Quantum Chromodynamics

Gregory Soyez

IPhT (CNRS, CEA Saclay) and CERN

CERN-FermiLab HCP Summer School 2023

CERN, August 22-31 2023

In the standard model, QCD is
the fundamental theory of strong interactions

Our journey together

QCD exhibits many rich structures — reward: fun/exciting behaviours

QCD exhibits mand challenging strutures — reward: precision/accuracy

Part I: QCD basics

Lagrangian and degrees of freedom

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu$$

SU(3) gauge theory with fundamental d.o.f.

quarks (matter)
fundamental representation
3 **colours** (red, green, blue)

gluons (vectors)
adjoint representation
8 **colours** $(8 = 3^2 - 1)$

Lagrangian and degrees of freedom

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$D_\mu = \partial_\mu + igT^aA^a_\mu \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu$$

quarks carry a **flavour** index (f) + are **charged** (interact with photons)

6 quarks
3 families

q	first	second	third
$\frac{2}{3}$	u (up) ($m \approx 0$)	c (charm) ($m \approx 1.3$ GeV)	t (top) ($m \approx 173$ GeV)
-$\frac{1}{3}$	d (down) ($m \approx 0$)	s (strange) ($m \approx 0$)	b (bottom) ($m \approx 4.2$ GeV)

Lagrangian and degrees of freedom

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu \qquad\qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu$$

quarks carry a **flavour** index (f) + are **charged** (interact with photons)

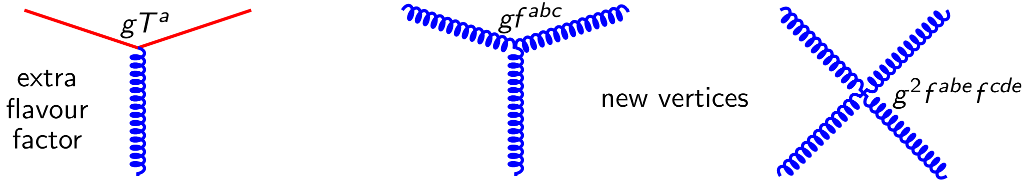
6 quarks
3 families
rich/complex structures

q	first	second	third
$\frac{2}{3}$	u (up) ($m \approx 0$)	c (charm) ($m \approx 1.3$ GeV)	t (top) ($m \approx 173$ GeV)
-$\frac{1}{3}$	d (down) ($m \approx 0$)	s (strange) ($m \approx 0$)	b (bottom) ($m \approx 4.2$ GeV)

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu \qquad\qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu$$

Non abelian theory : gluons interact! (complexity!)



Lagrangian and degrees of freedom

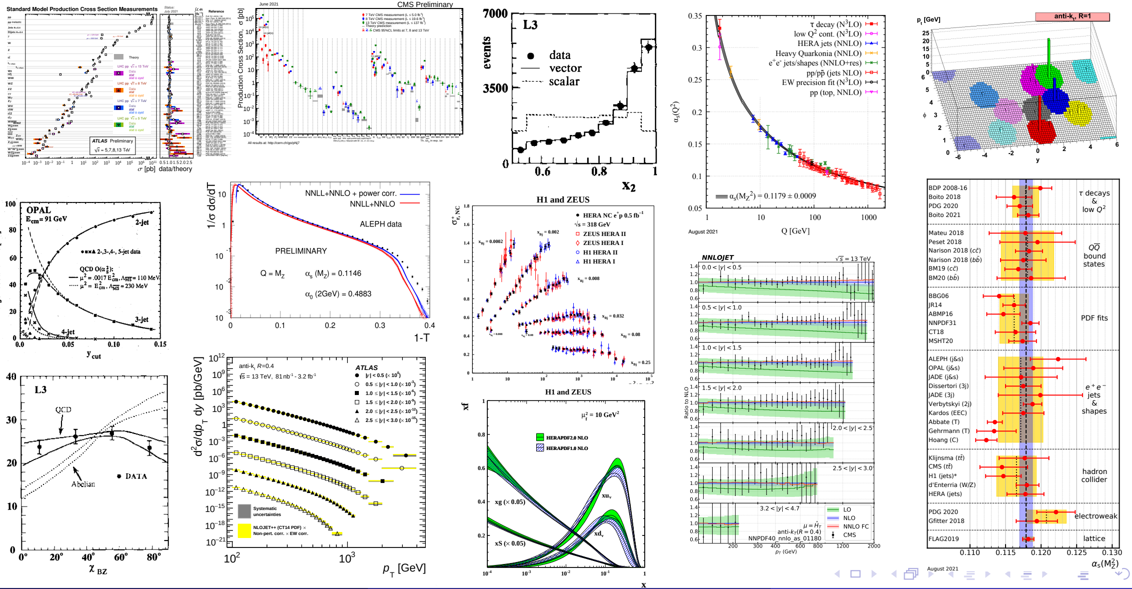
$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$D_\mu = \partial_\mu + igT^a A^a_\mu \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$$

$\theta F\tilde{F}$ term:

- CP violating
- corresponds to the **QCD axion** (link to BSM)
- experimental limit: $|\theta| \lesssim 10^{-10}$

Rich phenomenology



Topics covered (tentative)

- asymptotic freedom (UV divergences)

Topics covered (tentative)

- asymptotic freedom (UV divergences)
- *ee* $\rightarrow$ QCD
  - basic "validation" of QCD
  - structure of IR divergences
  - factorisation
  - IRC safety
  - resummations
  - jets

Topics covered (tentative)

- asymptotic freedom (UV divergences)
- $ee \to$ QCD
  - basic "validation" of QCD
  - structure of IR divergences
  - factorisation
  - IRC safety
  - resummations
  - jets
- QCD in deep-inelastic scattering
  PDFs (IR divergences)

Topics covered (tentative)

- asymptotic freedom (UV divergences)
- *ee* $\to$ QCD
  - basic “validation” of QCD
  - structure of IR divergences
  - factorisation
  - IRC safety
  - resummations
  - jets
- QCD in deep-inelastic scattering
  PDFs (IR divergences)
- *pp* collisions
  - collision anatomy/factorisation
  - basic kinematics
  - fixed-order complexity
  - quest for precision

Topics covered (tentative)

- asymptotic freedom (UV divergences)
- *ee* → QCD
  - basic "validation" of QCD
  - structure of IR divergences
  - factorisation
  - IRC safety
  - resummations
  - jets
- QCD in deep-inelastic scattering
  PDFs (IR divergences)
- *pp* collisions
  - collision anatomy/factorisation
  - basic kinematics
  - fixed-order complexity
  - quest for precision
- Monte Carlo generators
  - basic concept
  - fixed-order generators
  - parton showers
  - general-purpose generators

Topics covered (tentative)

- asymptotic freedom (UV divergences)
- $ee \to$ QCD
  - basic "validation" of QCD
  - structure of IR divergences
  - factorisation
  - IRC safety
  - resummations
  - jets
- QCD in deep-inelastic scattering
  PDFs (IR divergences)
- pp collisions
  - collision anatomy/factorisation
  - basic kinematics
  - fixed-order complexity
  - quest for precision
- Monte Carlo generators
  - basic concept
  - fixed-order generators
  - parton showers
  - general-purpose generators
- Outlook: "funny structures" in QCD

Before we get started...

Stop me whenever you want!

Better if you understsand even if it means not covering everything

Use your brain! (I will try to ask questions)

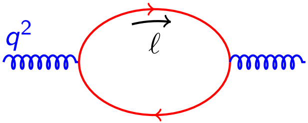
The philosophy to keep in mind is

- Why is this concept important/non-trivial?
- What are the past/current/future challenges?

I am happy/available to discuss during discussion sessions (except Friday/Saturday)

Part II: asymptotic freedom

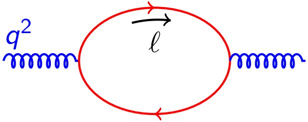
QCD (like QED) is a **renormalisable** gauge theory



$$: \quad \int^{\Lambda^2} \frac{d^4\ell}{(2\pi)^2}\delta(\ell^2)\delta((q-\ell)^2) \approx \beta_0\alpha_s \log\frac{\Lambda^2}{q^2}$$

UV renormalisation

QCD (like QED) is a **renormalisable** gauge theory



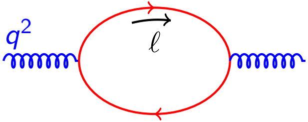
: $$\int^{\Lambda^2} \frac{d^4\ell}{(2\pi)^2}\delta(\ell^2)\delta((q-\ell)^2) \approx \beta_0\alpha_s \log\frac{\Lambda^2}{q^2}$$

Idea: absorb the UV (short distance) divergence in the definition of the coupling

$$\alpha_s^{\text{``bare''}} \to \alpha_s(q^2) = \alpha_s^{\text{``bare''}} + \beta_0(\alpha_s^{\text{``bare''}})^2 \log\frac{\Lambda^2}{q^2} + \dots$$

UV renormalisation

QCD (like QED) is a **renormalisable** gauge theory



: $$\int^{\Lambda^2} \frac{d^4\ell}{(2\pi)^2}\delta(\ell^2)\delta((q-\ell)^2) \approx \beta_0\alpha_s \log\frac{\Lambda^2}{q^2}$$

Idea: absorb the UV (short distance) divergence in the definition of the coupling

$$\alpha_s^{\text{“bare”}} \to \alpha_s(q^2) = \alpha_s^{\text{“bare”}} + \beta_0(\alpha_s^{\text{“bare”}})^2 \log\frac{\Lambda^2}{q^2} + \dots$$

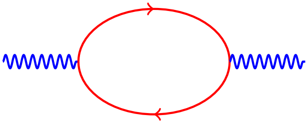
Renormalisation-group equation (consistency condition)

$$\mu^2\partial_{\mu^2}\alpha_s(\mu^2) = -\beta_0\alpha_s^2(\mu^2) + \dots \overset{\text{all orders}}{=} \beta(\alpha_s) \qquad (\beta \text{ function})$$

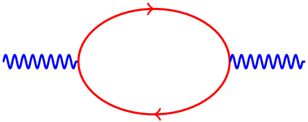
Generic renormalisation strategy: absorb UV divergences in physical parameters of the Lagrangian (typically coupling and masses)

Asymptotic freedom (1/3)

QED



QED

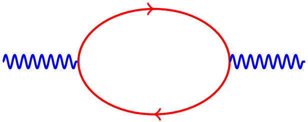


$\beta_{\text{QED}} > 0$

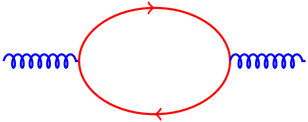
$\mu^2 \nearrow \quad \Rightarrow \quad \alpha_{\text{elm}} \nearrow$

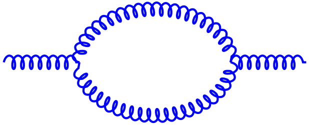
vacuum fluctuations screen electric charge

QED



QCD





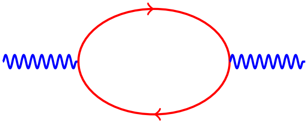
$$\beta_{\mathrm{QED}} > 0$$

$$\mu^2 \nearrow \quad \Rightarrow \quad \alpha_{\mathrm{elm}} \nearrow$$

vacuum fluctuations screen electric charge

Asymptotic freedom (1/3)

QED

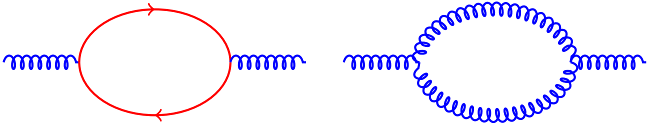


$$\beta_{\text{QED}} > 0$$

$$\mu^2 \nearrow \quad \Rightarrow \quad \alpha_{\text{elm}} \nearrow$$

vacuum fluctuations screen electric charge

QCD



$$\beta_{\text{QCD}} < 0 \qquad (\beta_0 = \frac{11 C_A - 4 n_f T_R}{12\pi})$$

$$\mu^2 \nearrow \quad \Rightarrow \quad \alpha_s \searrow$$

ASYMPTOTIC FREEDOM

Asymptotic freedom (2/3)

- **QCD becomes increasingly perturbative at larger energy scales**

Asymptotic freedom (2/3)

- **QCD becomes increasingly perturbative at larger energy scales**
- in particular: α_s does not diverge in the UV (chance of "remaining" a fundamental theory)

- **QCD becomes increasingly perturbative at larger energy scales**
- in particular: α_s does not diverge in the UV (chance of "remaining" a fundamental theory)
- At 1 loop:

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + 2\alpha_s(\mu_0^2)\beta_0 \log\frac{\mu}{\mu_0^2}} = \frac{1}{\beta_0 \log\frac{\mu}{\Lambda_{\text{QCD}}}}$$

$\Lambda_{\text{QCD}} \equiv$ Landau pole ($\sim 100 - 200$ MeV): α_s diverges in the IR
QCD becomes non-perturbative

Asymptotic freedom (2/3)

- **QCD becomes increasingly perturbative at larger energy scales**
- in particular: α_s does not diverge in the UV (chance of "remaining" a fundamental theory)
- At 1 loop:

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + 2\alpha_s(\mu_0^2)\beta_0 \log \frac{\mu}{\mu_0^2}} = \frac{1}{\beta_0 \log \frac{\mu}{\Lambda_{\mathrm{QCD}}}}$$

$\Lambda_{\mathrm{QCD}} \equiv$ Landau pole ($\sim 100 - 200$ MeV): α_s diverges in the IR
QCD becomes non-perturbative

- evaluating a process $\Rightarrow$ choosing a **renormalisation scale**, μ_R^2, to evaluate α_s

typically: (commensurate with) the hardest scale in the process

Asymptotic freedom (2/3)

- **QCD becomes increasingly perturbative at larger energy scales**
- in particular: α_s does not diverge in the UV (chance of "remaining" a fundamental theory)
- At 1 loop:

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + 2\alpha_s(\mu_0^2)\beta_0 \log\frac{\mu}{\mu_0^2}} = \frac{1}{\beta_0 \log\frac{\mu}{\Lambda_{\text{QCD}}}}$$

$\Lambda_{\text{QCD}} \equiv$ Landau pole ($\sim 100 - 200$ MeV): α_s diverges in the IR
QCD becomes non-perturbative

- evaluating a process $\Rightarrow$ choosing a **renormalisation scale**, μ_R^2, to evaluate α_s

typically: (commensurate with) the hardest scale in the process

- if one knows $\beta(\alpha_s)$, ... to all orders the choice of μ_R^2 is irrelevant

Asymptotic freedom (2/3)

- **QCD becomes increasingly perturbative at larger energy scales**
- in particular: α_s does not diverge in the UV (chance of "remaining" a fundamental theory)
- At 1 loop:

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + 2\alpha_s(\mu_0^2)\beta_0 \log \frac{\mu}{\mu_0^2}} = \frac{1}{\beta_0 \log \frac{\mu}{\Lambda_{\text{QCD}}}}$$

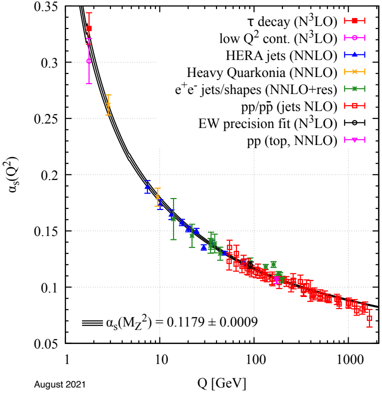
  $\Lambda_{\text{QCD}} \equiv$ Landau pole ($\sim 100 - 200$ MeV): α_s diverges in the IR
  QCD becomes non-perturbative

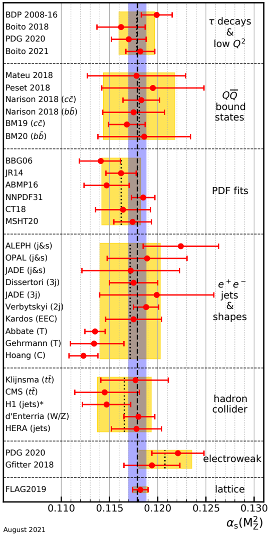
- evaluating a process $\Rightarrow$ choosing a **renormalisation scale**, μ_R^2, to evaluate α_s

  typically: (commensurate with) the hardest scale in the process

- if one knows $\beta(\alpha_s)$, ... to all orders the choice of μ_R^2 is irrelevant
- at a given fixed order α_s^n, leftover effects of $\mathcal{O}(\alpha_s^{n+1})$ (renormalisation scale uncertainty)

Asymptotic freedom (3/3)





QCD β known until 5 loops (β_4)

Theory gives dependence on scale

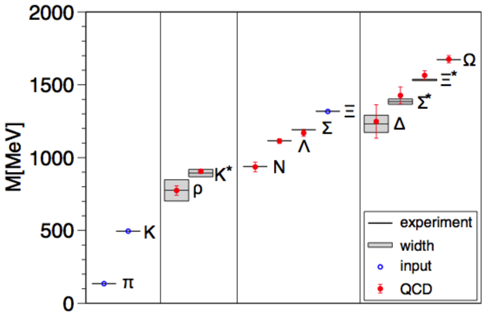
Measurement needed for $\alpha_s(Q_0)$

Several ways to do this!

Part III: hadrons and confinement

(VERY) Brief overview

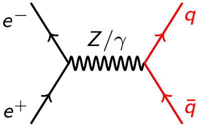
- In the IR, QCD becomes non perturbative
- Confinement property:
  one observes colourless hadrons (mesons& baryons)
  not quarks and gluons
- Generally poorly understood



- Typical approach: Lattice QCD. Good for static questions, dynamics more delicate
- Some analytic *models*
- Some numerical (Monte-Carlo) *models* (more later)

Part IV: e^+e^- collisions
basics

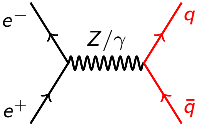
$ee \to \gamma/Z \to q\bar{q}$



$$\sigma_{e^+e^-\to q\bar{q}} = N_c \left(\sum_f e_f^2\right) \sigma_{e^+e^-\to\mu^+\mu^-}$$

What do we learn from this?

$ee \to \gamma/Z \to q\bar{q}$

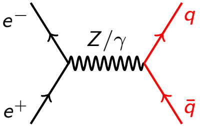


$$\sigma_{e^+e^-\to q\bar{q}} = N_c \left(\sum_f e_f^2\right) \sigma_{e^+e^-\to\mu^+\mu^-}$$

What do we learn from this?

- factor $\sum_f e_f^2$: count the number of "active" quarks
- factor N_c: count the number of colours (for each quark)

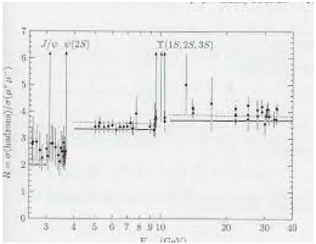
$ee \to \gamma/Z \to q\bar{q}$

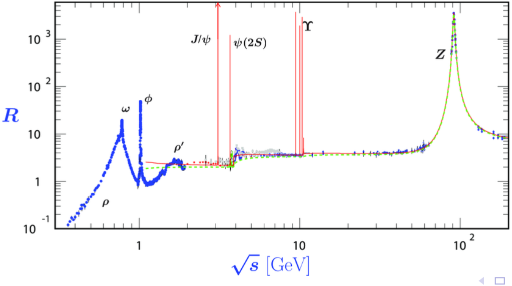


$$\sigma_{e^+e^-\to q\bar{q}} = N_c \left(\sum_f e_f^2\right) \sigma_{e^+e^-\to\mu^+\mu^-}$$

What do we learn from this?

- factor $\sum_f e_f^2$: count the number of "active" quarks
- factor N_c: count the number of colours (for each quark)





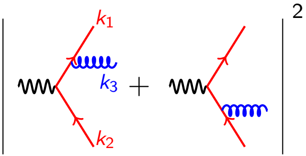
$R \overset{n_f=3}{=} 2$

$\overset{n_f=4}{=} \frac{10}{3}$

$\overset{n_f=5}{=} \frac{11}{3}$

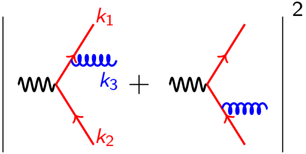
Is this exact?

$ee \to \gamma/Z \to q\bar{q}g$



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

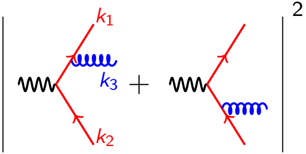
$ee \to \gamma/Z \to q\bar{q}g$



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

- $e_q^2 N_c$: as before (electromagnetic + $N_c = 3$ flavours)

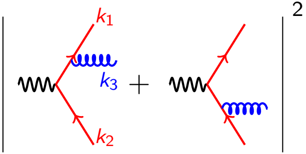
$ee \to \gamma/Z \to q\bar{q}g$



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\text{elm}}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

- $e_q^2 N_c$: as before (electromagnetic + $N_c = 3$ flavours)
- α_s: QCD interaction! We usually take $\alpha_s = \alpha_s(\sqrt{s})$

$ee \to \gamma/Z \to q\bar{q}g$

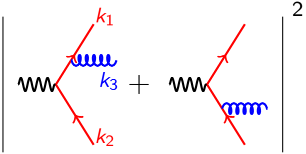


$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

- $e_q^2 N_c$: as before (electromagnetic + $N_c = 3$ flavours)
- α_s: QCD interaction! We usually take $\alpha_s = \alpha_s(\sqrt{s})$
- C_F: fundamental SU(3) constant (Casimir)

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3} \qquad C_A = N_c = 3 \qquad (T^a_{AC} T^a_{CB} = C_F \delta_{AB};\ f^{abc} f^{abd} = C_A \delta^{cd})$$

$ee \to \gamma/Z \to q\bar{q}g$

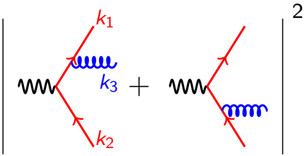


$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\text{elm}}}{s}\, e_q^2 N_c \alpha_s C_F \, \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

- $e_q^2 N_c$: as before (electromagnetic + $N_c = 3$ flavours)
- α_s: QCD interaction! We usually take $\alpha_s = \alpha_s(\sqrt{s})$
- C_F: fundamental SU(3) constant (Casimir)

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3} \qquad C_A = N_c = 3 \qquad (T^a_{AC} T^a_{CB} = C_F \delta_{AB};\ f^{abc} f^{abd} = C_A \delta^{cd})$$

- $\frac{\cdots}{\cdots}$: kinematic factor (more about this later)



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

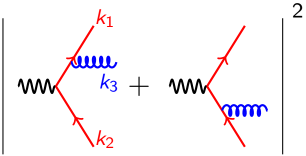
- $e_q^2 N_c$: as before (electromagnetic + $N_c = 3$ flavours)
- α_s: QCD interaction! We usually take $\alpha_s = \alpha_s(\sqrt{s})$
- C_F: fundamental SU(3) constant (Casimir)

$$C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3} \qquad C_A = N_c = 3 \qquad (T^a_{AC}T^a_{CB} = C_F\delta_{AB};\ f^{abc}f^{abd} = C_A\delta^{cd})$$

Helpful rewrite: $x_i = \frac{2E_i}{\sqrt{s}}$
$x_1 + x_2 + x_3 = 2,\ 0 \le x_i \le 1$

$$\frac{d^2\sigma}{dx_1 dx_2} = (\sigma_{ee\to\mu\mu}) \times (e_q^2 N_c) \times \frac{\alpha_s C_F}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

$ee \to \gamma/Z \to q\bar{q}g$

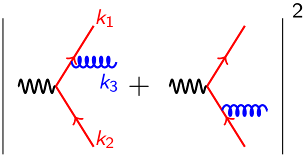


$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

$$\frac{d\sigma}{dx_1 dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

Does anything look strange/weird/suspicious/odd?

$ee \to \gamma/Z \to q\bar{q}g$



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

$$\frac{d\sigma}{dx_1 dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Does anything look strange/weird/suspicious/odd?

(logarithmic) IR divergences when

- $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$
- $x_1 \to 1$ or $x_2 \to 1$

Part V: e^+e^- collisions

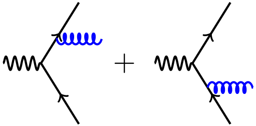
IR behaviour

- We first focus on the simplest observable: the inclusive cross-section $e^+e^- \to$ QCD

$ee \to \gamma/Z \to$ QCD

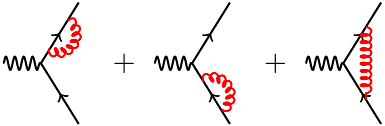
- We first focus on the simplest observable: the inclusive cross-section $e^+e^- \to$ QCD
- Issue: we have missed some diagrams!!

Real



$\mathcal{M} \propto g$, $\mathcal{M}^* \propto g$

Virtual

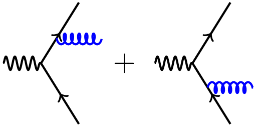


$\mathcal{M} \propto g^2$, $\mathcal{M}^* \propto 1$

$ee \to \gamma/Z \to$ QCD

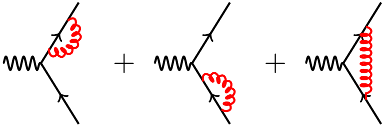
- We first focus on the simplest observable: the inclusive cross-section $e^+e^- \to$ QCD
- Issue: we have missed some diagrams!!

Real



$\mathcal{M} \propto g$, $\mathcal{M}^* \propto g$

Virtual



$\mathcal{M} \propto g^2$, $\mathcal{M}^* \propto 1$

needs a regulator

here: dimreg $d = 4 - 2\varepsilon$

$$\sigma^{(\text{real})}_{q\bar{q}g} = (e_q^2 N_c)\sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon) \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + \cdots\right]$$

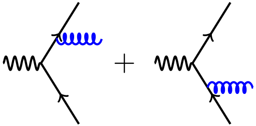
$$\sigma^{(\text{virt})}_{q\bar{q}(g)} = (e_q^2 N_c)\sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon) \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \cdots\right]$$

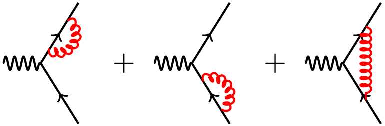
$ee \to \gamma/Z \to$ QCD

- We first focus on the simplest observable: the inclusive cross-section $e^+e^- \to$ QCD
- Issue: we have missed some diagrams!!

Real

Virtual





$\mathcal{M} \propto g$, $\mathcal{M}^* \propto g$

needs a regulator

here: dimreg $d = 4 - 2\varepsilon$

$\mathcal{M} \propto g^2$, $\mathcal{M}^* \propto 1$

$$\sigma^{(\text{real})}_{q\bar{q}g} = (e_q^2 N_c)\sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon)\left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + \cdots\right]$$

$$\sigma^{(\text{virt})}_{q\bar{q}(g)} = (e_q^2 N_c)\sigma_0 \frac{\alpha_s C_F}{2\pi} T(\varepsilon)\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \cdots\right]$$

$$R = \frac{\sigma_{ee\to\text{QCD}}}{\sigma_{ee\to\mu^+\mu^-}} = \left(\sum_f e_f^2\right) N_c \left[1 + \frac{3}{4}\frac{\alpha_s C_F}{\pi} + \mathcal{O}(\alpha_s^2)\right]$$

Fundamental property of (perturbative) QCD

KLN theorem

At each order of the perturbation theory, the divergences of the real and virtual contributions (to the squared amplitude) cancel

Kinoshita-Lee-Nauenberg (QCD) — Bloch-Nordsieck (QED)

Fundamental property of (perturbative) QCD

KLN theorem

At each order of the perturbation theory, the divergences of the real and virtual contributions (to the squared amplitude) cancel

Kinoshita-Lee-Nauenberg (QCD) — Bloch-Nordsieck (QED)

Wonderful! (given enough pen, paper, courage, ...) we can compute R at an arbitrary order in pQCD!

Can we actually compute more than a single number? (at a given $\sqrt{s}$)

Fundamental property of (perturbative) QCD

KLN theorem

At each order of the perturbation theory, the divergences of the real and virtual contributions (to the squared amplitude) cancel

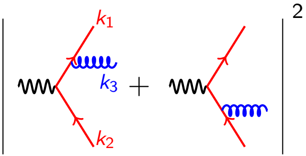
Kinoshita-Lee-Nauenberg (QCD) — Bloch-Nordsieck (QED)

Wonderful! (given enough pen, paper, courage, ...) we can compute R at an arbitrary order in pQCD!

Can we actually compute more than a single number? (at a given $\sqrt{s}$)

Let us first give these divergences a closer look...

Soft and collinear limit



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

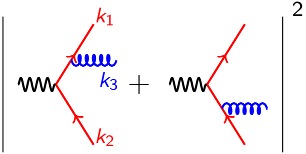
$$\frac{d\sigma}{dx_1\,dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

(logarithmic) IR divergences when

- $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$
- $x_1 \to 1$ or $x_2 \to 1$

When does this happen?

Soft and collinear limit



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

$$\frac{d\sigma}{dx_1\,dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

(logarithmic) IR divergences when

- $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$
- $x_1 \to 1$ or $x_2 \to 1$

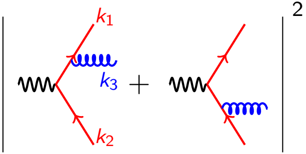
$$k_1.k_3 = E_1 E_3(1-\cos\theta_{13})$$

$$1 - x_2 = \tfrac{1}{2}x_1 x_3(1-\cos\theta_{13})$$

When does this happen?

Does this help?

Soft and collinear limit



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

$$\frac{d\sigma}{dx_1\,dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

(logarithmic) IR divergences when

- $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$
- $x_1 \to 1$ or $x_2 \to 1$

$$k_1.k_3 = E_1 E_3 (1-\cos\theta_{13})$$

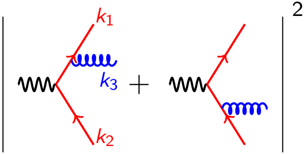
$$1 - x_2 = \tfrac{1}{2} x_1 x_3 (1-\cos\theta_{13})$$

When does this happen?

Does this help?

- $E_3 \to 0$: soft limit

Soft and collinear limit



$$|\mathcal{M}|^2 = \frac{256\pi^3\alpha_{\rm elm}}{s}\, e_q^2 N_c \alpha_s C_F \frac{(p_1.k_1)^2+(p_1.k_2)^2+(p_2.k_1)^2+(p_2.k_2)^2}{(k_1.k_3)(k_2.k_3)}$$

$$\frac{d\sigma}{dx_1\,dx_2} = (\sigma_{ee\to\mu\mu})(e_q^2 N_c)\frac{\alpha_s C_F}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

(logarithmic) IR divergences when

- $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$
- $x_1 \to 1$ or $x_2 \to 1$

$$k_1.k_3 = E_1 E_3(1-\cos\theta_{13})$$

$$1-x_2 = \tfrac{1}{2}x_1 x_3(1-\cos\theta_{13})$$

When does this happen?

Does this help?

- $E_3 \to 0$: soft limit
- θ_{13} or $\theta_{23} \to 0$: collinear limit

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon
- The kinematics of the $q\bar{q}$ pair is unaffected by the radiation of the gluon (eikonal limit)
- In this limit:

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times (8\pi\alpha_s C_F)\frac{(k_1.k_2)}{(k_1.k_3)(k_3.k_2)}$$

Can be rewritten

$$d\Phi_3|\mathcal{M}_{q\bar{q}g}|^2 \approx d\Phi_2|\mathcal{M}_{q\bar{q}}|^2 \times \frac{dE_3}{E_3}d\Omega_3\frac{(1-\cos\theta_{12})}{(1-\cos\theta_{13})(1-\cos\theta_{23})}$$

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon
- The kinematics of the $q\bar{q}$ pair is unaffected by the radiation of the gluon (eikonal limit)
- In this limit:

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times (8\pi\alpha_s C_F)\frac{(k_1.k_2)}{(k_1.k_3)(k_3.k_2)}$$

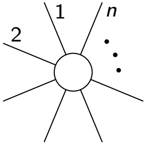
- **factorisation between** $ee \to q\bar{q}$ **and** g **radiation from** $q\bar{q}$ (the **antenna formula**)

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon
- The kinematics of the $q\bar{q}$ pair is unaffected by the radiation of the gluon (eikonal limit)
- In this limit:

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times (8\pi\alpha_s C_F)\frac{(k_1.k_2)}{(k_1.k_3)(k_3.k_2)}$$

- **factorisation between** $ee \to q\bar{q}$ **and** g **radiation from** $q\bar{q}$ (the **antenna formula**)
- Generalises: radiation of a soft gluon from n QCD legs (q or g)



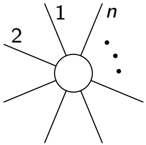
$$|\mathcal{M}_{n+1}|^2 = |\mathcal{M}_n|^2 \times \sum_{ij} 8\pi\alpha_s(\mathbf{T}_i.\mathbf{T}_j)\frac{(k_i.k_j)}{(k_i.k_{n+1})(k_{n+1}.k_j)}$$

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon
- The kinematics of the $q\bar{q}$ pair is unaffected by the radiation of the gluon (eikonal limit)
- In this limit:

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times (8\pi\alpha_s C_F)\frac{(k_1.k_2)}{(k_1.k_3)(k_3.k_2)}$$

- **factorisation between** $ee \to q\bar{q}$ **and** g **radiation from** $q\bar{q}$ (the **antenna formula**)
- Generalises: radiation of a soft gluon from n QCD legs (q or g)



$$|\mathcal{M}_{n+1}|^2 = |\mathcal{M}_n|^2 \times \sum_{ij} 8\pi\alpha_s(\mathbf{T}_i.\mathbf{T}_j)\frac{(k_i.k_j)}{(k_i.k_{n+1})(k_{n+1}.k_j)}$$

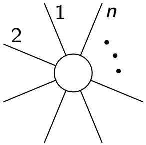
- Growing complexity due to colour, except at large N_c (more later)

Soft limit: $E_3 \to 0$

- One radiates an (arbitrarily) soft gluon
- The kinematics of the $q\bar{q}$ pair is unaffected by the radiation of the gluon (eikonal limit)
- In this limit:

$$|\mathcal{M}_{q\bar{q}g}|^2 = |\mathcal{M}_{q\bar{q}}|^2 \times (8\pi\alpha_s C_F)\frac{(k_1.k_2)}{(k_1.k_3)(k_3.k_2)}$$

- **factorisation between** $ee \to q\bar{q}$ **and** g **radiation from** $q\bar{q}$ (the **antenna formula**)
- Generalises: radiation of a soft gluon from n QCD legs (q or g)



$$|\mathcal{M}_{n+1}|^2 = |\mathcal{M}_n|^2 \times \sum_{ij} 8\pi\alpha_s(\mathbf{T}_i.\mathbf{T}_j)\frac{(k_i.k_j)}{(k_i.k_{n+1})(k_{n+1}.k_j)}$$

- Growing complexity due to colour, except at large N_c (more later)
- Physically: a soft gluon only sees colour lines

Collinear limit: $\theta_{13} \to 0$

- Change of variable: $x_{1,2} \to \theta \equiv \theta_{13}, z \equiv x_3$, and take the limit $\theta \ll 1$

Collinear limit: $\theta_{13} \to 0$

- Change of variable: $x_{1,2} \to \theta \equiv \theta_{13}, z \equiv x_3$, and take the limit $\theta \ll 1$
- Result:

$$d\sigma_{ee\to q\bar{q}g} = \sigma_{ee\to q\bar{q}} \times \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z} dz \frac{d\theta^2}{\theta^2}$$

- Again **factorisation** between $ee \to q\bar{q}$ and $q \to qg$
- Again a logarithmic divergence

Collinear limit: $\theta_{13} \to 0$

- Change of variable: $x_{1,2} \to \theta \equiv \theta_{13}, z \equiv x_3$, and take the limit $\theta \ll 1$
- Result:

$$d\sigma_{ee\to q\bar{q}g} = \sigma_{ee\to q\bar{q}} \times \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z} dz \frac{d\theta^2}{\theta^2}$$

- Again **factorisation** between $ee \to q\bar{q}$ and $q \to qg$
- Again a logarithmic divergence
- DGLAP/Altarelli-Parisi splitting function:

$$P_{gq}(z) = \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z}$$

represents a probability distribution for $q \to gq$ with energy sharing z and $1-z$

Collinear limit: $\theta_{13} \to 0$

- Change of variable: $x_{1,2} \to \theta \equiv \theta_{13}, z \equiv x_3$, and take the limit $\theta \ll 1$
- Result:

$$d\sigma_{ee\to q\bar{q}g} = \sigma_{ee\to q\bar{q}} \times \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z} dz \frac{d\theta^2}{\theta^2}$$

- Again **factorisation** between $ee \to q\bar{q}$ and $q \to qg$
- Again a logarithmic divergence
- DGLAP/Altarelli-Parisi splitting function:

$$P_{gq}(z) = \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z}$$

  represents a probability distribution for $q \to gq$ with energy sharing z and $1-z$
- Physically: collinear physics is local: $q \overset{\theta \ll 1}{\to} qg$ does not see the rest of the process

Collinear limit: $\theta_{13} \to 0$

- Change of variable: $x_{1,2} \to \theta \equiv \theta_{13}, z \equiv x_3$, and take the limit $\theta \ll 1$
- Result:

$$d\sigma_{ee\to q\bar{q}g} = \sigma_{ee\to q\bar{q}} \times \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z} dz \frac{d\theta^2}{\theta^2}$$

- Again **factorisation** between $ee \to q\bar{q}$ and $q \to qg$
- Again a logarithmic divergence
- DGLAP/Altarelli-Parisi splitting function:

$$P_{gq}(z) = \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z}$$

represents a probability distribution for $q \to gq$ with energy sharing z and $1-z$

- Physically: collinear physics is local: $q \overset{\theta\ll 1}{\to} qg$ does not see the rest of the process
- One recognises the soft $z \to 0$ divergence

Splendid! We understand a bit better IR divergences in QCD...

... however, we still have divergences!

Is there any hope to compute anything (other than R) in (perturbative) QCD?

Part VI: IRC safety

IRC safety: perturbative calculability

Question

can we compute an observable v in (perturbative) QCD?

Answer: IRC safety

Yes, provided it is **insensitive** to (arbitrarily) soft emissions and collinear branchings

IRC safety: perturbative calculability

Question

can we compute an observable v in (perturbative) QCD?

Answer: IRC safety

Yes, provided it is **insensitive** to (arbitrarily) soft emissions and collinear branchings

Logic

We can then apply the KLN theorem
(reals and virtuals are separately infinite but finite together)

Say that for n particles, v is given by $v_n(\Phi_n) \equiv v_n(k_1, \ldots, k_n)$

The distribution for v is therefore

$$\frac{1}{N}\frac{dN}{dv} = \sum_n \int d\Phi_n |M_n(\Phi_n)|^2 \, \delta(v - v_n(\Phi_n))$$

Works for (almost) everything
(could even consider output of ML)

IRC safety means:

IRC safety: conditions

Say that for n particles, v is given by $v_n(\Phi_n) \equiv v_n(k_1, \dots, k_n)$

The distribution for v is therefore

$$\frac{1}{N}\frac{dN}{dv} = \sum_n \int d\Phi_n |M_n(\Phi_n)|^2 \, \delta(v - v_n(\Phi_n))$$

Works for (almost) everything
(could even consider output of ML)

IRC safety means:

- soft-safe/IR-safe:

$$\lim_{E_i \to 0} v_n(k_1, \dots, k_i, \dots, k_n) = v_{n-1}(k_1, \dots, \cancel{k_i}, \dots, k_n)$$

IRC safety: conditions

Say that for n particles, v is given by $v_n(\Phi_n) \equiv v_n(k_1, \dots, k_n)$

The distribution for v is therefore

$$\frac{1}{N}\frac{dN}{dv} = \sum_n \int d\Phi_n |M_n(\Phi_n)|^2 \, \delta(v - v_n(\Phi_n))$$

Works for (almost) everything
(could even consider output of ML)

IRC safety means:

- soft-safe/IR-safe:

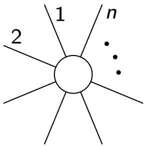
$$\lim_{E_i \to 0} v_n(k_1, \dots, k_i, \dots, k_n) = v_{n-1}(k_1, \dots, \cancel{k_i}, \dots, k_n)$$

- collinear-safe:

$$\lim_{\theta_{ij} \to 0} v_n(k_1, \dots, k_i, \dots, k_j, \dots, k_n) = v_{n-1}(k_1, \dots, \cancel{k_i}, \dots, \cancel{k_j}, \dots, k_n, k_i + k_j)$$

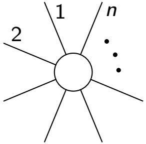
IRC safety: conditions illustrated

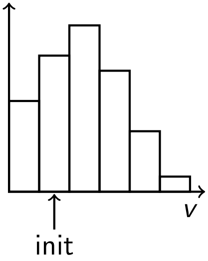
initial
n-particle configuration



IRC safety: conditions illustrated

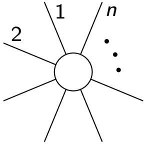
initial
n-particle configuration



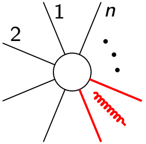


IRC safety: conditions illustrated

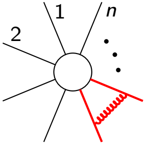
initial
n-particle configuration



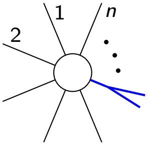
IR(real)



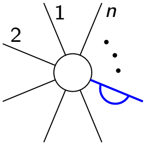
IR(virt)

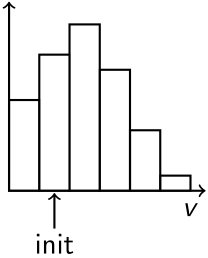


coll(real)



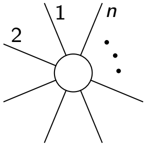
coll(virt)



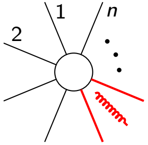


IRC safety: conditions illustrated

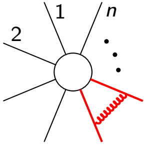
initial
n-particle configuration



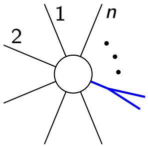
IR(real)



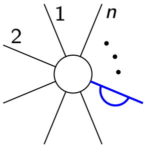
IR(virt)

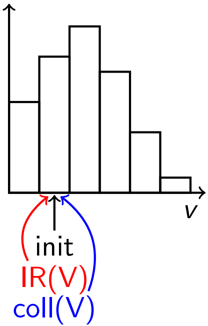


coll(real)



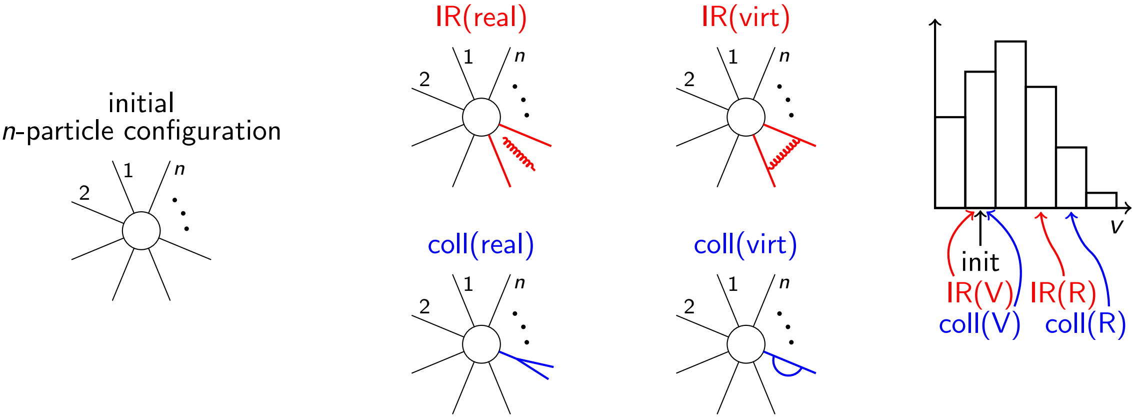
coll(virt)





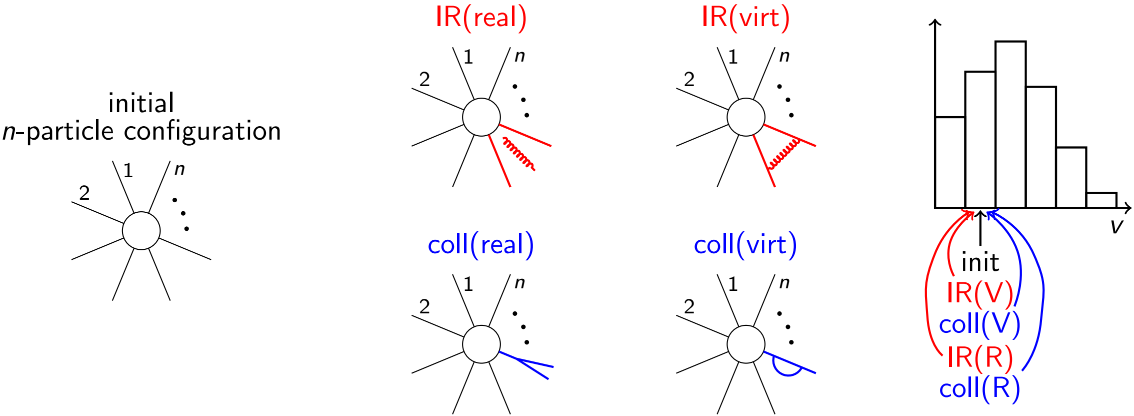
- virtual corrections: same bin as initial

IRC safety: conditions illustrated



- virtual corrections: same bin as initial
- unsafe: real in different bin [no local KLN cancellation]

IRC safety: conditions illustrated



- virtual corrections: same bin as initial
- unsafe: real in different bin [no local KLN cancellation]
- safe: real also in same bin [local KLN cancellation]

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity		

- multiplicity: simply count particles

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	

- multiplicity: simply count particles

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗

- multiplicity: simply count particles

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$		

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ		

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}		

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC		

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
E_{max}	✓	✗
Σ_θ	✗	✗
$n_{patches}$	✗	✓
EEC	✓	✓

- multiplicity: simply count particles
- $E_{max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- $n_{patches}$: split sphere in fixed regions, count how many contain at least 1 particle
- $EEC_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	✓
λ_D		

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$
- $\lambda_D = \sum_{i,j} E_i^2 E_j^2 \theta_{ij}$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	✓
λ_D	✓	

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$
- $\lambda_D = \sum_{i,j} E_i^2 E_j^2 \theta_{ij}$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	✓
λ_D	✓	✗

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$
- $\lambda_D = \sum_{i,j} E_i^2 E_j^2 \theta_{ij}$

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	✓
λ_D	✓	✗

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$
- $\lambda_D = \sum_{i,j} E_i^2 E_j^2 \theta_{ij}$

"**IRC-safety $\equiv$ perturbative calculability**"
$\Rightarrow$ make it a habit to check!

IRC safety: worked examples

observable	IR safe	collinear safe
multiplicity	✗	✗
$E_{\max}$	✓	✗
Σ_θ	✗	✗
n_{patches}	✗	✓
EEC	✓	✓
λ_D	✓	✗

- multiplicity: simply count particles
- $E_{\max} = \max_i E_i$
- $\Sigma_\theta = \sum_{i,j} \theta_{ij}$:
- n_{patches}: split sphere in fixed regions, count how many contain at least 1 particle
- $\text{EEC}_\alpha = \sum_{i,j} E_i E_j \theta_{ij}^\alpha$
- $\lambda_D = \sum_{i,j} E_i^2 E_j^2 \theta_{ij}$

"**IRC-safety $\equiv$ perturbative calculability**"
$\Rightarrow$ make it a habit to check!

Not always 100% trivial
$+$ more complex cases
($p_{t,\text{SoftDrop}}/p_{t,\text{jet}}$, old cone jets, z_g)

Part VII: final-state and jets

examples of standard IRC-safe observables

Event shapes (examples)

- Thrust:

$$T = \max_{|\vec{n}|=1} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

Notes:

- the "$\vec{n}$" achieving the min defines the "Thrust axis", $\vec{t}$
- defines two "hemispheres"
- radiation collimated around one axis: $T \approx 1$
- radiation spread uniformly: $T \approx 1/2$

Event shapes (examples)

- Thrust:

$$T = \max_{|\vec{n}|=1} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

- Thrust major (M), minor (m)

$$M = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}, \qquad m = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0,\vec{n}.\vec{t}_M=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

Event shapes (examples)

- Thrust:

$$T = \max_{|\vec{n}|=1} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

- Thrust major (M), minor (m)

$$M = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}, \quad m = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0,\vec{n}.\vec{t}_M=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

- Sphericity

$$S = \left(\frac{4}{\pi}\right)^2 \min_{|\vec{n}|=1} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}\right)^2$$

Event shapes (examples)

- Thrust:

$$T = \max_{|\vec{n}|=1} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

- Thrust major (M), minor (m)

$$M = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}, \quad m = \max_{|\vec{n}|=1,\vec{n}.\vec{t}=0,\vec{n}.\vec{t}_M=0} \frac{\sum_i |\vec{p}_i.\vec{n}|}{\sum_i |\vec{p}_i|}$$

- Sphericity

$$S = \left(\frac{4}{\pi}\right)^2 \min_{|\vec{n}|=1} \left(\frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}\right)^2$$

- C-parameter

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \qquad \text{with } \lambda \text{ eigenvalues of } \Theta_{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$$

Jets (1/2)

Idea

Most frequent branchings are either collinear or soft
$\Rightarrow$ expect most of the event's energy localised around a few axes
$\Rightarrow$ define jets as these few directions

Jets (1/2)

Idea

Most frequent branchings are either collinear or soft
$\Rightarrow$ expect most of the event's energy localised around a few axes
$\Rightarrow$ define jets as these few directions

(Historical) cone algorithm: find directions of energy flow

Event is n jets if all but a fraction ε of the $\sqrt{s}$ energy is in n cones of half-opening-angle δ (and not in $n-1$)

[Sterman, Weinberg, 1977]

Works but geometry makes it delicate to go to high orders in pQCD

Jets (2/2)

JADE

Iteratively:

1. Find the pair, p_i, p_j that minimises $m_{ij}^2 = (p_i + p_j)^2 = 2E_iE_j(1 - \cos\theta_{ij})$
2. Recombine $p_i, p_j \to p_{i+j} = p_i + p_j$ (i.e. from n to $n-1$ particles)

Stop when $m_{ij}^2 > y_{\rm cut} s$

Jets (2/2)

JADE

Iterately:

1. Find the pair, p_i, p_j that minimises $m_{ij}^2 = (p_i + p_j)^2 = 2E_iE_j(1 - \cos\theta_{ij})$
2. Recombine $p_i, p_j \to p_{i+j} = p_i + p_j$ (i.e. from n to $n-1$ particles)

Stop when $m_{ij}^2 > y_{\text{cut}}s$

Idea

Invert the QCD branching process

small m_{ij} when soft/collinear $\Rightarrow$ unlikely to be a new jet

JADE

Iteratively:

1. Find the pair, p_i, p_j that minimises $m_{ij}^2 = (p_i + p_j)^2 = 2E_iE_j(1 - \cos\theta_{ij})$
2. Recombine $p_i, p_j \to p_{i+j} = p_i + p_j$ (i.e. from n to $n-1$ particles)

Stop when $m_{ij}^2 > y_{\text{cut}}s$

Alternatives with more friendly behaviour

Durham/k_t: Same strategy with $d_{ij}^{(k_t)} = \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$

Cambridge: $d_{ij}^{(Cam)} = (1 - \cos\theta_{ij})$ (with Durham y_{cut} as a stopping condition)

Jets (2/2)

JADE

Iteratively:

1. Find the pair, p_i, p_j that minimises $m_{ij}^2 = (p_i + p_j)^2 = 2E_iE_j(1 - \cos\theta_{ij})$
2. Recombine $p_i, p_j \to p_{i+j} = p_i + p_j$ (i.e. from n to $n-1$ particles)

Stop when $m_{ij}^2 > y_{\text{cut}} s$

Alternatives with more friendly behaviour

Durham/k_t: Same strategy with $d_{ij}^{(k_t)} = \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$

Cambridge: $d_{ij}^{(Cam)} = (1 - \cos\theta_{ij})$ (with Durham y_{cut} as a stopping condition)

Note: two possible modes:

1. Count the number of jets for a fixed y_{cut}
2. Study the distributino of $y_{n-1,n}$, the transition beteen $n-1$ and n jets

Both allow strong tests of QCD (hold on a bit more before examples)

Part VIII: fixed-order and resummations

Example: JADE 3-jet rate

- take $\frac{d\sigma}{dx_1 dx_2}$ from above
- show that $m_{ij}^2 = E_i E_j (1 - \cos\theta_{ij}) = (1 - x_k)s \ (k \neq i, j) \Rightarrow$ 3 jets if $1 - x_i > y_{\text{cut}}, \ \forall\, i$

$$f_3^{(\text{JADE})} = \frac{\alpha_s C_F}{\pi} \left[\log^2 \frac{y}{1-y} + \frac{3}{2}(1-2y) \log \frac{y}{1-2y} + 2\text{Li}_2 \frac{y}{1-y} - \frac{\pi^2}{6} + \frac{5 - 12y - 9y^2}{4} \right]$$

What features do you recognise here?

Example: JADE 3-jet rate

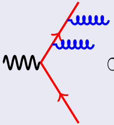
- take $\frac{d\sigma}{dx_1 dx_2}$ from above
- show that $m_{ij}^2 = E_i E_j (1 - \cos\theta_{ij}) = (1 - x_k)s$ $(k \neq i, j) \Rightarrow$ 3 jets if $1 - x_i > y_{\text{cut}}, \ \forall\, i$

$$f_3^{(\text{JADE})} = \frac{\alpha_s C_F}{\pi}\left[\log^2\frac{y}{1-y} + \frac{3}{2}(1-2y)\log\frac{y}{1-2y} + 2\text{Li}_2\frac{y}{1-y} - \frac{\pi^2}{6} + \frac{5-12y-9y^2}{4}\right]$$

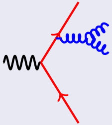
What features do you recognise here?

- Proportional to $\alpha_s C_F$, i.e. probes fundamental aspects of QCD

At $\mathcal{O}\left(\alpha_s^2\right)$, we get e.g. contributions sensitive to the non-abelian nature of QCD



$\propto C_F^2$



$\propto C_F C_A$

Example: JADE 3-jet rate

- take $\frac{d\sigma}{dx_1 dx_2}$ from above
- show that $m_{ij}^2 = E_i E_j (1 - \cos\theta_{ij}) = (1 - x_k)s \ (k \neq i, j) \Rightarrow$ 3 jets if $1 - x_i > y_{\text{cut}}, \ \forall\, i$

$$f_3^{(\text{JADE})} = \frac{\alpha_s C_F}{\pi} \left[\log^2 \frac{y}{1-y} + \frac{3}{2}(1-2y) \log \frac{y}{1-2y} + 2\text{Li}_2 \frac{y}{1-y} - \frac{\pi^2}{6} + \frac{5 - 12y - 9y^2}{4} \right]$$

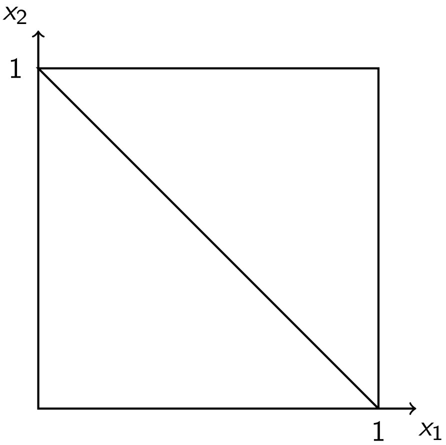
What features do you recognise here?

- Proportional to $\alpha_s C_F$, i.e. probes fundamental aspects of QCD
- When $y_{\text{cut}} \ll 1$:

$$f_3^{(\text{JADE})} \approx \frac{\alpha_s C_F}{\pi} \left[\log^2 y + \frac{3}{2} \log y \right]$$

Traces of the (logarithmic) IR behaviour of QCD

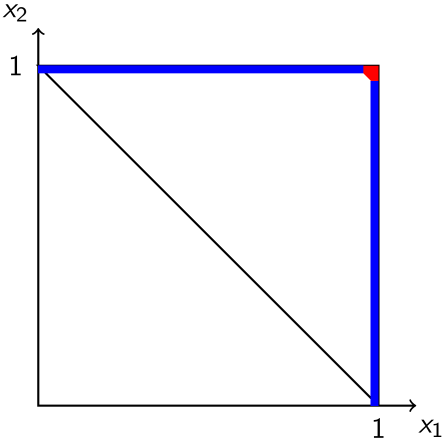
Example: JADE 3-jet rate



- Consider the x_1, x_2 phase-space

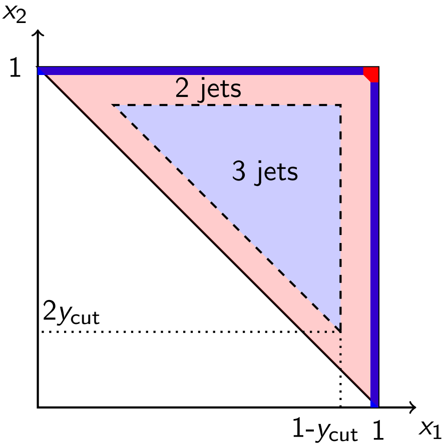
  Recall: $0 \leq x_i \leq 1$, $x_1 + x_2 + x_3 = 2$

Example: JADE 3-jet rate



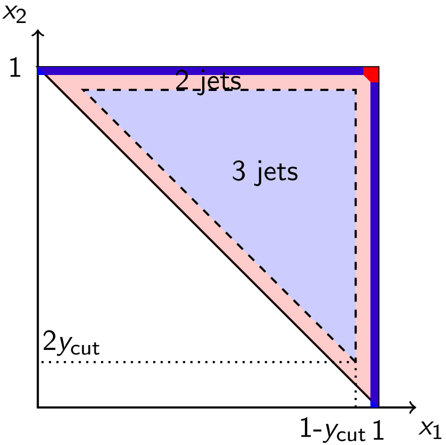
- Consider the x_1, x_2 phase-space
  Recall: $0 \leq x_i \leq 1$, $x_1 + x_2 + x_3 = 2$
- Soft and collinear divergences $x_{1,2} \to 1$
  IRC-safe observables should not get there!

Example: JADE 3-jet rate



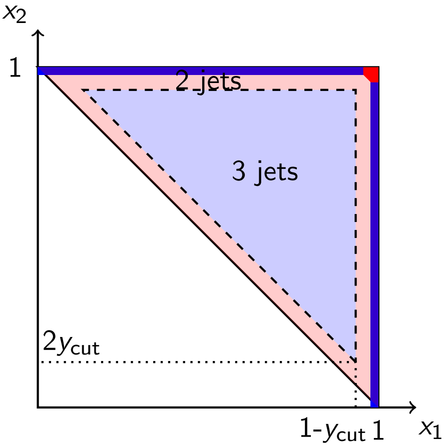
- Consider the x_1, x_2 phase-space
  Recall: $0 \le x_i \le 1$, $x_1 + x_2 + x_3 = 2$
- Soft and collinear divergences $x_{1,2} \to 1$
  IRC-safe observables should not get there!
- Jade f_3: $1 - x_i \ge y_{\rm cut}$
  $\Rightarrow$ IRC-safe

Example: JADE 3-jet rate



- Consider the x_1, x_2 phase-space
  Recall: $0 \leq x_i \leq 1$, $x_1 + x_2 + x_3 = 2$
- Soft and collinear divergences $x_{1,2} \to 1$
  IRC-safe observables should not get there!
- Jade f_3: $1 - x_i \geq y_{\rm cut}$
  $\Rightarrow$ IRC-safe
- However, when $y_{\rm cut} \ll 1$ one gets close to the log divergence

Example: JADE 3-jet rate



- Consider the x_1, x_2 phase-space
  Recall: $0 \leq x_i \leq 1$, $x_1 + x_2 + x_3 = 2$
- Soft and collinear divergences $x_{1,2} \to 1$
  IRC-safe observables should not get there!
- Jade f_3: $1 - x_i \geq y_{\text{cut}}$
  $\Rightarrow$ IRC-safe
- However, when $y_{\text{cut}} \ll 1$ one gets close to the log divergence
- Result: logs in observables
  double logs ($\log^2 y_{\text{cut}}$): both soft and collinear
  single logs ($\log y_{\text{cut}}$) : collinear

Organising the perturbative series

'Finite" y_{cut}

$(\alpha_s \log y_{\text{cut}} \ll 1,\ \alpha_s \ll 1)$

$y_{\text{cut}} \ll 1$

$(\alpha_s L \equiv \alpha_s \log y_{\text{cut}} \sim 1,\ \alpha_s \ll 1)$

Organising the perturbative series

'Finite" y_{cut}

$(\alpha_s \log y_{\text{cut}} \ll 1,\ \alpha_s \ll 1)$

$$f_2 = 1 + \alpha_s f^{(1)}(y) + \alpha_s^2 f^{(2)}(y) + \alpha_s^3 f^{(3)}(y) + \dots$$

$y_{\text{cut}} \ll 1$

$(\alpha_s L \equiv \alpha_s \log y_{\text{cut}} \sim 1,\ \alpha_s \ll 1)$

$$f_2 = (1 + C(\alpha_s))e^{g_1(\alpha_s L)L + g_2(\alpha_s L) + g_3(\alpha_s L)\alpha_s + \dots}$$

Organising the perturbative series

'Finite" y_{cut}

$(\alpha_s \log y_{\mathrm{cut}} \ll 1,\ \alpha_s \ll 1)$

$$f_2 = 1 + \underbrace{\alpha_s f^{(1)}(y)}_{\text{LO}} + \underbrace{\alpha_s^2 f^{(2)}(y)}_{\text{NLO}} + \underbrace{\alpha_s^3 f^{(3)}(y)}_{\text{NNLO}} + \dots$$

"standard" perturbation theory

$y_{\mathrm{cut}} \ll 1$

$(\alpha_s L \equiv \alpha_s \log y_{\mathrm{cut}} \sim 1,\ \alpha_s \ll 1)$

$$f_2 = (1+C(\alpha_s))\, e^{\underbrace{g_1(\alpha_s L)}_{\text{LL}} L + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{g_3(\alpha_s L)}_{\text{NNLL}} \alpha_s + \dots}$$

"resummed" perturbation theory

Organising the perturbative series

‘Finite” y_{cut}

$(\alpha_s \log y_{\text{cut}} \ll 1,\ \alpha_s \ll 1)$

$$f_2 = 1 + \underbrace{\alpha_s f^{(1)}(y)}_{\text{LO}} + \underbrace{\alpha_s^2 f^{(2)}(y)}_{\text{NLO}} + \underbrace{\alpha_s^3 f^{(3)}(y)}_{\text{NNLO}} + \dots$$

“standard” perturbation theory

Statue-of-the-art: NLO
Increasingly many NNLO
A few N^{3}LO

$y_{\text{cut}} \ll 1$

$(\alpha_s L \equiv \alpha_s \log y_{\text{cut}} \sim 1,\ \alpha_s \ll 1)$

$$f_2 = (1+C(\alpha_s))e^{\overbrace{g_1(\alpha_s L)}^{\text{LL}} L + \overbrace{g_2(\alpha_s L)}^{\text{NLL}} + \overbrace{g_3(\alpha_s L)}^{\text{NNLL}} \alpha_s + \dots}$$

“resummed” perturbation theory

Statue-of-the-art: NLL
Increasingly many NNLL
A few N^{3}LL

Organising the perturbative series

'Finite" $y_{\rm cut}$

$(\alpha_s \log y_{\rm cut} \ll 1,\ \alpha_s \ll 1)$

$$f_2 = 1 + \underbrace{\alpha_s f^{(1)}(y)}_{\text{LO}} + \underbrace{\alpha_s^2 f^{(2)}(y)}_{\text{NLO}} + \underbrace{\alpha_s^3 f^{(3)}(y)}_{\text{NNLO}} + \dots$$

"standard" perturbation theory

Statue-of-the-art: NLO
Increasingly many NNLO
A few N^3LO

$y_{\rm cut} \ll 1$

$(\alpha_s L \equiv \alpha_s \log y_{\rm cut} \sim 1,\ \alpha_s \ll 1)$

$$f_2 = (1+C(\alpha_s))e^{\overbrace{g_1(\alpha_s L)}^{\text{LL}} L + \overbrace{g_2(\alpha_s L)}^{\text{NLL}} + \overbrace{g_3(\alpha_s L)}^{\text{NNLL}} \alpha_s + \dots}$$

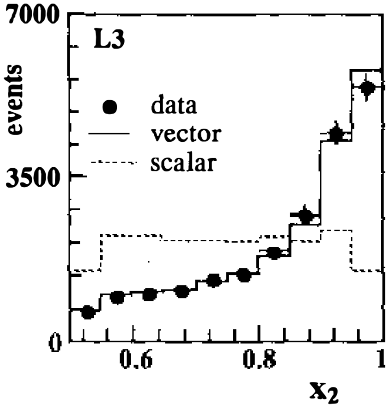
"resummed" perturbation theory

Statue-of-the-art: NLL
Increasingly many NNLL
A few N^3LL

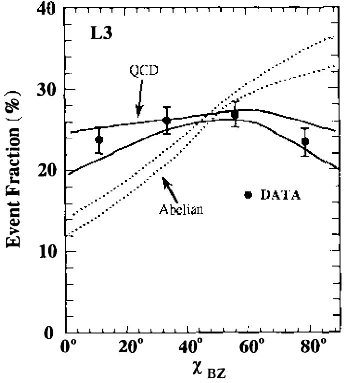
If only one thing to remember

Calculations are valid (i) up to a given accuracy, (ii) in certain limits

Evidence for a gluon



Evidence for a non-abelian theory



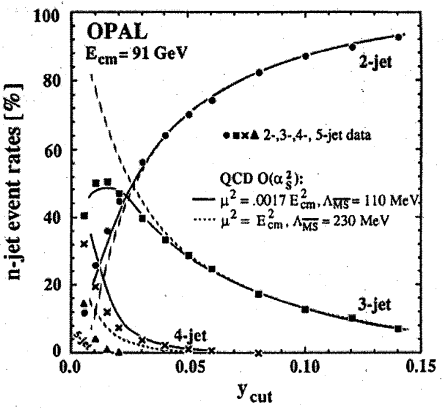
The gluon was discovered through 3-jet events

$$\frac{N_c}{C_F} = 2.55 \pm 0.55 \pm 0.4 \pm 0.2 \qquad \text{exp.:}2.25 \qquad \text{abelian:}0$$

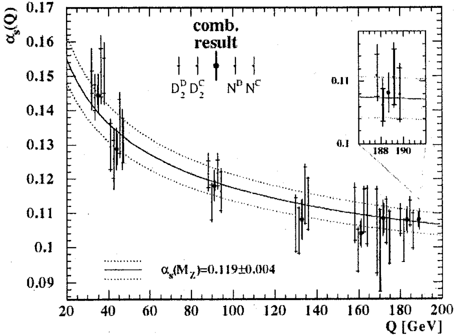
$$\frac{T_R}{C_F} = 0.1 \pm 2.4 \qquad \text{exp.:}1.875 \qquad \text{abelian:}15$$

Examples at LEP: testing QCD

JADE jet rates at OPAL

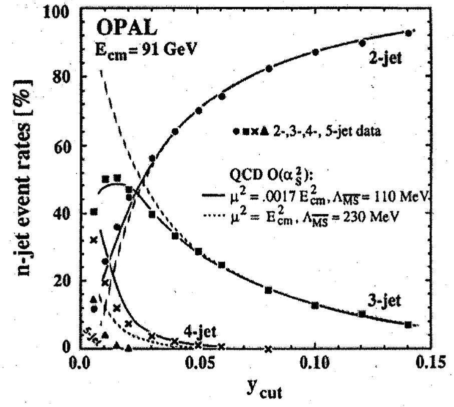


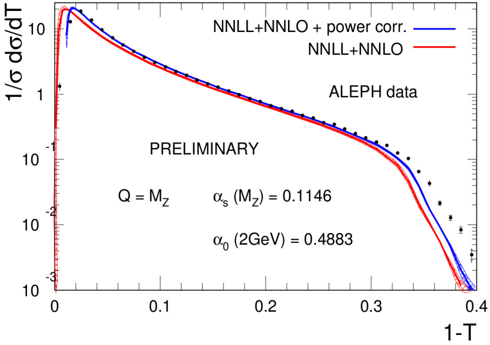
α_s from k_t/Durham jet rates



Examples at LEP: testing QCD

JADE jet rates at OPAL





Improved through the years

High accuracy requires (non-perturbative) power corrections

Part IX: DIS and PDFs

Hadrons in the initial state

2 cases to consider:

- *ep* collisions (Deep Inelastic Scattering (DIS)): HERA, EIC, ...
  can also do *eA* (not covered here)

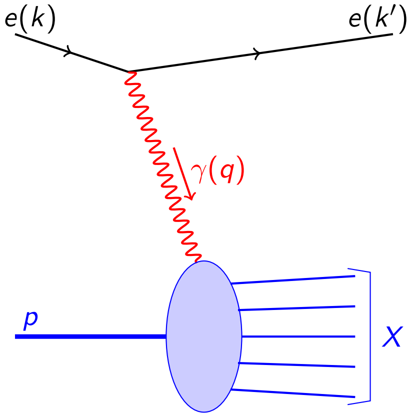
- *pp* collisions: LHC, Tevatron, FCC-*hh*, etc...
  can also do *pA* or *AA* (not covered here)

Hadrons in the initial state

2 cases to consider:

- *ep* collisions (Deep Inelastic Scattering (DIS)): HERA, EIC, ...
  can also do *eA* (not covered here)
  We will use this to discuss the basic physics of hadronic beams
- *pp* collisions: LHC, Tevatron, FCC-*hh*, etc...
  can also do *pA* or *AA* (not covered here)
  We will use this to discuss a few aspects of LHC physics and future challenges

DIS kinematics: $ep \to eX$ ($X \equiv$anything)



Kinematic variables:

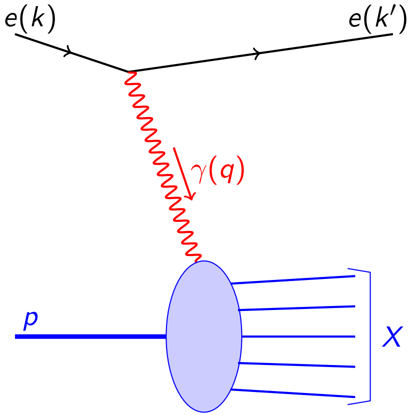
$$s = (p+k)^2 \qquad W = (p+q)^2$$

$$\nu = p.q \qquad y = \frac{p.q}{p.k} = \frac{2\nu}{s}$$

$$Q^2 = -q^2 (> 0) \qquad x = \frac{Q^2}{2\nu}$$

Idea: use the photon to probe the proton
large Q^2 $\Rightarrow$ small distance $\sim 1/Q$

DIS kinematics: $ep \to eX$ ($X \equiv$anything)



Kinematic variables:

$$s = (p+k)^2 \qquad W = (p+q)^2$$

$$\nu = p.q \qquad y = \frac{p.q}{p.k} = \frac{2\nu}{s}$$

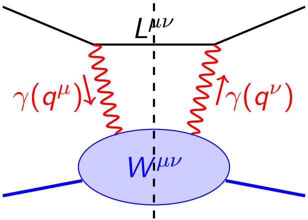
$$Q^2 = -q^2 (> 0) \qquad x = \frac{Q^2}{2\nu}$$

2 degrees of freedom (negleting azimuth):
energy (E') and angle (θ) of outgoing electron

$$Q^2 = 4EE' \cos^2(\theta/2)$$

$$x = \frac{EE' \cos^2(\theta/2)}{P(E - E' \sin^2(\theta/2)}$$

Structure functions



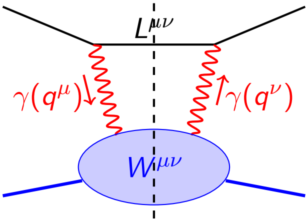
$$|\mathcal{M}|^2 = L_{\mu\nu} W^{\mu\nu} \qquad \text{(generic Lorentz structure)}$$

$$L_{\mu\nu} \equiv \text{lepton tensor} \qquad \text{(calculable from first principles)}$$

$$W^{\mu\nu} \equiv \text{hadron tensor} \qquad \text{(contains the proton structure)}$$

$$= -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1 + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right)\frac{F_2}{\nu}$$

Structure functions



$$|\mathcal{M}|^2 = L_{\mu\nu} W^{\mu\nu} \qquad \text{(generic Lorentz structure)}$$

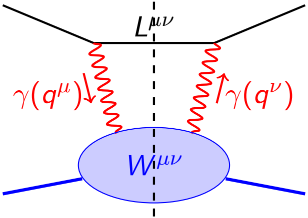
$$L_{\mu\nu} \equiv \text{lepton tensor} \qquad \text{(calculable from first principles)}$$

$$W^{\mu\nu} \equiv \text{hadron tensor} \qquad \text{(contains the proton structure)}$$

$$= -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1 + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right)\frac{F_2}{\nu}$$

- $F_{1,2}(x, Q^2)$ are the **proton structure functions** (also $F_L = F_2 - 2xF_1$)

Structure functions



$$|\mathcal{M}|^2 = L_{\mu\nu} W^{\mu\nu} \qquad \text{(generic Lorentz structure)}$$

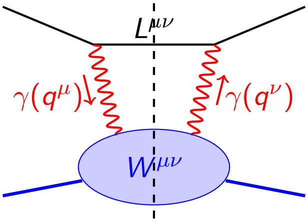
$$L_{\mu\nu} \equiv \text{lepton tensor} \qquad \text{(calculable from first principles)}$$

$$W^{\mu\nu} \equiv \text{hadron tensor} \qquad \text{(contains the proton structure)}$$

$$= -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1 + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right)\frac{F_2}{\nu}$$

- $F_{1,2}(x, Q^2)$ are the **proton structure functions** (also $F_L = F_2 - 2xF_1$)
- One can also have the exchange of a Z boson (neutral currents)

Structure functions



$$
\begin{aligned}
|\mathcal{M}|^2 &= L_{\mu\nu} W^{\mu\nu} && \text{(generic Lorentz structure)}\\
L_{\mu\nu} &\equiv \text{lepton tensor} && \text{(calculable from first principles)}\\
W^{\mu\nu} &\equiv \text{hadron tensor} && \text{(contains the proton structure)}\\
&= -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) F_1 + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right)\frac{F_2}{\nu}
\end{aligned}
$$

- $F_{1,2}(x, Q^2)$ are the **proton structure functions** (also $F_L = F_2 - 2xF_1$)
- One can also have the exchange of a Z boson (neutral currents)
- One can also have charged currents with a $W^\pm$ exchange (e.g. $e^\pm p \to \nu X$)
  This introduces a 3rd structure function $F_3(x, Q^2)$

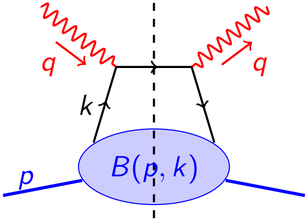
Working hypothesis: photon scatters on point-like particle

Frame with boosted proton:

$$p \equiv (0, 0, P, P)$$

$$n \equiv (0, 0, \frac{1}{2P}, \frac{1}{2P})$$

$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$



large $Q^2 \Rightarrow \quad \delta\left((q+k)^2\right) \approx \frac{1}{2\nu}\delta(\xi - x)$ and $F_2 = e_q^2 x q(x)$

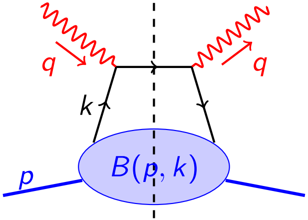
$\left(q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}(\not{n} B(p,k)) \delta(\xi - x)\right)$

Parton model: PDFs

Working hypothesis: photon scatters on point-like particle

Frame with boosted proton:

$$p \equiv (0, 0, P, P)$$
$$n \equiv (0, 0, \frac{1}{2P}, \frac{1}{2P})$$
$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$



large $Q^2 \Rightarrow \quad \delta\left((q+k)^2\right) \approx \frac{1}{2\nu}\delta(\xi - x)$ and $F_2 = e_q^2 x q(x)$ $\quad (q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}(\not{n} B(p,k))\delta(\xi - x))$

- Photon scatters on a "quark" carrying a fraction x of the proton's longitudinal momentum

Parton model: PDFs

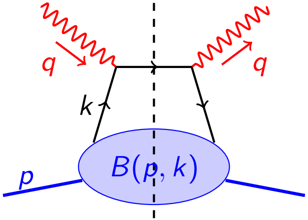
Working hypothesis: photon scatters on point-like particle

Frame with boosted proton:

$$p \equiv (0, 0, P, P)$$

$$n \equiv (0, 0, \frac{1}{2P}, \frac{1}{2P})$$

$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$



large $Q^2 \Rightarrow \quad \delta\left((q+k)^2\right) \approx \frac{1}{2\nu}\delta(\xi - x)$ and $F_2 = e_q^2 x q(x)$ $\quad (q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}(\not{n} B(p,k))\delta(\xi - x))$

- Photon scatters on a "quark" carrying a fraction x of the proton's longitudinal momentum
- **q(x)** $\equiv$ **Parton Distribution Function**: density of quarks q with momentum fraction x

Parton model: PDFs

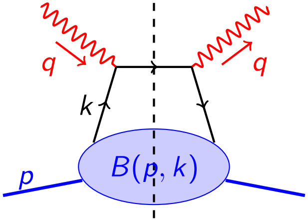
Working hypothesis: photon scatters on point-like particle

Frame with boosted proton:

$$p \equiv (0, 0, P, P)$$

$$n \equiv (0, 0, \frac{1}{2P}, \frac{1}{2P})$$

$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

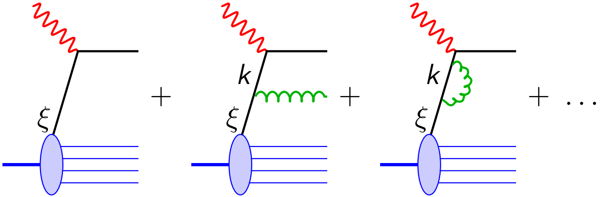


large $Q^2 \Rightarrow \quad \delta\left((q+k)^2\right) \approx \frac{1}{2\nu}\delta(\xi - x)$ and $F_2 = e_q^2 x q(x)$ $\quad (q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}(\not{n} B(p,k))\delta(\xi - x))$

- Photon scatters on a "quark" carrying a fraction x of the proton's longitudinal momentum
- **q(x)** $\equiv$ **Parton Distribution Function**: density of quarks q with momentum fraction x
- Bjorken scaling: $F_2(x, Q^2) \equiv F_2(x)$, independent of Q^2 ((very) roughly true)
- Callan-Gross relation: $F_L = F_2 - 2xF_1 = 0$ (in practice: $\ll F_2$) means quarks are spin $\frac{1}{2}$

QCD effects

The quark can radiate gluons (real or virtual):



Explicit calculation gives:

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \int^{Q^2} \frac{d|k^2|}{|k^2|}\right] \equiv e_q^2 x q_0(\xi) \left[1 + \frac{\alpha_s}{2\pi}(\text{divergent})\right]$$

How do we proceed?

QCD evolution

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \int^{Q^2} \frac{d|k^2|}{|k^2|}\right] \equiv e_q^2 x q_0(\xi) \left[1 + \frac{\alpha_s}{2\pi}(\text{divergent})\right]$$

Idea:

1. introduce a regulator μ^2
2. absorb the divergence in the PDF: the "bare" $q_0(x)$ becomes $q(x,\mu^2)$

We get:

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q(\xi,\mu^2) \left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2}\right] \equiv e_q^2 x q(x, Q^2)$$

QCD evolution

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2}\right] \equiv e_q^2 x q(x, Q^2)$$

Important consequences:

QCD evolution

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2} \right] \equiv e_q^2 x q(x, Q^2)$$

Important consequences:

1. F_2 does depend on Q^2 (Bjorken scaling violated by QCD)

QCD evolution

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\mu^2}\right] \equiv e_q^2 x q(x, Q^2)$$

Important consequences:

1. F_2 does depend on Q^2 (Bjorken scaling violated by QCD)
2. require that $F_2(x, Q^2)$ does not depend on the specific choice of μ^2 yields

$$\boxed{Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}\right) q(\xi, Q^2)} \qquad P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

QCD evolution

$$F_2 = e_q^2 x \int_x^1 \frac{d\xi}{\xi} q(\xi, \mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\mu^2}\right] \equiv e_q^2 x q(x, Q^2)$$

Important consequences:

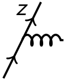
1. F_2 does depend on Q^2 (Bjorken scaling violated by QCD)
2. require that $F_2(x, Q^2)$ does not depend on the specific choice of μ^2 yields

$$\boxed{Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}\right) q(\xi, Q^2)} \qquad P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

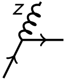
3. PDFs remain essentially non-perturbative but their Q^2 dependence is predicted by QCD

QCD evolution

In practice: all flavour combinations

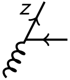


$$P_{qq} = C_F \frac{1+z^2}{1-z}$$



$$P_{gq} = C_F \frac{1+(1-z)^2}{z}$$

$$P_{gg} = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]_{(+\text{virt})}$$



$$P_{qg} = \frac{1}{2}[z^2 + (1-z)^2]$$

$$\mu^2 \partial_{\mu^2} \begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{qq}(\xi) & P_{qg}(\xi) \\ P_{gq}(\xi) & P_{gg}(\xi) \end{pmatrix} \begin{pmatrix} q(\frac{x}{\xi},\mu^2) \\ g(\frac{x}{\xi},\mu^2) \end{pmatrix}$$

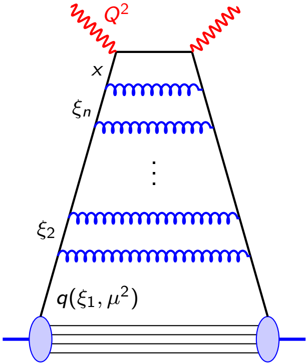
Comments:

- This is the **DGLAP equation**
- $\mu \equiv \mu_F$ is the factorisation scale
- "P"'s are the Altarelli-Parisi (or DGLAP) splitting functions
- Trace of the soft divergence at $z = 0, 1$
  (other equations to handle them: BFKL,...)

QCD evolution

- we had a (IR) divergence; we absorbed it in the PDFs; we are left with $\log(Q^2/\mu^2)$
- DGLAP is an all-order treatment (resummation) of $(\alpha_s \log(Q^2/\mu^2))^n$:

$$q(x,Q^2) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu^2}\right)^n \int_x^1 \frac{d\xi_n}{\xi_n} P(\frac{x}{\xi_n}) \int_{\xi_n}^1 \frac{d\xi_{n-1}}{\xi_{n-1}} P(\frac{\xi_n}{\xi_{n-1}}) \cdots \int_{\xi_2}^1 \frac{d\xi_1}{\xi_1} P(\frac{\xi_1}{\xi_2}) q(\xi_1,\mu^2)$$

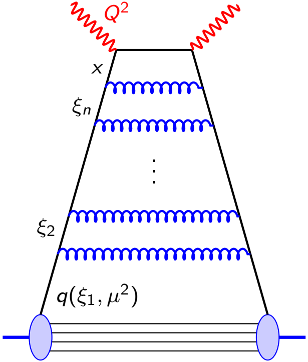


QCD evolution

- we had a (IR) divergence; we absorbed it in the PDFs; we are left with $\log(Q^2/\mu^2)$
- DGLAP is an all-order treatment (resummation) of $(\alpha_s \log(Q^2/\mu^2))^n$:

$$q(x,Q^2) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu^2}\right)^n \int_x^1 \frac{d\xi_n}{\xi_n}P(\frac{x}{\xi_n})\int_{\xi_n}^1 \frac{d\xi_{n-1}}{\xi_{n-1}}P(\frac{\xi_n}{\xi_{n-1}})\cdots\int_{\xi_2}^1 \frac{d\xi_1}{\xi_1}P(\frac{\xi_1}{\xi_2})q(\xi_1,\mu^2)$$

- What we did here is the "leading logarithmic" order
  Often also referred to as the "strongly ordered limit"
  $\alpha_s^n log\frac{Q^2}{\mu^2}$ comes from $\mu^2 \ll |k_1^2| \ll \cdots \ll Q^2$



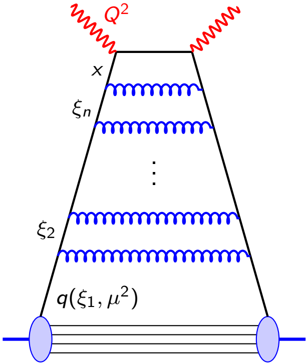
- we had a (IR) divergence; we absorbed it in the PDFs; we are left with $\log(Q^2/\mu^2)$
- DGLAP is an all-order treatment (resummation) of $(\alpha_s \log(Q^2/\mu^2))^n$:

$$q(x,Q^2) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu^2}\right)^n \int_x^1 \frac{d\xi_n}{\xi_n} P\left(\frac{x}{\xi_n}\right) \int_{\xi_n}^1 \frac{d\xi_{n-1}}{\xi_{n-1}} P\left(\frac{\xi_n}{\xi_{n-1}}\right)\cdots \int_{\xi_2}^1 \frac{d\xi_1}{\xi_1} P\left(\frac{\xi_1}{\xi_2}\right) q(\xi_1,\mu^2)$$

- What we did here is the "leading logarithmic" order
- Fundamental factorisation theorem: this remains true at all orders

$$\mu^2\partial_{\mu^2}\begin{pmatrix} q(x,\mu^2) \\ g(x,\mu^2)\end{pmatrix} = \int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq}(\xi,\alpha_s) & P_{qg}(\xi,\alpha_s) \\ P_{gq}(\xi,\alpha_s) & P_{gg}(\xi,\alpha_s)\end{pmatrix}\begin{pmatrix} q(\frac{x}{\xi},\mu^2) \\ g(\frac{x}{\xi},\mu^2)\end{pmatrix}$$

$$P(z,\alpha_s) = \underbrace{\frac{\alpha_s}{2\pi}P^{(1)}(z)}_{\text{LL/LO}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 P^{(2)}(z)}_{\text{NLL/NLO}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^3 P^{(3)}(z)}_{\text{NNLL/NNLO}} + \dots$$



PDF status

- State-of-the-art: NNLL known, N^3LL almost known (in moment space)

PDF status

- State-of-the-art: NNLL known, N^3LL almost known (in moment space)
- Practical approach:
  1. Take an initial condition for all quarks and gluons at an initial scale Q_0: $q_f(x, Q_0^2; \vec{a})$, $g(x, Q_0^2; \vec{a})$ (with $\vec{a}$ a set of free parameters)
  2. Solve DGLAP to get $q_f(x, Q^2; \vec{a})$, $g(x, Q^2; \vec{a})$ at all Q^2
  3. Fit the free parameters $\vec{a}$ to experimental data (F_2, $F_2^{c,b}$, F_L, pp jets, pp $t\bar{t}$, ...)

PDF status

- State-of-the-art: NNLL known, N^{3}LL almost known (in moment space)
- Practical approach:
  1. Take an initial condition for all quarks and gluons at an initial scale Q_0: $q_f(x, Q_0^2; \vec{a})$, $g(x, Q_0^2; \vec{a})$ (with $\vec{a}$ a set of free parameters)
  2. Solve DGLAP to get $q_f(x, Q^2; \vec{a})$, $g(x, Q^2; \vec{a})$ at all Q^2
  3. Fit the free parameters $\vec{a}$ to experimental data (F_2, $F_2^{c,b}$, F_L, pp jets, pp $t\bar{t}$, ...)
- Several subtleties: form of the init cdt, treatment of heavy quarks, data included, treatment of uncertainties, ...

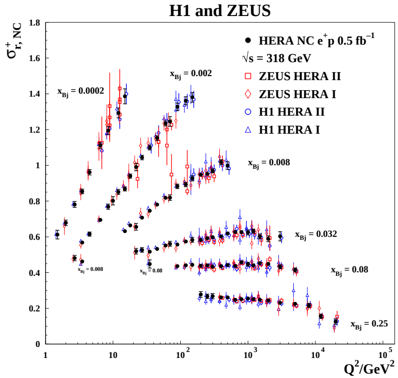
PDF status

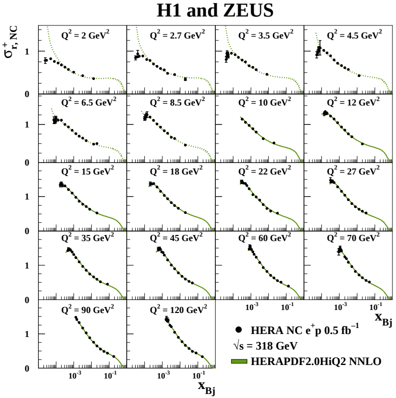
- State-of-the-art: NNLL known, N^3LL almost known (in moment space)
- Practical approach:
  1. Take an initial condition for all quarks and gluons at an initial scale Q_0: $q_f(x, Q_0^2; \vec{a})$, $g(x, Q_0^2; \vec{a})$ (with $\vec{a}$ a set of free parameters)
  2. Solve DGLAP to get $q_f(x, Q^2; \vec{a})$, $g(x, Q^2; \vec{a})$ at all Q^2
  3. Fit the free parameters $\vec{a}$ to experimental data (F_2, $F_2^{c,b}$, F_L, pp jets, pp $t\bar{t}$, ...)
- Several subtleties: form of the init cdt, treatment of heavy quarks, data included, treatment of uncertainties, ...
- Effort (still ongoing!!) from several groups: CTEQ/CT, MRST/MSTW/MMHT/MSHT, NNPDF, ...
  1438 PDF sets available from LHAPDF (link)

PDF status

$$\sigma_{\text{red}} = F_2 + y^2[1 + (1-y)^2]F_L$$

[HERAPDF2.0]



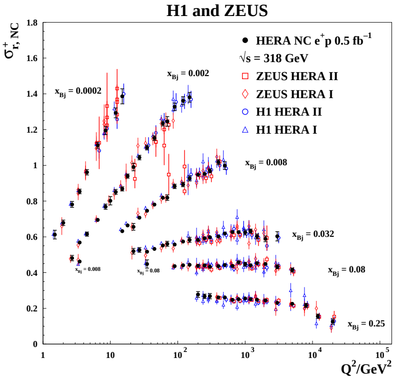


- Rapid rise at small x
- flatter at large x (Bj. scaling)

Well reproduced by DGLAP fit ($Q^2 \geq 10$ GeV^2)

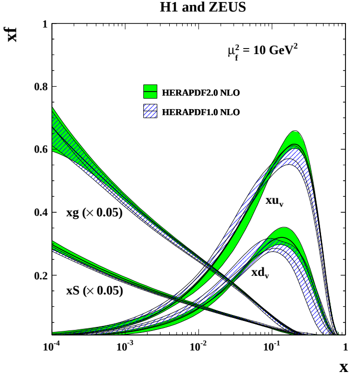
PDF status

$$\sigma_{\text{red}} = F_2 + y^2[1 + (1 - y)^2]F_L$$



[HERAPDF2.0]

Resulting PDFs



- Rapid rise at small x
- flatter at large x (Bj. scaling)

Gluon (and sea quarks) rise at small x

Part X: recap
divergences in QCD

Recap: divergences in QCD

1. UV divergences:

2. IR divergences in the initial state:

3. IR divergences in the final state:

Recap: divergences in QCD

1. UV divergences: absorbed in parameters of the QCD Lagrangian
   QCD is renormalisable
   Renormalisation Group Equation for the dependence of α_s and masses on the renormalisation scale

2. IR divergences in the initial state:

3. IR divergences in the final state:

Recap: divergences in QCD

1. UV divergences: absorbed in parameters of the QCD Lagrangian
   QCD is renormalisable
   Renormalisation Group Equation for the dependence of α_s and masses on the renormalisation scale

2. IR divergences in the initial state: absorbed in PDFs
   Depencence on the factorisation scale through the DGLAP equation

3. IR divergences in the final state:

Recap: divergences in QCD

1. UV divergences: absorbed in parameters of the QCD Lagrangian
   QCD is renormalisable
   Renormalisation Group Equation for the dependence of α_s and masses on the renormalisation scale

2. IR divergences in the initial state: absorbed in PDFs
   Depencence on the factorisation scale through the DGLAP equation

3. IR divergences in the final state: cancel between "real" and "virtual" contributions as long as the observable is infrared-and-collinear safe

Recap: divergences in QCD

Comments:

- All divergences are logarithmic
- Intimately connected to calculability in perturbative QCD:
  - kernels of the RGE and PDFs calculable order by order
  - IRC-safe observables calculable up to non-perturbative corrections $\propto \left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^{\#}$
- For a hard scale Q, perturbative expansion in powers of $\alpha_s(Q)$ (LO, NLO, NNLO, ...)
- For disparate scales, say Q and vQ ($v \ll 1$), perturbative expansion in powers of $\alpha_s(Q)\log^2 v$ or $\alpha_s(Q)\log v$ (LL, NLL, NNLL, ...)

Part XI: QCD at hadronic colliders

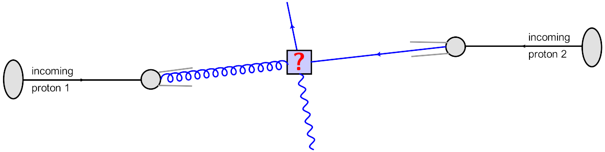
Foreword

- Most of the fundamental concepts are as in *ee* and DIS
- More busy environment due to hadronic beams
- Simply discuss the main differences with what we discussed earlier

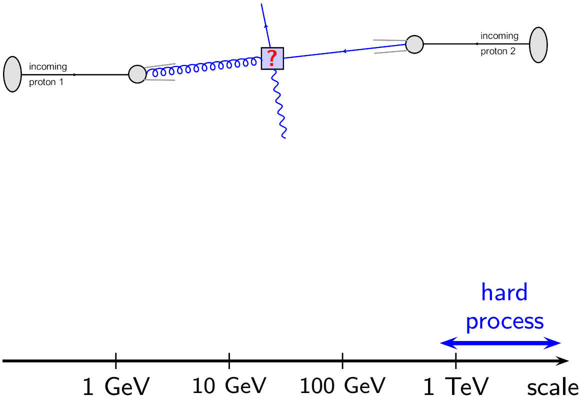
Colliders study fundamental interactions at high energy

Master formula:

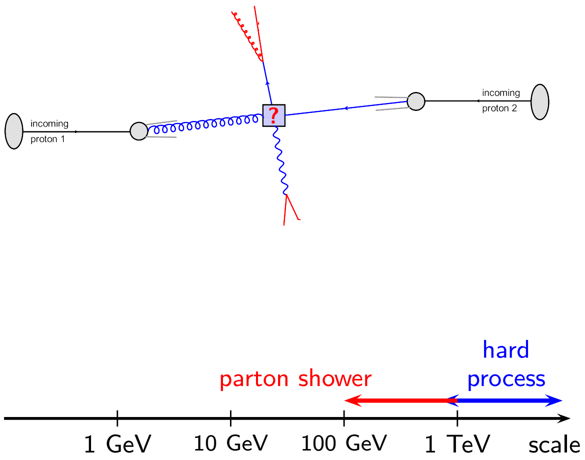
$$\sigma = \int dx_1 dx_2 \underbrace{f_a(x_1, Q) f_b(x_2, Q)}_{\text{PDFs}} \underbrace{\hat{\sigma}(x_1, x_2, Q)}_{\text{partonic x-sect.}}$$



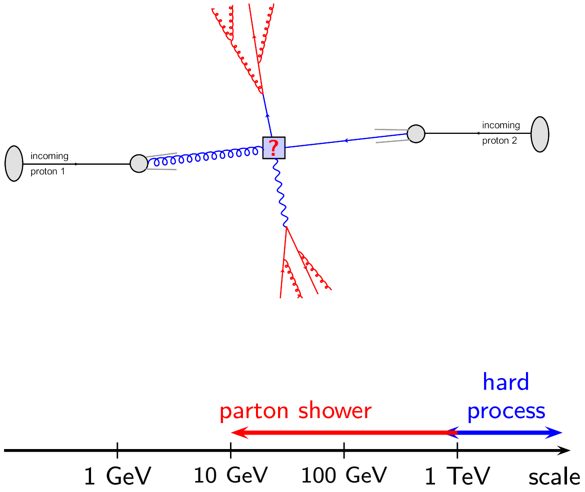
Anatomy of a high-energy collision



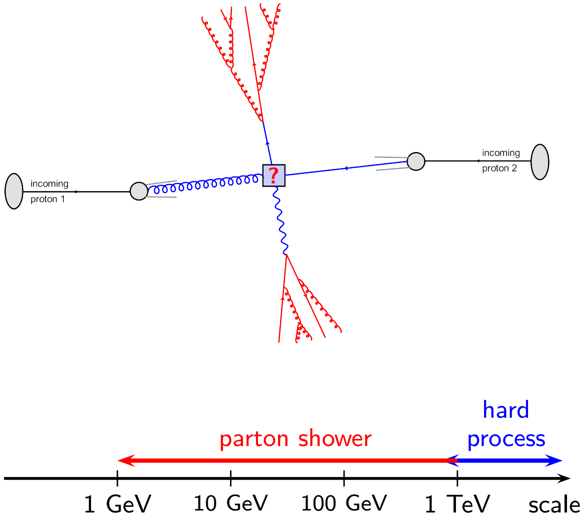
Anatomy of a high-energy collision



Anatomy of a high-energy collision



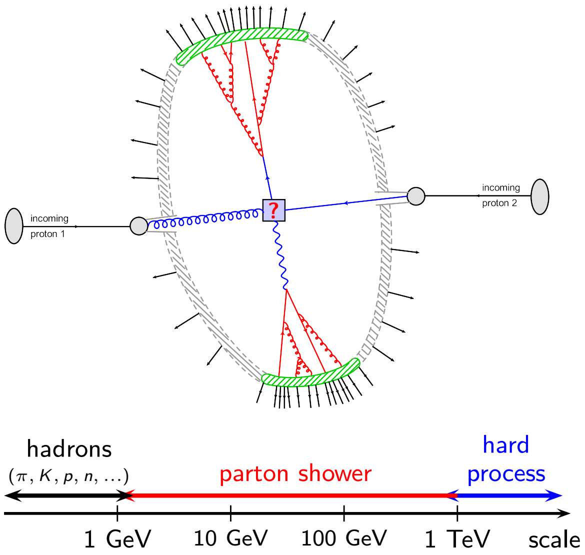
Anatomy of a high-energy collision



Hard + branchings

- perturbative QCD
- under solid control
- predictive, systematically improvable theory with genuine uncertainty estimates

Anatomy of a high-energy collision



Hard + branchings

- perturbative QCD
- under solid control
- predictive, systematically improvable theory with genuine uncertainty estimates

Hadronisation and UE/MPI

- NON-perturbative
- needs modelling
- model-dependent

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$\begin{aligned} p^\mu &\equiv (p_x, p_y, p_z, E) \\ &\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\ &\overset{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y) \end{aligned}$$

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E+p_z}{E-p_z}$$

ee collisions

$$\begin{aligned} p^\mu &\equiv (p_x, p_y, p_z, E) \\ &\equiv (p \sin\theta \cos\varphi, p \sin\theta \sin\varphi, p \cos\theta, E) \\ &\overset{m=0}{\equiv} p(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta, 1) \end{aligned}$$

Use spherical coordinates: E, θ, φ

Kinematics

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$
\begin{aligned}
p^\mu &\equiv (p_x, p_y, p_z, E) \\
&\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\
&\overset{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y)
\end{aligned}
$$

Use cylindrical coordinates: p_t, y, ϕ

$$
m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E+p_z}{E-p_z}
$$

- p_t is the **transverse momentum**
  (m_t is the transverse mass)

Kinematics

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$
\begin{aligned}
p^\mu &\equiv (p_x, p_y, p_z, E) \\
&\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\
&\stackrel{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y)
\end{aligned}
$$

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E+p_z}{E-p_z}$$

- p_t is the **transverse momentum**
  (m_t is the transverse mass)
- y is the **rapidity**

Kinematics

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$\begin{aligned} p^\mu &\equiv (p_x, p_y, p_z, E) \\ &\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\ &\stackrel{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y) \end{aligned}$$

- p_t is the **transverse momentum**
  (m_t is the transverse mass)
- y is the **rapidity**
- "energy-like" and geometrical

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2} \log \frac{E + p_z}{E - p_z}$$

The "partonic" collision can (usually) happen for a range of x_1, x_2

$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$\begin{aligned} p^\mu &\equiv (p_x, p_y, p_z, E) \\ &\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\ &\overset{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y) \end{aligned}$$

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E+p_z}{E-p_z}$$

- p_t is the **transverse momentum** (m_t is the transverse mass)
- y is the **rapidity**
- "energy-like" and geometrical
- transverse and longitudinal

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$\begin{aligned} p^\mu &\equiv (p_x, p_y, p_z, E) \\ &\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\ &\stackrel{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y) \end{aligned}$$

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E+p_z}{E-p_z}$$

- p_t is the **transverse momentum**
  (m_t is the transverse mass)
- y is the **rapidity**
- "energy-like" and geometrical
- transverse and longitudinal
- Pseudo-rapidity $\eta = -\log\tan\theta/2$

Kinematics

The "partonic" collision can (usually) happen for a range of x_1, x_2
$\Rightarrow$ the centre-of-mass of the hard collision is boosted compared to the lab frame

pp collisions

$$
\begin{aligned}
p^\mu &\equiv (p_x, p_y, p_z, E) \\
&\equiv (p_t \cos\phi, p_t \sin\phi, m_t \sinh y, m_t \cosh y) \\
&\overset{m=0}{\equiv} p_t(\cos\phi, \sin\phi, \sinh y, \cosh y)
\end{aligned}
$$

Use cylindrical coordinates: p_t, y, ϕ

$$m_t = \sqrt{p_t^2 + m^2} \qquad y = \frac{1}{2}\log\frac{E + p_z}{E - p_z}$$

- p_t is the **transverse momentum**
  (m_t is the transverse mass)
- y is the **rapidity**
- "energy-like" and geometrical
- transverse and longitudinal
- Pseudo-rapidity $\eta = -\log\tan\theta/2$
  - $y = \eta \quad \Leftrightarrow \quad m = 0$
  - Δy boost invariant, not $\Delta\eta$
  - Prefer y over η!

Jets

Strategy similar to *ee* except for:

- choice of kinematic variables
- UE/MPI

  $\Rightarrow$ extra hadronic activity

  $\Rightarrow$ jet radius R limiting the spatial extent of jets

Jets

Strategy similar to *ee* except for:

- choice of kinematic variables
- UE/MPI
  - ⇒ extra hadronic activity
  - ⇒ jet radius R limiting the spatial extent of jets

Pairwise recombination algorithms

Repeat the following until everything is clustered

1. Compute distances between all particles
$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \left[\Delta y_{ij}^2 + \Delta\phi_{ij}^2\right]$$
$$d_{iB} = p_{ti}^{2p} R^2$$
2. Find smallest of all distances
3. If d_{ij}: remove p_i and p_j and replace by $p_i + p_j$
   If d_{iB}: call i a jet

Jets

Strategy similar to *ee* except for:

- choice of kinematic variables
- UE/MPI
  - $\Rightarrow$ extra hadronic activity
  - $\Rightarrow$ jet radius R limiting the spatial extent of jets

Pairwise recombination algorithms

Repeat the following until everything is clustered

1. Compute distances between all particles
$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \left[\Delta y_{ij}^2 + \Delta\phi_{ij}^2\right]$$
$$d_{iB} = p_{ti}^{2p} R^2$$
2. Find smallest of all distances
3. If d_{ij}: remove p_i and p_j and replace by $p_i + p_j$
   If d_{iB}: call i a jet

3 typical cases:

1. $p = 1$: k_t algorithm (cf. *ee*)
2. $p = 0$: Cambridge/Aachen algorithm (cf. *ee*)
3. $p = -1$: anti-k_t algorithm (default at the LHC)

Jets

Strategy similar to *ee* except for:

- choice of kinematic variables
- UE/MPI
  - $\Rightarrow$ extra hadronic activity
  - $\Rightarrow$ jet radius R limiting the spatial extent of jets

For completeness: cone algorithms

- Idea of "dominant directions of energy flow" in the event
- Extensively used at the Tevatron (CDF MidPoint, D0 MidPoint, JetClu, ...)
- All the cone algorithms used at the Tevatron are IRC unsafe!
- One IRC-safe option: SISCone (not extensively used in practice)

Pairwise recombination algorithms

Repeat the following until everything is clustered

1. Compute distances between all particles
$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p})\,[\Delta y_{ij}^2 + \Delta\phi_{ij}^2]$$
$$d_{iB} = p_{ti}^{2p} R^2$$
2. Find smallest of all distances
3. If d_{ij}: remove p_i and p_j and replace by $p_i + p_j$
   If d_{iB}: call i a jet

3 typical cases:

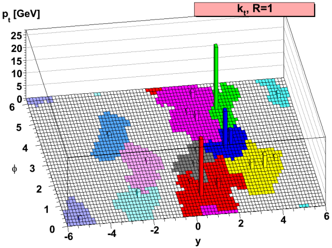
1. $p = 1$: k_t algorithm (cf. *ee*)
2. $p = 0$: Cambridge/Aachen algorithm (cf. *ee*)
3. $p = -1$: anti-k_t algorithm (default at the LHC)

Jets

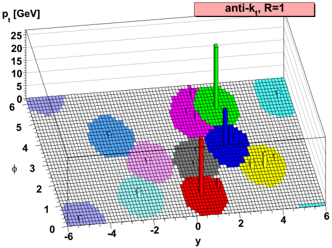
Example event at the LHC

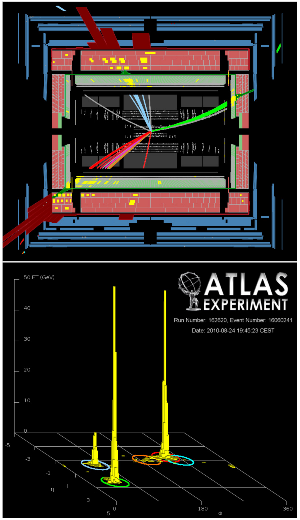
compared to others, hard anti-k_t are circles

k_t



anti-k_t





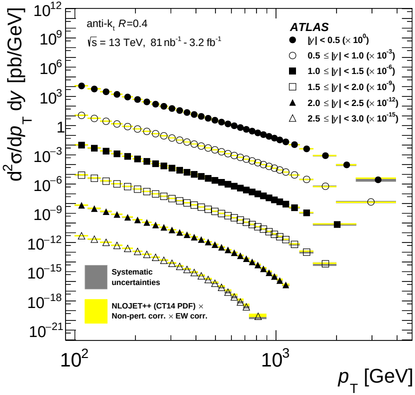
One typically uses $R = 0.4$ (R up to 0.8-1 in specific cases)

Example: (inclusive) jet cross-section

Jet ≡ parton

(At leat in the context of hadron colliders)

Jets are IRC-safe proxies to "hard partons" from the initial collision



- Ubiquitous at colliders: used in almost all measurements and searches
- Only well defined if one specifies
  - Which jet definition is used
  - Which cuts are applied

Jet $\equiv$ parton

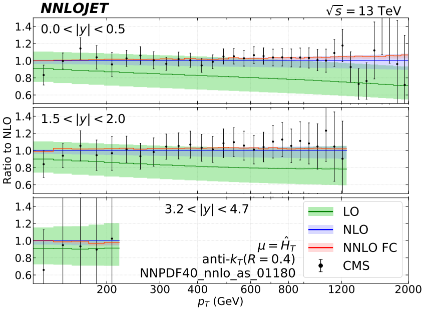
(At leat in the context of hadron colliders)

Jets are IRC-safe proxies to "hard partons" from the initial collision

- Ubiquitous at colliders: used in almost all measurements and searches
- Only well defined if one specifies
  - Which jet definition is used
  - Which cuts are applied

Example: (inclusive) jet cross-section

comparison to NNLO QCD



LO$\to$NLO$\to$NNLO: reduction of the uncertainties

[thanks to A.Huss]

QCD challenges

The LHC takes us through an amazing journey at the forefront of our knowledge
This implies a series of challenges

Things (briefly) discussed

- precision needed! (Including $\hat{\sigma}$, PDFs, α_s,...)
- large range of processes and multiplicities challenge for precision
- large range of scales $\Rightarrow$ requires resummations
- Need for good non-perturbative models

Things not (really) discussed

- A vast and rich heavy-ion program
- Everything amplified at future colliders Valid for both FCC-*ee* (+*ee* friends) and FCC-*hh*!

QCD challenges

The LHC takes us through an amazing journey at the forefront of our knowledge
This implies a series of challenges

Things (briefly) discussed

- precision needed! (Including $\hat{\sigma}$, PDFs, α_s,...)
- large range of processes and multiplicities
  challenge for precision
- large range of scales $\Rightarrow$ requires resummations
- Need for good non-perturbative models

Things not (really) discussed

- A vast and rich heavy-ion program
- Everything amplified at future colliders
  Valid for both FCC-*ee* (+*ee* friends) and FCC-*hh*!

If only one message to take home

A top-notch knowledge/understanding of QCD is

1. **interesting per se!** (part of a physicist's job to understand fundamental interactions)
   If time left: examples of fun structures emerging from QCD
2. **primordial for the whole programme of collider physics**

Fixed-order calculations: Amplitudes

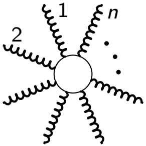
Two main ingredients/difficulties:

1. The amplitude $\mathcal{M}$ itself
2. Cancelling the divergences between real and virtual emissions

Complexity increases with:

1. The number of loops (LO, NLO, NNLO, ...)
2. The number of external (coloured) legs
   Including initial-state ones

tree-level
n-gluon
amplitude



n	#diagrams
4	4
5	25
6	220
7	2485
8	34300
9	559405
10	10525900

Rough estimate:
1 extra loop $\approx$ 2 extra legs

[thanks to S.Abreu and B.Page]

Fixed-order calculations: Amplitudes

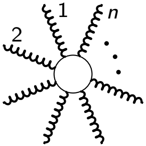
Two main ingredients/difficulties:

1. The amplitude $\mathcal{M}$ itself
2. Cancelling the divergences between real and virtual emissions

Complexity increases with:

1. The number of loops (LO, NLO, NNLO, ...)
2. The number of external (coloured) legs
   Including initial-state ones

tree-level
n-gluon
amplitude



n	#diagrams
4	4
5	25
6	220
7	2485
8	34300
9	559405
10	10525900

Rough estimate:
1 extra loop $\approx$ 2 extra legs

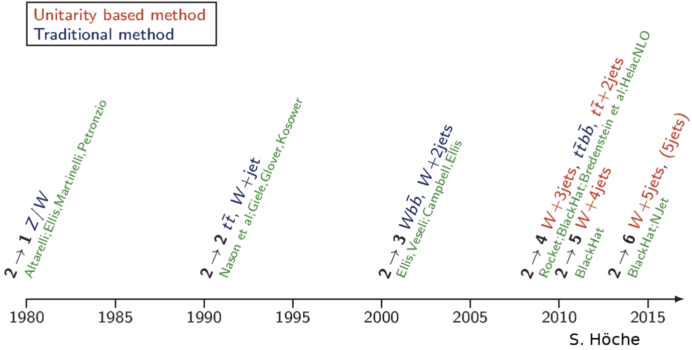
[thanks to S.Abreu and B.Page]

Field of **amplitudes** (born $\sim$ 15 years ago) meant to
study and compute amplitudes without going through Feynman graphs

The NLO revolution

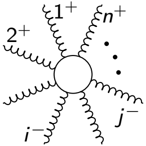
About 10 years ago: NLO made (almost) automated

The NLO revolution



Many core tools developed:

- Spinor-helicity formalism
  $\Rightarrow$ compact expressions
- Example: Parke-Taylor



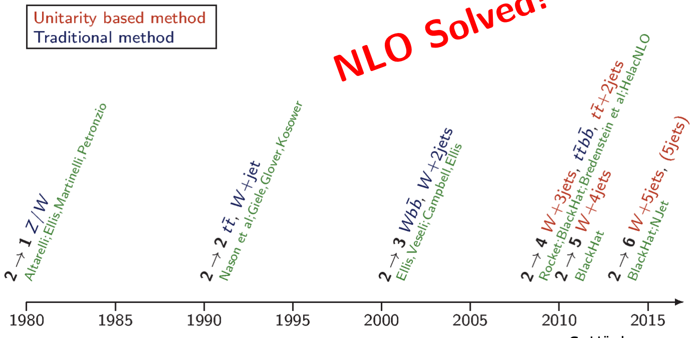
$$= \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Generalised unitarity
  Loops from trees and cuts
- ... and many others
  BCFW, double-copy, bootstrap, alphabet&symbols,...

The NLO revolution

About 10 years ago: NLO made (almost) automated

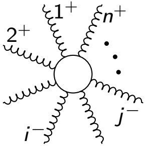
The NLO revolution



S. Höche

Many core tools developed:

- Spinor-helicity formalism
  $\Rightarrow$ compact expressions
- Example: Parke-Taylor

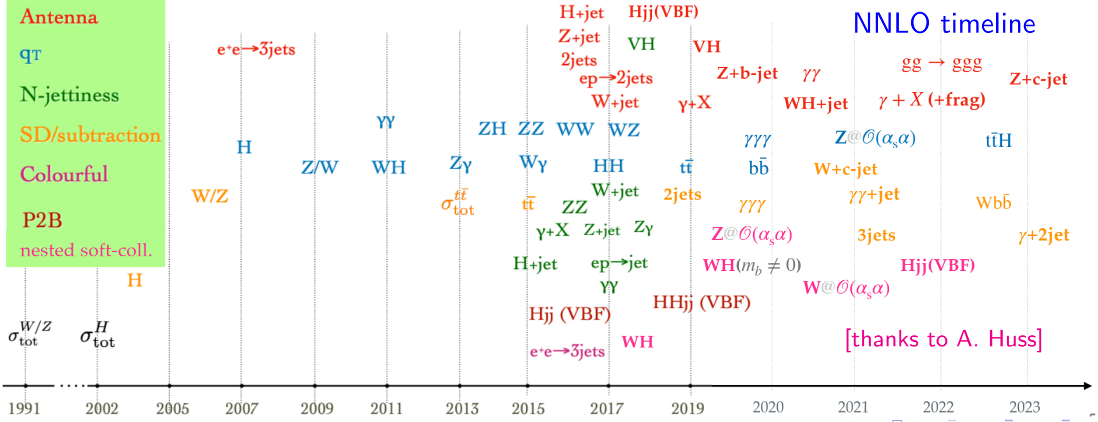


$$= \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Generalised unitarity
  Loops from trees and cuts
- ... and many others
  BCFW, double-copy, bootstrap, alphabet&symbols,...

Amplitudes: towards NNLO and beyond

- Deep understanding on the structure of amplitudes, rooted in field theory
- Often developed in $N = 4$ SUSY which has a higher degree of symmetry than QCD
- Now extending to NNLO (even N^{3}LO): current state-of-the-art: $2 \to 3$ at 2 loops



All-order calculations: Resummations

Two main approaches

1. "direct" calculation in QCD
2. effective field theory approach: Soft Collinear Effective Theory

My (rough and personal) take on this: SCET super efficient for systematic improvements (e.g. reaching high accuracy); direct calculation often nice to highlight underlying physics mechanisms

All-order calculations: Resummations

Two main approaches

1. "direct" calculation in QCD
2. effective field theory approach: Soft Collinear Effective Theory

My (rough and personal) take on this: SCET super efficient for systematic improvements (e.g. reaching high accuracy); direct calculation often nice to highlight underlying physics mechanisms

State-of-the-art

- NLL (almost) automated
- for *ee*, NNLL (almost) automated
- N^{3}LL for specific cases
- Collinear physics easier than soft emissions

All-order calculations: Resummations

Two main approaches

1. "direct" calculation in QCD
2. effective field theory approach: Soft Collinear Effective Theory

My (rough and personal) take on this: SCET super efficient for systematic improvements (e.g. reaching high accuracy); direct calculation often nice to highlight underlying physics mechanisms

State-of-the-art

- NLL (almost) automated
- for *ee*, NNLL (almost) automated
- N^3LL for specific cases
- Collinear physics easier than soft emissions

Soft emissions

- complicated geometrical and colour structures
- Field-theory progress (webs,...); connected to amplitudes
- Some observables (like a jet veto for jets with $|y| < y_{\rm cut}$ in H studies) are only sensitive to a part of the (geometrical) phase-space
  $\Rightarrow$ "non-global" logs difficult to resum
- Usually appear at NLL: OK at large N_c, tough beyond
  Recent progress: subleading correction at large N_c

All-order calculations: Resummations

Two main approaches

1. "direct" calculation in QCD
2. effective field theory approach: Soft Collinear Effective Theory

My (rough and personal) take on this: SCET super efficient for systematic improvements (e.g. reaching high accuracy); direct calculation often nice to highlight underlying physics mechanisms

State-of-the-art

- NLL (almost) automated
- for ee, NNLL (almost) automated
- N^{3}LL for specific cases
- Collinear physics easier than soft emissions

Matching

Quite often include **matched** predictions $N^pLO + N^qLL$

Idea: get the best of both limits:

- exact N^pLO α_s expansion (when logs are small)
- N^qLL resummation when logs are large
- avoiding double counting
  requires log expansion at fixed order; several "matching" schemes

All-order calculations: Resummations

Two main approaches

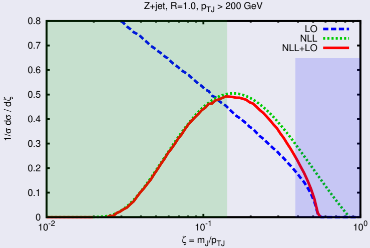
1. "direct" calculation in QCD
2. effective field theory approach: Soft Collinear Effective Theory

My (rough and personal) take on this: SCET super efficient for systematic improvements (e.g. reaching high accuracy); direct calculation often nice to highlight underlying physics mechanisms

State-of-the-art

- NLL (almost) automated
- for *ee*, NNLL (almost) automated
- N^3LL for specific cases
- Collinear physics easier than soft emissions

Matching

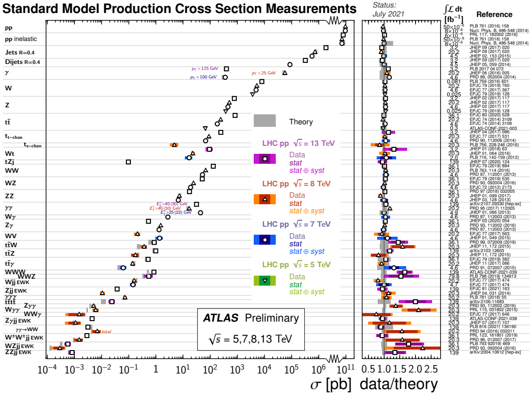


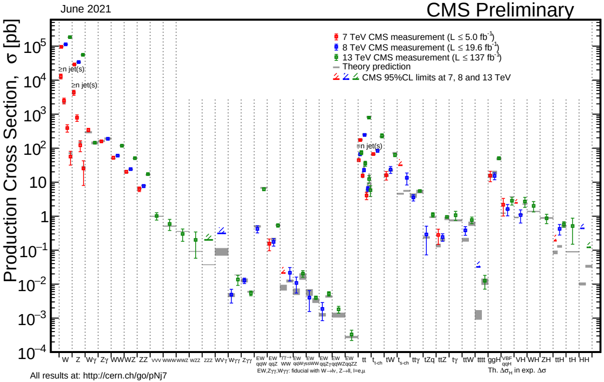
finite/large $v = m/p_t$
matched agrees with fixed-order

small $v = m/p_t$ (large log)
matched agrees with resummed

Contributed to big achievements at the LHC

Long list of standard-model measuirements





Also played a critical role in BSM searches

Highly challenging perspective

From a pheno QCD standpoint (i.e. besides experimental aspects/challenges)

- requires more precise determination of α_s
- requires high fixed-order accuracy (likely at least N^{3}LO)
- requires high resummation accuracy (likely at least N^{3}LL)
- requires mixed QCD+EW corrections with high accuracy
- requires excellent control over non-perturbative effects

Part XII: Monte Carlo event generators

Generic approach

Typical calculations take the following form:

$$\mathcal{O} = \sum_n \int d\Phi_n \, |\mathcal{M}(k_1, \dots, k_n)|^2 \, \mathcal{O}_n(k_1, \dots, k_n)$$

Even if we have the amplitudes analytically, this is still highly complex:

- real-virtual cancellations
- PDFs for hadronic beams
- often complex observables and cuts
- resummations sensitive to all n
- one can have non-perturbative hadronisation/MPI or detector simulations

Generic approach

Typical calculations take the following form:

$$\mathcal{O} = \underbrace{\sum_n \int d\Phi_n \, |\mathcal{M}(k_1, \ldots, k_n)|^2}_{\text{Monte Carlo sampling}} \underbrace{\mathcal{O}_n(k_1, \ldots, k_n)}_{\text{end-user's}}$$

Even if we have the amplitudes analytically, this is still highly complex:

- real-virtual cancellations
- PDFs for hadronic beams
- often complex observables and cuts
- resummations sensitive to all n
- one can have non-perturbative hadronisation/MPI or detector simulations

Idea of Monte Carlo generators

1. Provide a numerical sampling of the phase-space and amplitudes
2. hand over $k_1, \ldots, k_n$ to the end user
3. let the end user compute the obserable

Typical calculations take the following form:

$$\mathcal{O} = \underbrace{\sum_n \int d\Phi_n \, |\mathcal{M}(k_1, \dots, k_n)|^2}_{\text{Monte Carlo sampling}} \underbrace{\mathcal{O}_n(k_1, \dots, k_n)}_{\text{end-user's}}$$

Even if we have the amplitudes analytically, this is still highly complex:

- real-virtual cancellations
- PDFs for hadronic beams
- often complex observables and cuts
- resummations sensitive to all n
- one can have non-perturbative hadronisation/MPI or detector simulations

Idea of Monte Carlo generators

1. Provide a numerical sampling of the phase-space and amplitudes
2. hand over $k_1, \dots, k_n$ to the end user
3. let the end user compute the obserable

Key gain: works with any observable

Different types of MC generators

$$\mathcal{O} = \underbrace{\sum_n \int d\Phi_n \left|\mathcal{M}(k_1, \dots, k_n)\right|^2}_{\text{Monte Carlo sampling}} \mathcal{O}_n(k_1, \dots, k_n)$$

Different types of MC generators

$$\mathcal{O} = \underbrace{\sum_{n \in \mathrm{N}^k\mathrm{LO}}}_{\text{finite sum}} \int d\Phi_n \, |\mathcal{M}(k_1, \dots, k_n)|^2 \; \mathcal{O}_n(k_1, \dots, k_n)$$

- Fixed order

$$\mathcal{O} = \underbrace{\sum_{\text{all } n}}_{\text{infinite sum}} \int d\Phi_n \, |\mathcal{M}(k_1,\dots,k_n)|^2 \; \mathcal{O}_n(k_1,\dots,k_n)$$

- Fixed order or all orders

Different types of MC generators

$$\mathcal{O} = \sum_n \underbrace{\int d\Phi_n}_{\text{sampled}} \underbrace{|\mathcal{M}(k_1,\dots,k_n)|^2}_{\text{weight}} \mathcal{O}_n(k_1,\dots,k_n)$$

- Fixed order or all orders
- Weighted

Different types of MC generators

$$\mathcal{O} = \sum_n \underbrace{\int d\Phi_n \, |\mathcal{M}(k_1, \ldots, k_n)|^2}_{\text{sampled}} \mathcal{O}_n(k_1, \ldots, k_n)$$

- Fixed order or all orders
- Weighted or unweighted

Fixed-order MC generators

- Require a finite range of multiplicities
  E.g. dijets:
  - LO$\equiv \mathcal{O}(\alpha_s^2)$: $2 \to 2$ (tree level)
  - NLO$\equiv \mathcal{O}(\alpha_s^3)$: $2 \to 3$ (tree level), $2 \to 2$ (1-loop)
  - NNLO$\equiv \mathcal{O}(\alpha_s^4)$: $2 \to 4$ (tree level), $2 \to 3$ (1-loop), $2 \to 2$ (2-loops)

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

  - Overall finite but the $|\mathcal{M}_{\text{real}}|^2$ integration diverges and $|\mathcal{M}_{\text{virt}}|^2$ has poles (in ε)

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

  - Overall finite but the $|\mathcal{M}_{\text{real}}|^2$ integration diverges and $|\mathcal{M}_{\text{virt}}|^2$ has poles (in ε)
  - Common idea: factorise the phase space $d\Phi_{n+1} = d\Phi_n d\Phi_1$
  - and introduce a subtraction term S ($I = \int d\Phi_1 S$)

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}(|\mathcal{M}_{\text{real}}|^2 + S)\mathcal{O}_{n+1} + \int d\Phi_n(|\mathcal{M}_{\text{virt}}|^2 - I)\mathcal{O}_n$$

such that each term is separately finite

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

  - Overall finite but the $|\mathcal{M}_{\text{real}}|^2$ integration diverges and $|\mathcal{M}_{\text{virt}}|^2$ has poles (in ε)
  - Common idea: factorise the phase space $d\Phi_{n+1} = d\Phi_n d\Phi_1$
  - and introduce a subtraction term S ($I = \int d\Phi_1 S$)

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}(|\mathcal{M}_{\text{real}}|^2 + S)\mathcal{O}_{n+1} + \int d\Phi_n(|\mathcal{M}_{\text{virt}}|^2 - I)\mathcal{O}_n$$

  such that each term is separately finite

- Many methods: Catani-Seymour, FKS, slicing, projection to Born, sector decomposition,...

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

  - Overall finite but the $|\mathcal{M}_{\text{real}}|^2$ integration diverges and $|\mathcal{M}_{\text{virt}}|^2$ has poles (in ε)
  - Common idea: factorise the phase space $d\Phi_{n+1} = d\Phi_n d\Phi_1$
  - and introduce a subtraction term S ($I = \int d\Phi_1 S$)

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}(|\mathcal{M}_{\text{real}}|^2 + S)\mathcal{O}_{n+1} + \int d\Phi_n(|\mathcal{M}_{\text{virt}}|^2 - I)\mathcal{O}_n$$

  such that each term is separately finite

- Many methods: Catani-Seymour, FKS, slicing, projection to Born, sector decomposition,...
- Usually a weighted approach with negative weights

Fixed-order MC generators

- Require a finite range of multiplicities
- Main challenge: each n is separately infinite

$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}|\mathcal{M}_{\text{real}}|^2\mathcal{O}_{n+1} + \int d\Phi_n|\mathcal{M}_{\text{virt}}|^2\mathcal{O}_n = \text{finite}$$

  - Overall finite but the $|\mathcal{M}_{\text{real}}|^2$ integration diverges and $|\mathcal{M}_{\text{virt}}|^2$ has poles (in ε)
  - Common idea: factorise the phase space $d\Phi_{n+1} = d\Phi_n d\Phi_1$
  - and introduce a subtraction term S ($I = \int d\Phi_1 S$)

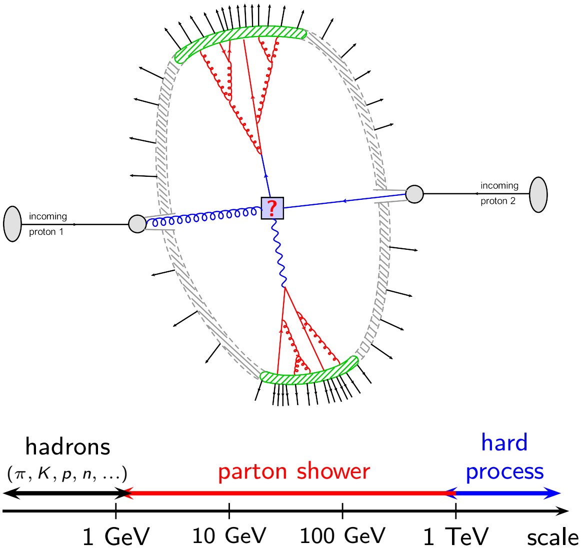
$$\frac{d\sigma_{\text{pure NLO}}}{d\mathcal{O}} = \int d\Phi_{n+1}(|\mathcal{M}_{\text{real}}|^2 + S)\mathcal{O}_{n+1} + \int d\Phi_n(|\mathcal{M}_{\text{virt}}|^2 - I)\mathcal{O}_n$$

  such that each term is separately finite

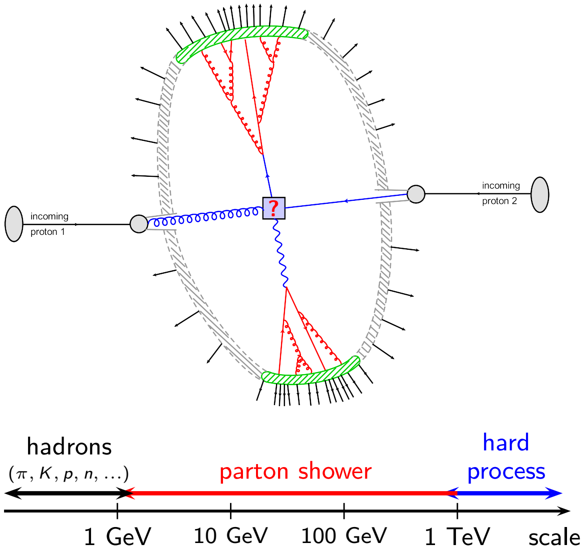
- Many methods: Catani-Seymour, FKS, slicing, projection to Born, sector decomposition,...
- Usually a weighted approach with negative weights
- Recall: the observable needs to be IRC-safe!

Generic-purpose MC generators (GPMC)

Idea: generate the full event



Generic-purpose MC generators (GPMC)

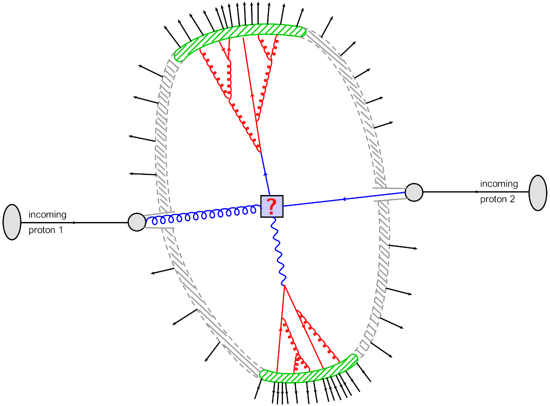


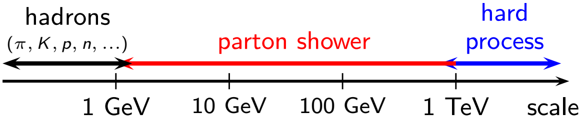
Idea: generate the full event

Several ingredients

- Hard process
- Parton shower
- hadronisation
- hadron decays
- MPI/UE

Generic-purpose MC generators (GPMC)



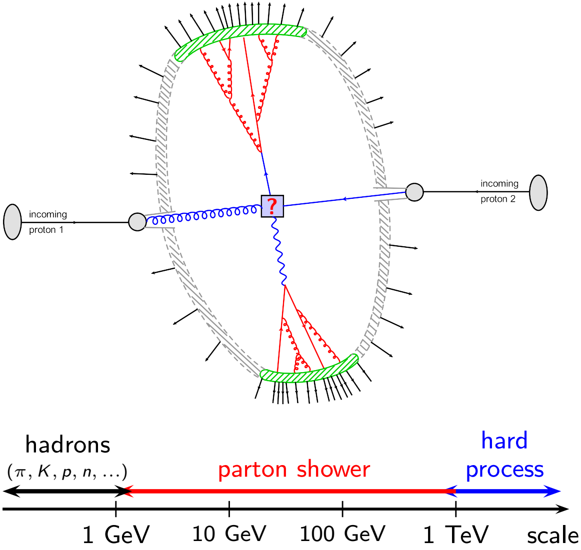


Idea: generate the full event

Several ingredients

- Hard process
  perturbative QCD, fixed order
- Parton shower
- hadronisation
- hadron decays
- MPI/UE

Generic-purpose MC generators (GPMC)

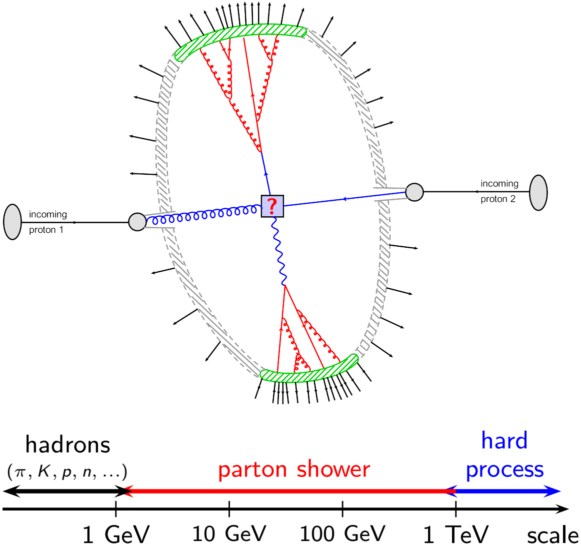


Idea: generate the full event

Several ingredients

- Hard process
  perturbative QCD, fixed order
- Parton shower
  perturbative QCD, all orders
- hadronisation
- hadron decays
- MPI/UE

Generic-purpose MC generators (GPMC)

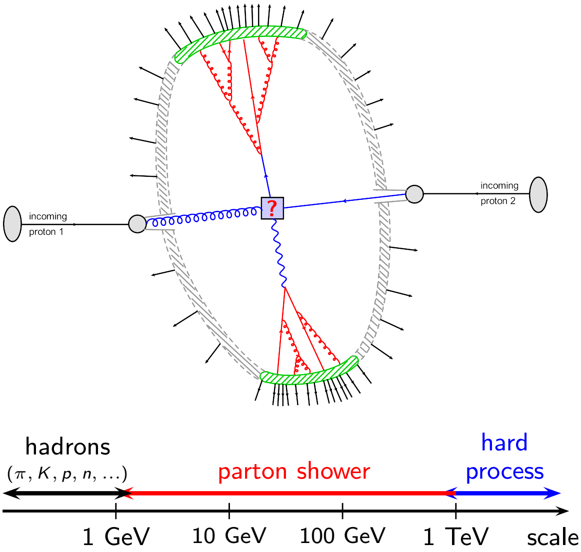


Idea: generate the full event

Several ingredients

- Hard process
  perturbative QCD, fixed order
- Parton shower
  perturbative QCD, all orders
- hadronisation
  non-perturbative, modelled
- hadron decays
  non-perturbative, modelled
- MPI/UE
  non-perturbative, modelled

Generic-purpose MC generators (GPMC)



Idea: generate the full event

Several ingredients

- Hard process
  perturbative QCD, fixed order
- Parton shower
  perturbative QCD, all orders
- hadronisation
  non-perturbative, modelled
- hadron decays
  non-perturbative, modelled
- MPI/UE
  non-perturbative, modelled

Q: How to estimate uncertainties?

The workhorses

Herwig, PYTHIA and Sherpa offer convenient frameworks
for LHC physics studies, covering all aspects above,
but with slightly different history/emphasis:

PYTHIA (successor to JETSET, begun in 1978):
originated in hadronization studies,
still special interest in soft physics.

Herwig (successor to EARWIG, begun in 1984):
originated in coherent showers (angular ordering),
cluster hadronization as simple complement.

Sherpa (APACIC++/AMEGIC++, begun in 2000):
had own matrix-element calculator/generator
originated with matching & merging issues.

[slide from T. Sjöstrand, 2016]

The workhorses

Herwig, PYTHIA and Sherpa offer convenient frameworks
for LHC physics studies, covering all aspects above,
but with slightly different history/emphasis:

PYTHIA (successor to JETSET, begun in 1978):
originated in hadronization studies,
still special interest in soft physics.

Herwig (successor to EARWIG, begun in 1984):
originated in coherent showers (angular ordering),
cluster hadronization as simple complement.

Sherpa (APACIC++/AMEGIC++, begun in 2000):
had own matrix-element calculator/generator
originated with matching & merging issues.

[slide from T. Sjöstrand, 2016]

Super useful!

- full events
- can compute basically anything you want
- can feed to detector simulations

The workhorses

Herwig, PYTHIA and Sherpa offer convenient frameworks for LHC physics studies, covering all aspects above, but with slightly different history/emphasis:

PYTHIA (successor to JETSET, begun in 1978): originated in hadronization studies, still special interest in soft physics.

Herwig (successor to EARWIG, begun in 1984): originated in coherent showers (angular ordering), cluster hadronization as simple complement.

Sherpa (APACIC++/AMEGIC++, begun in 2000): had own matrix-element calculator/generator originated with matching & merging issues.

[slide from T. Sjöstrand, 2016]

Torbjörn Sjöstrand — Status and Developments of Event Generators — slide 7/28

Super useful!

- full events
- can compute basically anything you want
- can feed to detector simulations

Watch out!

- building blocks have their own limitations
- different observables sensitive differently to each ingredient
- sometimes one expects MC to disagree with data

The workhorses

Herwig, PYTHIA and Sherpa offer convenient frameworks
for LHC physics studies, covering all aspects above,
but with slightly different history/emphasis:

PYTHIA (successor to JETSET, begun in 1978):
originated in hadronization studies,
still special interest in soft physics.

Herwi
origin
cluster hadronization as simple complement.

probably the most used theoretical tool in particle physics

Sherpa (APACIC++/AMEGIC++, begun in 2000):
had own matrix-element calculator/generator
originated with matching & merging issues.

[slide from T. Sjöstrand, 2016]

Torbjörn Sjöstrand   Status and Developments of Event Generators   slide 7/28

Super useful!

- full events
- can compute basically anything you want
- can feed to detector simulations

Watch out!

- building blocks have their own limitations
- different observables sensitive differently to each ingredient
- sometimes one expects MC to disagree with data

Part XIII: Monte Carlo event generators
parton showers

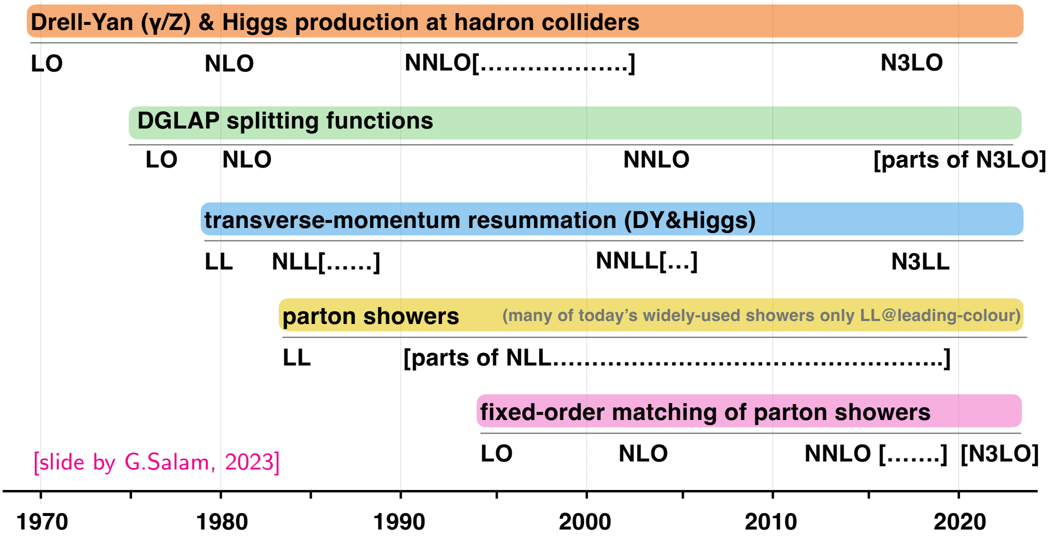
Basic comments

Role

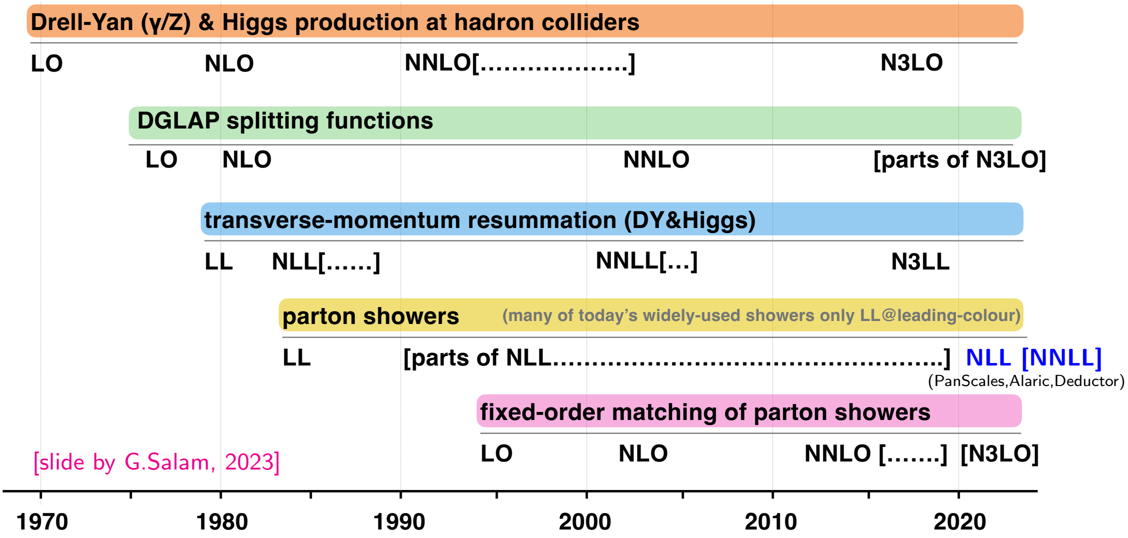
perturbative QCD connecting the scale of the hard process to the scale where non-perturbative hadronisation happens

- This is achieving resummations
- accuracy should be counted as LL, NLL, ...
- Keep in mind: not an exact α_s expansion...
- ... unless matched with exact fixed order (briefly discussed later)

selected collider-QCD accuracy milestones



selected collider-QCD accuracy milestones



Example 1: radioactive emissions

Toy model

a particle emits photons at a rate ω (per unit time)
Probability to have n emissions over a time T:

$$P_n(T) = \frac{(\omega T)^n}{n!} e^{-\omega T}$$

Simulation strategy

1. start at $t = t_0 = 0$
2. recursively select next emission time t_{n+1} acording to $R(t_{n+1}) = \omega e^{-\omega(t_{n+1}-t_n)}$
3. until reaching a cut-off time $t_{\rm cut}$

Logic

- Factor $e^{-\omega\Delta t} \equiv P_0(\Delta t_{n+1})$ (no emissions between t_n and t_{n+1}, often called Sudakov)
- Factor ω: emission rate at t_{n+1}

link to file

```
class Emission{
public:
  Emission(double t_in=0) : t(t_in){}
  double t;
};

class Event{
public:
  Event(){}
  vector<Emission> emissions;
};

Event generate_event(double omega, double tcut){
  Event ev;
  double t = 0.0;

  while (true){
    double u = ((double) rand()/RAND_MAX);
    t += -log(1-u)/omega;

    if (t>tcut) return ev;
    ev.emissions.push_back(Emission(t));
  }
  return ev;
}
```

```
average multiplicity = 0.00998358   exp: 0.01
mult. dispersion     = 0.100402     exp: 0.1
```

Example 2: toy abelian shower

Toy model

a particle emits photons with angl θ and momentum fraction $z \geq z_{\text{cut}}$ at a rate

$$dP = \frac{\alpha}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$$

Simulation strategy

Say "time" $= t = \log(\theta_{\max}/\theta)$; start at $t = t_0 = 0$

Emitter with mom fraction x (starting with $x = 1$)

Recursively

1. select next emission time t_{n+1} according to $R(t_{n+1}) = \frac{\alpha}{\pi} \log \frac{1}{z_{\text{cut}}} e^{-S}$
   $S = \left[\frac{\alpha}{\pi} \log \frac{1}{z_{\text{cut}}}\right] (t_{n+1} - t_n)$
2. generate the z fraction uniformly in $\ln z$
   emission takes zx, emitter $(1-z)x$

until a cut-off time $t_{\text{cut}} = \log(\theta_{\max}/\theta_{\min})$

link to file

```
class Emission{
public:
  Emission(double t_in, double x_in) : t(t_in), x(x_in){}
  double t, x;
};

class Event{
public:
  Event() : x_lead(1.0) {}
  vector<Emission> emissions;
  double x_lead;
  void add_emission(double t, double z){
    emissions.push_back(Emission(t,x_lead*z));
    x_lead *= (1-z);
  }
};

Event generate_event(double alpha, double zcut,
                     double theta_max, double theta_min){
  Event ev;

  double t      = 0.0;
  double tmax   = log(theta_max/theta_min);

  double lnzcut = log(1/zcut);
  double omega  = alpha/M_PI*lnzcut;

  while (true){
    double u = ((double) rand()/RAND_MAX);
    t += -log(1-u)/omega;

    if (t>tmax) return ev;

    double v = ((double) rand()/RAND_MAX);
    double z = exp(-v*lnzcut);
    ev.add_emission(t,z);
  }

  return ev;
}
```

Example 2: toy abelian shower

Toy model

a particle emits photons with angl θ and momentum fraction $z \geq z_{\text{cut}}$ at a rate

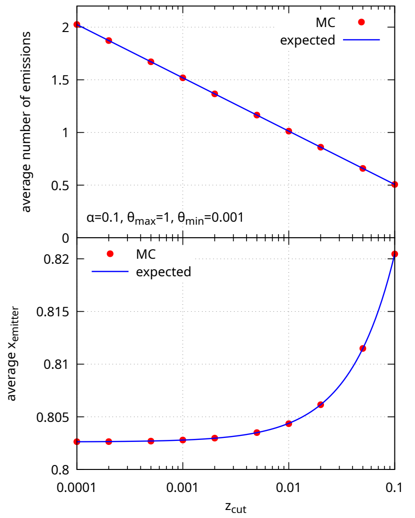
$$dP = \frac{\alpha}{\pi}\frac{dz}{z}\frac{d\theta}{\theta}$$

Simulation strategy

Say "time" $= t = \log(\theta_{\max}/\theta)$; start at $t = t_0 = 0$
Emitter with mom fraction x (starting with $x = 1$)
Recursively

1. select next emission time t_{n+1} according to
$$R(t_{n+1}) = \frac{\alpha}{\pi}\log\frac{1}{z_{\text{cut}}}e^{-S}$$
$$S = \left[\frac{\alpha}{\pi}\log\frac{1}{z_{\text{cut}}}\right](t_{n+1} - t_n)$$
2. generate the z fraction uniformly in $\ln z$
emission takes zx, emitter $(1-z)x$

until a cut-off time $t_{\text{cut}} = \log(\theta_{\max}/\theta_{\min})$



excellent agreement

Example 2: toy abelian shower revisited

Shower evolution variable

Previous example:

- $t = \log(1/\theta) \equiv$ shower evolution variable
- $\ln z \equiv$ auxiliary variable

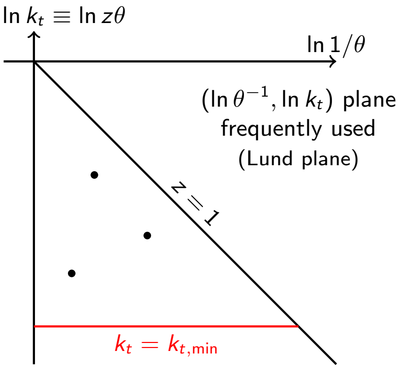
The rate can also be rewritten as $dP = \frac{\alpha}{\pi}\frac{dv}{v}\frac{dz}{z}$

- $\ln v = \ln(z\theta^{\beta+1})$ as the shower variable
- $\ln z$ as the auxiliary variable

Strategy

Same as before but

- no need for a z_{cut}
- $\theta_{\min} \to v_{\min}$
- $\beta = 0 \Rightarrow v \approx k_t$ (standard choice)
- One can impose a cut $k_t \geq k_{t,\min}$



Example 2: toy abelian shower revisited

Shower evolution variable

Previous example:

- $t = \log(1/\theta) \equiv$ shower evolution variable
- $\ln z \equiv$ auxiliary variable

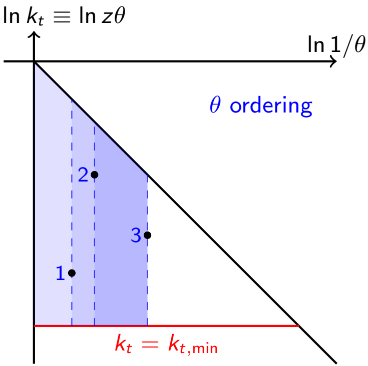
The rate can also be rewritten as $dP = \frac{\alpha}{\pi}\frac{dv}{v}\frac{dz}{z}$

- $\ln v = \ln(z\theta^{\beta+1})$ as the shower variable
- $\ln z$ as the auxiliary variable

Strategy

Same as before but

- no need for a z_{cut}
- $\theta_{\min} \to v_{\min}$
- $\beta = 0 \Rightarrow v \approx k_t$ (standard choice)
- One can impose a cut $k_t \geq k_{t,\min}$



Shower evolution variable

Previous example:

- $t = \log(1/\theta) \equiv$ shower evolution variable
- $\ln z \equiv$ auxiliary variable

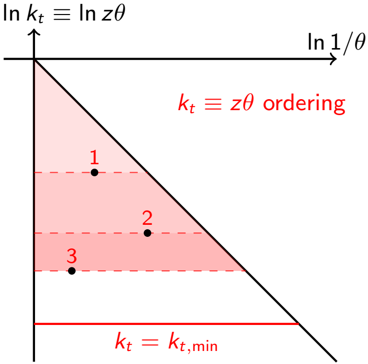
The rate can also be rewritten as $dP = \frac{\alpha}{\pi}\frac{dv}{v}\frac{dz}{z}$

- $\ln v = \ln(z\theta^{\beta+1})$ as the shower variable
- $\ln z$ as the auxiliary variable

Strategy

Same as before but

- no need for a z_{cut}
- $\theta_{\min} \to v_{\min}$
- $\beta = 0 \Rightarrow v \approx k_t$ (standard choice)
- One can impose a cut $k_t \geq k_{t,\min}$



Example 2: toy abelian shower revisited

Shower evolution variable

Previous example:

- $t = \log(1/\theta) \equiv$ shower evolution variable
- $\ln z \equiv$ auxiliary variable

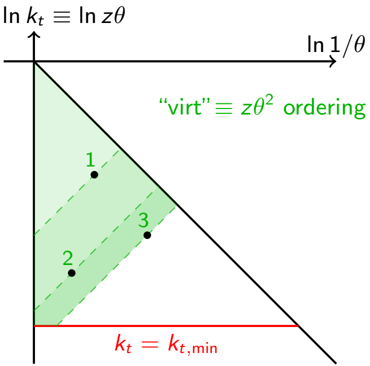
The rate can also be rewritten as $dP = \frac{\alpha}{\pi}\frac{dv}{v}\frac{dz}{z}$

- $\ln v = \ln(z\theta^{\beta+1})$ as the shower variable
- $\ln z$ as the auxiliary variable

Strategy

Same as before but

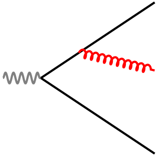
- no need for a z_{cut}
- $\theta_{\min} \to v_{\min}$
- $\beta = 0 \Rightarrow v \approx k_t$ (standard choice)
- One can impose a cut $k_t \geq k_{t,\min}$



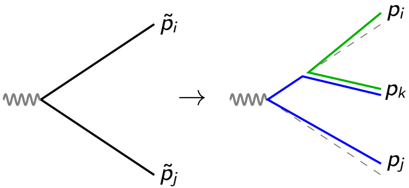
Towards QCD showers

Mostly two types of showers:

- Angular-ordered showers:
  mostly as before but after a branching both daughter partons can branch further
- Dipole shower ($v_{\beta\geq 0}$-ordered): large N_c



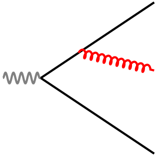
viewed as



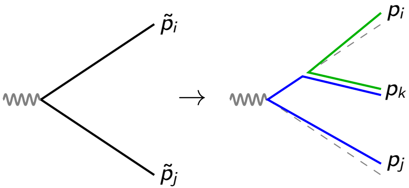
Towards QCD showers

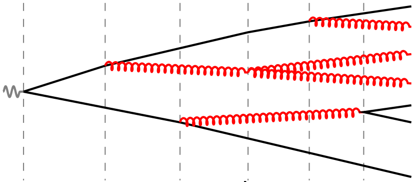
Mostly two types of showers:

- Angular-ordered showers:
  mostly as before but after a branching both daughter partons can branch further
- Dipole shower ($v_{\beta\geq 0}$-ordered): large N_c

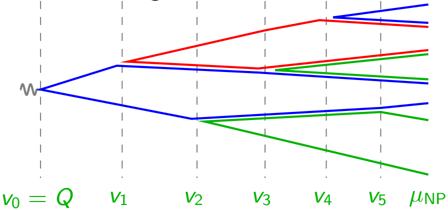


viewed as





generated as



Towards QCD showers

Angular-ordered shower

✓ correct collinear physics
respects QCD angular ordering: $\theta_{n+1} < \theta_n$,
the final-state equivalent of DGLAP

✓ full N_c

✗ soft-gluon pattern difficult
In particular: struggle with non-global logs

Dipole shower ($v_{\beta\geq 0}$-ordered)

✓ soft-gluon by construction
dipoles easily get the antenna pattern

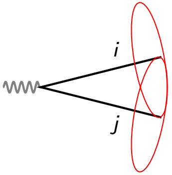
✓ collinear physics not too delicate to get

✗ delicate to go beyond leading N_c

Notes on angular ordering:

- fundamental property of QCD
- often referred to as "colour coherence"
- only valid after azimuthal averaging (connected to spin correlations)
- Relatively simple to show for soft emissions from an antenna:

$$\int d^2\theta_k \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{kj})} \propto \int^{\theta_{ij}} \frac{d\cos\theta_{ik}}{1-\cos\theta_{ik}} + \int^{\theta_{ij}} \frac{d\cos\theta_{jk}}{1-\cos\theta_{jk}}$$



Towards QCD showers

Angular-ordered shower

- ✓ correct collinear physics
  respects QCD angular ordering: $\theta_{n+1} < \theta_n$,
  the final-state equivalent of DGLAP
- ✓ full N_c
- ✗ soft-gluon pattern difficult
  In particular: struggle with non-global logs

Dipole shower ($v_{\beta\geq 0}$-ordered)

- ✓ soft-gluon by construction
  dipoles easily get the antenna pattern
- ✓ collinear physics not too delicate to get
- ✗ delicate to go beyond leading N_c

In principle...

techniques similar to what we used above should get us NLL accuracy

In practice...

- angular-ordering struggles with NGLs
- dipole showers can have nasty recoil issues

Matching

As for the analytic calculations, ideally we want both

1. fixed-order accuracy
2. resummation accuracy

in a single event simulation framework.

Matching

As for the analytic calculations, ideally we want both

1. fixed-order accuracy
2. resummation accuracy

in a single event simulation framework.

Rely on **matching techniques**

- Idea: generate a few "exact" (at fixed-order) hard emissions then let the shower take over
  i.e. connect the fixed-order hard-scale and broad scale range of the shower
- Delicate point: avoid double-counting
  (i.e. the fixed-order and shower should not spoil the other part's accuracy)
- Delicate point: not trivial to avoid negative weights
- Fairly automated at NLO through aMC@NLO, POWHEG or using a MiNLO approach
- Several recent NNLO approaches: MiNNLO$_{\text{PS}}$, UNLOPS, GenEvA

Part XIV: selected extra topics

Jet substructure

Idea

Instead of considering a "jet" as a particle (with a p_t, y, ϕ and mass), look at the internal dynamics of the jet constituents

- Originated in the study of boosted boson decay

  Take a $X \equiv W/Z/H$ decaying hadronically. The $q\bar{q}$ opening angle scales like m_X/p_t (Lorentw boost).
  At large p_t this is smaller than the jet radius so X is seen as a single jet.

  Techniques must be devived to separate X from QCD backgroungs
- Now applications in many directions including searches (e.g. diboson excess from run-I), precision calculations and measurements, Deep Learning, heavy-ions, ...
- Long list of tools designed (SoftDrop, mMDT, N-subjettiness, ...)
- Two families of modern tools with active research:
  Energy Correlation Functions and Lund Plane techniques
- Check out these lecture notes of the BOOST conference series for more

Amplitudes beyond QCD

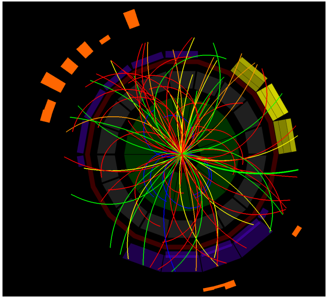
The main domain of usage of the amplitude results is QCD.

However:

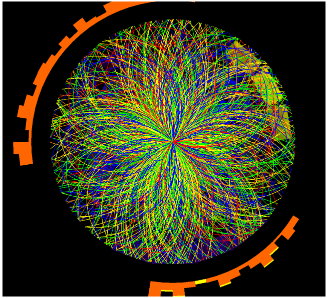
- many studies directly touch our fundamental understanding of quantum field theory, with new structures emerging regularly
- One recent application is the use of amplitude techniques to compute gravitational waves. This is based on a "double-copy" relation: gravity$\approx$Yang-Mills$\times$Yang-Mills
  Roughly on par with Post-Newtonian approach to in-mergers

Heavy-ion collisions

Alice *pp* event



Alice *PbPb* event



- Substantially more complex!!
- increased Underlying Event
- Quark-Gluon plasma interacting "with itself" and with high-energy particles (hard probes)
- QGP behaves as a perfect liquid
- Complex interaction with jets
- See Liliana's lectures!

Various interesting behaviours/scaling properties

Many interesting behaviours of QCD are still regularly discovered

- some equations describing soft gluon emissions show properties common to the evolution of populations in stat phys
- some equations describing interactions of hard jets with the QGP exhibit wave turbulence
- some substructure observables show behaviours independent of α_s
- the QGP behavesas a perfect fluid
- amplitudes show remarkable signs of simplicity/symmetries
- Casimir scaling for a large family of quark/gluon discriminants
- ...

Note that all of the above are true only in appropriate limits

Conclusions

A top-notch knowledge/understanding of QCD is

1. **interesting per se!**
   (part of a physicist's job to understand fundamental interactions)
2. **primordial for the whole programme of collider physics**
   searches AND measurements!

Still a lot to do at the (HL-)LHC and for future colliders