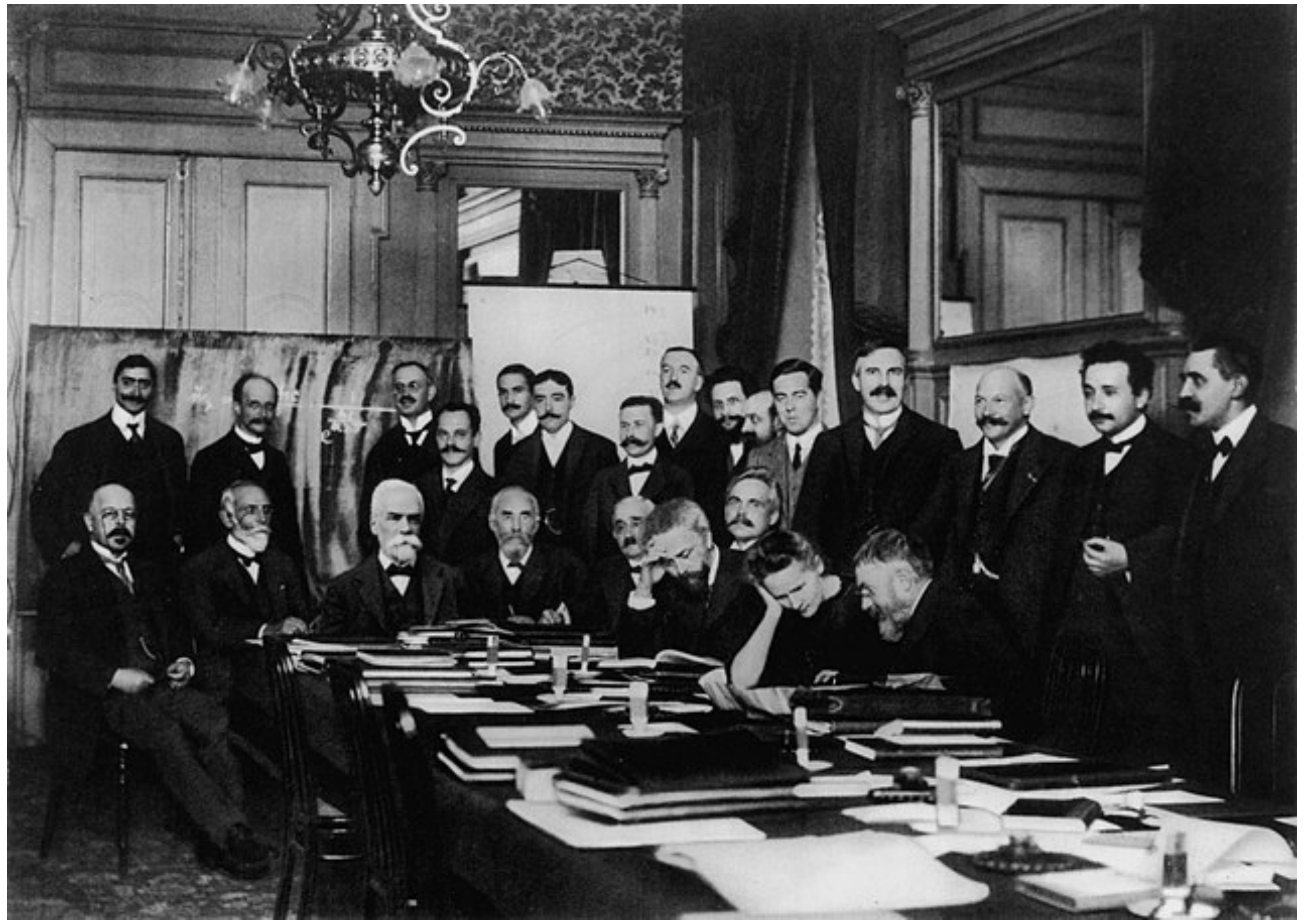
Non Linear Quantum Mechanics

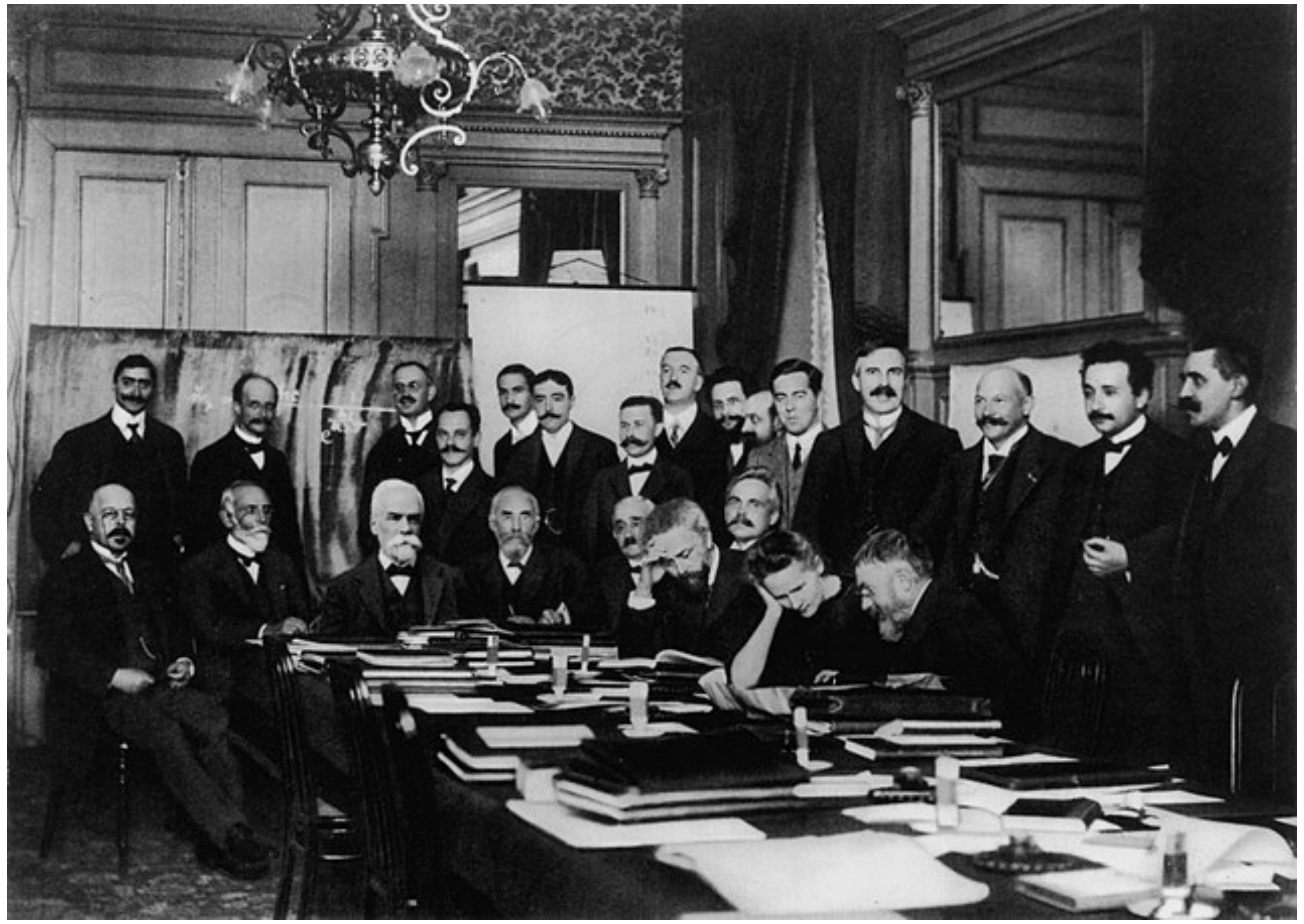
Surjeet Rajendran

Non Linear Quantum Mechanics?



Theory built on observations in the 1900s
Why should it be “the absolute truth”?

Non Linear Quantum Mechanics?



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What?

Two Postulates of Quantum Mechanics

Probability

Linearity

Which?

Probability

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Finite system has a finite set of energies

Continuous observables and symmetries

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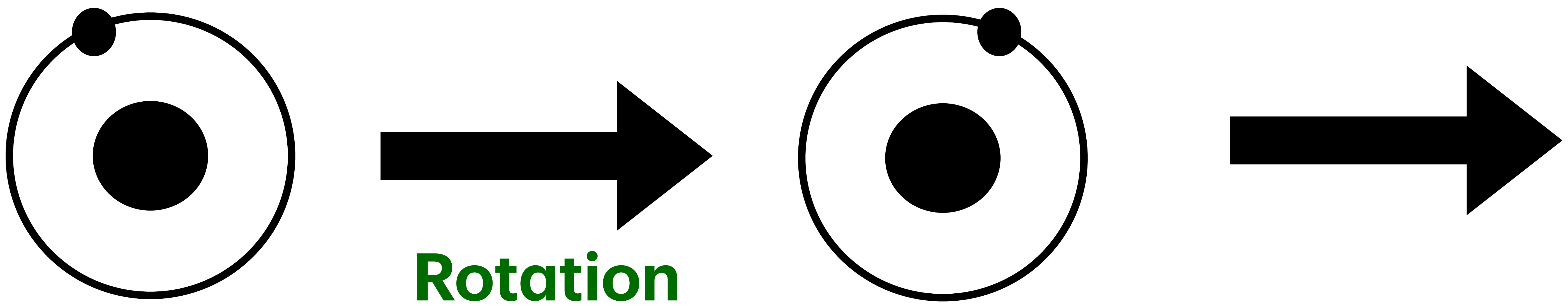

Infinite Degeneracy

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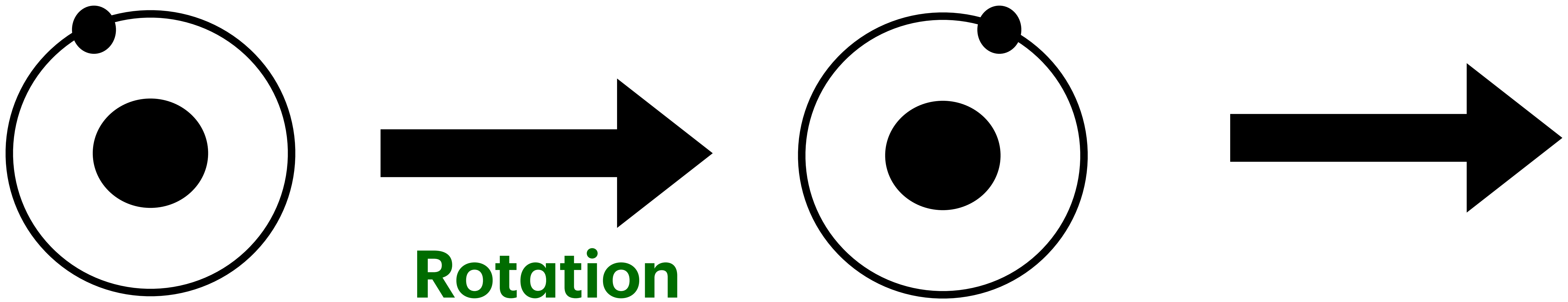

Infinite Degeneracy

Quantum Mechanics

Sacrifice Determinism.

Preserve finite set of energy states, continuous symmetries and observables

Bell Inequalities, Kochen-Specker, SSC Theorems

Causality and Entanglement

Trial Non-Linear Term

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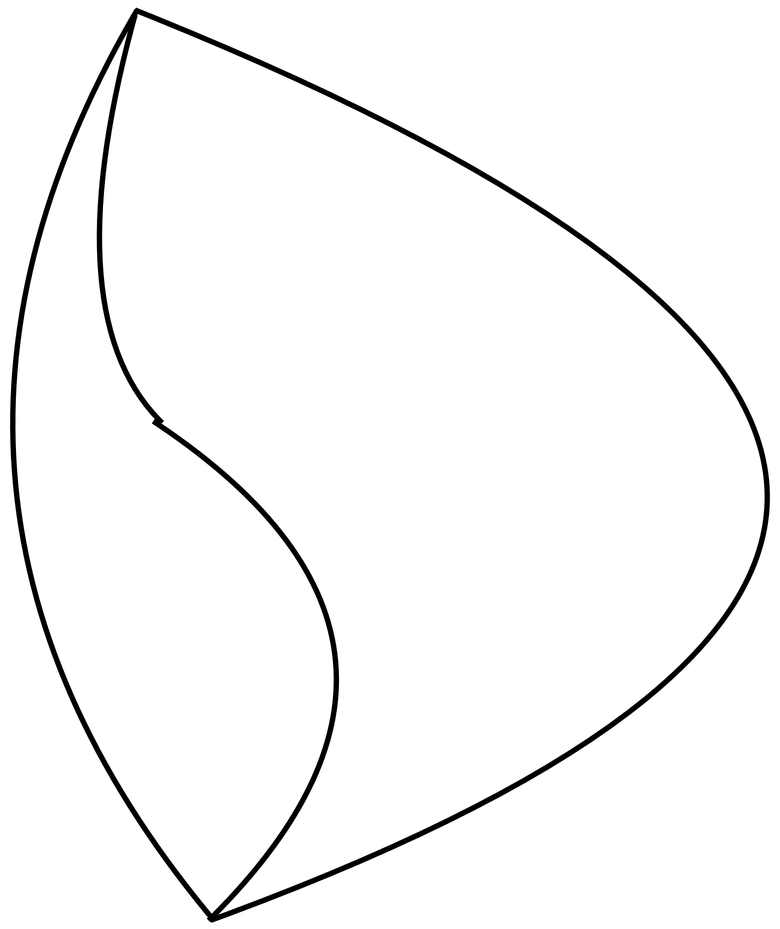
Apply some local operation on x: $a_i(x)$ -> U $a_i(x)$

Does it instantly change the time evolution of y?

YES
Not causal

Linearity

Electron Coupled to Electromagnetism



Electron paths do not interact via electromagnetism

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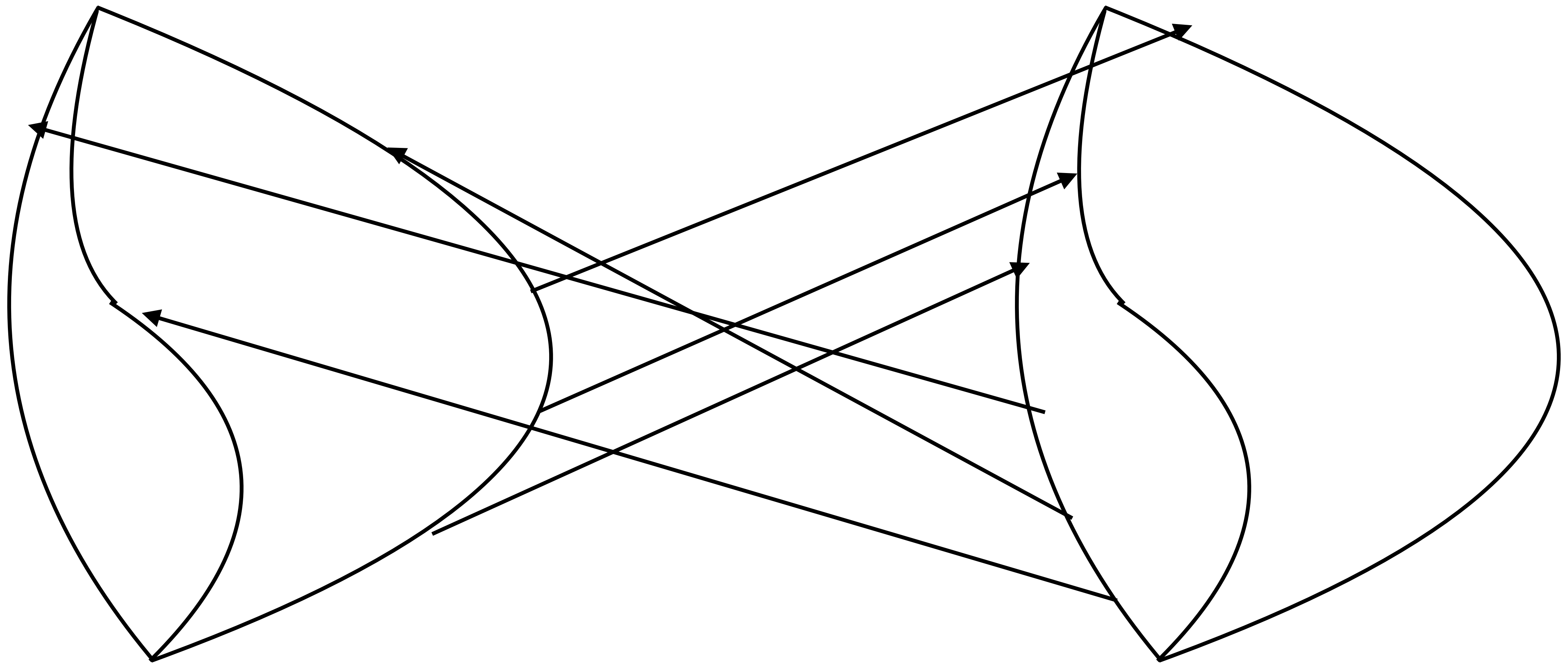

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Paths of two electrons interact causally (QFT)

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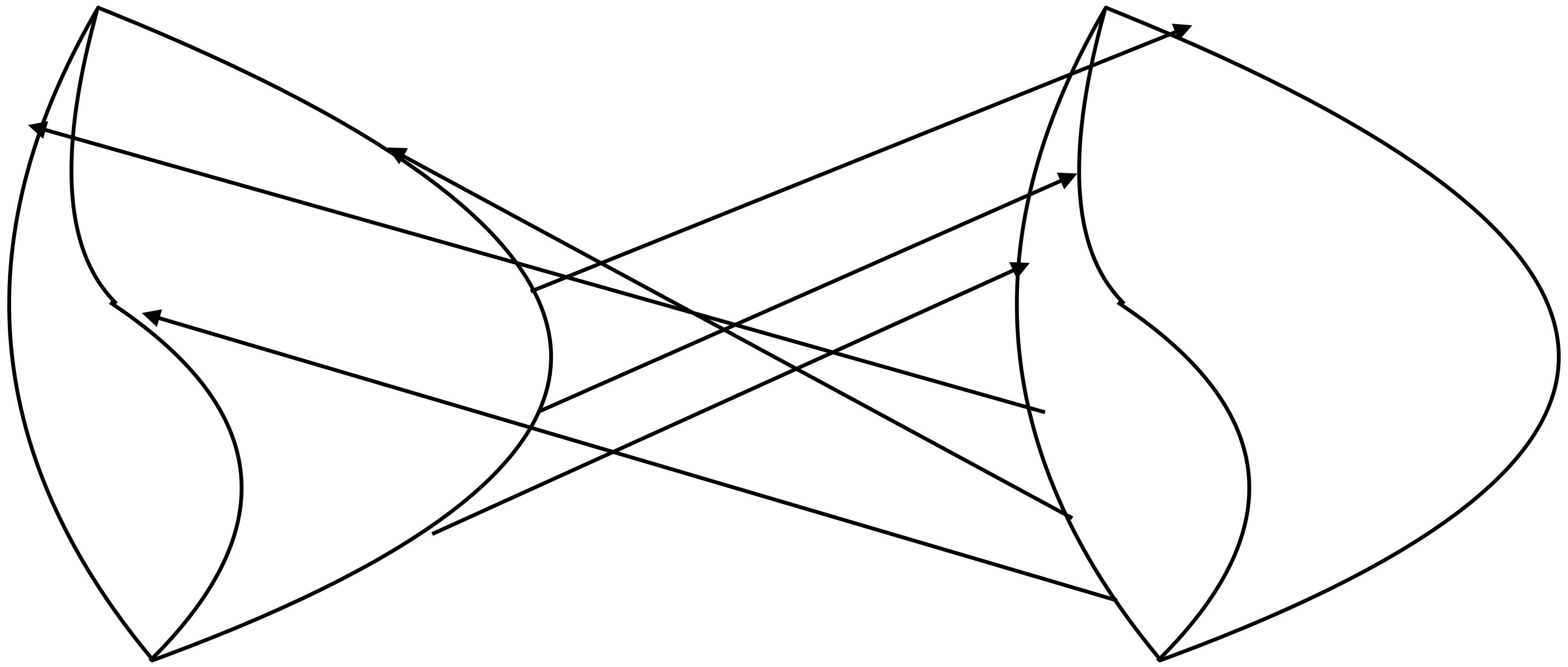

Electron paths do not interact via electromagnetism

Paths of two electrons interact causally (QFT)

Why can't path talk to itself?

Natural Language:
Quantum Field Theory

The Framework

The Schrodinger Picture of Quantum Field Theory

$|\chi(t)\rangle$ **Quantum State of Fields (e.g. in Fock states)**

$\phi(x)$ **Time Independent Operators**

$$H = \int d^3x \, \mathcal{H}\left(\phi(x), \pi(x)\right)$$

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Non-linearities in the operators but not in the state

The Framework

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Linear QFT: $S \supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

Non-Linear QFT: $S_{NL} \supset \epsilon \left(\int d^3x \, \langle \chi(t) | \phi(x) | \chi(t) \rangle \, \langle \chi(t) | \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

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Obeys all the rules

Higher order in states - leads to state dependent quantum evolution

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Analyze non-linearity perturbatively

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

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Causality: Non-linearity enters via expectation value. At lowest order, causal from QFT.
Causal background field for all higher orders

Gauge Theories and Gravitation

Linear QFT Lagrangian, Shift bosonic field by expectation value

To Path Integral, add:

$$e^{iS_0 + i\int d^4x (e((A_\mu + \epsilon_\gamma \langle \chi | A_\mu | \chi \rangle))J^\mu + \epsilon_{\tilde{\gamma}} \langle \chi | F_{\mu\nu} | \chi \rangle F^{\mu\nu})}$$

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Background Field

Gravitation

$$e^{iS_0 + i\int d^4x(\epsilon_G \langle\chi|g_{\mu\nu}|\chi\rangle \partial^\mu\phi\partial^\nu\phi)}$$

Single Particle

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Suppose we have a ψ particle - how does its wave-function evolve?

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Straightforward Computation of Expectation Value

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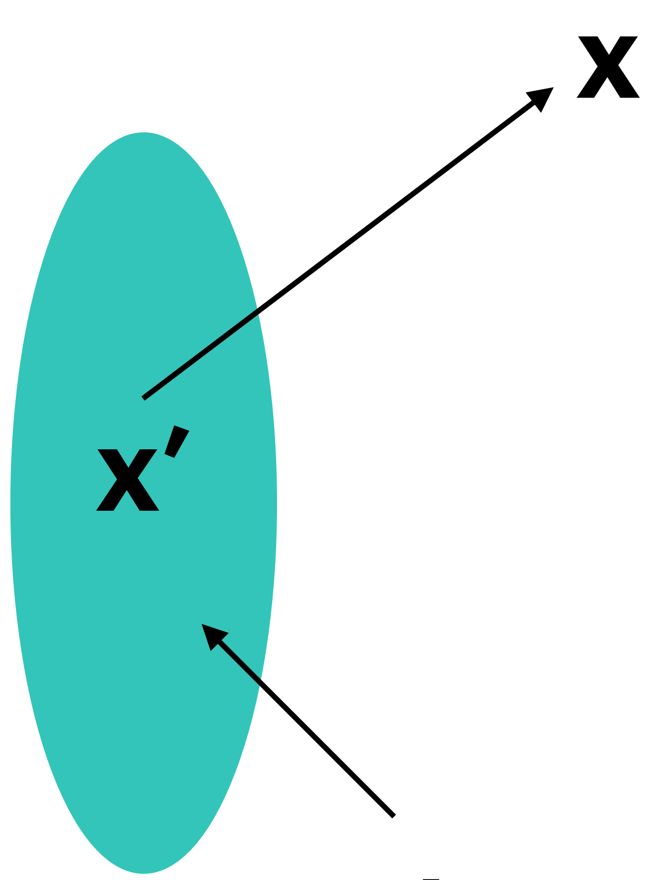

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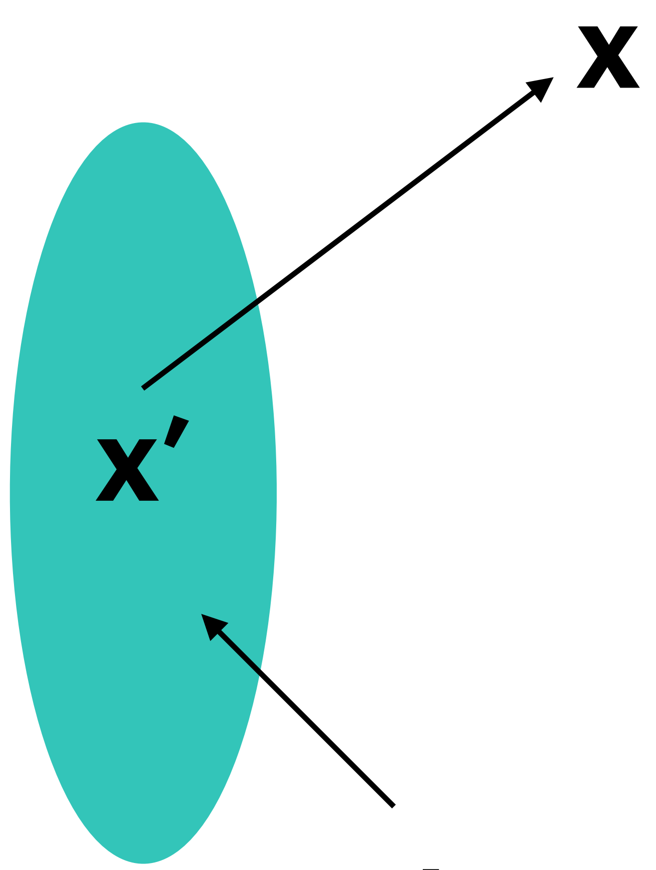

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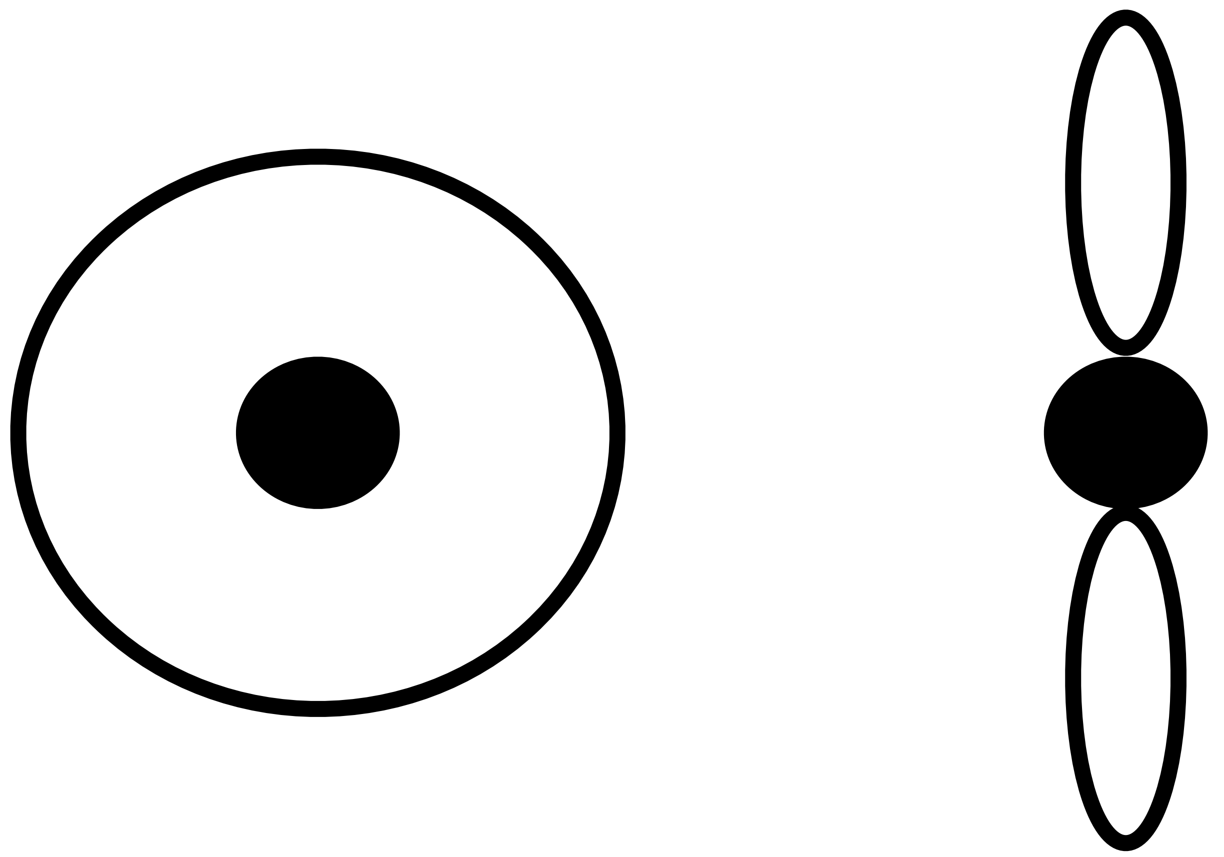
Causal Green's Function

Constraints

What does this do to the Lamb Shift?



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Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

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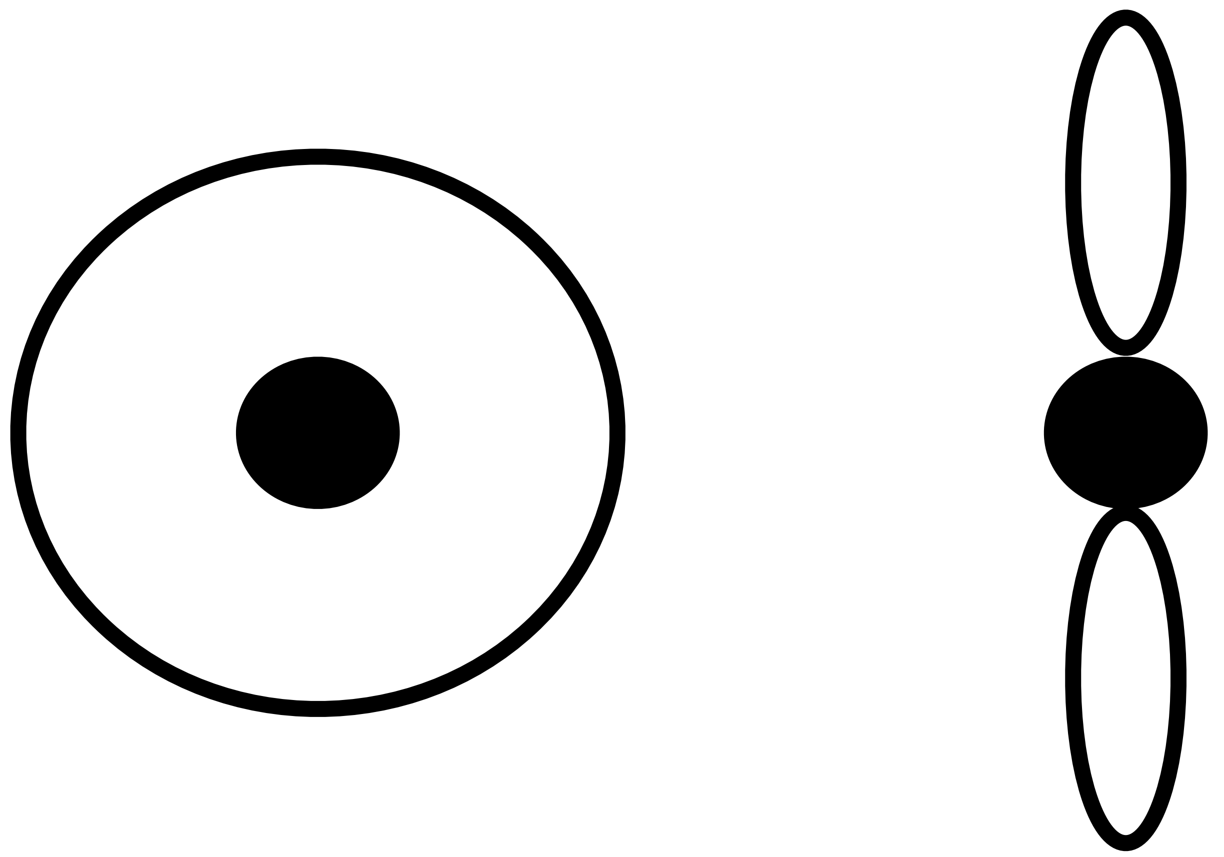

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BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

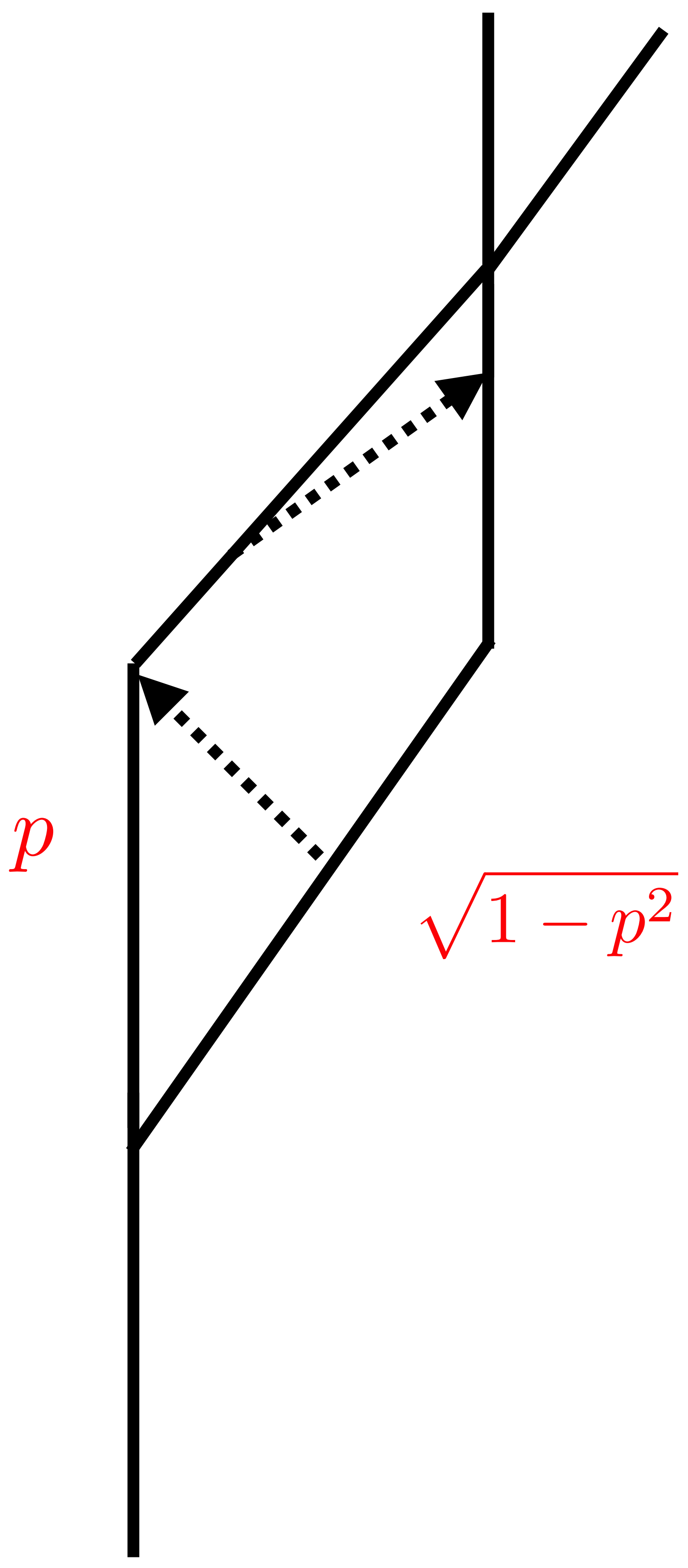
$\varepsilon < 10^{-2}$

Similarly, kills possible bounds on QCD and gravity

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function



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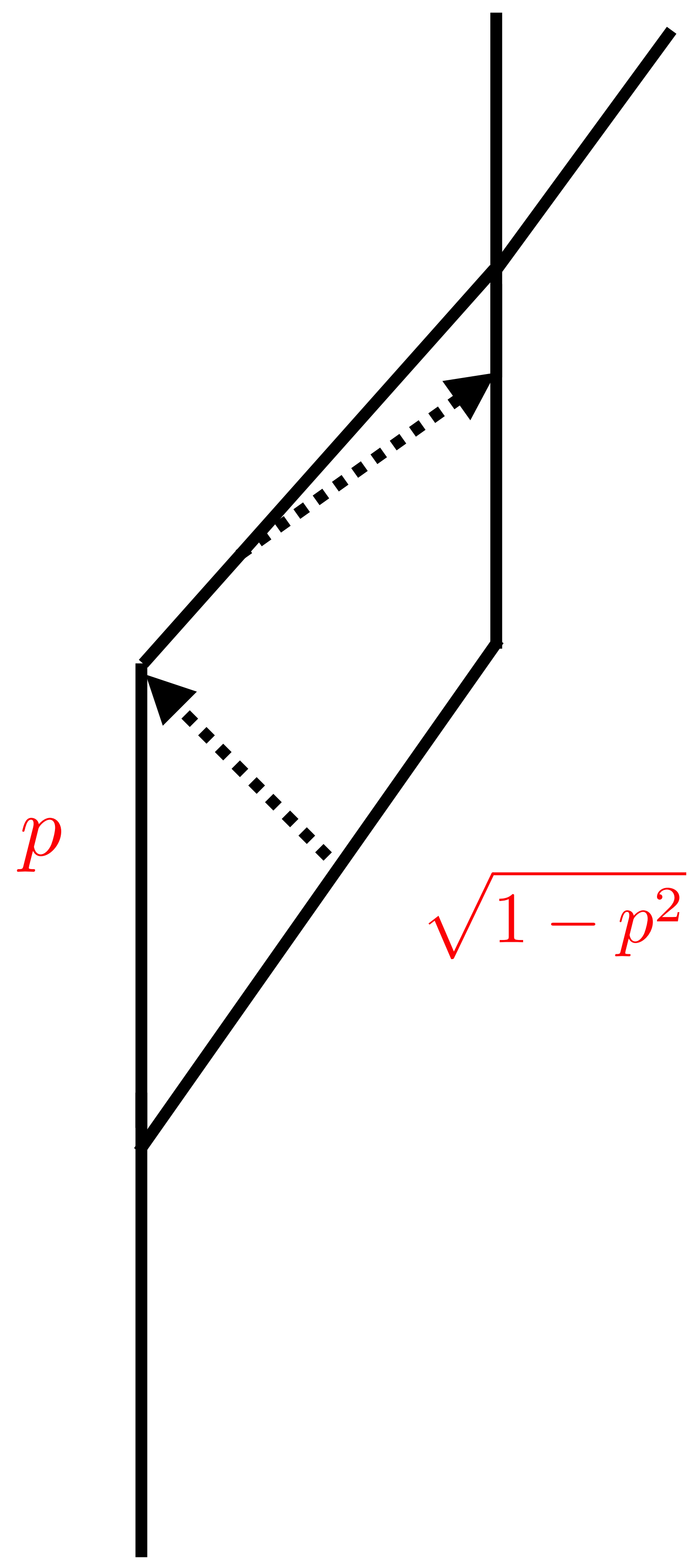

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Gives rise to phase shift that depends on the intensity p of the split

Use intensity dependence to combat systematics

Conclusions

1. Quantum Field Theory can be generalized to include non-linear, state dependent time evolution

2. Conventional tests of quantum mechanics in atomic and nuclear systems do NOT probe causal non-linear quantum mechanics

3. Straightforward set of experimental tests possible to probe non-linear quantum mechanics

4. Motivation to test other extensions as well - e.g. Lindblad Decoherence