $\to$
by expanding and colliding vacuum bubbles of new phase

Stochastic Gravitational Wave (GW) background
as a gravitational probe for New Physics

$$\frac{n_B - n_{\overline{B}}}{s} \sim 10^{-11}$$

Why strong FOPTs?

Sakharov'67

(i) B violation

(ii) C and CP violation

(iii) Departure from thermal equilibrium $\to$ **strong 1^{st}-order PT**

Nucleation of expanding broken-phase vacuum bubbles $\to$ sphaleron suppression

$$\frac{\phi(T_c)}{T_c} \gtrsim 1.1 \qquad \to \qquad 1^{st} \text{ order PT} \qquad (m_h \lesssim 50 \text{ GeV})$$

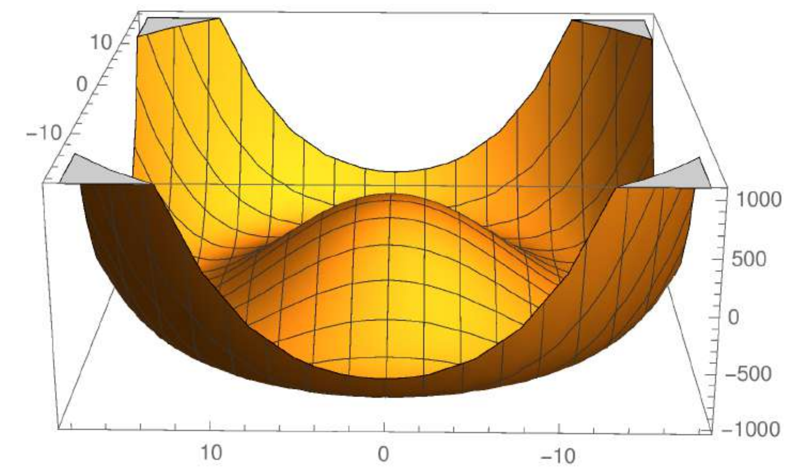
[Kajantie et al 1996; Csikor 1999]

Extra scalars modify the Higgs potential enabling to produce strong EWPTs

Basics of SFOPTs

Consider a the scalar potential: $V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$

$\mu^2 < 0$ and $\lambda > 0$

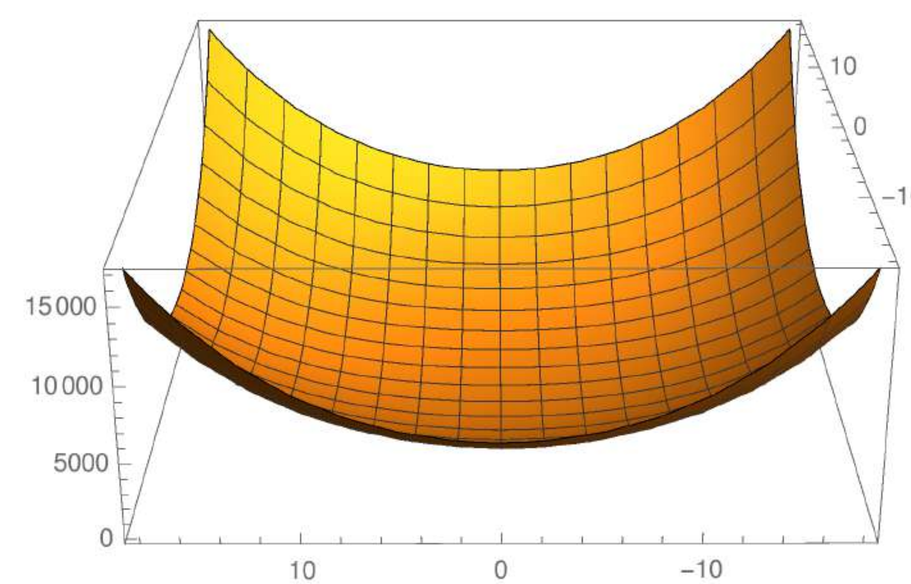


Add thermal corrections:

$$V(\phi, T) = (\mu^2 + C_\phi T^2)\phi^* \phi + \lambda (\phi^* \phi)^2$$

For $C_\phi > 0$, after a certain $T > 0$, $\mu_{eff} \equiv \mu^2 + C_\phi T^2 > 0$

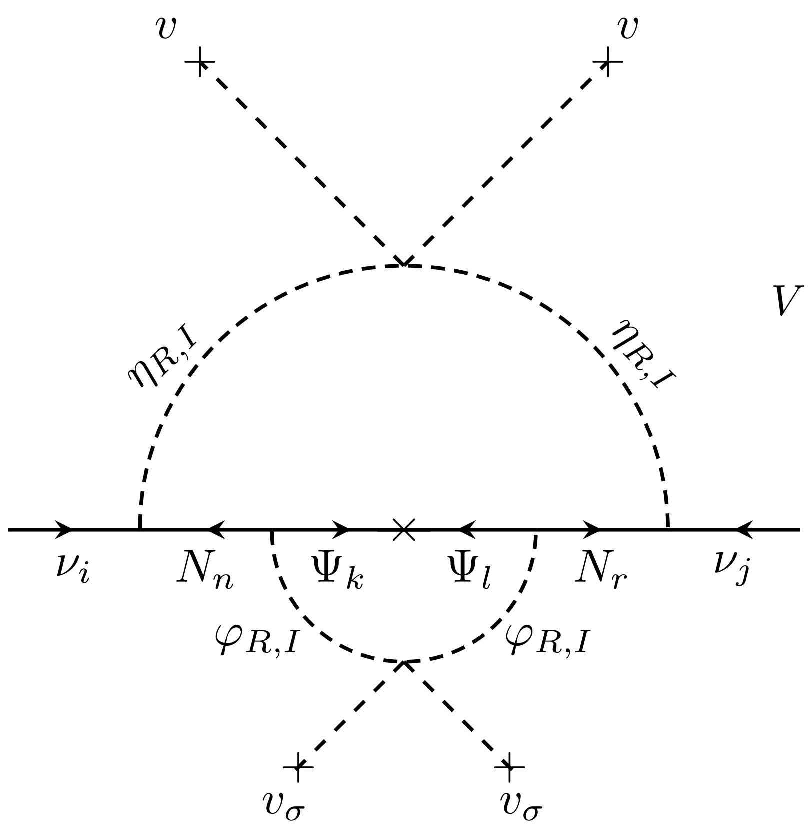
Restored symmetry



Example I: GWs in a scotogenic model

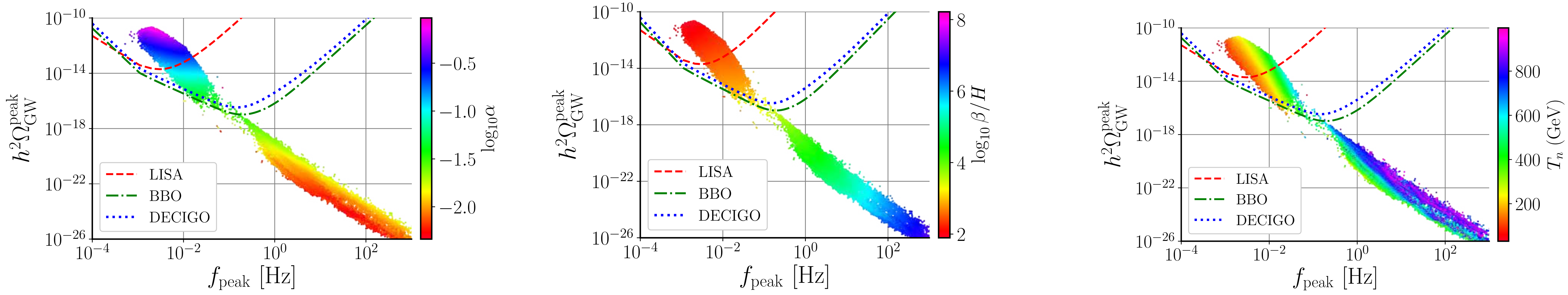
	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$\mathcal{Z}_2$
Φ	$\mathbf{1}$	$\mathbf{2}$	$1/2$	0	1
σ	$\mathbf{1}$	$\mathbf{1}$	0	$-1/2$	1
η	$\mathbf{1}$	$\mathbf{2}$	$1/2$	0	-1
φ	$\mathbf{1}$	$\mathbf{1}$	0	$-1/2$	-1

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$\mathcal{Z}_2$
$\ell_{i,L}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$1/2$	1
$\ell_{i,R}$	$\mathbf{1}$	$\mathbf{1}$	-1	$1/2$	1
$N_{k,R}$	$\mathbf{1}$	$\mathbf{1}$	0	$1/2$	-1
$\Psi_{k,R}$	$\mathbf{1}$	$\mathbf{1}$	0	0	1



Bonilla, Carcamo Hernandez, Goncalves, Vishnudath, Morais, Pasechnik, arXiv: 2305.01964

$$V = -\mu_\Phi^2(\Phi^\dagger\Phi) + \mu_\eta^2(\eta^\dagger\eta) + \mu_\varphi^2(\varphi^*\varphi) - \mu_\sigma^2(\sigma^*\sigma) - \mu_{sb}^2\left(\sigma^2 + h.c\right) + \lambda_1(\Phi^\dagger\Phi)(\Phi^\dagger\Phi) + \lambda_2(\eta^\dagger\eta)(\eta^\dagger\eta)$$
$$+ \lambda_3(\varphi^*\varphi)(\varphi^*\varphi) + \lambda_4(\sigma^*\sigma)(\sigma^*\sigma) + \lambda_5(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_6(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \frac{\lambda_7}{2}\left[(\Phi^\dagger\eta)^2 + \text{h.c.}\right]$$
$$+ \lambda_8(\Phi^\dagger\Phi)(\sigma^*\sigma) + \lambda_9(\Phi^\dagger\Phi)(\varphi^*\varphi) + \lambda_{10}(\eta^\dagger\eta)(\sigma^*\sigma) + \lambda_{11}(\eta^\dagger\eta)(\varphi^*\varphi) + \lambda_{12}(\varphi^*\varphi)(\sigma^*\sigma)$$
$$+ \frac{\lambda_{13}}{2}\left[\varphi^2\left(\sigma^*\right)^2 + \text{h.c.}\right],$$



Example II: GWs in a 6D Majoron seesaw model

Addazi, Marcianò, Morais, Pasechnik, Viana, Yang, arXiv: 2304.02399

$$V_0(H,\sigma) = V_{\rm SM}(H) + V_{\rm 4D}(H,\sigma) + V_{\rm 6D}(H,\sigma) + V_{\rm soft}(\sigma)$$

$$V_{\rm SM}(H) = \mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2\,,$$

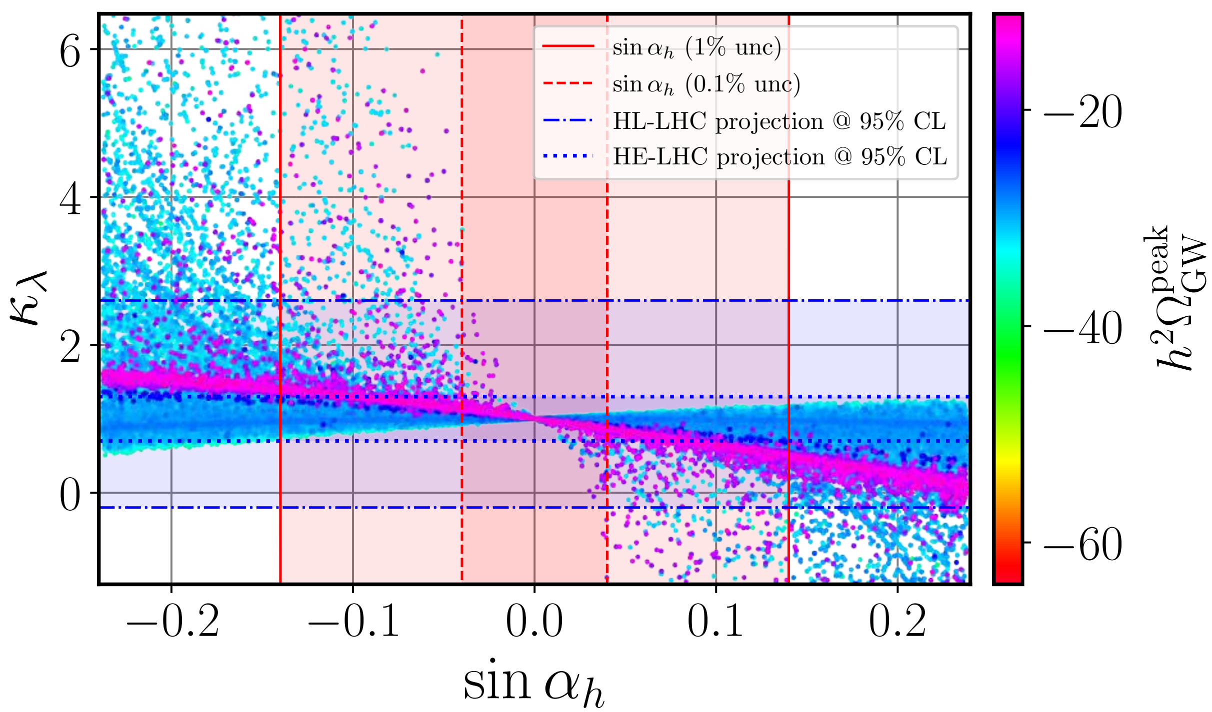
10 TeV < Λ < 1000 TeV ⟶ heavy neutrino mass scale

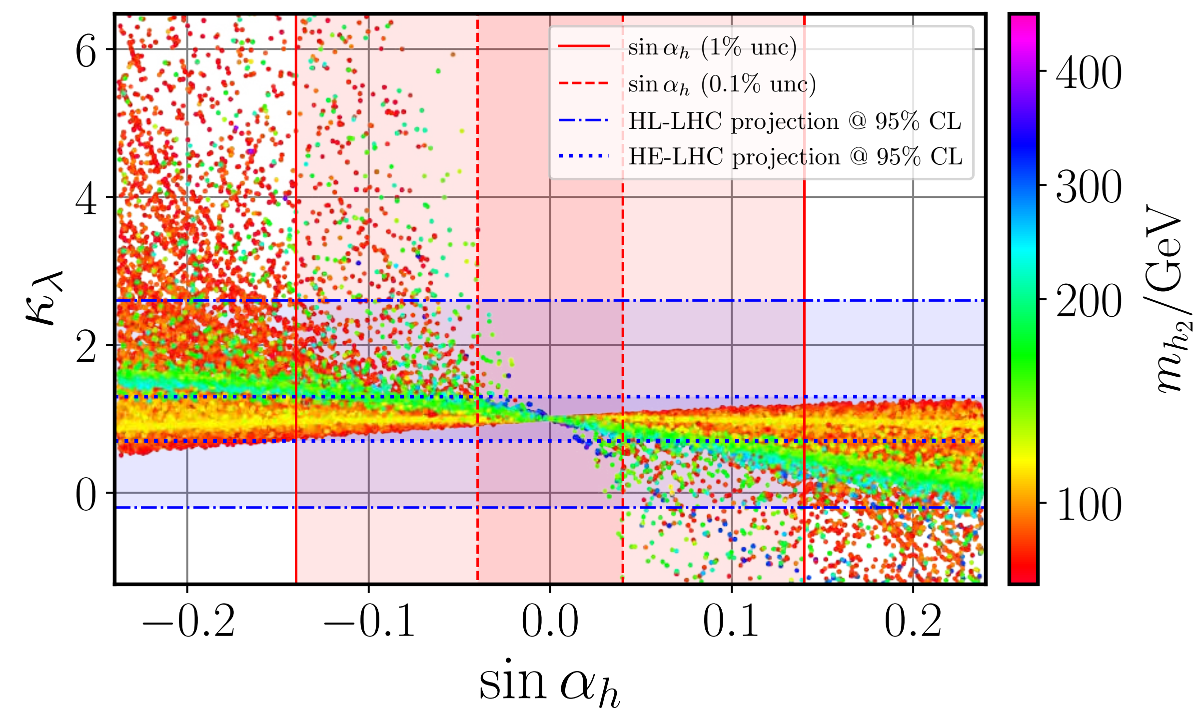
$$V_{\rm 4D}(H,\sigma) = \mu_\sigma^2 \sigma^\dagger\sigma + \lambda_\sigma(\sigma^\dagger\sigma)^2 + \lambda_{\sigma h} H^\dagger H \sigma^\dagger\sigma\,,$$

$$V_{\rm 6D}(H,\sigma) = \frac{\delta_0}{\Lambda^2}(H^\dagger H)^3 + \frac{\delta_2}{\Lambda^2}(H^\dagger H)^2\sigma^\dagger\sigma + \frac{\delta_4}{\Lambda^2}H^\dagger H(\sigma^\dagger\sigma)^2 + \frac{\delta_6}{\Lambda^2}(\sigma^\dagger\sigma)^3\,,$$

$$V_{\rm soft}(\sigma) = \frac{1}{2}\mu_b^2\left(\sigma^2 + \sigma^{*2}\right)\,.$$

δ_2 and δ_4 allow co-existence of $\Gamma_{\rm Higgs}^{\rm invisible}$ and SFOPTs





$$\kappa_\lambda \equiv \lambda_{h_1h_1h_1}/\lambda_{\rm SM}\,,\quad \lambda_{\rm SM} = 3m_{h_1}^2/v_h$$

Magenta band (LISA) / green band favour $0 < \kappa_\lambda < 2$ and $m_{h_2} \approx (200 \pm 50)$ GeV

Illustrates the potential interplay between collider and SGWB interplay

Quark masses and mixing

Quark Yukawa interactions:

$$\bar{Q}_{Li}\Gamma_{ij}\phi d_{Rj} + \bar{q}_{Li}\Delta_{ij}\tilde{\phi}u_{Rj} + h.c. \to \bar{d}_{Li}(M_d)_{ij}d_{Rj} + \bar{u}_{Li}(M_u)_{ij}u_{Rj} + h.c.$$

Mass matrices: $M_d = \Gamma\frac{v}{\sqrt{2}}$ $\quad M_u = \Delta\frac{v^*}{\sqrt{2}}$

Diagonalisation $\quad V_{dL}^\dagger M_d V_{dR} = D_d, \; V_{uL}^\dagger M_u V_{uR} = D_u$

also diagonalises physical Yukawa interactions — no tree-level FCNCs mediated by Higgs

but yields non-trivial charged current:

$$\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} \to \bar{u}_{Li}\gamma^\mu W_\mu^+ V_{ij} d_{Lj}\,, \qquad V_{ij} = V_{uL}^\dagger V_{dL} \neq \delta_{ij}$$

4 parameters: 3 angles + 1 phase

More than one Higgs "generation": NHDM

NHDM quark Yukawa sector: $$\sum_a \left(\bar{Q}_{Li} \Gamma^{(a)}_{ij} \phi_a d_{Rj} + \bar{q}_{Li} \Delta^{(a)}_{ij} \tilde{\phi}_a u_{Rj} \right) + h.c.$$

Separate textures can be simple, constrained by flavour symmetries that leave traces in quarks masses and mixing

VEV alignment:

$\langle \phi^0_a \rangle = v_a/\sqrt{2}$

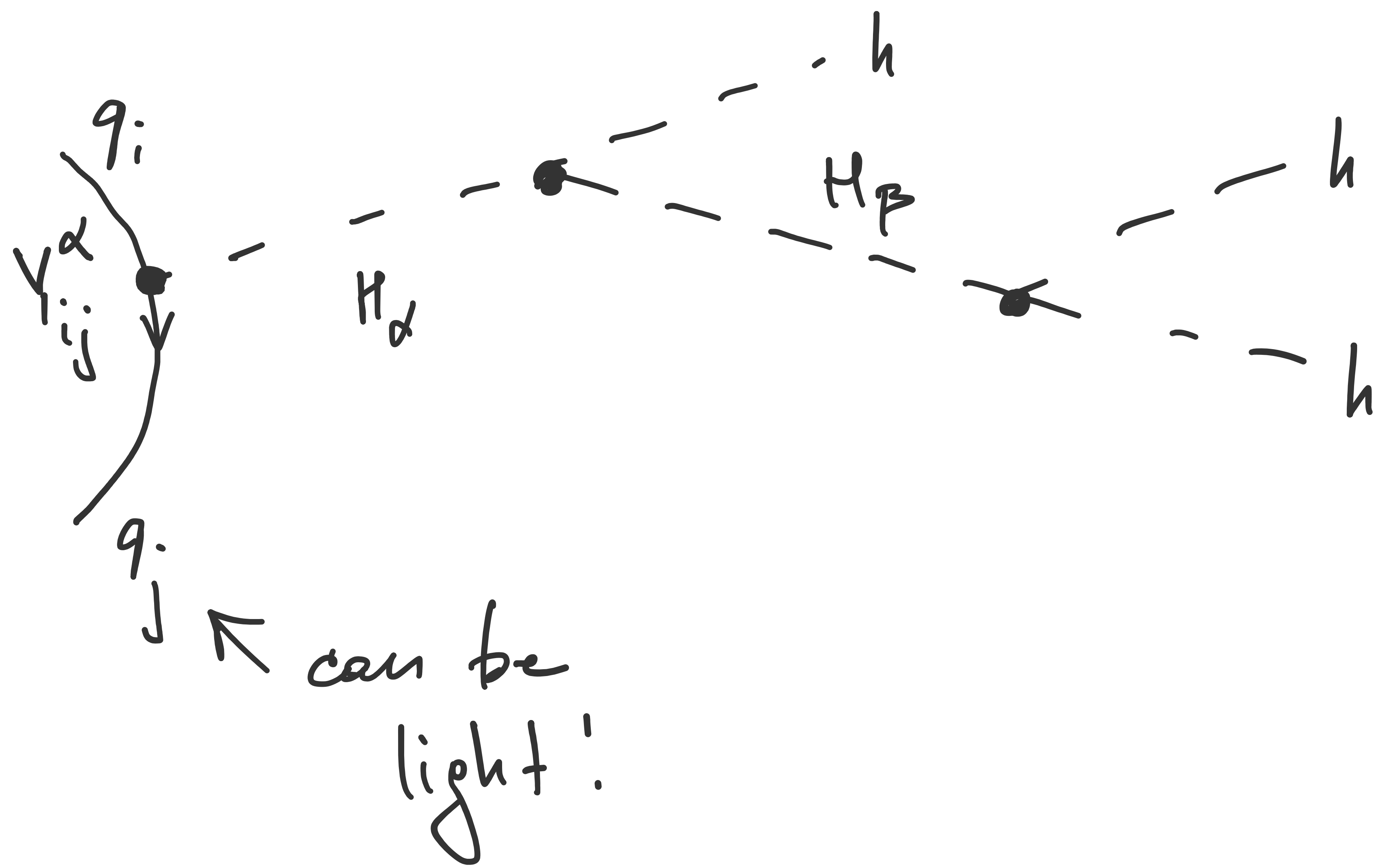
$$M_d = \frac{1}{\sqrt{2}} \sum_a \Gamma^{(a)} v_a \,, \qquad M_u = \frac{1}{\sqrt{2}} \sum_a \Delta^{(a)} v^*_a$$

Consequences:

- Generic textures lead to potentially dangerous tree-level FCNCs that can be eliminated by natural flavour conservation via discrete symmetries [Pachos 1977; Weinberg, Glashow 1977];
- A relative phase in v_a, CP-symmetry can be spontaneously broken for real textures [Branco 1979; T.D. Lee 1973]

Connection to multi-Higgs production pheno

The lowest-order resonant diagram for hhh-production in multi-Higgs models involves light quarks and non-trivial flavour structures



U(1) x U(1) Three Higgs doublet model

- **The most constraining realisable Abelian symmetry of 3HDM**
  Keus, King, Moretti 2014; Ivanov, Keus, Vdovin, 2012

- **Promote this symmetry to be a family symmetry of the fermion sector**
  Camargo-Molina, Mandal, Pasechnik, Wessén, JHEP 03 (2018) 024

softly broken

$\mathrm{U(1)_X} \times \mathrm{U(1)_Z}$

- No tree-level FCNCs
- Cabbibo-like mixing at tree-level
- Fermion mass hierarchies are partly explained by hierarchy of VEVs
- New scalar states couple dominantly to the second quark family (exotic collider signatures)

$$V_0 = -\sum_{i=1}^{3} \mu_i^2 |H_i|^2 + \sum_{i,j=1}^{3} \left(\frac{\lambda_{ij}}{2} |H_i|^2 |H_j|^2 + \frac{\lambda'_{ij}}{2} |H_i^\dagger H_j|^2 \right) , \qquad V_{\rm soft} = \sum_{i=1}^{3} \frac{1}{2} (m_{ij}^2 H_i^\dagger H_j + \mathrm{c.c})$$

All the parameters can be taken real

The model is CP-conserving

$$\lambda_{ij} = \lambda_{ji} \,, \quad \lambda'_{ij} = \lambda'_{ji} \,, \quad m_{ij}^2 = m_{ji}^2 \,,$$

$$\lambda'_{11} = \lambda'_{22} = \lambda'_{33} = 0 \,, \quad m_{11}^2 = m_{22}^2 = m_{33}^2 = 0 \,.$$

Yukawa sector

$$\mathcal{L}^{\rm q}_{\rm Yukawa} = \sum_{i,j=1}^{2} \left\{ y^{\rm d}_{ij} \bar{d}^i_{\rm R} H_1^\dagger Q^j_{\rm L} - y^{\rm u}_{ij} \bar{u}^i_{\rm R} \tilde{H}_2^\dagger Q^j_{\rm L} \right\} + y_{\rm b} \bar{b}_{\rm R} H_3^\dagger Q^3_{\rm L} - y_{\rm t} \bar{t}_{\rm R} \tilde{H}_3^\dagger Q^3_{\rm L} + {\rm c.c.}$$

Lepton sector is SM-like (couple to H_3 only)

Fixed!

	$\mathrm{U}(1)_{\rm Y}$	$\mathrm{U}(1)_{\rm X}$	$\mathrm{U}(1)_{\rm Z}$
H_1	$\frac{1}{2}$	-1	$-\frac{2}{3}$
H_2	$\frac{1}{2}$	1	$\frac{1}{3}$
H_3	$\frac{1}{2}$	0	$\frac{1}{3}$
$Q^{1,2}_{\rm L}$	$\frac{1}{6}$	γ	δ
$Q^3_{\rm L}$	$\frac{1}{6}$	β	α
$u^{1,2}_{\rm R}$	$\frac{2}{3}$	$1+\gamma$	$\frac{1}{3}+\delta$
$t_{\rm R}$	$\frac{2}{3}$	β	$\frac{1}{3}+\alpha$
$d^{1,2}_{\rm R}$	$-\frac{1}{3}$	$1+\gamma$	$\frac{2}{3}+\delta$
$b_{\rm R}$	$-\frac{1}{3}$	β	$-\frac{1}{3}+\alpha$

$v_3 \gg v_{1,2}$ ⇒

heavy third generation
no tree level FCNCs
Cabibbo mixing enforced

The model is treatable fully analytically in this limit!

Dim-6 operators:

$$\bar{d}^{1,2}_{\rm R} \left(H_i^\dagger Q^3_{\rm L}\right)\left(H_j^\dagger H_k\right), \quad \bar{u}^{1,2}_{\rm R} \left(\tilde{H}_i^\dagger Q^3_{\rm L}\right)\left(H_j^\dagger H_k\right)$$

$$\bar{b}_{\rm R} \left(H_i^\dagger Q^{1,2}_{\rm L}\right)\left(H_j^\dagger H_k\right), \quad \bar{t}_{\rm R} \left(\tilde{H}_i^\dagger Q^{1,2}_{\rm L}\right)\left(H_j^\dagger H_k\right)$$

⇓

Full CKM?

$$(\beta-\gamma, \alpha-\delta) \notin \{(-1,-1), (-1,0), (0,0), (1,0), (1,1), (2,1)\}$$

Physical Higgs interactions

$$\xi \equiv \frac{\sqrt{v_1^2 + v_2^2}}{v_3}$$

$$\xi \ll 1$$

SM-like Higgs: $$\mathcal{L} \supset \sum_q \frac{m_q}{v_3} \bar{q} q \, h_{125} + \mathcal{O}(\xi)$$

Additional scalars' Yukawa couplings:

$$\mathcal{L} \supset \cos\theta_\mathrm{C} \frac{\sqrt{2} m_\mathrm{s}}{v_1} \bar{s}_\mathrm{R} c_\mathrm{L} H_\mathrm{a}^- - \cos\theta_\mathrm{C} \frac{\sqrt{2} m_\mathrm{c}}{v_2} \bar{c}_\mathrm{R} s_\mathrm{L} H_\mathrm{b}^+ + \text{c.c.} + \mathcal{O}(\xi)$$
$$+ \frac{m_\mathrm{s}}{v_1} \bar{s} s h_\mathrm{a} - \frac{m_\mathrm{c}}{v_2} \bar{c} c h_\mathrm{b} + \mathrm{i} \frac{m_\mathrm{s}}{v_1} \bar{s} \gamma^5 s A_\mathrm{a} - \mathrm{i} \frac{m_\mathrm{c}}{v_2} \bar{c} \gamma^5 c A_\mathrm{b} + \mathcal{O}(\xi) \, ,$$

New scalars are mostly produced via $c\bar{s}$ fusion!

Main focus so far: $c\bar{s} \to H^+ \to W^+ h_{125}$ $\quad m_{H^\pm} > 200 \text{ GeV}$

Analysis

$$pp \to H^\pm \to W^\pm h_{125} \to \ell^\pm + \not{E}_T + b\bar{b}$$

- implement the model-independent Lagrangian to FeynRules (leading order);
- generate UFO for MadGraph, use NNPDF for S/B event generation;
- use Pythia6 for showering/hadronisation of generated events;
- detector simulation via Delphes employing FastJet for jet clustering (anti-kT);
- for the multivariate analysis, we use Boosted Decision Tree Algorithm.

Selection criteria:
one charged lepton $\ell = \{e, \mu\}$ and at least two jets + missing transverse energy

b-tagging on the two leading-p_T jets

reduces B, but also affects S

Two S categories:

- 1b-tag: In this category, we demand at least one b-tagged jet among the two leading p_T jets.
- 2b-tag: In this category, we demand that both the two leading p_T jets are b-tagged. This category is a subset of the 1b-tag category.

Charged Higgs search in cs fusion channel

$$pp \to H^\pm \to W^\pm h_{125} \to \ell^\pm + \not{E}_T + b\bar{b}$$

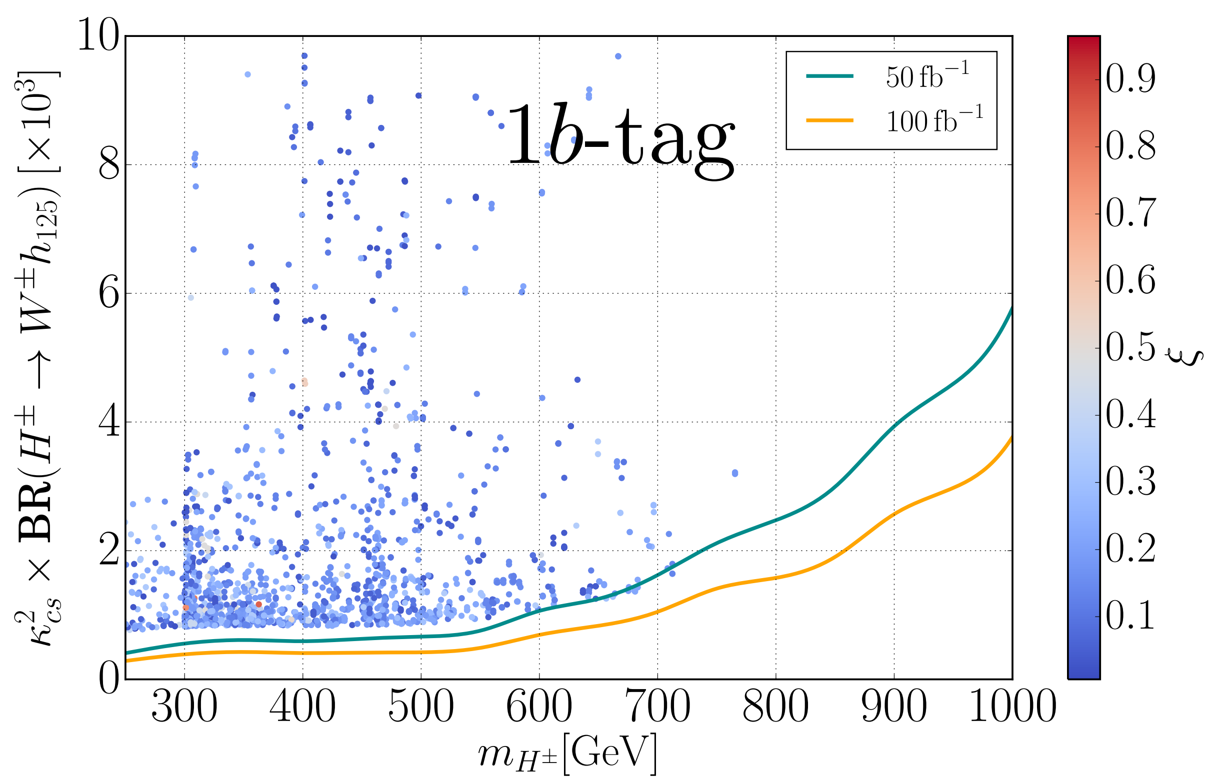
Parton-level CSs for typical backgrounds (no cuts!): LHC ($\sqrt{s} = 13$ TeV)

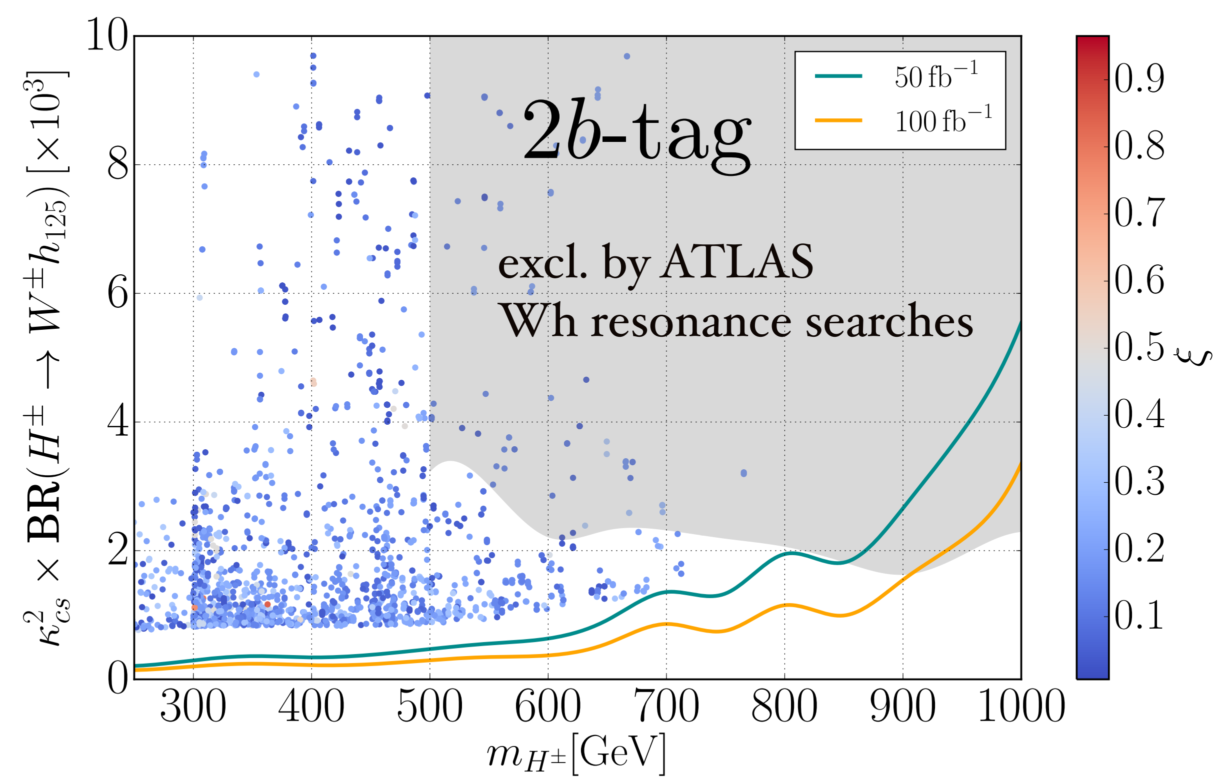
Process	$W + n\ j$	Wbj	$Wb\bar{b}$	$t\bar{t} + n\ j$	tj	tb	tW	WW	WZ	Wh_{125}
x-sec (pb)	1.53×10^5	308.9	41.7	431.3	174.6	2.6	54.0	67.8	25.4	1.1

Selection cuts:

- Lepton: $p_T(\ell) > 25$ GeV, $|\eta(\ell)| < 2.5$
- Jet: $p_T(J) > 25$ GeV, $|\eta(J)| < 4.5$
- Missing transverse energy: $\not{E}_T > 25$ GeV
- ΔR separation: $\Delta R(J_1, J_2) > 0.4$, $\Delta R(\ell, J) > 0.4$

5σ discovery contours





CKM suppression of FCNCs in down-sector

Branco-Grimus-Lavoura (BGL): symmetry suppressed tree-level FCNCs first realised in the context of 2HDM

Impose a family symmetry: $Q_{L1} \to e^{i\theta} Q_{L1}, \quad p_{R1} \to e^{2i\theta} p_{R1}, \quad \Phi_2 \to e^{i\theta} \Phi_2,$

Allowed textures:

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} \times & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \qquad \Delta_2 = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

up-quark mass form:

$$M_p = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

$$N_u = \mathrm{diag}\left(-\frac{m_{u_1}}{\tan\beta}\,,\, m_{u_2}\tan\beta\,,\, m_{u_3}\tan\beta\right)$$

No FCNCs in the up-sector!

CKM: $V = V_L^\dagger U_L$

$$U_L \equiv \begin{pmatrix} V(1,1) & V(1,2) & V(1,3) \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

$$(N_d)_{aa} = m_a \left(\tan\beta - \frac{|V_{1a}|^2}{\sin\beta\cos\beta}\right),$$

$$(N_d)_{ab} = -\frac{V_{1a}^* V_{1b}}{\sin\beta\cos\beta} m_b \quad (a \neq b)$$

CKM-suppressed FCNCs in the down-sector!

BGL-like 3HDM: scalar sector

Das, Ferreira, Morais, Padilla-Gay, Pasechnik, Rodrigues, JHEP 11 (2021) 079

Impose a family symmetry:
$$\mathrm{U}(1): \ \phi_1 \to e^{i\alpha}\phi_1 \,, \qquad \phi_3 \to e^{i\alpha}\phi_3 \,.$$
$$\mathbb{Z}_2: \ \phi_1 \to -\phi_1 \,, \qquad \phi_2 \to \phi_2 \,, \qquad \phi_3 \to \phi_3 \,.$$

CP symmetry:
$$\phi_1 \to \phi_1^* \,, \qquad \phi_2 \to \phi_2^* \,, \qquad \phi_3 \to \phi_3^*$$

Invariant potential:

$$\begin{aligned}
V_0(\phi_1,\phi_2,\phi_3) =\ & \mu_1^2\left(\phi_1^\dagger\phi_1\right) + \mu_2^2\left(\phi_2^\dagger\phi_2\right) + \mu_3^2\left(\phi_3^\dagger\phi_3\right) + \lambda_1\left(\phi_1^\dagger\phi_1\right)^2 \\
& + \lambda_2\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\left(\phi_3^\dagger\phi_3\right)^2 + \lambda_4\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_5\left(\phi_1^\dagger\phi_1\right)\left(\phi_3^\dagger\phi_3\right) \\
& + \lambda_6\left(\phi_2^\dagger\phi_2\right)\left(\phi_3^\dagger\phi_3\right) + \lambda_7\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \lambda_8\left(\phi_1^\dagger\phi_3\right)\left(\phi_3^\dagger\phi_1\right)
\end{aligned}$$

Soft-breaking potential:

$$V_{\rm soft}(\phi_1,\phi_2,\phi_3) = \mu_{12}^2\phi_1^\dagger\phi_2 + \mu_{13}^2\phi_1^\dagger\phi_3 + \mu_{23}^2\phi_2^\dagger\phi_3 + \text{h.c.}\,, \qquad V = V_0 + V_{\rm soft}$$

Higgs doublets:
$$\phi_k = \begin{pmatrix} w_k^+ \\ \frac{1}{\sqrt{2}}(v_k + h_k + i z_k) \end{pmatrix}, \qquad (k = 1,2,3)$$

$$v_1 = v\sin\beta_1\cos\beta_2\,, \quad v_2 = v\sin\beta_2\,, \quad v_3 = v\cos\beta_1\cos\beta_2\,, \quad v = \sqrt{v_1^2+v_2^2+v_3^2}$$

BGL-like 3HDM: Yukawa sector

family symmetry:

$$\mathrm{U}(1): \; Q_{L3} \to e^{i\alpha} Q_{L3}\,, \quad p_{R3} \to e^{2i\alpha} p_{R3}\,,$$

$$\mathbb{Z}_2: \; Q_{L3} \to -Q_{L3}\,, \quad p_{R3} \to -p_{R3}\,, \quad n_{R3} \to -n_{R3}$$

Yukawa Lagrangian:

$$\mathscr{L}_Y = -\sum_{k=1}^{3} \left[\bar{Q}_{La} (\Gamma_k)_{ab}\, \phi_k\, n_{Rb} + \bar{Q}_{La} (\Delta_k)_{ab}\, \tilde{\phi}_k\, p_{Rb} + \text{h.c.} \right]$$

Allowed textures:

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & 0 \end{pmatrix}, \quad \Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2, \Delta_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3, \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

Up/down mass matrices:

$$M_p = \frac{1}{\sqrt{2}} \sum_{k=1}^{3} \Delta_k v_k = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_n = \frac{1}{\sqrt{2}} \sum_{k=1}^{3} \Gamma_k v_k = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}$$

In the alignment limit, no FCNCs from SM Higgs state:

$$\begin{pmatrix} H_0 \\ H_1' \\ H_2' \end{pmatrix} = \mathcal{O}_\beta \cdot \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_Y^{H_0} &= -\frac{H_0}{v} \left[\bar{n}_L \left(\frac{1}{\sqrt{2}} \sum_{k=1}^{3} \Gamma_k v_k \right) n_R + \bar{p}_L \left(\frac{1}{\sqrt{2}} \sum_{k=1}^{3} \Delta_k v_k \right) p_R + \text{h.c.} \right] \\ &= -\frac{H_0}{v} \left[\bar{d}_L D_d d_R + \bar{u}_L D_u u_R + \text{h.c.} \right] . \end{aligned}$$

BGL-like 3HDM: tree-level FCNCs

CP-even BSM scalars interact with down-quarks as:

$$\mathscr{L}_Y^{H_1',H_2'} = -\frac{H_1'}{v}\bar{d}_L N_{d1} d_R - \frac{H_2'}{v}\bar{d}_L N_{d2} d_R + \text{h.c.}$$

FCNC matrices:

$$N_{d1} = \frac{v}{\sqrt{2}v_{13}} U_L^\dagger (\Gamma_1 v_3 - \Gamma_3 v_1) U_R\,,$$

$$N_{d2} = U_L^\dagger \left[\frac{v_2}{v_{13}}\frac{1}{\sqrt{2}}\left(\Gamma_1 v_1 + \Gamma_3 v_3\right) - \frac{v_{13}}{v_2}\frac{1}{\sqrt{2}}\Gamma_2 v_2\right] U_R$$

Bi-diagonalising matrices in the up-sector have block-diagonal form:

$$V_L = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad\qquad (U_L)_{3A} = V_{3A}$$

Textures have the following structure:

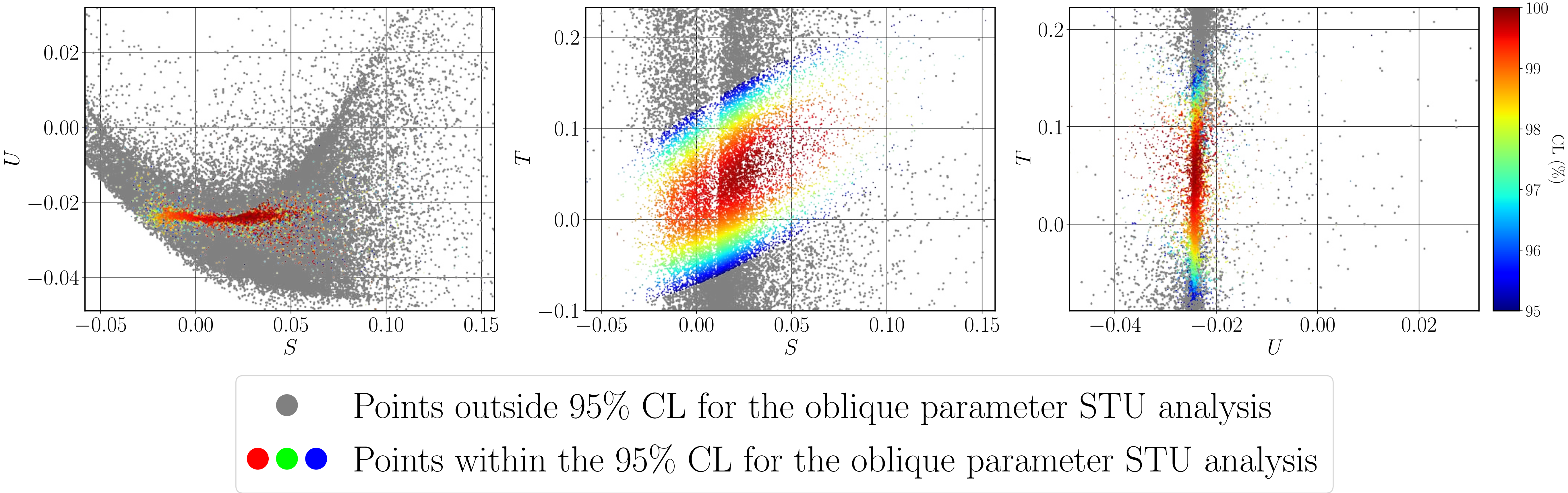
$$\Gamma_3 = (\Gamma_3)_{33} P\,, \qquad \frac{1}{\sqrt{2}}\left(\Gamma_1 v_1 + \Gamma_3 v_3\right) = P\, M_d \qquad\qquad P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

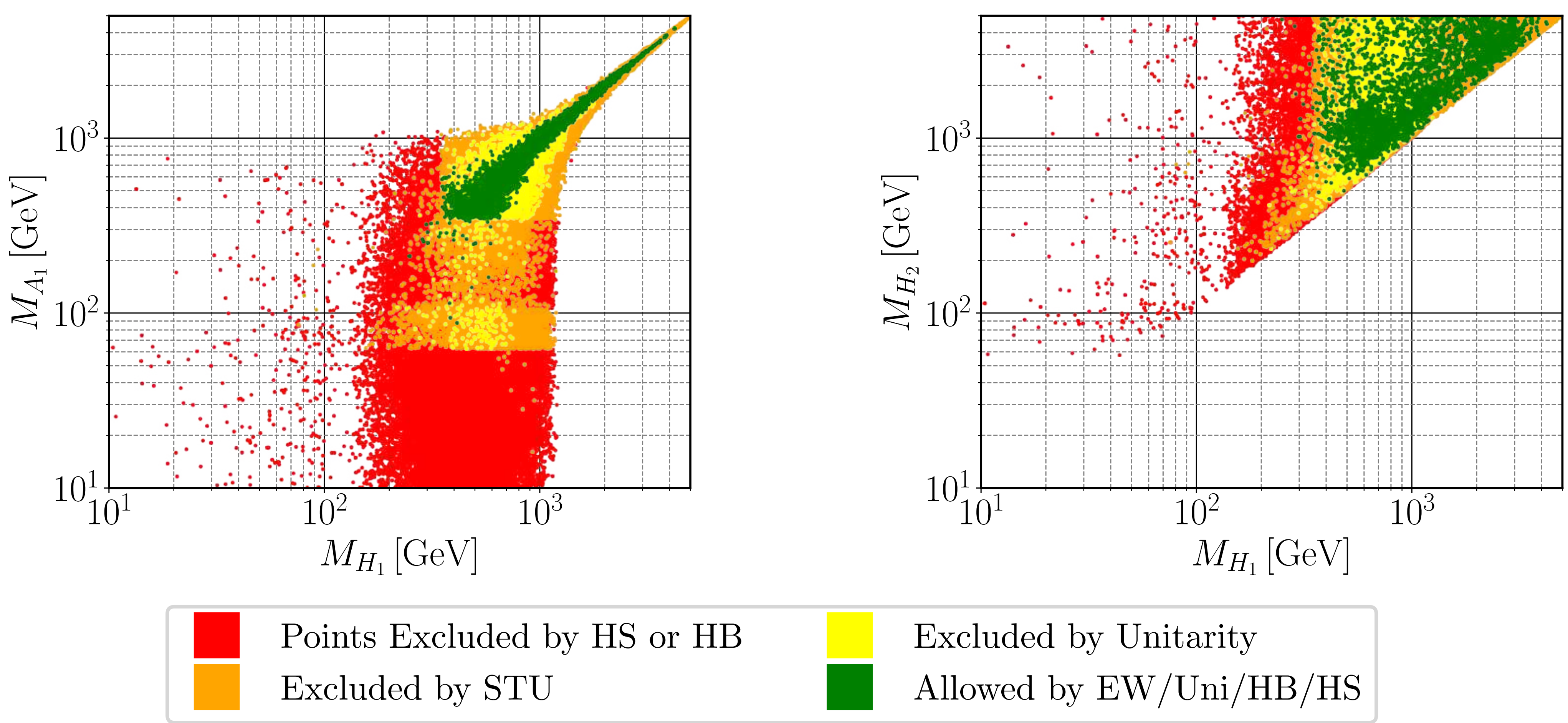
physical
FCNC interactions:

$$(N_{d1})_{AB} = \frac{v\, v_3}{v_1 v_{13}} V_{3A}^* V_{3B} (D_d)_{BB} - \frac{1}{\sqrt{2}}\frac{v\, v_{13}}{v_1}(\Gamma_3)_{33} V_{3A}^* (U_R)_{3B}\,,$$

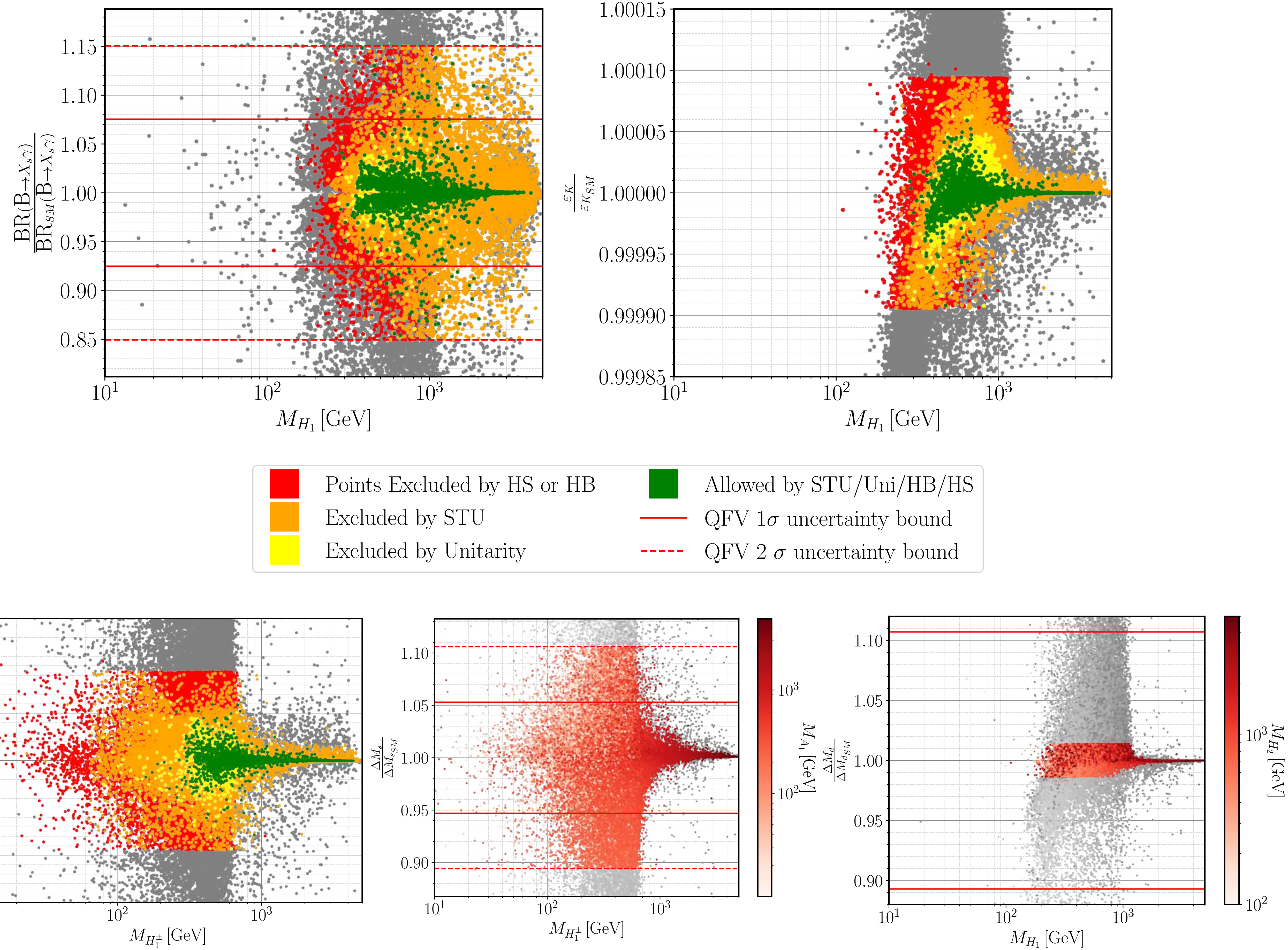
$$(N_{d2})_{AB} = \frac{v_{13}}{v_2}(D_d)_{BB}\delta_{AB} + \left(\frac{v_{13}}{v_2} + \frac{v_2}{v_{13}}\right) V_{3A}^* V_{3B} (D_d)_{BB}\,.$$

BGL-like 3HDM: numerical results

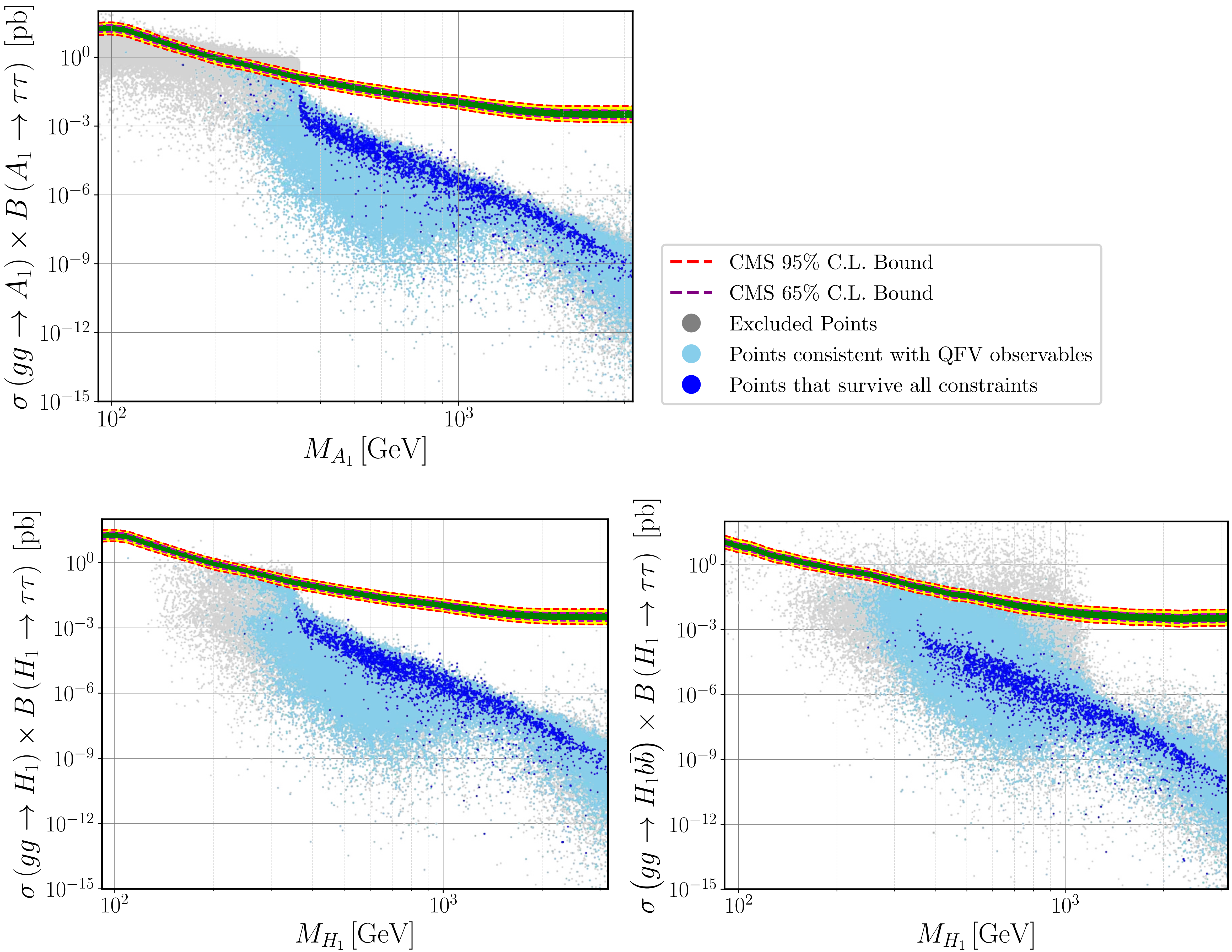




FCNC observables



Predictions for the LHC searches



Generalised CP transformation

Requirements on multi-Higgs model-building:

- making as few assumptions on top of the SM as possible;
- obtaining a model that satisfies all experimental constraints without introducing an excessive number of free parameters;
- providing predictions testable at current/future measurements

Let us take the basic assumption:

The minimal multi-Higgs-doublet model implementing a CP-symmetry of higher order without producing any accidental symmetry.

CP is not uniquely defined in QFT
[Feinberg, Weinberg 1959]

The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent

In a NHDM, ϕ_i, $i = 1, \dots, N$ the transformation

$$J_X : \quad \phi_i(\mathbf{x}, t) \xrightarrow{CP} \mathcal{CP}\, \phi_i(\mathbf{x}, t)\, \mathcal{CP}^{-1} = X_{ij} \phi_j^*(-\mathbf{x}, t), \quad X_{ij} \in U(N)$$

can play a role of the "CP-transformation"

[Grimus, Rebelo 1997; Branco, Lavoura, Silva 1999]

CP transformations of order-k

When one says "the model is CP-conserving", one may refer to any form of GCP symmetry (when all CP-odd observables are zero)

Applying GCP twice: family transformation

$$\phi_i(\mathbf{x},t) \to (\mathcal{CP})^2\phi_i(\mathbf{x},t)(\mathcal{CP})^{-2} = (XX^*)_{ij}\phi_j(\mathbf{x},t)$$

Using the redefinition freedom in the choice of the basis of scalar fields, one can bring X to the block-diagonal form, with blocks being either phases, or 2x2 matrices:

$$\begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & e^{i\alpha} \\ e^{-i\alpha} & 0 \end{pmatrix}$$

$(J_X)^2 = XX^* \neq \mathbb{I}$ CP-transformation is not of order-2

For smallest (even) integer k: $(J_X)^k = \mathbb{I}$ GCP-transformation of order-k

$k = 2^p, \text{ with } p \geq 1$ CP2 CP4 CP8 etc

CP-4 Three Higgs Doublet Model

How many different global symmetries can one impose upon a given BSM model?

Due to a basis redefinition freedom, different symmetries may be related by basis choices: for 2HDM a symmetry w.r.t. $\phi_1 \leftrightarrow \phi_2$ is equivalent, in a different basis, to the usual $\mathbb{Z}_2$-symmetry: $\phi_1 \to \phi_1$ and $\phi_2 \to -\phi_2$

In 2HDM, only six symmetry classes exist, not related by basis choices [Ivanov 2008]

In 2HDM, three different choices for X are possible leading to usual CP2 x accidental symmetries — what is the minimal set up (with CP4) that does not lead to those?

There exists only one 3HDM with CP4, which does not lead to accidental symmetries [Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016]

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}$$

$$J^2 = XX^* = \mathrm{diag}(1,\,-1,\,-1) \neq \mathbb{I} \qquad\qquad J^4 = \mathbb{I}$$

CP-4 3HDM potential

$$V = V_0 + V_1$$

$$\begin{aligned} V_0 =\ & -m_{11}^2(\phi_1^\dagger\phi_1) - m_{22}^2(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2\left[(\phi_2^\dagger\phi_2)^2 + (\phi_3^\dagger\phi_3)^2\right] \\ & +\ \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3) + \lambda_3'(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ & +\ \lambda_4\left[(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1)\right] + \lambda_4'(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2)\,, \end{aligned}$$

with all parameters real, and

$$V_1 = \lambda_5(\phi_3^\dagger\phi_1)(\phi_2^\dagger\phi_1) + \lambda_8(\phi_2^\dagger\phi_3)^2 + \lambda_9(\phi_2^\dagger\phi_3)(\phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3) + h.c.$$

with real λ_5 and complex $\lambda_8,\ \lambda_9$

Most general charge-preserving VEVs:

$$\sqrt{2}\langle\phi_i^0\rangle = (v_1,\ v_2e^{i\gamma_2},\ v_3e^{i\gamma_3}) \equiv (v_1,\ uc_\psi e^{i\gamma_2},\ us_\psi e^{i\gamma_3})\,, \quad v_1 > 0,\ u \equiv \sqrt{v_2^2 + v_3^2}$$

$$\phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}h_1^+ \\ v_1 + h_1 + ia_1\end{pmatrix},\ \phi_2 = \frac{e^{i\gamma_2}}{\sqrt{2}}\begin{pmatrix}\sqrt{2}h_2^+ \\ v_2 + h_2 + ia_2\end{pmatrix},\ \phi_3 = \frac{e^{i\gamma_3}}{\sqrt{2}}\begin{pmatrix}\sqrt{2}h_3^+ \\ v_3 + h_3 + ia_3\end{pmatrix}$$

$$\sqrt{v_1^2 + u^2} \equiv v = 246\ \text{GeV} \qquad\qquad \gamma_3 = -\gamma_2 \equiv -\gamma$$

Higgs alignment limit

Traditional choice for the Higgs basis (only first doublet gets a VEV):

$$\Phi_i = \begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_3\end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}\rho_1 + iG^0\\ \rho_2 + i\eta_2\\ \rho_3 + i\eta_3\end{pmatrix} \qquad \begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_3\end{pmatrix} = \begin{pmatrix}c_\beta & s_\beta c_\psi & s_\beta s_\psi\\ 0 & -s_\psi & c_\psi\\ s_\beta & -c_\beta c_\psi & -c_\beta s_\psi\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2 e^{-i\gamma}\\ \phi_3 e^{i\gamma}\end{pmatrix}$$

$$\sqrt{2}\langle\phi_i^0\rangle = (v_1,\ v_2 e^{i\gamma},\ v_3 e^{-i\gamma}) \equiv v\,(c_\beta, s_\beta c_\psi e^{i\gamma}, s_\beta s_\psi e^{-i\gamma})$$

$$\varphi_a = (h_1, h_2, h_3, a_1, a_2, a_3) \qquad \Phi_a = (G^0, \rho_1, \rho_2, \rho_3, \eta_2, \eta_3)$$

$$\Phi_a = P_{ab}\varphi_b \qquad \mathcal{M}^H = P\mathcal{M}P^T = \begin{pmatrix}0 & 0 & 0_4\\ 0 & m^2_{H_{125}} & 0_4\\ 0_4 & 0_4 & \mathcal{M}^H_{4\times 4}\end{pmatrix}$$

$$m^2_{11} = m^2_{22}\,, \qquad m^2_{H_{125}} = 2m^2_{11}$$

No FCNCs occur in H_{125} interactions with fermions

Alignment without decoupling: the remaining scalars can have any mass

CP4-symmetric Yukawa sector

- **CP4 can be extended to the Yukawa sector and must be spontaneously broken leading to particular patterns in the flavour sector**

Ferreira, Ivanov, Jimenez, Pasechnik, Serodio JHEP 01 (2018) 065

$$\psi_i \to Y_{ij}\psi_j^{CP}, \quad \psi^{CP} = \gamma^0 C \bar{\psi}^T \qquad -\mathcal{L}_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \phi_a^* + h.c.$$

CP4 invariance: $$(Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^*$$

Explicitly:
$$(Y^L)^\dagger \Gamma_1 Y^d = \Gamma_1^*, \quad i(Y^L)^\dagger \Gamma_2 Y^d = \Gamma_3^*, \quad -i(Y^L)^\dagger \Gamma_3 Y^d = \Gamma_2^*,$$
$$(Y^L)^\dagger \Delta_1 Y^u = \Delta_1^*, \quad -i(Y^L)^\dagger \Delta_2 Y^u = \Delta_3^*, \quad i(Y^L)^\dagger \Delta_3 Y^u = \Delta_2^*$$

With an appropriate change of the basis, all matrices can be brought to the form:

$$Y = \begin{pmatrix} 0 & e^{i\alpha} & 0 \\ e^{-i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CP4-symmetric Yukawa textures

- **Case** A**.** $e^{i\alpha_L} = 1$ and $e^{i\alpha_d} = 1$,

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{12}^* & g_{11}^* & g_{13}^* \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_{2,3} = 0$$

- **Case** B_1**.** $e^{i\alpha_L} = i$ and $e^{i\alpha_d} = 1$,

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}$$

- **Case** B_2**.** $e^{i\alpha_L} = 1$ and $e^{i\alpha_d} = i$,

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}$$

- **Case** B_3**.** $e^{i\alpha_L} = i$ and $e^{i\alpha_d} = i$,

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}$$

FCNCs

In order to reproduce correct mass/mixing patterns and correct CPV, CP4 symmetry must be spontaneously broken

Tree-level Higgs-mediated FCNCs are unavoidable in CP4-3HDM, if not from the SM-like Higgs in the alignment limit, but definitely from other scalars

Quark mass forms (in the real VEV basis):

$$M_d^0 = \frac{v}{\sqrt{2}}(\Gamma_1 c_\beta + \Gamma_2 s_\beta c_\psi + \Gamma_3 s_\beta s_\psi)\,, \quad M_u^0 = \frac{v}{\sqrt{2}}(\Delta_1 c_\beta + \Delta_2 s_\beta c_\psi + \Delta_3 s_\beta s_\psi)$$

Relation between the gauge and Higgs bases:

$$\Gamma_1 \phi_1^0 + \Gamma_2 \phi_2^0 + \Gamma_3 \phi_3^0 = \frac{\sqrt{2}}{v}(H_1^0 M_d^0 + H_2^0 N_{d2}^0 + H_3^0 N_{d3}^0)$$

$$\begin{aligned} N_{d2}^0 &= M_d^0 \cot\beta - \frac{v}{\sqrt{2}s_\beta}\Gamma_1 = -M_d^0 \tan\beta + \frac{v}{\sqrt{2}c_\beta}(\Gamma_2 c_\psi + \Gamma_3 s_\psi)\,, \\ N_{d3}^0 &= \frac{v}{\sqrt{2}}(-\Gamma_2 s_\psi + \Gamma_3 c_\psi)\,. \end{aligned}$$

Turning to the mass basis:

$$D_d = V_{dL}^\dagger M_d^0 V_{dR} = \mathrm{diag}(m_d, m_s, m_b)\,,$$

Physical FCNC matrices:

$$N_{d2} = V_{dL}^\dagger N_{d2}^0 V_{dR}\,,$$ etc

Reverse engineering: inversion

Zhao, Ivanov, Pasechnik, Zhang JHEP 04 (2023) 116

$$
\begin{tikzcd}
\Gamma_i,\ \Delta_i \arrow[r, "v_i"'] \arrow[dr] & M_d^0,\ M_u^0 \arrow[r, "\text{diag}"'] \arrow[l, bend right, "\text{inversion}"'] & m_q,\ V_{\text{CKM}} \arrow[d, "{V_{dL}, V_{dR}, V_{uL}, V_{uR}}"] \\
 & N_d^0,\ N_u^0 \arrow[r] & N_d,\ N_u
\end{tikzcd}
$$

One can show that in the CP4-3HDM the procedure is invertible:

starting from physical quark parameters $m_q,\ V_{\text{CKM}}$ and parametrising the quark rotation matrices, we are able to calculate the FCNC matrices for physical quark couplings in the Higgs basis

> is it possible to achieve, within CP4 3HDM, sufficiently small FCNC couplings which would satisfy all the neutral meson oscillation constraints for a 1 TeV Higgs boson without relying on additional cancellation?

Viable benchmark scenarios

$$\frac{1}{v}\bar{d}_{Li}\,(N_d)_{ij}d_{Rj} + h.c = \bar{d}_i\left(A_{ij} + iB_{ij}\gamma^5\right)d_j\,,$$

$$A = \frac{N_d + N_d^\dagger}{2v}\,, \quad iB = \frac{N_d - N_d^\dagger}{2v}\,.$$

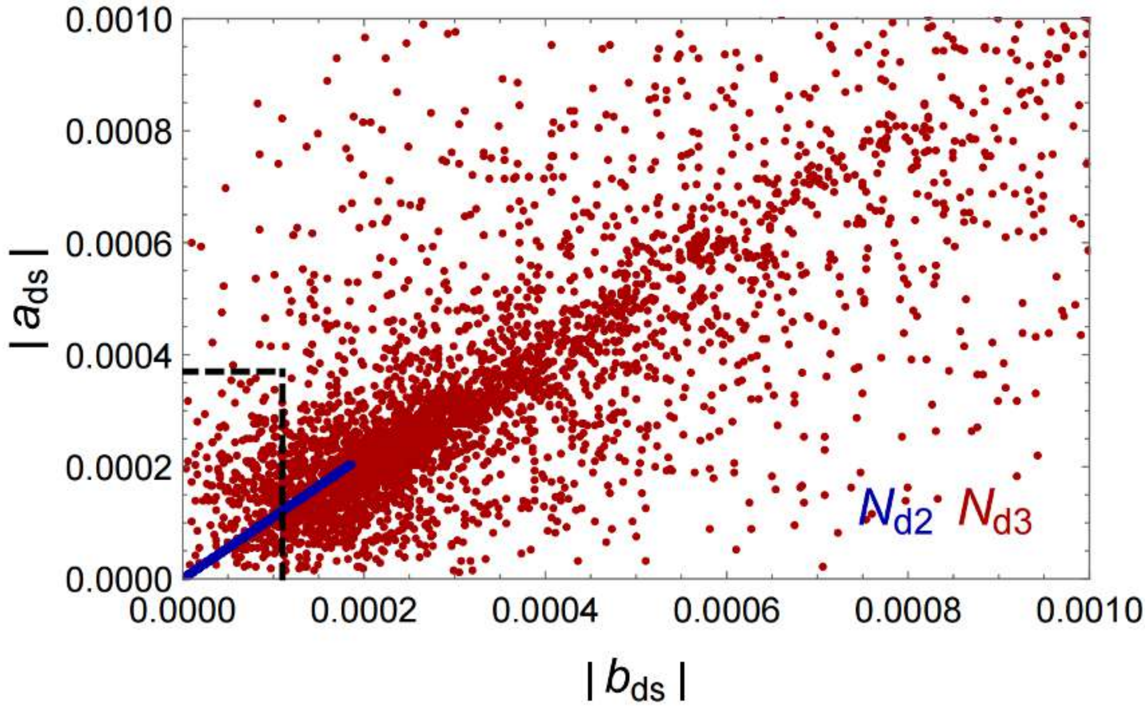
$$\begin{array}{rll} K^0 - \overline{K^0}: & |a_{ds}| < 3.7\times 10^{-4}\,, & |b_{ds}| < 1.1\times 10^{-4}\,, \\ B^0 - \overline{B^0}: & |a_{db}| < 9.0\times 10^{-4}\,, & |b_{db}| < 3.4\times 10^{-4}\,, \\ B_s^0 - \overline{B_s^0}: & |a_{sb}| < 45\times 10^{-4}\,, & |b_{sb}| < 17\times 10^{-4}\,, \\ D^0 - \overline{D^0}: & |a_{uc}| < 5.0\times 10^{-4}\,, & |b_{uc}| < 1.8\times 10^{-4}\,. \end{array}$$

Only two benchmark scenarios pass all four meson constraints:

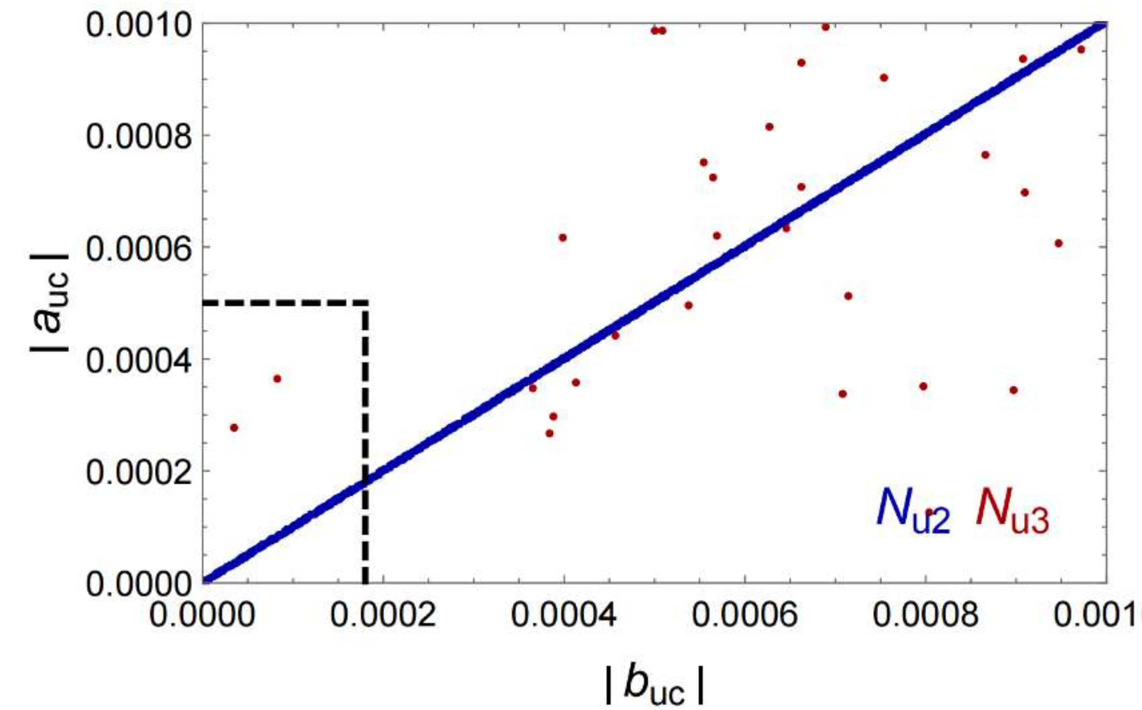
- Benchmark scenario (A, B_2), in which the down-quark sector is completely free from FCNC. The only constraints arise from the D-meson oscillations and can be easily satisfied as the magnitude of FCNCs can be parametrically suppressed.
- Benchmark scenario (B_1, B_1), in which both up and down-quark sectors exhibit FCNCs but their magnitudes are small if the quark rotation matrices are close to the block-diagonal form.

Numerical results: case (B1,B1)

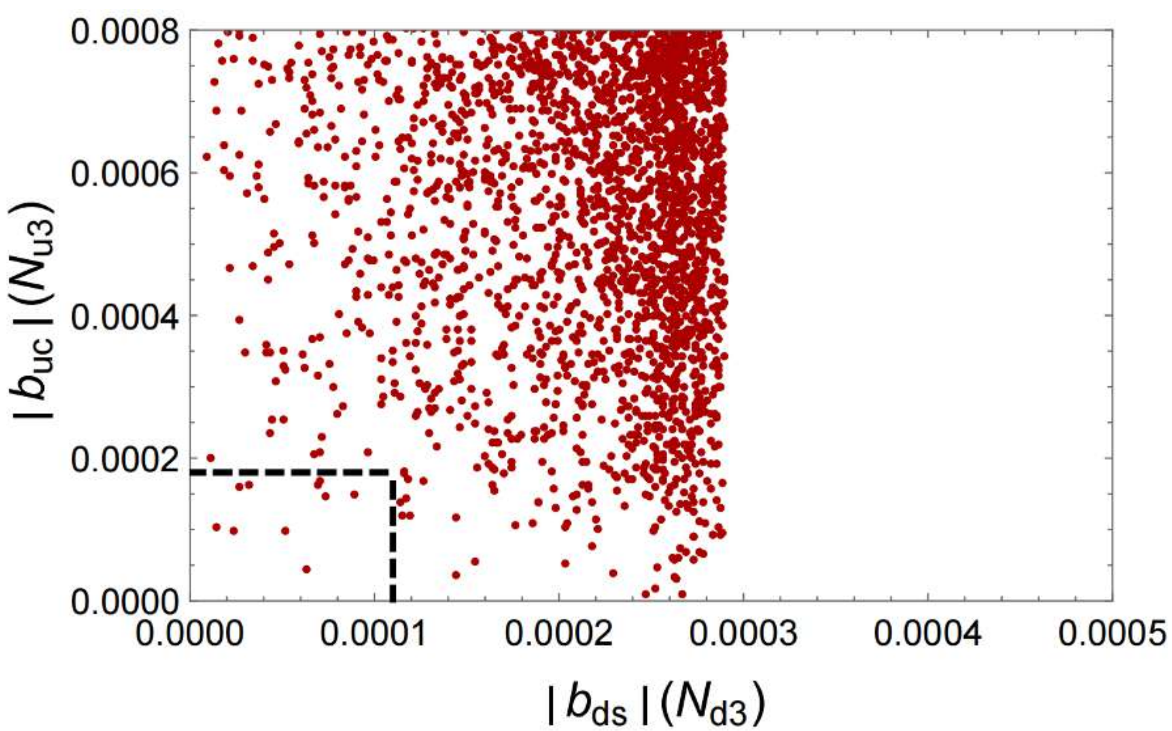
case (B_1, B_1), $\tan\beta = 1$, full scan



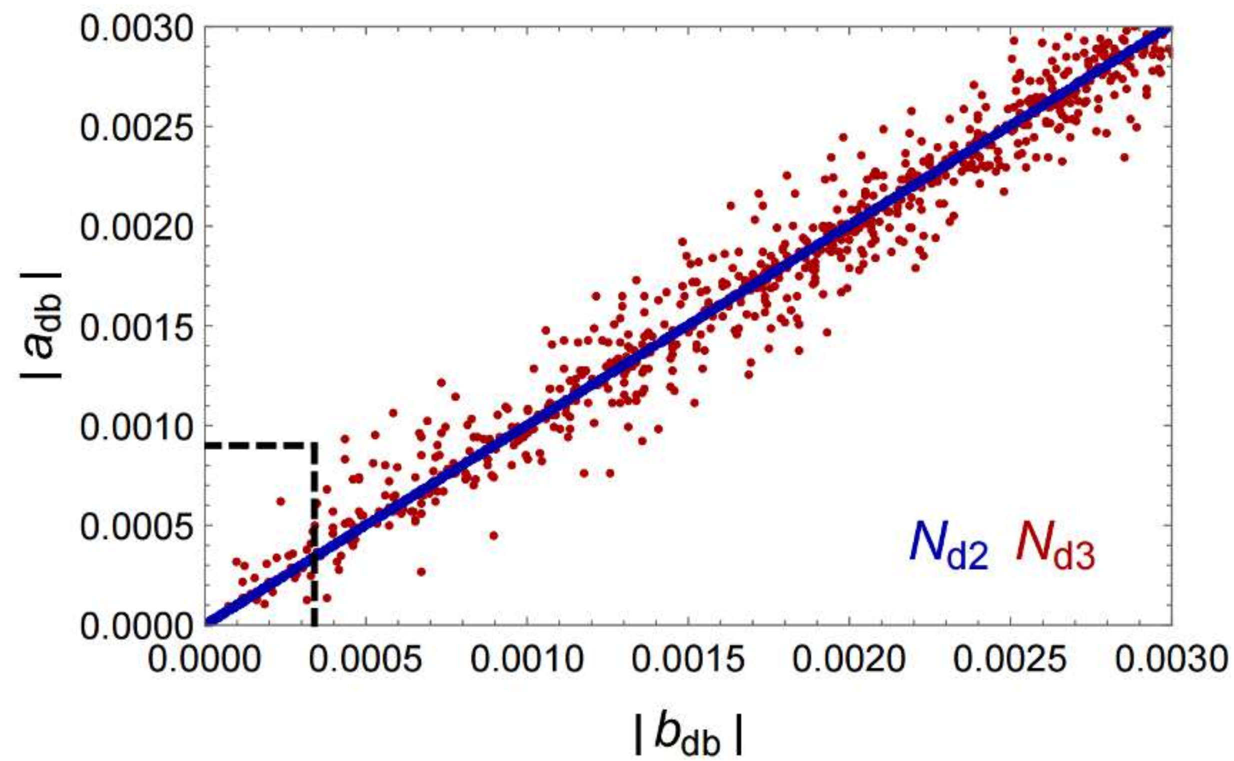
case (B_1, B_1), $\tan\beta = 1$, full scan



case (B_1, B_1), $\tan\beta = 1$, restricted scan



case (B_1, B_1), $\tan\beta = 1$, full scan



D-meson oscillations place the strongest constraints in the generic scan

Summary

- additional scalars offers way to resolve some of the long-standing issues of the SM framework

- multi-scalar models offer rich phenomenology at colliders, in neutrino physics and in cosmology

- flavour and high-CP symmetries enable to generate very specific patterns in mass, mixing and FCNC hierarchies

- search for suitable UV complete theories giving rise to such models is under way