QED corrections at lepton colliders: collinear factorisation vs YFS

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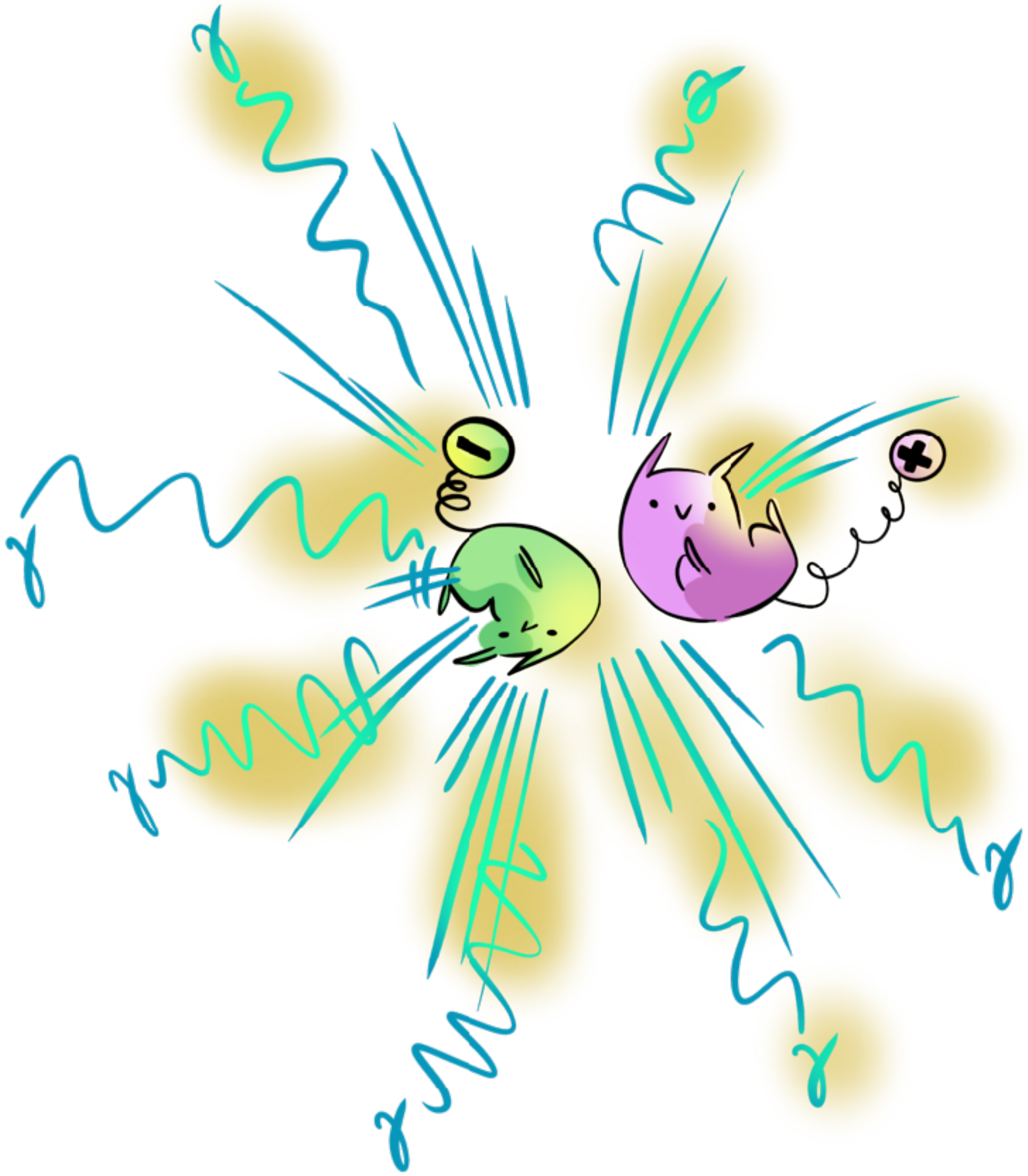
Parton Showers for future e^+e^- colliders, CERN, 24.04-28.04.23

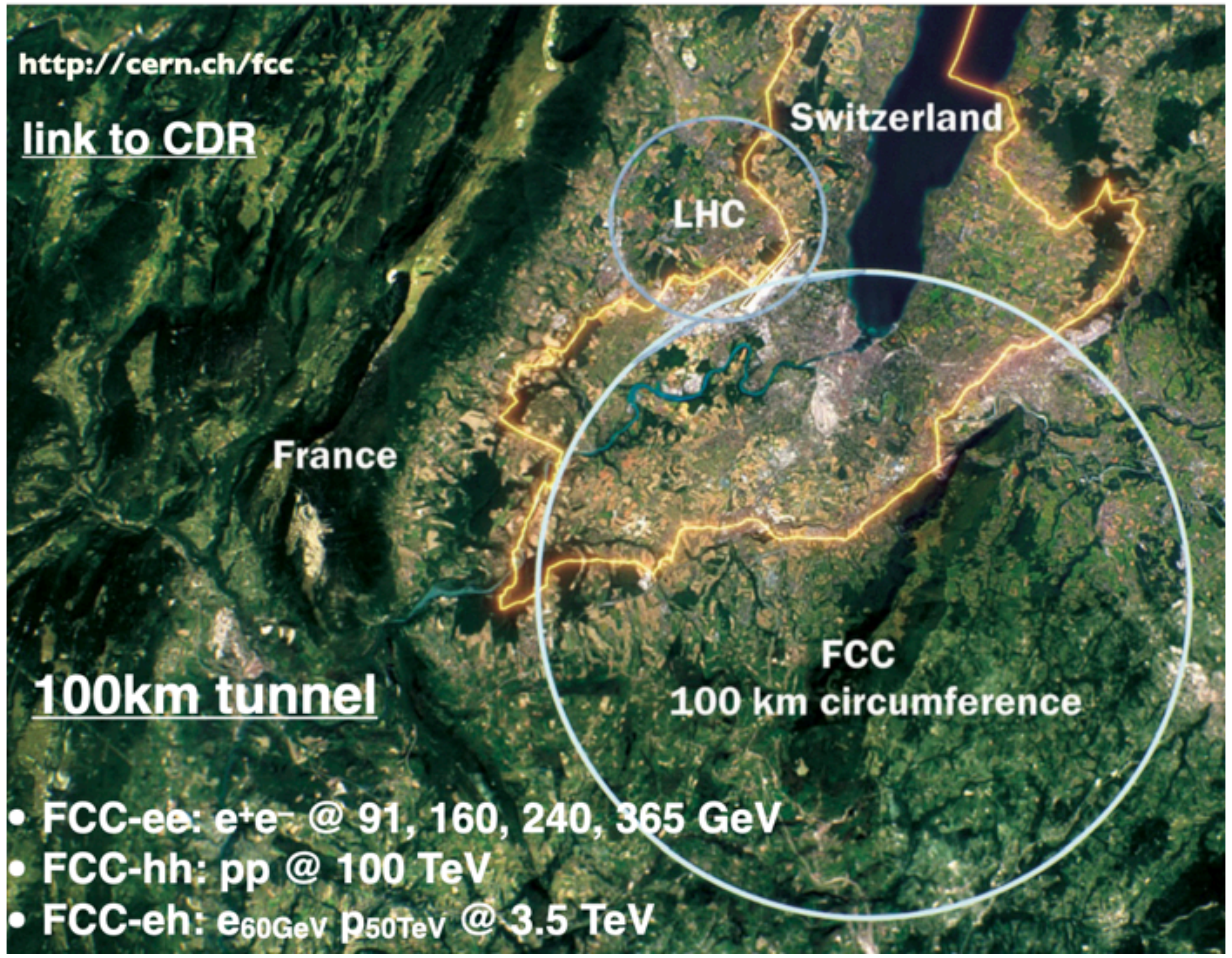
Introduction

- Initial-state radiation (ISR) is an intrinsic "feature" of lepton colliders
- QED theoretical basis are well established, but conceptual and technical progress still needed to reach target precision of future colliders
- The LEP legacy is not enough. The many tools and ideas developed at the LHC can offer solutions useful for e^+e^- environment



Why?

Future Circular Collider



M. Mangano, "Why FCC?"
Theory Colloquium, 15 June 2022, CERN
https://indico.cern.ch/event/1155782/

Electro weak precision observables

	Experiment uncertainty			Theory uncertainty
	Current	**CEPC**	**FCC-ee**	**Current**
M_W[MeV]	15	0.5	0.4	4
Γ_Z[MeV]	2.3	0.025	0.025	0.4
$R_b[10^{-5}]$	66	4.3	6	10
$\sin^2\theta^{l}_{\rm eff}[10^{-5}]$	16	< 1	0.5	4.5

Summary slides of week 1
"Precision calculations for future e+e– colliders: targets and tools"
7-17 June 2022, CERN
https://indico.cern.ch/event/1140580/

Disclaimer

I will talk about QED initial-state radiation (ISR)
(with some bias towards the collinear factorisation approach…)

For more details, refer to the 2021 Snowmass white paper

Initial state QED radiation aspects for future e^+e^- colliders

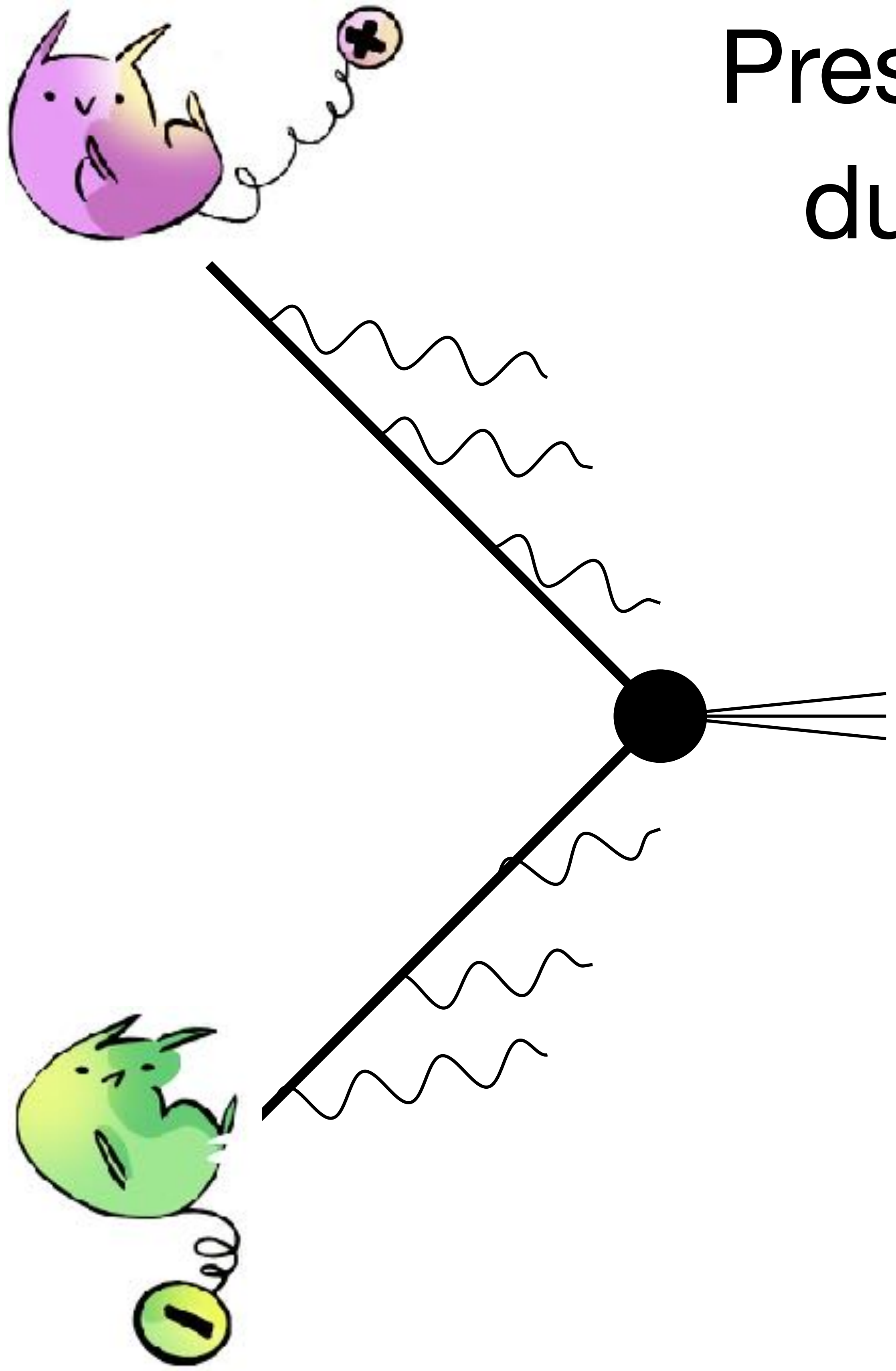
https://arxiv.org/abs/2203.12557

Conveners: **S. Frixione,**[1] **E. Laenen**[2,3]

C.M. Carloni Calame,[4] **A. Denner,**[5] **S. Dittmaier,**[6] **T. Engel,**[7,8] **L. Flower,**[9] **L. Gellersen,**[10] **S. Hoeche,**[11] **S. Jadach,**[12] **M.R. Masouminia,**[13] **G. Montagna,**[4,14] **O. Nicrosini,**[4] **F. Piccinini,**[4] **S. Plätzer,**[15,16] **A. Price,**[17] **J. Reuter,**[18] **M. Rocco,**[7] **M. Schönherr,**[9] **A. Signer,**[7,8] **T. Sjöstrand,**[10] **G. Stagnitto,**[8] **Y. Ulrich,**[9] **R. Verheyen,**[20] **L. Vernazza,**[21,2] **A. Vicini,**[22] **B.F.L. Ward,**[23] **M. Zaro**[24]

Initial state radiation (ISR)

Presence in the cross section $d\sigma_{e^+e^-}$ of **potentially large logarithms,** due to **collinear and/or soft photon emissions** in the initial state



Soft **Collinear**

$$X \simeq \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^{n} \sum_{j=0}^{n} c_{n,i,j}\, l^i L^j \qquad l = \log \frac{Q^2}{\langle E_\gamma^2 \rangle} \qquad L = \log \frac{Q^2}{m_e^2}$$

b: power of the α in the Born process, m_e: electron mass

Q^2: typical hard scale of the process e.g. c.o.m. energy squared s

Basically all precision observables at e^+e^- colliders affected by ISR!

Numerology: production of $Z \to l\bar{l}$

Example stolen from S. Frixione

$$\sqrt{Q^2} = m_Z$$

$$L = 24.18 \implies \frac{\alpha}{\pi} L = 0.06$$

$$0 \le m_{ll} \le m_Z\,, \quad \ell = 8.29 \implies \frac{\alpha}{\pi}\ell = 0.02$$

$$m_Z - 1\ \text{GeV} \le m_{ll} \le m_Z\,, \quad \ell = 13.66 \implies \frac{\alpha}{\pi}\ell = 0.034$$

$$\sqrt{Q^2} = 500\ \text{GeV}$$

$$L = 24.59 \implies \frac{\alpha}{\pi} L = 0.068$$

$$0 \le m_{ll} \le m_Z\,, \quad \ell = 1.46 \implies \frac{\alpha}{\pi}\ell = 0.0036$$

$$m_Z - 1\ \text{GeV} \le m_{ll} \le m_Z\,, \quad \ell = 4.51 \implies \frac{\alpha}{\pi}\ell = 0.01$$

Collinear

$$L = \log \frac{Q^2}{m_e^2}$$

Soft

$$l = \log \frac{Q^2}{\langle E_\gamma^2 \rangle}$$

ANNALS OF PHYSICS: **13**: 379–452 (1961)

The Infrared Divergence Phenomena and High-Energy Processes*

D. R. YENNIE†

School of Physics, University of Minnesota, Minneapolis, Minnesota

S. C. FRAUTSCHI‡

Department of Physics, University of California, Berkeley, California

AND

H. SUURA

Department of Physics, Nihon University, Tokyo, Japan

A general treatment of the infrared divergence problem in quantum electrodynamics is given. The main feature of this treatment is the separation of the infrared divergences as multiplicative factors, which are treated to all orders of perturbation theory, and the conversion of the residual perturbation expansion into one which has no infrared divergence, and hence no need for an infrared cutoff. In the infrared factors, which are exponential in form, the in-

GRIBOV V. N. and LIPATOV L. N.
Sow. J . Nucl. Phys., 15 (1972) 438

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$$D_{\rm GL}(x,Q^2)=\frac{\exp\left[(1/2)\eta(3/4-\gamma_{\rm E})\right]}{\Gamma(1+(1/2)\eta)}\,\frac{1}{2}\,\eta(1-x)^{(1/2)\eta-1}$$

$$\left(\eta=\frac{2\alpha}{\pi}\log\frac{Q^2}{m^2}\right)$$

Yennie-Frautschi-Suura (YFS):
priority to **soft** logarithms

Collinear factorisation:
priority to **collinear** logarithms

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Collinear factorisation:
priority to **collinear** logarithms

YFS in a nutshell $\left\{ e^+(p_1) + e^-(p_2) \to X(p_X) + \sum_{i=0}^{n_\gamma} \gamma(k_n) \right\}_{n_\gamma=0}^{\infty}$

For a short introduction to YFS, see e.g. 2203.10948

Soft all-order universal factor

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \, \tilde{S}(k_i) \, \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{S}(k_j) \tilde{S}(k_k)} + \cdots \right)$$

Real photons emissions in the resolved region 1-$\Theta(\Omega)$

Soft-finite process-specific remainder (built out of matrix elements and eikonals)

Virtual photon emissions B

$$Y(\Omega) = 2\alpha \left[B + \tilde{B}(\Omega) \right]$$

Integrated real photon emissions $\tilde{B}$ over unresolved region $\Theta(\Omega)$

Example:

$e^+e^- \to f\bar{f}$

$$Y_e(\Omega_I; p_1, p_2) = \gamma_e \ln \frac{2E_{min}}{\sqrt{2p_1p_2}} + \frac{1}{4}\gamma_e + Q_e^2 \frac{\alpha}{\pi}\left(-\frac{1}{2} + \frac{\pi^2}{3}\right), \qquad \gamma_e = 2Q_e^2 \frac{\alpha}{\pi}\left(\ln \frac{2p_1p_2}{m_e^2} - 1\right),$$

$$Y_f(\Omega_F; q_1, q_2) = \gamma_f \ln \frac{2E_{min}}{\sqrt{2q_1q_2}} + \frac{1}{4}\gamma_f + Q_f^2 \frac{\alpha}{\pi}\left(-\frac{1}{2} + \frac{\pi^2}{3}\right), \qquad \gamma_f = 2Q_f^2 \frac{\alpha}{\pi}\left(\ln \frac{2q_1q_2}{m_f^2} - 1\right),$$

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$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} \mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma \, \tilde{S}(k_i) \, \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{S}(k_j)\,\tilde{S}(k_k)} + \cdots \right)$$

- Nice physical picture emerging from YFS master formula, *exclusive* in photons
- Mapping $\mathscr{R}$ required to evaluate matrix elements with a fixed number of photons, e.g. matrix correction for one real photon $\tilde{\beta}_1$

$$\sum_{i=1}^{2} p_i = \sum_{j=3}^{N+2} q_j + \sum_{k=1}^{n_\gamma} k_k \quad \longrightarrow \quad \sum_{i=1}^{2} \mathcal{R} p_i = \sum_{j=3}^{N+2} \mathcal{R} q_j + k_1 \, .$$

- Fixed-order part includes collinear logarithms $\log(Q^2/m_f^2)$ up to some order in α (note the explicit dependence on quark masses when quarks are radiators).
- The β functions are not standard, but systematically improvable

Variants and implementations of YFS

Two main variants:

- EEX: *exclusive* exponentiation, matrix element level (original YFS)
- CEEX: coherent exclusive exponentiation, amplitude level, including interference

  Jadach, Ward, Was hep-ph/0006359

Implementations of YFS:

- Family of software: KKMC-ee, KORAL[W/Z], BH[LUMI/WIDE], YFS[WW3/ZZ]

  All EEX, except KKMC-ee having both EEX and CEEX

  Jadach, Placzek, Richter-Was, Skrzypek, Ward, Was

  hep-ph/9912214, 1307.4037, hep-ph/9906277, hep-ph/9705430 +various Comp. Phys. Comm 1992-2001
- Also Sherpa has an implementation of YFS EEX

  Krauss, Price, Schönherr 2203.10948

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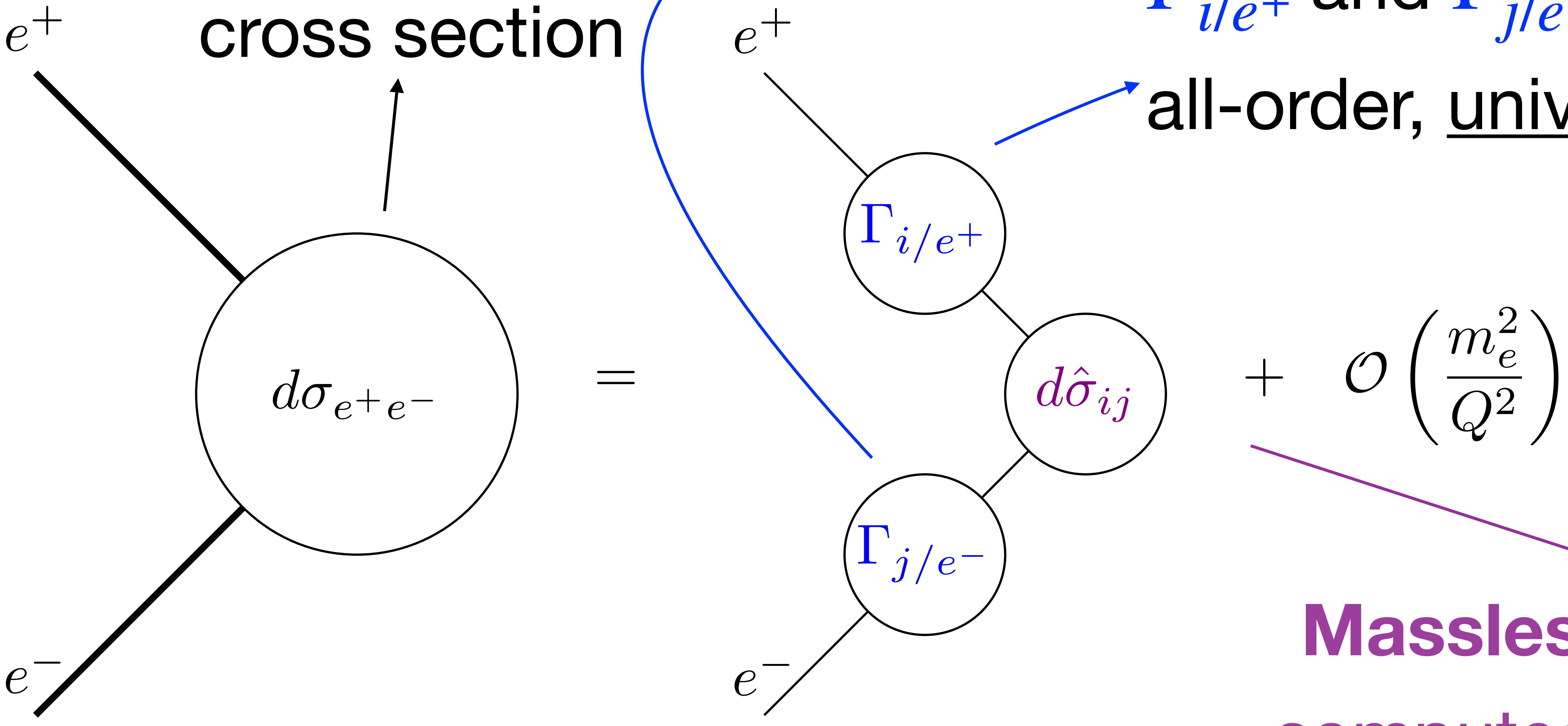
Collinear factorisation:
priority to **collinear** logarithms

Collinear factorisation

Massive particle-level cross section



QED Parton Distribution Functions (PDFs)

Γ_{i/e^+} and Γ_{j/e^-}, *partons* $i, j = e^+, e^-, \gamma, \ldots$

all-order, <u>universal</u>, resumming $\log(Q^2/m_e^2)$

Massless parton-level cross section $d\hat{\sigma}_{ij}$

computed order by order, process-specific

$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/e^+}(z_+, \mu^2, m_e^2) \, \Gamma_{j/e^-}(z_-, \mu^2, m_e^2) \, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

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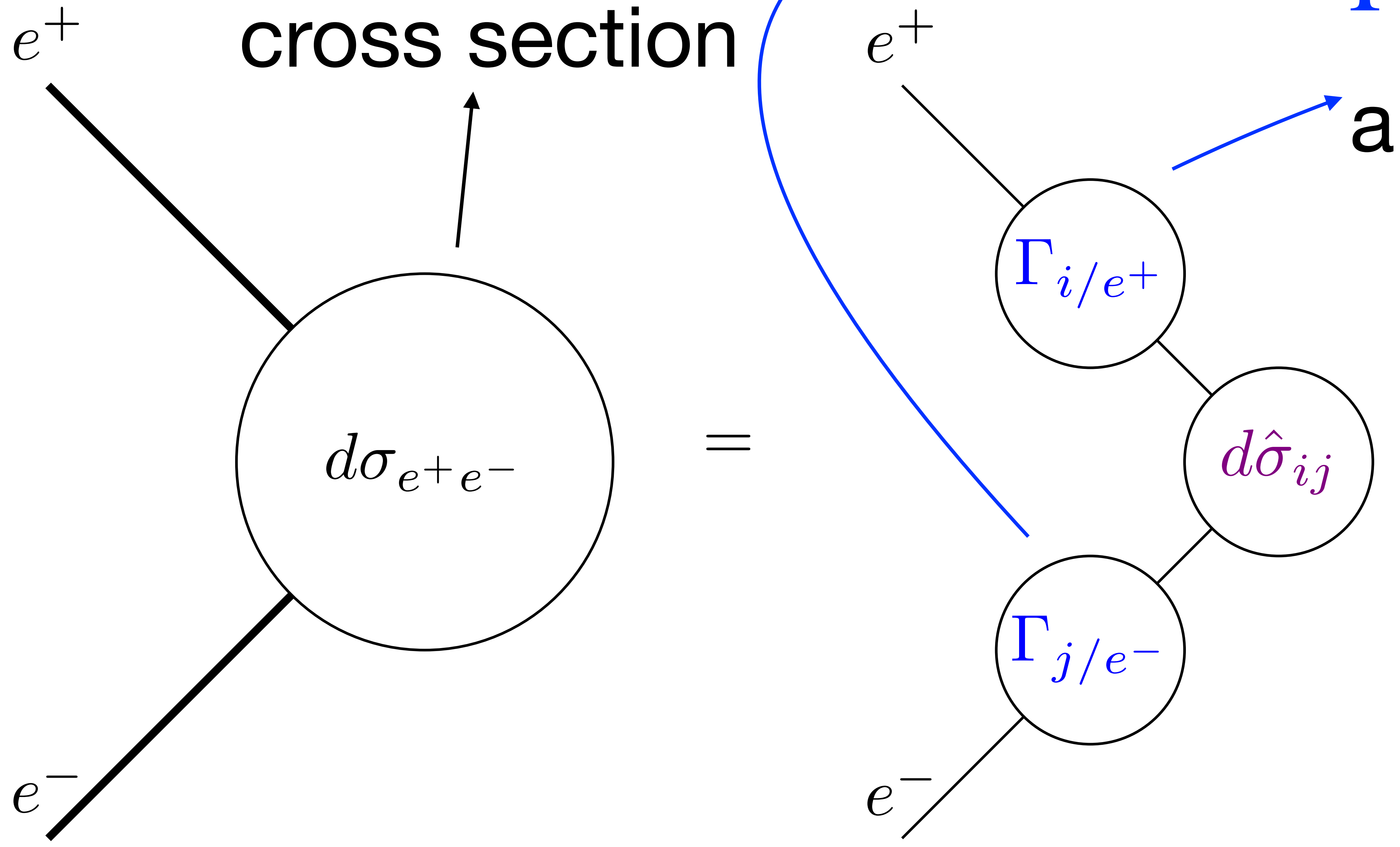

$+ \quad \mathcal{O}\left(\frac{m_e^2}{Q^2}\right)$

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all-order, universal, resumming $\log(Q^2/m_e^2)$

leading log (LL): $(\alpha L)^k$

next-to-leading log (NLL): $\alpha(\alpha L)^{k-1}$

…

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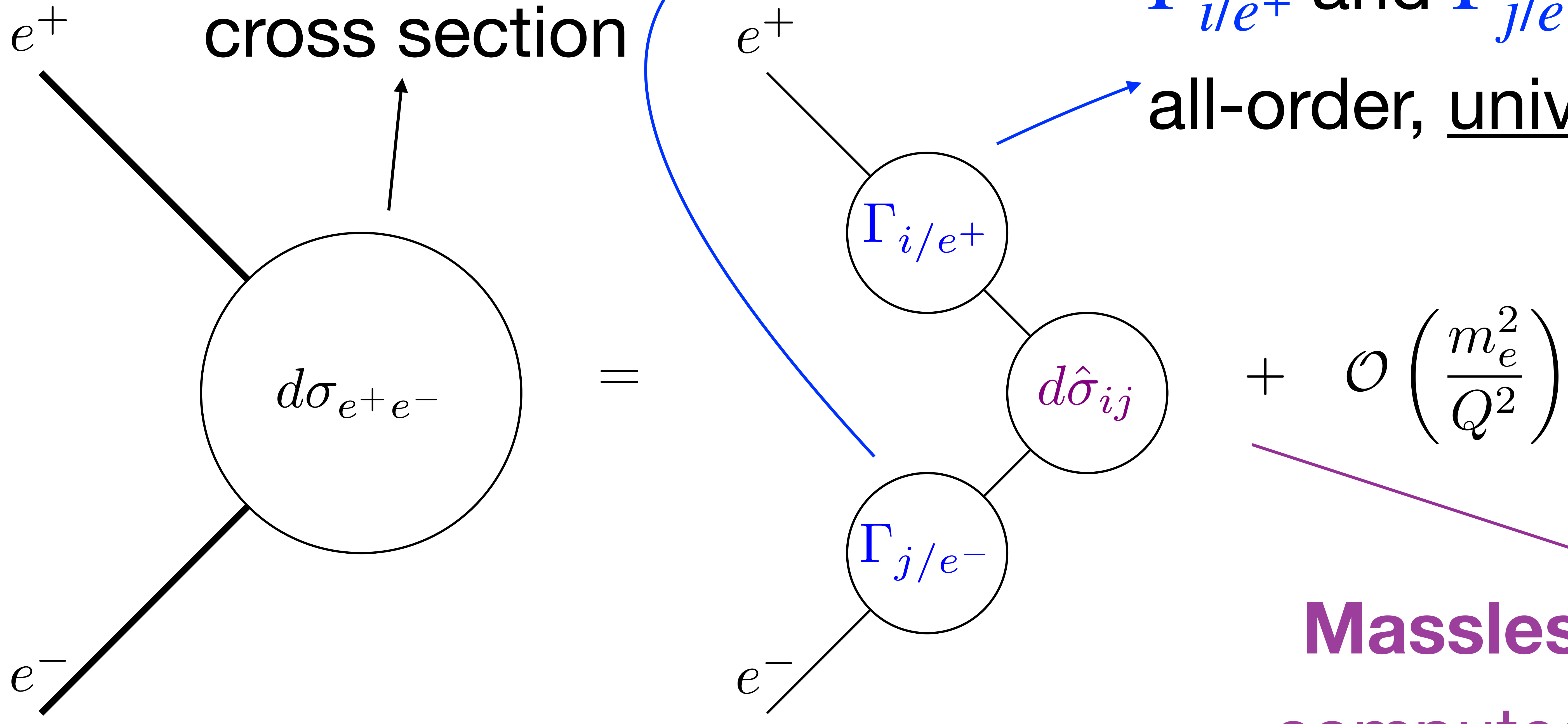
Collinear factorisation

Charge-conjugation implies $\Gamma_{\alpha/e^-} = \Gamma_{\bar{\alpha}/e^+} \equiv \Gamma_\alpha$

Massive particle-level cross section



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Evolution operator formalism

Collinear Logarithms resumed by mean of DGLAP equation:

$$\frac{\partial \Gamma(z,\mu^2)}{\partial \log \mu^2} = \frac{\alpha(\mu)}{2\pi}[\mathbb{P} \otimes \Gamma](z,\mu^2)$$

In Mellin space, $f_N = \int_0^1 \mathrm{d}z\, z^{N-1} f(z)$, it becomes multiplicative

$$\Gamma_N(\mu^2) = \boxed{\mathbb{E}_N(\mu^2,\mu_0^2)}\,\boxed{\Gamma_N(\mu_0^2)}$$

Evolution operator

Initial condition
(fully perturbative in QED!)

We end up with an **equation for the evolution operator**

$$\frac{\partial \mathbb{E}_N(\mu^2,\mu_0^2)}{\log \mu^2} = \frac{\alpha(\mu)}{2\pi}\left[\mathbb{P}_N^{[0]} + \frac{\alpha(\mu)}{2\pi}\mathbb{P}_N^{[1]}\right]\mathbb{E}_N(\mu^2,\mu_0^2) + \mathcal{O}(\alpha^2)$$

Here α fixed, but LL with α running also available

QED PDFs $\Gamma_\alpha(z, \mu^2)$ at LL

Well-known LL result for Γ_{e^-}, evolving $\Gamma(z, \mu_0^2) = \delta(1-z)$ at scale $\mu_0^2 \simeq m_e^2$:

$$\Gamma_{e^-}^{\rm LL}(z, \mu^2) = \frac{\exp\left[(3/4 - \gamma_E)\eta\right]}{\Gamma(1+\eta)}\eta(1-z)^{-1+\eta} - \frac{1}{2}\eta(1+z) + \mathcal{O}(\alpha^2), \qquad \eta = \frac{\alpha}{\pi}\log\frac{\mu^2}{m_e^2} \equiv \frac{\alpha}{\pi}L$$

All-order large-z bulk

Gribov, Lipatov 1972

Obtained by exploiting $z \to 1 \iff N \to \infty$ in N-space and then invert back to z-space

Fixed-order all-z terms (known up to high order)

Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini

Obtained by recursively solving the DGLAP equation or by fixed order calculations

PDFs are *inclusive* in photon emissions

(but of course exclusive in the photons from the hard process $\mathrm{d}\hat{\sigma}_{ij}$)

NLL-accurate QED PDFs

Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265

LL accuracy is insufficient and systematics not well defined at LL (e.g. which α?)

- NLO initial conditions at scale $\mu_0^2 = m_e^2$ **evolved at NLL up to to** μ^2 **with all fermion families** (lepton and quarks), in a variable flavour number scheme.
- PDFs in **three renormalisation schemes:** $\overline{\mathrm{MS}}$ (where α runs), $\alpha(m_Z)$ and G_μ (where α is fixed); **two factorisation schemes:** $\overline{\mathrm{MS}}$ and Δ (DIS-like, with NLO initial condition maximally simplified).
- Solution with numerical evolution, plus a **switch to analytical expressions for** $z \to 1$, where the electron PDF Γ_{e^-} features an integrable singularity.
- **Photon-initiated partonic contributions** (through the photon PDF Γ_γ) naturally included in the collinear framework at NLL.

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$$\Gamma_{e^-}^{[0],\overline{\mathrm{MS}}}(z,\mu_0^2) = \Gamma_{e^-}^{[0],\Delta}(z,\mu_0^2) = \delta(1-z)$$

$$\Gamma_{e^-}^{[1],\overline{\mathrm{MS}}}(z,\mu_0^2) = \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ \quad , \quad \Gamma_{e^-}^{[1],\Delta}(z,\mu_0^2) = \log\frac{\mu_0^2}{m^2}\left[\frac{1+z^2}{1-z}\right]_+$$

Evolution operator and short-distance cross section modified, such that $\hat{\sigma}_N(\mu^2)\, E_N(\mu^2,\mu_0^2)\, \Gamma_N(\mu_0^2)$ independent on the fact. scheme (up to NLO)

Large-z analytical expressions for Γ_{e^-}

$$\Gamma_{e^-}^{\rm NLL}(z,\mu^2) = \frac{e^{-\gamma_E \xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)} \xi_1 (1-z)^{-1+\xi_1} h(z,\mu^2)$$

$$\xi_1 = 2t + \mathcal{O}(\alpha^2)$$

$$\hat{\xi}_1 = \frac{3}{2} t + \mathcal{O}(\alpha^2)$$

$$t = \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)}$$

$$h^{\overline{\rm MS}}(z,\mu^2) = 1 + \frac{\alpha(\mu_0)}{\pi}\left[\left(\log\frac{\mu_0^2}{m^2} - 1\right)\left(A(\xi_1) + \frac{3}{4}\right) - 2B(\xi_1) + \frac{7}{4} + \left(\log\frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1)\right)\underline{\log(1-z)} - \underline{\log^2(1-z)}\right]$$

$$h^{\Delta}(z,\mu^2) = \frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi}\log\frac{\mu_0^2}{m^2}\left(A(\xi_1) + \log(1-z) + \frac{3}{4}\right)$$

$$A(\xi_1) = \frac{1}{\xi_1} + \mathcal{O}(\xi_1)$$

$$B(\xi_1) = -\frac{\pi^2}{6} + 2\zeta_3 \xi_1 + \mathcal{O}(\xi_1^2)$$

Logarithmic terms artefacts of the $\overline{\rm MS}$ fac. scheme, **absent in the Δ scheme.**

Here shown in the $\overline{\rm MS}$ ren. scheme and with a single-fermion family; evolution with multiple fermion families with their mass thresholds and different ren. schemes (e.g. $\alpha(m_Z)$, G_μ) amount to a redefinition of ξ_1 and $\hat{\xi}_1$.

NLL-accurate QED PDFs

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$$\alpha_R = \alpha_{\overline{\text{MS}}}(m_Z) - \Delta_{\overline{\text{MS}}\to R}\,\alpha^2_{\overline{\text{MS}}}(m_Z) + \mathcal{O}(\alpha^3)$$

Modified evolution to reabsorb the running of alpha, leading to $\mathcal{O}(\alpha^3)$ w.r.t. $\overline{\text{MS}}$ results → naively neglecting the running of α leads to $\mathcal{O}(\alpha^2)$ differences w.r.t. $\overline{\text{MS}}$

$$\mathbb{P}_R^{[0,k]} = \mathbb{P}_{\overline{\text{MS}}}^{[0,k]}$$

$$\mathbb{P}_R^{[1,k]} = \mathbb{P}_{\overline{\text{MS}}}^{[1,k]} + \left(2\pi b_0^{(k)} \log\frac{\mu^2}{m_{k+1}^2} + D^{(k)}\right)\mathbb{P}_{\overline{\text{MS}}}^{[0,k]}$$

$$D^{(k)} = 2\pi \sum_{i=k+1}^{M} b_0^{(i)} \log\frac{m_i^2}{m_{i+1}^2} + 2\pi\Delta_{\overline{\text{MS}}\to R}$$

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- Solution built out of a numerical evolution, with a **switch to analytical expressions for** $z \to 1$, where the electron PDF Γ_{e^-} features a power-like integrable singularity.

Public code eMELA: https://github.com/gstagnit/eMELA

Numerical evolution in Mellin space with a discretised path-ordered product.

Runtime evaluation too slow → grids in LHAPDF format

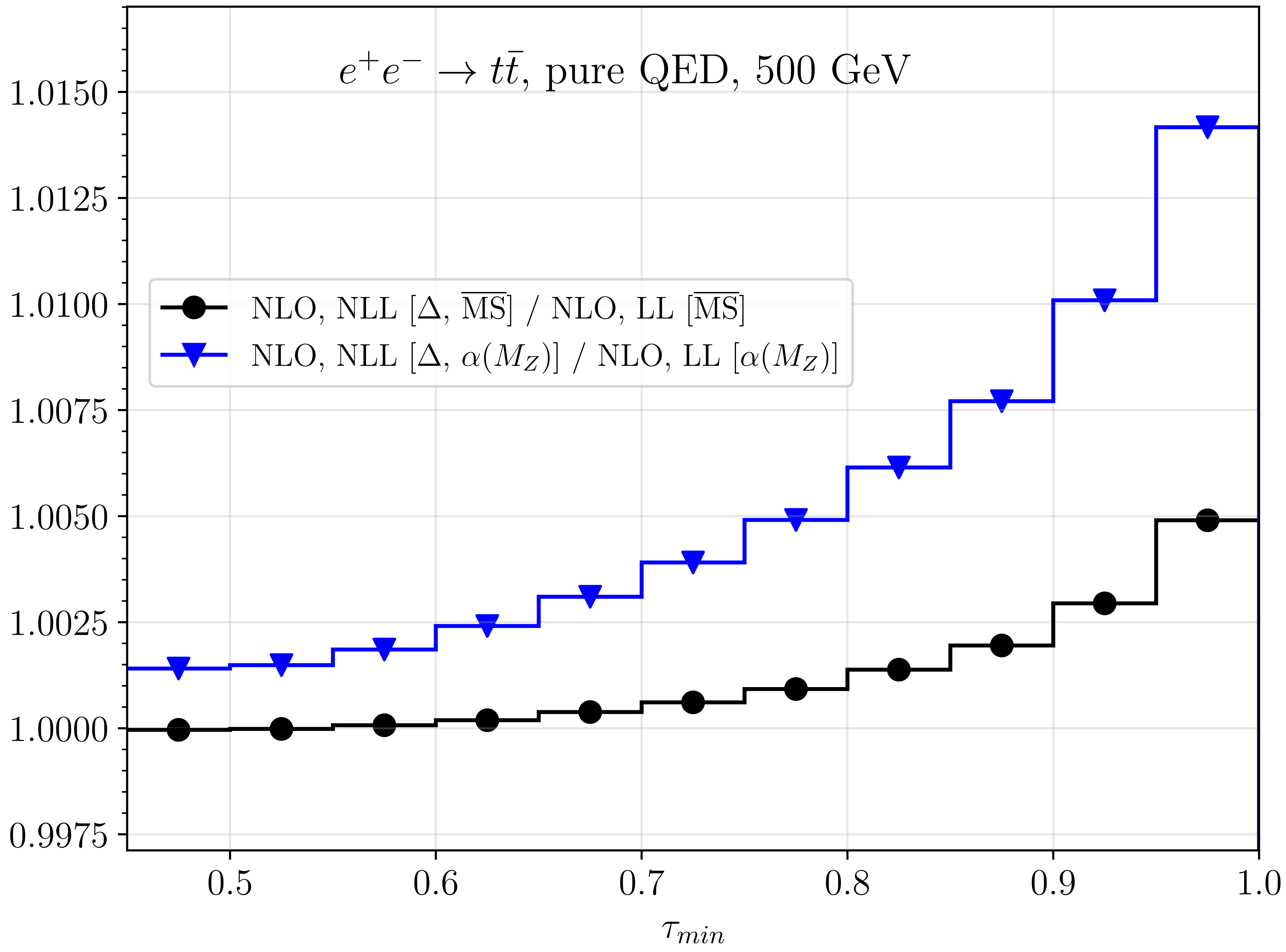
Even with grids, eMELA always switches to the analytical solution for $z \to 1$

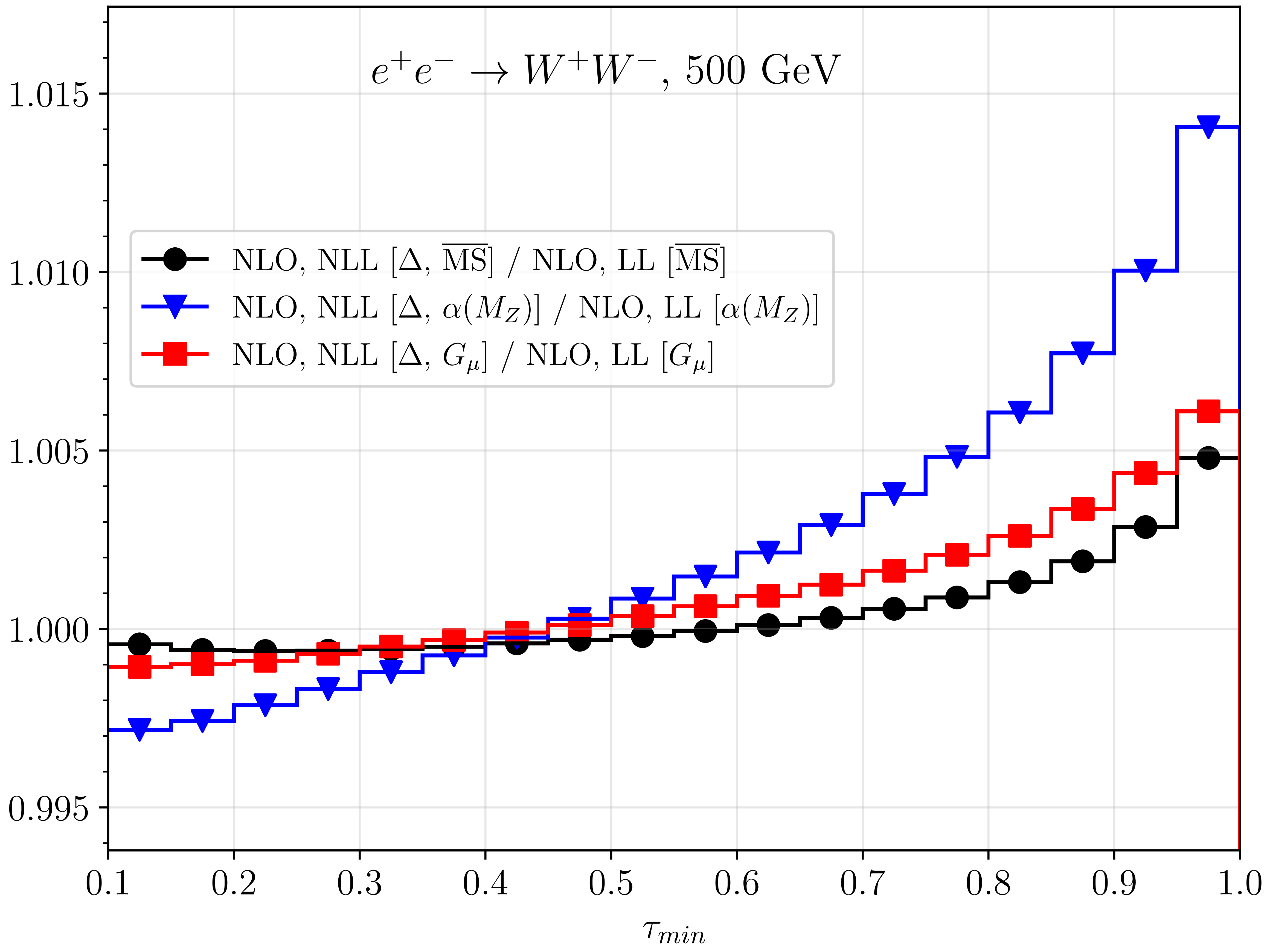
Studies on physical cross sections

- Computed in the MG5_aMC framework, at NLO (EW) + NLL in e^+e^- collisions: https://github.com/mg5amcnlo/mg5amcnlo/tree/3.0.1-lepcoll
- Processes:
  - $e^+e^- \to q\bar{q}(\gamma)$ [pure QED, with real and virtual radiation limited to initial state]
  - $e^+e^- \to W^+W^-(X)$ [full EW]
  - $e^+e^- \to t\bar{t}(X)$ [full EW] and $e^+e^- \to t\bar{t}(X)$ [pure QED]
- $\mu = \sqrt{s} = 500$ GeV (qualitatively similar results in the range 50-500 GeV)
- We focus on the cumulative cross section:

$$\sigma(\tau_{min}) = \int d\sigma \, \Theta(\tau_{min} \leq M^2_{p\bar{p}}/s) \, , \quad p = q, t, W^+$$

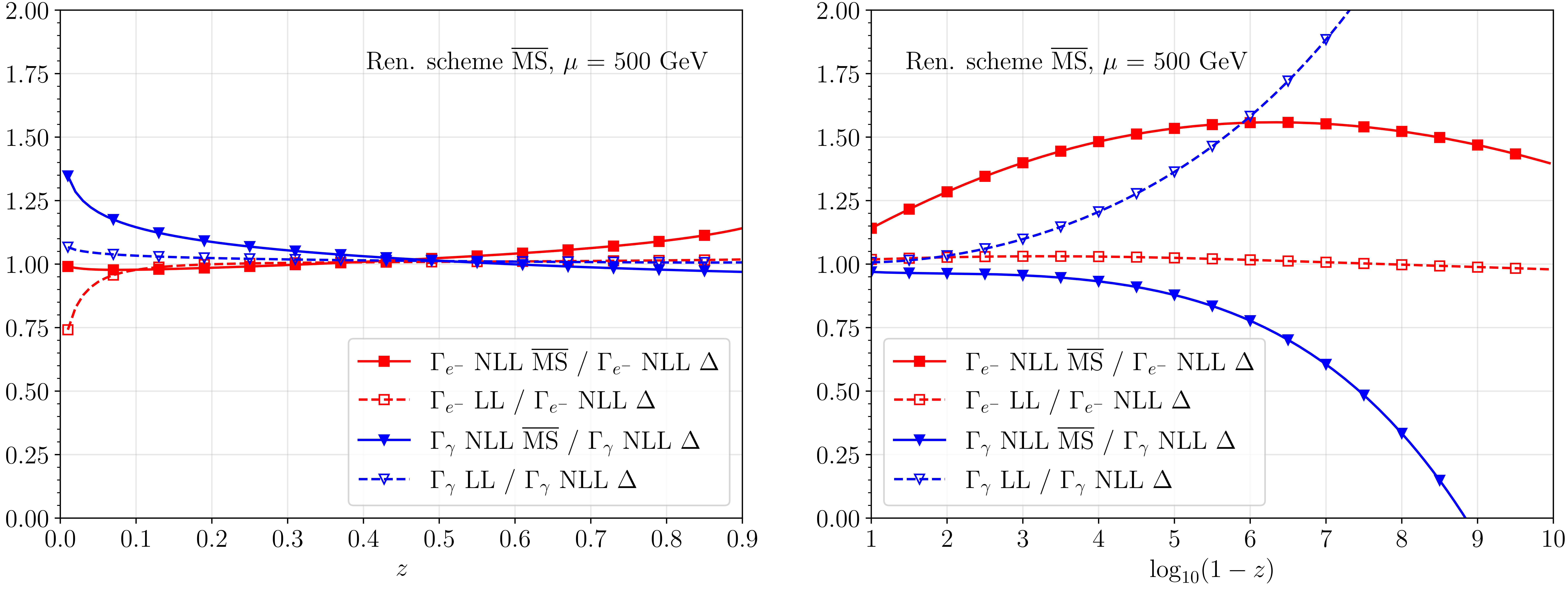
Impact of NLL





Non trivial pattern, impossible to account in some universal manner.
NLL-accurate PDFs are phenomenologically important for precision studies.

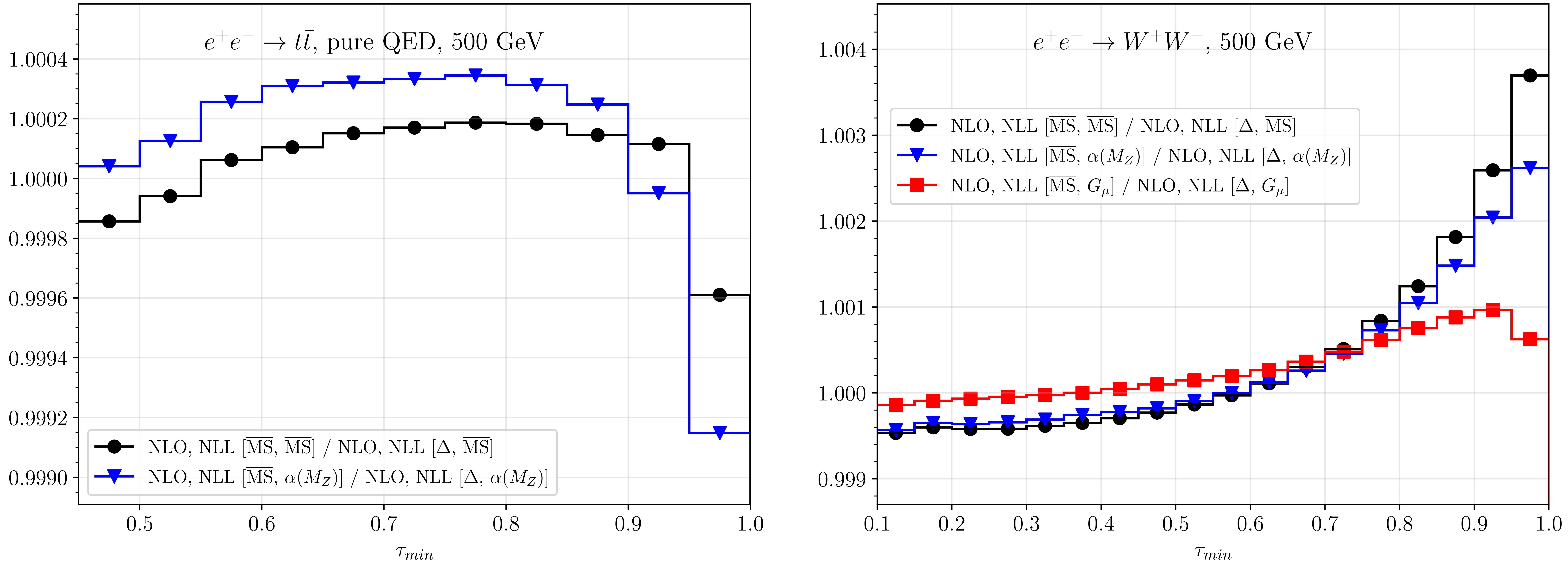
Dependence on factorisation scheme



<u>At the PDF level</u>, $\mathcal{O}(1)$ difference between $\overline{\text{MS}}$ and Δ scheme.

Electron at NLL in the Δ scheme closer to the LL value.

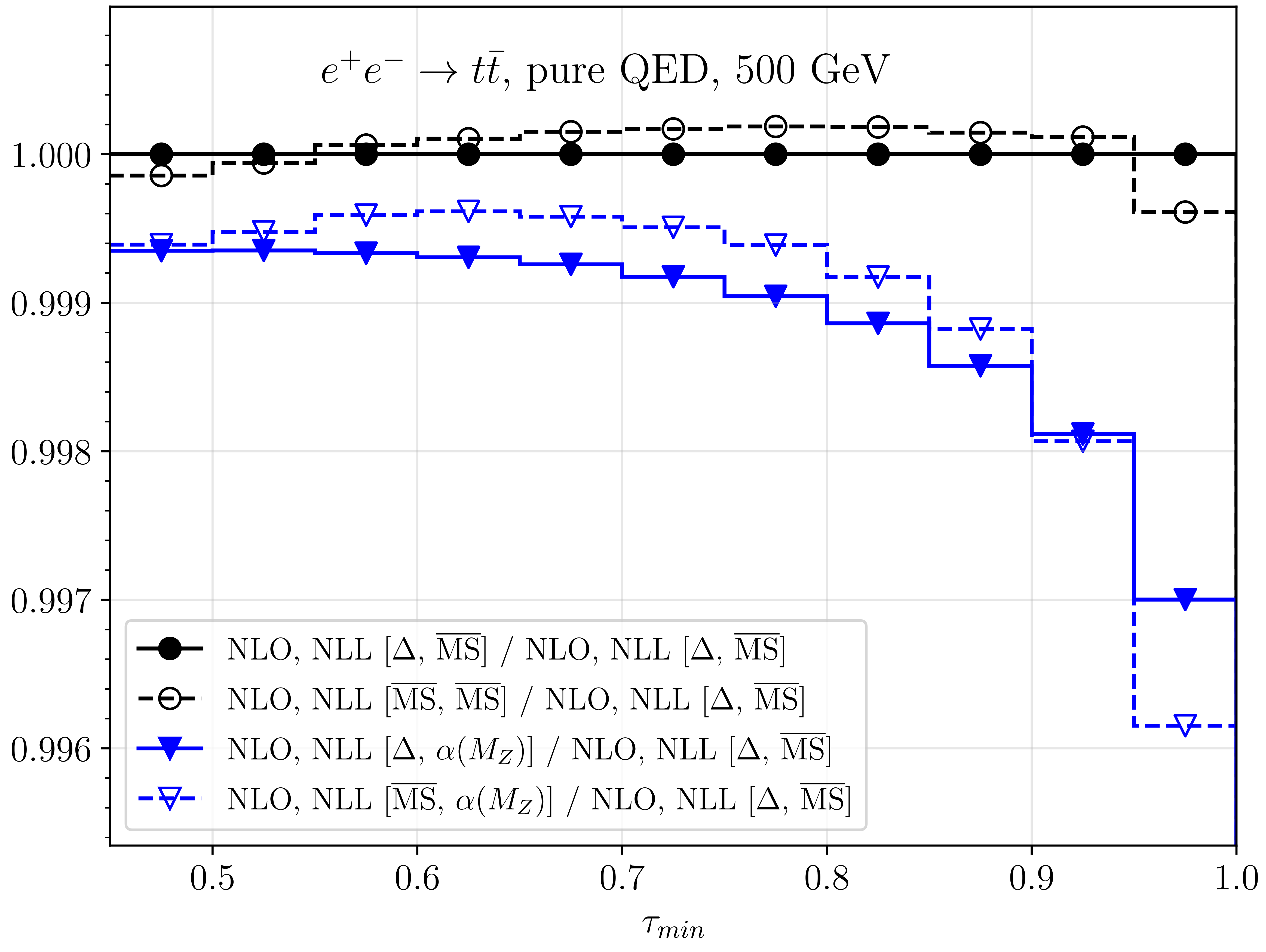
Dependence on factorisation scheme

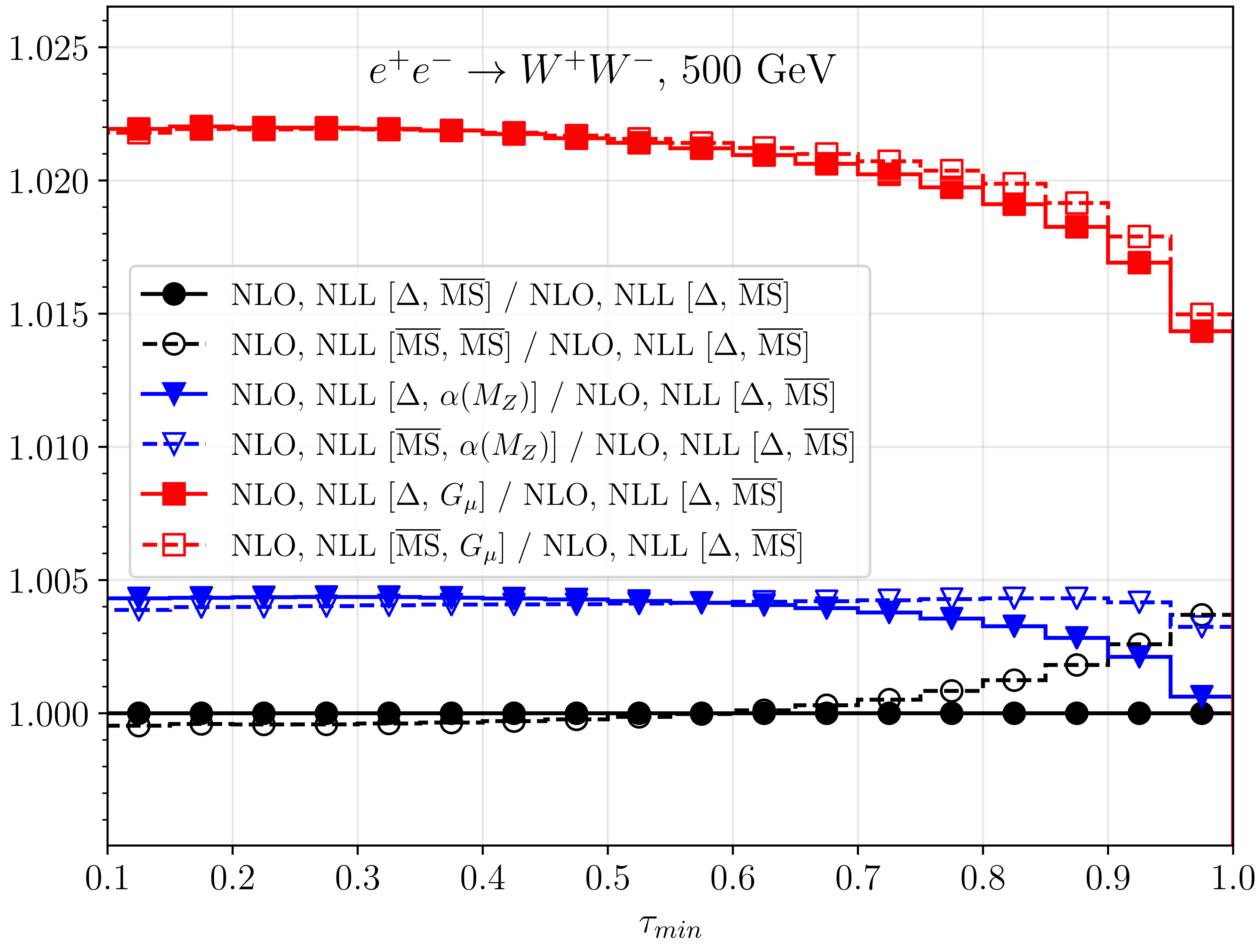


At the cross section level, $\mathcal{O}(10^{-4} - 10^{-3})$ difference between fact. schemes.

Large cancellations in the $\overline{\text{MS}}$ fact. scheme.

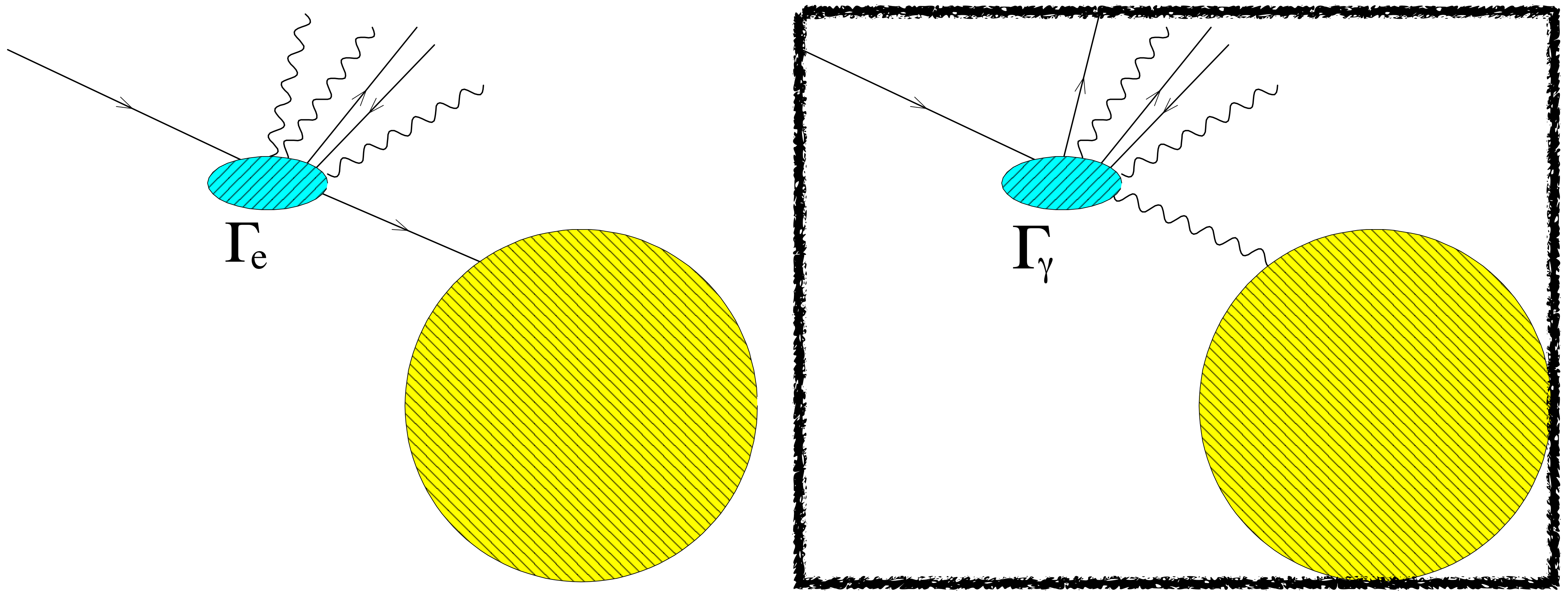
Dependence on renormalisation scheme

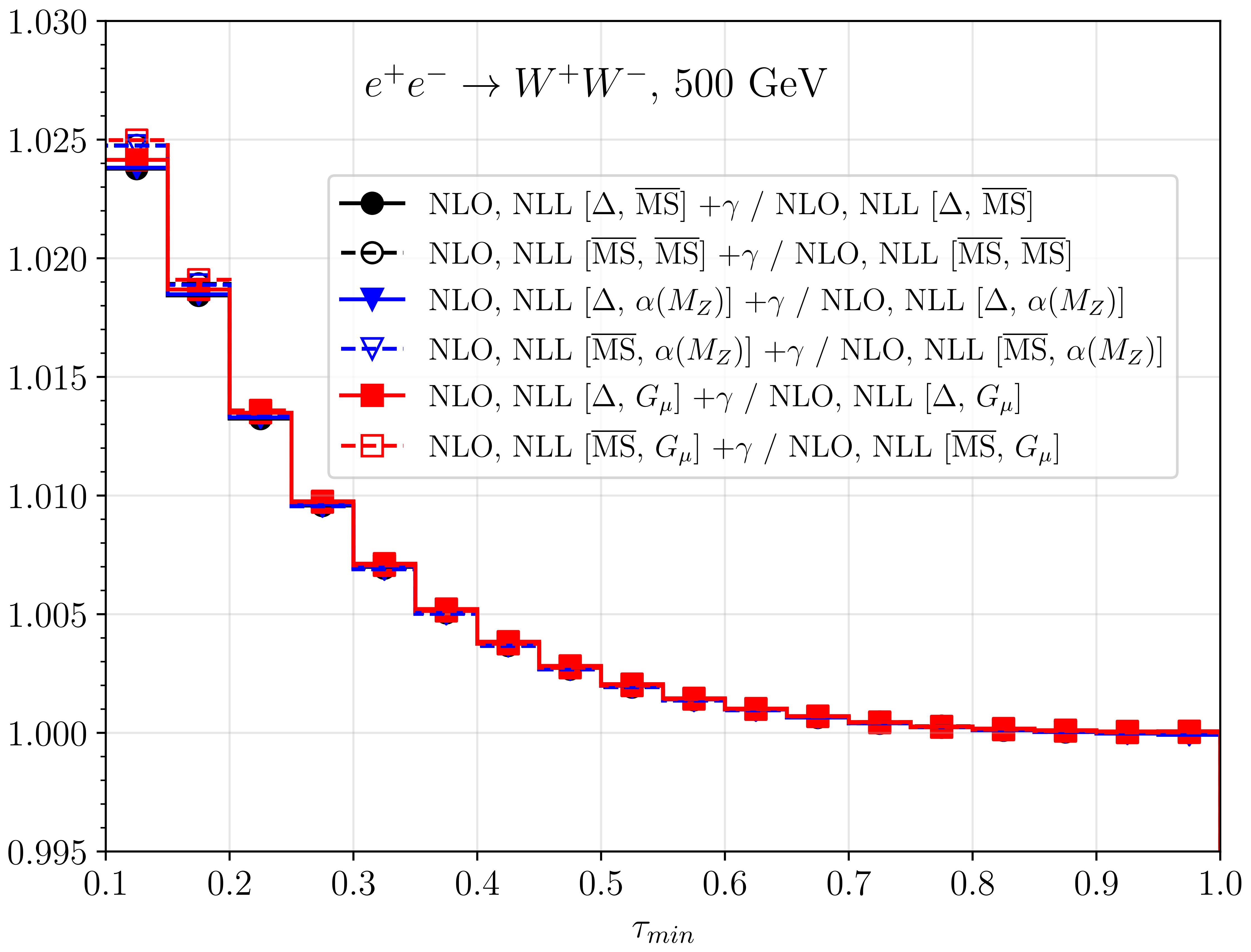




Ren. scheme dependence significantly **larger** than the fact. scheme one.
Mostly a normalisation effect.

Impact of photon-induced contributions





- At LO, i.e. $\mathcal{O}(\alpha^2)$, both W^+W^- and $t\bar{t}$ feature a $\gamma\gamma$ channel.
- Photon PDF Γ_γ only suppressed by a power of α w.r.t. Γ_{e^-}, and peaked at small-z values.

Both effects can lead to **physical effects** e.g. W^+W^- at small τ_{min}.

Beamstrahlung effects

Frixione, Mattelaer, Zaro, Zhao 2108.10261

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

$$\mathcal{B}_{kl}(y_+, y_-) \approx \sum_{n=1}^{N} b_{n,kl}^{(e^+)}(y_+) \, b_{n,kl}^{(e^-)}(y_-)$$

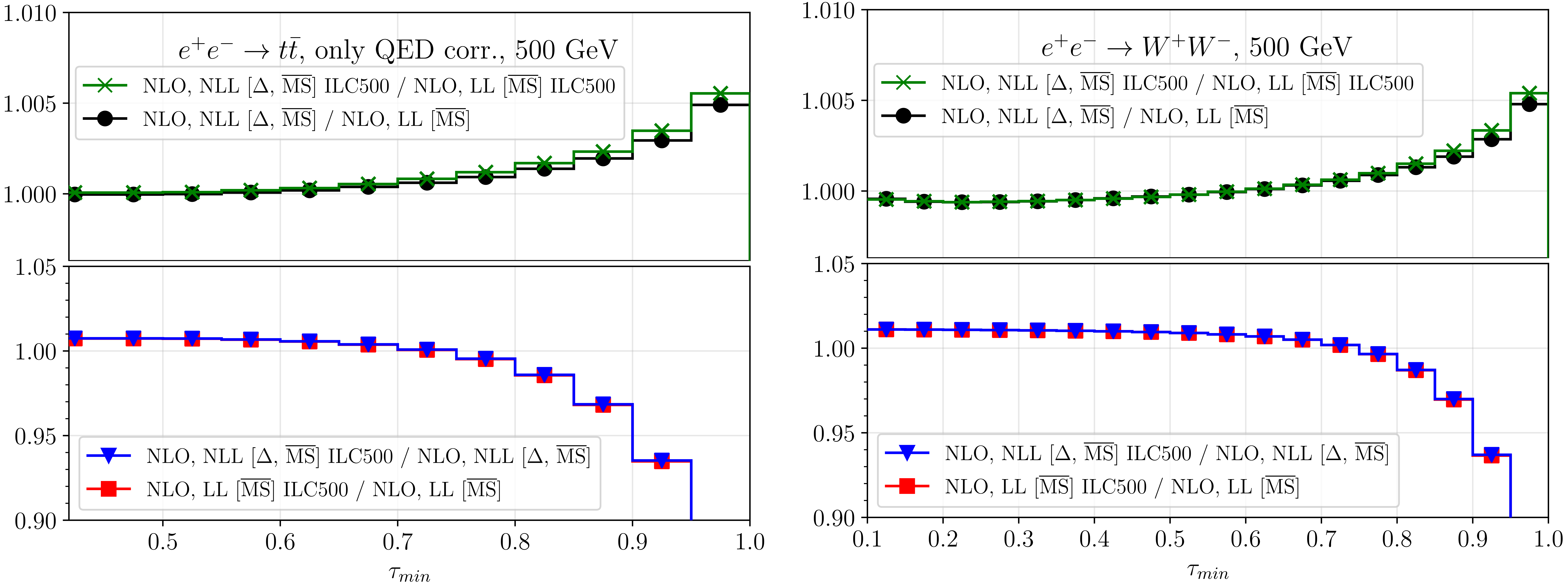
Parameters in b determined through fit to GuineaPig simulations

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{n=1}^{N} \sum_{ijkl} \int dx_+ dx_- \, \phi_{i/k,n,kl}^{(e^+)}(x_+, \mu^2, m^2) \, \phi_{j/l,n,kl}^{(e^-)}(x_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(x_+ P_{e^+}, x_- P_{e^-}, \mu^2, m^2) \,,$$

We can store in the grids also beamstrahlung!

$$\phi_{i/k,n,kl}^{(e^\pm)}(x, \mu^2, m^2) = \int dy \, dz \, \delta(x - yz) \, b_{n,kl}^{(e^\pm)}(y) \, \Gamma_{i/k}(z, \mu^2, m^2)$$

Example of beamstrahlung (ILC500)



Beamstrahlung effects have a clearly visible impact,
however affecting in the same way predictions at NLO+LL and at NLO+NLL

Other implementations of collinear factorisation

- For specific processes, logarithmic corrections calculated up $\mathcal{O}(\alpha^6)$

  see e.g. $e^+e^- \to \gamma^*/Z$ Ablinger, Blümlein, De Freitas, Schönwald 2004.04287
- `Pythia` and `Sherpa`: QED shower with LL PDFs
- `Whizard`: LL PDFs (NLL implementation in progress)

  Fixed order NLO EW see e.g. Bredt, Kilian, Reuter, Stienemeier 2208.09438

  Beam dynamics with interface to GuineaPig
- `RacoonWW`, `Racoon4f` $[e^+e^- \to W^+W^-(\to 4f)]$: analytic LL PDFs

  Denner, Dittmaier, Roth, Wackeroth, Wieders e.g. hep-ph/0006307, hep-ph/0502063
- `BabaYaga`: QED Parton Shower, next slides

QED Parton Shower

see for instance review in 0912.0749

Introduction of a cutoff $x_+ = 1 - \epsilon$, with $\epsilon \ll 1$, to regularise splitting kernels:

$$P_+(z) = \theta(x_+ - z)P(z) - \delta(1-z)\int_0^{x_+} \mathrm{d}x\, P(x)$$

By introducing a Sukadov form factor: $\Pi(s_1, s_2) = \exp\left(-\frac{\alpha}{2\pi}\int_{s_2}^{s_1}\frac{\mathrm{d}s'}{s'}\int_0^{x_+}\mathrm{d}z\, P(z)\right)$

one can recast the evolution equation in an iterative integral form:

$$D(x,s) = \sum_{n=0}^{\infty}\prod_{i=1}^{n}\left\{\int_{m_e^2}^{s_{i-1}}\frac{\mathrm{d}s_i}{s_i}\Pi(s_{i-1},s_i)\frac{\alpha}{2\pi}\int_{x/(z_1\cdots z_{i-1})}^{x_+}\frac{\mathrm{d}z_i}{z_i}P(z_i)\right\}\Pi(s_n, m_e^2)D\left(\frac{x}{z_1\cdots z_n}, m_e^2\right)$$

which can be solved by means of a MC algorithm

QED Parton Shower

see for instance review in 0912.0749

It allows for exclusive photon emission in the context of collinear factorisation.

Photon energies dictated by distribution in z, whereas angles are generated independently according to the YFS formula, valid in the soft limit:

$$\cos\theta_l \propto -\sum_{i,j=1}^{N} \eta_i \eta_j \frac{1-\beta_i\beta_j\cos\theta_{ij}}{(1-\beta_i\cos\theta_{il})(1-\beta_j\cos\theta_{jl})}$$

with η_i a charge factor and β_i the speed of the emitting particle.

Algorithm adopted in BabaYaga $[e^+e^- \to e^+e^-, e^+e^- \to \mu^+\mu^-, e^+e^- \to \gamma\gamma]$

hep-ph/0003268, hep-ph/0103117, hep-ph/0312014, hep-ph/0801.3360, hep-ph/0607181

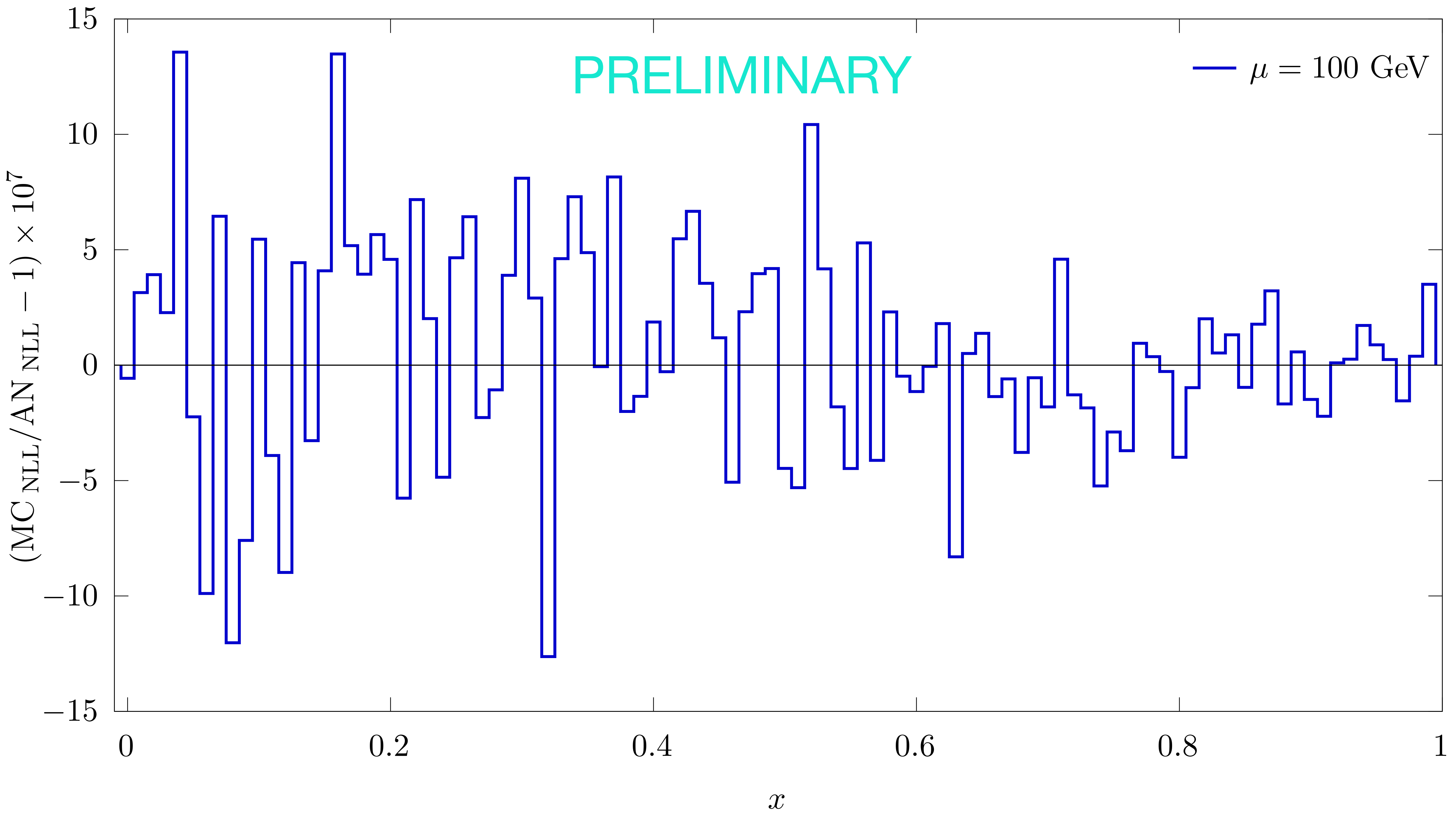
Balossini, Bignamini, Carloni Calame, Lunardini, Montagna, Nicrosini, Piccinini

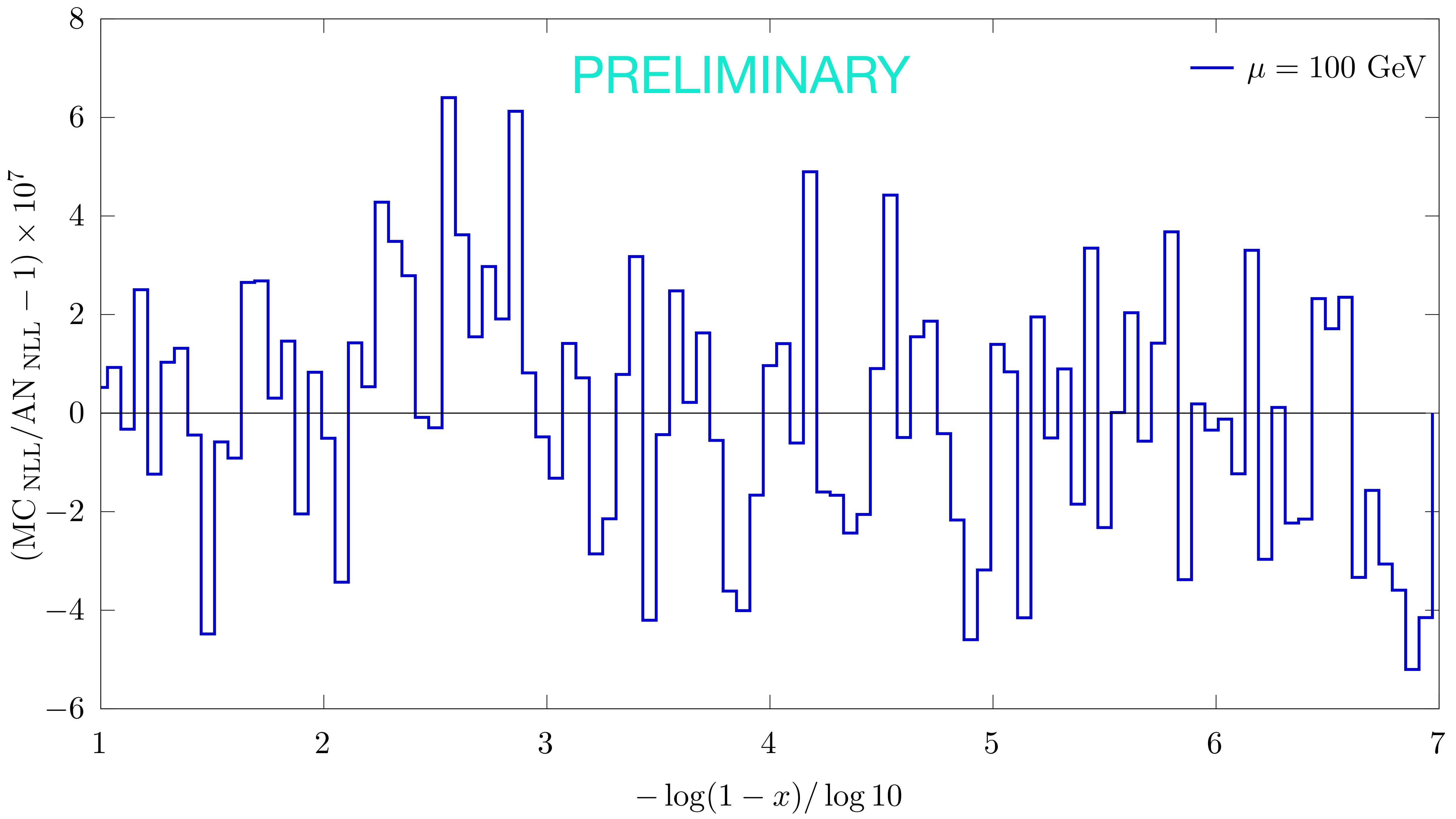
BabaYaga also includes a matching to NLO QED in the short distance cross section

Towards a NLL QED Parton Shower

C. M. Carloni Calame, M. Chiesa, S. Frixione, G. Montagna, F. Piccinini, GS

With a NLL iterative solution, we recover the known (non-singlet) NLL PDFs





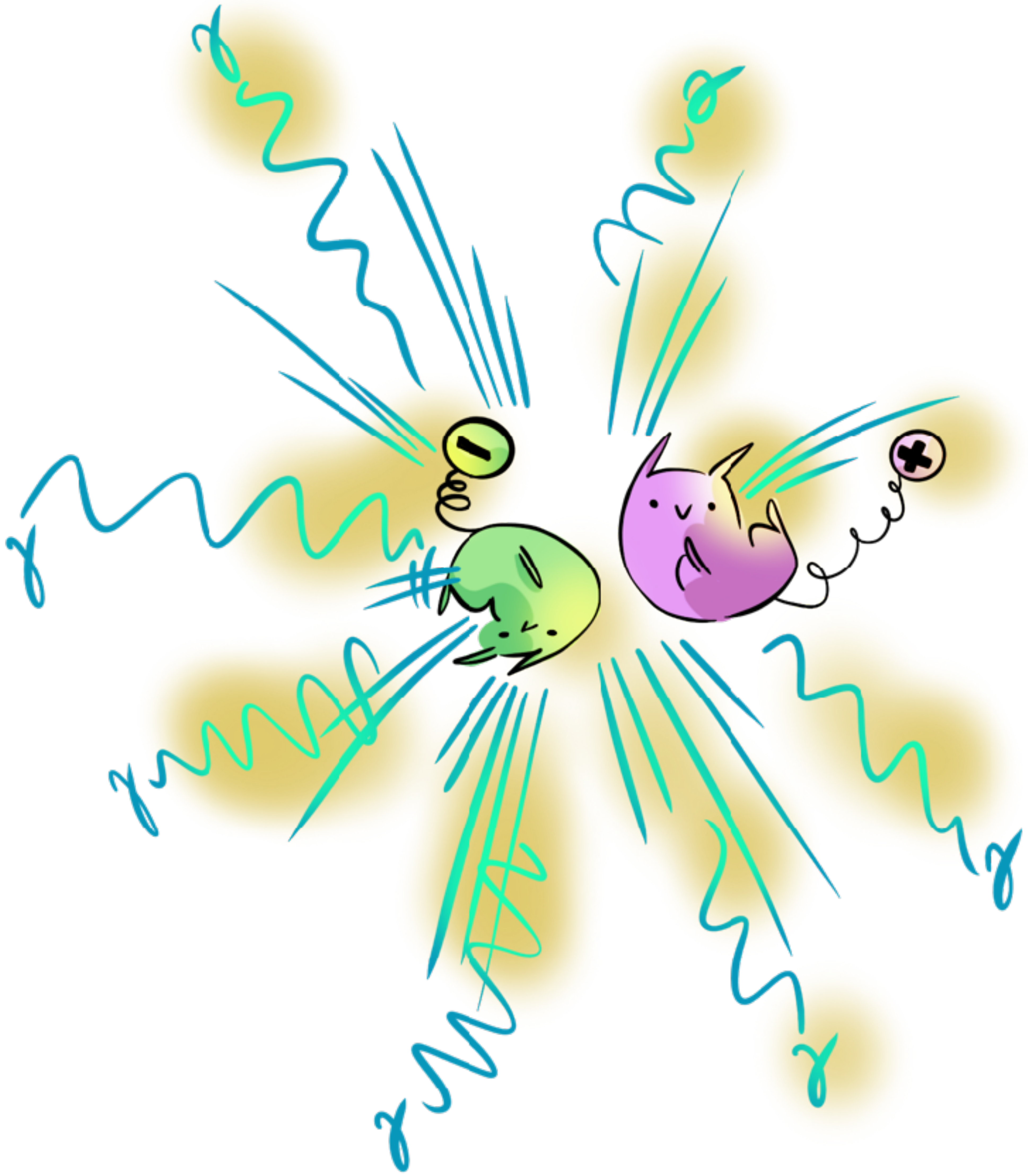
WIP towards exclusive kinematics of final-state photons and singlet components

~~Introduction~~
Outlook

- Initial-state radiation (ISR) is an intrinsic “feature” of lepton colliders

- QED theoretical basis are well established, but conceptual and technical progress still needed to reach target precision of future colliders

- The LEP legacy is not enough. The many tools and ideas developed at the LHC can offer solutions useful for e^+e^- environment



Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Registration open here:
https://indico.psi.ch/event/13708

Jun 7 – 9, 2023
University of Zurich
Europe/Zurich timezone

Enter your search term

Satellite event to the 5th WorkStop/ThinkStart in Zurich
(https://indico.psi.ch/event/13707),
with the aim to collect some community input and form new collaborations.