

Quantum Computing in High Energy Physics

Examples from CERN



Sofia.Vallecora@cern.ch

Outline

- Introduction
 - Bell Inequalities & Quantum Technologies
- Quantum Computing
 - Qubits and Quantum Computers architectures
 - Quantum algorithms
- The CERN Quantum Technology Initiative
- Quantum Machine Learning and Applications at CERN
 - Anomaly Detection
 - Beam Optimisation in linear accelerators
- Summary & Outlook

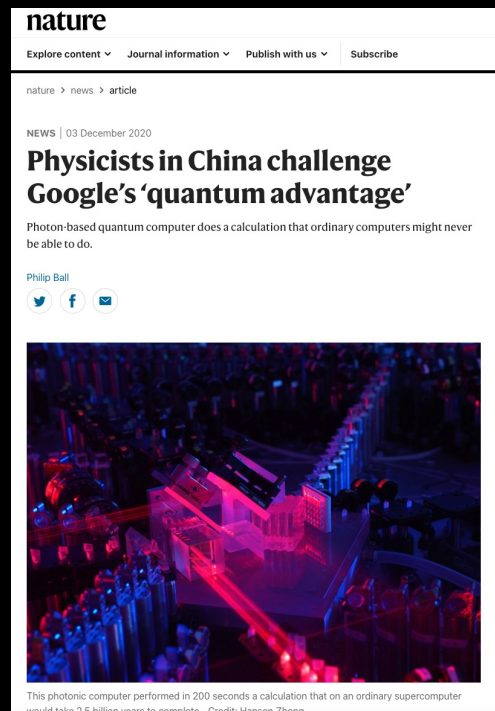
Hype and Potential...

2019: Google



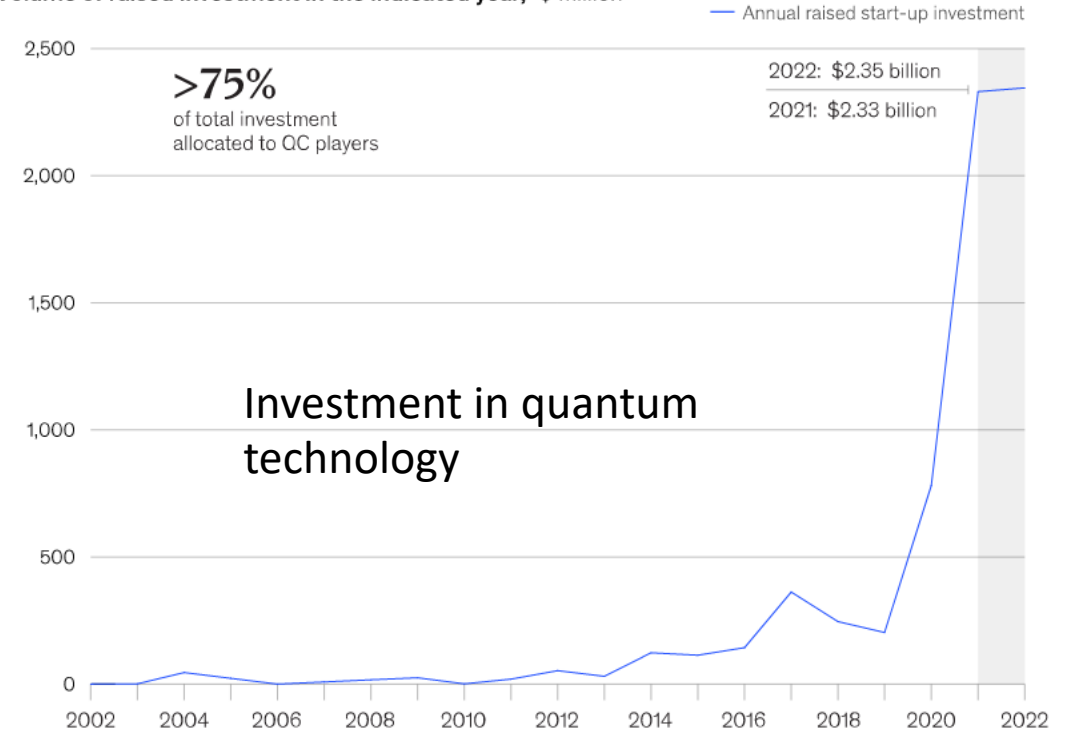
<https://www.nature.com/articles/s41586-019-1666-5>

2020: Hefei
National Lab



<https://www.nature.com/articles/d41586-020-03434-7>

Volume of raised investment in the indicated year,¹ \$ million



¹Based on public investment data recorded in PitchBook; actual investment is likely higher.
Source: PitchBook

Source: McKinsey 2023

<https://www.mckinsey.com/capabilities/mckinsey-digital/our-insights/quantum-technology-sees-record-investments-progress-on-talent-gap>



Photo: Royal Society

Alain Aspect

Université Paris-Saclay &
École Polytechnique, France



Photo: Peter Lyons

John F. Clauser

J.F. Clauser & Assoc.,
USA



Photo: Sepp Drehslinger

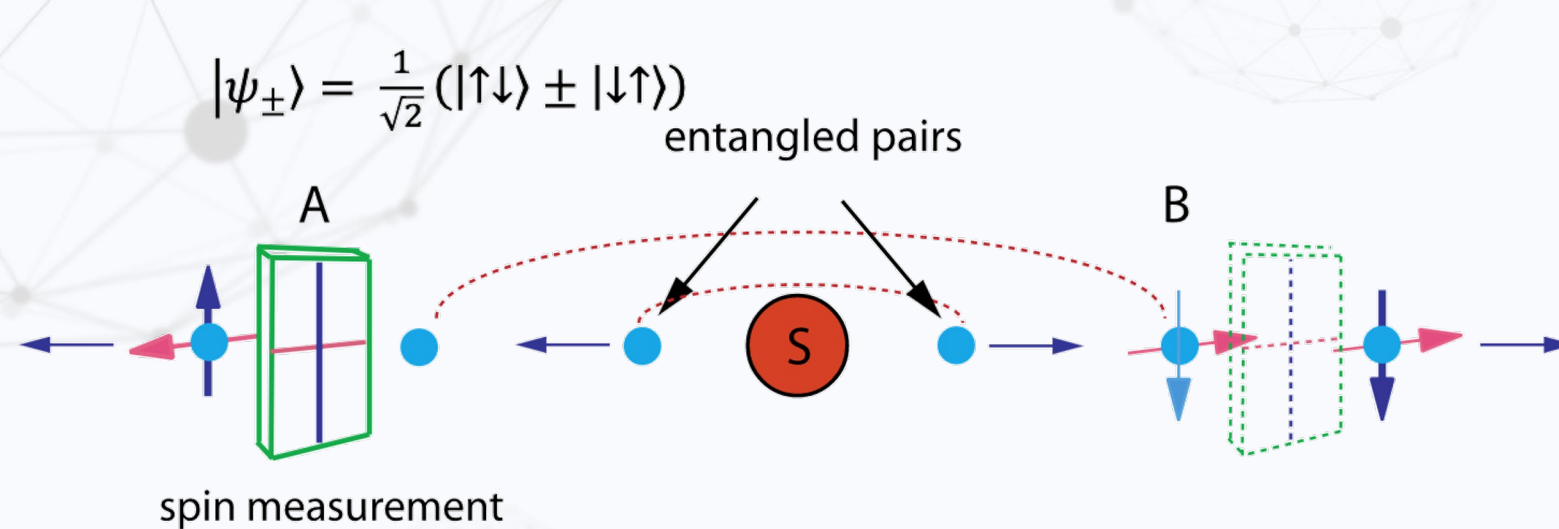
Anton Zeilinger

University of Vienna,
Austria

*"för experiment med sammanflätade fotoner som påvisat brott mot Bell-olikheter och
banat väg för kvantinformatikvetenskap"*

*"for experiments with entangled photons, establishing the violation of Bell inequalities and
pioneering quantum information science"*

Einstein Podolsky Rosen Paradox



$$[S_i, S_j] = i\hbar\epsilon^{ijk} S_k$$

For arbitrary **a, b**
measurement directions
the probability to measure
opposite spin values is

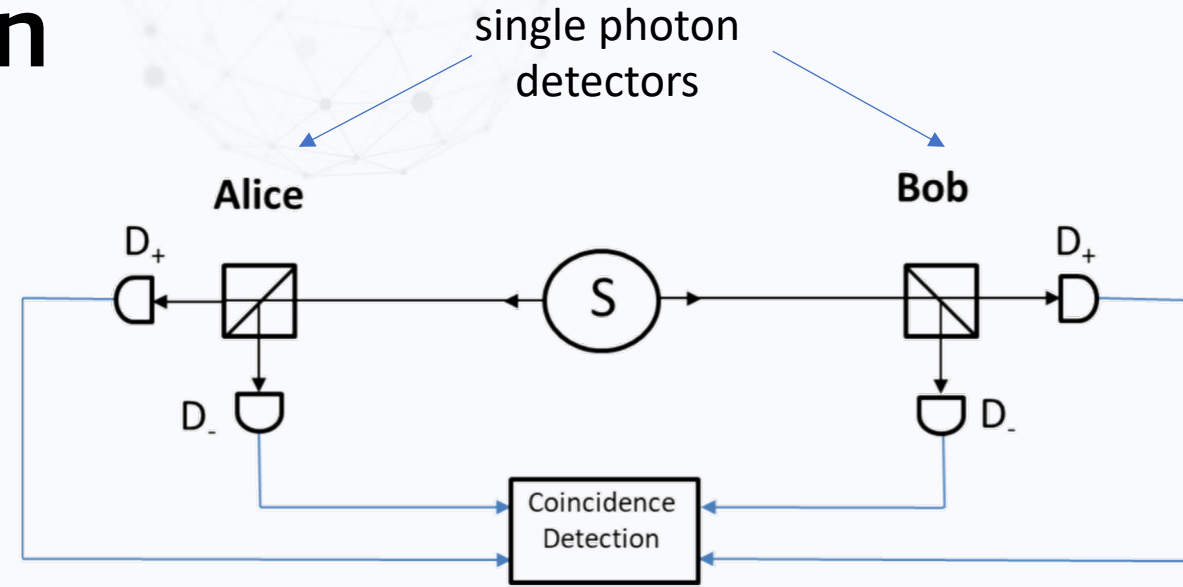
$$\frac{1}{2}(1 + \vec{a} \cdot \vec{b})$$

- 1935: EPR paradox **questions completeness** of Quantum Mechanics

‘From this follows that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality’

Bell Inequalities and the second quantum revolution

- 1964: Bell inequality are a mathematical proof that no theory based on **local hidden variables** (realism) can reproduce QM results
- 1969: Clauser, Horne, Shimony (CHSH) formulation
- 1972 Freedman & Clauser experiment
- 1976-1982 Aspects experiments
- Measure the correlation across the possible outcomes of multiple measurements in A and B
- «Classical» (local realist) theory predicts bound correlation
- It is possible to design experimental conditions in which QM violates the bound



Second Quantum Revolution
Use distant entangled photons as a quantum resource

- No clone theorem
- Quantum networks and teleportation

Many particles entanglement

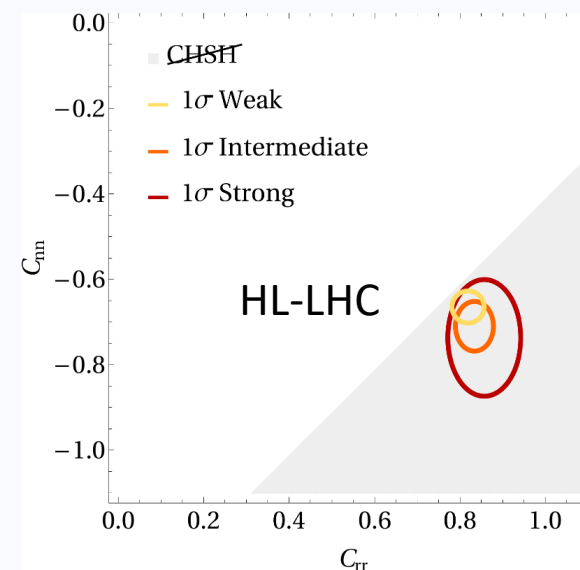
- Computing and cryptography

Bell Inequalities tests at colliders

- Multiple tests at colliders over the years at different energy scales
- Recent proposals at the LHC / HL-LHC:
 - Fabbrichesi et al. "Testing Bell inequalities at the LHC with top-quark pairs." *Physical Review Letters* 127.16 (2021): 161801.
 - Severi et al. "Quantum tops at the LHC: from entanglement to Bell inequalities." *The European Physical Journal C* 82.4 (2022): 285
 - Aguilar-Saavedra and Casas. "Improved tests of entanglement and Bell inequalities with LHC tops." *The European Physical Journal C* 82.8 (2022): 666
 - Aguilar-Saavedra et al. "Testing entanglement and Bell inequalities in $H \rightarrow Z Z$." *Physical Review D* 107.1 (2023): 016012
 - Barr, Alan J. "Testing Bell inequalities in Higgs boson decays." *Physics Letters B* 825 (2022): 136866

$t\bar{t}$ production at hadron colliders:

- Leading mechanism generates top spins that are highly correlated
- Tops decay semi-weakly before spin is randomised
- Charged lepton from $t \rightarrow Wb$, $W \rightarrow \ell \nu$ is 100% correlated with top spin



Severi, Claudio, et al.
"Quantum tops at the LHC:
from entanglement to Bell
inequalities." *The European
Physical Journal C* 82.4
(2022): 285

Quantum Technology

Second quantum revolution

Use quantum mechanics principles to develop new technology

“Artificial” quantum states

Now

First Quantum Revolution

Max Planck black-body radiation

Transistor, laser, atomic clock, computers, optical fibre communication, ...

2000

1900



Dowling & Milburn. "Quantum technology: the second quantum revolution." Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences 361.1809 (2003): 1655-1674.

Quantum Computing:

From Quantum Mechanics to Computer Science

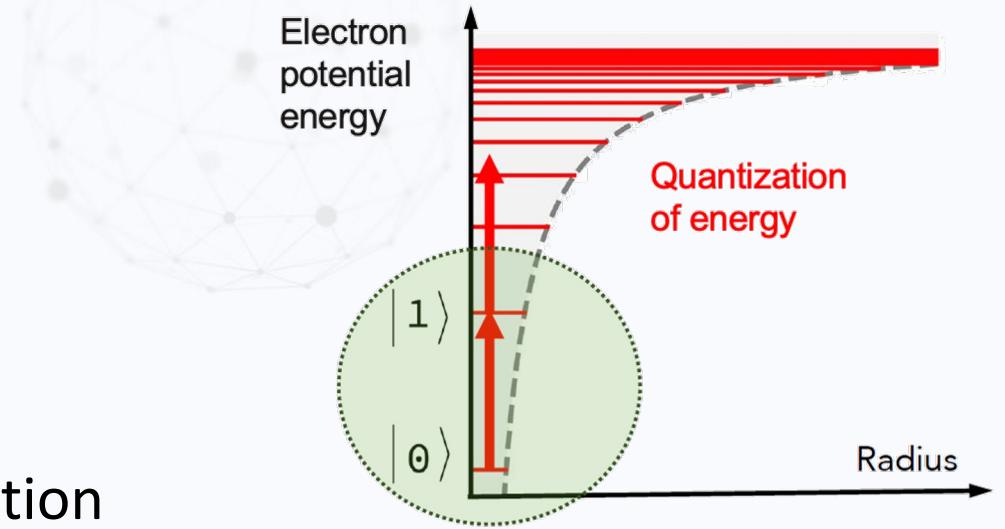
Qubit: Quantum Bit

- Basic Unit of Quantum Computation representation
 - **Classical bits are binary “0 or 1”**
- Quantum Mechanics predicts **superposition states (exponential storage information)**
- **Dirac notation** is used to describe quantum states

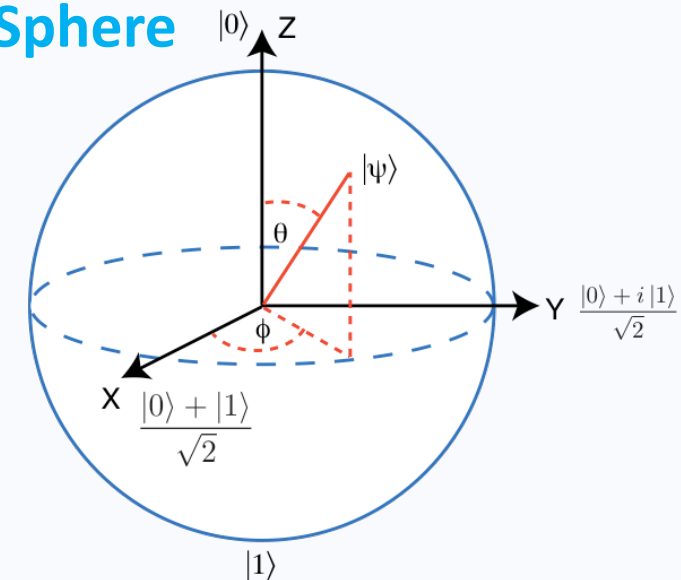
$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

$$\alpha, \beta \in \mathbb{C} \quad |\alpha|^2 + |\beta|^2 = 1$$

Interest in **multi level** representations: qutrits..



Bloch Sphere



$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \quad 10$$

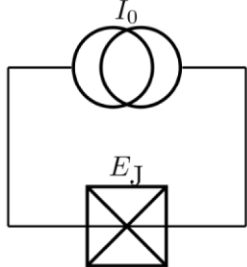
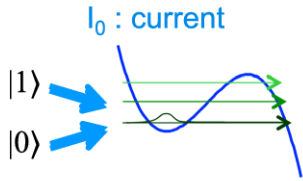
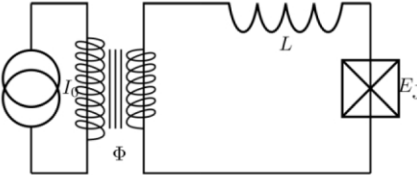
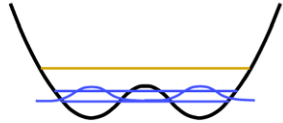
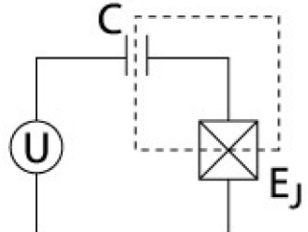
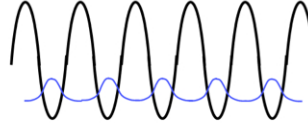
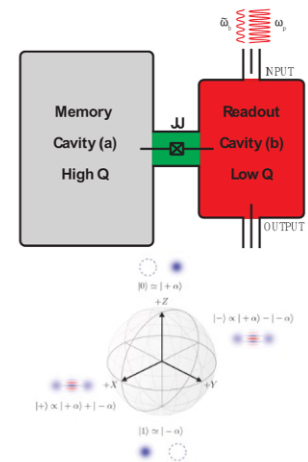



Quantum Computing

Principles of quantum mechanics enhance computations

- **Superposition** leads to parallelism → **exponential speedup?**
- **Entanglement** → **non linear correlation and classical simulability?**
- Operations (gates) are unitary transformations → **reversible computing?**
- Output is the result of a quantum state measurement according to Born rule → **stochastic computation ?**
- **No-cloning theorem** → **information security**
- **Quantum state coherence and isolation** → **computation stability and errors**
- **Qubit state collapses** → **reproducibility?**

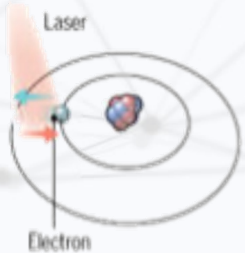
Superconducting Qubits

Ezratty, O. Perspective on superconducting qubit quantum computing. Eur. Phys. J. A 59, 94 (2023).
<https://doi.org/10.1140/epja/s10050-023-01006-7>

	phase qubit	flux qubit	charge qubit - transmon	cat-qubits
	 I_0 : current 	 L : inductance 	 U : tension 	
	Josephson junctions handle the qubit degree of liberty			Josephson junctions prepare, couple and correct the cat-qubits
 0> and 1> qubits	two energy levels in a potential well	two superconducting current directions	two levels of charge of Cooper pairs	pairs of entangled microwave photons in a cavity
quantum gates	micro-waves	magnetic field	micro-waves	micro-waves
qubits readout	resonator and micro-waves	magnetometer (SQUID)	resonator and micro-waves	resonator and micro-waves
commercial vendors	abandoned			

Multiple technologies

Trapped ions



Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.

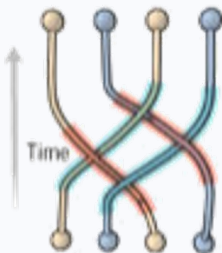
Longevity (seconds)	>1000
Logic success rate	99.9%
Number entangled	14

Company support

ionQ

- Pros**
 - Very stable. Highest achieved gate fidelities.
- Cons**
 - Slow operation. Many lasers are needed.

Topological qubits



Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.

Longevity (seconds)	N/A
Logic success rate	N/A
Number entangled	N/A

Company support

Microsoft, Bell Labs

- Pros**
 - Greatly reduce errors.
- Cons**
 - Existence not yet confirmed.

Silicon quantum dots



These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.

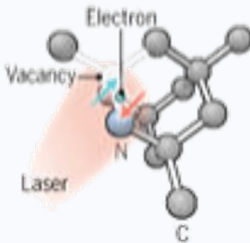
Longevity (seconds)	0.03
Logic success rate	~99%
Number entangled	2

Company support

Intel

- Pros**
 - Stable. Build on existing semiconductor industry.
- Cons**
 - Only a few entangled. Must be kept cold.

Diamond vacancies



A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

Longevity (seconds)	10
Logic success rate	99.2%
Number entangled	6

Company support

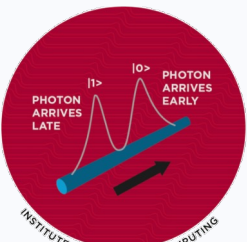
Quantum Diamond Technologies

- Pros**
 - Can operate at room temperature.
- Cons**
 - Difficult to entangle.

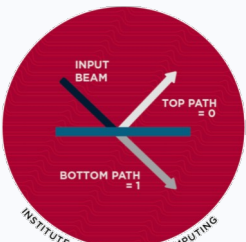
Polarization States



Time qubits



Path Qubits:

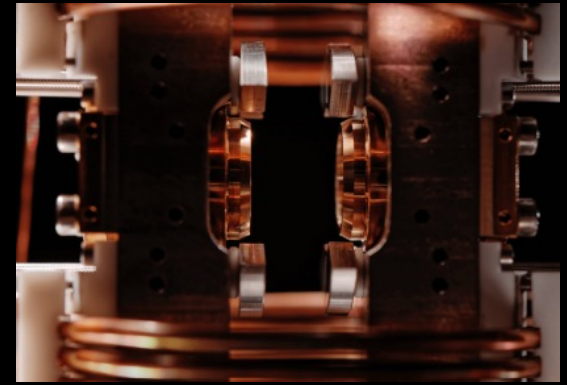


See **Institute of Quantum Computing, U. of Waterloo**, <https://uwaterloo.ca/institute-for-quantum-computing/quantum-101/quantum-information-science-and-technology/what-qubit#Spin>

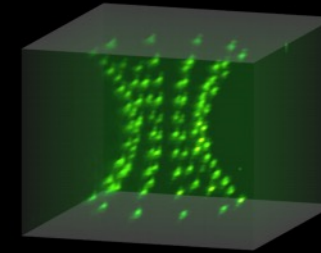
Neutral atom arrays

- Configurable arrays of **single neutral atoms**
- 2 energy levels represent the qubit states
- Use **lasers** to control position and the state of the atom
 - assemble and read-out registers made of **hundreds of qubits**
 - **fully programmable quantum processing**
- **High connectivity**
- Specific computation cycle because the **register is not permanently built**
 - register preparation
 - quantum processing
 - register readout

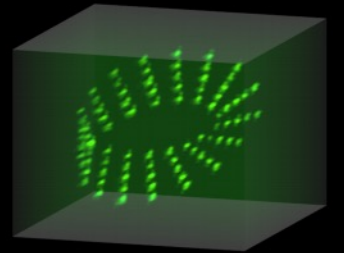
D. Barredo et al., "Synthetic three-dimensional atomic structures assembled atom by atom." [arXiv:1712.02727](https://arxiv.org/abs/1712.02727), 2017.



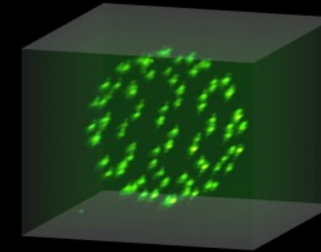
a Hyperboloid (90 sites)



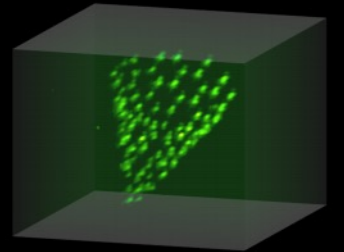
b Möbius strip (85 sites)



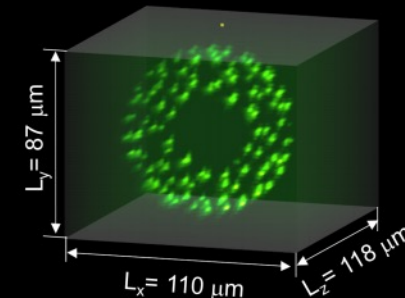
c C₈₄ fullerene-like (84 sites)



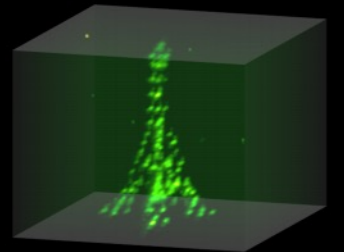
d Cone (100 sites)



e Torus (120 sites)

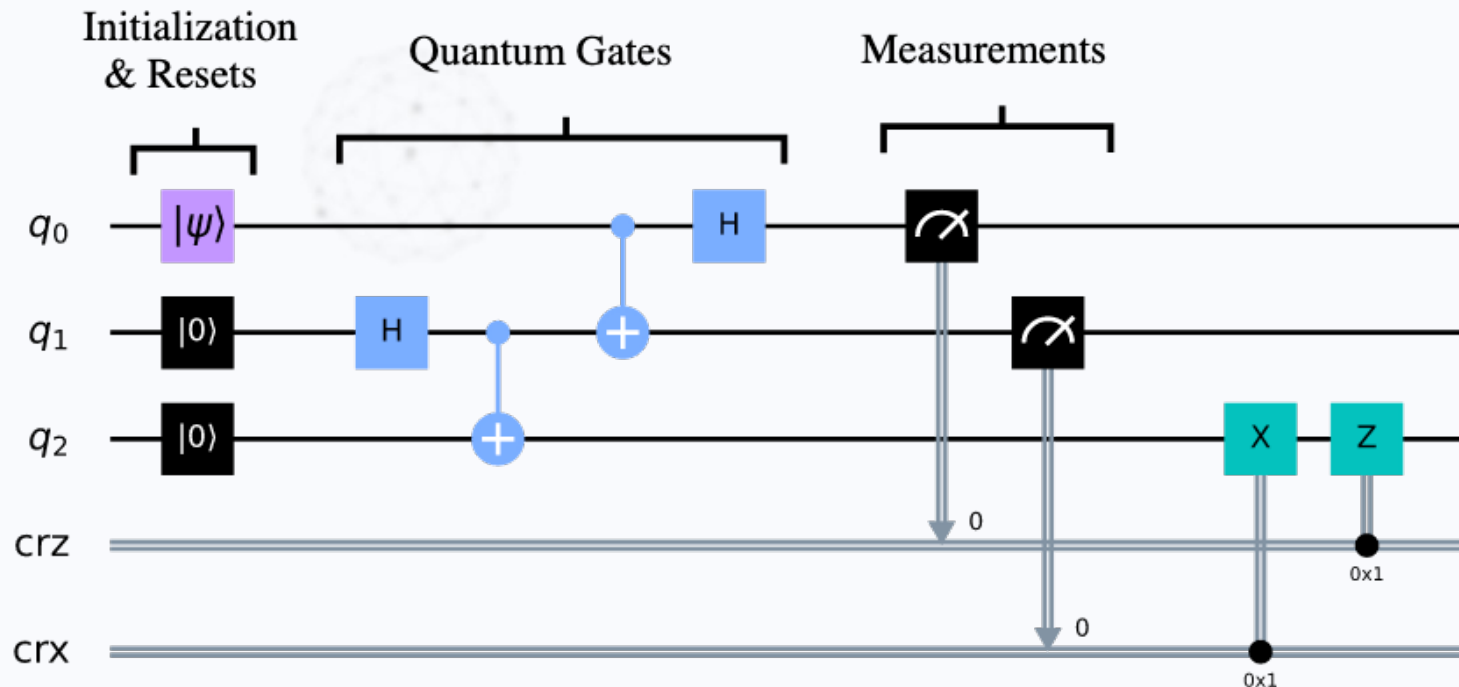


f Eiffel tower (126 sites)



Gates and circuits

- Operations on qubits are **unitary** matrices describing Schrodinger state evolution
 - **Reversible operations**
 - Input and output states have the **same dimension**
 - Some classical gates (or , and, nand, xor...) **cannot be implemented directly**
 - Can **simulate** any classical computation with small overhead

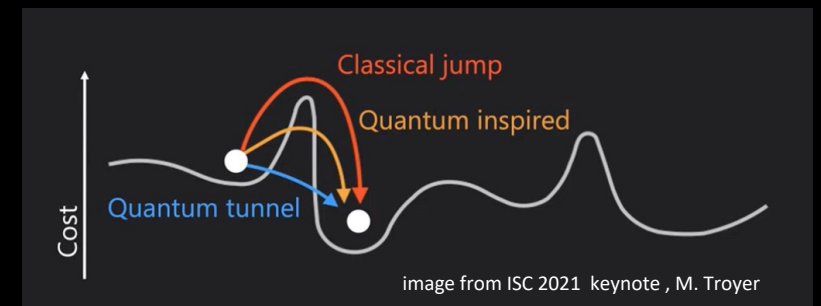
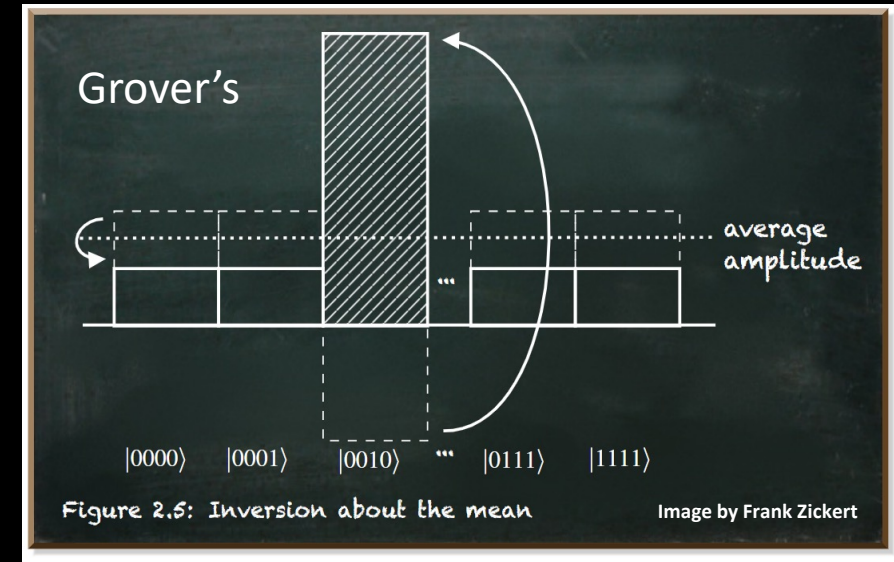


Quantum Algorithms

A collection on <http://quantumalgorithmzoo.org>

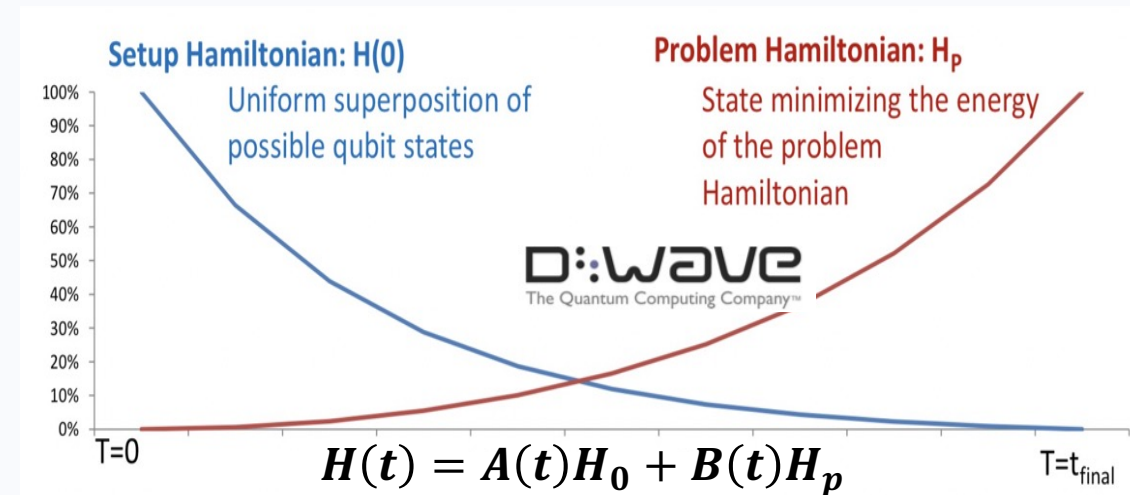
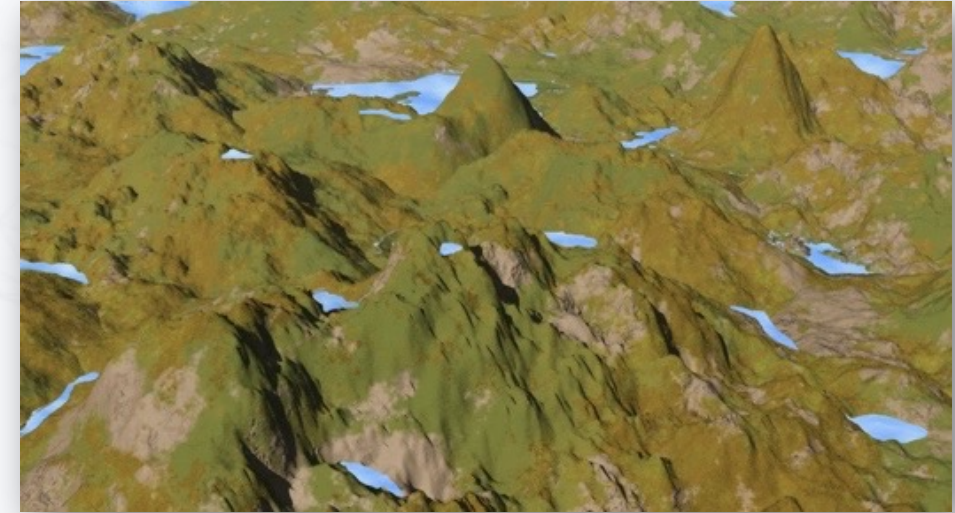
- Multiple algorithms have been studied
 - Shor algorithm for **prime factorization**
 - Grover algorithm for unsorted DB **searches**
 - Quantum **Fourier Transform**
 - ...
- Quantum-inspired algorithms (emulate quantum effects on classical hardware)
- Quantum Machine Learning
- Challenge is re-thinking **algorithms design** and define fair **benchmarking** and **comparison** to classical algorithms

<https://quantum-computing.ibm.com/composer/docs/iqx/guide/shors-algorithm>



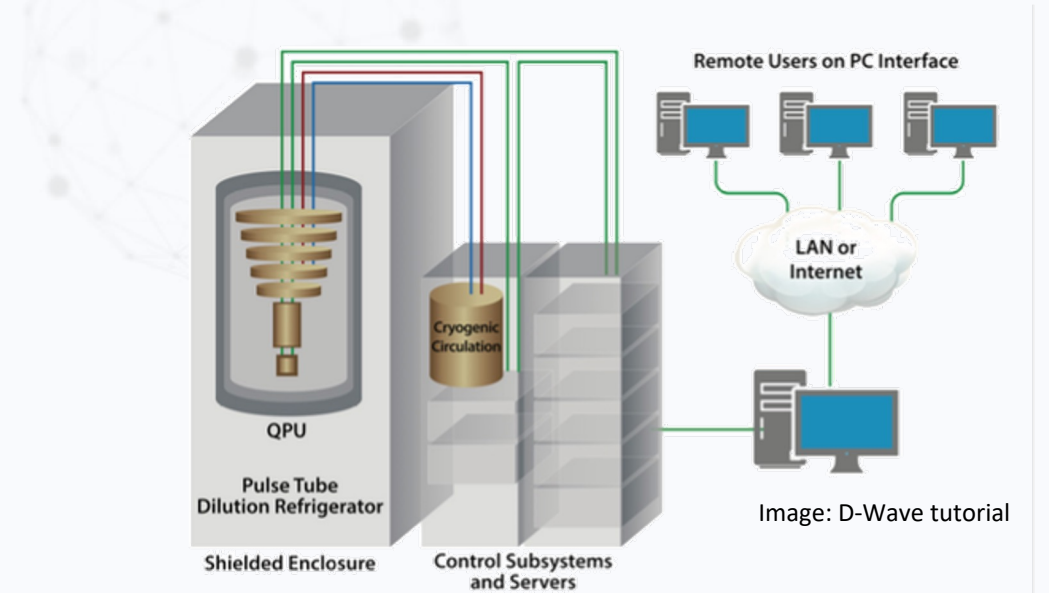
Quantum Annealing

- **Annealing for optimization problems**
 - Smoothly evolve probability of being at any given coordinate with time.
 - Probability increases around the coordinates of deep valleys
- Quantum systems based on **superconducting qubits**
- **D-Wave Advantage**: 5436 qubits - 15 connection (Pegasus)
 - **Quantum superposition**: scan simultaneously multiple coordinates
 - **Quantum tunneling**: reduces risk of local minima (tunnel through hills)
 - **Quantum entanglement**: discover correlations between the coordinates that lead to deep valleys.

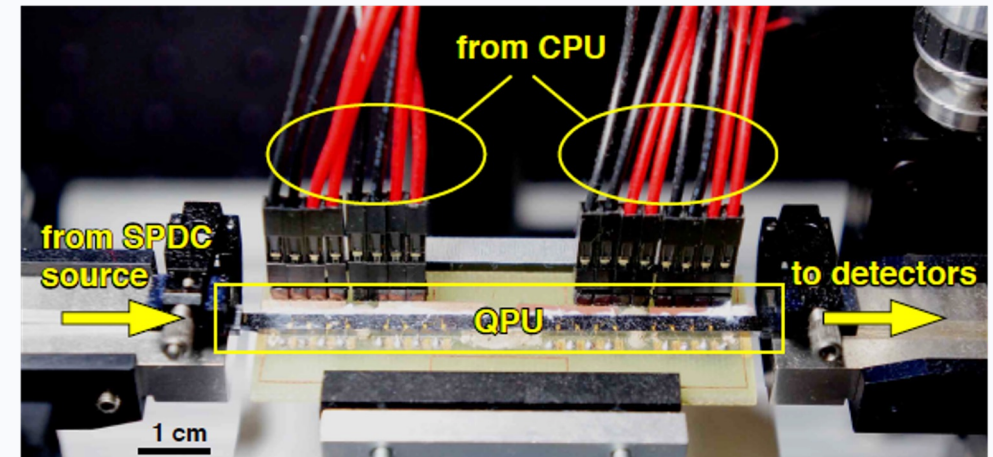


Today's challenges

- **Noisy Intermediate-Scale Quantum** devices
 - Limitations in terms of **stability** and **connectivity**
 - **De-coherence**, measurement errors or gate level errors
 - Specific **error mitigation techniques**
 - **Circuit optimisation**
 - Prefer algorithms **robust against noise**
- Quantum computers initially integrated in **hybrid quantum-classical infrastructure**
 - Engineering, cooling, I/O
 - Hybrid algorithms, QPU as accelerators



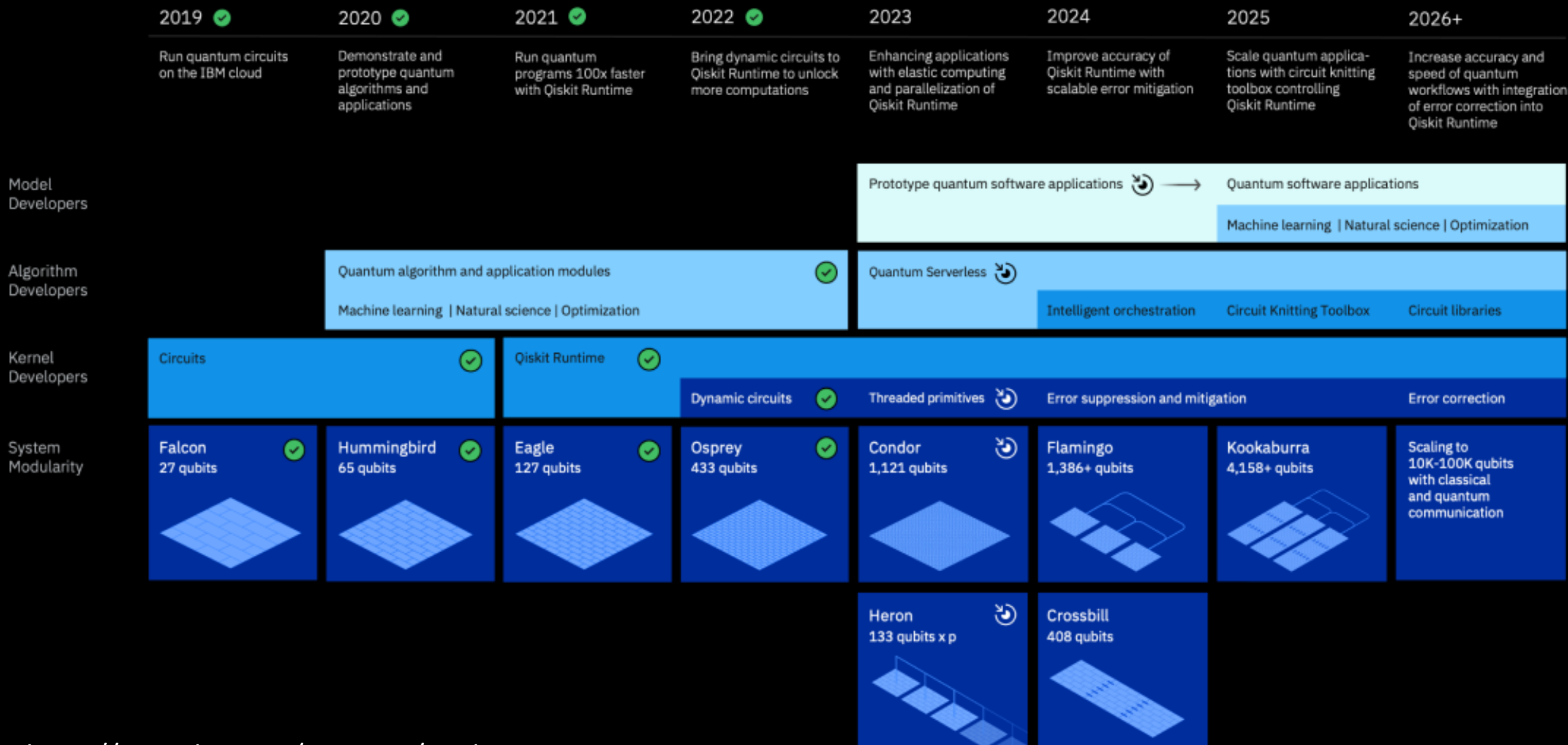
Peruzzo, A. "A variational eigenvalue solver on a quantum processor." *arXiv preprint arXiv:1304.3061* (2013).




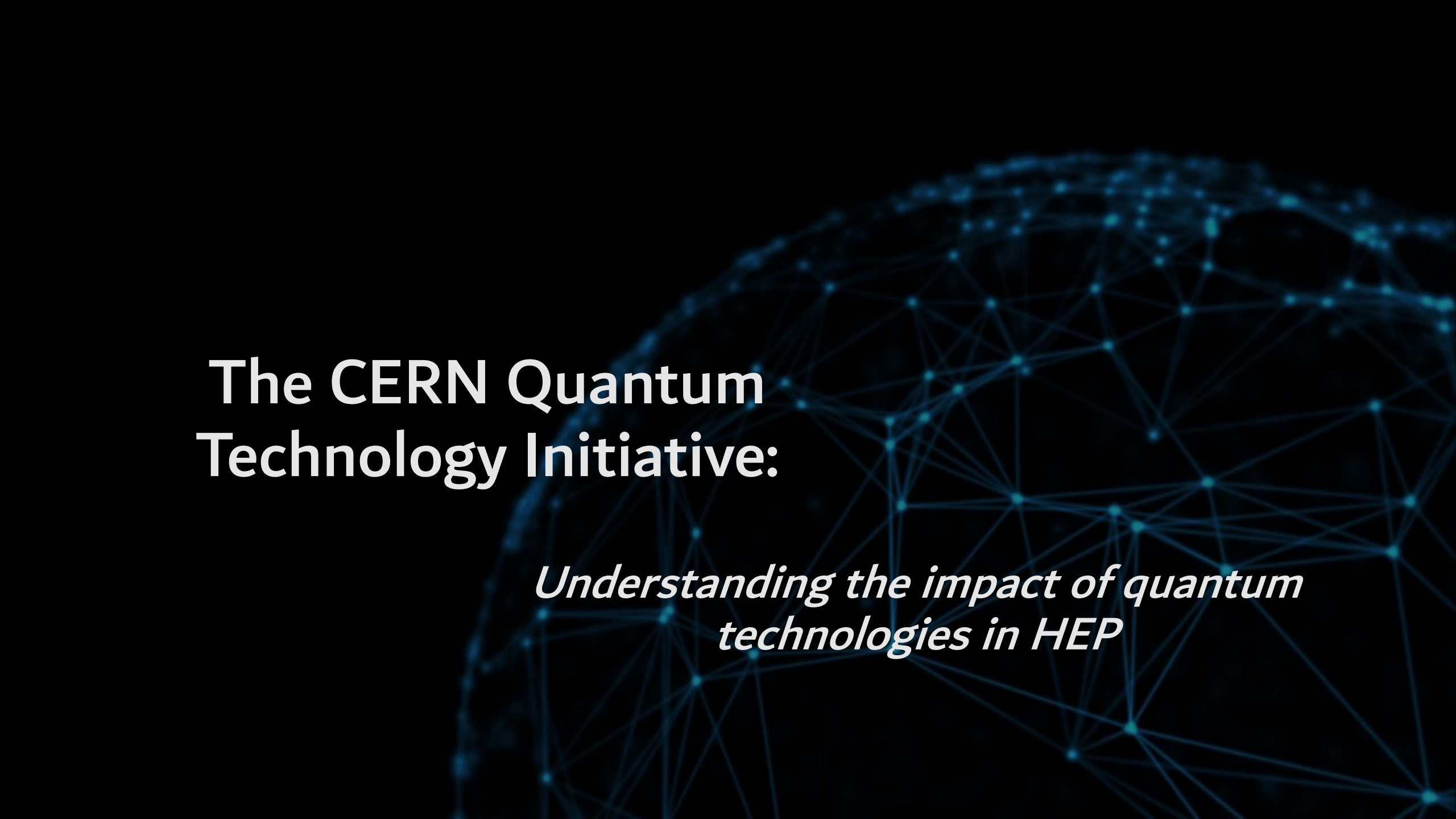


What the future brings

An example quantum roadmap from IBM



- 
- How do we define advantage?
 - Speed-up and complexity
 - Sample efficiency
 - Representational power
 - Energy efficiency???
 - Evaluate performance on realistic use cases



The CERN Quantum Technology Initiative:

*Understanding the impact of quantum
technologies in HEP*

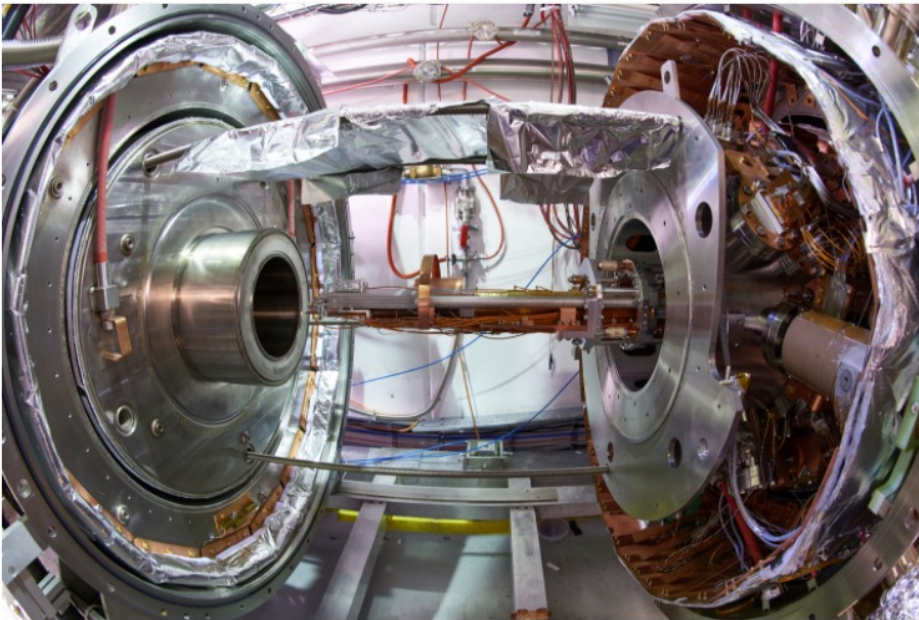
The CERN QTI launched in 2020

Voir en [français](#)

CERN meets quantum technology

The CERN Quantum Technology Initiative will explore the potential of devices harnessing perplexing quantum phenomena such as entanglement to enrich and expand its challenging research programme

30 SEPTEMBER, 2020 | By Matthew Chalmers



The AEGIS 1T antimatter trap stack. CERN's AEGIS experiment is able to explore the multi-particle entangled nature of photons from positronium annihilation, and is one of several examples of existing CERN research with relevance to quantum technologies. (Image: CERN)

Quantum simulation and HEP theory applications

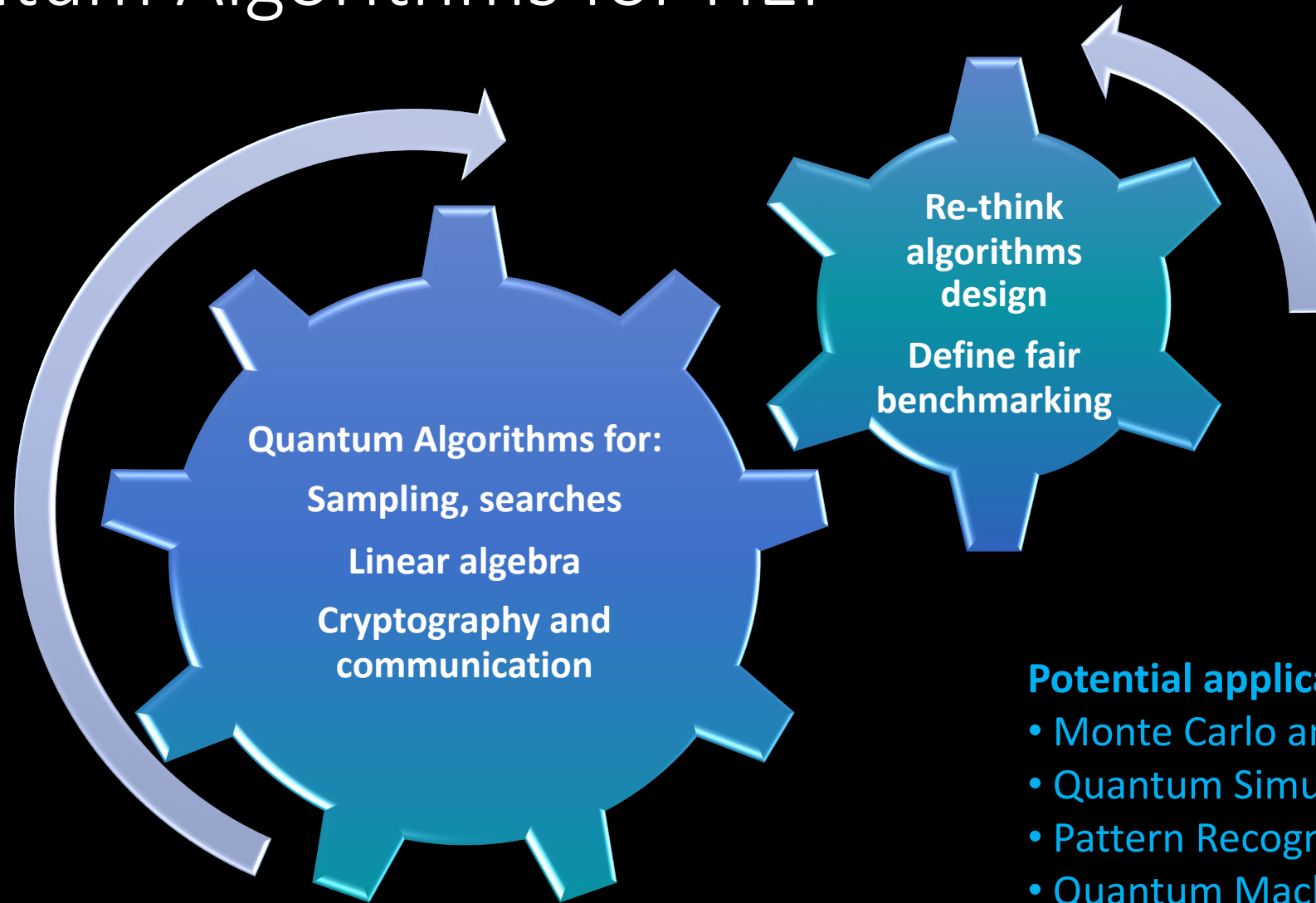
Quantum Computing

Quantum Sensing

Quantum Communication

QTI Roadmap: <https://doi.org/10.5281/zenodo.5553774>²³

Quantum Algorithms for HEP

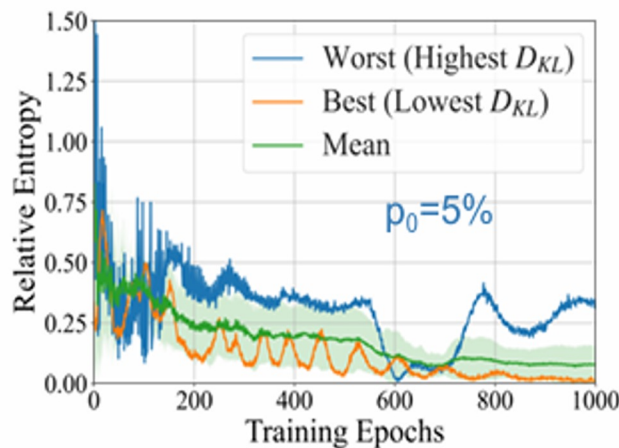


Potential applications:

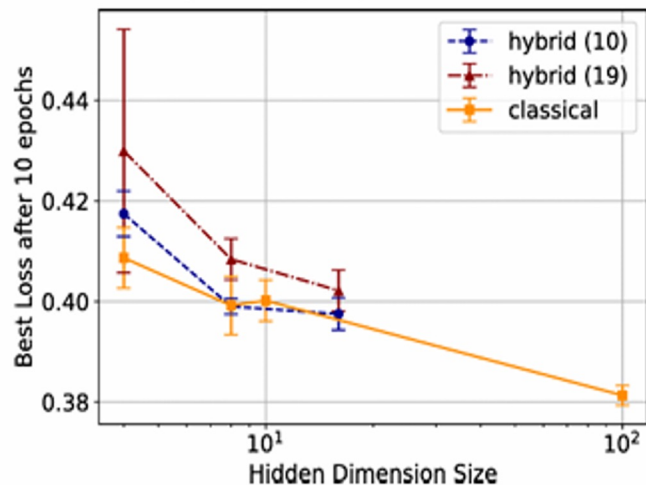
- Monte Carlo and Event Generation
- Quantum Simulation
- Pattern Recognition
- Quantum Machine Learning

QC @CERN

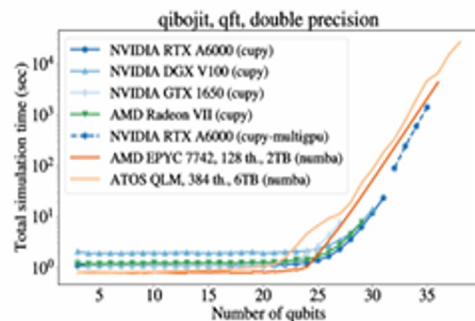
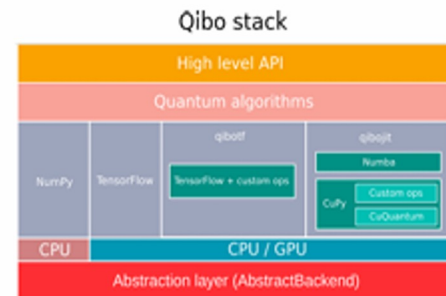
Borras, Kerstin, et al. "Impact of quantum noise on the training of quantum Generative Adversarial Networks." *arXiv preprint arXiv:2203.01007* (2022).



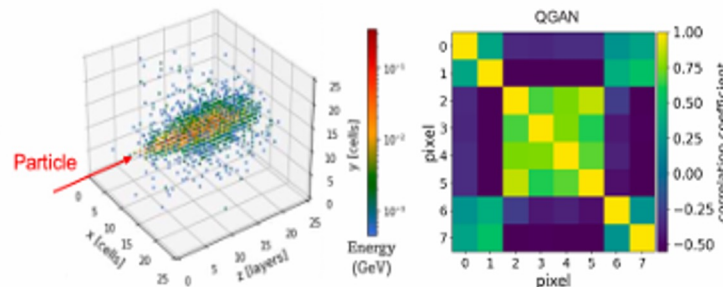
Tüysüz, Cenk, et al. "Hybrid quantum classical graph neural networks for particle track reconstruction." *Quantum Machine Intelligence* 3.2 (2021): 1-20.



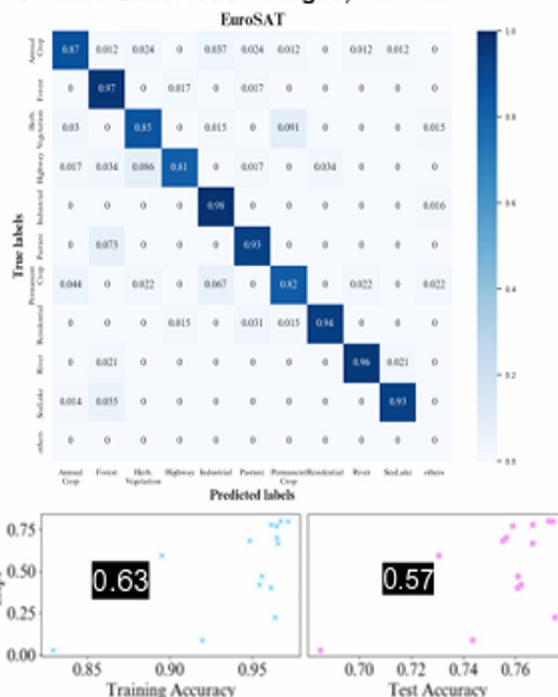
E.Stavros et al., Quantum simulation with just-in-time compilation, Quantum 2022



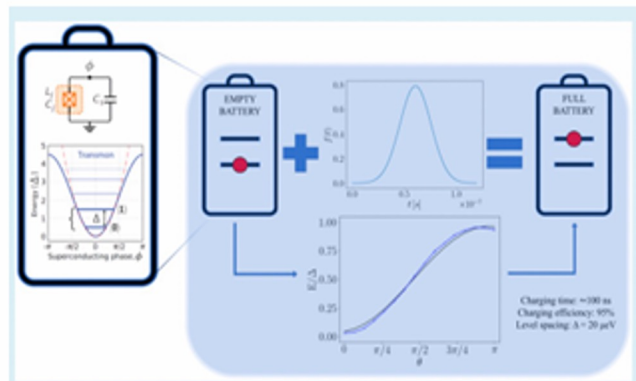
F.Rehm, Full Quantum GAN Model for HEP Detector Simulations, ACAT22



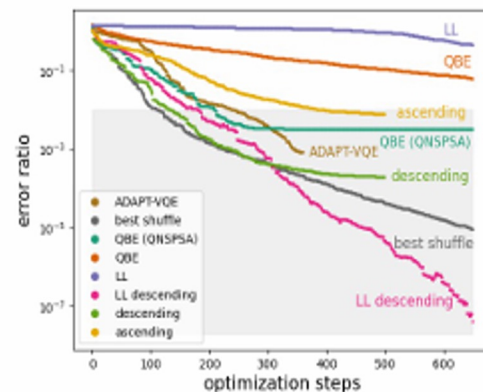
S.Chang, et al, Hybrid Quantum-Classical Networks for Reconstruction and Classification of Earth Observation Images, ACAT22



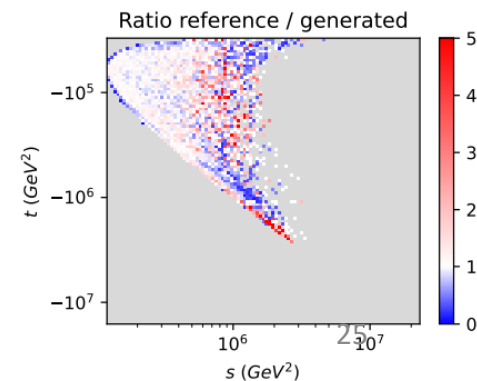
G. Gemme, M. Grossi et al, IBM Quantum Platforms: A Quantum Battery Perspective, Batteries 8, 43 (2022)



O. Kiss, Quantum computing of the ^6Li nucleus via ordered unitary coupled cluster, 10.1103/PhysRevC.106.034325



Bravo-Prieto, Carlos, et al. "Style-based quantum generative adversarial networks for Monte Carlo events." *Quantum* 2022





Quantum Machine Learning :

Some basic concepts

QML in HEP

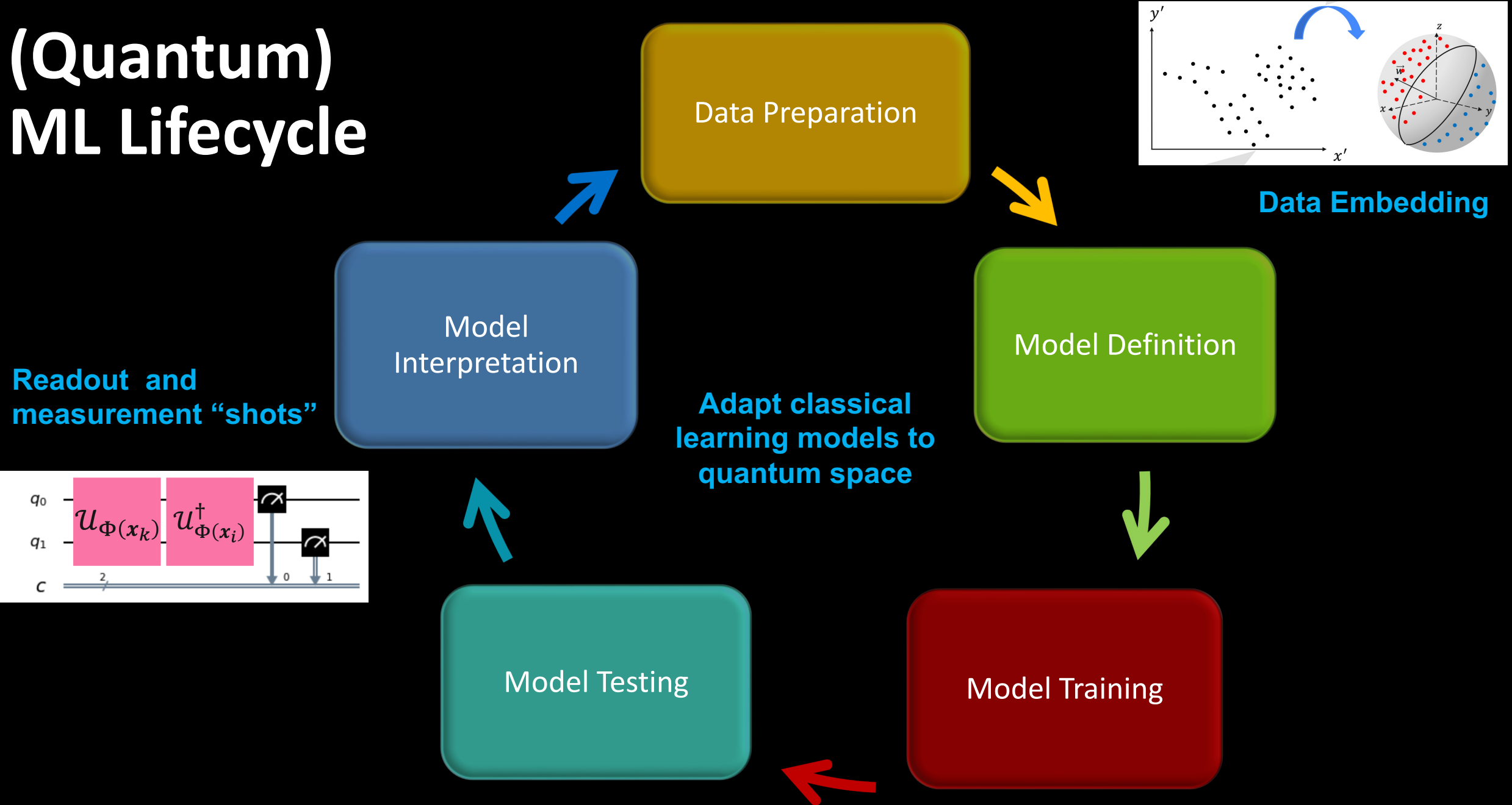
- Does it make sense to use QML in HEP?
- How do we understand when it is *useful* ?
- Which are the QML models we can leverage?

		Type of Algorithm	
		Classical	Quantum
Type of Data	Classical	CC	CQ
	Quantum	QC	QQ

Classical Intractability:

- No established recipe for classical data
- Compromise between algorithm **expressivity** vs **trainability** and **generalization**

(Quantum) ML Lifecycle



The advantage of many known QML algorithms is impeded today by I/O bottleneck

Models

Variational algorithms (ex. QNN)

Gradient-free or gradient-based optimization

Data Embedding can be learned

Ansatz design can leverage data symmetries¹

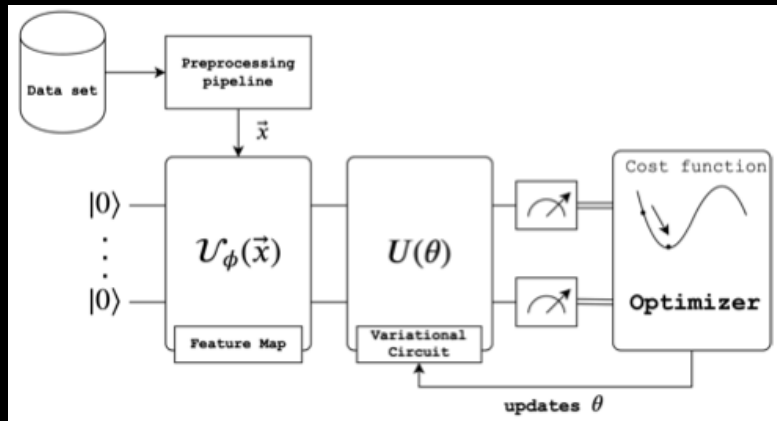


Image credit
SwissQuantumHub

Representer theorem:

Implicit models achieve **better accuracy**³

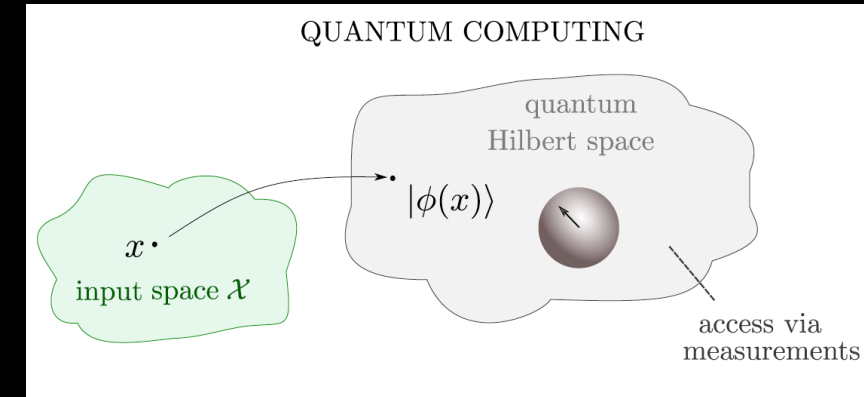
Explicit models exhibit **better generalization** performance

Kernel methods (ex. QSVM)

Feature maps as quantum kernels

Classical **kernel-based training** (convex losses)

Identify classes of kernels that relate to specific data structures²



Energy-based ML (ex. QBM)

Build network of stochastic binary units and optimise their energy.

QBM has quadratic energy function that follows the Boltzman distribution (Ising Hamiltonian)

Image credit M. Schuld

¹ Bogatskiy, Alexander, et al. "Lorentz group equivariant neural network for particle physics." PMLR, 2020.

² Glick, Jennifer R., et al. "Covariant quantum kernels for data with group structure." *arXiv:2109.03406* (2021).

³ Jerbi, Sofiene, et al. "Quantum machine learning beyond kernel methods." *arXiv preprint arXiv:2110.13162* (2021).



Quantum Machine Learning examples:

Anomaly Detection

New Physics at the LHC

So far only negative results in direct (model dependent) searches

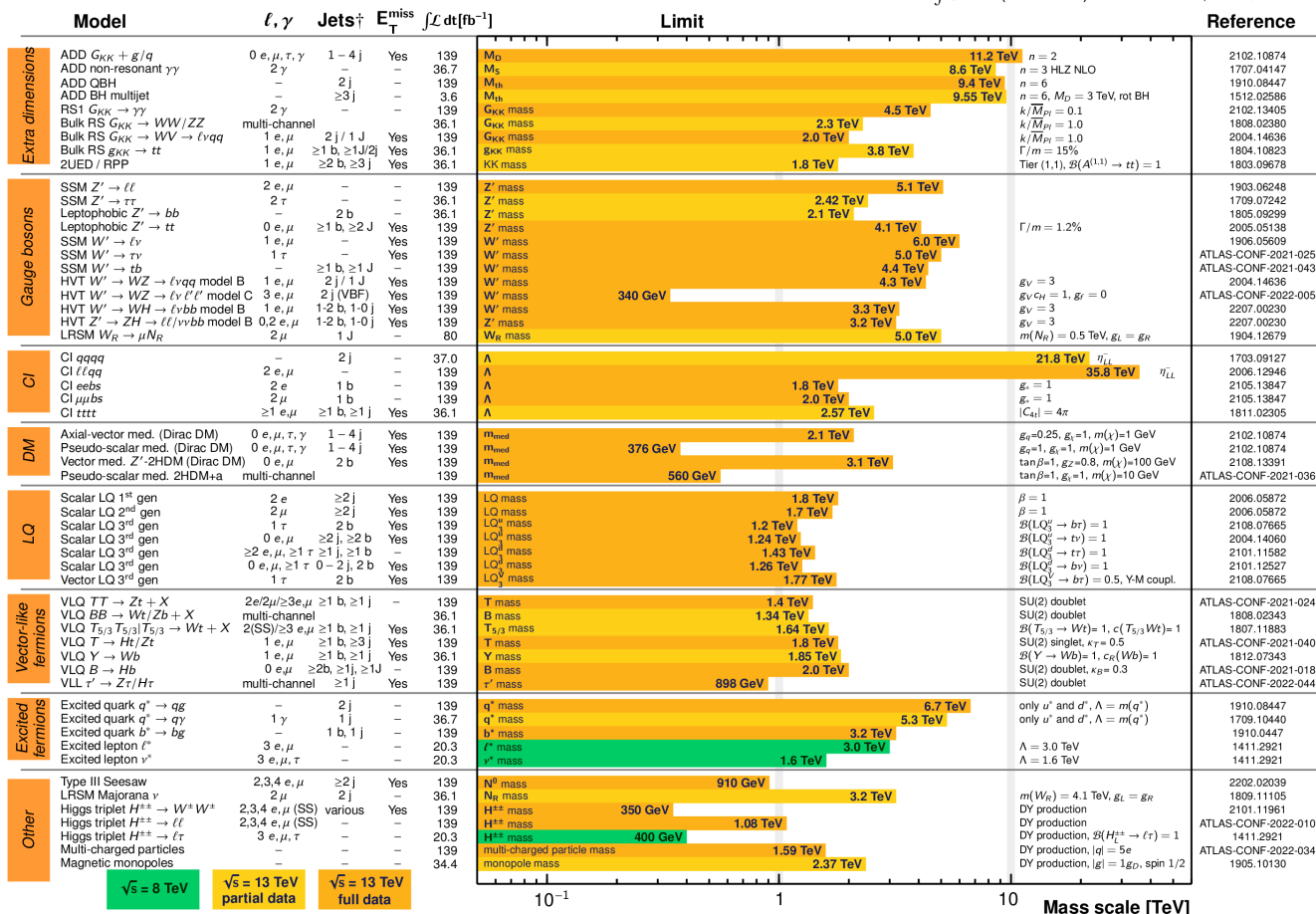
ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: July 2022

ATLAS Preliminary

$$\int \mathcal{L} dt = (3.6 - 139) \text{ fb}^{-1}$$

$$\sqrt{s} = 8, 13 \text{ TeV}$$



*Only a selection of the available mass limits on new states or phenomena is shown.

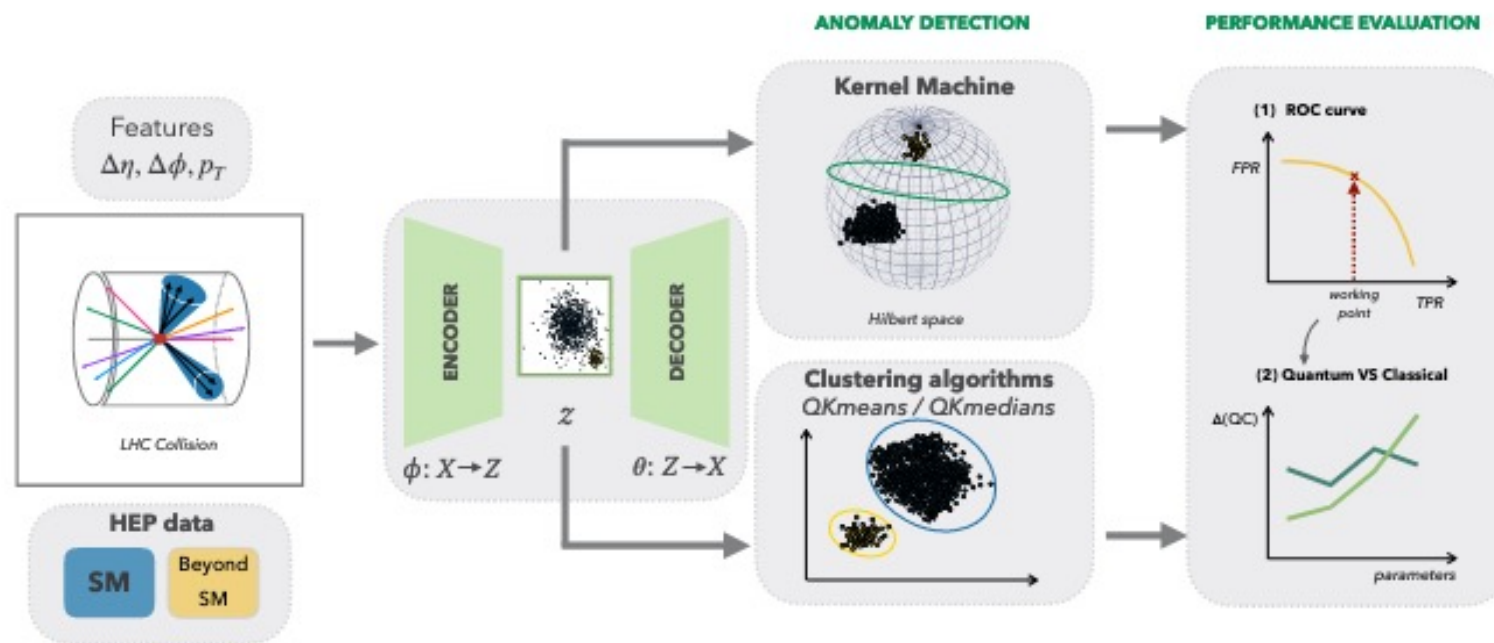
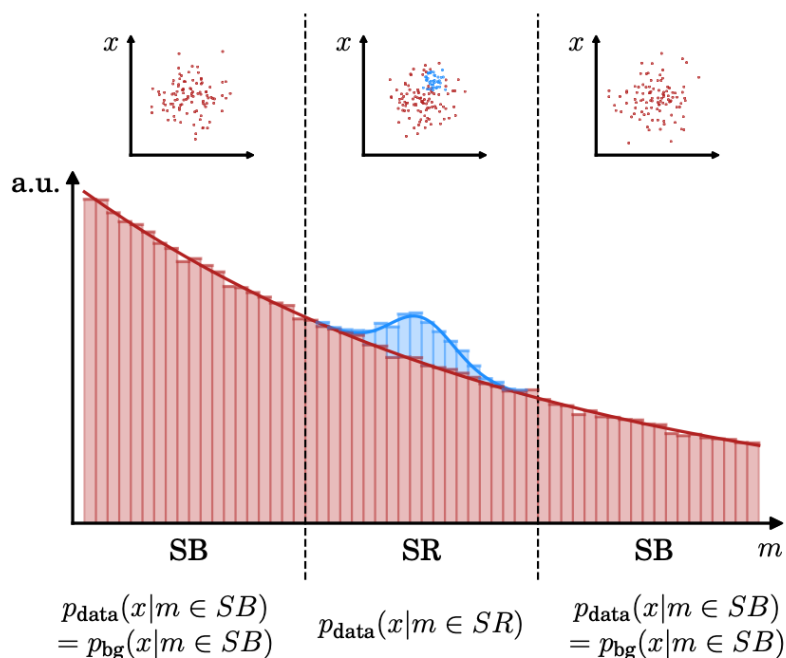
[†]Small-radius (large-radius) jets are denoted by the letter j (J).

How to insure we do not miss potential discoveries?

We can design model agnostic searches!

Unsupervised learning for Anomaly Detection

Anomaly detection can point to new physics at the LHC



Standard Model jets

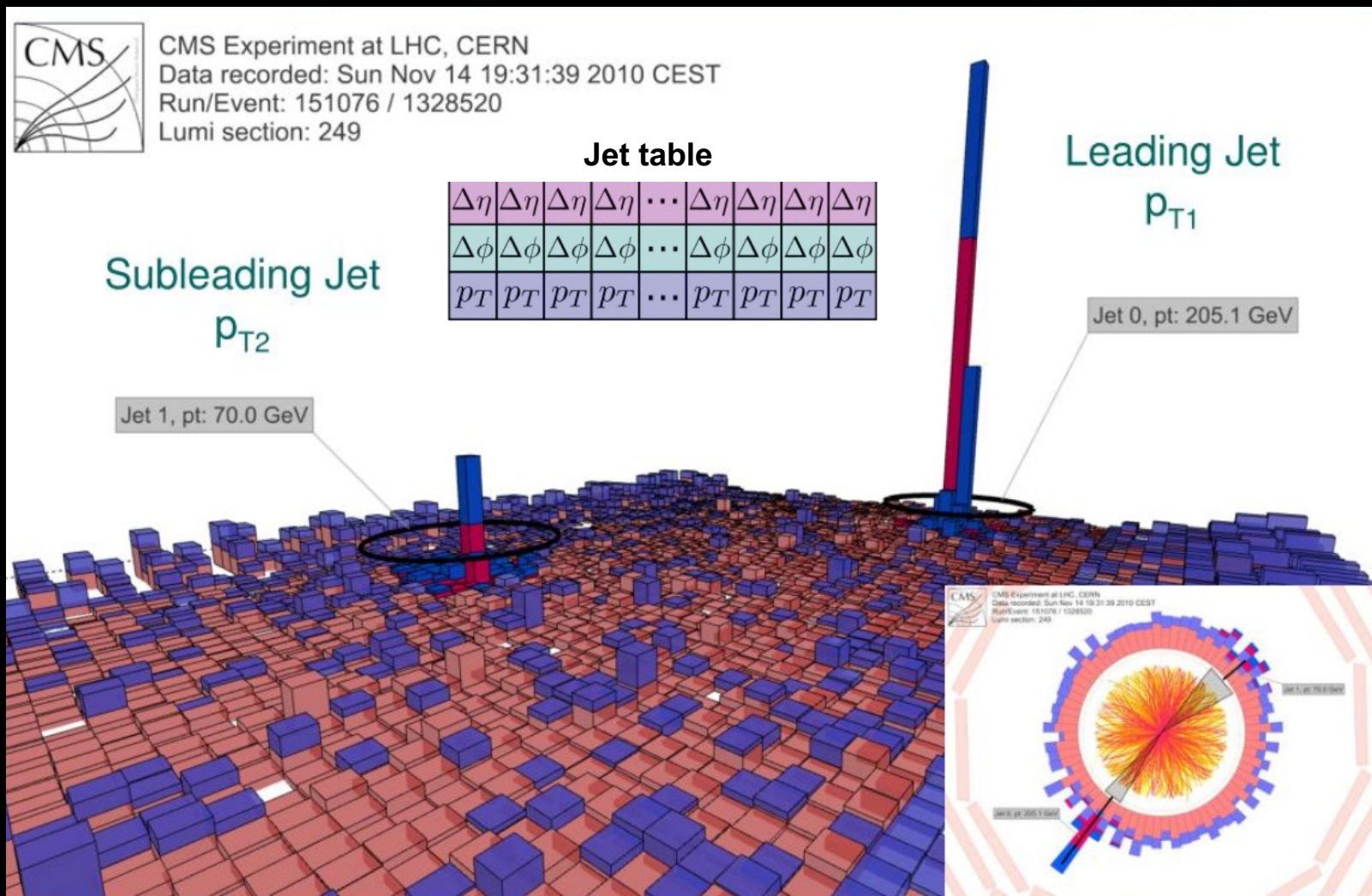
Simulate QCD multi-jets at the LHC

Build jet from 100 highest pt particles

Apply realistic event selection

Convolutional AutoEncoder
learns the jet internal structure

$$\mathbb{R}^{300} \rightarrow \mathbb{R}^{\ell}, \ell = 4, 8, 16$$



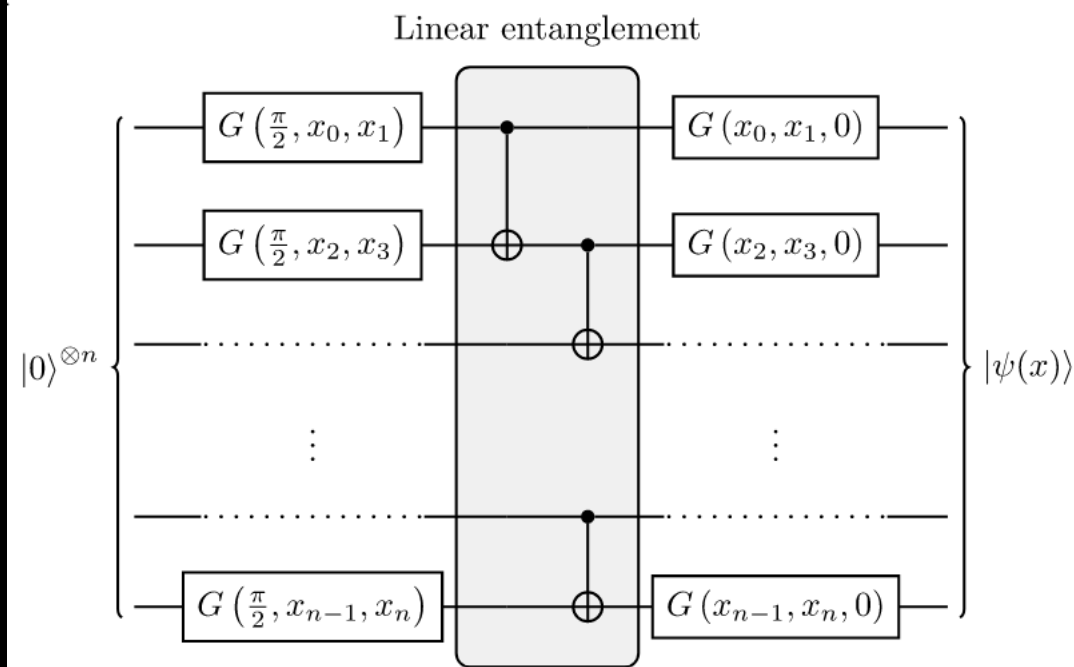
Unsupervised kernel machine

Find the hyperplane that maximizes the distance of the data from the origin of the feature vector space

Upper bound on fraction of anomalies in training data at 0.01 (at most 1% QCD training data are falsely flagged)

$$k(x_i, x_j) := \text{tr}[\rho(x_i)\rho(x_j)] = |\langle 0|U^\dagger(x_i)U(x_j)|0\rangle|^2$$

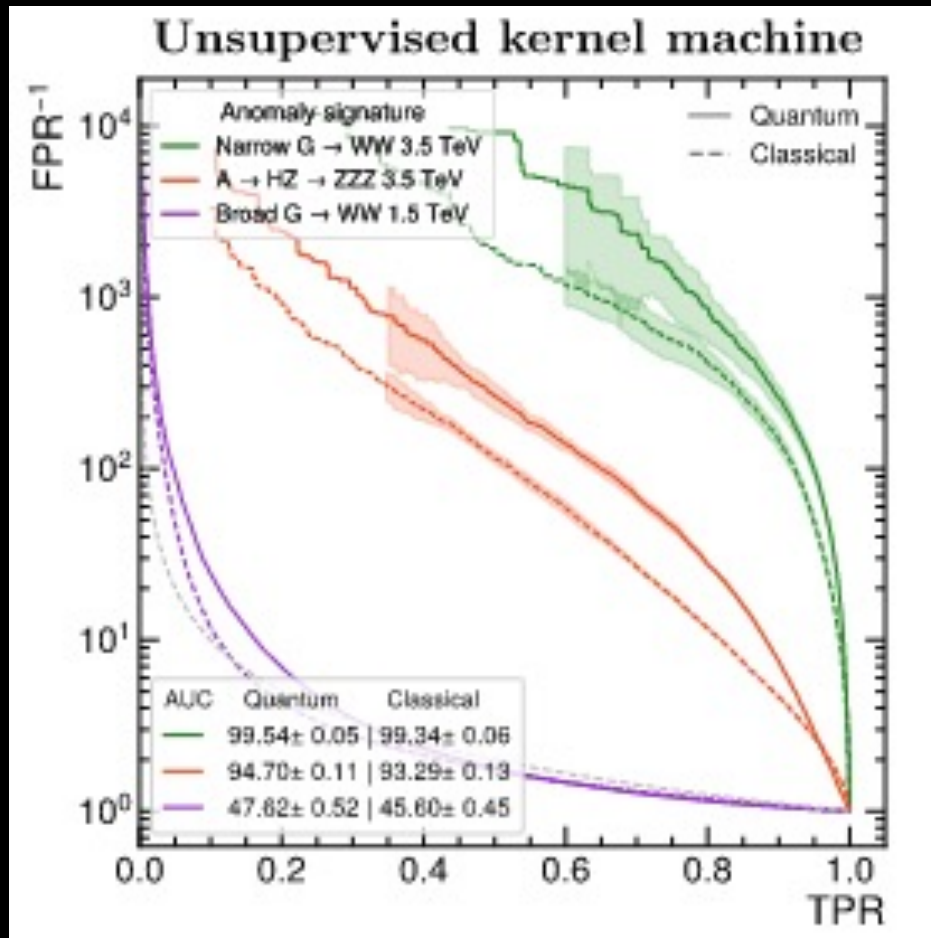
$$\rho(x_i) := U(x_i) |0\rangle \langle 0| U^\dagger(x_i)$$



$$\min_{w \in \mathcal{F}, \xi \in \mathbb{R}^\ell, \rho \in \mathbb{R}} \frac{1}{2} \|w\|^2 + \frac{1}{\nu \ell} \sum_i \xi_i - \rho$$

$$\text{subject to } w \cdot \Phi(x_i) \geq \rho - \xi_i, \xi_i \geq 0, \forall i, \quad \nu \in (0, 1)$$

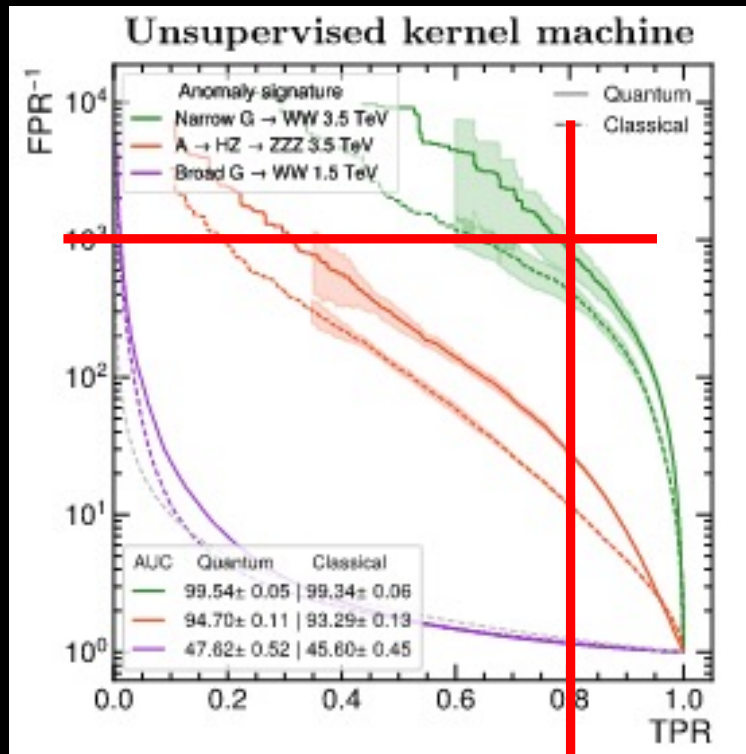
Results



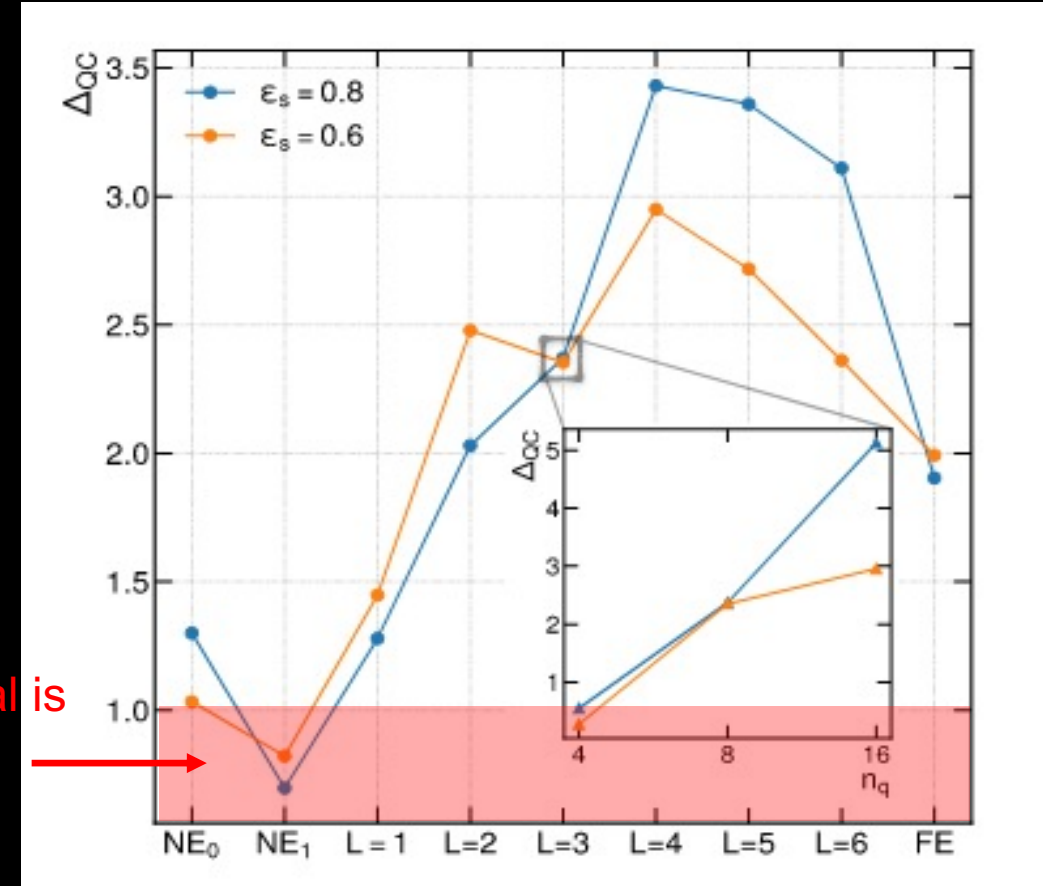
Is this an «advantage»
we can use?

Quantum anomaly detection in the latent space
of proton collision events at the LHC
Vasileios Belis *et al.*, *arXiv:2301.10780*.

In reality....



Classical is better



Higher is better

Increasing entanglement & expressivity

Quantum anomaly detection in the latent space of proton collision events at the LHC
Vasileios Belis *et al.*, *arXiv:2301.10780*.



Quantum Machine Learning examples:

Reinforcement Learning

Reinforcement learning

in a nutshell

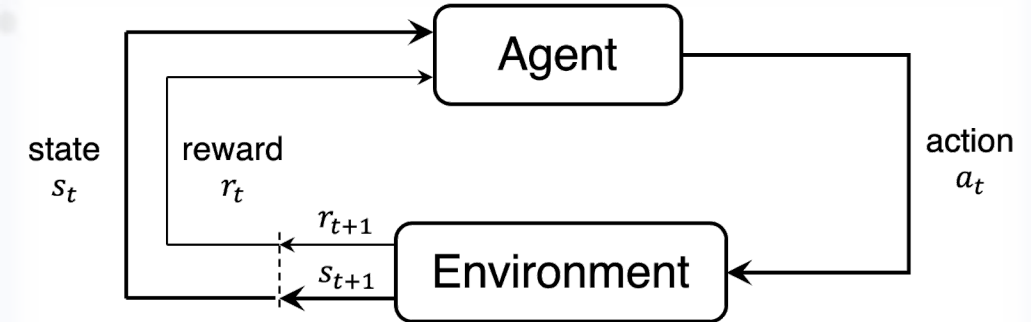
Trial-and-error learning

- Agent **takes actions** in environment and **collects rewards**

Q-learning

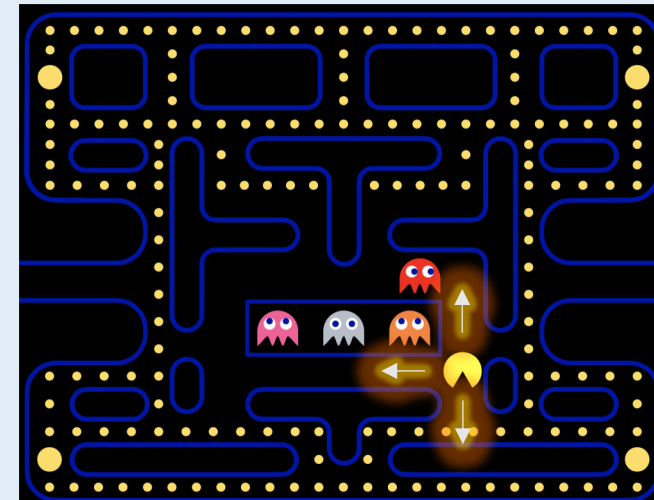
- Estimate **return using Q-function** $Q(s, a)$
- **Learn iteratively** using **collected interactions**
- Once trained, **select action greedily**

$$a = \arg \max_a Q(s, a)$$



RL book: Sutton & Barto

Example: Pacman



State

where am I? Where are ghosts, snacks, cookies?

Actions

up, down, left, right

Reward

food (+), ghosts (-)

Return

how much food am I going to eat over time

Free-energy based RL (FERL)

RL performance depends on type of Q-function approximator

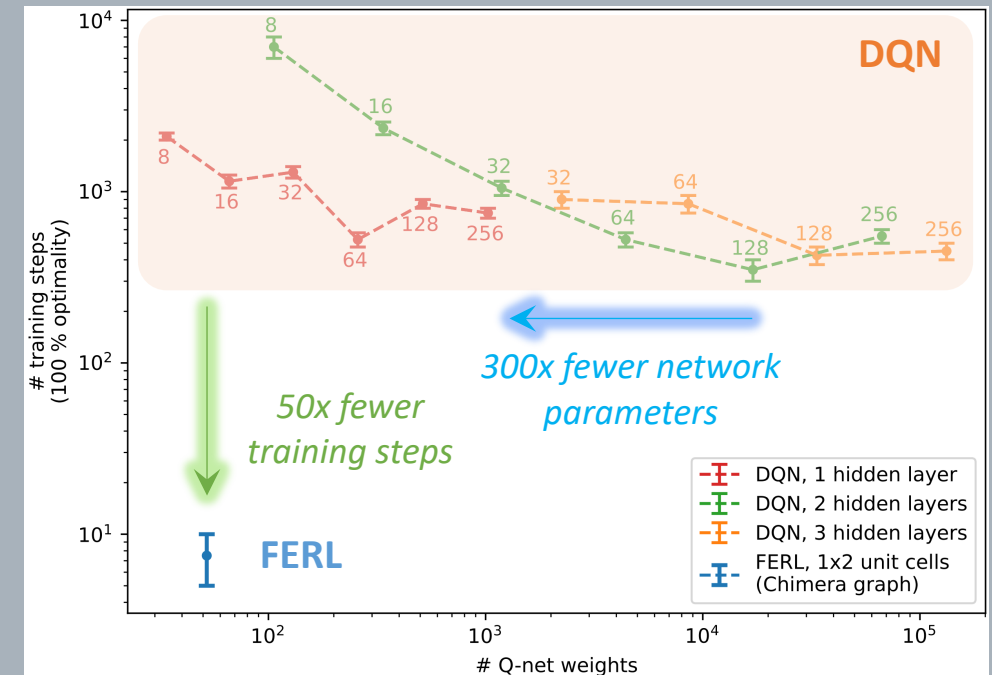
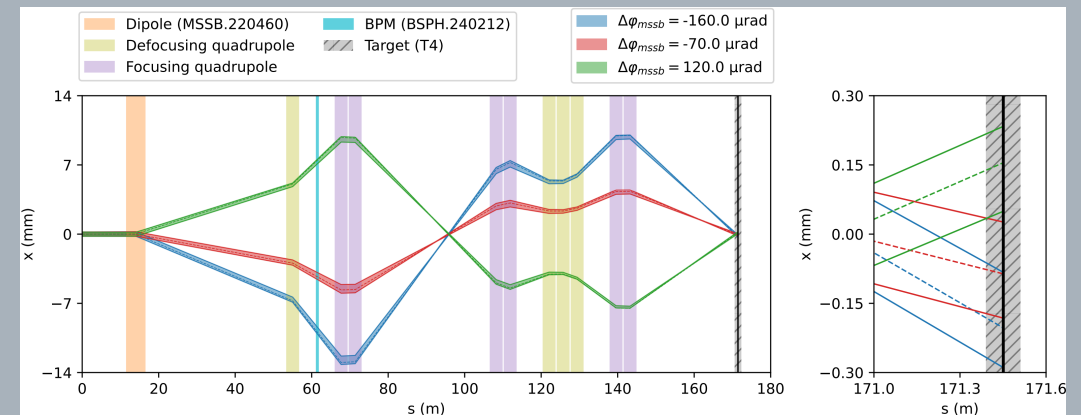
- Classical Deep Q-learning (DQN)
Feed-forward neural net
- Free-energy based RL (FERL)
Quantum Boltzmann machine (QBM)

Key concept: sample-efficiency

- Relevant for **particle accelerator control**
given cost of beam time (*online training*)

1st study: 1D beam steering

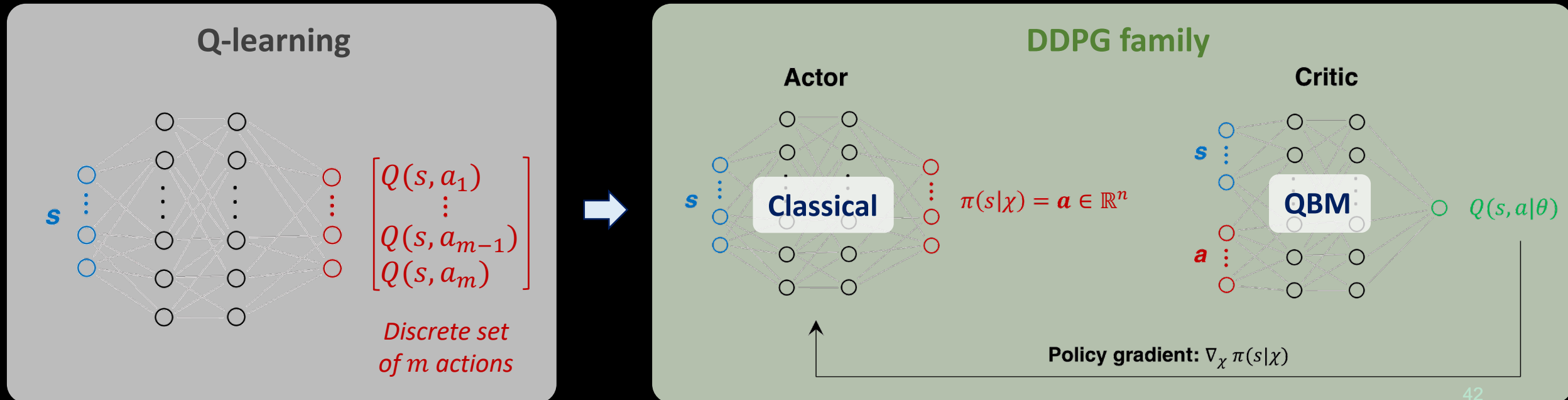
CERN North Area transfer line (discrete action space)



Developing a hybrid actor-critic scheme

Accelerator optimization requires **continuous action space** → **develop hybrid actor-critic algorithm**

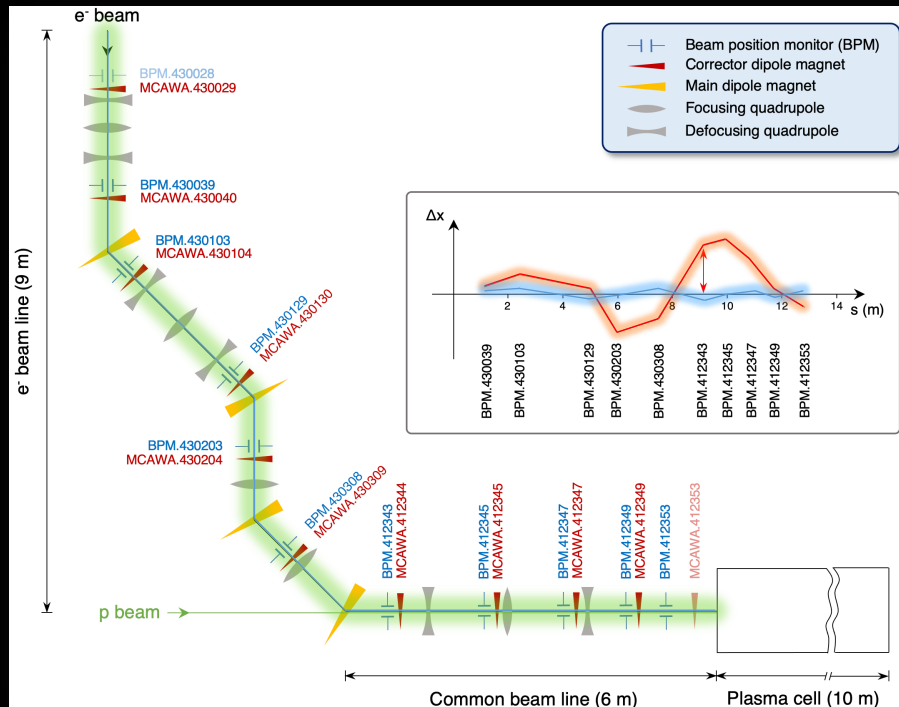
➤ **QBM replaces classical critic net**



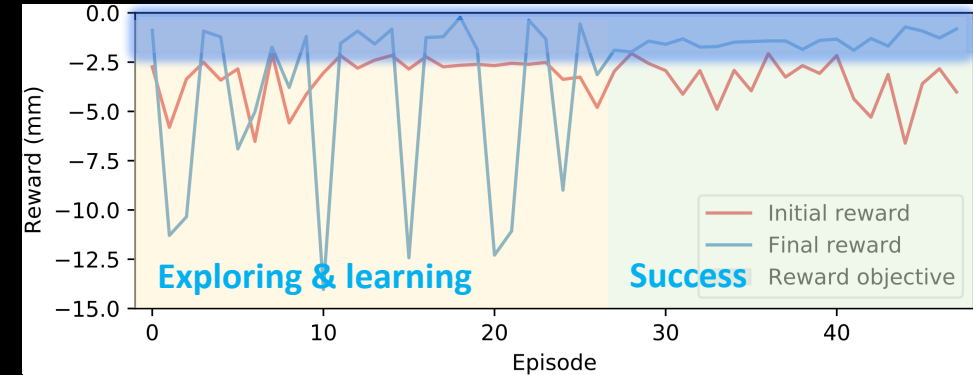
2nd study: 10D continuous beam steering

Environment: e⁻ beam line of AWAKE

- **Action:** deflection angles at 10 correctors
 - **State:** beam positions at 10 BPMs
 - **Objective:** minimize beam trajectory rms
- ➡ **reward:** negative rms from 10 BPMs



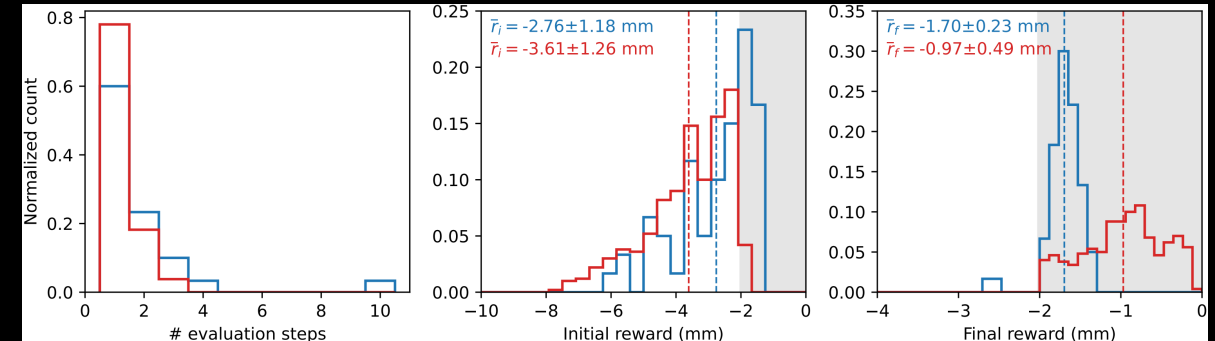
Training: on D-Wave Advantage quantum annealer (QA)



Objective

Evaluation: on actual beam line

Real vs. simulated QA

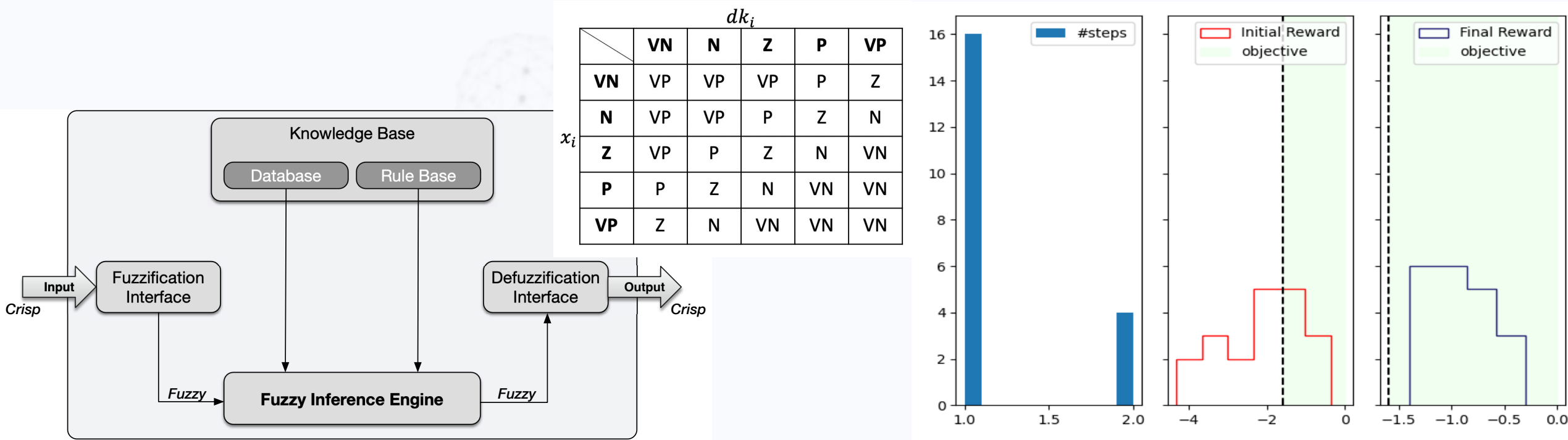


- Agent minimizes rms in **1 step** in **60 %** cases
- Hyperparameter tuning with **simulated QA**

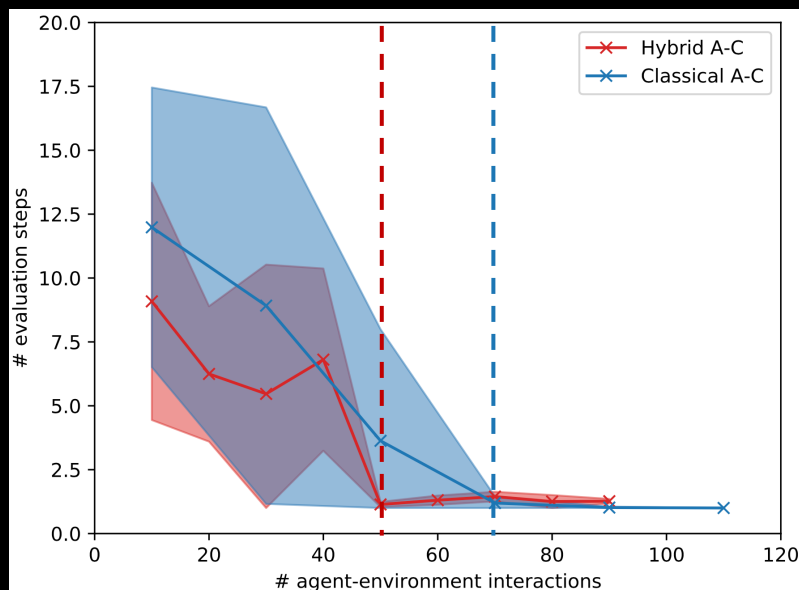
1-slide excursion: quantum fuzzy logic controller

- **Alternative control algorithm** to RL
- **Fuzzy Logic** is used to develop control systems **based on linguistic rules** ➡ **highly interpretable**
- **Quantum Fuzzy Control System** (*G. Acampora, R. Schiattarella, A. Vitiello*)
Exploit **exponential advantage** in computing fuzzy rules on quantum computers
- **Successfully evaluated on AWAKE beam line, no training required**

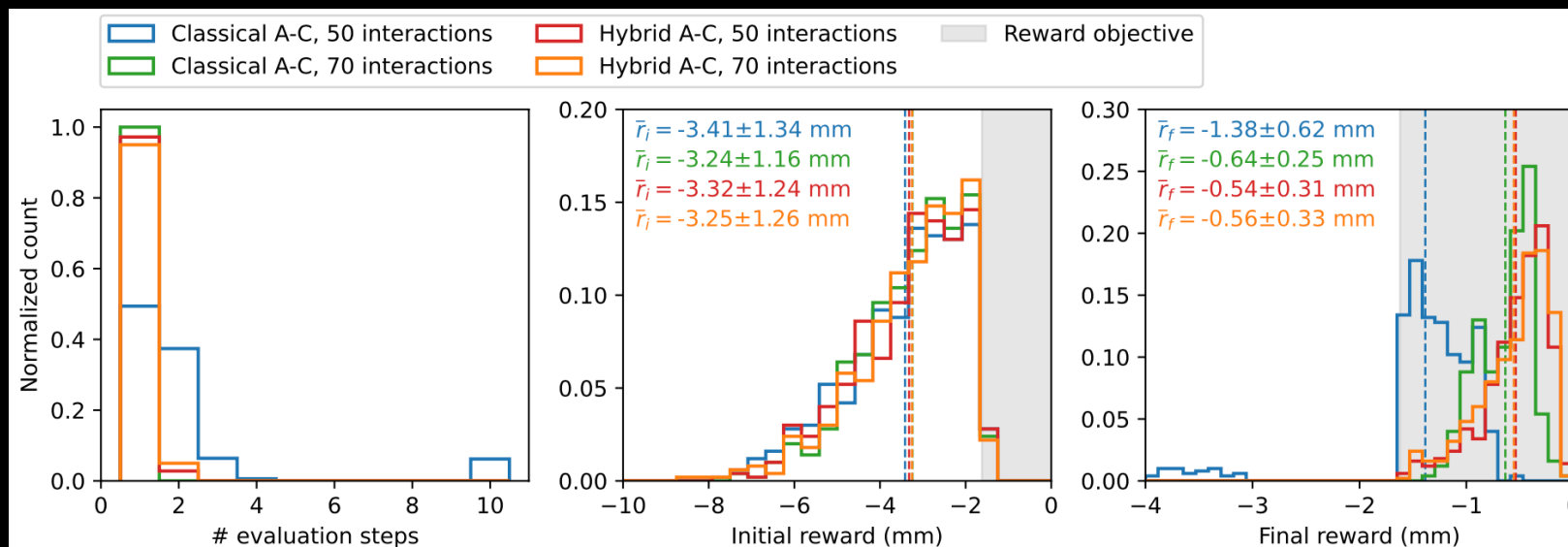
Evaluation: on AWAKE beam line
Objective reached typically in 1 step



2nd study: 10D continuous beam steering



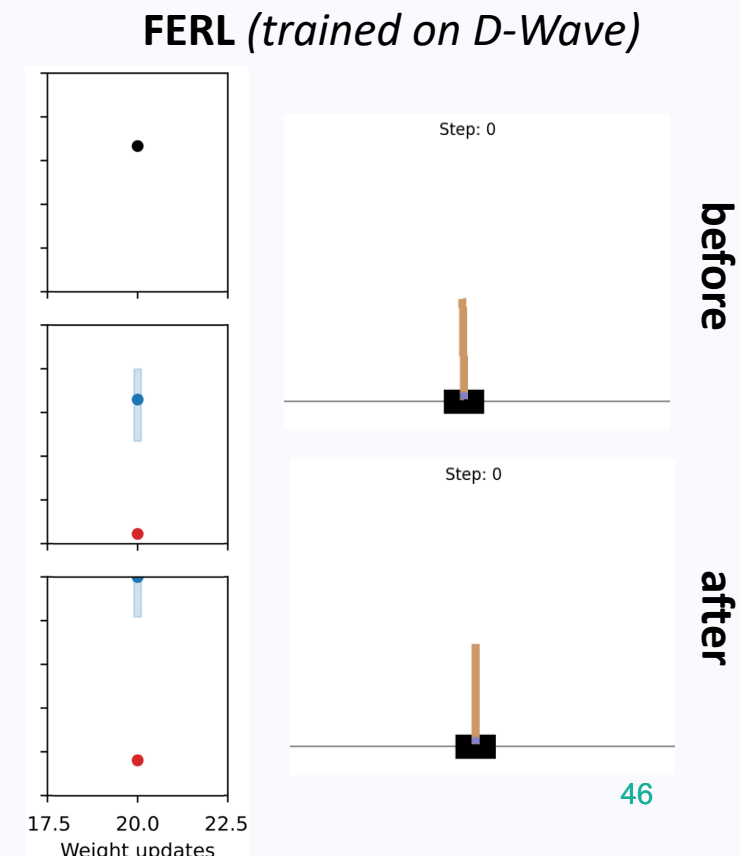
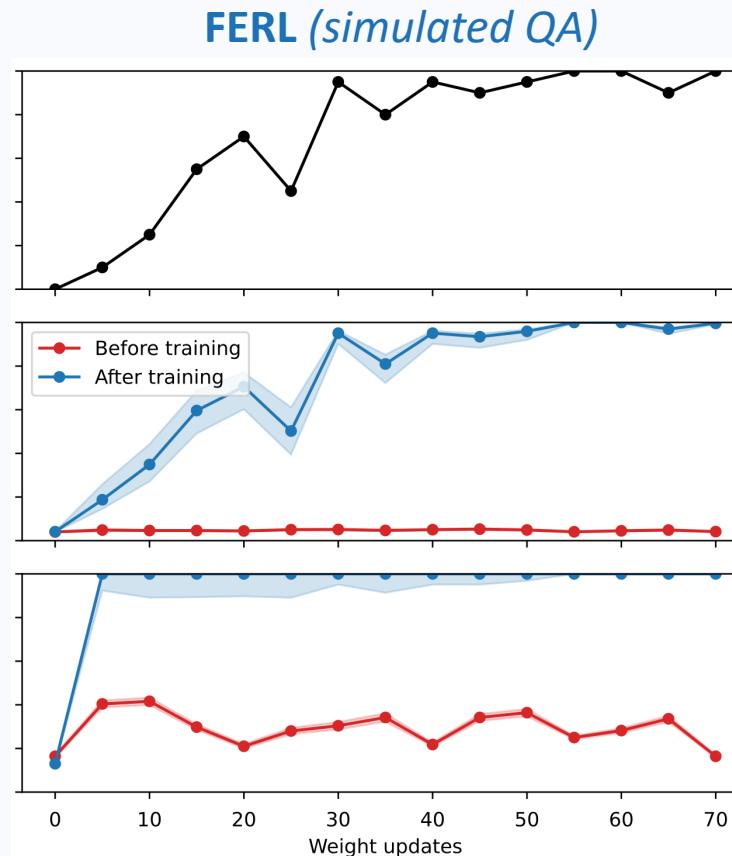
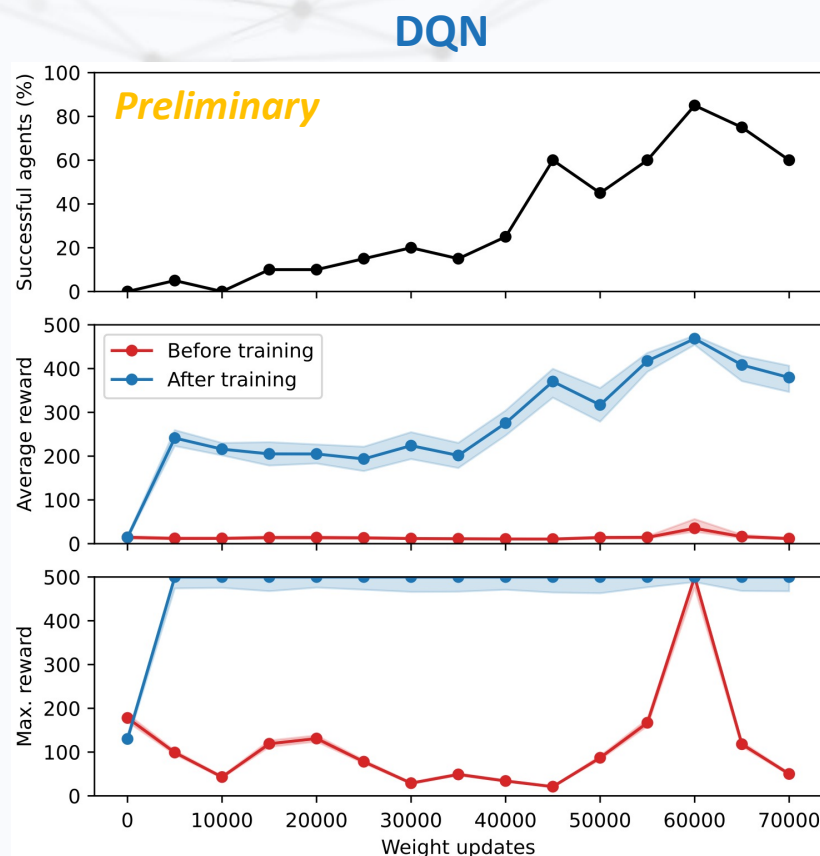
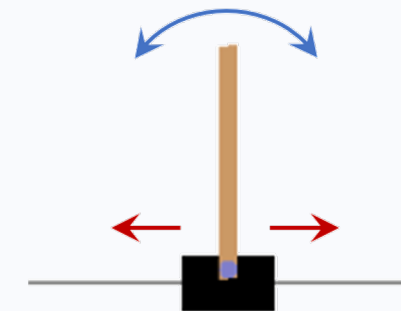
- **Hybrid actor-critic (A-C) works**
- **Minor improvement in terms of sample efficiency**
50 vs 70 interactions
- **Very few interactions sufficient for both approaches**
- **Dynamics potentially too simple (linear)**
➔ Move towards more complex RL benchmarks



3rd study: Cartpole-v1

Discrete action problem, non-linear dynamics

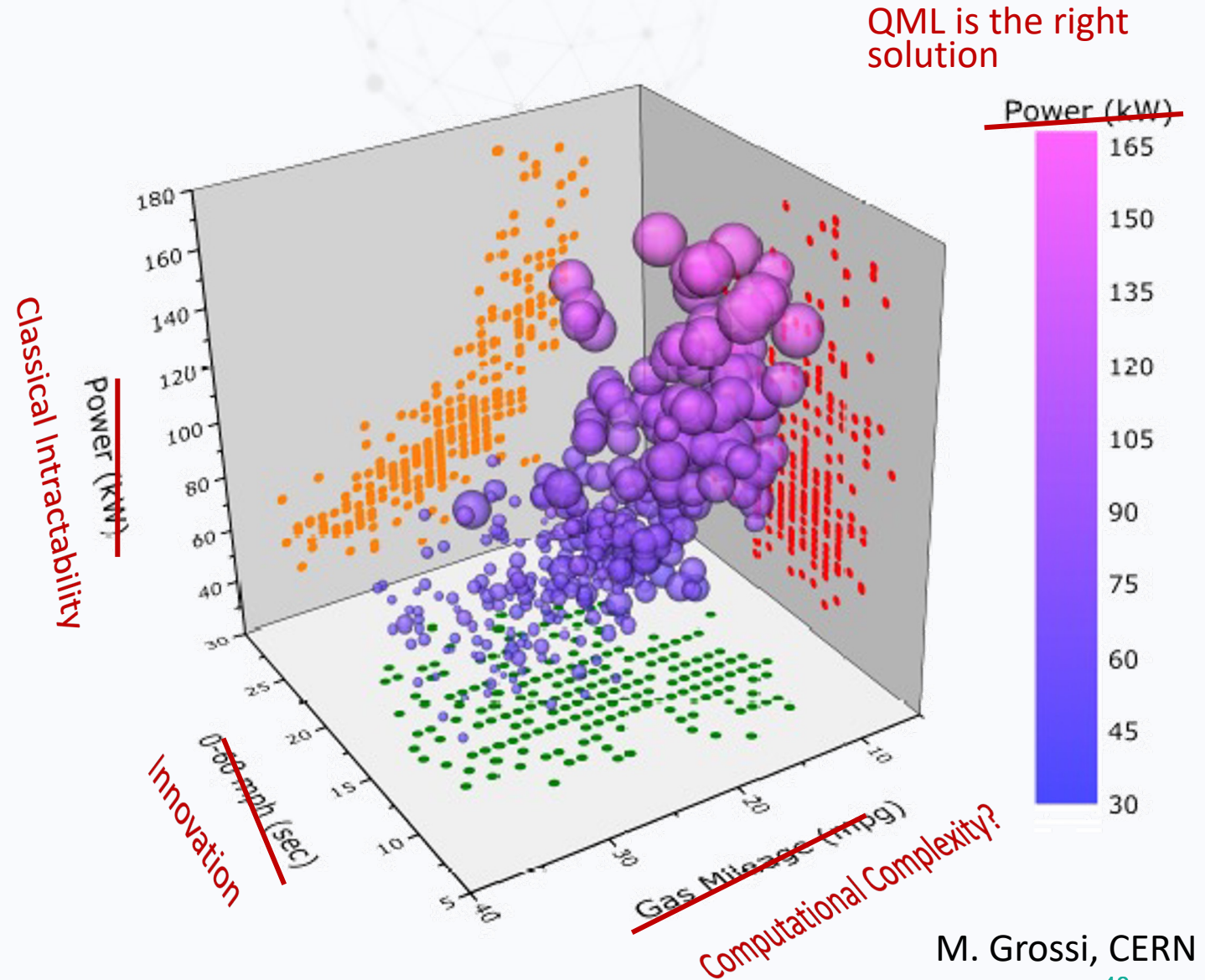
- **Cartpole-v1**: official [OpenAI gym env](#) from classic control problems domain
- Continuous state (4D), **discrete action** (*right, left*) problem with **non-linear dynamics**
- Terminate episodes after **max. 500 steps**
- **Big gain in sample-efficiency and robustness** for FERL vs DQN



Outlook and open questions

- Quantum technologies could be revolutionary in terms of computing
- HEP provides challenges to Quantum Computing
 - **What are the most promising applications?**
 - How do we define performance and validate results on **realistic use cases?**
- Experimental data has high dimensionality
 - Can we train Quantum Machine Learning algorithms effectively?
 - Can we reduce the impact of **data reduction** techniques?
- Experimental data is shaped by physics laws
 - Can we leverage them to build better algorithms?

QML Exclusion Region in HEP?



M. Grossi, CERN

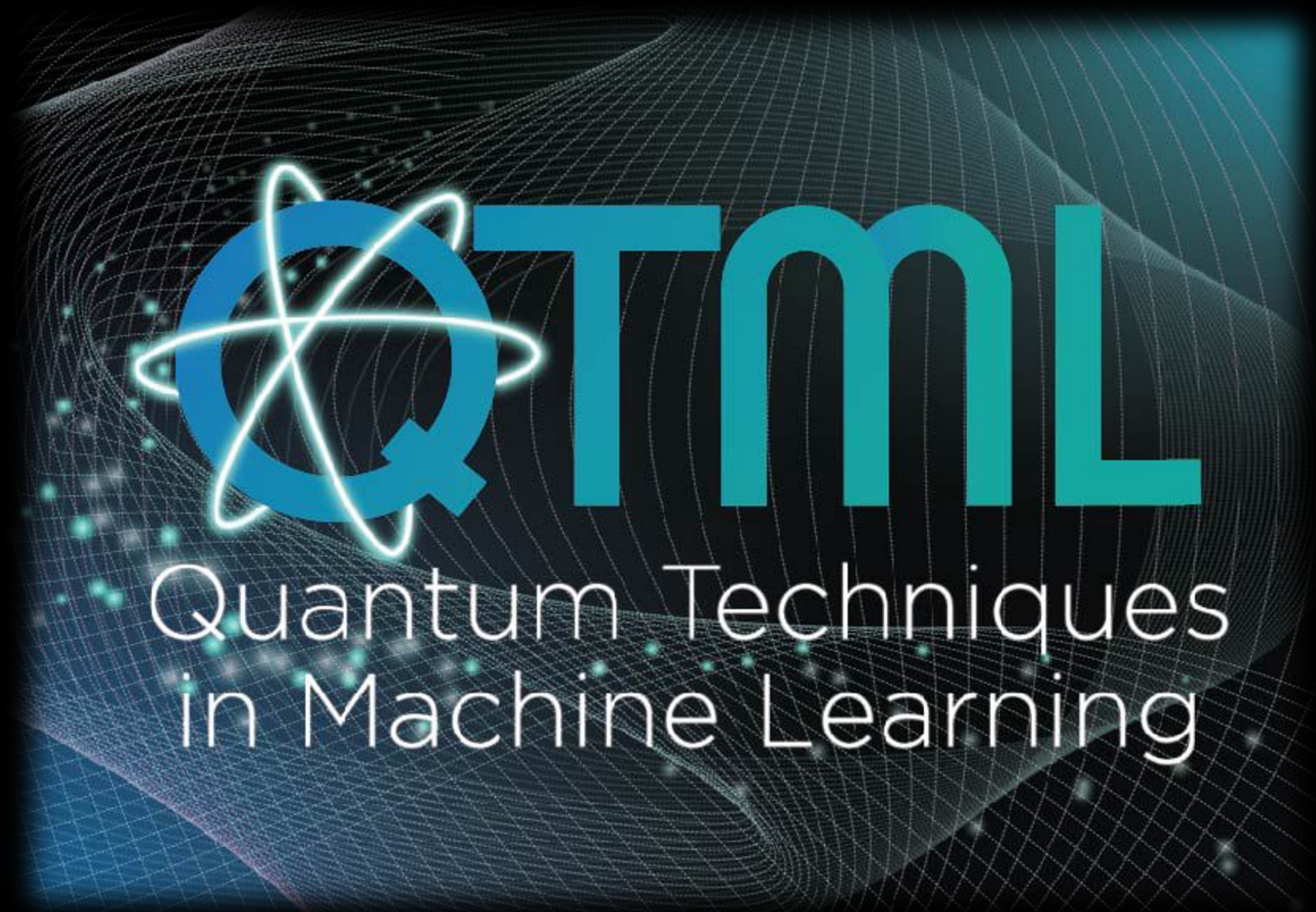
Lectures and Hands-On at CERN

- «A practical Introduction to quantum computing», Elias Combarro
<https://indico.cern.ch/event/970903/>
- «Introduction to quantum computing », Heather Grey
<https://indico.cern.ch/event/870515/>
- A set of two hands-on (introduction) sessions for summer students (2023 openlab summer student lectures)
<https://indico.cern.ch/event/1293871/>
<https://indico.cern.ch/event/1293874/>

Thank you!

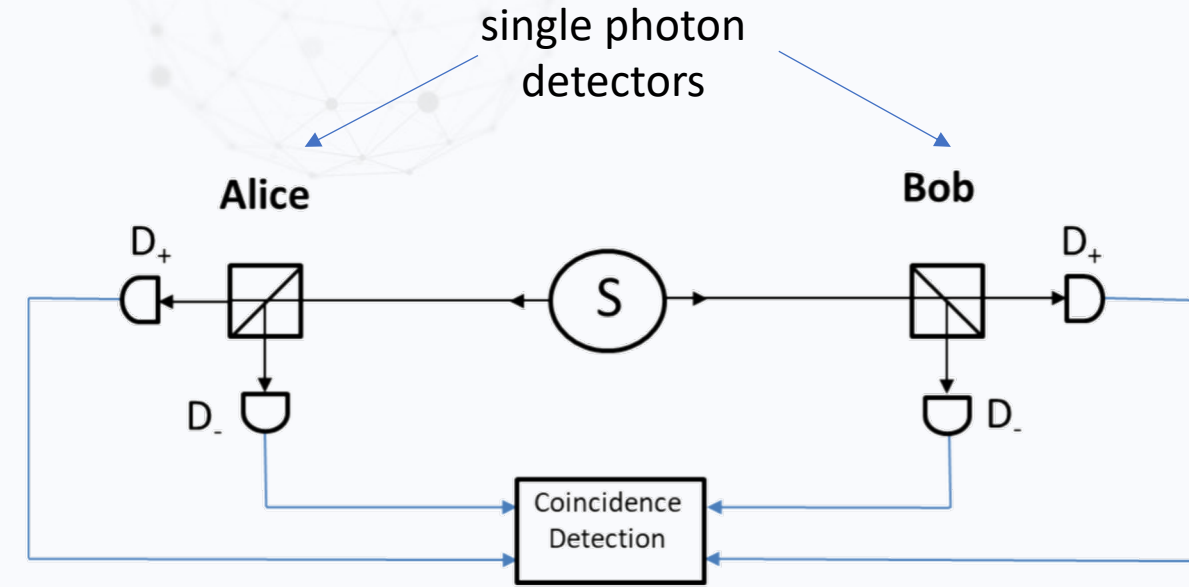
November 20th-24th, 2023
@CERN

Sofia.Vallecorsa@cern.ch



Bell Inequalities

- 1964: Bell inequality are a mathematical proof that no theory based on **local hidden variables** (realism) can reproduce QM results
- 1969: Clauser, Horne, Shimony (CHSH) formulation



- Measure the correlation $E(a, b) = (N_{++} - N_{+-} - N_{-+} + N_{--}) / (N_{++} + N_{+-} + N_{-+} + N_{--})$
- Where N_{++} , N_{+-} , N_{-+} , and N_{--} are the number of coincidence events corresponding to the simultaneous detection

NB in a realist theory the measurement outcome is «known» even if the measurement is not performed

Bell Inequalities

For experimental outputs of the form $A_1 = \pm 1$

$$A_1(B_1 + B_2) + A_2(B_1 - B_2) = \pm 2,$$

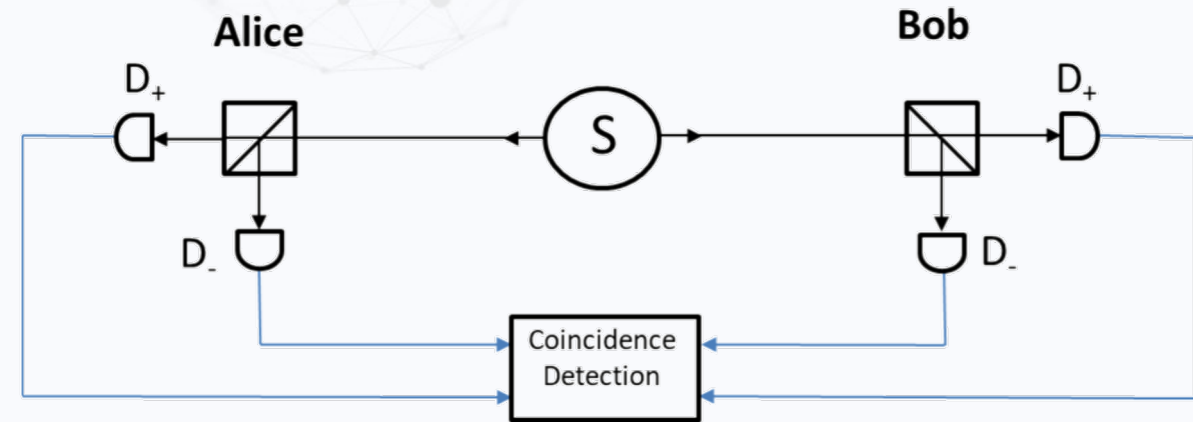
since $B_1 + B_2 = \pm 2$ and $B_1 - B_2 = 0$ or v.v.

Performing the experiment many times, the correlation is calculated using the ensemble average over the measurements

$$E(a_1, b_1) = \langle A_1 \cdot B_1 \rangle,$$

$$S = |E(a_1, b_1) + E(a_1, b_2) + E(a_2, b_1) - E(a_2, b_2)|$$

Classical theory: $S < 2$



Quantum mechanics: $S = 2\sqrt{2}$

since $E(a_1, b_1) = -\mathbf{a}_1 \cdot \mathbf{b}_1$ for $|\psi_-\rangle$

and experiment directions can be chosen so that:

$$\mathbf{a}_1 \cdot \mathbf{b}_1 = \mathbf{a}_1 \cdot \mathbf{b}_2 = \mathbf{a}_2 \cdot \mathbf{b}_1 = -\mathbf{a}_2 \cdot \mathbf{b}_2 = 1/\sqrt{2}$$