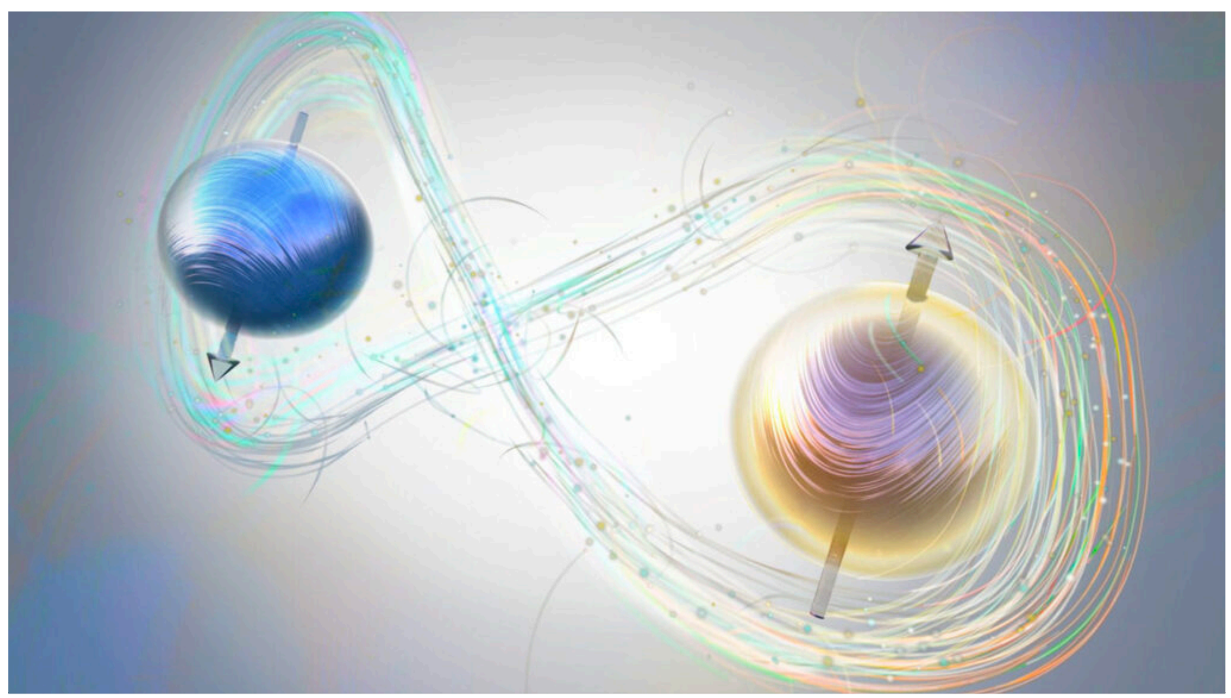
Entanglement and Bell's Inequalities with Boosted Top Quarks

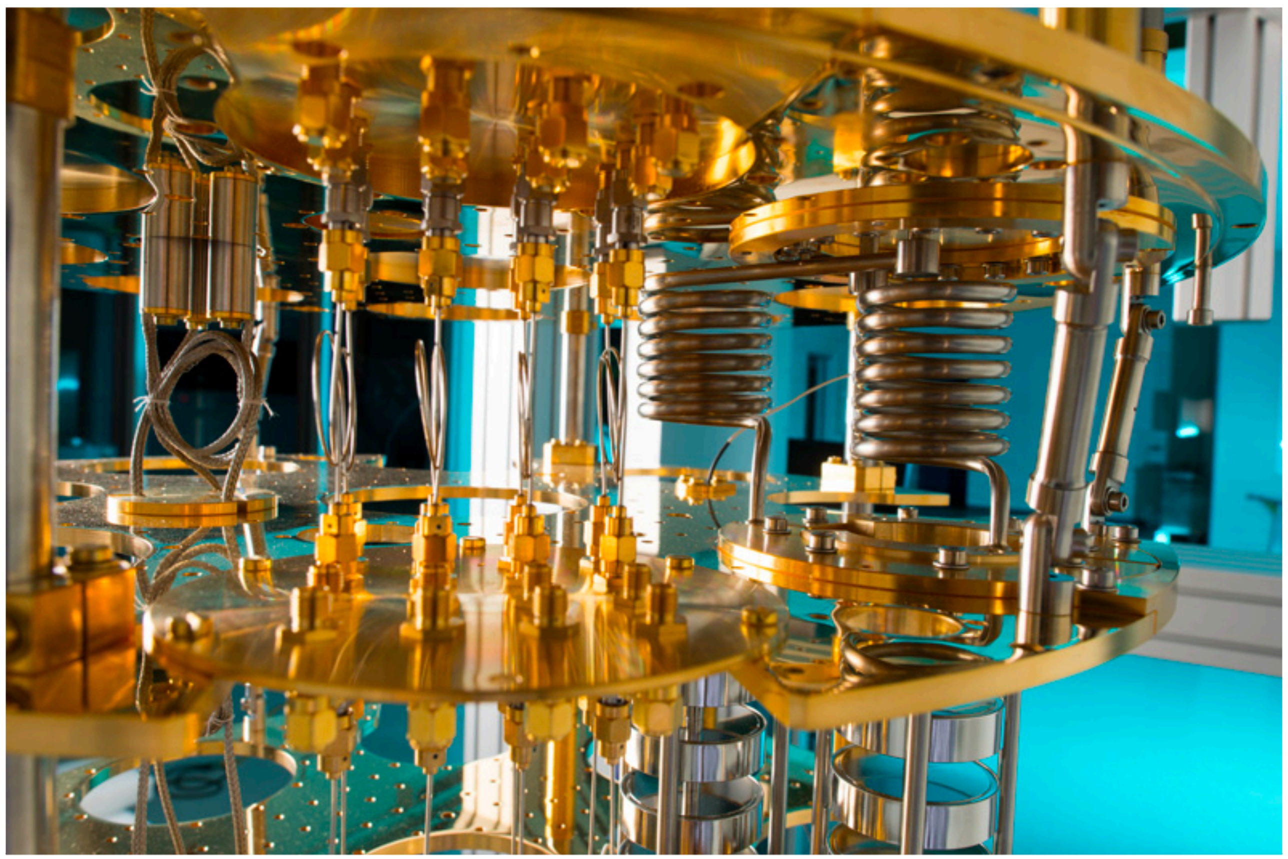
Dorival Gonçalves

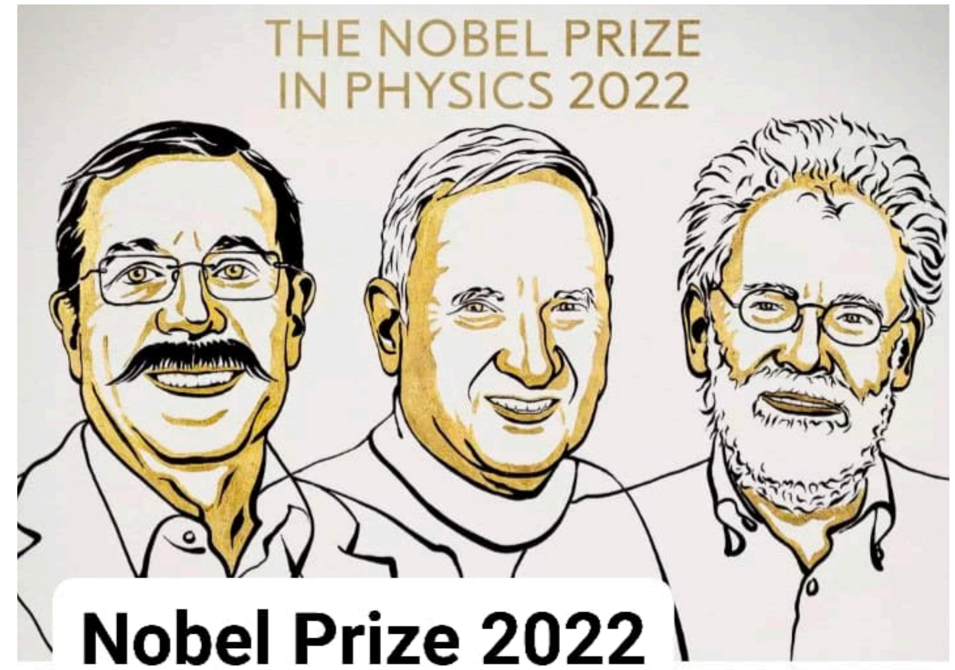
arXiv:2305.07075: Z. Dong, DG, KC Kong, A. Navarro



Top 2023 workshop - Sep 28, 2023

Motivation





Nobel Prize 2022

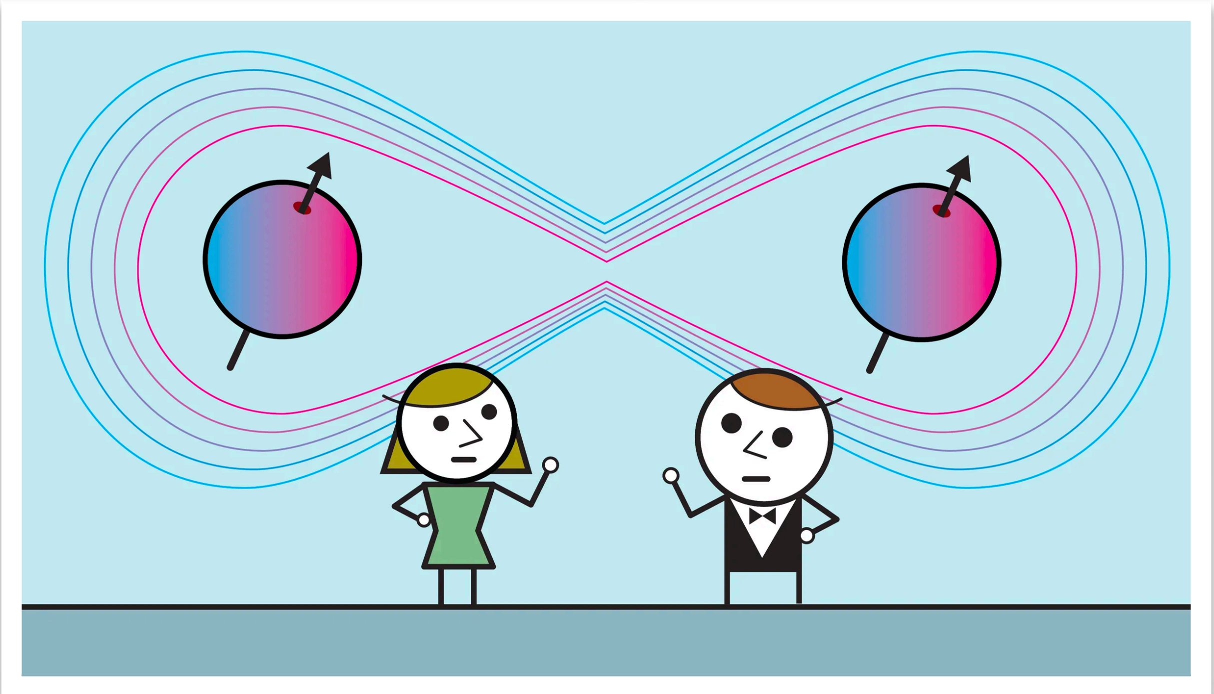
Nobel Prize in Physics awarded to Aspect, Clauser and Zeilinger for work in quantum mechanics

The Royal Swedish Academy of Sciences has decided to award the 2022 Nobel Prize in Physics to Alain Aspect, John F Clauser and Anton Zeilinger, according to an official tweet. They have been awarded the Nobel Prize for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science.

- QI theory provides a set of tools designed to probe the inner behavior of QM. While these phenomena have been widely tested at low energies, their study at higher energy scales has not be undertaken

Motivation

- LHC can provide a unique environment to study entanglement and violation of Bell's inequalities at the highest energy available today



- Top quark pair production is an optimal candidate for these studies

  See as well talks by Claudio Severi, Kun Cheng, Oliver Baker, Giulia Negro (CMS), Yoav Afik (ATLAS)

Top quark as a one qubit system

- As a spin-1/2 particle, the most general spin density matrix for the top quark is

$$\rho = \frac{\mathbb{I} + B_i \sigma_i}{2}$$

- Characterized by three parameters: $B_i = \langle \sigma_i \rangle = tr(\sigma_i \rho)$

→ However, the top quark decays. In general, spin information could be lost by hadronization or spin decorrelation effects

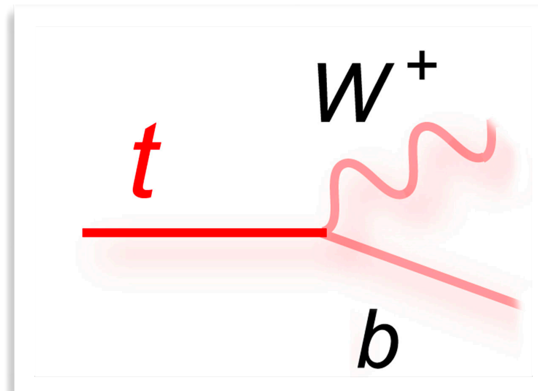
Top Quark is Special

- Decays before it hadronizes or its spin flips

$$\tau_{top} \approx 5 \times 10^{-25} s$$

$$\tau_{had} \approx 2 \times 10^{-24} s$$

$$\tau_{flip} \approx 10^{-21} s$$



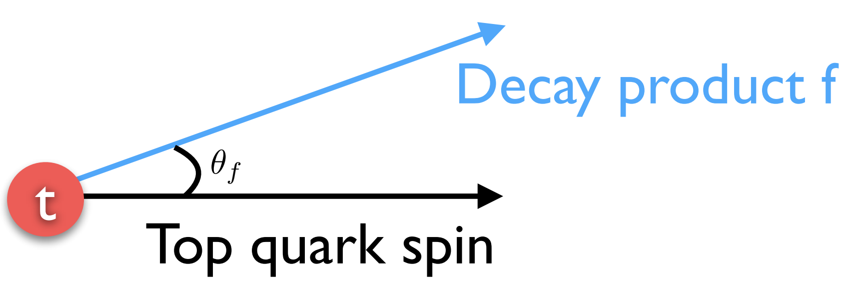
Bottom quark is several orders of magnitude behind

- Top polarization directly observable via angular distributions of its decay products

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$$

	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	1	-0.4	-0.3

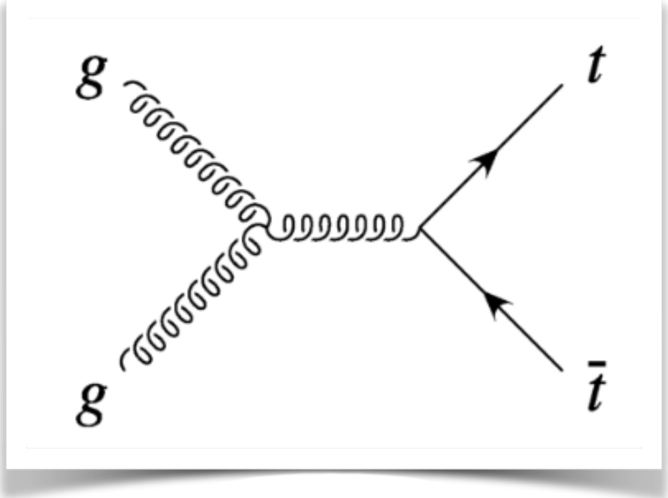
Spin analyzing power: maximum for charged leptons and down-type quarks



Top quark pair production as a two qubit system

- The most general two-qubit system can be represented by

$$\rho = \frac{\mathbb{I}\otimes\mathbb{I} + \left(B_i \sigma_i \otimes \mathbb{I} + \bar{B}_i \mathbb{I}\otimes\sigma_i\right) + C_{ij}\sigma_i\otimes\sigma_j}{4}$$



- Characterized by 15 parameters parameters: B_i, $\bar{B}_i$, and C_{ij}

$B_i = \langle \sigma_i \otimes \mathbb{I} \rangle$ → Polarizations

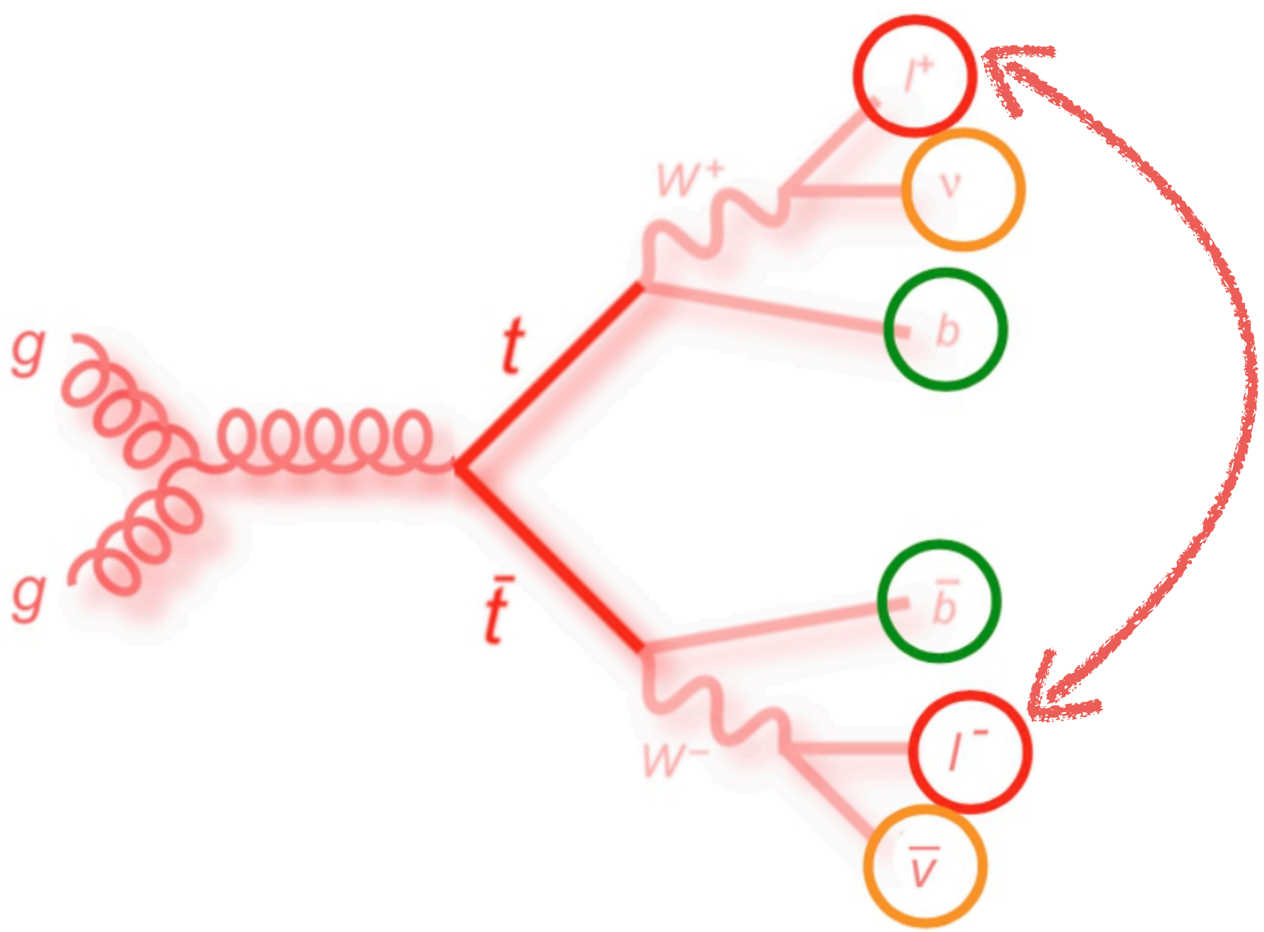
$\bar{B}_i = \langle \mathbb{I} \otimes \sigma_i \rangle$ → Polarizations

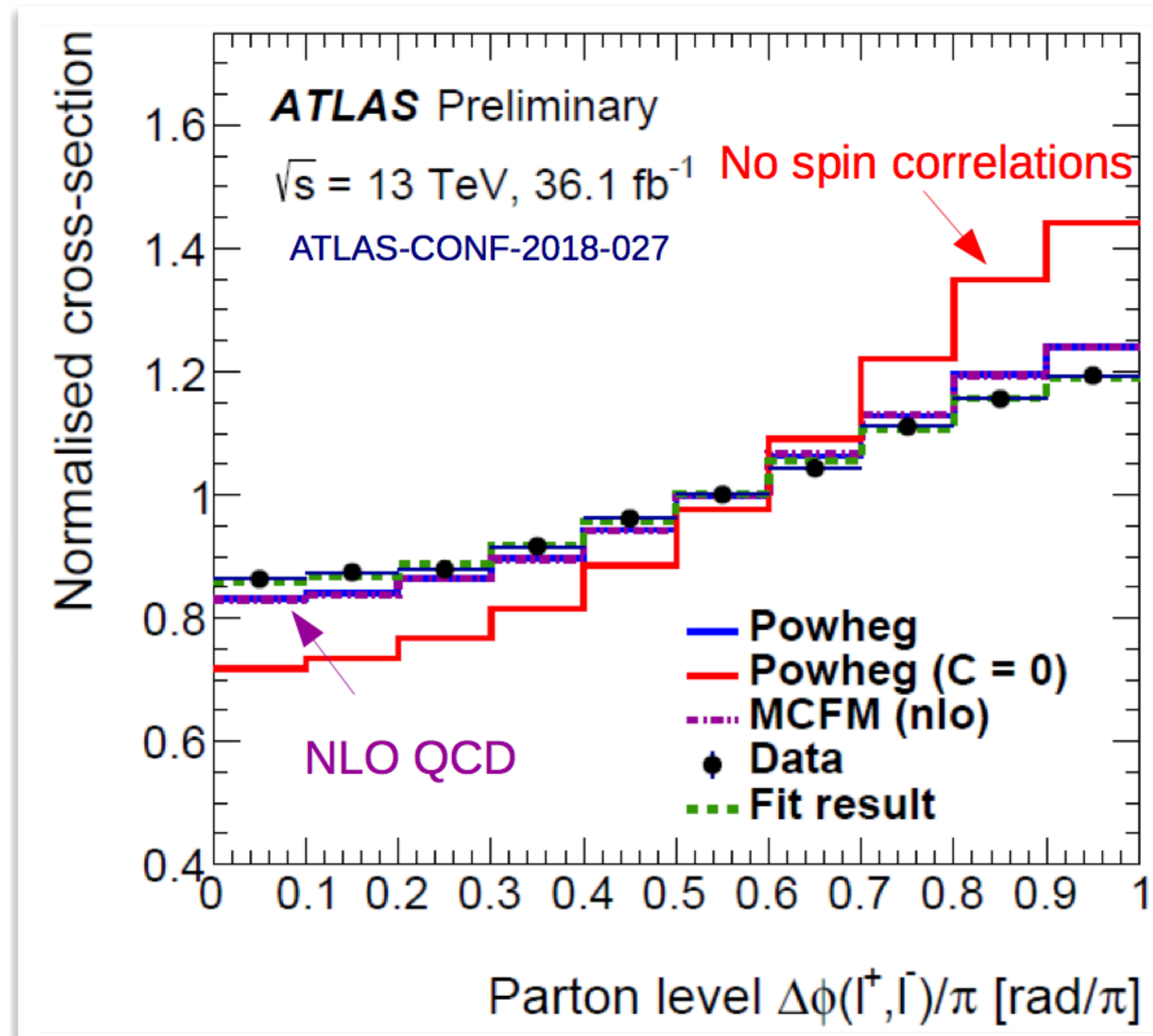
$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$ → Spin correlations

- P and CP invariance under $t\bar{t}$ production $\rightarrow B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$
- Things further simplify in the helicity basis: only non-vanishing parameters are the diagonal terms C_{ii} and one off-diagonal term $C_{12} \simeq C_{21}$ Bernreuther, Heisler, Si '15 Frederix, Tsinikos, Vitos '21

Top quark pair production as a two qubit system

- We are probing the spin correlations of top and anti-top since the Tevatron era and continue to study them at the LHC in different forms: relevant precision observable





Parke, Mahlon ’95

Top quark pair production as a two qubit system

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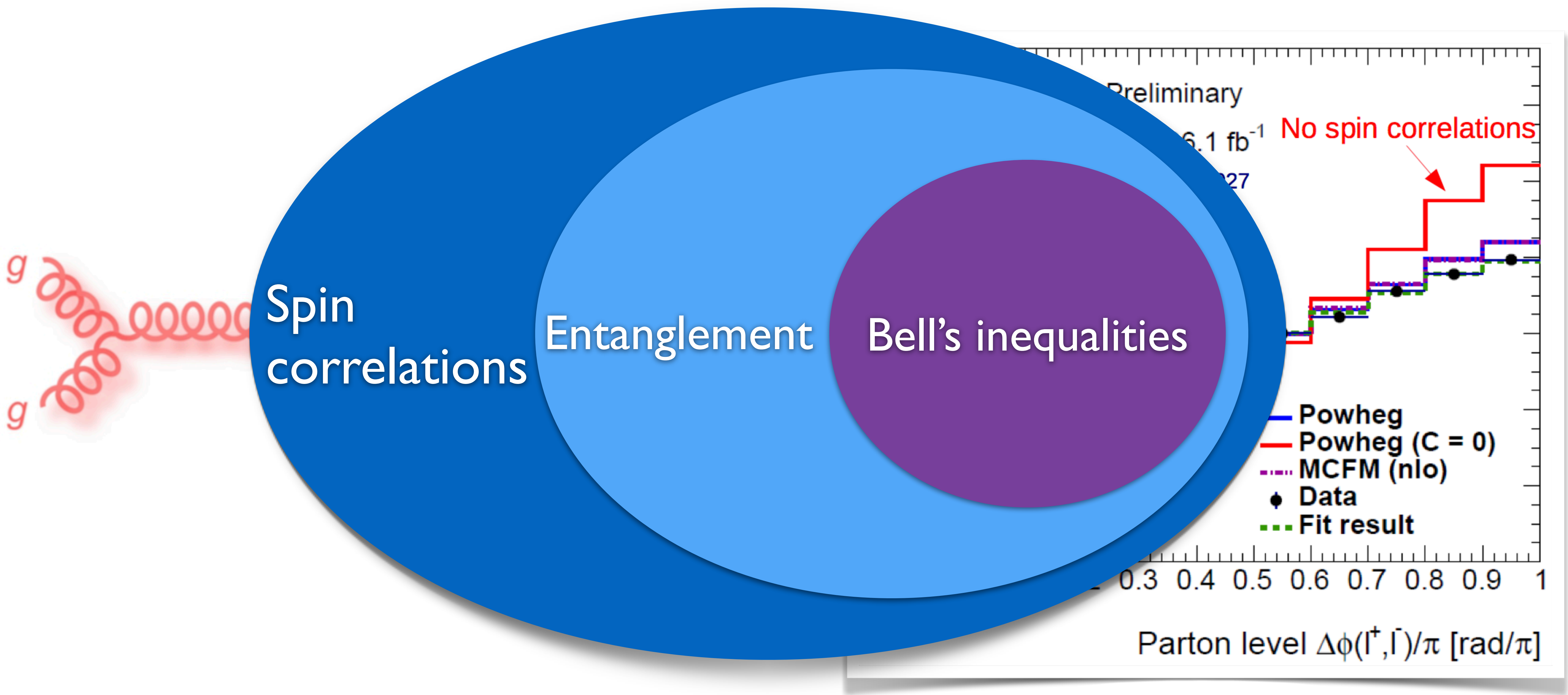


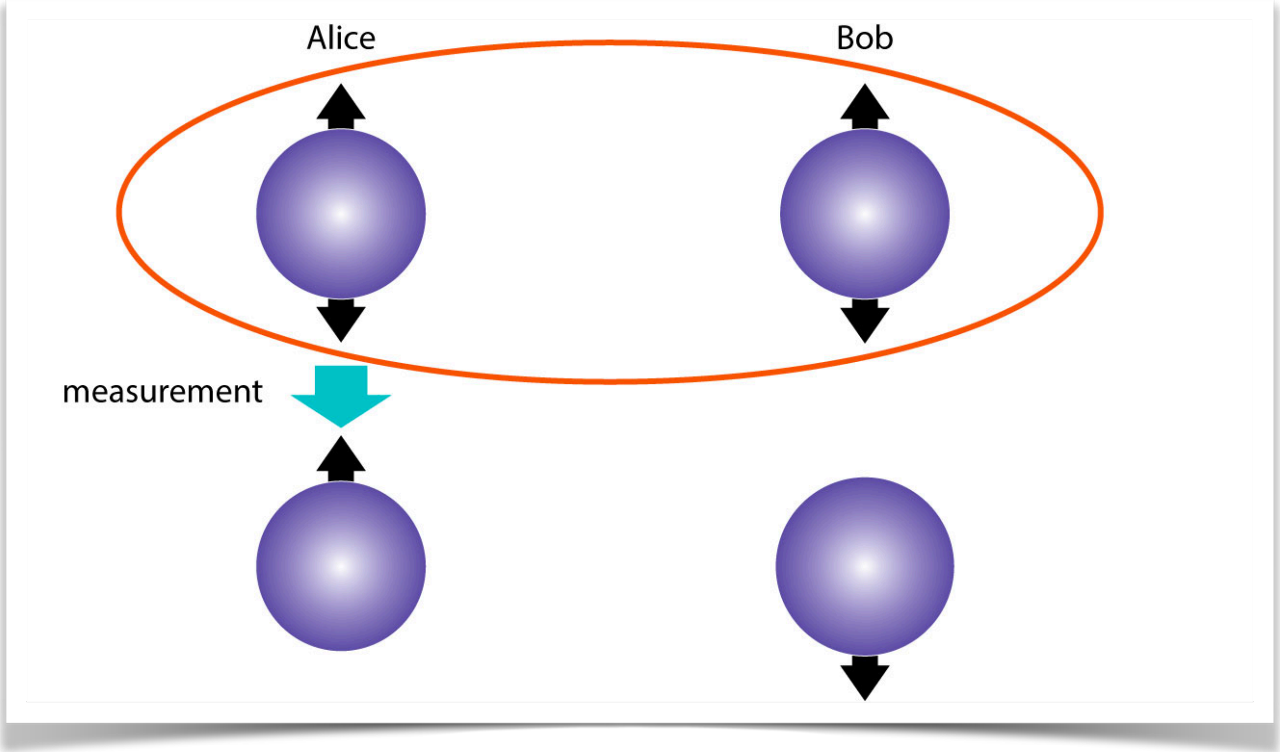
Parke, Mahlon '95

Quantum Entanglement

- A quantum state of two subsystems A and B is separable when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

→ If the state is not separable, it is named *entangled*



Measurement in one subsystem immediately affect the other, even if they are causally disconnected

Quantum Entanglement

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- The Peres-Horodecki criterion provides a necessary and sufficient condition for entanglement in two-qubit systems:

Peres '96; Horodecki '97

Take the transpose of indices associated only to Bob (or Alice)

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} results in a non-negative operator

→ If ρ^{T2} displays at least one negative eigenvalue, the system is entangled

Quantum Entanglement

- Since for the $t\bar{t}$ system $B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$ (P and CP) and $C_{13} \simeq C_{23} \simeq 0$ (helicity basis), we obtain some simple sufficient conditions for entanglement

Afik, Nova '20
Severi, Boschi, Maltoni, Sioli '21
Saavedra, Casas '22

$$|C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

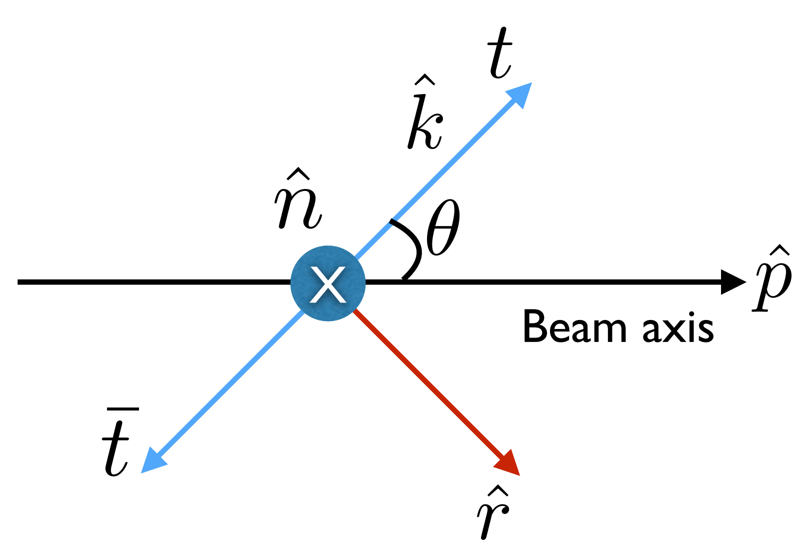
$$|C_{kk} - C_{rr}| + C_{nn} - 1 > 0$$

- Helicity basis

$\hat{k} =$ top quark direction

$\hat{r} = \text{sign}(\cos\theta)(\hat{p} - \cos\theta\hat{k})/\sin\theta$

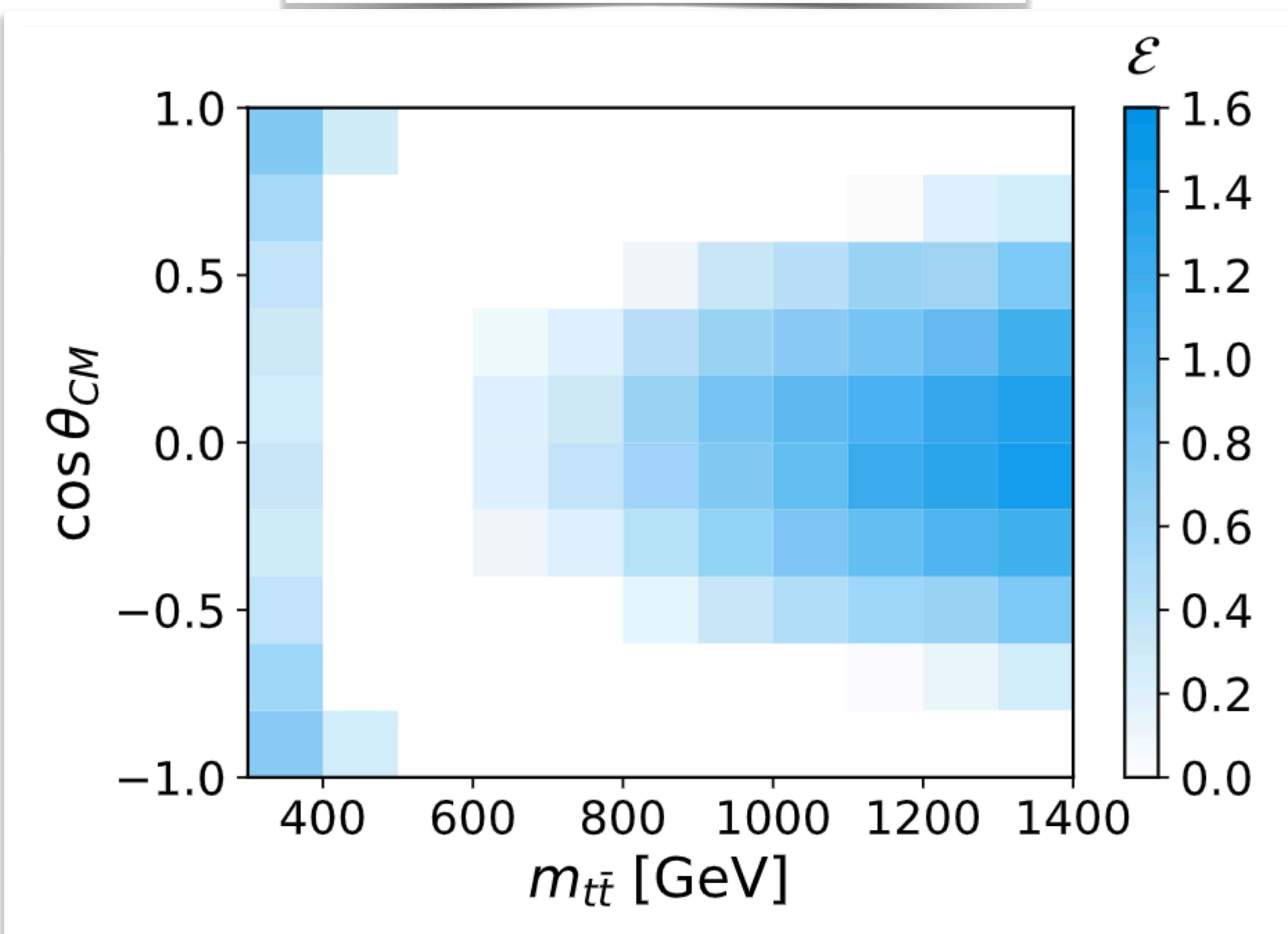
$\hat{n} = \hat{k} \times \hat{r}$



$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_a^i \, d\cos\bar{\theta}_b^j} = \frac{1}{4}\left(1 + \beta_a\beta_b C_{ij}\cos\theta_a^i\cos\bar{\theta}_b^j\right)$$

Quantum Entanglement

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$



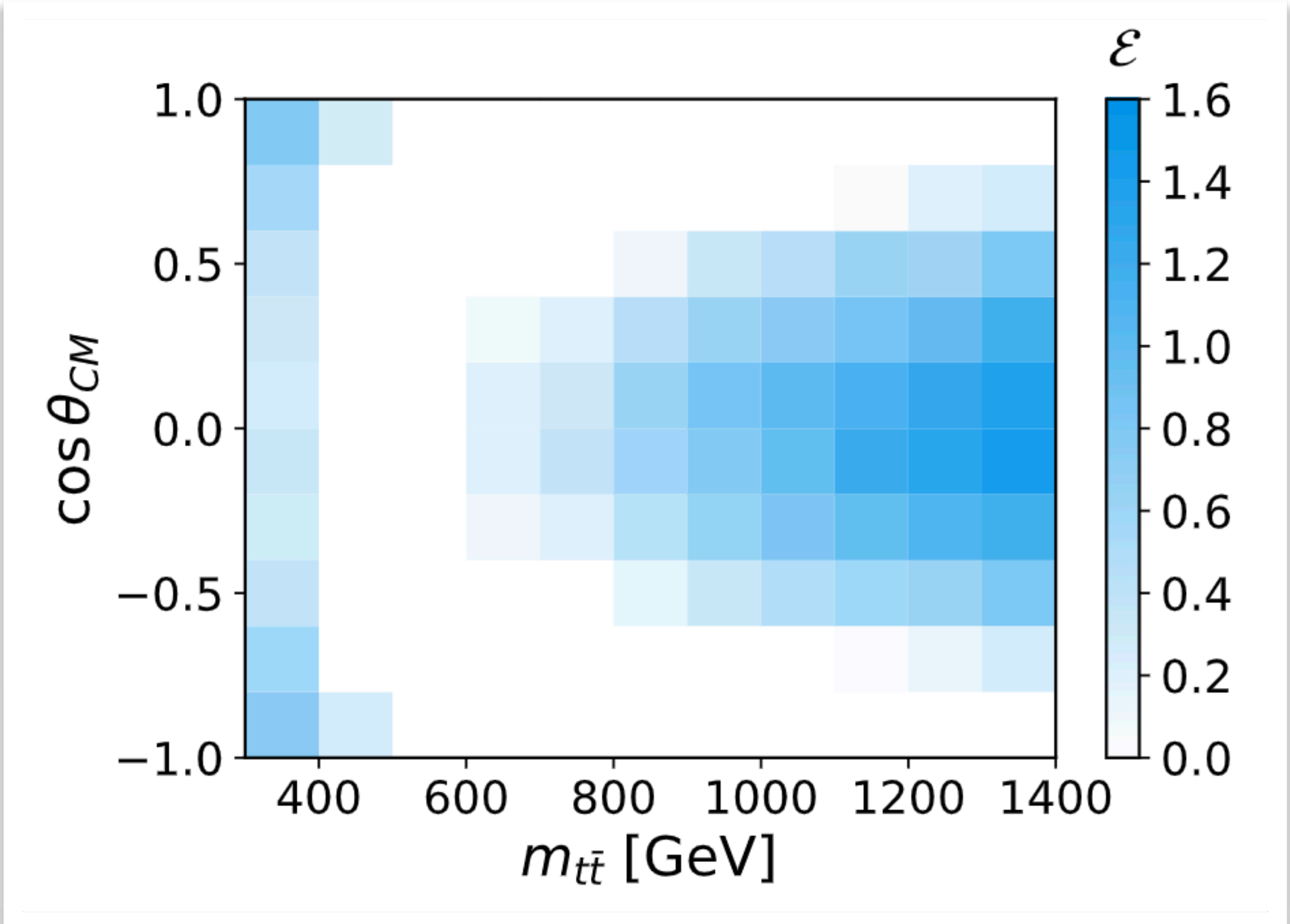
Dong, DG, Kong, Navarro '23

➡ Top pair displays entanglement in two disconnected regimes: **threshold** and **boosted** regions

Three reasons to study the boosted semileptonic tt final state

- i) Semi-leptonic top pair has an event rate higher than that of the dileptonic process, making it a more effective probe for the high-energy regime

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

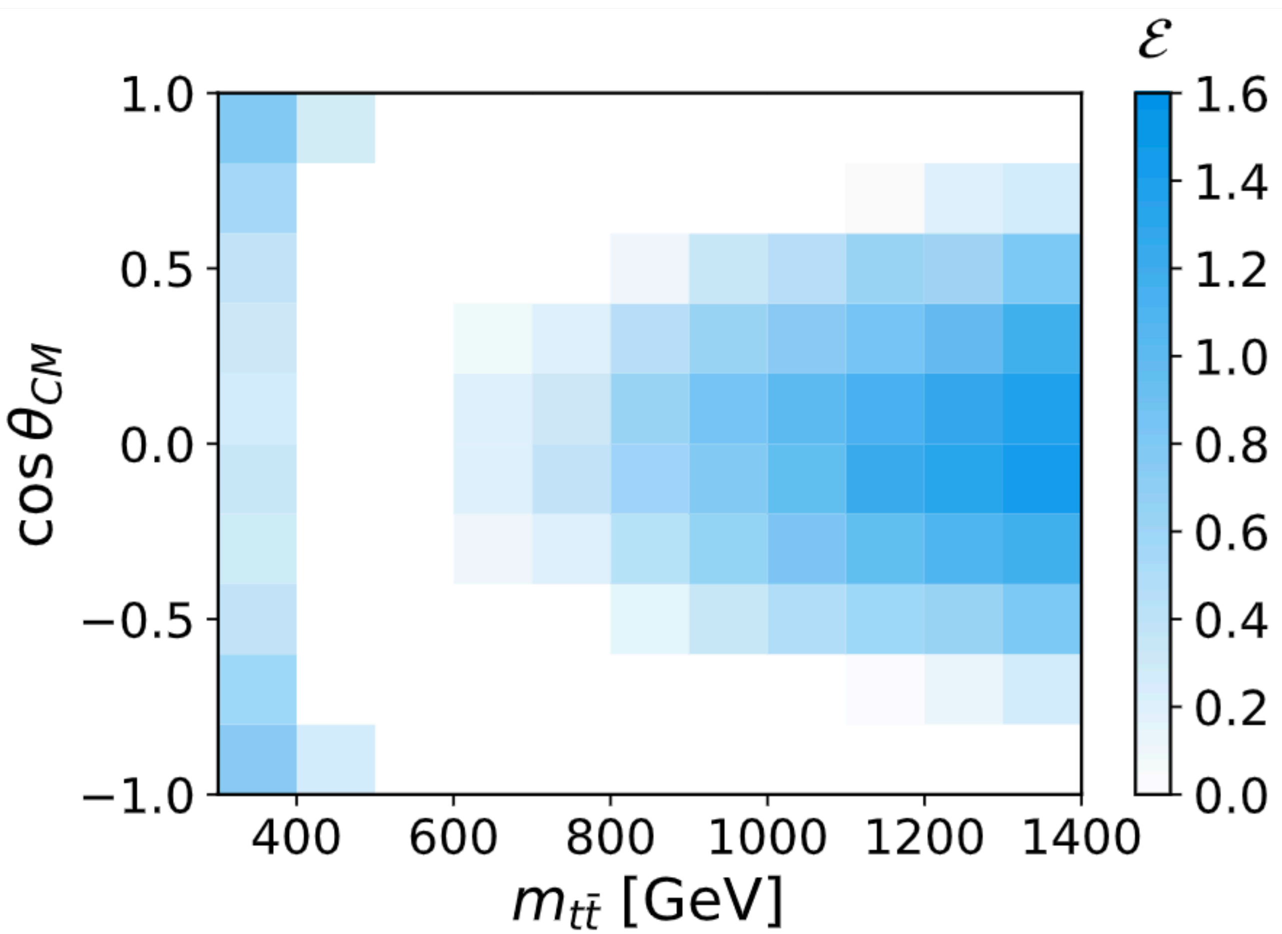


Dong, DG, Kong, Navarro ‘23

Three reasons to study the boosted semileptonic tt final state

- ii) Boosted top quark regime nicely matches with both entanglement and CHSH probes. High transverse momentum results in large top tagging efficiency and small fake rate for top tagging (e.g., HEPTopTagger)

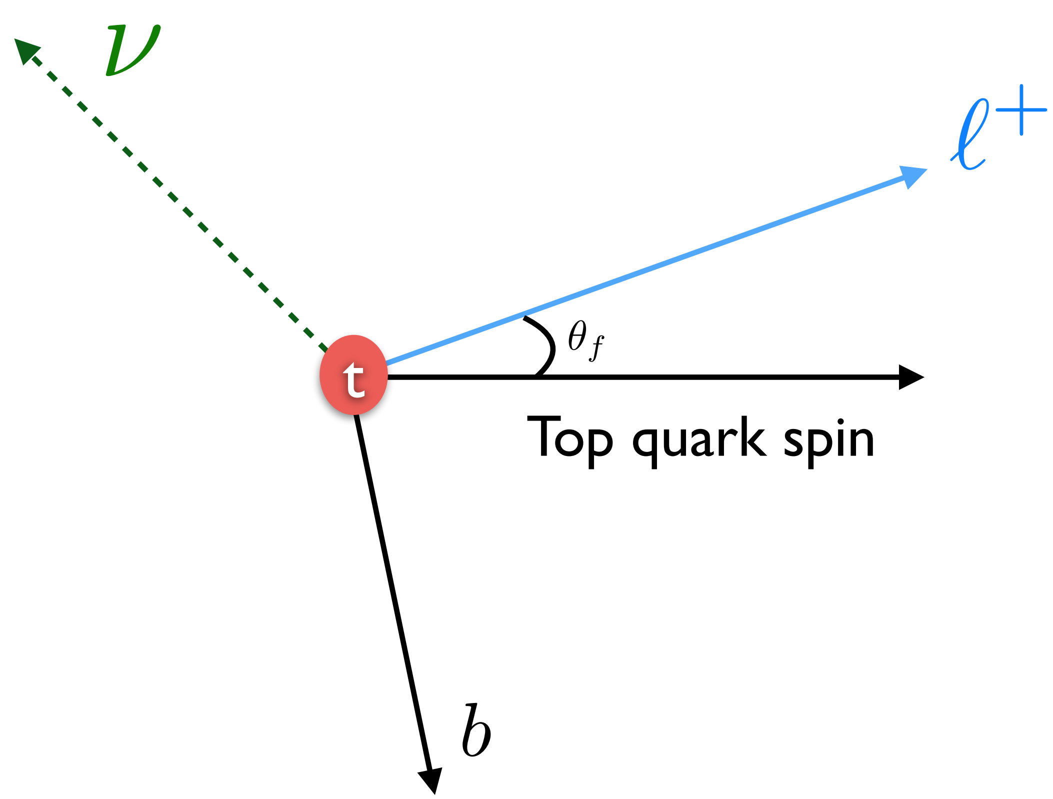
$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$



Dong, DG, Kong, Navarro '23

Three reasons to study the boosted semileptonic tt final state

- iii) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction



$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{1.0}\ \cos\theta_f)$$

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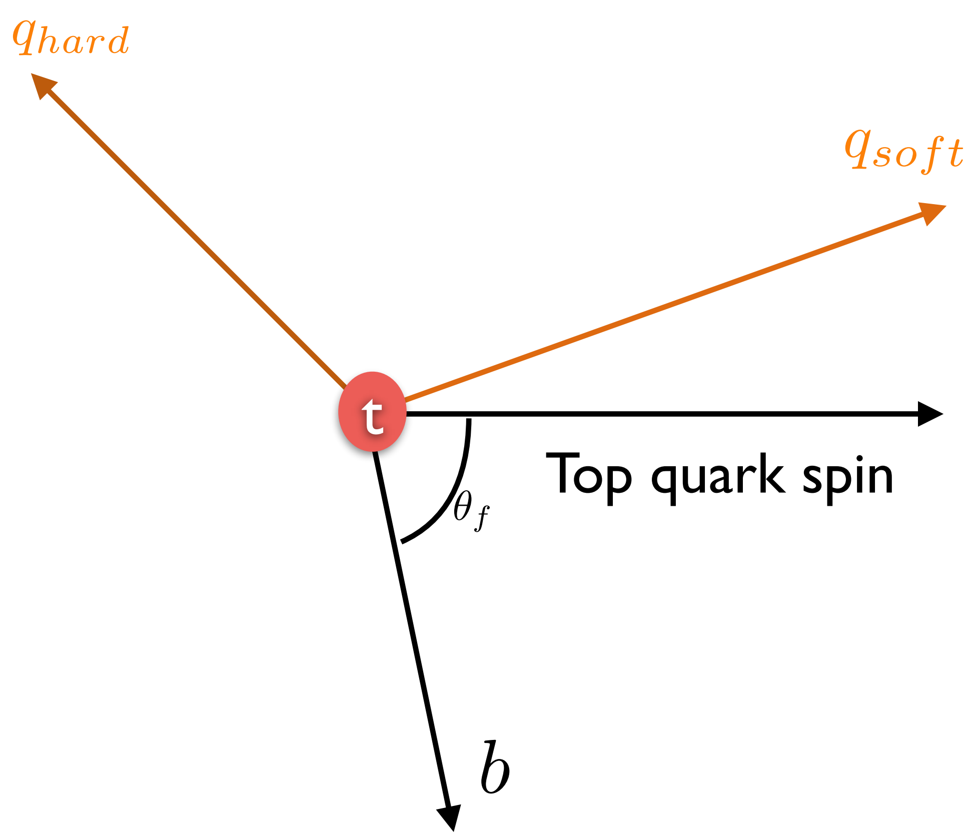

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 \; \mathbf{-0.4} \; \cos\theta_f)$$

Three reasons to study the boosted semileptonic tt final state

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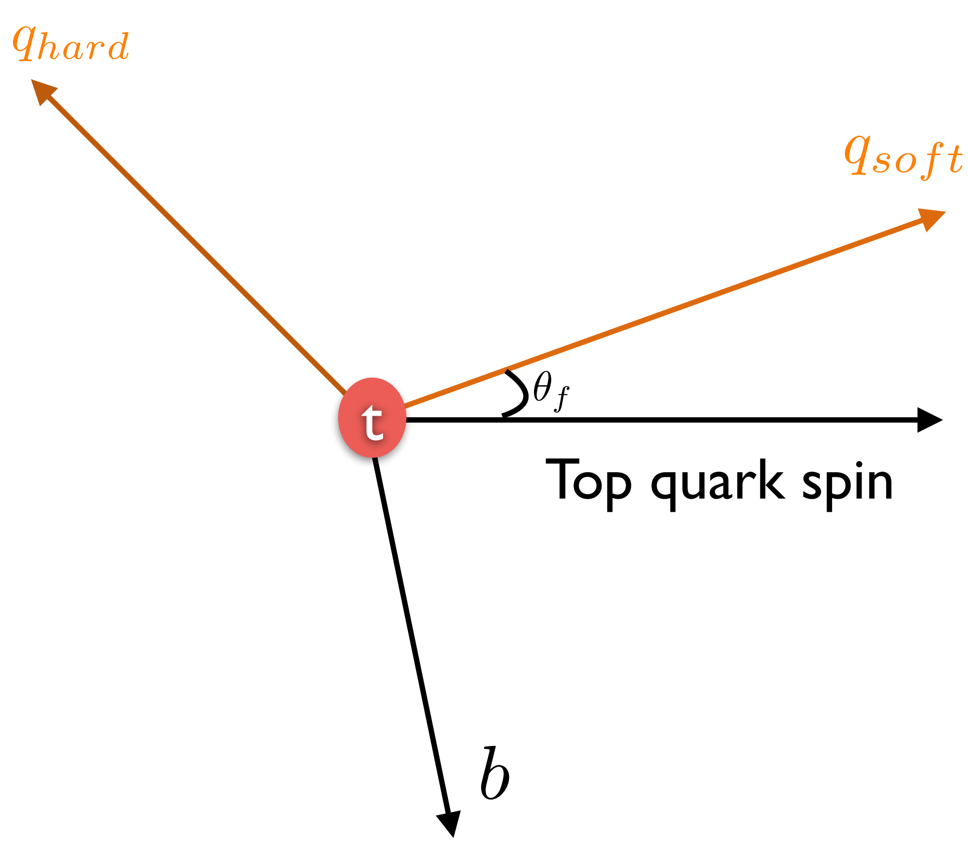
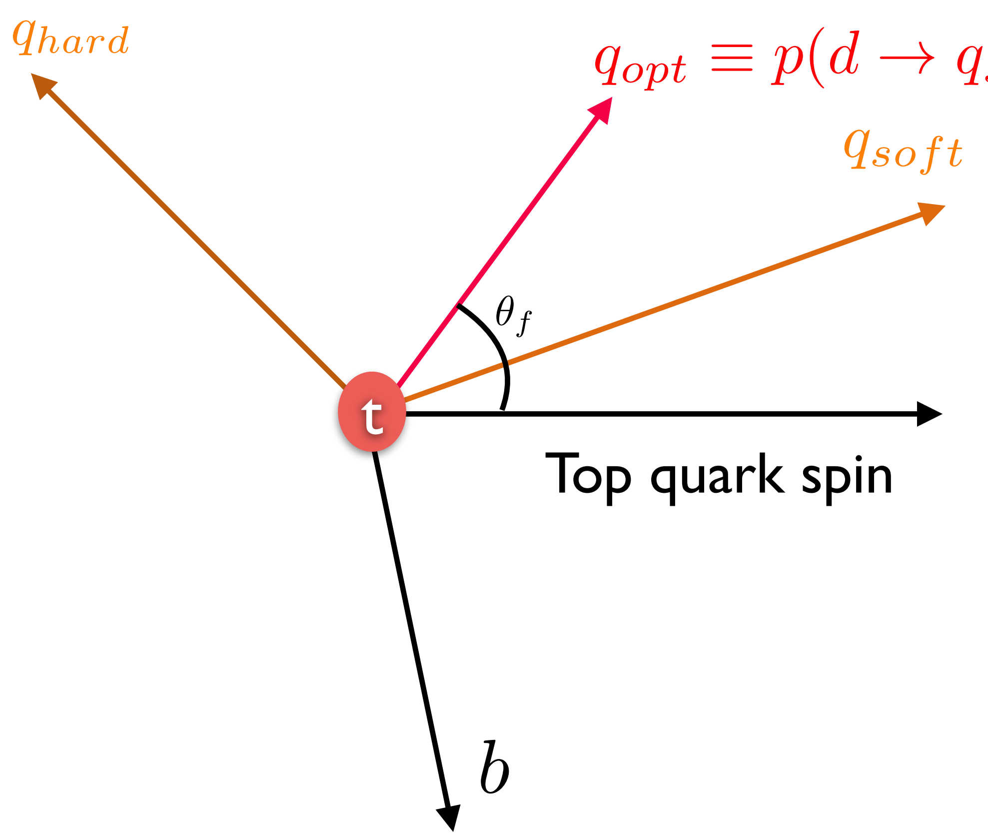


$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{0.5}\cos\theta_f)$$

Three reasons to study the boosted semileptonic tt final state

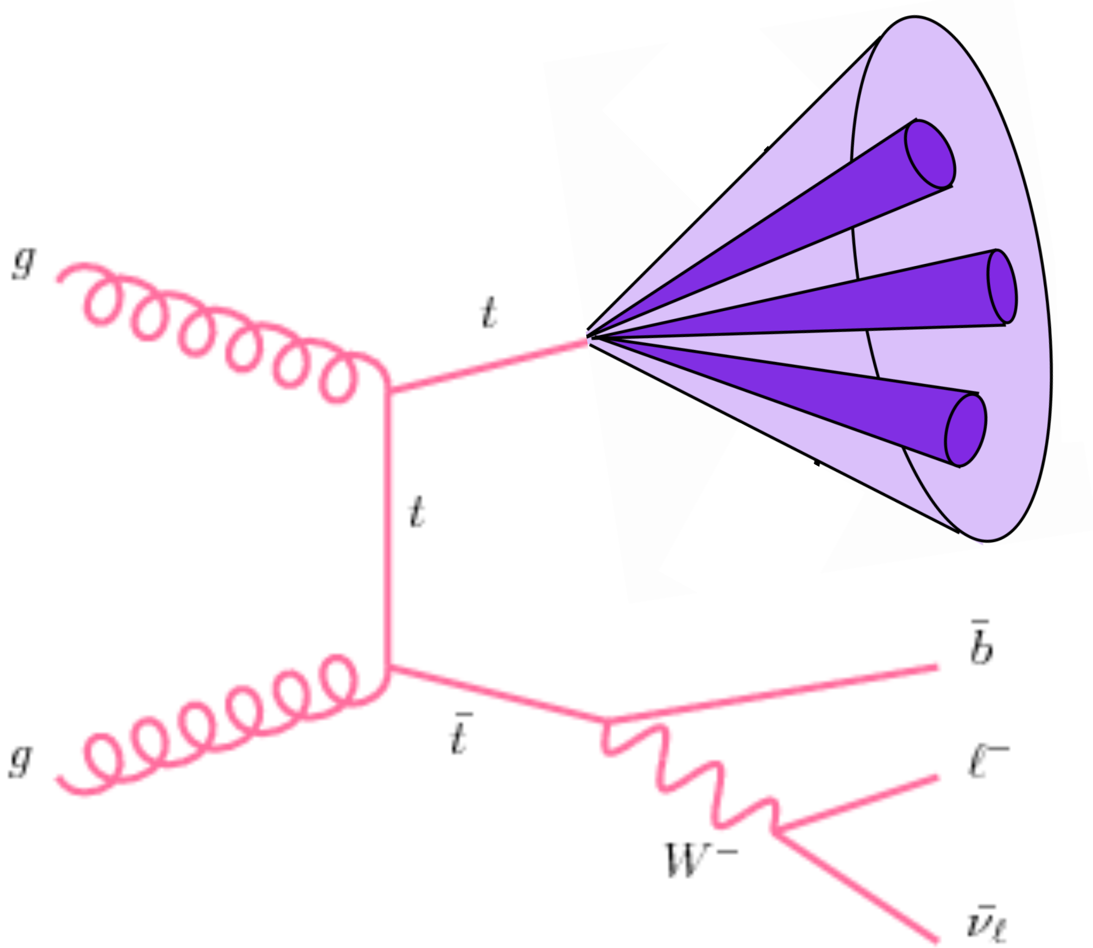
- iii) In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify the optimal hadronic direction



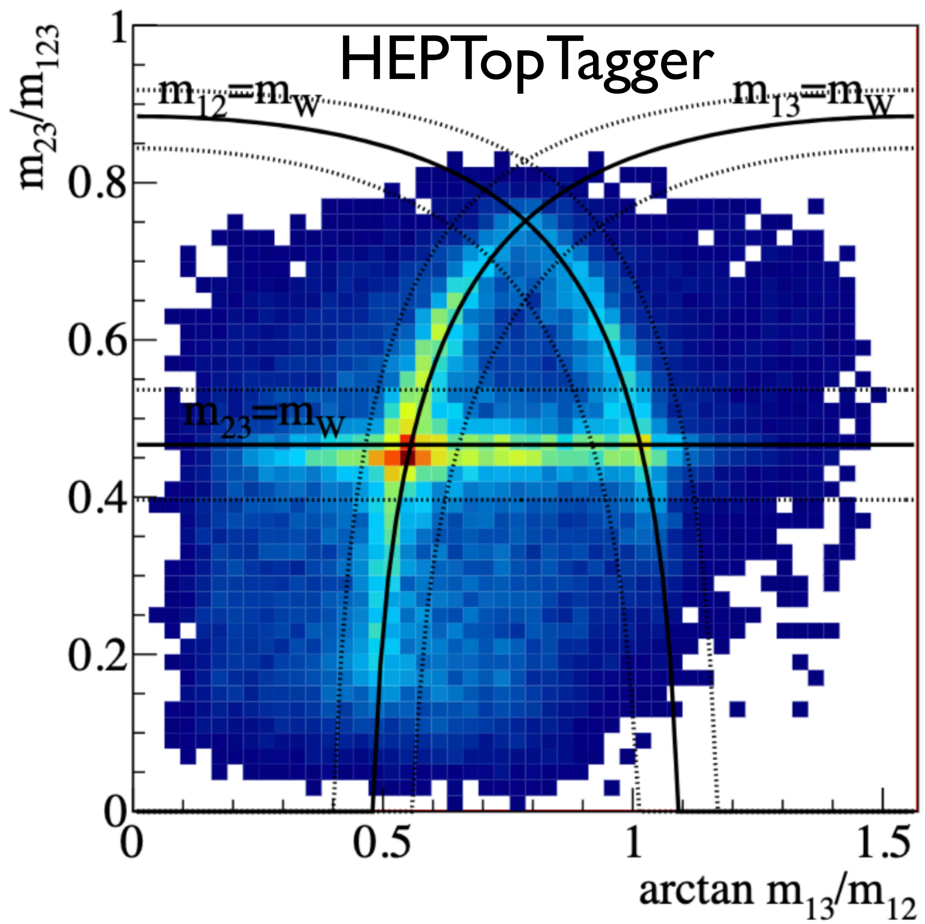
$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{0.64}\cos\theta_f)$$

Analysis

- Some key ingredients for the analysis:

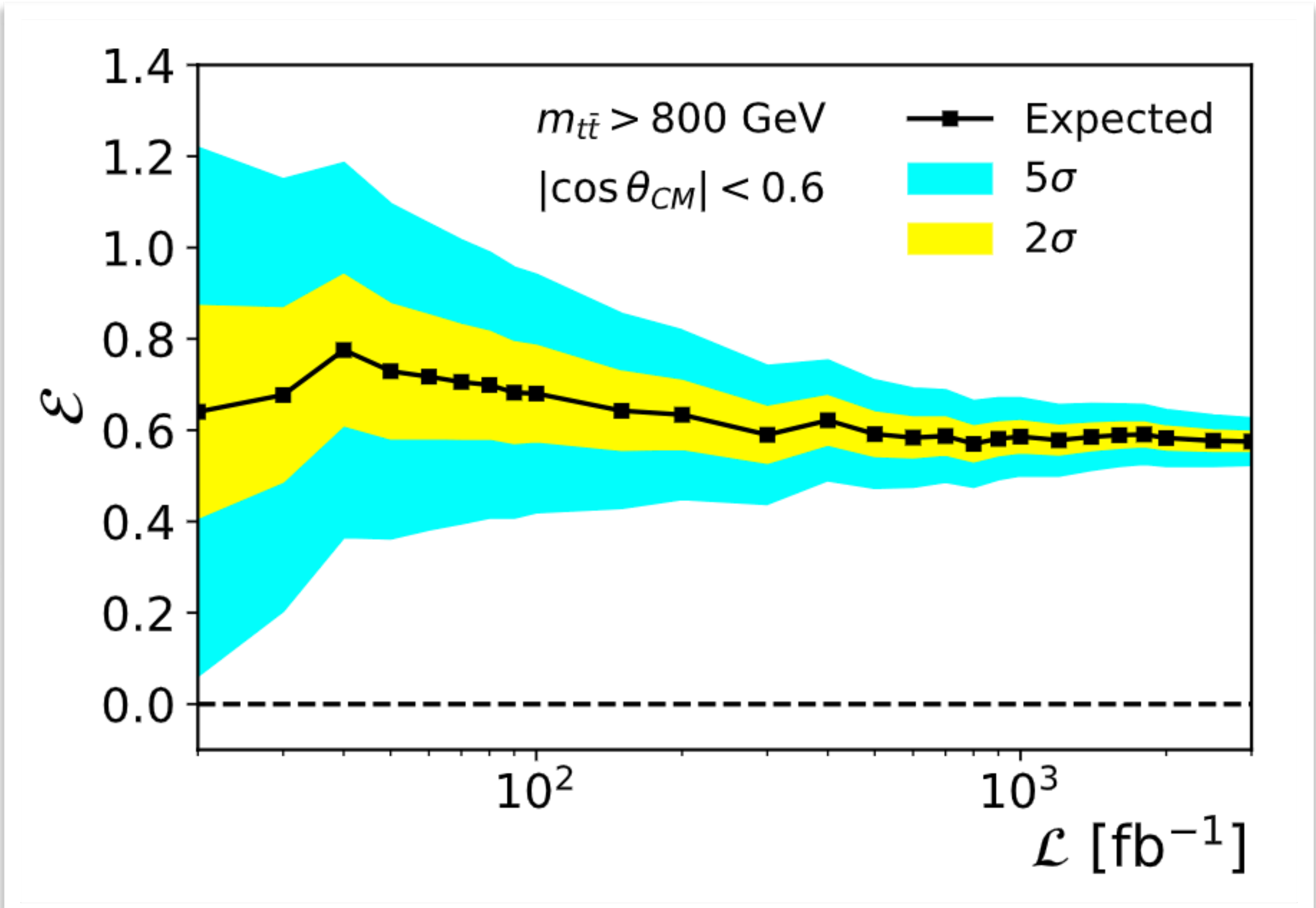


- Boosted top tagging (HEPTopTagger)
  Plehn, Spannowsky, Takeuchi, Zerwas '10
- Lorentz Boost Network: determines which of the subjects is b-tagged
- Proxy for down-quark: optimal hadronic polarimeter



LHC Projections

- Entanglement: $\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$



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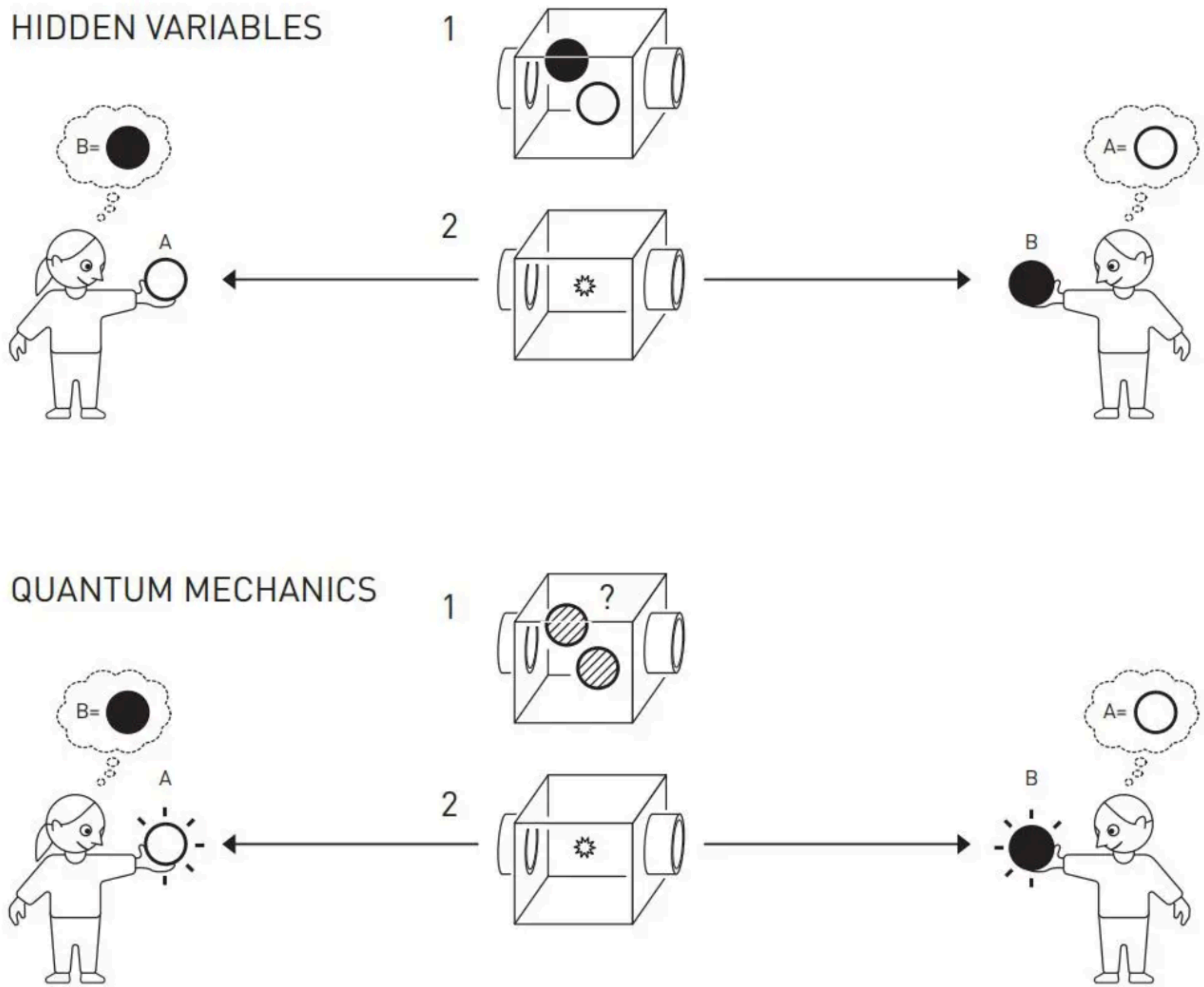
→ Current dataset can probe entanglement at 5-sigma level!

Bell's Inequalities

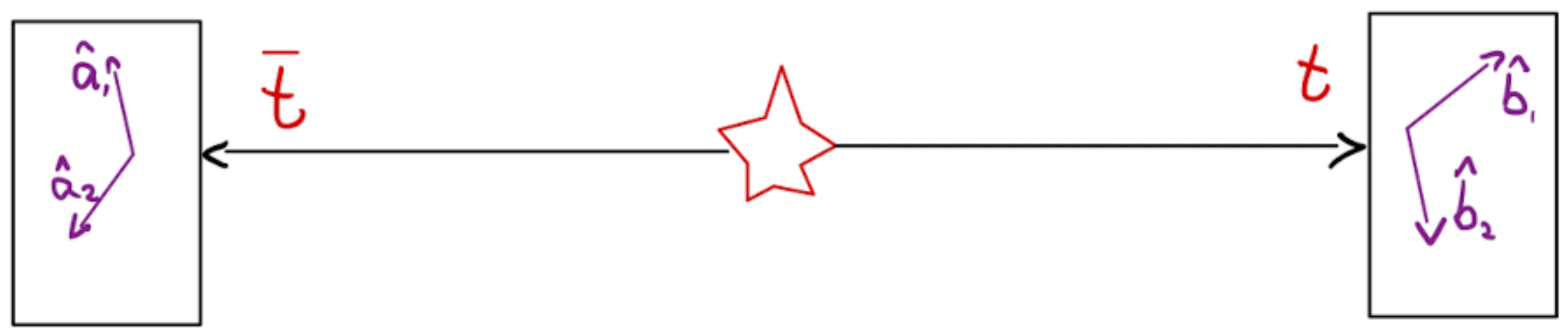


Bell's Inequalities

- Violation of Bell-type inequalities demonstrates that there is no hidden variable theory capable of encoding the generated entanglement. QM cannot be explained by classical laws
- Bell's inequality can be distilled in a simpler form: CHSH inequality

Clauser, Horne, Shimony, Holt '69

$$|\langle A_1B_1\rangle + \langle A_2B_1\rangle + \langle A_1B_2\rangle - \langle A_2B_2\rangle| \leq 2$$



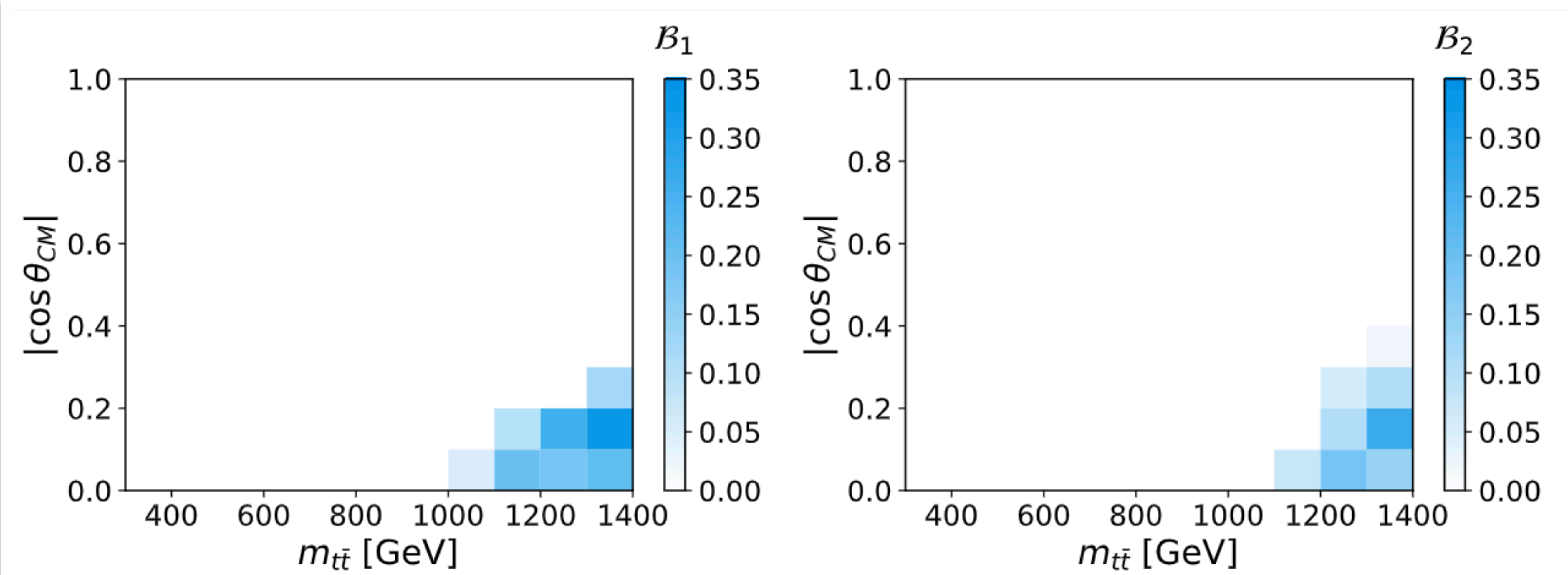
Observer A

Observer B

Bell's Inequalities

- Bell/CHSH inequalities: $\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$
  $\mathcal{B}_2 \equiv |C_{kk} + C_{rr}| - \sqrt{2} > 0,$

Afik, Nova '20
Severi, Boschi, Maltoni, Sioli '21
Saavedra, Casas '22



Dong, DG, Kong, Navarro '23

→ Bell/CHSH violation studies well match **boosted** top pair searches

LHC Projections

- Bell/CHSH inequalities: $\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$
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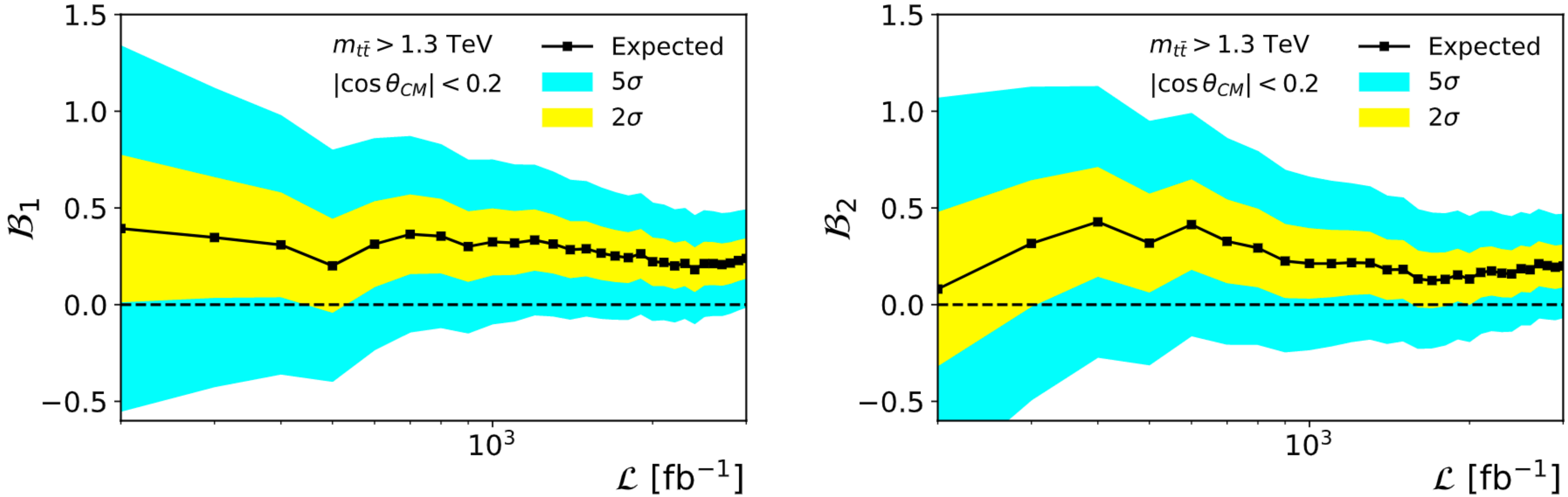

Dong, DG, Kong, Navarro '23

CHSH loopholes in top pair production

Measurement of loophole free Bell violation is more delicate than probing entanglement

Severi, Boschi, Maltoni, Sioli '21; Afik, Nova '22

- **Free-will loophole**: There is no external intervention in choosing measurements. ✗
  However, directions of final state leptons and q_{opt} are randomized ✓
- **Causally disconnected** at the boosted regime ✓ (statistical level) ✗
- **Detection loophole**: We examine a subset of events, potentially biasing results ✗

→ It is not surprising that the LHC can only probe weak violation of Bell's inequalities. LHC wasn't designed for this. In fact, it took decades to prepare loophole-free setups, with the first measurement in 2015-2018, leading to the 2022 Nobel Prize

Summary

- LHC provides a unique opportunity to study entanglement and violation of Bell's inequalities at high energy scales

- Top pair production at the LHC forms a two-qubit system: quantum tomography

- Boosted semileptonic top pair searches well match entanglement and CHSH studies:
  - High event rate
  - Boosted top tagging aid subject and light quark matching
  - Optimal hadronic polarimeter

- Promising measurements:
  - Entanglement can be probed with current LHC dataset
  - Bell/CHSH violation may be probed at 4-5 sigma level at the HL-LHC
  - Bell violation results can be statistically boosted by ATLAS+CMS combination and combination with dileptonic tops

Work in collaboration with

Zhongtian Dong (KU)

KC Kong (KU)

Alberto Navarro (OSU)

Backup

- Since for the $t\bar{t}$ system $B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$ (P and CP) and $C_{13} \simeq C_{23} \simeq 0$ (helicity basis), we obtain some simple sufficient conditions for entanglement

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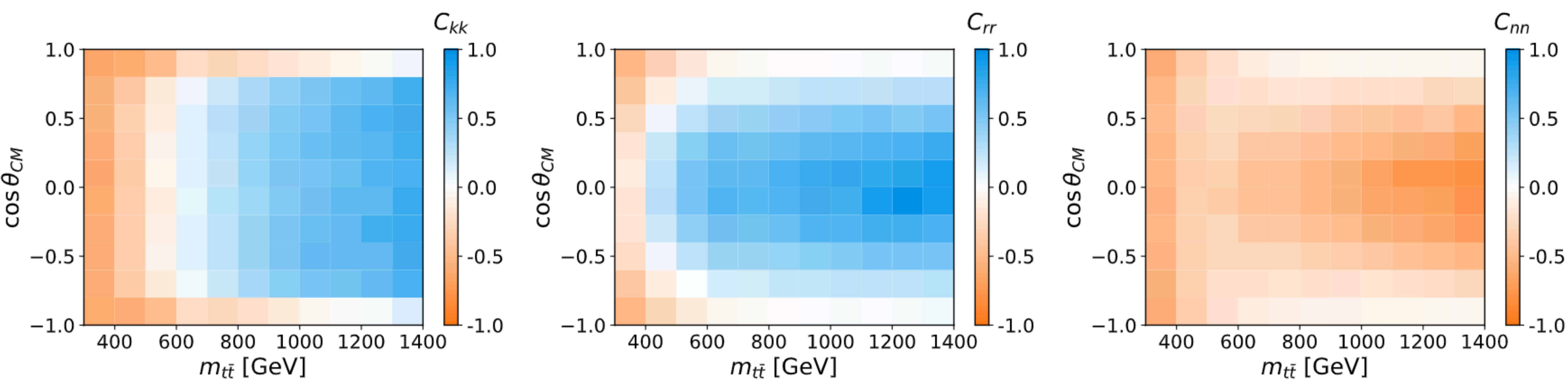

$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$