Thoughts on experimental QCD studies at future e^+e^- colliders

(Selected topics)

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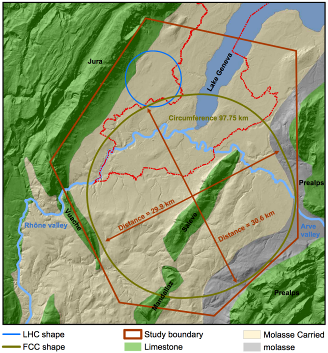
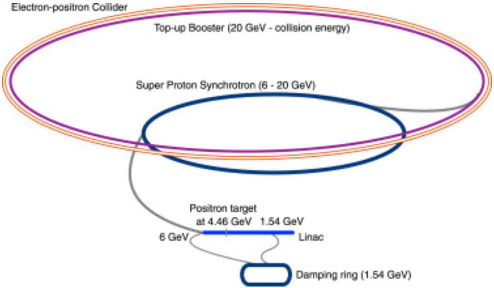
Parton Showers for Future e^+e^- collides, CERN, Geneva, 24th of April 2023

FCC-*ee*





Source: Ref. [1, 2].

- e^+e^- collider in CERN, 100 km [3] long, with 2(+) IP.
- $\sqrt{s} = (20 GeV) M_Z - M_{ZH} (M_{t\bar{t}})$
- Data taking from 203X?: $Z+WW+ZH = 9$ years, $t\bar{t} = 6$ years
- A lot of physics [4].

FCC-*ee*: numbers and more

- State of the art detector(s) desing.
- Aiming at 10^{-5} precision for EW observables, 10^{-3} precision for QCD observables.
- 10^6 ZH and $t\bar{t}$ events, 10^8 WW events; 5×10^{12} Z events, $100000 \times LEP$.
- 10^5 extra-clean digluon events from $e^+e^- \rightarrow Z(\mu\mu)H \rightarrow gg$.
- 10^{11} of $c\bar{c}$, $b\bar{b}$ pairs for HF physics.

Even conceptually different physics from LEP.

Content

- ~~Higgs sector~~ – Please see the great talks on the subject on FCC workshops and many articles, e.g. Ref. [1].
- ~~Electroweak physics in detail~~ – See above.
- ~~BSM searches~~ – See above.
- **Some thoughts about data analysis for QCD/Hadronic final state.**
- **Hadronic final state and QCD:**
  - **QCD analyses**
  - **Flavour physics**
  - **MCEG models**

Some thoughts about data analysis.

Wish list of physicists for FCC-*ee* era[1]

- pQCD: higher& even higher order corrections and **fast** for all relevant processes.
- Parton showers: better precision and **fast**.
- Resummation: see above.
- MCEG: all the above for calculations in MC and better precision/parameters for the modelling.

Analysis: we know **we will** have precision via larger statistics → guaranteed progress on **intensity&precision** frontiers. **Data diversity frontier** as a suggestion: get as much as possible data that machine can deliver in terms of different beams, beam energies etc. And concentrate on the **Analysis quality**.

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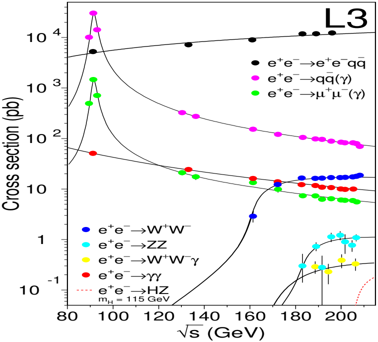
[1] Apart from the discoveries of new phenomena

Data diversity frontier

- Perform measurements not only at $\sqrt{s} = M(Z), M(WW), M(ZH), M(t\bar{t})$. But also at $\sqrt{s} = 20 - 91 GeV$. Make FCC-*ee* not only super-LEP, but also super-(PETRA+TRISTAN). See Ref. [5] for details.



- Use as much as possible events discarded at LEP and elsewhere, e.g. $e^+e^- \to \gamma hadrons$, e.g. $e^+e^- \to \gamma\mu^-\mu^+$.

Analysis quality frontier, part I

Things that will surge/appear at FCC-*ee* :

- Use state of the art statical methods. **Providing correlations should be mandatory**.
- State of the art simulations. In the best case **all processes for individual analyses** should be simulated within one MCEG. Most likely it will be possible.
- Most likely the “unfolding” approach will be replaced with the “folding” approach, at least for some analyses. So the theory predictions will be passed though a detector “model” and compared to the data at detector level. **Actually, this the right way.**
- Sophisticated tracking and particle ID.
- At least some reconstruction methods will be based on machine learning and similar approaches.
- Real-time analysis.

Analysis quality frontier, part II

Things that will decline/disappear at FCC-*ee*:

- Complex LHC-like triggering. High throughput data acquisition system designed for Z peak data probably will be able to process **all** collisions at other energies. **Remember: any data are good and we want it all. Be prepared to understand** $e^+e^- \to \textit{anything}$ **at different energies.** .
- Approaches that drop some info from event, e.g. compression or recording only selected objects [6]. With $\approx 20-50$ physical objects per event there is no much things to drop or compress.
- Over-complicated tagging techniques aimed at high efficiency. High statistics will allow for simple (even inefficient) triggers with $\approx 100\%$ purity, e.g. full reconstruction of B decays in $e^+e^- \to b\bar{b}$ events.
- Using MC to derive the correlations in data. New methods should come, see e.g. Ref. [7].
- Background subtraction.

QCD and hadronic final state.

$\sigma_{tot}(e^+e^- \to hadrons)$

6 *44. Plots of cross sections and related quantities*

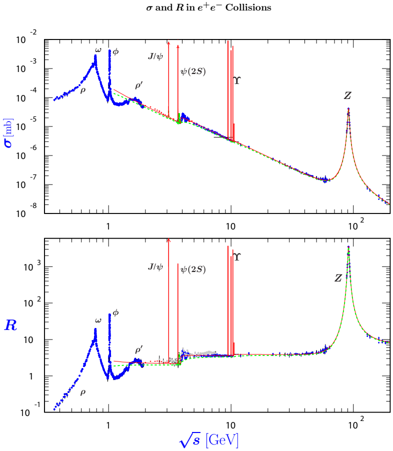


Figure 44.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000) (Erratum *ibid.* **B634**, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. Corresponding computer-readable data files are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010.)

Source: Ref. [8]

- The measurement of $\sigma(e^-e^-) \to hadrons$ is the ultimate way to measure α_s
- FCC-*ee* has capacity to run the measurements between $\approx$ 30 and 360 GeV and supersede all the measurements in this region within some weeks.

Classical event shapes

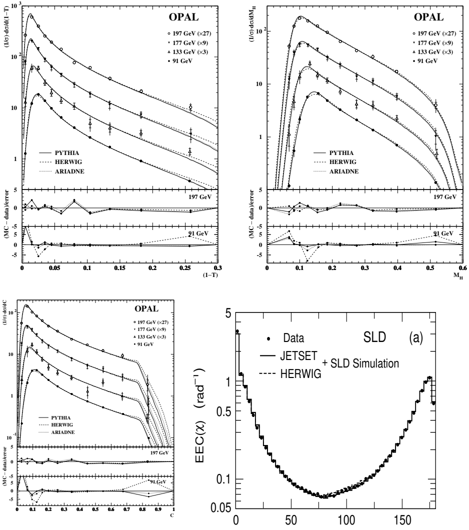
- Infrared safe quantities defined via momenta or energy.
- Either one scalar/event, e.g. thrust

$$T = \max_{\vec{n}} \left(\frac{\sum_i |p_i \cdot \vec{n}|}{\sum_i |p_i|} \right)$$

Or distributions/event as e.g. energy-energy correlations [9]:

$$\frac{dEEC(\chi)}{d\chi} = \sum_{ij} \frac{E_i E_j}{E_{tot}^2} \delta(\cos(\chi_{ij}) - \cos(\chi))$$

- Measured at all e^+e^- colliders.
- Can be calculated in pQCD and often can be resummed.



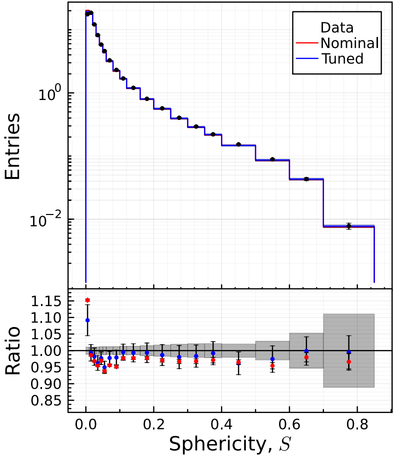
Sources: Refs. [10], [11].

Classical event shapes at FCC-*ee* is a must

Points to pay attention to:

- Consistent measurements of event shapes with all the classical definitions.
- Correlations between the bins in of distributions **and** between the distributions should be provided. Especially important for the complex event shapes such as EEC/AEEC.
- The measurements should be done at all possible energy points **Note: was not the case for LEP!**.
- The measurements should not be a patch-work of different analyses of one event shape/thesis. **Learn form HERA, see e.g. Ref. [12]**.

Classical event shapes at FCC-*ee*: benefits



Source: Ref. [13].

Use case:

- Compare to the existing measurements at LEP/PETRA/SLD/TRISTAN.
- Possibly re-tune MCEGs only on FCC data and compare the predictions to data from previous colliders.
- Classical analyses and MCEG tunes. **Each and every MCEG tune used some of them.**

Exotic event shapes

It is not **that** hard to invent a special type of event shape.

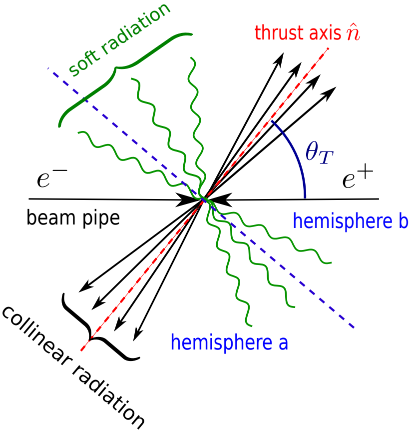
- Event shapes with orientation of the initial final state.
- Event shapes with grooming.
- Event shapes with flavour tagging [14].
- . . .

However, there are the following problems:

- If the event shape is more differential or contains more information in comparison to its 'classical' counterpart, the perturbative calculations and the resummation might be not available.
- If the event shape definition on the contrary, drops some information in the definition, less information will be available in the output.

Oriented event shapes

Oriented event shapes take into account e.g. the beam direction.



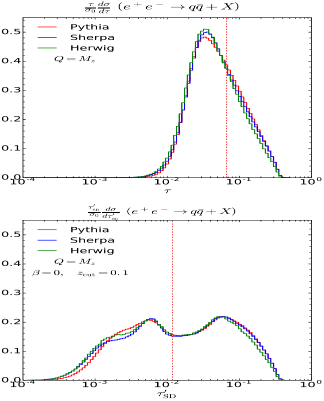
I.e. for the thrust T this could result in 2-D distribution $\frac{1}{\sigma}\frac{d^2\sigma}{dTd\theta_T}$ Figure: Ref. [15].

Very few measurements exist. A good review is given in Ref. [16].

- Previous measurements of event shapes were extrapolated to the 4π solid angle of the detector.
- One of the reasons – an absence of good theory calculations **actually** for $e^+e^- \to partons$. Most theory predictions used $Z/\gamma \to partons$. Post-LEP progress: Refs. [15] [16]. Precision?

Groomed event shapes

The definition for e.g. soft-drop thrust is given in Ref. [17]. Briefly: drop some “soft” “pseudojets” from clustering and recalculate thrust.



Source: Ref. [17].

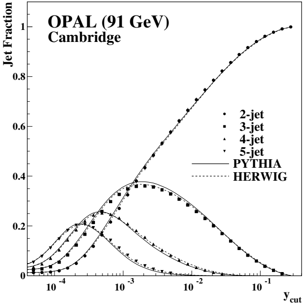
- Less objects cannot result in “more information” in the output.
- Low multiplicity and energy: not much to drop.
- The Ref. [17] is quite moderate in estimations: only qual. are given on the improvements and the main uncertainty source in the α_S extraction.
- The pQCD predictions [18] and the resummation [17] already exist, however more th. studies are needed.

Exotic event shapes at FCC-*ee*: benefits

- For some complex “more differential’ event shapes the benefits depend on the developments in the theory. Same true for massive event shapes. A breakthrough in theory is needed to match the data precision.
- For the event shapes with more complex definitions things are not clear so far. Some recent honest attempts to study the topic give mixed results. Personal thought: it is not obvious for me that e.g. groomed event shapes are “good” defined observables if one considers decays.

Jets

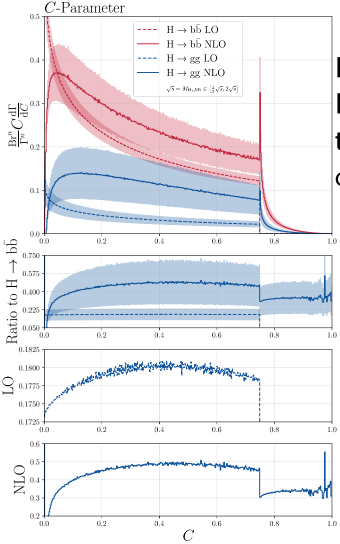
Essentially jets are the relatives of events shapes and all the considerations are applicable.



Source: Ref. [19]

- Jet algorithms with more parameters require more efforts for the predictions, esp. resummation.
- Not to forget: multiplicity. LEP: up to 6 jets. FCC-*ee*? 7? 8? Larger multiplicity means higher orders in the pQCD predictions. As of now we have 7 jets NLO [20]. One will have to take into account the $QCD \times QED$ corrections in the theory predictions and in MC. A breakthrough in MC is needed to match the data precision.
- The correlation between jet measurements should not be forgotten.

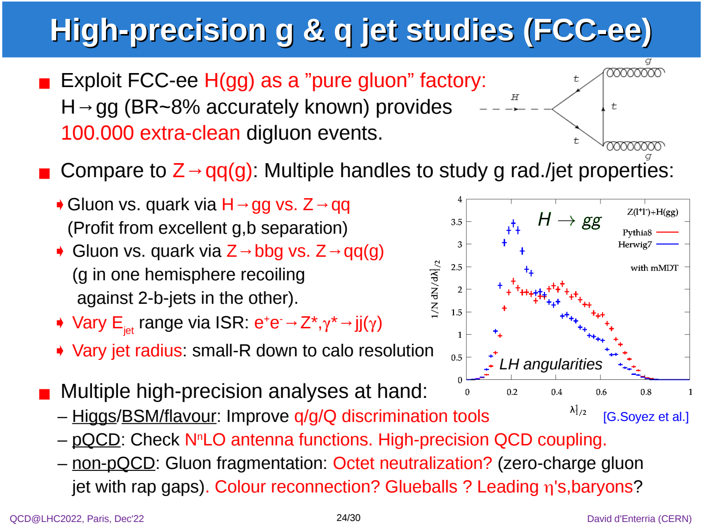
Exotic sources for HFS



High statistics → QCD analyses with exotic HFS sources for e.g. **parton shower tuning**. Experimentally there should be no complications at all.

- $e^+e^- \rightarrow ZH \rightarrow \mu\mu gg$ as a source for the events with HFS **only** from gluons. See e.g. [21]. Figure from Ref. [21]. Possible input for $e^+e^- \rightarrow H \rightarrow gg$.
- $e^+e^- \rightarrow VV \rightarrow q_1 q_2 q_3 q_4$. Anyone?
- Events with rare topologies, e.g. $e^+e^- \rightarrow b\bar{b}g$, where $b\bar{b}$ pair recoils against g.

Exotic sources for HFS: credits to David



High-precision g & q jet studies (FCC-ee)

- Exploit FCC-ee H(gg) as a "pure gluon" factory: H → gg (BR~8% accurately known) provides 100.000 extra-clean digluon events.
- Compare to Z → qq(g): Multiple handles to study g rad./jet properties:
  - Gluon vs. quark via H → gg vs. Z → qq (Profit from excellent g,b separation)
  - Gluon vs. quark via Z → bbg vs. Z → qq(g) (g in one hemisphere recoiling against 2-b-jets in the other).
  - Vary E_{jet} range via ISR: $e^+e^- \to Z^*,\gamma^* \to jj(\gamma)$
  - Vary jet radius: small-R down to calo resolution
- Multiple high-precision analyses at hand:
  - Higgs/BSM/flavour: Improve q/g/Q discrimination tools
  - pQCD: Check N^{n}LO antenna functions. High-precision QCD coupling.
  - non-pQCD: Gluon fragmentation: Octet neutralization? (zero-charge gluon jet with rap gaps). Colour reconnection? Glueballs ? Leading η's,baryons?

[G.Soyez et al.]

QCD@LHC2022, Paris, Dec'22

24/30

David d'Enterria (CERN)

τ and heavy flavour.

$\tau \rightarrow$ *hadrons*, heavy flavour decays, PID

FCC-*ee* will provide:

- A lot (10^{11}) of τ.
- A lot of *b* and *c*-hadrons. Some orders of magnitude more than LHCb or BELLE-II [3].
  - A lot of data for excited *b* and *c* states.
  - A lot of data for *b* and *c* decays.

Another expected feature of FCC-*ee* analyses is a nice PID, so:

- Will be there more expensive, PID-aware tracking? It will be crucial for flavour physics.
- But PID is "contagious": if PID is adopted in tracking it will be propagated to particle flow, calculation of jets and event shapes. Are we ready for that?

HF fragmentation functions

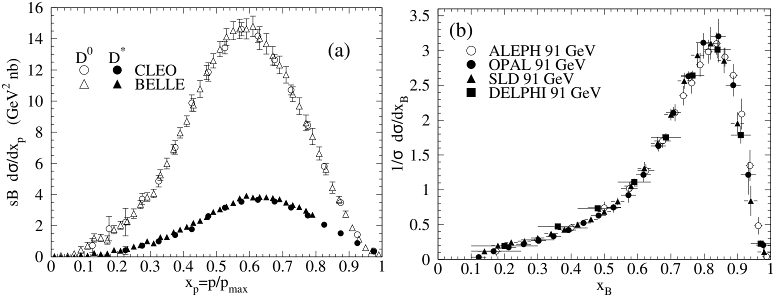
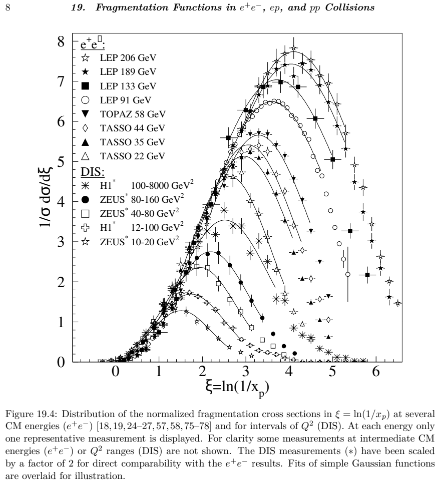


Figure 19.12: (a) Efficiency-corrected inclusive cross-section measurements for the production of D^0 and D^{*+} in e^+e^- measurements at $\sqrt{s} \approx 10.6$ GeV, excluding B decay products [288] [289]. (b) Measured e^+e^- fragmentation function of b quarks into B hadrons at $\sqrt{s} \approx 91$ GeV [292].

Source: Ref. [22].

FCC-*ee* will have enough statistics to measure precisely the fragmentation functions for many hadrons. The *c* or *b* jet tagging can be very inefficient, one can aim at highest purity even for **excited states**.

Also non-HF fragmentation functions



8 ***19. Fragmentation Functions in e^+e^-, ep, and pp Collisions***

Figure 19.4: Distribution of the normalized fragmentation cross sections in $\xi = \ln(1/x_p)$ at several CM energies (e^+e^-) [18, 19, 24–27, 57, 58, 75–78] and for intervals of Q^2 (DIS). At each energy only one representative measurement is displayed. For clarity some measurements at intermediate CM energies (e^+e^-) or Q^2 ranges (DIS) are not shown. The DIS measurements (∗) have been scaled by a factor of 2 for direct comparability with the e^+e^- results. Fits of simple Gaussian functions are overlaid for illustration.

- Test fragmentation universality.
- Test fragmentation models.
- Note the measurements for $\sqrt{s} = 20 - 90\,GeV$.
- One can even extract α_S[23].
- Do measurements for identified hadrons.

Source: Ref. [22].

Other topics

With a huge statistics one can measure:

- $g \to c\bar{c}$
- $g \to b\bar{b}$
- $e^+e^- \to e^+e^- b\bar{b}$, $e^+e^- \to e^+e^- c\bar{c}$ and get F_2^γ, see e.g. Ref [24].

MCEGs for HFS

MCEGs for FCC-*ee* (as of now)

	MEs	Shower	Internal merging	Hadronization
Herwig [25]	Internal, libraries, LHE	QTilde, dipoles	+	Cluster/Lund8 via TheP8I
KKMC [26]	Internal, LHE	External	+?	Lund6/Lund8
Sherpa [27]	Internal, libraries, LHE?	CSS, DIRE	+	AHADIC/Lund6/Lund8
Pythia8 [28]	Internal, libraries?, LHE	Pt, DIRE, Vincia	-	Lund8
Whizard [29]	Internal, libraries, LHE	External?	+	Lund6/Lund8/HERWIG6?

"?" stands for "Please correct me if the actual situation is different".

There are many good MCEGS with a lot of features. These are modern (mostly C++) with a high level of modality. If we extrapolate their future using the JETSET/PYTHIA5 as an example, one can assume to see those at FCC.

MCEG problems

. . . are quite universal.

- Not all MCEGs can handle processes “classical” processes in e^+e^- colliders. The processes with resolved photon, ISR, polarisation are not always simulated correctly because there are not people to check them.
- Problems when the quark masses should be taken into account in hadronization and in the showers.
- The LHC era generators sometimes fail showering or hadronization at lowest scales and multiplicities, e.g. $\sqrt{s} \approx 20 GeV$.

How MCEGs can be ready for FCC-*ee*

- Implement the missing processes and check them against the still existing MCEGs of LEP era. In the best case when there are still people proficient in F77.
- Validate the MCEGs against theory predictions and each other **at sub-permil level**.
- Improve the precision: a lot of work ahead.
- "Easy" improvements: decays – BELLE&LHCb are taking data.
- Hadronization: Rivet [30]&tuning packages are doing a great job in validation of MCEGs against the old e^+e^- data. However, 1) there are **too few models for hadronization**[2] 2) the quality of the data suitable for the turning is poor 3) no new data (EIC?) suitable hadronization tunning is foreseen. Hard to expect a breakthrough before FCC-*ee*. Biggest challenge.

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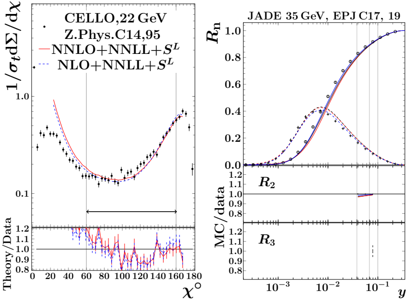
[2] Will ML help?

Hadronization in $e^+e^- \to$ *hadrons*

- The modern MCEG models are for $\sqrt{s} \approx M_Z$, but not trustable for other energies[31][32] and lower scales.
- This is an artefact: the models were tuned with LEP data at $\sqrt{s} \approx M_Z$ or LHC data, where the tuning does not give very certain results.
    - The recent efforts to re-use the PETRA, TRISTAN and PEP data [33] had limited success due to huge data uncertainties.



With enough data away from Z peak, MCEG models can be re-tuned to describe the hadronization better at all energies.

An extension of FCC-e^+e^- physics program for MCEGs

Understanding the importance of data below Z peak for MCEGs, the great FCC-e^+e^- CDR can be extended and include data taking in range $\sqrt{s} = 20 - 91\,\text{GeV}$[3]

$$\text{FCC-}e^+e^- = \text{Higgs factory} + \text{SuperLEP}$$
$$+ \text{SuperTRISTAN} + \text{SuperPEP} + \text{SuperPETRA}$$

Two **non-excluding** options are available:

- Dedicated: Dedicated runs with lowered beam energy.
- $e^+e^-\gamma$: γ tagging of radiative events $e^+e^- \to hadrons + \gamma$.

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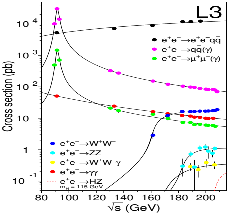
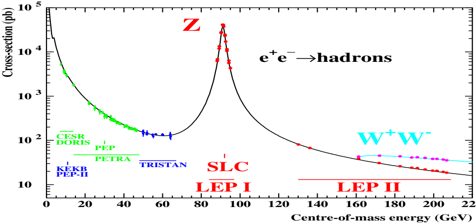
[3] The lower bound depends on the actual capabilities of the machine.

Two options for data collection

- Dedicated: Perfect data, fast to collect – $10^7 - 10^9$ background free events/day (see backups) – supersedes data collected at **all** previous colliders in one day.
- $e^+e^-\gamma$: Lower data quality and numerous issues (see backups). But with and advanced FCC-*ee* detector (see backups) this option can be extremely valuable.





A perfect scenario: dedicated runs with ≈ 10 equidistant energy points in range $20 - 91$ GeV with $10^7 - 10^8$ events each.

Conclusions

- FCC-*ee* will be a crucial machine for the understanding of QCD/HFS.
- The improvements in the understanding of QCD with FCC-*ee* data will go beyond the "larger statistics".
- But, the increased statistics on itself will make possible completely new studies.
- **Breakthroughs are needed in theory, MCEGs and hadronization modeling to exploit the full potential of FCC-*ee* data.** A lot of work ahead.

Backups and discussion

Origin of systematics uncertainties related to $e^+e^- \rightarrow VV$

The $e^+e^{\rightarrow}VV$ processes can be simulated and calculated quite precisely since a long time.

Nevertheless, the measurements of the $e^+e^{\rightarrow}Z/\gamma \rightarrow hadrons$ with $e^+e^- \rightarrow VV$ still have related uncertainties. This is related to the way the measurements of $e^+e^{\rightarrow}Z/\gamma \rightarrow hadrons$ are done:

- Measure events with hadrons in final state, e.g. event shapes.
- Apply cuts to to reduce the amount of $e^+e^- \rightarrow V_1V_2, V_1 \rightarrow q_1q_2, V_2 \rightarrow l\nu$ (semileptonic) and $e^+e^- \rightarrow V_1V_2, V_1 \rightarrow q_1q_2, V_2 \rightarrow q_2q_3$ (allhadronic) events
- Subtract from the distributions after the cuts the "MC-simulated" reminder of $e^+e^- \rightarrow V_1V_2, V_1 \rightarrow q_1q_2, V_2 \rightarrow l\nu$ and $e^+e^- \rightarrow V_1V_2, V_1 \rightarrow q_1q_2, V_2 \rightarrow q_2q_3$ events
- ...

The systematics related to this procedure will exist even in the case of perfect modelling of $e^+e^- \rightarrow V_1V_2$ processes.

Costs in terms of money, time and manpower for below-Z measurements

No detector amendments needed.	$=$0€ extra for detector construction
Running time for dedicated runs would be couple days with lower energy consumption.	$\approx$0€ extra for running
The changes of beam energies would require readjustments of some magnets (but not the main ring).	Some manpower and time (some weeks?)
The data is of same type as the data at and above Z and would fit into any software/analysis for higher energy.	$\approx$0€ extra for computing and physics

Costs in terms of money, time and manpower are tiny.

Methodology of measurements of QCD observables: $e^+e^-\gamma$ vs. dedicated runs

$e^+e^-\gamma$

- Measure γ energy.
- Calculate the CM boost assuming γ comes from ISR.
- Alternatively to the points above do a kinematic fit of the hadronic final state to gen the energy of γ.
- Boost the event to the calculated CM.
- Calculate observables from the boosted hadronic final state.

Dedicated

- Make sure the CM energy is close to nominal using cuts.
- Calculate observables from hadronic final state.

The measurement of γ and the boost procedure bring additional uncertainties. The performance of these methods could be insufficient for the desired accuracy of the measurements.

FCC detector for $e^+e^-\gamma$

While at previous e^+e^- experiment the $e^+e^-\gamma$ events produced much less precise data sets for QCD measurements, the FCC-*ee* detectors would be a major improvement.

- The low angle limit for detector acceptance can be lowered to much lower angle than at LEP: the detector/machine interface has been set at 100mrad, so tracking and e/gamma acceptance should be good down to about 10 degrees or even less.
- Modern vertex detectors should ensure superior reconstruction of the event kinematics.

A dedicated study is needed!

$e^+e^-\gamma$ vs. dedicated runs: Point 0

- …

- Even if one registers hard γ the $e^+e^-\gamma$ process cannot be described in theory as γ plus $e^+e^- \to$ *hadrons* at lower scale. This is a significant theoretical distinction. To be on pair with the dedicated runs, $\alpha_s^3 \times \alpha_{EW}$ calculations are needed.

$e^+e^-\gamma$ vs. dedicated runs: Point 1

- It will take time to change the beam energy for dedicated runs.

- True. But it is acceptable to sacrifice a tiny fraction of running time to take a better data and better physics.

Need input from accelerator physicists and engineers.

$e^+e^-\gamma$ vs. dedicated runs: Point 2

- The $\sqrt{s} = 20 - 91\,\text{GeV}$ data can be taken during high energy runs using $e^+e^-\gamma$ anyway.

- The sys. uncertainties of such data will be much higher.
- Will take much more time to collect.
- Adjusting detector/reconstruction for such data could take even more time.
- Potential problems with acceptance of highly boosted events.
- Such data are not suitable for many analyses and calibration.
- If there will be two e^+e^- colliders in the future, the project with dedicated runs will be able to get the precious data much faster.

$e^+e^-\gamma$ vs. dedicated runs: Point 3

- There will be enough data from $e^+e^-\gamma$ anyway.

- Not really and not of good quality, see L3 [34] and OPAL [35] at LEPI:

Type	$\sqrt{s}$, GeV	$\langle\sqrt{s}\rangle$, GeV	Int. Lumi (pb)	Selection Eff.(%)	Purity(%)	Sel. Events
Reduced Centre-of-Mass Energy	30–50	41.4	142.4	48.3	68.4	1247
	50–60	55.3	142.4	41.0	78.0	1047
	60–70	65.4	142.4	35.2	86.0	1575
	70–80	75.7	142.4	29.9	89.0	2938
	80–84	82.3	142.4	27.4	90.5	2091
	84–86	85.1	142.4	27.5	87.0	1607
Z pole	91.2	91.2	8.3	98.5	99.8	248100

$\alpha_S(M_Z)_{41\ \mathrm{GeV}} = 0.1418 \pm 0.0053(\mathrm{stat.}) \pm 0.0030(\mathrm{exp.syst.}) \pm 0.0055(\mathrm{hadr.}) \pm 0.0085(\mathrm{theory.})(NLO)$
$\alpha_S(M_Z)_{55\ \mathrm{GeV}} = 0.1260 \pm 0.0047(\mathrm{stat.}) \pm 0.0056(\mathrm{exp.syst.}) \pm 0.0066(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(NLO)$
...V.S.
$\alpha_S(M_Z)_{91\ \mathrm{GeV}} = 0.1210 \pm 0.0008(\mathrm{stat.}) \pm 0.0017(\mathrm{exp.syst.}) \pm 0.0040(\mathrm{hadr.}) \pm 0.0052(\mathrm{theory.})(NLO)$

E_γ [GeV]	Events	$\sqrt{s'}_{Mean}$[GeV]	Background [%]		
			Non-rad. MH		$\tau\tau$
			Likelihood	Isolated tracks	
10-15	1560	78.1± 1.7	6.0± 0.7	6.2± 0.9	0.9± 0.2
15-20	954	71.8± 1.9	3.1± 0.5	4.9± 0.8	1.0± 0.3
20-25	697	65.1± 2.0	2.6± 0.6	6.3± 1.1	0.9± 0.4
25-30	513	57.6± 2.3	5.1± 1.1	7.9± 1.4	1.1± 0.5
30-35	453	49.0± 2.6	4.5± 1.1	9.6± 1.6	0.7± 0.4
35-40	376	38.5± 3.5	5.2± 1.2	13.1± 1.9	0.8± 0.5
40-45	290	24.4± 5.3	10.4± 2.3	12.9± 1.7	0.8± 0.5

$\alpha_S(M_Z)_{comb} = 0.1182 \pm 0.0015(\mathrm{stat.}) \pm 0.0038(\mathrm{exp.syst.}) \pm 0.0070(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(NLO)$

+specific problems: hadronization, systematics, statistics.

$e^+e^-\gamma$ vs. dedicated runs: Point 4

It is interesting to admit the differences between the hadronization uncertainties of results from OPAL [35]

$0.1182 \pm 0.0015(\mathrm{stat.}) \pm 0.0038(\mathrm{exp.syst.}) \pm 0.0070(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(\mathit{NLO})$

and JADE [36]:

$0.1172 \pm 0.0006(\mathrm{stat.}) \pm 0.0020(\mathrm{exp.syst.}) \pm 0.0035(\mathrm{hadr.}) \pm 0.0030(\mathrm{theory.})(\mathit{NNLO} + \mathit{NLLA})$

	Year	Type	$\sqrt{s}$	Hadr. unc.	Exp. syst. unc .
JADE	2008	Low energy	12-46	0.0035	0.0020
OPAL	2007	Radiative	10-45	0.0070	0.0038

$e^+e^-\gamma$ vs. dedicated runs: Scaling the L3 $e^+e^-\gamma$ case to FCC-*ee* $e^+e^-\gamma$

L3 [34]:

Type	$\sqrt{s}$, GeV	$\langle\sqrt{s}\rangle$, GeV	Lumi (pb)	Selection Eff.(%)	Purity(%)	Sel. Events	FCC $e^+e^-\gamma$
Reduced	30–50	41.4	142.4	48.3	68.4	1247	2.8×10^9
Centre-	50–60	55.3	142.4	41.0	78.0	1047	2.4×10^9
of-	60–70	65.4	142.4	35.2	86.0	1575	3.6×10^9
Mass	70–80	75.7	142.4	29.9	89.0	2938	6.7×10^9
Energy	80–84	82.3	142.4	27.4	90.5	2091	3.7×10^9
	84–86	85.1	142.4	27.5	87.0	1607	3.6×10^9
Z pole	91.2	91.2	8.3	98.5	99.8	248100	10^{12}

With a tighter selection from OPAL, the number of FCC $e^+e^-\gamma$ events would be order of magnitude smaller.

The dedicated runs could obtain such amount of data in some days or even hours.

Results from $e^+e^- \rightarrow$ *hadrons*

Determination[4]	Type	Data and procedure	Ref.
0.1175 ± 0.0025	Non-global	ALEPH 3-jet rate (NNLO+MChad)	[38]
0.1199 ± 0.0059	fit	JADE 3-jet rate (NNLO+NLL+MChad)	[39]
0.1224 ± 0.0039	+MChad	ALEPH event shapes (NNLO+NLL+MChad)	[40]
0.1172 ± 0.0051		JADE event shapes (NNLO+NLL+MChad)	[36]
0.1189 ± 0.0041		OPAL event shapes (NNLO+NLL+MChad)	[41]
$0.1164\ ^{+0.0028}_{-0.0026}$	Global fit	Thrust (NNLO+NLL+anlhad)	[42]
$0.1134\ ^{+0.0031}_{-0.0025}$	+anlhad	Thrust (NNLO+NNLL+anlhad)	[43]
0.1135 ± 0.0011		Thrust (SCET NNLO+N^{3}LL+anlhad)	[44]
0.1123 ± 0.0015		C-parameter (SCET NNLO+N^{3}LL+anlhad)	[45]
0.11750 ± 0.00287	Global fit	EEC (NNLO+N^{2}LL+MChad+NLO$_{m_b}$)	[32]
0.11881 ± 0.00131	+MChad	2-jet rate (N^{3}LO+N^{3}LL+MChad+N^{2}LO$_{m_b}$)	[31]

Global fits and wide $\sqrt{s}$ range → best precision.
The discrepancy between the analytic and MC hadronization should be clarified.

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[4] Credits to Ref. [37]

Groomed event shapes

The “good” range for the fits is defined as everything on the right of $\tau = 6 \times 10^{-2}$. 70-80% of the cross-section lies in that region. But the quote from Ref. [17]. is “ By looking at the vertical lines on the cross-section plots (left-hand side of Fig. 2), which indicate where non-perturbative corrections reach the 10% level, we see that for the un-groomed case, only a third of the cross-section is in the perturbative region, while this fraction nearly doubles in the case of soft-drop thrust “

Bibliography I

[1] Alipour Tehrani, N. and others,
FCC-ee: Your Questions Answered,
CERN Council Open Symposium on the Update of European Strategy for Particle Physics, Blondel, A. and Janot, P. (eds.).
(2019).
Also in preprint 1906.02693.
`arXiv:1906.02693`.

[2] Niemi, Arto and Penttinen, Jussi-Pekka,
Availability and critical systems of the Future Circular Electron-Positron Collider.
Nucl. Instrum. Meth. A **963**, 163759 (2020).

[3] FCC, Abada, A. and others,
FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1.
Eur. Phys. J. C **79**, 474 (2019).

[4] FCC, Abada, A. and others,
FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2.
Eur. Phys. J. ST **228**, 261 (2019).

[5] A. Verbytskyi et al.,
High-precision $\alpha_S(M_Z)$ determinations from future FCC-*ee* $e^+e^- \rightarrow$ *hadrons* data below the Z peak, 2020,
`https://www.snowmass21.org/docs/files/summaries/EF/SNOWMASS21-EF5_EF4_Andrii_Verbytskyi-208.pdf`.

[6] Benson, Sean and Gligorov, V. V. and Vesterinen, Mika Anton and Williams, Mike,
The LHCb Turbo Stream.
J. Phys. Conf. Ser. **664**, 082004 (2015).

Bibliography II

[7] A. Verbytskyi,
Studies of correlations between measurements of jet observables.
JINST **12**, P04013 (2017).
`arXiv:1609.06898`.

[8] Particle Data Group, Olive, K. A. and others,
Review of Particle Physics.
Chin. Phys. C **38**, 090001 (2014).

[9] C. Basham et al.,
Energy correlations in electron-positron annihilation: testing QCD.
Phys.Rev.Lett. **41**, 1585 (1978).

[10] OPAL, Abbiendi, G. and others,
Measurement of event shape distributions and moments in e+ e- —> hadrons at 91-GeV - 209-GeV and a determination of alpha(s).
Eur. Phys. J. C **40**, 287 (2005).
`arXiv:hep-ex/0503051`.

[11] SLD Collaboration, K. Abe et al.,
Measurement of α_S from energy-energy correlations at the Z^0 resonance.
Phys. Rev. **D50**, 5580 (1994).
`arXiv:hep-ex/9405006`.

[12] H1, ZEUS, Abramowicz, H. and others,
Combination of differential $D^{*\pm}$ cross-section measurements in deep-inelastic ep scattering at HERA.
JHEP **09**, 149 (2015).
`arXiv:1503.06042`.

Bibliography III

[13] La Cagnina, Salvatore and Kroninger, Kevin and Kluth, Stefan and Verbytskyi, Andrii,
A Bayesian tune of the Herwig Monte Carlo event generator.
(2023).
`arXiv:2302.01139`.

[14] OPAL, Akers, R. and others,
Determination of event shape distributions and alpha-s(b) from Z0 —> b anti-b events at LEP.
Z. Phys. C **65**, 31 (1995).

[15] Mateu, Vicent and Rodrigo, Germán,
Oriented Event Shapes at $\mathrm{N}^3\mathrm{LL} + O(\alpha_S^2)$.
JHEP **11**, 030 (2013).
`arXiv:1307.3513`.

[16] Gehrmann, T. and Glover, E. W. N. and Huss, A. and Niehues, J. and Zhang, H.,
NNLO QCD corrections to event orientation in e^+e^- annihilation.
Phys. Lett. B **775**, 185 (2017).
`arXiv:1709.01097`.

[17] J. Baron, S. Marzani and V. Theeuwes,
Soft-Drop Thrust.
JHEP **08**, 105 (2018).
`arXiv:1803.04719`.
[erratum: JHEP05,056(2019)].

[18] Kardos, Adam and Somogyi, Gábor and Trócsányi, Zoltán,
Soft-drop event shapes in electron–positron annihilation at next-to-next-to-leading order accuracy.
Phys. Lett. B **786**, 313 (2018).
`arXiv:1807.11472`.

Bibliography IV

[19] OPAL, Abbiendi, G. and others,
Determination of alpha(s) using jet rates at LEP with the OPAL detector.
Eur. Phys. J. C **45**, 547 (2006).
`arXiv:hep-ex/0507047`.

[20] Becker, Sebastian and Goetz, Daniel and Reuschle, Christian and Schwan, Christopher and Weinzierl, Stefan,
NLO results for five, six and seven jets in electron-positron annihilation.
Phys. Rev. Lett. **108**, 032005 (2012).
`arXiv:1111.1733`.

[21] Coloretti, Guglielmo and Gehrmann-De Ridder, Aude and Preuss, Christian T.,
QCD predictions for event-shape distributions in hadronic Higgs decays.
JHEP **06**, 009 (2022).
`arXiv:2202.07333`.

[22] Particle Data Group, M. Tanabashi et al.,
Review of Particle Physics.
Phys. Rev. **D98**, 030001 (2018).

[23] Perez-Ramos, Redamy and DEnterria, David,
α_S from soft QCD jet fragmentation functions,
pp. 94–99.
(2019),
doi:10.22323/1.365.0005.

[24] OPAL, Abbiendi, G. and others,
Measurement of the charm structure function $F2(c)(gamma)$ of the photon at LEP.
Phys. Lett. B **539**, 13 (2002).
`arXiv:hep-ex/0206021`.

Bibliography V

[25] J. Bellm et al.,
Herwig 7.0/Herwig++ 3.0 release note.
Eur. Phys. J. **C76**, 196 (2016).
arXiv:1512.01178.

[26] Jadach, S. and Ward, B. F. L. and Was, Z. and Yost, S. A. and Siodmok, A.,
Multi-photon Monte Carlo event generator KKMCee for lepton and quark pair production in lepton colliders.
Comput. Phys. Commun. **283**, 108556 (2023).
arXiv:2204.11949.

[27] E. Bothmann et al.,
Event generation with SHERPA 2.2.
(2019).
arXiv:1905.09127.

[28] Bierlich, Christian and others,
A comprehensive guide to the physics and usage of PYTHIA 8.3 (2022).
.
arXiv:2203.11601.

[29] Kilian, Wolfgang and Ohl, Thorsten and Reuter, Jurgen,
WHIZARD: Simulating Multi-Particle Processes at LHC and ILC.
Eur. Phys. J. C **71**, 1742 (2011).
arXiv:0708.4233.

[30] A. Buckley et al.,
Rivet user manual.
Comput. Phys. Commun. **184**, 2803 (2013).
arXiv:1003.0694.

Bibliography VI

[31] A. Verbytskyi et al.,
High precision determination of α_s from a global fit of jet rates.
JHEP **08**, 129 (2019).
arXiv:1902.08158.

[32] A. Kardos et al.,
Precise determination of $\alpha_S(M_Z)$ from a global fit of energy–energy correlation to NNLO+NNLL predictions.

Eur. Phys. J. **C78**, 498 (2018).
arXiv:1804.09146.

[33] C. Bierlich et al.,
Robust Independent Validation of Experiment and Theory: Rivet version 3.
SciPost Phys. **8**, 026 (2020).
arXiv:1912.05451.

[34] L3 Collaboration, P. Achard et al.,
Studies of hadronic event structure in e^+e^- annihilation from 30 GeV to 209 GeV with the L3 detector.
Phys. Rept. **399**, 71 (2004).
arXiv:hep-ex/0406049.

[35] OPAL, G. Abbiendi et al.,
Measurement of α_s with Radiative Hadronic Events.
Eur. Phys. J. **C53**, 21 (2008).
arXiv:0902.1128.

[36] JADE Collaboration, S. Bethke et al.,
Determination of the Strong Coupling α_s from hadronic Event Shapes with $\mathcal{O}(\alpha_s^3)$ and resummed QCD predictions using JADE Data.
Eur. Phys. J. **C64**, 351 (2009).
arXiv:0810.1389.

Bibliography VII

[37] G.P. Salam,
The strong coupling: a theoretical perspective,
in From My Vast Repertoire . . . : Guido Altarelli's Legacy, eds. Levy, Aharon and Forte, Stefano and Ridolfi, Giovanni, pp. 101–121.
2019,
doi:10.1142/97898132380530_007.

[38] G. Dissertori et al.,
Precise determination of the strong coupling constant at NNLO in QCD from the three-jet rate in electron-positron annihilation at LEP.
Phys. Rev. Lett. **104**, 072002 (2010).
`arXiv:0910.4283`.

[39] JADE Collaboration, J. Schieck et al.,
Measurement of the strong coupling α_S from the three-jet rate in e^+e^- - annihilation using JADE data.
Eur. Phys. J. **C73**, 2332 (2013).
`arXiv:1205.3714`.

[40] G. Dissertori et al.,
Determination of the strong coupling constant using matched NNLO+NLLA predictions for hadronic event shapes in e^+e^- annihilations.
JHEP **08**, 036 (2009).
`arXiv:0906.3436`.

[41] OPAL Collaboration, G. Abbiendi et al.,
Determination of α_s using OPAL hadronic event shapes at $\sqrt{s} = 91 - 209$ GeV and resummed NNLO calculations.
Eur. Phys. J. **C71**, 1733 (2011).
`arXiv:1101.1470`.

Bibliography VIII

[42] R.A. Davison and B.R. Webber,
Non-Perturbative Contribution to the Thrust Distribution in e+ e- Annihilation.
Eur. Phys. J. **C59**, 13 (2009).
arXiv:0809.3326.

[43] T. Gehrmann, G. Luisoni and P.F. Monni,
Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution.
Eur. Phys. J. **C73**, 2265 (2013).
arXiv:1210.6945.

[44] R. Abbate et al.,
Thrust at N^3LL with power corrections and a precision global fit for $\alpha_S(M_Z)$.
Phys.Rev. **D83**, 074021 (2011).
arXiv:1006.3080.

[45] A. Hoang et al.,
Precise determination of α_s from the C-parameter distribution.
Phys. Rev. **D91**, 094018 (2015).
arXiv:1501.04111.