The status of the logarithmic accuracy of dipole showers

Melissa van Beekveld

within **PanScales**: *Mrinal Dasgupta*, Frederic Dreyer, Basem El Menoufi, *Silvia Ferrario Ravasio*, Keith Hamilton, *Jack Helliwell, Alexander Karlberg*, Rok Medves, *Pier Monni,* Gavin Salam, Ludovic Scyboz, *Alba Soto-Ontoso*, *Gregory Soyez*, Rob Verheyen

THE ROYAL SOCIETY

erc

Science & Technology Facilities Council

UNIVERSITY OF OXFORD

Progress in improving the PS accuracy

- **Assessing the logarithmic accuracy of a shower**
  Herwig [1904.11866, 2107.04051], Deductor [2011.04777], Forshaw, Holguin, Plätzer [2003.06400]
  PanScales [1805.09327, 2002.11114], Alaric [2110.05964], …

- **Triple collinear / double soft splittings**
  Dulat, Höche, Krauss, Gellersen, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964]
  Li & Skands [1611.00013], Löschner, Plätzer, Simpson Dore [2112.14454], …

- Matching to fixed-order **see Alexander's talk**
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], …
  NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], …
  NNNLO; Prestel [2106.03206], Bertone, Prestel [2202.01082]

- Colour (and spin) correlations **see Simon's talk**
  Forshaw, Holguin, Plätzer, Sjödahl [1201.0260, 1808.00332, 1905.08686, 2007.09648, 2011.15087]
  Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955], Plätzer & Ruffa [2012.15215]
  PanScales [2011.10054, 2103.16526, 2111.01161], …

- Electroweak corrections
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], …

Disclaimer: list is not exhaustive

Addressing the accuracy of a parton shower

For a given observable, one may address the question of accuracy systematically

At fixed order

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + \dots$$

At all orders using analytic resummation

$$\Sigma^{\text{NLL}}(\lambda \equiv \alpha_s L) = \exp(\underbrace{\frac{1}{\alpha_s} g_1(\lambda)}_{\mathcal{O}(1/\alpha_s)} \quad + \quad \underbrace{g_2(\lambda)}_{\mathcal{O}(1)} \quad + \quad \dots) \qquad \Sigma^{\text{NDL}}(\xi \equiv \alpha_s L^2) = h_1(\xi) + \sqrt{\alpha_s} h_2(\xi) + \dots$$

in resummation regime where $\alpha_s L = \mathcal{O}(1)$

How to design showers that are NLL/NDL accurate for *all* observables?

PanScales NLL/NDL correctness requirements

Resummation

Require single-logarithmic accuracy for suitably defined observables

- global event shapes ($\alpha_s^n L^n$) Probe the structure of double-log Sudakov resummation in the shower
- parton distribution / fragmentation functions ($\alpha_s^n L^n$) Probe the hard-collinear region
- non-global observables ($\alpha_s^n L^n$) Probe the soft wide-angle region
- particle/jet multiplicity ($\alpha_s^n L^{2n-1}$) Probe nested emissions in the soft and collinear regions

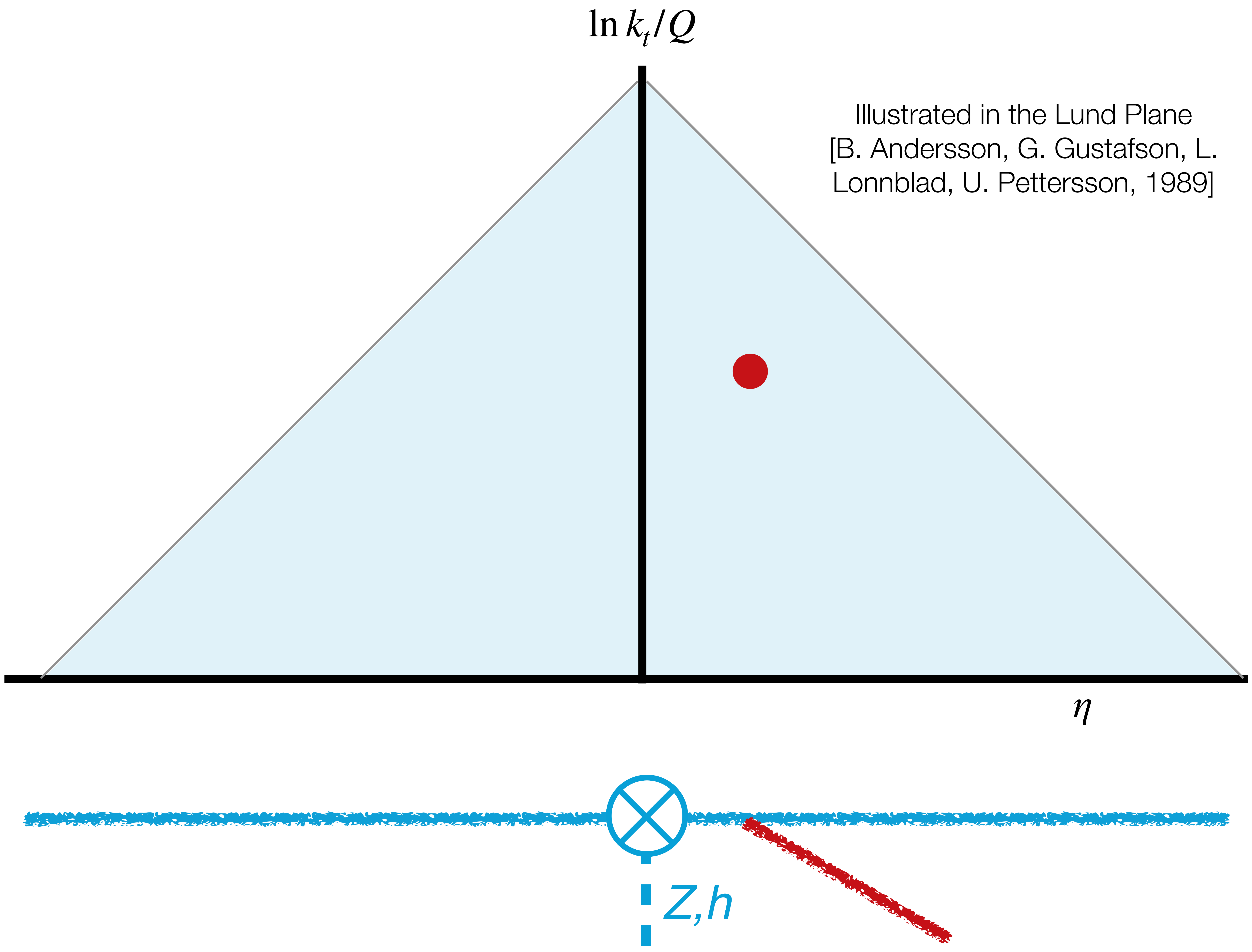
Test the basic underlying concept

Require correctness of effective matrix elements generated by the shower for well-separated emissions (only thing one can do if a resummation cannot be formulated)

Testing the underlying concept

- QCD amplitudes factorise in soft and collinear limits
- Shower has factorised $1 \to 2$ splitting kinematics implemented
- Shower must reproduce the factorised amplitude when emissions are ‘sufficiently’ independent

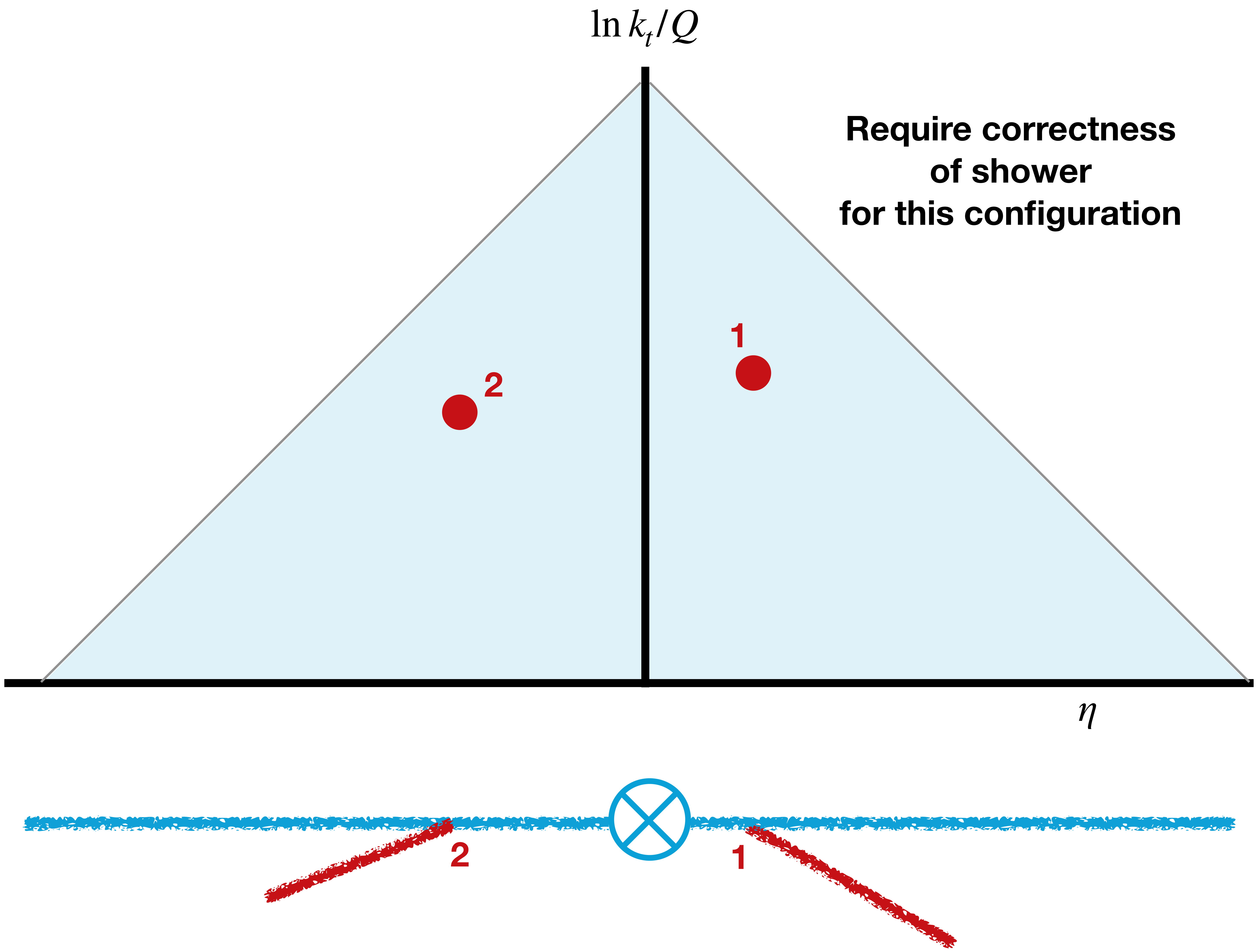
This means that any particle emitted after particle 1 may not influence the kinematics of particle 1!



Testing the underlying concept

- QCD amplitudes factorise in soft and collinear limits
- Shower has factorised $1 \to 2$ splitting kinematics implemented
- Shower must reproduce the factorised amplitude when emissions are ‘sufficiently’ independent

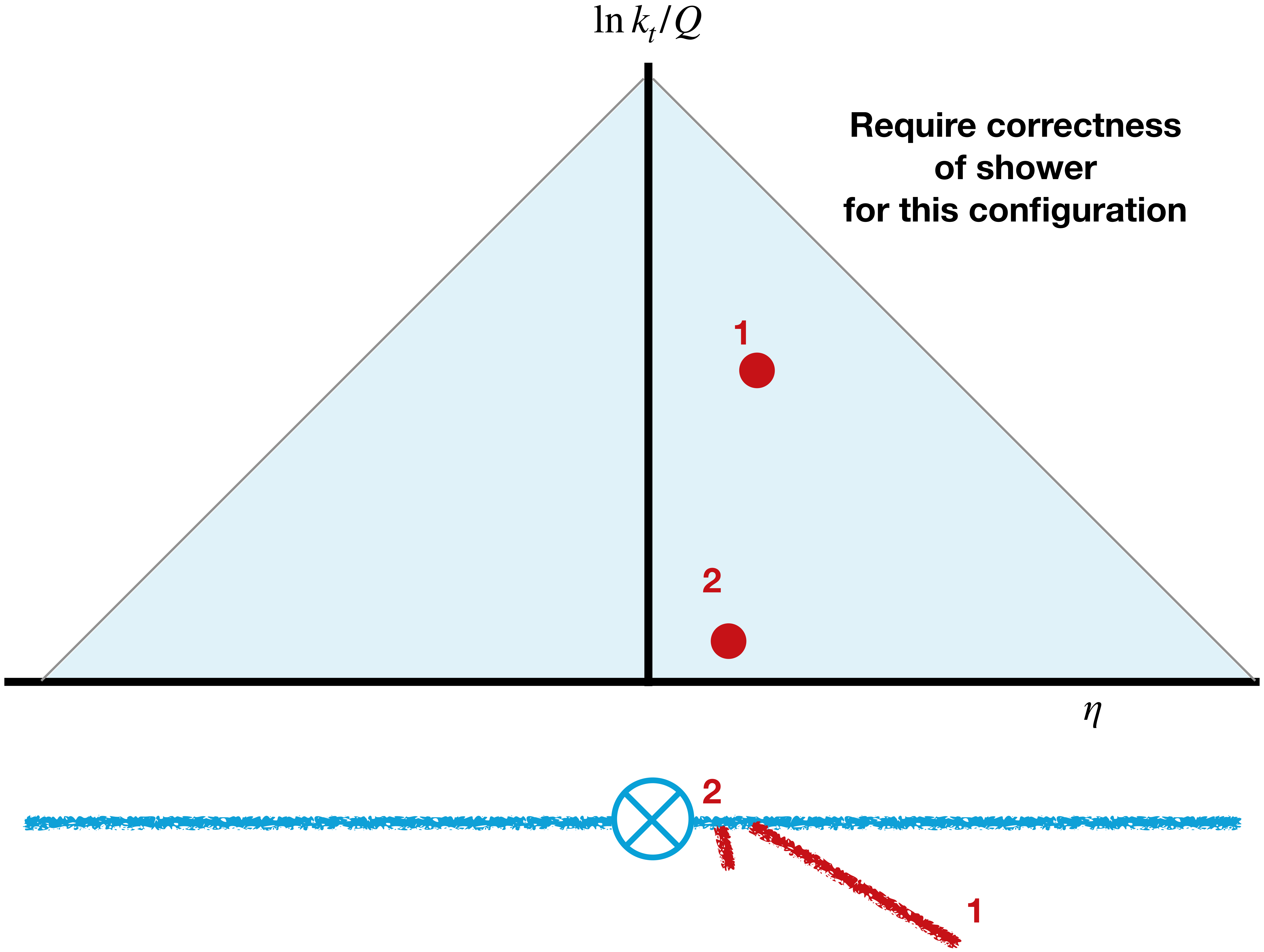
This means that any particle emitted after particle 1 may not influence the kinematics of particle 1!



Testing the underlying concept

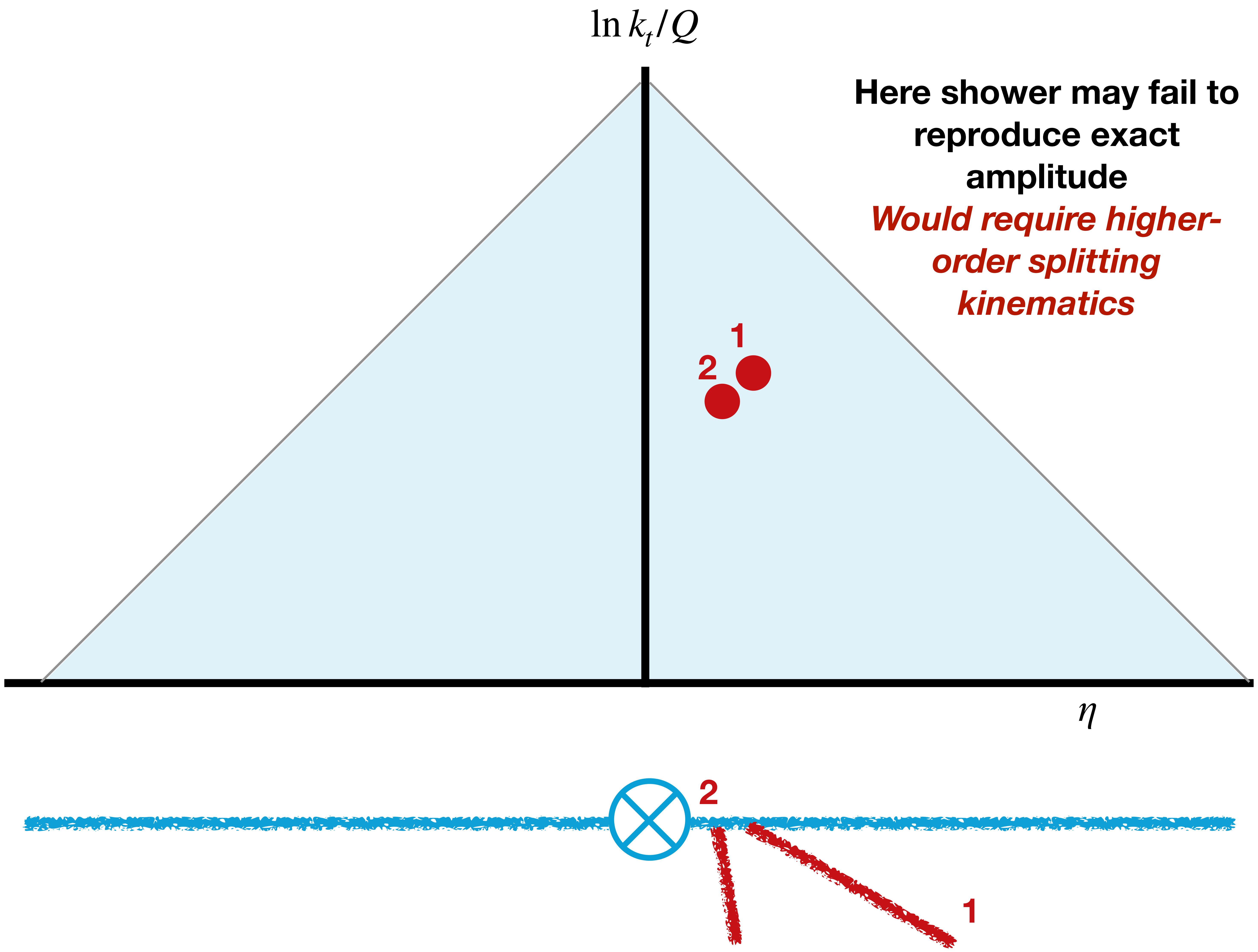
- QCD amplitudes factorise in soft and collinear limits
- Shower has factorised $1 \to 2$ splitting kinematics implemented
- Shower must reproduce the factorised amplitude when emissions are 'sufficiently' independent

This means that any particle emitted after particle 1 may not influence the kinematics of particle 1!



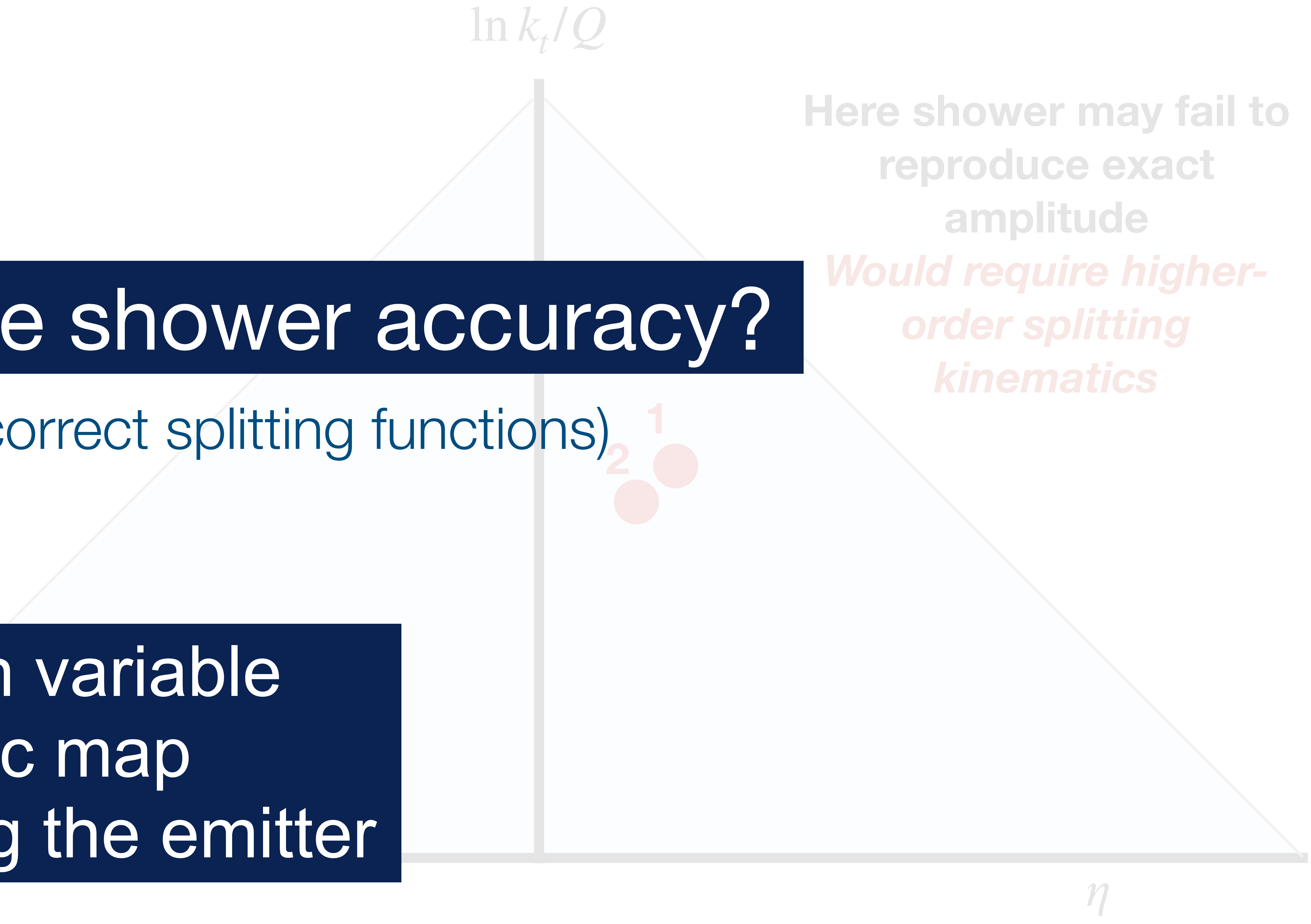
Testing the underlying concept

- QCD amplitudes factorise in soft and collinear limits
- Shower has factorised $1 \to 2$ splitting kinematics implemented
- Shower must reproduce the factorised amplitude when emissions are ‘sufficiently’ independent



Testing the underlying concept

- QCD amplitudes factorise in soft and collinear limits
- Shower has factorised $1 \to 2$ splitting kinematics implemented
- Shower must reproduce the factorised amplitude when emissions are 'sufficiently' independent



Here shower may fail to reproduce exact amplitude
Would require higher-order splitting kinematics

What determines the shower accuracy?

(apart from having the correct splitting functions)

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

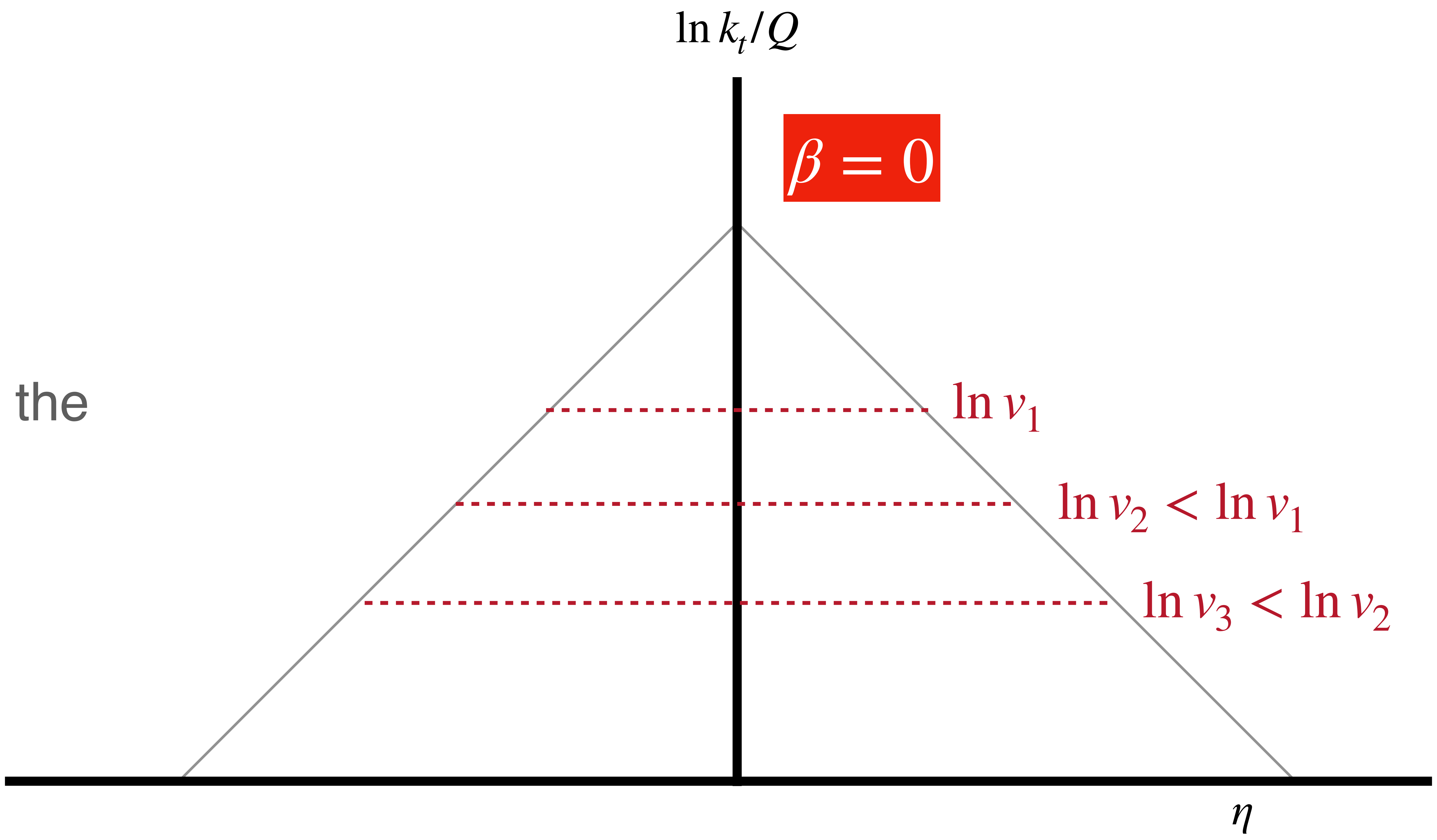
What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

A parton shower orders emissions

The evolution variable v tells us which emissions come first, and which later in the showering process

We use the definition $v \simeq k_t e^{-\beta|\eta|}$



Transverse-momentum ordered with $\beta_{PS} = 0$

Choice for most dipole parton showers

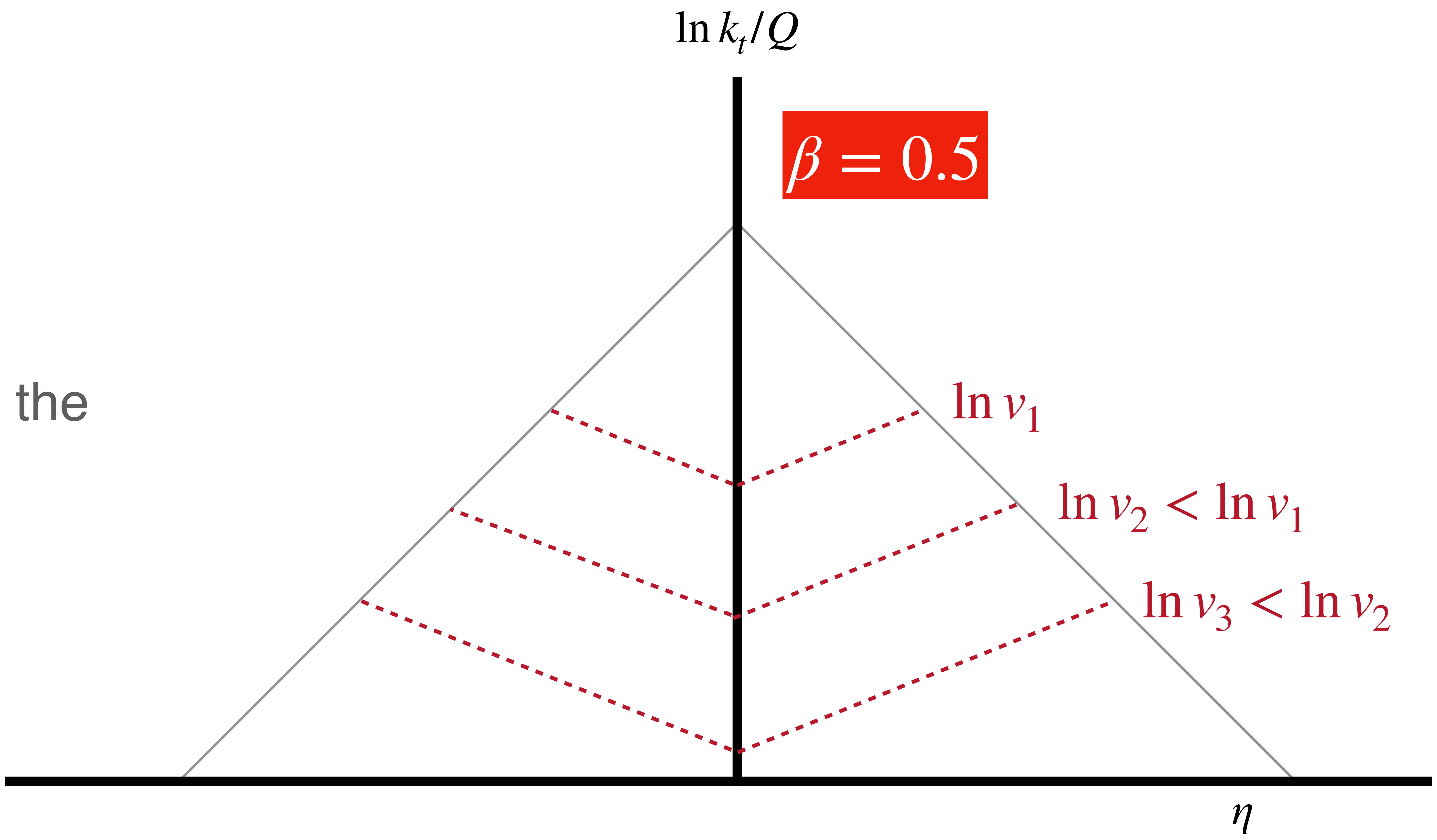
What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

A parton shower orders emissions

The evolution variable v tells us which emissions come first, and which later in the showering process

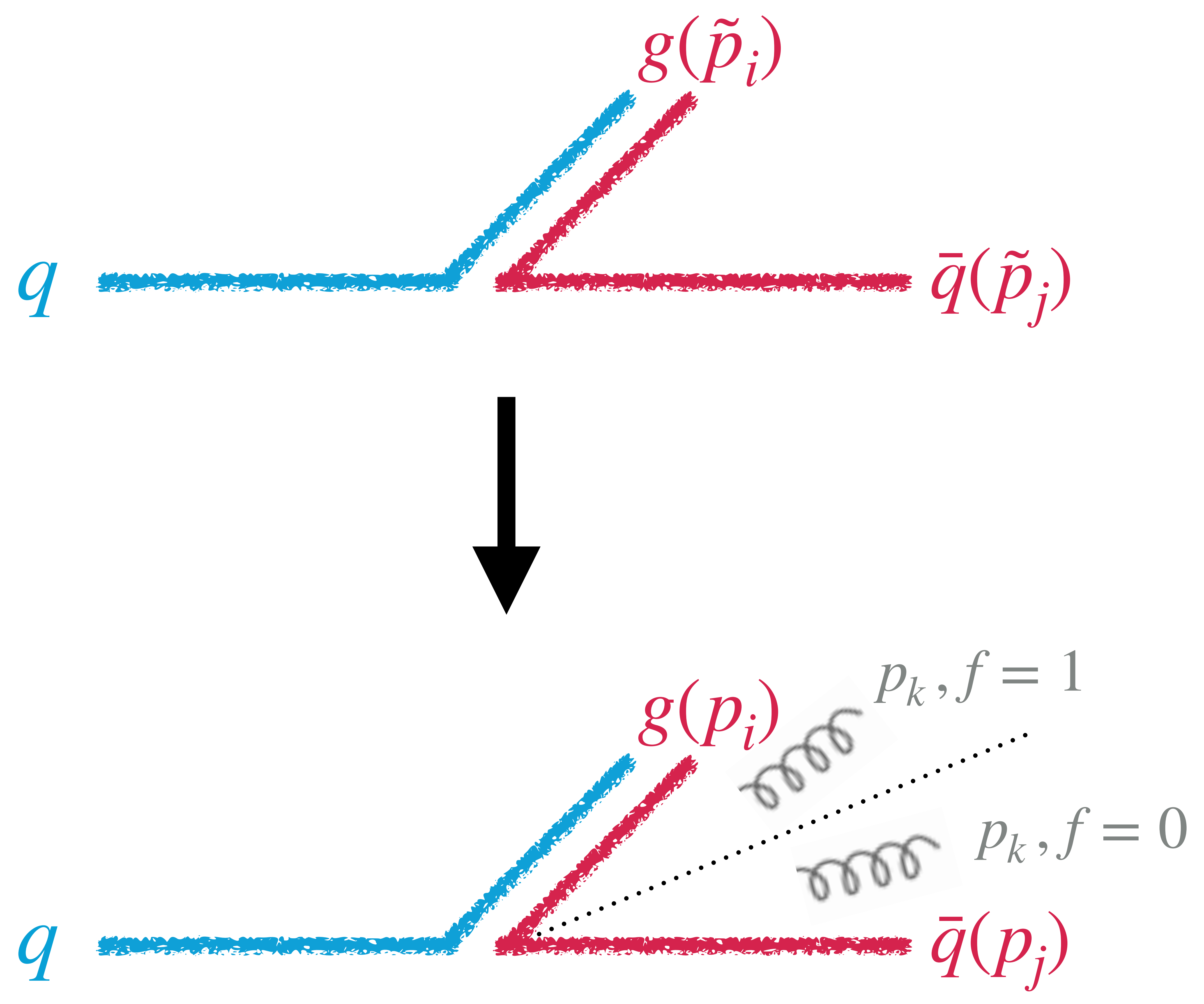
We use the definition $v \simeq k_t e^{-\beta|\eta|}$



Introduce some angular dependence with $\beta > 0$

Angular-ordering (e.g. as implemented in Herwig) will not be considered here

What determines the shower accuracy?



Local kinematic map

$$p_i = a_i\tilde{p}_i + b_i\tilde{p}_j + fk_\perp$$

$$p_j = a_j\tilde{p}_i + b_j\tilde{p}_j + (1-f)k_\perp$$

$$p_k = a_k\tilde{p}_i + b_k\tilde{p}_j + k_\perp$$

Mapping coefficients depend on

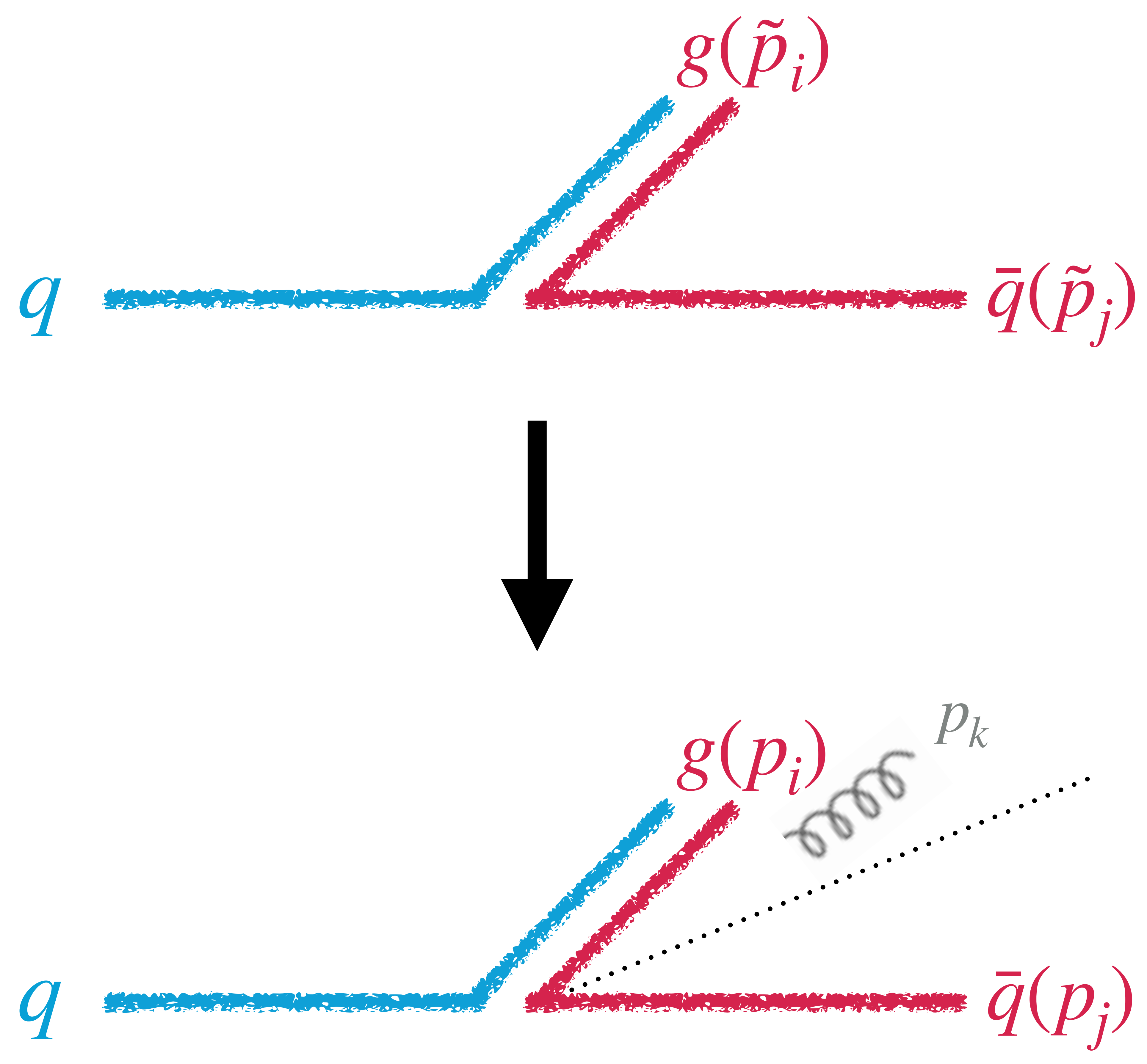
- Evolution variable $\ln v$
- Rapidity η

Dipole: step function for f

Antenna: smooth transition for f

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter



Global kinematic map

$$p_i = a_i \tilde{p}_i$$

$$p_j = b_j \tilde{p}_j$$

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp$$

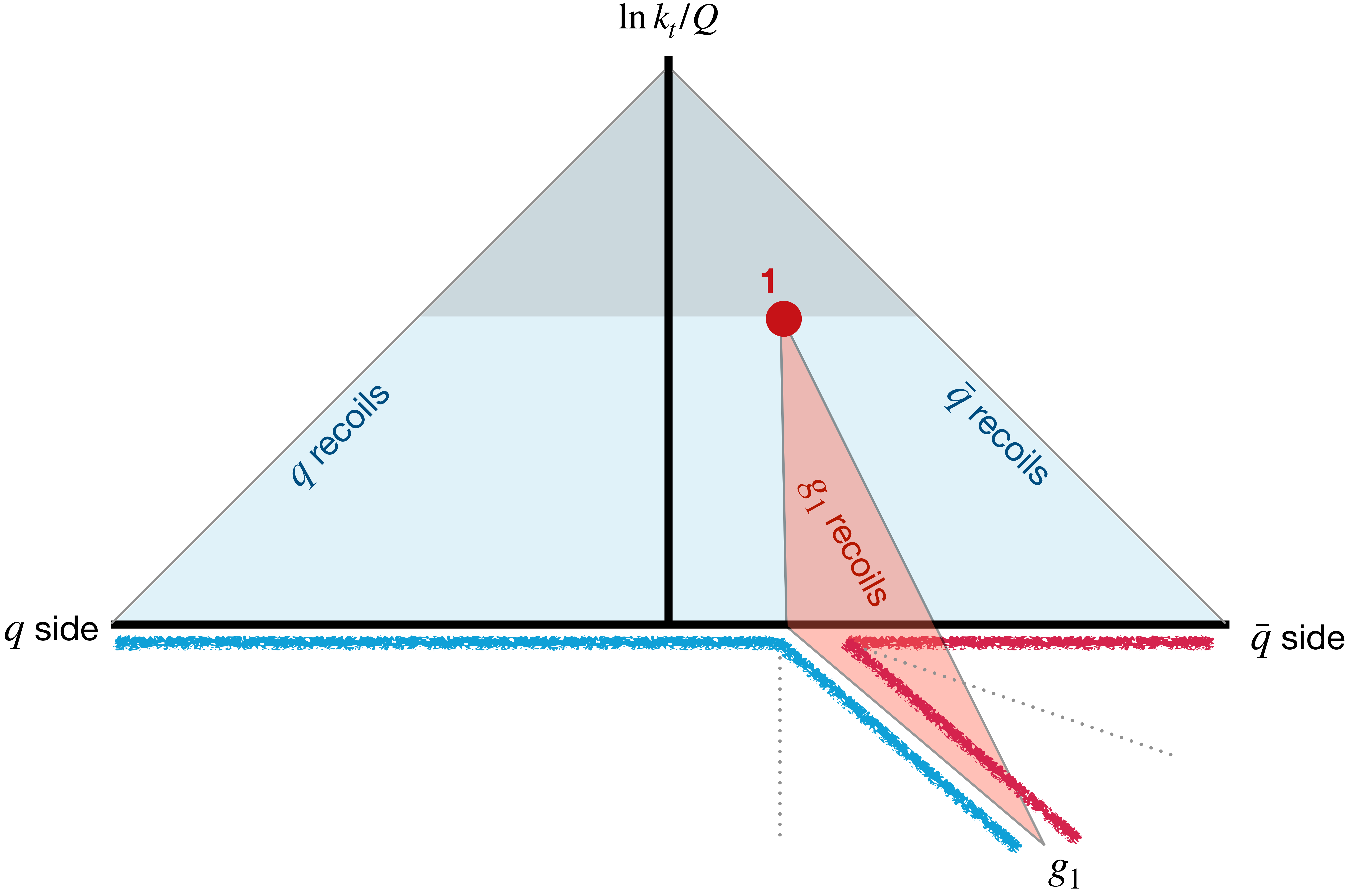
Boost (part of) event after each emission to restore momentum conservation

Choice: global in some/all $+/-$ and $\perp$ components

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

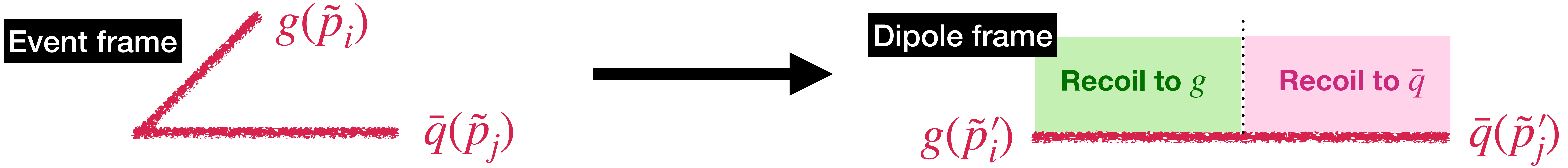
Expected attribution of recoil



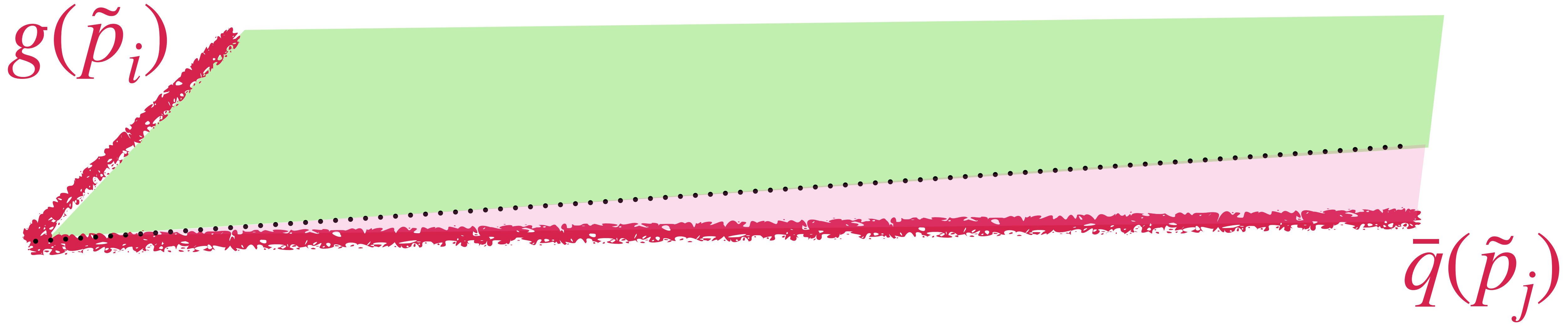
What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. **Choosing the emitter**

Standard dipole showers distinguish the emitter from the spectator at $\eta = 0$ in the CM dipole frame



Boosting back to the event frame…



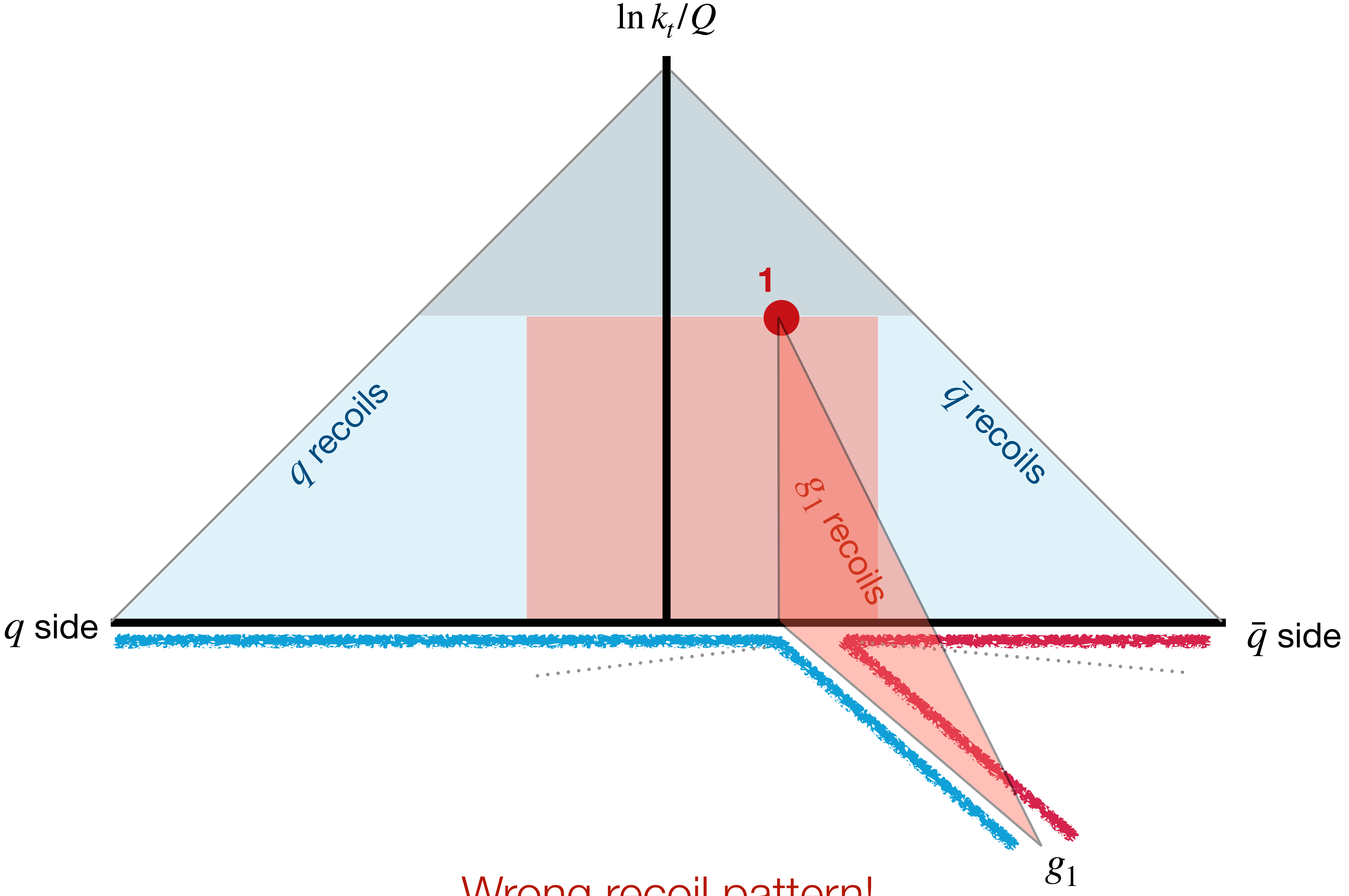
Leads to an incorrect (and quite unphysical) recoil picture!

Physical attribution of recoil

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

Recoil attribution for transverse-momentum ordered local shower (choosing emitter in dipole frame)

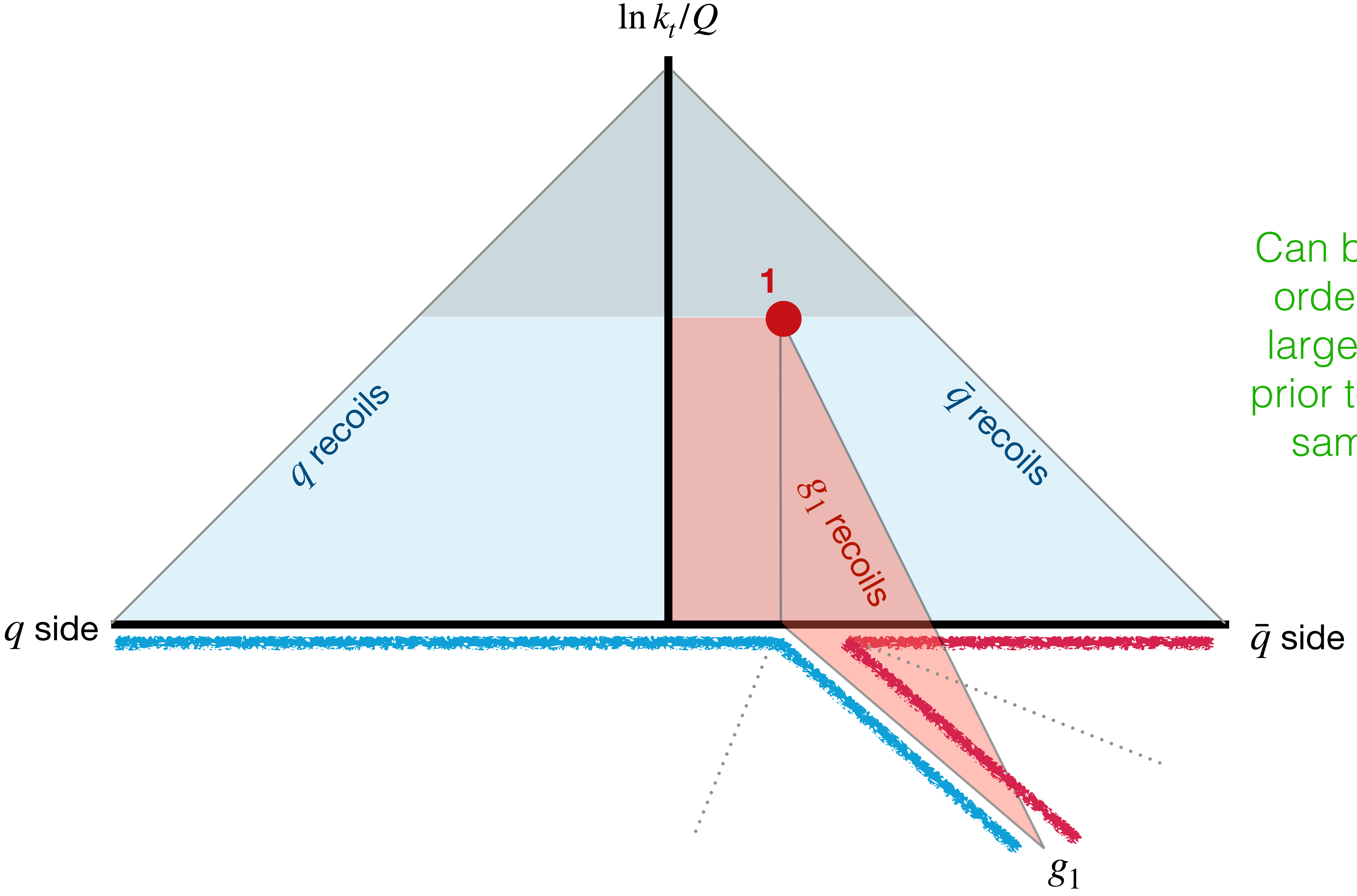


Wrong recoil pattern!

What determines the shower accuracy?

1. Evolution variable
2. Kinematic map
3. Choosing the emitter

Recoil attribution for transverse-momentum ordered local shower (choosing emitter in event frame)



Can be fixed using a different ordering variable, such that large-angle emissions come prior to small-angle ones (with same k_t), or a global map

Less wrong, but still not correct recoil pattern!

The current status of possibly NLL-accurate dipole showers

Possible NLL dipole-shower solutions for e^+e^-

		Ordering	Kinematic map: Dipole-local	Kinematic map: Global	Tests
PanScales showers [2002.11114]	**PanLocal** (Dipole and antenna)	$0 < \beta < 1$	$+, -, \perp$		Fixed- and all-order numerical tests for different observables for e^+e^- and pp (colour singlet)
	PanGlobal	$0 \leq \beta < 1$	$+, -$	$\perp$	
Alaric [2208.06057]		$\beta = 0$	$+$	$-, \perp$	Numerical tests for global event shapes
Deductor [2011.04777]	**Deductor k_t**	$\beta = 0$	$+$ (Also formulation with $+, -, \perp$)	$-, \perp$	Analytical and to some extent numerical for thrust
	Deductor Λ	$\beta = 1$	$+$	$-, \perp$	
Manchester-Vienna [2003.06400]		$\beta = 0$	$+$	$-, \perp$	Analytical for thrust and multiplicity

Showers also differ on the implementation of the splitting functions and how the global imbalance is redistributed

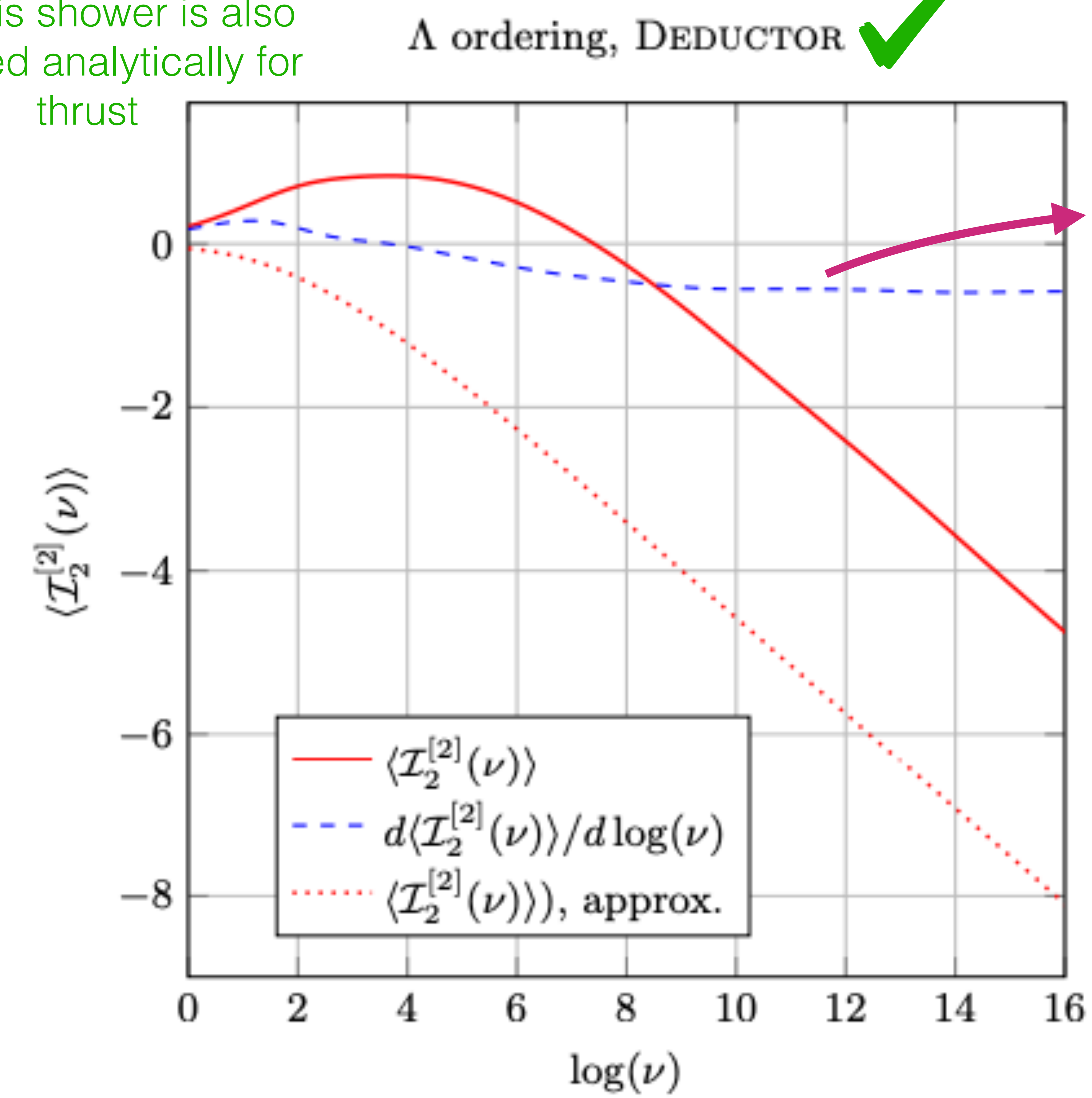
All have different approaches to assess NLL accuracy

Nagy, Soper [2011.04777]

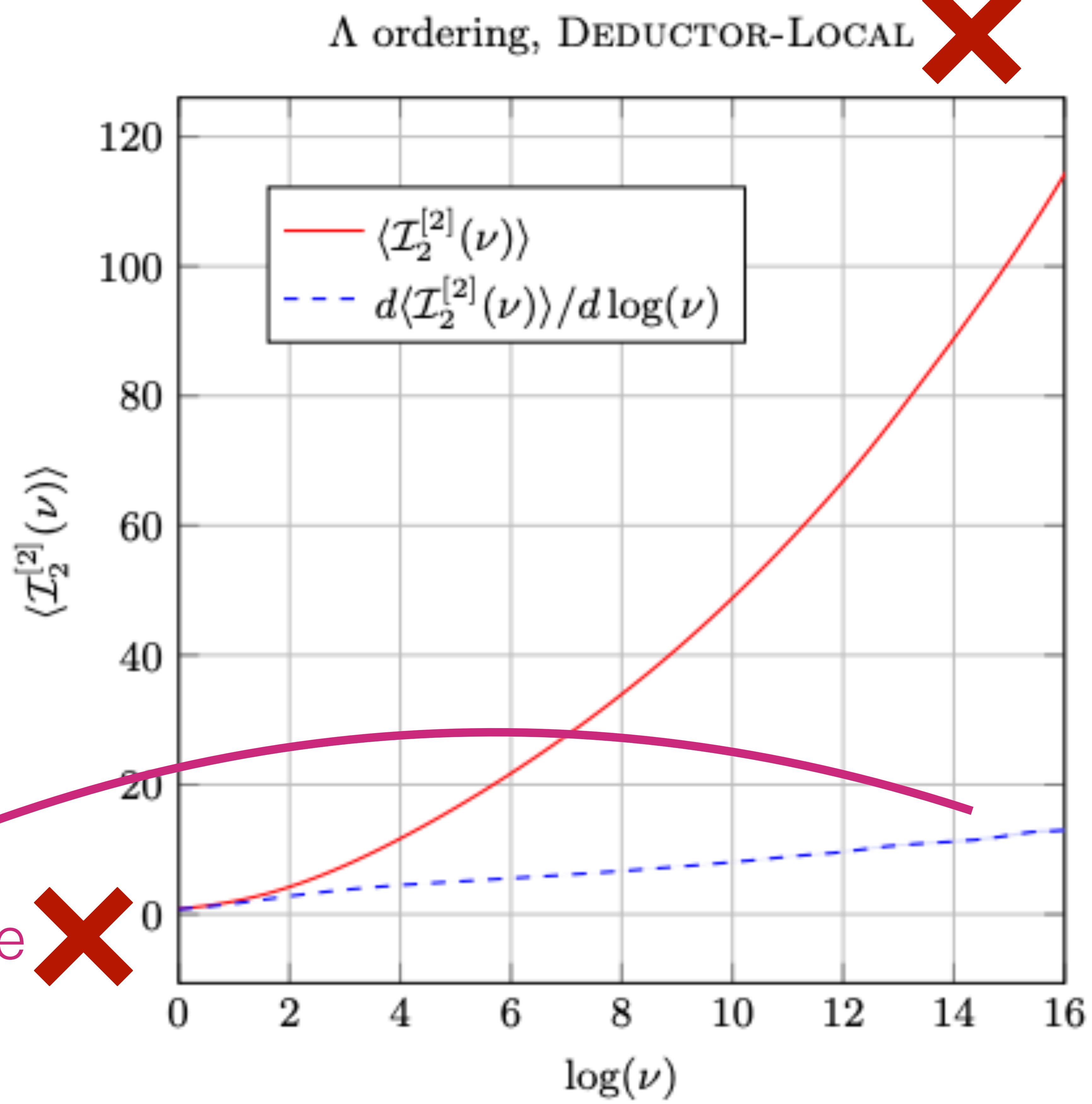
Examine behaviour of higher-order shower operators $I_n^{[k\geq 2]}(\nu)$ to the Laplace-space thrust

$I_n^{[k\geq 2]}(\nu)$ may not contain L^{n+1} / L^n for an LL/NLL shower

n.b. this shower is also checked analytically for thrust



Approaches a constant
→ no L^3 or L^2 term



A residual L^2 dependence

Interesting and important to perform the all-order checks, and to see other observables...

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

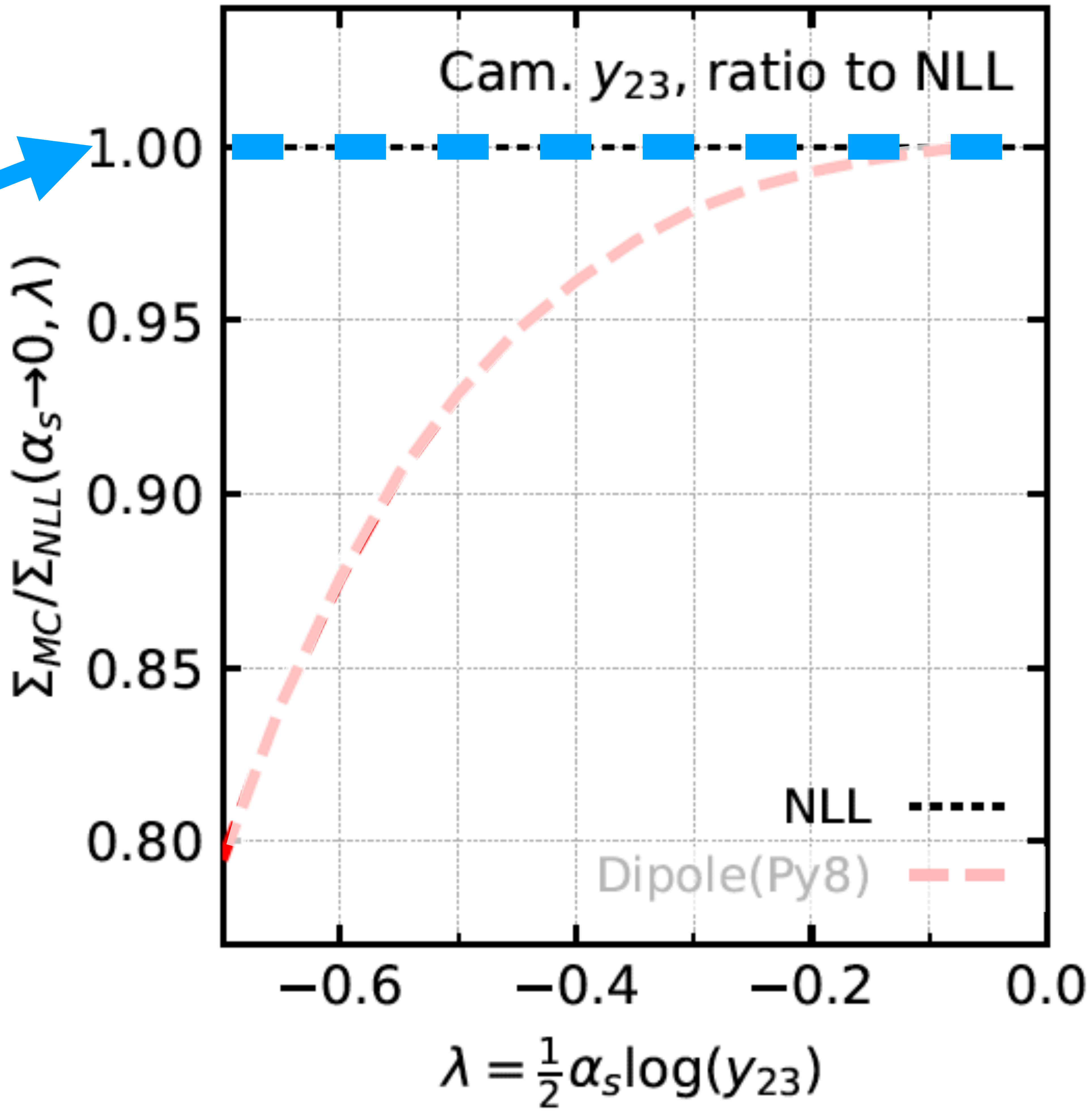
Consider e.g. Cambridge y_{23}

Observable with standard resummation at NLL of the form

$$\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s) = \exp\left[-L g_1(\lambda) + g_2(\lambda)\right]$$

with $\lambda = \alpha_s \ln \sqrt{y_{23}}$

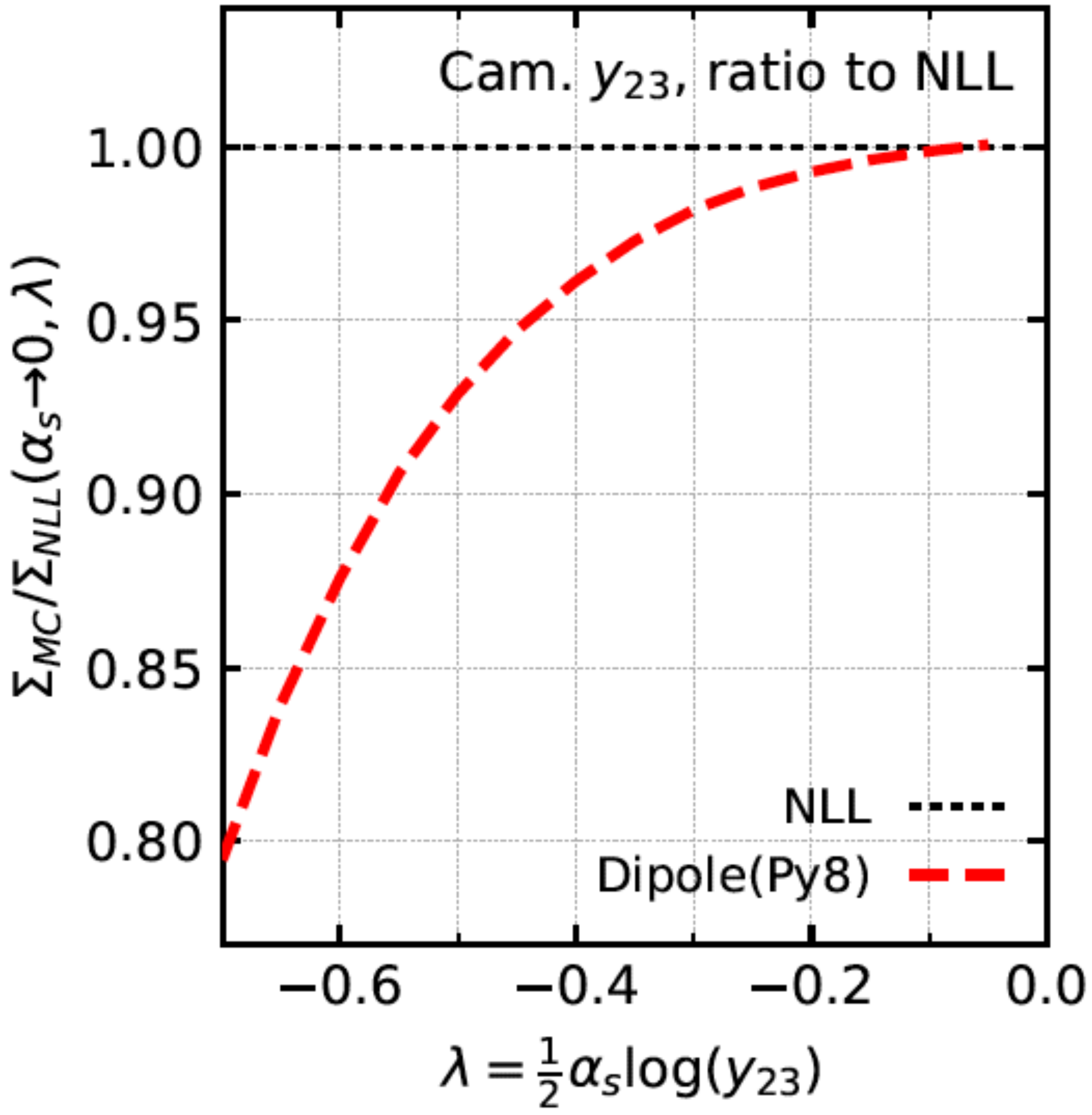
Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \overset{!}{=} 1$



Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \overset{!}{=} 1$

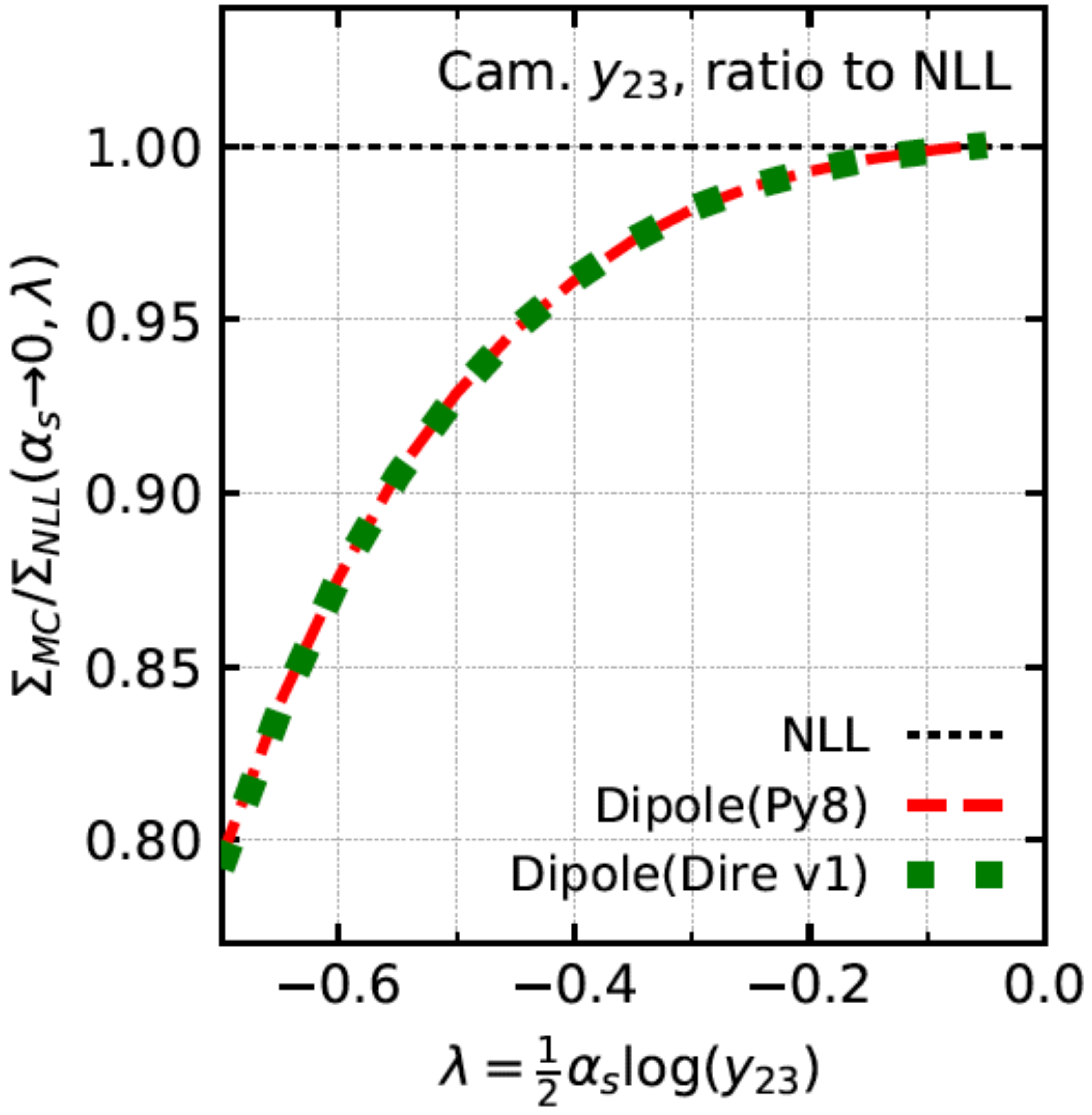
- Pythia8 deviates from NLL ✗



Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \overset{!}{=} 1$

- Pythia8 deviates from NLL ✗
- Dire looks identical to Pythia8 ✗

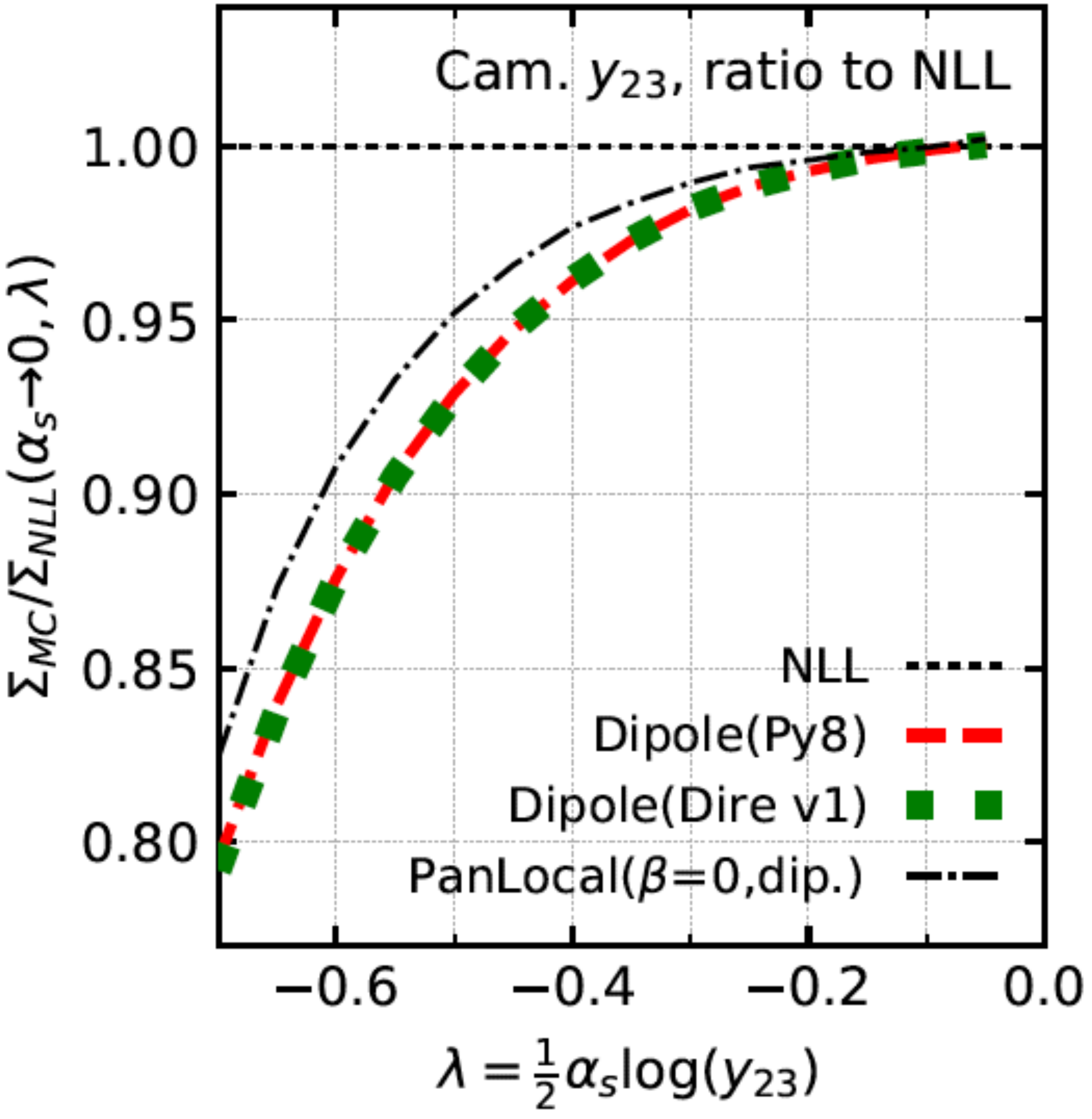


PanScales

Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \stackrel{!}{=} 1$

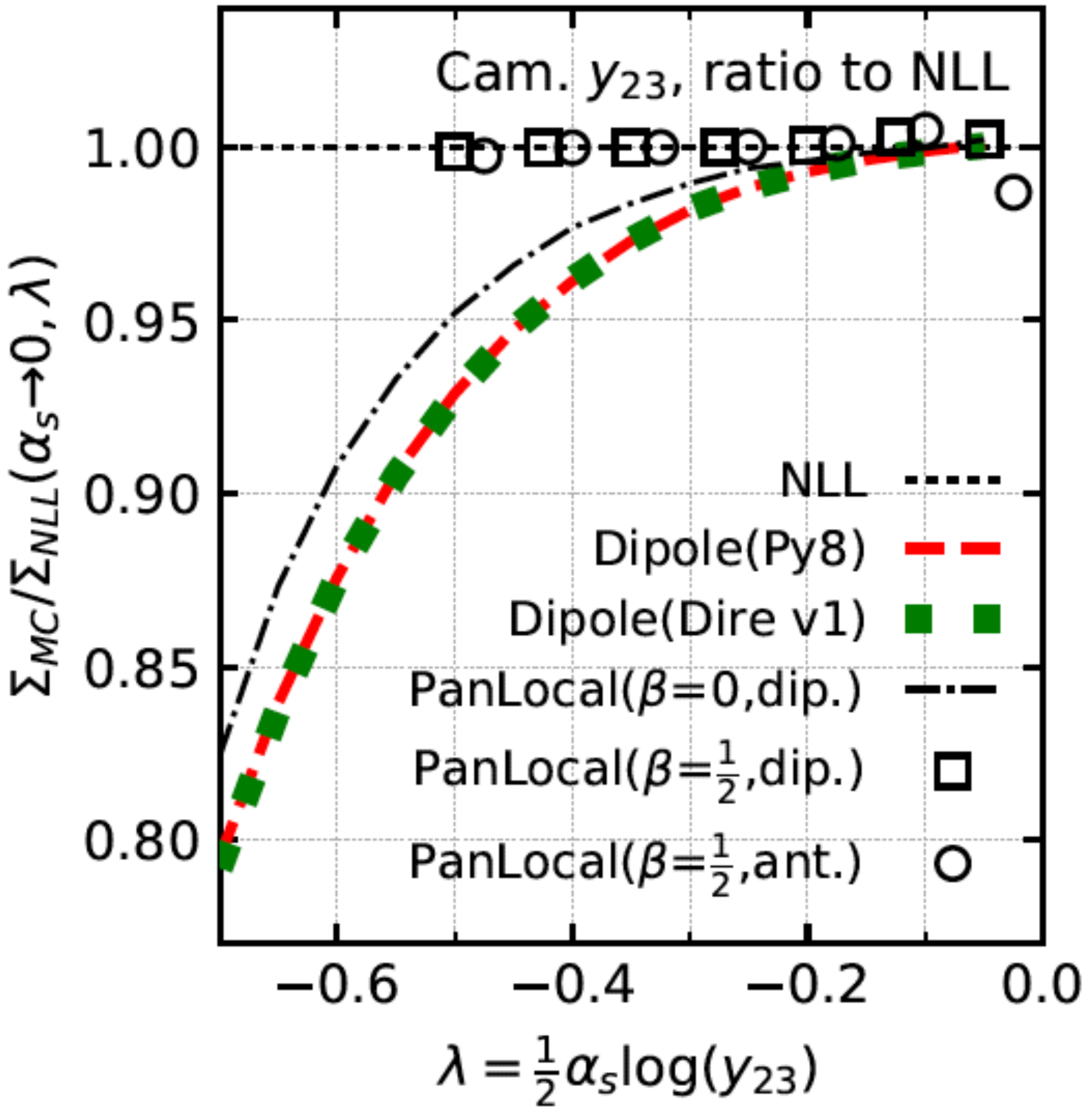
- Pythia8 deviates from NLL ✗
- Dire looks identical to Pythia8 ✗
- PanLocal($\beta = 0$) softens the issue, ✗ but not NLL accurate



Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \overset{!}{=} 1$

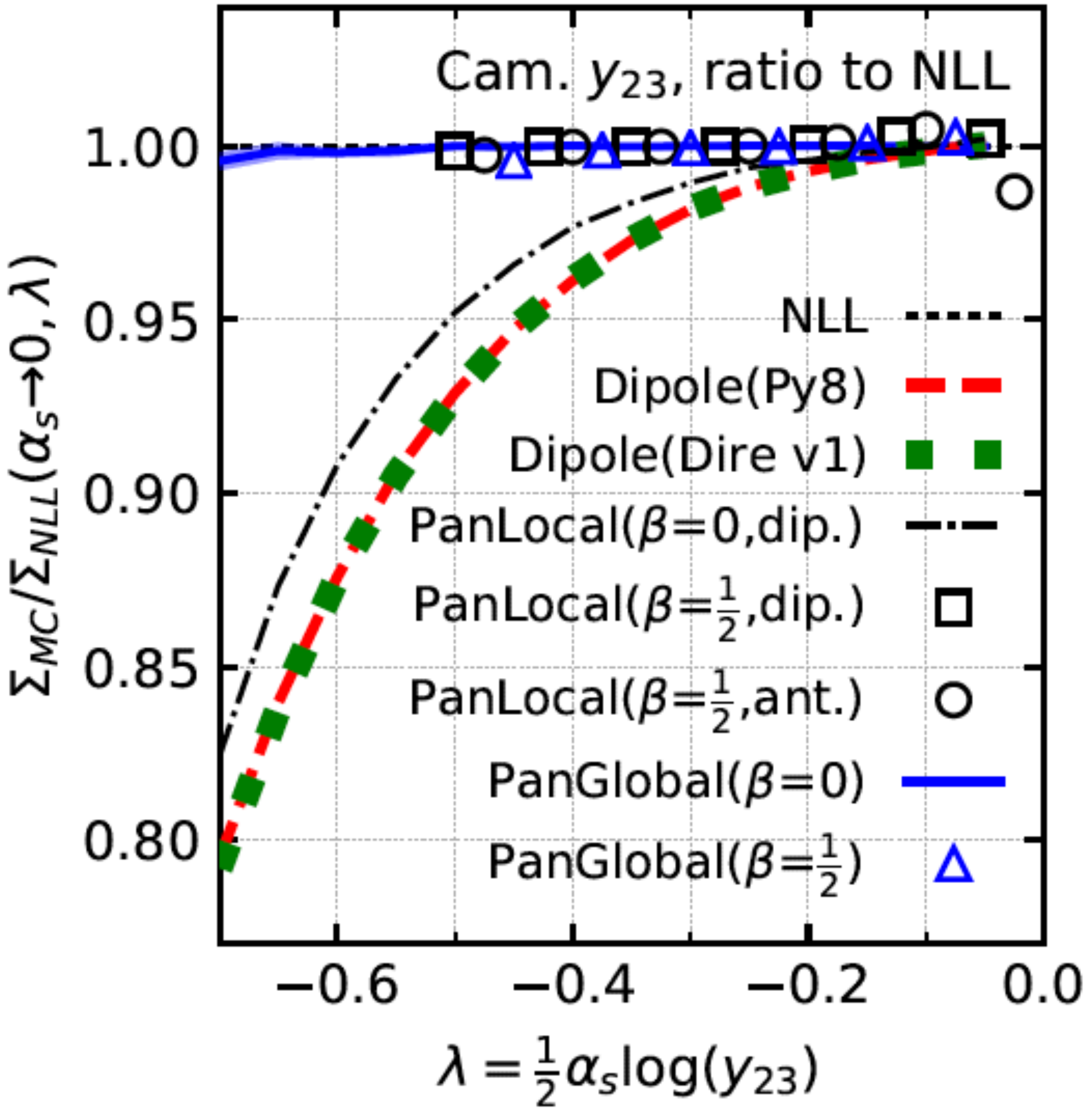
- Pythia8 deviates from NLL ✗
- Dire looks identical to Pythia8 ✗
- PanLocal($\beta = 0$) softens the issue, but not NLL accurate ✗
- PanLocal($0 < \beta < 1$) works ✓



Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Test $\lim_{\alpha_s \to 0} \frac{\Sigma_{\mathrm{PS}}(\lambda, \alpha_s)}{\Sigma_{\mathrm{NLL}}(\lambda, \alpha_s)} \overset{!}{=} 1$

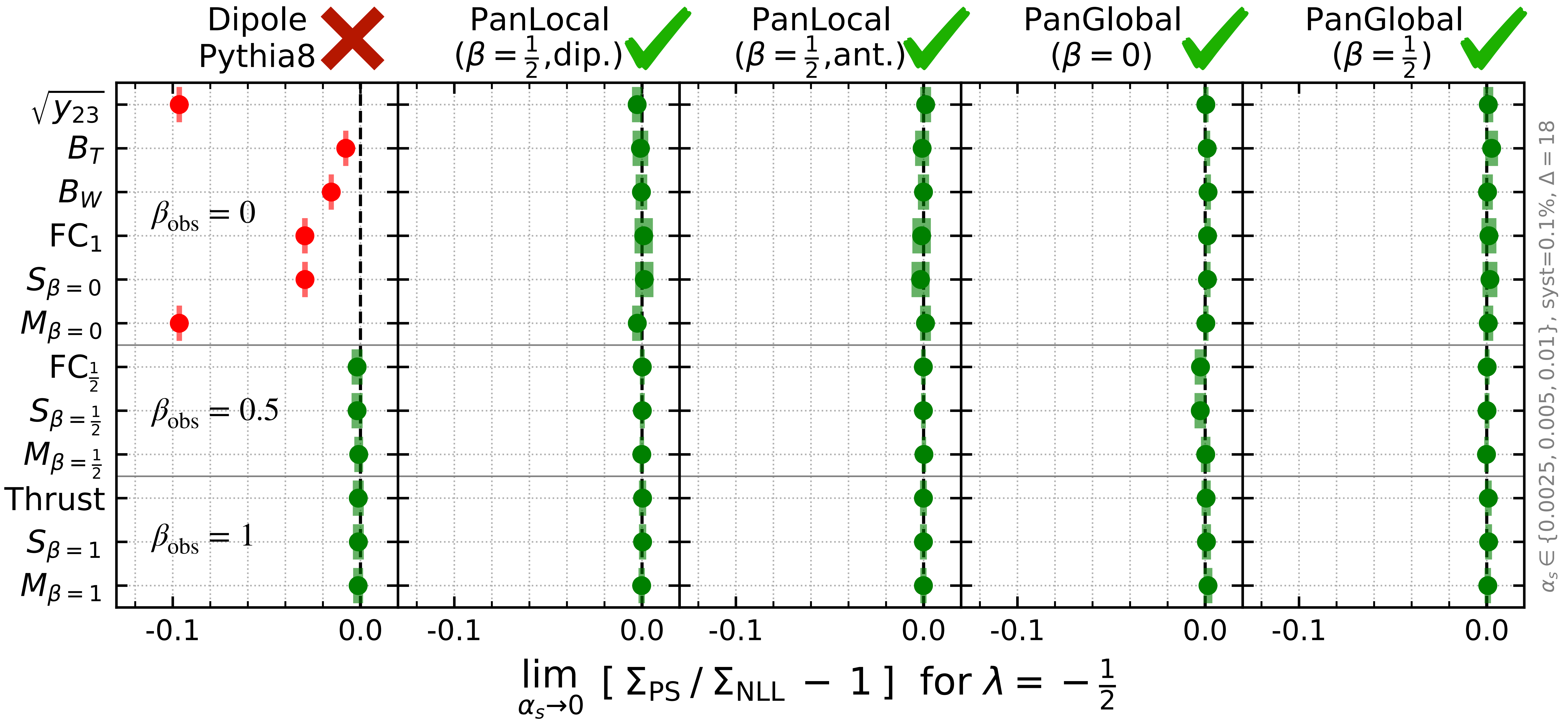
- Pythia8 deviates from NLL ✗
- Dire looks identical to Pythia8 ✗
- PanLocal($\beta = 0$) softens the issue, ✗ but not NLL accurate
- PanLocal($0 < \beta < 1$) works ✓
- PanGlobal($0 \leq \beta < 1$) works ✓



$$FC_{1-\beta} \sim S_\beta = \sum_{i \notin q\bar{q}} p_{\perp,i} e^{-\beta|\eta_i|}$$

$$M_\beta = \max_{i \notin q\bar{q}} \left[p_{\perp,i} e^{-\beta|\eta_i|}\right]$$

Tests for all types of global observables performed at full colour ($C_F = 4/3,\ C_A = 3$, NODS)
$\alpha_s^{(\mathrm{CMW})}$ with 2-loop running at $\lambda = -0.5$



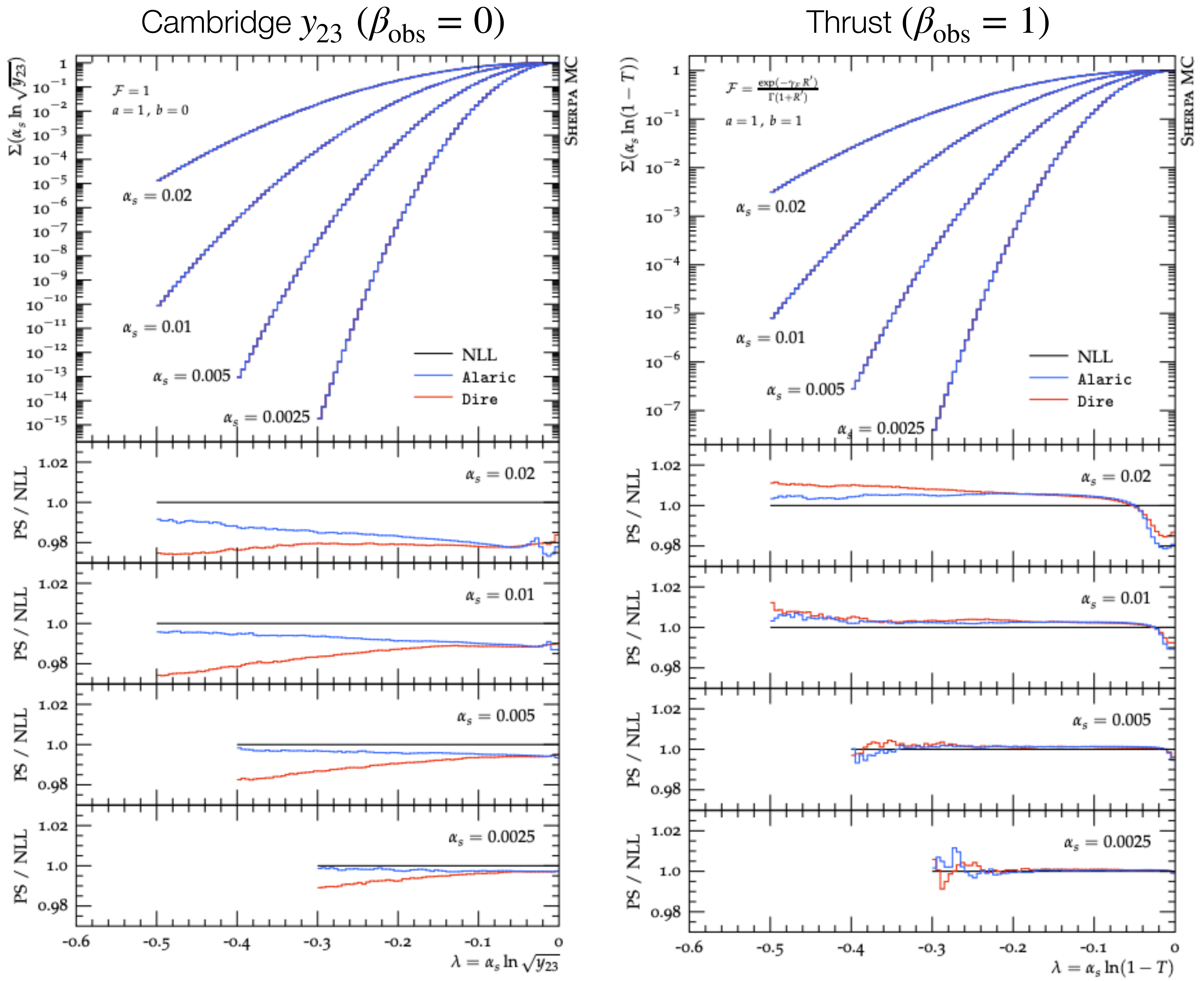
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

Alaric

Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

Tests performed at leading colour
$(2C_F = C_A = 3)$

Testing method similar to PanScales but fixed coupling, no K_{CMW}, no $\alpha_s \to 0$ extrapolation



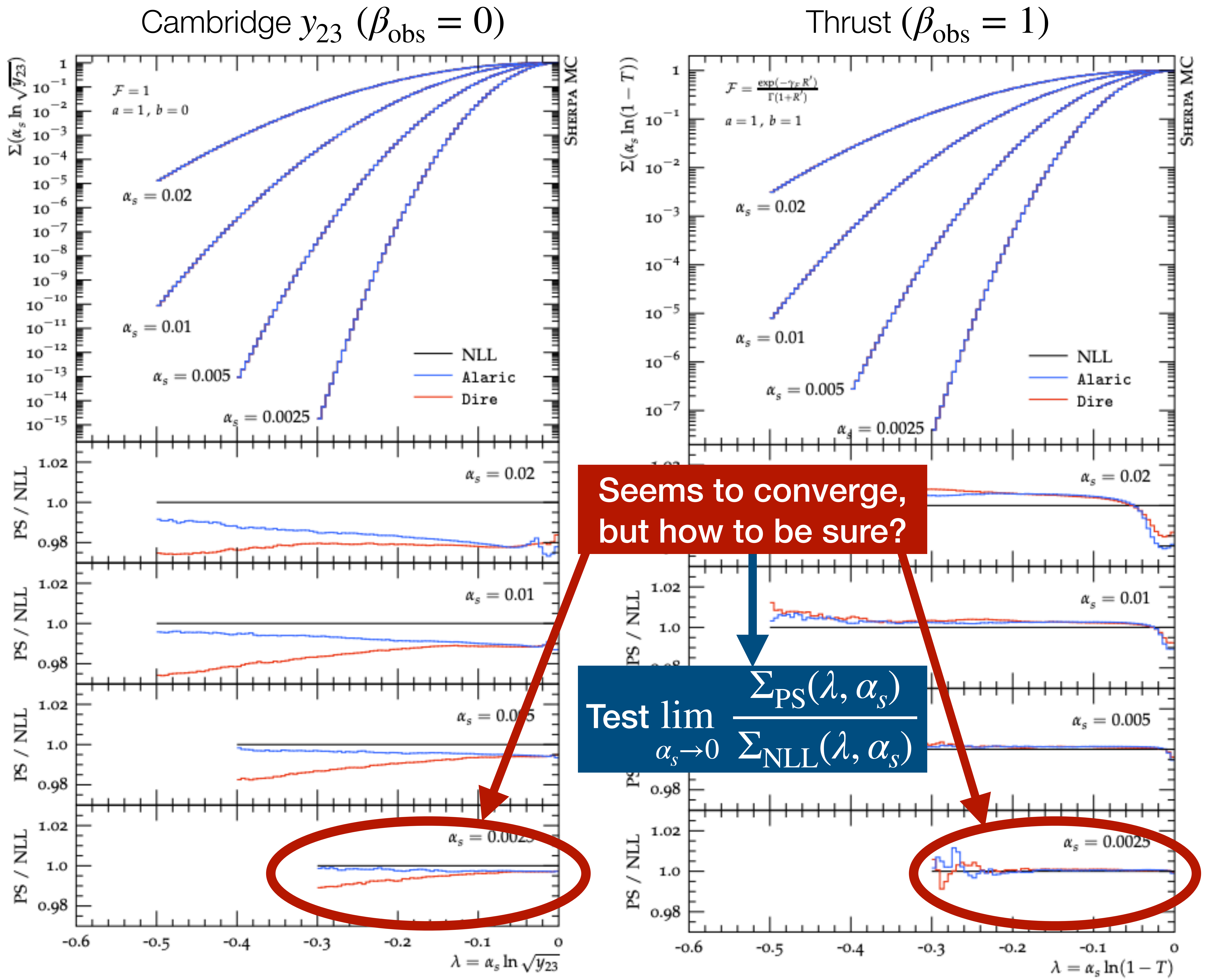
$+\Delta\psi_{12},\ B_T,\ FC_1,\ FC_{1/2},\ M_H$

Alaric

Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

Tests performed at leading colour
$(2C_F = C_A = 3)$

Testing method similar to PanScales but fixed coupling, no K_{CMW}, no $\alpha_s \to 0$ extrapolation

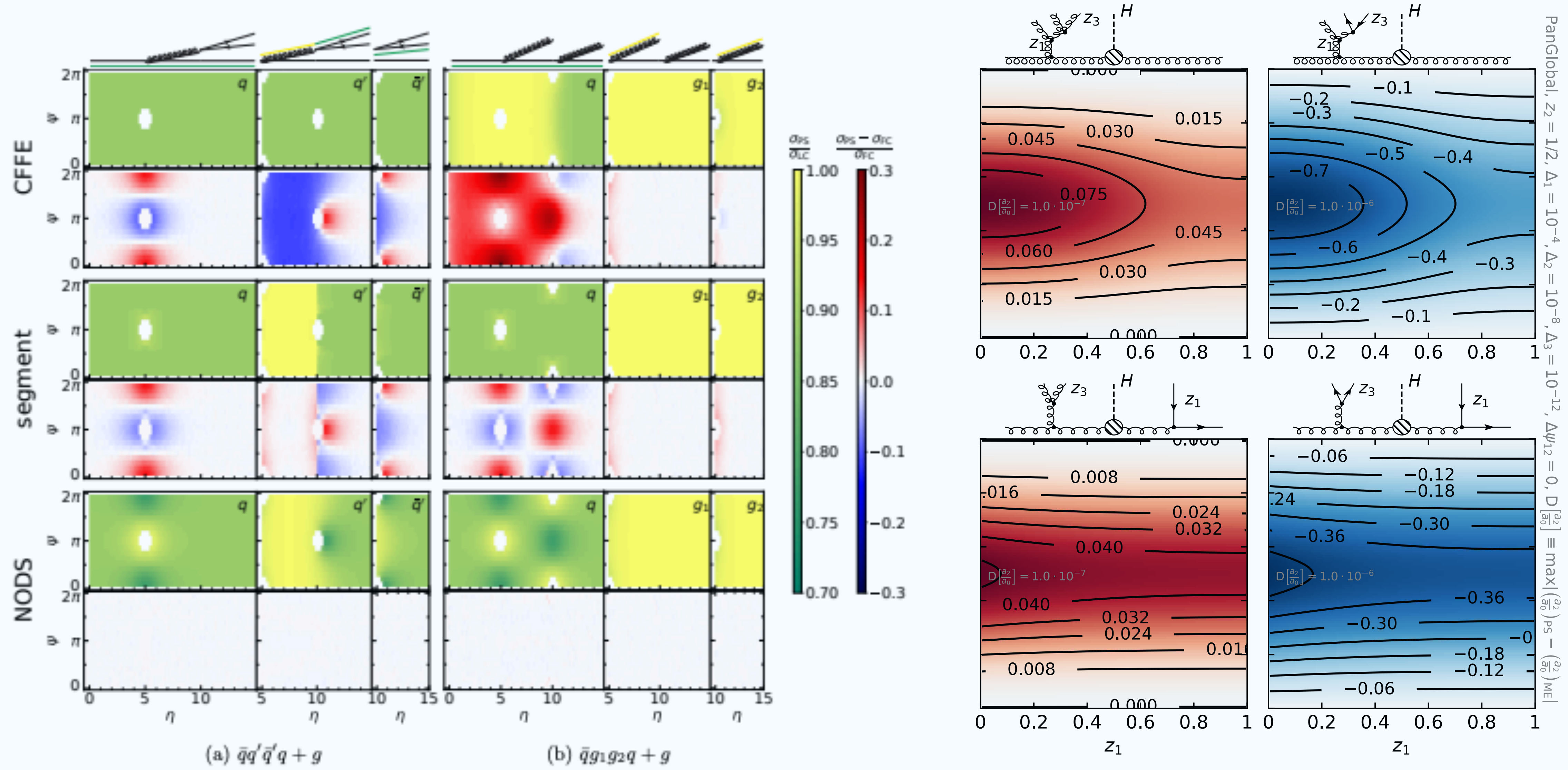


[n.b. recoil effects were also analyzed analytically]

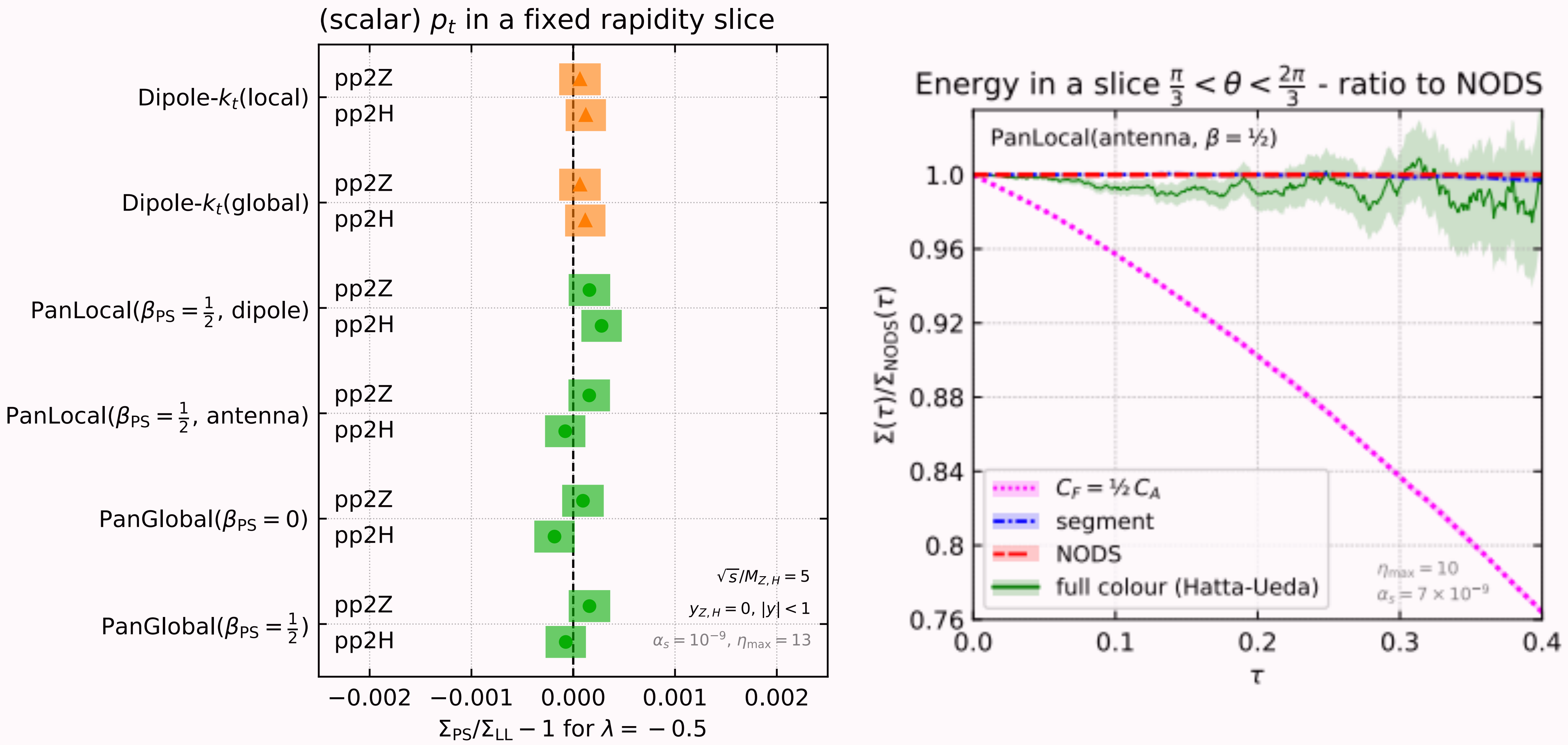
But there is more to test!

[2002.11114, 2103.16526, 2011.10054, 2111.01161, 2205.02237, 2207.09467]

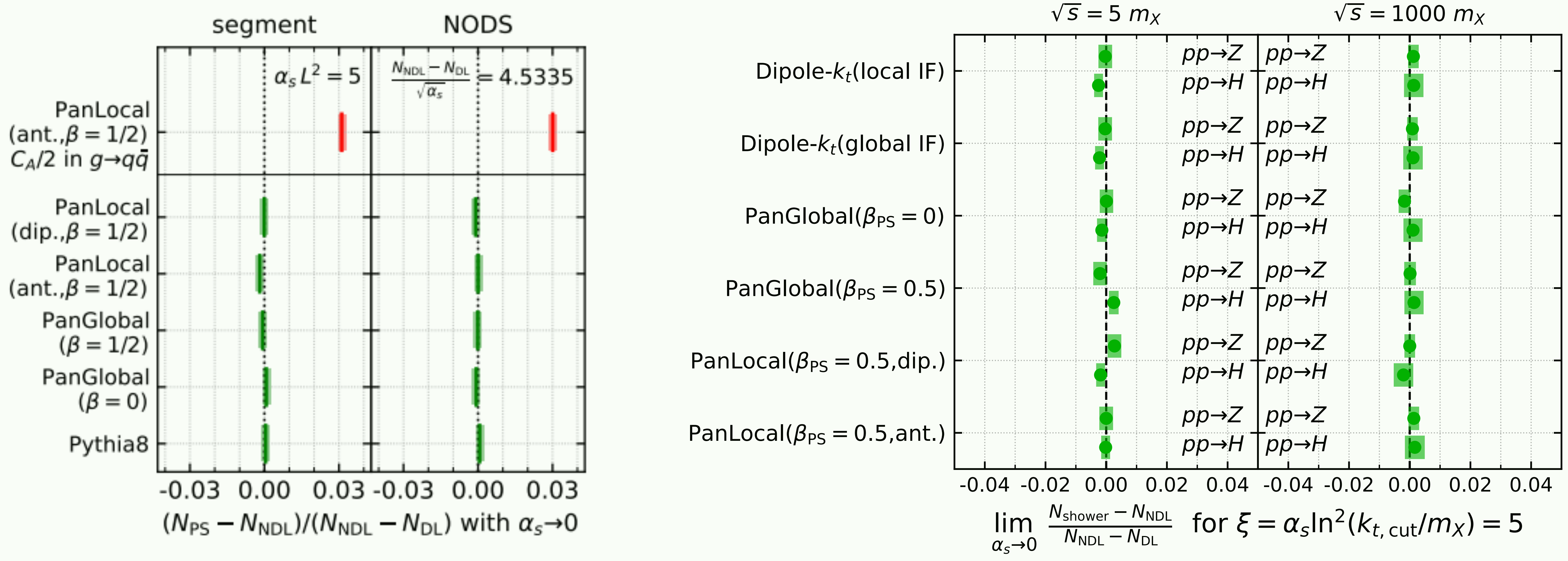
Fixed-order checks



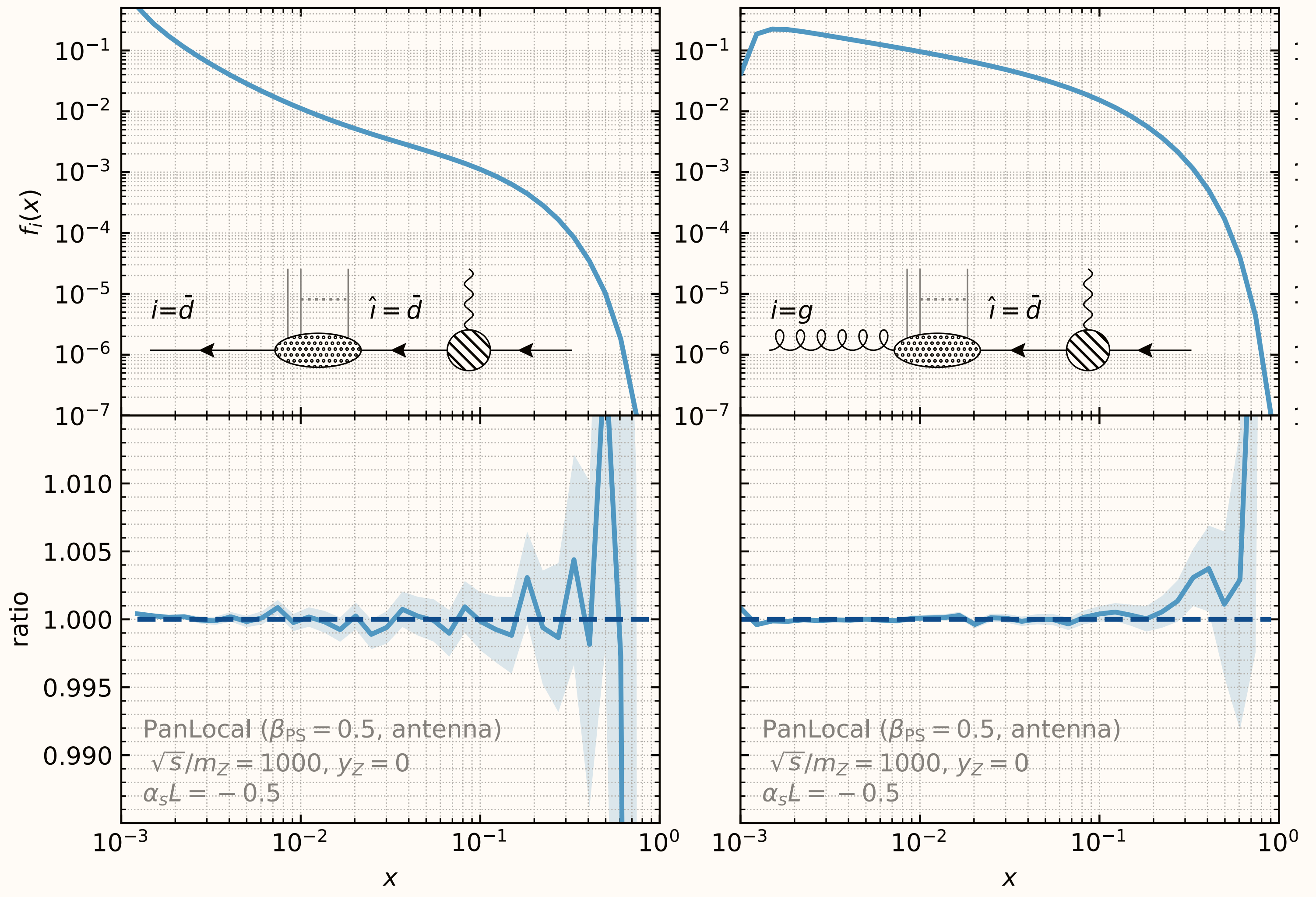
Non-global observables



Multiplicity



DGLAP evolution



Including higher-logarithmic effects

Including higher-logarithmic effects

Discussion so far is based on the factorisation in a single unresolved limit

What about double-unresolved configurations?

Triple-collinear splitting functions

Catani, Grazzini [9810389, 9908523]

$$|M_{1,2,3,\ldots,k,\ldots}(p_1,p_2,p_3,\ldots)|^2 \xrightarrow{123-\mathrm{coll}} \left(\frac{8\pi\mu^{2\varepsilon}\alpha_s}{s_{123}}\right)^2 \mathcal{T}^{ss'}_{123,\ldots}(p_{123},\ldots)\, P^{ss'}_{123}(p_1,p_2,p_3)$$

Double-soft emissions

Campbell, Glover [9710255]
Catani, Grazzini [9908523]

$$|M_{1,2,3,\ldots,n}(p_1,p_2,p_3,\ldots,p_n)|^2 \xrightarrow{12-\mathrm{soft}} \left(4\pi\mu^{2\varepsilon}\alpha_s\right)^2 \sum_{i,j=3}^{n} \mathcal{I}_{ij}(p_1,p_2)\, |M^{(i,j)}_{3,\ldots,n}(p_3,\ldots,p_n)|^2$$

These corrections need to be included to get to NNLL/NNDL accuracy

Dasgupta, El-Menoufi [2109.07496]

Analytic ingredients - new hard collinear terms

One important and new ingredient for a fully differential shower is $B_2(z)$

Consider the Sudakov for transverse-momentum resummation

Parisi, Petronzio [NPB 154 (1979) 427-440]

$$S(Q,b) = \exp\left(-\int_{\bar{b}^2/b^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_s(q^2))\right]\right)$$

Both obey a perturbative expansion in α_s

$$A(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_n \qquad B(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_n$$

$A_1,\ B_1,\ A_2$ are observable independent
(they only depend on the emitting particle)

Dasgupta, El-Menoufi [2109.07496]

Analytic ingredients - new hard collinear terms

One important and new ingredient for a fully differential shower is $B_2(z)$

Consider the Sudakov for transverse-momentum resummation

Parisi, Petronzio [NPB 154 (1979) 427-440]

$$S(Q,b) = \exp\left(-\int_{\bar{b}^2/b^2}^{Q^2} \frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_s(q^2))\right]\right)$$

Both obey a perturbative expansion in α_s

$$A(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_n \qquad B(\alpha_s) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_n$$

B_2 is observable-dependent, i.e. for a quark emitter

$$B_2^q = -\gamma_q^{(2)} + C_F b_0 X_v$$

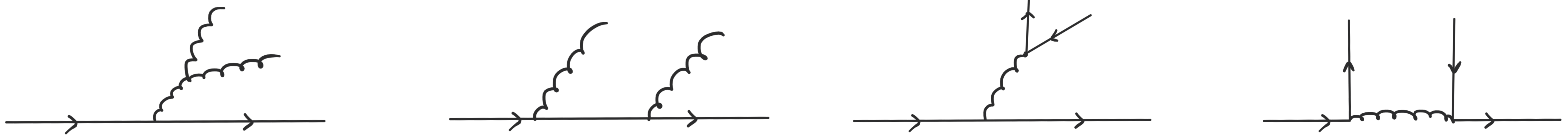
Catani, de Florian, Grazzini [0008184, 0407241]

$B_2^{q/g}$ needs to be included in a differential manner $\rightarrow$ $B_2^{q/g}(z)$

$B_2(z)$ for quark channels

1. Integrate the triple-collinear contributions over 2 energies and 1 angular variable $(\theta, \rho, k_T, \ldots)$

Non-abelian $C_F C_A$ | Abelian C_F^2 | $g \to q^{(\prime)}\bar{q}^{(\prime)}$ splitting $C_F T_R n_F$ | Identical quarks interference $C_F\left(C_F - \frac{C_A}{2}\right)$



2. Isolate the pure NNLL terms (subtract iterated LO splittings and K_{CMW} contributions)

+ virtual corrections

Result: $B_2^q(z)$ differential in z, θ for all channels

$$\int_0^1 \mathrm{d}z \left[B_2^{q,C_F C_A}(z) + B_2^{q,C_F^2}(z) + B_2^{q,C_F T_R n_F}(z) + B_2^{q,\mathrm{id}}(z)\right] = -\gamma_q^{(2)} + C_F b_0 X_v = B_2$$

Observable-dependence depends on the scale of the coupling through the angular variable that is fixed

To be done: get $B_2^g(z)$, implement this in a shower, understand cross-talk with double-soft…

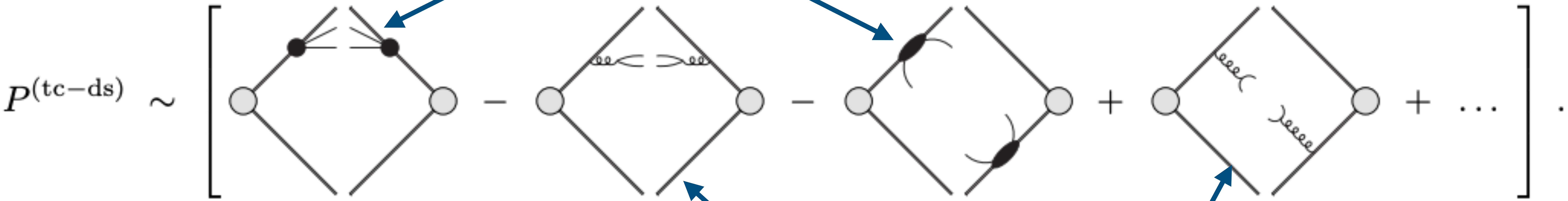
Dulat, Gellersen, Höche, Prestel [1705.00742, 1805.03757, 2110.05964]

Implementing higher-order splitting kernels

Consider quark-pair emissions in the triple-collinear (tc) and double-soft (ds) limits

Need to remove overlapping singularities and contributions obtained by LO iteration

Complete MEs in the tc and ds limits
(latter with a minus sign to remove the double counting)



Diagrams obtained iterating LO splittings

Result is fully finite through introduction of integrated subtraction terms and factorization counter terms

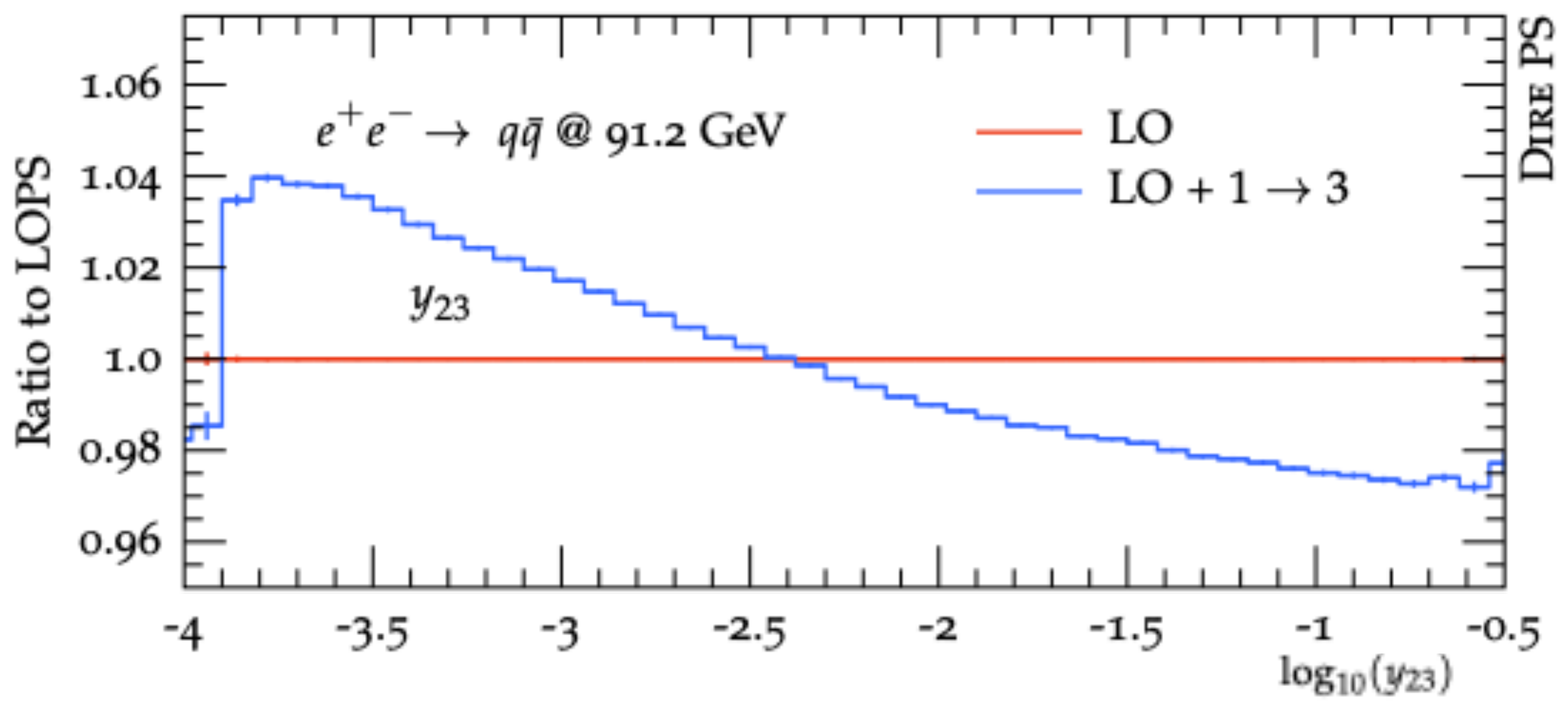
Generate emissions using the $1 \rightarrow 3$ branching kernels in a $2 \rightarrow 4$ ‘tripole’

Note that this is not an NNLL shower, i.e. the kinematic map has the issues pointed out before

Implementing higher-order splitting kernels

- Dire with soft-subtracted triple-collinear $q \rightarrow qq\bar{q}$ splittings

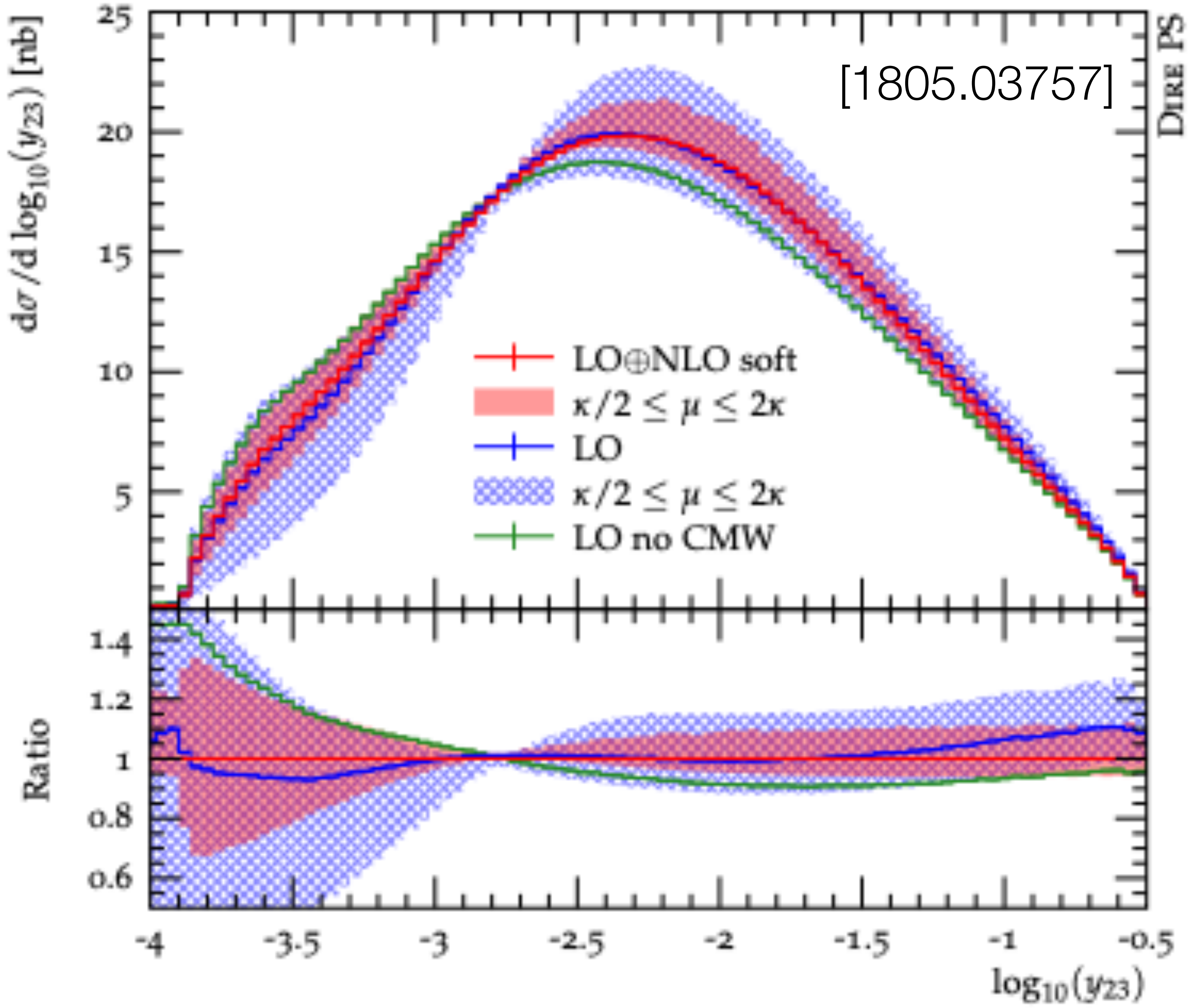
- K_{CMW} included in the coupling (not in differential form)



tc corrections shift the y_{23} distribution wrt the LO shower

---

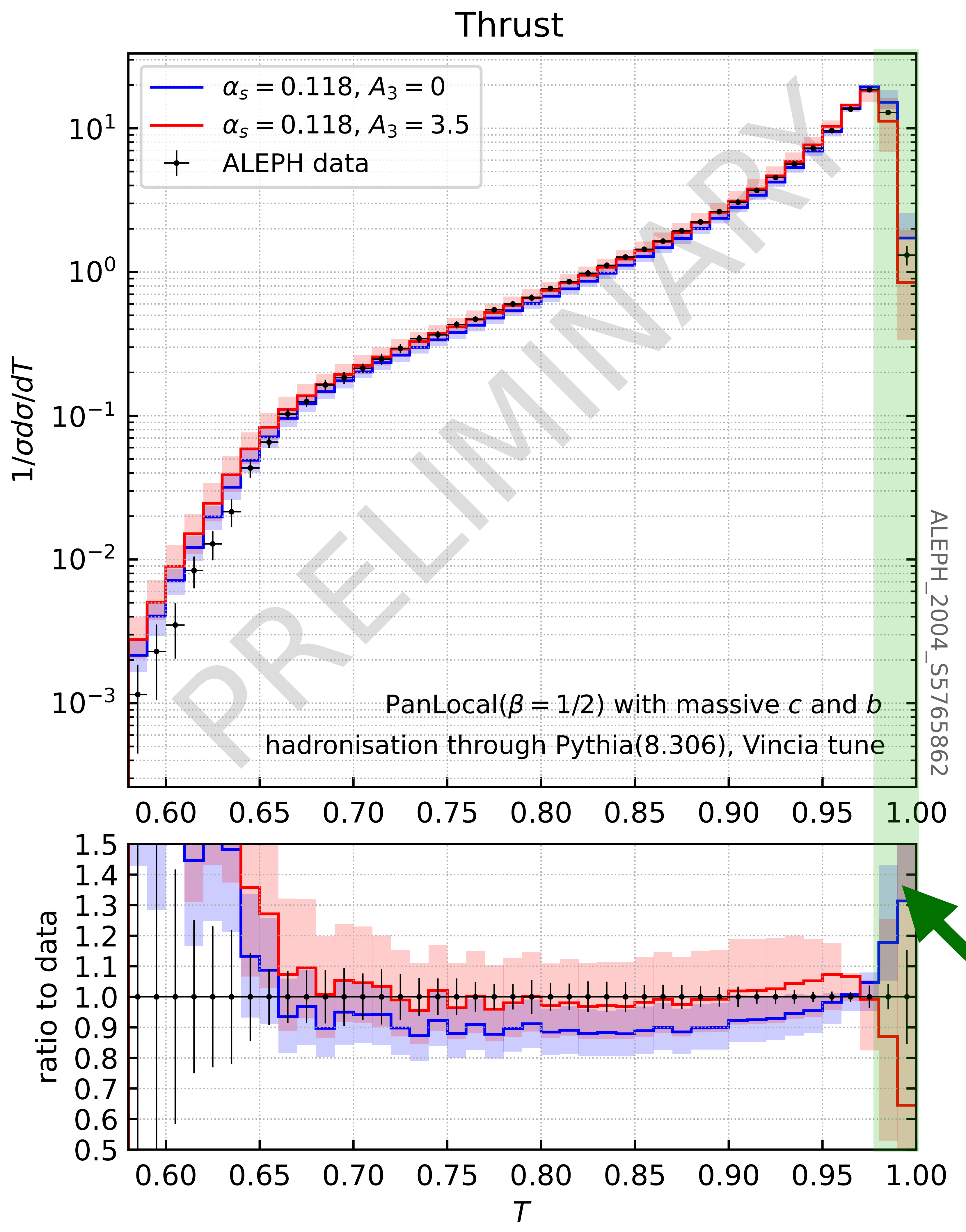
- Dire with only double-soft corrections (all channels)



ds corrections have a similar effect as the soft-subtracted tc terms on the y_{23} distribution

Towards phenomenology

Towards LEP phenomenology



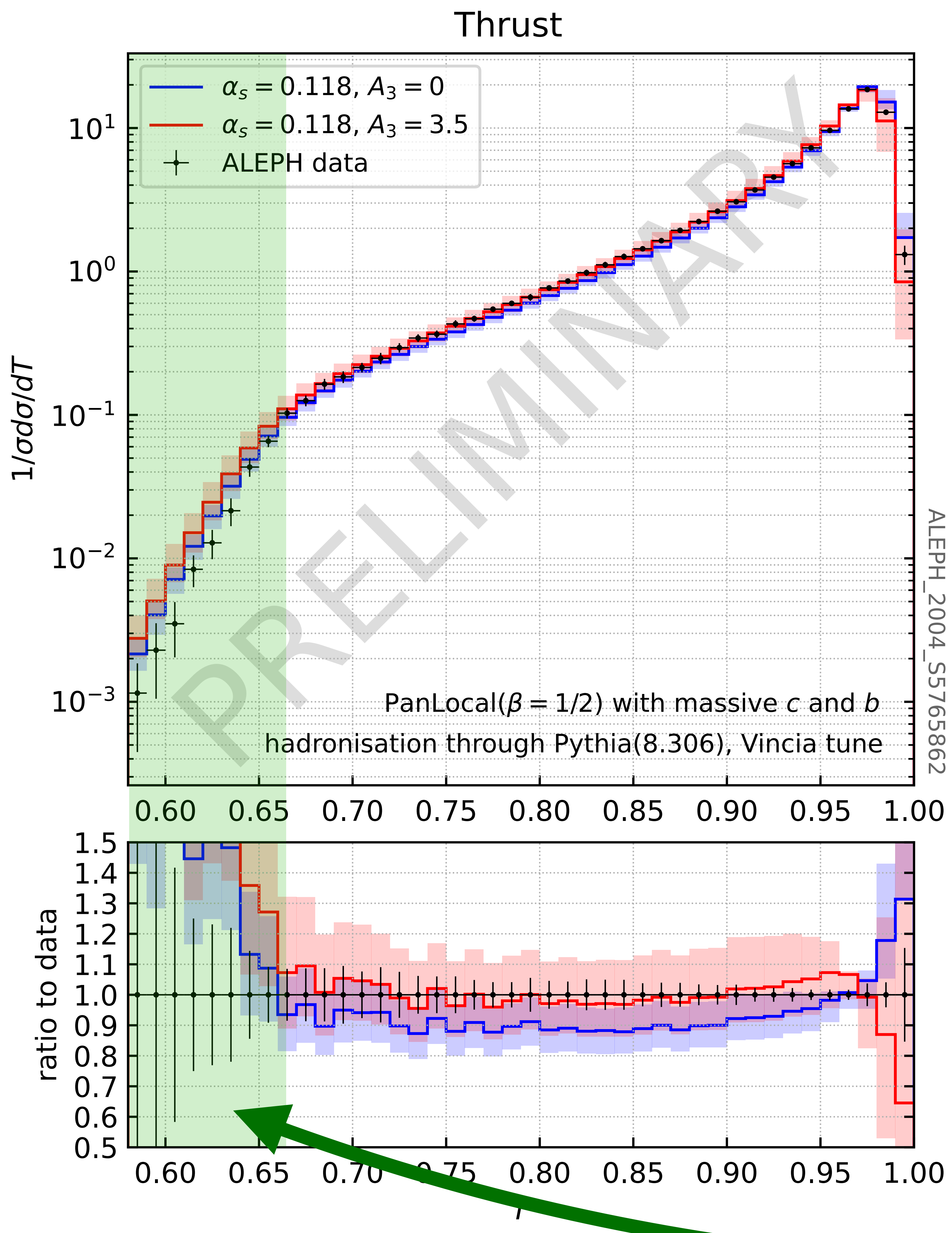
- PanLocal($\beta = 0.5$) dipole shower
- Heavy quarks ($m_c = 1.5$ GeV, $m_b = 4.8$ GeV)
- Matching to NLO
- Renormalisation-scale uncertainties included

$$\alpha_s^{(\mathrm{CMW})} = \alpha_s(x_r\mu_{r,0})\left(1 + \frac{K_{\mathrm{CMW}}\alpha_s(x_r\mu_{r,0})}{2\pi} + 2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

- Enhanced coupling - $\alpha_s = \alpha_s^{(CMW)} + A_3\alpha_s^3$
- Hadronisation from Pythia8 with the Vincia tune

Hadronisation region
(tuning of the shower is needed)

Towards LEP phenomenology



- PanLocal($\beta = 0.5$) dipole shower
- Heavy quarks ($m_c = 1.5$ GeV, $m_b = 4.8$ GeV)
- Matching to NLO
- Renormalisation-scale uncertainties included

$$\alpha_s^{(\mathrm{CMW})} = \alpha_s(x_r\mu_{r,0})\left(1 + \frac{K_{\mathrm{CMW}}\alpha_s(x_r\mu_{r,0})}{2\pi} + 2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

- Enhanced coupling - $\alpha_s = \alpha_s^{(CMW)} + A_3\alpha_s^3$
- Hadronisation from Pythia8 with the Vincia tune

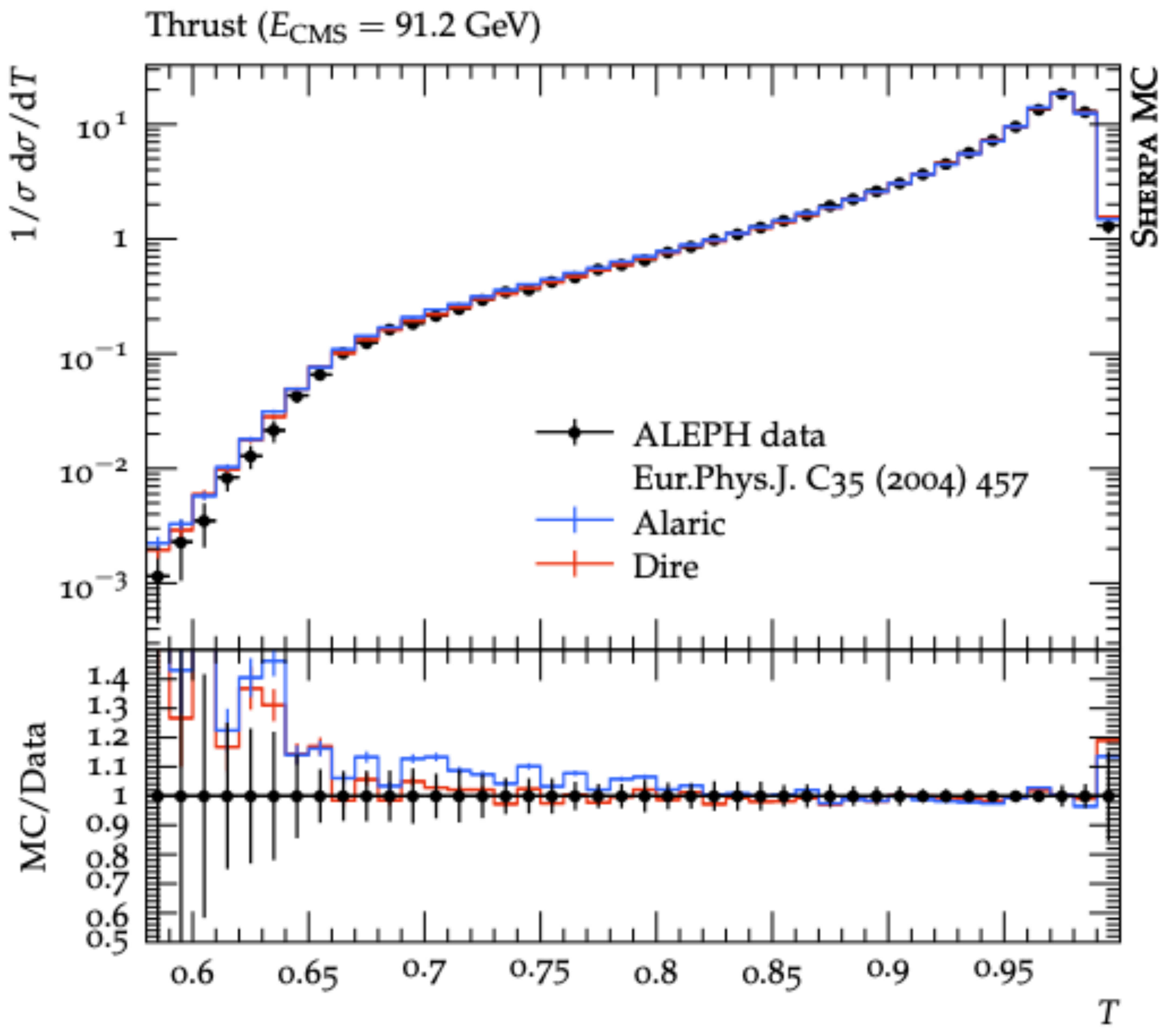
Poor description in the 4-jet region - need for 2-jet at NNLO?

Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

Towards LEP phenomenology

- No NLO matching, no masses
- CMW scheme with flavour thresholds
- Thresholds at $m_c = 1.42$ GeV, $m_b = 4.92$ GeV
- Hadronisation from Pythia8 with default parameters except
  Alaric: PARJ(21) = 0.3, PARJ(41) = 0.4, PARJ(42) = 0.36
  Dire: PARJ(21) = 0.3, PARJ(41) = 0.4, PARJ(42) = 0.45

Qualitatively similar features observed as for the PanLocal shower



Conclusions

- Parton showers will continue to play an indispensable role in any (future) particle physics experiment

- NLL showers for massless partons in e^+e^- collisions from several groups are now available
  - Including massive partons is a natural next step

- But what about the step to NNLL?
  - We need to understand the logarithmic structure
  - We need to have reference calculations, e.g.
    - Next-to-leading non-global logarithms Banfi, Dreyer and Monni [2104.06416]
    - NNDL multiplicity Medves, Soto-Ontoso, Soyez [2205.0286]
    - NNLL groomed jet observables Anderle, Dasgupta, El-Menoufi, Helliwell, Guzzi [2007.10355, 2211.03820]

- And what about QED/EW radiation?

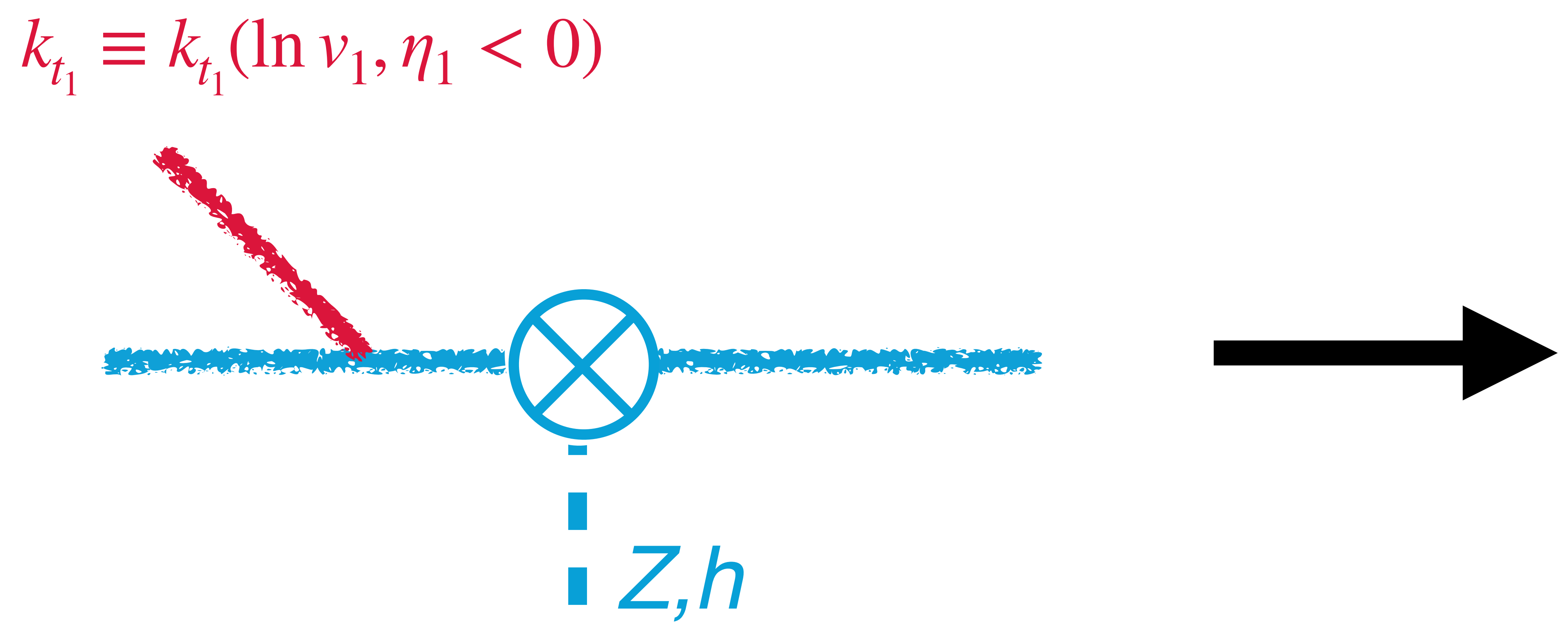
Back up

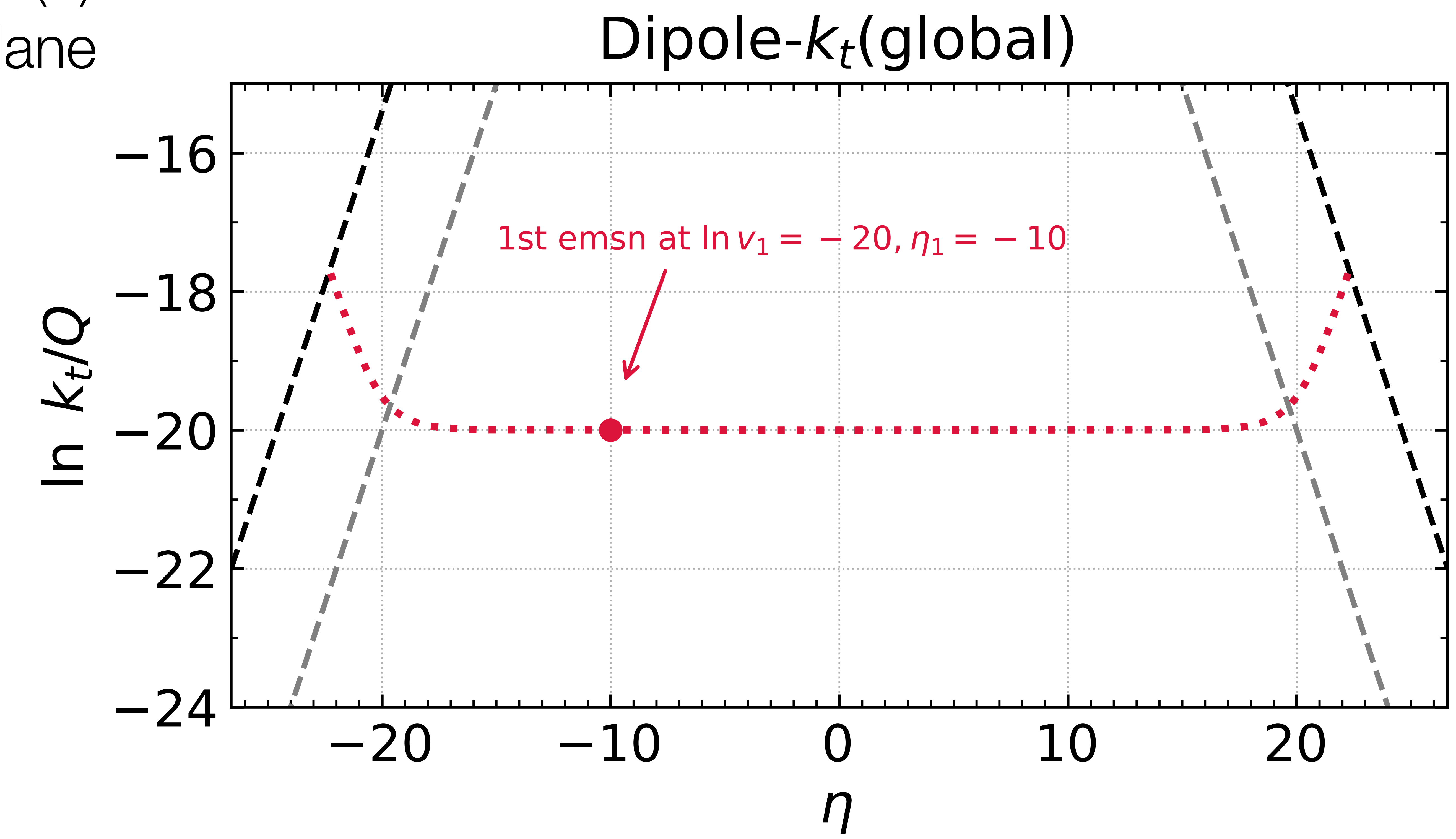
Mapping between λ and physical quantities

Q [GeV]	$\alpha_s(Q)$	$p_{t,\min}$ [GeV]	$\xi = \alpha_s L^2$	$\lambda = \alpha_s L$	τ
91.2	0.1181	1.0	2.4	-0.53	0.27
91.2	0.1181	3.0	1.4	-0.40	0.18
91.2	0.1181	5.0	1.0	-0.34	0.14
1000	0.0886	1.0	4.2	-0.61	0.36
1000	0.0886	3.0	3.0	-0.51	0.26
1000	0.0886	5.0	2.5	-0.47	0.22
4000	0.0777	1.0	5.3	-0.64	0.40
4000	0.0777	3.0	4.0	-0.56	0.30
4000	0.0777	5.0	3.5	-0.52	0.26
20000	0.0680	1.0	6.7	-0.67	0.45
20000	0.0680	3.0	5.3	-0.60	0.34
20000	0.0680	5.0	4.7	-0.56	0.30

Fixed-order checks also give powerful information

Choice of evolution variable (1) + kinematic map (2) determine *phase-space contours* in the Lund plane

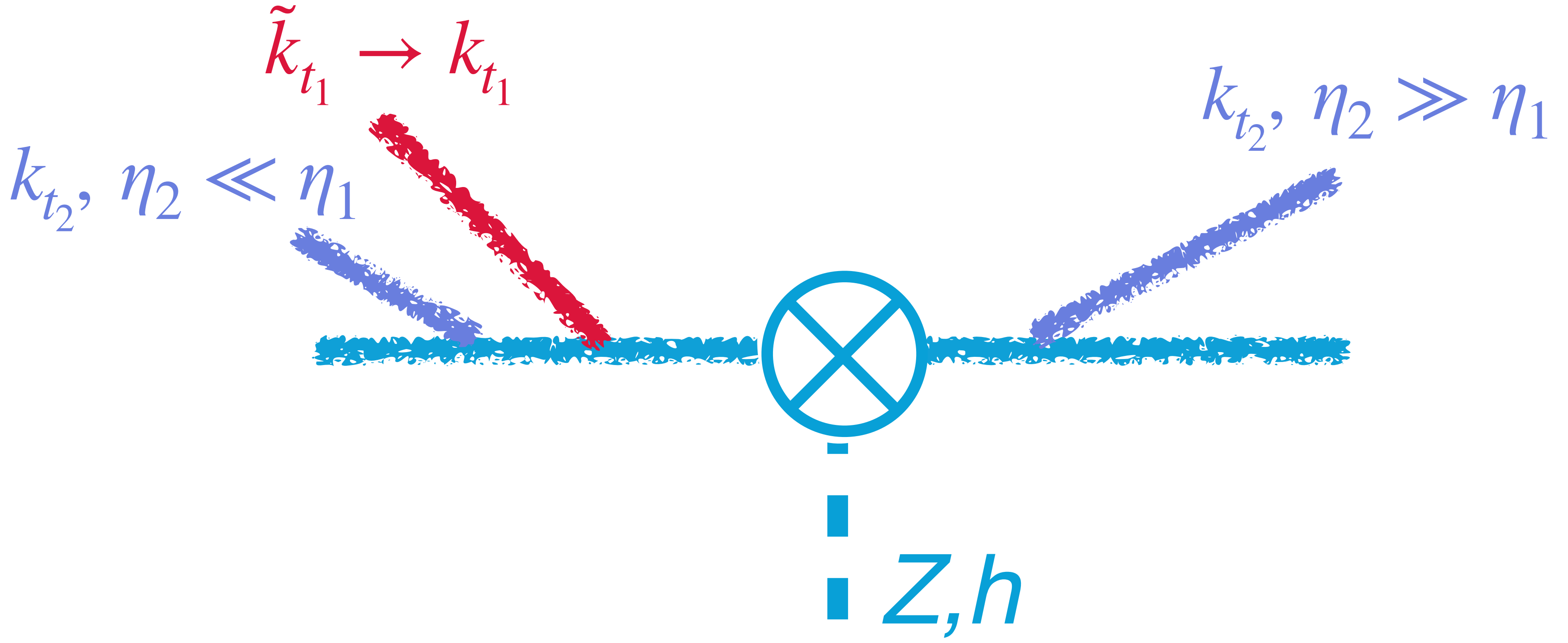


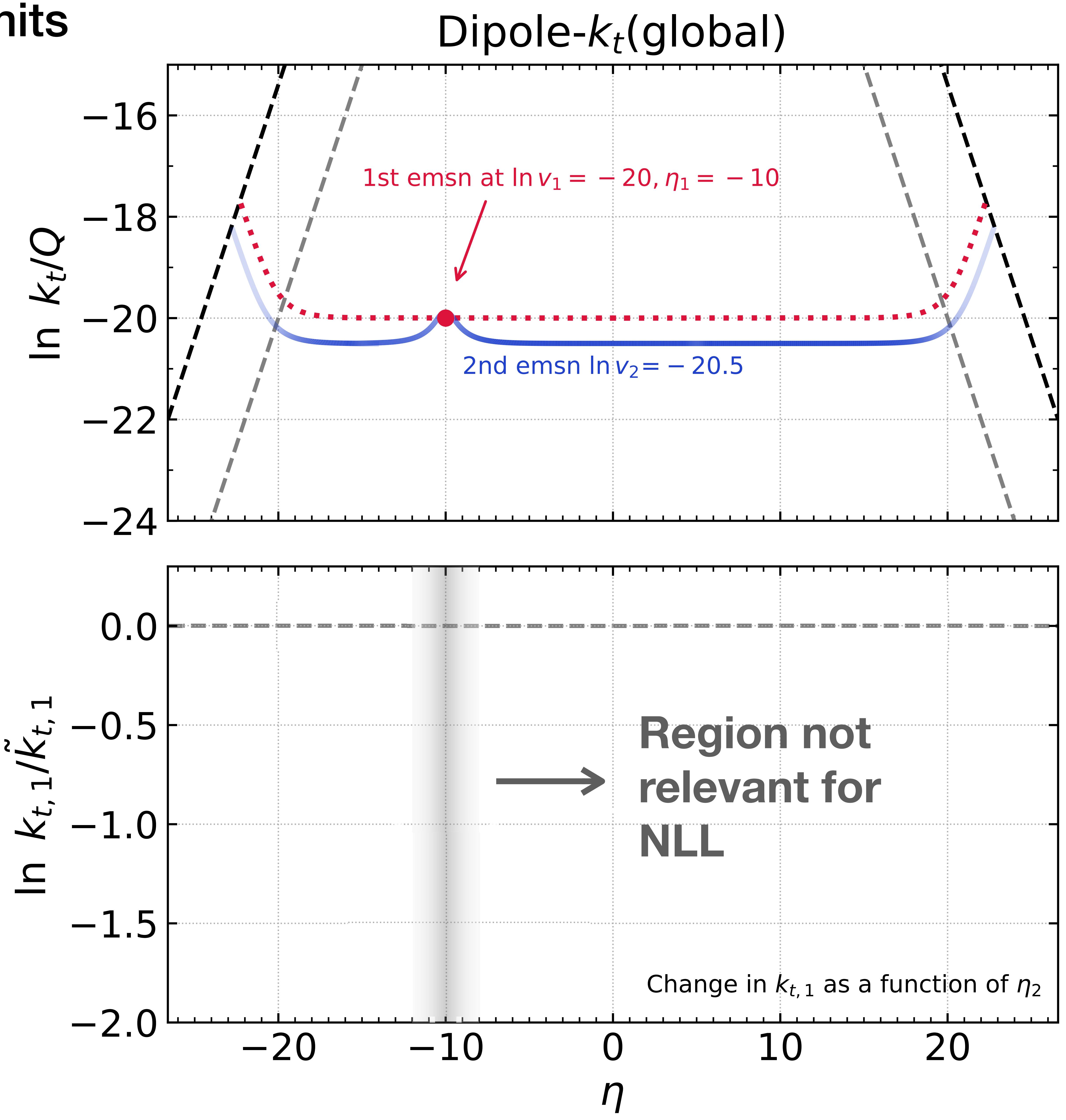


van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]

Amplitudes factorise in the soft and collinear limits

How does a **second** emission affect the **first** emission's momentum?

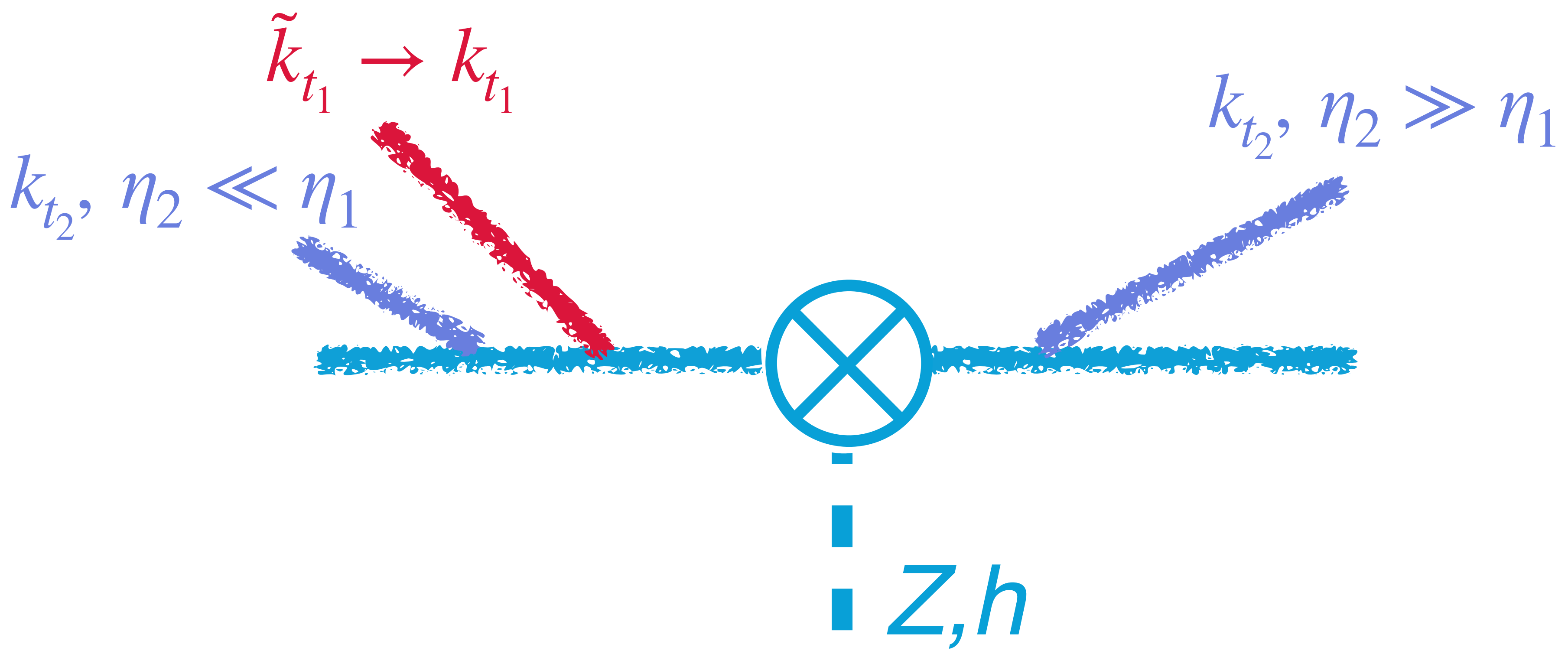


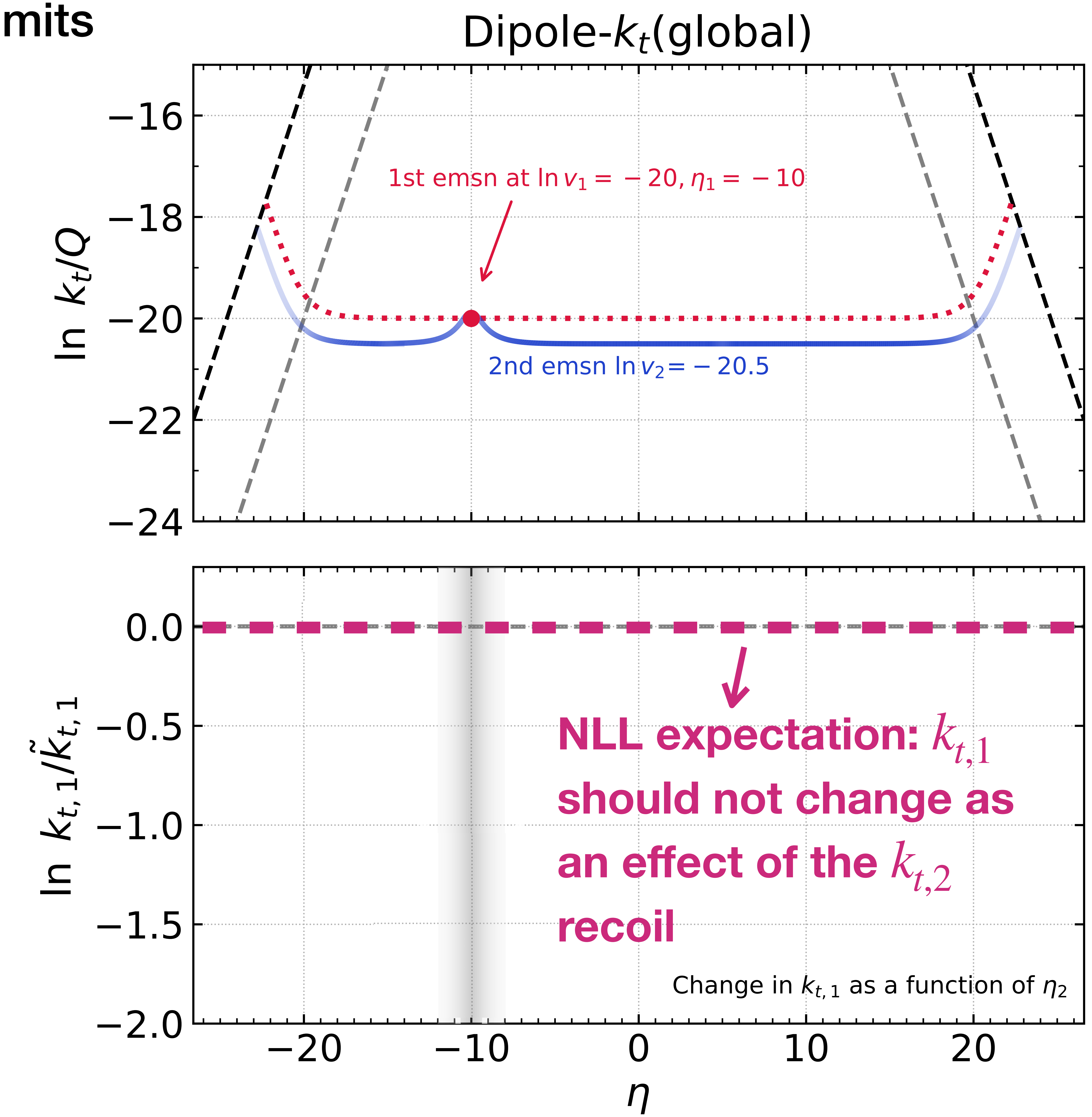


van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]

Amplitudes factorise in the soft and collinear limits

How does a **second** emission affect the **first** emission's momentum?

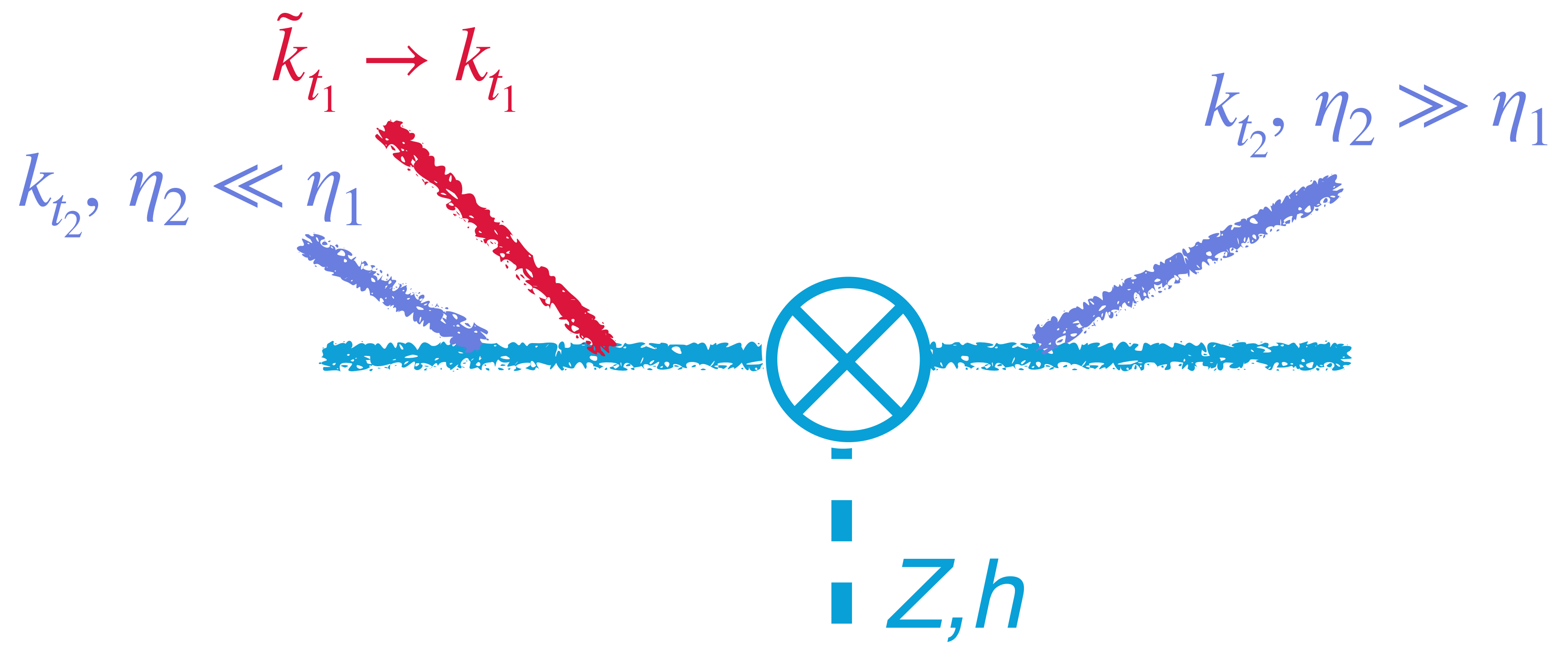




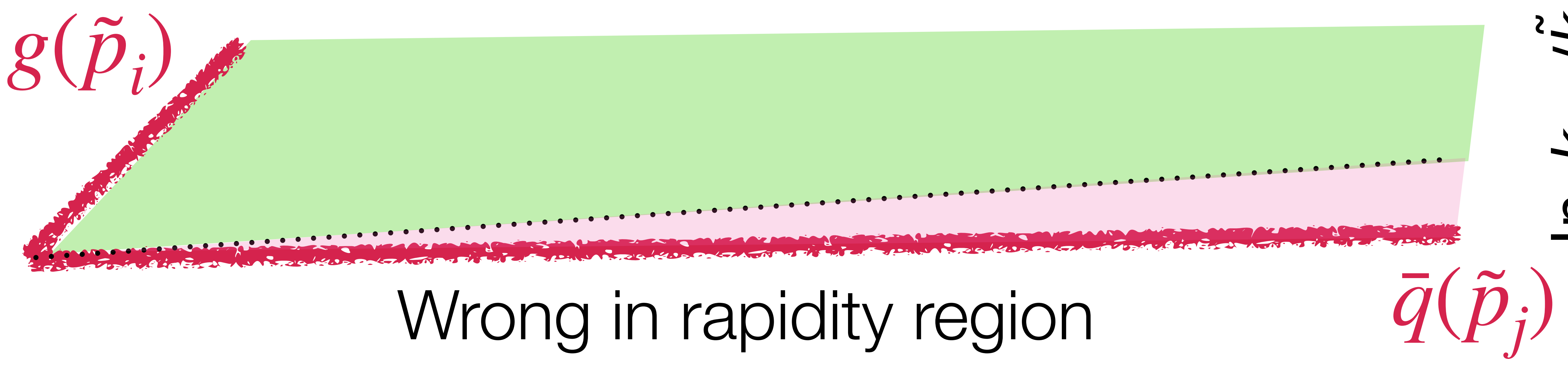
van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]

Amplitudes factorise in the soft and collinear limits

How does a **second** emission affect the **first** emission's momentum?

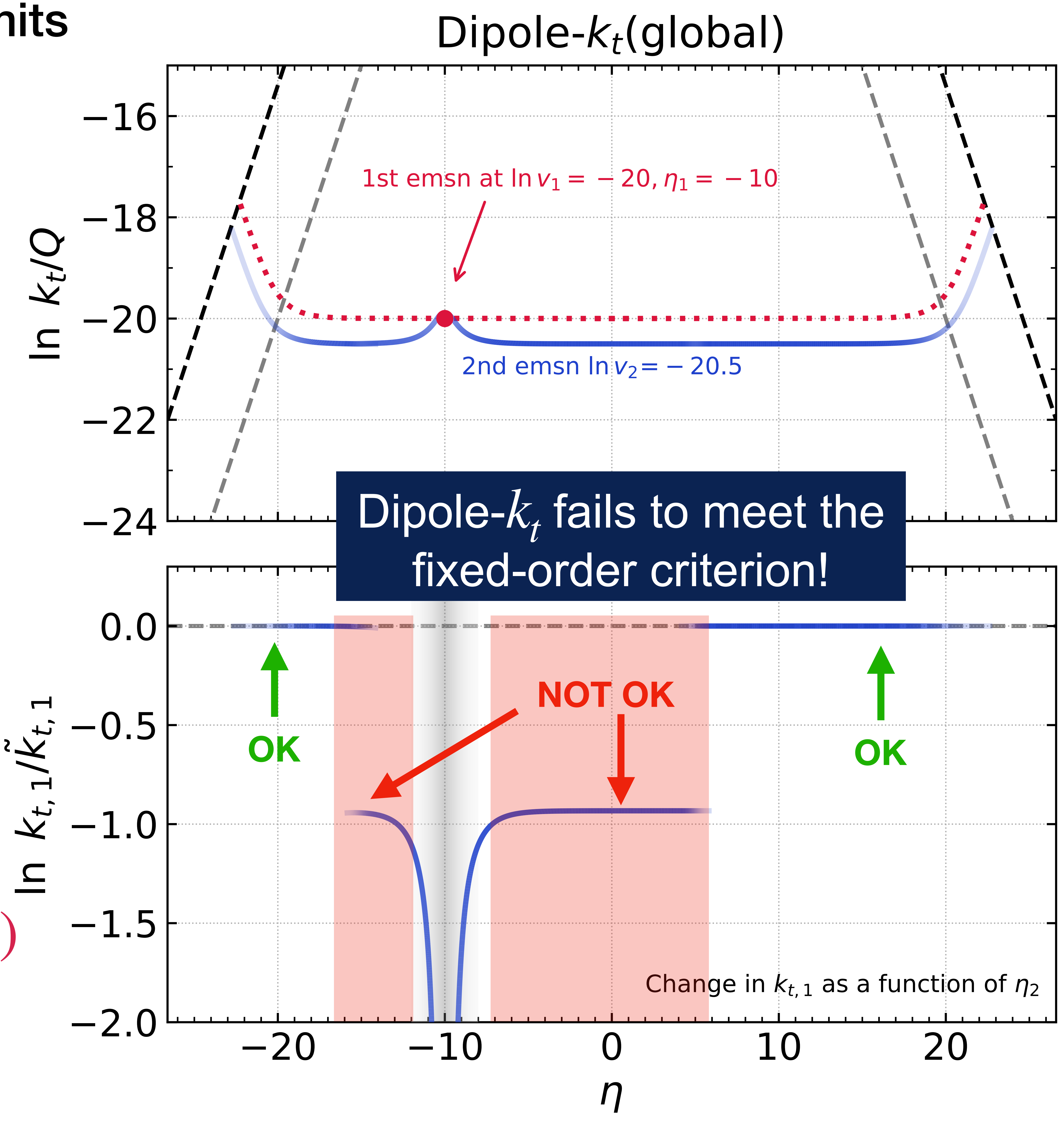


Direct consequence of CM dipole separation



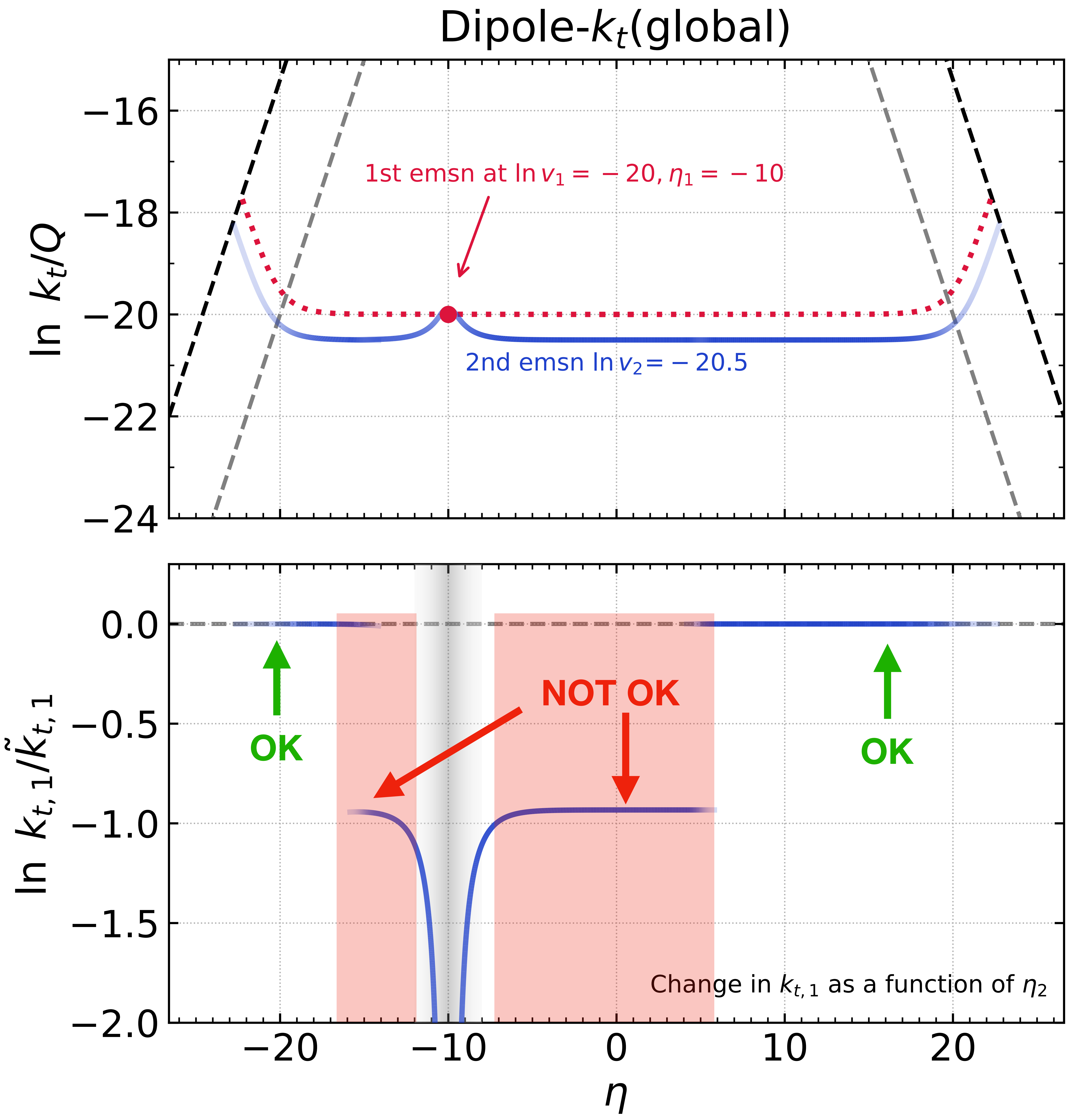
Wrong in rapidity region

$$\frac{1}{2}\left(\eta_1 + \ln\frac{k_{t_1}}{Q}\right) < \eta_2 < \frac{1}{2}\left(\eta_1 - \ln\frac{k_{t_1}}{Q}\right)$$



van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]

Such showers may introduce spurious logarithms at higher orders, which could get masked/come with a tiny coefficient for some observables



van Beekveld, Ferrario Ravasio, Salam, Soto-Ontoso, Soyez, Verheyen [2205.02237]

Deductor

Consider the showered thrust variable in Laplace space, written as $\tilde{g}(\nu) = (1 \,|\, I(\nu) \,|\, \rho_H)$

$I(\nu)$ contains a sum of powers of Laplace-space splitting operators S for a first-order parton shower

$$I(\nu) = \sum_{k=1}^{\infty} I^{[k]}(\nu)$$

In general, the action of S on a partonic state is complicated (includes e.g. the map)

i.e. in the soft-collinear approximation (no change in the kinematics)

$$\begin{aligned}
&\mathcal{S}^{[1,0]}(yQ^2)\big|\{p,f,c,c'\}_m\big) \\
&\approx -\sum_{l=1}^{m}\sum_{\substack{k=1\\k\neq l}}^{m}[T_l\otimes T_k^\dagger + T_k\otimes T_l^\dagger]\big|\{c,c'\}_m\big) \\
&\quad\times \int\frac{d\phi}{2\pi}\int\frac{dz}{1-z}\,\frac{\alpha_s(\lambda_R(1-z)yQ^2/a_l)}{2\pi} \\
&\quad\times \Theta\!\left(\frac{a_l y}{\vartheta(l,k)} < 1-z < 1\right) \\
&\quad\times \big|\{\hat{p},\hat{f}\}_{m+1}\big)\,.
\end{aligned}$$

Nagy, Soper [2011.04777]

Deductor

Consider the showered thrust variable in Laplace space, written as $\tilde{g}(\nu) = (1 \,|\, I(\nu) \,|\, \rho_H)$

$I(\nu)$ contains a sum of powers of Laplace-space splitting operators S for a first-order parton shower

$I(\nu) = \sum_{k=1}^{\infty} I^{[k]}(\nu)$ expanding the coupling gives us $I^{[k]}(\nu) = \sum_{n=k}^{\infty} \left[\frac{\alpha_s(Q^2/\nu)}{2\pi} \right]^n I_n^{[k]}(\nu)$

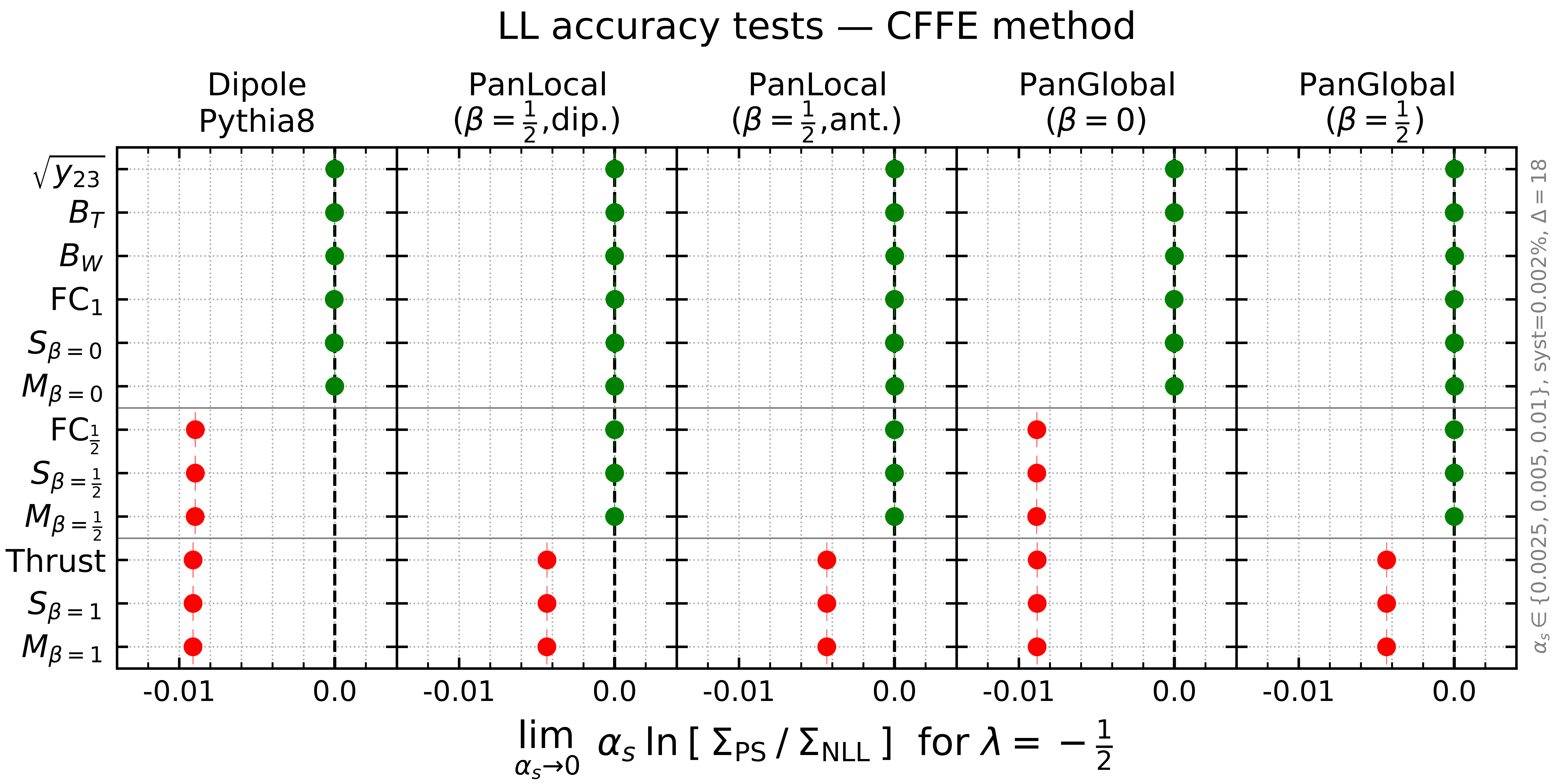
where $I_n^{[k]}$ may contain at most L^{n+1}

Relating this to an analytic calculation, LL and NLL contributions $(\alpha_s^n L^{n+1},\ \alpha_s^n L^n)$ should be fully generated by the exponentiation of the first-order operator $(I^{[1]}(\nu))$

Higher-order expansions may not spoil this picture

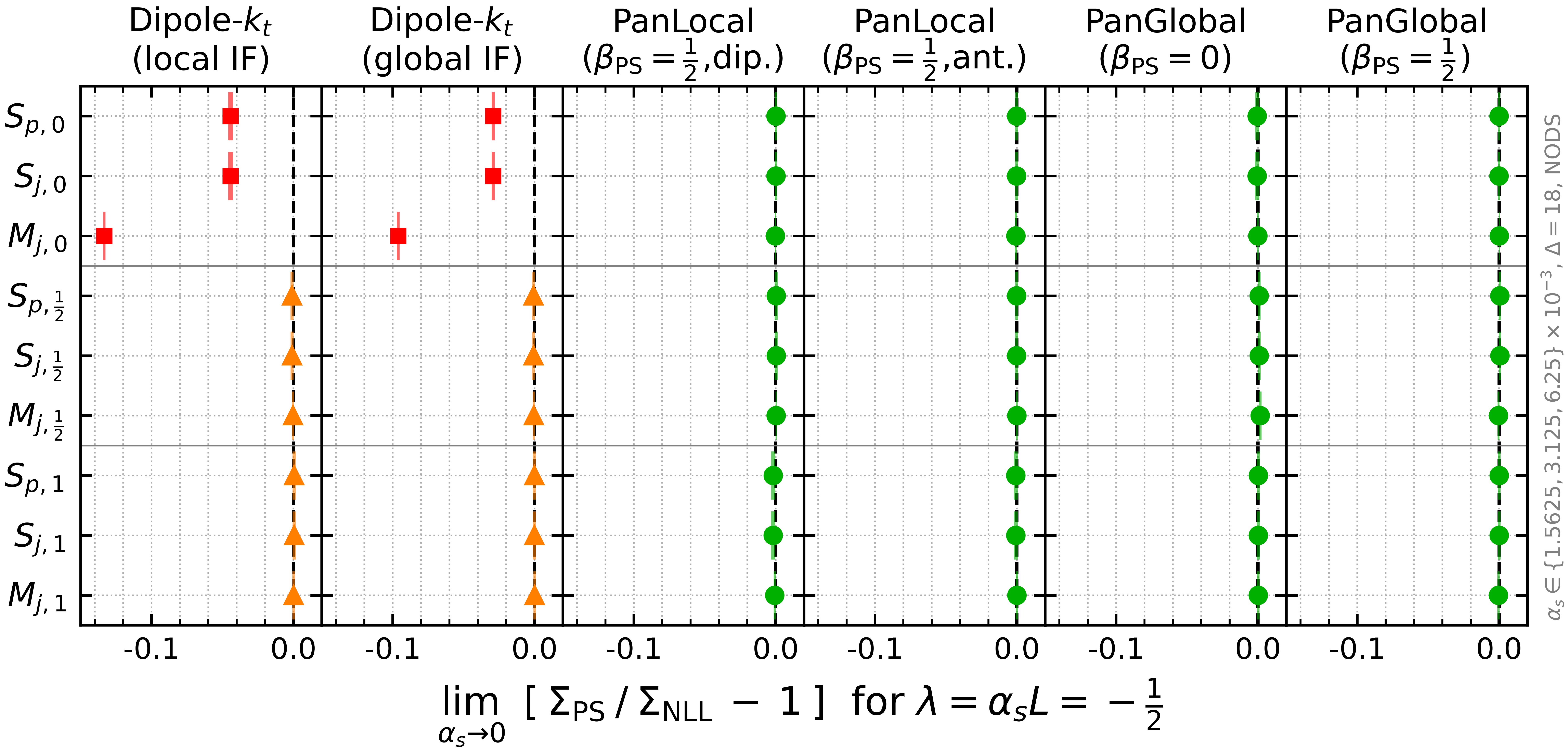
$\rightarrow$ $I_n^{[k\geq 2]}(\nu)$ may not contain L^{n+1} nor L^n

LL tests for CFFE

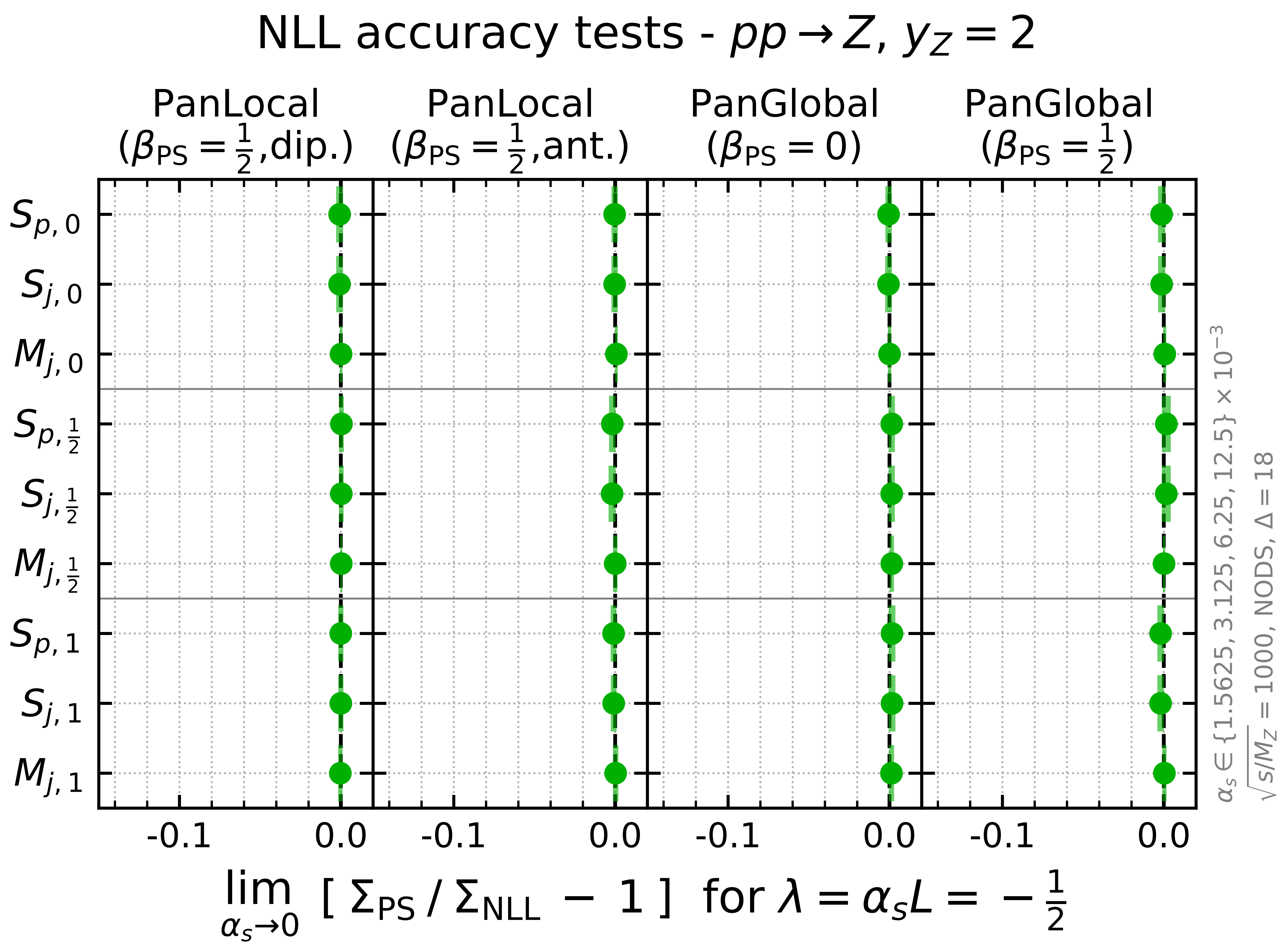


Global event shapes for pp

$$S_{p/j,\beta} = \sum_{i \in f/\text{jets}} p_{\perp,i}\, e^{-\beta|\eta_i|} \qquad M_{j,\beta} = \max_{i \in \text{jets}} \left[p_{\perp,i}\, e^{-\beta|\eta_i|} \right]$$



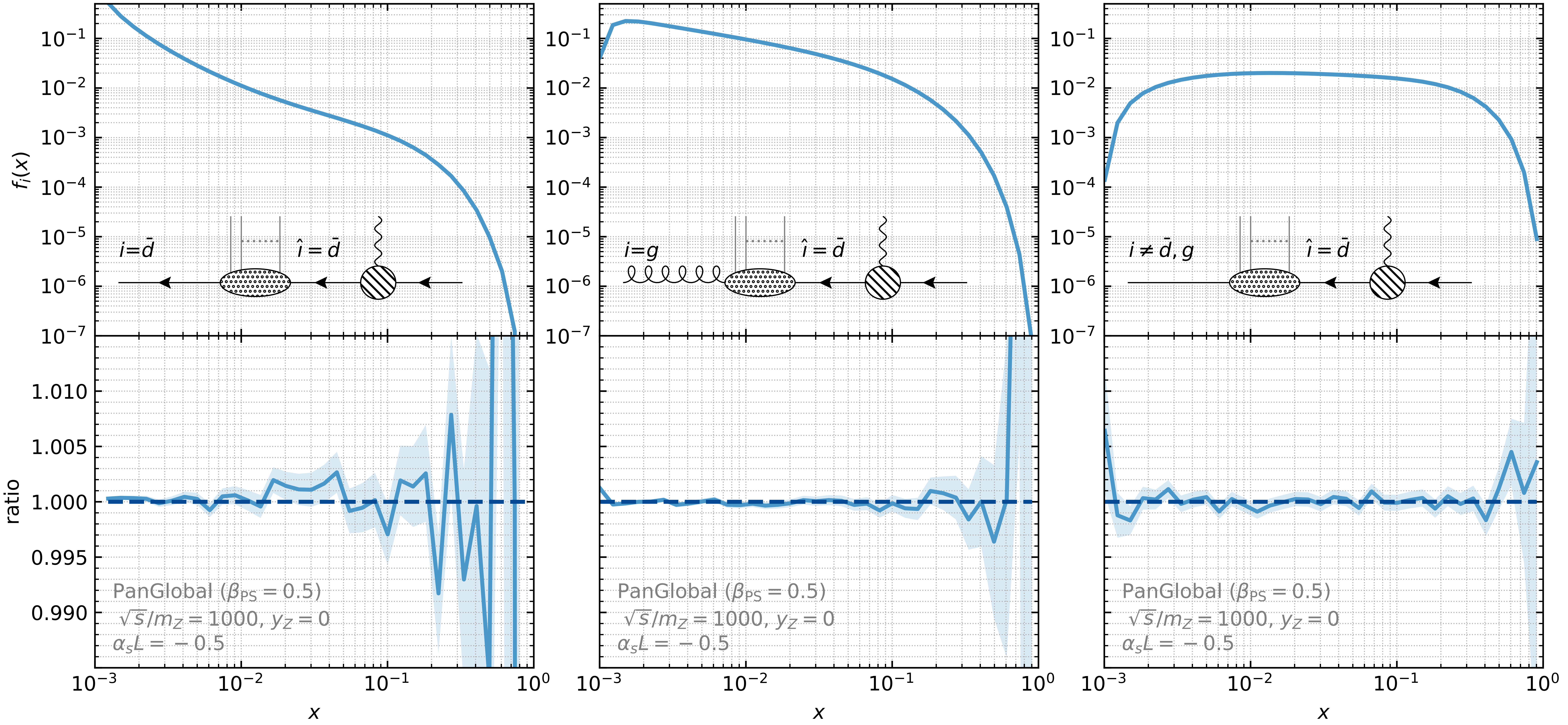
Global event shapes for $y_Z \neq 0$



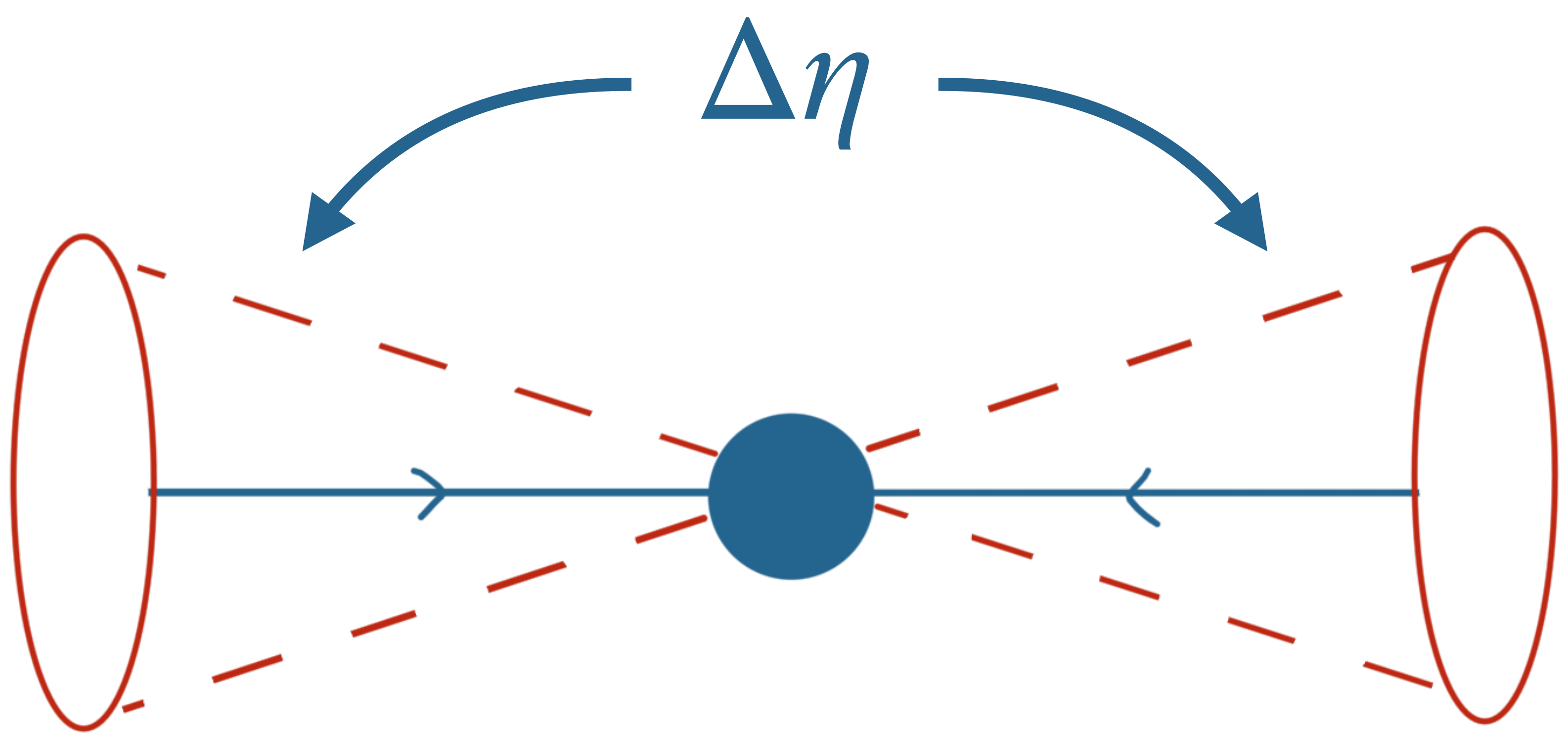
Parton distribution functions

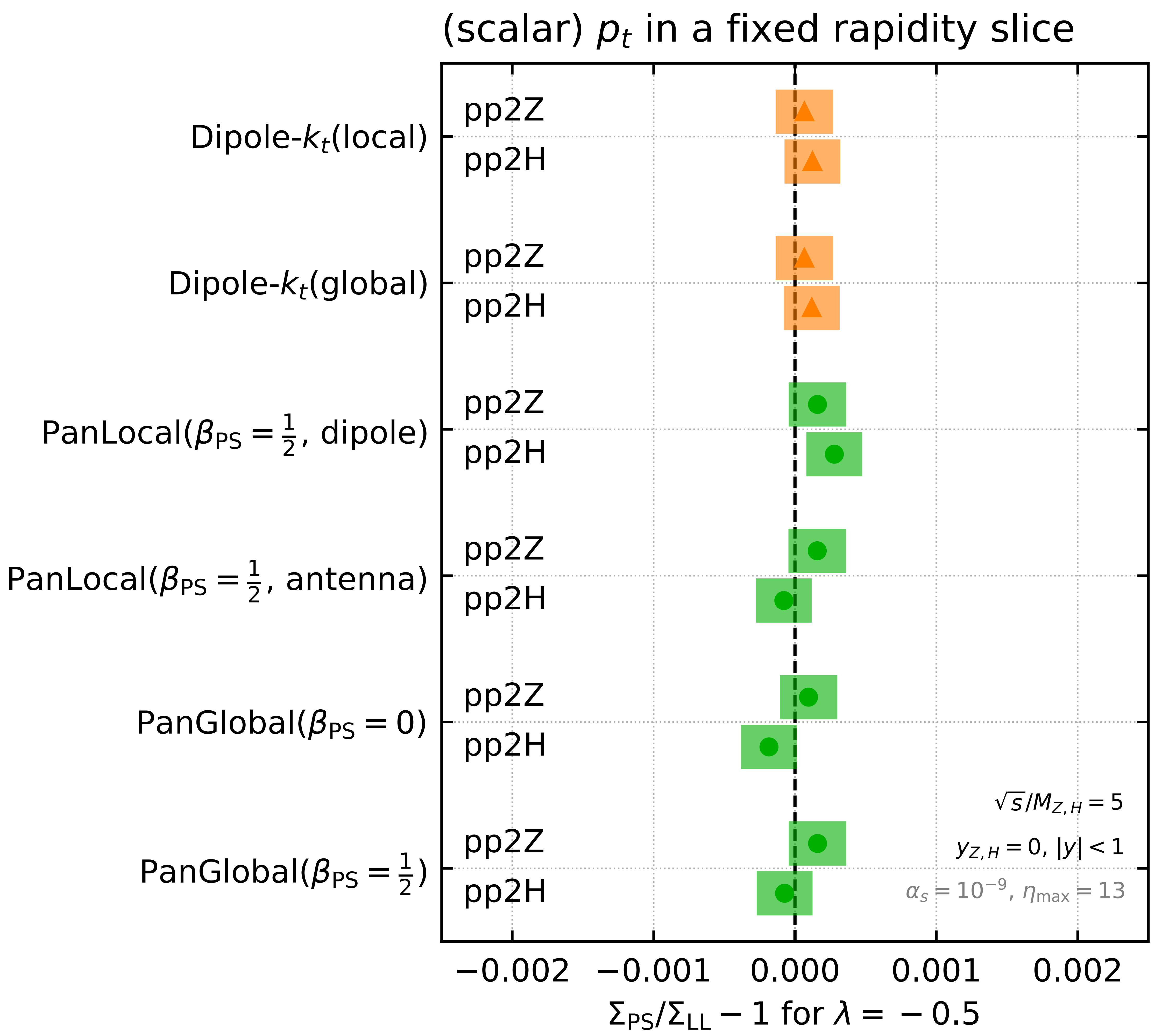
DGLAP expectation

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma_i}{\mathrm{d}x} = \frac{1}{f_{\hat{i}}(\hat{x}, m_Z^2)} \int_{\hat{x}}^{1} \frac{\mathrm{d}z}{z} D_{\hat{i}i}(z, \alpha_s L) f_i\left(\frac{\hat{x}}{z}, p_{t,\mathrm{cut}}^2\right) \delta\left(\frac{\hat{x}}{z} - x\right)$$

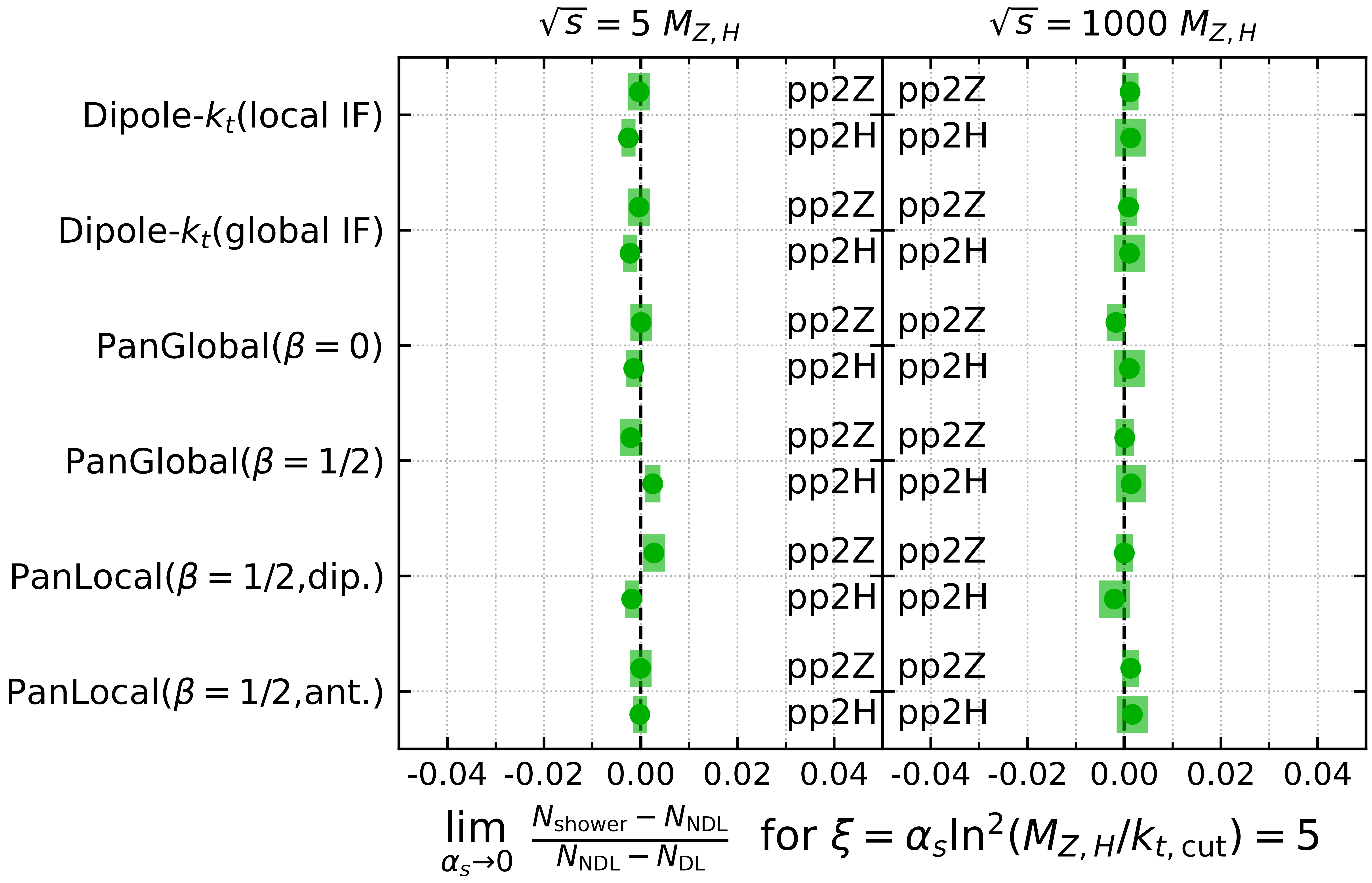


Non-global observable





Particle multiplicity



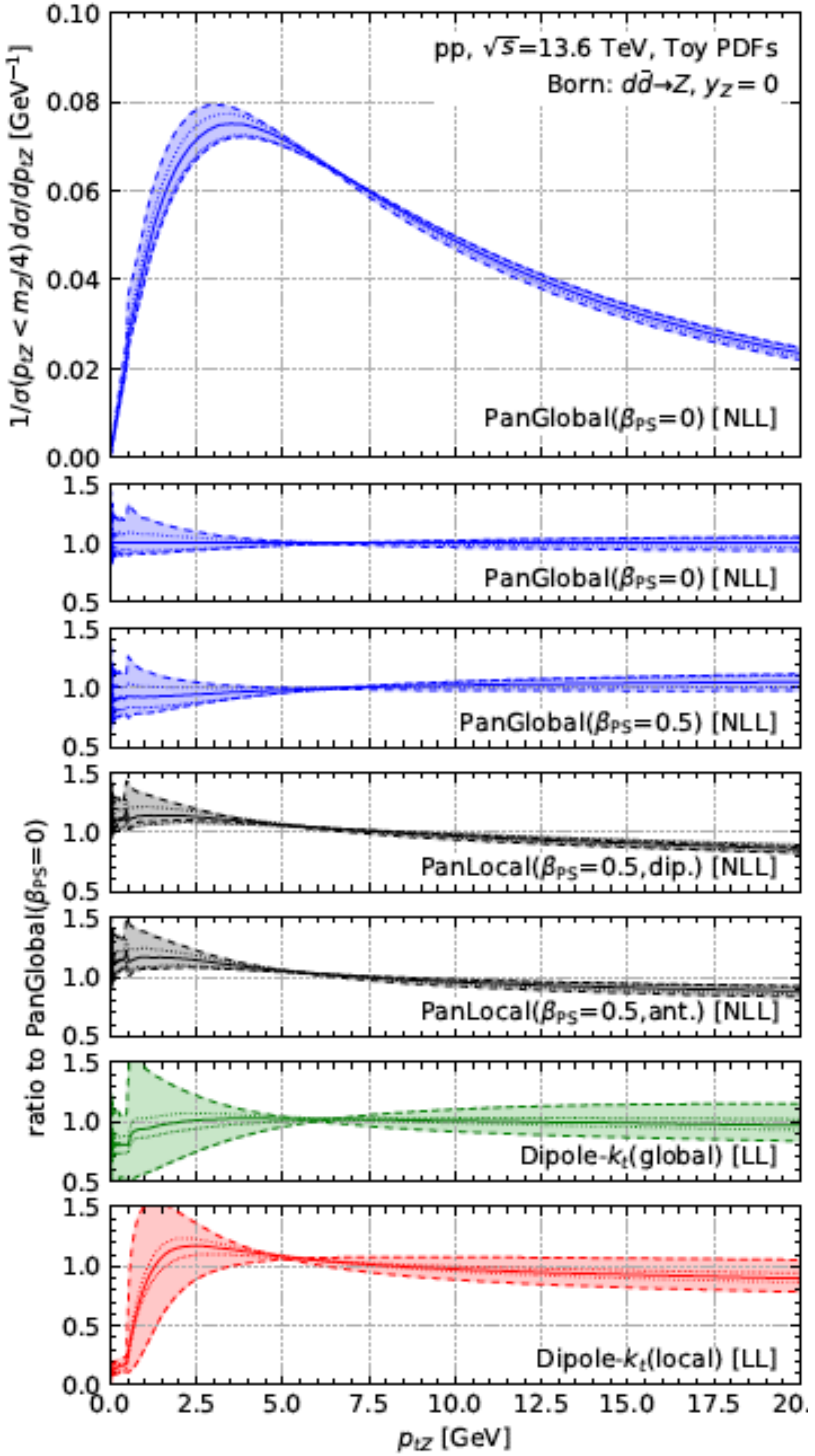
van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso, Soyez, Verheyen [2207.09467]

Towards LHC phenomenology - p_{tZ}

- Consider Z production at the LHC
- Toy setup (fixed underlying born)
- Toy PDFs
- Uncertainty estimated from μ_R, μ_F variations

$$\alpha_s^{(\mathrm{CMW})} = \alpha_s(x_r\mu_{r,0})\left(1 + \frac{K_{\mathrm{CMW}}\alpha_s(x_r\mu_{r,0})}{2\pi} + 2\alpha_s(x_r\mu_{r,0})b_0(1-z)\ln x_r\right)$$

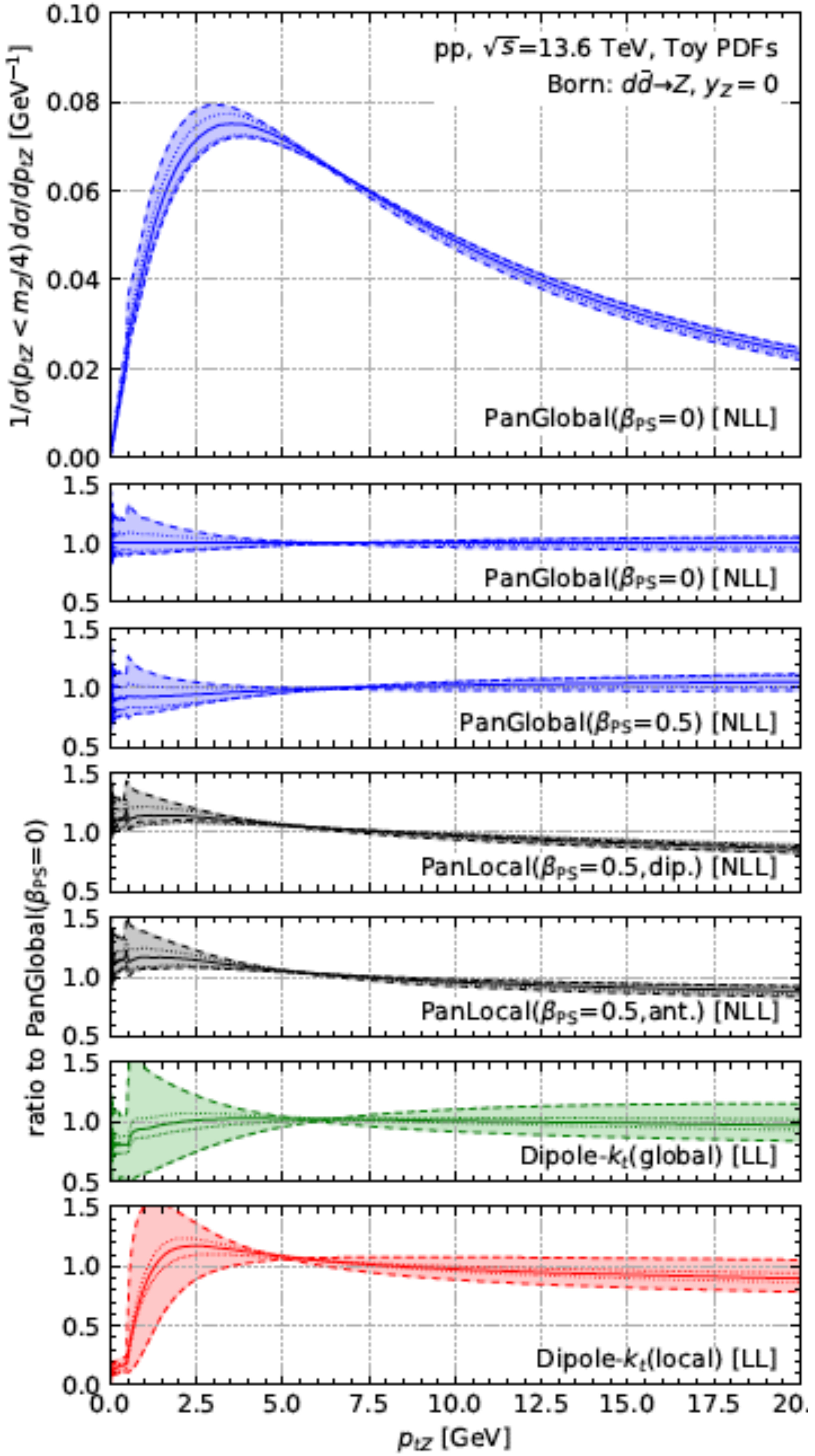
only included for NLL showers
to compensate scale uncertainty for soft emissions



Towards LHC phenomenology - p_{tZ}

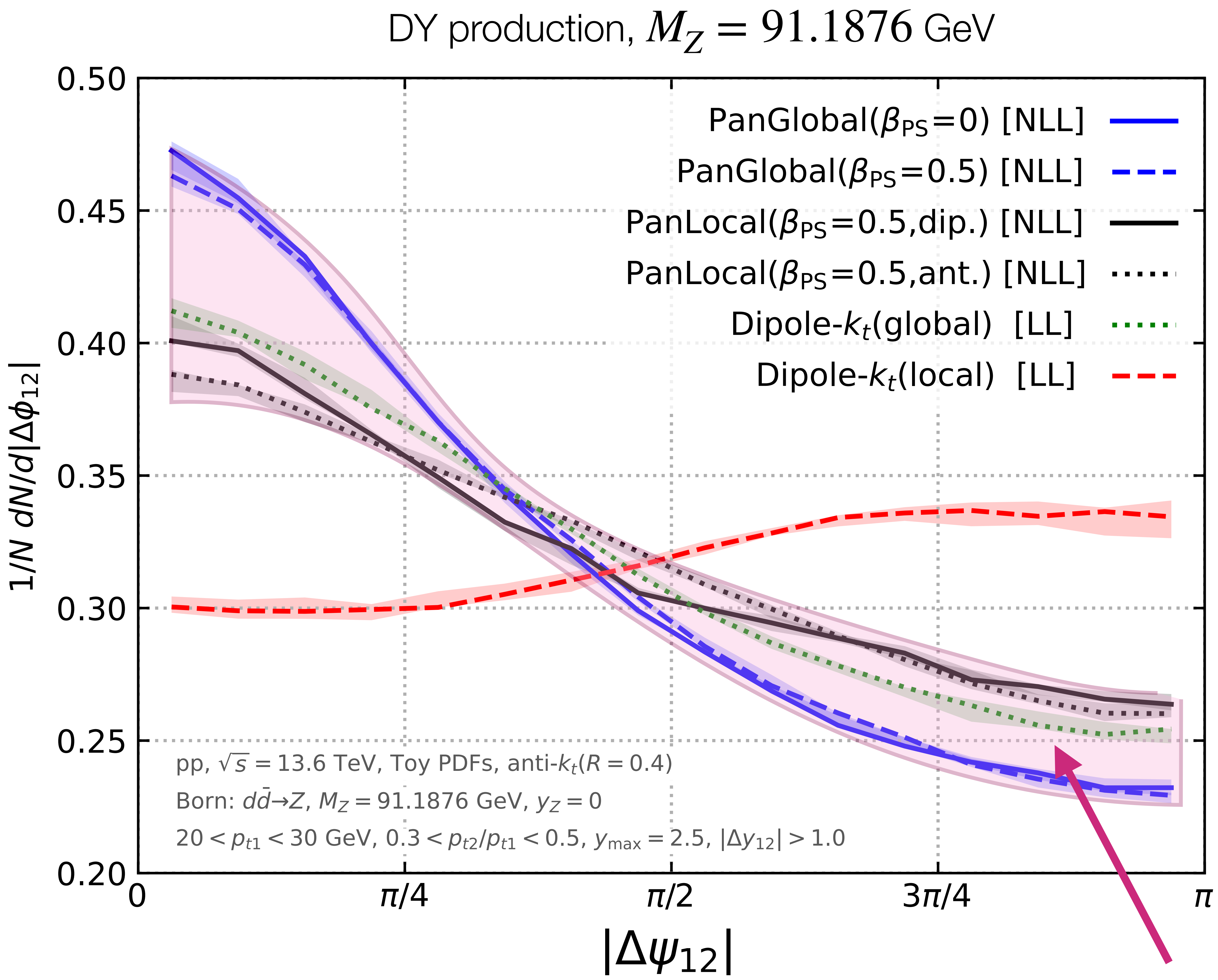
- Consider Z production at the LHC
- Toy setup (fixed underlying born)
- Toy PDFs
- Uncertainty estimated from μ_R, μ_F variations

Differences are relatively small except at very small p_{tZ}
(related to the absence of azimuthal cancelations)



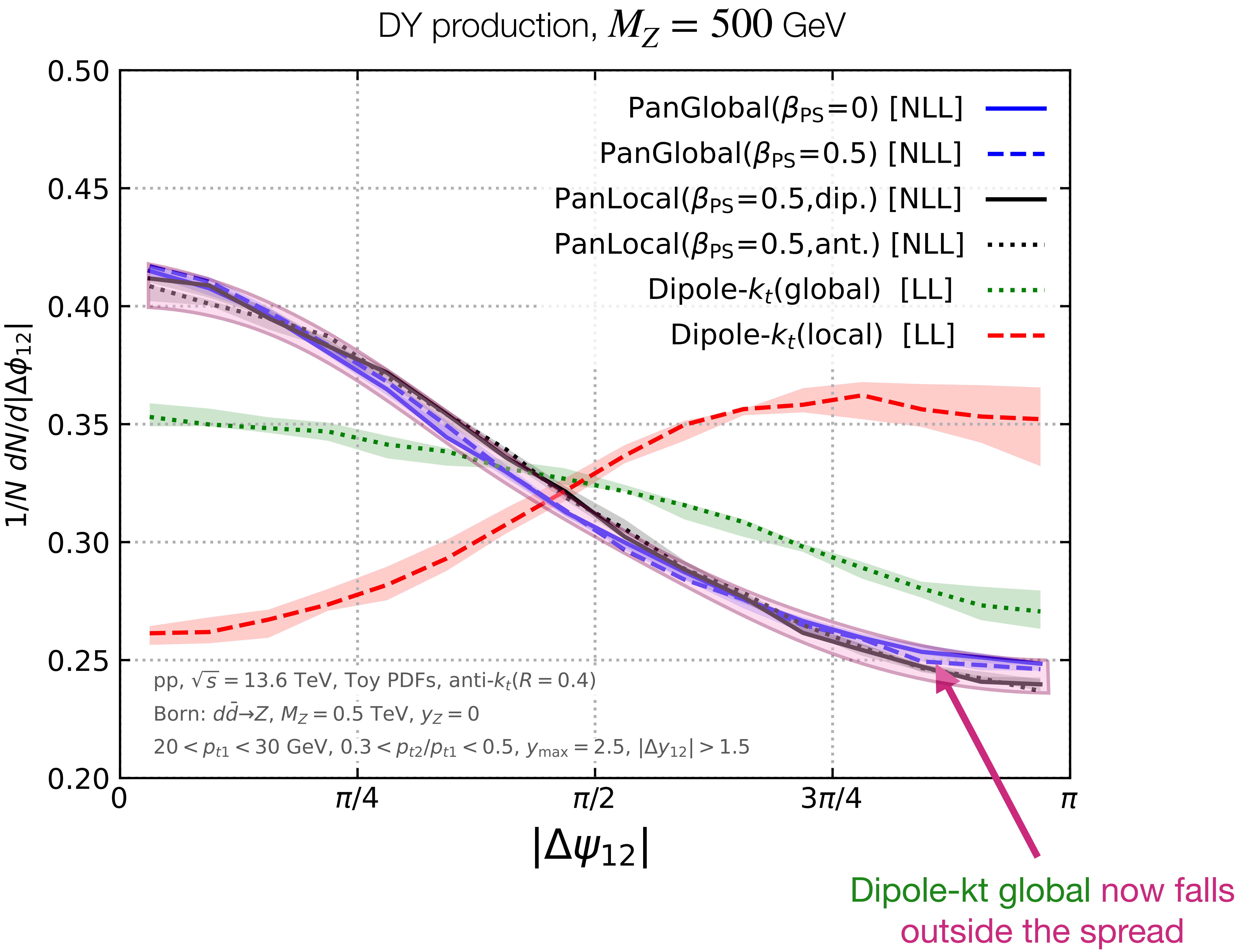
Towards LHC phenomenology - $\Delta\psi_{12}$

- Consider Z production at the LHC
- Toy setup (fixed underlying born)
- Toy PDFs
- Jets clustered with anti-k_t $(R = 0.4)$
- Set $|\Delta y_{12}| > 1$



Towards LHC phenomenology - $\Delta\psi_{12}$

- Consider Z production at the LHC
- Toy setup (fixed underlying born)
- Toy PDFs
- Jets clustered with anti-k_t ($R = 0.4$)
- Set $|\Delta y_{12}| > 1.5$



A standard dipole shower: **dipole-k_t**

1. Evolution variable: transverse momentum (k_t)

2. Kinematic map:

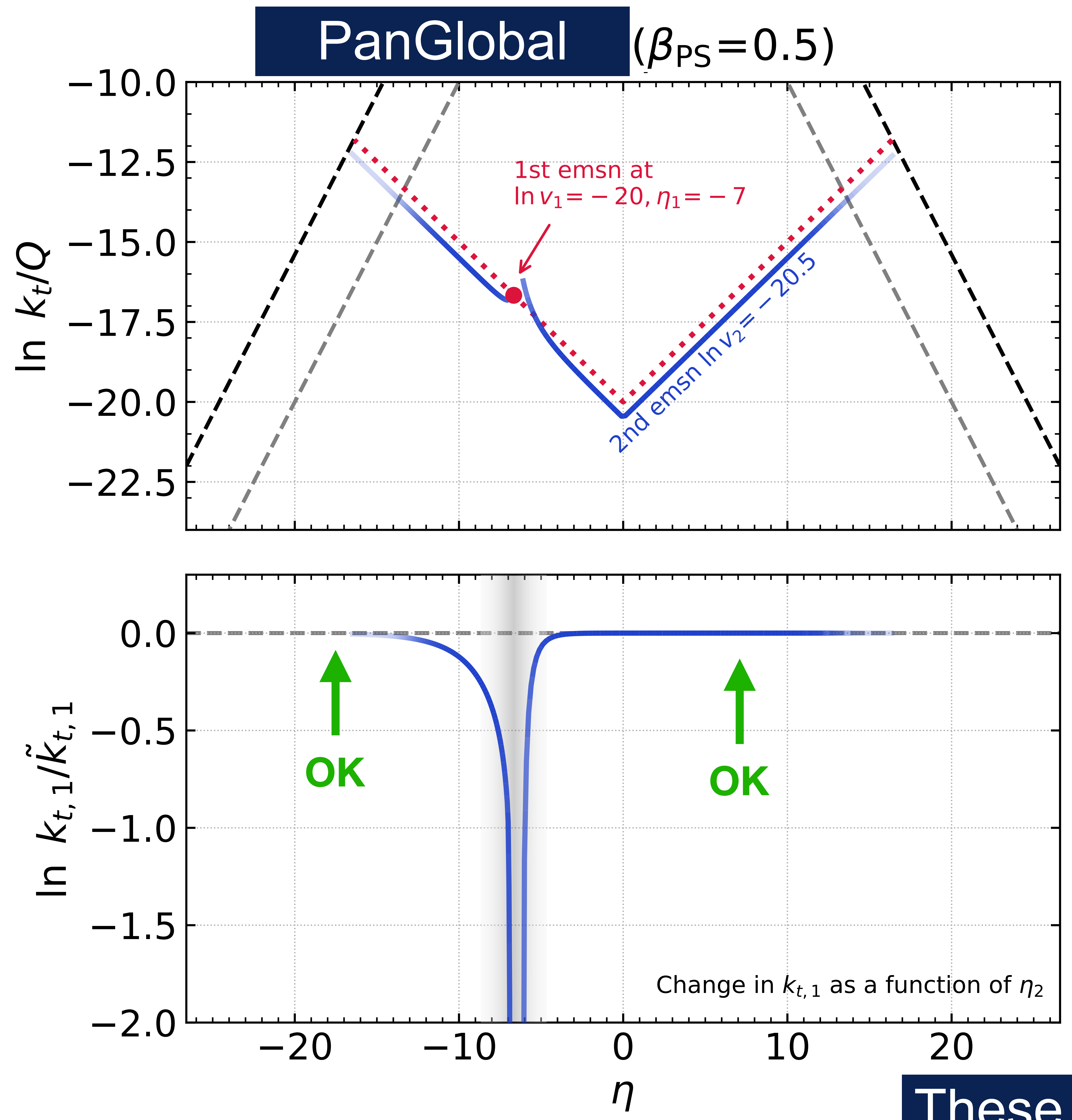
   a) Local Dates back to Gustafson, Petterson [Nucl. Phys. B 306 (1988)], Catani, Seymour [hep-ph/9605323], many variations available

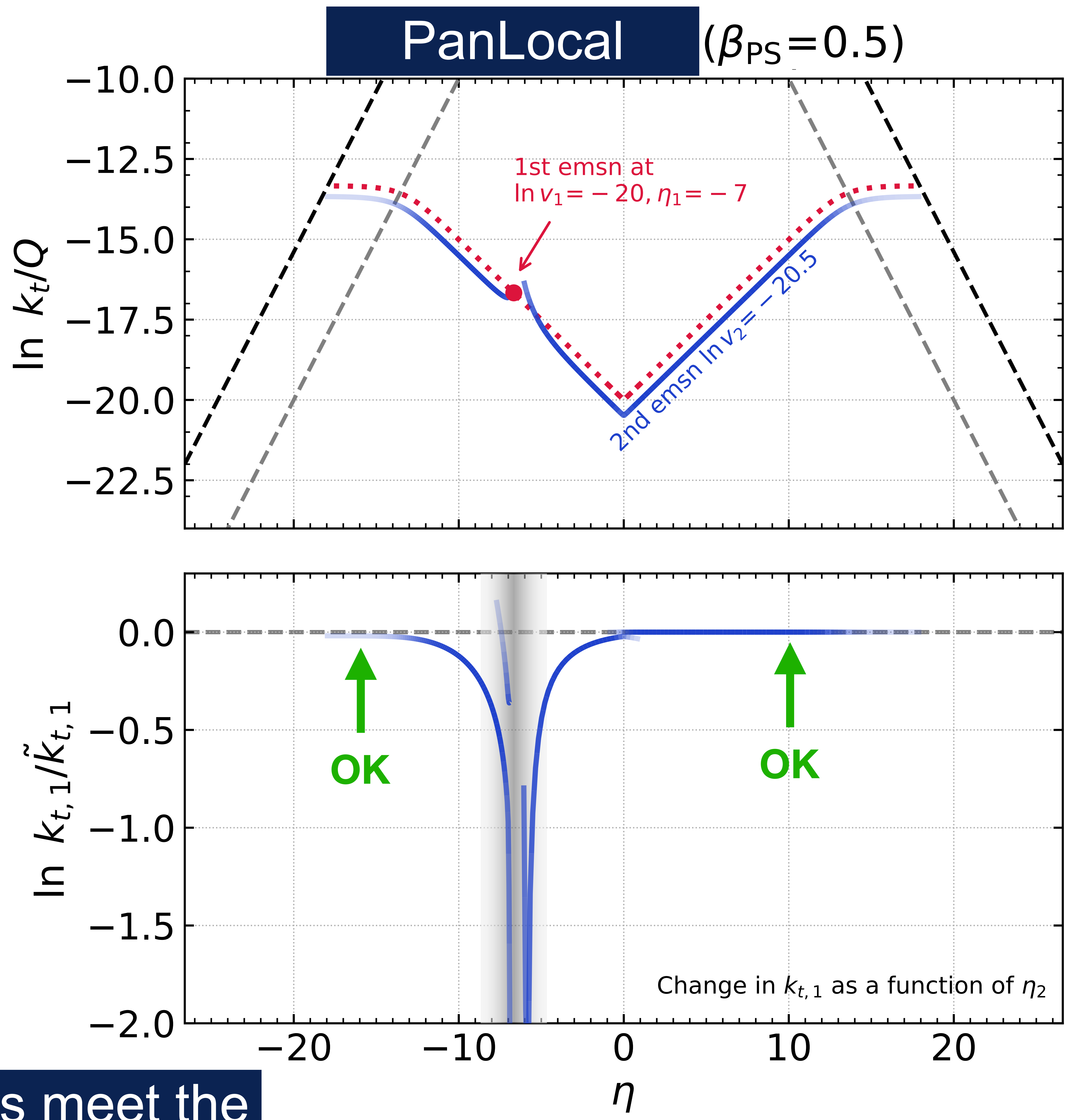
   For every emission the momentum is locally conserved
   This means that the e.g. the Z-boson p_t almost never gets rescaled
   → not in line with the NLL prediction Plätzer, Gieseke [0909.5593], Nagy, Soper [0912.4534]

   b) Global Plätzer, Gieseke [0909.5593], Höche, Prestel [1506.05057] [Pythia8 & Deductor have different solutions]

   The Z-boson absorbs the k_t imbalance induced by the global map through a boost
   Claimed to fix the Z-p_t distribution

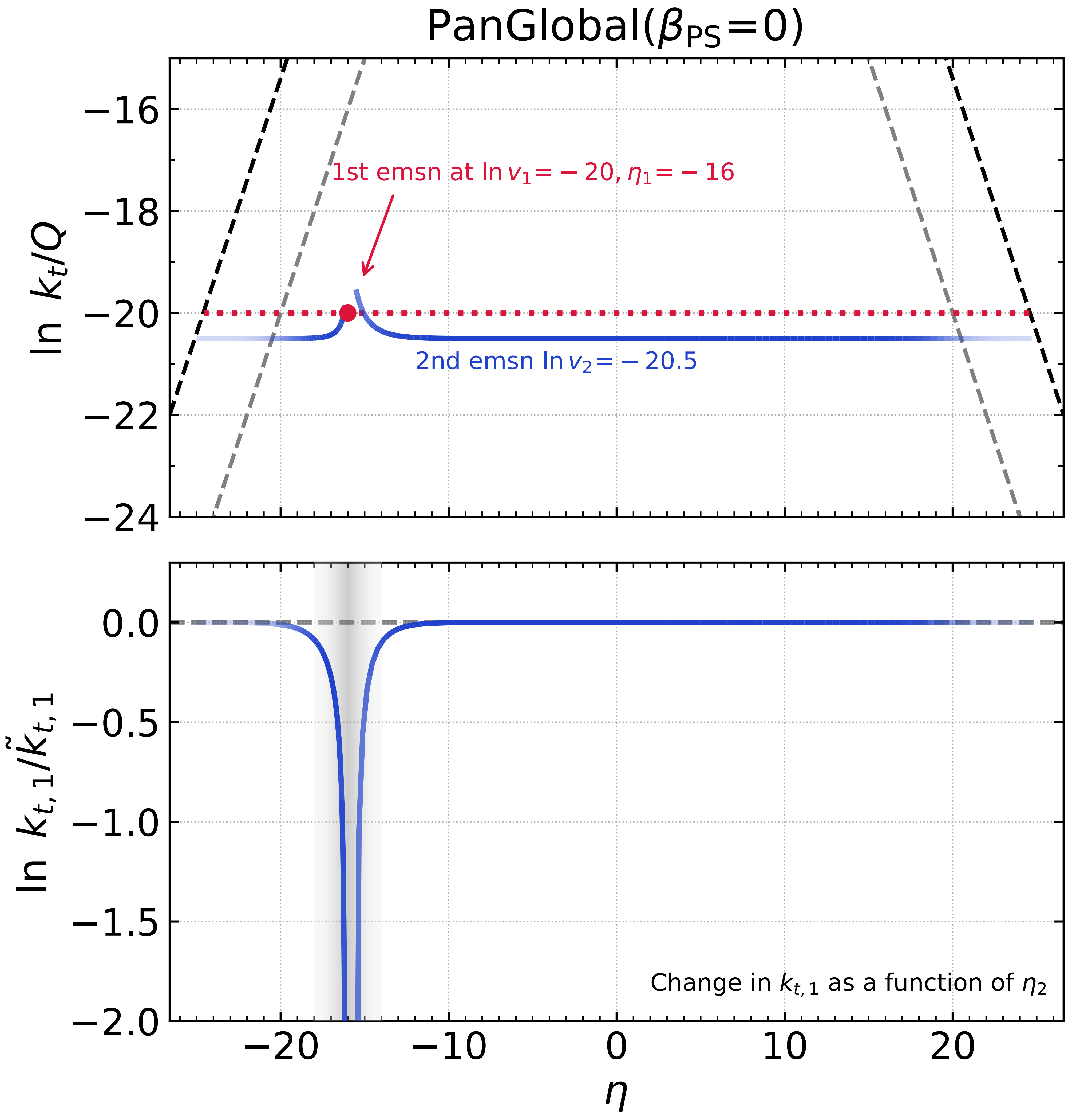
3. Attribution of recoil: dipole CM frame



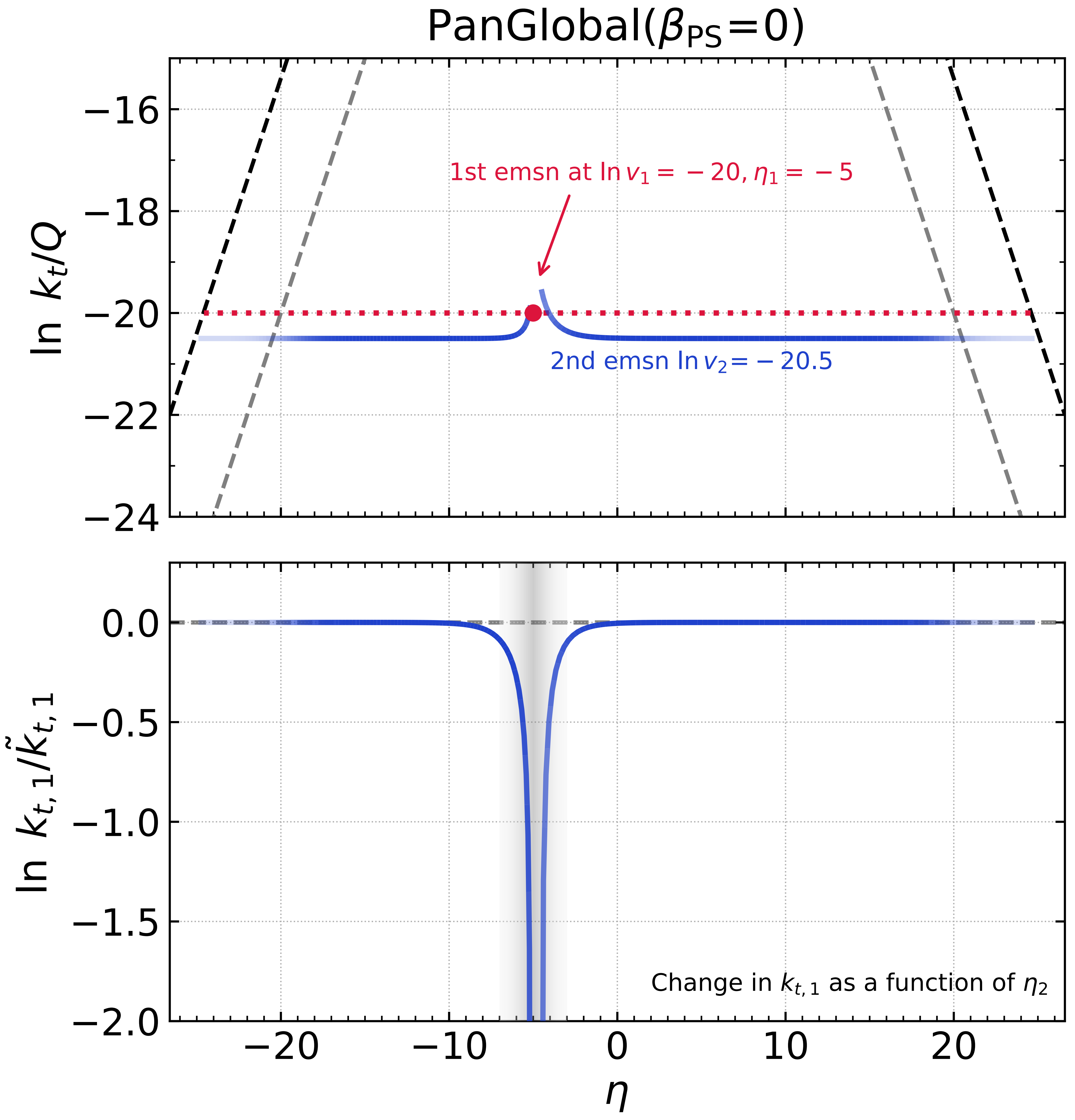


These showers meet the fixed-order criterion

PanGlobal

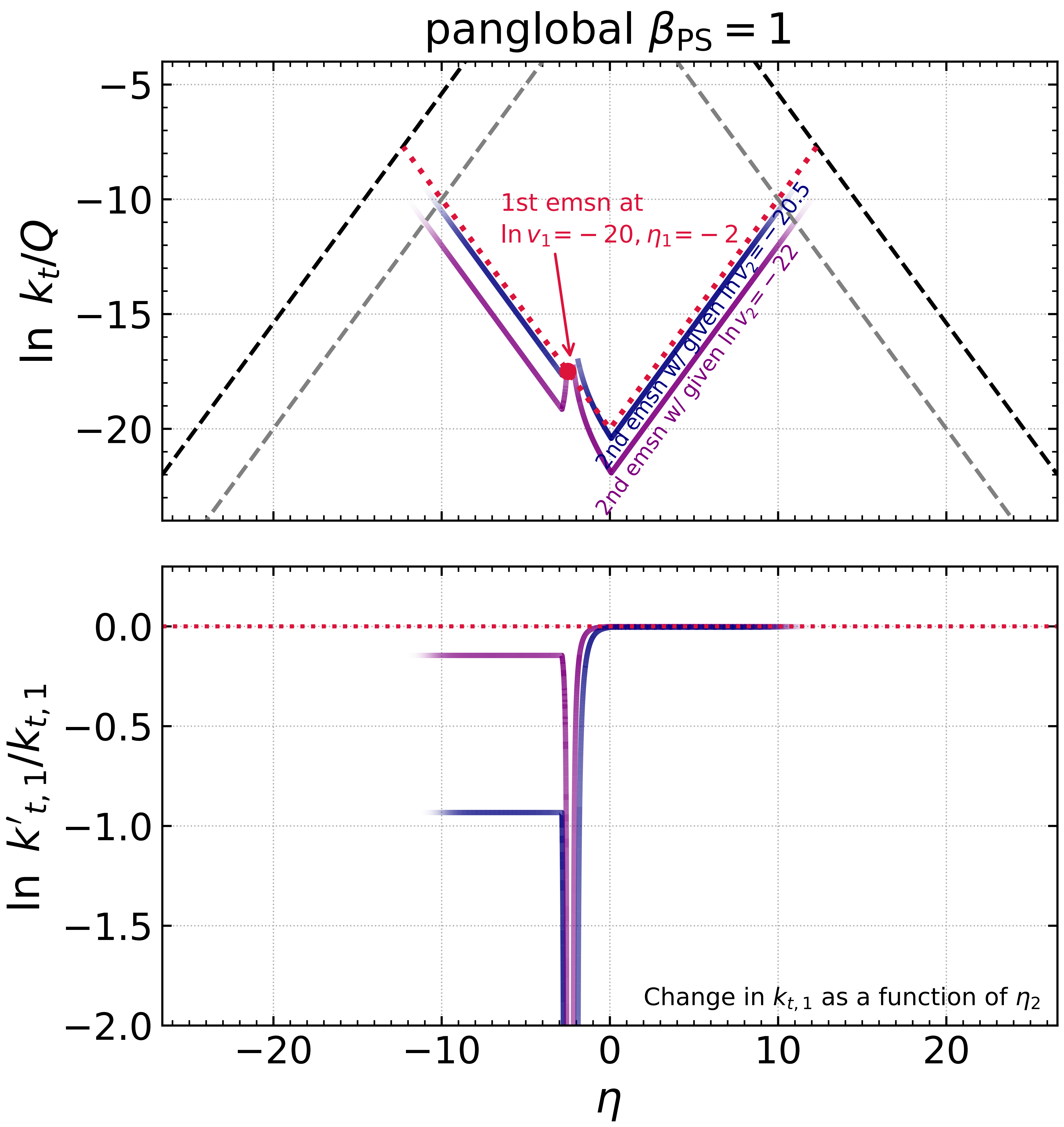


PanGlobal(β_{PS}=0)
1st emsn at $\ln v_1 = -20, \eta_1 = -16$
2nd emsn $\ln v_2 = -20.5$
Change in $k_{t,1}$ as a function of η_2
$\ln k_t/Q$
$\ln k_{t,1}/\tilde{k}_{t,1}$
η



PanGlobal(β_{PS}=0)
1st emsn at $\ln v_1 = -20, \eta_1 = -5$
2nd emsn $\ln v_2 = -20.5$
Change in $k_{t,1}$ as a function of η_2
$\ln k_t/Q$
$\ln k_{t,1}/\tilde{k}_{t,1}$
η

Issue for $\beta_{\mathrm{PS}} = 1$



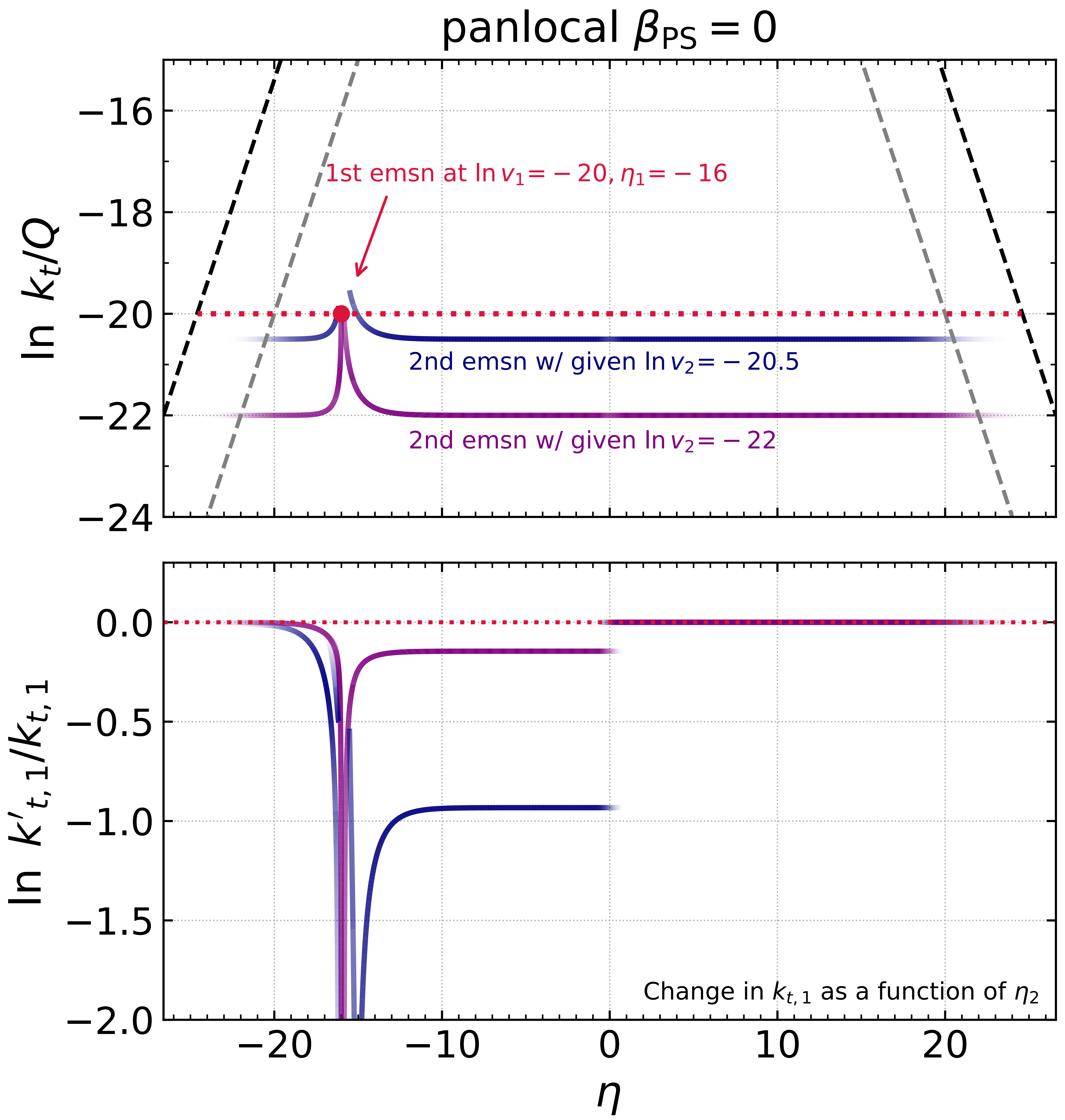
- For IF dipoles, momentum of first emission is rescaled by $b_j = 1 - \beta_k$ in map
- For $\beta = 1$ this equates to $1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}} \frac{v}{Q}$ and becomes independent of $\bar{\eta}$
- Consider change in first emitted parton:

$$p_{k,1} = \tilde{p}_j \to b_j p_{k,1} = \left(1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}} \frac{v_2}{Q}\right) p_{k,1}$$

- With $\frac{\tilde{s}_i}{\tilde{s}_{ij}} = \frac{2\tilde{p}_i \cdot Q}{2\tilde{p}_i \cdot \tilde{p}_j} = \frac{1}{b_{k,1}}$ and $b_{k,1} = \beta_{k,1} = \frac{v_1}{Q}$

$$\frac{k_{\perp,1}}{k_{\perp,1\ \text{after}\ 2}} = \left(1 - \frac{v_2}{v_1}\right)$$

PanLocal issue for $\beta_{\rm PS} = 0$



- Recoil is taken from the first gluon even when emissions are separated in rapidity
- Separation of dipole in event CM frame is not enough to cure dipole-showers with local maps from locality issue, the transverse momentum ordering is problematic here
- Only when emissions are ordered in angle ($\beta_{\rm PS} > 0$) we solve this
- Then commensurate k_t emissions are ordered in angle, so they take their recoil from the hard system (after boost)

Colour tests

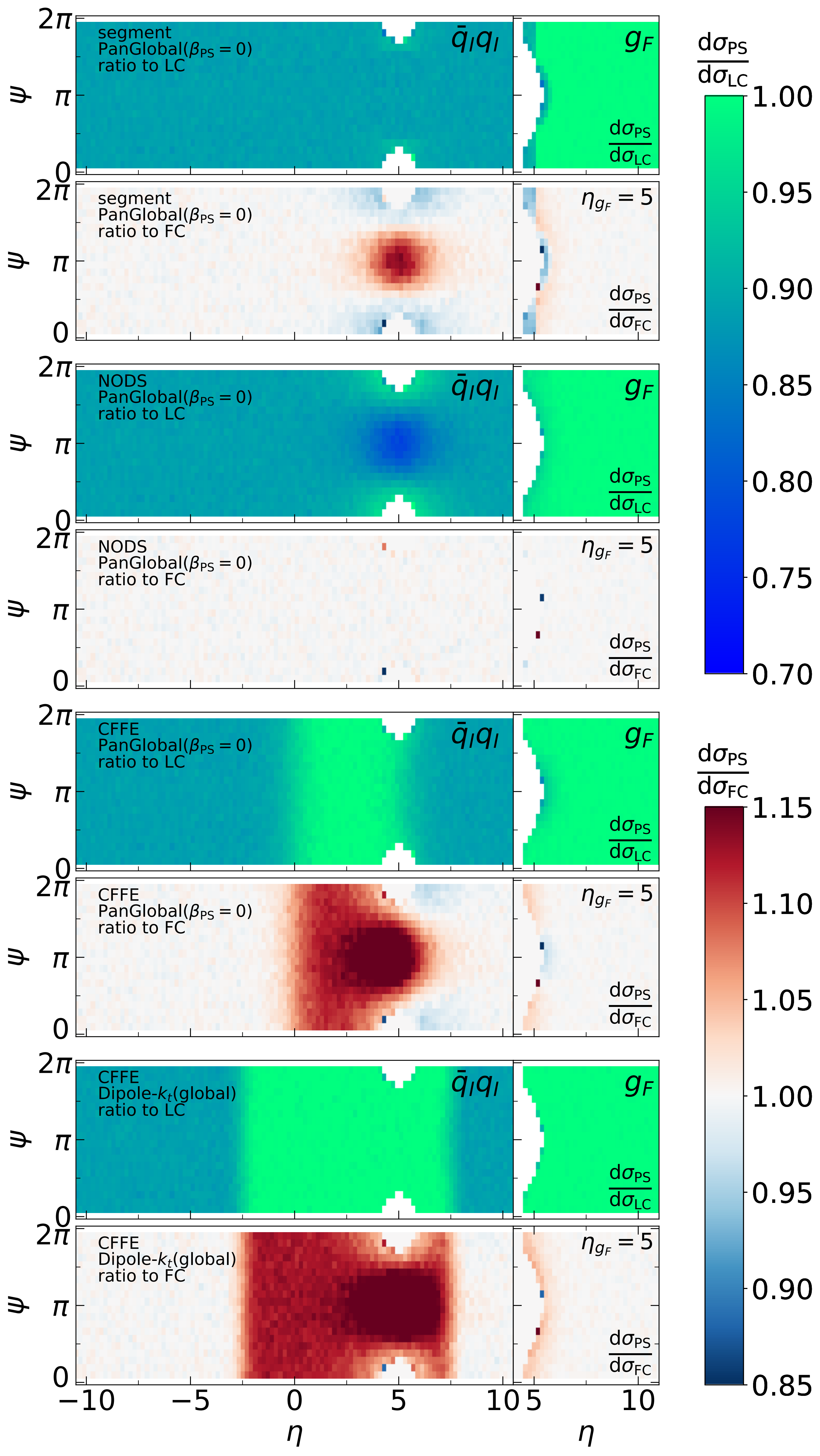
Test of the differential matrix element

Here primary $\bar{q}q$ Lund plane and the new g Lund leaf

LC = leading colour (standard)
FC = full colour

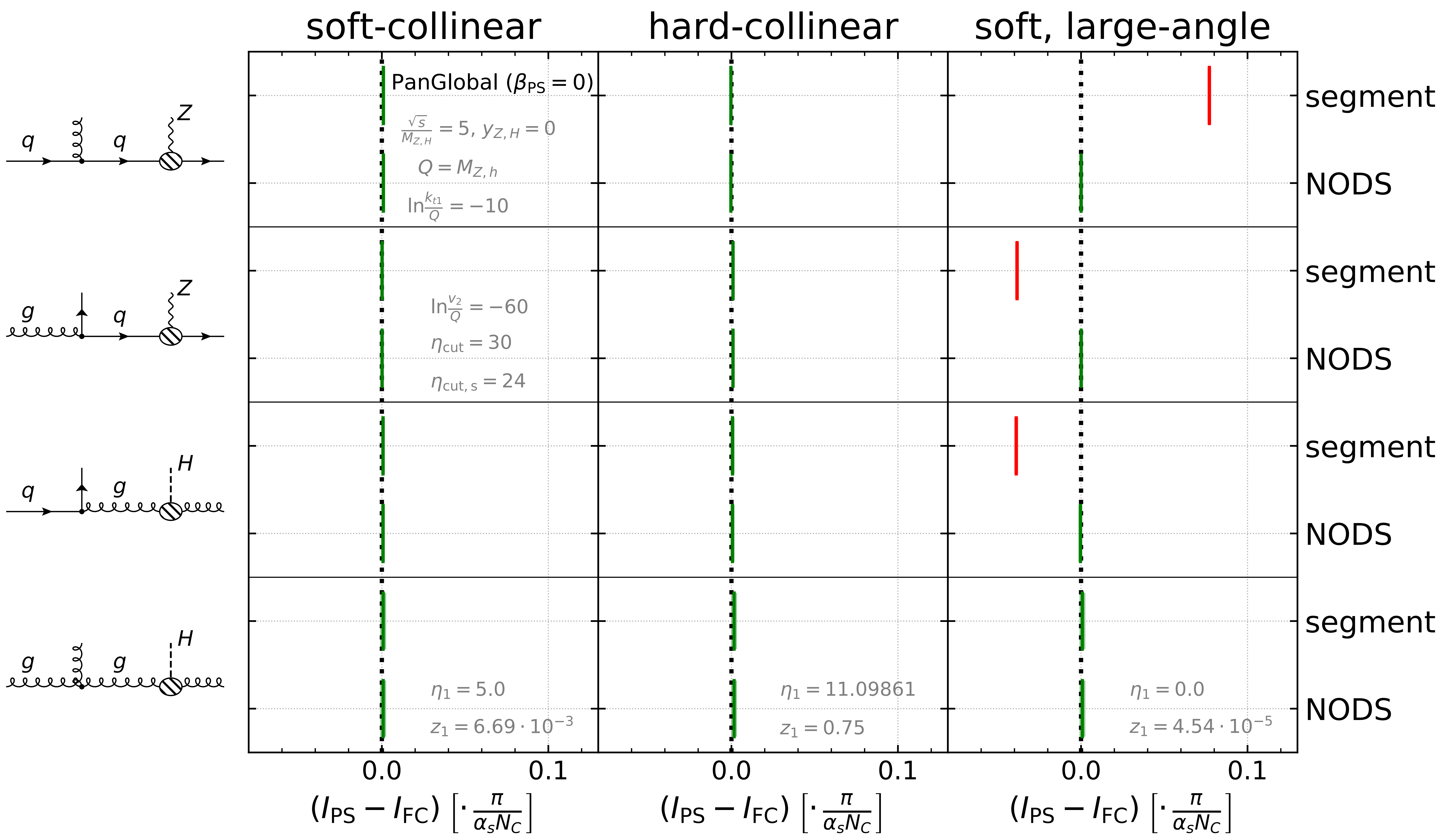
CFFE = standard colour treatment

Segment and NODS two ways to improve the colour handling in the PanScales showers



Colour tests

$$I_{\rm FC}^{Zg_1} \equiv \int \frac{d\Omega}{2\pi} \frac{|\mathcal{M}_{q\bar{q}g_1g_2}|^2}{|\mathcal{M}_{q\bar{q}g_1}|^2}$$

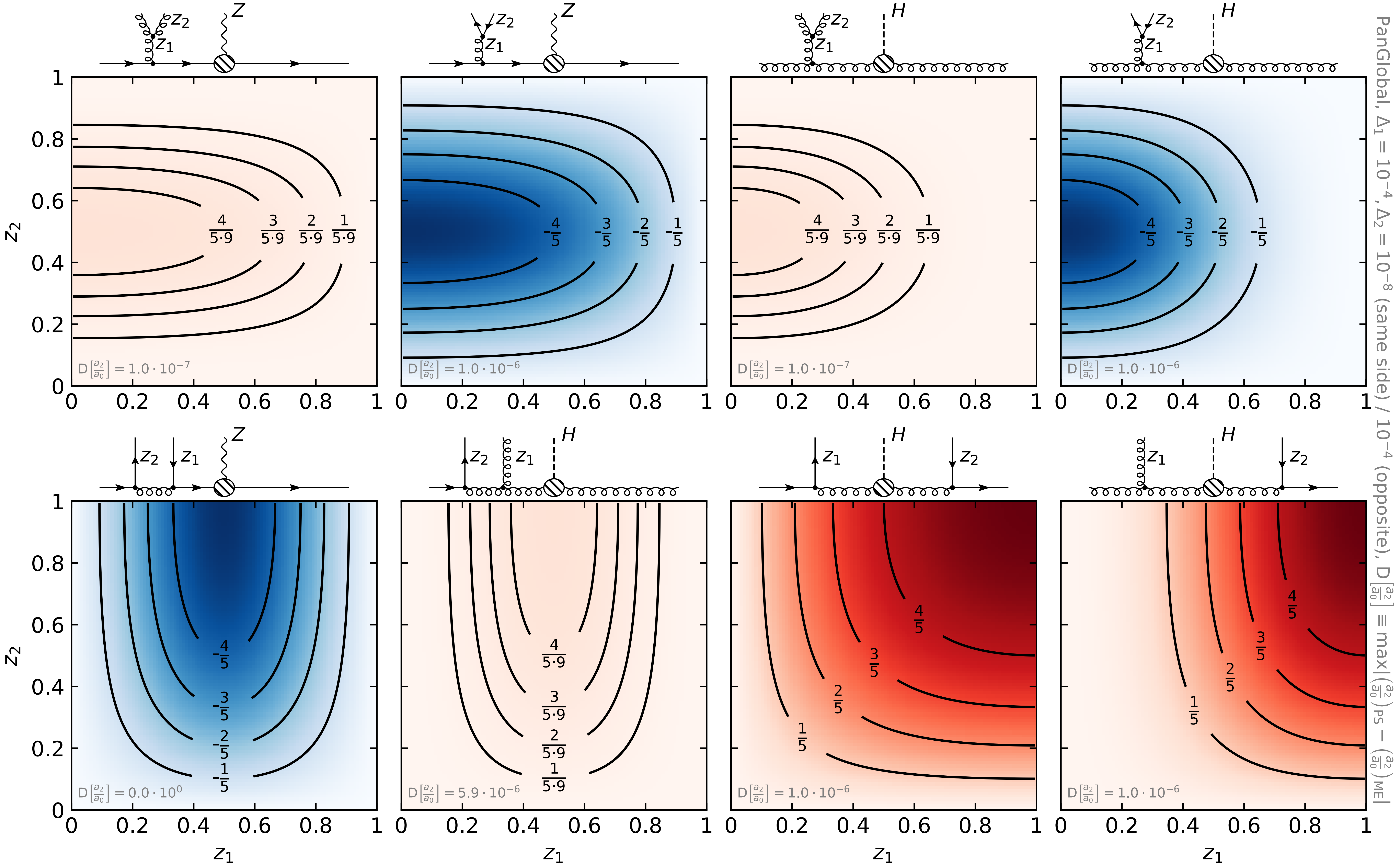


Test of the integrated rate of emissions

Spin tests

$$\frac{d\sigma}{d\Delta\psi_{ij}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{ij})\right)$$

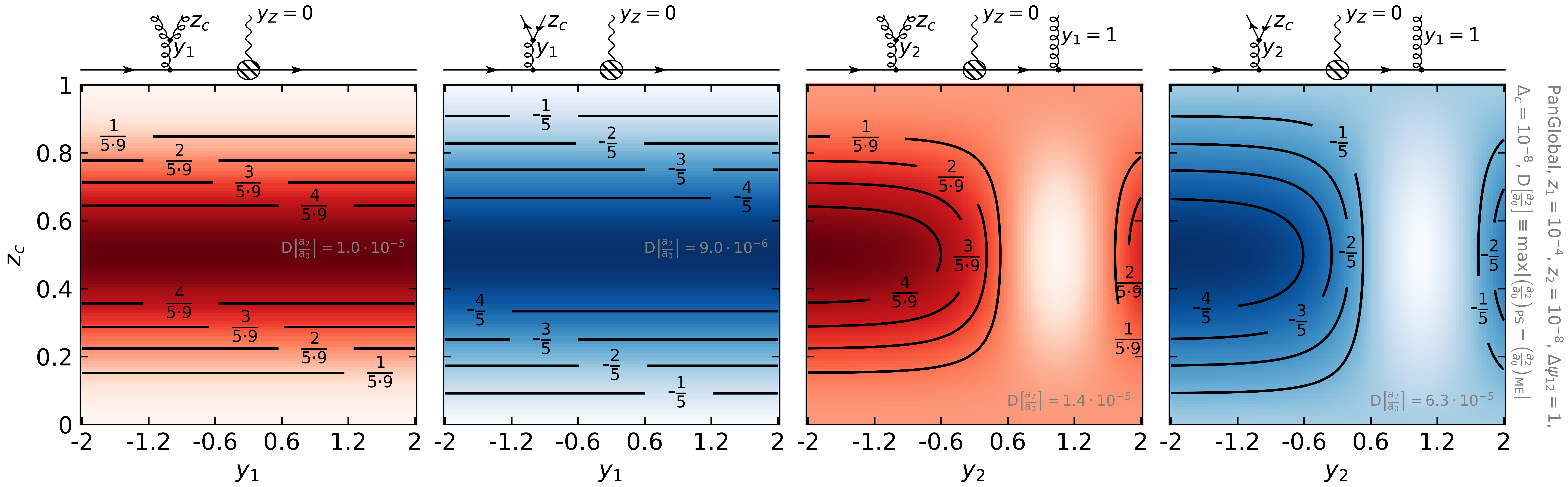
Two collinear emissions



Spin tests

$$\frac{d\sigma}{d\Delta\psi_{ij}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{ij})\right)$$

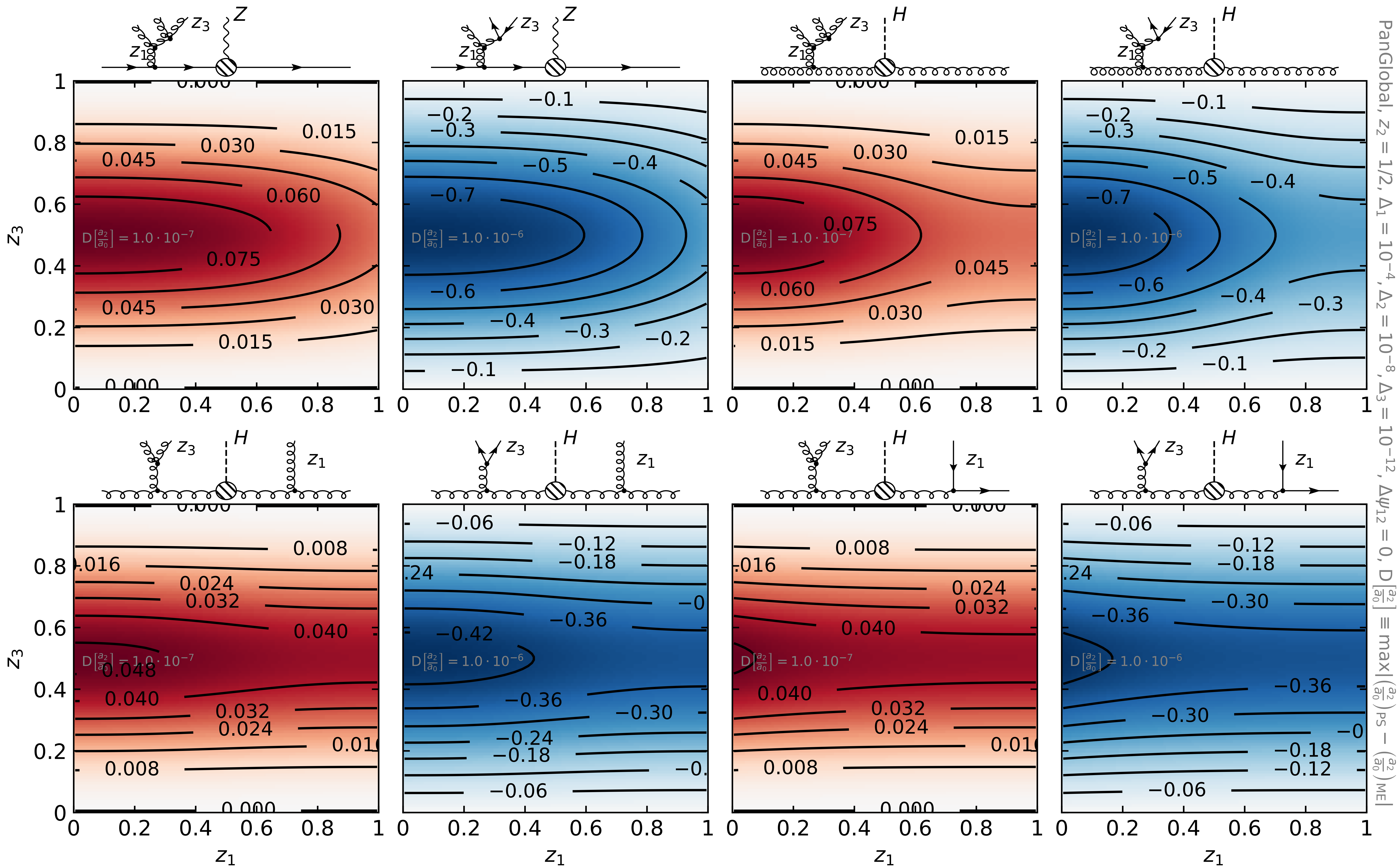
One collinear, one soft emission



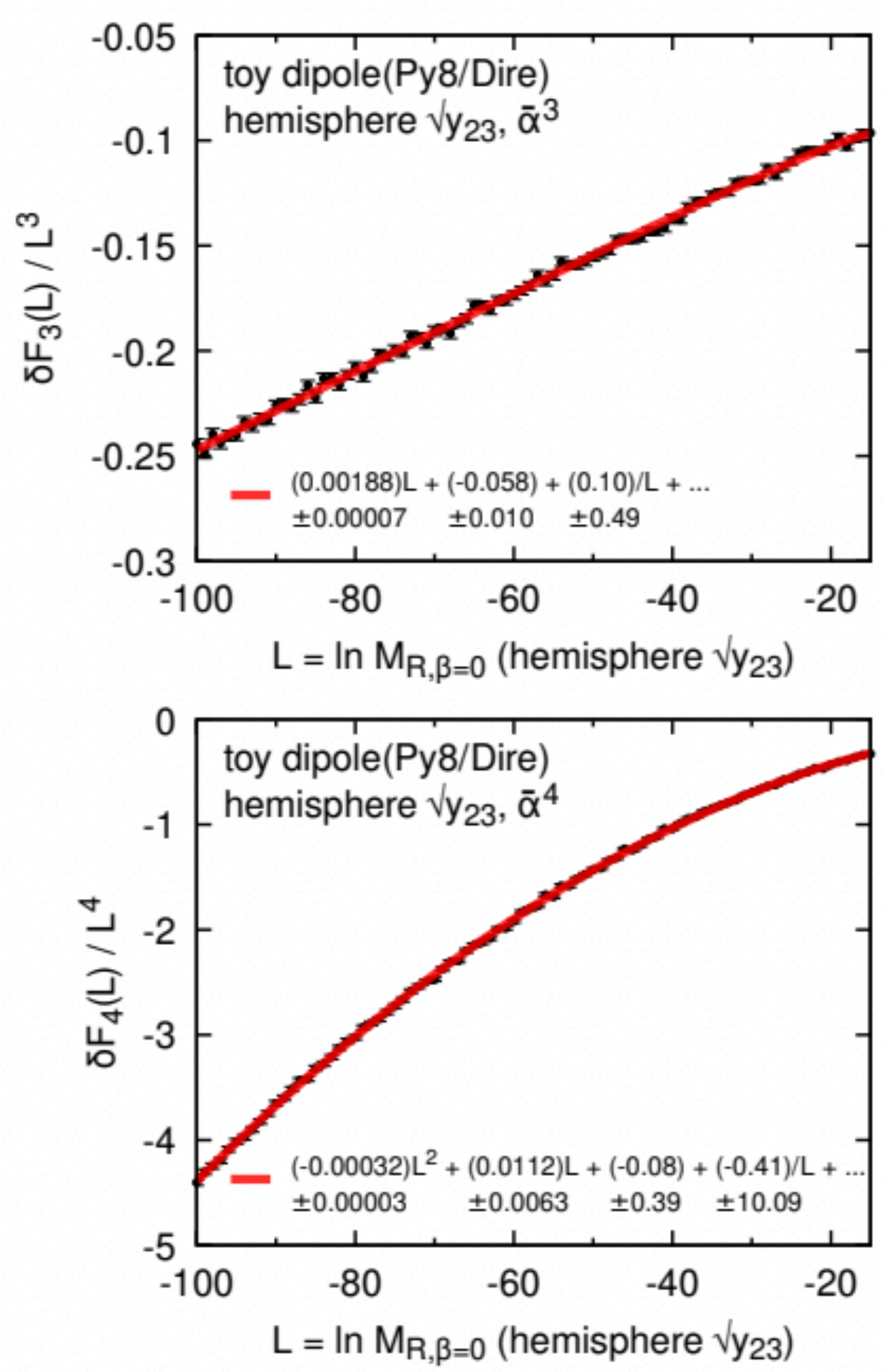
Spin tests

$$\frac{d\sigma}{d\Delta\psi_{13}} \propto a_0 \left(1 + \frac{a_2}{a_0}\cos(2\Delta\psi_{13}) + \frac{b_2}{a_0}\sin(2\Delta\psi_{13})\right)$$

Three collinear emissions

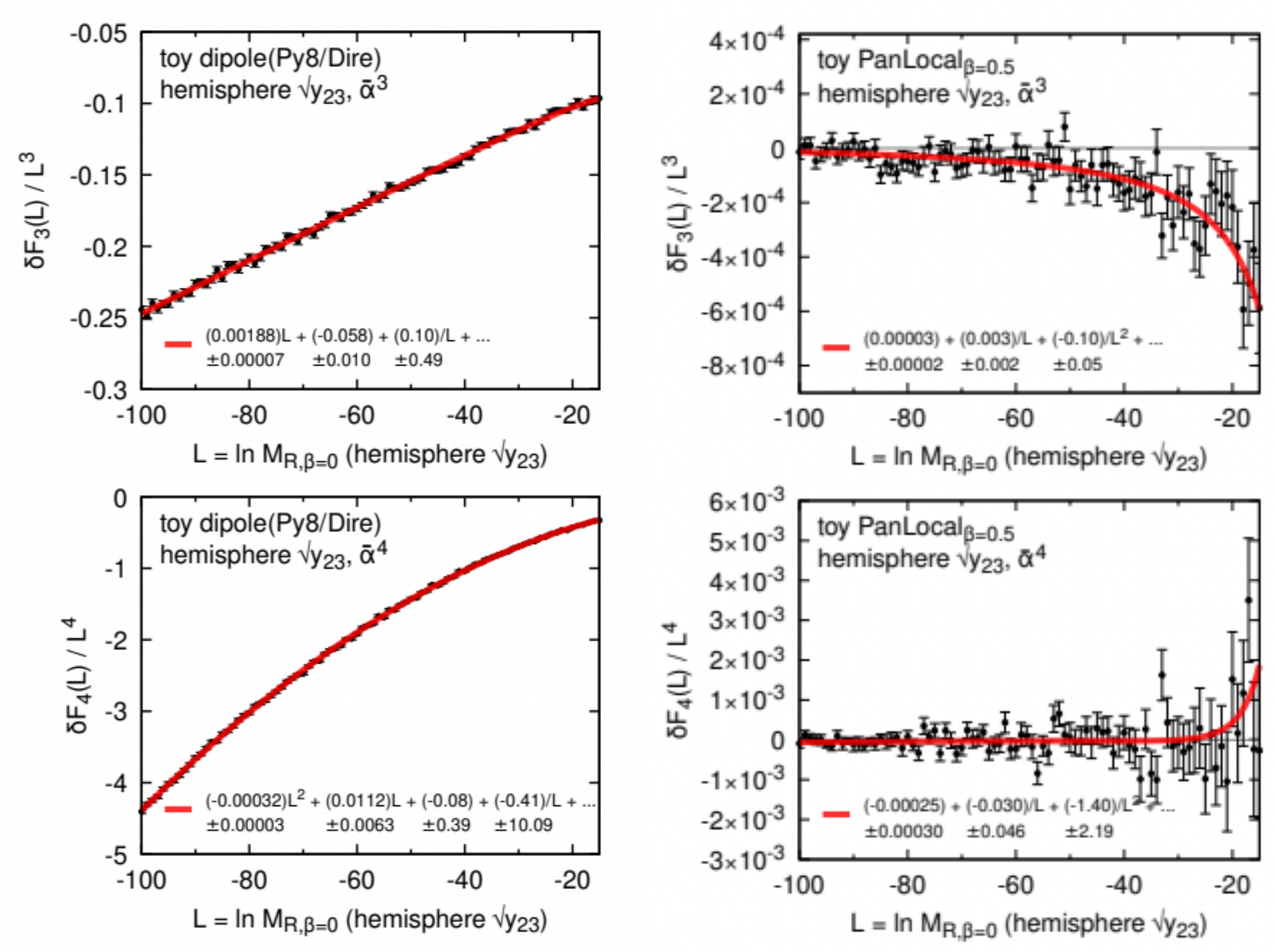


Super-leading logarithms



- Consider $M_{R,0}$, max $p_\perp$ of emissions in the right hemisphere (sensitive to super-leading logs at $\mathcal{O}(\alpha_s^3)$)
- Take toy-model approach with only soft primary emissions and fixed coupling
- Take difference between CEASAR result and toy shower $\delta F_n(L)$, n = order in α_s, where $F = \sum \alpha_s^n F_n$ has terms of $\alpha_s^n L^m$ with $m \leq n$
- Clearly a discrepancy at fixed-order for standard dipole showers
- Vanishes at all orders because it is numerically comparable to the NNLL terms -> orange points

Super-leading logarithms

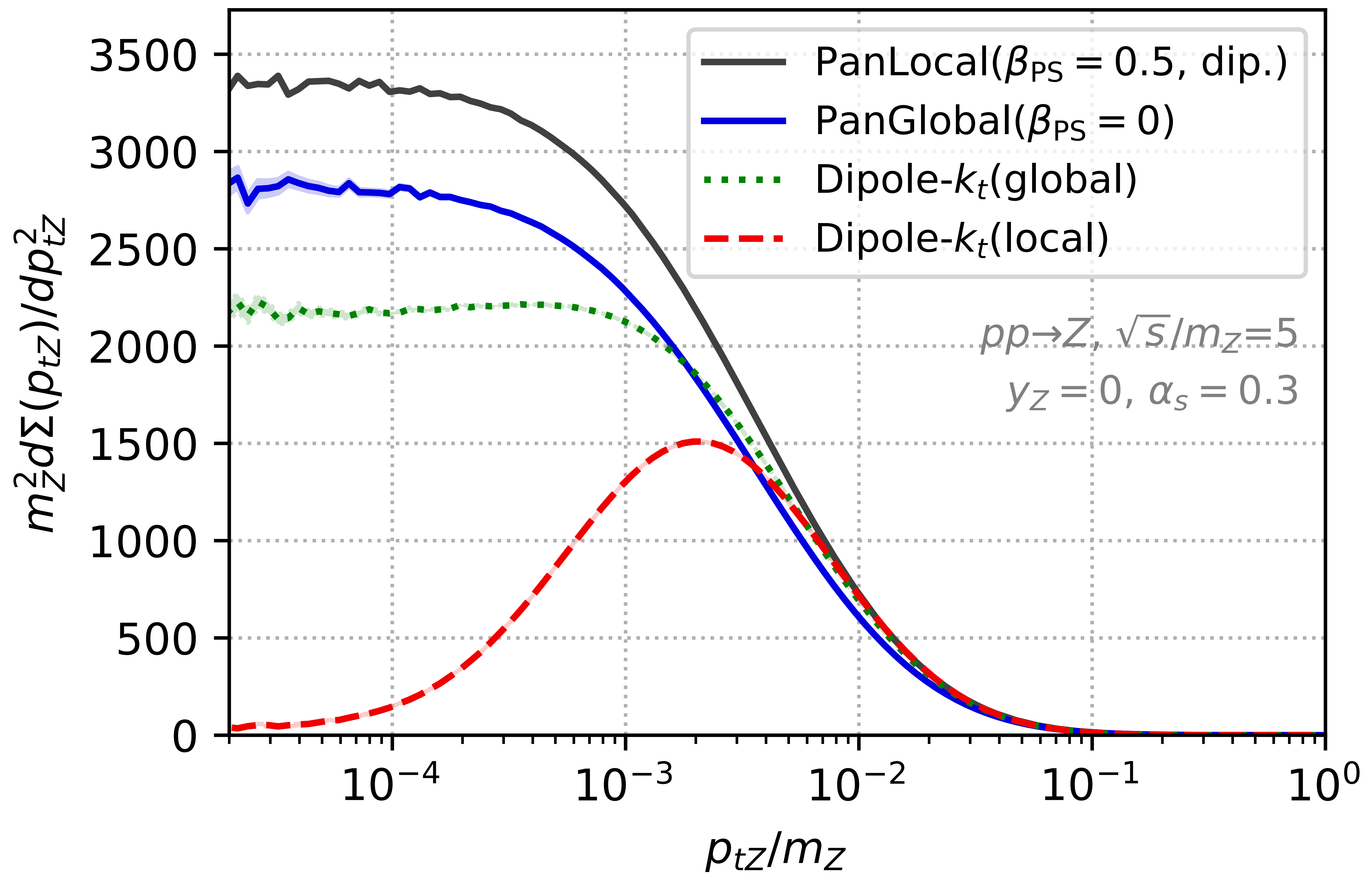


- Discrepancy not there for PanScales family of showers

Transverse momentum of the Z boson

Scaling at small p_t

The Sudakov suppression is compensated by azimuthal cancellations at small p_t
Leads to a power-law fall-off



Parisi, Petronzio [NPB 154 (1979) 427-440]

$$\frac{d\Sigma}{dp_{tZ}^2} = \int_0^\infty \frac{db}{2} b\, J_0(bp_{tZ})\, \Sigma_V(b_0/b)$$

$p_{tZ} \to 0$ normalisation of $\Sigma(p_{tZ})$