Accelerators Part I

Tatiana Pieloni (Tatiana.Pieloni@epfl.ch)

Laboratory for Particle Accelerator Physics, EPFL

References and accessible Reading Material

available on the internet:

P. Schmüser & J. Rossbach, Basic course on accelerator optics:

https://cds.cern.ch/record/247501/files/p17.pdf

F. Tecker, Longitudinal Dynamics material:

https://arxiv.org/pdf/1601.04901.pdf

Book, H.Wiedemann, Particle Accelerators, download pdf !:

https://link.springer.com/book/10.1007%2F978-3-319-18317-6

CERN Accelerator School (CAS) proceedings homepage (huge!)
http://cas.web.cern.ch/cas/CAS_Proceedings.html

books, papers:

S.Peggs, T.Satogata, *Introduction to Accelerator Dynamics*, Cambridge University Press, 2017

A. Wolski, *Beam Dynamics in high energy particle accelerators*, Imperial College Press, 2014

A. W. Chao, M. Tigner, *Handbook of Accelerator Physics and Engineering*, World Scientific 1999

E. D. Courant and H. S. Snyder, Annals of Physics: **3**, 1-48 (1958)

*** with focus on linear beam optics**

Contents:

- Particle types and relativity for accelerators
- Accelerator components: Dipole, quadrupoles magnets, accelerating RF cavities…
- Transverse plane (x,y) → Guiding and focusing beams
  - Particle motion in linear approximation
  - Invariant of motion and Emittance
  - Beam Optics: beta functions, beams sizes, Beam Tunes
- Longitudinal plane (s,t) → Acceleration
  - Synchronous motion
  - Synchrotrons and LHC injection complex
- Hadron Accelerators: Synchrotrons
  - Beam production
  - Magnets
  - Luminosity
  - Collective effects

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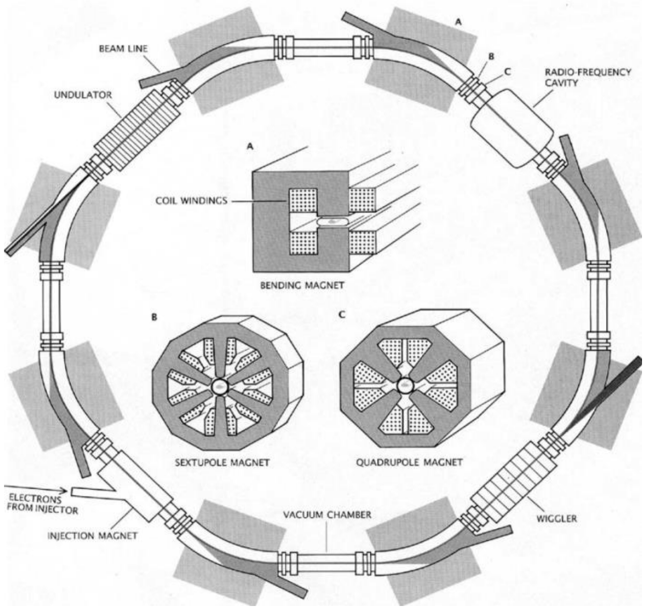
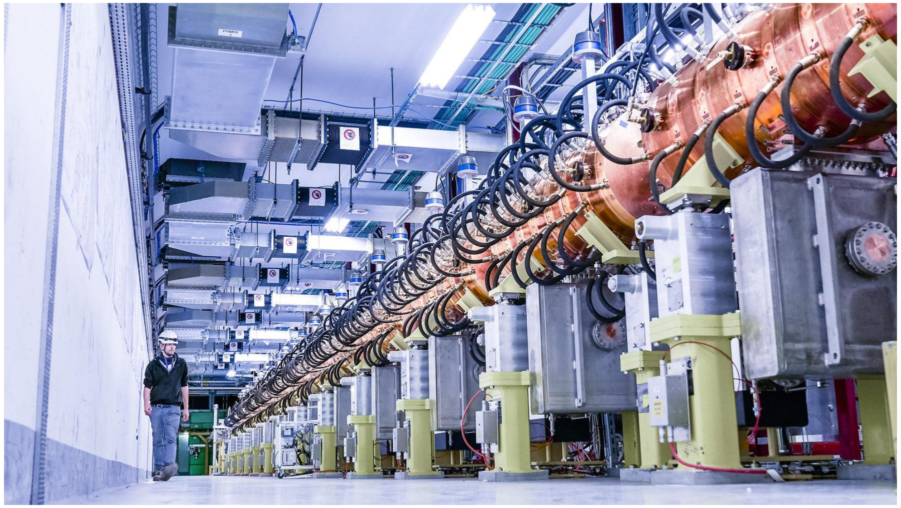
Introduction to accelerators and particle Dynamics

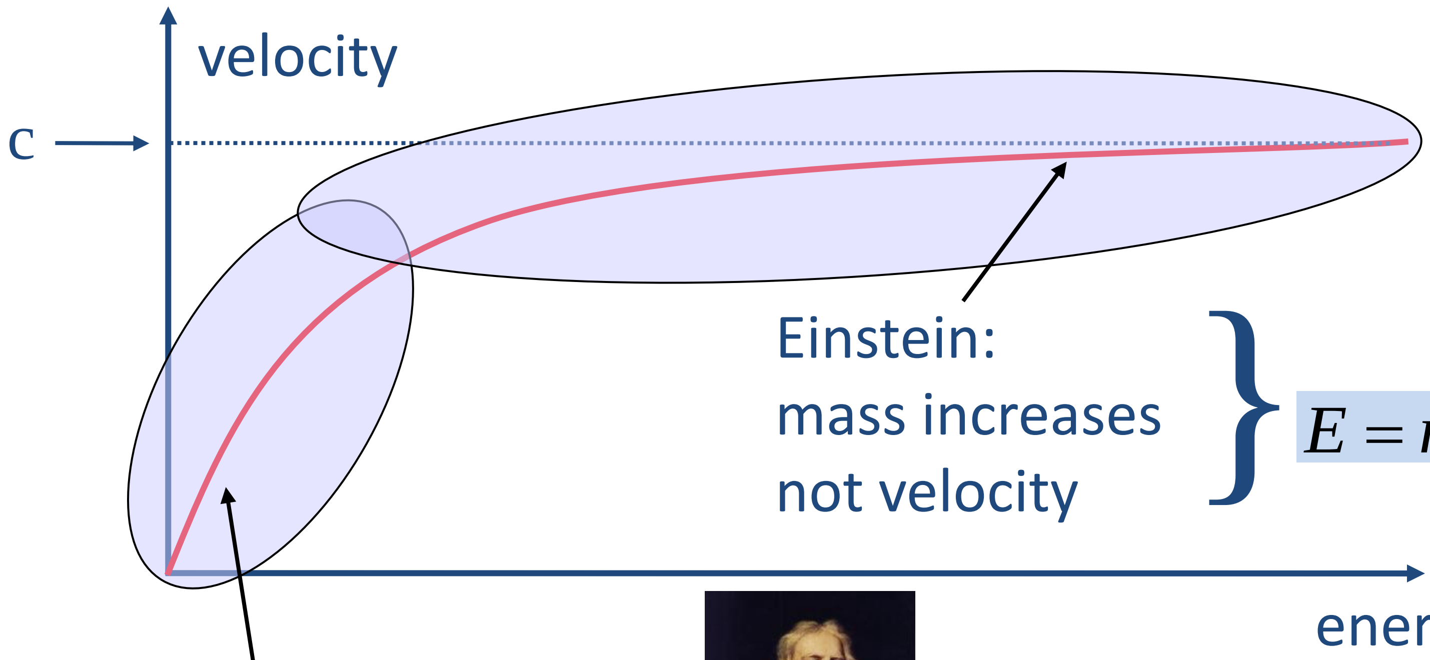
Accelerator = series of elements for **beam guiding** (bending, focusing) and **acceleration of particles**

- **guiding fields** must ensure stability of circulating particles on designed trajectory
- often arranged in a **closed loop** (ring) → acceleration occurs at every turn
- or in a periodic **"straight" sequence** (linacs) → acceleration all along the length



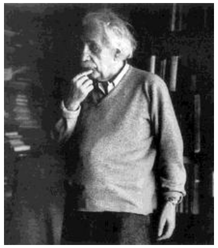


Accelerating particles → Towards Relativity



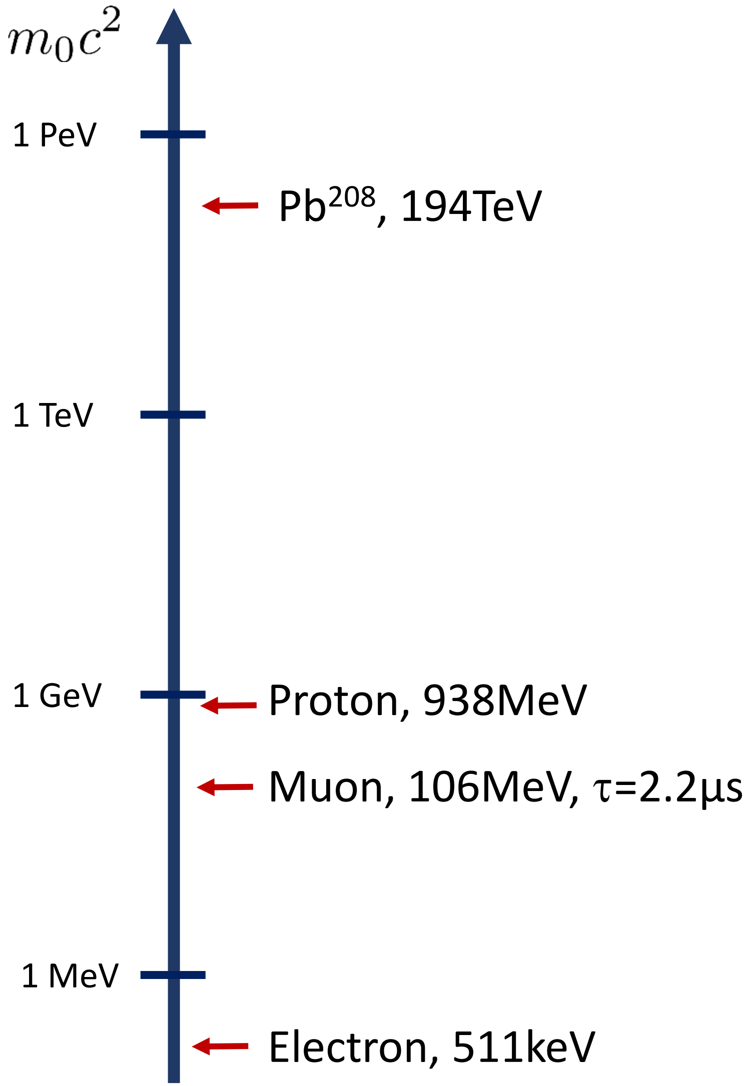
Einstein:
mass increases
not velocity

$$E = mc^2$$



Newton: $E = \frac{1}{2}mv^2$

Particles to Accelerate



Wide range of rest masses from electron to heavy ions

The accelerators differ vastly, e.g.

- particle speed in cavities
- synchrotron radiation power
- activation by losses
- requirements for vacuum

Accelerator design depends on particle type and properties Energy

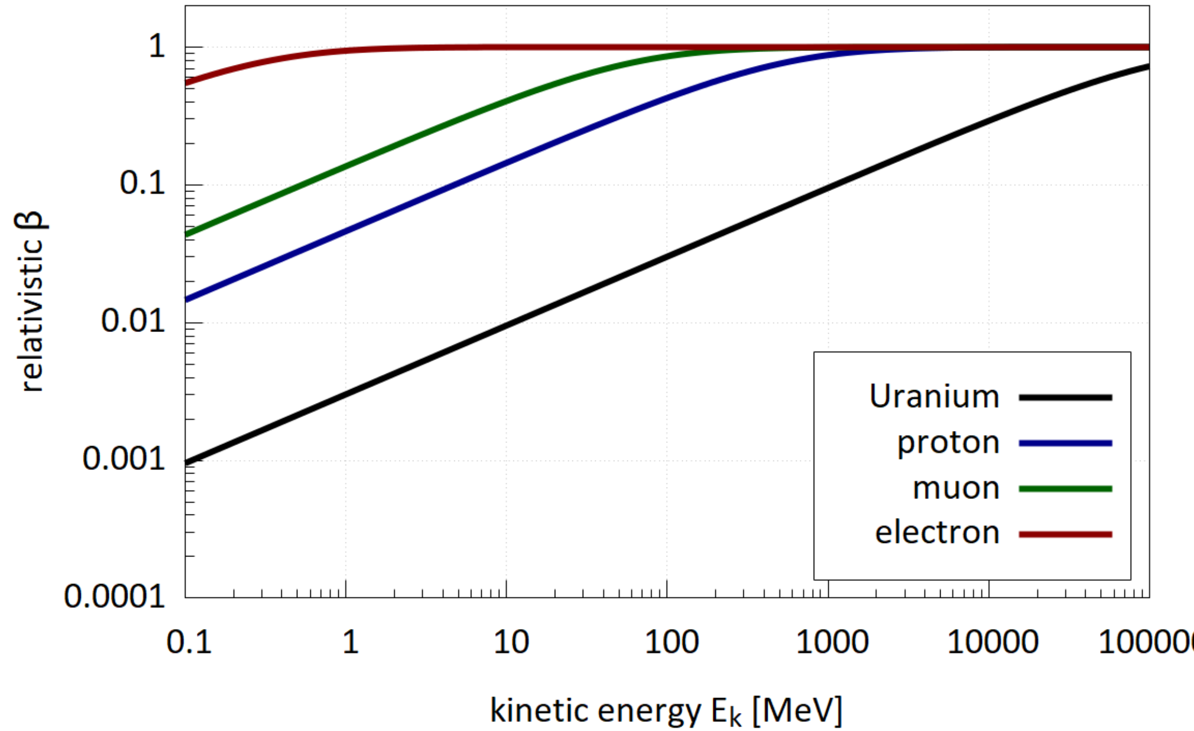
Speed of different particles vs energy

relativistic energy-momentum relation:

$$E = \sqrt{m_0^2c^4 + c^2p^2}$$
$$= m_0c^2 + E_k$$

$$\gamma = \frac{E}{m_0c^2} = 1 + \frac{E_k}{m_0c^2}$$
$$\beta = \sqrt{1 - 1/\gamma^2}$$

Relativistic electrons at ~ MeV
Relativistic protons ~ GeV



E_k [MeV]	γ	β	p [MeV/c]
590	1.63	0.79	1207

numerical example for protons
LHC injection energy 450 GeV ultra relativistic beam β~1

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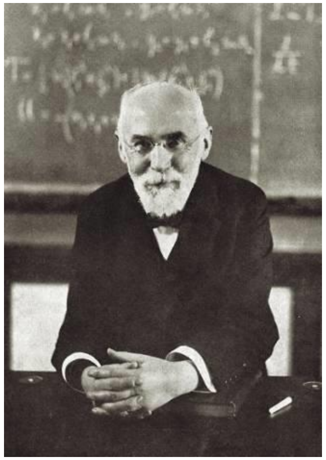
Guiding charged particles: Lorentz Force

$$\vec{F} = e\vec{E} + e\vec{v} \times \vec{B}$$ (charge = e)



H.A.Lorentz
1853-1928

electric field

energy gain: $\Delta E_k = eU$

Longitudinal Motion

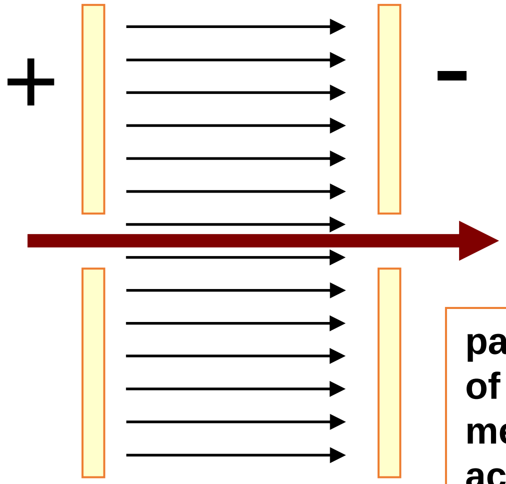
Parallel to the direction of motion.
Used to accelerate charged particles.

magnetic field

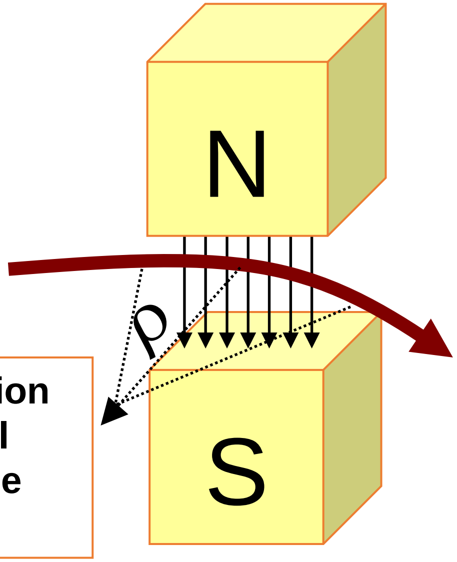
bending: $B\rho = p/e, \ \Delta E_k = 0$

Transverse Motion

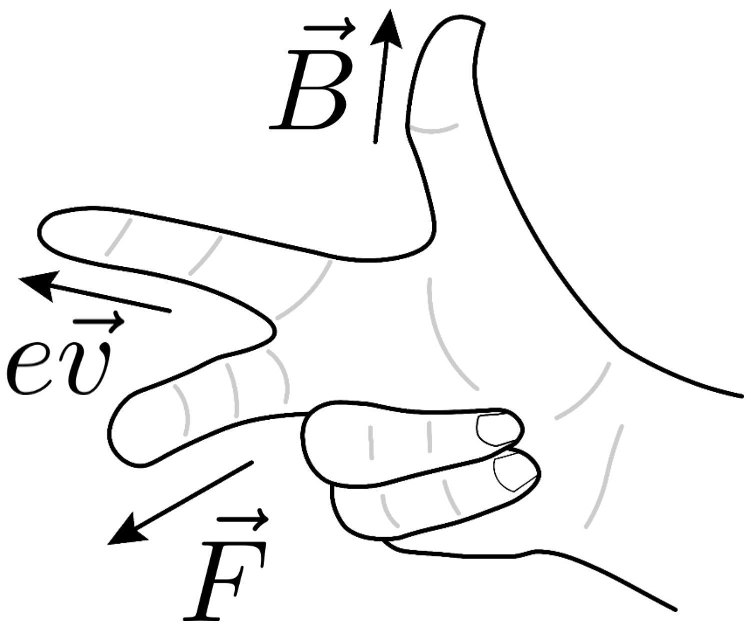
Perpendicular to the direction of motion.
Used to keep circulating orbit and beam steering.



path bending and acceleration of particles are fundamental mechanisms of each particle accelerator

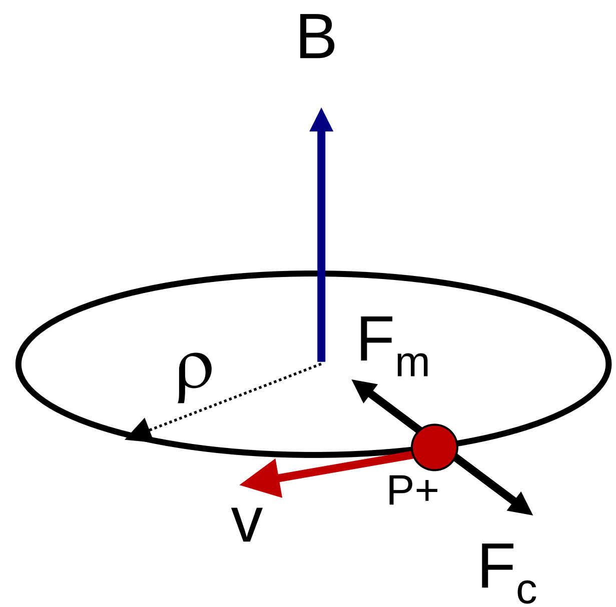


Lorentz Force – getting it right



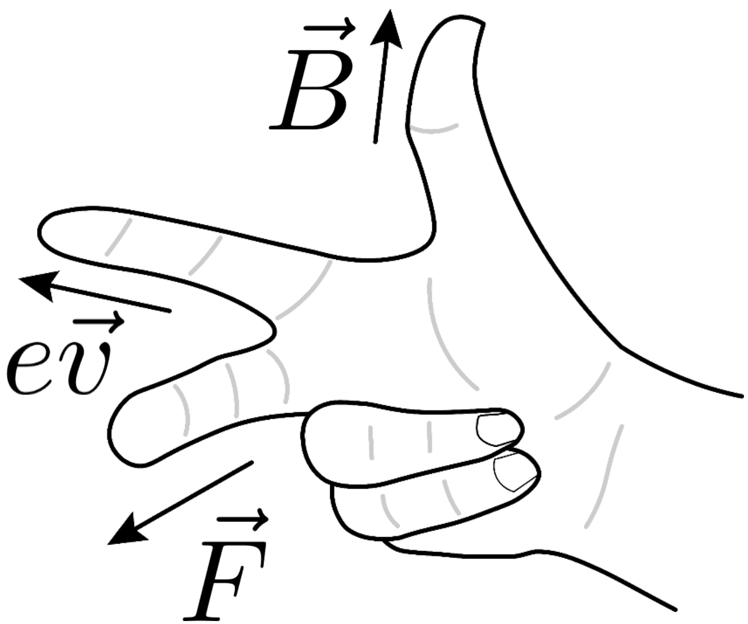
$$B\rho[\mathrm{Tm}] \quad = \quad 3.3356 \cdot \beta E_{\mathrm{tot}}[\mathrm{GeV}]$$

[see appendix for derivation]



Tevatron p-pbar versus LHC pp collider…. Why?

Lorentz Force – getting it right



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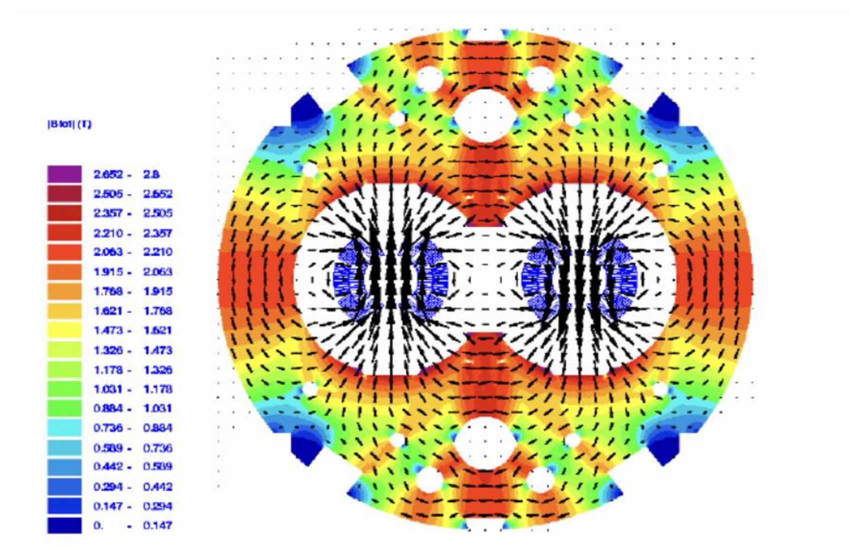


Tevatron p-pbar collider → same B field→difficult to have pbar beams
LHC p-p collider → opposite B field → complex magnet design so called 2 in 1

Comparison E and B field

Bending radius for protons in B and E:

example: electric and magnetic force on protons

$$\vec{F_E} = e \cdot \vec{E}, \quad \vec{F_B} = e \cdot \vec{v} \times \vec{B}$$

table: bending radius, varying E_k

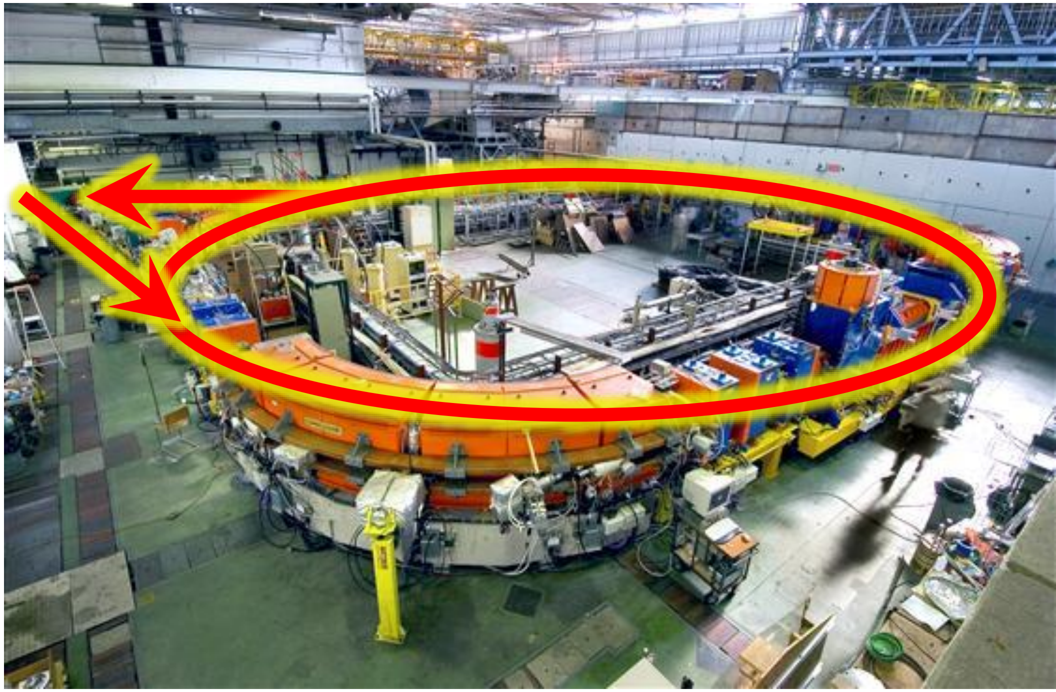
E_k	B = 1T	E = 10MV/m
60 keV	35 mm	12 mm
1 MeV	140 mm	200 mm
1 GeV	5.6 m	150 m

Magnetic fields are used exclusively to bend and focus ultra-relativistic particles

Accelerators in fundamental Particle Physics Research

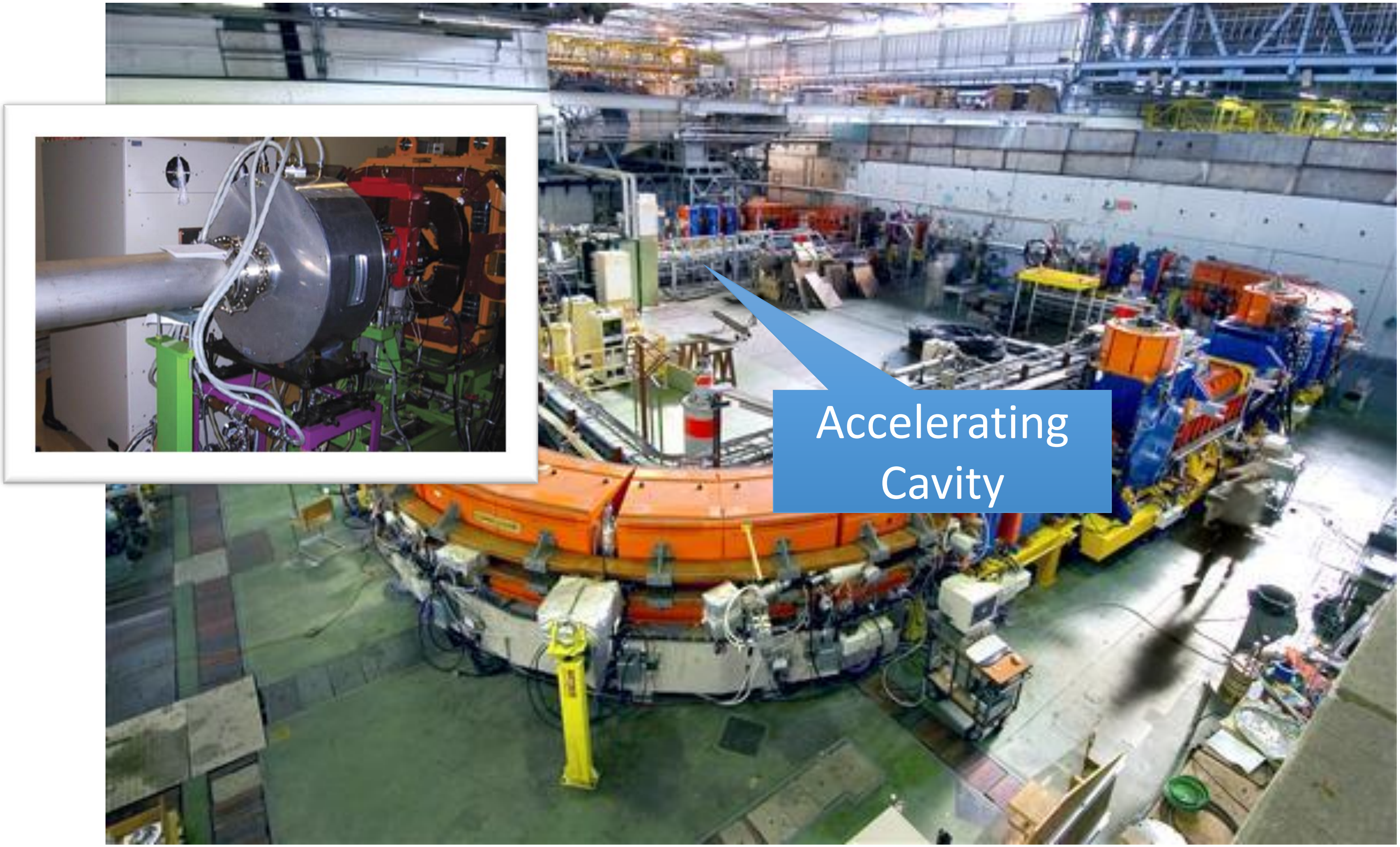
- High Energy → Acceleration
- High Luminosity → Guiding and focusing high intensity beams

LEIR
Low **E**nergy **I**on **R**ing



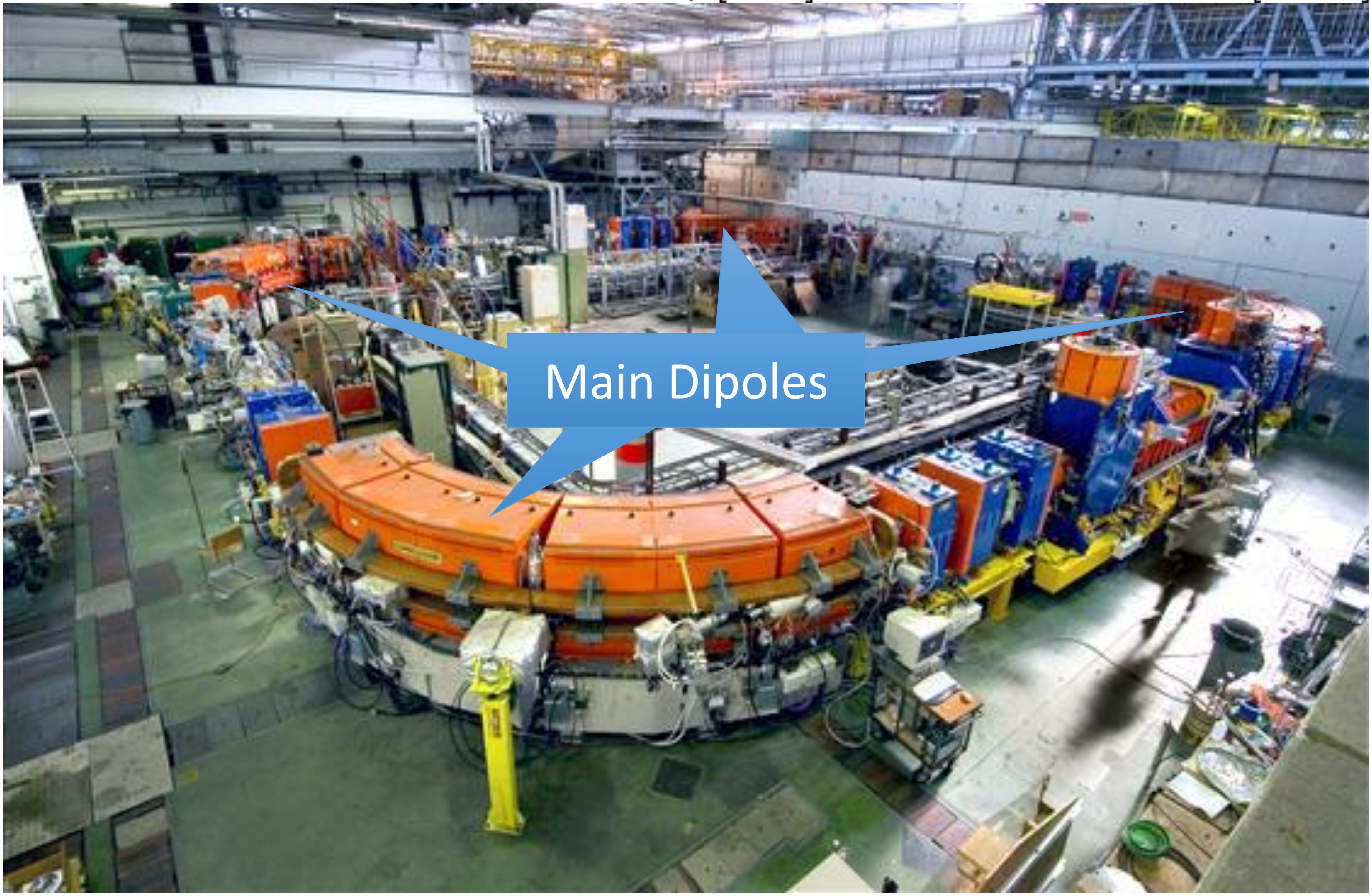
Accelerate Particles

$$\Delta E_k = eU$$



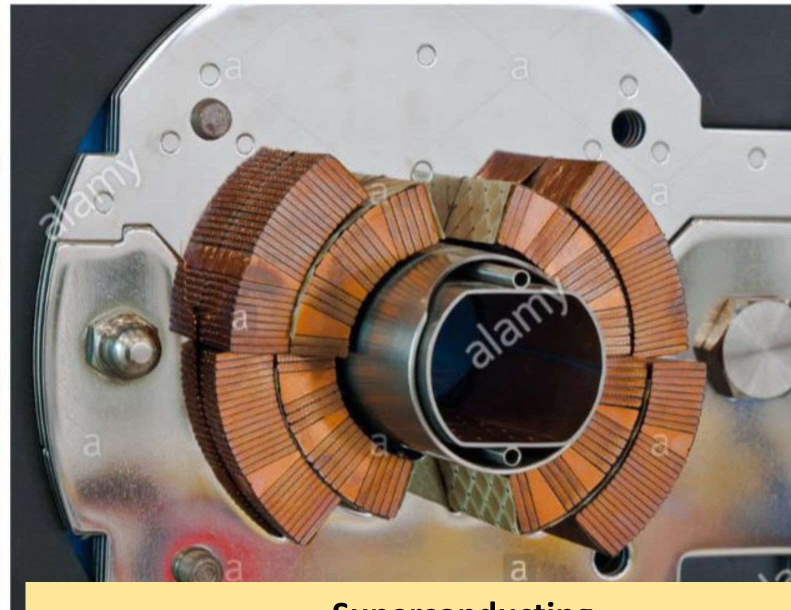
Make Particles Circulate

$$B\rho[\mathrm{Tm}] \quad = \quad 3.3356 \cdot \beta E_{\mathrm{tot}}[\mathrm{GeV}]$$



Bending Magnet and magnetic rigidity

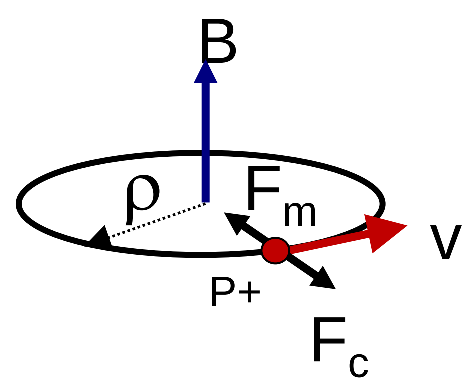
Iron dominated
Field defined by the geometry of poles
→ 2 flat poles



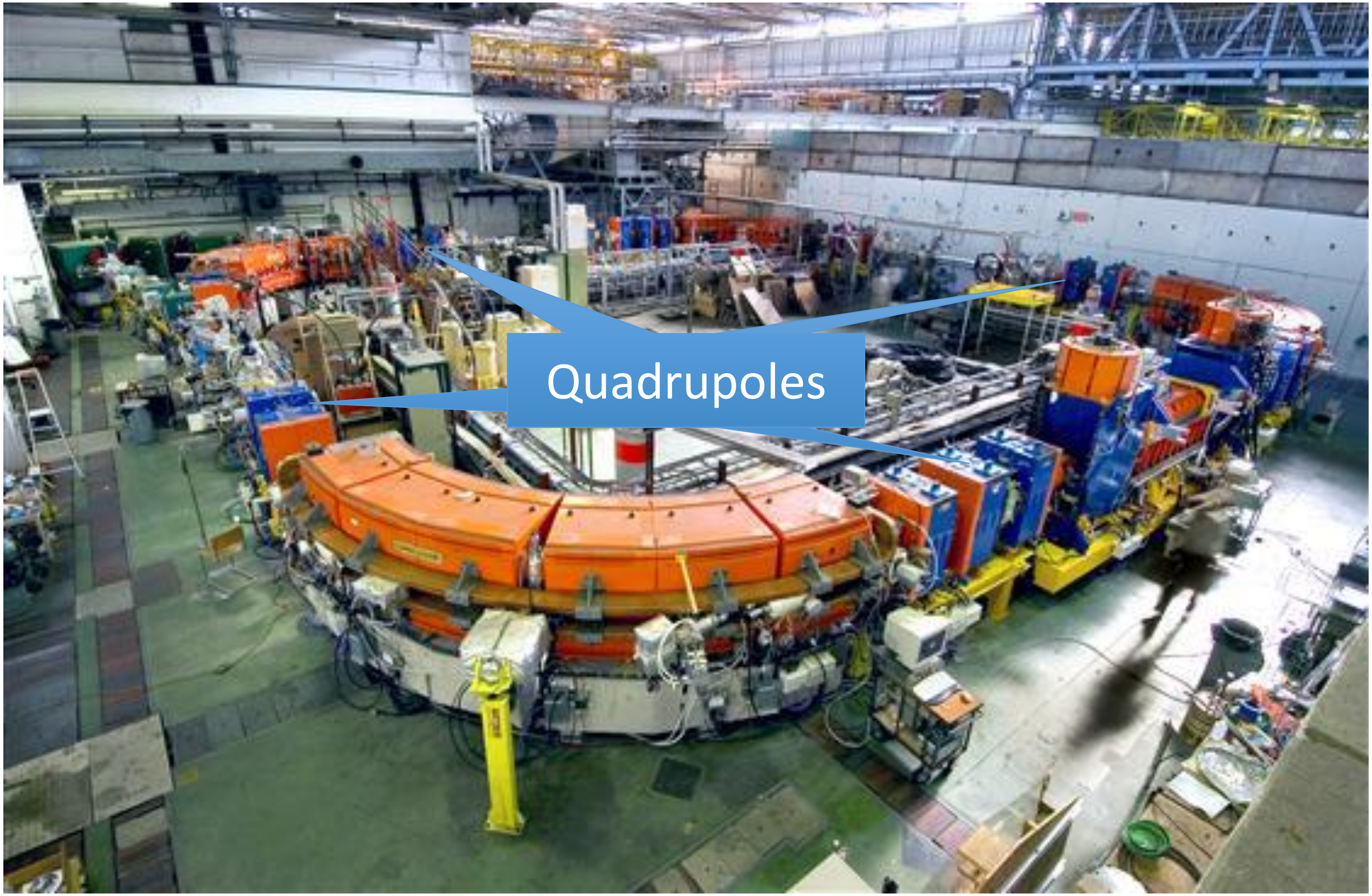
Superconducting
Field defined by the geometry of coils
→ Current distribution Cosϕ

Magnetic rigidity: $$B\rho = \frac{p}{e}$$

- accelerate beams → increase B
- at fixed B: higher p → increase bending angle…

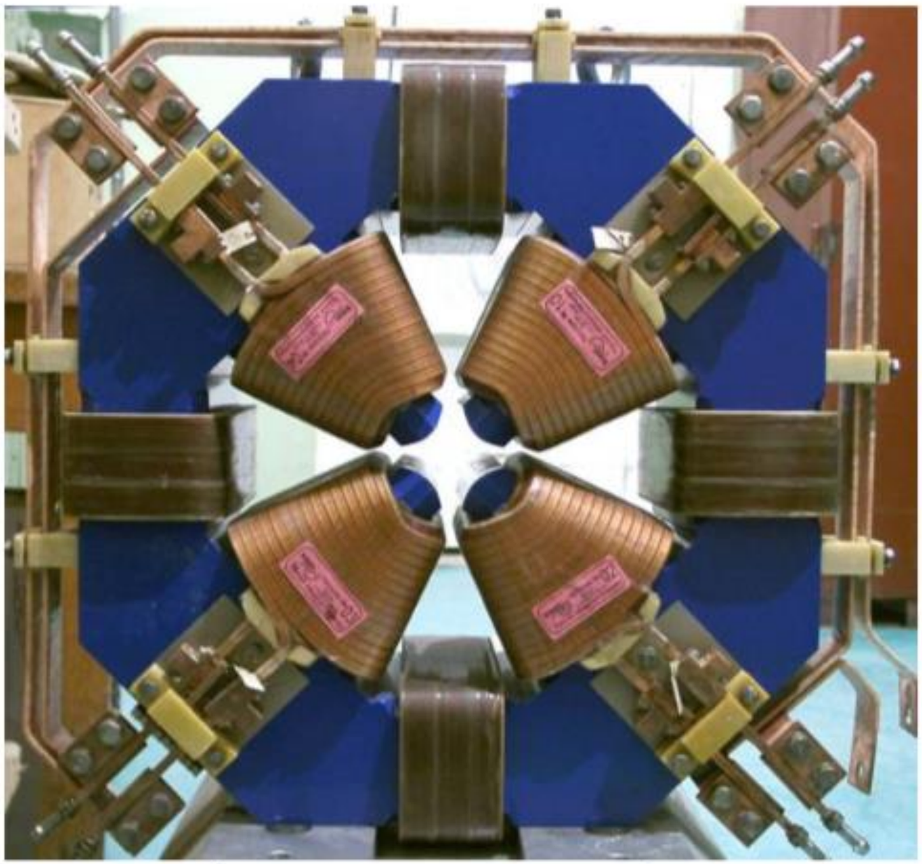


Focusing the Particles

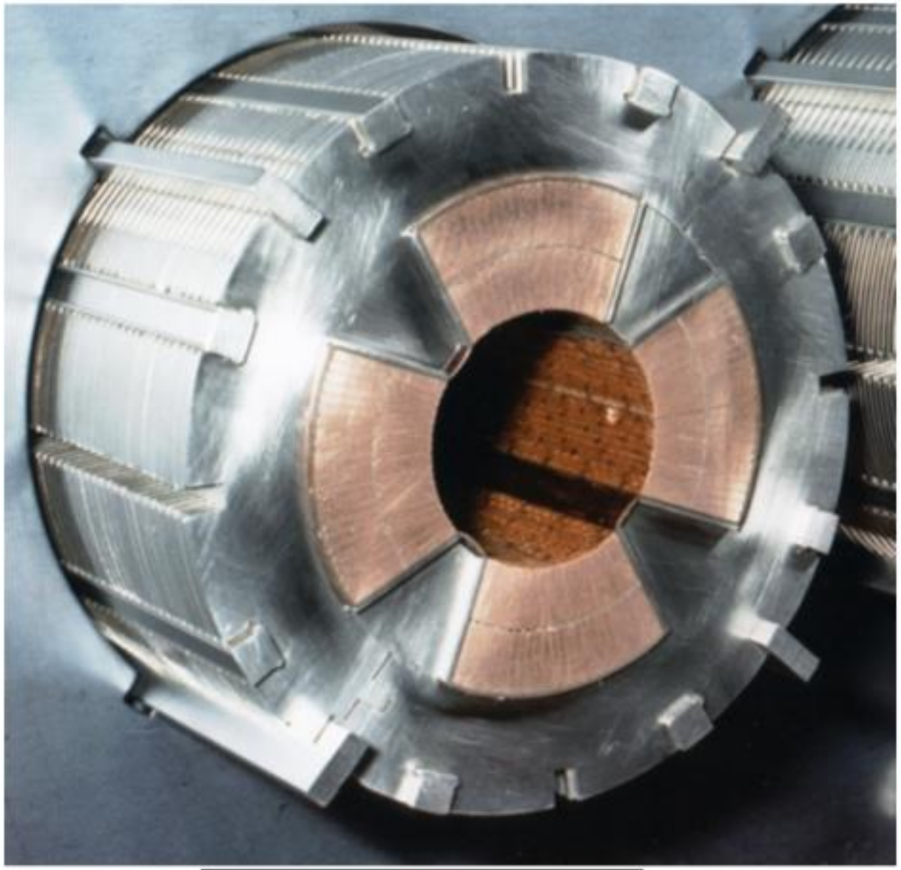


Quadrupole Magnet - Focusing Element

Quadrupole magnets:



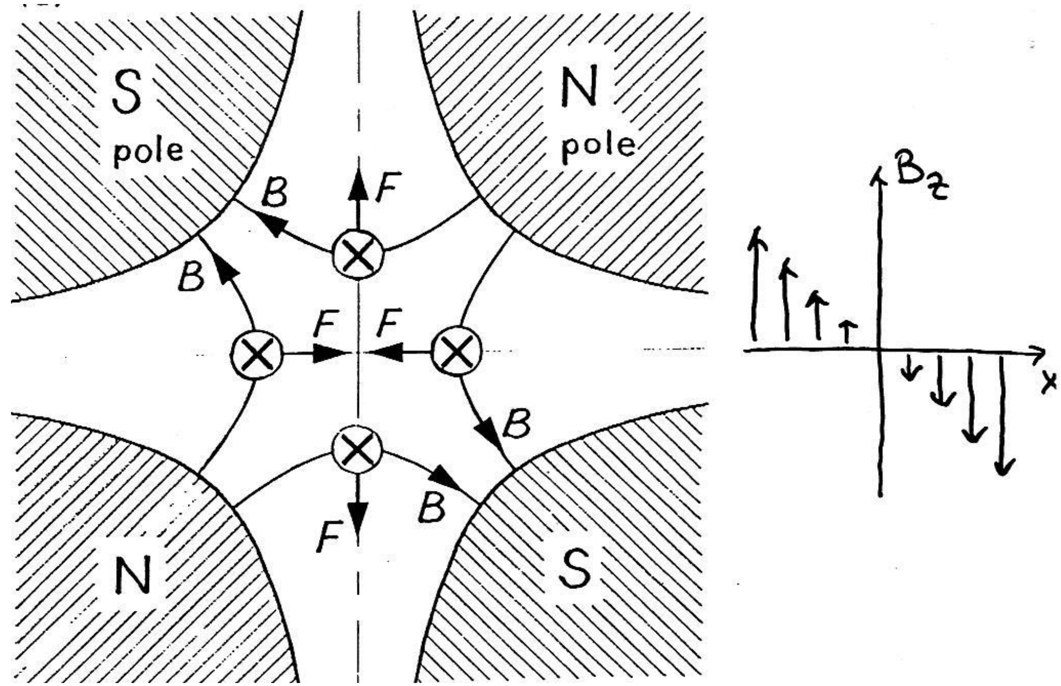
Iron dominated:
field determined by
geometry of poles
→ 4 hyperbolic poles



Superconducting:
field determined by
geometry of coils
→ $j(\phi) \sim \cos 2\phi$

Quadrupole magnets

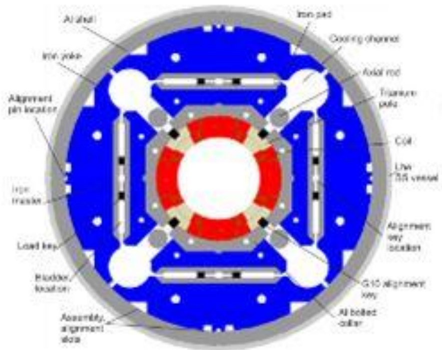
- Focusing in one plane
- Defocusing in the other plane



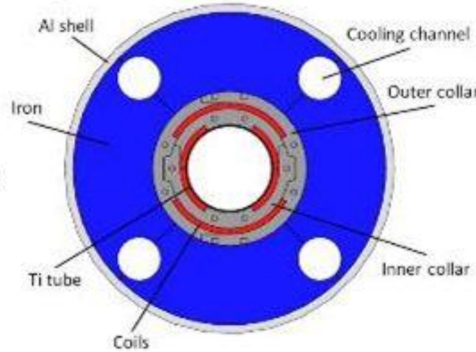
$$\nabla \times \boldsymbol{B} = 0 \rightarrow \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

Gradient g

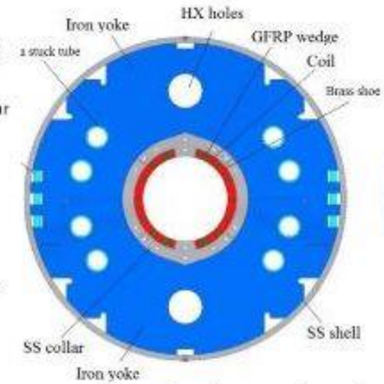
HiLumi LHC magnet zoo



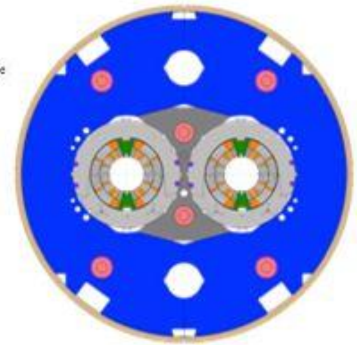
Triplet QXF (LARP and CERN)



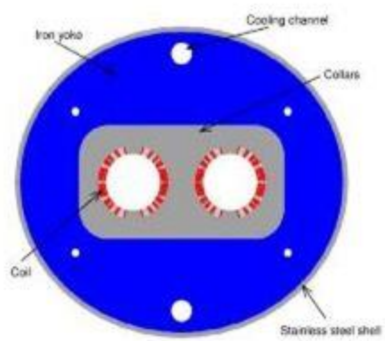
Orbit corrector (CIEMAT)



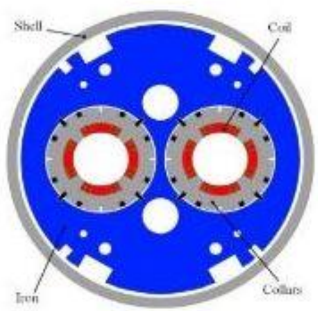
Separation dipole D1 (KEK)



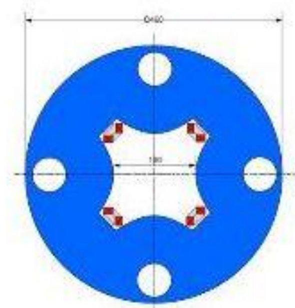
11 T dipole (CERN)



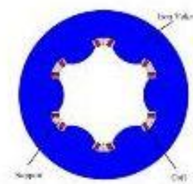
Recombination dipole D2 (INFN design)



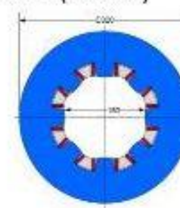
Q4 (CEA)



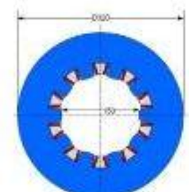
Skew quadrupole (INFN)



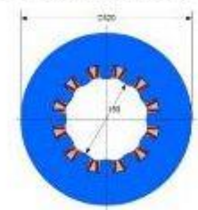
Sextupole (INFN)



Octupole (INFN)



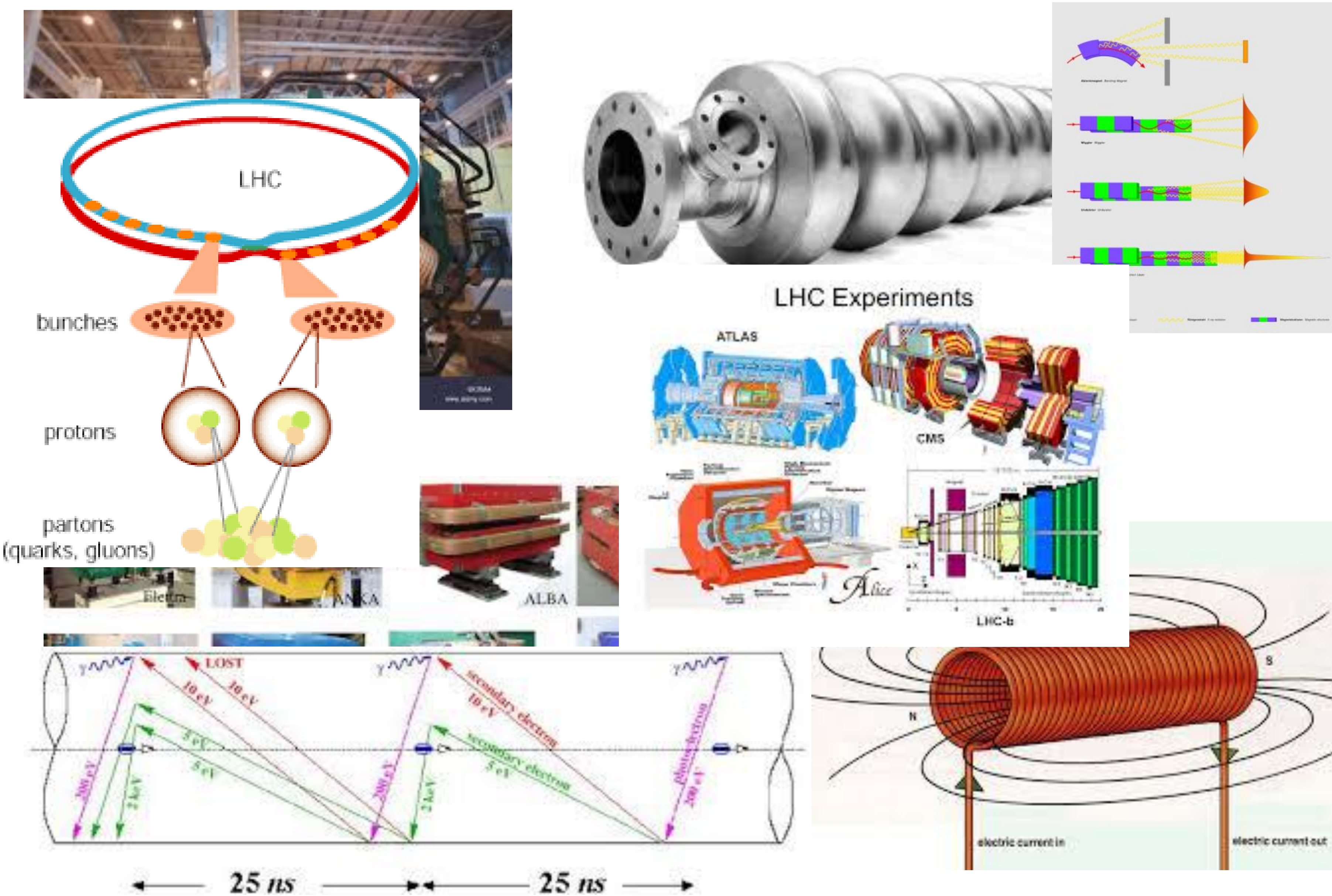
Decapole (INFN)



Dodecapole (INFN)

Overall, about 150 magnets are needed

Accelerator elements



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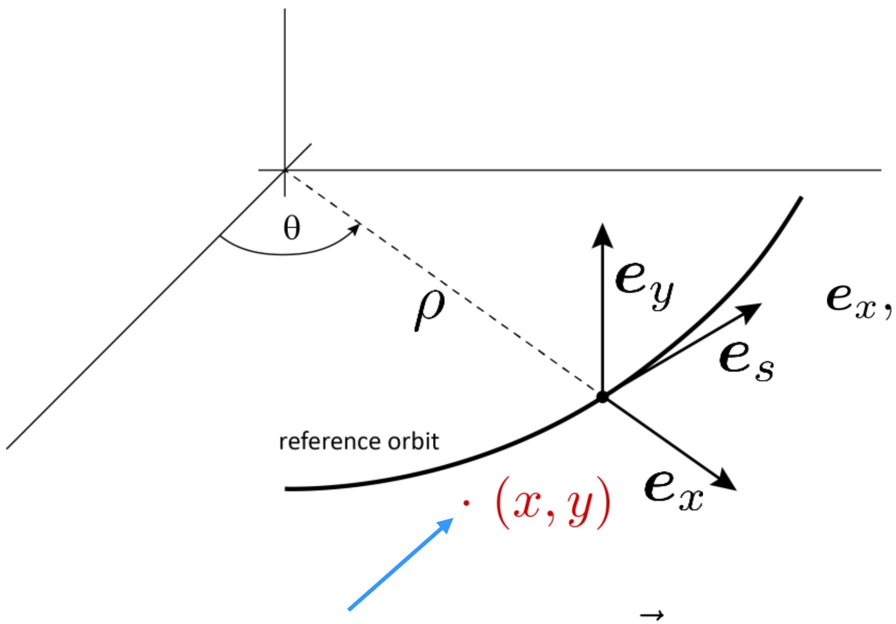
Curvelinear Coordinate System

aim: derive a set of equations that describe the motion of a single particle wrt. a curved coordinate system around the reference orbit of a beam, (x, y)



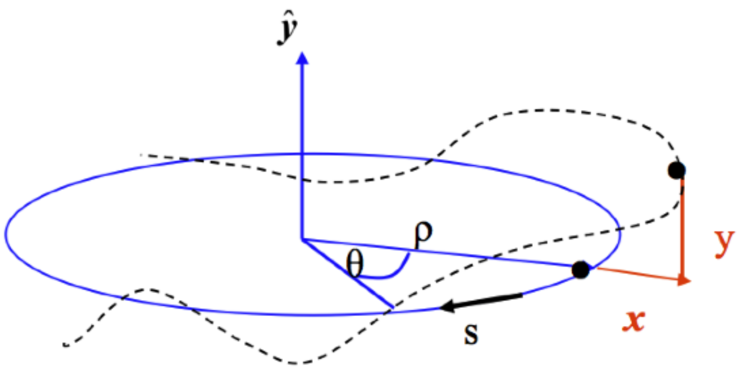
$\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_s$ unit vectors

particle coordinate: $\vec{R} = r\boldsymbol{e}_x + y\boldsymbol{e}_y, \;\; r \equiv \rho + x$

$$x, y \ll \rho$$

see also: Frenet-Serret coordinates, e.g. Wiedemann chap 4.3

Deriving the Equation of Motion in x-plane (see Appendix)



Frenet-Serret coordinate system

the effect of the curved coordinate system, i.e. the moving unit vectors e_x, e_s must be *included in the calculation*

starting with general equation of motion: $$\frac{d\vec{p}}{dt} = \gamma m_0 \ddot{\vec{R}} = \vec{F}$$

$$B_y = B_0 + gx, \quad B_x = gy$$ dipole and quadrupole field

$$\frac{1}{\rho} = \frac{eB_0}{\gamma m_0 v}$$ orbit curvature

$$g \equiv \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$ Quadrupole field gradient sign convention!

$$k = \frac{eg}{\gamma m_0 v}$$ k - value

$$x'' + (\frac{1}{\rho^2} + k)\ x = 0$$

k - value

curved coordinates

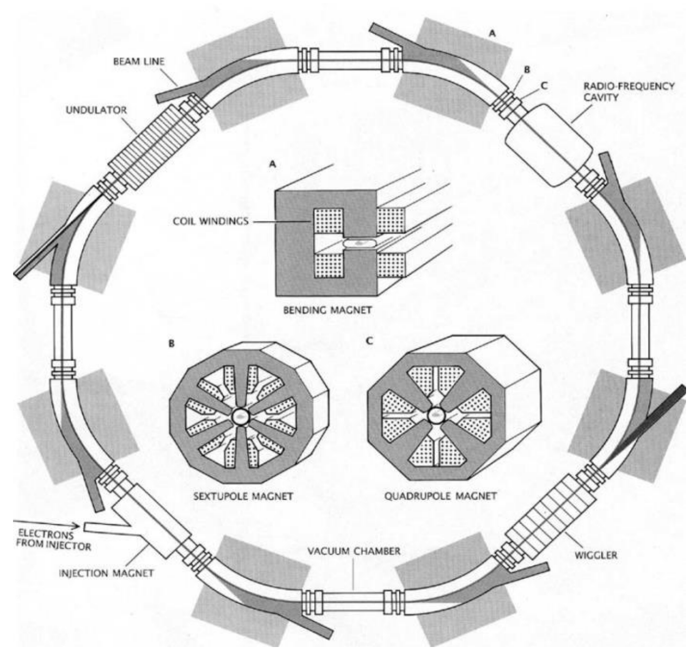
off momentum term

derivative w.r.t. path-length *s*, not time *t*

Equation of Motion in x and y planes for designed momentum:

$$x'' + (\frac{1}{\rho^2} + k)\; x = 0$$

$$y'' - ky = 0$$



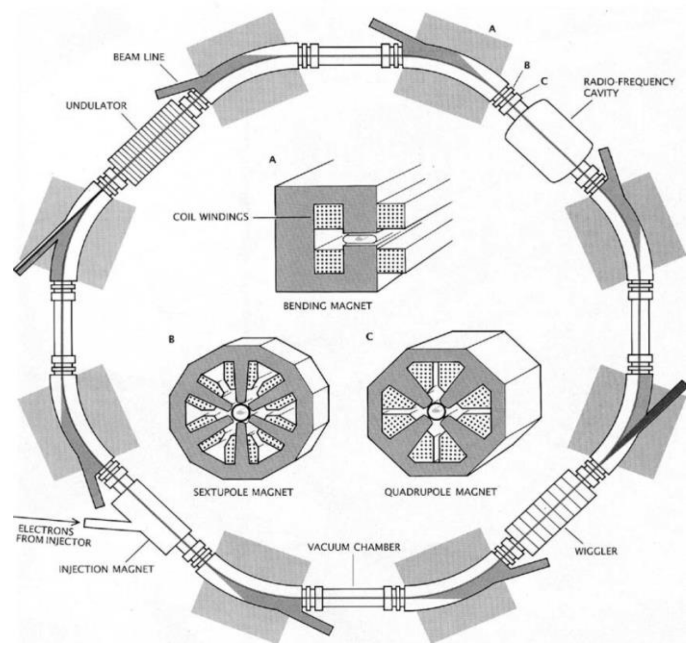
Equation of Motion in x and y planes for designed momentum: generalized form

$$x'' + (\frac{1}{\rho^2} + k)\ x = 0$$

$$y'' - ky = 0$$

generalised form:

$$x'' + K_x(s)x = 0$$
$$y'' - K_y(s)y = 0$$



*see also Wiedemann sec. 1.5.8

Equation of Motion in x and y planes for designed momentum: generalized form

$$x'' + (\frac{1}{\rho^2} + k)\ x = 0$$

$$y'' - ky = 0$$

generalised form:

$$x'' + K_x(s)x = 0$$
$$y'' - K_y(s)y = 0$$

Differential Equation valid for:

- drift spaces
- Quadrupoles ($k \neq 0$)
- combined function magnets ($k \neq 0$, $1/\rho \neq 0$)
- on-momentum particles ($\Delta p = 0$)

we discuss solutions of different cases of this equations in single accelerator magnets, depending on K(s) and ρ(s)

*see also Wiedemann sec. 1.5.8

Equation of Motion in x and y planes for designed momentum: off momentum particles

$$
\begin{aligned}
x'' + \left(\frac{1}{\rho^2} + k\right) x &= \frac{1}{\rho}\frac{\Delta p}{p_0} \\
y'' - ky &= 0
\end{aligned}
$$

generalised form:

$$
x'' + K_x(s) x = \frac{1}{\rho(s)}\frac{\Delta p}{p_0}
$$

$$
y'' - K_y(s) y = 0
$$

Differential Equation valid for:

- drift spaces
- Quadrupoles ($k \neq 0$)
- combined function magnets ($k \neq 0$, $1/\rho \neq 0$)
- on-momentum particles ($\Delta p \neq 0$, first order)

we discuss solutions of different cases of this equations in single accelerator magnets, depending on K(s), ρ(s) and **Δp**

*see also Wiedemann sec. 1.5.8

Summary on Approximations used

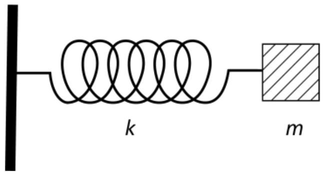
- small displacements $x \ll \rho, y \ll \rho, \ddot{s} \approx 0$ **(paraxial optics)**
- only dipole and quadrupole magnets **(linear field changes)**
- design orbit lies in a plane, horizontal **(flat accelerator)**
- no coupling between motion in hor. and vert. plane **(upright magnets)**
- small momentum deviations $\Delta p/p_0 \sim 10^{-4}$ **(quasi monochromatic beam)**
- **in general: no quadratic or higher order terms (linear beam optics)**

Next Step: Solving the Equation of Motion

$$x'' + K_x(s)x = 0$$
$$y'' - K_y(s)y = 0$$

Piecewise Solution of Equation

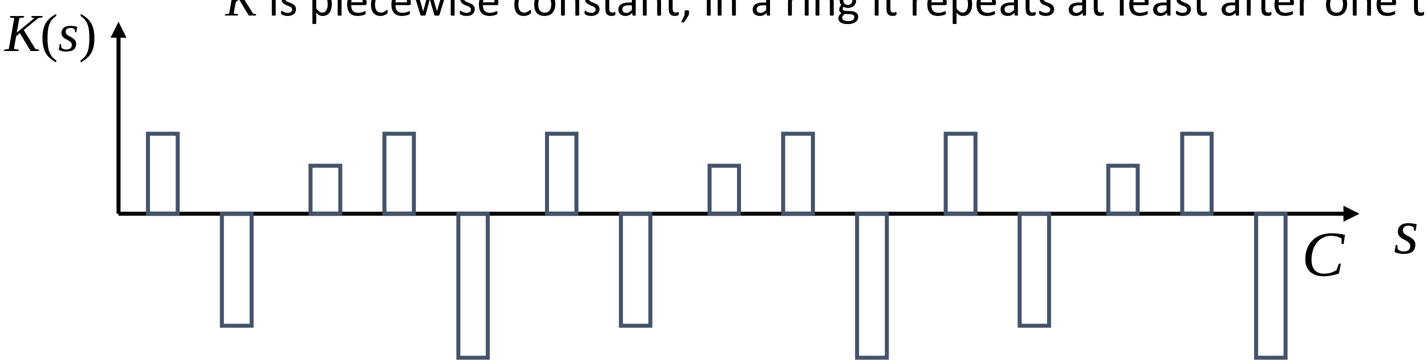
$$\boxed{x'' + K(s)x \;=\; 0}$$



For ON MOMENTA particles →
general form of equation similar to harmonic oscillator with three cases: K=0, K<0, K>0

$$m\ddot{x} + kx = 0,\;\; \omega = \sqrt{\frac{k}{m}}$$

K is piecewise constant, in a ring it repeats at least after one turn (s=C)



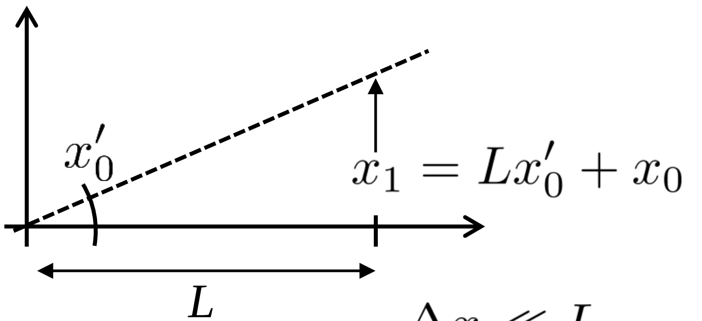
Drift Space

On momentum particles (Δp = 0) moves straight

$$\boxed{x'' + K(s)x \quad = \quad 0}$$

1) K=0 → Drift Space

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$



$\Delta x \ll L$

Focusing Quadrupole

On momentum particles (Δp = 0)

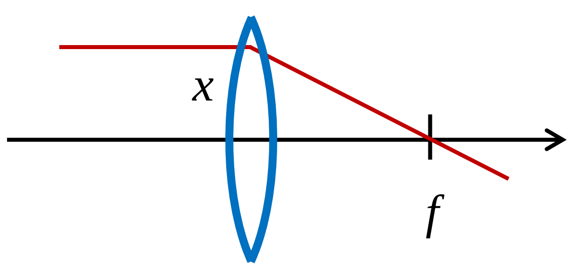
$$\boxed{x'' + K(s)x \quad = \quad 0}$$

2) K>0: Focusing Quadrupole

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\mathrm{out}} = \begin{pmatrix} \cos(\sqrt{K}L) & \sin(\sqrt{K}L)/\sqrt{K} \\ -\sin(\sqrt{K}L)\sqrt{K} & \cos(\sqrt{K}L) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\mathrm{in}}$$

thin lens approximation: $$K = \frac{1}{Lf}, \quad \lim_{L\to 0}\left(\sin\left(\sqrt{L/f}\right)\frac{1}{\sqrt{Lf}}\right) = \frac{1}{f}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\mathrm{out}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\mathrm{in}}$$



Defocusing Quadrupole

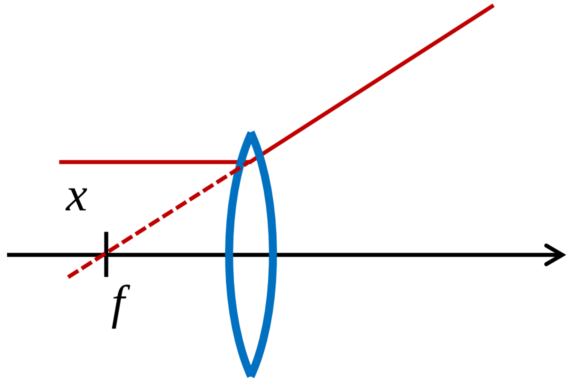
3) K<0: Defocusing Quadrupole

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} \cosh(\sqrt{|K|}L) & \sinh(\sqrt{|K|}L)/\sqrt{|K|} \\ \sinh(\sqrt{|K|}L)\sqrt{|K|} & \cosh(\sqrt{|K|}L) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$

thin lens approximation: $$K = \frac{1}{Lf}, \quad \lim_{L\to 0}\left(\sin\left(\sqrt{L/f}\right)\frac{1}{\sqrt{Lf}}\right) = \frac{1}{f}$$

thin lens approximation for defocusing quad:

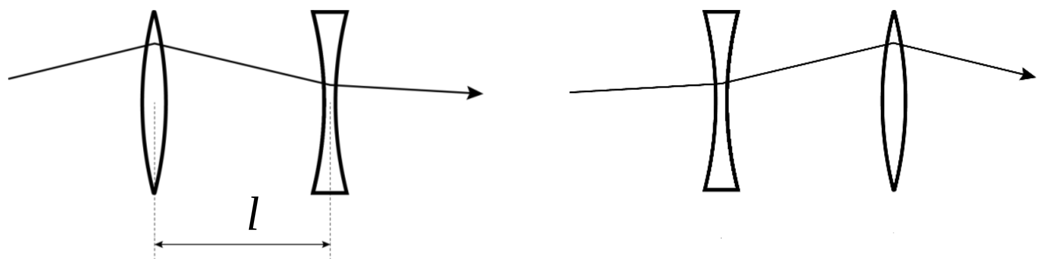
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}}$$



Alternating gradient sequence → **net focusing effect!**

concatenation of particle transport through a series of elements:

$$\boldsymbol{M} = \boldsymbol{M}_n \ldots \boldsymbol{M}_2 \cdot \boldsymbol{M}_1$$ (***M*** = transport matrix 2x2)



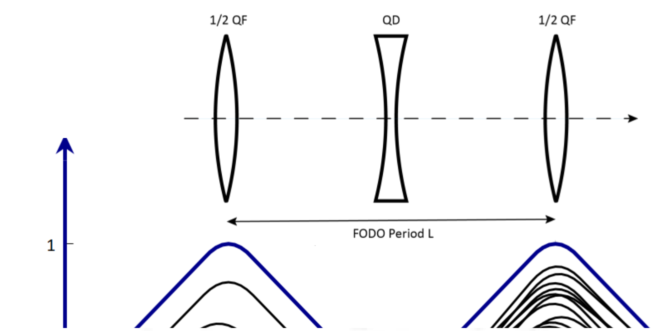
$$M_{doublet} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + \frac{l}{f} & l \\ -\frac{1}{f^*} & 1 - \frac{l}{f} \end{pmatrix}$$

$$f^* = \frac{f^2}{l} > 0$$ $\rightarrow \mathrm{M}_{\mathrm{doublet}}$ is always focusing

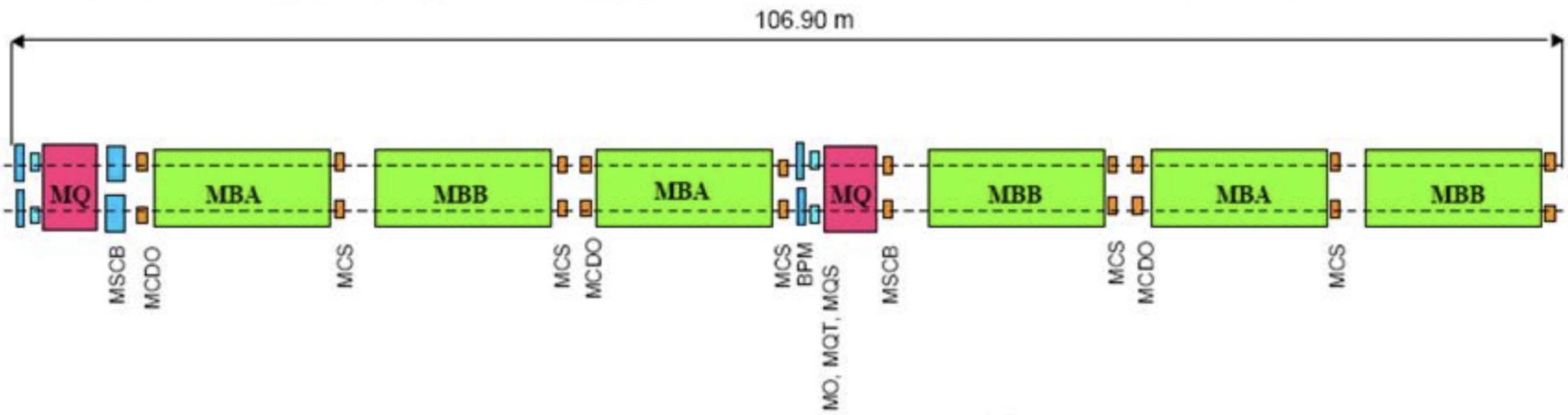
FODO Cell

$$\boldsymbol{M}_{\mathrm{FODO}} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} 1 - \frac{L^2}{8f^2} & L\left(1 + \frac{L}{4f}\right) \\ -\frac{1}{f^*} & 1 - \frac{L^2}{8f^2} \end{pmatrix}, \quad \frac{1}{f^*} = \frac{L}{4f^2}\left(1 - \frac{L}{4f}\right)$$



Unit sequence of magnets used to build an accelerator

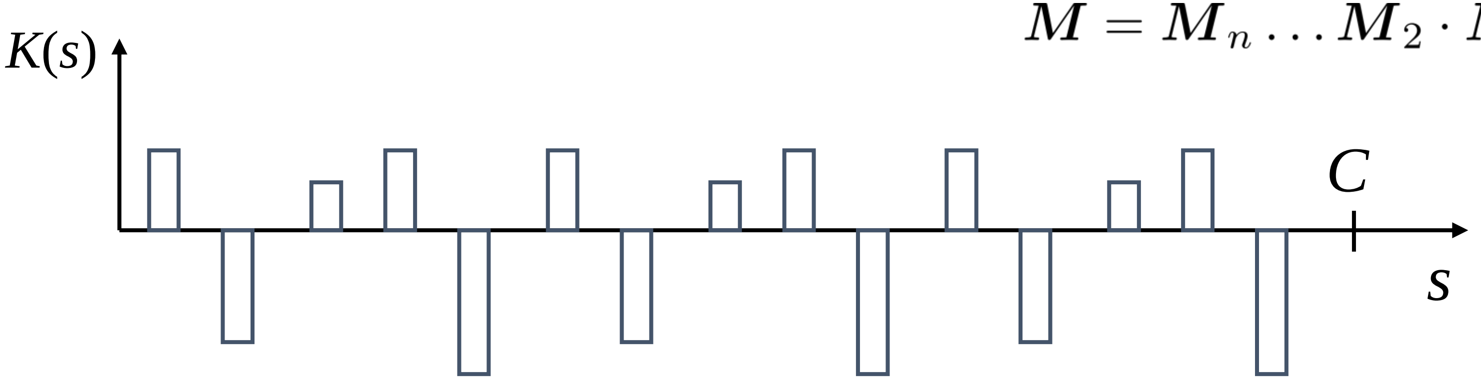
Alternating gradients → net focusing!



Summary Matrix Treatment

- equation of motion is piecewise solved for constant K(s)
- coordinates x, x' are transported by multiplication with a 2x2 matrix
- matrixes can be concatenated → particle transport over many turns
- defocusing and focusing quadrupoles are combined in overall focusing doublets
- linear motion in a ring is stable over n turns if stability conditions are fulfilled ($|\mathrm{Tr}\,\mathbf{M}|<2$)

$$\boldsymbol{M} = \boldsymbol{M}_n \dots \boldsymbol{M}_2 \cdot \boldsymbol{M}_1$$

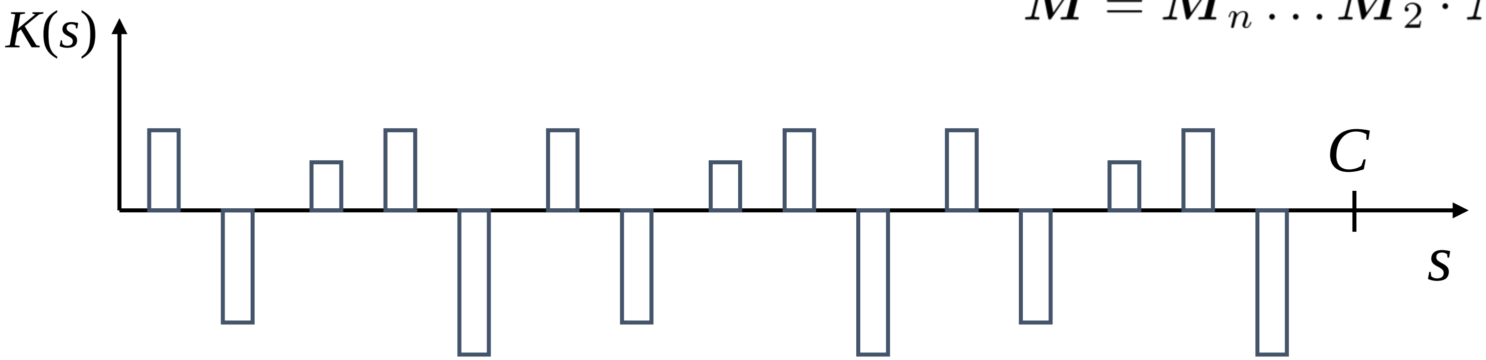


The two dialects of Accelerator Physics

Summary Matrix Treatment

- equation of motion is piecewise solved for constant K(s)
- coordinates x, x' are transported by multiplication with a 2x2 matrix
- matrixes can be concatenated → particle transport over many turns
- defocusing and focusing quadrupoles are combined in overall focusing doublets
- linear motion in a ring is stable over n turns if stability conditions are fulfilled ($|\mathrm{Tr}\,\mathbf{M}|<2$)
- The motion can be parametrized (Courant-Schneider Parametrization) → introduce optical function β function

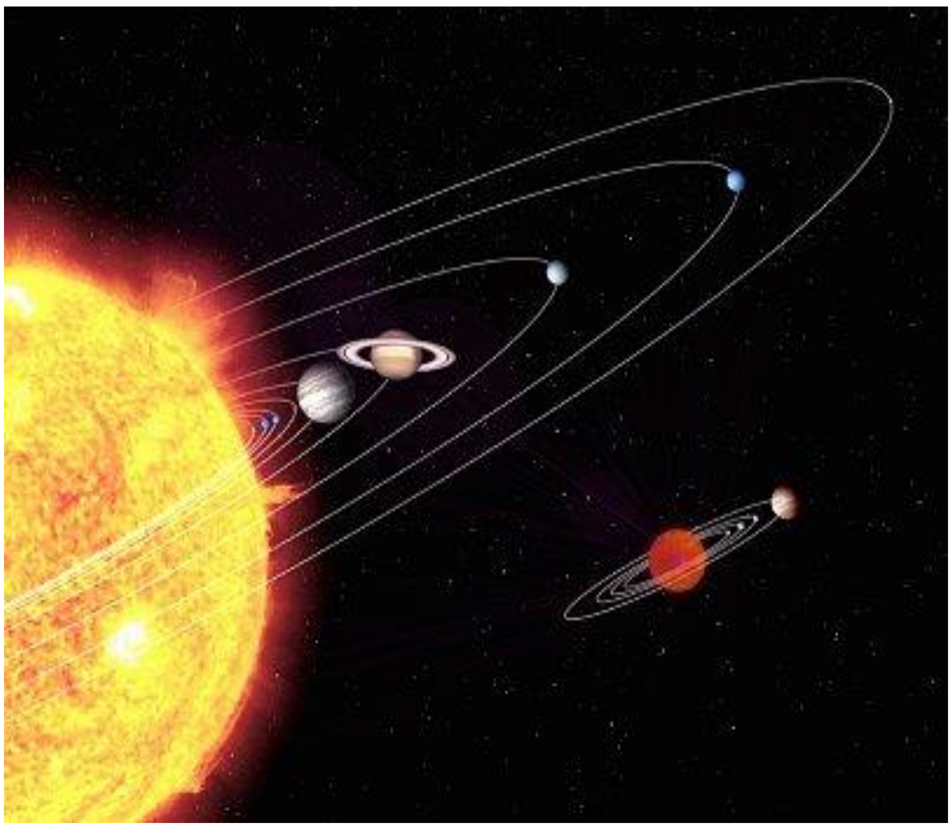
$$\boldsymbol{M} = \boldsymbol{M}_n \ldots \boldsymbol{M}_2 \cdot \boldsymbol{M}_1$$

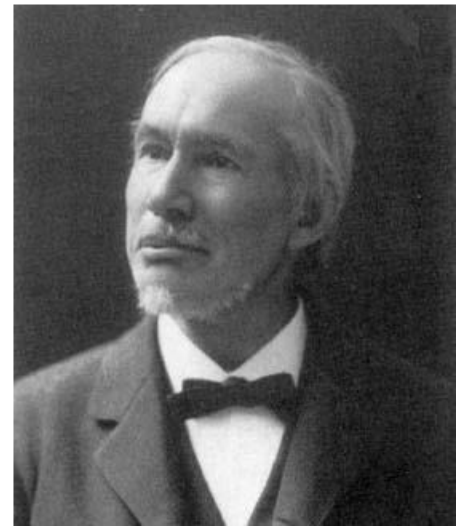


Hill equation

- First used by an astronomer G. Hill in his studies of the motion of the moon, a motion under the influence of periodically changing forces

$$x'' + K(s) \cdot x = 0$$





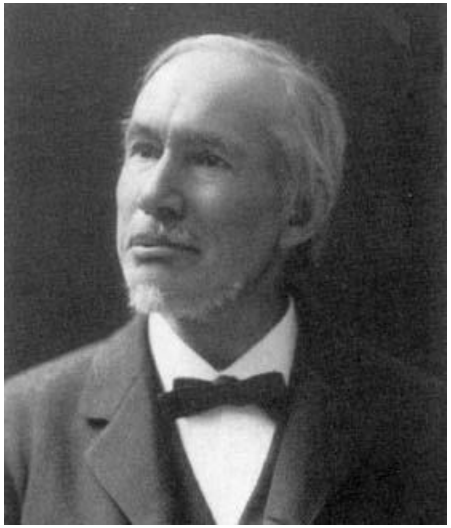
1838 -- 1914

$$K(s) = K(s + C)$$

Periodic over one full revolution
C = 29 days

Hill equation

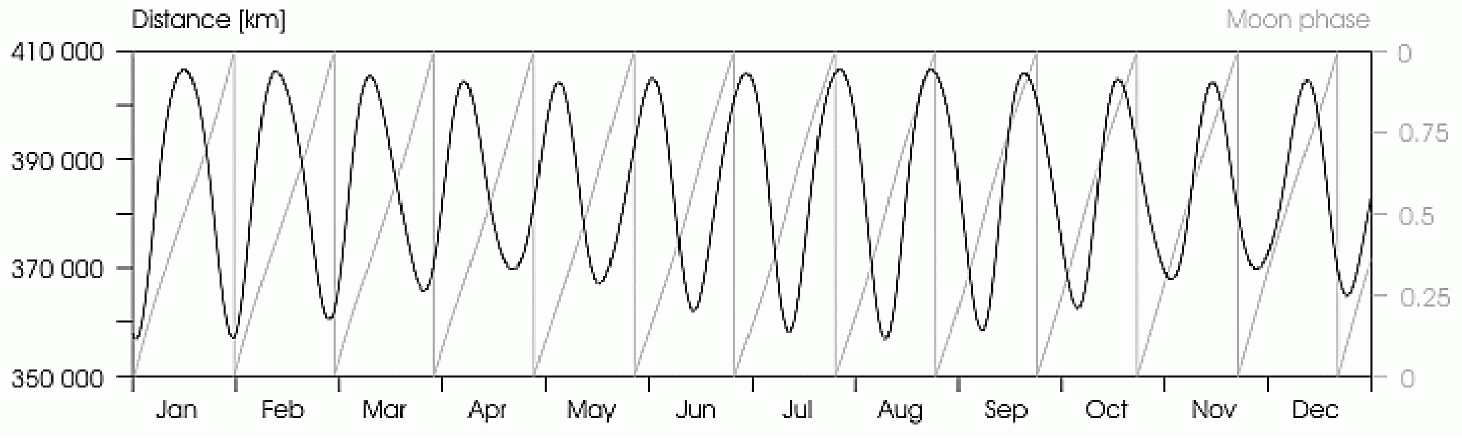
- First used by an astronomer G. Hill in his studies of the motion of the moon, a motion under the influence of periodically changing forces



Solution is of the type:

$$\boxed{u(s) = A\sqrt{\beta(s)}\cos\left[\phi(s)\right]}$$

Pseudo-harmonic oscillator



$$x'' + K(s) \cdot x = 0$$

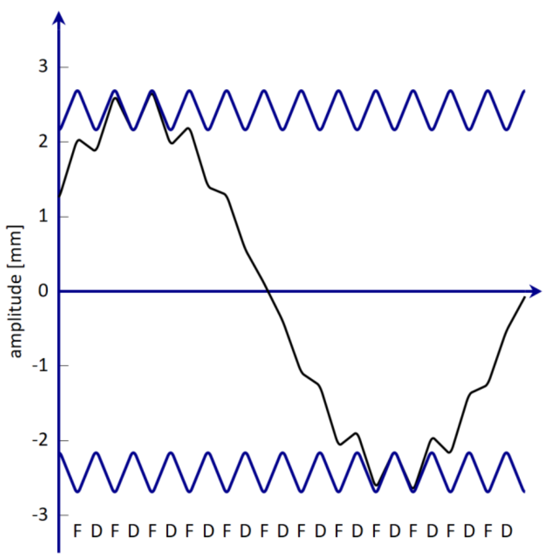
$$K(s) = K(s + C)$$

Hill: Solution for periodic *K*

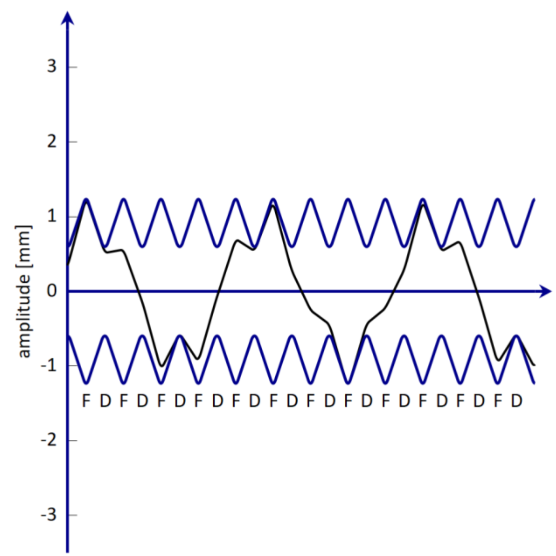
$$x(s) = A\sqrt{\beta(s)}\cos(\varphi(s) - \varphi_0),\ \ \varphi(s) = \int_{t=s_0}^{s} \frac{dt}{\beta(t)}$$

$\rightarrow$ the **beta function is a scaling factor** for the amplitude of orbit oscillations and their **local wavelength**

A, φ_0 are constants of motion



weak quads

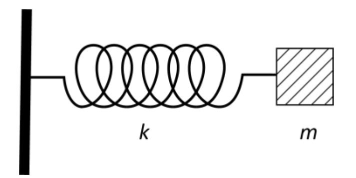


$$A\sqrt{\beta(s)}$$

strong quads

Comparison to Classical Harmonic Oscillator

$$\boxed{\begin{gathered}\ddot{u} + \omega^2 u = 0 \\ u(t) = A\cos\omega t,\ \ \omega = \sqrt{\frac{k}{m}}\end{gathered}}$$



amplitude is fixed: $A = \mathrm{const}$

phase grows linear with time: $\sqrt{\frac{k}{m}}t$

conserved (energy): $\frac{k}{2}u^2 + \frac{m}{2}\dot{u}^2 = \frac{k}{2}A^2$

Hill Equation (pseudo harmonic equation)

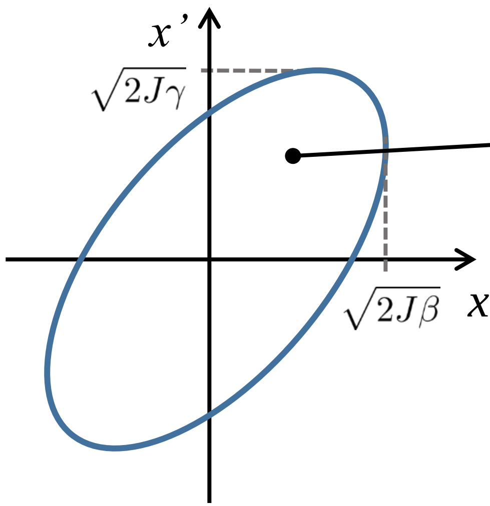
$$\boxed{\begin{aligned} x(s) &= \sqrt{2J\beta}\cos(\varphi) \\ x'(s) &= -\sqrt{\frac{2J}{\beta}}\left(\alpha\cos(\varphi) + \sin(\varphi)\right) \end{aligned}}$$

amplitude varies: $x(s) \propto \sqrt{\beta(s)}$

phase increases monotonically
but growth rate varies as 1/β : $d\varphi = \dfrac{ds}{\beta(s)}$

conserved (action): $\gamma x^2 + 2\alpha x x' + \beta x'^2 = 2J = \text{const}$

Conserved action : invariant on motion for single Particle



$$\text{area} = 2\pi J = \pi(\gamma x^2 + 2\alpha x x' + \beta x'^2)$$

phase space area is preserved

Closer look to Equation of Motion

Initial conditions for the amplitude and phase.

Particle Action

Initial Phase

$$x(s) = \boxed{\sqrt{2J\beta(s)}}\, cos(\phi(s) + \phi_0)$$

Beta-function

Phase-advance

Periodic functions.

Amplitude of the oscillation => Beam Envelope

Defined by the BEAM

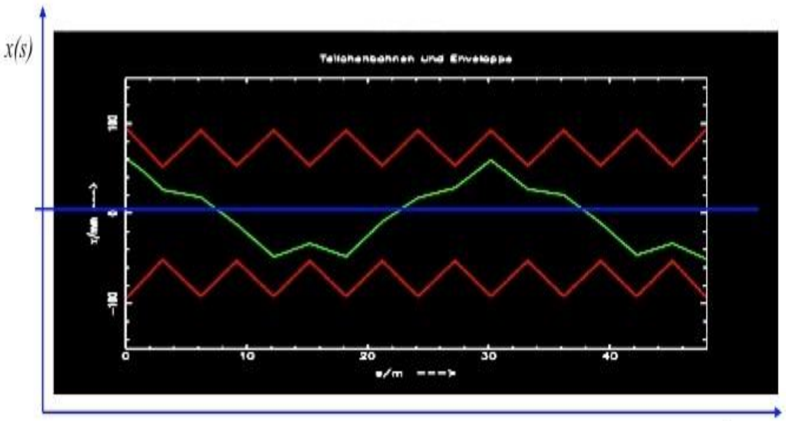
Defined by the LATTICE

Beta Function (1)

The Beta-function is a periodic function entirely defined by the lattice (the magnets).

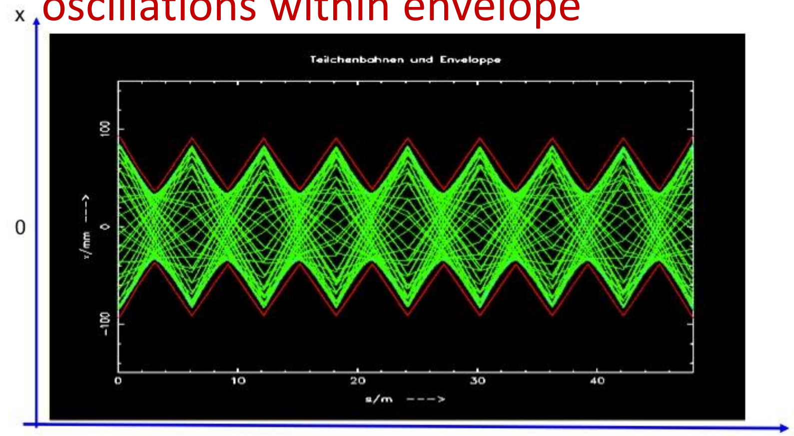
This function is calculated by means of accelerator design software codes. An examples of this is the **Methodical Accelerator Design (MAD-X)** that describes particle accelerators, simulate beam dynamics and optimise the optics.
In case you want to play http://cern.ch/madx

Beta-function → beam envelope



Trajectory of a single particle

Turn, after turn, after turn…betatron oscillations within envelope

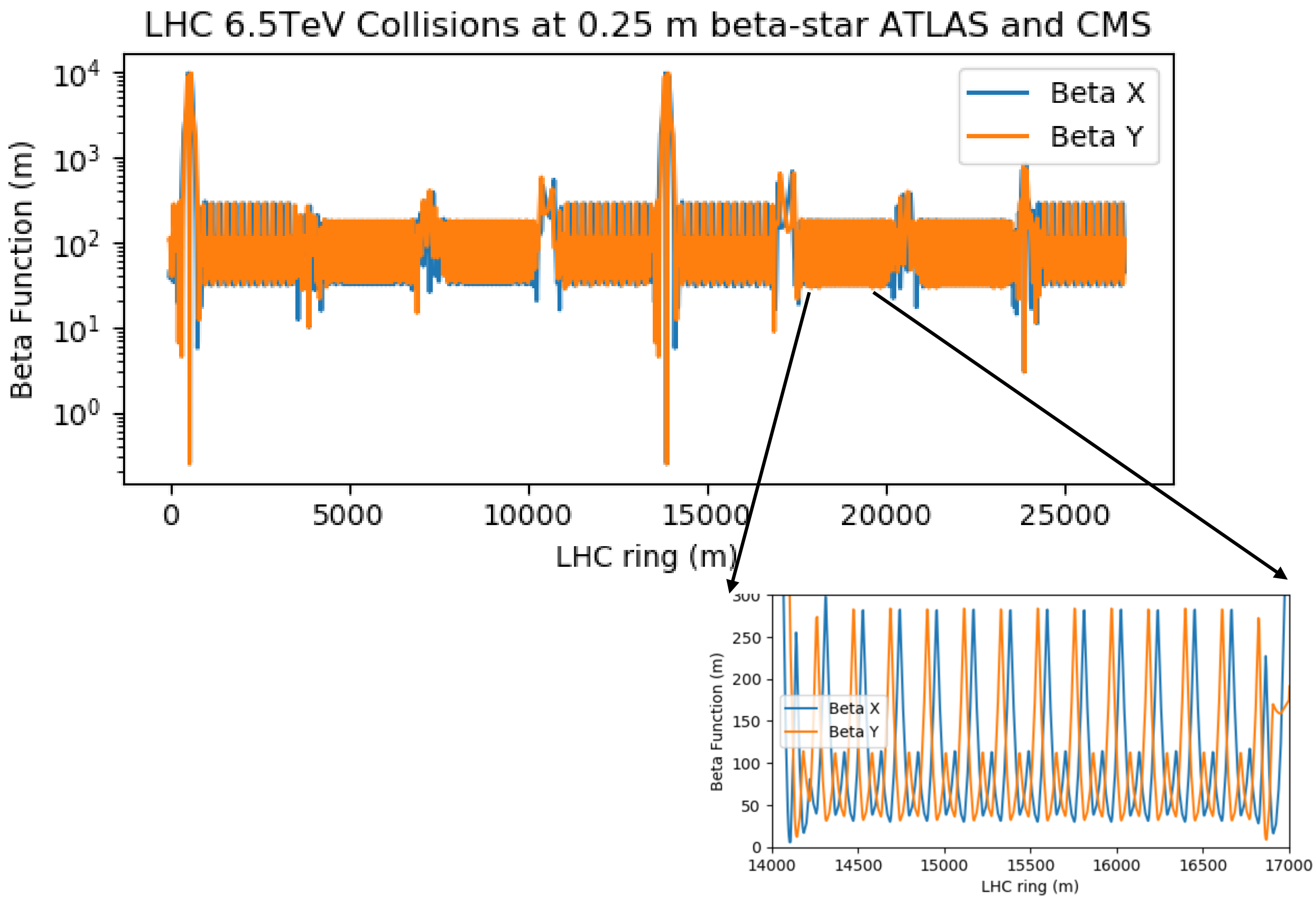


Trajectory of a many particles defining the beam envelope

LHC beams contain about 3×10^{14} protons/beam

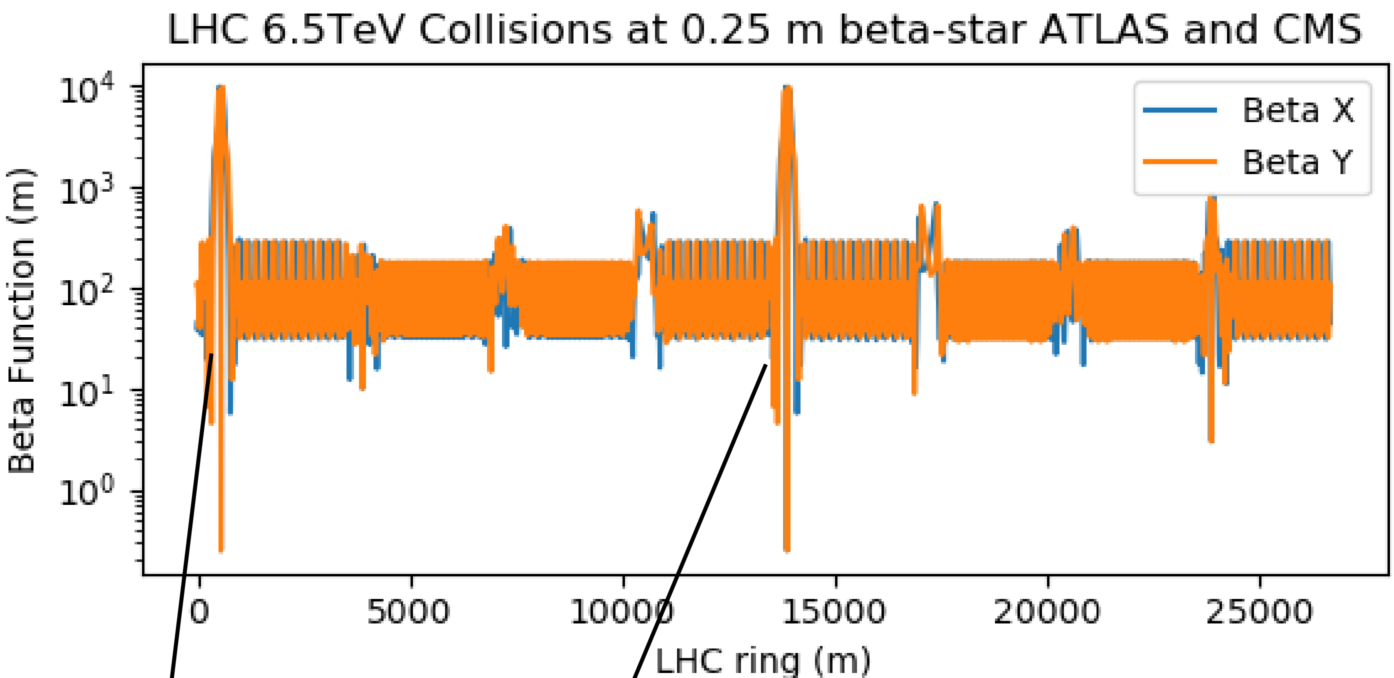
Beta Function at LHC

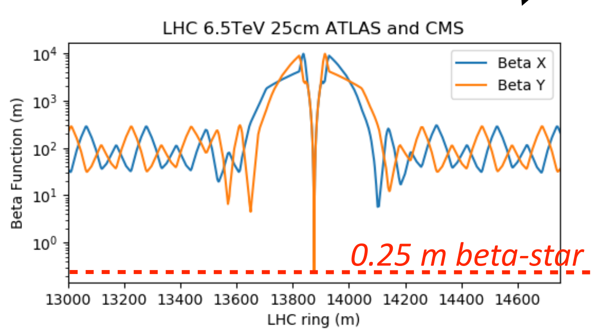
Examples of real optics used in the LHC at the very small beta-star of 0.25 m in ATLAS and CMS.



Beta Function at LHC

Examples of real optics used in the LHC at the very small beta-star of 0.25 m in ATLAS and CMS.





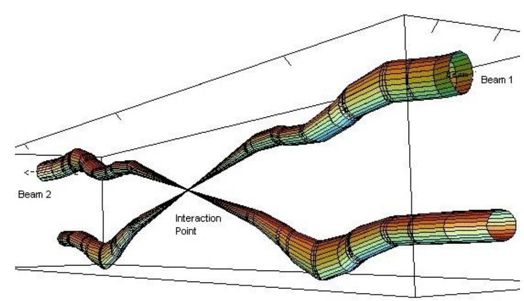
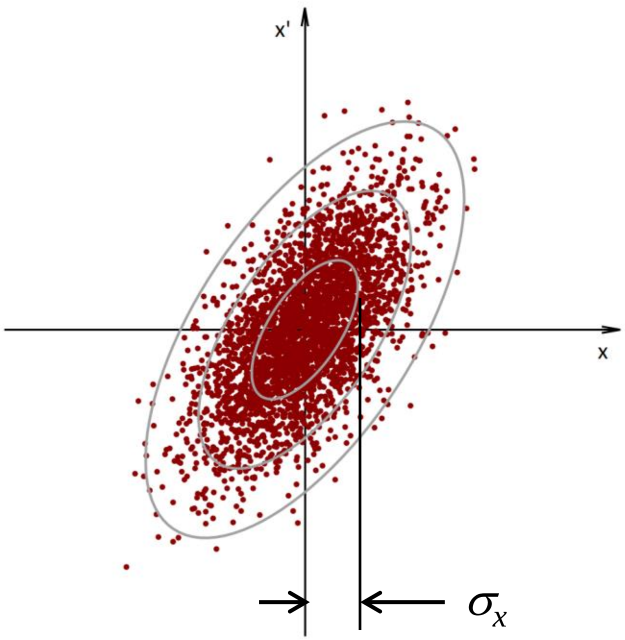


Image credit: J.Jowett

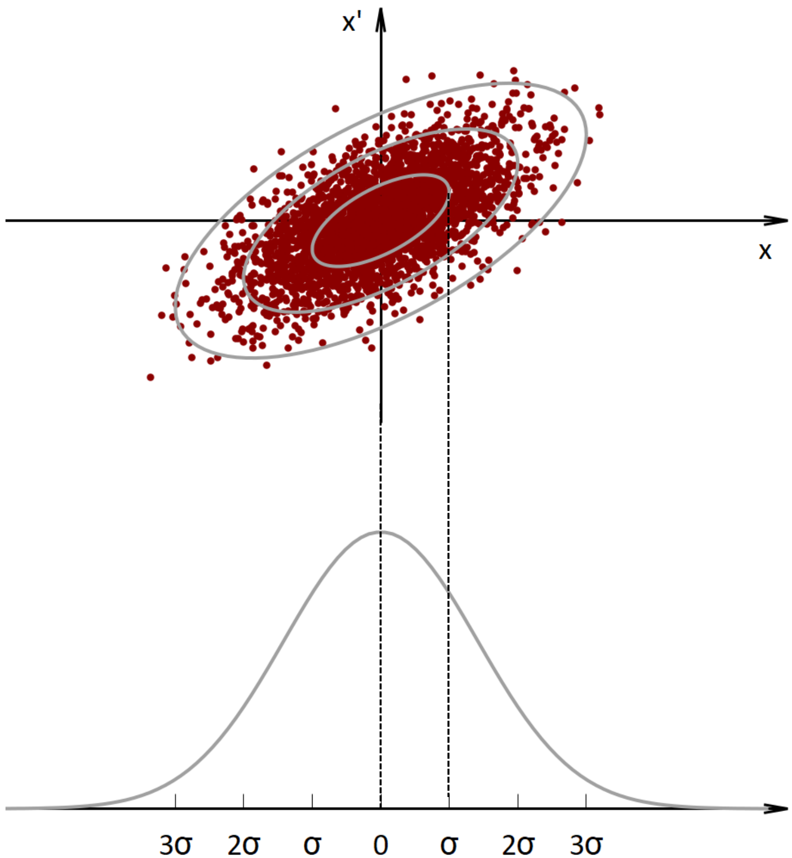
Beam Emittance



- single particles are associated with a particular ellipse
- In a bunch we have many particles 10^{11}
- emittance ε is the average value of particle action J
- Beam Emittance is a property of the beam.

$$\boxed{\varepsilon = \langle J \rangle}$$

Beam Emittance



beam emittance as statistical property:

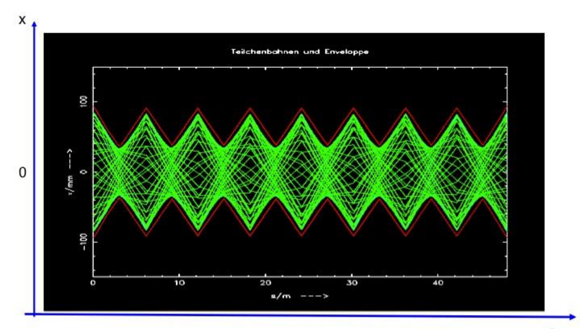
$$\varepsilon_x = \sqrt{< x^2 >< x'^2 > - < xx' >^2}$$

projected Gaussian distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}\sqrt{\beta_x \varepsilon_x}} \exp\left(-\frac{x^2}{2\beta_x \varepsilon_x}\right)$$

Beam size is known all along the ring:

$$\sigma_{x,y}(s) = \sqrt{\epsilon_{x,y}\beta_{x,y}(s)}$$

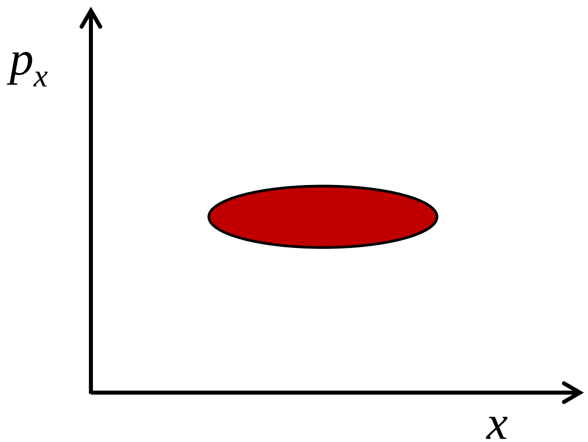


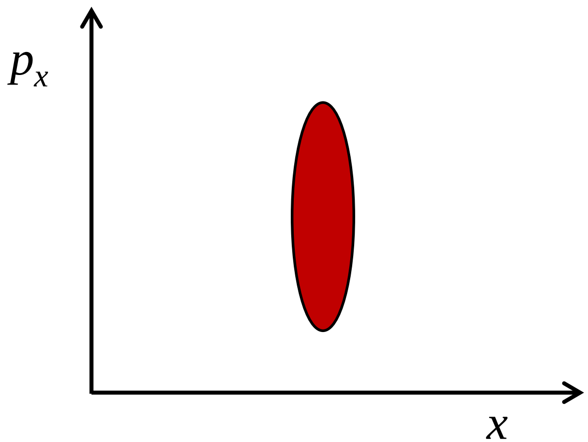
see Wiedemann sec. 8.1.3

Conservation of Emittance

Beams subject to conservative forces as in our accelerator (without dissipative forces i.e. synchrotron radiation) → preserve the phase space density over time

The phase space density behaves like an incompressible liquid.

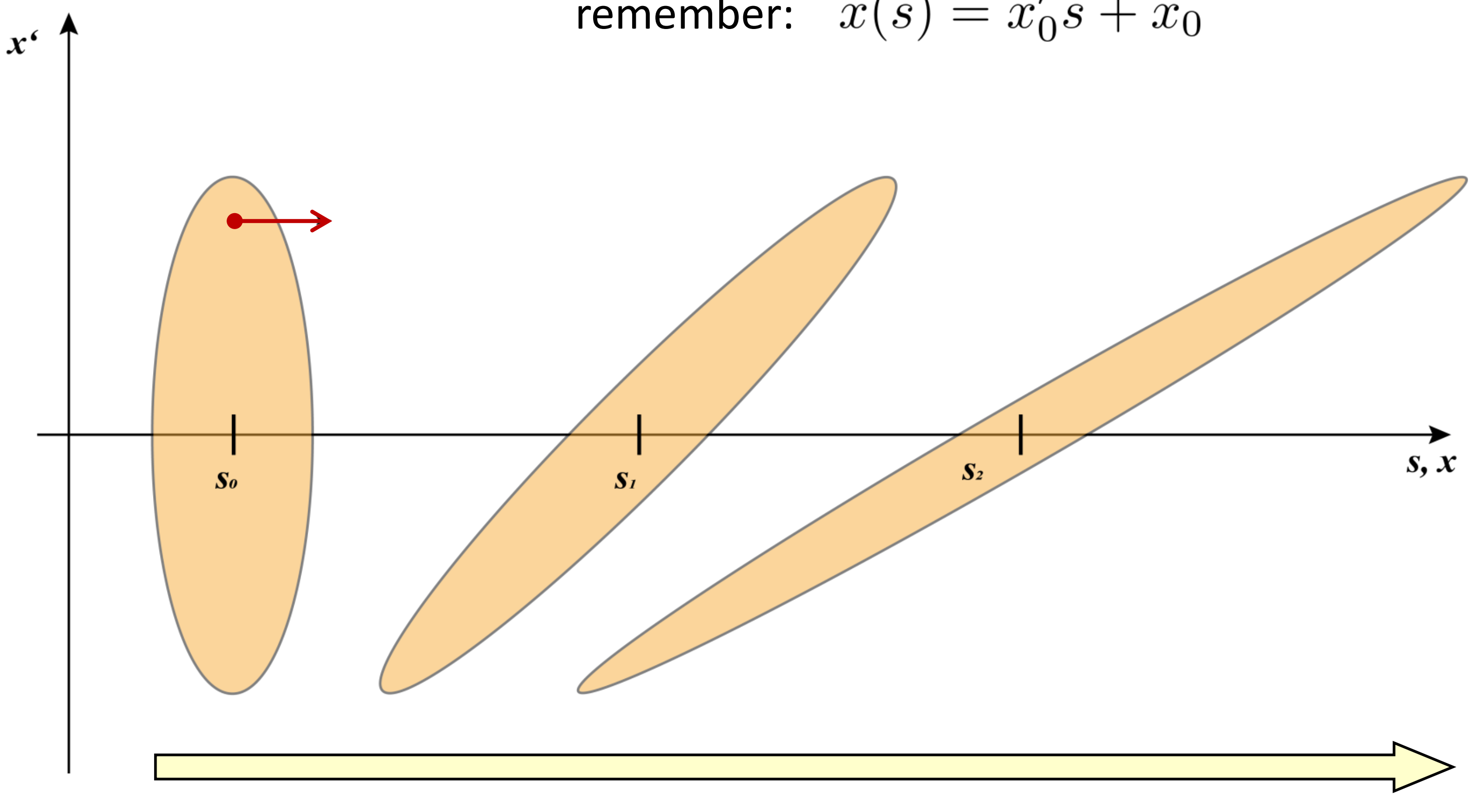




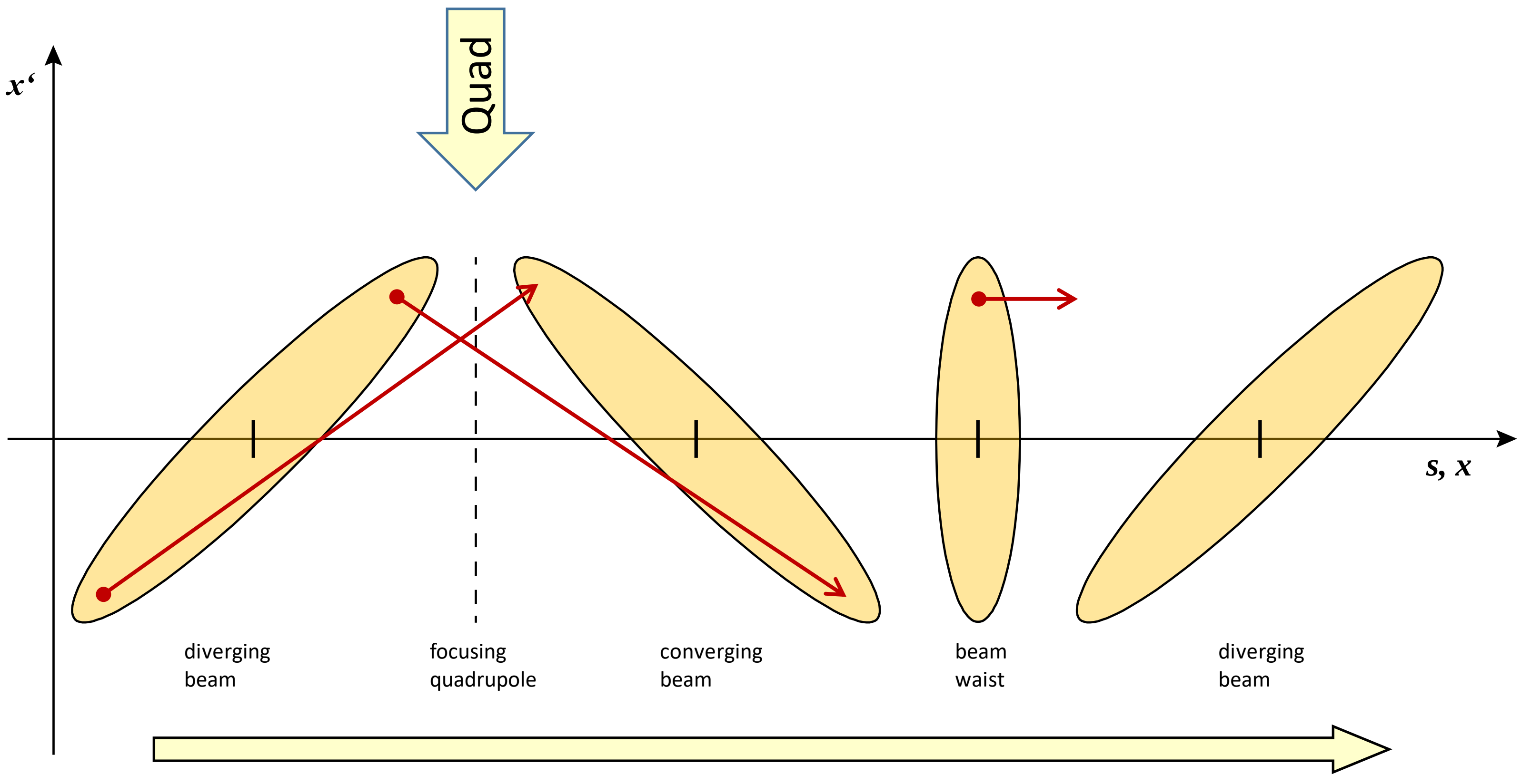
with a given emittance a beam can be made small with large angular spread, or can have small angular spread with a large size

Phase Space Ellipse in Drift Space

remember: $x(s) = x_0' s + x_0$



Phase Space Ellipse after focusing

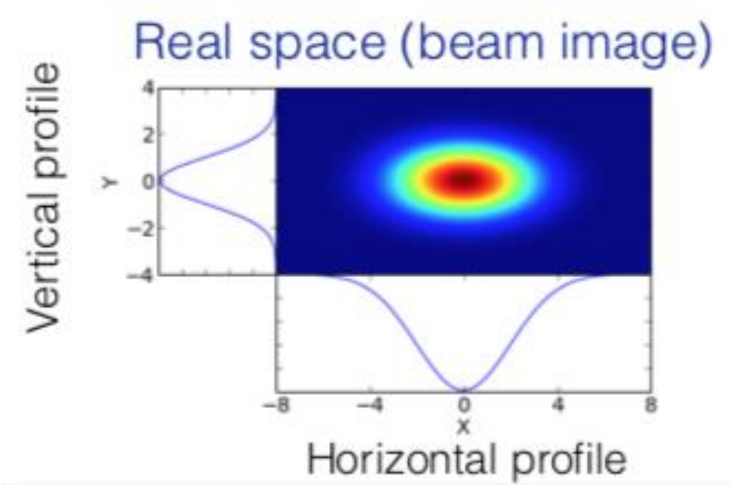


Beam transverse size

Beam Emittance is a property of the beam.

Together with the beta-function gives the complete definition of the beam size (standard deviation).

$$\sigma_x(s) = \sqrt{\epsilon\beta_x(s)}$$



Emittance cannot be changed by focusing/defocusing but it shrinks with beam energy.

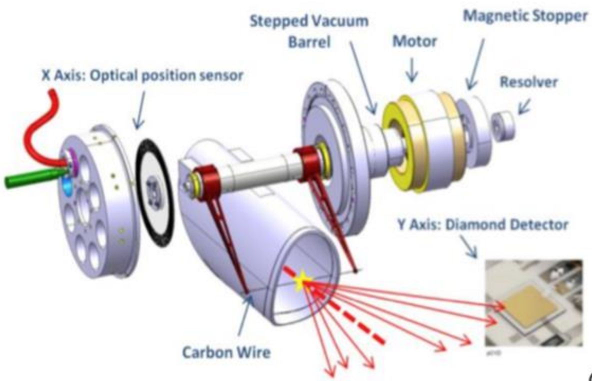
Normalized Emittance is constant with energy

$$\epsilon_n = \beta_{\rm rel}\gamma_{\rm rel}\epsilon$$

Beam size and Emittance measurements

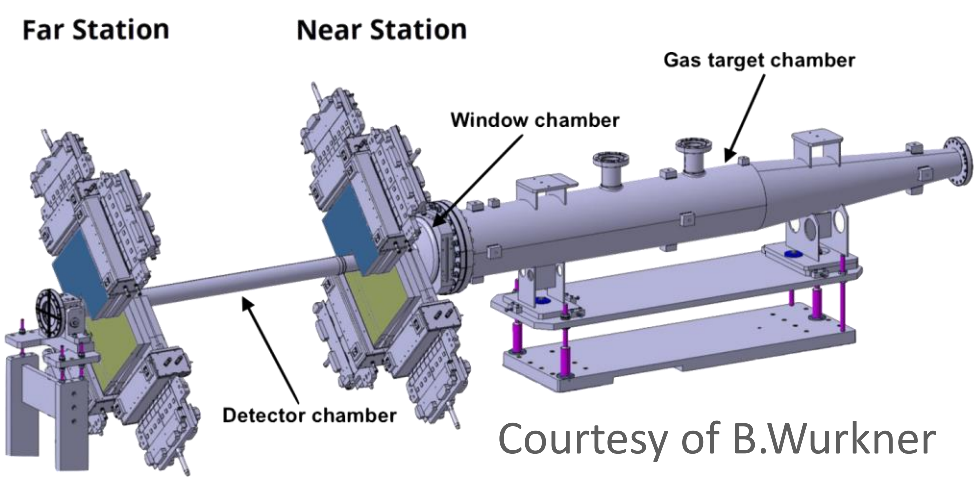
Different mechanisms are used to measure the transverse beam size (and de-convolute it to global emittance).

Some interact with the beam, they can only be used at low intensities or low energies, like fast rotations wire scanners.

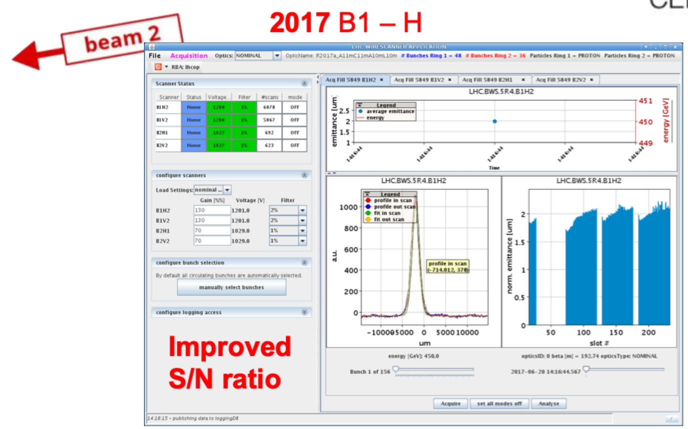


CERN

Other measure the induced ionisation in the rest gas, like ionisation profile monitors or synchrotron radiation, like LHC BSRT.



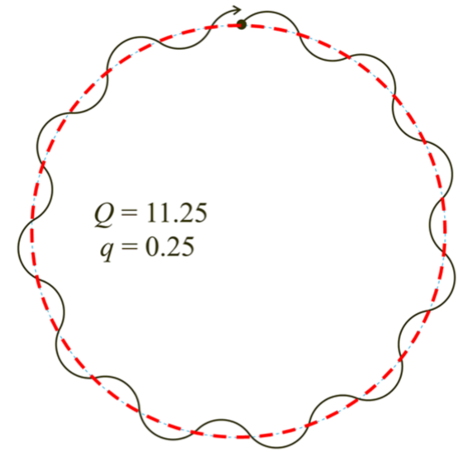
Courtesy of B.Wurkner



Betatron Tune

LHC:
Qx = 64.31
Qy = 59.32

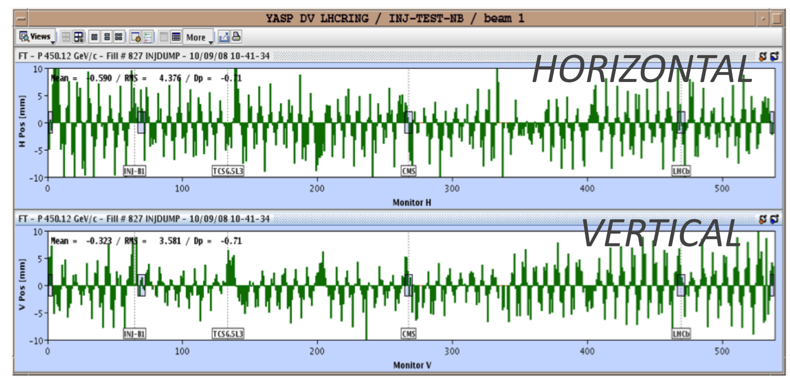
$$X(s) = \sqrt{\epsilon\beta_x(s)}cos(\phi(s) + \phi_0)$$



Number of complete oscillations per turn:

$$Q_x = \frac{1}{2\pi}\oint\frac{\mathrm{d}s}{\beta_x(s)}$$

x: horizontal tune
y: vertical tune



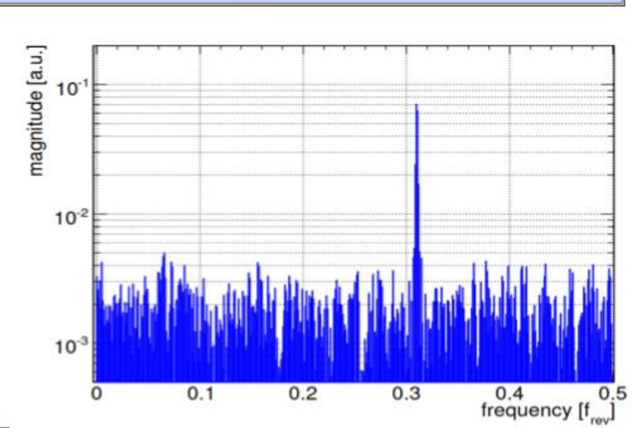
Integer tune:

Seen in orbit response by ~550 dual plane Beam Position Monitors (BPM Electrodes)

Fractional Tune:

Turn-by-turn signal on single electrode after a small beam excitation (kick)

Fast Fourier transform (FFT) of oscillation data gives resonant frequency



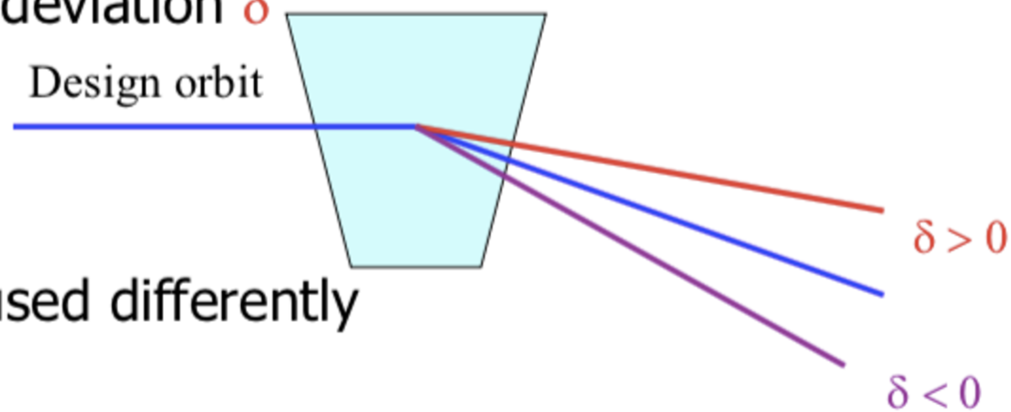
Off momentum particles

What happens to a particle with energy deviation δ travelling in the accelerator magnetic elements?

Off momentum particles

What happens to a particle with energy deviation δ travelling in the accelerator magnetic elements?

Particle with an energy deviation δ



- Will be bent and focused differently
- The equation of motion: non-homogeneous Hill equation

Off momentum particles: Dispersion

$$\delta = \frac{\Delta p}{p_0}$$

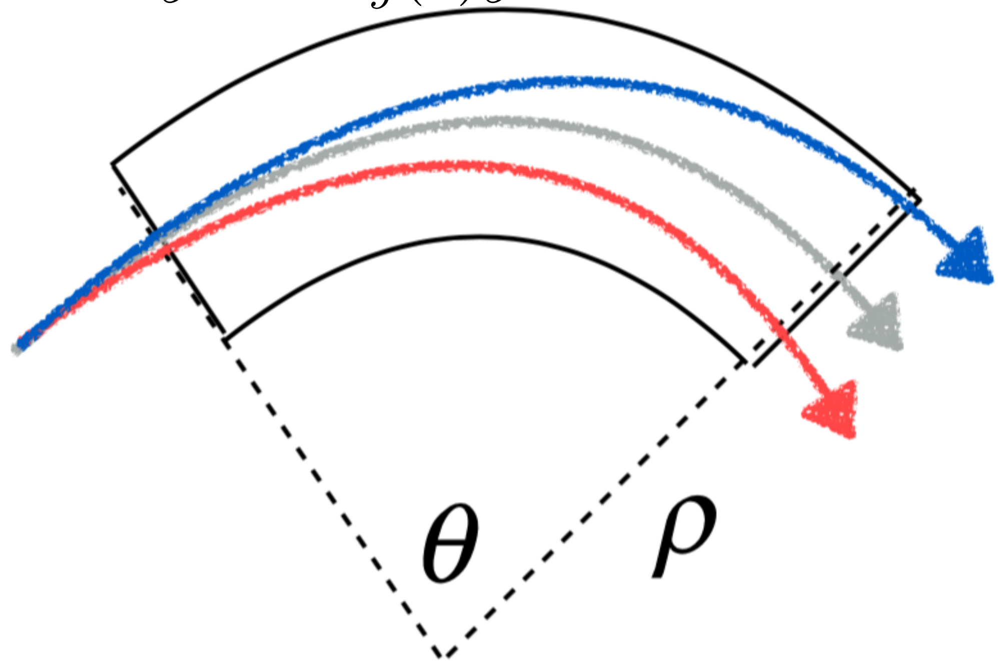
$$x'' + K_x(s)x = \frac{1}{\rho(s)}\frac{\Delta p}{p_0}$$

$$y'' - K_y(s)y = 0$$

Bending in a dipole changes with the particle energy…

Particles will move on different orbit!

$$B\rho = p/e$$



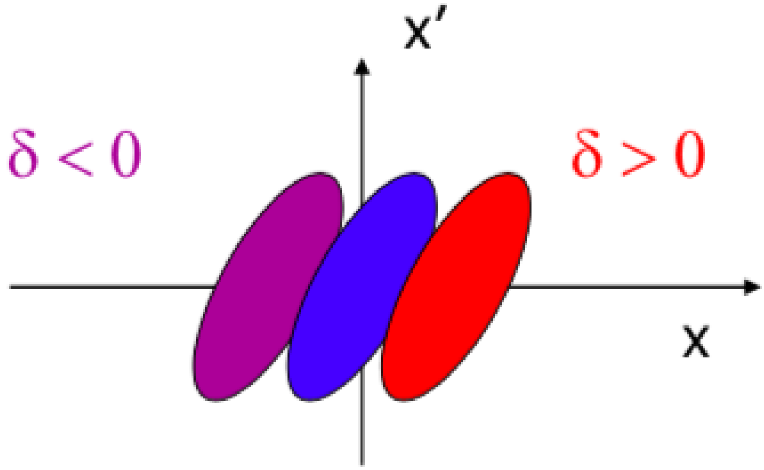
Particle deviation from ideal orbit

$$\boxed{x = x_\beta + x_\varepsilon = x_\beta + D(s)\cdot\delta}$$

D(s) - dispersion function

Beam size

- When the beam energy spread is δ



$$\boxed{\sigma^2 = \sigma_\beta^2 + \sigma_\varepsilon^2 = \varepsilon \cdot \beta + D^2\delta^2}$$

Contents:

- Particle types and relativity for accelerators
- Accelerator components: Dipole, quadrupoles magnets, accelerating RF cavities…
- Transverse plane (x,y) → Guiding and focusing beams
  - Particle motion in linear approximation
  - Invariant of motion and Emittance
  - Beam Optics: beta functions, beams sizes, Beam Tunes
- Longitudinal plane (s,t) → Acceleration
  - Synchronous motion
  - Synchrotrons and LHC injection complex
- Hadron Accelerators: Synchrotrons
  - Beam production
  - Magnets
  - Luminosity
  - Collective effects

Acceleration

- Why we would like to accelerate particles?
  - Reach of higher energetic collisions (ions, protons and leptons)
  - Compensate for energy loss due to emission of synchrotron radiation (leptons)

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = e(\underbrace{\vec{E}} + \underbrace{\vec{v} \times \vec{B}})$$

Longitudinal Motion

Parallel to the direction of motion.

Used to accelerate charged particles.

Transverse Motion

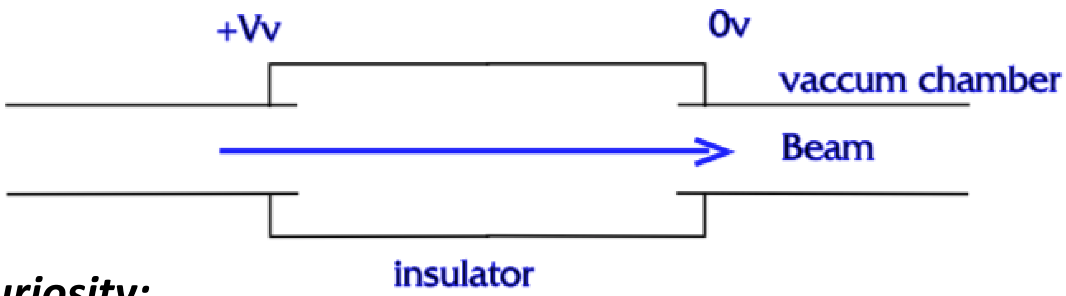
Perpendicular to the direction of motion.

Used to keep circulating orbit and beam steering.

Acceleration has to be done by an electric field in the direction of the motion

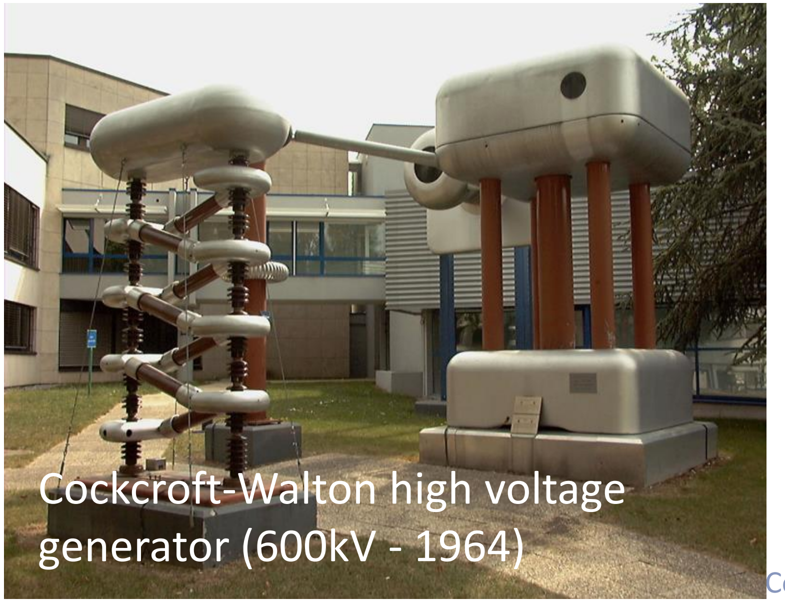
Electrostatic acceleration

Simplest way to generate an electric field in the motion direction: voltage difference



Gain on kinetic energy is proportional to V (the potential)

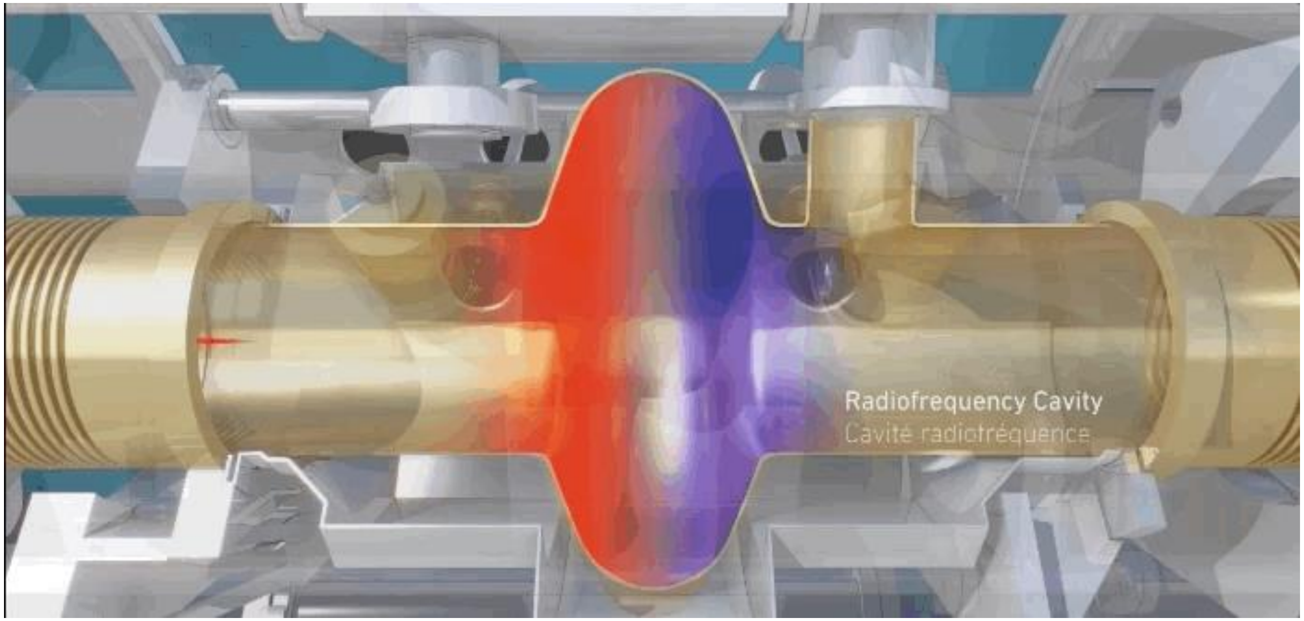
Curiosity:
The energy unit (electron Volt): 1 eV is the energy that 1 elementary charge e gains when it is accelerated in a voltage of 1 Volt.


Cockcroft-Walton high voltage generator (600kV - 1964)

Electrostatic machines are still used at lower energy, as a 1st stage of acceleration, radiotherapy, particle source, etc.

Limitations:
Max. Voltage ~ 10MV due to insulation problems.

Radio-frequency acceleration



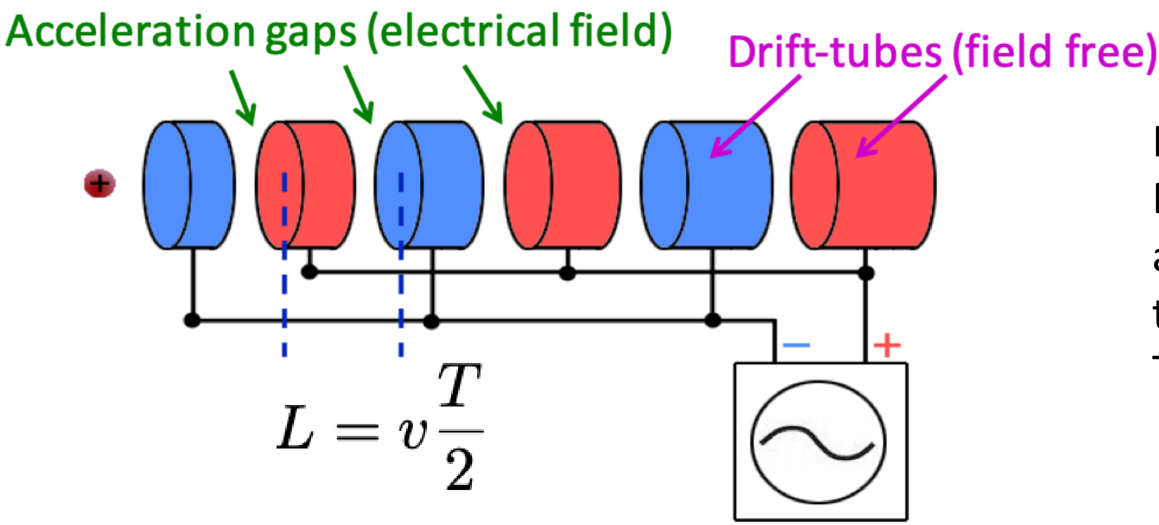
Apply an E-field which is reversed while the particle travels inside the tube
→ it gets accelerated at each passage.

Build the acceleration with one or more series of drift tubes with gaps in between them.

Could accelerate in linear and circular machines

Only particles synchronized with RF will be accelerated → particles are bunched in packages

LINAC: linear accelerator



$$L = v\frac{T}{2}$$

For non-relativistic particles → Distance (L) between the acceleration gaps needs to fulfil the synchronism condition with T the period of the RF oscillator.

Bunched Beam

$$\uparrow v \quad \Longrightarrow \quad \uparrow L$$

Energy gain:

$$E = neV_{\mathrm{RF}} \sin\phi_{\mathrm{s}}$$

n : number of gaps
e : charge
V_{RF} : applied voltage
ϕ_{s} : synchronous phase

RF field break down

High gradient limits : field levels of **10-100 MV/m.**

Electrons in surface are emitted (field emission), vacuum arcs may form and the field breaks down. Eventually the break down processes may damage the structure.

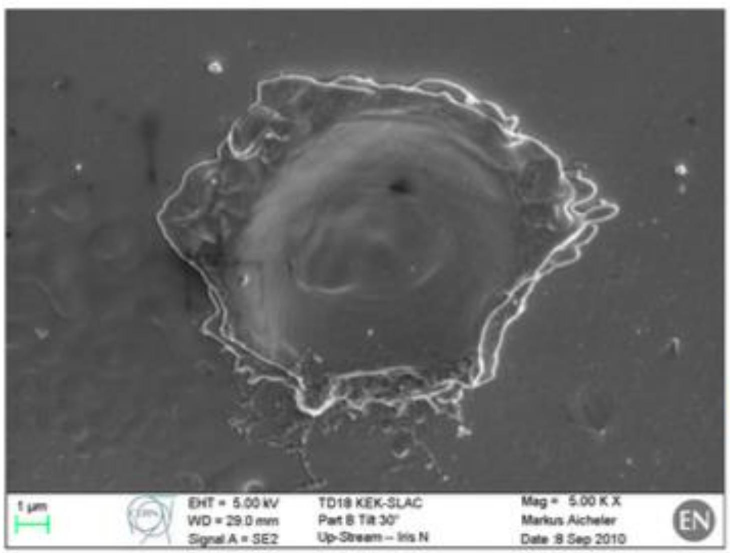


Figure 4.1.: Electron micrograph by Markus Aicheler [4] of the crater left behind from a breakdown on the iris of a TD18 accelerating structure.

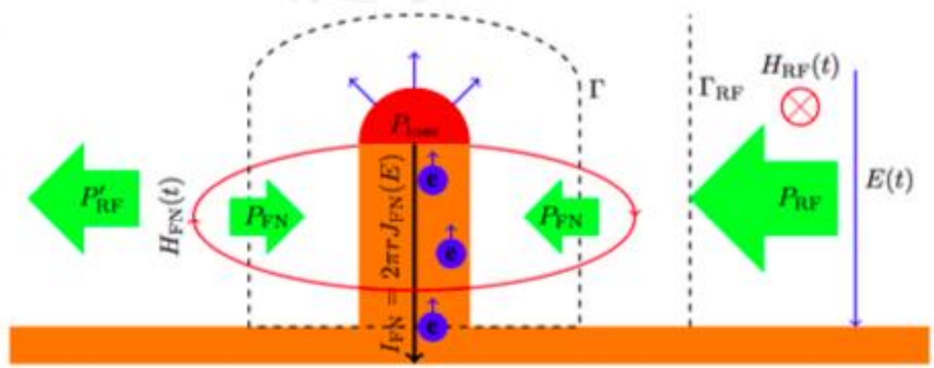
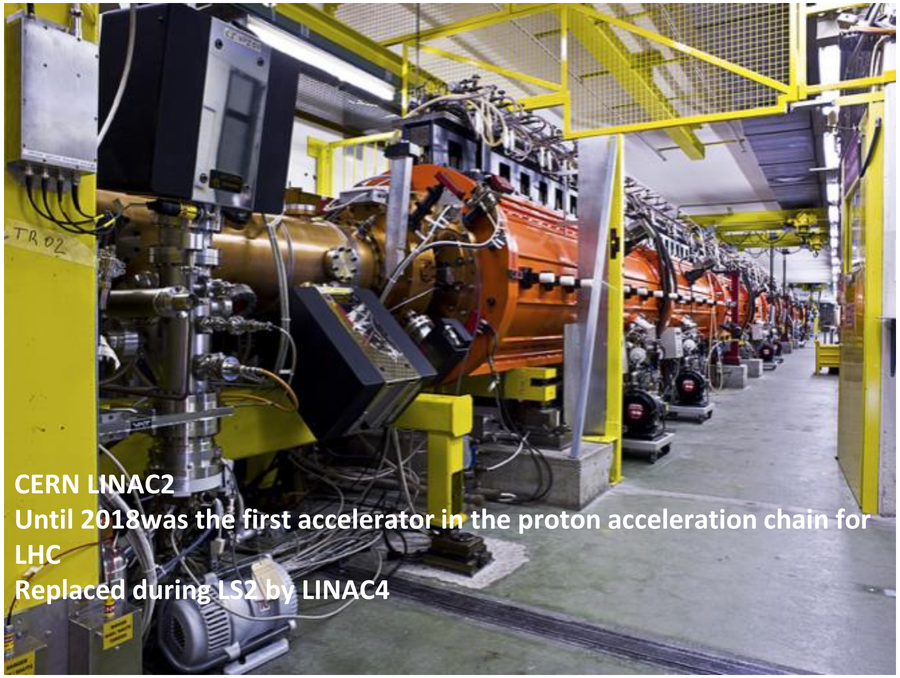


Figure 4.3.: Power flows around a field emitter tip in an RF cavity.

From LINAC to Circular Machines

LINACs are today the first stage in many accelerator complexes

Limited by the particle energy reach due to length and single pass

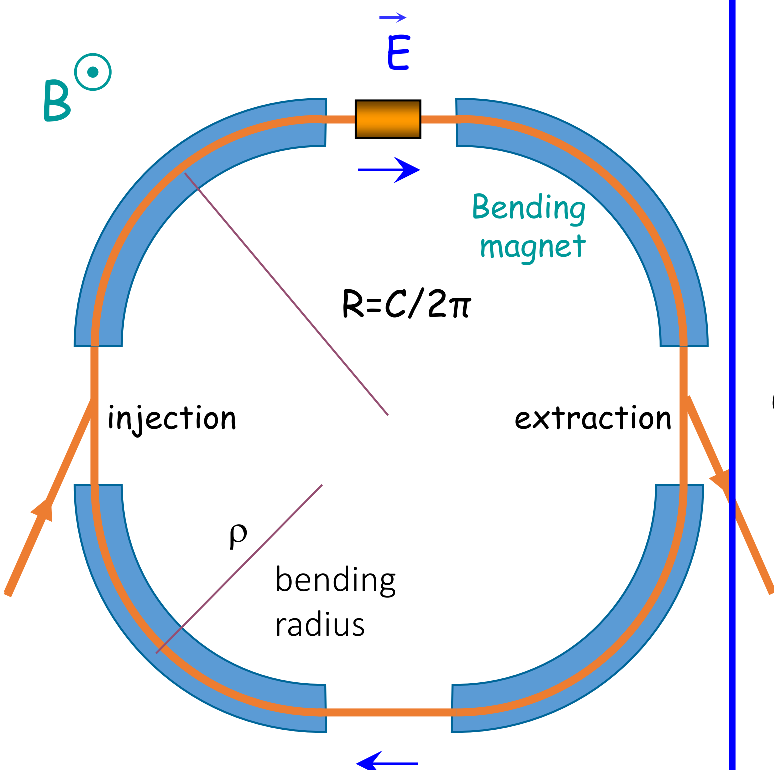


Circular Accelerators

Use of circular structures in order to apply over and over the accelerating fields.
Particles are bend onto circular trajectories → **Many passages through RF structure**

The Synchrotron: acceleration

The synchrotron is a synchronous accelerator since there is a synchronous RF phase for which the energy gain fits the increase of the magnetic field at each turn. That implies the following operating conditions:



$e\hat{V}\sin\phi \longrightarrow$ **Energy gain per turn**

$\phi = \phi_s = cte \longrightarrow$ **Synchronous particle**

$\omega_{RF} = h\omega \longrightarrow$ **RF synchronism (h - harmonic number : # of RF cycles per revolution)**

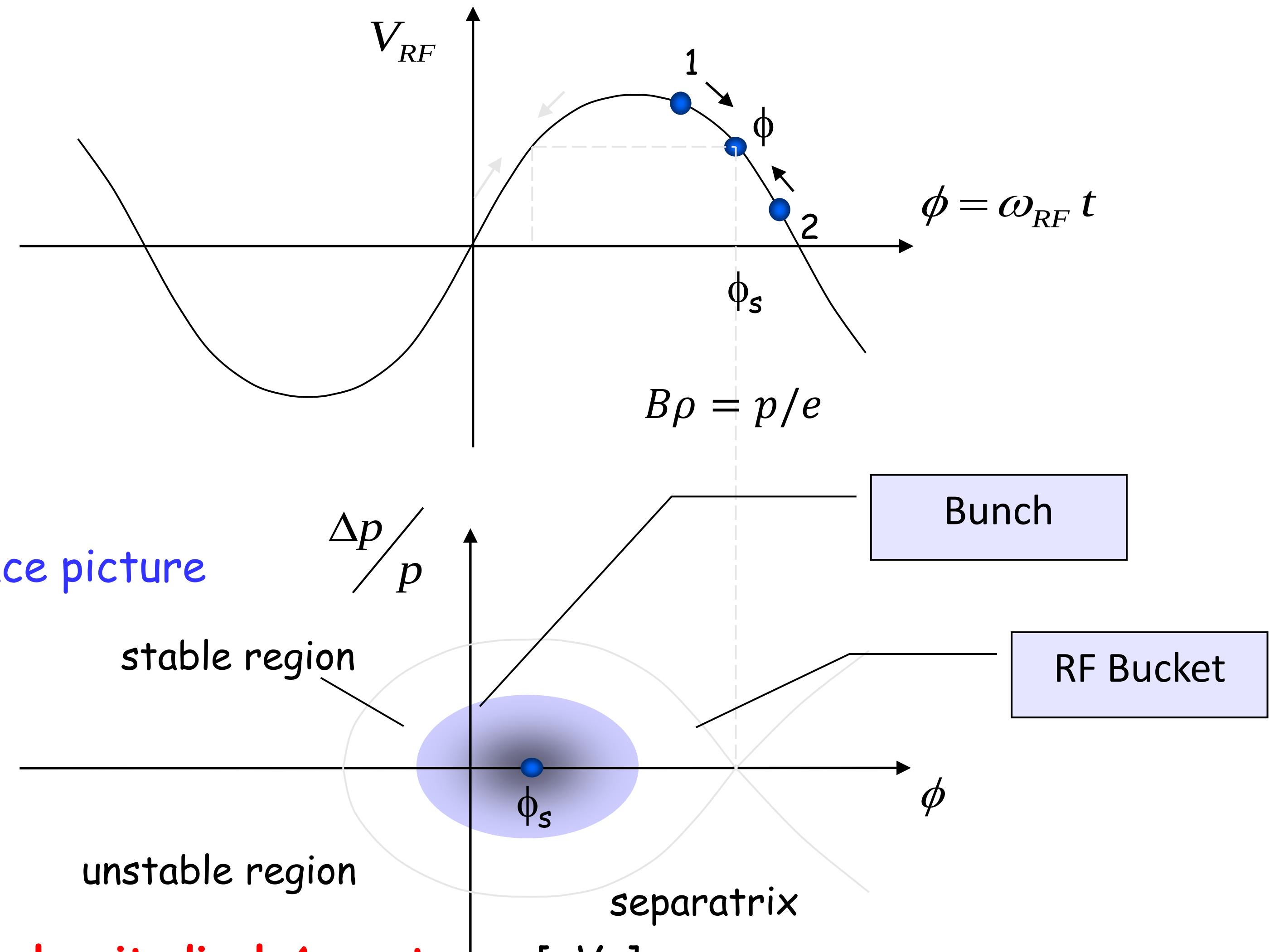
$\rho = cte \quad R = cte \longrightarrow$ **Constant orbit**

$B\rho = P/e \Rightarrow B \longrightarrow$ **Variable magnetic field**

If v≈c, ω hence ω_{RF} remain constant (ultra-relativistic)
LHC case fRF = 400 MHz and frev = 11kHz = c / 27 Km h ~ 35640

Synchrotron oscillations (with acceleration)

Case with acceleration *B increasing* $\gamma > \gamma_t$



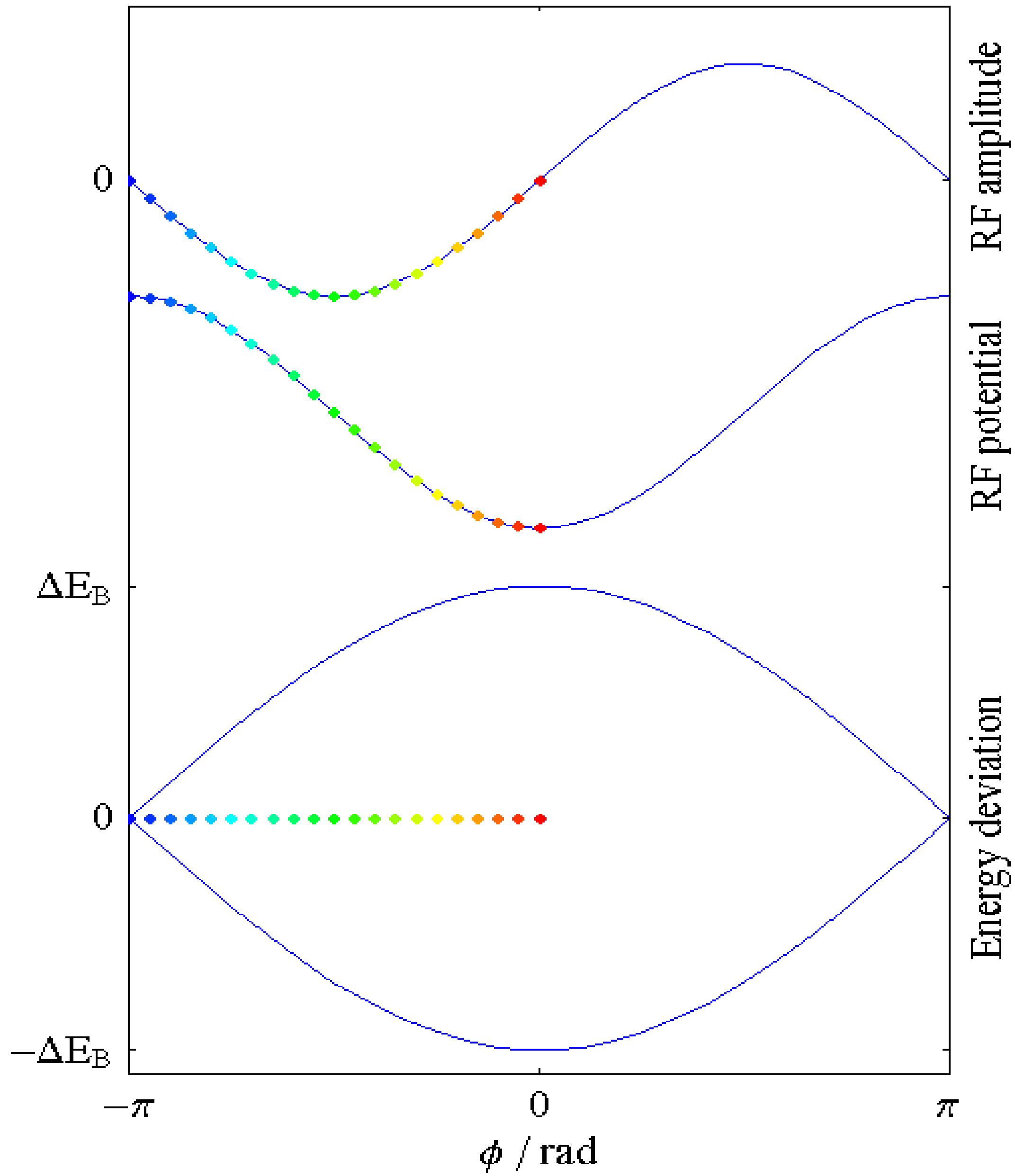
- Bucket area = **longitudinal Acceptance** [eVs]
- Bunch area = **longitudinal beam emittance** = $4\pi\, \sigma_E\, \sigma_t$ **[eVs]**

Synchrotron motion in phase space

The restoring force is non-linear.

$\Rightarrow$ speed of motion depends on position in phase-space

(here shown for a stationary bucket)



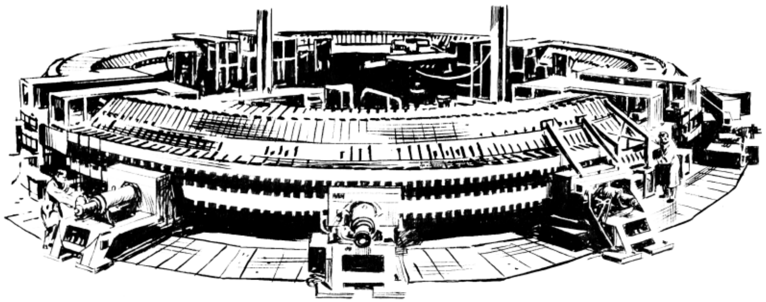
Contents:

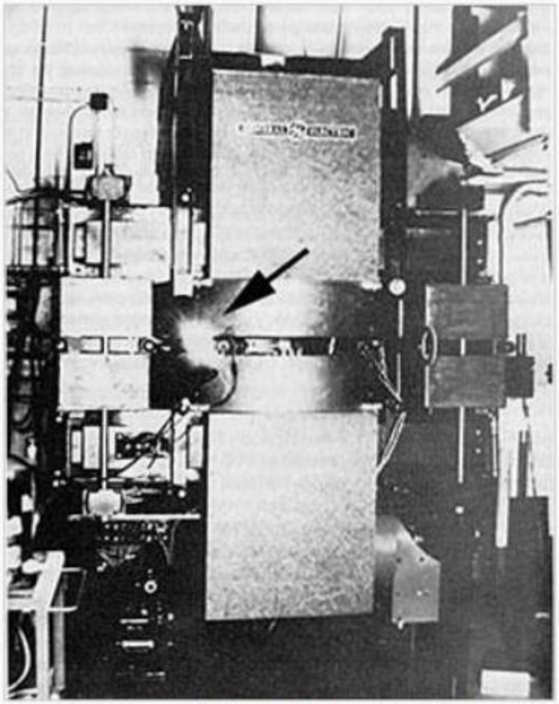
- Particle types and relativity for accelerators
- Accelerator components: Dipole, quadrupoles magnets, accelerating RF cavities…
- Transverse plane (x,y) → Guiding and focusing beams
  - Particle motion in linear approximation
  - Invariant of motion and Emittance
  - Beam Optics: beta functions, beams sizes, Beam Tunes
- Longitudinal plane (s,t) → Acceleration
  - Synchronous motion
  - Synchrotrons and LHC injection complex
- Hadron Accelerators: Synchrotrons
  - Beam production
  - Magnets
  - Luminosity
  - Collective effects

EPFL

Synchrotrons

- **First Synchrotron the Cosmotron** at Brookhaven Laboratory 1952 3 GeV protons (288 magnets)
- In 1947 General Electric's Research Lab observed for the first time **Synchrotron radiation** → electromagnetic radiation emitted by charged particles travelling at relativistic speeds, forced to take a curved path by a magnetic field (Synchrotron light sources for spectroscopy and crystallography)





General Electric synchrotron accelerator built in 1946, the origin of the discovery of synchrotron radiation. The arrow indicates the evidence of radiation.

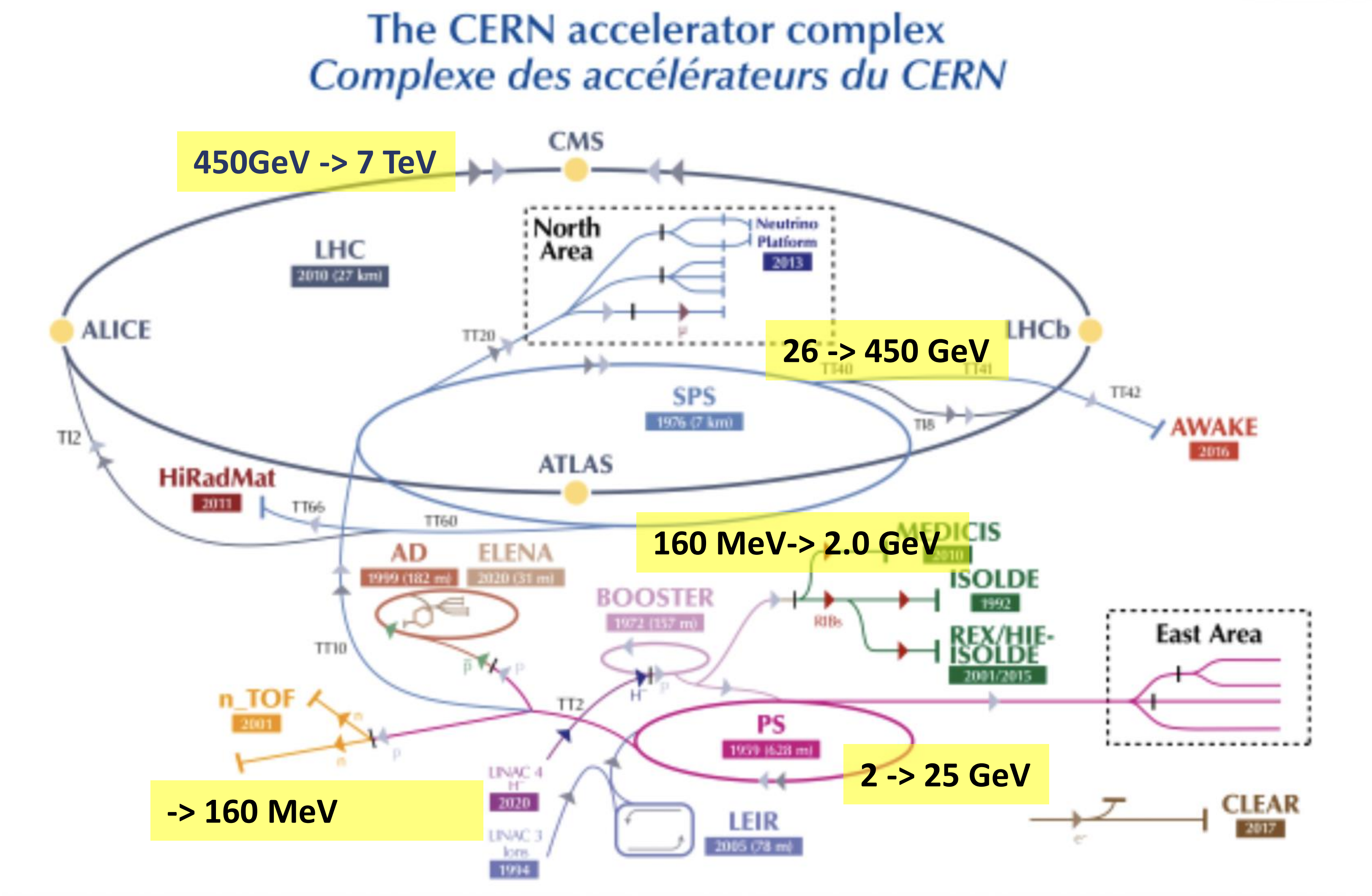
Large scale accelerators since components can be divided in different sections

Synchrotrons

1959 construction of the first “larger” synchrotron machines

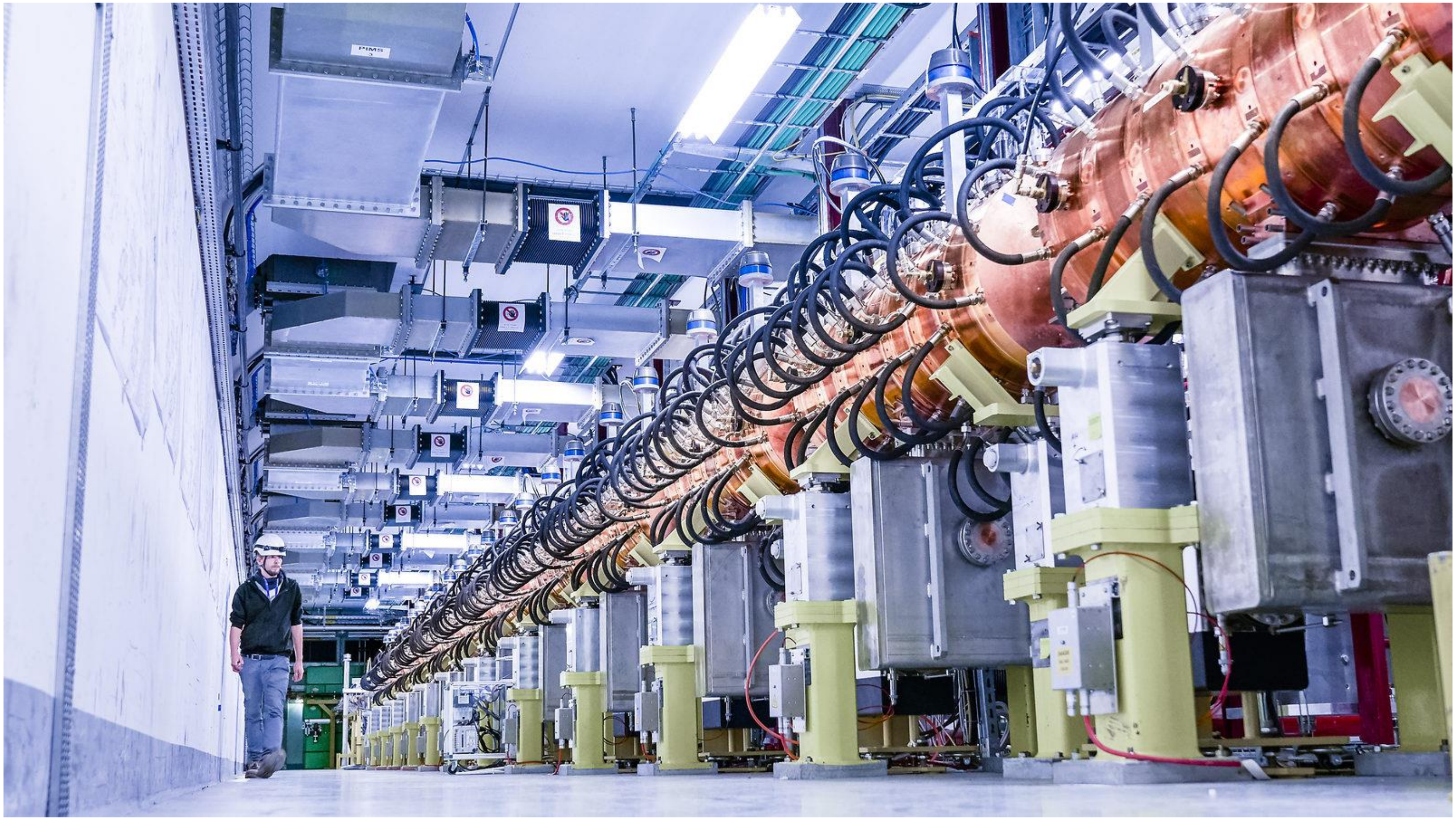
- CERN-PS (Proton Synchrotron): 60 year still in operation, still in use for the injection to the LHC.
- BNL-AGS (Alternating Gradient Synchrotron)

The CERN Accelerator Complex



LINAC 4

160 MeV (90 meter linac)

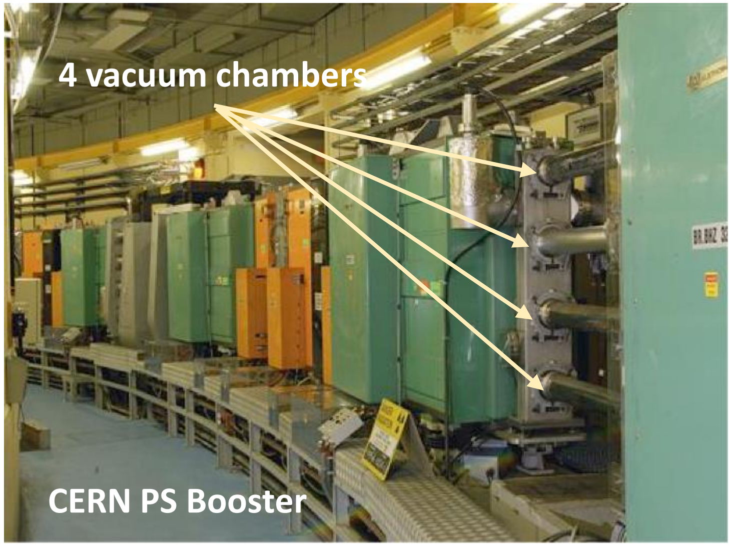


PS Booster

1st Synchrotron in the chain with 4 superposed rings

Circumference of 157m

Increases proton energy from **160 MeV to 2 GeV in 1.2s**



LINAC 4 pulse is distributed vertically in the 4 rings. Bunches are built as multi-turn PSB injection. Keeping charge density constant every injection in a different phase-space defining the **transverse emittance.**

- **ISOLDE**: High-Intensity 10-13 turns are injected = large transverse emittance
- **LHC**: 2-3 injected turns = small transverse emittance

After acceleration they will be combined and transferred to the PS.

PS: Protons Synchrotron

The oldest operating synchrotron at CERN (since 1959)

Circumference of 628 m

4 x PSB ring

Accelerates from 2 GeV to a range of energies up to 26 GeV depending on the user

- East area: 24GeV
- SPS: 14GeV or 26GeV
- AD: 26 GeV
- n-TOF: 20 GeV

Cycle length goes from 1.2s to 3.6s

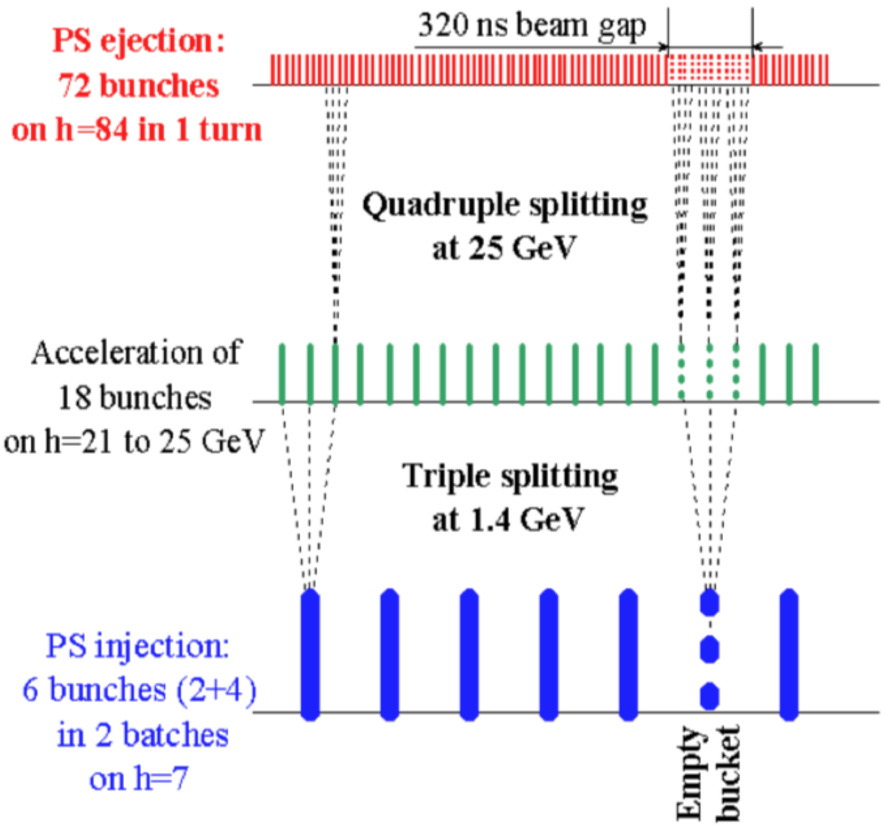
Various types of extractions: fast, slow and multi-turn (MTE)

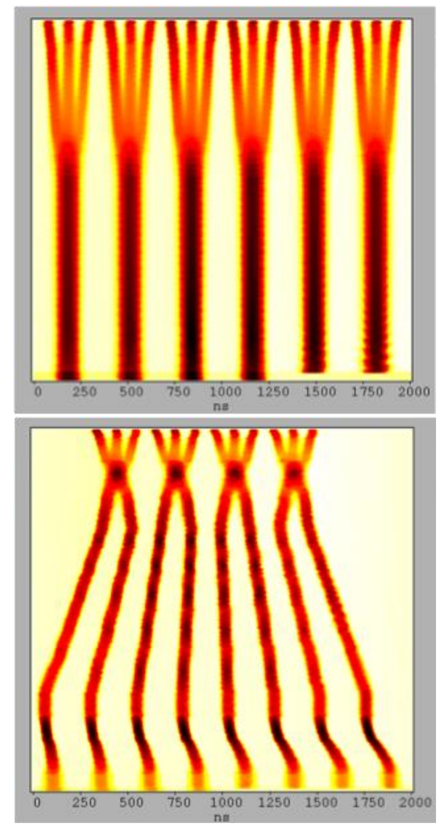
Many different RF cavities: 10 MHz, 13/20 MHz, 40 MHz, 80 MHz,200 MHz

LHC filling and Bunch Splitting in PS

Changing RF frequency we change the harmonic number h

$$\omega_{RF} = h\omega$$





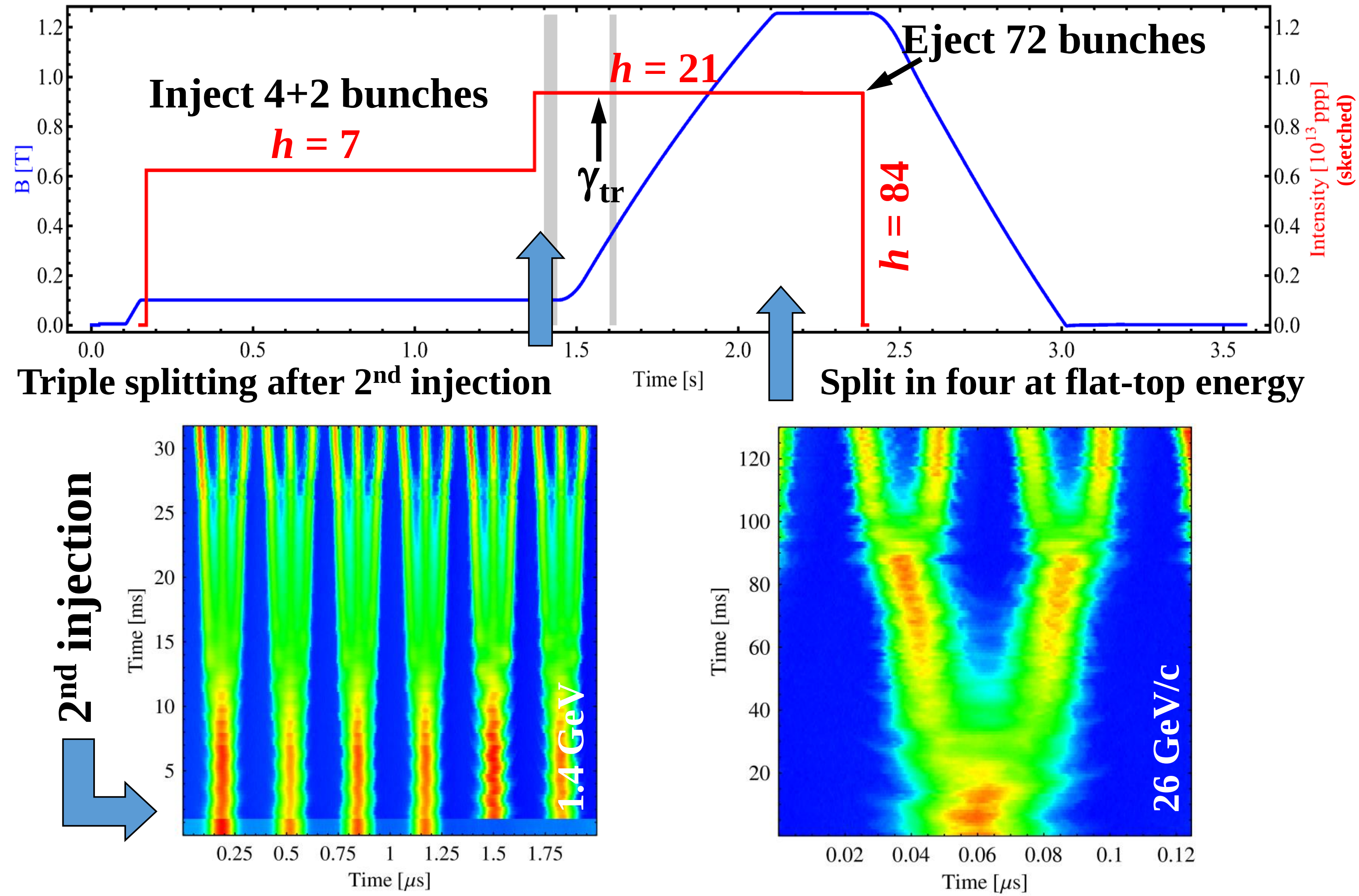
Standard: 72 bunches @ 25 ns

BCMS: 48 bunches @ 25 ns

Smaller Emittance

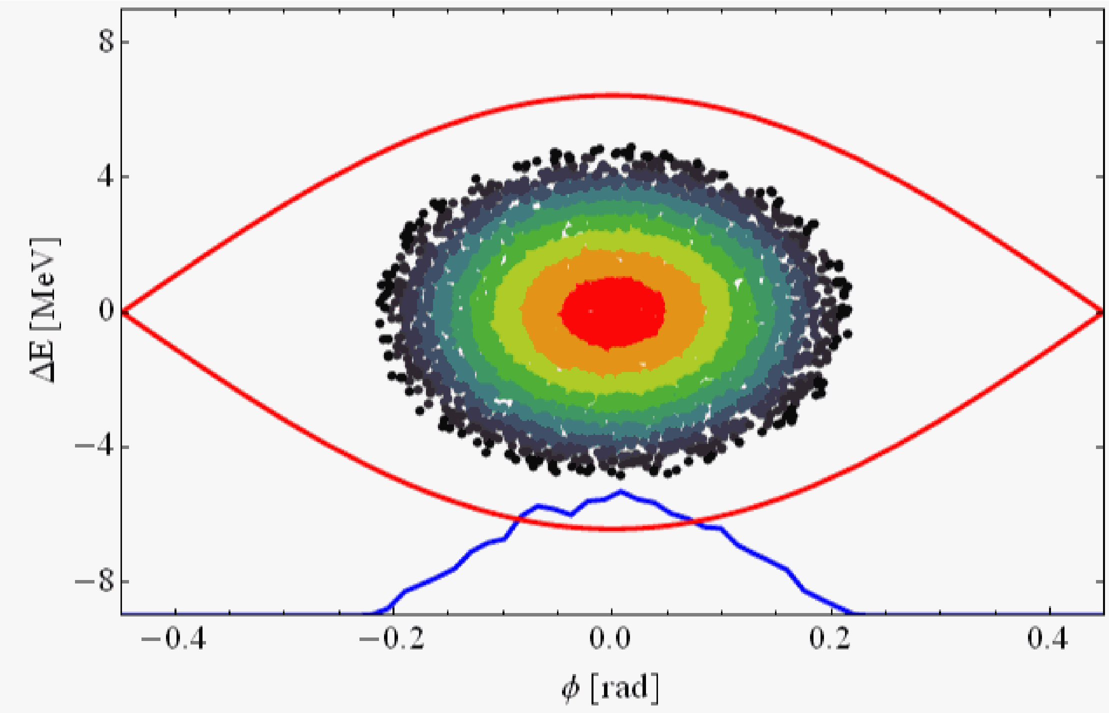
Image credit R.Garoby

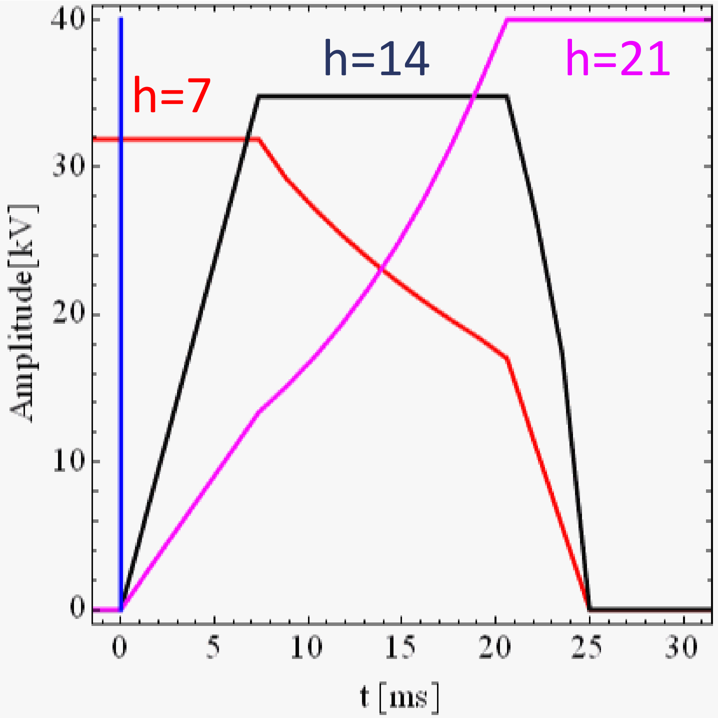
The LHC25 (ns) cycle in the PS



→ Each bunch from the Booster divided by 12 → 6 × 3 × 2 × 2 = 72

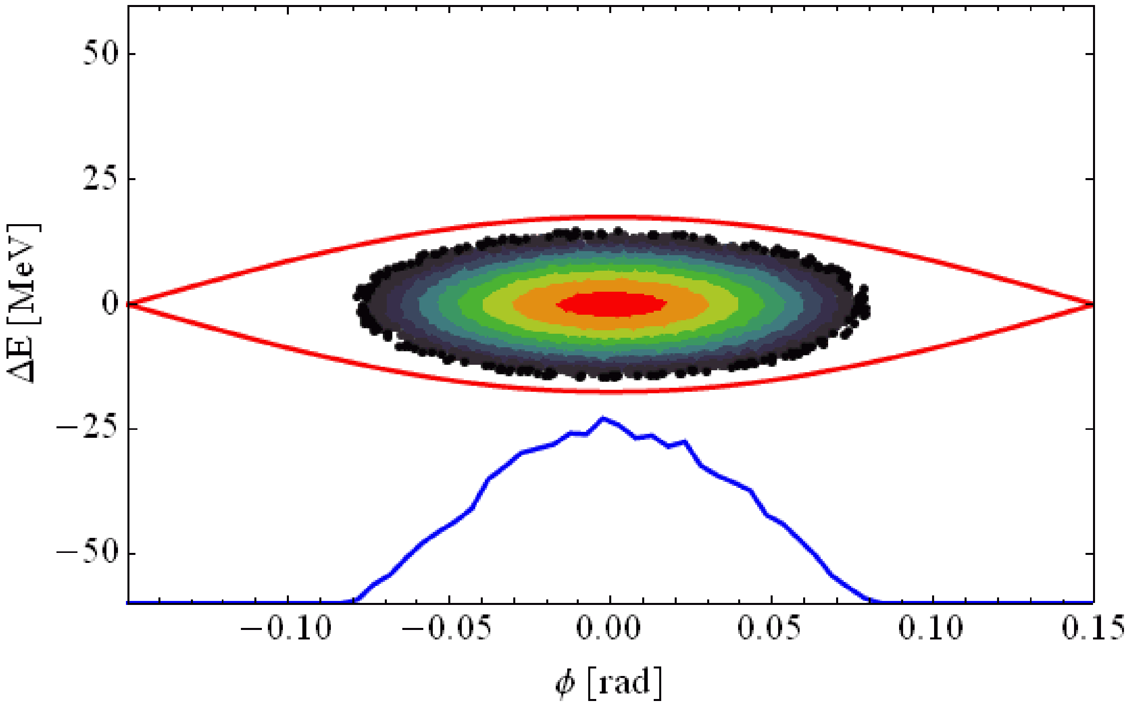
Triple splitting in the PS

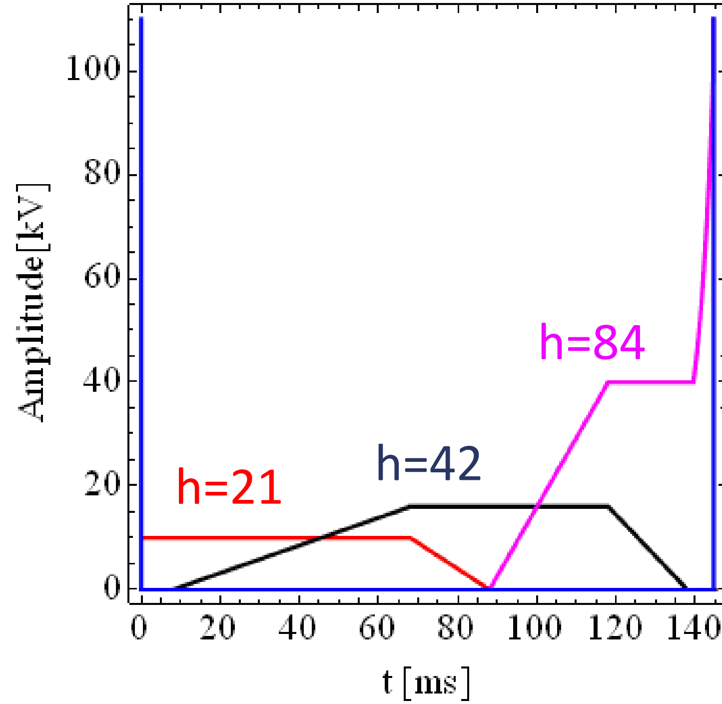




Two times double splitting in the PS

Two times double splitting and bunch rotation:





- Bunch is divided twice using RF systems at
  h = 21/42 (10/20 MHz) and h = 42/84 (20/40 MHz)

- Bunch rotation: first part h84 only + h168 (80 MHz) for final part

SPS

The first synchrotron in the LHC chain **at 30m underground.**

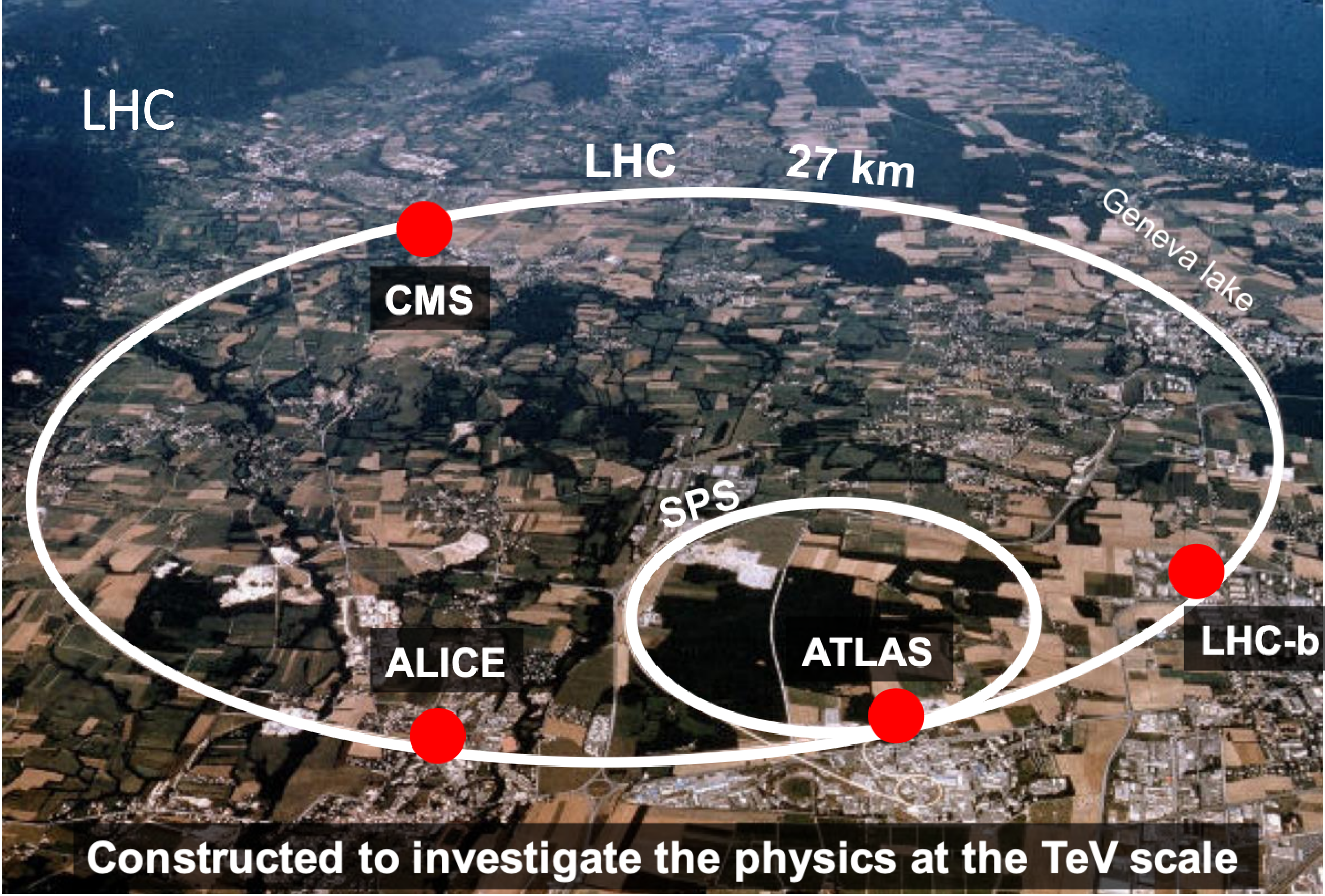
Circumference of 6.9km

11 x PS ring

Accelerates from 26GeV to up 450GeV

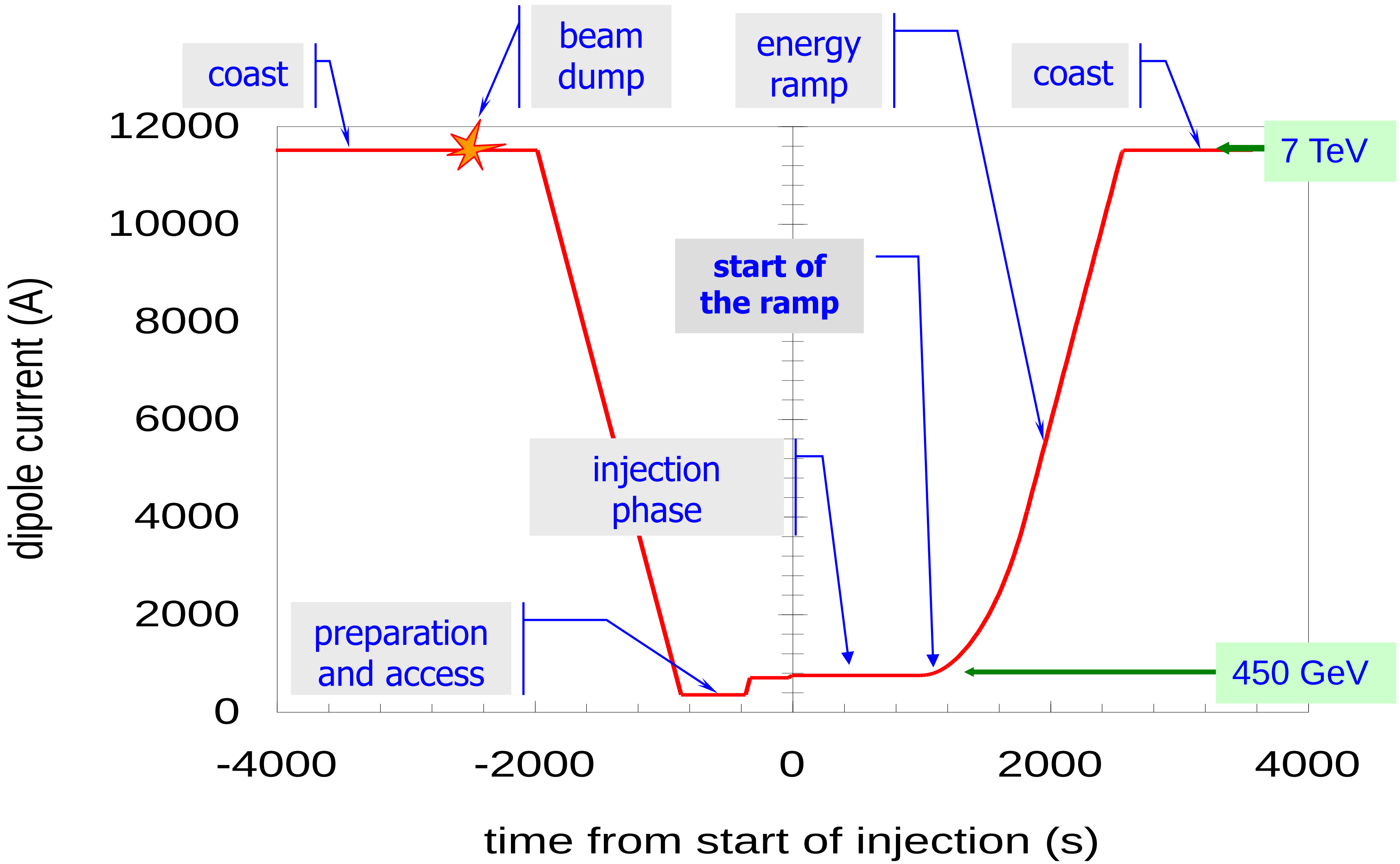
Store intensity up to 5e13 protons per cycle.

- Slow extraction to North Area
- Fast extraction to LHC, AWAKE and HiRadMat


LHC
LHC
27 km
Geneva lake
CMS
SPS
ALICE
ATLAS
LHC-b
Constructed to investigate the physics at the TeV scale

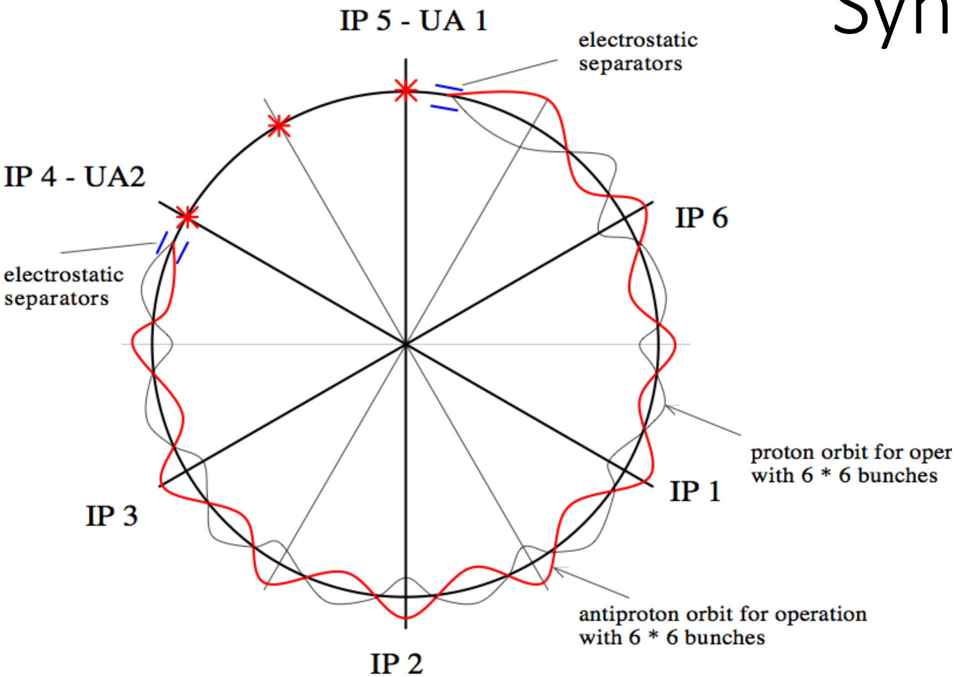
The Synchrotron – LHC Operation Cycle

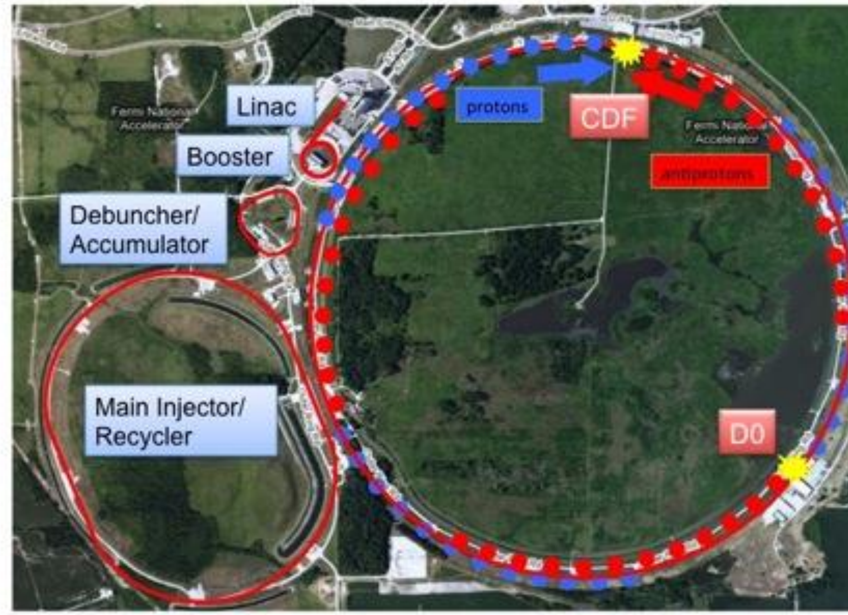
The magnetic field (dipole current) is increased during the acceleration.



Synchrotrons Colliders

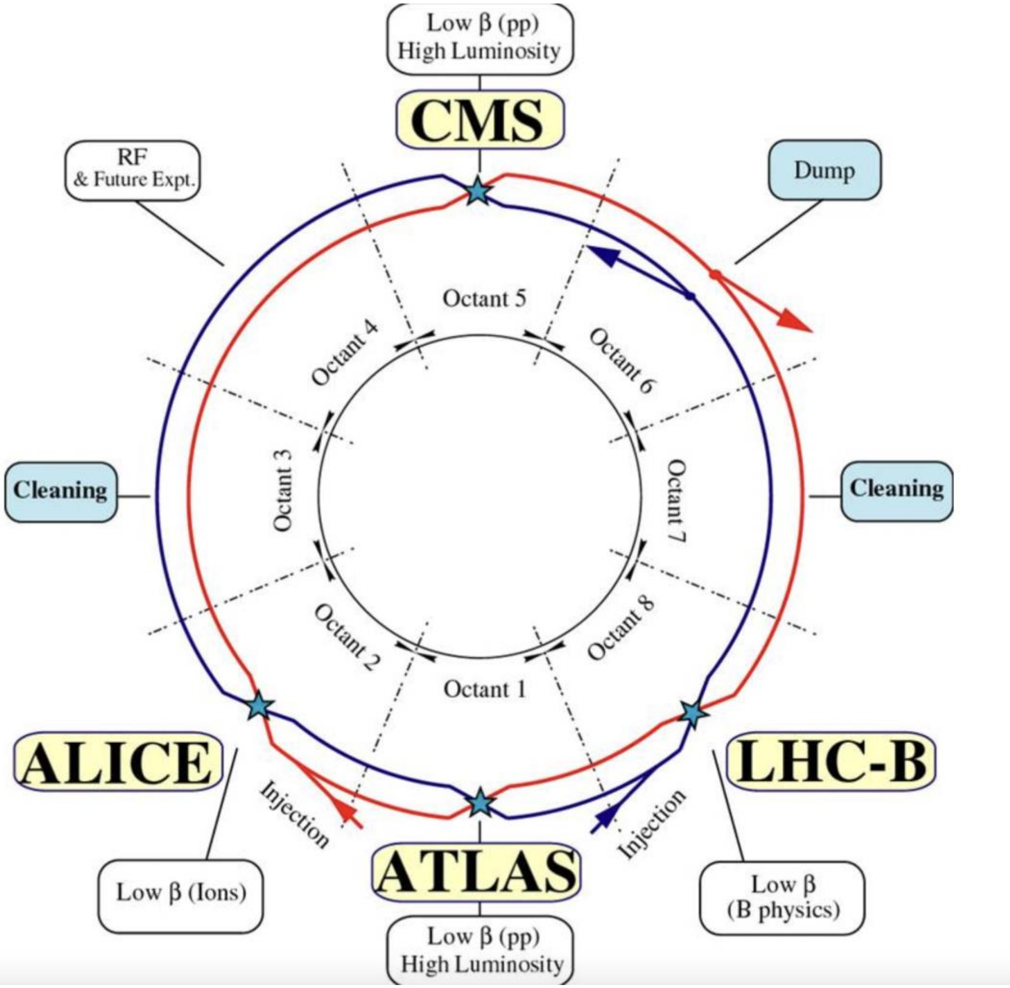
SPPbarS collider 6 bunches



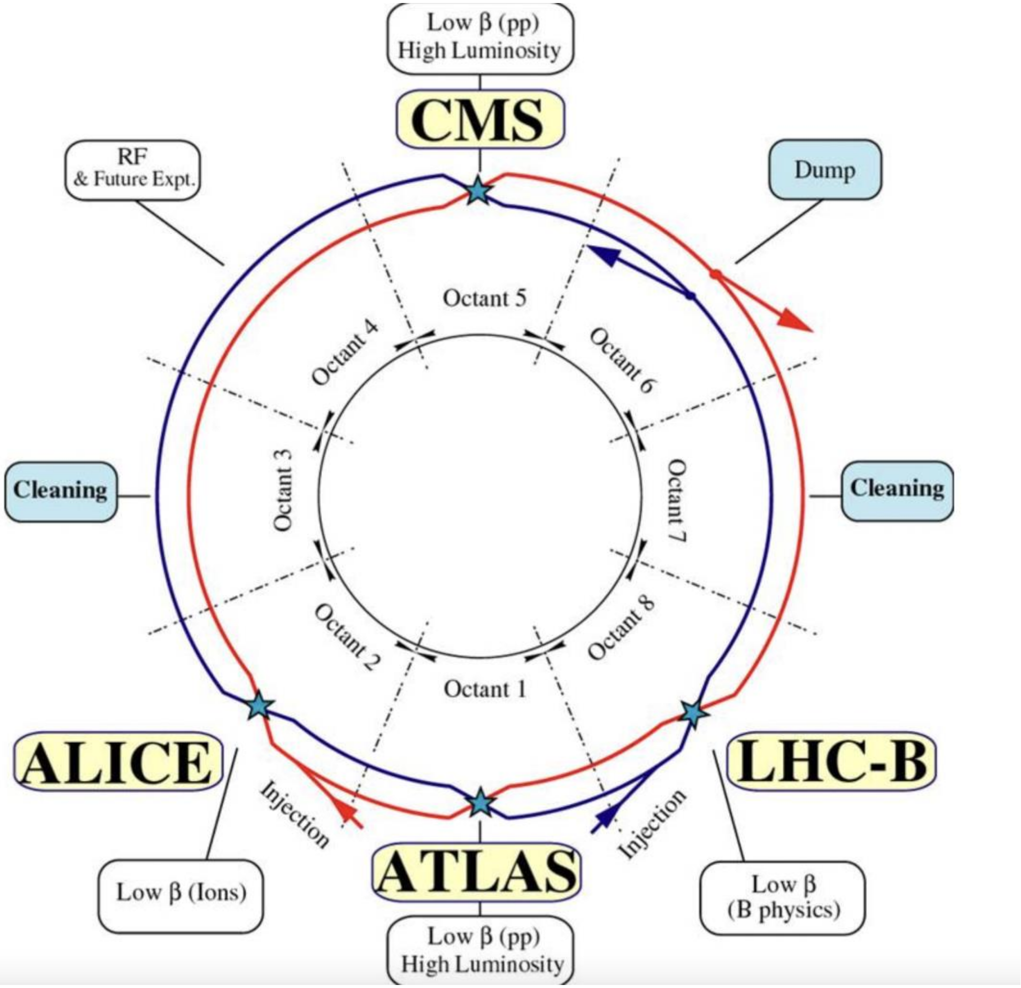


Tevatron: 36 bunches

RHIC: 110 bunches



1. Superconducting magnets
2. Cleaning and protection
3. Luminosity and Interaction Regions design
4. Collective effects



1. Superconducting magnets
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Superconducting magnets → LHC dipole field for 7 TeV protons

What is the needed dipole field to keep the protons circulating in the 27 km ring?

Magnetic rigidity $\rightarrow\ 0.3B[\mathrm{T}] \approx \frac{p[\mathrm{GeV}/c]}{\rho[\mathrm{m}]}$

The radius of the circumference cannot be just *27km/2π* as we need space for the detectors, RF, injection and extraction regions and collimation (so-called straight sections).
Approx. 2/3 of LHC ring are dedicated to the bending

$$\rho \approx 2.8\ \mathrm{m} \approx \frac{0.65 \times 26.7\ \mathrm{km}}{2\pi}$$

$$B[\mathrm{T}] \approx \frac{7000\mathrm{GeV}/\mathrm{c}}{0.3 \times 2.8\ \mathrm{m}} = 8.33\ \mathrm{T}$$

LHC Nominal dipole field 8.33 T

LHC super-conducting dipoles

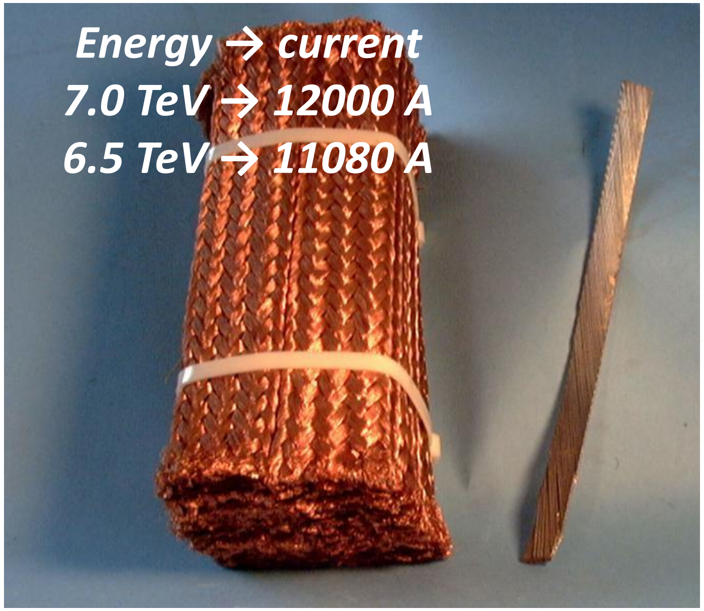
Previous machines use super-conducting magnets:

- Tevatron at FNAL 1987 - 2011: proton-antiproton collider
- HERA at Desy 1992 -2007: hadron-electron collider
- RHIC at BNL 2000 - present : relativistic heavy-ion collider

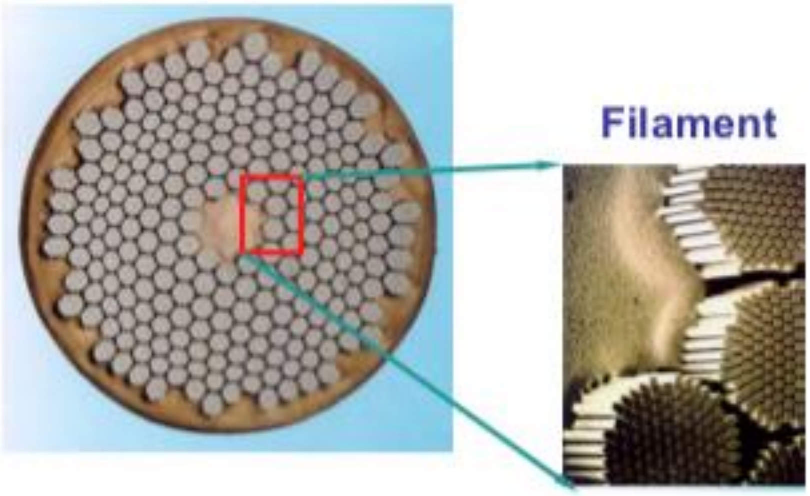
All used NbTi cooled with He at 4.2K with a maximum B-field ~ 5 Tesla

LHC also uses Nb-Ti (Cu clap) used but **to push the performance they are cooled to 1.9K using super-fluid He**.

With the drawback that a **very small energy deposition** (by beam interaction in the surroundings) or **the slightest microscopic movement of the conductor** could create a **magnet quench** (loosing super-conductivity). *unless the fault was detected quickly and the current turned off.*



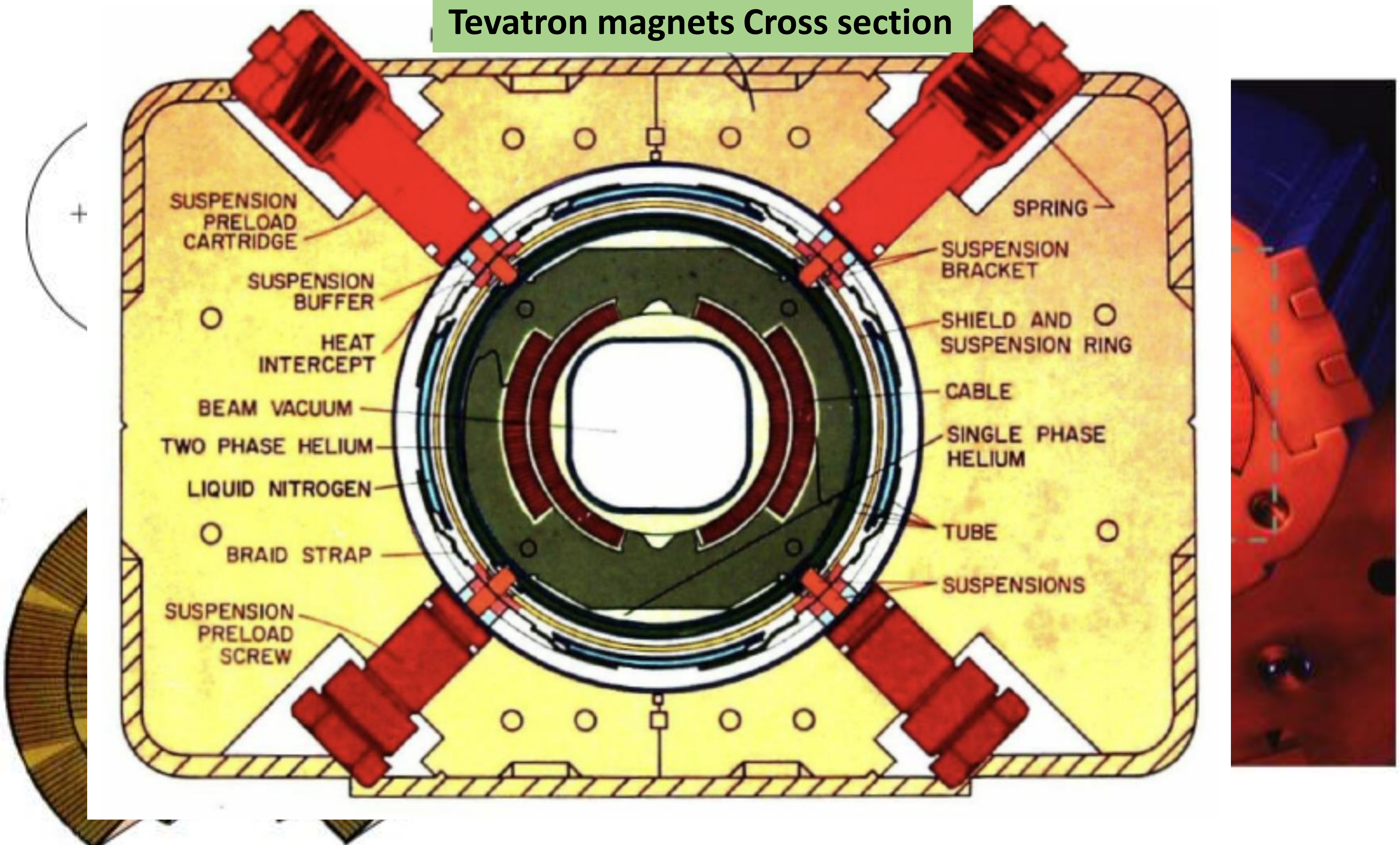
Niobium-Titanium Rutherford cable



Total superconducting cable required 1200 tonnes which translates to around 7600 km of cable.

The cable is made up of strands which is made of filaments, total length of filaments would go 5 times to the sun and back with enough left over for a few trips to the moon.

LHC dipole



$$cos\theta$$

LHC cross-section

Re-use the LEP tunnel constrained the size of the magnet using the **two-in-one design**.

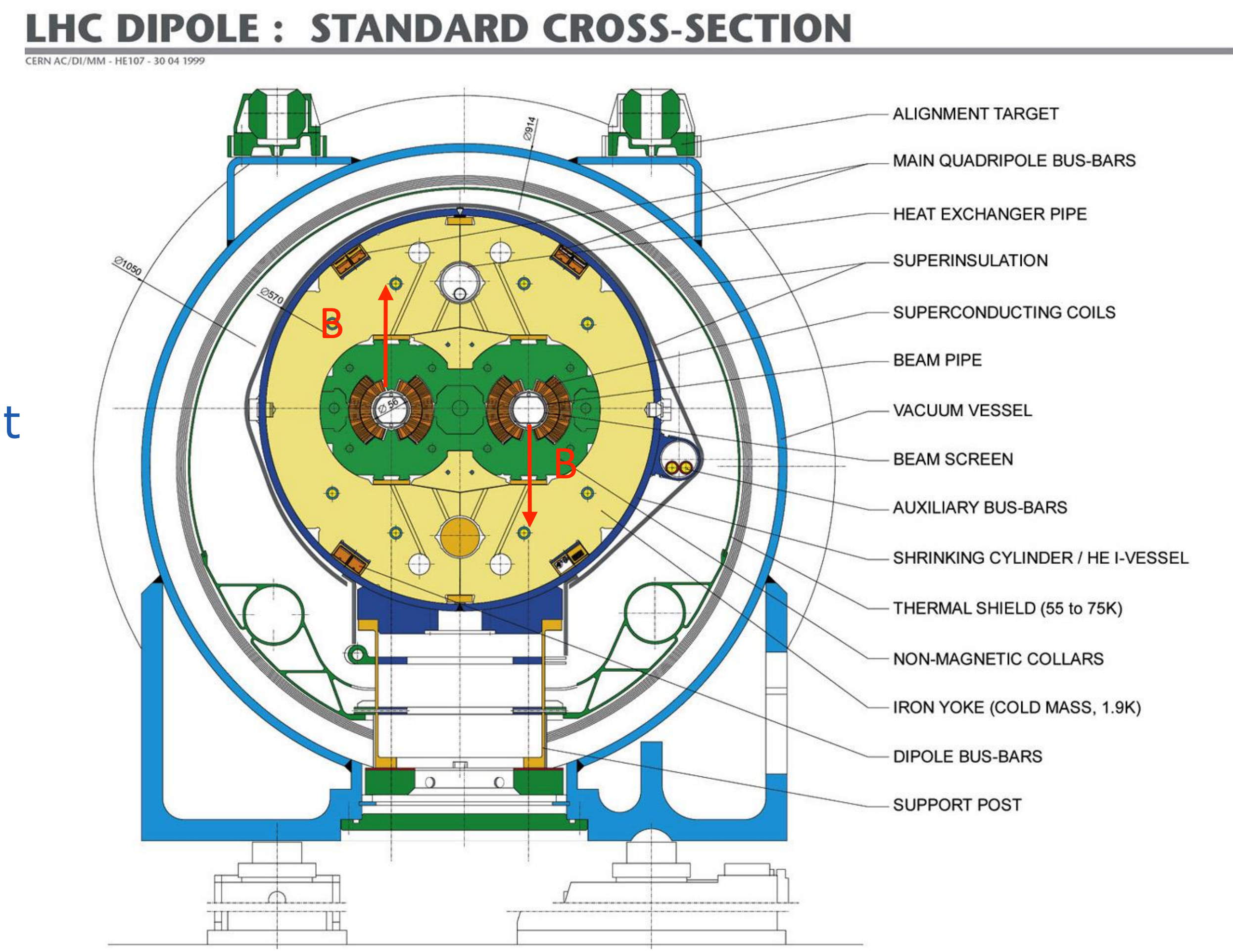
Two beam channels in a common cold mass cryostat and magnetic flux in opposite sense.

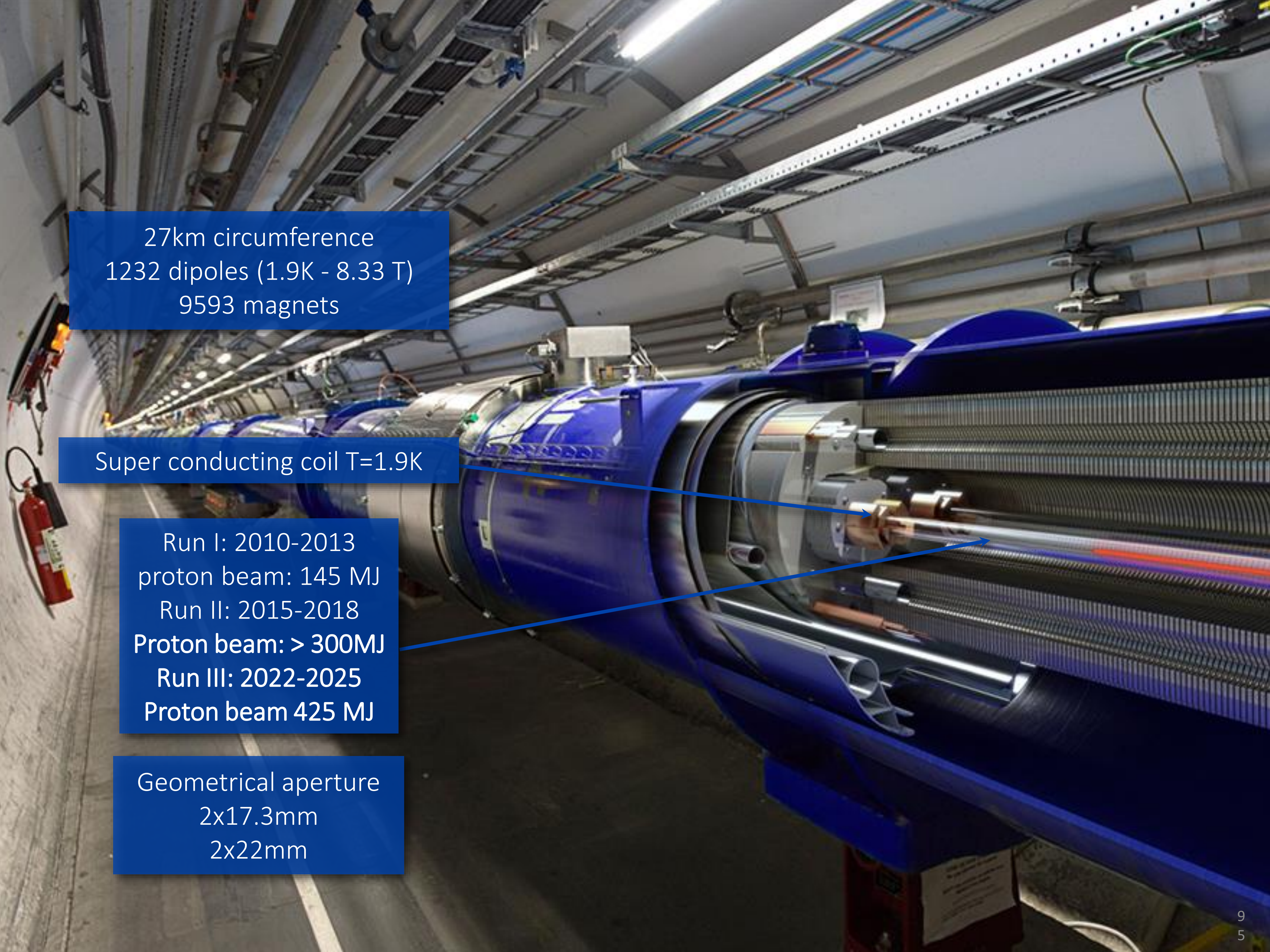
Complex design.

Dimensions of the dipole beam screen are:

22 mm horizontal

17 mm vertical



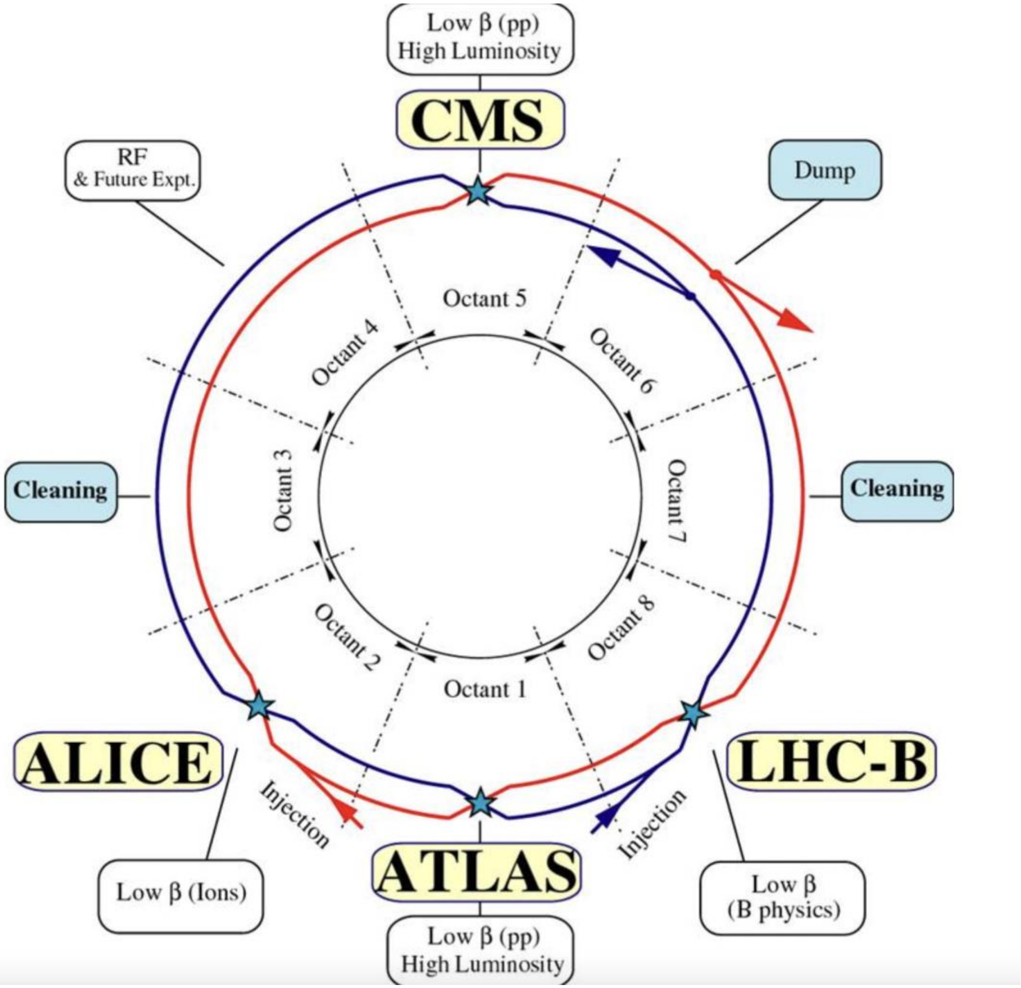


27km circumference
1232 dipoles (1.9K - 8.33 T)
9593 magnets

Super conducting coil T=1.9K

Run I: 2010-2013
proton beam: 145 MJ
Run II: 2015-2018
Proton beam: > 300MJ
Run III: 2022-2025
Proton beam 425 MJ

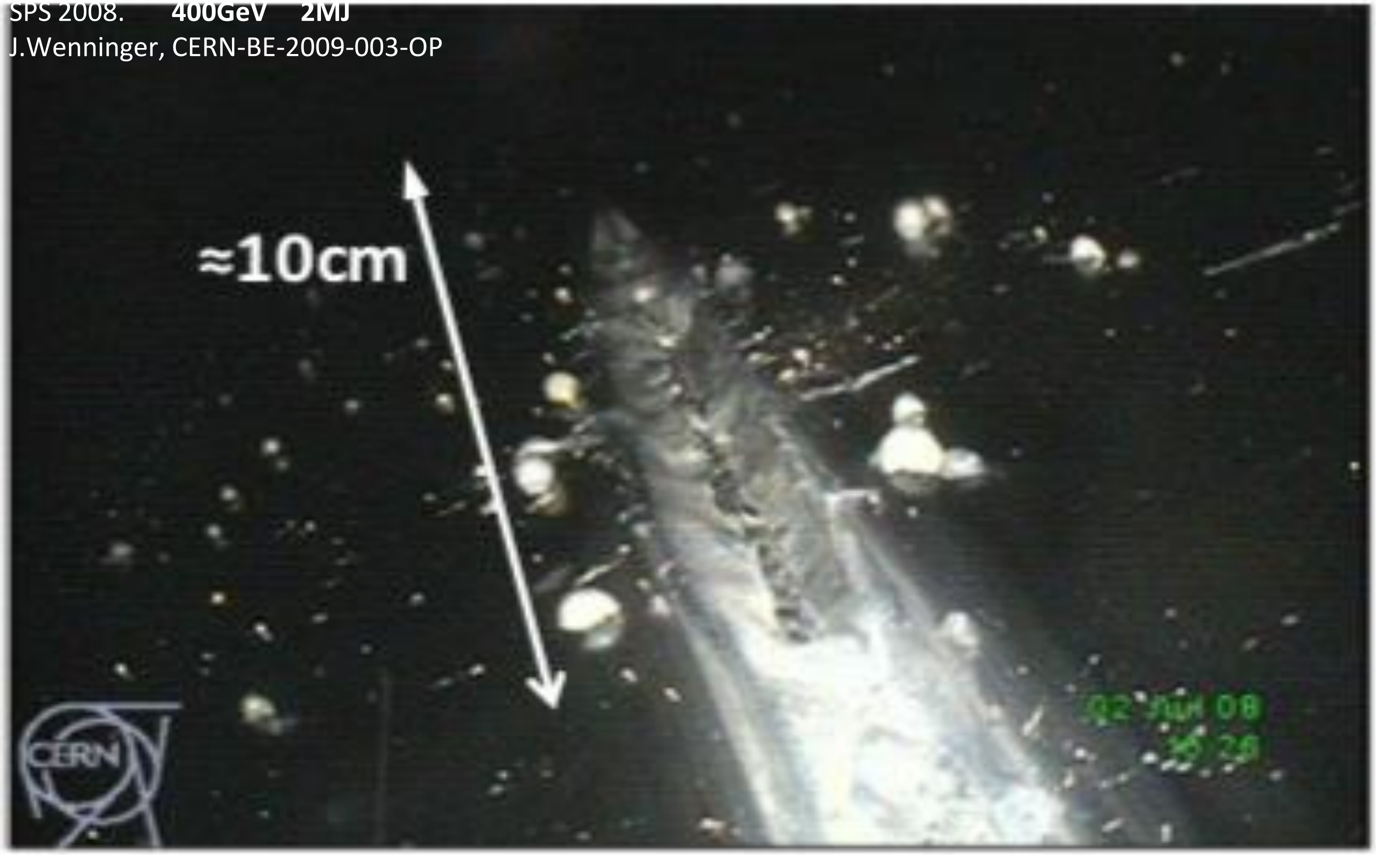
Geometrical aperture
2x17.3mm
2x22mm



1. Superconducting magnets
2. Cleaning and protection
3. Luminosity and Interaction Regions design
4. Collective effects

Comparison of the 3 LHC Running Periods

Energy depositions **at 6.5TeV ~ 100 mJ/cm³ risk to initiate a quench.**



Stored en ... 3 TeV)

A quench without damage will require ~ 10 hours of cool down time to recover the cryogenic conditions. With damage > 3 months.

At 6.8 TeV with about 3e14 proton beams, a tiny fraction of beam, 0.00002%, could quench a magnet (~ 6e7protons)

Beam Losses at LHC

- A tiny fraction of the full beam is enough to damage equipment
- Therefore, a very control of beam losses is mandatory to ensure safe LHC operation

Normal Losses

They can be minimised but **cannot be avoided completely**
Due to beam dynamics: particle diffusion, scattering processes, instabilities.
Due to Operational variations: orbit, tune, chromaticity changes during ramp, squeeze, collision.

Collimation *system (smallest aperture) is designed to* ***catch*** *increased* ***beam losses up to 500kW over 10sec.***

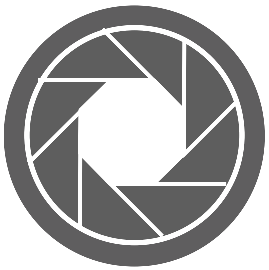
Beam Loss Measurements that extract the beam if exceed the specified max. loss rates.

Abnormal losses

Due to failure or irregular behaviour of accelerator components.

LHC Collimation System

LHC Collimation system guarantees that **losses will not reach the cold region.**



Like a diaphragm in a camera, collimators are the closest elements to the circulating beam concentrating the losses in the collimation regions.

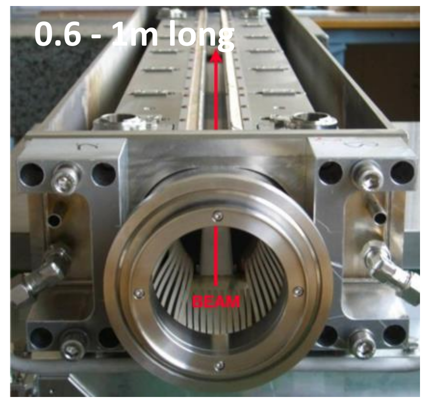
Collimator Design

Two parallel jaws in a vacuum tank at different orientations.

Jaw material depends on its functionality:

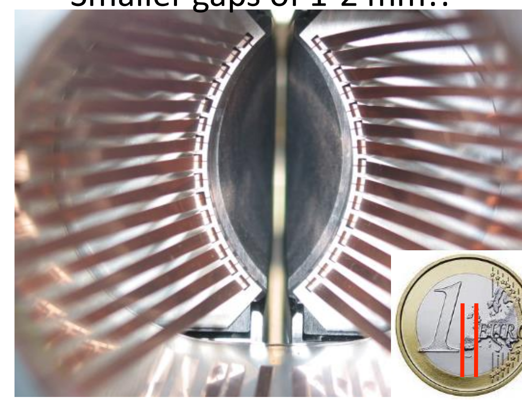
- **Carbon** (primary and secondary collimators)
- **Copper and Tungsten** (absorbers and tertiary collimators)

Movable jaws, controlling **gap and jaw angle with precision of 5 microns**



LHC Collimator with vacuum tank opened

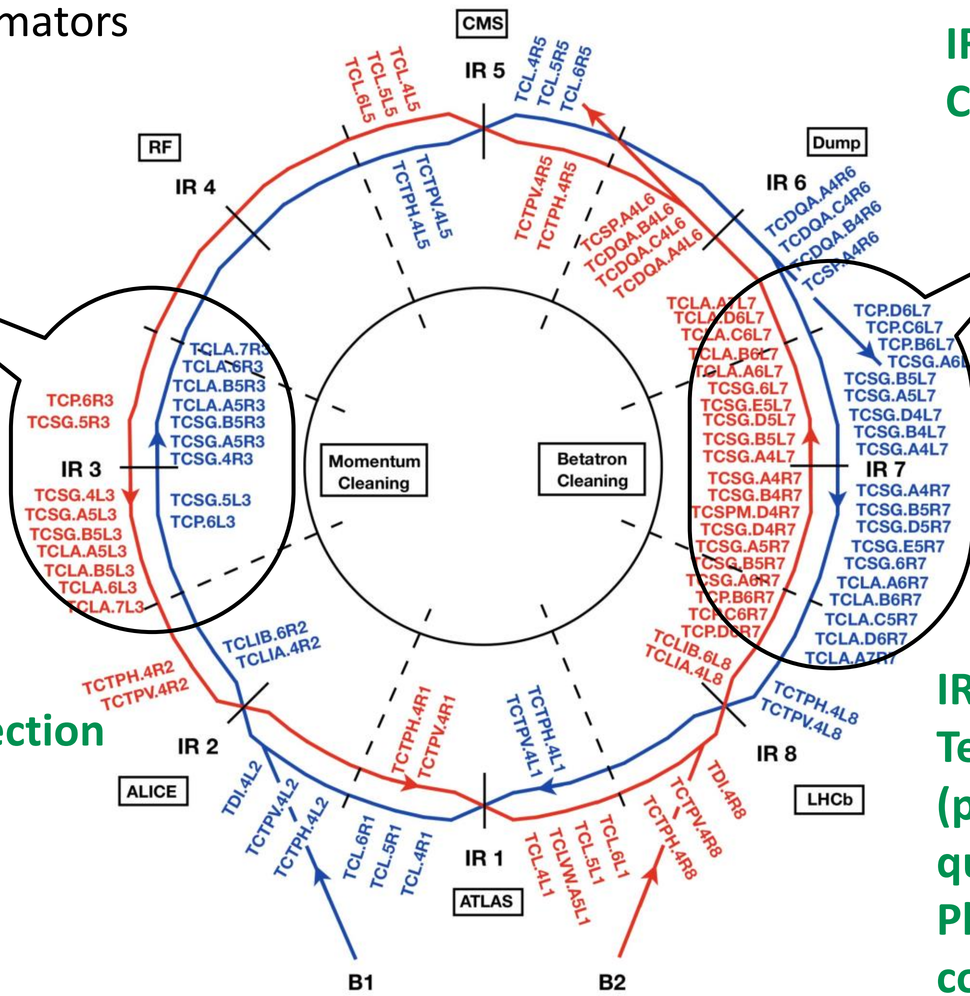
Smaller gaps of 1-2 mm!!



LHC Collimation System

108 Movable Collimators

Momentum cleaning: particles with different momentum are absorbed in this area



IR6 Extraction Collimators

Main Transverse **Betatron** Beam Halo Cleaning

IR2/IR8 Injection Protection collimators

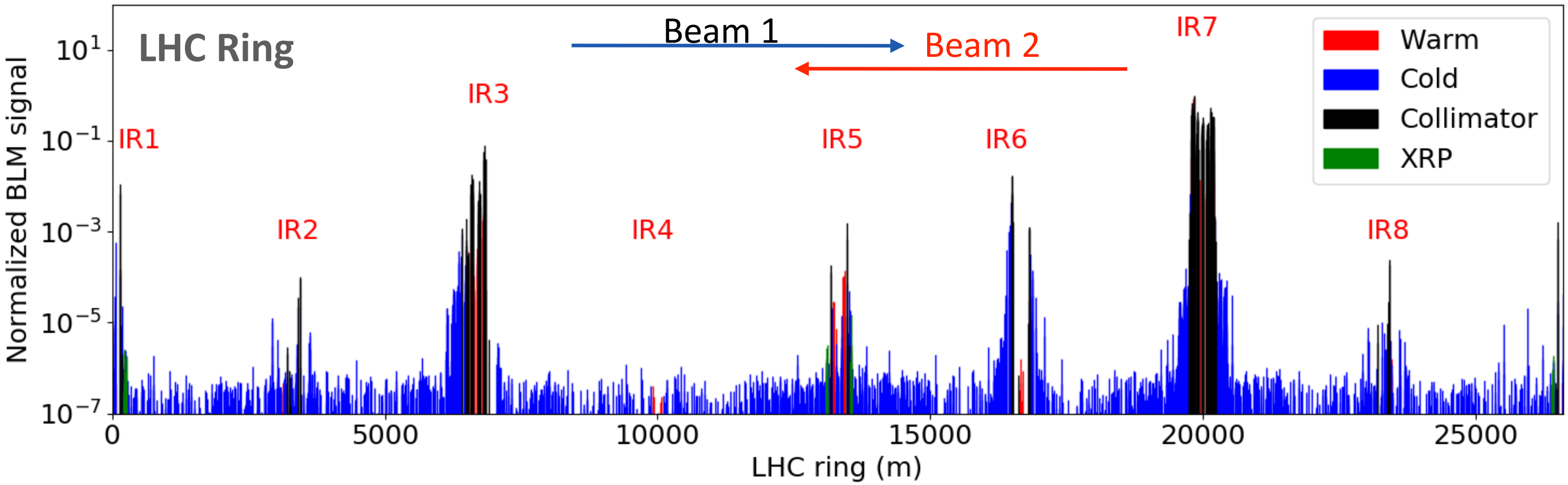
IR1,IR2,IR5,IR8 Tertiary Collimators (protecting the triplet quadrupoles) Physics Debris collimators

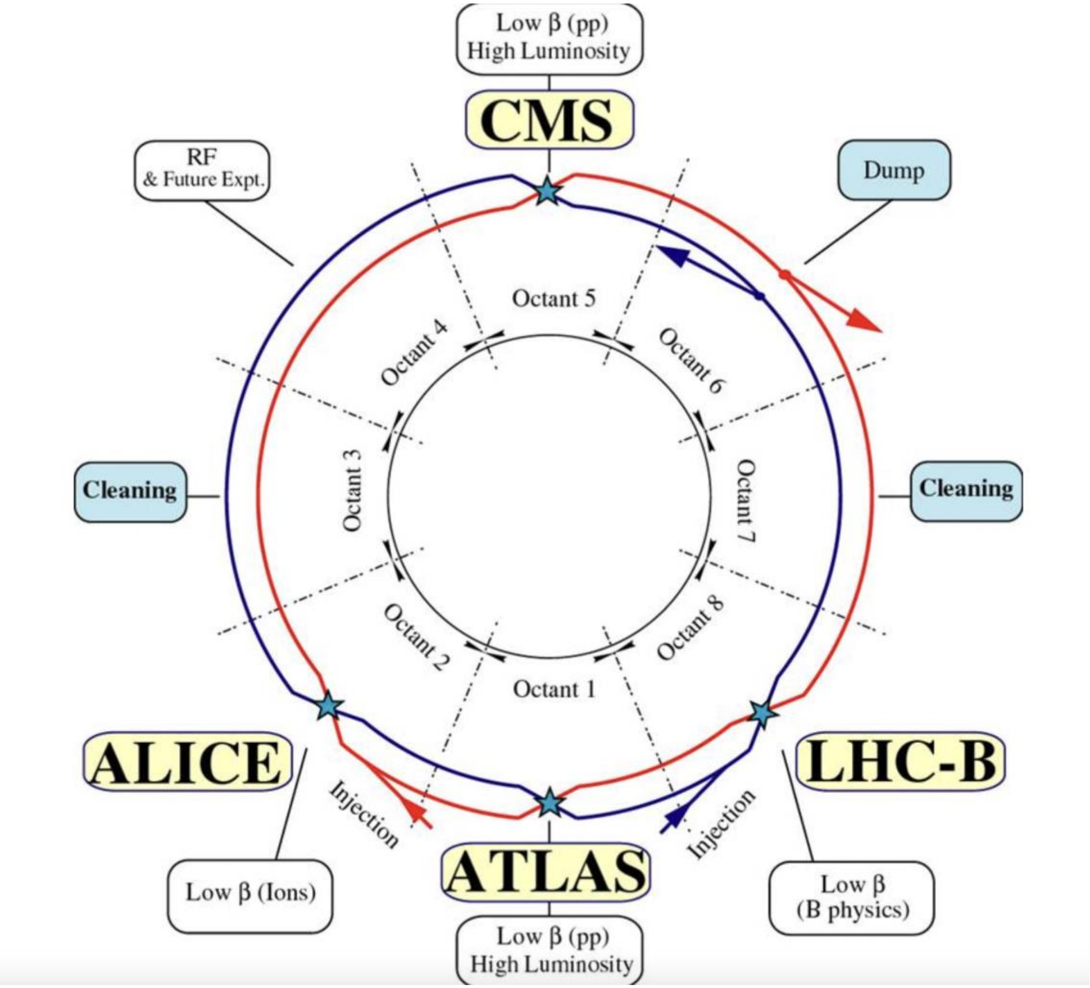
LHC Beam Loss Monitoring

Approximately 4000 Beam Loss Detectors (ionization chambers) distributed along the LHC covering critical locations:

- Losses in the cold area: dipoles, quadrupoles, etc.
- Losses at injection and extraction: transfer lines
- Losses down stream each collimator.

Losses are concentrated in warm regions





1. Superconducting magnets
2. Cleaning and protection
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Luminosity

For accelerator people **this IS the quantity used to optimise the machine.**

The higher the luminosity the better.

$$N_{\text{event}} = L\sigma_{\text{event}}$$

Accelerator

Nature

Number of particles per bunch

Number of bunches

$$L = \frac{N_{\text{b}}^2 n_{\text{b}} f_{\text{rev}} \gamma_r}{4\pi\epsilon_n \beta^*} F$$

Geometric Reduction factor

$$F = 1/\sqrt{1 + \left(\frac{\theta_c \sigma_z}{2\sigma^*}\right)^2}$$

Transverse Emittance

Beta-star

Crossing Angle

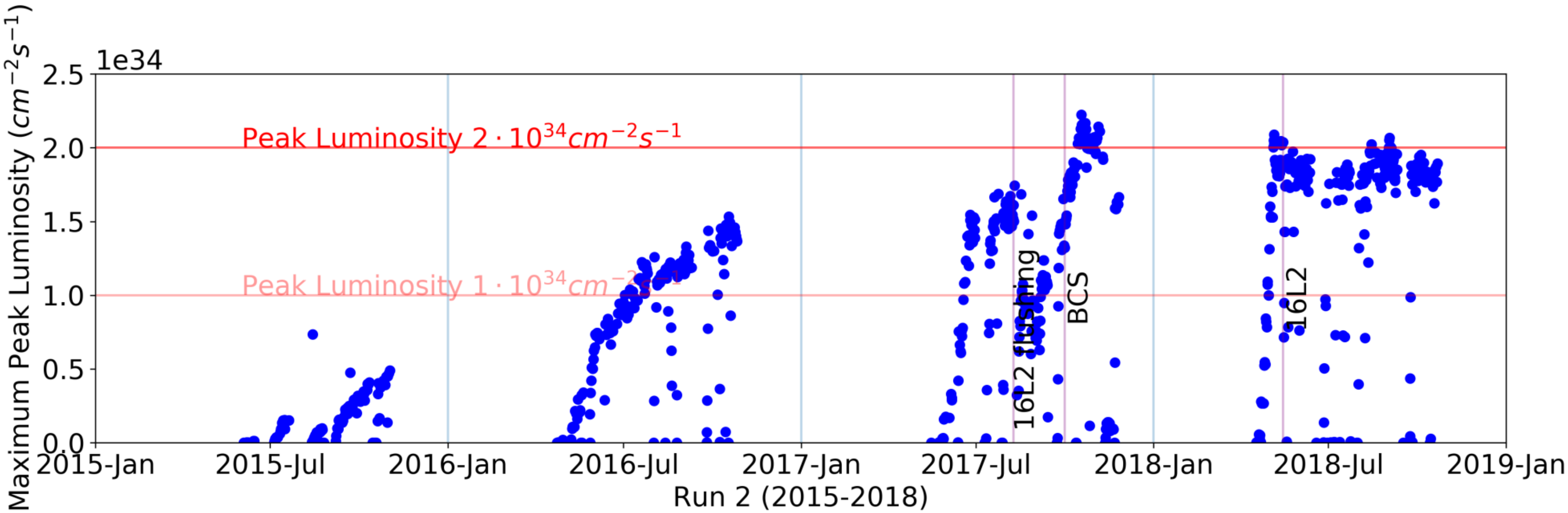
Bunch length

LHC nominal parameters

Table 2.1: LHC beam parameters relevant for the peak luminosity

		Injection	Collision
Beam Data			
Proton energy	[GeV]	450	7000
Relativistic gamma		479.6	7461
Number of particles per bunch		1.15×10^{11}	
Number of bunches		2808	
Longitudinal emittance (4σ)	[eVs]	1.0	2.5^a
Transverse normalized emittance	[μm rad]	3.5^b	3.75
Circulating beam current	[A]	0.582	
Stored energy per beam	[MJ]	23.3	362
Peak Luminosity Related Data			
RMS bunch length[c]	cm	11.24	7.55
RMS beam size at the IP1 and IP5[d]	μm	375.2	16.7
RMS beam size at the IP2 and IP8[e]	μm	279.6	70.9
Geometric luminosity reduction factor F[f]		-	0.836
Peak luminosity in IP1 and IP5	[$cm^{-2}sec^{-1}$]	-	1.0×10^{34}
Peak luminosity per bunch crossing in IP1 and IP5	[$cm^{-2}sec^{-1}$]	-	3.56×10^{30}

Peak luminosity



How the increase of peak luminosity was achieved?

LHC Runs Challenges

Energy

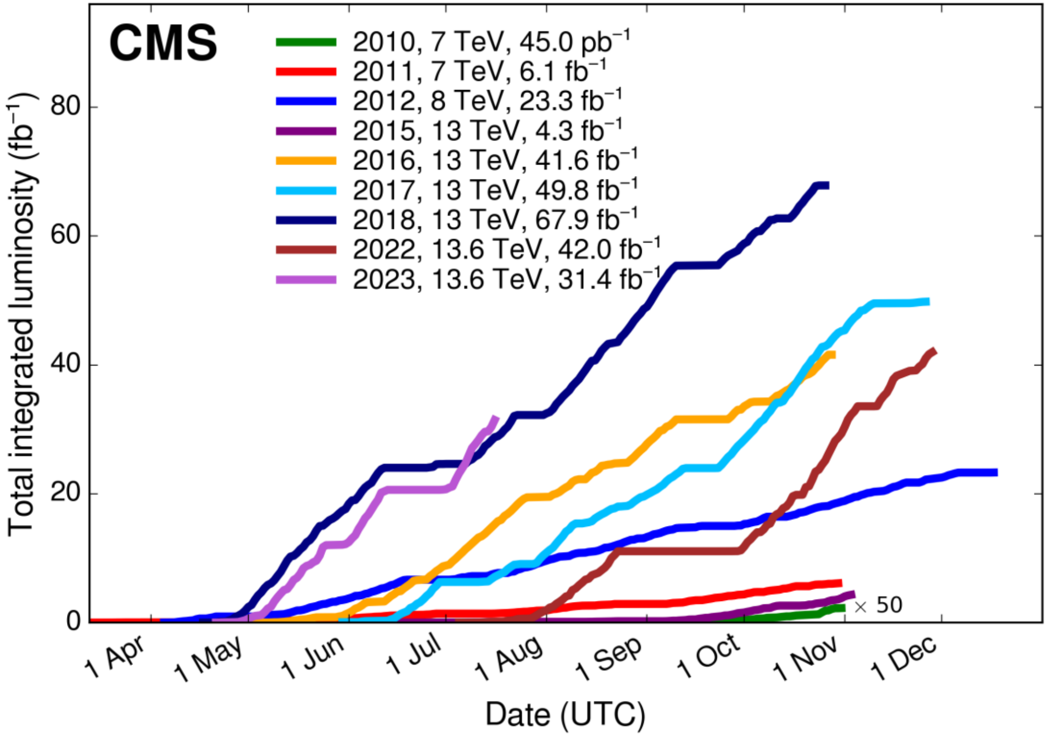
- Lower quench margins
- Lower tolerance to beam loss
- Hardware closer to maximum (beam dumps, power converters etc.)

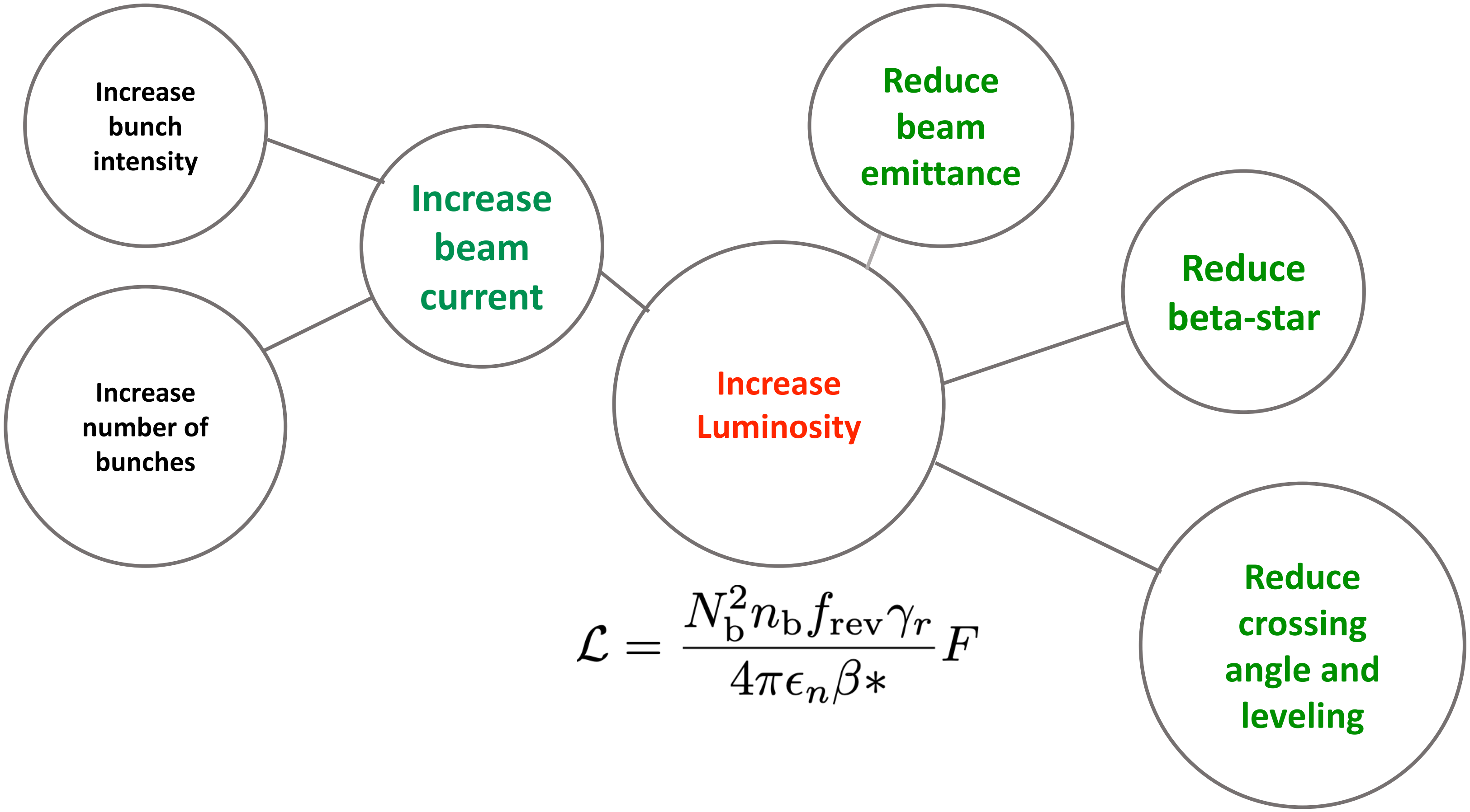
25 ns

- Electron-cloud
- UFOs
- More long range collisions
- Larger crossing angle, higher beta*
- Higher total beam current
- Higher intensity per injection

Smaller Beta-star

- Smaller machine aperture
- Tighter collimator settings
- Higher beam losses



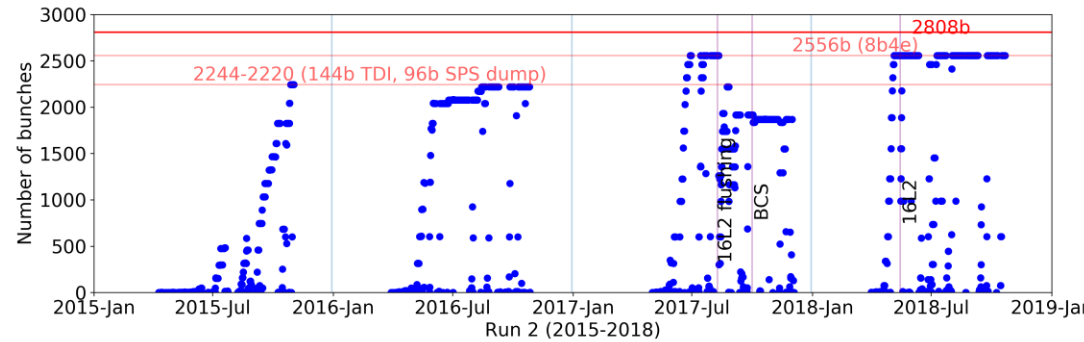


$$\mathcal{L} = \frac{N_{\rm b}^2 n_{\rm b} f_{\rm rev} \gamma_r}{4\pi \epsilon_n \beta *} F$$

Increase of beam current

Number of bunches

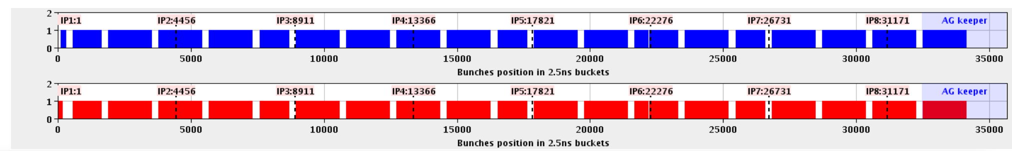
Early 2015 went from 50ns bunch spacing to 25ns.

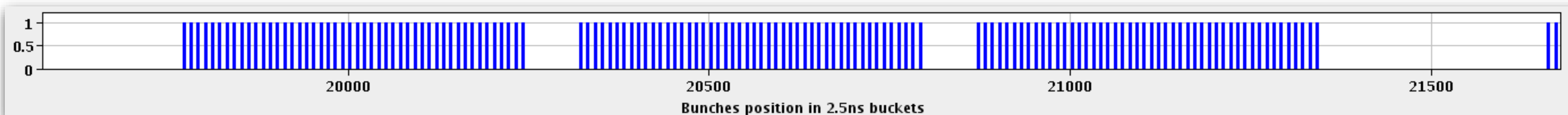


Example 2017

144 bunches SPS batch (max 2556b)

Based on 48 PS batch x 3





Number of protons/bunch

With past LINAC 2 (50 MeV max energy)

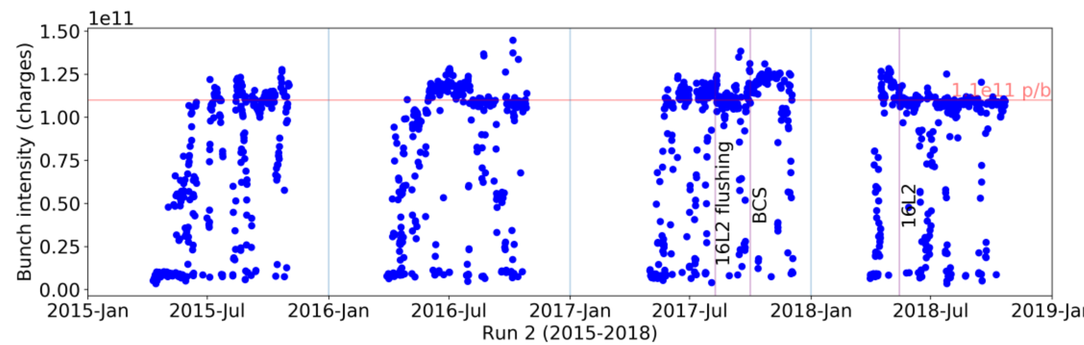
Average 1.1e11p/b in 2018

Peak ~ 1.5e11p/b

With LINAC 4 (160 MeV)

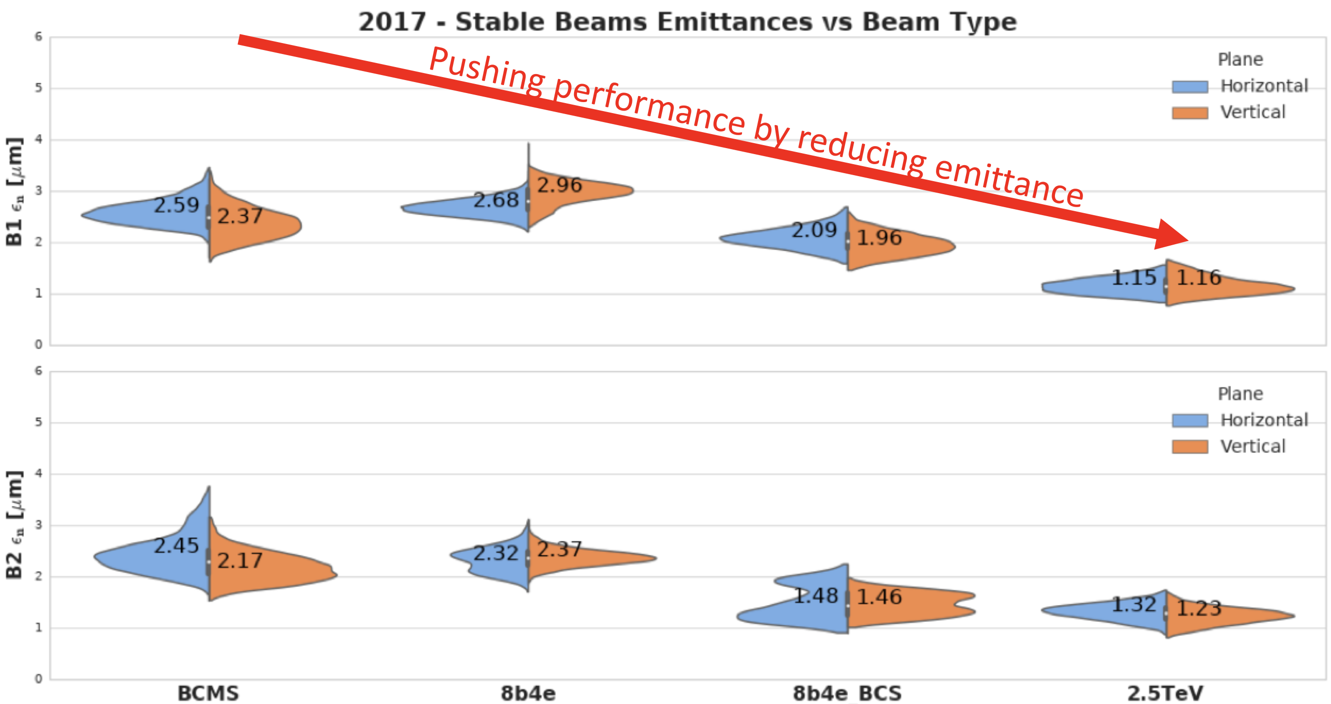
Average 1.6 e11p/b in RUN III

Peak ~ 2.2e11p/b → ready for HL-LHC era



Reduction of beam emittance

Different bunch splitting and merging in PS gives a **push on beam brightness** (reduction of emittance)

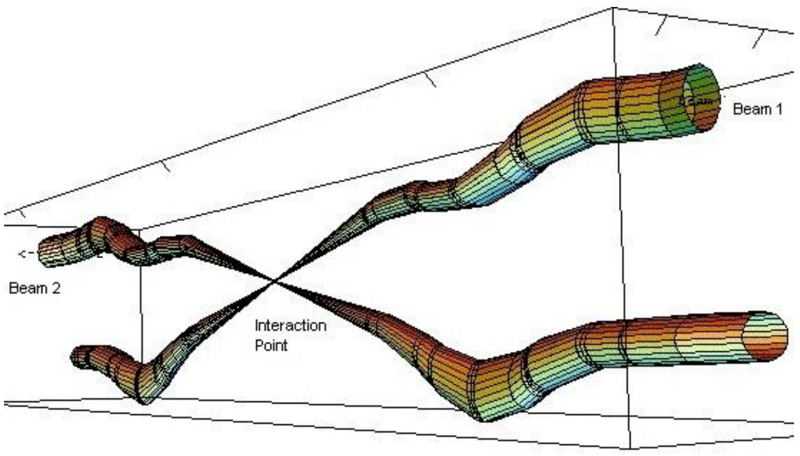


***Higher peak luminosity** at the cost of **higher pile-up** due to reduced number of bunches*

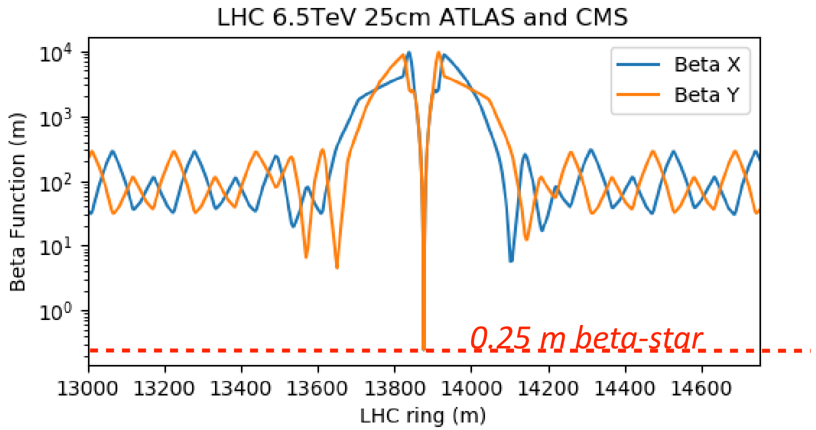
Beta-star

Reduction of beta-star in ATLAS/CMS over Run 2:

- *2015:* **80cm**
- *2016:* **40 cm**
  First time below Nominal values
- *2017:* **40cm → 30cm**
- *2018-2023:* **Dynamic squeeze in Stable Beams: 30cm → 27 cm → 25 cm**
- **2029 15 cm**



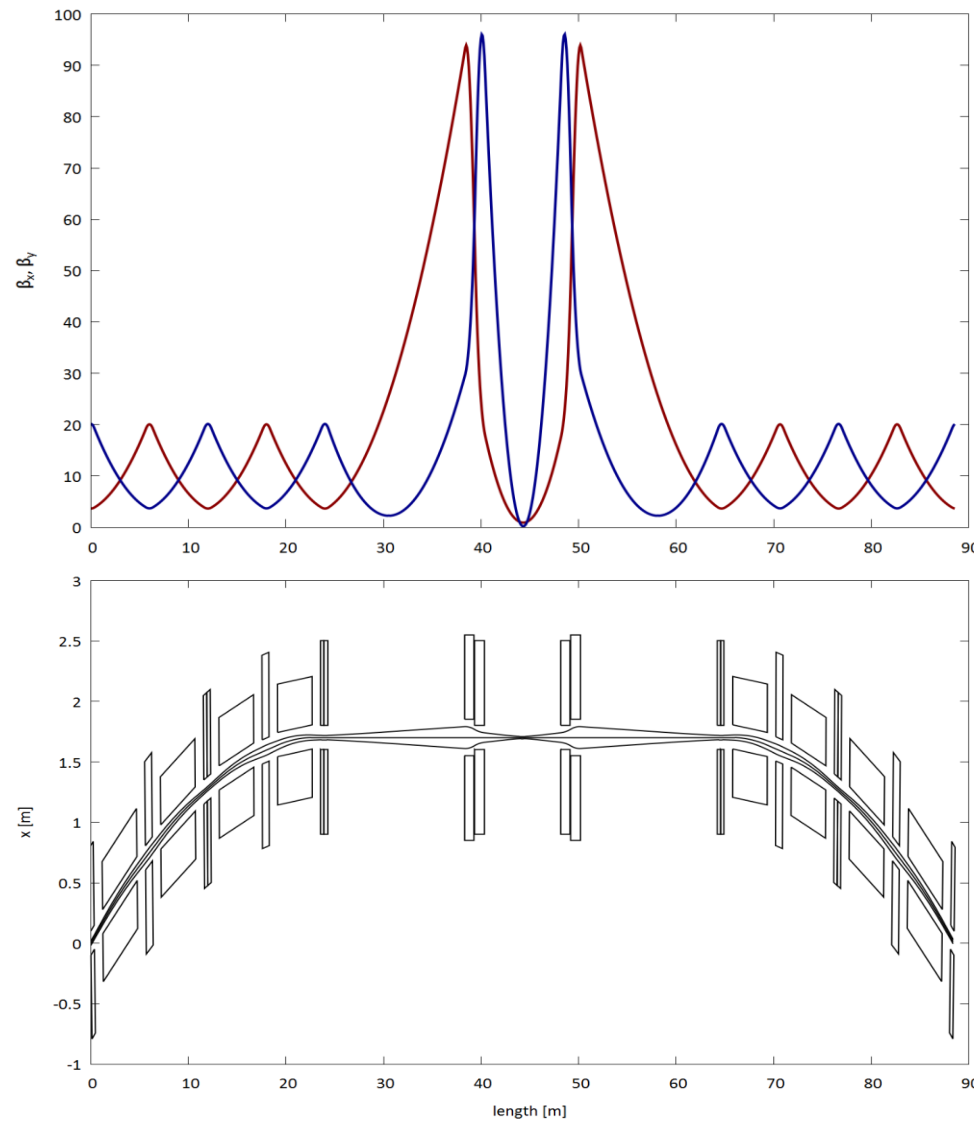
concept sketch: using a quadrupole doublet it is possible to focus particles in the horizontal and vertical planes simultaneously through the interaction point



Low Beta Insertion

the most simple IR configuration

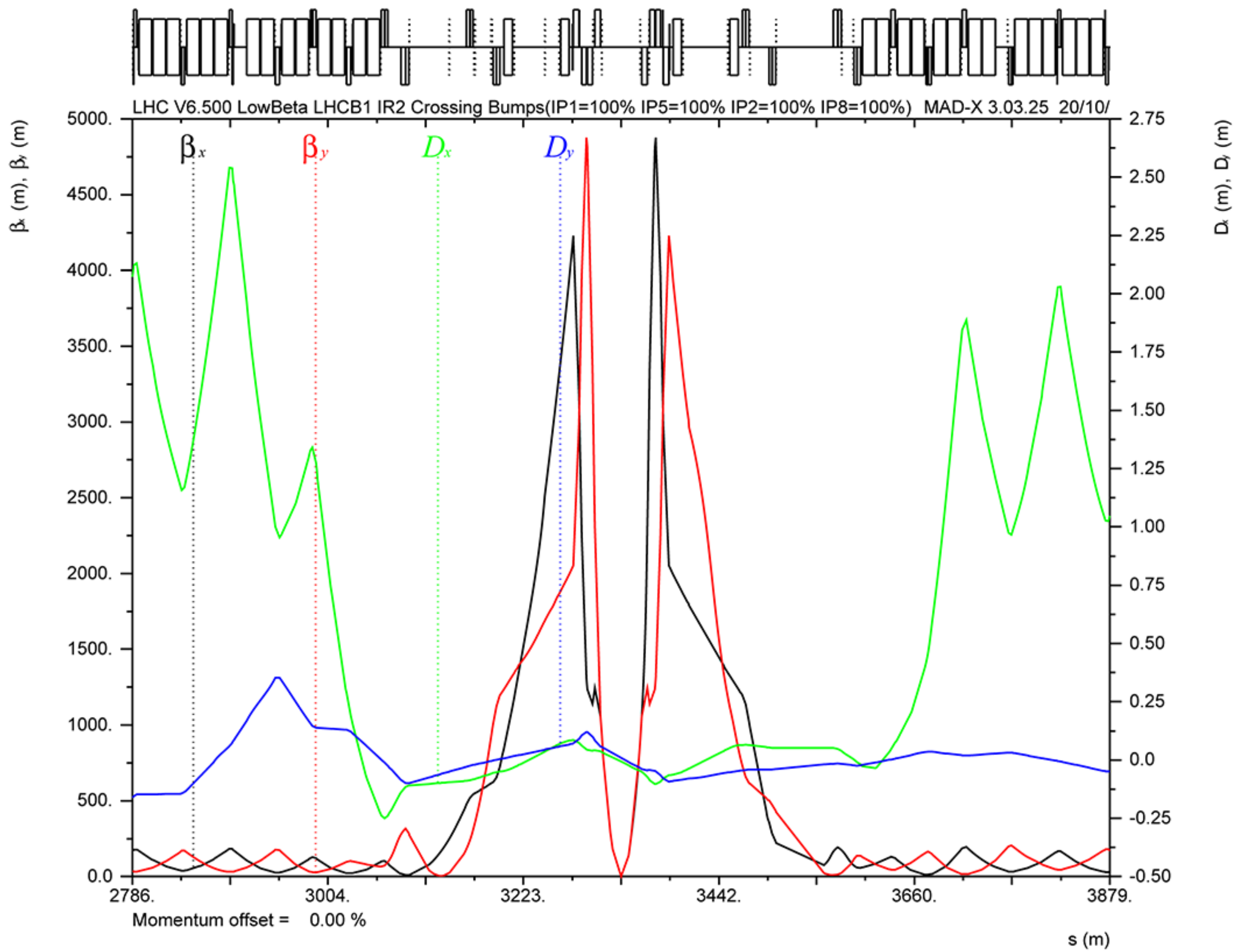
- doublet focusing
- large beta function in doublet
  $\rightarrow$ aperture limitation for ring



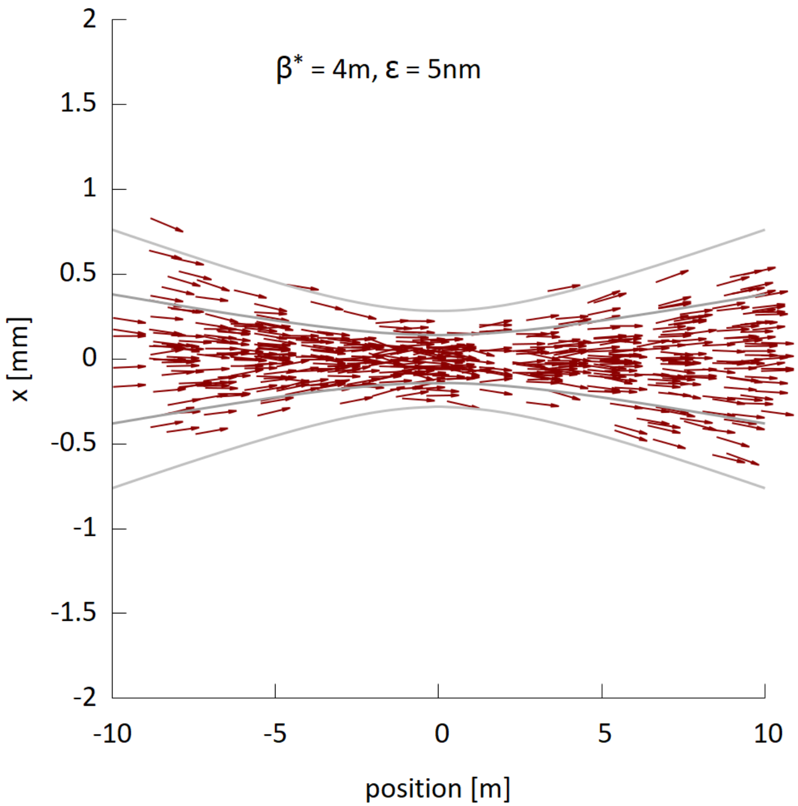
see also Wiedemann
sec. 10.2.4

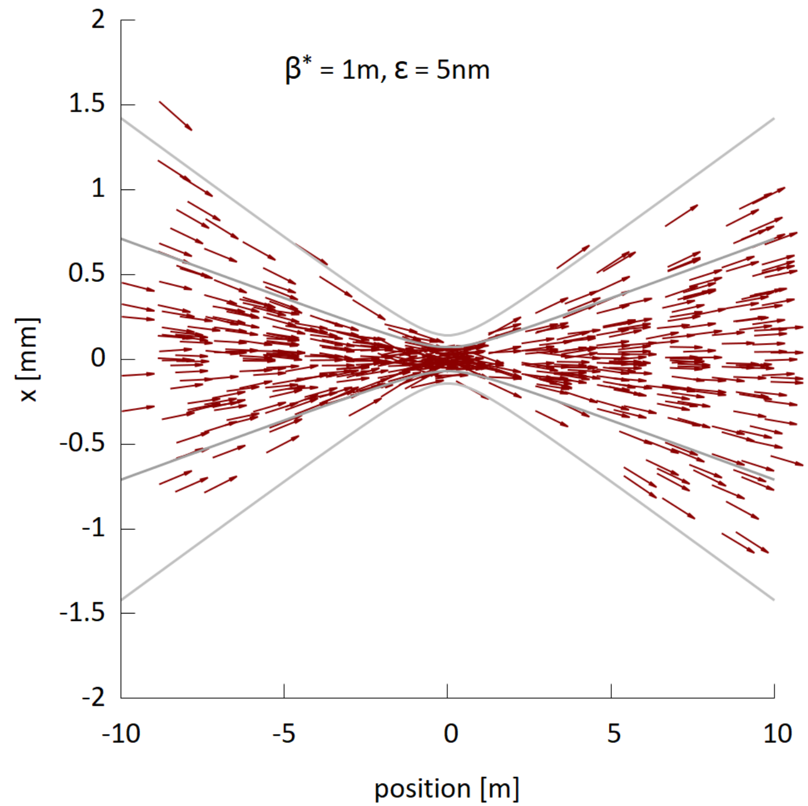
Low Beta Insertion – Example of LHC

LHC interaction region with Low-Beta + D.S.



Beam Waist (e.g. interaction point collider)



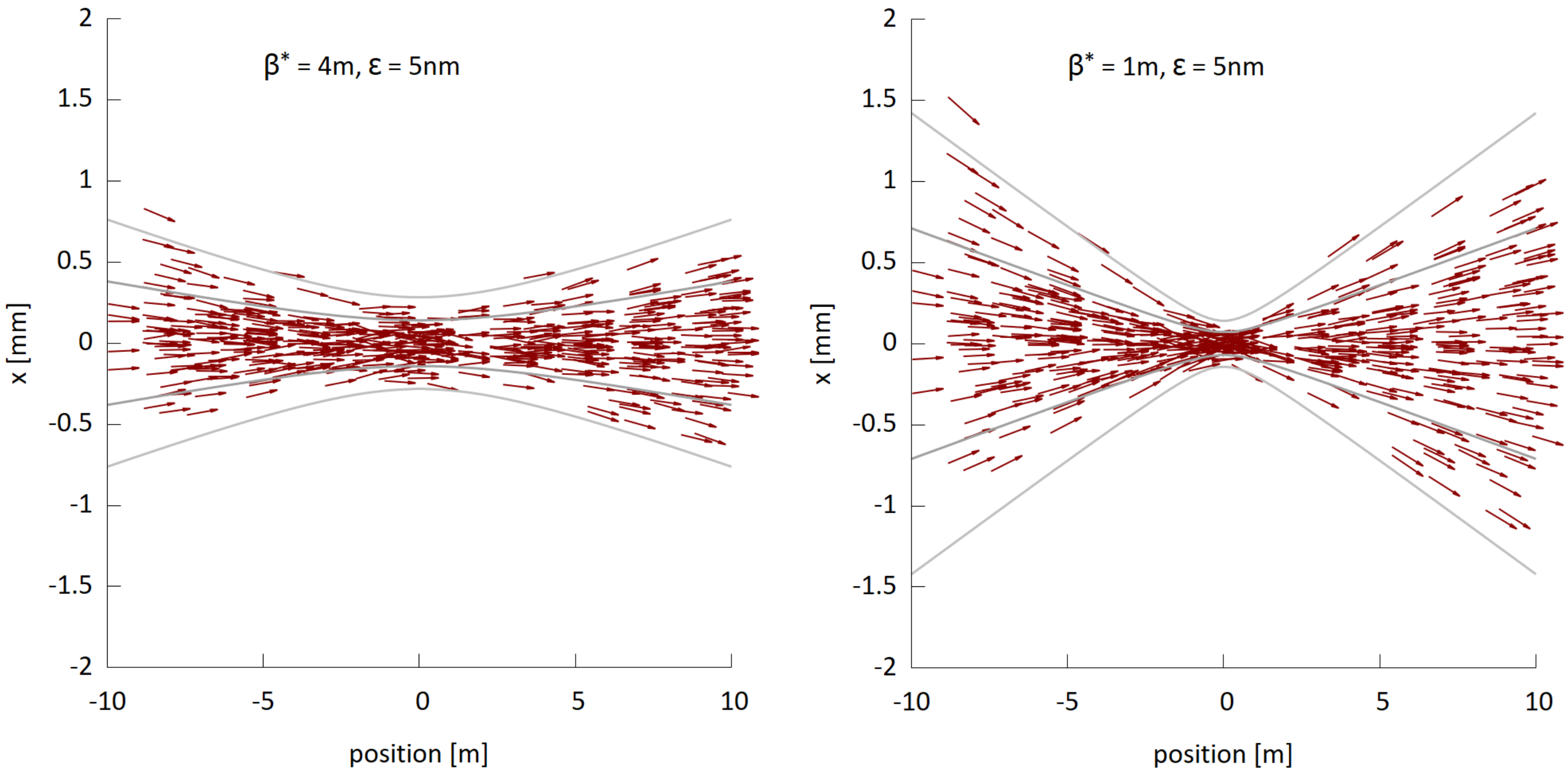


$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

$$\sigma_{\text{rms}} = \sqrt{\varepsilon \beta^*}, \;\; \sigma'_{\text{rms}} = \sqrt{\frac{\varepsilon}{\beta^*}}$$

β* = Beta function at waist

Beam Waist (e.g. interaction point in collider)

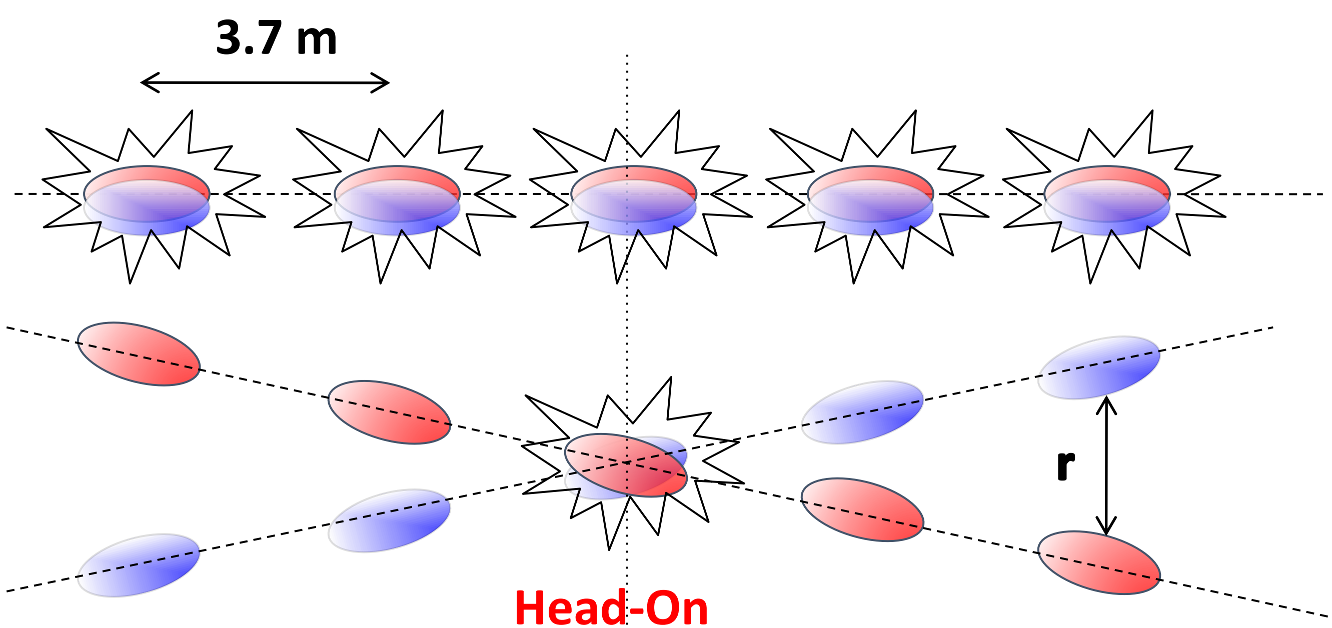


Crossing angle operation

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

Num. of maximum bunches $n_b = 2808$

Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments



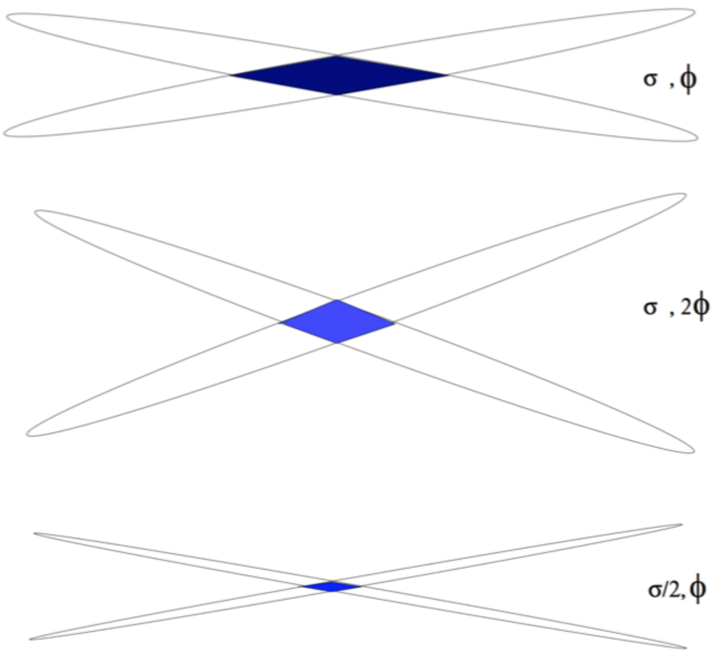
A finite crossing angle has to be applied to avoid multiple collision points

Luminosity Geometric reduction factor

Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot \mathcal{S}$$

S is the geometric reduction factor



$$\sigma_s >> \sigma_{x,y}$$

Always valid for LHC and HL-LHC
σ_x = 17-7 μm, σ_s = 7.5 cm

$$S \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

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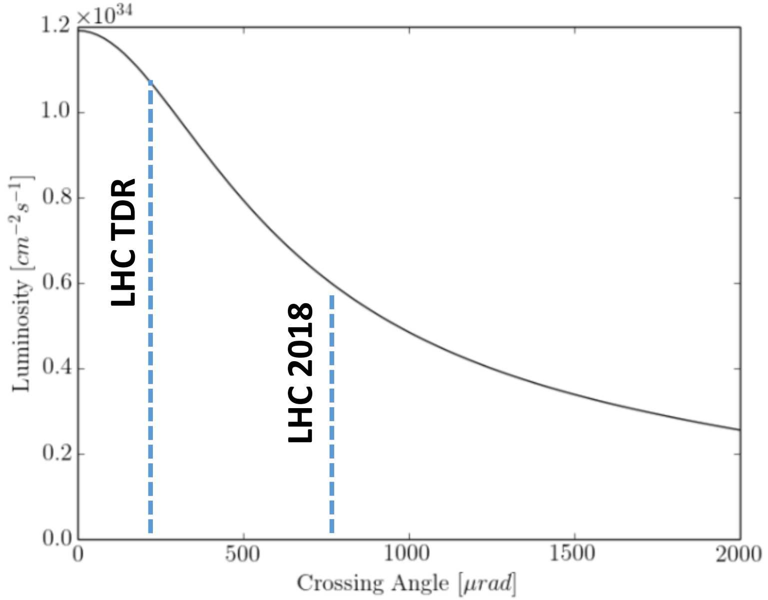
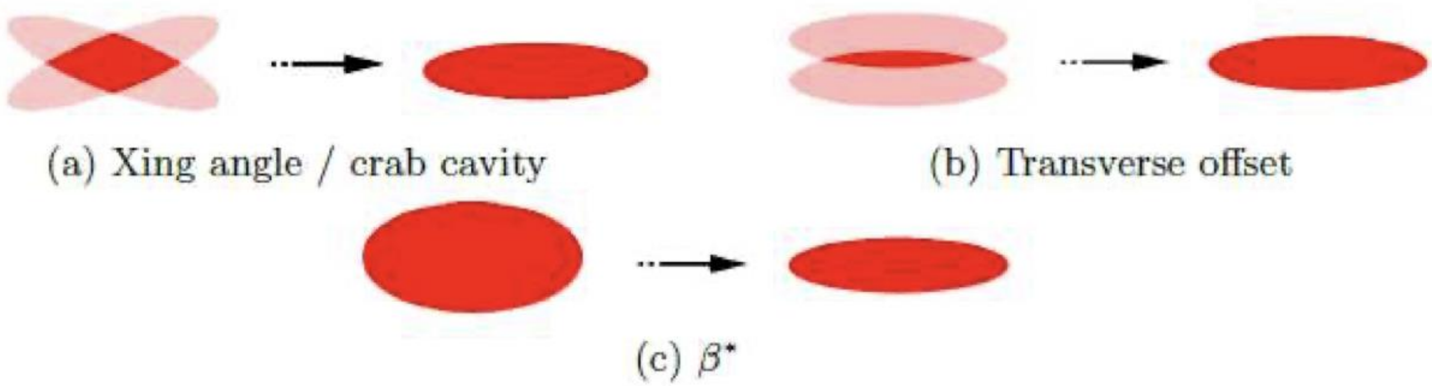


LHC design: ϕ = 285 μrad, σ_x = 17 μm, σ_s = 7.5 cm, S=0.84
LHC 2018: ϕ = 320 μrad, σ_x = 9.3 μm, σ_s = 7.5 cm, S=0.61

Luminosity Levelling at LHC



a) Crossing angle levelling
Modification of large local orbit bump

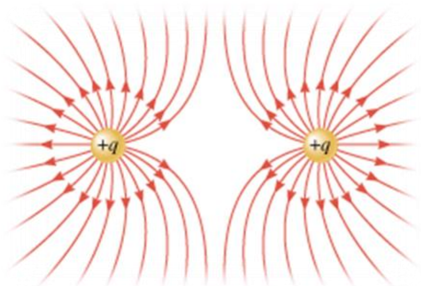
b) Separation Levelling
Adding a small transverse offset (local orbit bump) to the beams.
It is the simplest way of implementing the levelling

c) Beta* levelling
Requires modification of the beta function at IP
Complex but very effective also in reducing beam-beam long range effects

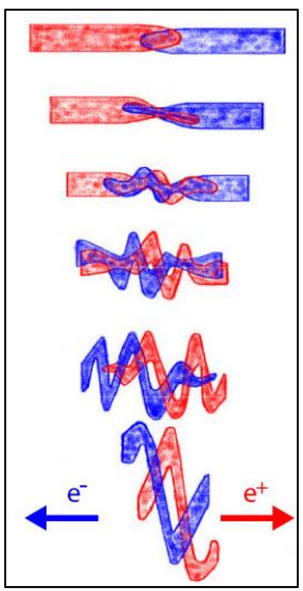
Reference

Collective effects:

But … these particles are electrically charged, and hence are **sources of additional EM fields themselves**.

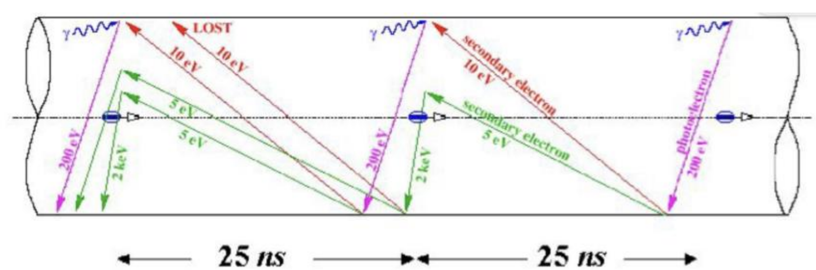


- They 'speak' to each other via these EM fields.
- They are **not independent**, but influence each other motion



Beam-beam effects: electromagnetic interaction of the beams

Self induced Wake fields



Electron Clouds

Contents:

- Particle types and relativity for accelerators
- Accelerator components: Dipole, quadrupoles magnets, accelerating RF cavities…
- Transverse plane (x,y) → Guiding and focusing beams
  - Particle motion in linear approximation
  - Invariant of motion and Emittance
  - Beam Optics: beta functions, beams sizes, Beam Tunes
- Longitudinal plane (s,t) → Acceleration
  - Synchronous motion
  - Synchrotrons and LHC injection complex
- Hadron Accelerators: Synchrotrons
  - Beam production
  - Magnets
  - Luminosity
  - Collective effects

Future Accelerators

The near and far future accelerators:

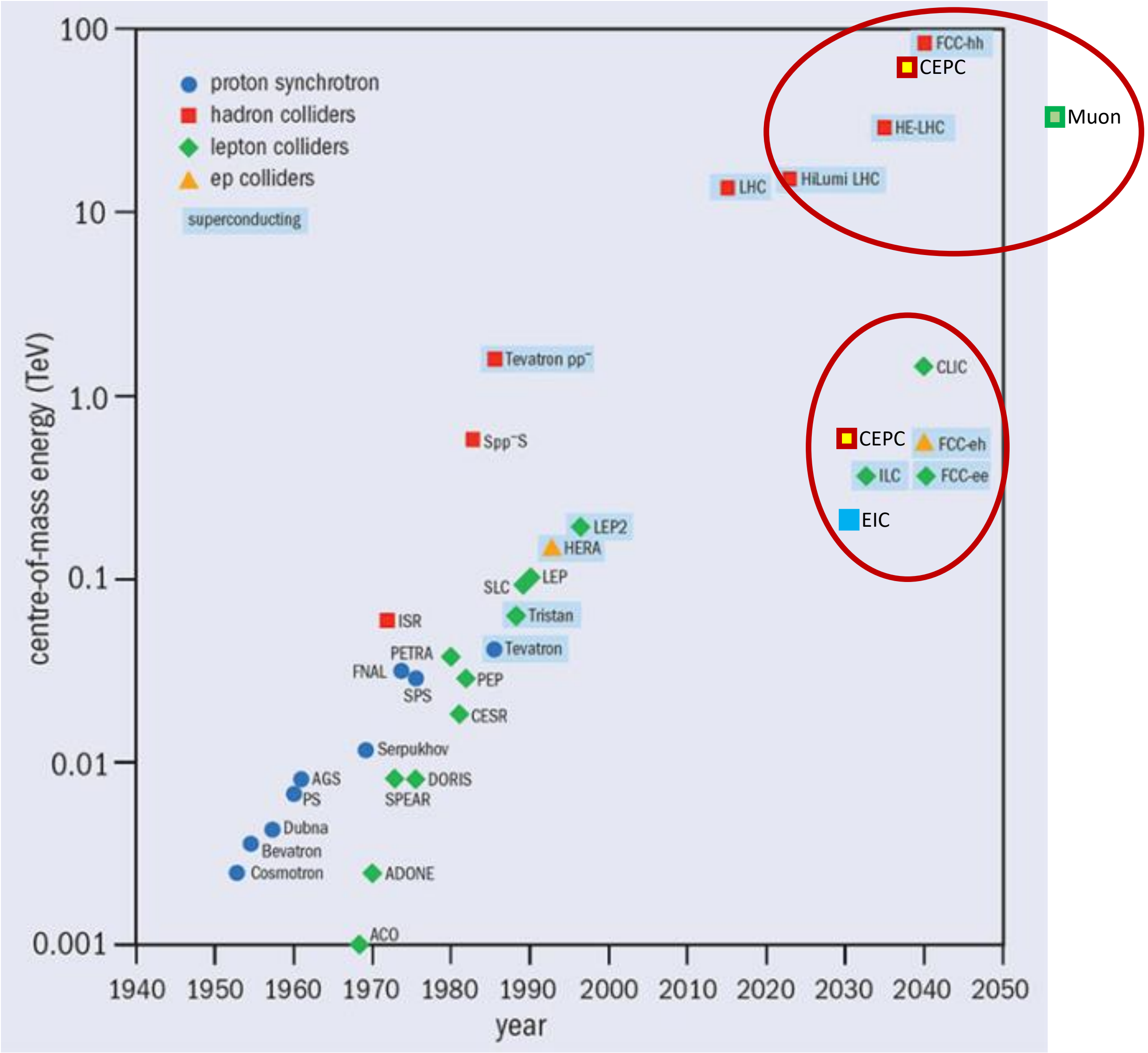
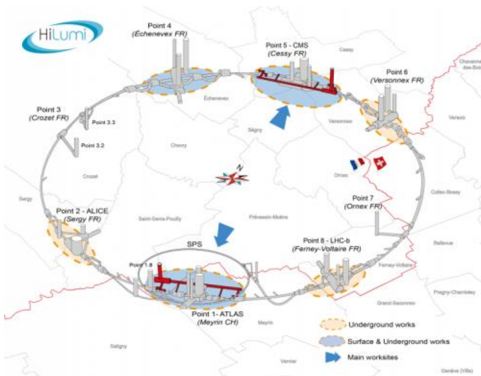
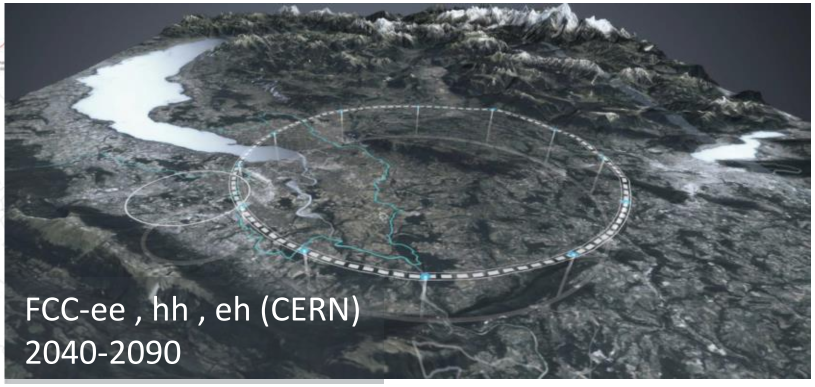


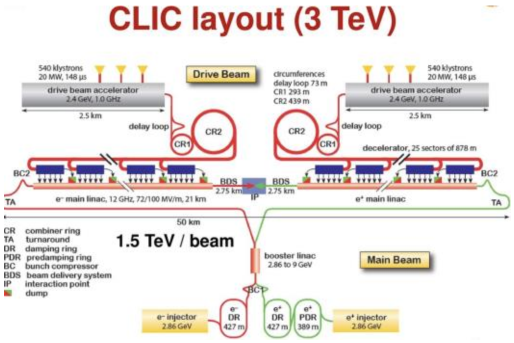
Image credit L.Rossi, P Lebrun

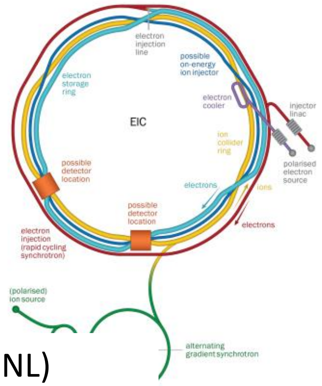
HEP Landscape - Colliders

HL-LHC (CERN)
Installation 2026
Commissioning 2029




FCC-ee , hh , eh (CERN)
2040-2090



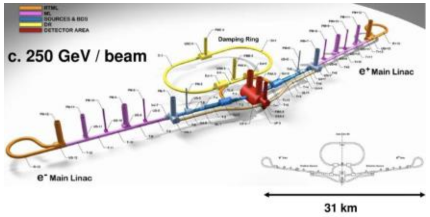


EIC (BNL)
In construction
CD4 June 2030


SppC (IHEP)
> 2040

μ Collider
> 2040

ILC



Very active and motivating moment→ several feasibility studies

HL-LHC

The main goal is to increase luminosity by a factor of 5 to 10 in order to observe rare physics processes.

250 fb^{-1} per year $\leftarrow$ 2 x LHC 4 years of Run II

3000 fb^{-1} in 12 years

This will be accomplished with a series of upgrades

Injectors Upgrade (LIU)

Higher brightness beams
More intensity less emittance

LHC Upgrade

Increase of luminosity

HL-LHC established as project in summer 2010
Described in HL-LHC book and the HL-LHC design report

HI-LHC Upgrade

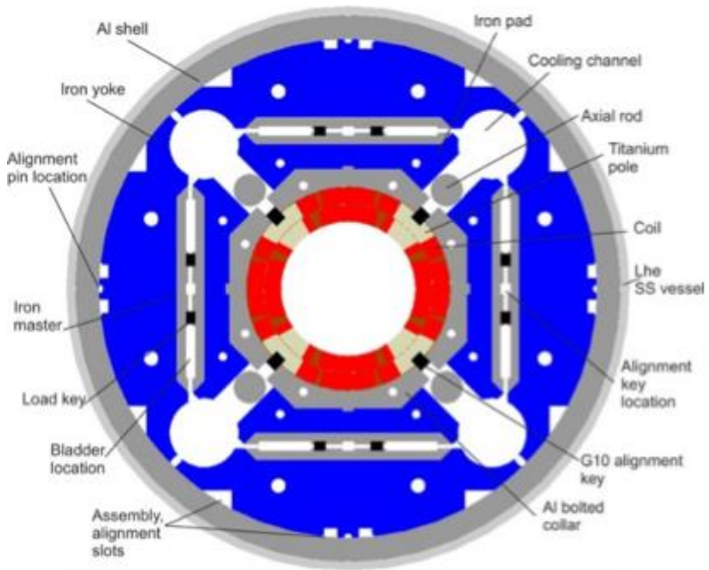
- LHC Upgrade of IR ATLAS/CMS inner triplets (quadrupoles)
- Upgrade of Collimation System
- Crab cavities for beam rotation
- 11 Tesla magnet + connection cryostat
- Cold powering
- Machine protection
- …

IR ATLAS/CMS

HL-LHC baseline smaller beta-star 15 cm

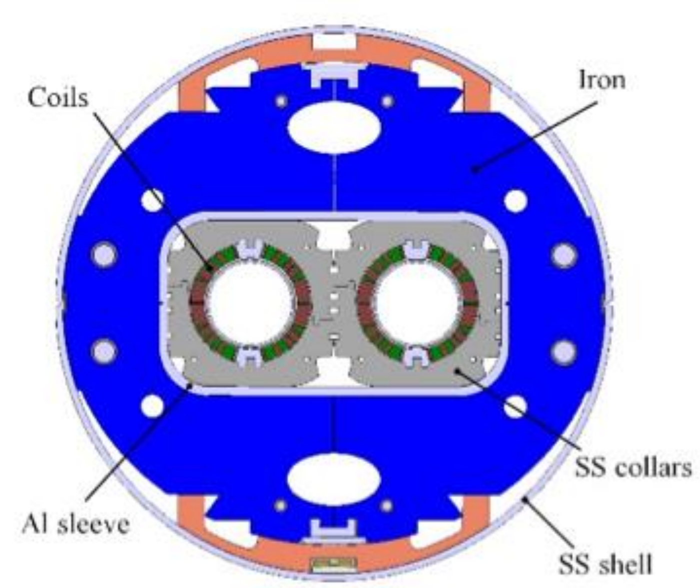
Replace 1.2 km of the 27 km LHC ring

Super conductive large aperture triplet quadrupoles with use of novel Nb3Sn magnet technology



Triplet [G. Ambrosio, P. Ferracin et al.]

Super conductive separation/recombination dipoles D2 with B field same direction.



D2 [P. Fabbricatore, S. Farinon, et al.]

HL-LHC Collimators

Study of more robust materials for collimation and reduce impedance.

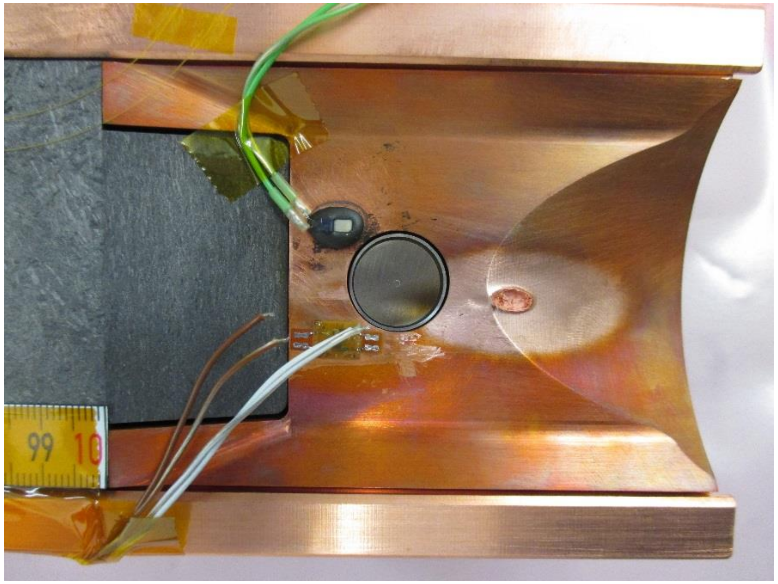
During LS2

Replacement of existing primary collimators and 8 secondary collimators with higher-electrically-conductive material MoGr.

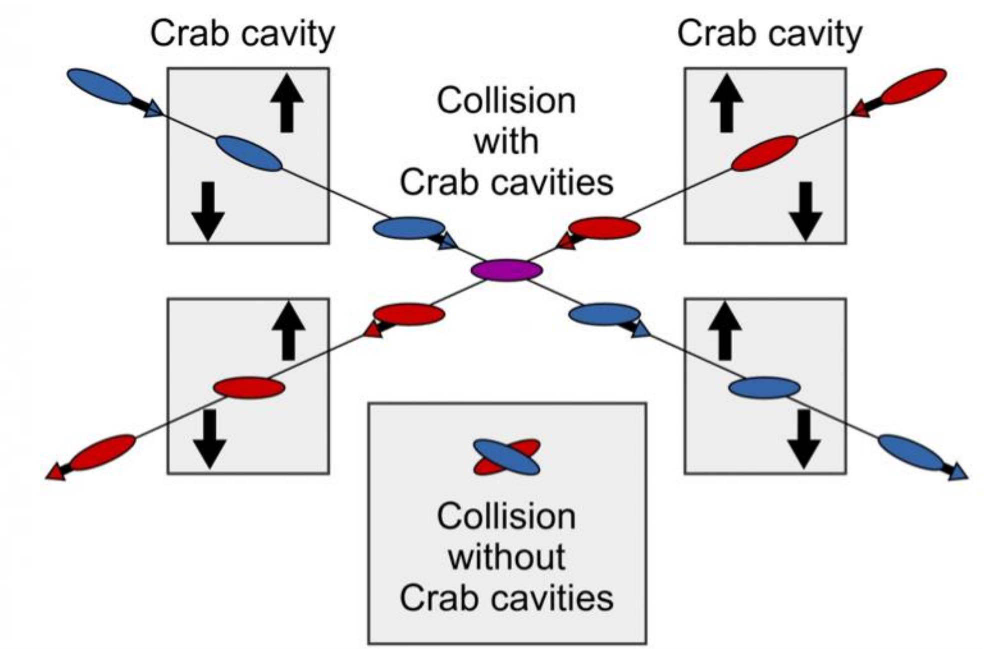
Addition of 4 collimators in the dispersion suppression region

→ shorter magnets 11T Dipole 11-m (Nb_3Sn technology)

Impact of 288 proton bunches on copper-allow (left) and MoGr (right)

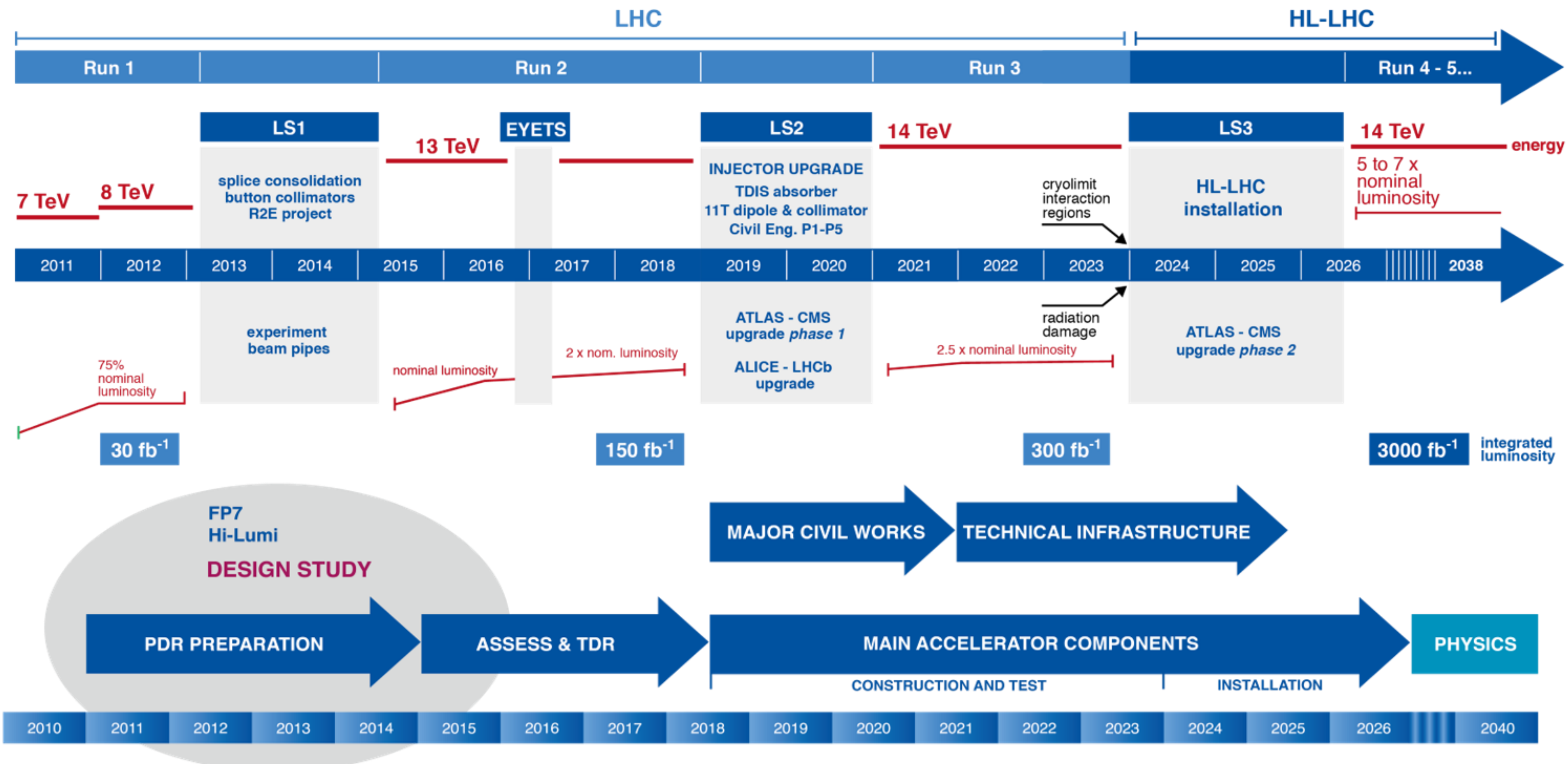


HL-LHC Crab-cavities



Crab cavities will reduce the effect of the geometrical factor on the luminosity

HL-LHC timeline

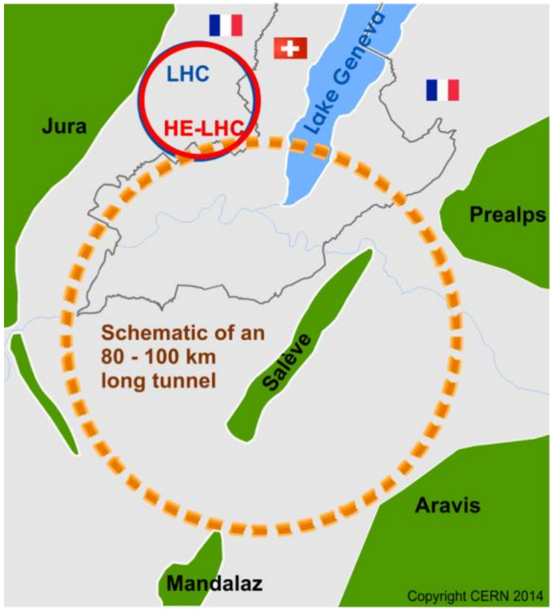


HL-LHC parameter table

Parameters	Nominal LHC (Design report)	LHC 2018 max value	HL-LHC (standard)	HL-LHC 8b+4e[12]	HL-LHC (Ultimate)
Beam energy in collision [TeV]	7	6.5	7	7	7
N_b	1.15E+11	1.15E+11	2.2E+11	2.2E+11	2.2E+11
n_b	2808	2556	2760	1972	2760
Number of collisions in IP1 and IP5 [1]	2808	2544	2748	1967	2748
N_{tot}	3.2E+14	2.9E+14	6.1E+14	4.3E+14	6.1E+14
beam current [A]	0.58	0.52	1.1	0.79	1.1
x-ing angle [μrad]	285	320 ==> 260	500	470[10]	500
beam separation [σ] [11]	9.4	10.3 ==> 6.8	10.5	10.5[10]	10.5
β^* [m]	0.55	0.30 ==> 0.25	0.15	0.15	0.15
ε_n [μm]	3.75	2 ==> 2.5	2.50	2.20	2.50
r.m.s. bunch length [m]	7.55E-02	8.25E-02	7.61E-02	7.61E-02	7.61E-02
Total loss factor R0 without crab-cavity			0.342	0.342	0.342
Total loss factor R1 with crab-cavity [13]			0.716	0.749	0.716
Virtual Luminosity with crab-cavity: Lpeak*R1/R0 [$cm^{-2} s^{-1}$] [13]			1.70E+35	1.44E+35	1.70E+35
Luminosity [$cm^{-2} s^{-1}$] or Leveling luminosity for HL-LHC	1.00E+34	2.00E+34	5.0E+34[5]	3.82E+34	7.5E+34[5]
Events / crossing (with leveling and crab-cavities for HL-LHC) [8]	27	55	131	140	197
Peak line density of events [event/mm] (max over stable beams)	0.21	0.38	1.3	1.3	1.9
Leveling time [h] (assuming no emittance growth) [8, 13]	-		7.2	7.2	3.5

Proj. leader L. Rossi talk 8th annual collaboration meeting October 2018

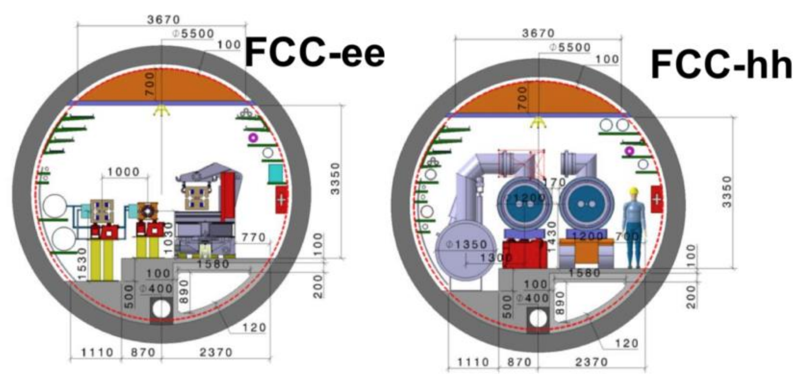
Future Circular Collider (FCC)



Study of a hadron **collider** with a **centre-of-mass energy of the order of 100 TeV** in a new tunnel of **80-100 km circumference**

Start as *e+e-* collider FCC-ee → Higgs Factory
Ecom of 90 - 365 GeV
Luminosity ~ 17 x 10^{34} cm-2s-1
Beta-star ~ 1 mm

Second stage *pp* collider FCC-hh → Energy frontier
Ecom of 50 - 100 TeV **16 T ⇒ 100 TeV *pp* in 100 km**
Luminosity ~ 3 x 10^{34} cm-2s-1

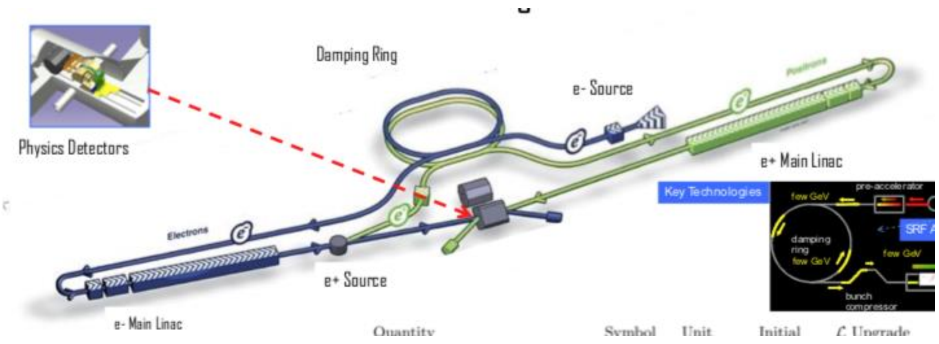


[FCC-Condeptual Design Reports]

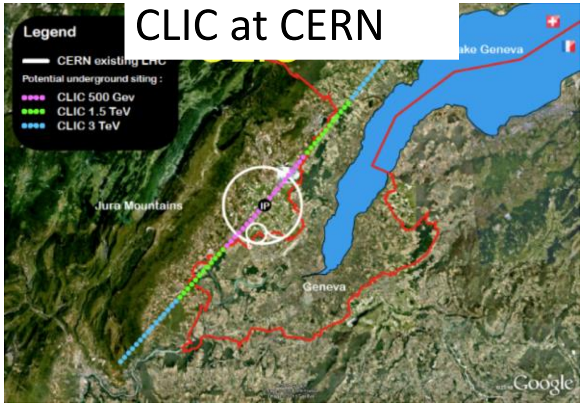
Linear Colliders ILC/CLIC

Two linear accelerators facing each other

ILC in Japan



CLIC at CERN



Both propose a staged implementation of e+e- collider

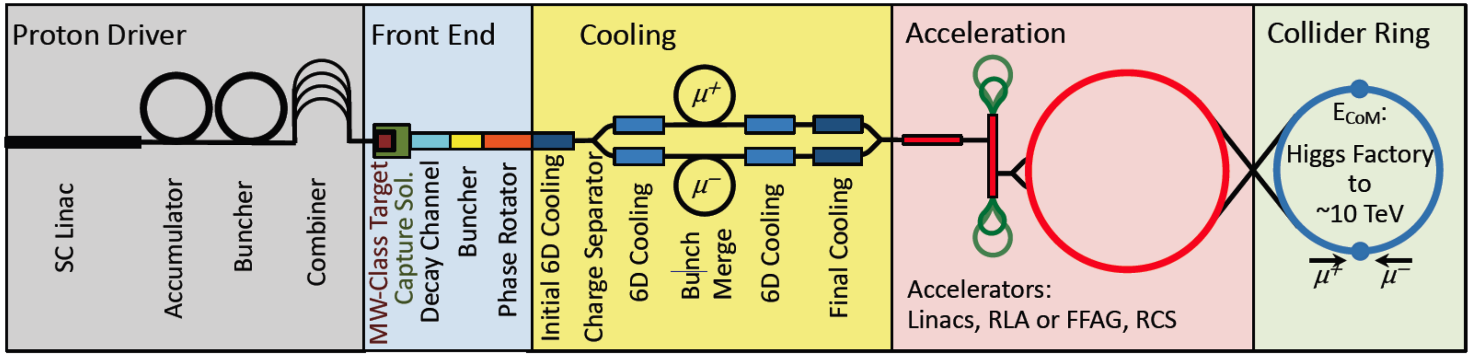
Ecom = 0.25 - 1 TeV
Luminosity 1.35 x 10^{34} cm^{-2}s^{-1}

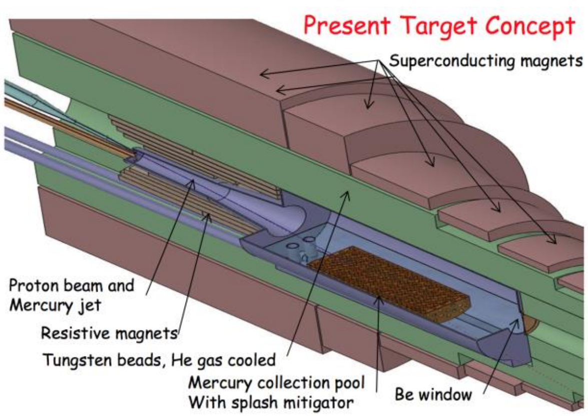
Ecom = 0.5 TeV - 3 TeV
Luminosity 1.3 - 5.9 x 10^{34} cm^{-2}s^{-1}

More about ILC: https://ilchome.web.cern.ch

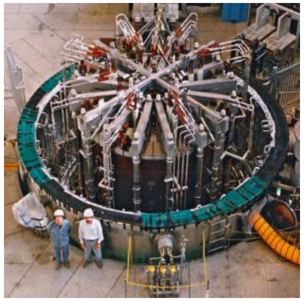
More about CLIC: https://clic.cern

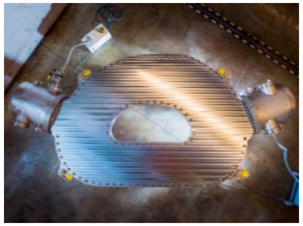
Muon Collider Study

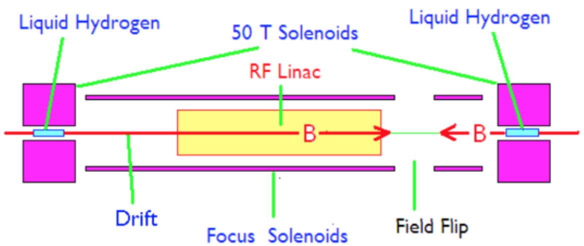




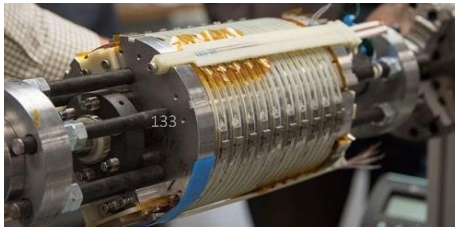
20 T, 150 mm, 100 kW







30...50 T, 50 mm



Appendix: Magnetic Rigidity (proton)

Lorentz force $\vec{F}_B = e \cdot \vec{v} \times \vec{B}$

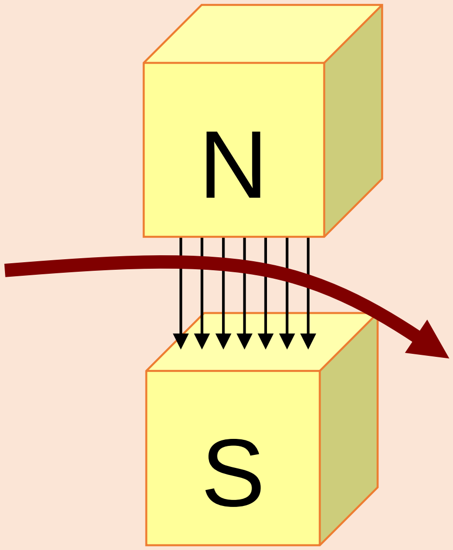
B, v perpendicular $F_B = evB$

centrifugal force $F_c = -m\frac{v^2}{\rho}$

$$F_B + F_c = 0 \longrightarrow evB = m\frac{v^2}{\rho}$$

$$B\rho = \frac{mv}{e}$$

Magnetic rigidity $\boxed{B\rho = \frac{p}{e}}$



B = magnetic field
ρ = local bending radius
p = momentum
e = elementary charge

Appendix: Magnetic Rigidity in Practical Units

$$B\rho = \frac{p}{e} = \frac{mv}{e} = \beta\gamma\frac{m_0 c}{e}$$

$$= \beta\gamma \; \frac{m_0 c^2}{ce}$$

$$= \beta \; \frac{E_{\mathrm{tot}}}{ce}$$

$$= \beta \; \frac{10^9}{c} E_{\mathrm{tot}}[\mathrm{GeV}]$$

$$\downarrow$$

$$B\rho[\mathrm{Tm}] \approx 3.3356 \cdot E_k[\mathrm{GeV/c}]$$

$$B\rho[\mathrm{Tm}] = 3.3356 \cdot p[\mathrm{GeV/c}]$$

B = magnetic field
ρ = local bending radius
p = momentum
e = elementary charge
E_k = kinetic energy

total energy:

$$E_{\mathrm{tot}} = E_k + m_0 c^2$$

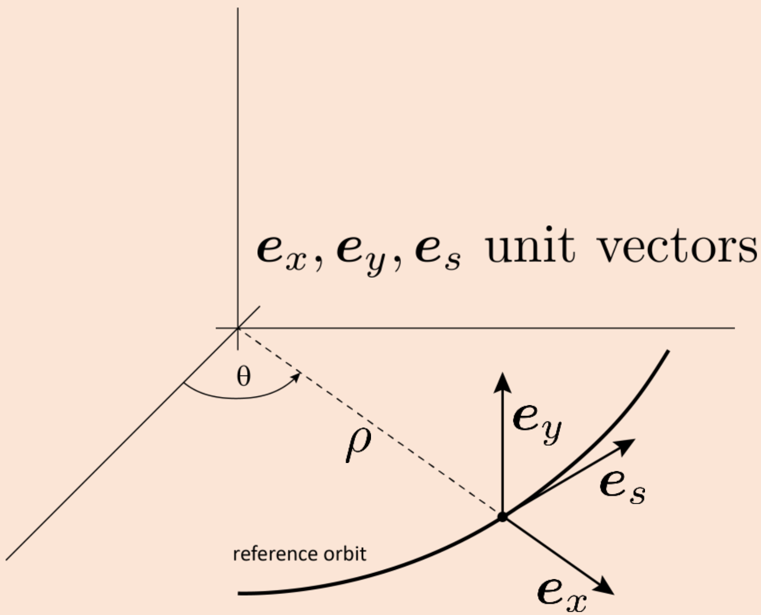
approximations:

$$\beta \approx 1, cp \approx E_k$$
$$\text{for } E_k \gg m_0 c^2$$

see also Wiedemann, p.101, eq.5.6

Appendix, Derivation: Equation of Motion I

starting with general equation of motion: $\frac{d\vec{p}}{dt} = \gamma m_0 \ddot{\vec{R}} = \vec{F}$



$$\begin{aligned}
\vec{R} &= r\boldsymbol{e}_x + y\boldsymbol{e}_y, \;\; r \equiv \rho + x \\
\dot{\vec{R}} &= \dot{r}\boldsymbol{e}_x + r\dot{\boldsymbol{e}}_x + \dot{y}\boldsymbol{e}_y \\
\dot{\vec{R}} &= \dot{r}\boldsymbol{e}_x + r\dot{\theta}\boldsymbol{e}_s + \dot{y}\boldsymbol{e}_y \\
\ddot{\vec{R}} &= \ddot{r}\boldsymbol{e}_x + (2\dot{r}\dot{\theta} + r\ddot{\theta})\boldsymbol{e}_s + r\dot{\theta}\dot{\boldsymbol{e}}_s + \ddot{y}\boldsymbol{e}_y \\
\ddot{\vec{R}} &= (\ddot{r} - r\dot{\theta}^2)\boldsymbol{e}_x + (2\dot{r}\dot{\theta} + r\ddot{\theta})\boldsymbol{e}_s + \ddot{y}\boldsymbol{e}_y
\end{aligned}$$

used here: $\dot{\boldsymbol{e}}_x = \dot{\theta}\boldsymbol{e}_s, \;\; \dot{\boldsymbol{e}}_s = -\dot{\theta}\boldsymbol{e}_x$

comment: the main purpose here is to correctly treat the effect of the curved coordinate system, i.e. the moving unit vectors e_x, e_s

Derivation: Equation of Motion II

right side of equation, the force:

$$\begin{aligned}\vec{F} &= e\vec{v}\times\vec{B}\\ \vec{v}\times\vec{B} &= \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_s \\ v_x & v_y & v_s \\ B_x & B_y & 0 \end{vmatrix}\\ &= -v_s B_y \boldsymbol{e}_x + v_s B_x \boldsymbol{e}_y + (v_x B_y - v_y B_x)\boldsymbol{e}_s\end{aligned}$$

assumptions:

- no B_s
- $B_x(y=0) = 0$

use:

$$B_y = B_0 + gx$$
$$B_x = gy$$

$$g \equiv \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$$

result: two equations hor/vert from *x*,*y* components:

$$\begin{aligned}\gamma m_0(\ddot{r} - r\dot{\theta}^2) &= -ev_s(B_0 + gx)\\ \gamma m_0 \ddot{y} &= ev_s g y\end{aligned}$$

in literature g has varying sign conventions
Wiedemann, Table 6.2: $g = +dB_y/dx$
Schmüser/Hillert: $g = -dB_y/dx$

Derivation: Equation of Motion III

introduce path length *s* as independent variable:

$$
\begin{aligned}
\gamma m_0(\ddot{r} - r\dot{\theta}^2) &= -ev_s(B_0 + gx) \\
\gamma m_0 \ddot{y} &= ev_s g y
\end{aligned}
$$

$$
\begin{aligned}
x'' &= \frac{1}{r} - \frac{e}{\gamma m_0 v}(B_0 + gx) \\
y'' &= \frac{e}{\gamma m_0 v} g y
\end{aligned}
$$

use:

$$v_s = r\dot{\theta} \approx v$$

$$\ddot{r} = \ddot{x}$$

$$\ddot{x} = v^2 x'', \; x'' \equiv \frac{\partial^2 x}{\partial s^2}$$

$$\ddot{y} = v^2 y'', \; y'' \equiv \frac{\partial^2 y}{\partial s^2}$$

Derivation: Equation of Motion IV

$$
\begin{aligned}
x'' &= \frac{1}{r} - \frac{e}{\gamma m_0 v}(B_0 + gx) \\
y'' &= \frac{e}{\gamma m_0 v} gy
\end{aligned}
$$

$$
\begin{aligned}
x'' &= \frac{1}{\rho}\left(1 - \frac{x}{\rho}\right) - kx - \frac{1}{\rho\left(1 + \frac{\Delta p}{p_0}\right)} \\
&= -\left(\frac{1}{\rho^2} + k\right)x + \frac{1}{\rho}\frac{\Delta p}{p_0} \\
y'' &= ky
\end{aligned}
$$

use:

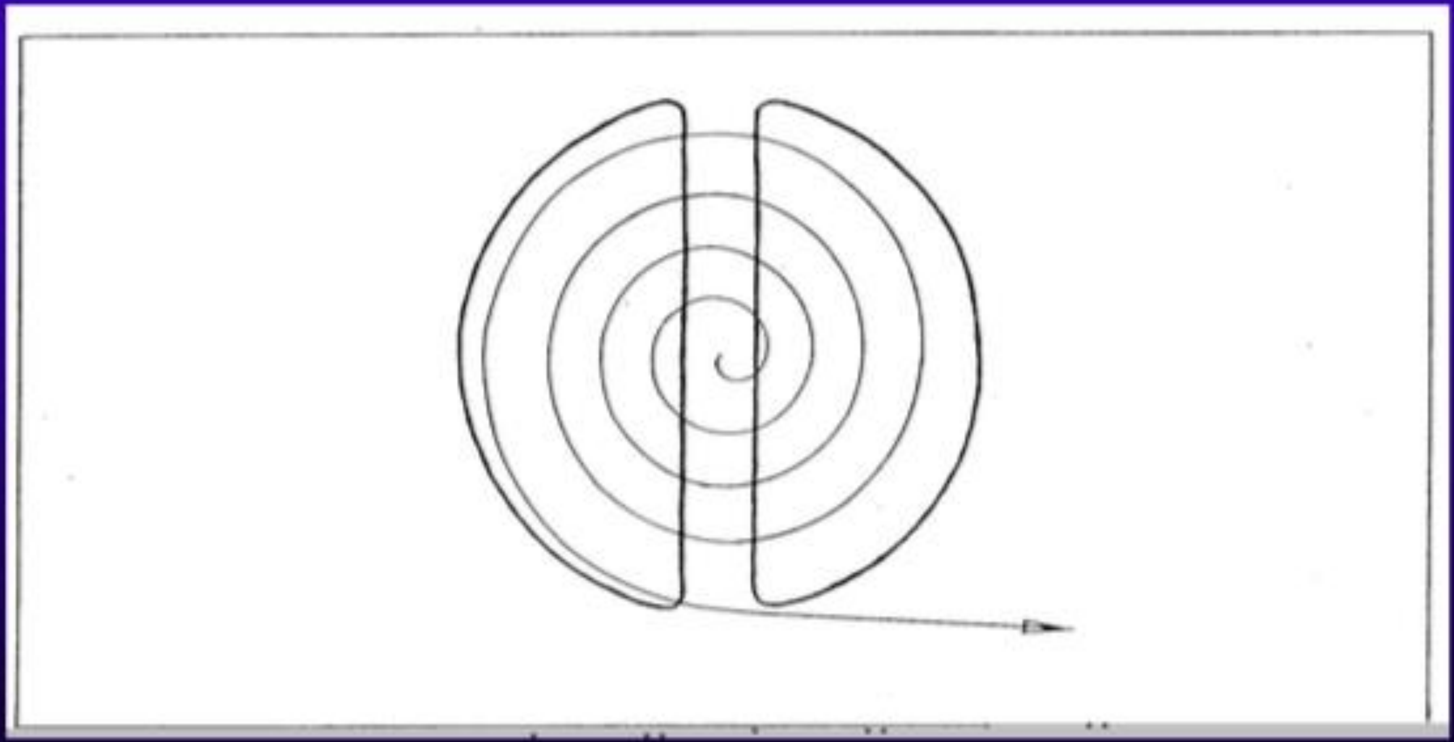
$$\frac{1}{r} = \frac{1}{\rho + x} \approx \frac{1}{\rho}\left(1 - \frac{x}{\rho}\right)$$

$$\frac{eB_0}{\gamma m_0 v} = \frac{eB_0}{p} = \frac{1}{\rho}$$

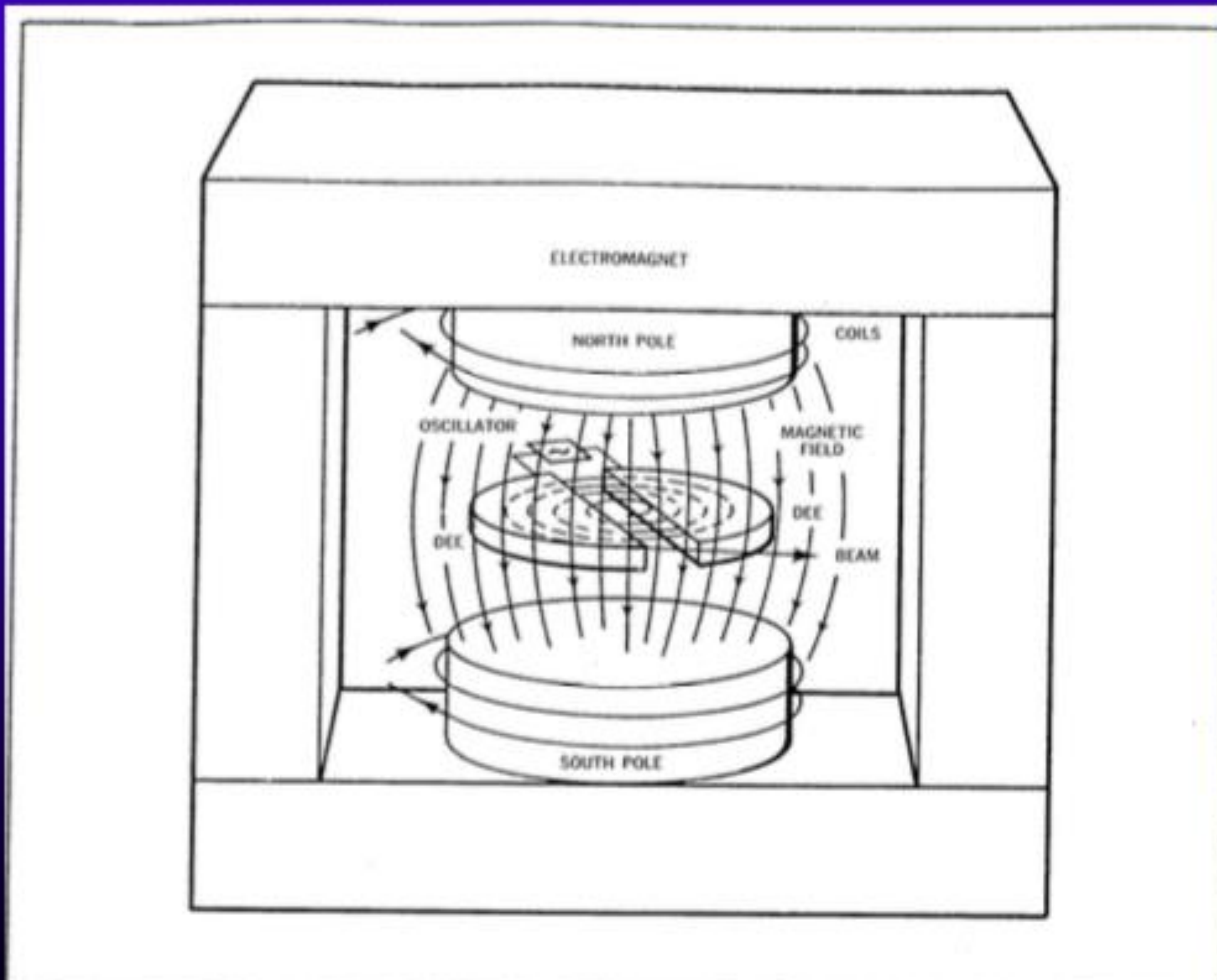
$$p = p_0\left(1 + \frac{\Delta p}{p_0}\right)$$

$$k = \frac{eg}{\gamma m_0 v}$$

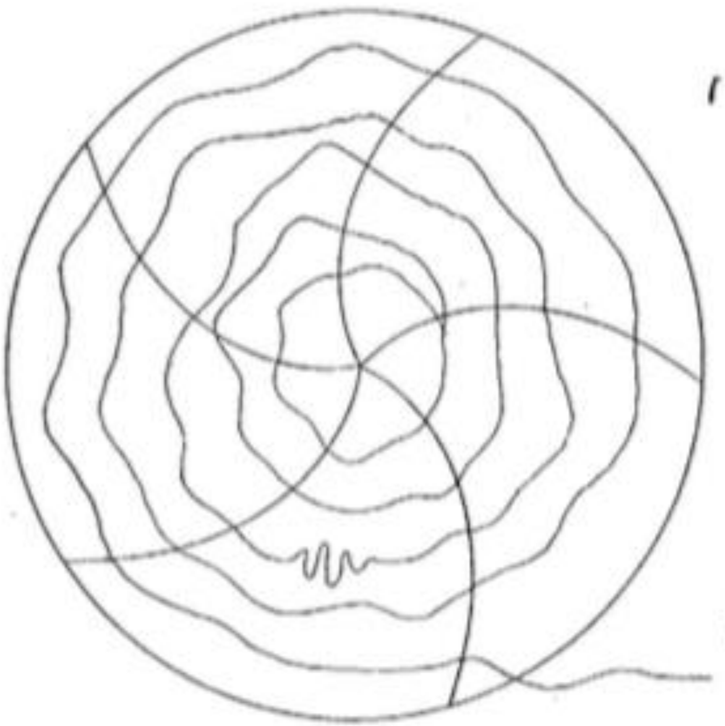
The Cyclotron as seen by the inventor



as seen by the LBL booklet 1967



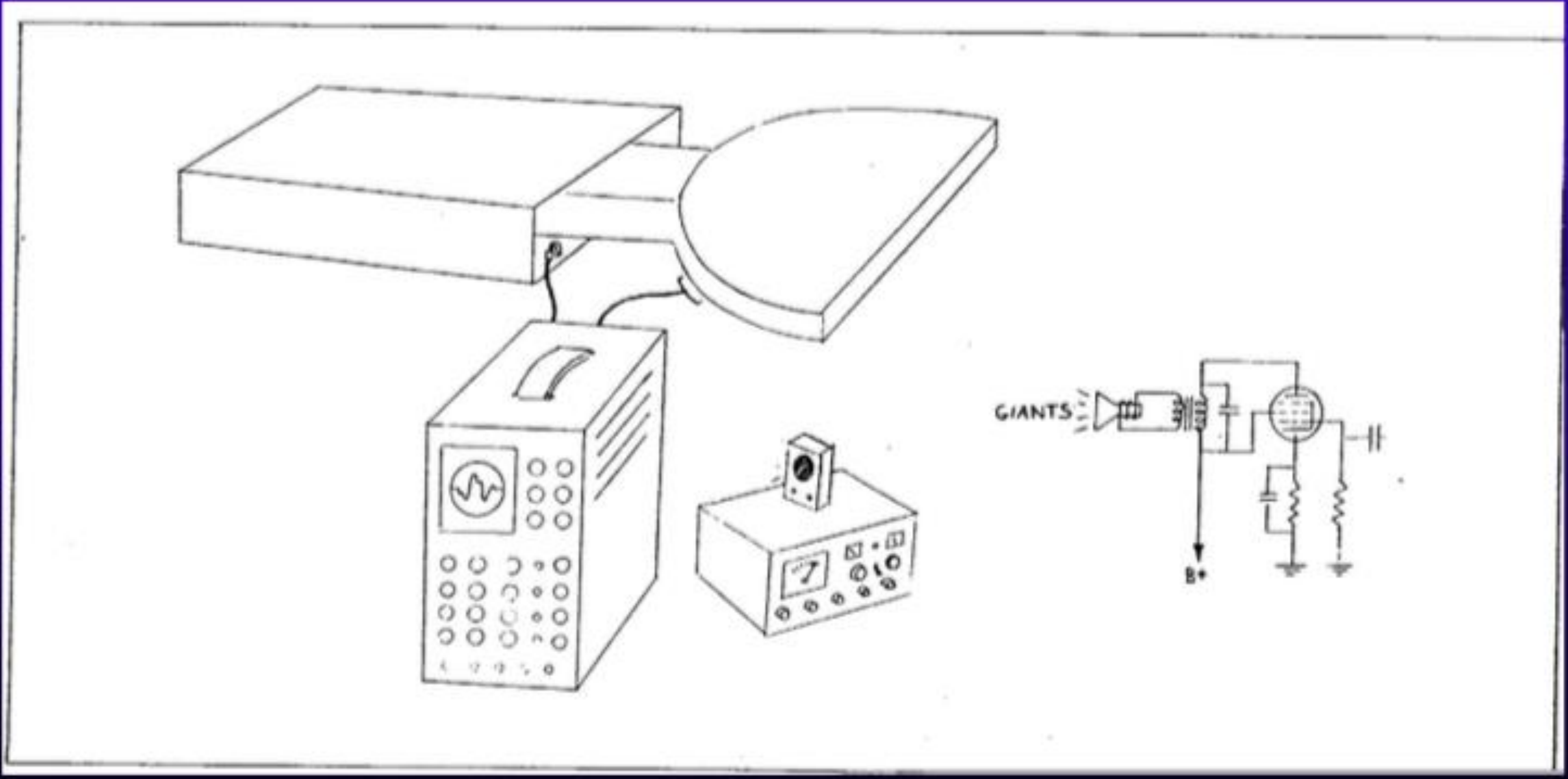
as seen by the theoretical physicist



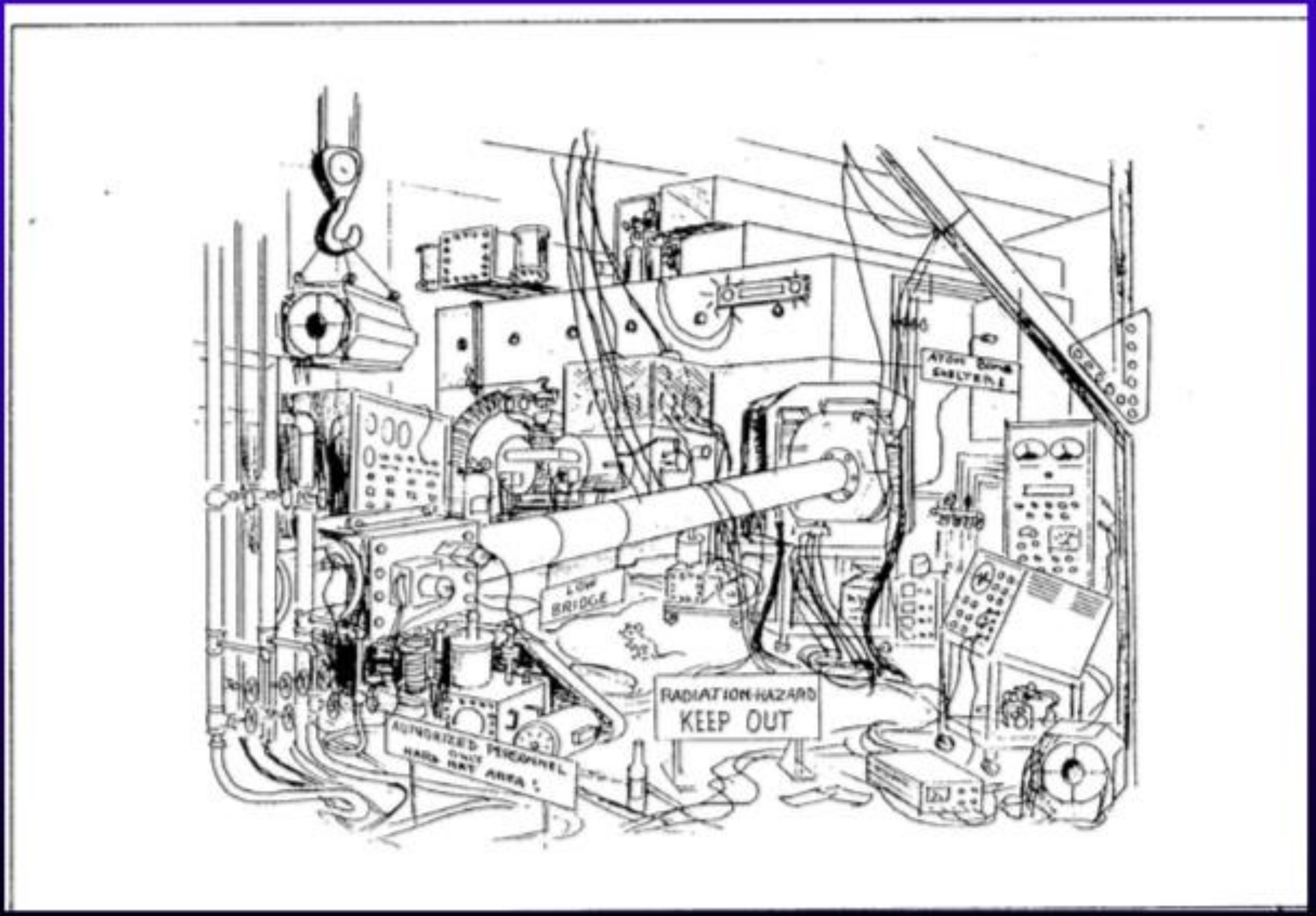
$$r = r_0\Big[1 + \left(\frac{frw}{c}\right)\cos(3\theta + \delta_0 + \delta_1 r) + \left(\frac{frw}{c}\right)^2\cos(5\theta + \delta_3 - \delta_5 r^2) + \left(\frac{frw}{c}\right)^3\cos(7\theta + \delta_7 - \delta_9 r^3) + \text{------}\Big] \times \left\{\frac{e^{7/5\, r^2 \ln Z}}{1 + \left(\frac{a}{b}\right)^{7/4}}\right\}$$

$$\frac{d\phi}{dt} = \left[\sin(\omega t - k\phi) - \sin k\theta - \tfrac{3}{5} f f_1 f_2 f_3'\right]\frac{eV_0}{2\pi} - \omega$$

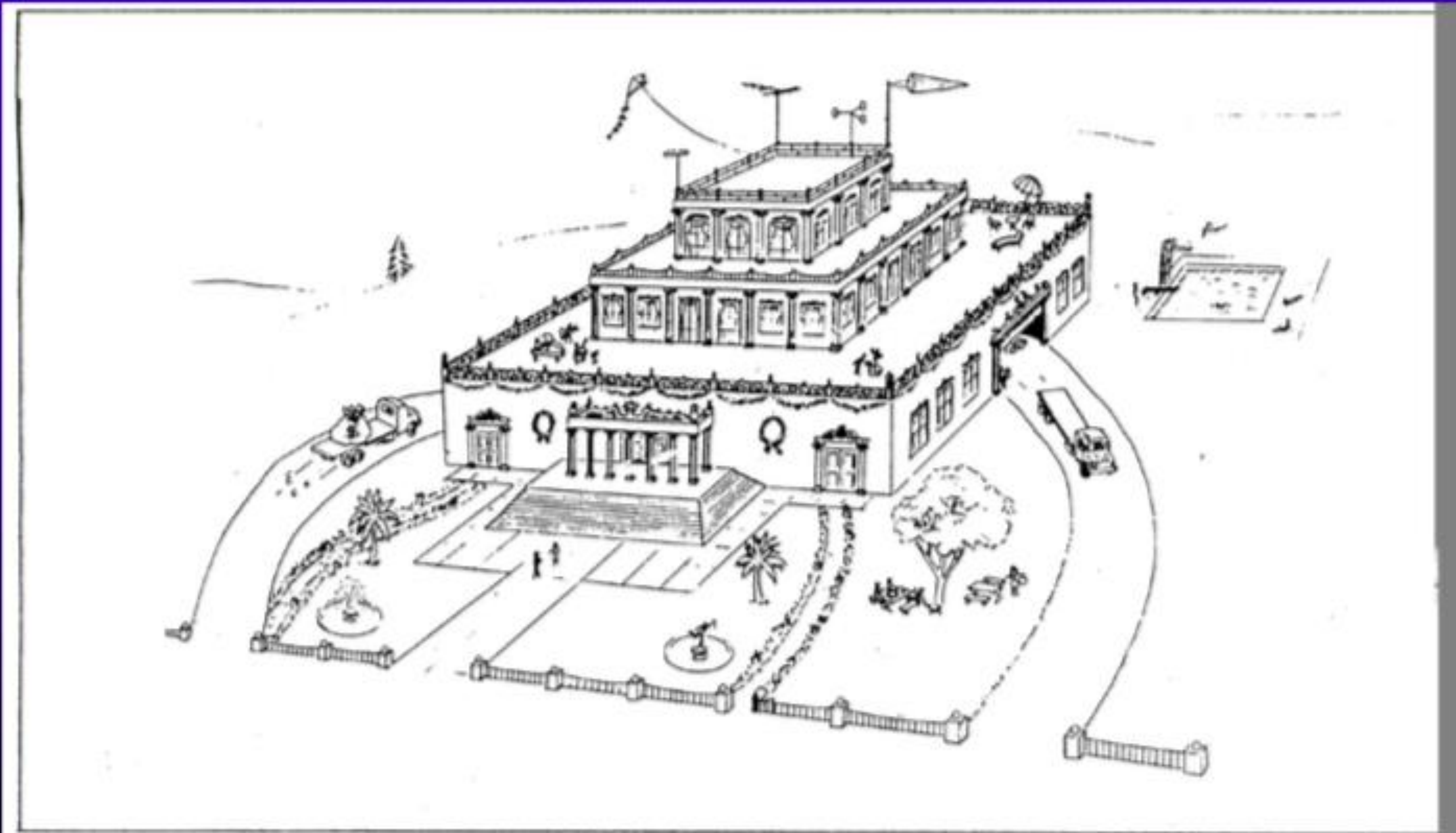
by the electrical engineer



as seen by the visitor

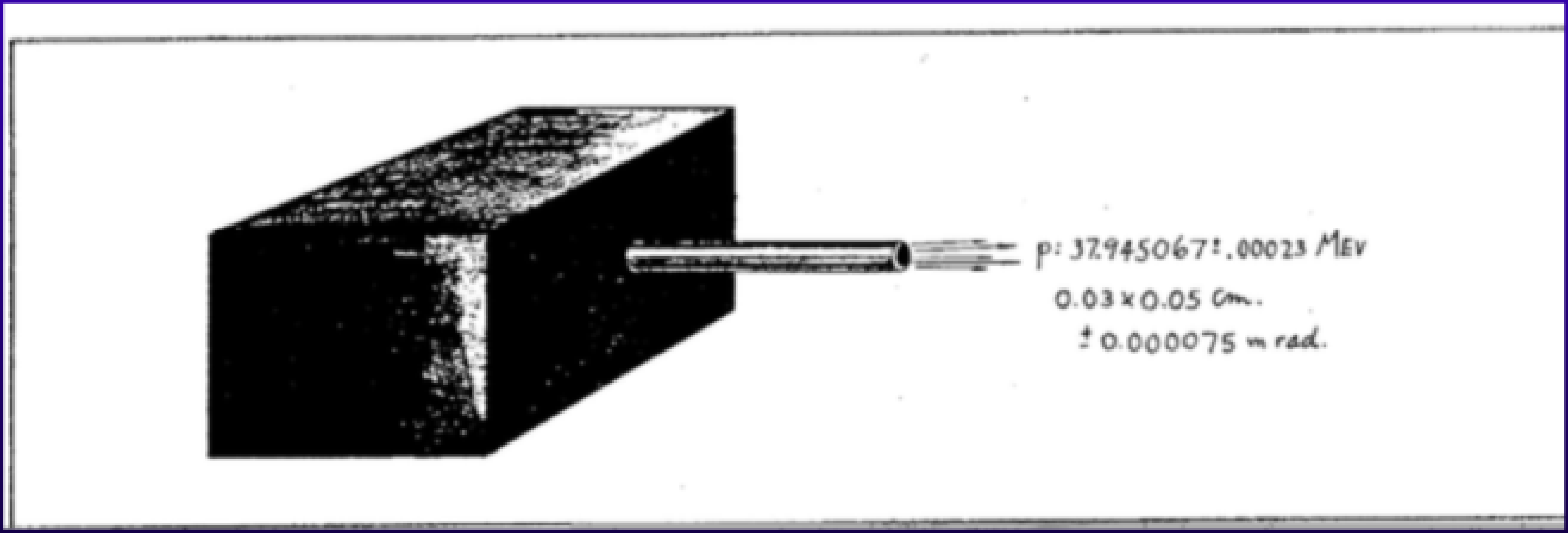


by the government funding agency



as seen by the laboratory director

by the experimental physicist



The cyclotron as seen by the student