Top Tagging and Machine Learning

Sung Hak Lim
Rutgers University

Top 2023,
Traverse City, MI, USA

Sep. 2023

Based on 1807.03312, 1904.02092, 2003.11787, 2010.13469
+ work in progress with A. Furuichi and M. M. Nojiri

Boosted Top Jet Tagging and Machine Learning

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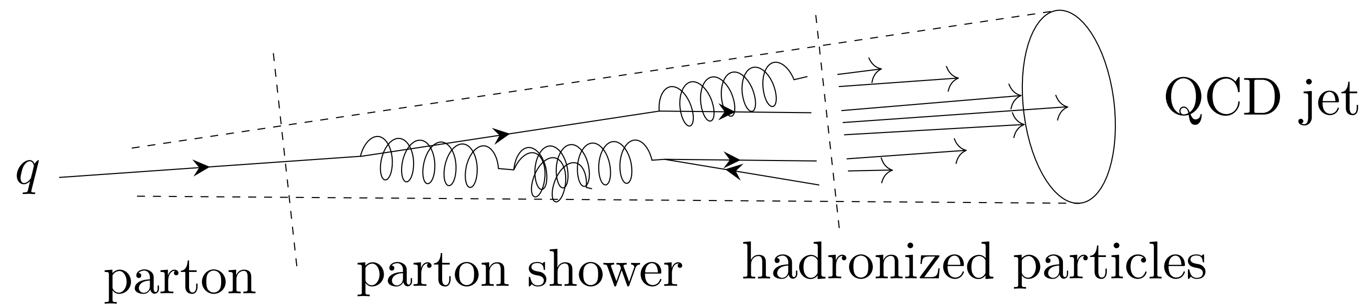
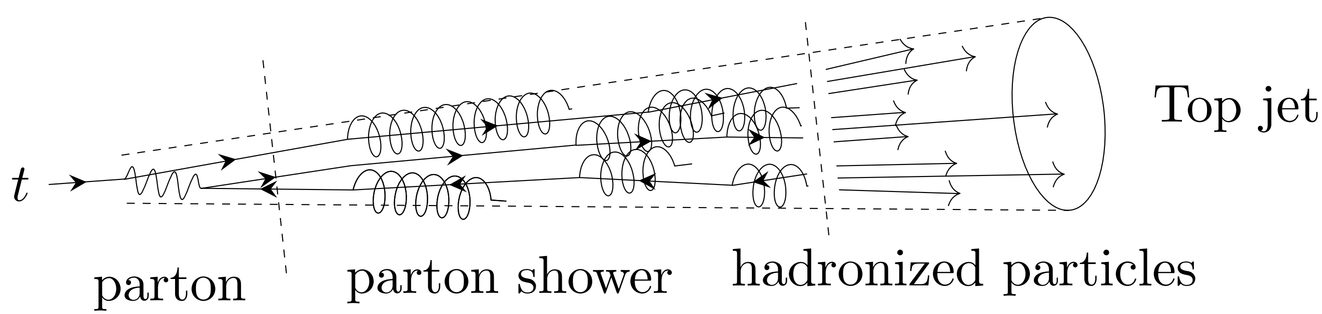
What is a Jet?

At a hadron-hadron collider, such as LHC, a collimated particle cluster called a jet appears when a colored parton is produced.

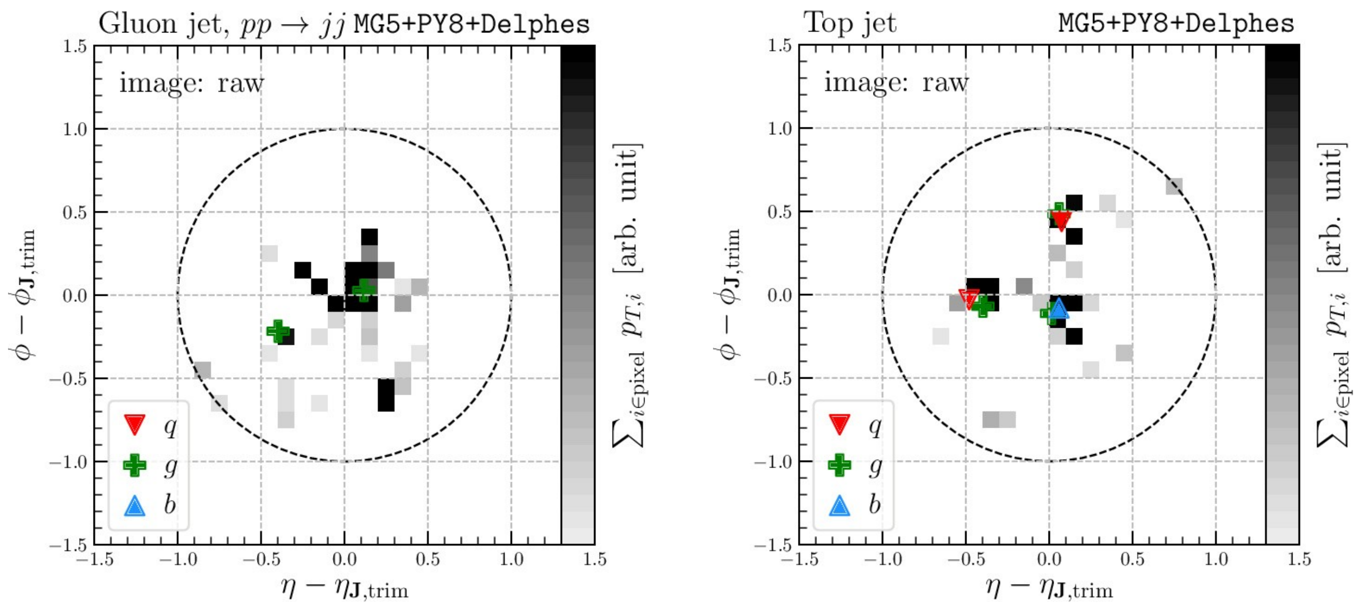


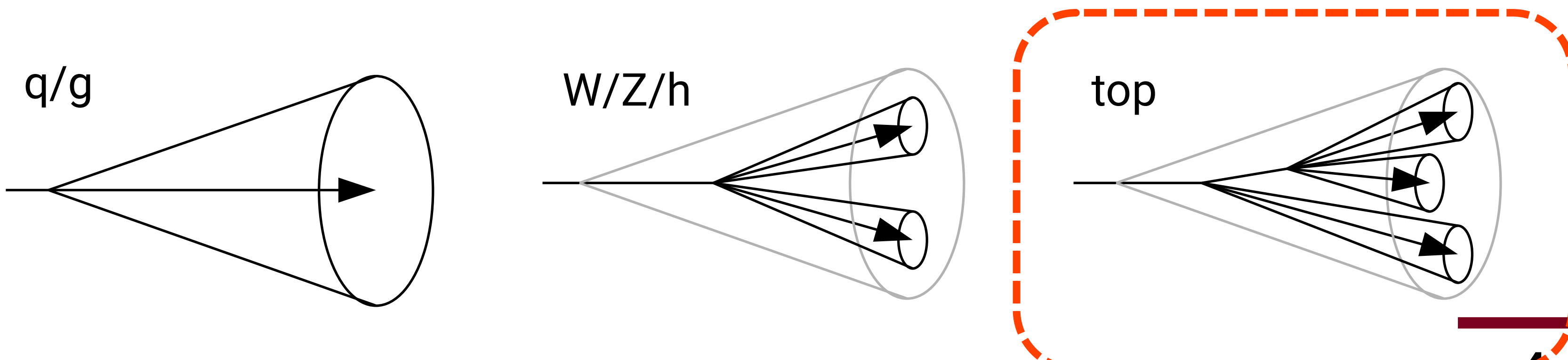
Since we see more and more multi TeV center-of-mass energy events at colliders, boosted heavy particles are produced. They can form a single collimated cluster of particles similar to the QCD jets.



Jets have substructure!

In order to distinguish non-trivial jets from the QCD jets,
we need to check features of jets called substructure:.

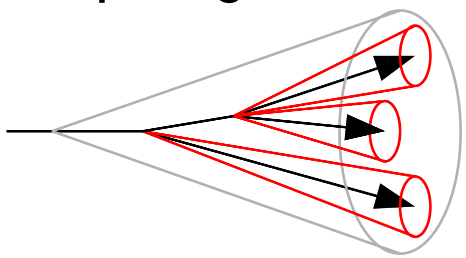




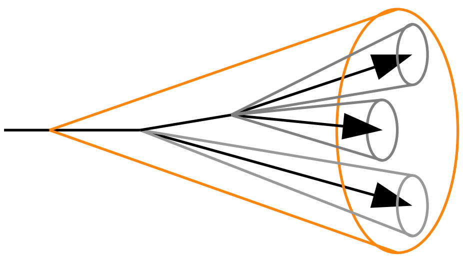
Top Jet Substructures

Boosted Top jets is very rich in characteristic features. Good example for tagger study and benchmarking various ML architectures!

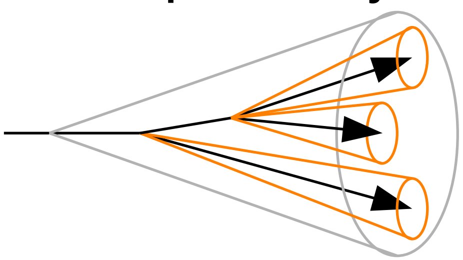
Three-prong



Color triplet

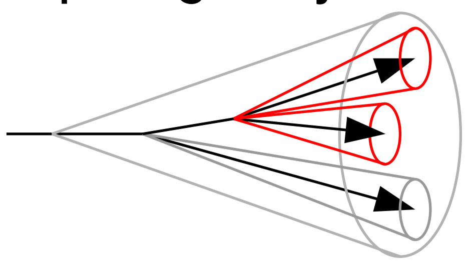


Color triplet subjets

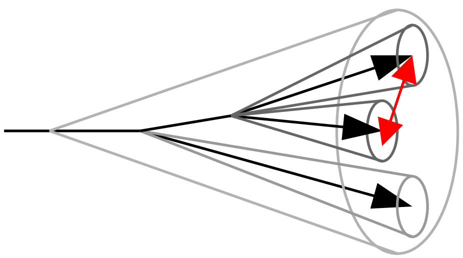


Top jet also have **W boson jet** inside.

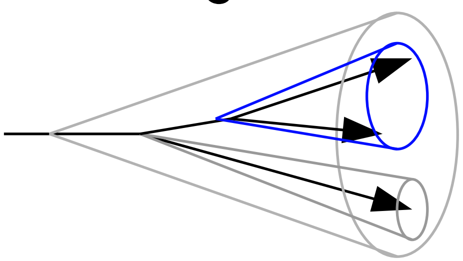
Two-prong subjet inside



Color connection

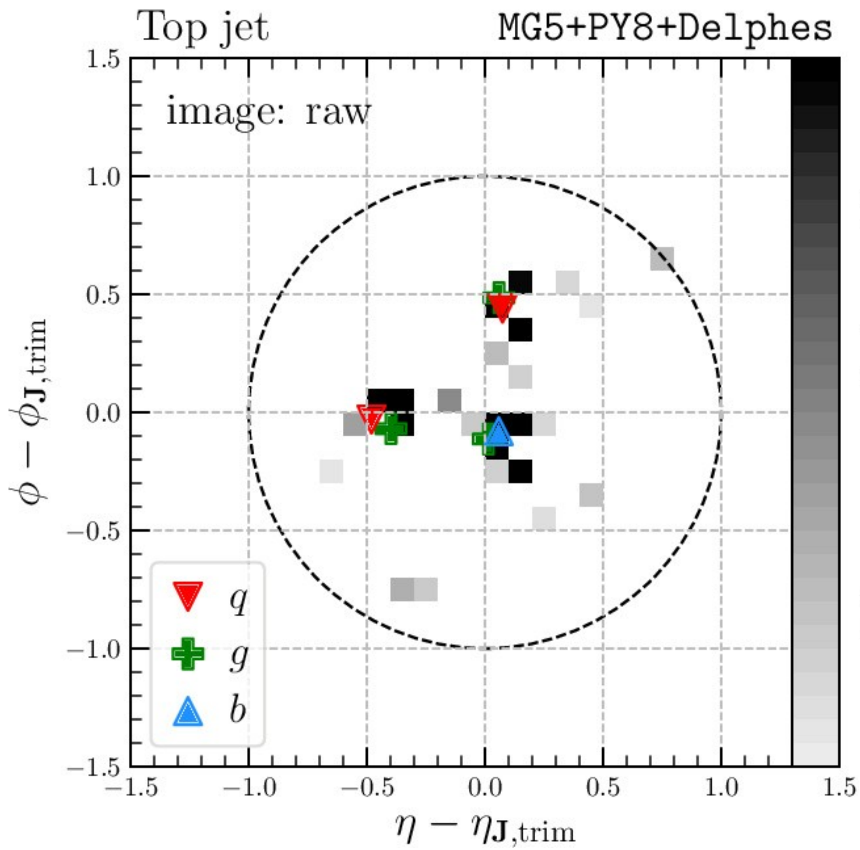


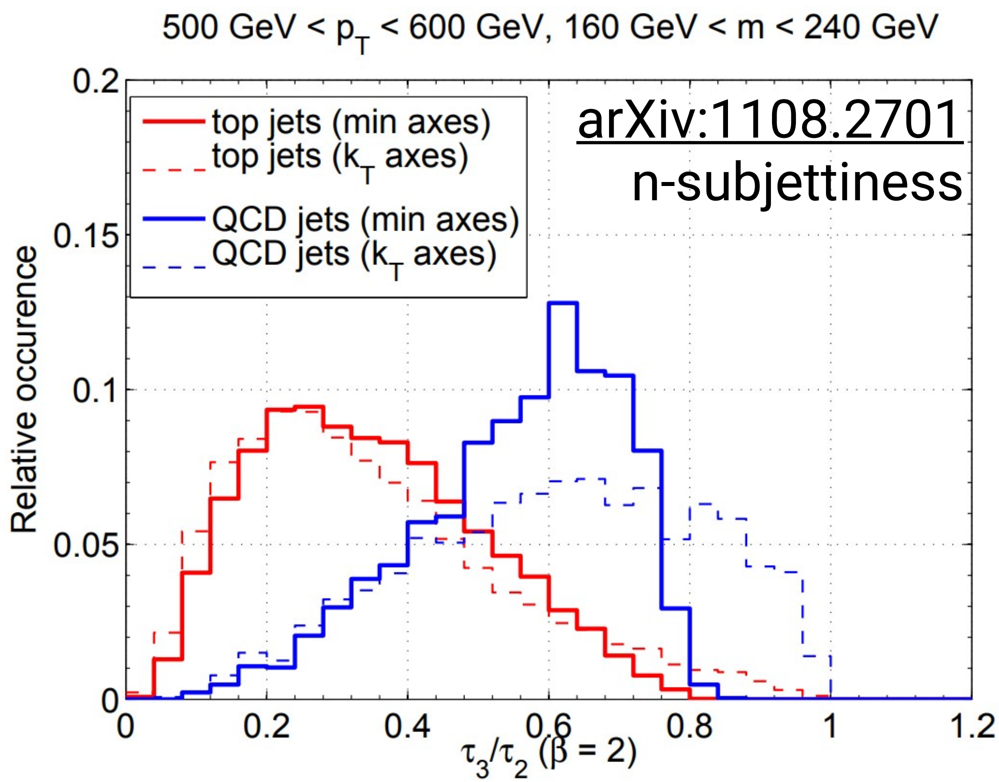
Color singlet



Jet tagger requires understanding of these patterns either implicitly or explicitly. Machine learning is very useful tool for analyzing these patterns!

There are **two** approaches for building ML based jet taggers:





Directly analyze
Low-Level Feature (LLF)

Construct physics-motivated
High-Level Features (HLF)

Use CNN/GNN/Transfomers to analyze LLF
- ParticleNet
- ParticleTransformer
- LorentzNet, PELICAN (equivariant GNN/Transformer)

- Jet PT, mass, (basic kinematics)
- N-subjettiness
- Energy Flow Polynomials
- Constituent Multiplicities...

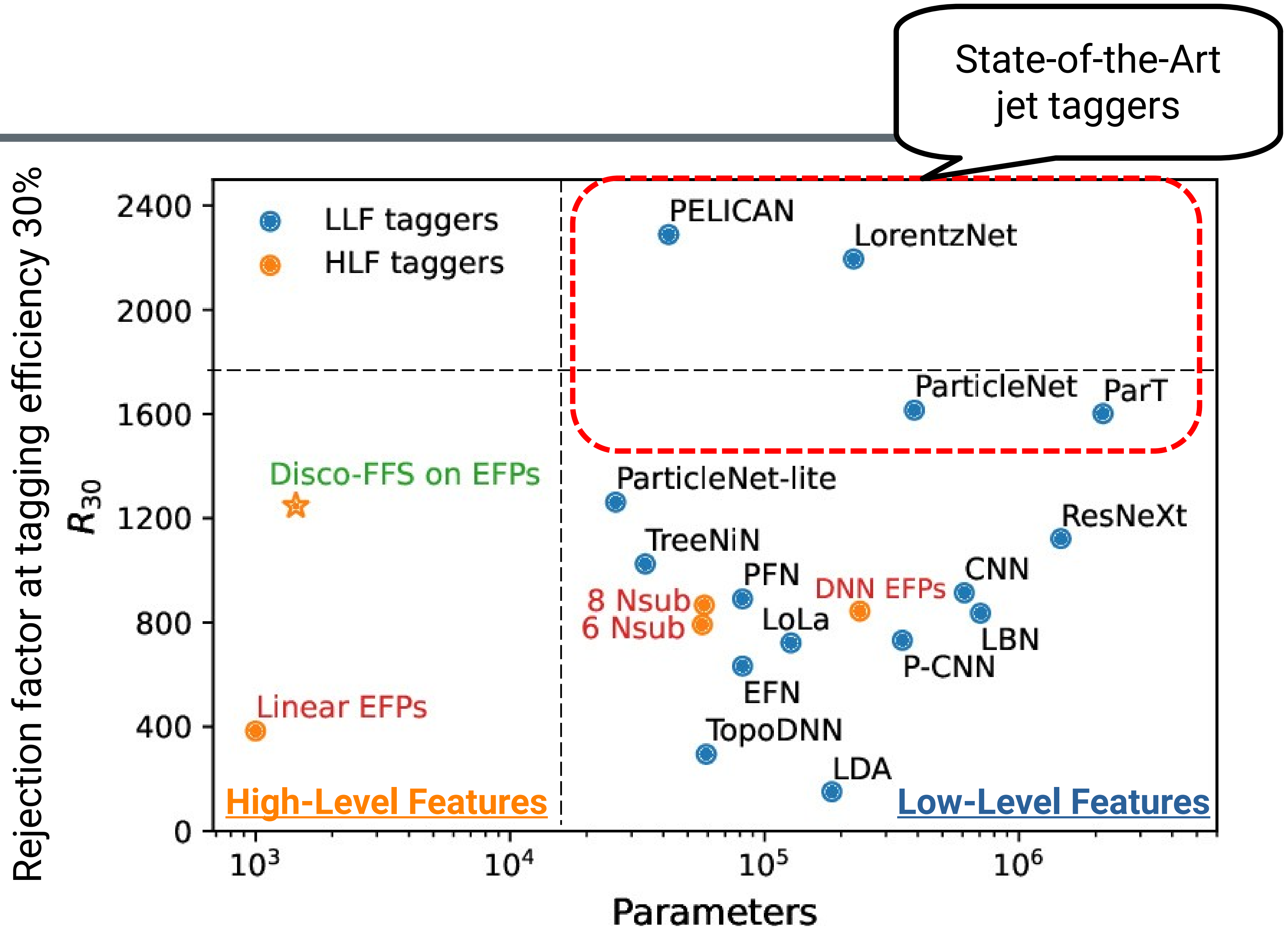
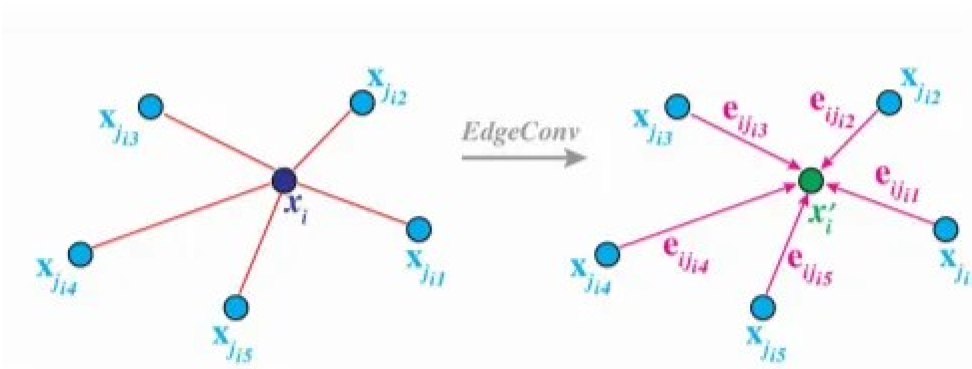
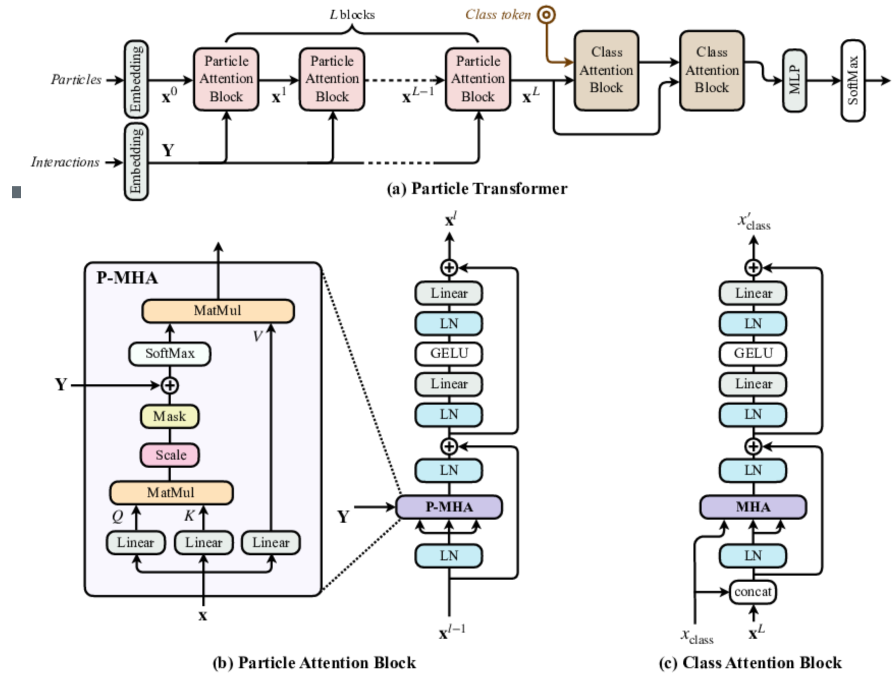


Figure from R. Das, G. Kasieczka, D. Shih, 2212.00046





GNN / Transformers are working great. But because they are general purpose low-level feature analysis tools, it is hard to understand outcome other than the fact that they estimated the classifier output (likelihood ratio) more precisely.

Can we build up a high-level feature based classifier equally performing well? YES!

Advantages:
- simpler network: less training uncertainty (at a cost of expressivity)
- interpretable (by understanding HLF inputs)

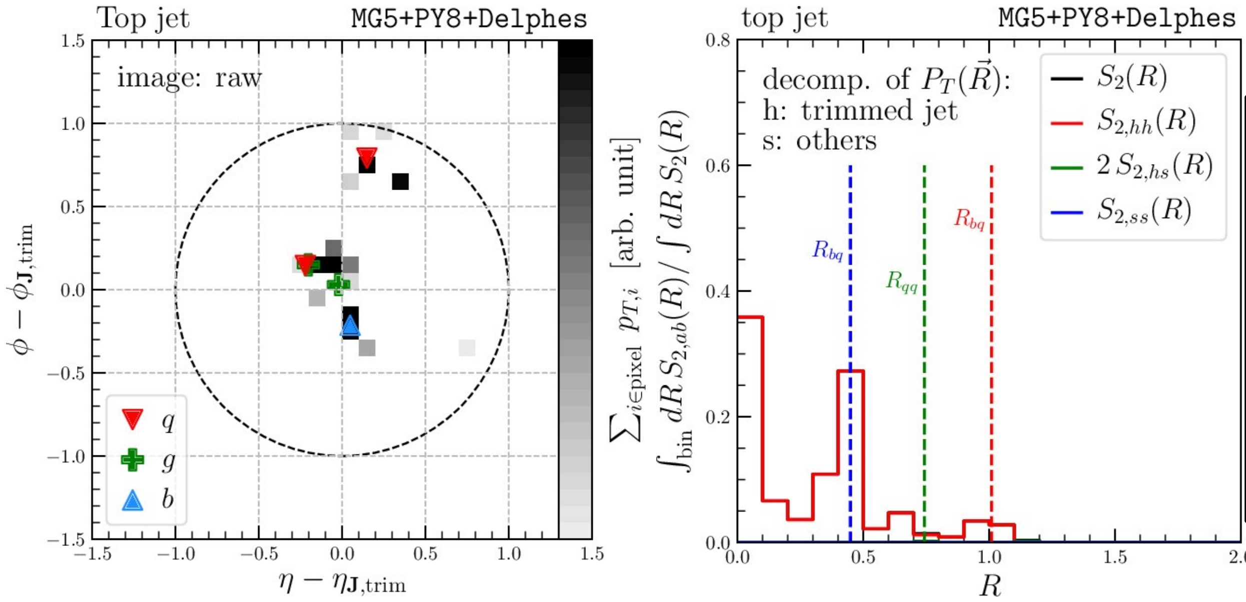
Two-point energy correlation spectrum

SHL, M. M. Nojiri, 1807.03312
A. Chakraborty, SHL, M. M. Nojiri,1904.02092
A. Chakraborty, SHL, M. M. Nojiri, M. Takeuchi, 2003.11787

Two-point energy correlation is an aggregated energy correlation between two constituents at a distance R.

$$S_{2,ab}(R) = \int d\vec{R}_1\, d\vec{R}_2\; P_{T,a}(\vec{R}_1) P_{T,b}(\vec{R}_2)\delta(R - R_{12})$$

$$P_{T,a}(\vec{R}) = \sum_{i\in \mathbf{J}_a} p_{T,i}\, \delta(\vec{R} - \vec{R}_i)$$



Two-point energy correlation captures
three characteristic angular scales
of three prong substructures.

IRC-safe energy correlator based Neural Networks

A. Chakraborty, **SHL**, M. M. Nojiri, M. Takeuchi,
2003.11787

We use the two point correlation S2 as inputs to MLP.
The resulting network is called Relation Network,
a type of GNN using only edge features.

First linear layer: $$\int dR\, S_2(R)\phi^e(R) = \sum_{i,j\in J} p_{T,i}p_{T,j}\phi^e(R_{ij})$$

Graph Networks

Relation Network

Utilizes edge features

$$F[\sum_{i,j\in J} \phi^e(p_i, p_j)]$$

Raposo, et al. (1702.05068),
Santoro, et al. (1706.01427)

IRC safety →

IRC-safe energy correlator based Networks

Relation Network

Utilizes two-point energy correlation

$$F[\sum_{i,j\in J} p_{T,i}p_{T,j}\phi^e(R_{ij})]$$

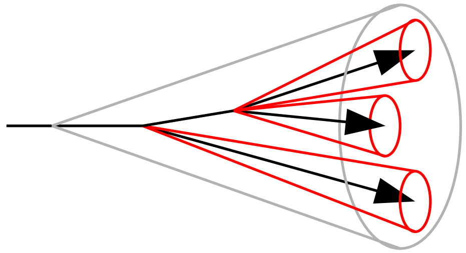
Chakraborty, **SHL**, Nojiri, and Takeuchi (2003.11787)

This network is able to analyze most of prong substructures and their correlations.

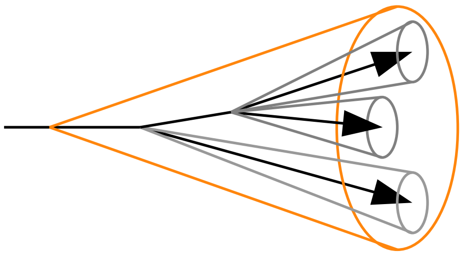
What else do we need?

Boosted Top jets is very rich in characteristic features

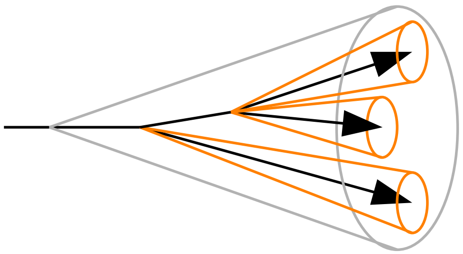
Three-prong



Color triplet

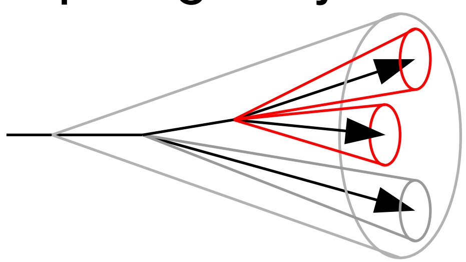


Color triplet subjets

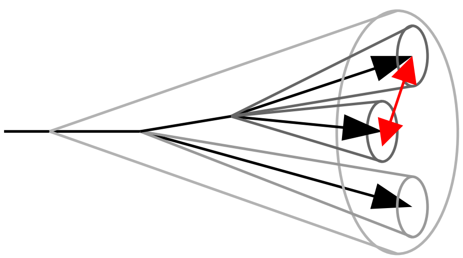


Top jet also have **<u>W boson jet</u>** inside.

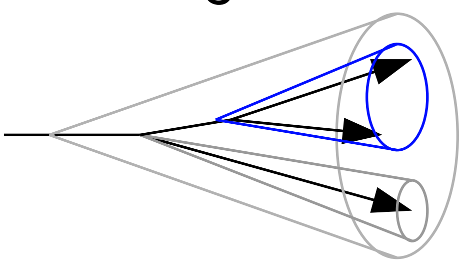
Two-prong subjet inside



Color connection



Color singlet



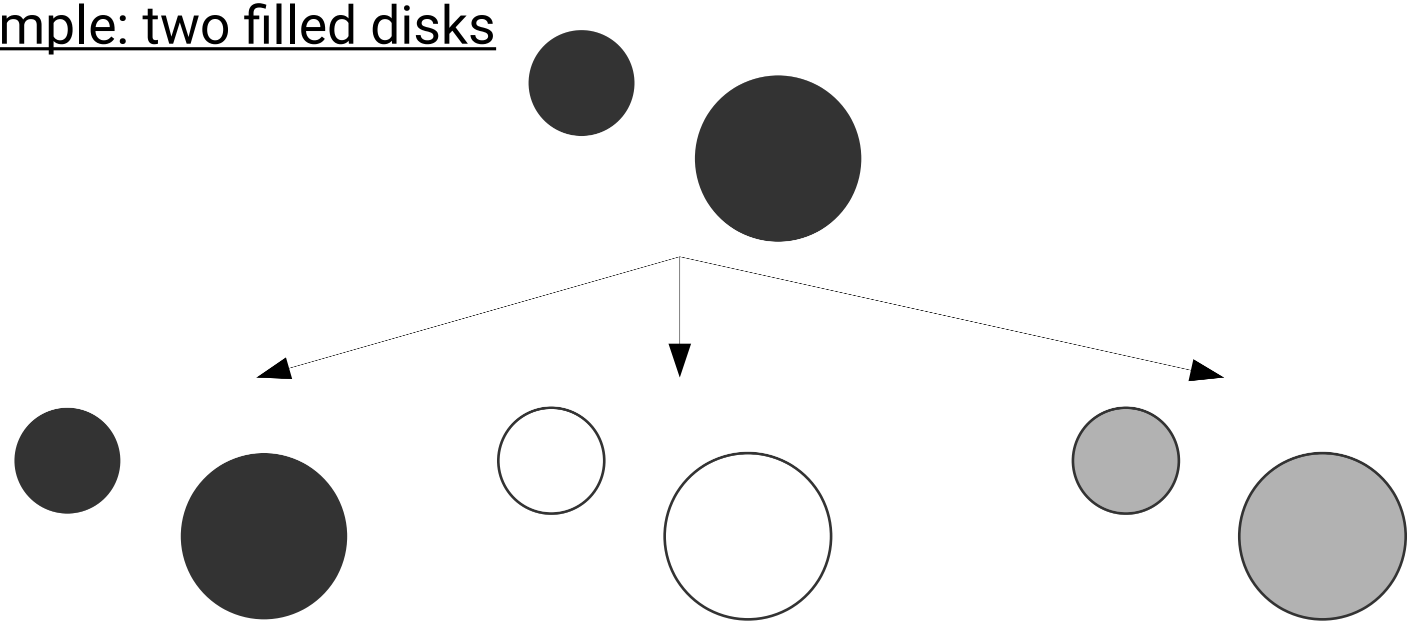
<u>Color charge senstive variable</u> → constituent multiplicity.

We will consider its generalization: **<u>Minkowkski Functionals</u>**

Minkowski Functionals

Minkowski functionals (MFs) are the basis of geometric measure (called valuation) of a given set.
For 2D object analysis, there are three MFs:

Example: two filled disks



Area

$$\mathrm{MF}_0 = \int_A d^2\vec{r}$$

Boundary length

$$\mathrm{MF}_1 = \frac{1}{2\pi}\int_{\partial A} d\vec{r}$$

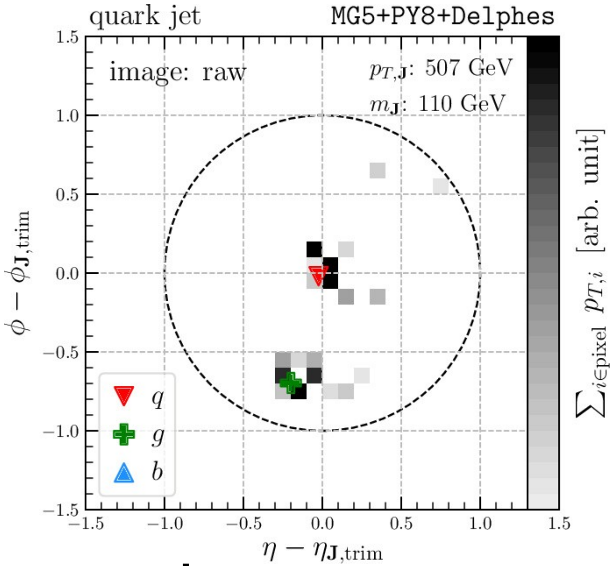
Euler characteristic

(Gauss-Bonnet)

$$\mathrm{MF}_2 = \frac{1}{2\pi^2}\int_{\partial A} \frac{1}{R} d\vec{r}$$

With these three numbers, we can describe all the geometric measures related to this 2D objects. (Hadwiger's theorem)

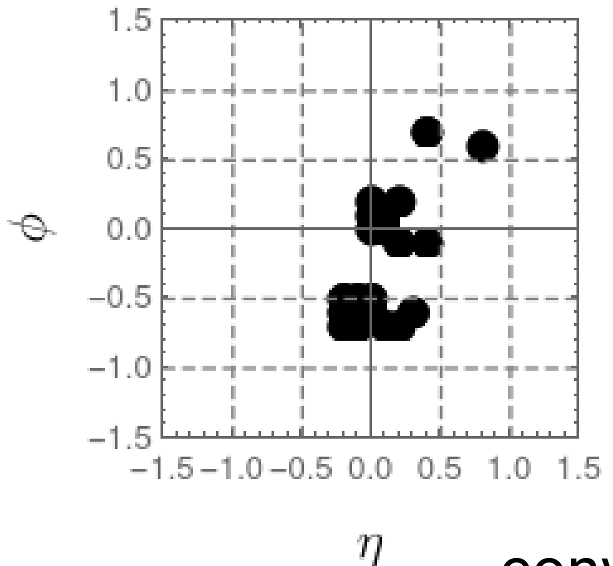
Mathematical Morphology: Minkowski Functionals and Dilation



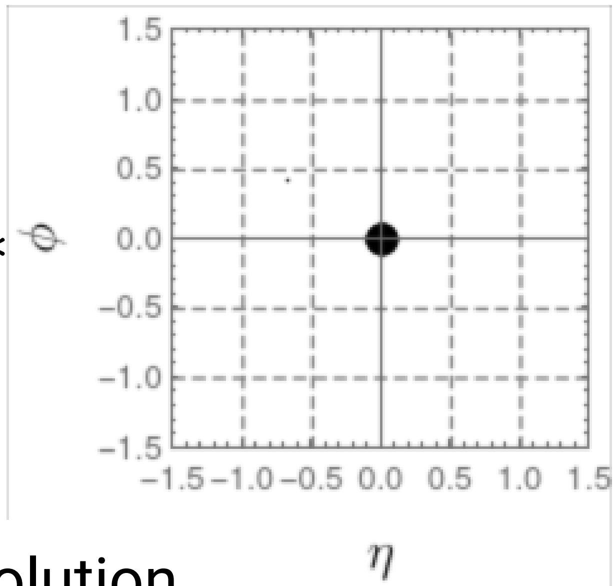
Binarize

For the jet constituents analysis,
we **binarize** the points using energy cutoffs and apply **dilation** on the binary image.

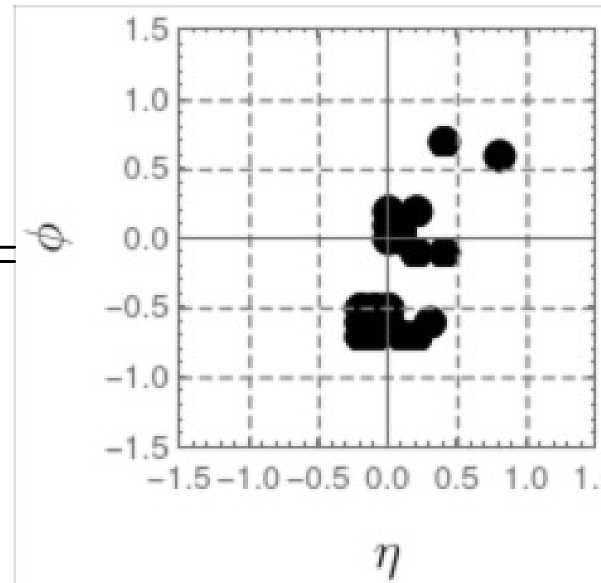
This morphological operation may be regarded as coarse-graining,
and it will allow us to systematically analyze the **geometry of jet constituents** when we use this together with the MFs.



$*$

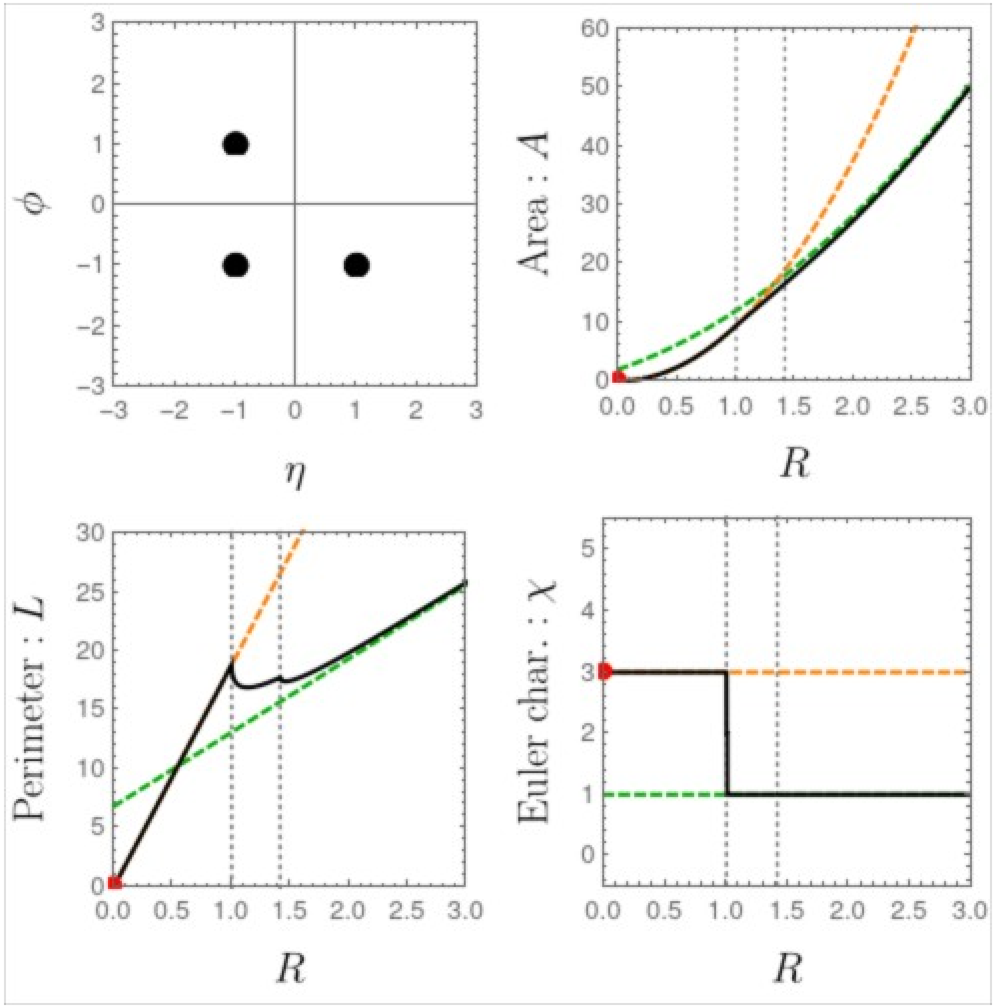


$=$



convolution

Mathematical Morphology and Minkowski Functionals



If we have more constituents, such behavior changes may happen multiple times.

Start:

Cech complex: three points

First change happens when the nearest-neighbor meets

$$R = 1$$

Cech complex: an L-shaped line

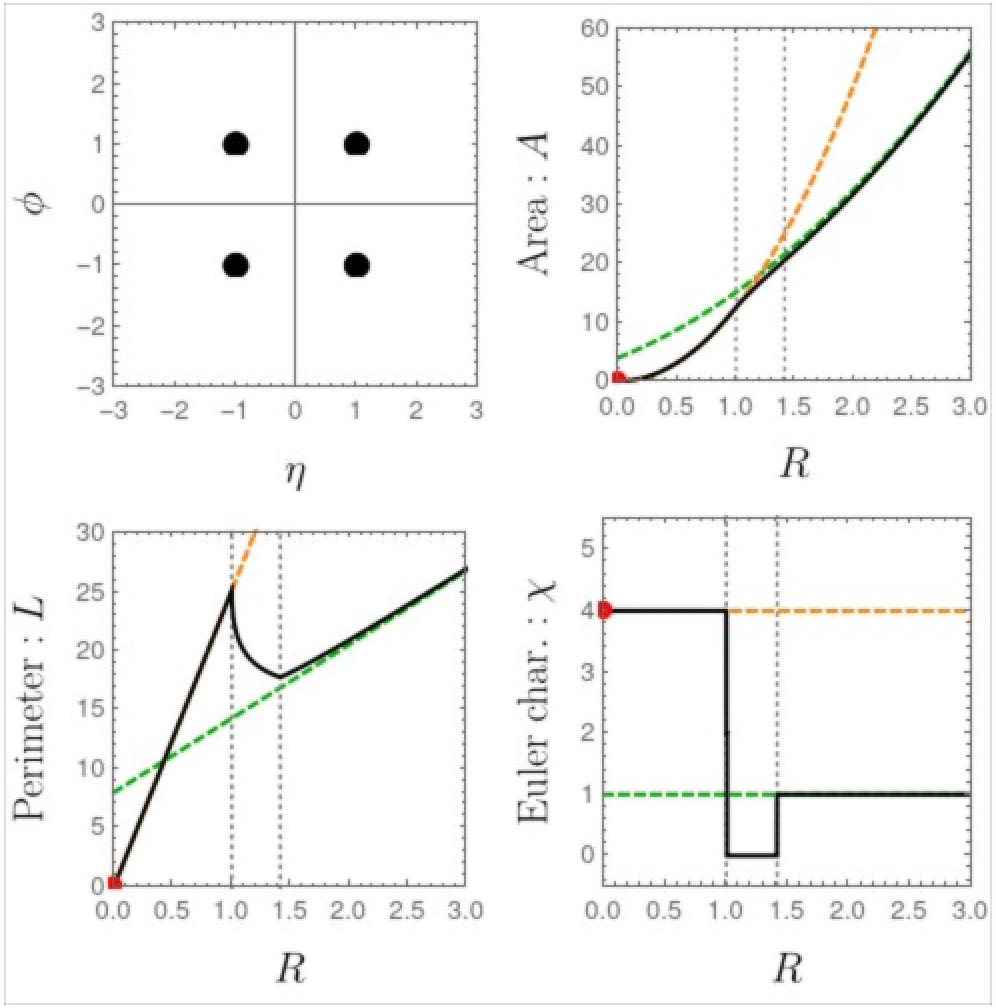
Second change happens when the next nearest-neighbor meets

$$R = \sqrt{2}$$

Cech complex: a right triangle

Orange: asymtote as r → 0
Green: asymtote as r → infinity

Mathematical Morphology and Minkowski Functionals



Orange: asymtote as r → 0
Green: asymtote as r → infinity

During the dilation, some peculiar topological structures may appear. For example, when a **hole** appears, the Euler characteristic can record that clearly.

Start: four constituents

$$\chi = 4$$

Cech complex: four dots

Hole appears:
cancels Euler characteristic by 1.

$$\chi = 1 - 1 = 0$$

Cech complex: a square

Hole disappears:

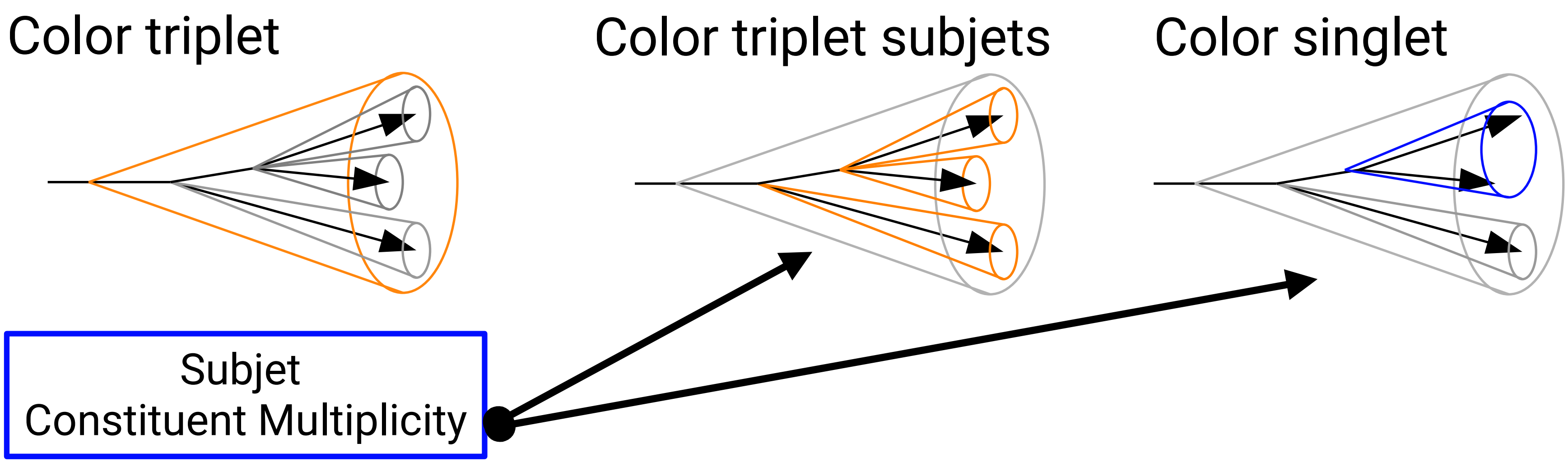
$$\chi = 1$$

Cech complex: a filled square

The topology of the jet constituents can be analyzed.

Subjet color charges

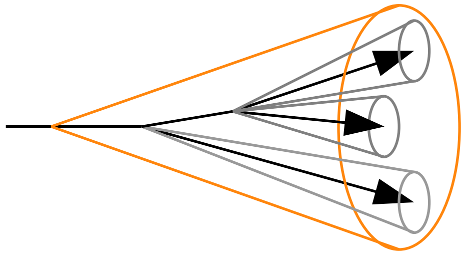
Minkowski functionals will deal with jet topology and color charges.
How about subjet color charges?



Subjet color charges

Minkowski functionals will deal with jet topology and color charges.
How about subjet color charges?

Color triplet

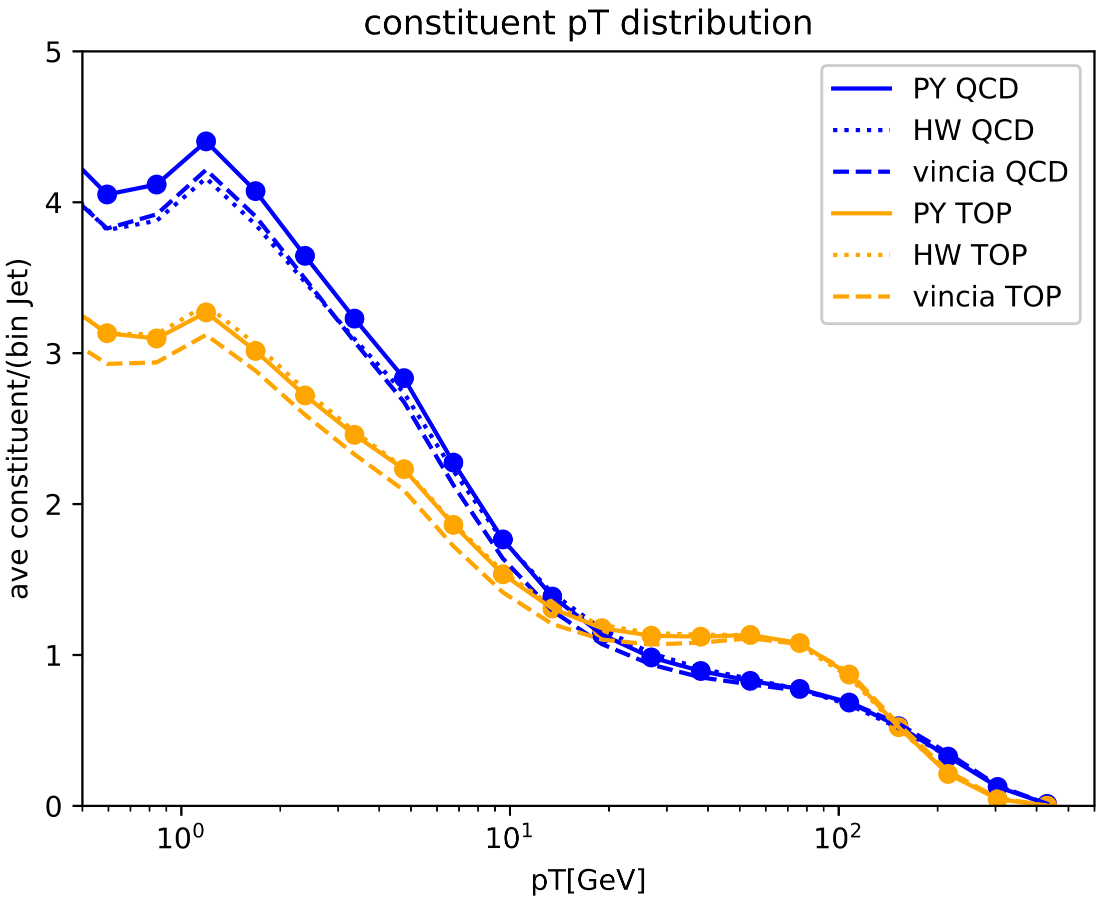


Color triplet subjets

Color singlet

Subjet
Constituent Multiplicity

Constituent PT
histogram:
capturing multiplicity
conditioned on pt



Each subjet has different pt so this also provides extra information.

(Combined) Analysis Model

Jet Kinematics
(PT, mass, ...)

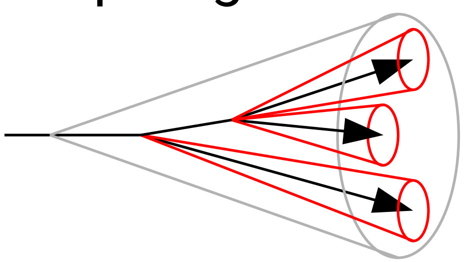
Generalization of
Constituent Multiplicity:
Minkowski Functionals
(Euler Char., Length, Area)

Two-Point
Energy Correlations S2
(Relation Network)

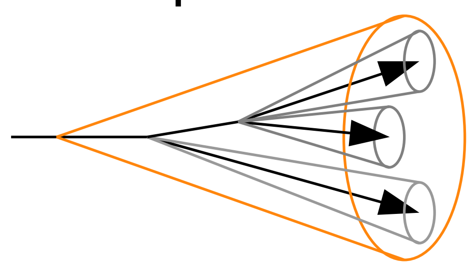
Subjet
Constituent Multiplicity
+ constituent PT histogram

We will consider a NN analysing all these features.
All Top jet features below are covered by these inputs!

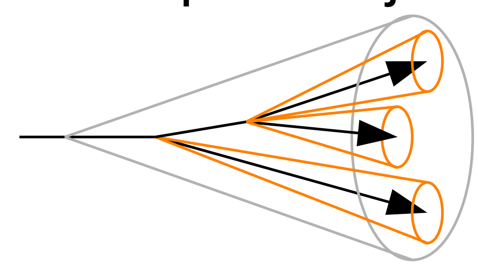
Three-prong



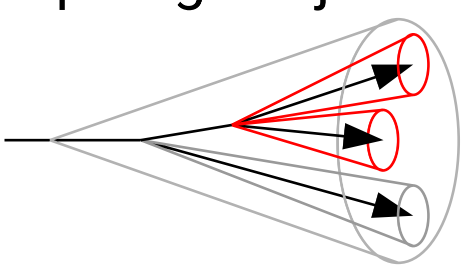
Color triplet



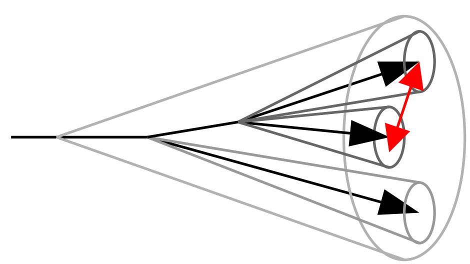
Color triplet subjets



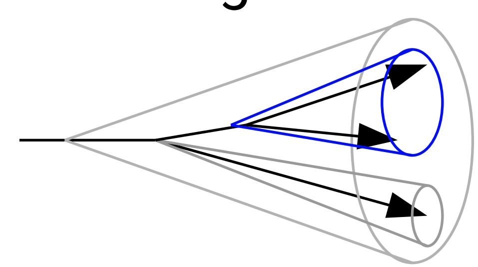
Two-prong subjet inside



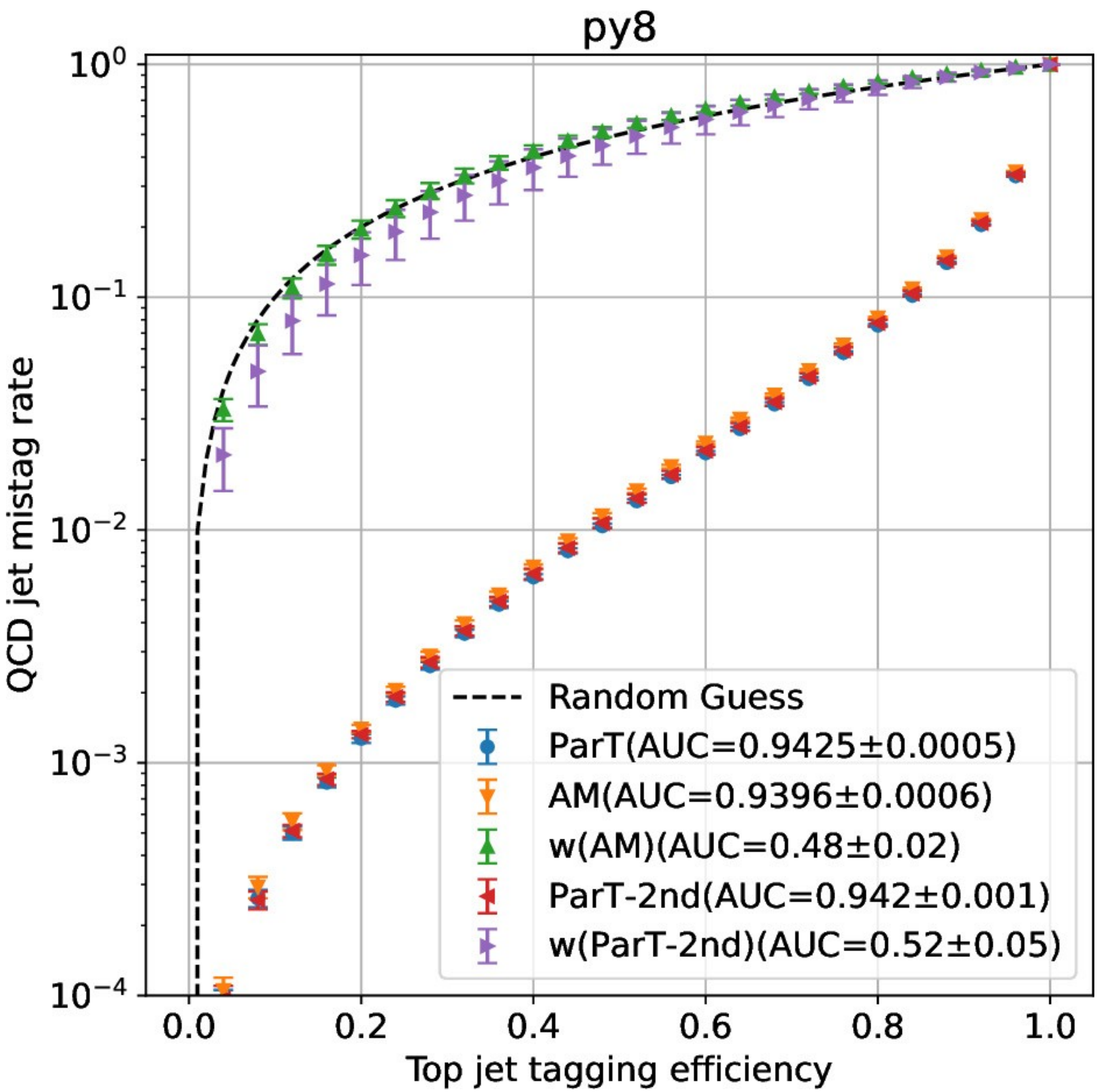
Color connection



Color singlet



ROC curve

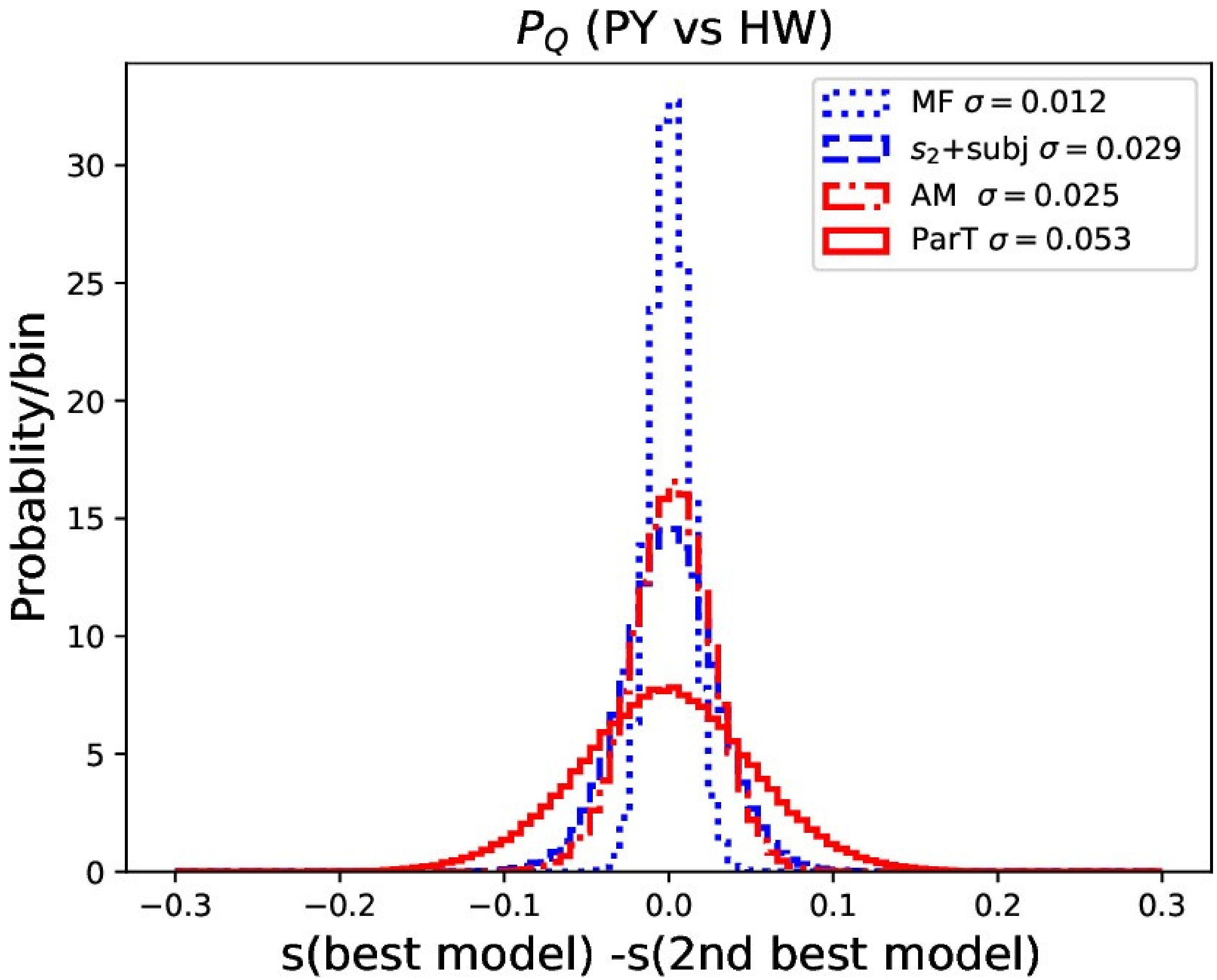


We compare the tagging performance of our analysis model to Particle Transformer working on pixellated jet constituents with HCAL resolution scale (0.1)

ROC curves are almost the same!

Low variance!

One Advantage of using high-level feature based networks is low variance of training compared to low-level feature based networks.



Less training uncertainty in classifier training.

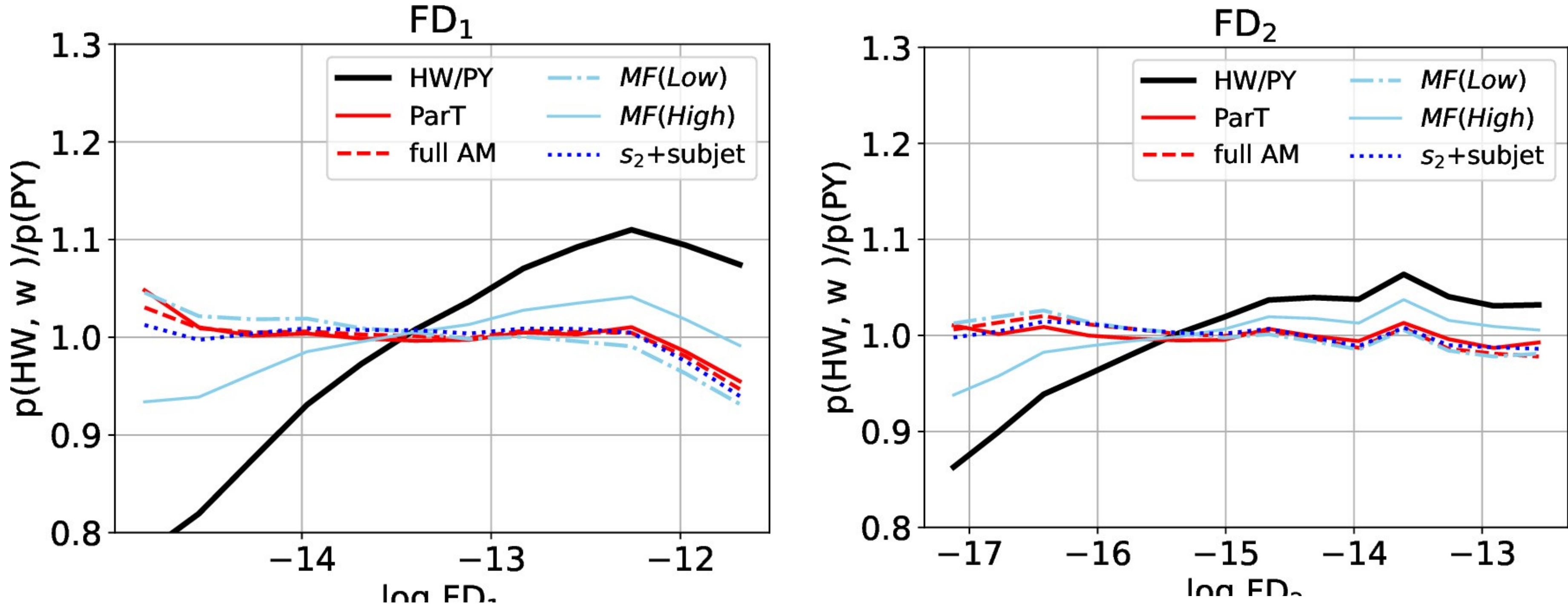
Classifier-based sample reweighting

Low variance - high performance classifier is especially useful when we use classifier for reweighting MC samples for the calibration.

$$\hat{y}(x) = \frac{1}{1 + \frac{p_{\text{Data}}(x)}{p_{\text{MC}}(x)}}$$

Less training uncertainty on likelihood ratio estimation
→ more accurate reweighing! Work in progress...

Reweighted energy flow polynomial distributions important in top tagging (found by DisCo method: 2212.00046)



Conclusion

- We introduced an analysis model for jet classfication using two-point energy correlations and Minkowski functionals.
- We showed than this High-Level Feature based classifier shows competetive performance compared to the state-of-the-art classifiers such as ParticleNet and ParticleTransformers. at HCAL resolution scale,
- Our method is more constrained setup than those SotA methods without losing tagging performance much, we have less training uncertainty.
- Less training uncertainty is valuable especially when using classifier as density-ratio estimator, and using it for re-weighting Monte Carlo generated samples.

Backups

IRC-safe energy correlator based Neural Networks

Graph Networks → IRC-safe energy correlator based Networks

Relation Network → (IRC safety) → Relation Network

Relation Network (Graph Networks)

Utilizes edge features

$$F[\sum_{i,j\in J} \phi^e(p_i, p_j)]$$

Raposo, et al. (1702.05068),
Santoro, et al. (1706.01427)

Relation Network (IRC-safe energy correlator based Networks)

Utilizes two-point energy correlation

$$F[\sum_{i,j\in J} p_{T,i} p_{T,j} \phi^e(R_{ij})]$$

Chakraborty, **SHL**, Nojiri, and Takeuchi (2003.11787)

Deep Sets (Particle Flow Network) → (IRC safety) → Energy Flow Network

Deep Sets (Particle Flow Network)

Utilizes vertex features

$$F[\sum_{i\in J} \phi^v(p_i)]$$

Zaheer, et al. (1703.06114)

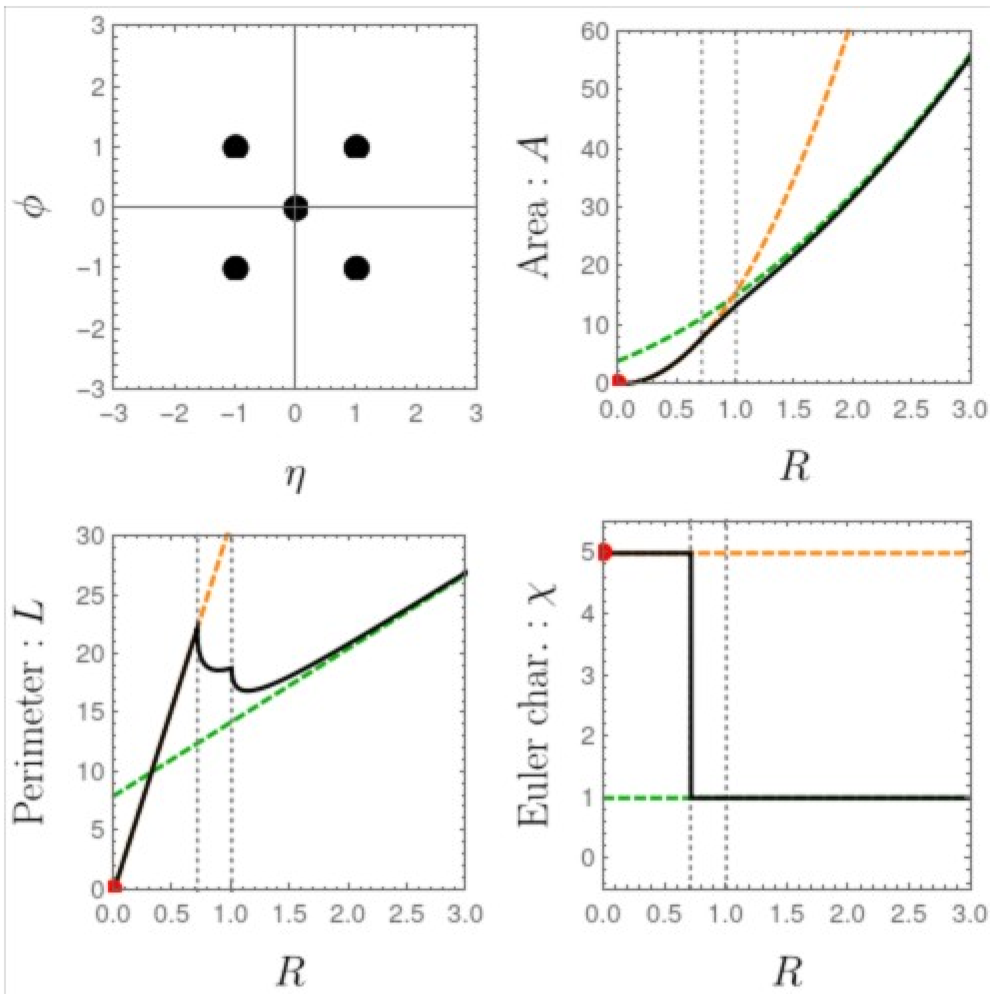
Energy Flow Network

Utilizes one-point energy correlation:
permutation invariant
energy-weighted linear sum of angular map

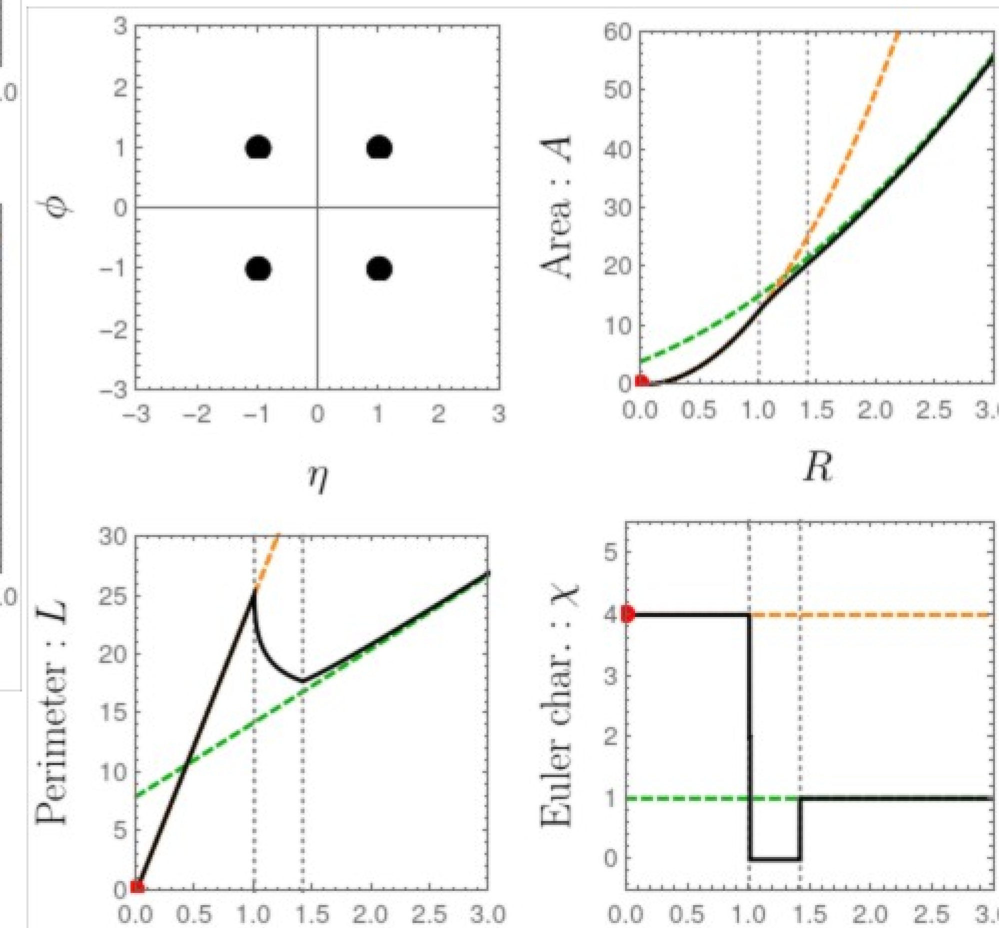
$$F[\sum_{i\in J} p_{T,i} \phi^v(\vec{R}_i)]$$

Komiske, Metodiev, and Thaler (1810.05165)

Mathematical Morphology and Minkowski Functionals



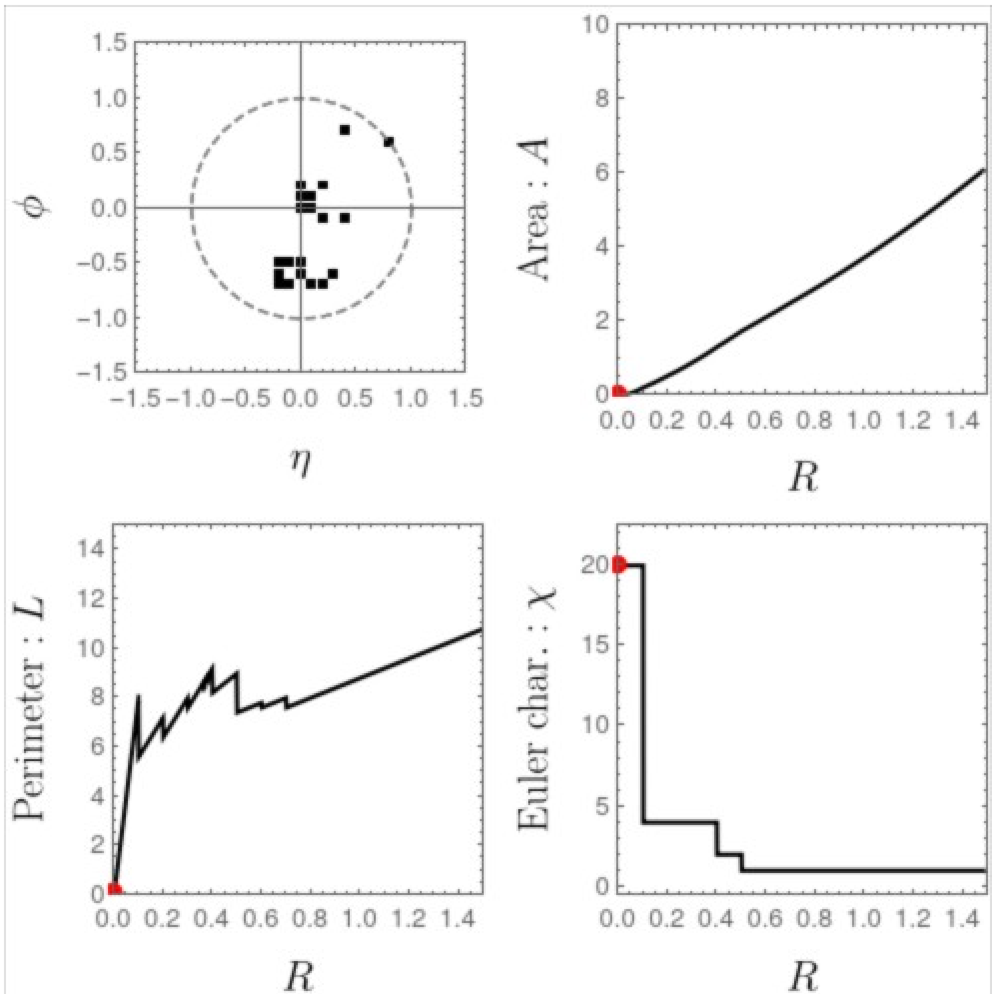
A similar point distribution with a different topology may exist, but the MFs could capture the fine difference.

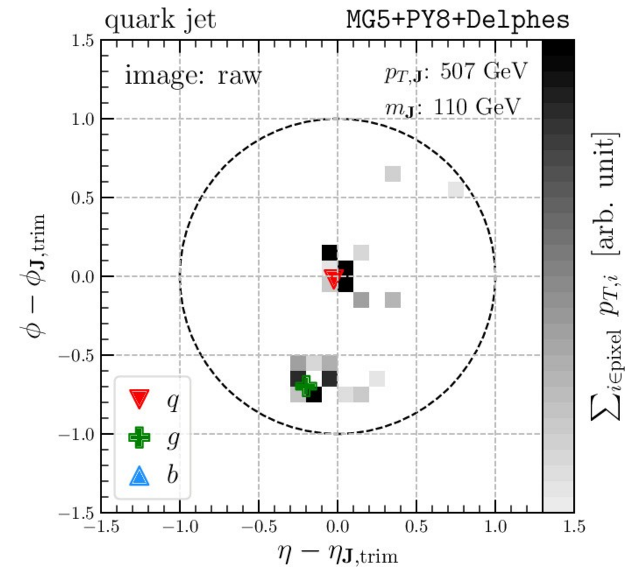


Orange: asymtote as r → 0
Green: asymtote as r → infinity

Morphological Analysis on (pixellated) Jet Image

In the case of the analysis on jet images,
we use squares for the dilation in order to preserve underlying geometry of the data.

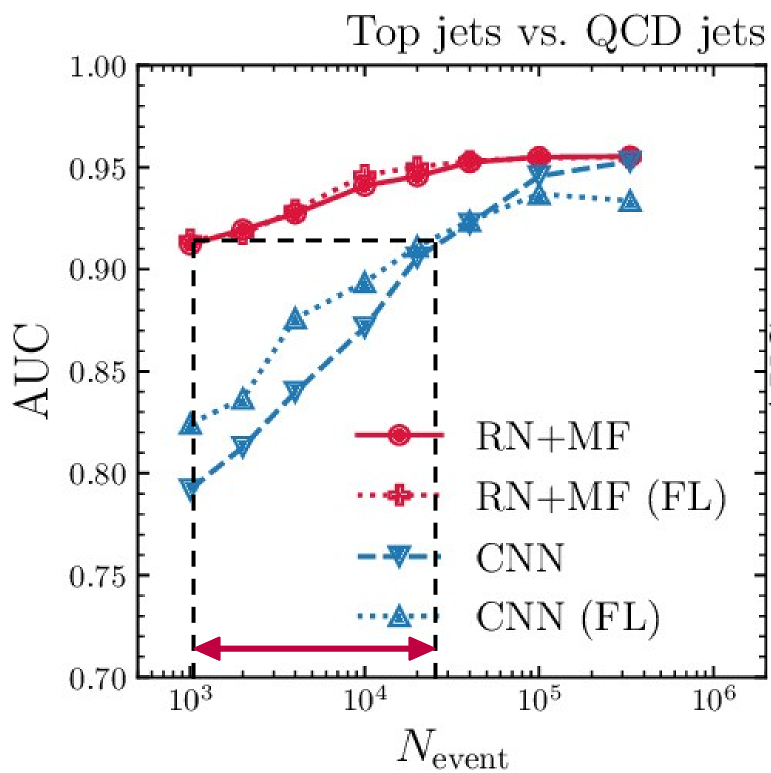


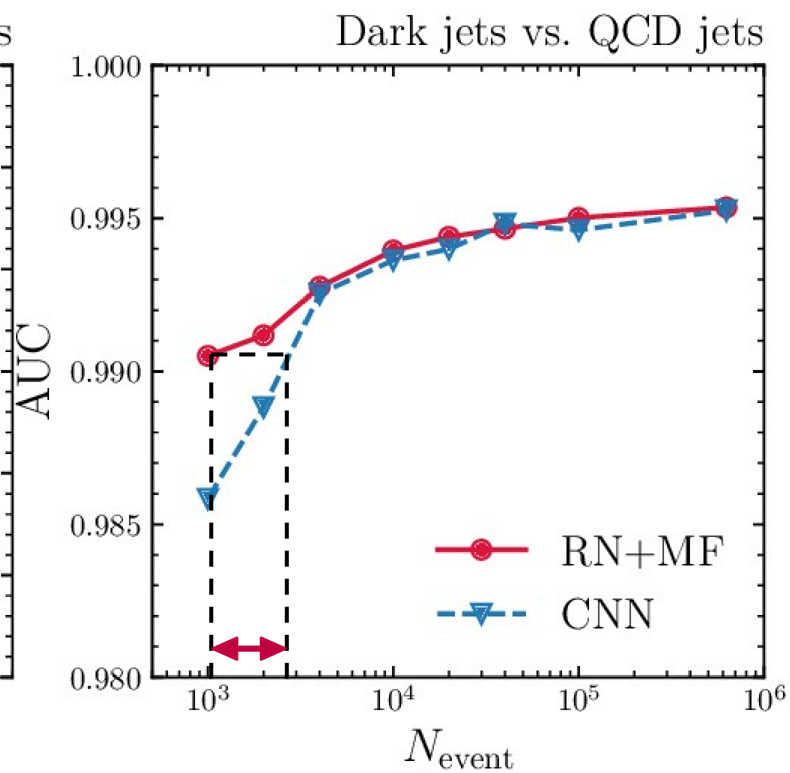


One interesting property of this setup is that
all the calculation steps of the MFs can be written in terms of **discrete convolutions.**

Constrained Architectures and Low-Shot Learning

We showed that our RN+MF has comparable performance to the CNN.
Moreover, it has advantages when **<u>the dataset is small</u>**,
because RN+MF is more constrained architecture than the CNN.





RN+MF is much less sample-demanding thanks to its constraints.

A smaller factor 3 gap is here, but this example has an exclusive phase space region parameterized by MFs.

Sample description

- All the SM jets are simulated by MG5+pythia8.3
- Dark jets are simulated by pythia8.3
- Top jet vs. QCD jet
  - Jet constituents: Delphes EFlows
  - PT $\in$ [500, 600] GeV
  - Mass $\in$ [150, 200] GeV
  - Leading pt anti-kt jets with radius 1.0
  - For top jets, all the originating b-quarks and quarks must be within jet radius 1.0 from the jet center.