Department of Theoretical Physics

Logarithmic accuracy of matching (and shower uncertainties)

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Parton Showers for future e^+e^- colliders
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Based on JHEP03 (2023)224 within the PanScales collaboration with
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NLO Matching - a solved problem?

- Event generators with NLO accuracy have become the *de facto* tool for particle collision simulations.
- There are a number of solutions available, going back more than 20 years, but by far the two most widely used are MC@NLO [Frixione, Webber '02] and POWHEG [Nason '04, Frixione, Nason, Oleari '07].
- Both were formulated at a time when parton showers had limited (i.e. leading) logarithmic accuracy.
- For this reason the concern was mainly to improve the fixed order side of things, without breaking the shower.
- With the advent of NLL showers (of which a number have emerged in recent years) it has become relevant to return to the question of formal shower accuracy in the context of NLO matching.
- Will discuss the two-body decay processes $\gamma^* \to q\bar{q}$ and $h \to gg$ in the following.

NLO Matching - revisited

- To understand the interplay between matching and logarithmic accuracy, it is instructive to discuss the example of event shapes, for which the probability of some observable $\mathcal{O}$ to have a value below e^L is given by

$$\Sigma(O < e^L) = (1 + C_1\alpha_s + \ldots)\, e^{\alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots}, \qquad L \ll -1\,.$$

- Here g_1 is responsible for LL terms ($\alpha_s^n L^{n+1}$), g_2 for NLL terms ($\alpha_s^n L^n$) and C_1 and g_3 for NNLL terms ($\alpha_s^n L^{n-1}$).
- Σ can also be written in terms of a double-logarithmic expansion

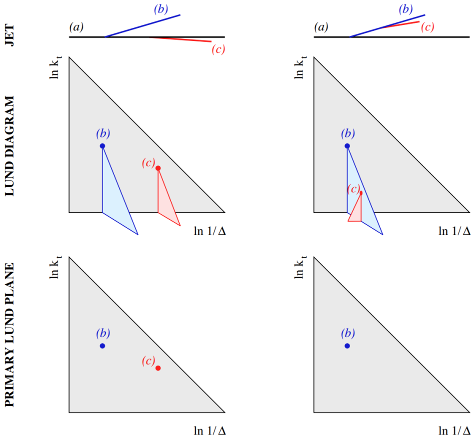
$$\Sigma(O < e^L) = h_1(\alpha_S L^2) + \sqrt{\alpha_S}\, h_2(\alpha_S L^2) + \alpha_S h_3(\alpha_S L^2) + \ldots, \qquad |L| \gg 1\,,$$

- with h_1 responsible for DL terms ($\alpha_S^n L^{2n}$), h_2 for NDL ($\alpha_S^n L^{2n-1}$), and h_3 for NNDL terms ($\alpha_S^n L^{2n-2}$).
- In analytic resummation C_1 is typically obtained through NLO matching, and its inclusion is enough to achieve NNDL for event shapes.

NLO Matching - revisited

- Hence, for event shapes there is an obvious logarithmic correspondence with NLO matching: A good NLO matching scheme should augment an NLL shower to NNDL.
- However, this is not the case in general.
- As is know from analytic resummation NLO matching is a necessary ingredients to achieve NNLL accuracy in general, since a term α_s contributes to the $\alpha_s^n L^{n-1}$ logarithmic tower.
- So instead of thinking of NLO matching as a way of achieving better fixed order accuracy we can think of it as a step towards having NNLL accurate event generators.
- In fact, if we try to understand matching from the point of view of the Lund Plane, this becomes even more clear...

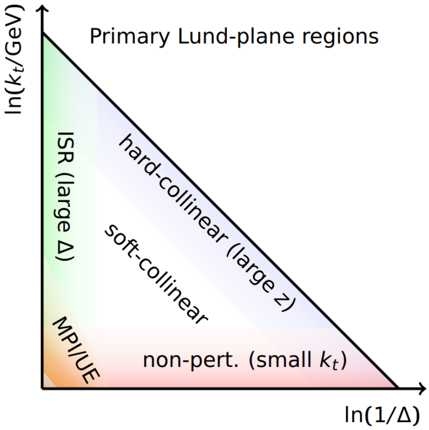
The Lund Plane



[1807.04758]

- Cluster the event with the Cambridge/Aachen algorithm, producing an angular ordered clustering sequence.
- Decluster the last clustering and record the transverse momentum and the opening angle of the declustering (plus other kinematics).
- Iterate along the hardest branch after each declustering to produce the *primary* Lund Plane.
- Following the softer branch produces the secondary, tertiary, etc Lund Plane.
- One can impose cuts easily on the declusterings (e.g. that they satisfy $z > z_{\text{cut}}$)

Logarithms in the Lund Plane



[1807.04758]

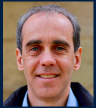
- The emission probability in the Lund Plane is then
$$\mathrm{d}\rho \sim \alpha_S \, \mathrm{d}\ln k_T \, \mathrm{d}\ln\theta$$
- Hence emissions that are well-separated in *both* directions are associated with *double logarithms* of the form $\alpha_S^n L^{2n}$
- Emissions separated along one direction are associated with *single logarithms* of the form $\alpha_S^n L^n$
- Emissions that are close in the Lund Plane are associated with a factor α_S^n
- An NLL accurate shower correctly describes kinematical configurations where *all* emissions are well-separated in the Lund Plane

The Lund Plane expansion

- A simple picture now emerges.
- In order to go *beyond NLL* we have to be able to describe configurations in the Lund Plane, where at most two emissions are close to each other.
- This in particular includes when an emission is close to the top of the Lund Plane (where the initial "hard" parton sits), but it also includes configurations with for instance two commensurate energy wide-angle emissions.
- We should therefore think of NLO matching as one of several corrections to the Lund Plane.
- And in particular if we want to think of uncertainties in a particular shower, we should probably think of all these contributions on a similar footing.
- Now that the motivation is hopefully clear, let us review various matching procedures with the view to understanding their impact on logarithmic accuracy...

The PanScales collaboration

Oxford



Gavin Salam

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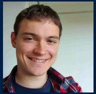

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Matching in a nut-shell

- **Multiplicative**: Modify the shower's first emission through a veto on $P_{\text{exact}}/P_{\text{shower}}$, which itself is expected to go to 1 in the infrared/collinear limit.
- **MC@NLO**: Supplement the shower events with a set of hard events, $P_{\text{exact}} - P_{\text{shower}}$, which vanish in the infrared/collinear limit.
- **POWHEG**: Handle the hardest emission generation with a special Hardest Emission Generator (HEG) that acheives NLO acuracy for the hardest emission.
- Not discussed here: **KrkNLO** which is similar in spirit to multiplicative matching and **MAcNLOPS** which is multiplicative when $P_{\text{exact}} < P_{\text{shower}}$ and MCNLO otherwise.

Let's match!

- The first matching procedure we consider is multiplicative matching (also often called Matrix Element Corrections). The hardest emission cross section can be written as

$$d\sigma_{\text{mult}} = \bar{B}(\Phi_B) \left[S_{\text{PS}}(v_\Phi^{\text{PS}}, \Phi_B) \times \frac{R_{\text{PS}}(\Phi)}{B_0(\Phi_B)} d\Phi \otimes \frac{R(\Phi)}{R_{\text{PS}}(\Phi)} \right] \times I_{\text{PS}}(v_\Phi^{\text{PS}}, \Phi)\,.$$

- With the parton shower Sudakov given by

$$S_{\text{PS}}(v, \Phi_B) = \exp\left[-\int_{v_\Phi^{\text{PS}} > v} \frac{R_{\text{PS}}(\Phi)}{B_0(\Phi_B)} d\Phi_{\text{rad}} \right],$$

- and the NLO normalisation factor written as

$$\bar{B}(\Phi_B) = B_0(\Phi_B) + V(\Phi_B) + \int R(\Phi) d\Phi_{\text{rad}}\,,$$

Multiplicative matching

- In practice the multiplicative matching can only work if $R(\Phi) \leqslant R_{\text{PS}}(\Phi)$ in order for the first emission probability to be bounded by 1.
- Since $R(\Phi)$ and $R_{\text{PS}}(\Phi)$ agree in the soft/collinear limits, the matching has no impact in these limits, and from a logarithmic point of view we therefore expect NLL accuracy to be retained.
- This type of matching has to be implemented directly inside the relevant shower code, and cannot be achieved with external tools.

MC@NLO matching

- In the MC@NLO scheme the hardest emission cross section takes the form

$$\mathrm{d}\sigma_{\mathrm{MC@NLO}} = \bar{B}_{\mathrm{PS}}(\Phi_{\mathrm{B}})\, S_{\mathrm{PS}}(v_{\Phi}^{\mathrm{PS}}, \Phi_{\mathrm{B}}) \times \frac{R_{\mathrm{PS}}(\Phi)}{B_0(\Phi_{\mathrm{B}})} \mathrm{d}\Phi \times I_{\mathrm{PS}}(v_{\Phi}^{\mathrm{PS}}, \Phi) + \\ + [R(\Phi) - R_{\mathrm{PS}}(\Phi)]\, \mathrm{d}\Phi \times I_{\mathrm{PS}}(v^{\max}, \Phi)\,,$$

- with

$$\bar{B}_{\mathrm{PS}}(\Phi_{\mathrm{B}}) = B_0(\Phi_{\mathrm{B}}) + V(\Phi_{\mathrm{B}}) + \int R_{\mathrm{PS}}(\Phi) \mathrm{d}\Phi_{\mathrm{rad}}\,.$$

- Interpretation: Generate events with the shower (modifying the normalisation) and supplement these with a set of finite hard events.
- Specifically, this ensures that the shower is preserved in the infrared and collinear regions.

POWHEG$_\beta$

- Now let us consider a simple version of POWHEG matching given by

$$d\sigma_{\text{POWHEG-simple}} = \bar{B}(\Phi_B)\, S_{\text{HEG}}(v_\Phi^{\text{HEG}}, \Phi_B) \times \frac{R_{\text{HEG}}(\Phi)}{B_0(\Phi_B)}\, d\Phi \times I_{\text{PS}}(v_\Phi^{\text{HEG}}, \Phi)\,.$$

- In this variant of POWHEG the HEG generates an event at a scale v_Φ^{HEG} that is then handed over to the shower, which continues showering starting at the same scale.
- In order to preserve leading logarithmic accuracy, the ordering variable of the HEG and the shower need to coincide in the simulatneously soft and collinear limit.
- This is for instance the case in standard transverse-momentum ordered `POWHEG-BOX`+Pythia8 usage.
- It would however not be the case if one were to use a $\beta = 1/2$ variant of one of the PanScales showers.

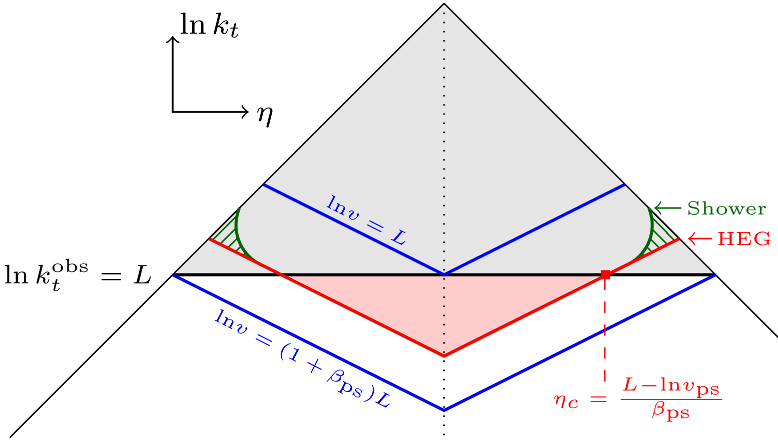
POWHEG$_\beta$

- One can however fairly easily modify the POWHEG ordering variable to have the necessary β dependence such that it coincides with the PanScales showers in the simultaneously soft and collinear limit

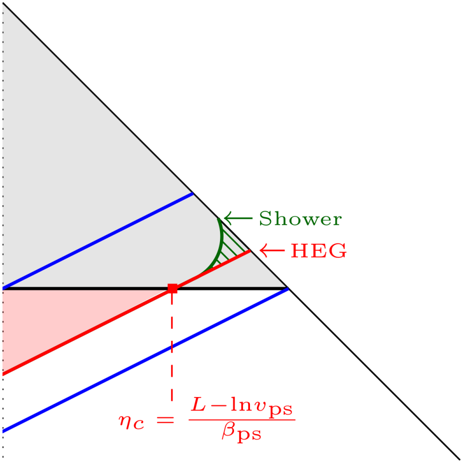
$$\bar{\eta} = -\ln\tan\left(\frac{\arccos y}{2}\right), \quad \ln v = \ln\frac{\sqrt{s}}{2} + \ln\sin\left[2\arctan e^{-\bar{\eta}}\right] + \ln\xi - \beta|\bar{\eta}|.$$

- Inside the PanScales framework we call this POWHEG$_\beta$.
- Even so there can still be mismatches in both the hard-collinear and soft wide-angle regions of the Lund Plane.
- This is something that has been known for some time [Corke, Sjöstrand '10], and is connected to the question of under-/double-counting in matching. It is mostly solved by the usual veto
- To address the logarithmic impact we again return to the Lund Plane...

POWHEG$_\beta$ and NNDL accuracy



POWHEG$_\beta$ and NNDL accuracy



- At DL accuracy the answer we are after is given by

$$\Sigma(O < e^L) = e^{-\bar{\alpha} L^2}, \quad \bar{\alpha} = \alpha_S$$

- If the shower and HEG contours line up everywhere, we would get that answer. If they disagree in the hard-collinear region, we instead get (neglecting terms beyond NNDL)

$$\Sigma(O < e^L) = e^{-\bar{\alpha} L^2}\left[1 + 2\left(e^{-\bar{\alpha}\beta L^2} - 1\right)\bar{\alpha}\Delta\right] \tag{1}$$

- Δ is the effective area of one shaded green region, which for PanLocal and $\gamma \to q\bar{q}$ is given by

$$\bar{\alpha}\Delta = \frac{2C_F \alpha_S}{\pi} \cdot \frac{4\pi^2 - 15}{24}.$$

- Since Δ is $\mathcal{O}(1)$ this gives rise to a tower $\propto \alpha_S(\bar{\alpha}_S L^2)^n$ in eq. (1), which breaks NNDL.

~~NLL~~

- While breaking of NNDL is not desirable, one could take the view that as long as NLL is not broken, the matching still achieved its goal.
- Eq. (1) gives the impression that NLL is not broken, as the term $\propto \alpha_S(\alpha_S L^2)^n$ vanishes when $\alpha_S \to 0$.
- However, if we take the logarithm of eq. (1) we get

$$\ln\Sigma = -\bar{\alpha}L^2 - \sum_{n=2}^{\infty} \frac{2\beta^{n-1}\Delta}{(n-1)!} \cdot \bar{\alpha}^n L^{2n-2} + \mathcal{O}(\bar{\alpha}^n L^{2n-3})\,.$$

  which fails to satisfy the exponentiation criterion, that there are no terms $\alpha_S^n L^m$ in $\ln\Sigma$ with $m > n+1$ (starting at $\mathcal{O}(\alpha_S^4)$).
- Alternatively one can view these terms as spurious super-leading logarithms induced by the matching.

~~NLL~~ - so what?

- Okay, we broke NLL, but in a very technical way. Maybe this breaking will not be very relevant for phenomenology, since the NLL breaking starts at $\mathcal{O}(\alpha_S^4)$ and the NNDL breaking a relative $\mathcal{O}(\alpha_S)$ in Σ?
- Hard to say without running the code, but one needs to keep in mind that there are other observables than event shapes, and that some of these could potentially be more sensitive to the problem.
- One such is the mass of the first SoftDrop ($\beta = 0$) splitting, which is sensitive to the hard-collinear region by construction, and does not have double-logarithmic terms. It has the following single-logarithmic structure

$$\partial_L \Sigma_{SD}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L}$$

- Taking the shower/HEG mismatch into account, one instead finds

$$\partial_L \Sigma_{SD}(L) = \bar{\alpha} c\, e^{\bar{\alpha} c L - \bar{\alpha}\Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2} (1 - e^{-\bar{\alpha}\Delta}),$$

- This again gives rise to terms $\alpha_S^n L^{2n-2}$ in the logarithm, but more importantly when $\alpha_S L^2 \sim 1$ the second term is only suppressed by a relative $\mathcal{O}(\sqrt{\alpha_S})$ compared to the first one, which is parametrically larger than the $\mathcal{O}(\alpha_S)$ effect for event shapes.

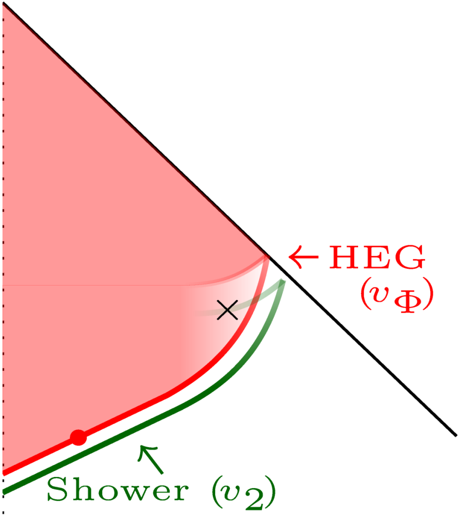
Solution to the problem

- The solution to the problem is actually well-known and already applied in typical POWHEG usage.
- After the HEG hands over the hardest emission, the shower should not start from v_Φ^{HEG} but rather from the maximum scale, and then veto *all* emissions with a hardness scale above v_Φ^{HEG}.
- We can write this procedure as

$$d\sigma_{\texttt{POWHEG}\text{-veto}} = \bar{B}(\Phi_{\text{B}})\, S_{\text{HEG}}(v_\Phi^{\text{HEG}}, \Phi_{\text{B}}) \times \frac{R_{\text{HEG}}(\Phi)}{B_0(\Phi_{\text{B}})}\, d\Phi \times I_{\text{PS}}(v^{\max}, \Phi | v_i^{\text{HEG}} < v_\Phi^{\text{HEG}}),$$

- As we shall see, this will be enough to restore NNDL accuracy, with a proviso having to do with gluon splittings...

Further subtleties



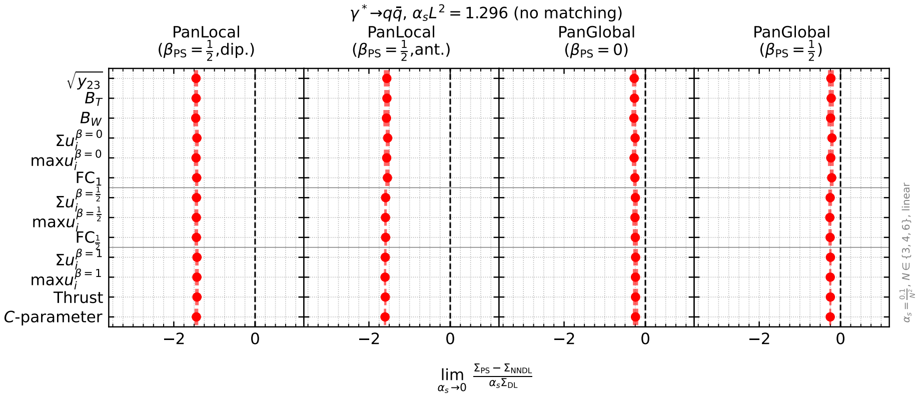
- Even when the contours are fully aligned there are issues associated with how dipole showers partition the $g \to gg(q\bar{q})$ splitting function.
- In PanScales we use

$$\frac{1}{2!}P_{gg}^{\text{asym}}(\zeta) = C_A\left[\frac{1+\zeta^3}{1-\zeta} + (2\zeta-1)w_{gg}\right],$$

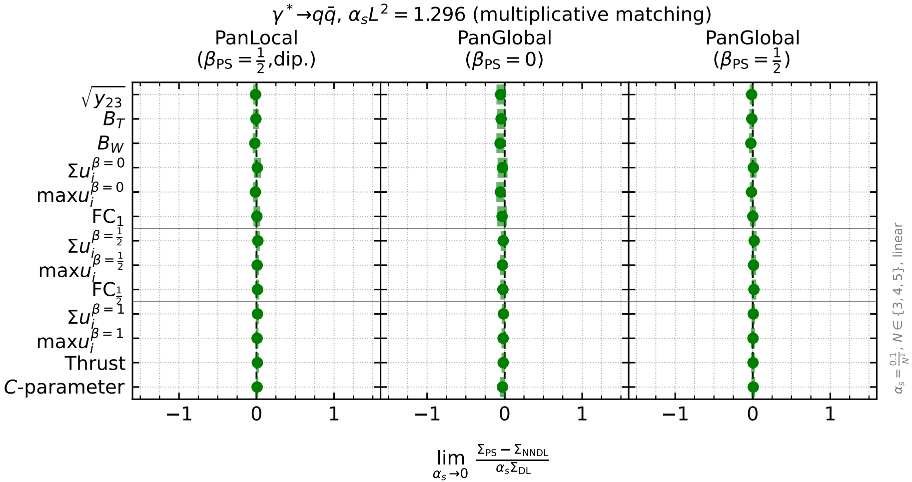
such that $P_{gg}^{\text{asym}}(\zeta) + P_{gg}^{\text{asym}}(1-\zeta) = 2P_{gg}(\zeta)$

- This partitioning takes place to isolate the two soft divergences in the splitting function ($zeta \to 0$ and $zeta \to 1$), but there is some freedom in how one handles the non-singular part.
- Similarly, in the HEG one needs to handle this issue, and in general if the shower and the HEG do not agree on this procedure, one can induce similar NNDL breaking to what was seen above.

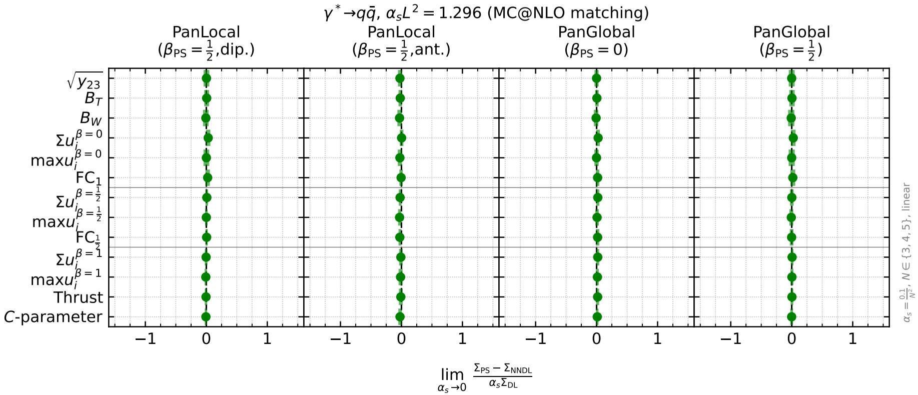
Showers without matching are not NNDL accurate



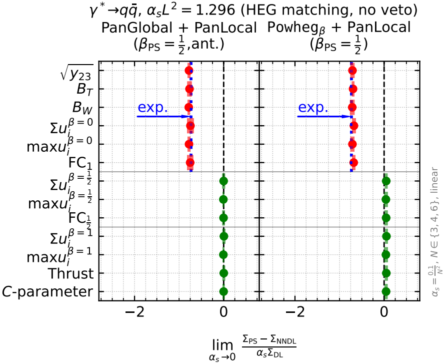
Multiplicative matching achieves NNDL accuracy

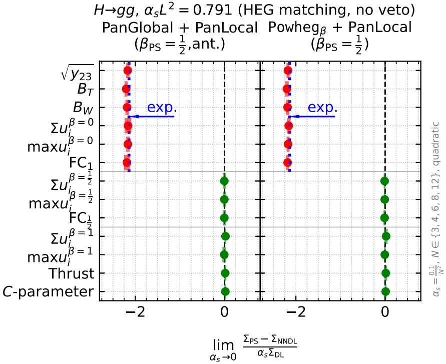


MC@NLO matching achieves NNDL accuracy

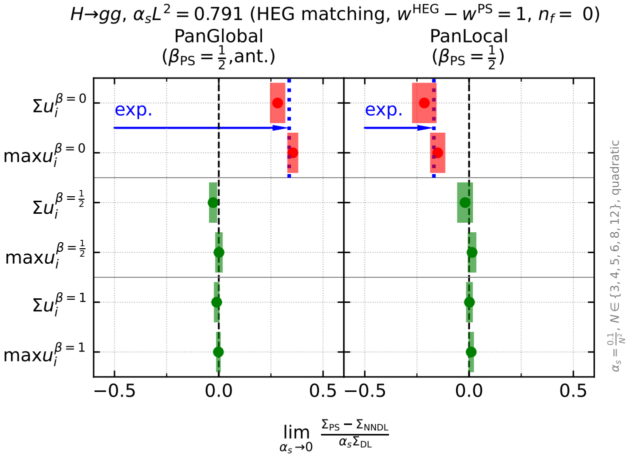


HEG-matching without a veto is not NNDL accurate

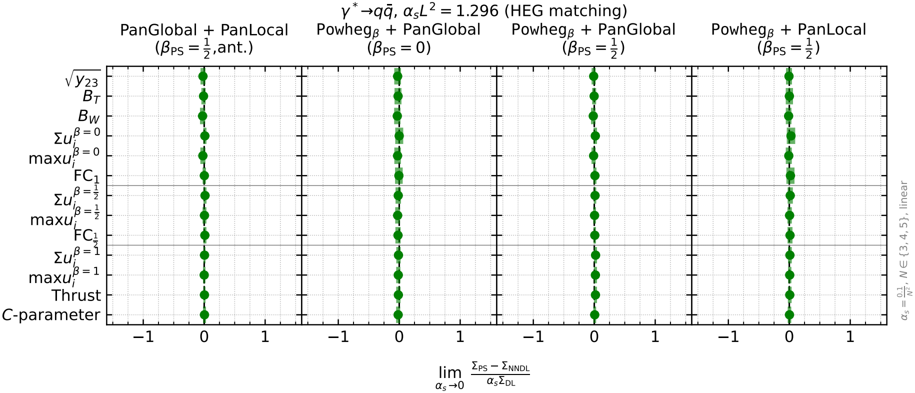




HEG-matching with $w^{\mathrm{HEG}} \,!= w^{\mathrm{PS}}$ is not NNDL accurate



Proper HEG-matching achieves NNDL accuracy



Phenomenological considerations

- Now that we have improved the logarithmic accuracy of our showers, we also want to assess the impact on phenomenology.
- However, in order to make a fair comparison, we need to understand their uncertainty.
- To this effect we include scale compensation, for an emission carrying away a momentum fraction z, given by[1]

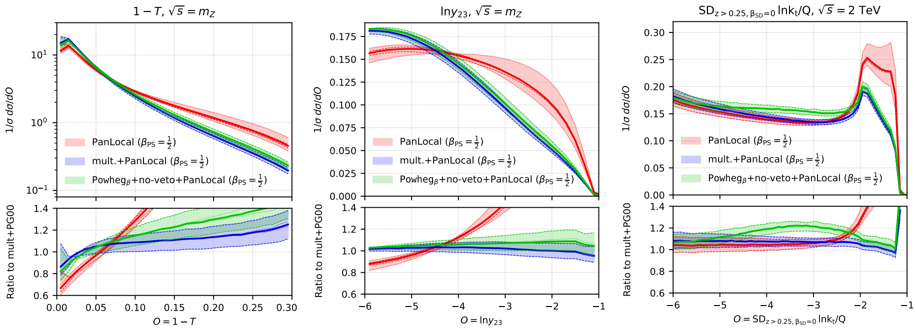
$$\alpha_S(\mu_R)\left(1+\frac{K\alpha_S(\mu_R)}{2\pi}+\frac{2(1-z)\beta_0\alpha_S(\mu_R)}{2\pi}\ln(x_R)\right), \quad \mu_R = x_R\mu_R^{\text{central}}.$$

  where the factor $1-z$ ensures that we only apply the scale compensation in the soft limit, and not the hard where the shower does include all the necessary ingredients. For showers that are not NLL we include the term proportional to K (CMW scheme) but omit the $1-z$ term.
- In order to assess missing terms in the hard matching region we take the emission strengt proportional to (unless matching that emission)

$$P_{\text{splitting}}(x_{\text{hard}}) = P^{(\text{default})}_{\text{splitting}} \times \left[1+(x_{\text{hard}}-1)\min\left(\frac{4\kappa_\perp^2}{Q^2},1\right)\right],$$

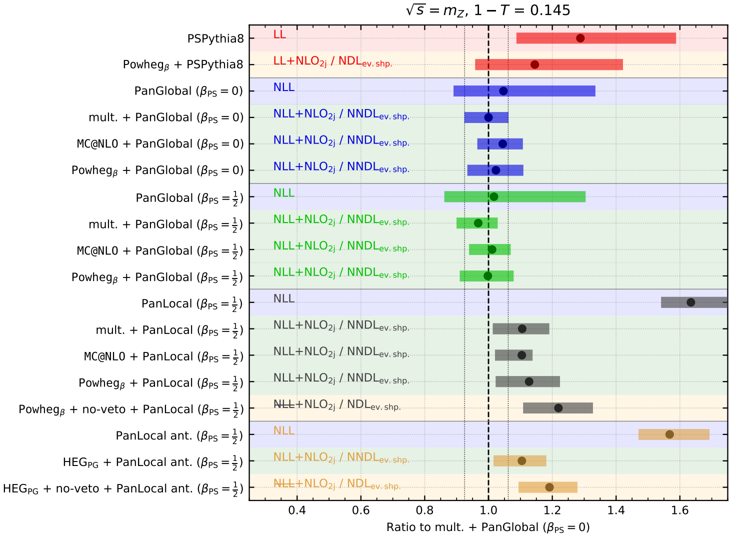
[1] Inspired by [Mrenna, Skands '16]

No matching vs multiplicative vs no veto

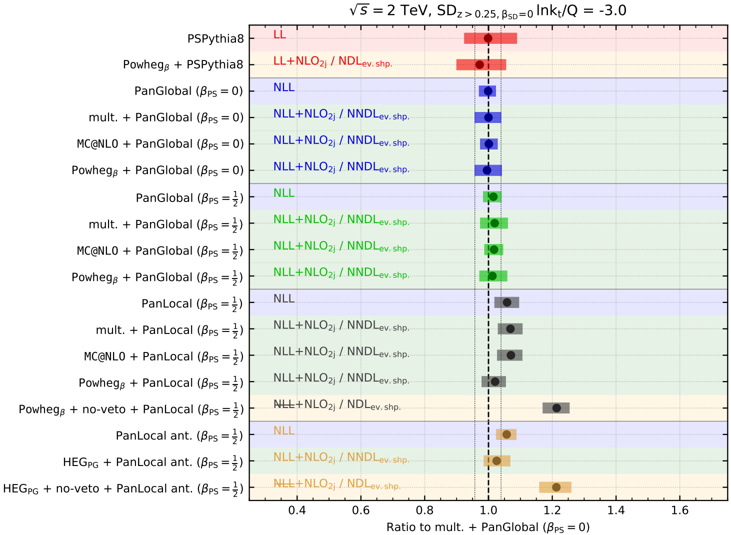


- Ratio is to PanGlobal $\beta=0$ with multiplicative matching
- Large effect of matching, with good agreement between showers after matching
- Omitting the veto in POWHEG leads to sizable effects in SD (expected), moderate effects in thrust (surprising as it is $\beta=1$) and little effect in $\sqrt{y_{23}}$ (disappointing as it is $\beta=0$).

Summary for thrust



Summary for SD



Outlook

- We have seen that standard NLO matching procedures can achieve NNDL accuracy for global event shapes.
- For multiplicative and MC@NLO matching this came straight out of the box, whereas with POWHEG care has to be taken to ensure that differences in the HEG and shower contours are correctly accounted for
- Although the veto procedure works, it might be worth thinking about designing POWHEG maps that mimic exactly the shower maps, in particular as the veto might become more delicate when showers achieve NNLL accuracy one day
- We have seen that we can reasonably well capture the uncertainty of our PanScales showers, but more work is needed in particular to also include uncertainties from other higher order sources (like the A_3 re-scaling discussed by Melissa yesterday)