Energy-Peak-Based Method to Measure Top Quark Mass via B-Hadron Decay Lengths

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In collaboration with K. Agashe, S. Airen, R. Franceschini, J. Incandela, D. Sathyan,
JHEP06 (2023) 021, arXiv:2212.03929

Top Quark Mass Measurements and Systematics

- ☐ Theoretical
  - ✓ Uncertainties in top quark (pair) production: Beyond SM (BSM) contribution (e.g., light supersymmetric top decaying into top [Czakon, Mitov, Papucci, Ruderman, Weiler, PRL113 201803; Cohen, Majewski, Ostdiek, Zheng, JHEP06 019]); PDF’s, higher-order effects, even in SM [e.g., top quark (mis-)modeling?!]; hadronization of bottom quark (cf. lepton from decay)

- ☐ Experimental
  - ✓ JES uncertainty for b-jet vs. using “cleaner” leptonic measurements

- ☐ Each method is insensitive to some systematics but is affected by others.

No “Best” Methods, Nevertheless…

- ☐ In my humble opinion, no best method or no “slam dunk” in top quark mass measurement ⇒ motivating new ideas, especially,
  - ✓ INDEPENDENT of details/modeling of production [based on kinematics of (only) decay, thus avoid (some) theoretical systematics] and/or
  - ✓ INSENSITIVE to some experimental systematics

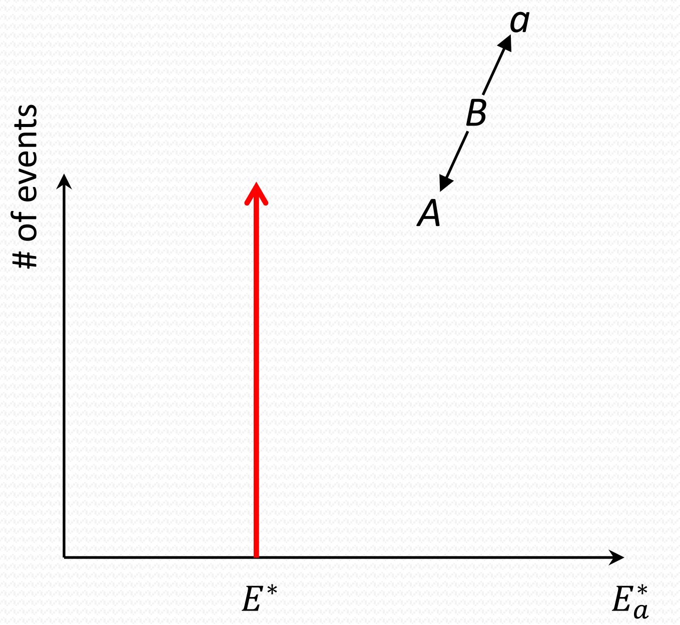
- ☐ Benefits of new ideas
  - ✓ Different methods have different sensitivity to systematics, i.e., complementarity
  - ✓ Good exercise/testbed for new physics signatures (e.g., pair-production, invisible decay products, multi-step decays, etc)
  - ✓ (Potentially) new handles in the search for new physics, e.g., b partner searches

Review: Energy-peak method

Energy Peak: 2-Body Decay Kinematics in the Rest Frame

For a simple 2-body decay of a heavy resonance *B* into *A* and ***massless*** visible *a*

Rest frame of particle *B*



- ☐ Energy of visible particle *a* is **monochromatic** and **simple** function of masses in the rest frame of particle *B*

$$E^* = \frac{m_B^2 - m_A^2}{2m_B}$$

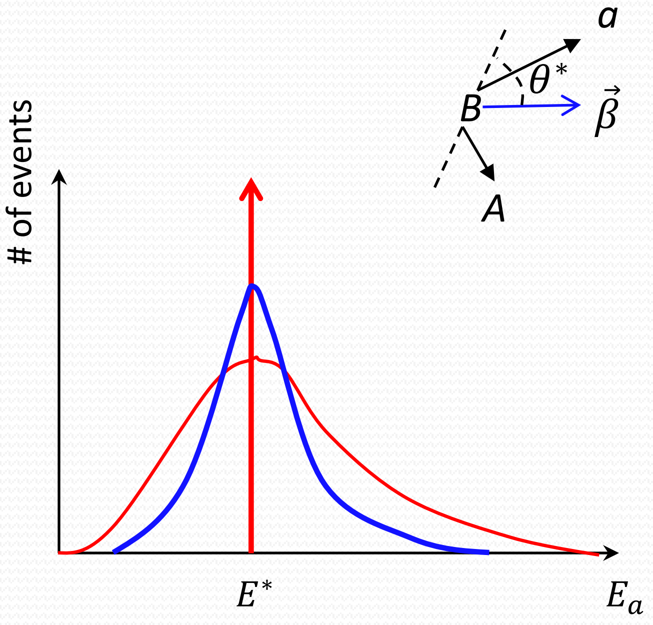
  - ✓ E^* : energy of visible particle measured in the rest frame of particle *B*

- ☐ E^* is measured, mass of *A* is known → **mass of *B* can be measured!** and vice versa
- ☐ Great to be on this special frame!

Energy Peak: 2-Body Decay Kinematics in the Lab Frame

Energy (not a Lorentz-invariant) of particle *a* should be Lorentz-transformed

Lab frame



- ☐ Depending on m_A and m_B *plus* ***unknown*** **boost factor** of particle B, β, and **emission angle** of particle *a* from the axis of $\vec{\beta}$

$$E = E^* \frac{1 + \beta \cos \theta^*}{\sqrt{1-\beta^2}}$$

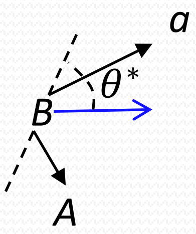
- ☐ No longer fixed energy of particle *a* in the lab frame, but a function of β, θ^* → becoming smeared due to variation in them → information **loss**?!

Peak of such an energy distribution
= rest-frame energy and "invariant"

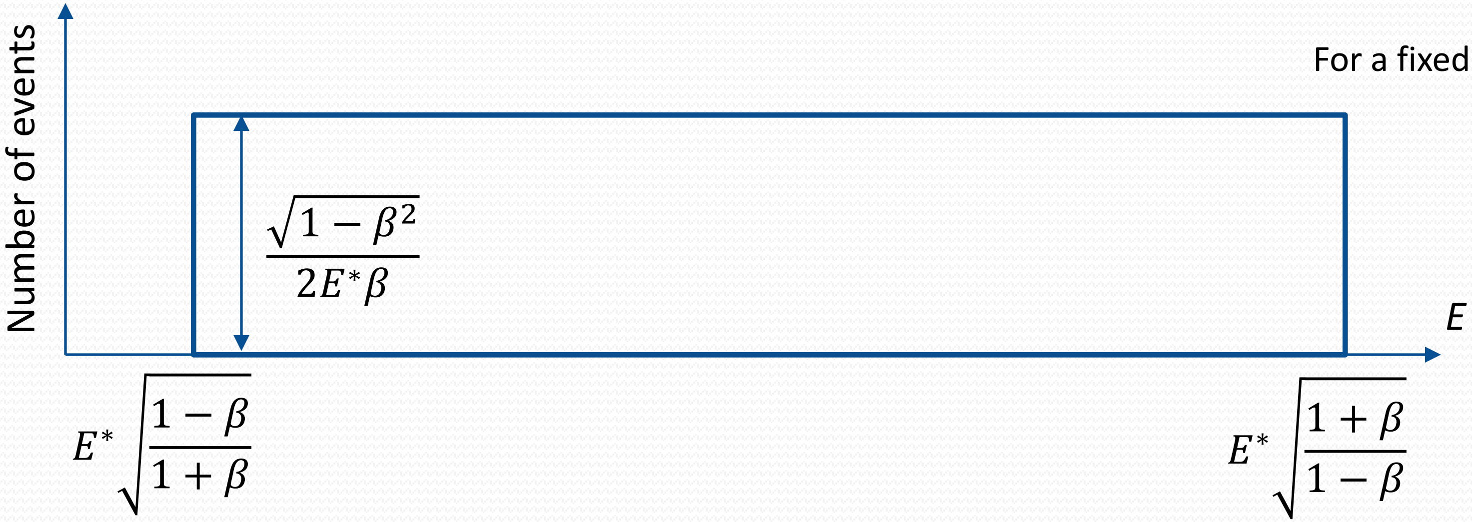
[Agashe, Franceschini, **DK**, PRD88 (2013) 057701]

Existence of the Energy Peak: Varying θ^* and Fixing β

- ☐ Lorentz transformation: $E = E^* \frac{1+\beta \cos\theta^*}{\sqrt{1-\beta^2}}$



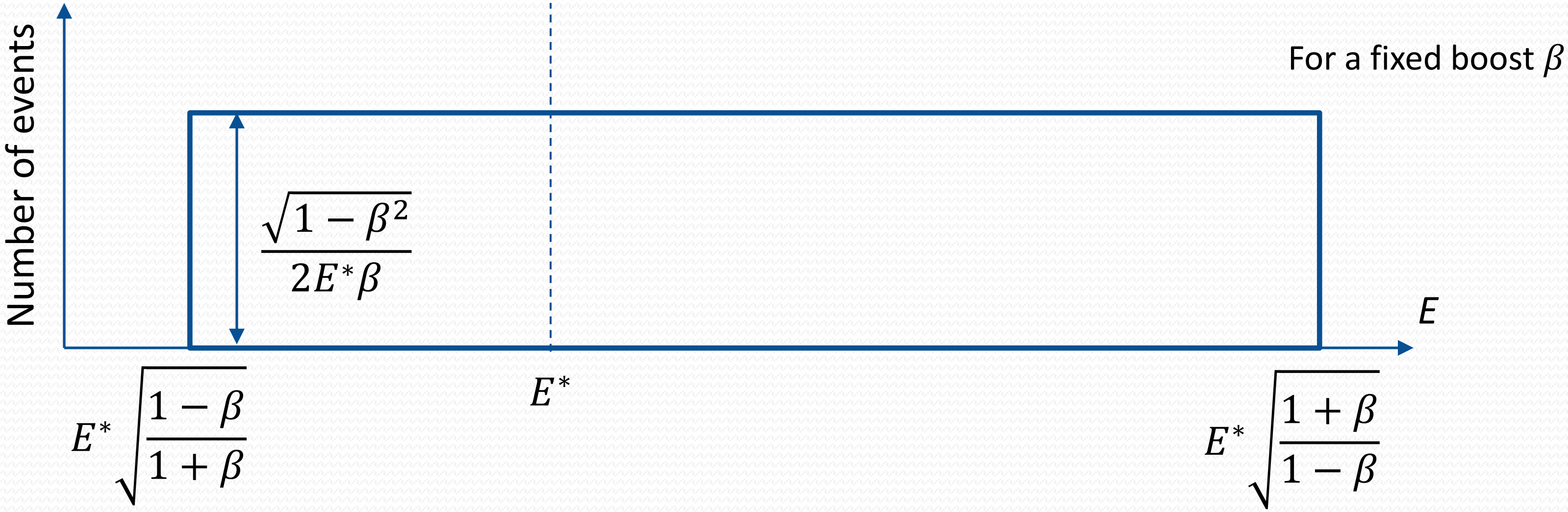
- ☐ Unpolarized/scalar parent particles
  - ✓ $\cos\theta^*$ becomes flat → *E* is also flat (simple chain rule)
  - ✓ Maximum (minimum) energy when particle a is emitted in the direction (anti-)parallel to the boost direction, i.e., $\cos\theta^* = 1(-1)$



Existence of the Energy Peak: Varying θ^* and Fixing β

- ☐ Lower bound (upper bound) smaller (bigger) than E^* (for **any** boost)

- ☐ **Asymmetric** on linear *E* and **symmetric** on logarithmic *E* (i.e., E^* is the geometric mean of the lower bound and the upper bound)



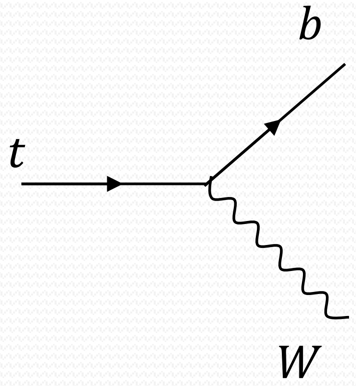
Existence of the Energy Peak: Varying θ^* and β

☐ Distribution in *E*: “Stacking up” rectangles weighted by all relevant boost factors

$$f(E) = \int_{\frac{1}{2}\left(\frac{E}{E^*}+\frac{E^*}{E}\right)}^{\infty} d\gamma \, \frac{g(\gamma)}{2E^*\sqrt{\gamma^2-1}}$$

E^* must be the unique peak which is **invariant irrespective of the top quark production details** that are encapsulated in the boost distribution!

Application: Top Quark Decay

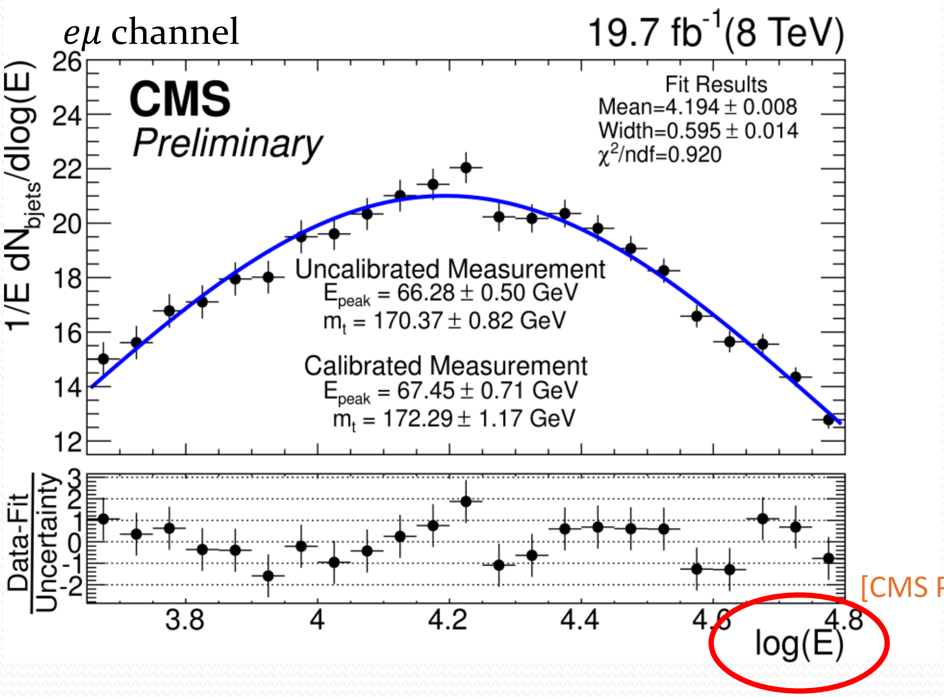


Using the peak in the energy distribution of b-quark-induced jets and the W mass from independent measurements, we can extract the top quark mass!

$$E_b^{\text{peak}} = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t} \cong \frac{m_t^2 - m_W^2}{2m_t}$$

The b quark mass is much smaller than the t and W masses, hence negligible.

Top Quark Mass Measurement of CMS



[CMS PAS TOP-15-002]

Energy spectrum should be symmetric w.r.t. E_b^* in log E: Gaussian fit near the peak region

$m_t = 172.29 \pm 1.17(\text{stat.}) \pm 2.66(\text{sys.})$ GeV ⇐ consistent with m_t from other methods

- b-jet energy peak at next-to-leading order [Agashe, Franceschini, **DK**, Schulze, EPJC76 636]
- B meson decay length method [Agashe, Airen, Francechini, Incandela, **DK**, Sathyan, 2212.03929]

Merits vs. Challenges

- ☐ Merits
  - ✓ (Quasi-)independent of top quark boost distribution or production details (only assumption: unpolarized production of top quarks) vs. Other methods assuming SM matrix elements.

    Prediction (m_t; theory) = data with theory = SM

    ⇒ Valid only if BSM "contamination" in top production is negligible.

  - ✓ Even with SM production only, the energy-peak method has reduced sensitivity to PDFs, high-order QCD effects (in production) [Agashe, Franceschini, **DK**, Schulze, EPJC76 636]

- ☐ Challenges
  - ✓ (b-induced)JES uncertainty

Energy-peak-based
B-hadron decay length method

Motivation of B-Hadron Decay Length Method

B-hadron decay length as "proxy" for

bottom quark energy instead of b-jet energy

to avoid the JES uncertainty

Main Idea

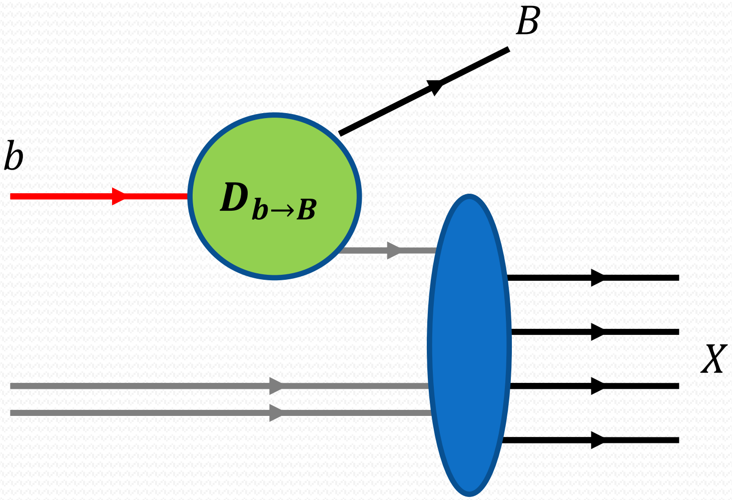
- Going from measured B-hadron decay length, L_B, to bottom quark energy E_b

1) $L_B \longrightarrow \tau_B^{\mathrm{lab}}$: Exponential decay law

2) τ_B^{lab} vs. $\tau_B^{\mathrm{rest}} \longrightarrow \gamma_B^{\mathrm{lab}}$ or E_B : B-hadron energy

3) $E_B \longrightarrow E_b$: Hadronization model

Hadronization



- Hadronization, $b \to b$ jet = B hadron + X

  : Fixed E_b still gives a distribution of E_B.
- Fragmentation function describes

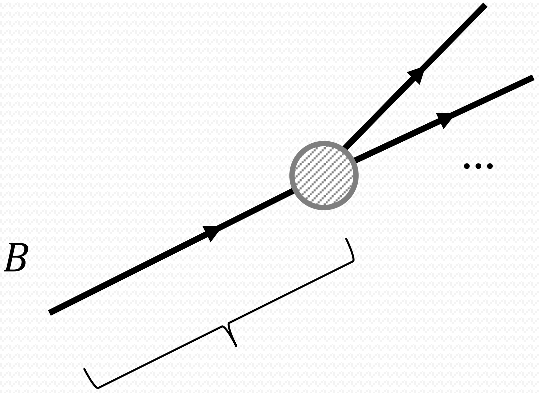
  probability density of $x = \frac{E_B}{E_b}$:

  $\int dx\, D(x;\ E_b) = 1$ for any (fixed) E_b

The probability density functions (pdf's) of E_B and E_b are related by

$$F(E_B) = \int dE_b\, f(E_b) D\left(\frac{E_B}{E_b};\ E_b\right)$$

From B-Hadron Energy to Mean Decay Lifetime/Length



λ_B: lab-frame mean decay length

$$\lambda_B = c\gamma_B\beta_B\tau_B^{\text{rest}} = c\,\frac{E_B}{m_B}\sqrt{1-\left(\frac{E_B}{m_B}\right)^2}\,\tau_B^{\text{rest}}$$

$$\approx c\,\frac{E_B}{m_B}\tau_B^{\text{rest}}$$

B hadrons are relativistic.

The pdf of the mean decay length is given by

$$g(\lambda_B) = \frac{F(E_B)}{\frac{d\lambda_B}{dE_B}} \approx F(E_B)\,\frac{m_B}{c\tau_B^{\text{rest}}}$$

Connection between Measured Decay Length and b-Quark Energy

$$
\begin{aligned}
G(L_B) &= \int d\lambda_B \, \frac{g(\lambda_B)}{\lambda_B} \exp\left(-\frac{L_B}{\lambda_B}\right) \\
&\approx \int dE_B \, \frac{F(E_B)}{E_B} \frac{m_B}{c\tau_B^{\mathrm{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\mathrm{rest}} E_B}\right) \\
&= \int dE_B dE_b f(E_b) D\left(\frac{E_B}{E_b};\; E_b\right) \frac{m_B}{E_B c\tau_B^{\mathrm{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\mathrm{rest}} E_B}\right)
\end{aligned}
$$

$G(L_B)$: pdf of B-hadron decay lengths

$f(E_b)$: pdf of b-quark energy which **contains m_t information**!

$D\left(\frac{E_B}{E_b};\; E_b\right)$: b-quark fragmentation function

τ_B^{rest}: mean decay lifetime of B-hadron in its rest frame

Earlier Implementations by CDF/CMS

- ☐ Earlier CDF/CMS implementations [Hill, Incandela, Lamb, PRD71 054029; CDF Collaboration, PRD75 071102; CMS Collaboration, CMS-PAS TOP-12-030] were SM-based.
  - Top quark boosts, hence pdf of E_b, i.e., $f(E_b)$ is computed using the SM matrix element with top quark mass as a parameter.
  - SM “fitting” function for the decay length distribution.

Fitting function

Top quark boosts from SM: model-dependence

$$G^{\text{fit,SM}}(L_B^{xy}; m_t) = \int dE_B dE_b f^{\text{SM}}(E_b) D\left(\frac{E_B}{E_b};\ E_b\right) \frac{m_B}{E_B c\tau_B^{\text{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

Transverse observable

(Unknown) parameter

New Ideas/Our Proposal

3D decay length to accommodate E_B correctly

$$G(L_B^{xyz}; m_t) = \int dE_B dE_b f(E_b) D\left(\frac{E_B}{E_b};\ E_b\right) \frac{m_B}{E_B c\tau_B^{\text{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

- We relate the B-hadron decay length distribution to m_t using the energy-peak observation (instead of SM production).

- We twice de-convolve the measured decay length distribution $G(L_B)$ to obtain the b-quark energy distribution $f(E_b)$ whose peak is a function of m_t.

Location of the $f(E_b)$ peak → $\frac{m_t^2 - m_W^2 + m_b^2}{2m_t}$

Fitting Function/pdf of b-Quark Energy

❑ General properties that a fitting function satisfies

- ✓ Even under $\frac{E}{E^*} \leftrightarrow \frac{E^*}{E}$
- ✓ Maximized at $E = E^*$
- ✓ Vanishing as $E \to 0, \infty$
- ✓ Converging to a δ-function in some limiting case

$$f(E) = \int_{\frac{1}{2}\left(\frac{E}{E^*}+\frac{E^*}{E}\right)}^{\infty} d\gamma \frac{g(\gamma)}{2E^*\sqrt{\gamma^2-1}}$$

❑ Our choice

$$f^{\text{fit,us}}(E_b) = \frac{1}{N(w)} \exp\left[-w\left(\frac{E_b}{E_b^*}+\frac{E_b^*}{E_b}\right)^{\nu}\right]$$

with $E_b^* = E_b^{\text{peak}}$

- $\nu = 1$ can allow for successful extraction of E_b^* [Agashe, Franceschini, DK, PRD88 (2013) 057701]
- CMS tested a variation in the log-E space [CMS PAS TOP-15-002]
- We choose $\boldsymbol{\nu = 0.3}$ to describe the tail part of the energy distribution more carefully.

Bottomline of Our Proposal

$$G^{\text{fit,us}}(L_B^{xyz}; E_b^{\text{peak}}, w) = \int dE_B dE_b \frac{1}{N(w)} \exp\left[-w\left(\frac{E_b}{E_b^{\text{peak}}} + \frac{E_b^{\text{peak}}}{E_b}\right)^{0.3}\right]$$

$$\times D\left(\frac{E_B}{E_b};\ E_b\right) \frac{m_B}{E_B c\tau_B^{\text{rest}}} \exp\left(-\frac{L_B m_B}{c\tau_B^{\text{rest}} E_B}\right)$$

$G^{\text{fit,us}}(L_B^{xyz}; E_b^{\text{peak}}, w)$: fitting function of measured B-hadron decay length distribution

Best-fit E_b^{peak}: used for $\boldsymbol{m_t}$ **determination**!

$D\left(\frac{E_B}{E_b};\ E_b\right)$: b-quark fragmentation function

τ_B^{rest}: mean decay lifetime of B-hadron in its rest frame

w: width of fitting function

$N(w)$: normalization factor

Simulation of Sample Data (Schematic)

- Finding/modeling pdf's
  - $f^{\text{fit,us}}(E_b)$ using MadGraph5@MC
  - $D\left(\frac{E_B}{E_b};\ E_b\right)$ using Pythia8 (as a shortcut and isolation of the uncertainty in D)

- Reweighting top quark p_T
  - One of the major systematics sources in m_t measurement using B-hadron decay lengths by CMS [CMS Collaboration, CMS-PAS-TOP-12-030]

new

$$\frac{\tilde{\omega}}{\omega} = 1 + \alpha(p_T^{\text{top}} - 200\ \text{GeV})\ \text{for}\ p_T^{\text{top}} < 400\ \text{GeV}$$

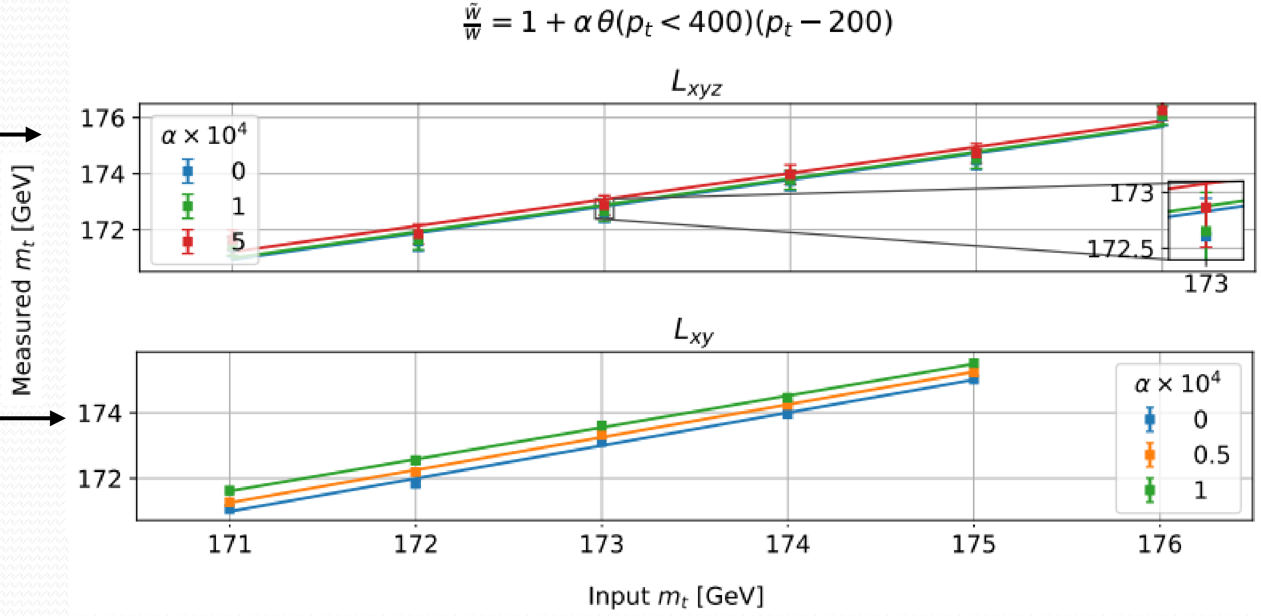
original

  - $f^{\text{fit,us}}\left(E_b;\ m_t^{\text{input}}\right) \to \tilde{f}^{\text{fit,us}}\left(E_b; m_t^{\text{input}}\right)$ due to reweighting

Results

Energy-peak-based method

CMS/SM Lxy method

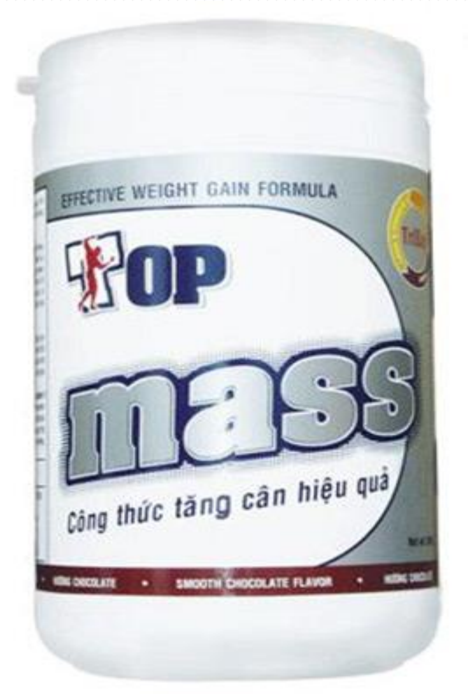


300/fb @LHC14TeV

- $\alpha = 10^{-4}$ (green lines) roughly corresponds to the theoretical uncertainty in top p_T spectrum (roughly moving the average top p_T by 0.5%) [CMS collaboration, PRD104 092013]. Lxy method shifts by ~ 600 MeV vs. Lxyz method by ~ 50 MeV. ⇒ Negligible error for the energy-peak-based method!!

Conclusions

- Review of the b-jet energy-peak method for m_t determination: (Quasi-)production model-independent (cf. others assume SM) but afflicted by the JES uncertainty.

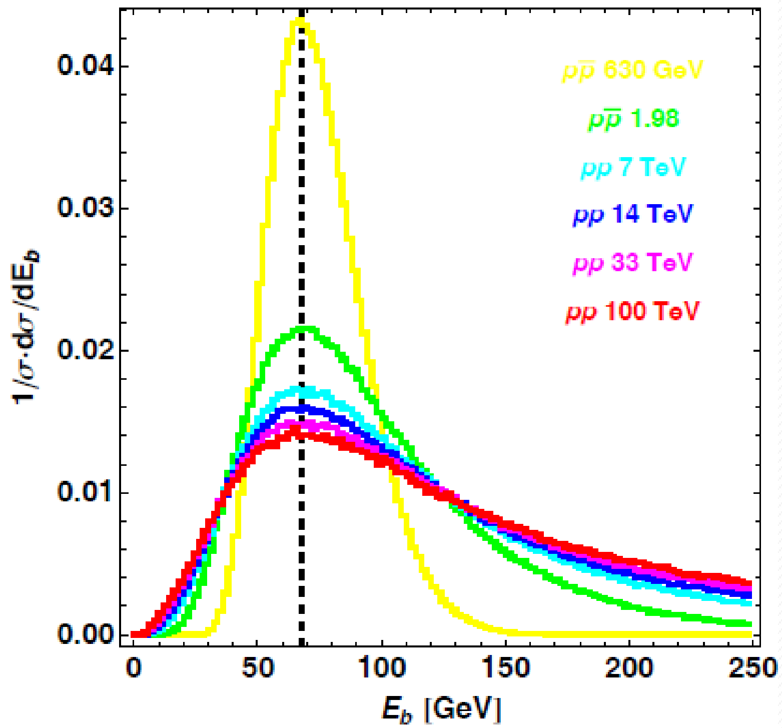


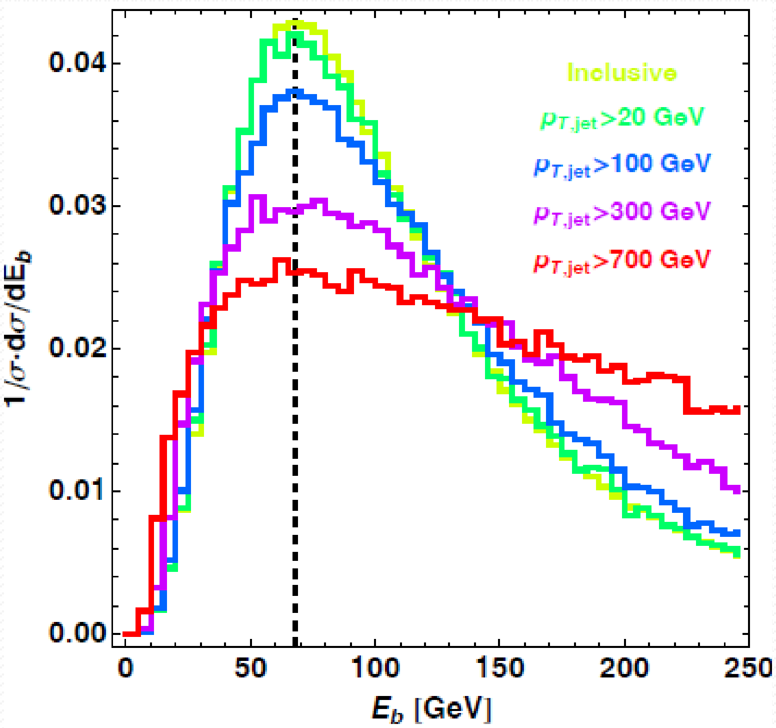
- We extend it to the B-hadron decay length method (correlated with bottom quark energy): circumventing the JES uncertainty, “replaced” by hadronization model/fragmentation function.
  - New systematics?: hadronization modeling (theoretical) and tracker resolution (experimental)

Thank you!

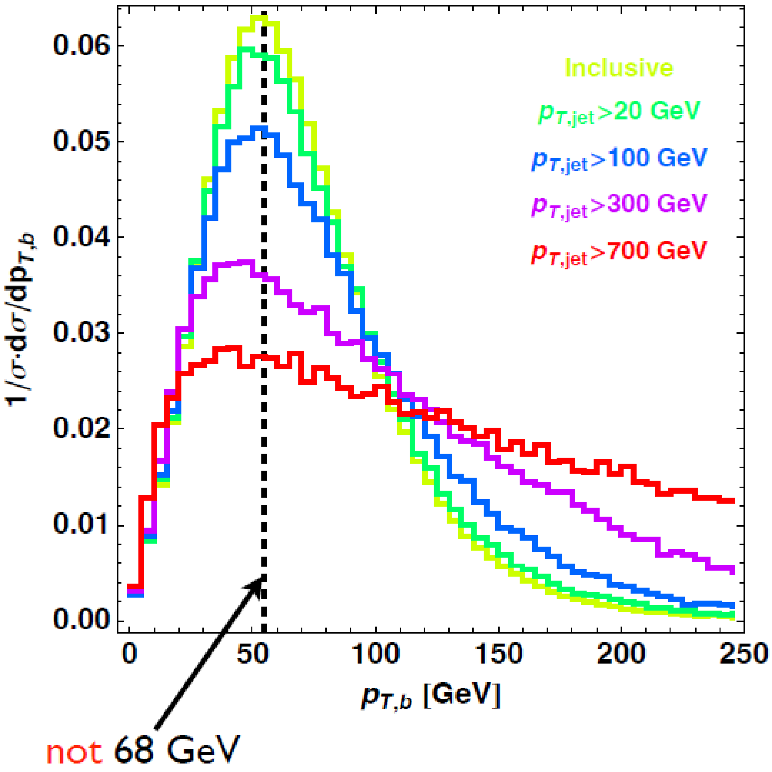
Back-up

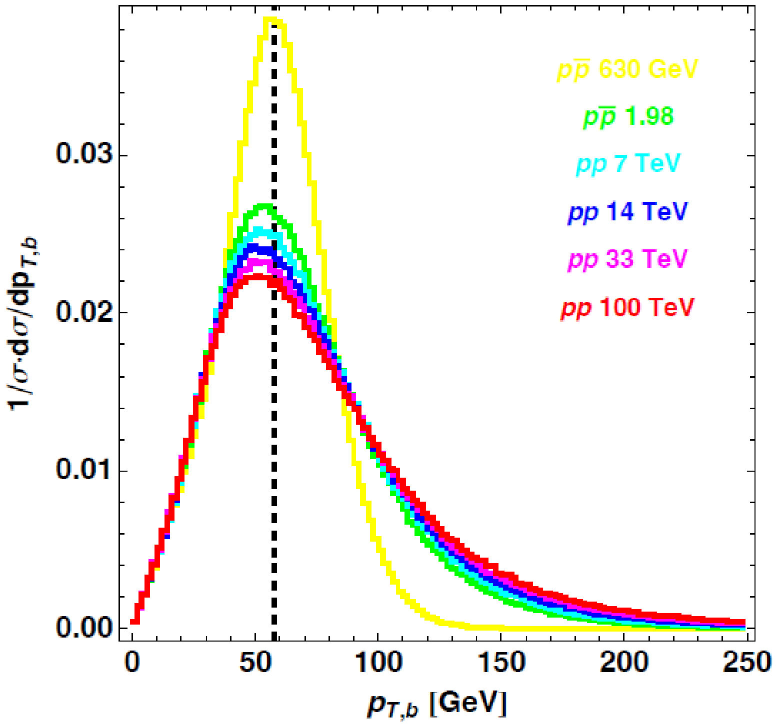
Invariance of the Energy Peak



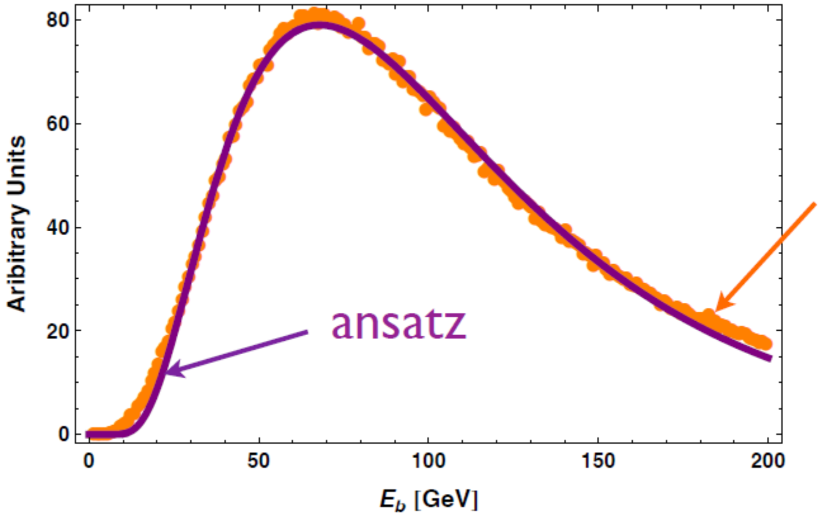


No Such Invariance for p_T





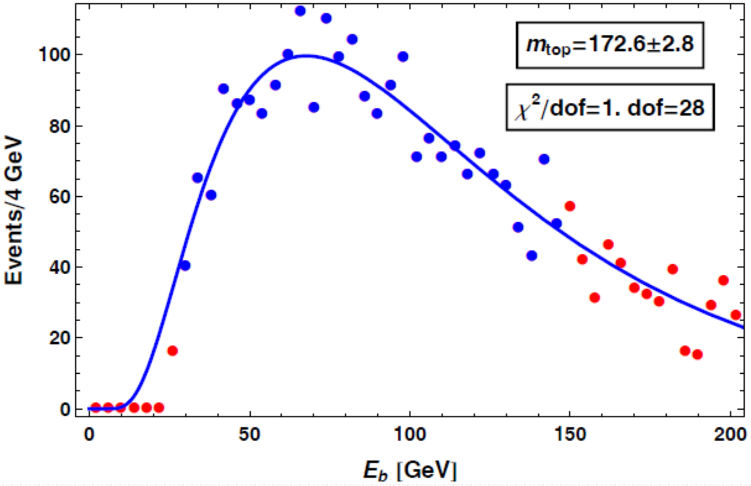
Performance of Our Fitting Function



ansatz

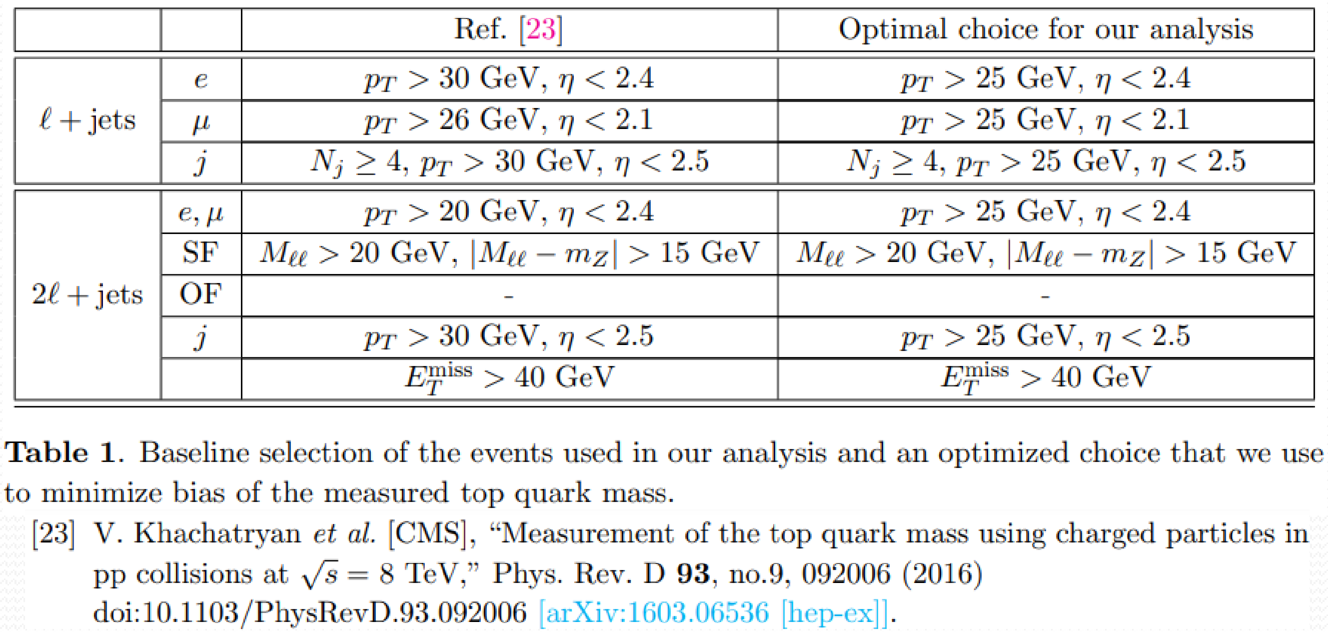
Madgraph (``infinite” statistics; no smearing)

(one pseudo-experiment shown)



(use only blue dots)

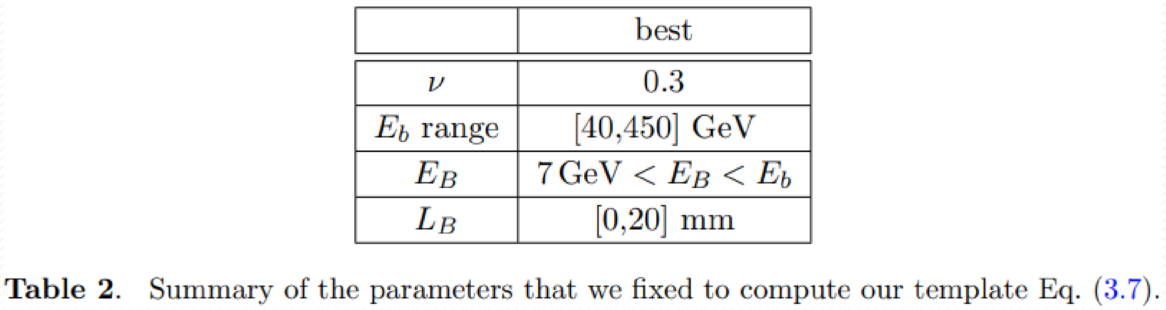
Selection Criteria and Parameter Choices



		Ref. [23]	Optimal choice for our analysis
$\ell+\text{jets}$	e	$p_T > 30$ GeV, $\eta < 2.4$	$p_T > 25$ GeV, $\eta < 2.4$
	μ	$p_T > 26$ GeV, $\eta < 2.1$	$p_T > 25$ GeV, $\eta < 2.1$
	j	$N_j \geq 4$, $p_T > 30$ GeV, $\eta < 2.5$	$N_j \geq 4$, $p_T > 25$ GeV, $\eta < 2.5$
$2\ell+\text{jets}$	e, μ	$p_T > 20$ GeV, $\eta < 2.4$	$p_T > 25$ GeV, $\eta < 2.4$
	SF	$M_{\ell\ell} > 20$ GeV, $\lvert M_{\ell\ell} - m_Z\rvert > 15$ GeV	$M_{\ell\ell} > 20$ GeV, $\lvert M_{\ell\ell} - m_Z\rvert > 15$ GeV
	OF	-	-
	j	$p_T > 30$ GeV, $\eta < 2.5$	$p_T > 25$ GeV, $\eta < 2.5$
		$E_T^{\text{miss}} > 40$ GeV	$E_T^{\text{miss}} > 40$ GeV

Table 1. Baseline selection of the events used in our analysis and an optimized choice that we use to minimize bias of the measured top quark mass.

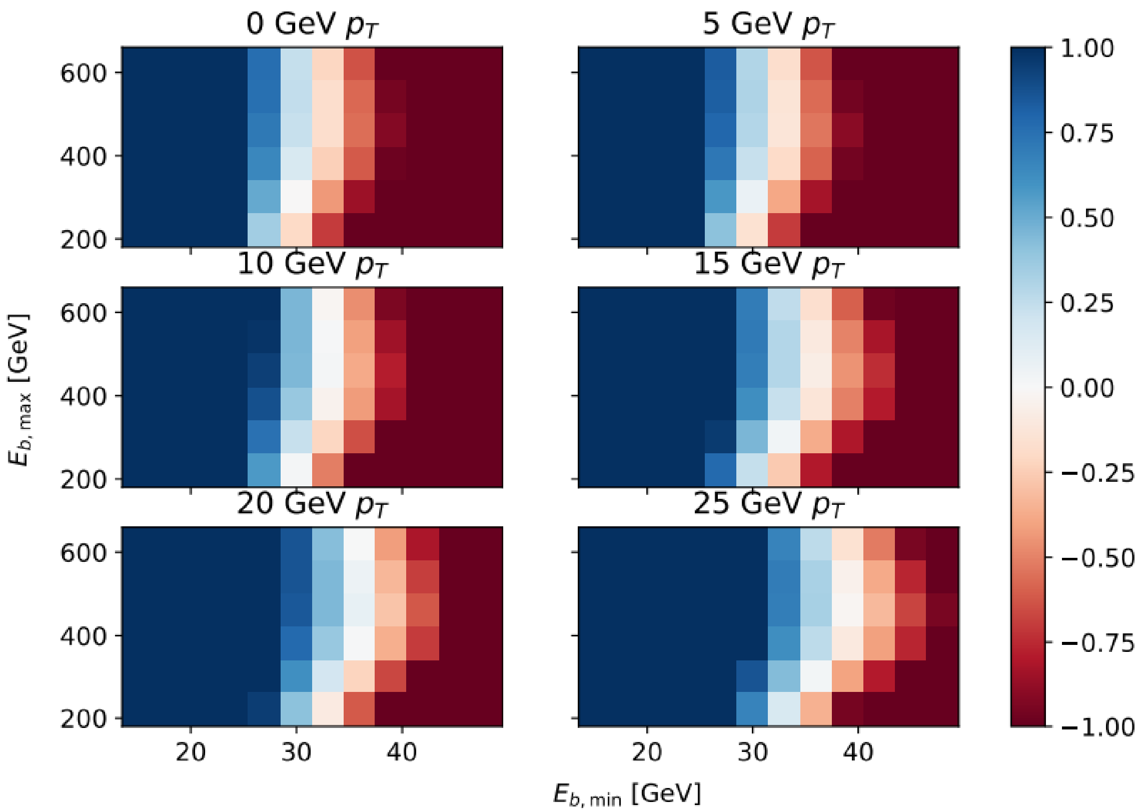
[23] V. Khachatryan *et al.* [CMS], "Measurement of the top quark mass using charged particles in pp collisions at $\sqrt{s} = 8$ TeV," Phys. Rev. D **93**, no.9, 092006 (2016) doi:10.1103/PhysRevD.93.092006 [arXiv:1603.06536 [hep-ex]].



	best
ν	0.3
E_b range	[40,450] GeV
E_B	$7\,\text{GeV} < E_B < E_b$
L_B	[0,20] mm

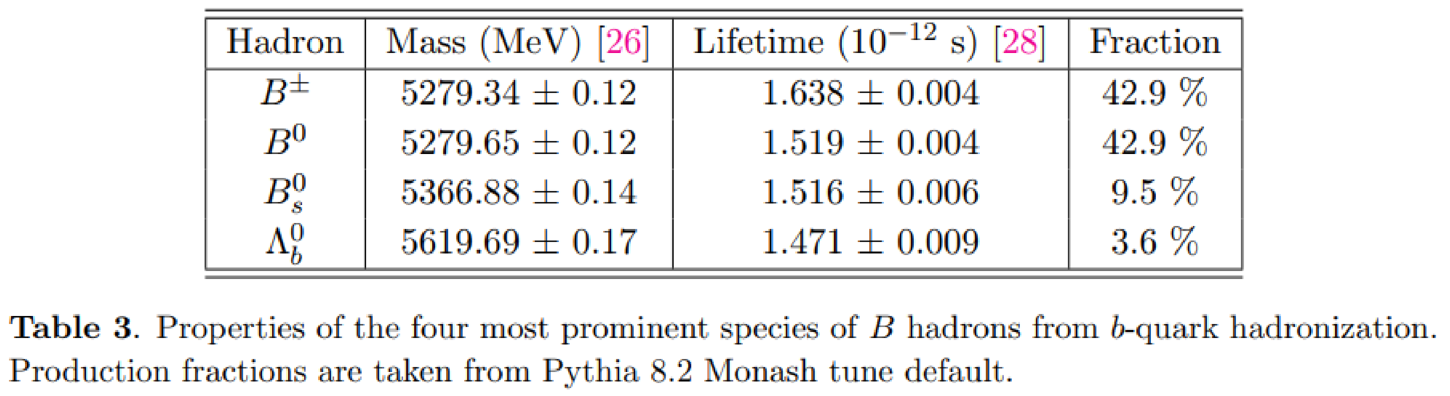
Table 2. Summary of the parameters that we fixed to compute our template Eq. (3.7).

b-quark Energy Spectrum Fit Range Dependence



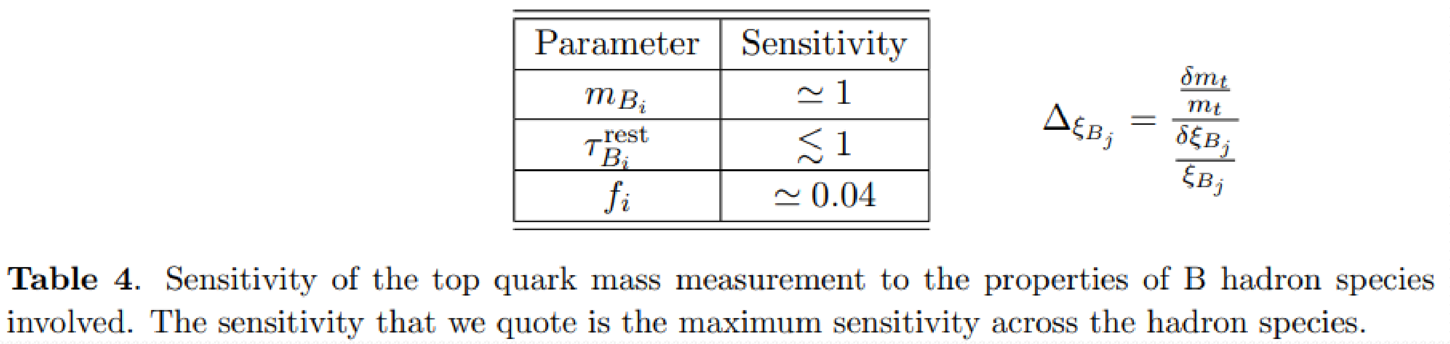
Bias (in GeV) as a function of the limits on the b-quark energy range in the E_b integral of Eq. (3.7). Subplots are titled by the common p_T cut on leptons and jets used for the selection of events.

Chosen B-Hadrons



Hadron	Mass (MeV) [26]	Lifetime (10^{-12} s) [28]	Fraction
$B^{\pm}$	5279.34 ± 0.12	1.638 ± 0.004	42.9 %
B^0	5279.65 ± 0.12	1.519 ± 0.004	42.9 %
B_s^0	5366.88 ± 0.14	1.516 ± 0.006	9.5 %
Λ_b^0	5619.69 ± 0.17	1.471 ± 0.009	3.6 %

Table 3. Properties of the four most prominent species of B hadrons from b-quark hadronization. Production fractions are taken from Pythia 8.2 Monash tune default.



Parameter	Sensitivity
m_{B_i}	$\simeq 1$
$\tau_{B_i}^{\text{rest}}$	$\lesssim 1$
f_i	$\simeq 0.04$

$$\Delta_{\xi_{B_j}} = \frac{\frac{\delta m_t}{m_t}}{\frac{\delta \xi_{B_j}}{\xi_{B_j}}}$$

Table 4. Sensitivity of the top quark mass measurement to the properties of B hadron species involved. The sensitivity that we quote is the maximum sensitivity across the hadron species.

Fragmentation Function Modeling Dependence

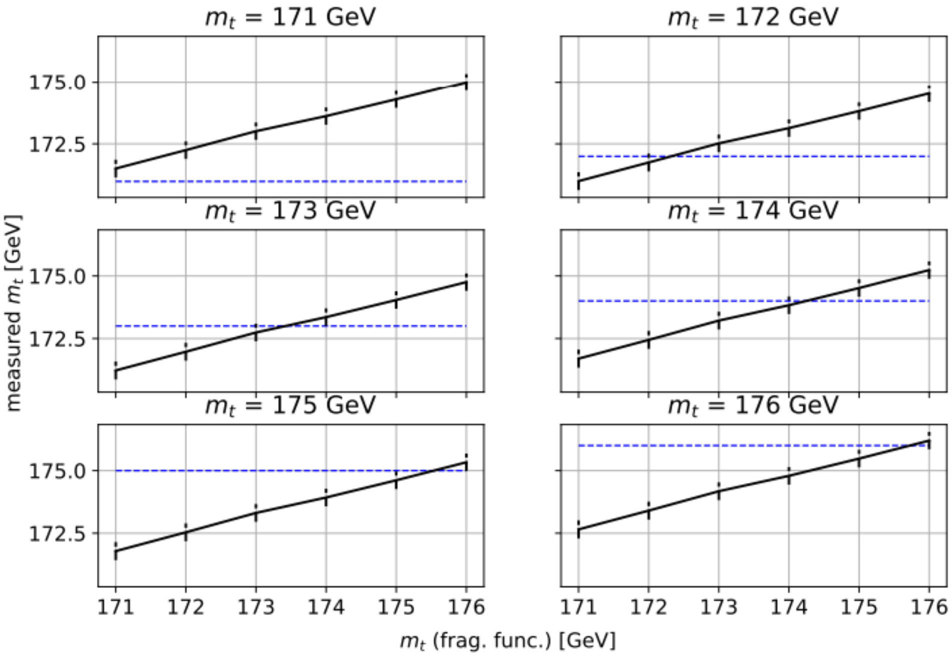


Figure 8. Effect on the extracted m_t from the change of the fragmentation function as parameterized by changing m_t in the data used to the MC truth on which the fragmentation functions is measured. The m_t used to measure the fragmentation is on the horizontal axis; the measurement is shown as a black line for each subplot corresponding to a correct m_t used to generate data. The blue line is shown as a reference, as it corresponds to an unbiased measurement.