Renormalons and Linear Power Corrections

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Outline

I will illustrate recent progress in the study of (linear) power suppressed corrections in collider processes, obtained in the framework of renormalon calculus.

- Absence of linear power corrections in certain collider observables involving massless partons.
- Implications for e^+e^--annihilation shape-variables in the 3-jet region.
- Fits to ALEPH data.
- The case of massive partons.
- Say something smart about parton shower generators.

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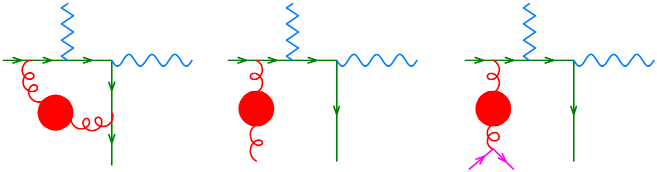
Recent progress

There have been recently new findings regarding the structure of linear power corrections in collider observables:

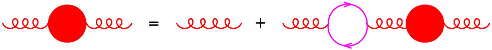
- In ref. (JHEP 01 (2022) 093, Caola, Ferrario-Ravasio, Limatola, Melnikov, P.N.) it was demonstrated that linear power corrections are absent in sufficiently inclusive observables, in a variety of processes, in the framework of renormalon calculus.
- The same findings opened the possibility to compute linear power corrections to shape variables in the 3-jet configuration (JHEP 12 (2022) 062, Caola,Ferrario-Ravasio,Limatola,Melnikov, Ozcelik,P.N.)

Renormalon calculus

Example: $q\gamma \to Z + q$ at large transverse momentum.
Representative graphs for: virtual, real with g, real with $q\bar{q}$:



where



It turns out that the result of the bubble sum can be linked to a calculation with a massive gluon. For an (IR safe) observable O, we have

$$\langle O\rangle = B_O - \int \mathrm{d}\lambda \frac{\mathrm{d}T_O(\lambda)}{\mathrm{d}\lambda}\frac{1}{\alpha_S}\overbrace{\left[\frac{1}{\pi b_0}\arctan\frac{\pi b_0\alpha_S}{1+b_0\alpha_S\log\lambda^2/\mu_C^2}\right]}^{\text{Beneke,98}}$$

Where

$$T_O(\lambda) = \overbrace{V_O(\lambda)+R_O(\lambda)}^{\text{result for a gluon with mass }\lambda} + \overbrace{\Delta_O(\lambda)}^{\text{Seymour,P.N.1995}},$$

$$\Delta_O(\lambda) = \frac{3\pi\lambda^2}{\alpha_S T_F}\int \mathrm{d}\Phi_{q\bar{q}} R_{q\bar{q}}(\Phi_{q\bar{q}})\delta(m_{q\bar{q}}^2-\lambda^2)\left[O(\Phi_{q\bar{q}})-O(\Phi_{g^*})\right]$$

It turns out that a linear term in λ in the expansion of $T(\lambda)$ around zero is associated with linear renormalons.

- The framework outlined above can be seen as the large (negative!) n_f limit of QCD.
- The full perturbative expansion is calculable in principle.
- The origin of renormalons is quite transparent.
- It has only been applied so far to processes not involving gluons at the Born level.

Applications to QCD require further assumptions:

- Use the result of the large n_f model as an indication of what happens in the full QCD.
- Replace $4T_f n_f \to -11C_A + 4T_f n_f$ at the end of the calculation to make numerical estimates.

The new findings

- The new findings (Caola,Ferrario-Ravasio,Limatola,Melnikov,P.N.) regard the absence of linear terms in λ for collider processes involving two external massless quarks.
- They reproduce old results: the absence of linear renormalons in $e^+e^- \to q\bar{q}$, in the DY total cross section (Beneke,Braun,1995) and rapidity distribution (Dasgupta,1999).
- New results: the absence of linear renormalons in the differential distribution of a Z boson produced in the process $q\gamma \to qZ$. (This has the structure of a Z recoiling against a hadronic jet, so it can be taken as an indication of the absence of linear renormalons in the differential distribution of a Z boson produced at the LHC).

Notice that

- Real and virtual corrections exhibit infrared divergences, leading to results that scale like λ^0 times logs of λ plus constant terms.
- We are after terms that scale like λ^1, so the leading soft approximation is not enough to get them right.

The new findings

We find that

- Virtual corrections do not generate linear terms.
- Real corrections do not generate linear terms, irrespective of the structure of subleading soft term.

The cancellation of linear terms in the real corrections is better seen if

- the real phase space is factorized into an underlying Born and a radiation phase space;
- the factorized mapping satisfies (in the soft limit) certain linearity conditions in the radiation variables;

Under these conditions, by integrating in the radiation variables at fixed underlying Born momenta, no linear terms in λ are generated.

It can be shown that the "dangerous" soft integrals have the form

$$\int \frac{\mathrm{d}^3\vec{k}}{k^0}\left[\frac{1}{p_q\cdot k\ p_{\bar{q}}\cdot k},\quad \frac{k^\mu}{p_q\cdot k\ p_{\bar{q}}\cdot k},\quad \frac{\lambda^2}{(p_q\cdot k)^2\ p_{\bar{q}}\cdot k}\right]$$

where $p_{q/\bar{q}}$ are the "underlying Born", and k is the radiation variables. By just performing the integrations we get no linear terms in λ.

This is non-trivial, since all terms, by scaling, could in principle lead to linear terms.

The linearity condition of the mapping guarantees that expanding the full real momenta $P_{q/\bar{q}}(p_{q/\bar{q}}, k)$ in powers of k we get at most terms like the middle one in the above integrands (plus eventually non-linear terms that vanish by azimuthal integration in the dipole rest frame.)

Mappings example

Dipole mappings that in the soft limit become

$$P_1 = p_1 - \frac{p_2 k}{2 p_1 p_2} p_1 + \frac{p_1 k}{2 p_1 p_2} p_2 - \frac{k}{2} + f(y) k_\perp$$
$$P_2 = p_2 - \frac{p_1 k}{2 p_1 p_2} p_2 + \frac{p_2 k}{2 p_1 p_2} p_1 - \frac{k}{2} - f(y) k_\perp$$

where y and $k_\perp$ are the rapidity and transverse momentum of the gluon in the dipole rest frame.

- They satisfy $P_{1/2}^2 = 0$, $P_1 + P_2 + k = p_1 + p_2$, and for $k = \gamma p_{1/2}$ we have $P_{1/2} = (1-\gamma) p_{1/2}$, $P_{2/1} = p_{2/1}$.
- They are linear in k, except possibly for the coefficient of $k_\perp$

Implications for shape variables

The findings illustrated so far have profound implications for shape variables. A cumulative cross section can be written as

$$\Sigma(v) = \int \mathrm{d}\sigma \theta(v - V(P))$$

where V is a shape variable, and we can rewrite

$$\Sigma(v) = \underbrace{\int \mathrm{d}\sigma \theta(v - V(p))}_{\text{no linear } \lambda \text{ terms}} + \int \mathrm{d}\sigma \underbrace{\left[\theta(v - V(P)) - \theta(v - V(p))\right]}_{\text{Vanishes in IR limit!}}$$

But now

- no linear corrections in the first term (can be integrated first in the radiation variables at fixed underlying Born momenta);
- The second term has an IR suppression: we only need to use the soft approximation for $\mathrm{d}\sigma$ to evaluate the linear λ terms.

Since we can rely upon the soft approximation:

- we can apply our result to the $e^+e^- \to q\bar{q}g$ process, that controls the distribution of shape variables in the 3-jet region.
- We evaluate the linear correction for the three radiating dipoles, qg, $\bar{q}g$ and $q\bar{q}$, by writing the soft emission in the eikonal approximation.
- In previous literature, non-perturbative corrections to shape variables were obtain by extrapolating their value from the 2-jet region
- The only exception: Luisoni,Monni,Salam 2021 computed the linear power correction to the C-parameter in the 3-jet symmetric limit ($c = 3/4$).

Chain of assumptions

- In the large n_f limit, in processes without gluons at the Born level: no linear terms in inclusive quantities. This holds, for example, for $e^+e^- \to q\bar{q}\gamma$.
- It follows that IR-finite, non-inclusive quantities have linear power corrections that can be computed using the soft approximation. In the $e^+e^- \to q\bar{q}\gamma$ example, we use the eikonal formula for the $q\bar{q}$ radiating dipole.
- Assume that this can be generalized to cross sections involving gluons at the Born level, like $e^+e^- \to q\bar{q}g$. It is enough to add the contributions of the three radiating dipoles qg, $\bar{q}g$ and $q\bar{q}$.

In the last step we assumes that linear terms cancels for inclusive quantities also in the full theory (in some sense), so we use the large n_f model to extrapolate to properties of the full theory.

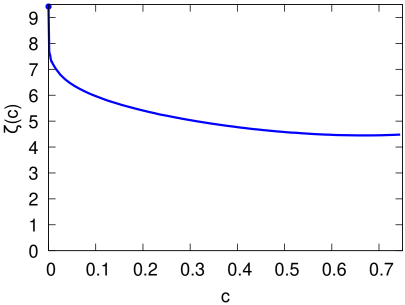
Application to e^+e^- shape variables in the 3-jet region:

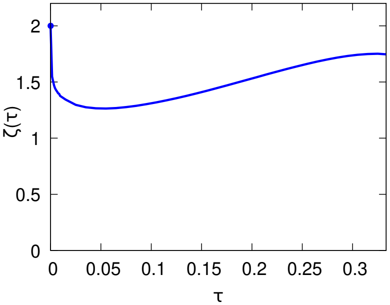
- Caola,Ferrario Ravasio,Limatola,Melnikov,Ozcelik,P.N.2022, formulation of the general method, application to C-parameter and thrust.
- Zanderighi,P.N.2023 added heavy jet mass, jet mass difference, y_3 and wide jet broadening; performed fits to ALEPH data.

Non-perturbative corrections can be parametrized as a shift in the perturbative cumulant distribution:

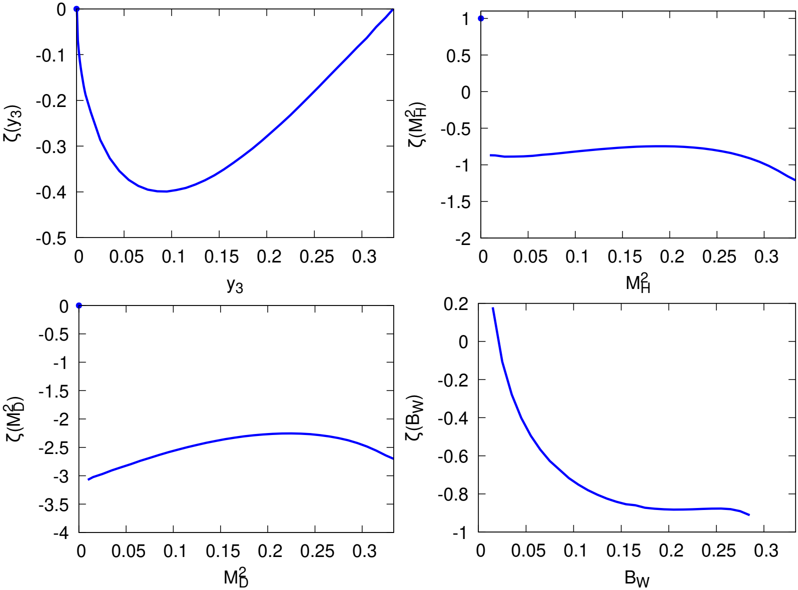
$$\Sigma(s) \longrightarrow \Sigma(s + H_{\mathrm{NP}}\zeta(s)), \quad \text{where} \quad \Sigma(s) = \int \mathrm{d}\sigma(\Phi)\theta(s - s(\Phi))$$

and $H_{\mathrm{NP}} \approx \Lambda/Q$ is a non-perturbative parameter that must be fitted to data.





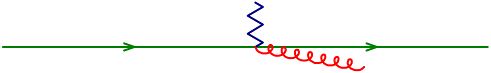
The dot in the plots represents the constant value that was used in earlier studies. The value of $\zeta(c)$ at the symmetric point $c = 3/4$ was also computed by Luisoni,Monni,Salam 2021.



(G.Zanderighi,P.N.2023) In some cases ζ is negative!

Rapid variations near $v = 0$

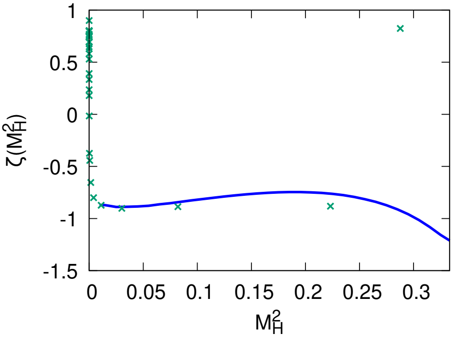
Near $v = 0$, the Born amplitude is dominated by the soft-collinear region.



$$\text{radiation} = \frac{C_A}{2} M_{\bar{q}g} + \frac{C_A}{2} M_{qg} + \left(C_F - \frac{C_A}{2} \right) M_{q\bar{q}}$$

but $M_{qg} \approx 0$, $M_{\bar{q}g} \approx M_{q\bar{q}}$, so the total is $\approx C_F M_{q\bar{q}}$.

Our $\zeta(v)$ functions, for $v \to 0$ MUST approach the 2-jet limit value; but up to single logs!, i.e. terms of relative order $1/|\log(v)|$.

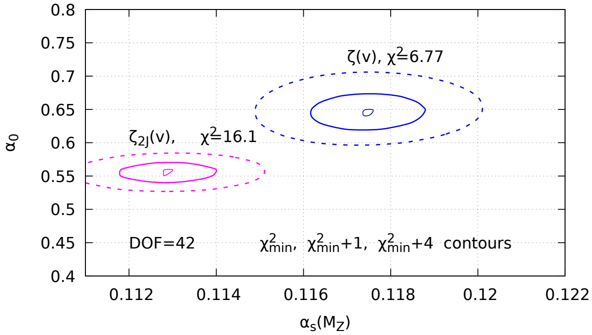


Insist on $v \to 0$ (quadruple precision, log scale histogram). Two-jet limit reached, but subleading terms are extremely important!

RESULTS

Simultaneous fit to C, t and y_3, both for our newly computed $\zeta(v)$, and, for comparison, with $\zeta(v) \to \zeta_{2J}(v) = \zeta(0)$ (traditional method for the computation of power corrections).

(we excluded variables with "bizarre" behaviour near the 2-jet limit)



The central value is at $\alpha_s(M_Z) = 0.1174$, $\alpha_0 = 0.64$. The "traditional" method leads to smaller values of α_s.

Fit details

Take v_i to span all bins of all shape variables considered; we define

$$\chi^2 = \sum_{ij} \Delta_i V_{ij}^{-1} \Delta_j, \quad \Delta_i = \left(\frac{1}{\sigma_{\rm exp}} \frac{{\rm d}\sigma_{\rm exp}(v_i)}{{\rm d}v_i} - \frac{1}{\sigma_{\rm th}} \frac{{\rm d}\sigma_{\rm th}(v_i)}{{\rm d}v_i} \right),$$

$$V_{ij} = \delta_{ij}(R_i^2 + T_i^2) + (1 - \delta_{ij}) C_{ij} R_i R_j + {\rm Cov}_{ij}^{\rm (Syst)}$$

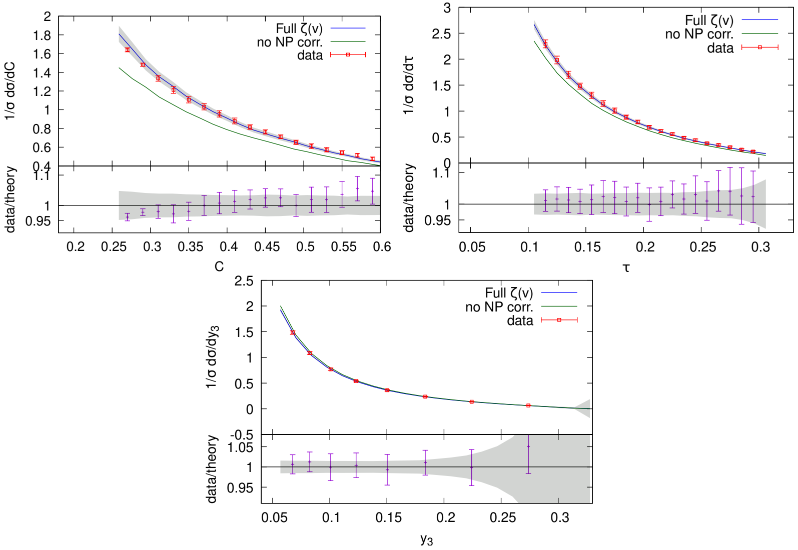
- R_i: statistical error
- T_i: theoretical error (scale variation plus error estimate of non-perturbative shift).
- C_{ij} statistical correlation (from Monte Carlo simulation)
- ${\rm Cov}_{ij}^{\rm (Syst)}$: systematics covariance matrix

Variation	$\alpha_s(M_Z)$	α_0	χ^2	$\frac{\chi^2}{N_{\rm deg}}$
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Ren. sc. $Q/4$	0.1180	0.60	6.1	0.14
Ren. sc. Q	0.1182	0.68	7.9	0.18
NP sch. (b)	0.1186	0.79	6.4	0.15
NP sch. (c)	0.1194	0.84	4.7	0.11
NP sch. (d)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
Std. scheme	0.1168	0.58	8.1	0.18
No hq corr.	0.1176	0.68	6.2	0.14
`Herwig 6`	0.1174	0.60	14.7	0.33
`Herwig 7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Alt. correl.	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.0	0.00
C, τ	0.1230	0.51	2.0	0.05

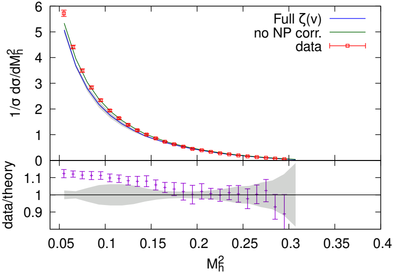
Several variations of setup parameters/methods lead to variations of the central value of order 1%. Among them

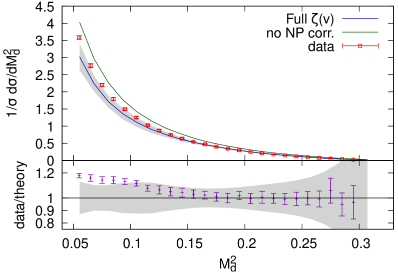
- Central ren. scale
- Ambiguity in implementation of NP corrections
- Treatment of correlation in systematic errors
- Treatment of hadron masses (P, D and std. schemes)

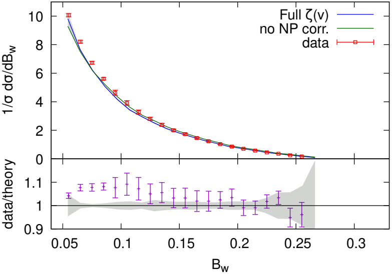
Quality of the fit for C, τ and y_3, using the new calculation of the non-perturbative effect (i.e. the full $\zeta(v)$ dependence.)



Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.







Good fit away from the two jet region.

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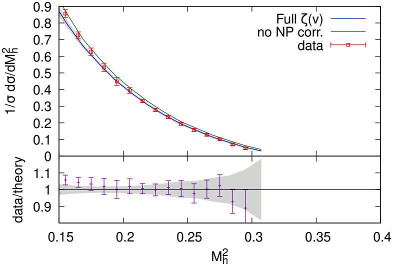
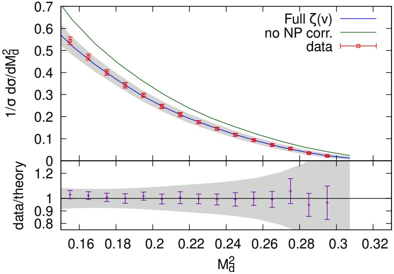
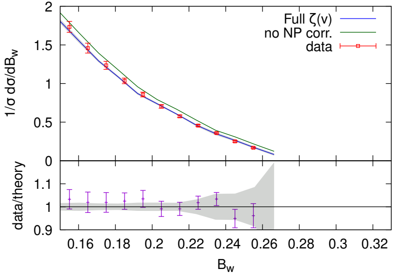
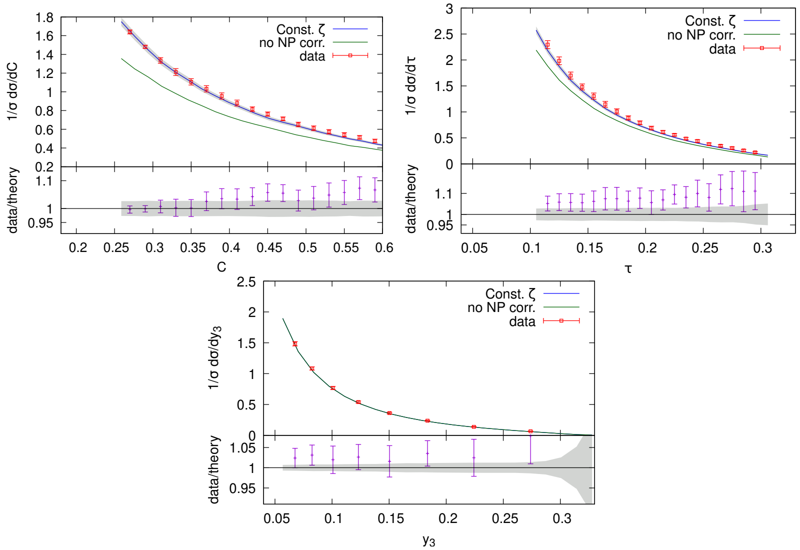




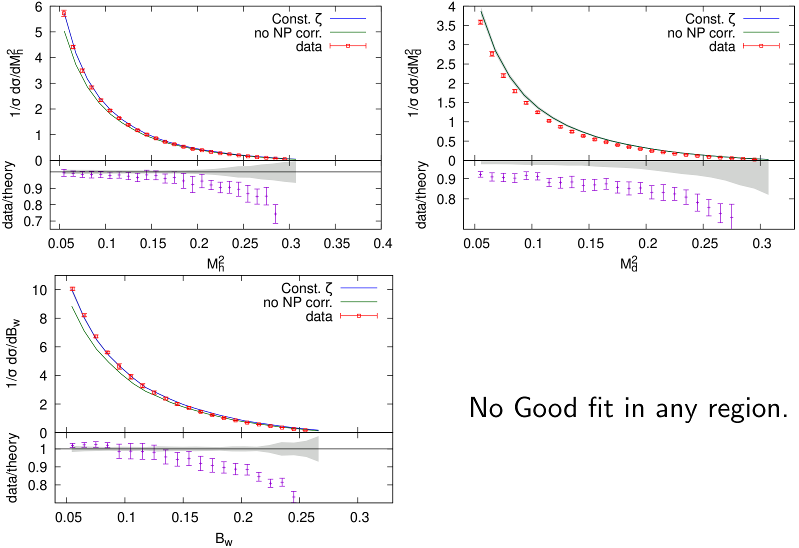


Good fit far away from the two jet region.

Quality of the fit for C, τ and y_3, obtained setting $\zeta(v) = \zeta(0)$.

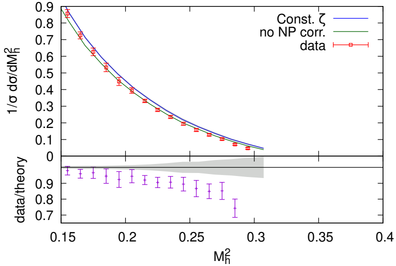


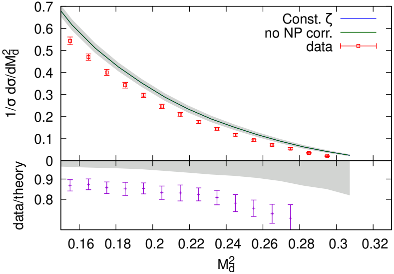
Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.

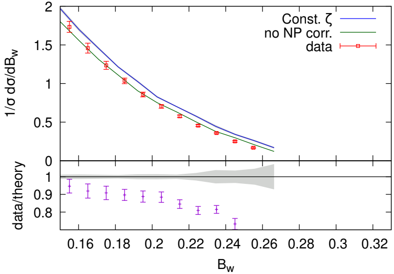


No Good fit in any region.

Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.







No Good fit in any region.

At the moment, including higher energy does not help (too poor statistics). However, for future e^+e^- colliders:

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})
FCC-ee-Z	4	88-95	150
FCC-ee-W	2	158-162	12
FCC-ee-H	3	240	5
FCC-ee-tt	5	345-365	1.5

I estimate the following Z/γ^* hadronic cross sections:

E_{CM}	σ (nb)	Num. had. events
91.2	33.1	5.0×10^{12}
160	0.026	0.31×10^{9}
240	0.009	0.45×10^{8}
350	0.0039	0.58×10^{7}

Even at the highest energy the number of events is not distant from what was collected at LEP1 (16×10^6 events).

Massive quarks

Makarov,Melnikov,Ozcelik,P.N.2023 and ongoing work.

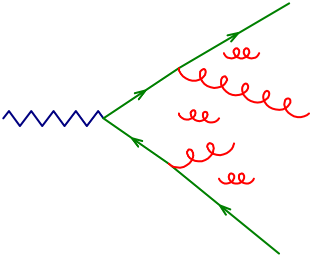
- Unlike the case of massless quarks, when heavy quarks are present terms linear in λ arise both in the virtual and real corrections.
- linear terms cancel in the total.
- The cancellation can be shown to takes place for generic processes by using the Low-Burnett-Kroll theorem, that allows one to relate the subleading soft terms to derivatives of the cross section with no soft emissions.
- As in the massless case, there are factorized form of the phase space in terms of an underlying Born and a radiation phase space such that the cancellation takes place at fixed underlying Born momenta.
- However, this does not happen for “generic” mappings, that are linear in the radiation momentum in the soft limit.

Relevance to parton showers

- It seems that parton shower concepts, like the factorization of phase space into an "underlying Born" and "radiation" factors play a very relevant role in the discussion of power corrections.
- The requirements on the mappings are satisfied by several recoil schemes: Catani-Seymour dipoles, PanLocal and PanGlobal mappings, for example.
- Are there advantages in using these recoil schemes in parton showers?

Relevance to parton showers

The hard question is: what is the interplay of multiple soft radiation and power corrections?



Dipole structure for radiating the gluer deeply modified by further soft radiation; are there corrections of order $\alpha_s(Q_{\mathrm{soft}})$ to the power corrections that we compute?

Do these corrections provide a realistic picture of leading power corrections in a shower framework?

The main obstacle in answering these questions is that the large n_{f} approach does not get along with soft gluon resummation ...

Conclusions

- Some new, intriguing results regarding linear power corrections in collider observables have been obtained.
- Some implications for shape variables in $e^+e^- \to$ hadrons have been discussed
- These results involve suggestively parton shower concepts, although, at the moment, it is not clear whether they can lead to progress in this framework.