

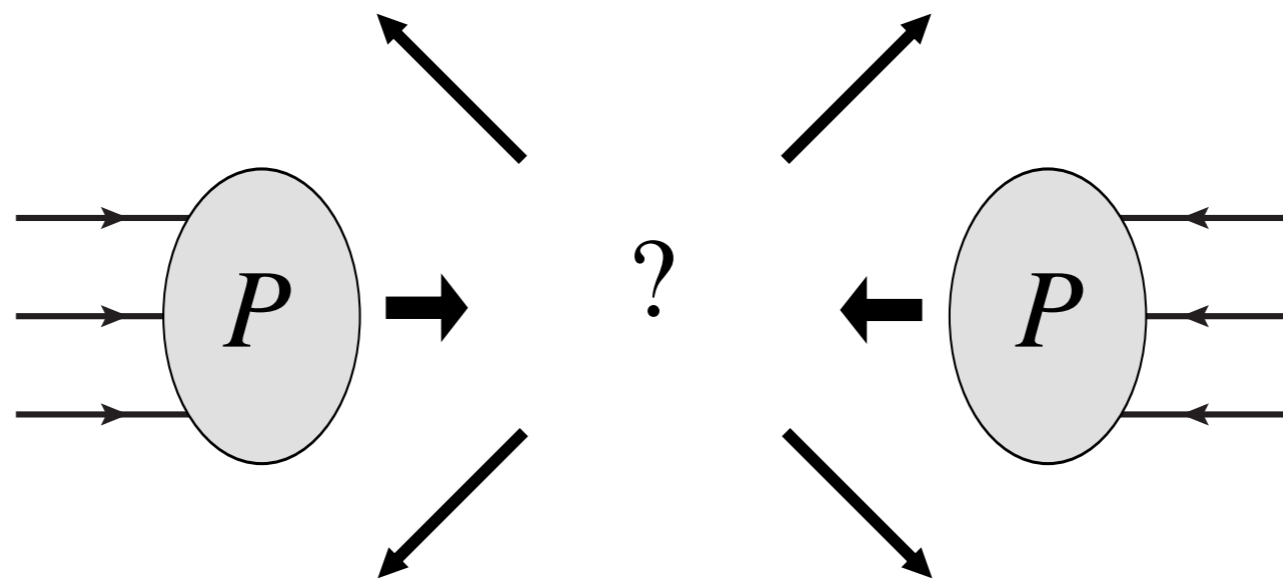
# Heavy-flavour jets: theory progress; and applications

Rhorry Gauld

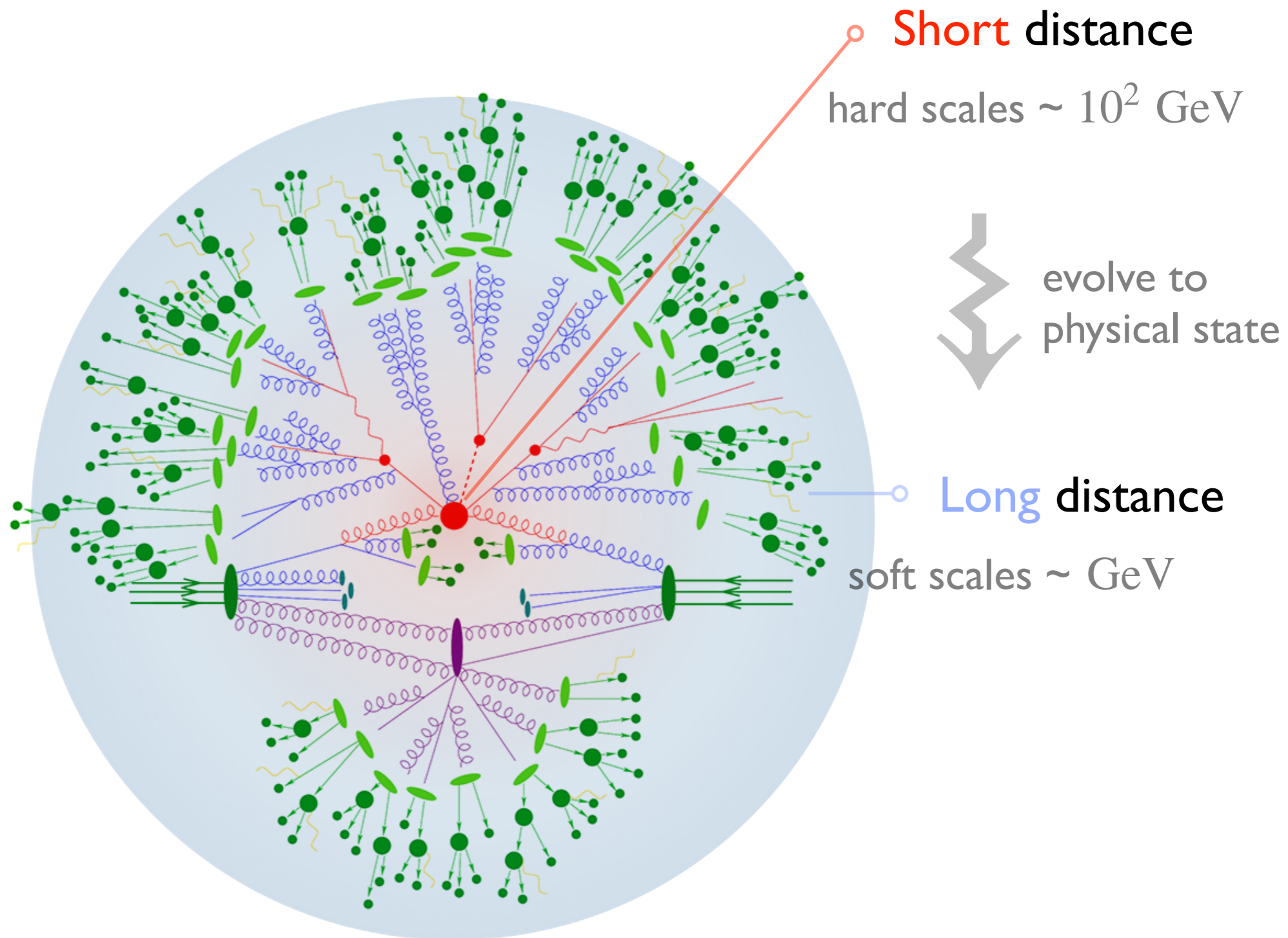
CERN QCD Seminar (04/04/23)



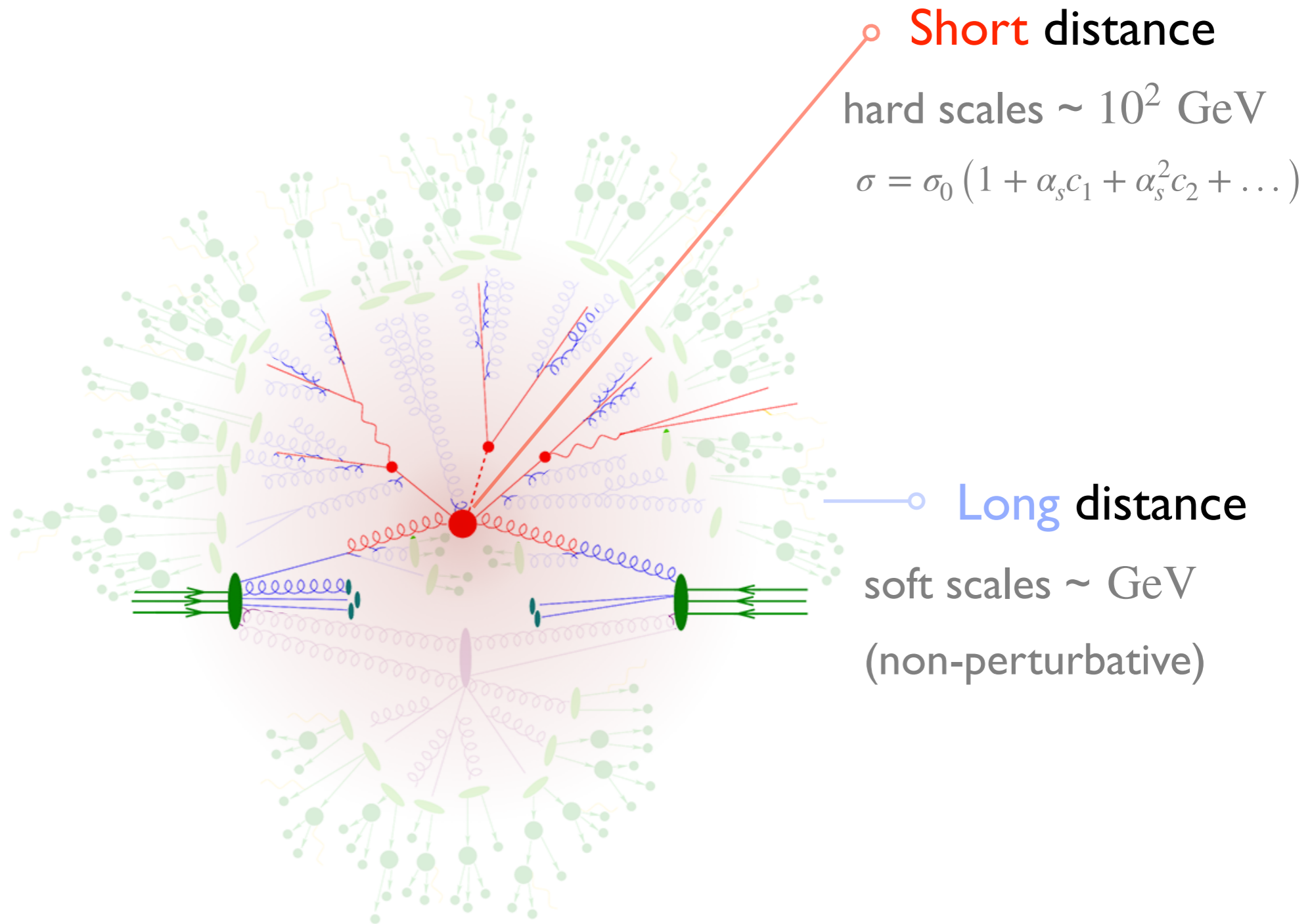
MAX-PLANCK-INSTITUT  
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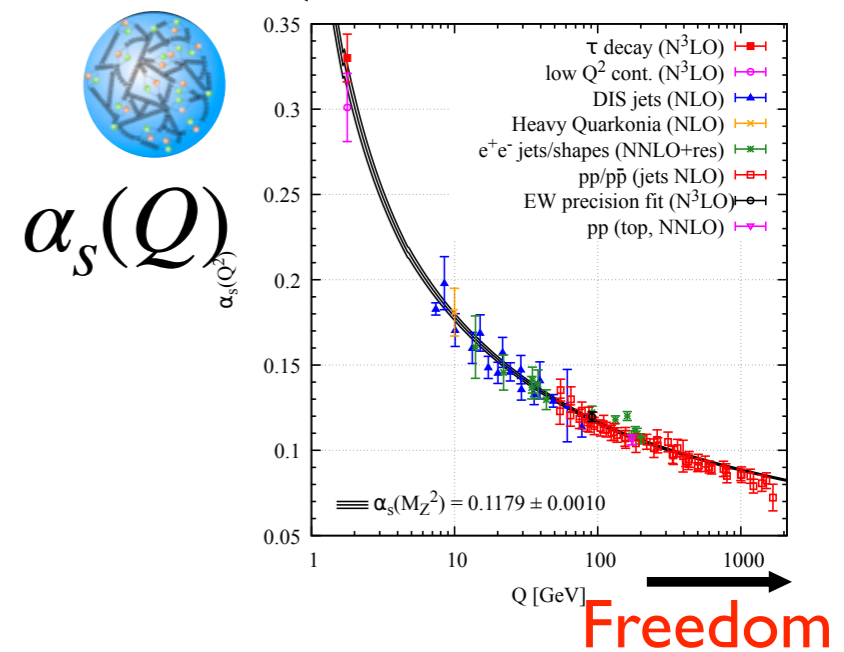
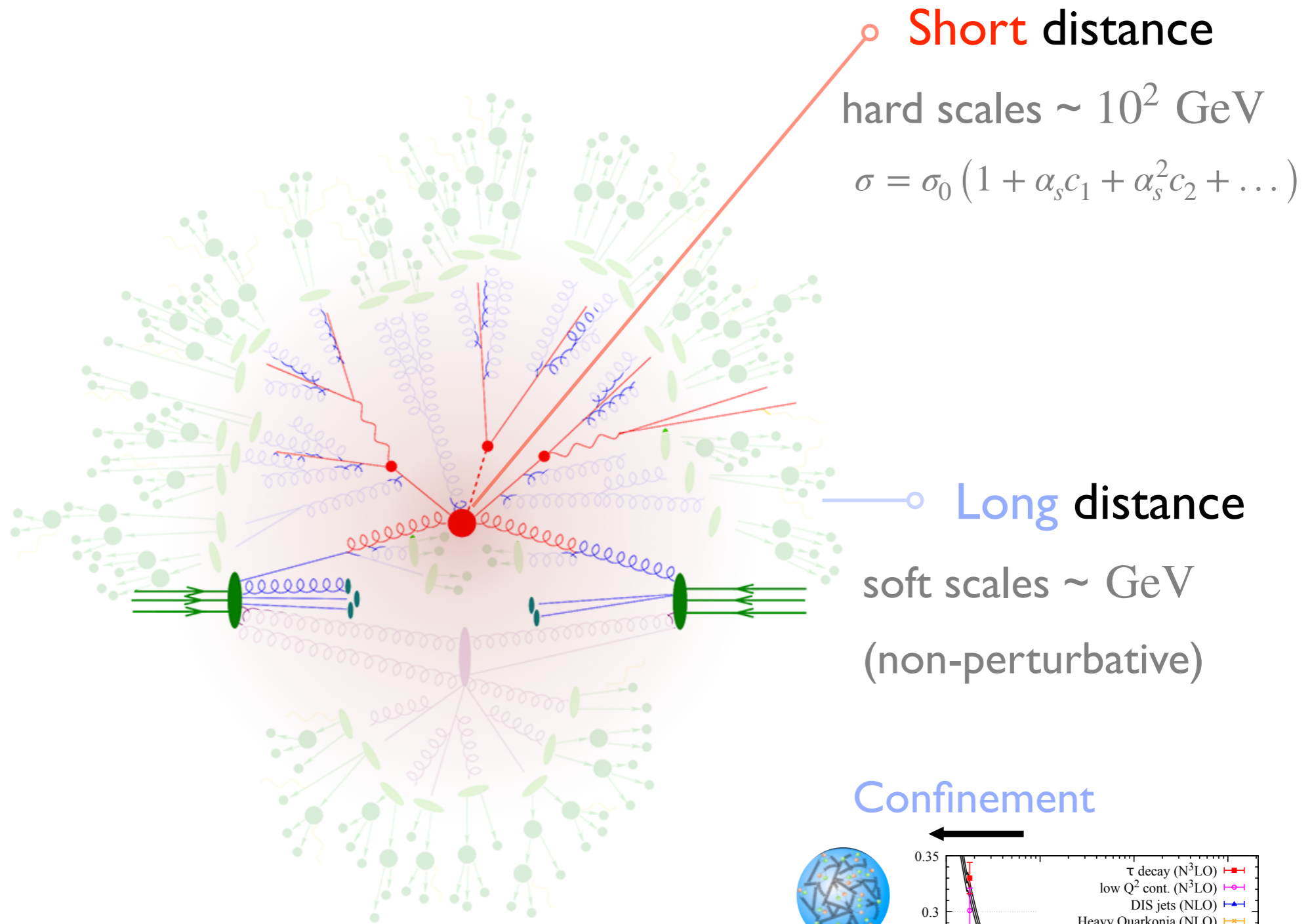
$$PP \rightarrow f + X$$



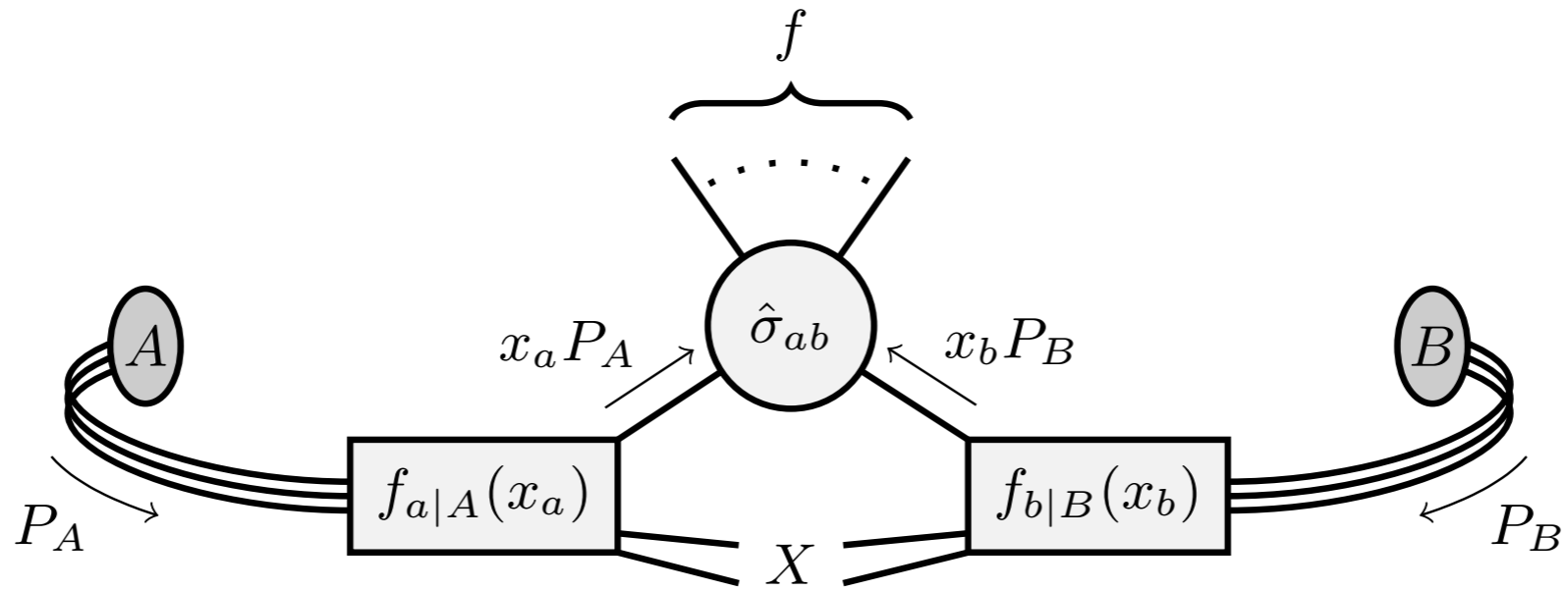
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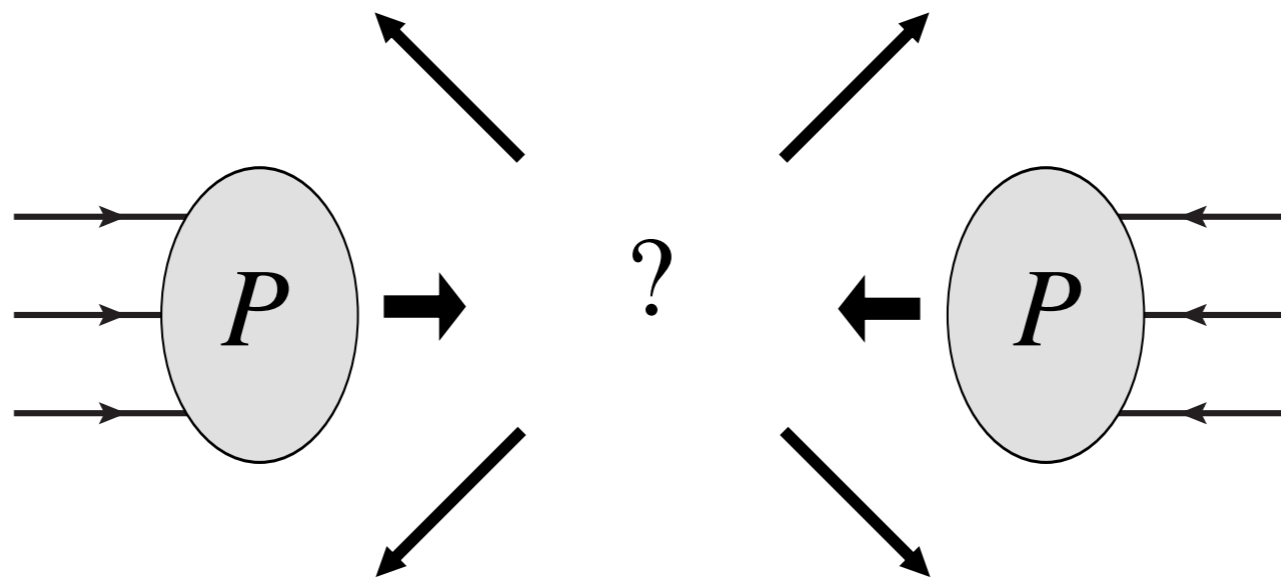


$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b \underbrace{f_{a|A}(x_a)} \underbrace{f_{b|B}(x_b)} \underbrace{\hat{\sigma}_{ab}(x_a, x_b)} (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

parton distribution functions (PDFs)  
non-perturbative, data-driven

hard scattering  
perturbation theory

hadronisation corrections, ...  
non-perturbative effects



$$PP \rightarrow f + X$$



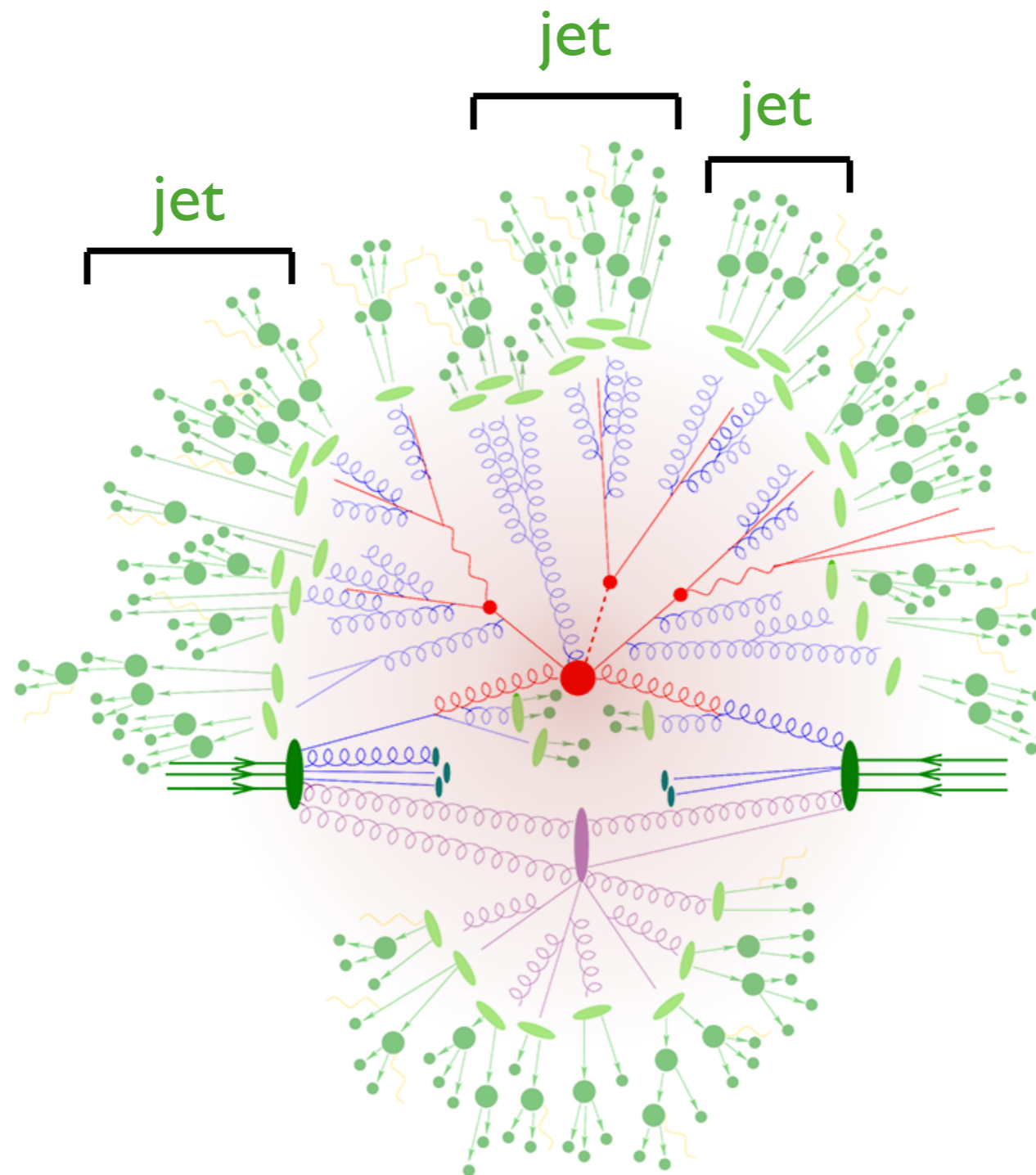
$f$  composed of :

- leptons
- hadrons
- photons
- missing  $E_T$
- jet
- ...

critical question: how to define  $f$  to best enable this comparison

$$d\sigma_{PP \rightarrow f+X}^{\text{data (meas.)}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{theory}}$$

e.g. flavour = c or b



leptons  
hadrons  
photons  
missing  $E_T$   
flavoured jet  
...

critical question: how to define  $f$  to best enable this comparison

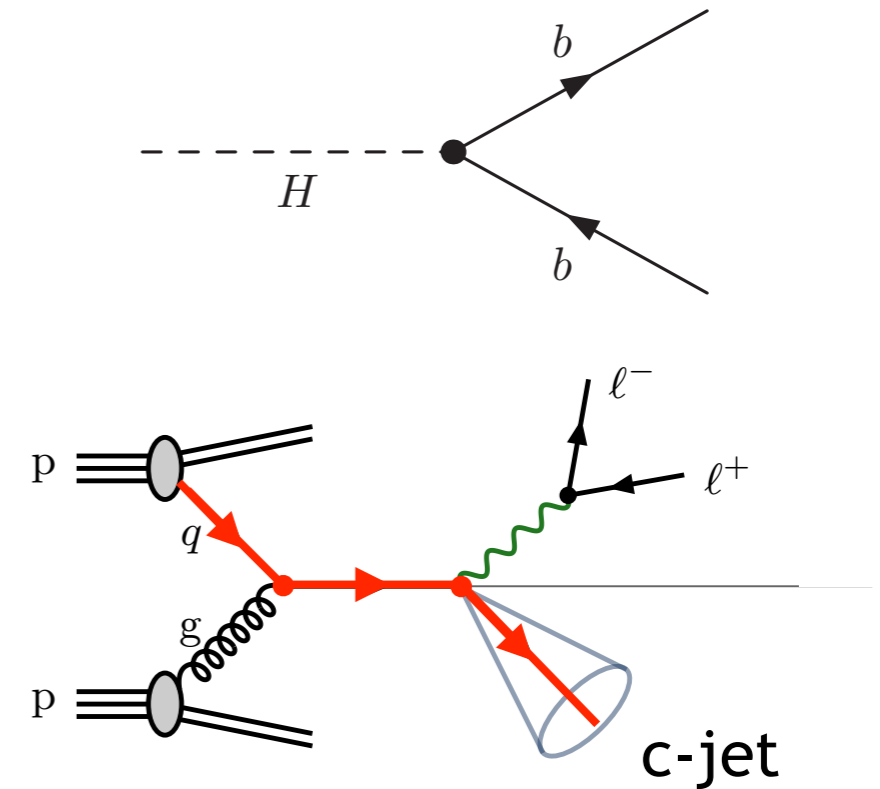
$$d\sigma_{PP \rightarrow f+X}^{\text{data (meas.)}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{theory}}$$





## ... and why the focus on flavoured jets?

- a) Higgs physics (hadronic decays)
- b) Top-quark physics ( $|V_{tb}| \sim 1$ )
- c) New physics searches (f-jet +  $E_T^{\text{miss}}$ )
- d) Gauge-boson + heavy-flavour
- ...



critical question: how to define  $f$  to best enable this comparison

$$d\sigma_{PP \rightarrow f+X}^{\text{data (meas.)}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{theory}}$$

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## Focus is on IRC (InfraRed and Collinear) safe observables:

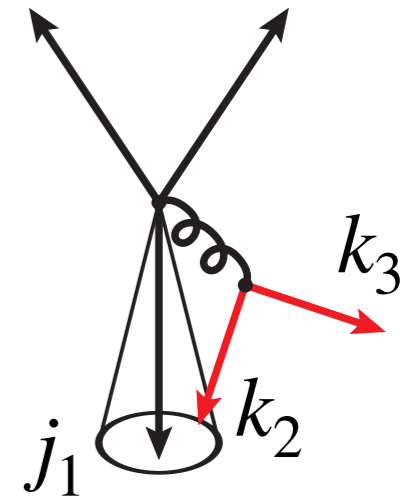
- Those not impacted by collinear splitting(s) or emission(s) of soft particles
  - Those calculated in terms of quarks and gluons where the  $m_q \rightarrow 0$  limit does not introduce singularities (Stermann, Weinberg '77)
- ➔ Can (reliably) use fixed-order perturbation theory

KLN theorem: (Kinoshita '62, Lee & Nauenberg '64)

- For such observables, a cancellation of IRC divergences between virtual and real emissions is ensured (order-by-order)
- IRC unsafe observables can be defined, all-order-resummation/factorisation theorems typically required (PDF evolution, obs. dependent resummation)

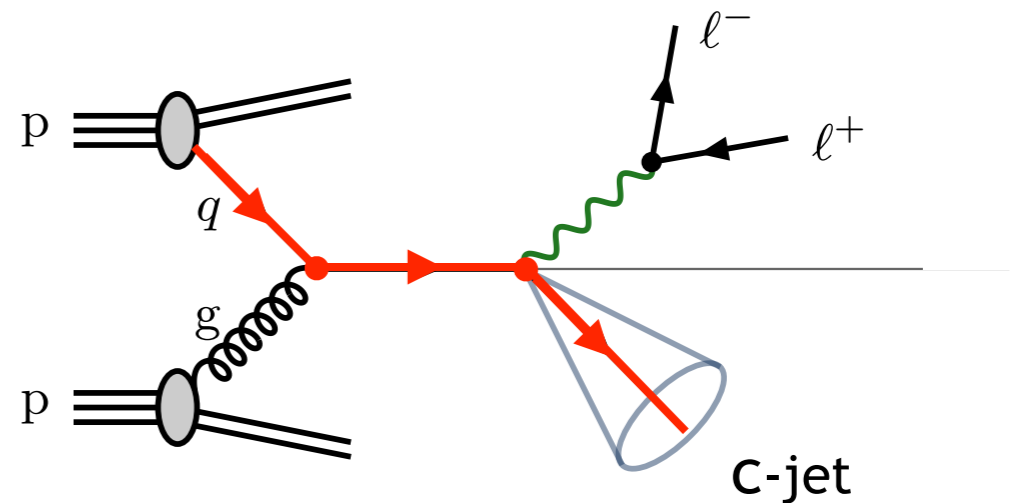
# 1) The problem of jet flavour (IRC safety)

- ▶ Stating the problem
- ▶ Recent solutions (algorithms)



# 2) Application: Z-boson + c-jet production at LHCb

- ▶ Motivation and context
- ▶ Phenomenological results



# 3) Experimental feasibility of flavoured jet algorithms

[Time permitting]

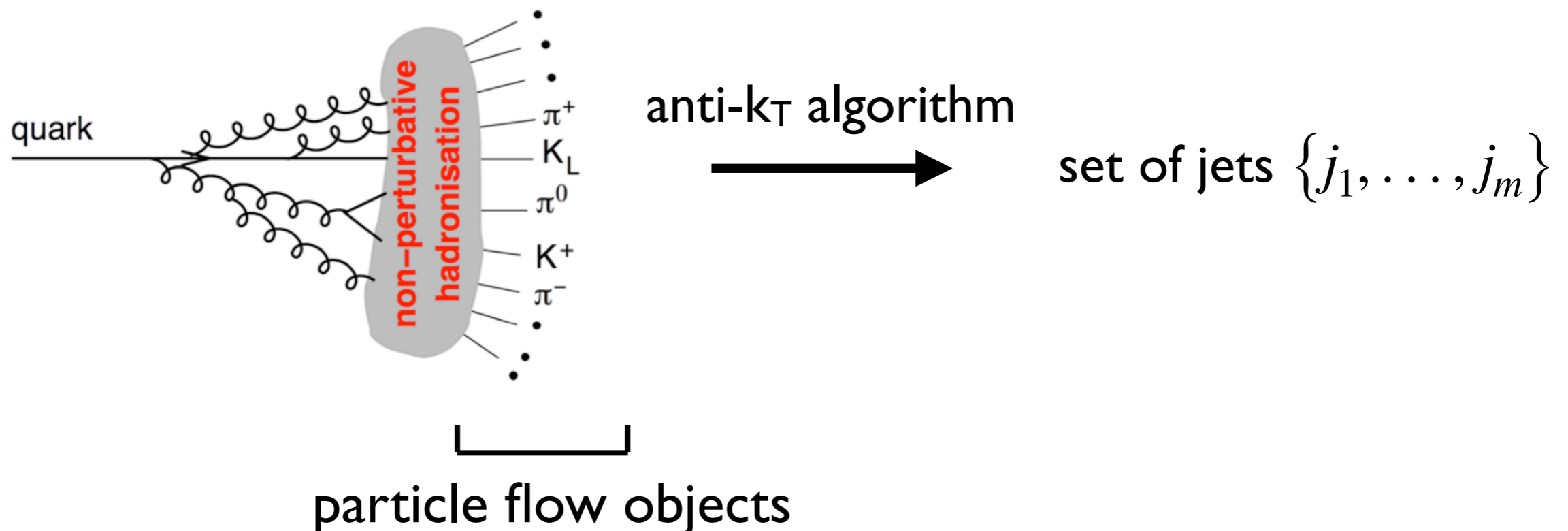
# Jets at the LHC

Experimentally: apply an algorithm to particle flow objects (Kaons, Pions,...)  
(e.g. ATLAS arXiv:1703.10485, CMS arXiv:1706.04965, LHCb arXiv:1310.8197)

The anti- $k_T$  algorithm (Cacciari, Salam, Soyez arXiv:0802.1189) applied to these objects

Simple version

➔ Reconstruct hadronic jets ( $\sim$ collimations of hadronic radiation)



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Or... initialise a list of particles (pseudo jets) from these objects

Introduce distance measures between particles (pseudo jets) and a Beam:

$$d_{ij} = \min \left( k_{Ti}^{2p}, k_{Tj}^{2p} \right) \frac{\Delta R_{ij}^2}{R^2} \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$
$$d_{iB} = k_{Ti}^{2p}$$

(Inclusive) clustering proceeds by identifying the min. distance:

- If it is  $d_{ij}$  combine particles  $ij$  (update list to contain combined particle)
- If it is  $d_{iB}$ , identify  $i$  as a jet and remove from list

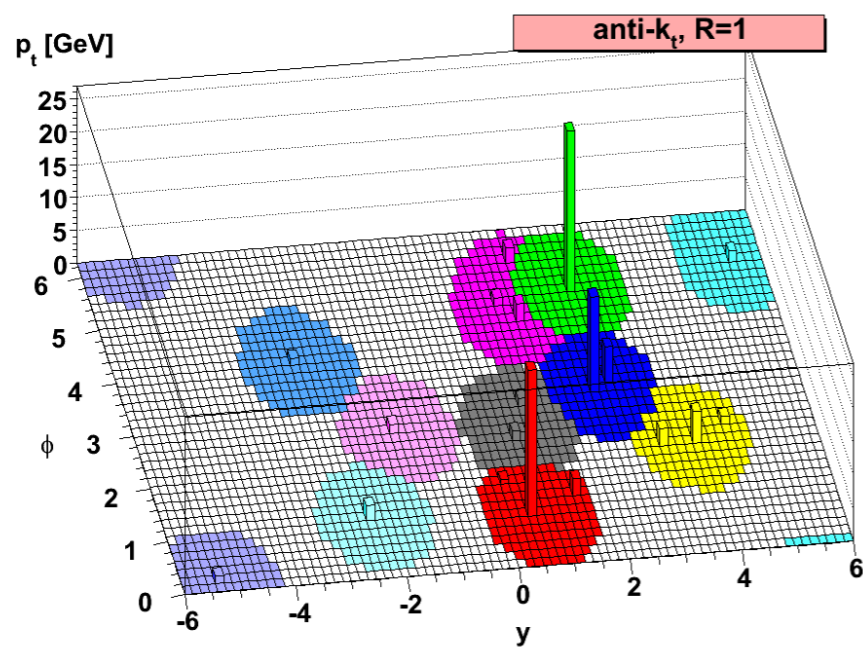
[repeat until list is empty]

# Jets at the LHC

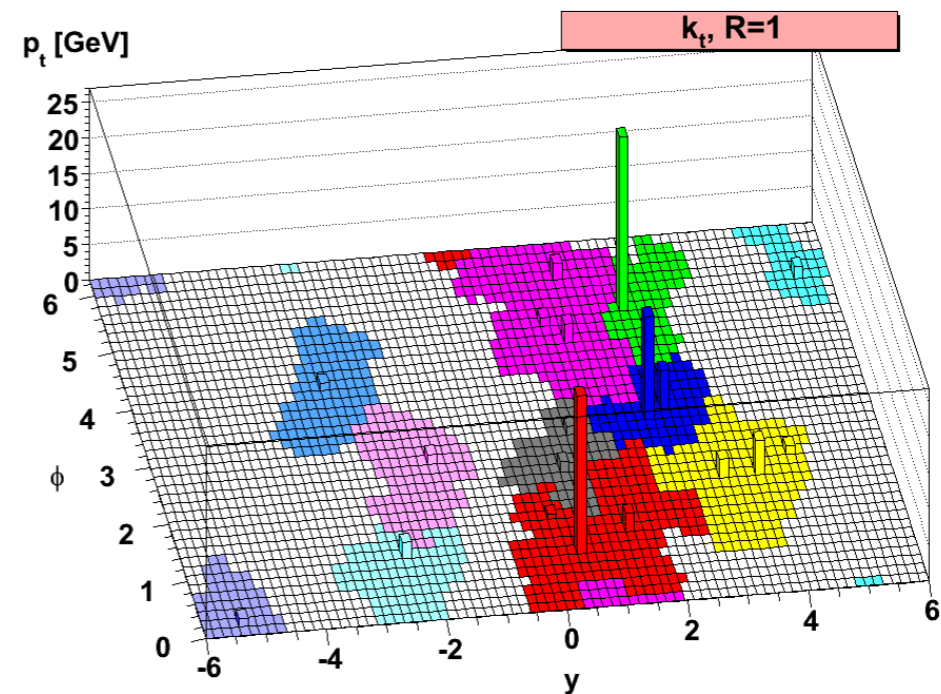
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The anti- $k_T$  algorithm (Cacciari, Salam, Soyez arXiv:0802.1189) applied to these objects

anti- $k_T$  has nice geometrical properties (used in all LHC analyses)



anti- $k_T$  ( $p=-1$ )



$k_T$  ( $p=1$ )

# Heavy-flavour jets at the LHC

Typical experimental approaches of defining jet flavour (truth/data level):

(ATLAS arXiv:1504.07670, CMS arXiv:1712.07158, LHCb arXiv:1504.07670)

- i) First identify flavour-blind anti- $k_T$  jets in a fiducial region
- ii) Tag these jets with flavour by the presence of 1 or more D/B hadrons

$$\Delta R(j, D/B) < 0.5$$

- iii) [ATLAS/LHCb] Apply  $p_T$  requirement to D/B hadron  $\sim p_T^{D/B} > 5 \text{ GeV}$

# Heavy-flavour jets at the LHC

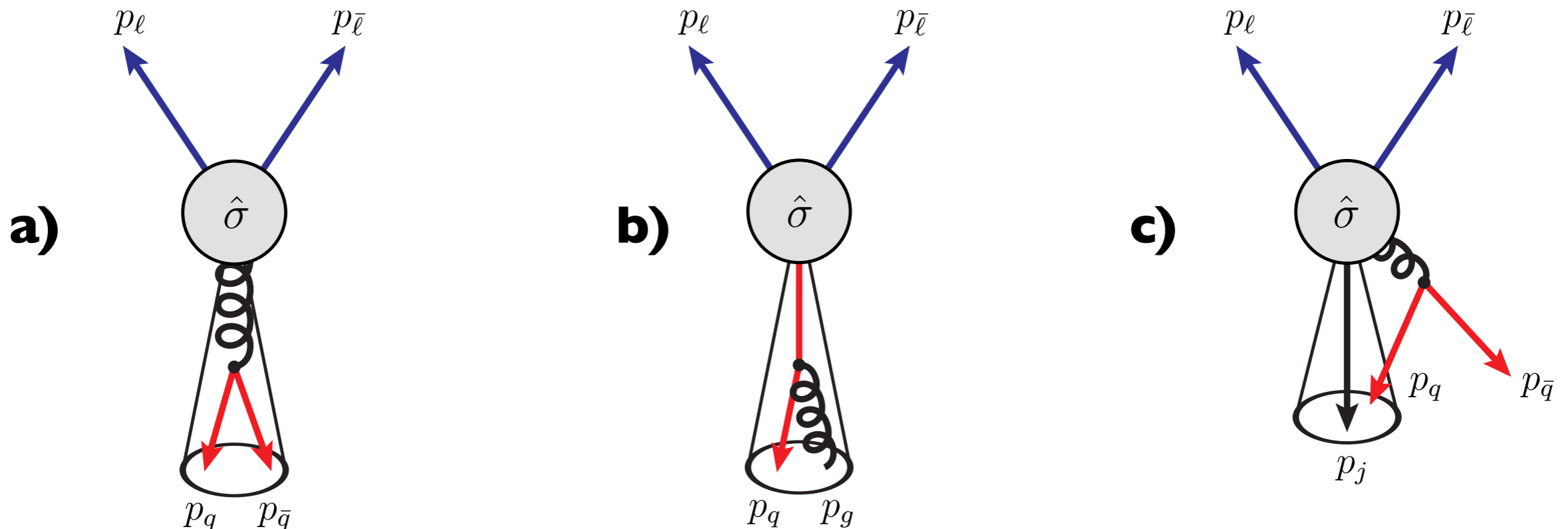
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# Solutions to this problem

## Comments/status

“The Flavour- $k_T$  algorithm”

(Banfi, Salam, Zanderighi: hep-ph/0601139)

$k_T$  jets

all-order IRC safe?

... theory progress on NNLO QCD jet calculations ( $VH$ ,  $t\bar{t}$  w/ decay,  $V + j$ )

Practical jet flavour through NNLO

(Caletti, Larkoski, Marzani, Reichelt: arXiv:2205.01109)

(substructure based)

Soft unsafe N3LO

Infrared-safe flavoured anti- $k_T$  jets

(Czakon, Mitov, Poncelet: arXiv:2205.11879)

approx. anti- $k_T$  jets

all-order IRC safe?

A dress of flavour to suit any jet

(**RG**, Huss, Stagnitto: arXiv:2208.11138)

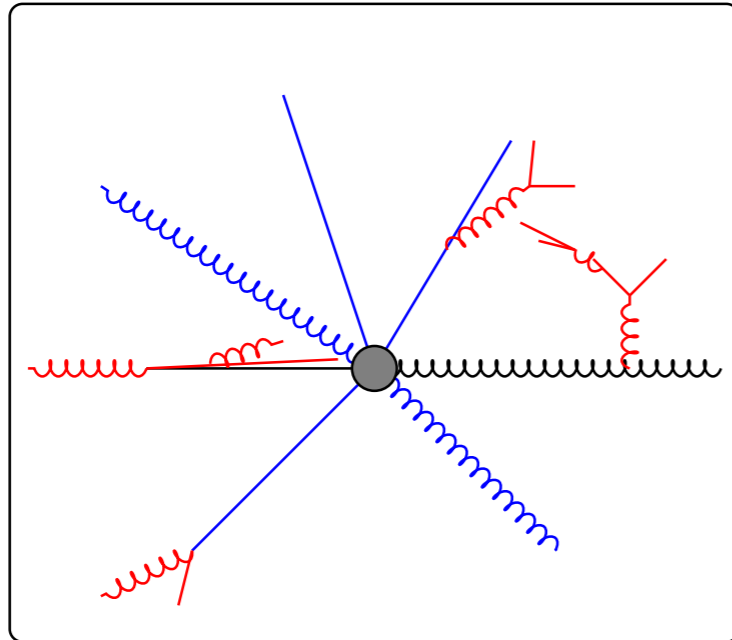
Tested at N3LO ( $e^+e^- \rightarrow$  jets)

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# Solutions to this problem

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A dress of flavour to suit any jet ( <b>RG</b> , Huss, Stagnitto: arXiv:2208.11138)	Tested at N3LO ( $e^+e^- \rightarrow$ jets) all-order IRC safe?
Flavoured jets with exact anti- $k_T$ kinematics (Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler: preliminary—Moriond last week)	(substructure based)

# Systematic framework to test IRC safety [numerical tests up to $\mathcal{O}(\alpha_s^6)$ ]



FSR-DS = double-soft  
ISR-DS  
FC = FS hard-collinear  
IC = IS hard-collinear

possibly nested

Set of hard jets  
 $\mathcal{J}_{\text{hard}} = \{(p_1, f_1), \dots\}$

!

Set of hard+IR jets  
 $\mathcal{J}_{\text{hard+IR}} = \{(\tilde{p}_1, \tilde{f}_1), \dots\}$

## Comments/status

$k_T$  jets  
all-order IRC safe?

$t, t\bar{t}$  w/ decay,  $V + j$ )

(substructure based)  
Soft unsafe N3LO

approx. anti- $k_T$  jets  
IC-IC unsafety?

A dress of flavour to suit any jet  
(**RG**, Huss, Stagnitto: arXiv:2208.11138)

Tested at N3LO ( $e^+e^- \rightarrow$  jets)  
adjustments required

Flavoured jets with exact anti- $k_T$  kinematics  
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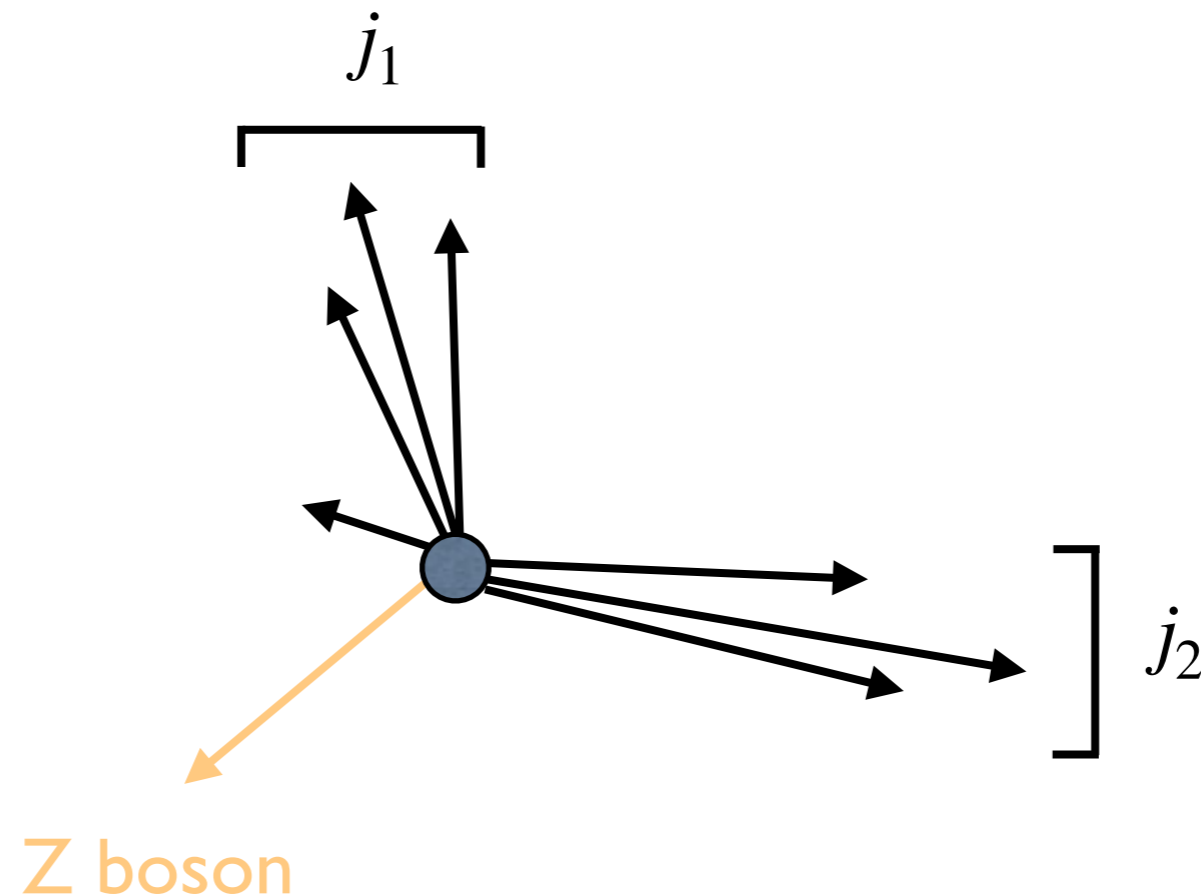
# A dress of **flavour** to suit any jet

(**RG**, Huss, Stagnitto arXiv:2208.11138)

**Our motivation:** A well defined flavour algorithm applicable to anti- $k_T$  jets  
(actually, any jet)

Also, applicable to heavy-flavour tagging in LHC analyses

Toy event



set of jets  $\{j_1, \dots, j_m\}$

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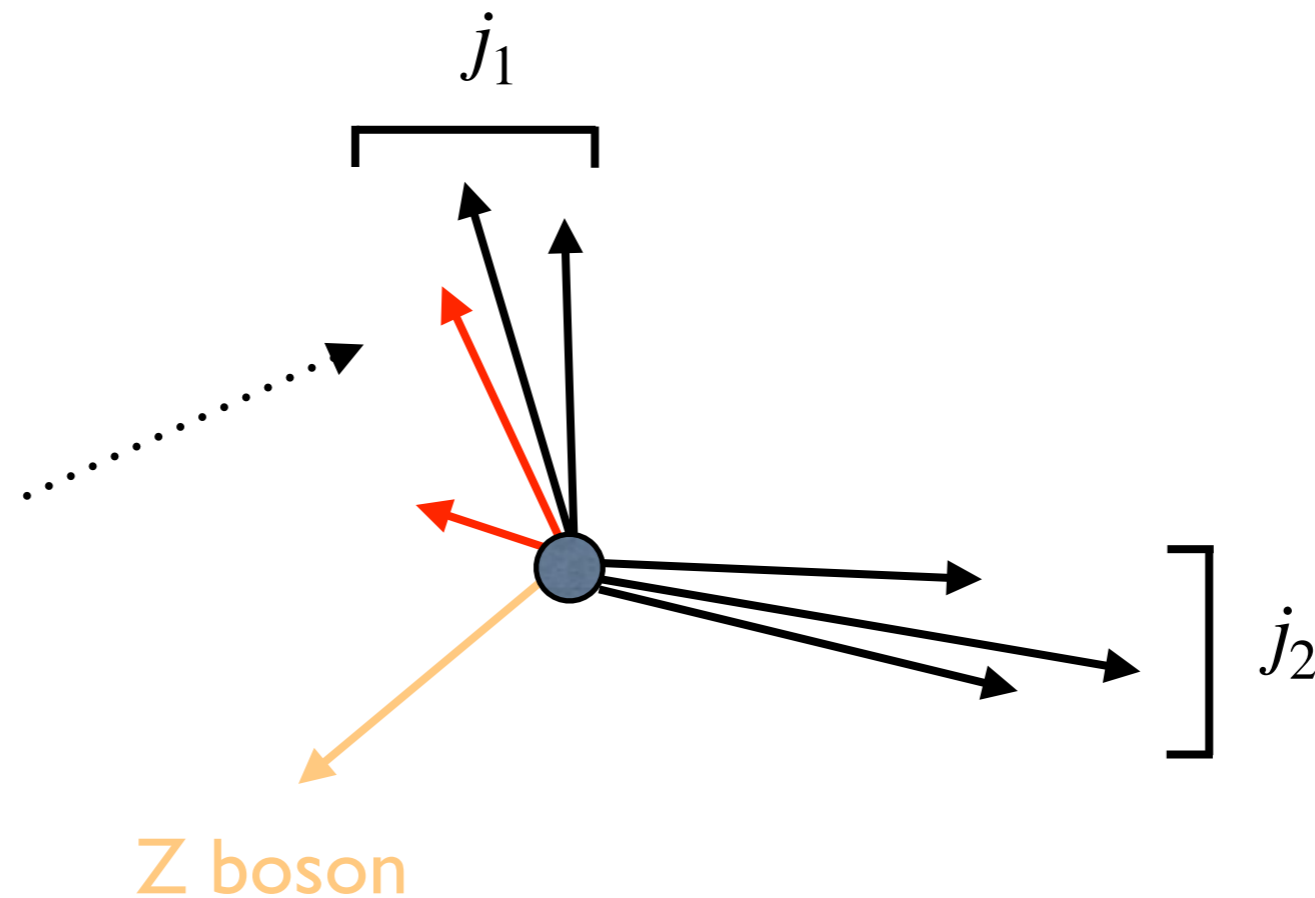
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Toy event

Flavoured particles

b-quark (theory)

secondary vertex (exp.)



set of jets  $\{j_1, \dots, j_m\}$

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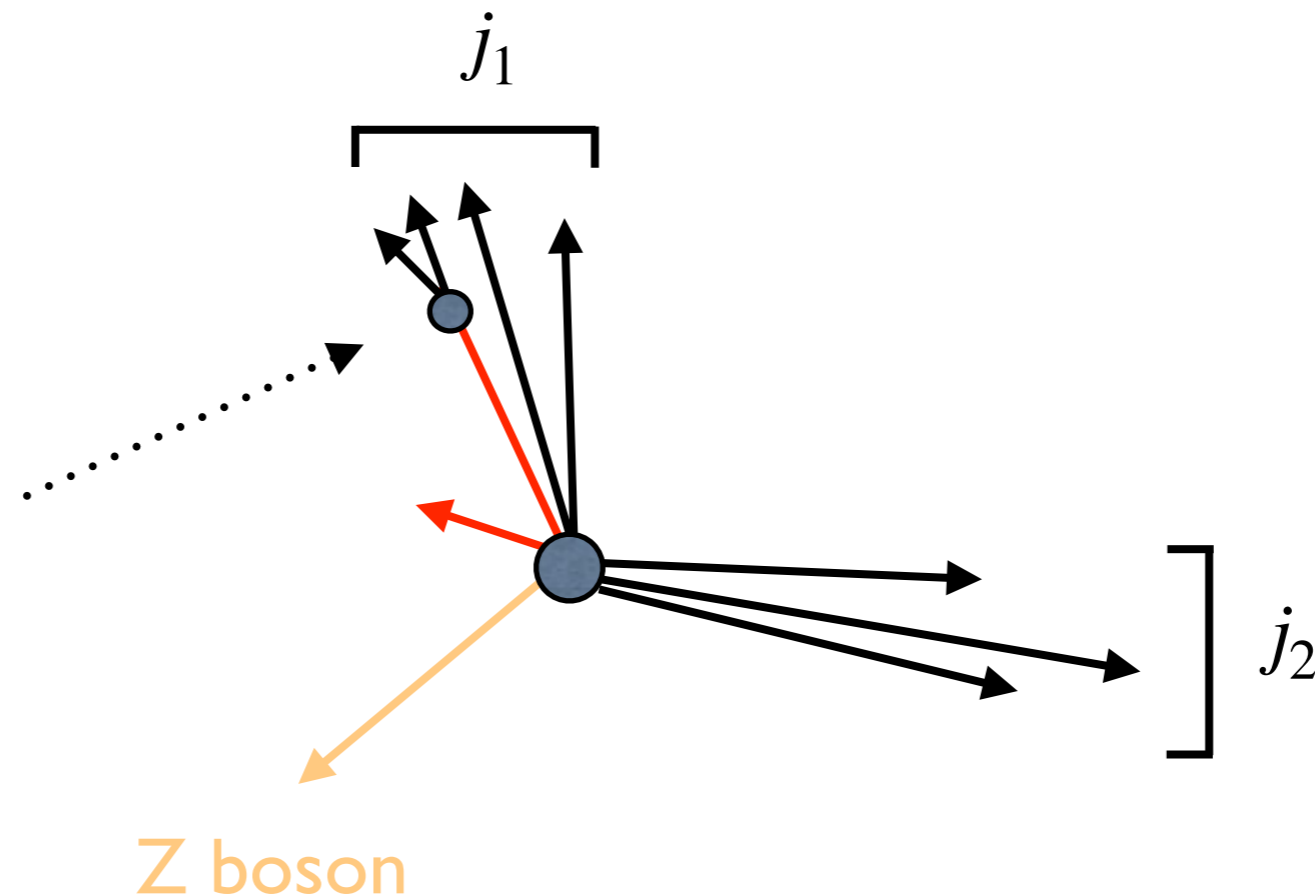
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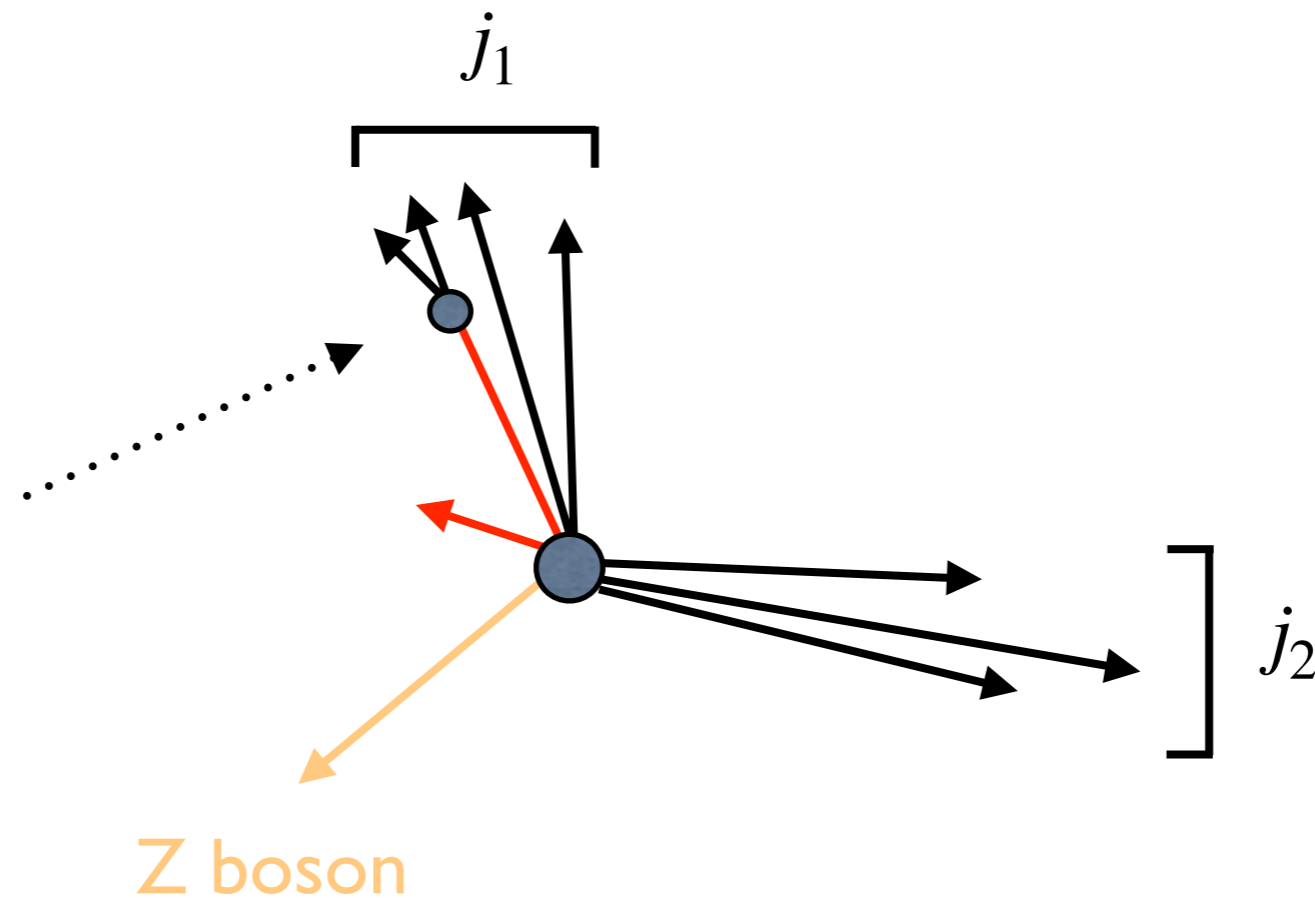
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Flavoured particles

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algorithm assigns  $\hat{f}_i$  to  $j_k$

set of jets  $\{j_1, \dots, j_m\}$  ← set of flavoured objects  $\{\hat{f}_1, \dots, \hat{f}_n\}$

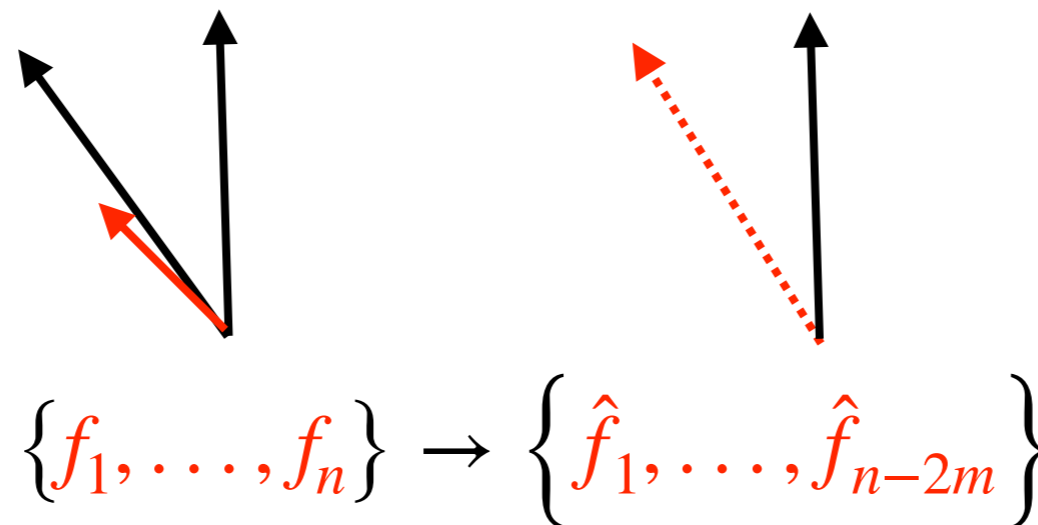
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# (1/3) collinear-safe flavoured objects

(**RG**, Huss, Stagnitto arXiv:2208.11138)

flavoured particles (quarks, hadrons) not collinear safe. Define new objects:

Dress the flavoured particles with collinear radiation  
(altering momenta but not flavour)



flavoured particles  $\rightarrow$  flavoured ‘clusters’  
(potentially annihilating  $m$  collinear  $f_i f_j$  pairs)



# (1/3) collinear-safe flavoured objects

(**RG**, Huss, Stagnitto arXiv:2208.11138)

flavoured particles (quarks, hadrons) not collinear safe. Define new objects:

- i) Initialise a list of all particles
- ii) Add to the list all flavoured particles, removing any overlap
- iii) Calculate the distances  $d_{ij} = \Delta R_{ij}^2$  between all particles
- iv) If  $d_{ij}^{\min} > \Delta R_{\text{cut}}^2$  terminate the clustering. Otherwise:
  1. (i & j flavourless) replace i & j in the list with combined object ij
  2. (i or j flavoured) combine i and j if:

$$\frac{\min(p_{T,i}, p_{T,j})}{p_{T,i} + p_{T,j}} > z_{\text{cut}} \left( \frac{\Delta R_{ij}}{R_{\text{cut}}} \right)^\beta \quad \text{[Soft-drop]}$$

(Larkoski et al. arXiv:1402.2657)

Otherwise:

(i & j flavoured) remove both from list

(i or j flavourless) remove only flavourless object

[Repeat until list empty, or no flavoured particles left]

## (2/3) Association criterion and counting

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}, \{\hat{f}_1, \dots, \hat{f}_n\}$

We introduce an **Association criterion** for  $\hat{f}_a$  with  $j_b$  (some possibilities):

- the flavoured particle  $f_a$  is a constituent of jet  $j_b$
- or  $\Delta R(\hat{f}_a, j_b) < R_{\text{tag}}$
- or Ghost association of  $\hat{f}_a$  (include direction of  $\hat{f}_a$  in anti- $k_T$  clustering)  
(association criterion required as not assumed that  $f_a$  is a stable particle)

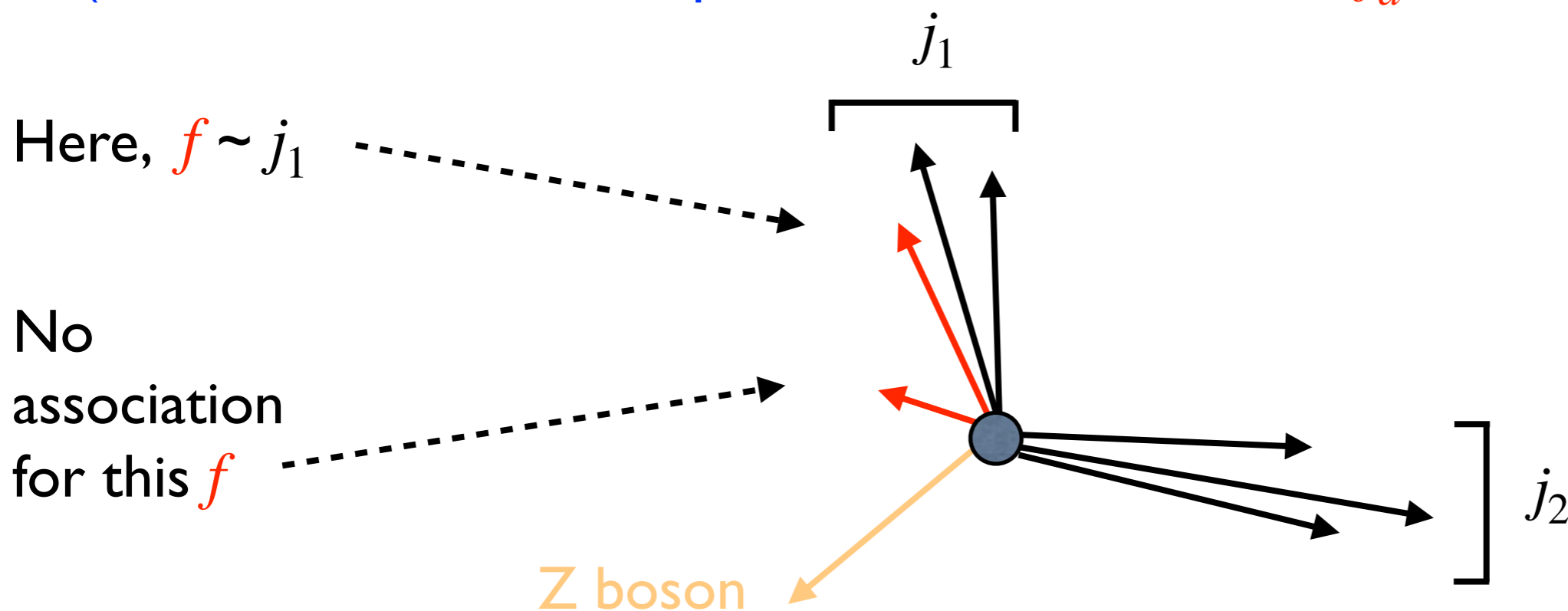
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Introduce a **Counting** or **Accumulation** for flavour:

- with charge info. ( $q$  vs  $\bar{q}$ ), then  $q = +1$  and  $\bar{q} = -1$  (net flavour is sum)
- if one cannot (e.g. experiment),  $q = \bar{q} = 1$  (net flavour is sum modulo 2)  
[i.e. jets with even number of  $q_i + \bar{q}_j$  are NOT flavoured]

# (3/3) The flavour dressing algorithm

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}, \{\hat{f}_1, \dots, \hat{f}_n\}$ , association, and counting rules

# (3/3) The flavour dressing algorithm

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}, \{\hat{f}_1, \dots, \hat{f}_n\}$ , association, and counting rules

Dressing algorithm:

- Calculate a set of distances between the flavoured objects, jets and beam:
  - [ff]  $d_{ab}$  between all all flavoured objects  $\hat{f}_a$  and  $\hat{f}_b$
  - [fj]  $d_{ab}$  between  $\hat{f}_a$  and  $j_b$  ONLY if there is an association
  - [fB]  $d_{aB}$  for all  $\hat{f}_a$  without a jet association

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  - [fB]  $d_{aB}$  for all  $\hat{f}_a$  without a jet association
- Find the minimum distance of all entries in the list
  - if it is an [fj] assign  $\hat{f}_a$  to  $j_b$  (removing entries involving  $\hat{f}_a$  from list)
  - otherwise just remove  $\hat{f}_a$  [fB] or  $\hat{f}_a$  and  $\hat{f}_b$  [ff] from the list

[repeat until list empty]

- The flavour of each jet is then just the accumulation of its flavour

# (3/3) The flavour dressing algorithm

(**RG**, Huss, Stagnitto arXiv:2208.11138)

We now have  $\{j_1, \dots, j_m\}, \{\hat{f}_1, \dots, \hat{f}_n\}$ , association, and counting rules

**Note:** Originally we used the distance measures proposed in flavour- $k_T$   
(Banfi, Salam, Zanderighi hep-ph/0601139)

$$d_{ab} = \Delta R_{ab}^2 \max \left( p_{T,a}^\alpha, p_{T,b}^\alpha \right) \min \left( p_{T,a}^{2-\alpha}, p_{T,b}^{2-\alpha} \right)$$

$$d_{aB_\pm} = \max \left( p_{T,a}^\alpha, p_{T,B_\pm}^\alpha(y_{\hat{f}_a}) \right) \min \left( p_{T,a}^{2-\alpha}, p_{T,B_\pm}^{2-\alpha}(y_{\hat{f}_a}) \right)$$

As pointed out by (Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler) alteration required

[e.g. unsafe configuration “IDS x FDS” encountered at N4LO]

(TBC: addressed with Jade distance and  $\beta < 2$ )

$$d_{ab} = 2p_a \cdot p_b$$



# Tests of the algorithm ( $e^+e^-$ )

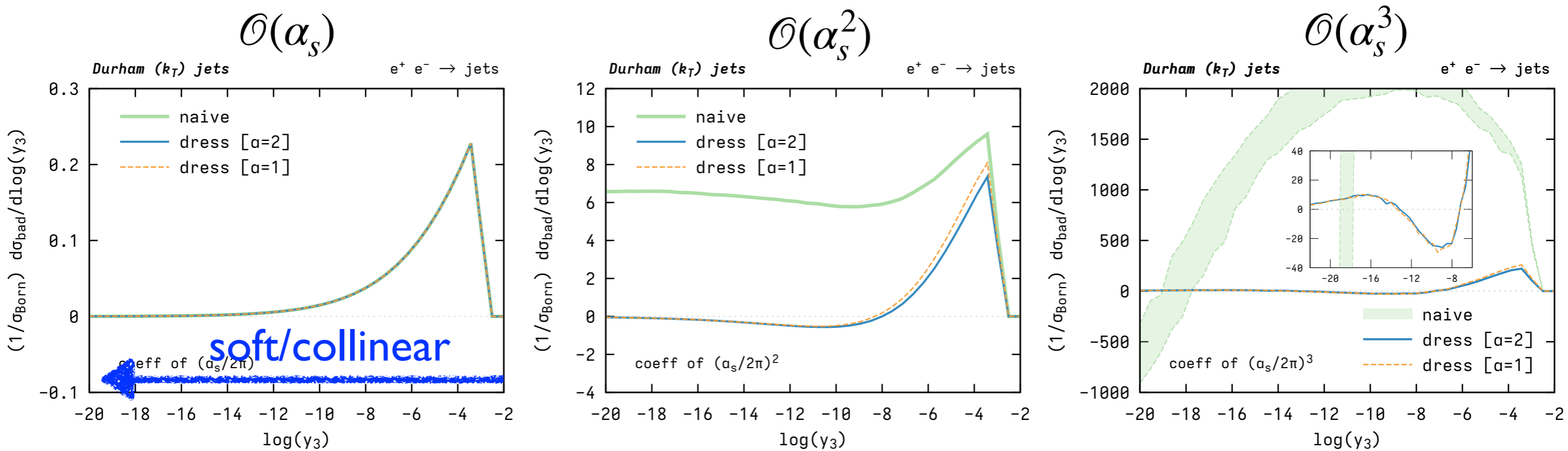
(**RG**, Huss, Stagnitto arXiv:2208.11138)

Consider the process  $e^+e^- \rightarrow 2$  jets at fixed-order using  $k_T$  algorithm

Look at ‘bad’ events (i.e. where we do not find 2 flavoured jets,  $e^+e^- \rightarrow q\bar{q}$ )

The ‘bad’ cross-section should vanish in the  $y_3 \rightarrow 0$  limit

( $y_3 \rightarrow 0$  corresponds to limit of extremely soft and/or collinear emissions)



These tests originally proposed/shown in the original flavour- $k_T$  study

(Banfi, Salam, Zanderighi hep-ph/0601139)

# Summary Part (I)

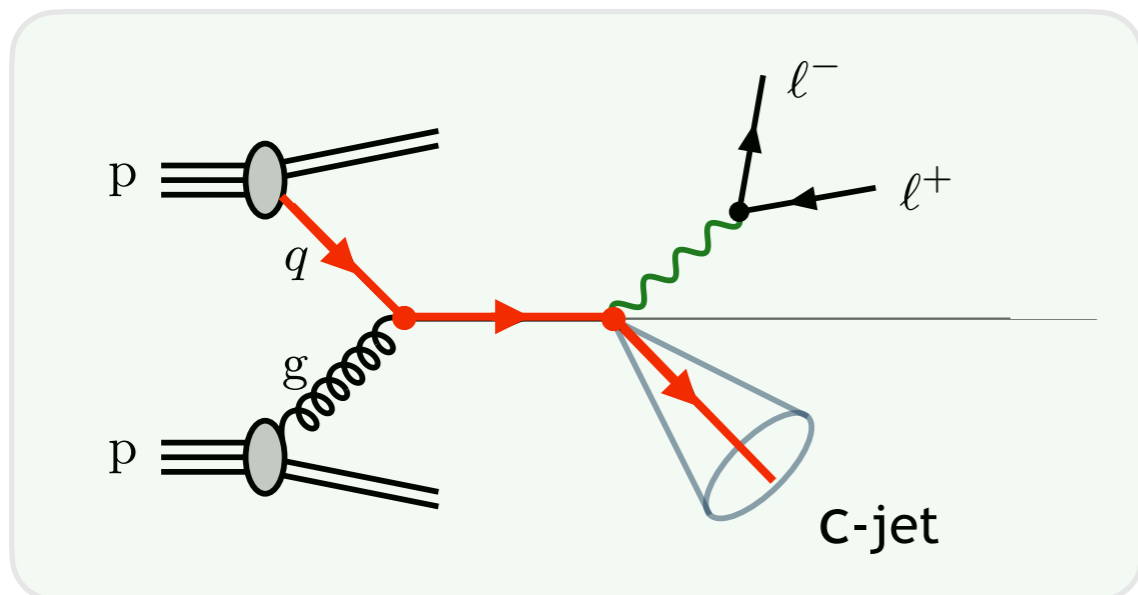
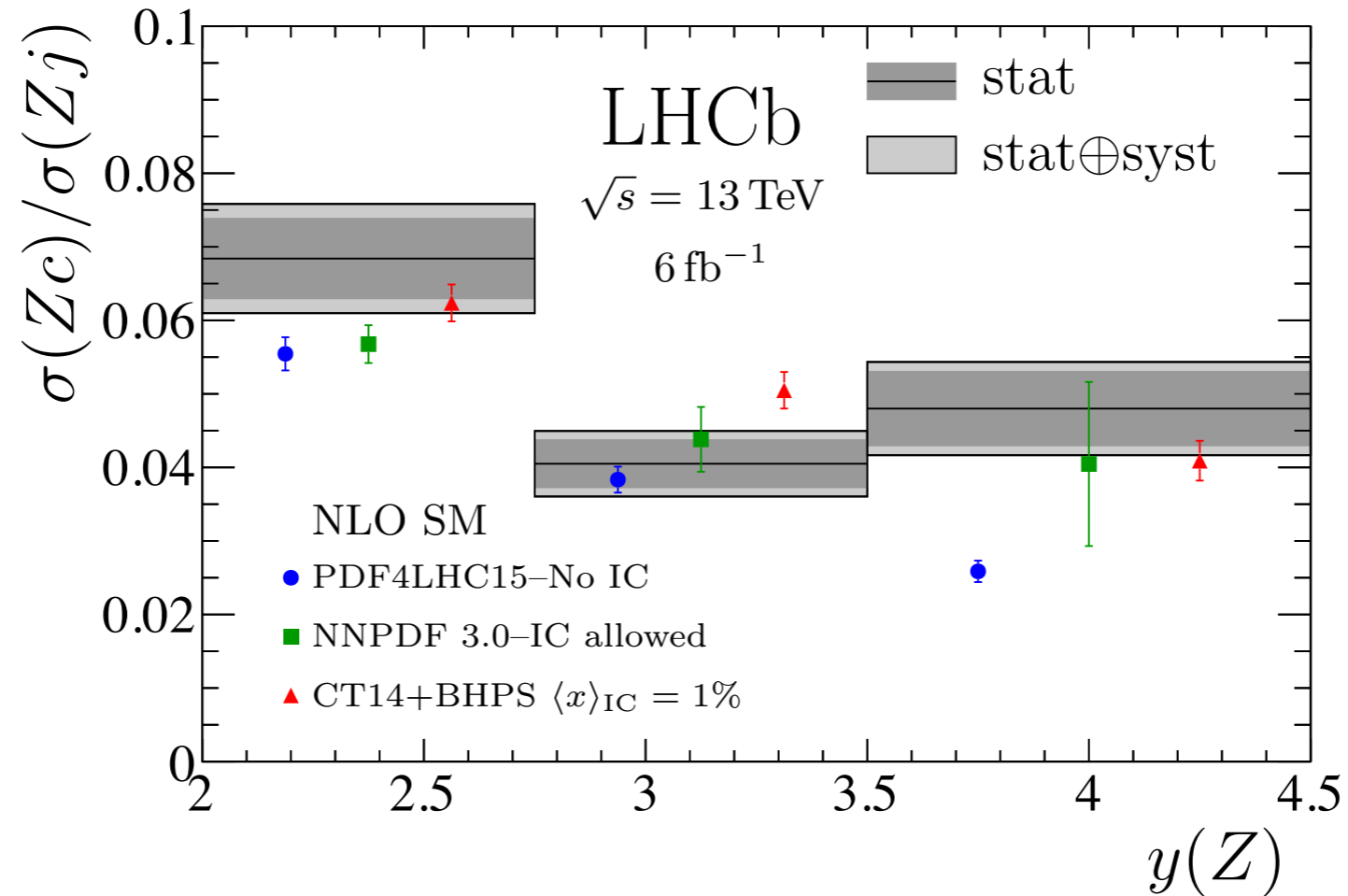
(reminder: I have detailed the “flavour-dressing” approach)

- i) Several theory motivated algorithms for jet flavour, with differences:
  - \* Reproduction of exact anti- $k_T$  kinematics (at parton level)
  - \* Fixed-order IRC safety (between N2LO and N6LO+)
  - \* Applicability at truth-level (parton) or measurement (unstable B/D hadrons)
  
- ii) An all-order proof of IRC safety very difficult
  - \* How many orders are needed? Differential N3LO feasible in future...
  
- iii) Experimental feasibility (or dependence on an unfolding correction)
  - \*) Size of this correction may be strongly algorithm/process dependent

$$d\sigma_{PP \rightarrow f+X}^{\text{data (meas.)}} \quad \text{vs} \quad d\sigma_{PP \rightarrow f+X}^{\text{theory}}$$

# Applications: Z+c-jet at LHCb

LHCb measurement (13 TeV), arXiv: 2109.08084



Forward kinematics:

$$x_{1(2)} \sim \frac{1}{\sqrt{s}} \left( m_T^Z e^{+(-)y_Z} + p_T^j e^{+(-)y_j} \right)$$

unique probe of large(small)  $x$

see Boettcher et al., arXiv: 1512.06666

# Applications: Z+c-jet at LHCb

LHCb measurement (13 TeV), arXiv: 2109.08084

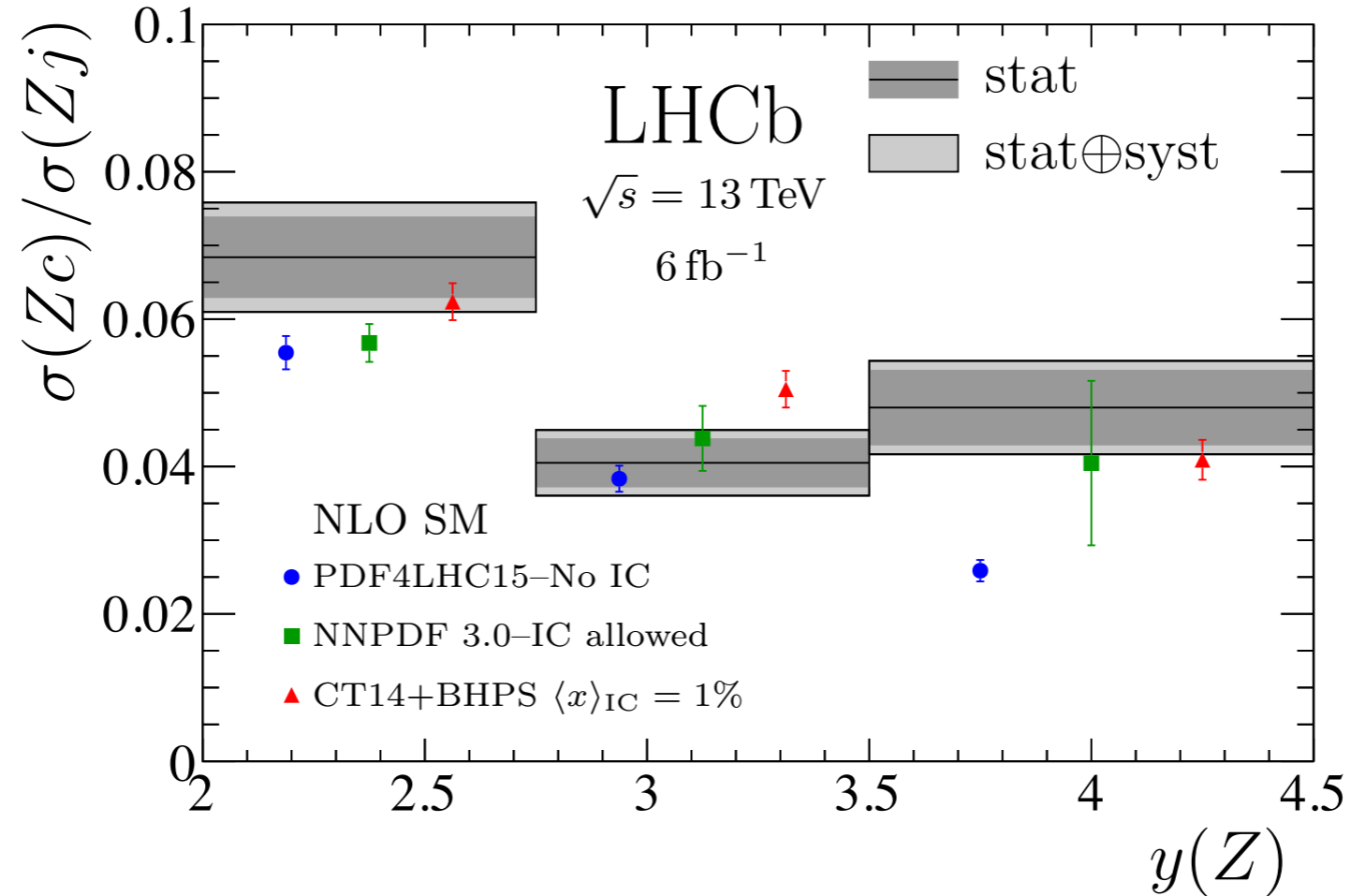


Table 1: Definition of the fiducial region.

Z bosons	$p_T(\mu) > 20 \text{ GeV}, 2.0 < \eta(\mu) < 4.5, 60 < m(\mu^+ \mu^-) < 120 \text{ GeV}$
Jets	$20 < p_T(j) < 100 \text{ GeV}, 2.2 < \eta(j) < 4.2$
Charm jets	$p_T(c \text{ hadron}) > 5 \text{ GeV}, \Delta R(j, c \text{ hadron}) < 0.5$
Events	$\Delta R(\mu, j) > 0.5$

# Applications: $Z+c$ -jet at LHCb

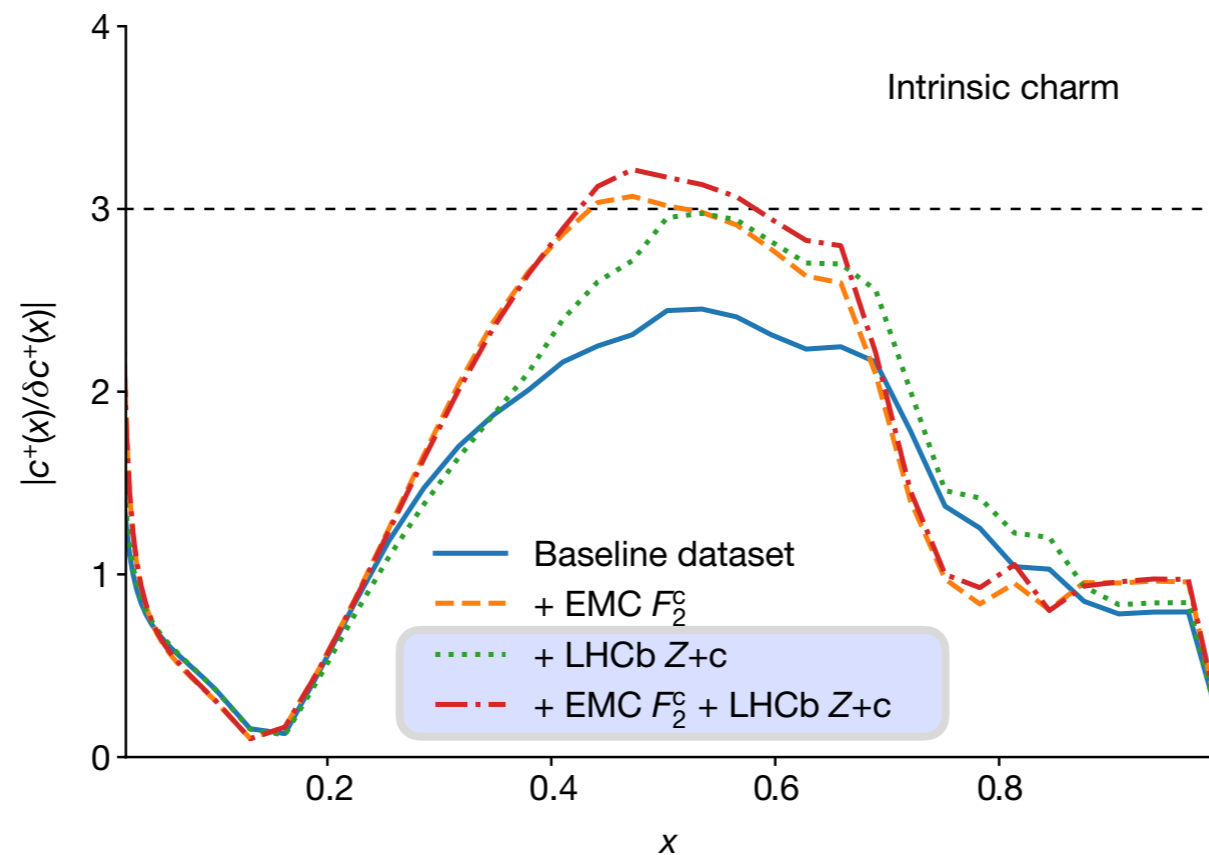
Article | [Open Access](#) | [Published: 17 August 2022](#)

## Evidence for intrinsic charm quarks in the proton

[The NNPDF Collaboration](#)

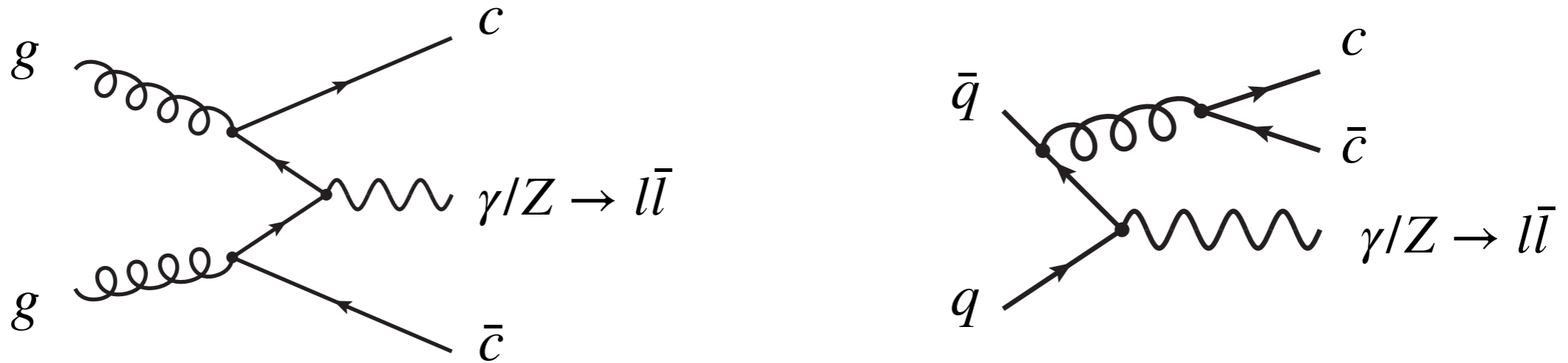
[Nature](#) **608**, 483–487 (2022) | [Cite this article](#)

**42k** Accesses | **7** Citations | **374** Altmetric | [Metrics](#)



(not observed by all pdf fitting groups, Guzzi et al. arXiv:2211.01387)

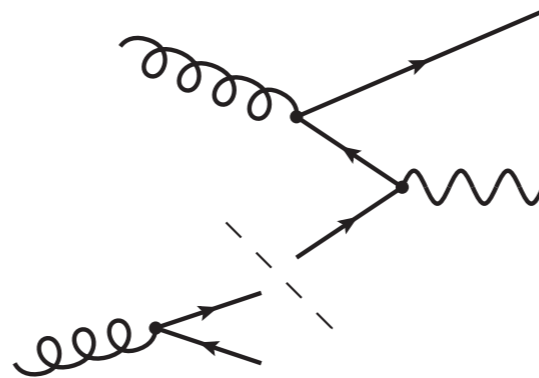
# Applications: Z+c-jet at LHCb



Calculated in the 3fs scheme (i.e.  $n_f^{\max} = 3$  in PDFs, and  $\alpha_s$  evolution)

$$d\sigma^{3fs} = d\sigma^{m_c=0} + d\sigma^{\ln[m_c]} + d\sigma^{m_c}$$

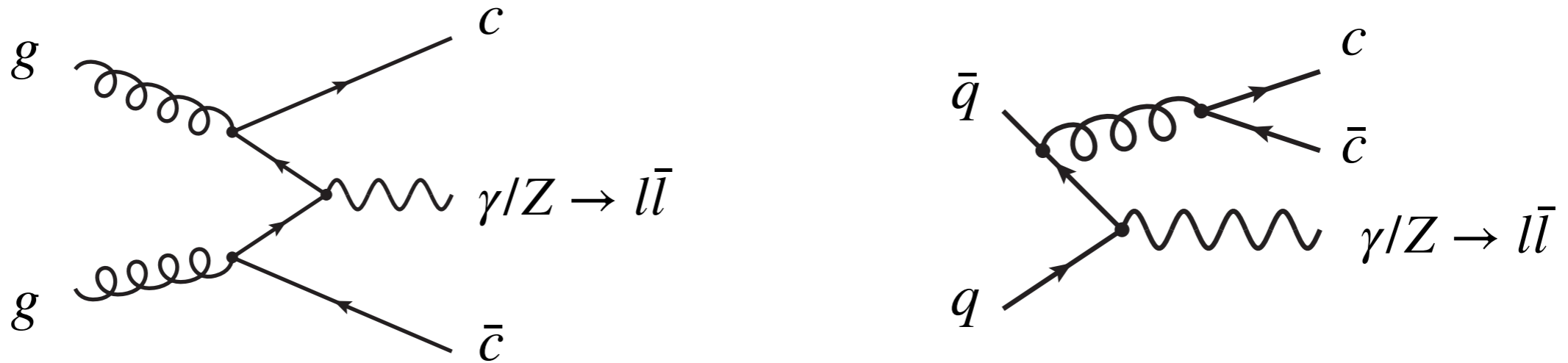
Massless component  
 $\mathcal{O}(\alpha_s^2 n_f)$  in 4fs



$\mathcal{O}(m_c^2)$  effects  
 (exact kinematics)

Note, initial-state mass singularities still there (even with IRC safe jet alg.)

# Applications: Z+c-jet at LHCb



Calculated in the 3fs scheme (i.e.  $n_f^{\max} = 3$  in PDFs, and  $\alpha_s$  evolution)

$$d\sigma^{3fs} = d\sigma^{m_c=0} + d\sigma^{\ln[m_c]} + d\sigma^{m_c}$$

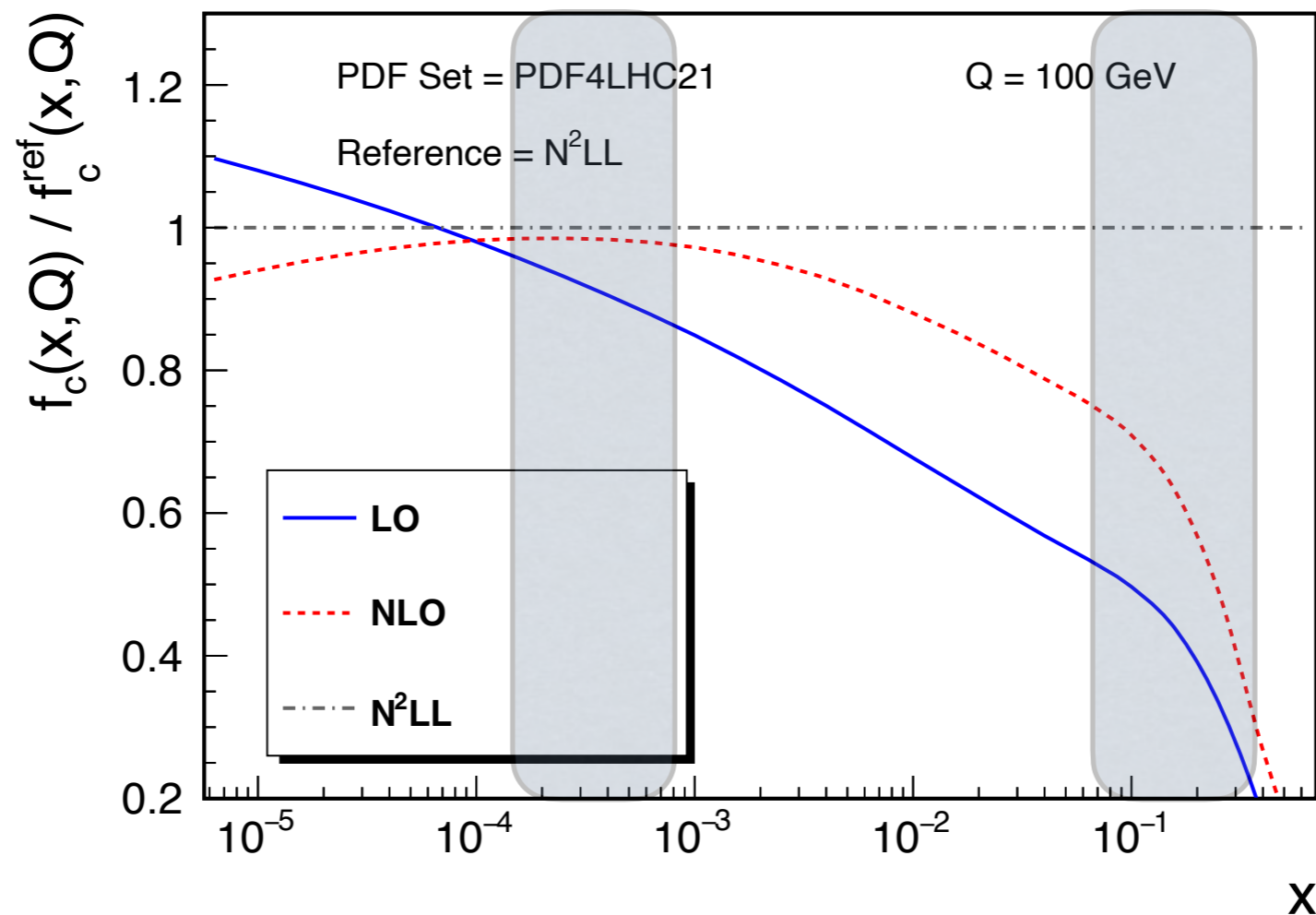
$$\mathbf{0.220 = +0.0364 \quad +0.203 \quad -0.019 \text{ [pb]}}$$

$$\mathbf{100\% = +16\% \quad +92\% \quad -8\%}$$

Note, initial-state mass singularities still there (even with IRC safe jet alg.)

# Applications: Z+c-jet at LHCb

The perturbative corrections are enormous: resummation critical  
(this class of logarithm resummed by PDF evolution)



$$\langle x_1 \rangle \sim 0.2$$

$$\langle x_2 \rangle \sim 6 \times 10^{-4}$$

LHCb cross-section: Leading Log (1st order) = 0.203pb, Leading Log (resummed) = 0.332pb

I am showing fixed-order pdf versus a resummed one (PDF evolution)

$$\alpha_s^m \ln^n[\mu_F^2/m_c^2], \quad m \geq n$$

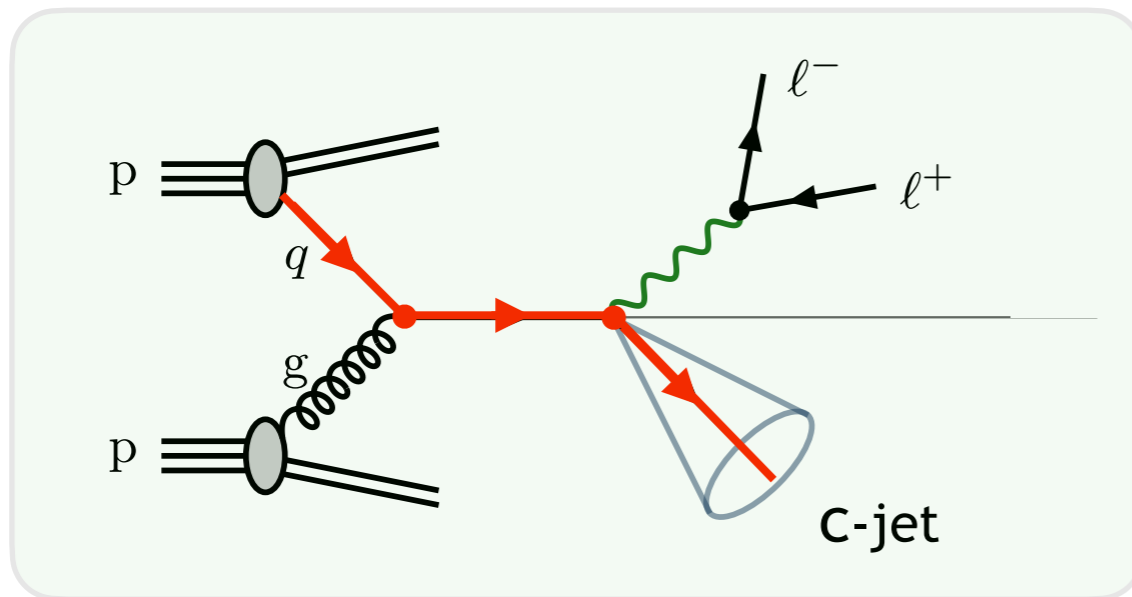
$$\text{Note! } \alpha_s \ln[m_Z^2/m_c^2] \approx 1.0$$



# Applications: Z+c-jet at LHCb

**RG**, Gehrman-De Ridder, Glover, Huss, Rodriguez Garcia, Stagnitto, arXiv:2302.12844

- ▶ Theory study based on SPS predictions (no MPI corrections)
- ▶ Consider a fiducial region matching that of the LHCb experiment



Introduce the constraint

$$p_T(Zj_c) \leq p_T(j_c)$$

Predictions are provided in a Massive - Variable Flavour Number Scheme

**RG**, Gehrman-De Ridder, Glover, Huss, Maier, arXiv:2005.03016, **RG**, arXiv:2107.01226

$$d\sigma^{M-VFNS} = d\sigma^{ZM-VFNS} + d\sigma^{pc}$$

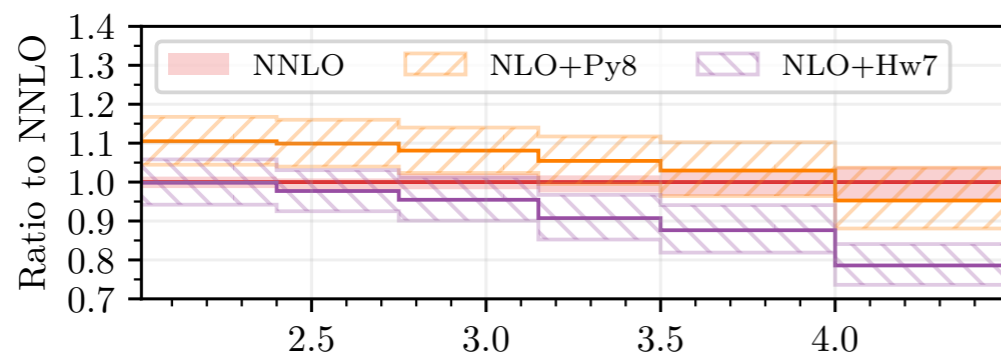
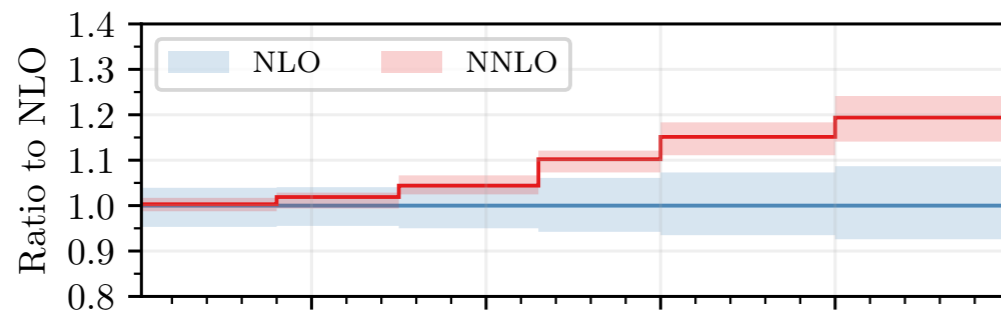
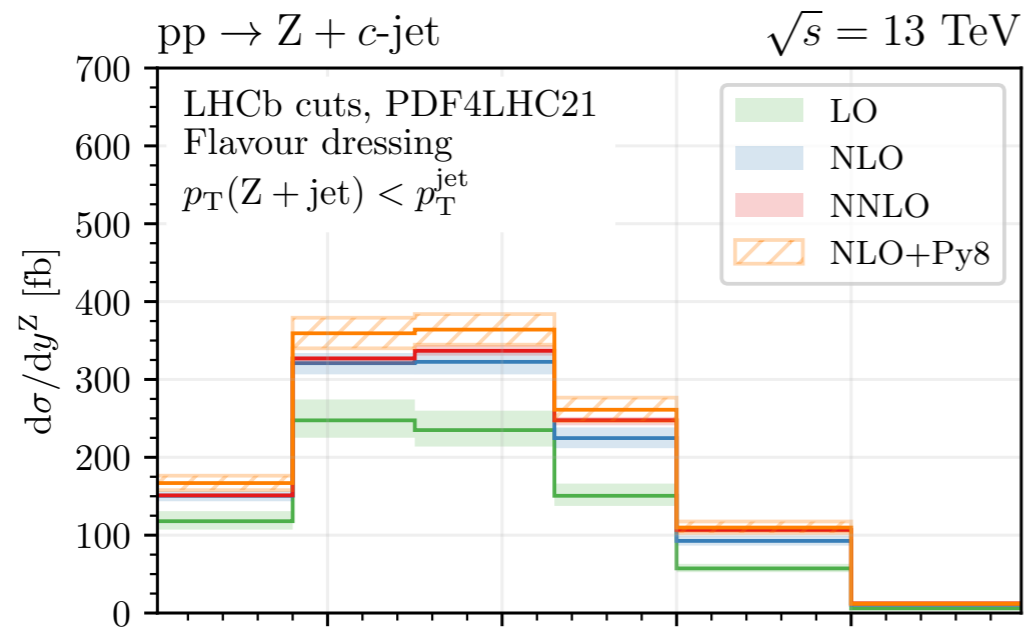
NNLO QCD predictions via the Z+jet antenna subtraction calculation

Gehrman-De Ridder, Gehrman, Glover, Huss, Morgan, arXiv:1507.02850

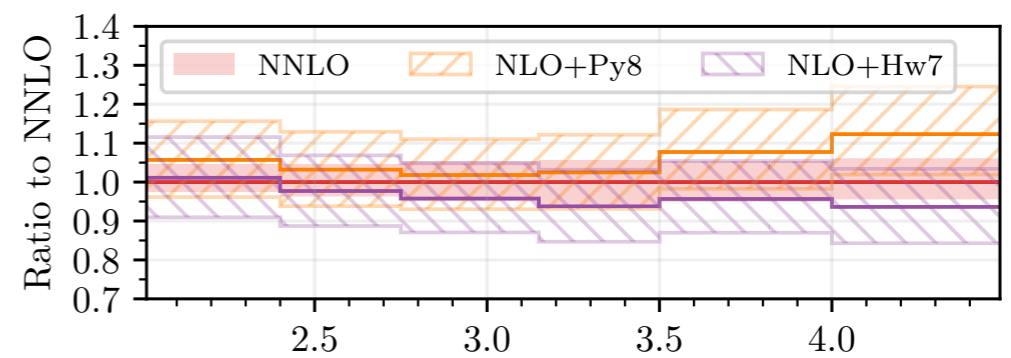
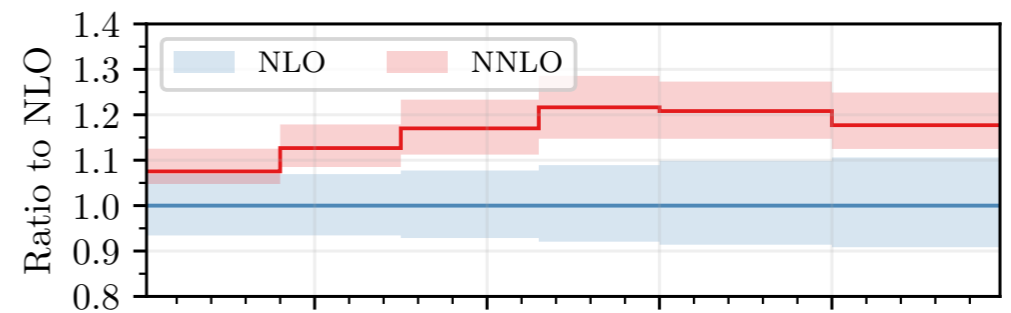
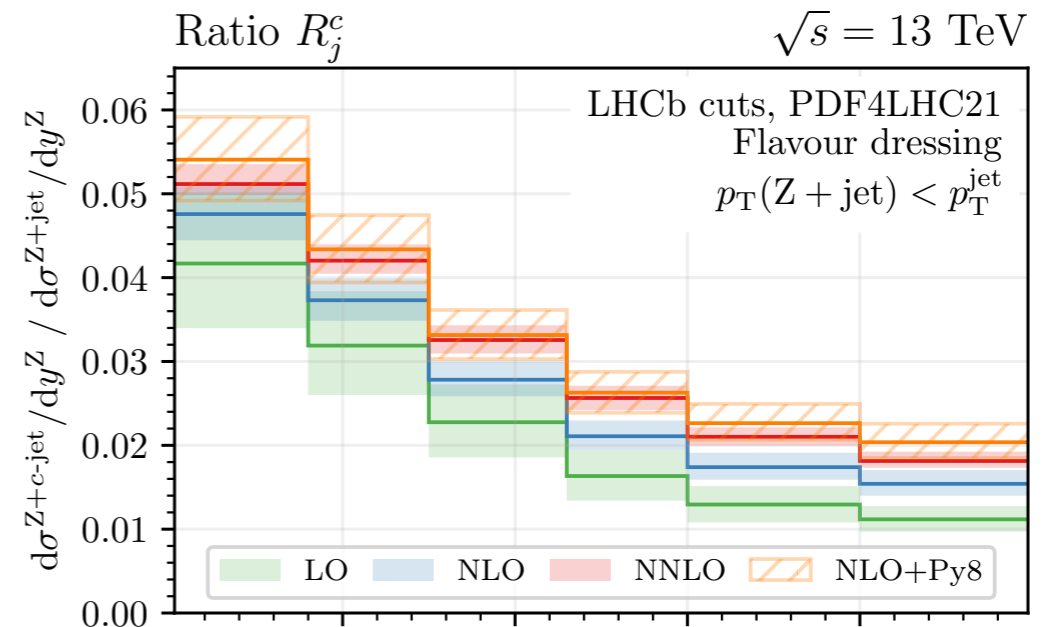
$\alpha_{G_\mu}$  scheme, 7-point scale variation around  $E_{T,Z}$ , and the PDF4LHC21 set

arXiv:2203.05506

# Applications: Z+c-jet at LHCb



$$\frac{d\sigma^{Z+c\text{-jet}}}{dy_Z}$$



$$\frac{d\sigma^{Z+c\text{-jet}}}{dy_Z} \bigg/ \frac{d\sigma^{\text{jet}}}{dy_Z}$$

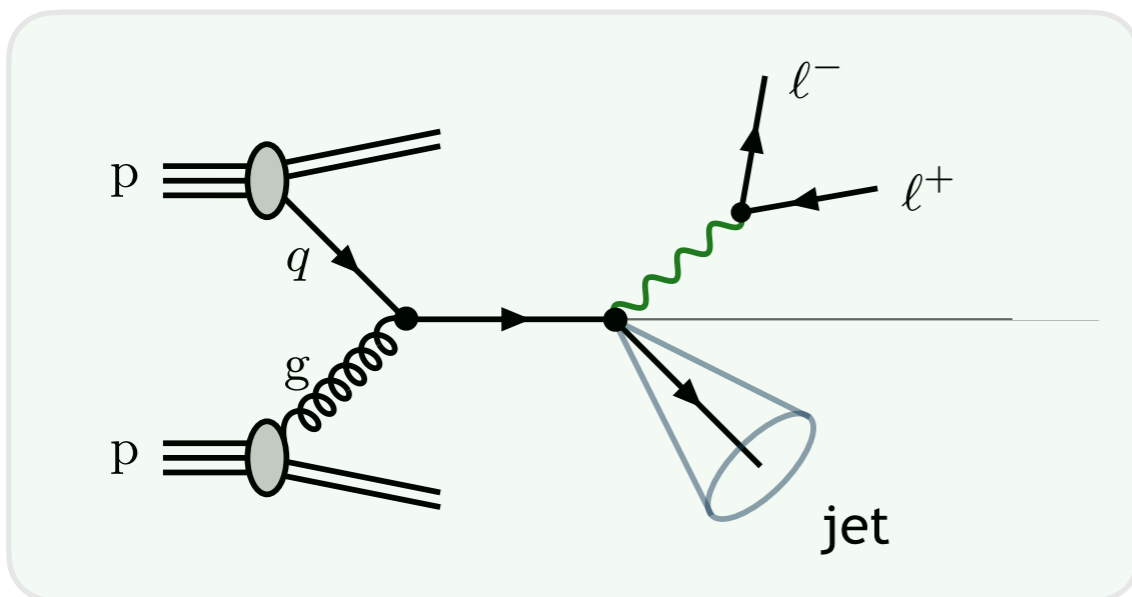
NNLO QCD corrections positive and grow with  $y_Z$

# Applications: Z+c-jet at LHCb ... MPI

Possibility for multiple hard interactions in a single pp-collision

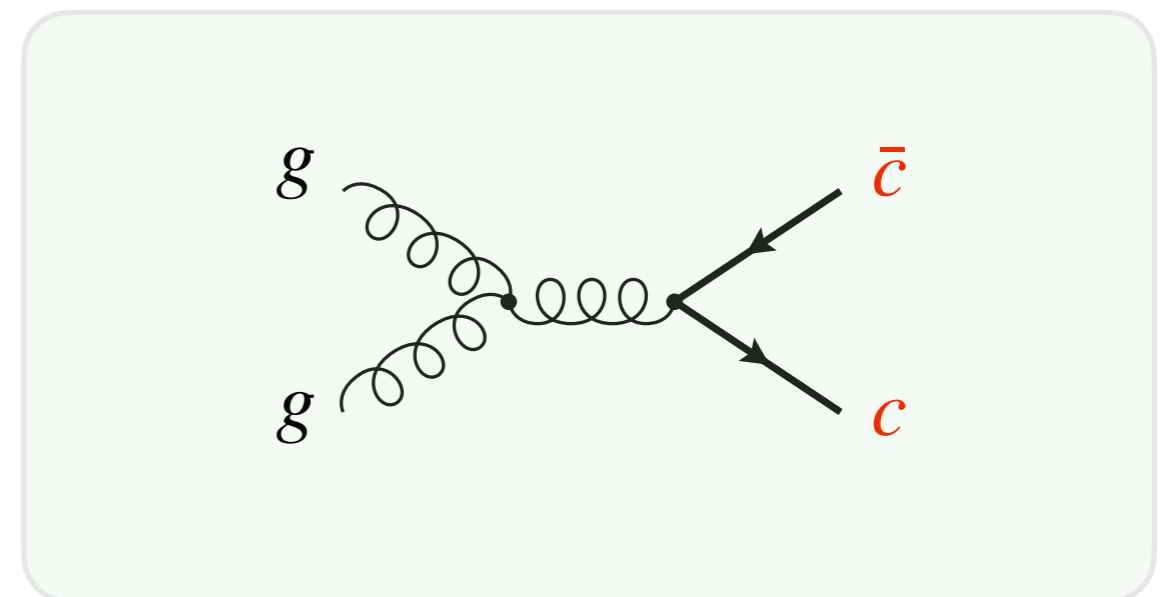
e.g. single-parton-scattering (SPS), double-parton-scattering (DPS), ...

Hard Process 1 (HP1) = Z+jet



The jet is flavour inclusive

Hard Process 2 (HP2) =  $c\bar{c}$



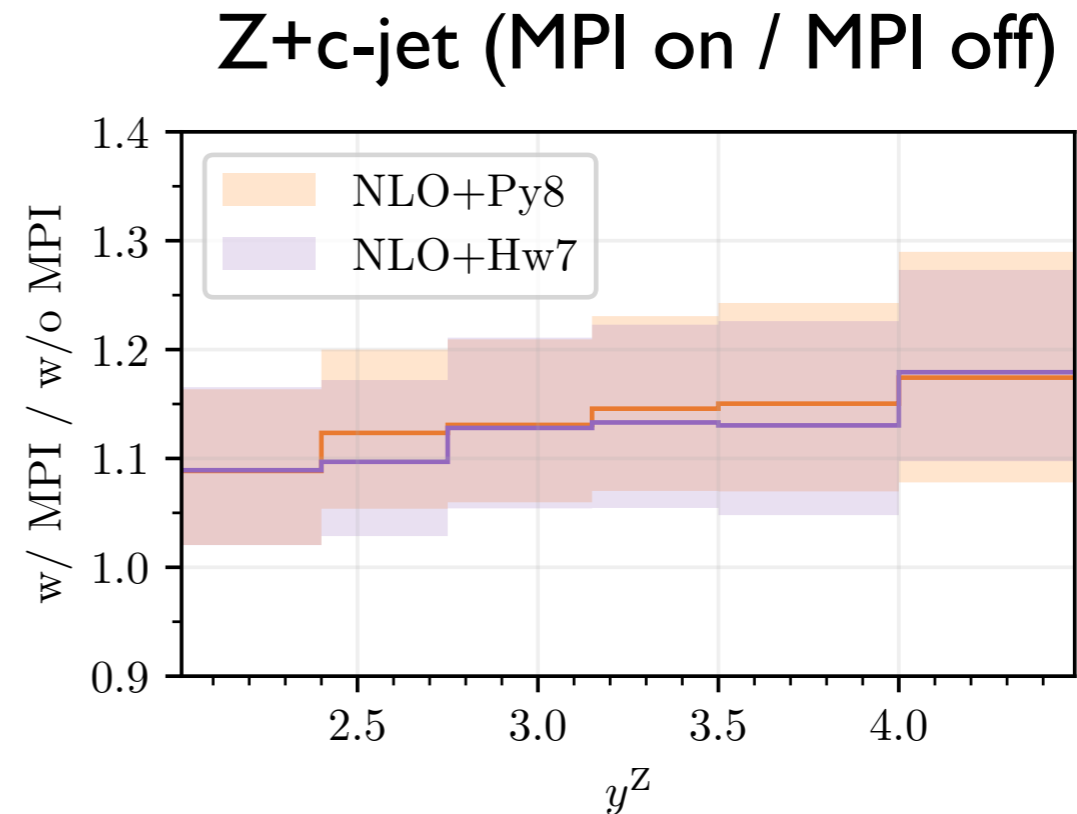
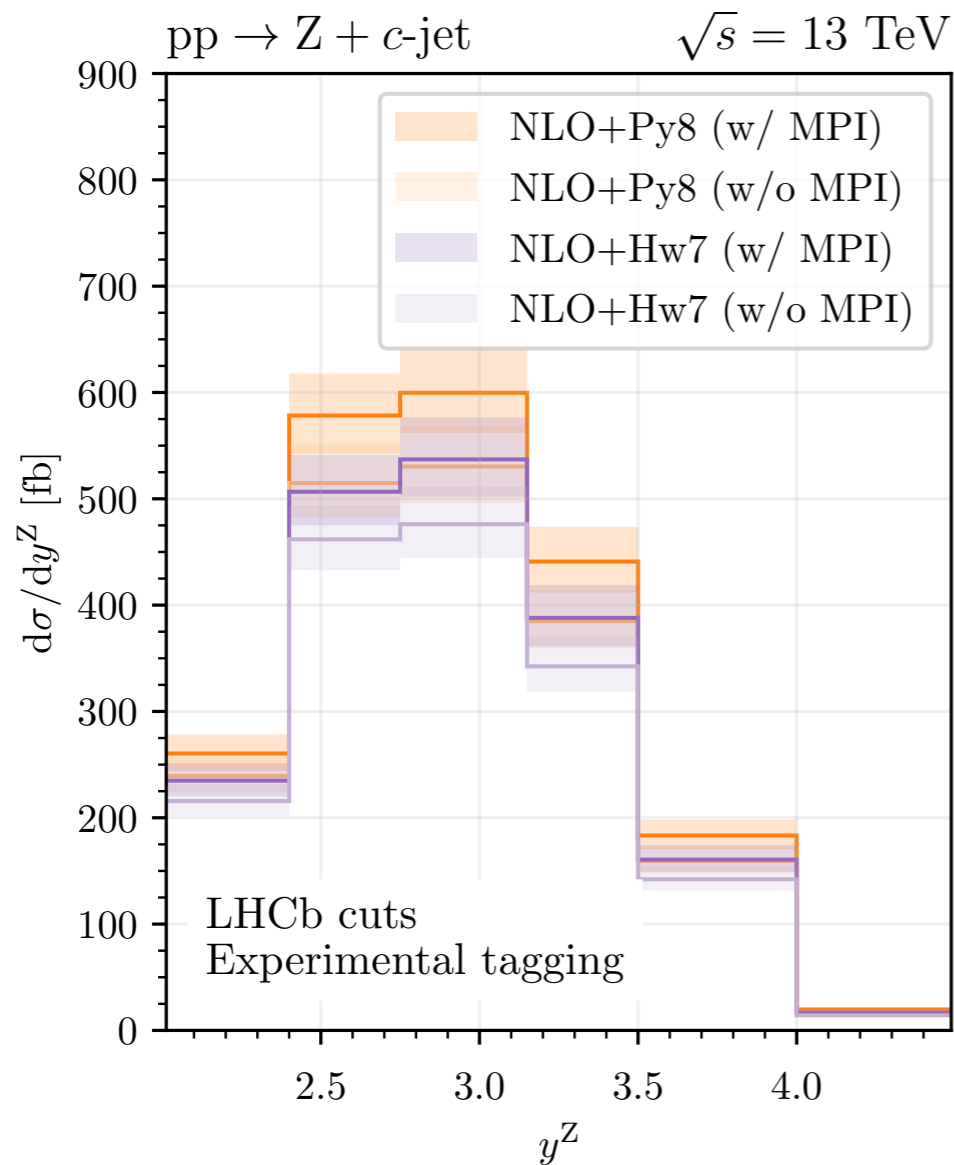
Large cross-section at LHCb

Probability that  $\Delta R(j_{HP1}, c_{HP2}) \leq 0.5$  leading to a charm tagged jet  
(small phase-space compensated by large  $c\bar{c}$  cross-section)

# Applications: Z+c-jet at LHCb ... MPI

Possibility for multiple hard interactions in a single pp-collision

e.g. single-parton-scattering (SPS), double-parton-scattering (DPS), ...



MPI correction required when the considered observable is sensitive to the combination of H1 and H2 (a genuine physics effect not described by SPS)

# Summary Part (2)

i) Precise theory predictions with anti- $k_T$  charm jets available

\* NNLO QCD accurate, additionally includes finite charm mass corrections

ii) RE: Use of LHC data in collinear PDF fits

\* The data should be IRC safe (necessarily to the considered fixed-order)

\* The role of the discussed MPI effects in data must be accounted for

iii) MPI effects in  $Z+c$ -jet need careful attention...

\* Note that the process  $pp \rightarrow c\bar{c}$  has theory uncertainties in excess of 50%

\* Pythia8 and Herwig MPI models this at LO with a single scale choice

\* Theoretically subtracting this component will introduce substantial uncertainty

Further work still required for a careful interpretation of this data...

# Closing remarks

## ... and why the focus on flavoured jets?

- a) Higgs physics (hadronic decays)
- b) Top-quark physics ( $|V_{tb}| \sim 1$ )
- c) New physics searches (f-jet +  $E_T^{\text{miss}}$ )
- d) Gauge-boson + heavy-flavour

...

critical question: how to define  $f$  to best enable this comparison

$$d\sigma_{\text{PP} \rightarrow \mathbf{f} + \mathbf{X}}^{\text{data (meas.)}} \quad \text{vs} \quad d\sigma_{\text{PP} \rightarrow \mathbf{f} + \mathbf{X}}^{\text{theory}}$$

(my opinion) still a lot of exciting work ahead

# Whiteboard

# Experimental feasibility of flavoured jet algorithms

Input particles to the jet algorithm are typically particle flow objects (i.e. not unstable B/D-hadron candidates such as secondary vertices)

Heavy-flavour candidates found by reconstructing secondary vertices (with a probability of being a light, charm or beauty object)

Most of these algorithms require the jets be built knowing the flavour of input particles (which for heavy flavours is not known at particle flow level)

Could experimentally consider a new set of inputs:  $\{PF\} + \{SV\} - \{\text{overlap}\}$

That would result in modified kinematics to the current anti- $k_T$  jets (that step not necessary for flavour-dressing approach)

Additionally, systematic treatment of flavour probabilities required (different flavour paths lead to different jet kinematics for flavour-[anti-] $k_T$ )

Or just rely on an unfolding to “truth level” (where B/D-hadrons stable)



# Tests of the algorithm ( $e^+e^-$ )

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Consider the process  $e^+e^- \rightarrow 2$  jets at fixed-order using  $k_T$  algorithm

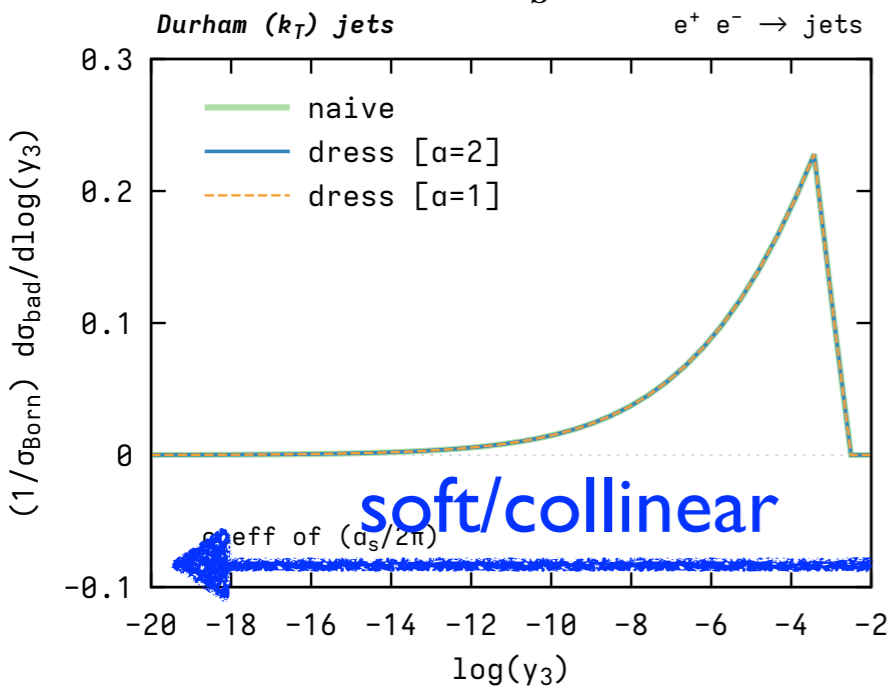
Look at ‘bad’ events (i.e. where we do not find 2 flavoured jets,  $e^+e^- \rightarrow q\bar{q}$ )

The ‘bad’ cross-section should vanish in the  $y_3 \rightarrow 0$  limit

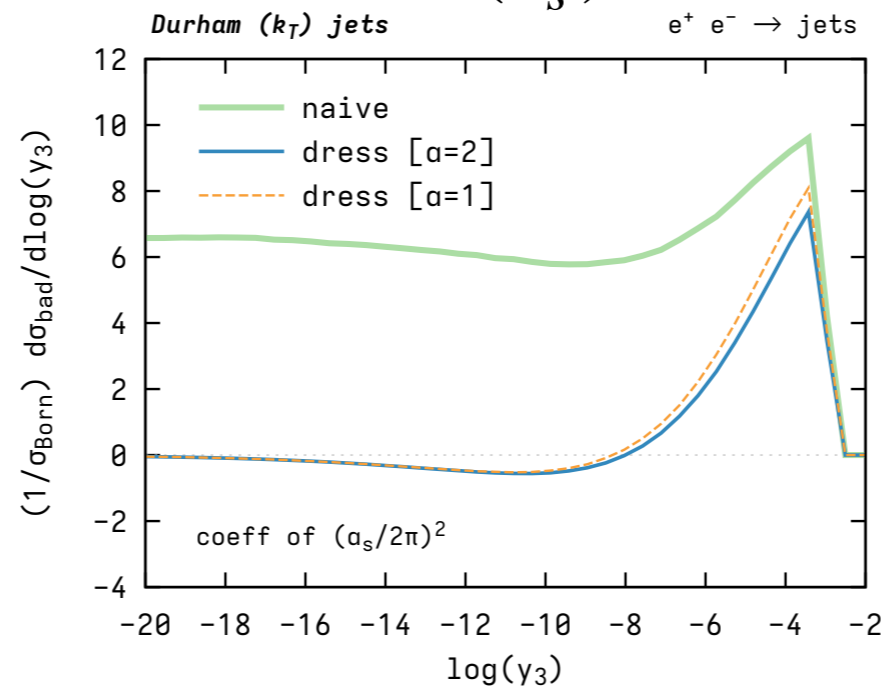
( $y_3$  defines the distance measure at which the event goes from 2 jet  $\rightarrow$  3 jet)

( $y_3 \rightarrow 0$  corresponds to limit of extremely soft and/or collinear emissions)

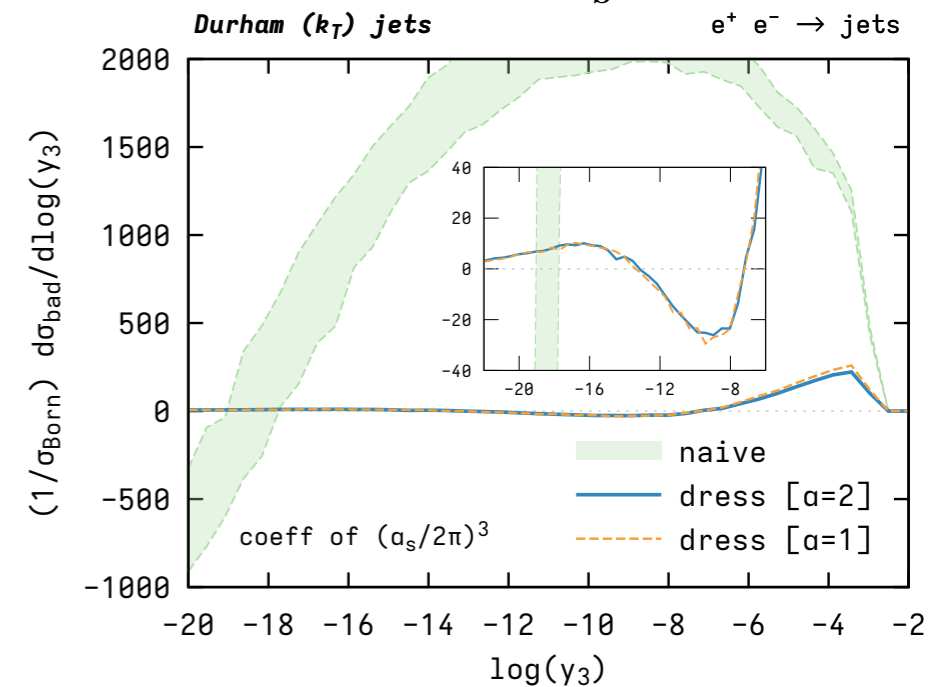
$\mathcal{O}(\alpha_s)$



$\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_s^3)$



These tests originally proposed/shown in the original flavour- $k_T$  study

(Banfi, Salam, Zanderighi hep-ph/0601139)

# Tests of the algorithm (pp)

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Can also perform all-order ‘sensitivity’ tests using Parton Shower framework

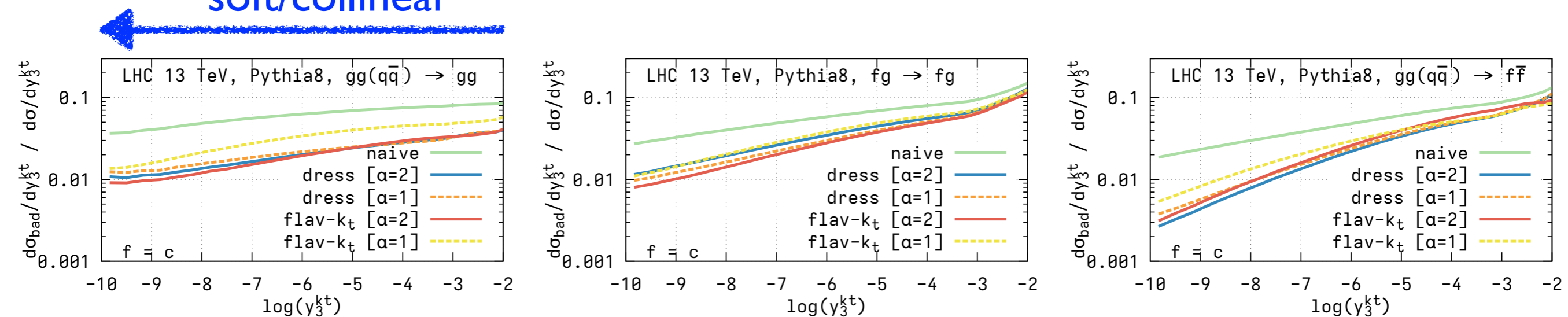
In this case study, also use resolution variable to probe IRC sensitive regions (here we study the behaviour, rather than the bad cross-section vanishing)

Here consider dijet events (exclusive  $k_T$  algorithm) with  $E_T \geq 1$  TeV

We use the resolution variable:  $y_3^{k_T} = d_3^{k_T} / (E_{T,1} + E_{T,2})$

(Buonocore et al. arXiv:2201.11519)

← soft/collinear



These tests originally proposed/shown in the original flavour- $k_T$  study

(Banfi, Salam, Zanderighi hep-ph/0601139)

# Application of the algorithm (pp)

(**RG**, Huss, Stagnitto arXiv:2208.11138)

Now consider the process  $pp \rightarrow Z + b - \text{jet}$  in Fiducial region (13 TeV, CMS-like)

(N)NLO at fixed-order w/ NNLOJET, **RG** et al. arXiv:2005.03016

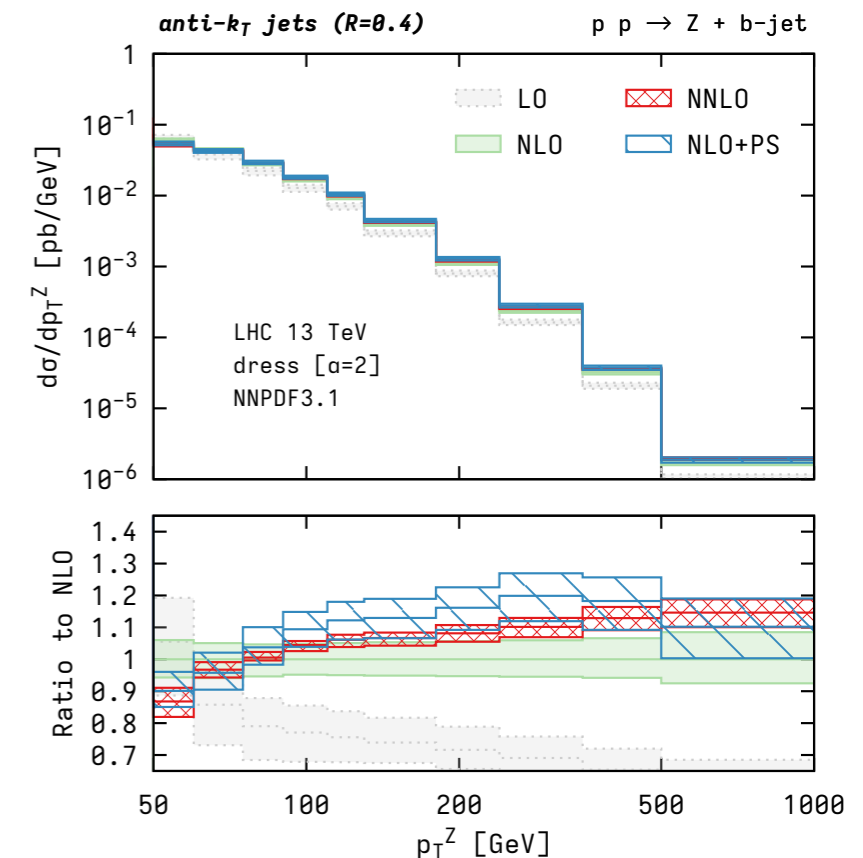
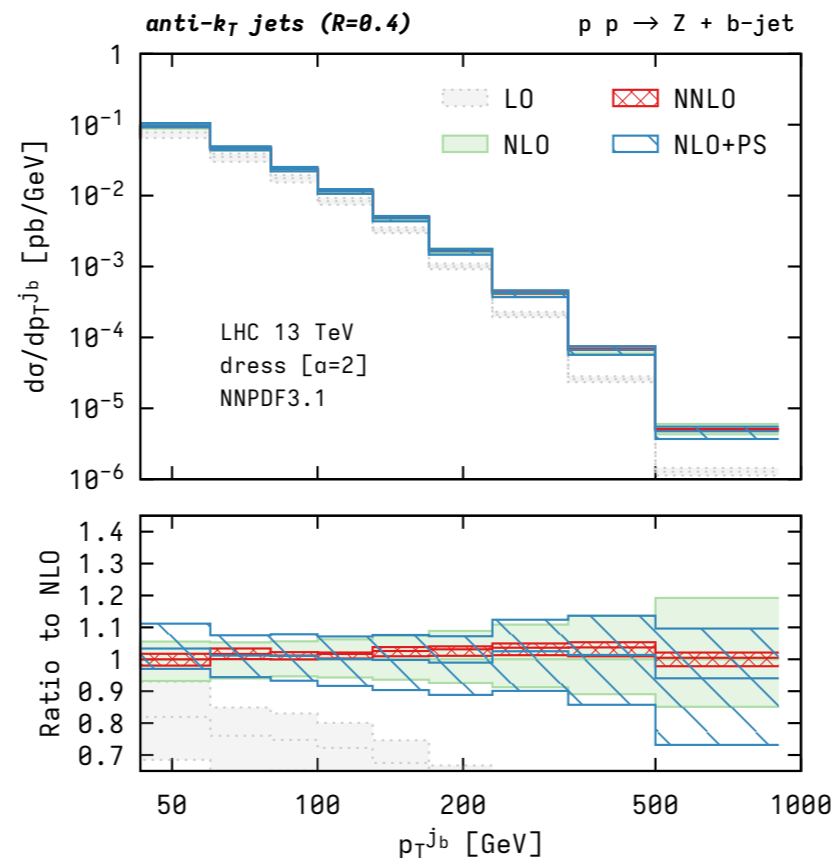
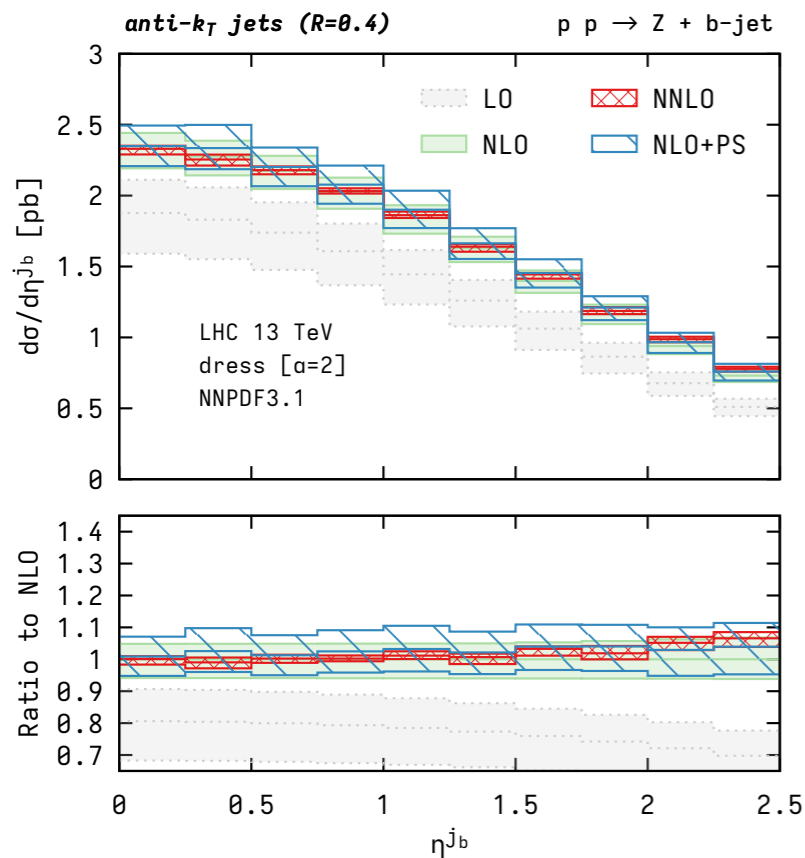
NLO+PS Hadron-level with aMC@NLO interfaced to Pythia8

Tests sensitivity to: all-order effects, hadronisation (also FO IRC safety in pp)

$\eta_{b\text{-jet}}$

$p_{T,b\text{-jet}}$

$p_{T,Z}$

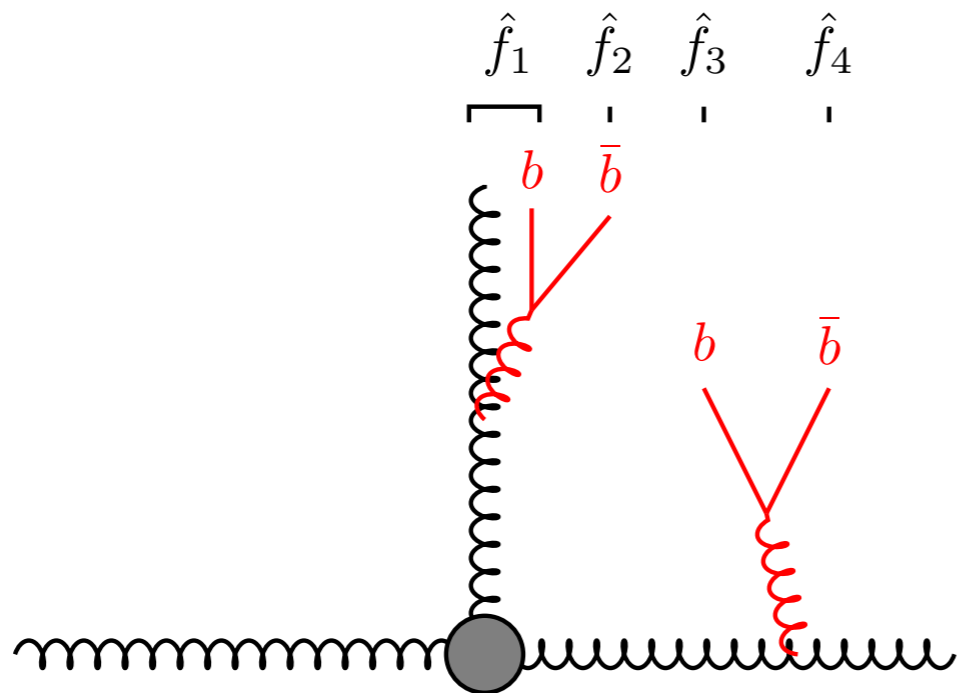


# Higher-order configurations / pitfalls

(Ludovic Scyboz, Moriond QCD)

## Examples of pitfalls / III

11 / 11



Hard event:

→ 1 flavourless jet

Hard+IR event:

1( $b$ ) accumulated into hard  $g$ ,  
but not 2( $\bar{b}$ )

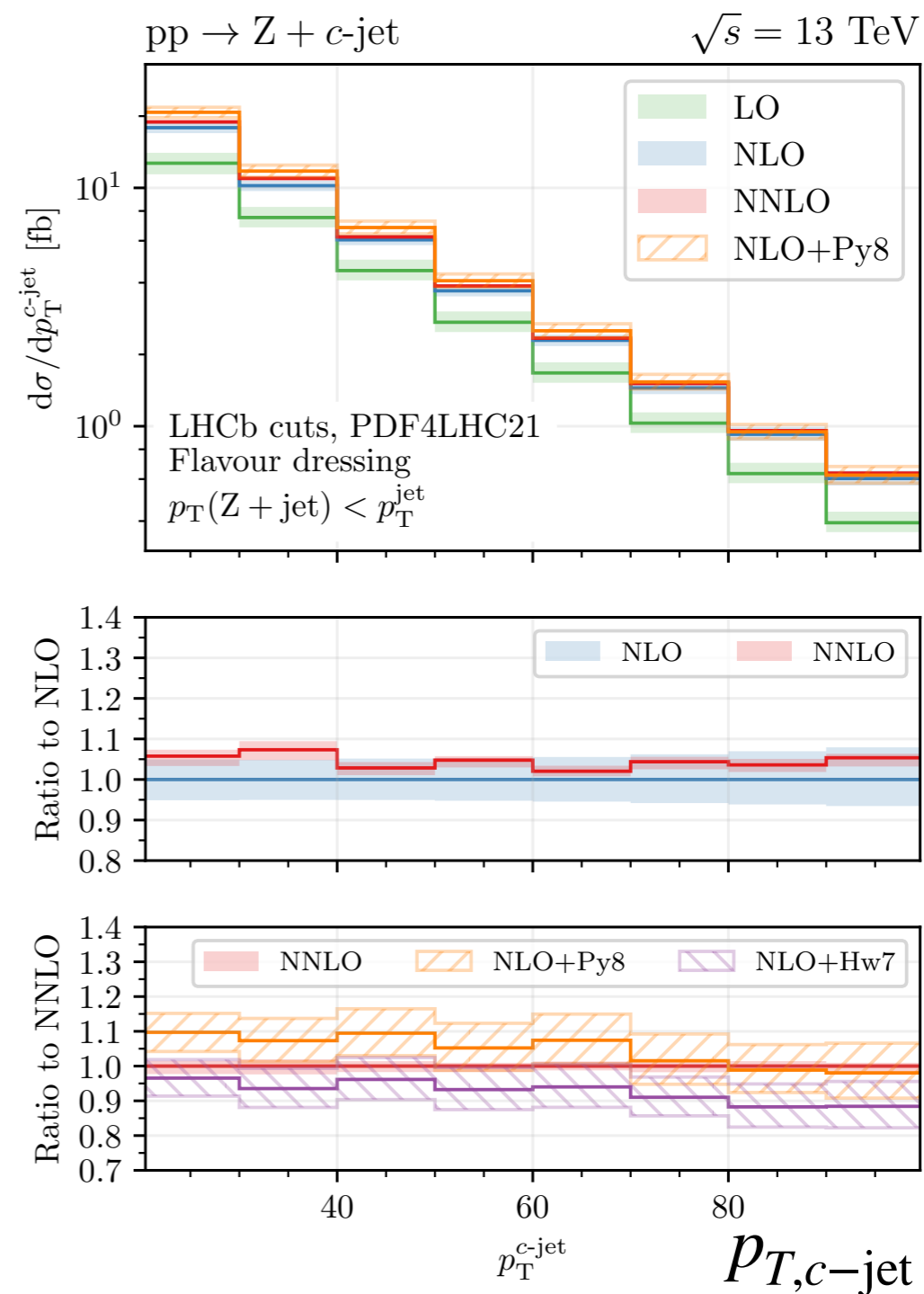
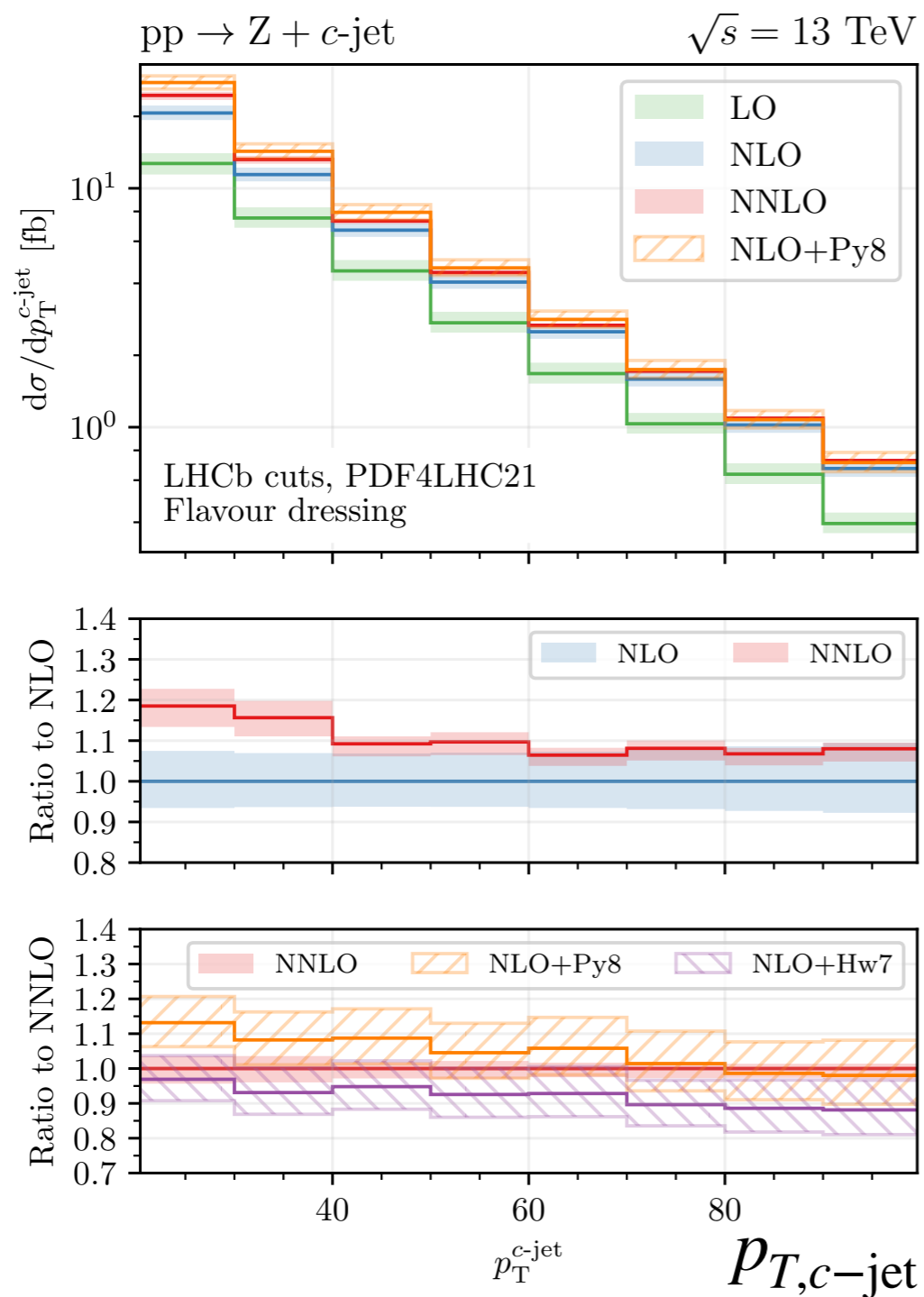
$\hat{f}_2$  and  $\hat{f}_3$  annihilate,  
but  $\hat{f}_1$  and  $\hat{f}_4$  do not

→ 1  $b$ -jet (+ 1  $\bar{b}$  beam jet)

- Some analytic/numerical understanding of the complicated interplay between distances (as a function of  $\alpha$  and  $\beta$ )  
→ suggests  $\alpha \cdot \beta < 2$  is fine for this configuration



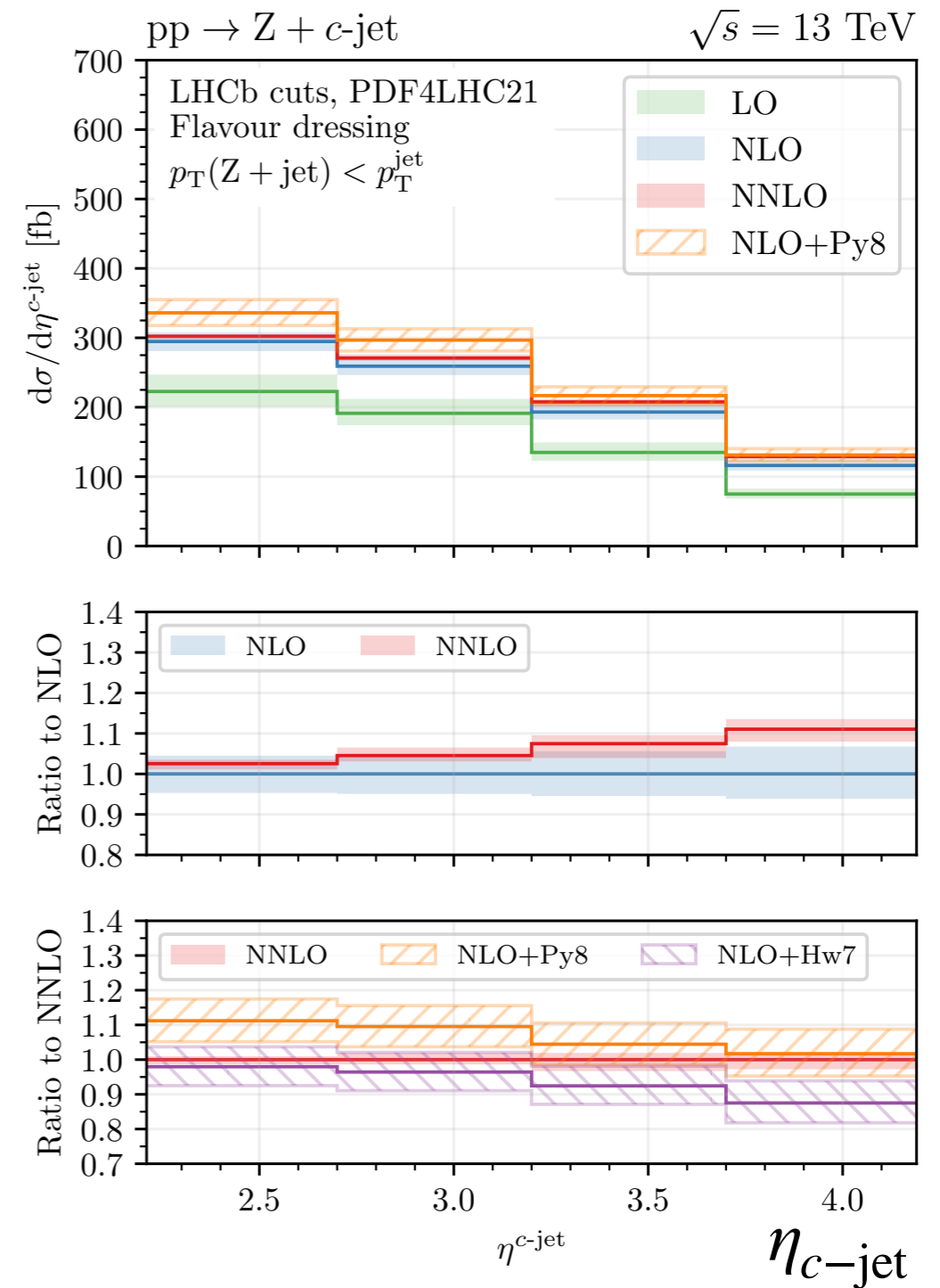
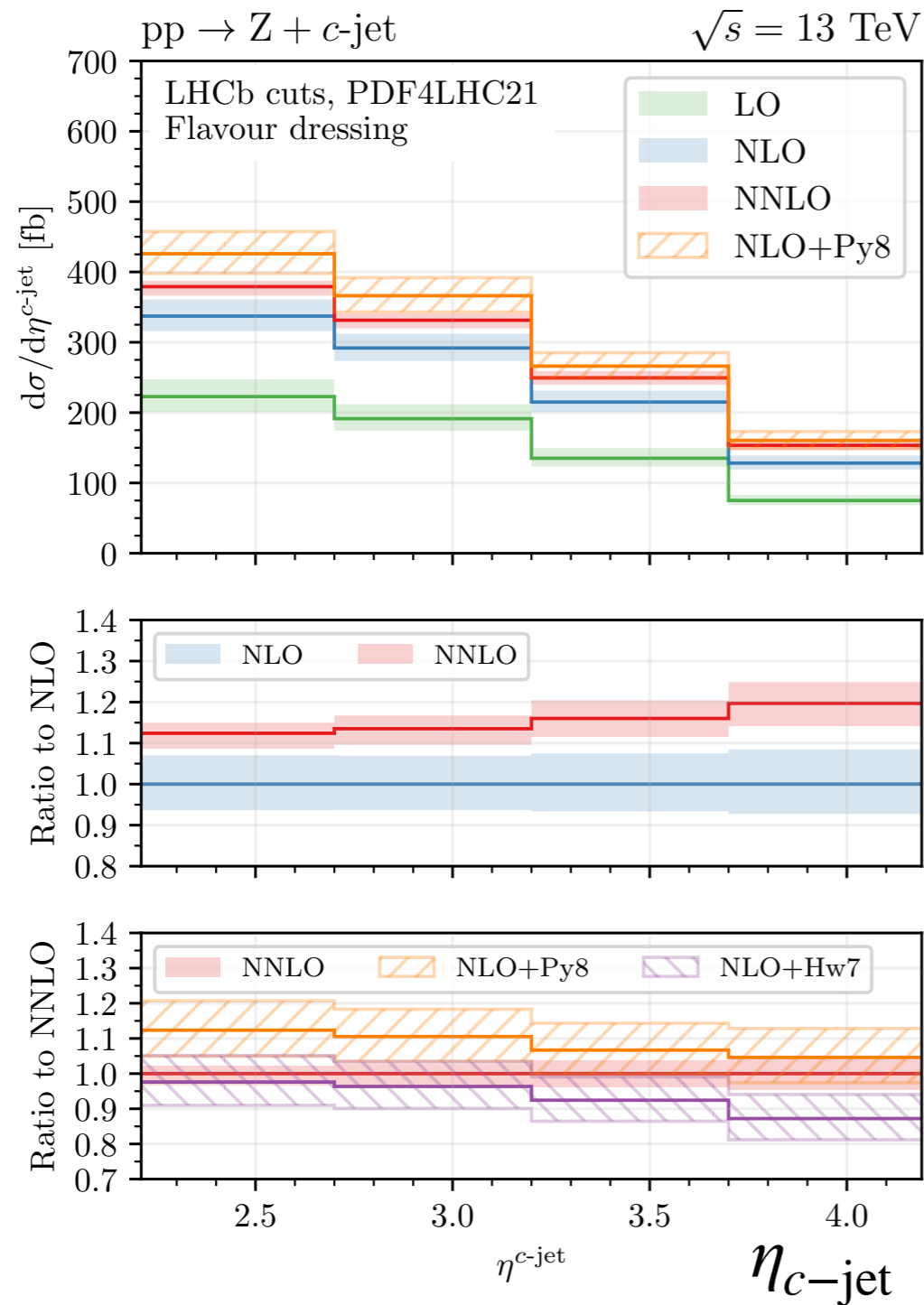
# NNLO QCD predictions for Z+c-jet production: $p_{T,c\text{-jet}}$



RHS includes:  $p_T(Zj_c) \leq p_T(j_c)$

NNLO QCD: reduced uncertainties, and consistent with NLO+PS

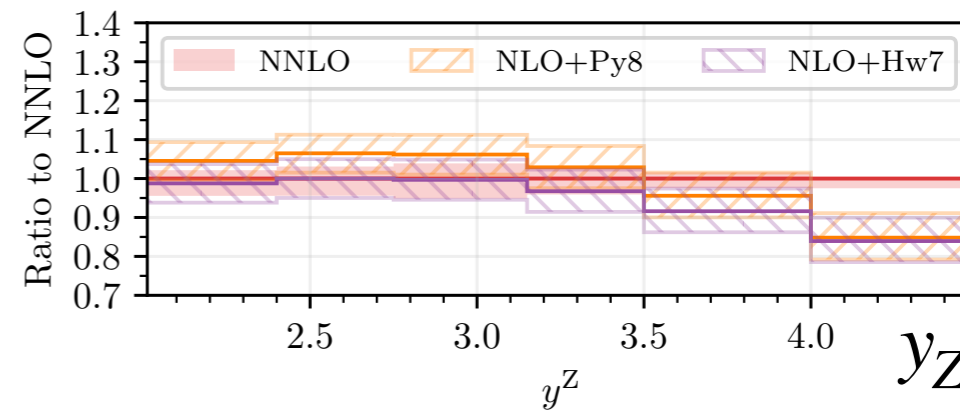
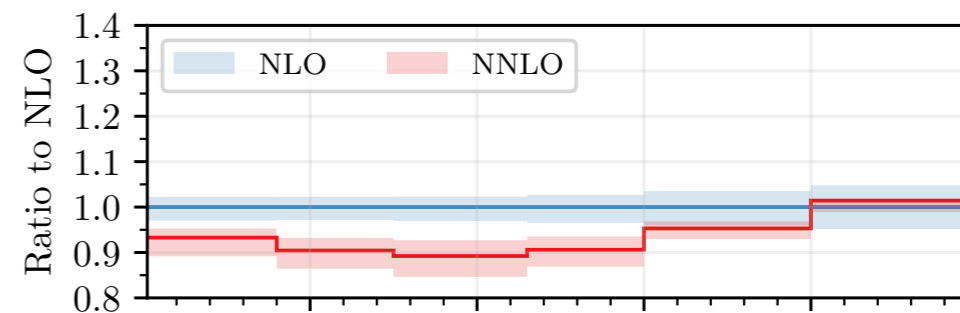
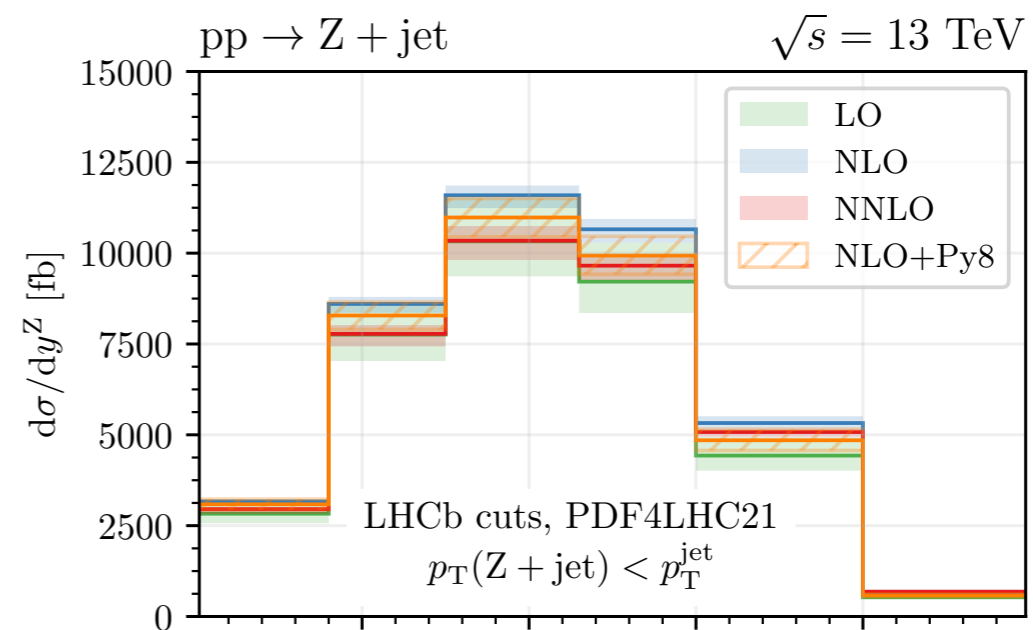
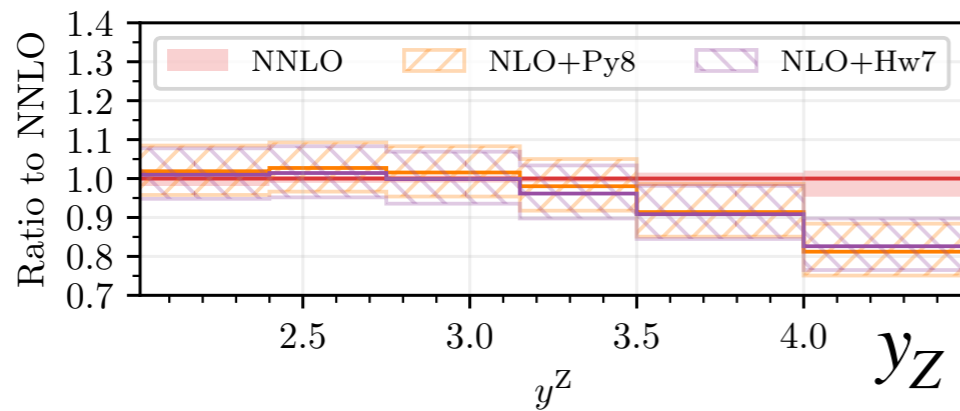
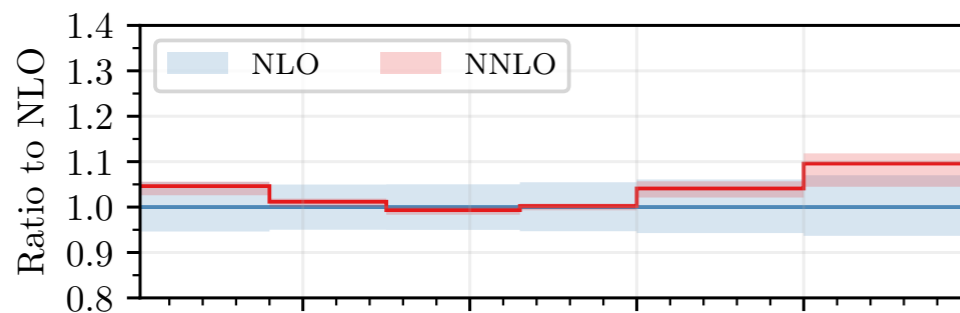
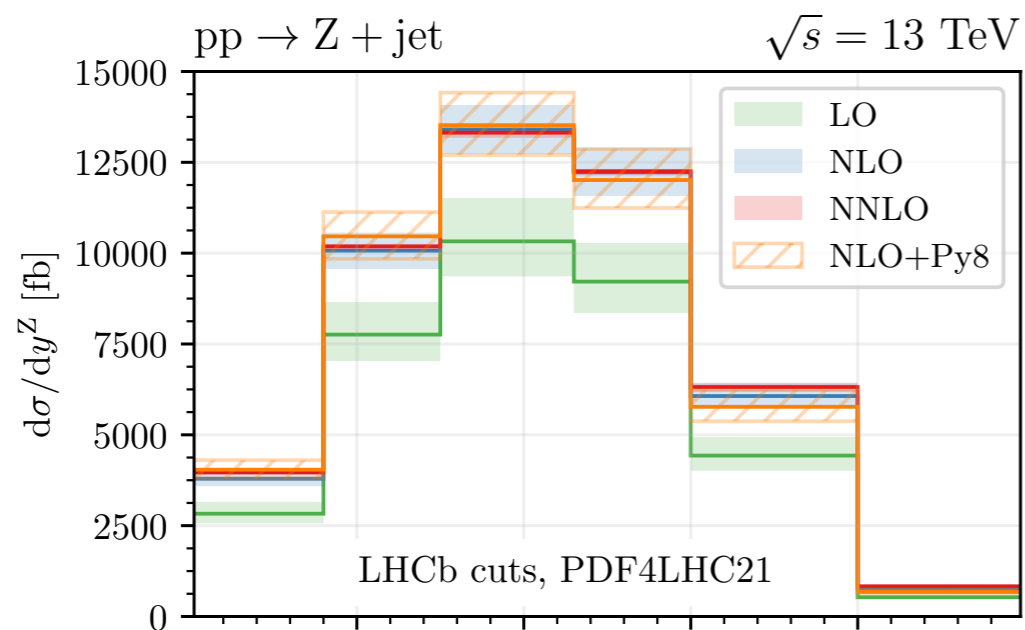
# NNLO QCD predictions for $\eta_{c\text{-jet}}$



RHS includes:  $p_T(Zj_c) \leq p_T(j_c)$

...

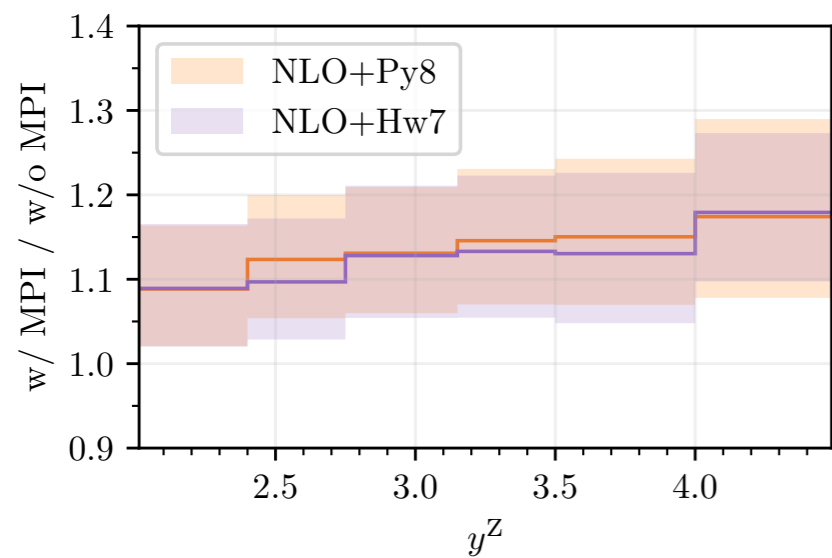
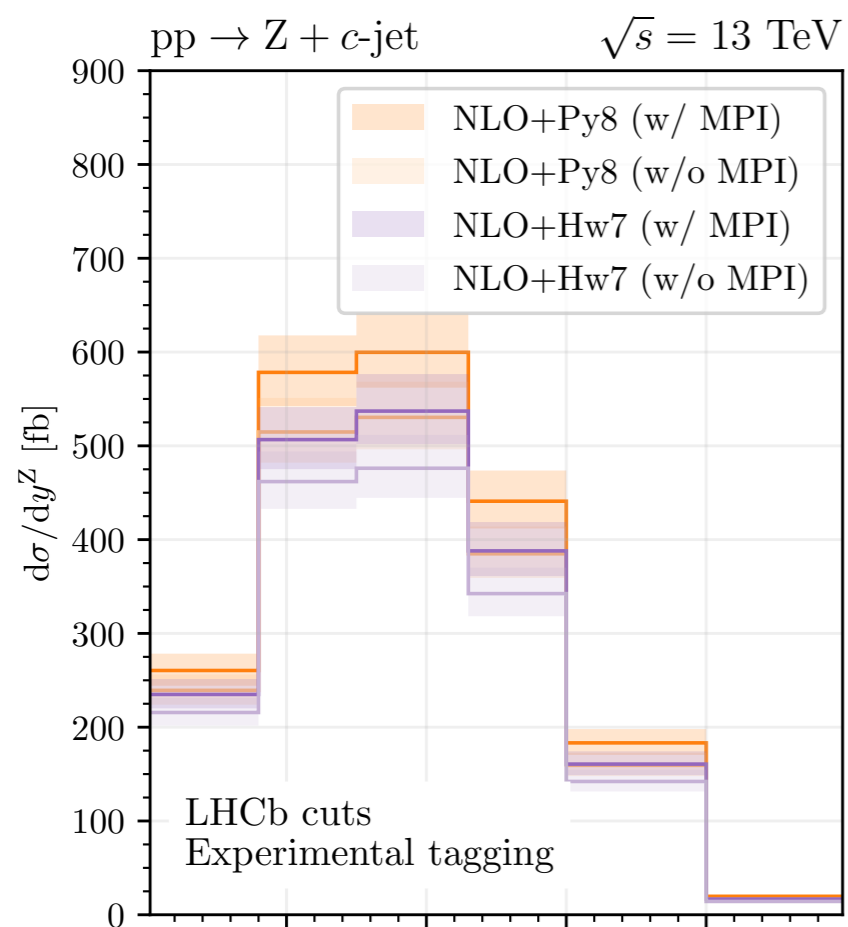
# NNLO QCD predictions for flavour inclusive process



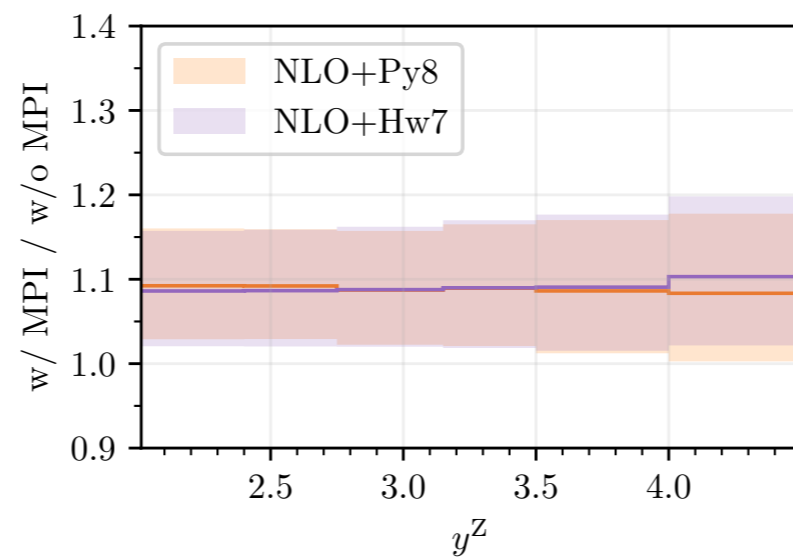
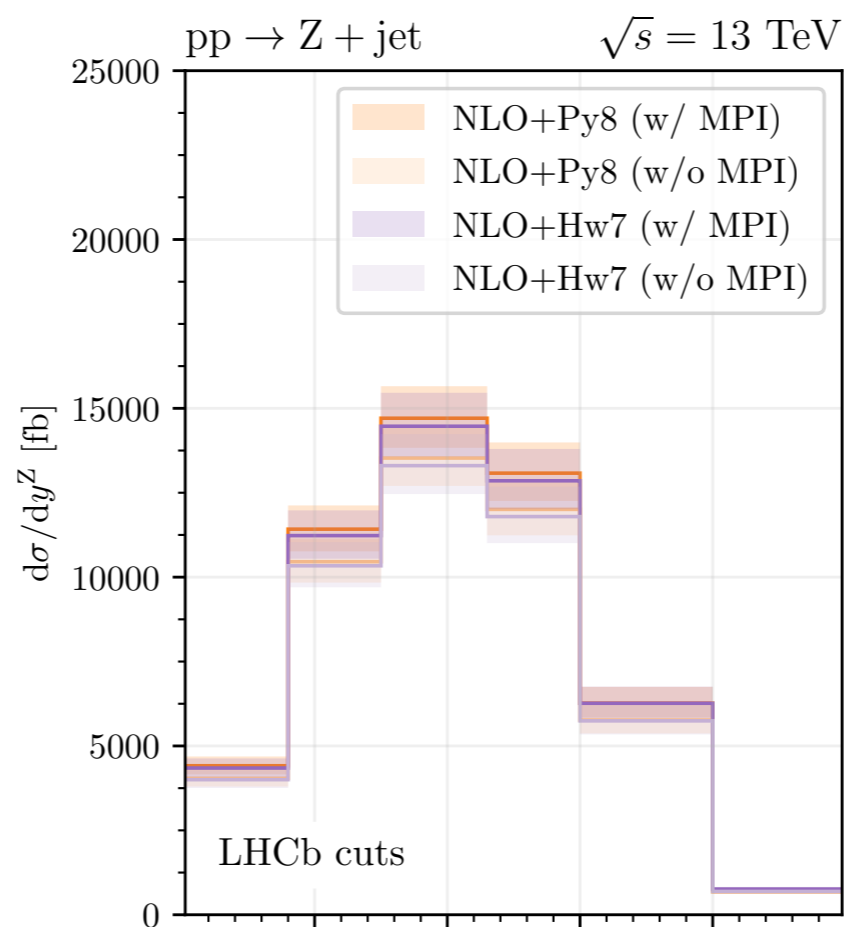
RHS includes:  $p_T(Zj_c) \leq p_T(j_c)$

...

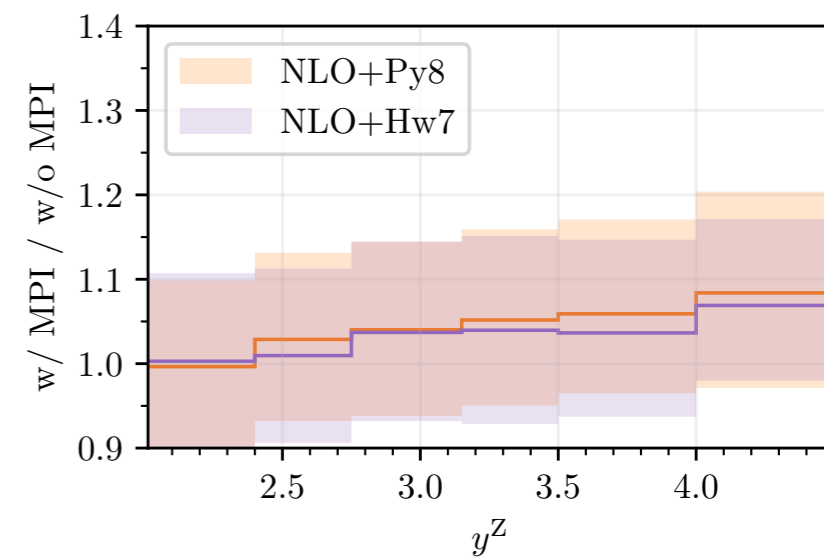
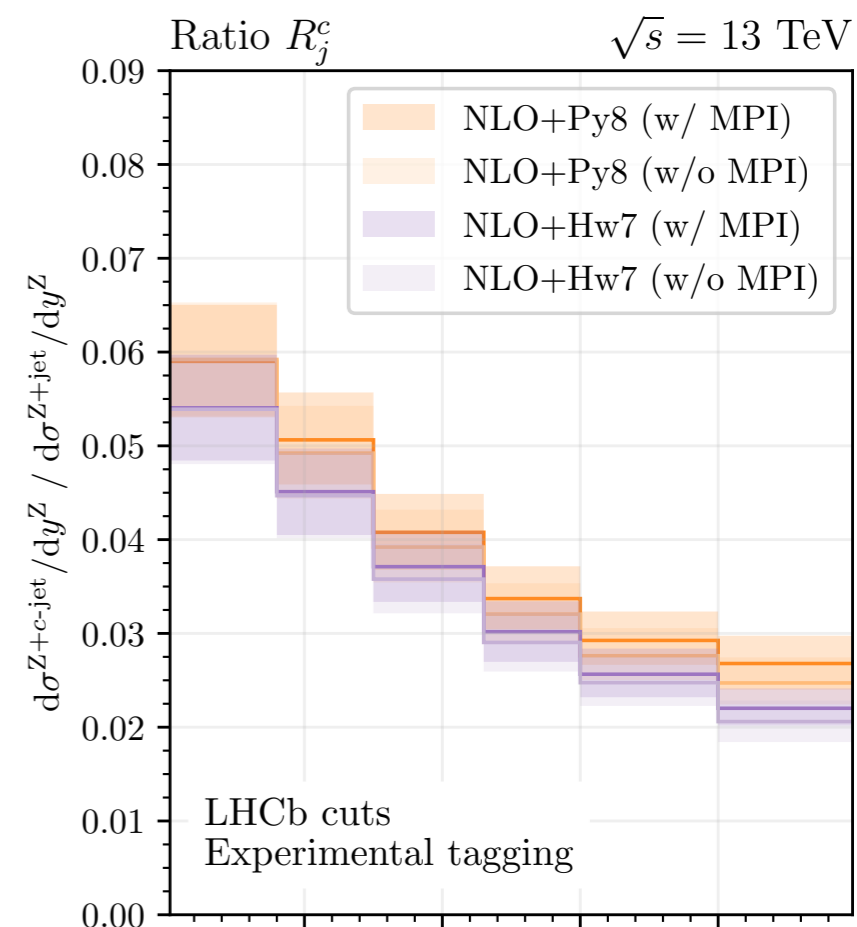
# MPI effects



Z+c-jet



Z+jet



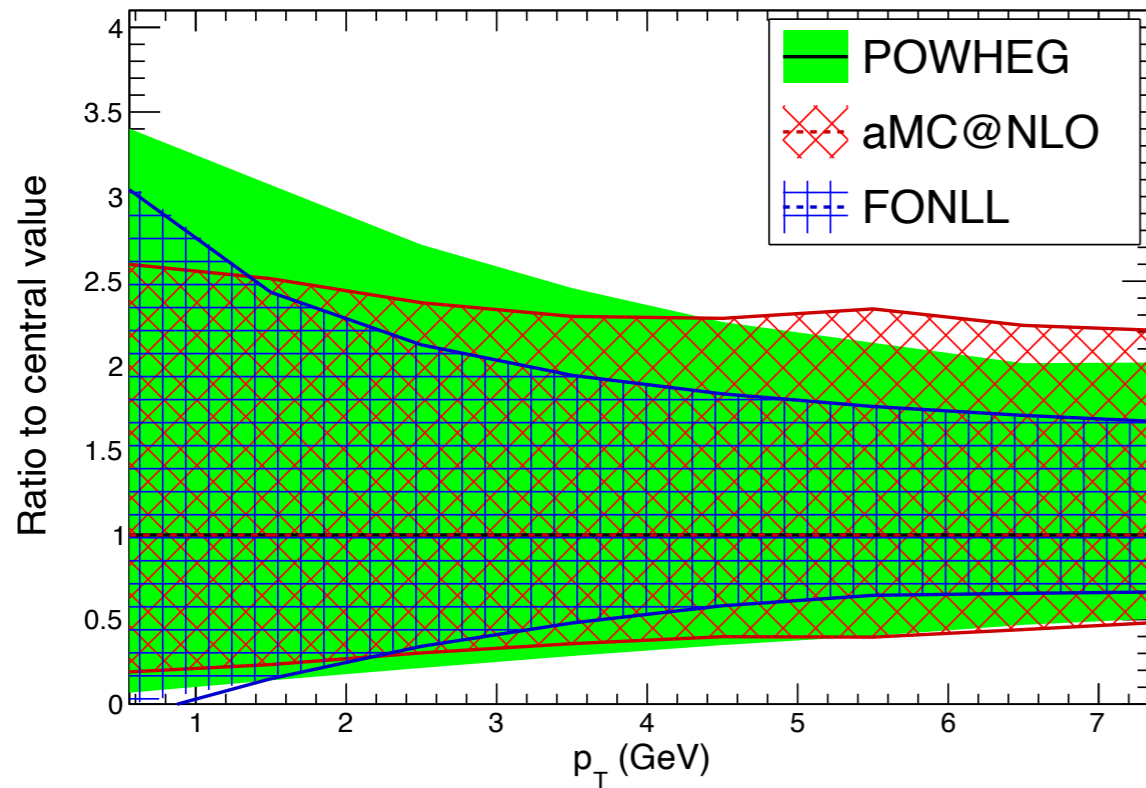
Ratio



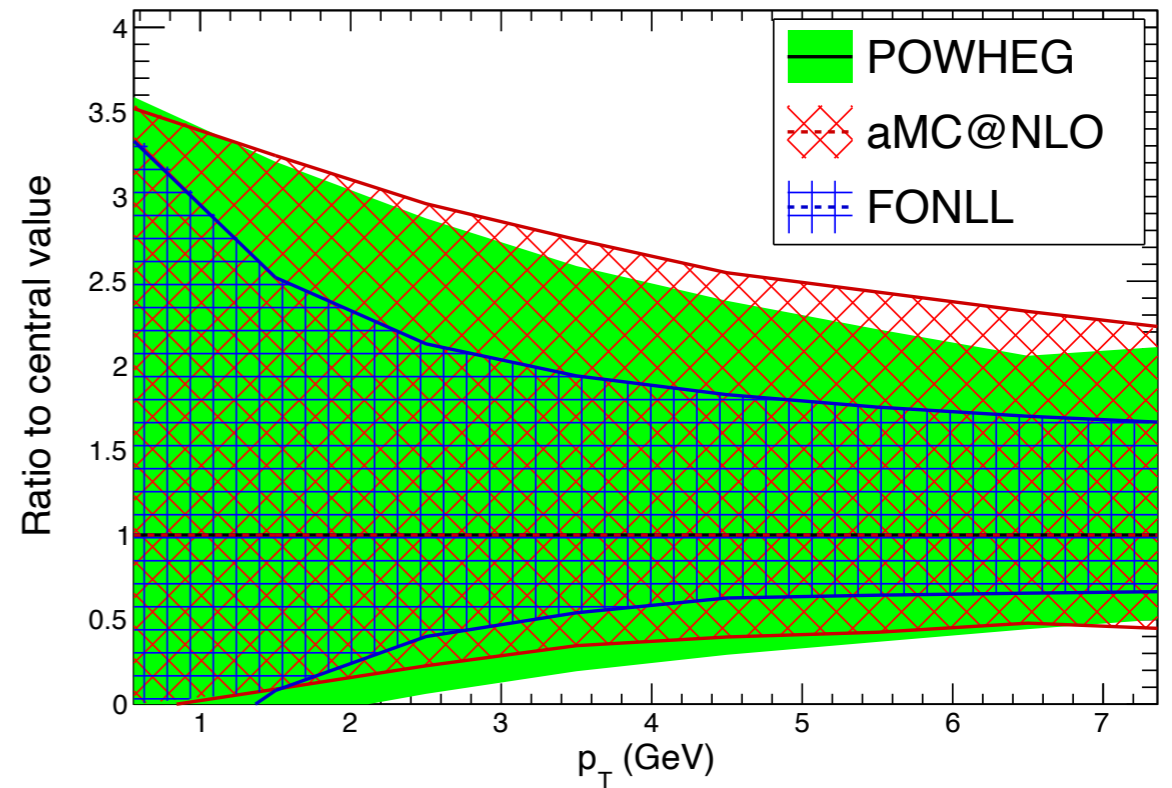
# Heavy-quark pair production

(RG et al., arXiv:1506.08025)

NNPDF3.0, scales+PDFs,  $D^0$  mesons,  $2.0 < y < 2.5$



NNPDF3.0, scales+PDFs,  $D^0$  mesons,  $3.5 < y < 4.0$



These are the theory uncertainties (PDF+scales) for D-cross section at LHCb

With a requirement of  $P_{T,c} > 5$  GeV QCD uncertainties  $\gg 50\%$  (at best)

The charm MPI component generates a  $\sim 15\%$  contribution to LHCb Z+c-jet  $\sigma$

Extracting the SPS component will lead to increased uncertainties ( $\gg 7.5\%$ )

# Ingredients of an NNLO computation

$$\begin{aligned}
 \sigma_{\text{NNLO}} = & \int_{\phi_{n+2}} d\sigma_{\text{NNLO}}^{RR} & \begin{array}{c} \text{Single unresolved} \\ \text{Double unresolved} \end{array} \\
 & + \int_{\phi_{n+1}} d\sigma_{\text{NNLO}}^{RV} & \begin{array}{c} \text{Single unresolved} \\ 1/\epsilon, 1/\epsilon^2 \end{array} \\
 & + \int_{\phi_{n+0}} d\sigma_{\text{NNLO}}^{VV} & 1/\epsilon, 1/\epsilon^2, 1/\epsilon^3, 1/\epsilon^4
 \end{aligned}$$

---


$$\Sigma = \text{Finite}$$

Non-trivial cancellation of IR divergences

# Ingredients of an NNLO computation

$$\begin{aligned}
 \sigma_{\text{NNLO}} = & \int_{\phi_{n+2}} \left( d\sigma_{\text{NNLO}}^{RR} - d\sigma_{\text{NNLO}}^S \right) \\
 & + \int_{\phi_{n+1}} \left( d\sigma_{\text{NNLO}}^{RV} - d\sigma_{\text{NNLO}}^T \right) \\
 & + \int_{\phi_{n+0}} \left( d\sigma_{\text{NNLO}}^{VV} - d\sigma_{\text{NNLO}}^U \right)
 \end{aligned}$$

mimic unresolved

explicit pole cancellation

---


$$\Sigma = \text{Finite} - 0$$

Each line individually finite, can be integrated in 4-d

# Ingredients of an NNLO computation (Antenna)

Gehrmann De-Ridder, Gehrmann, Glover '05

Primary challenge: dealing with flavour (see RG et al., arXiv:1907.05836)

$$d\hat{\sigma}_{ij,\text{NLO}} = \int_{n+1} [d\hat{\sigma}_{ij,\text{NLO}}^R - d\hat{\sigma}_{ij,\text{NLO}}^S] + \int_n [d\hat{\sigma}_{ij,\text{NLO}}^V - d\hat{\sigma}_{ij,\text{NLO}}^T], \quad (2.1)$$

Jet function acts on flavour and momenta of reduced MEs. In general  $(i, j, k) \xrightarrow[\text{momentum}]{\text{flavour}} (l, K)$

$$d\hat{\sigma}_{ij,\text{NLO}}^R = \mathcal{N}_{\text{NLO}}^R d\Phi_{n+1}(\{p_3, \dots, p_{n+3}\}; p_1, p_2) \frac{1}{S_{n+1}} \\ \times \left[ M_{n+3}^0(\{p_{n+3}\}, \{f_{n+3}\}) J_n^{(n+1)}(\{p_{n+1}\}, \{f_{n+1}\}) \right]. \quad (2.2)$$

The  $\sim$  functions denoted mapped (in soft/collinear limits) momenta/flavour sets

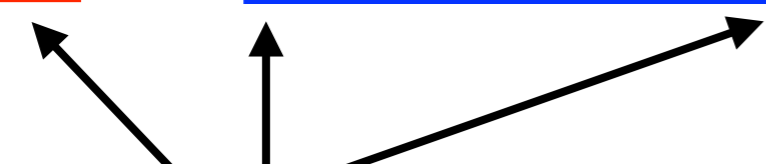
$$d\hat{\sigma}_{ij,\text{NLO}}^S = \mathcal{N}_{\text{NLO}}^R \sum_k d\Phi_{n+1}(\{p_3, \dots, p_{n+3}\}; p_1, p_2) \frac{1}{S_{n+1}} \\ \times \left[ X_3^0(\cdot, k, \cdot) M_{n+2}^0(\{\tilde{p}_{n+2}\}, \{\tilde{f}_{n+2}\}) J_n^{(n)}(\{\tilde{p}_n\}, \{\tilde{f}_n\}) \right], \quad (2.3)$$

# Massive - Variable Flavour Number Scheme

$$d\sigma^{M-VFNS} = d\sigma^{m=0} + \left( \underline{d\sigma^m} - \underline{d\sigma^{m \rightarrow 0}} \right)$$

Form of massive computation for IRC safe or QCD inclusive observables

**RG**, arXiv:2107.01226

$$\underline{d\sigma^m} = \underline{d\sigma^{m=0, n_f}} + \underline{d\sigma^{L[m]}} + d\sigma^{\mathcal{O}(m^2)}$$


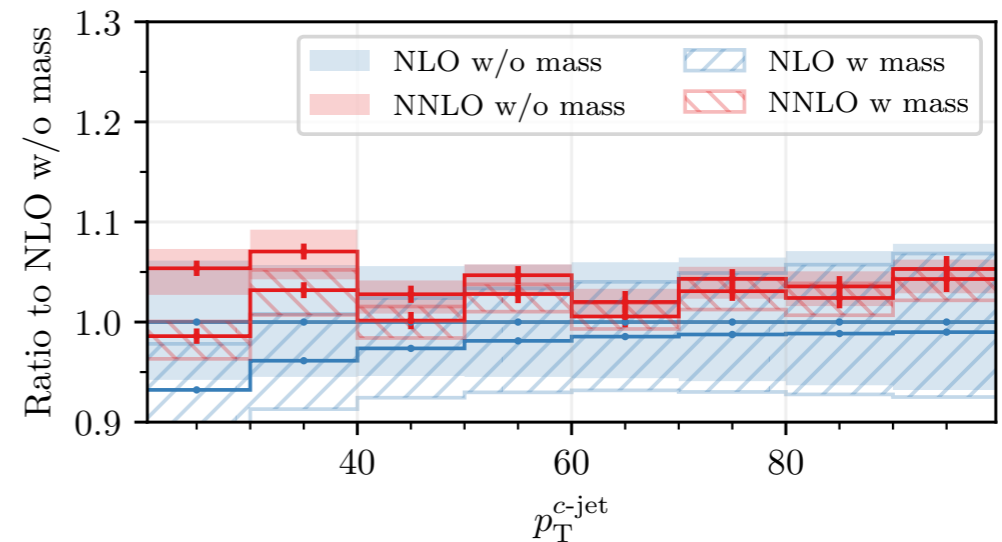
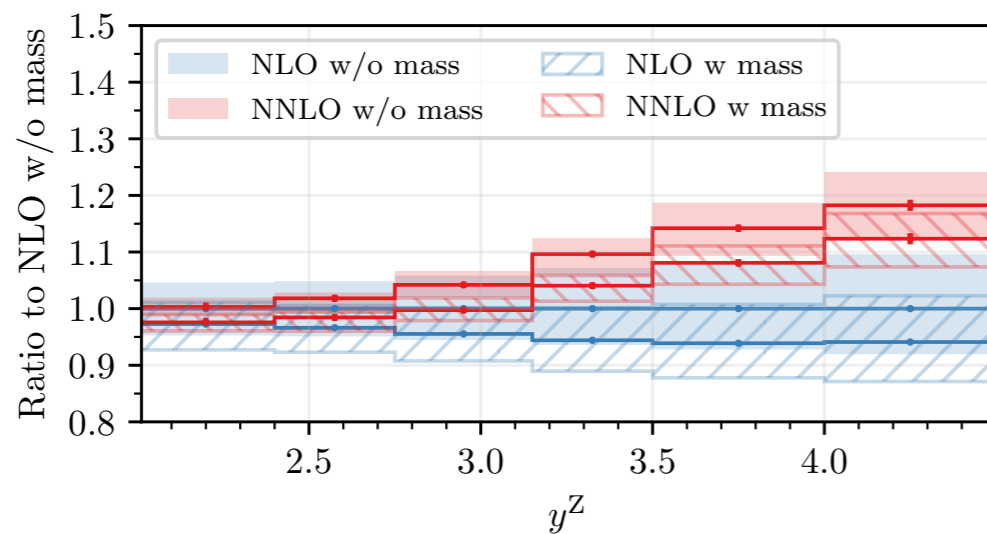
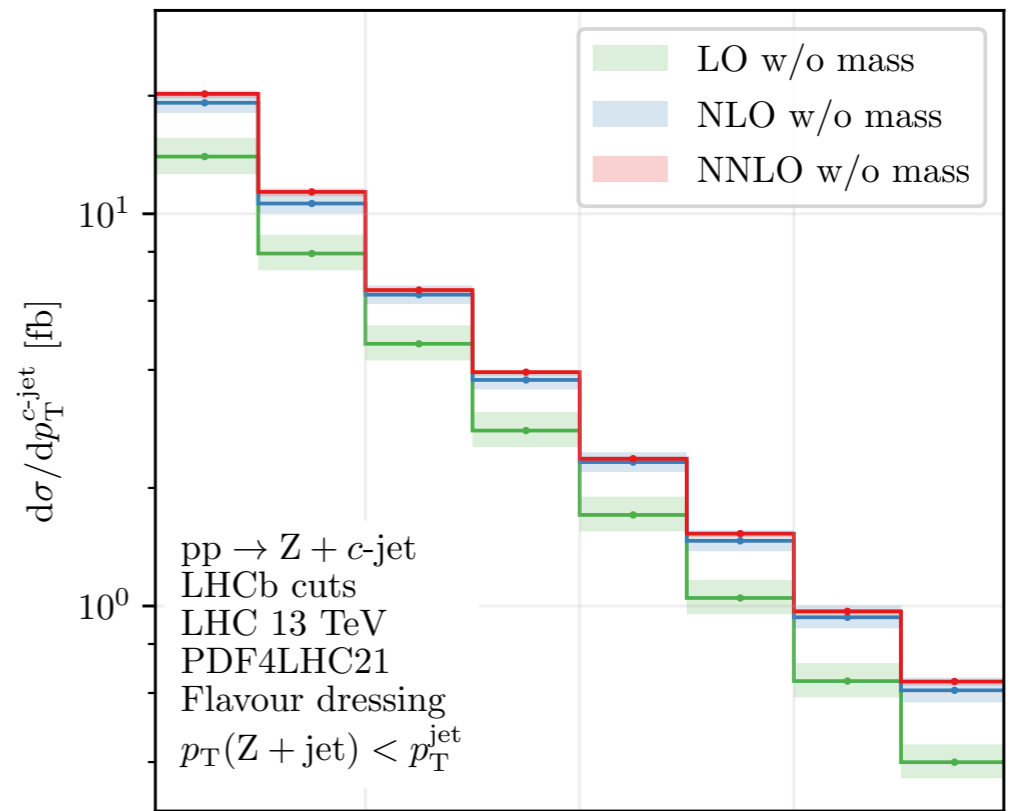
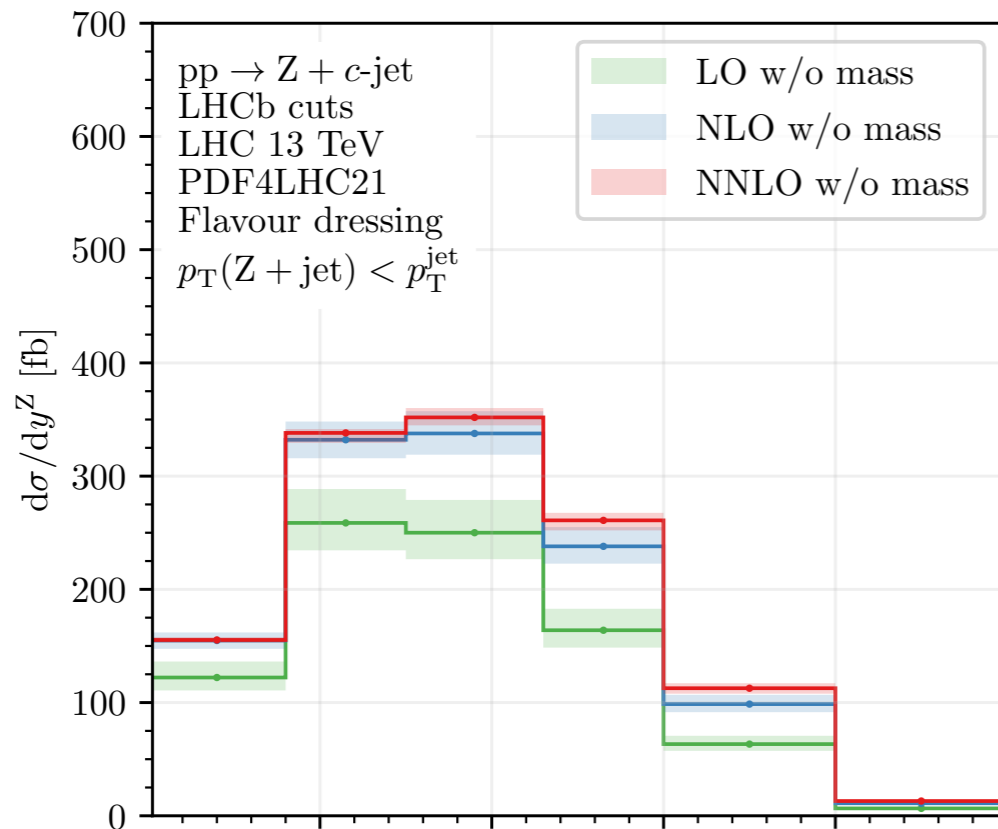
These terms can be directly calculated

(so the last term can be numerically extracted)

$$d\sigma^{M-VFNS} = d\sigma^{m=0} + d\sigma^{\mathcal{O}(m^2)}$$

# Massive - Variable Flavour Number Scheme

$$d\sigma^{M-VFNS} = d\sigma^{m=0} + (d\sigma^m - d\sigma^{m \rightarrow 0})$$



# Z+b-jet and unfolding

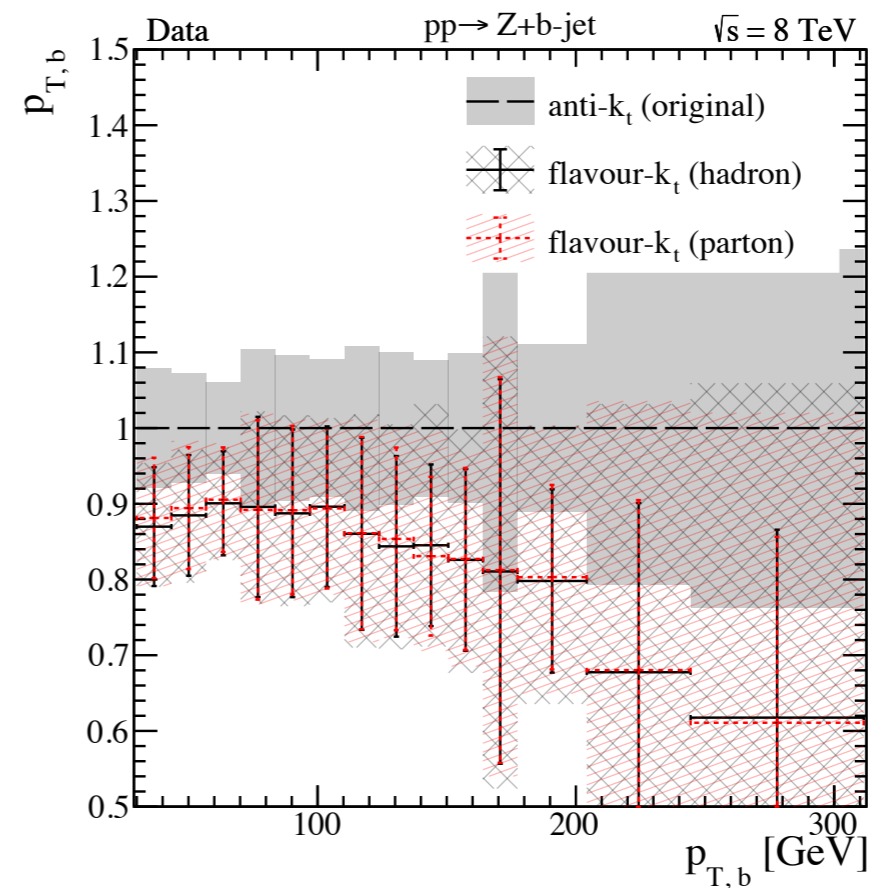
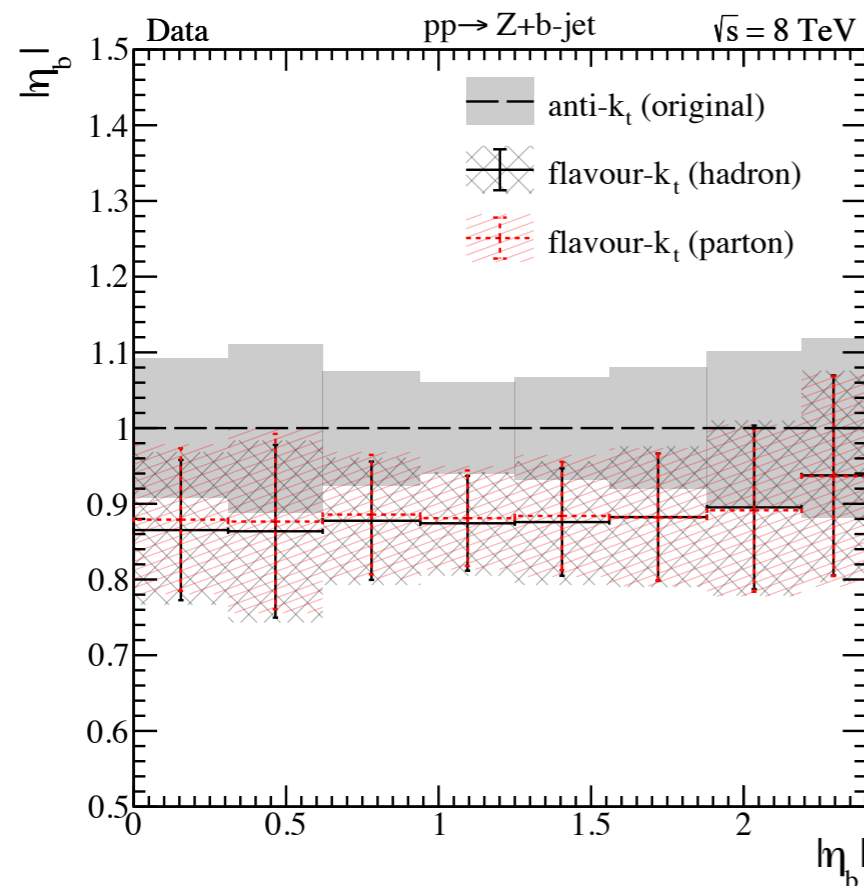
How to account for theory-experiment mismatch?

[Gauld, Gehrmann-De Ridder, Glover, Huss, Majer] *PRL* 125 (2020) 22, 222002

Use an NLO + Parton Shower prediction (which can evaluate both)

- 1) Prediction at parton-level, flavour- $k_T$  algorithm (**Theory**)
- 2) Prediction at hadron-level, anti- $k_T$  algorithm (**Experiment**)

Calculate an “Unfolding” correction from 2) Experiment  $\rightarrow$  1) Theory



We use RooUnfold (following the procedure used in the exp. analyses)

# f-jets @ NNLO as of 2021

$$d\hat{\sigma}_{ij \rightarrow \hat{X}} = d\hat{\sigma}_{ij \rightarrow \hat{X}}^{\text{LO}} + \alpha_s d\hat{\sigma}_{ij \rightarrow \hat{X}}^{\text{NLO}} + \alpha_s^2 d\hat{\sigma}_{ij \rightarrow \hat{X}}^{\text{NNLO}} + \dots$$

$V + (H \rightarrow b\bar{b})$	Ferrera et al. ( <a href="#">1705.10304</a> ), Caola et al. ( <a href="#">1712.06974</a> ), Gauld et al. ( <a href="#">1907.05836</a> )
$Z + b - \text{jet}$	Gauld et al. ( <a href="#">2005.03016</a> )
$W^\pm + c - \text{jet}$	Czakon et al. ( <a href="#">2011.01011</a> )
$t\bar{t}$ with PFF [B-hadrons]	Czakon et al. ( <a href="#">2102.08267</a> )

flavoured-jet algorithm applied

anti- $k_T$  algorithm applied (regulated by  $m_b$ , a tech. cut, or 'prescription')

$t\bar{t}$ with decay	Behring et al. ( <a href="#">1901.05407</a> ), Czakon et al. ( <a href="#">2008.11133</a> )
$t, \bar{t}$ (t-chan with decay)	Berger et al. ( <a href="#">1606.08463</a> , <a href="#">1708.09405</a> ), Campbell et al. ( <a href="#">2012.01574</a> )
$V + (H \rightarrow b\bar{b})$ [4fs]	Behring et al. ( <a href="#">2003.08321</a> )