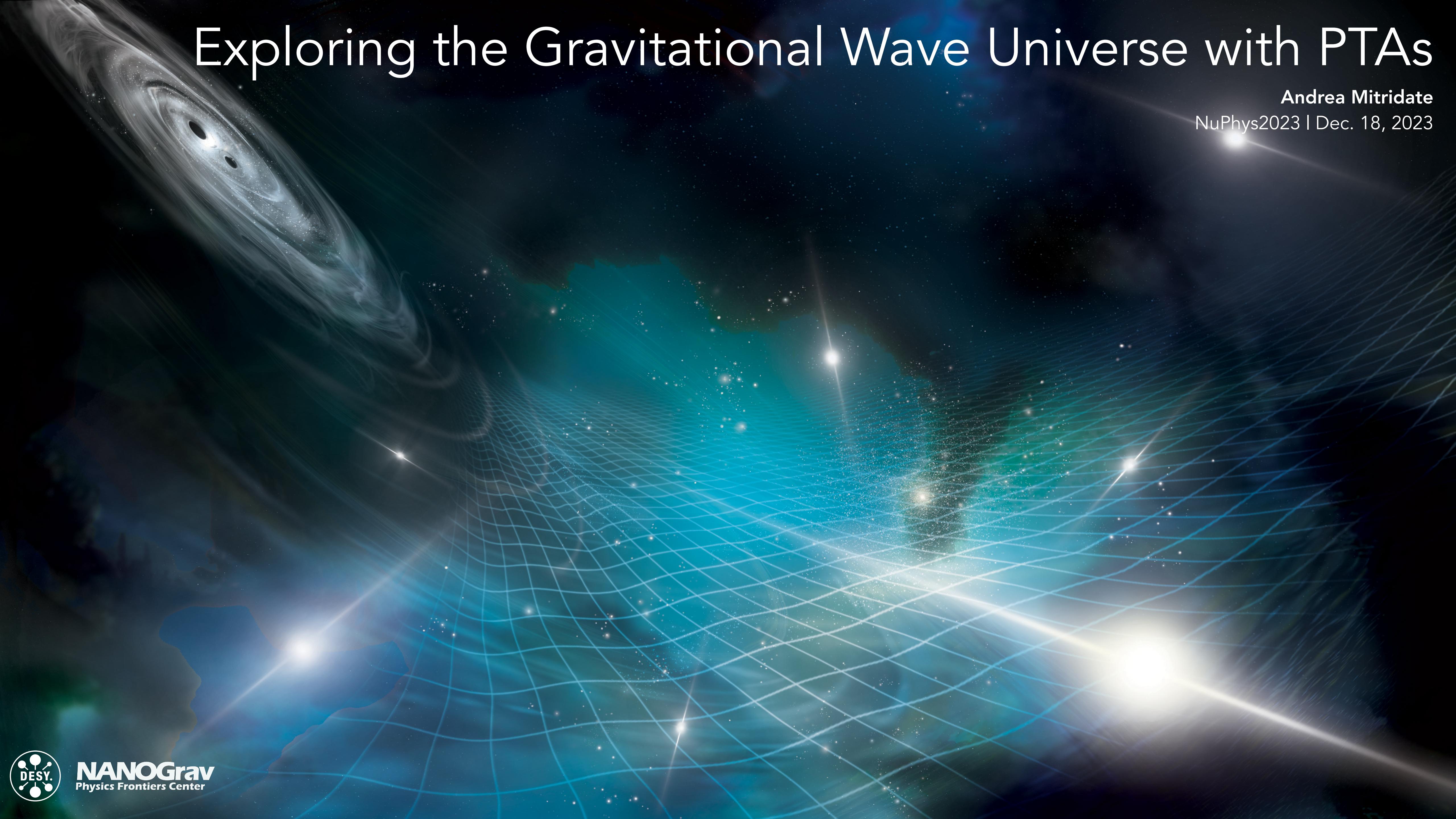


# Exploring the Gravitational Wave Universe with PTAs

Andrea Mitridate

NuPhys2023 | Dec. 18, 2023



**NANOGrav**  
Physics Frontiers Center

# PULSARS

**Rotation  
Axis**

$$\nu(t) = \nu_0 + \dot{\nu}_0 t$$

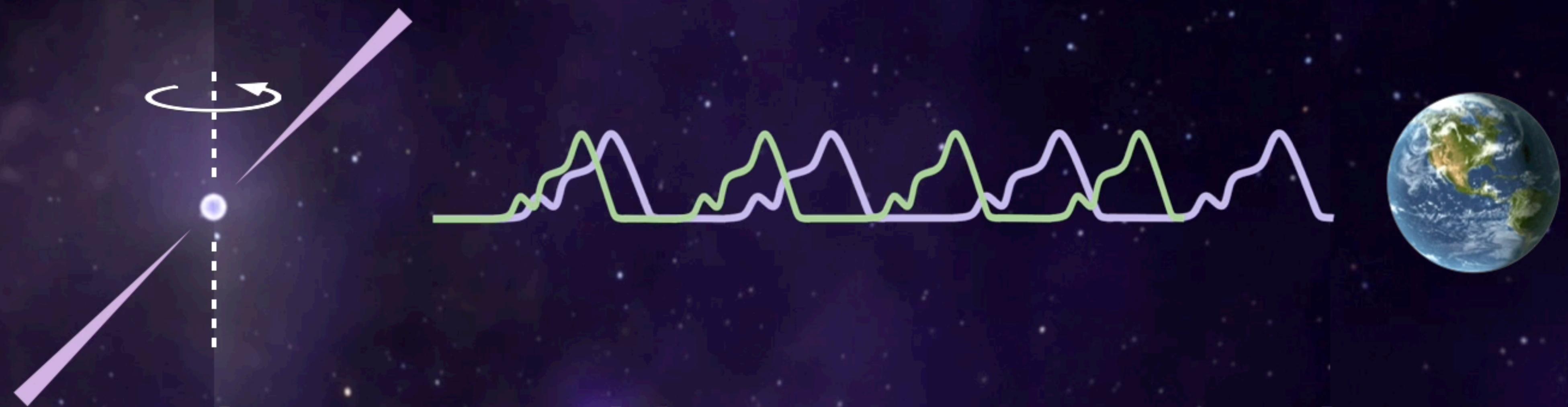
$$\dot{\nu}_0/\nu_0 \sim 10^{-23} - 10^{-20} \text{ Hz}$$

**Radiation Beams**

**Magnetic  
Field Axis**



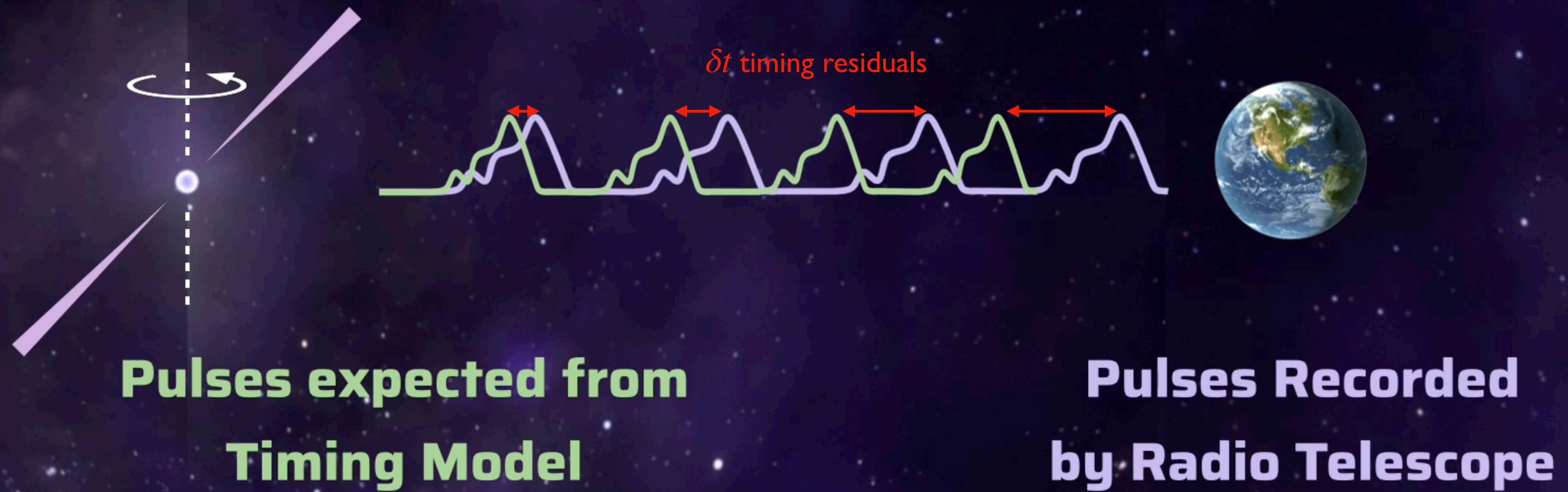
# TIMING RESIDUALS



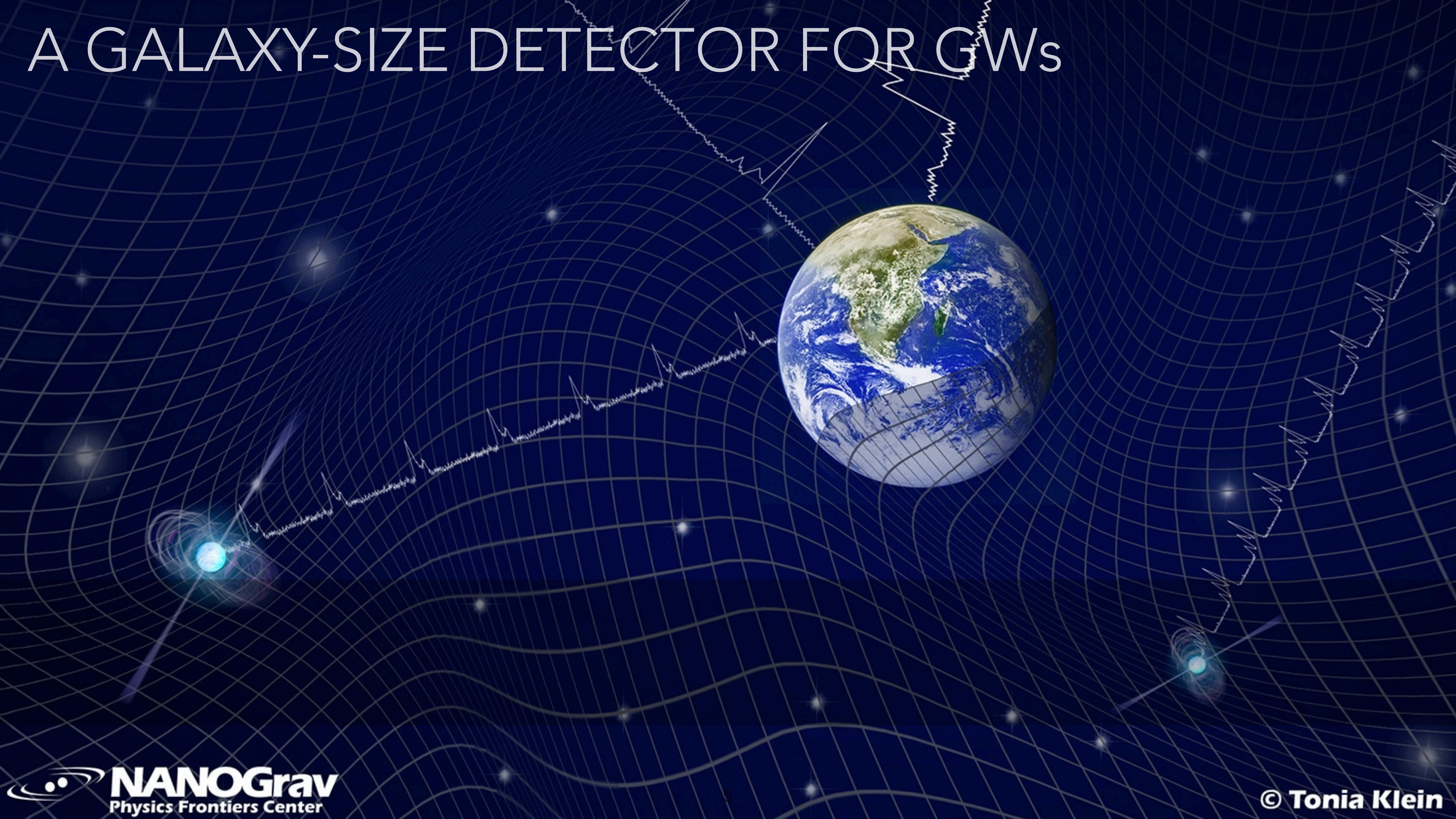
**Pulses expected from  
Timing Model**

**Pulses Recorded  
by Radio Telescope**

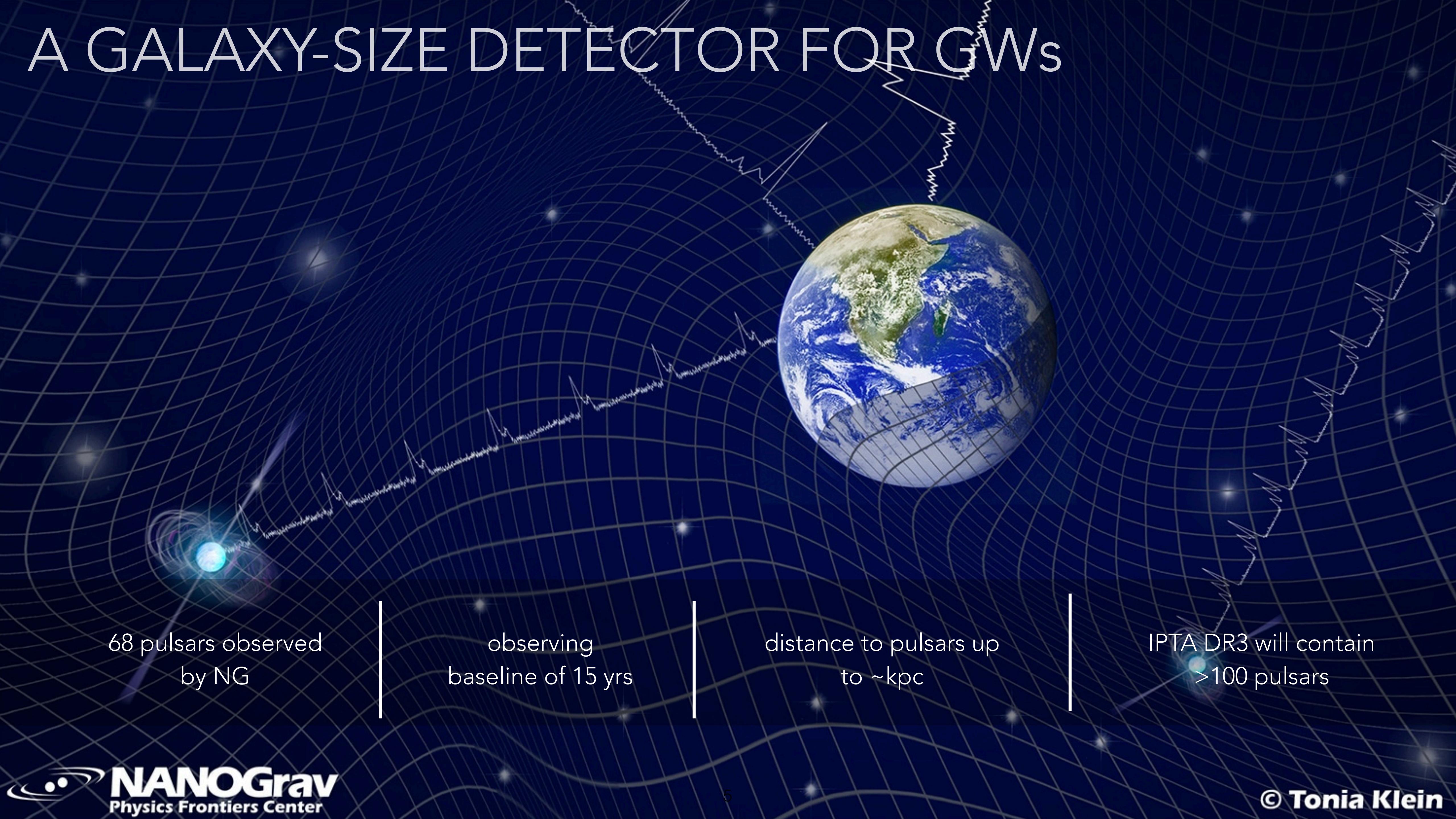
# TIMING RESIDUALS



# A GALAXY-SIZE DETECTOR FOR GWs

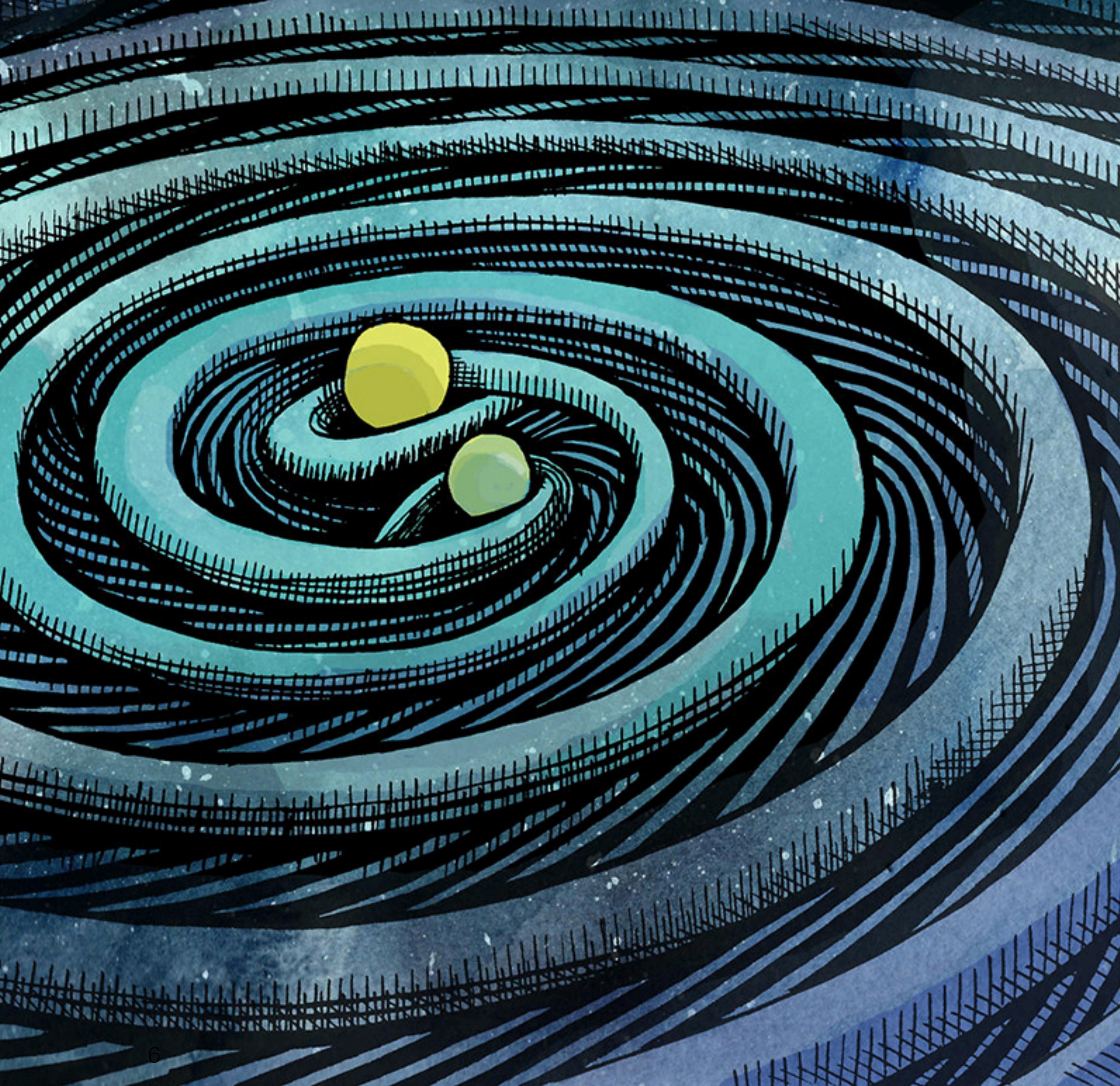
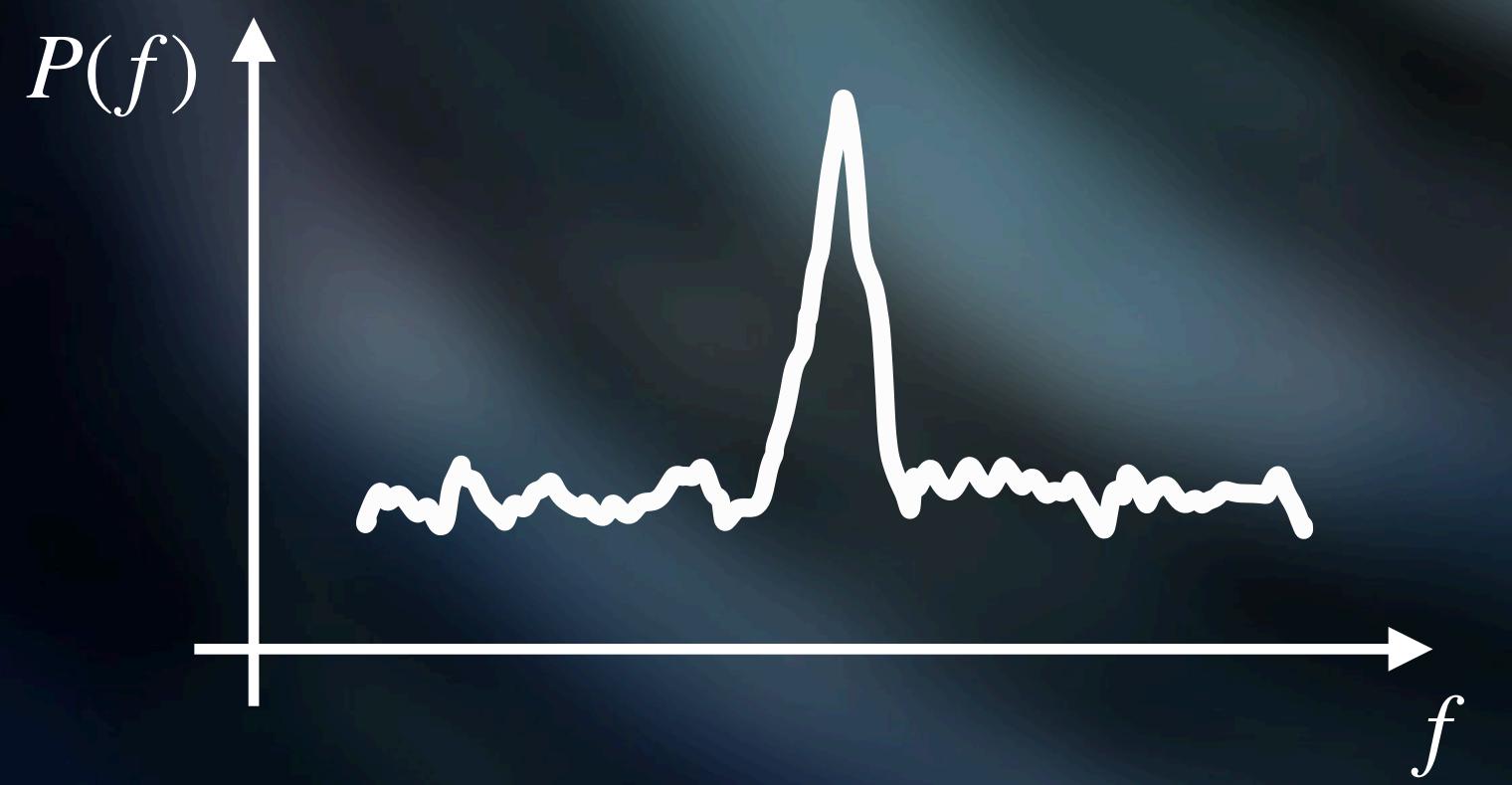


# A GALAXY-SIZE DETECTOR FOR GWs



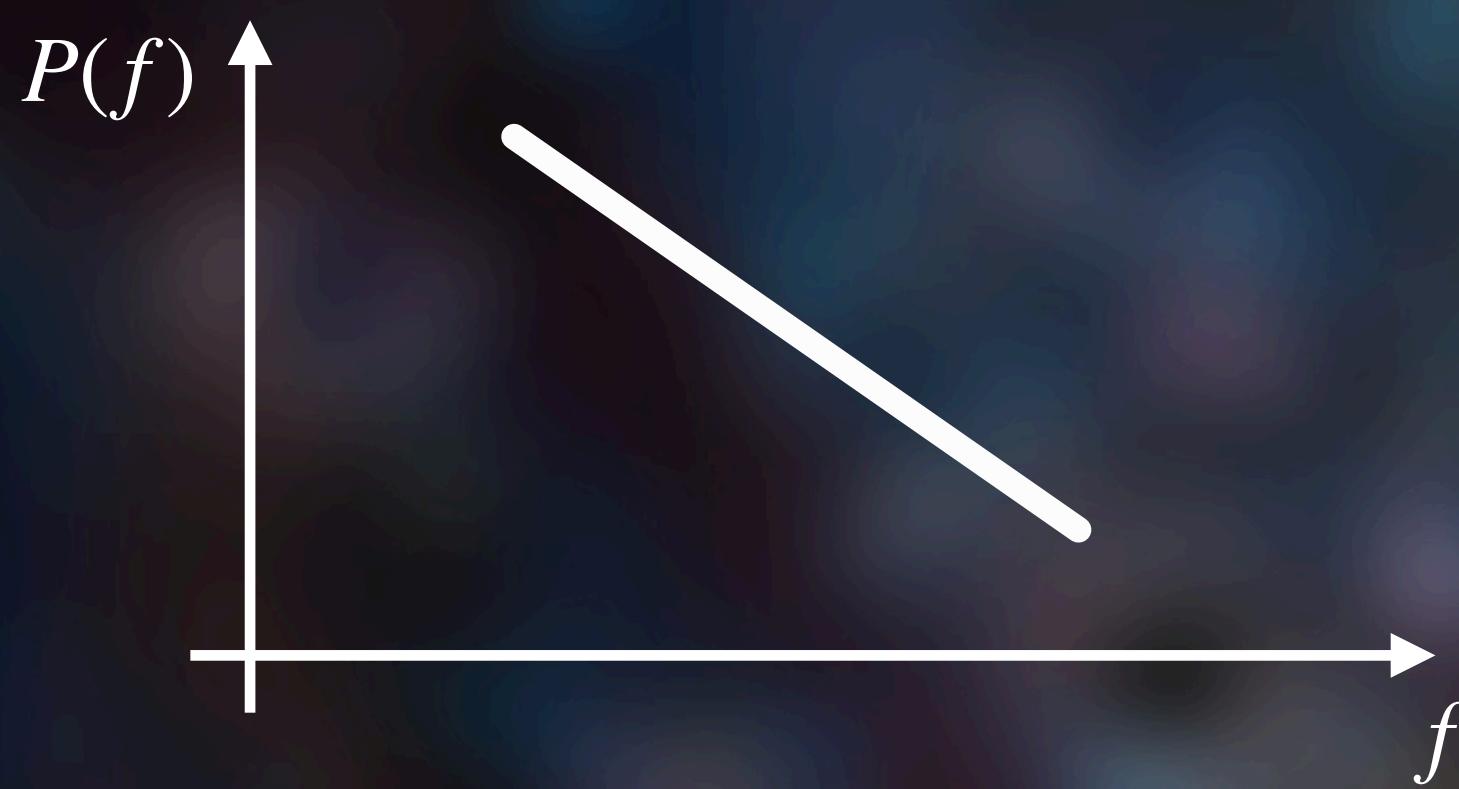
# CONTINUOUS WAVE

$$h_{ij}(t, \mathbf{x}) = \sum_{A=+, \times} e_{ij}^A(\hat{n}) \cos [\omega(t - \hat{n} \cdot \mathbf{x})]$$



# GW BACKGROUND

$$h_{ij}(t, \mathbf{x}) = \sum_{A=+, \times} \int df \int d^2\hat{n} \tilde{h}_A(f, \hat{n}) e_{ij}^A(\hat{n}) e^{-2\pi i f(t - \hat{n} \cdot \mathbf{x})}$$



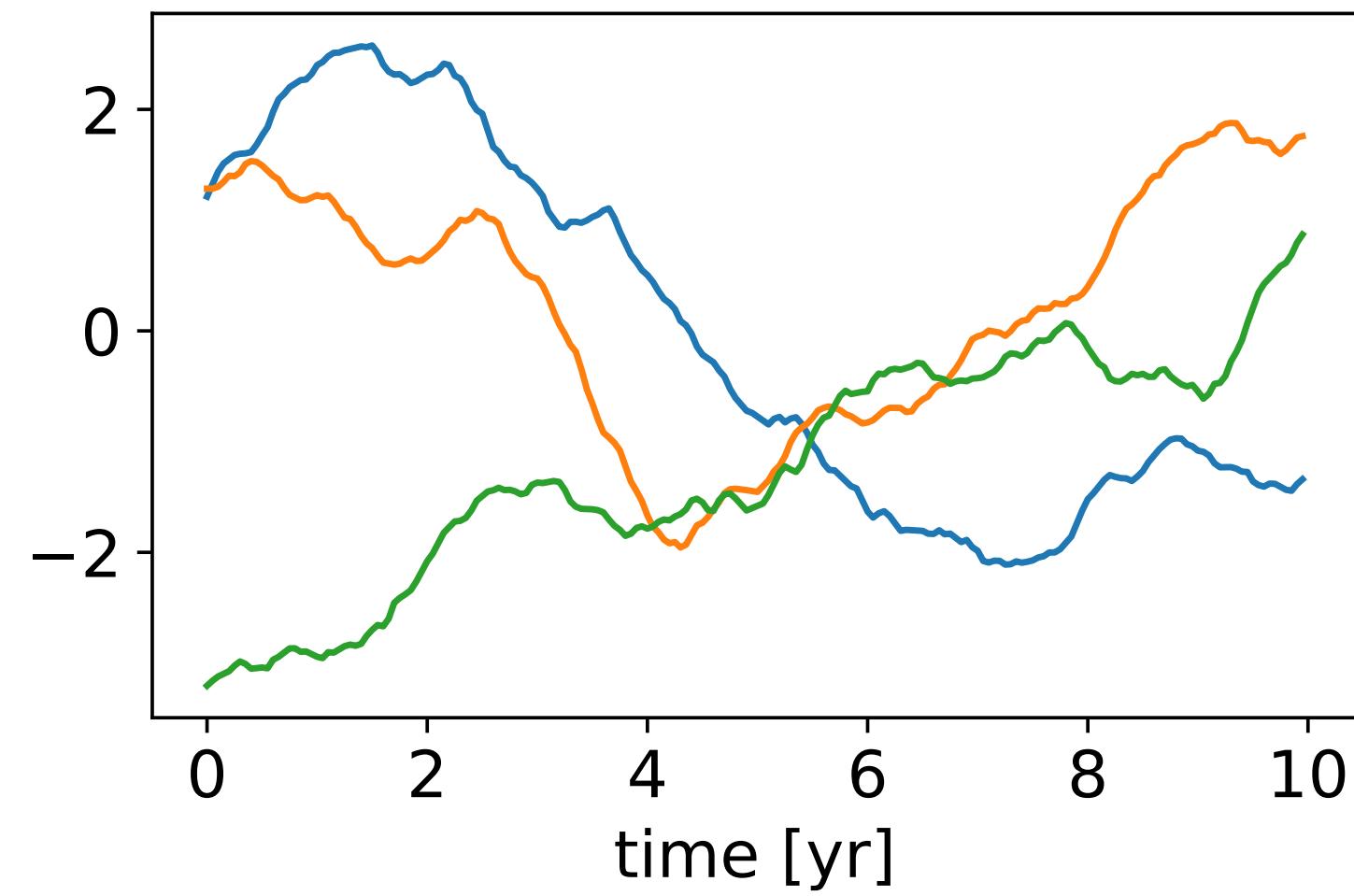
# GW BACKGROUND

$$h_{ij}(t, \mathbf{x}) = \sum_{A=+, \times} \int df \int d^2\hat{n} \tilde{h}_A(f, \hat{n}) e_{ij}^A(\hat{n}) e^{-2\pi i f(t - \hat{n} \cdot \mathbf{x})}$$



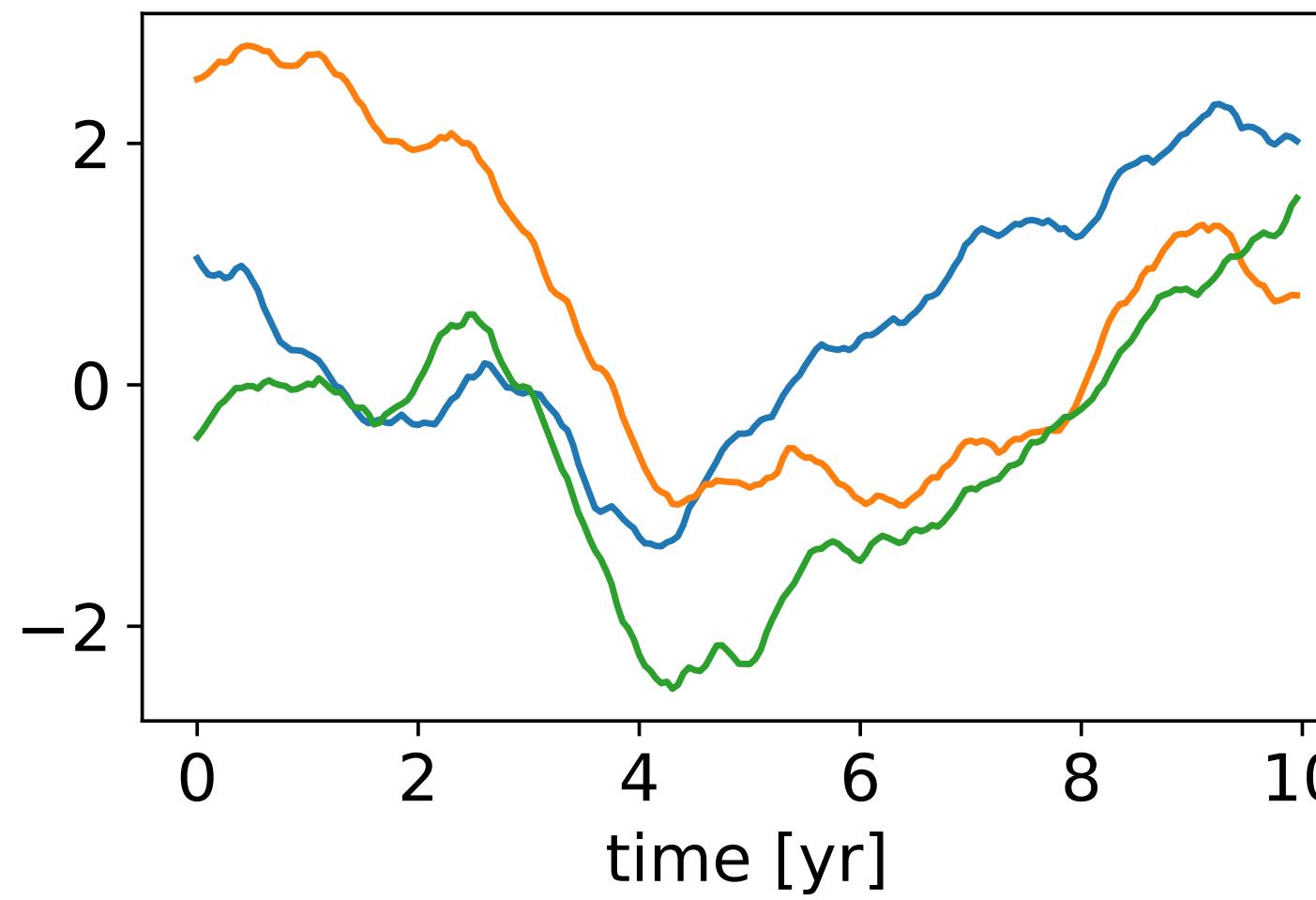
# CORRELATIONS EXAMPLE

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



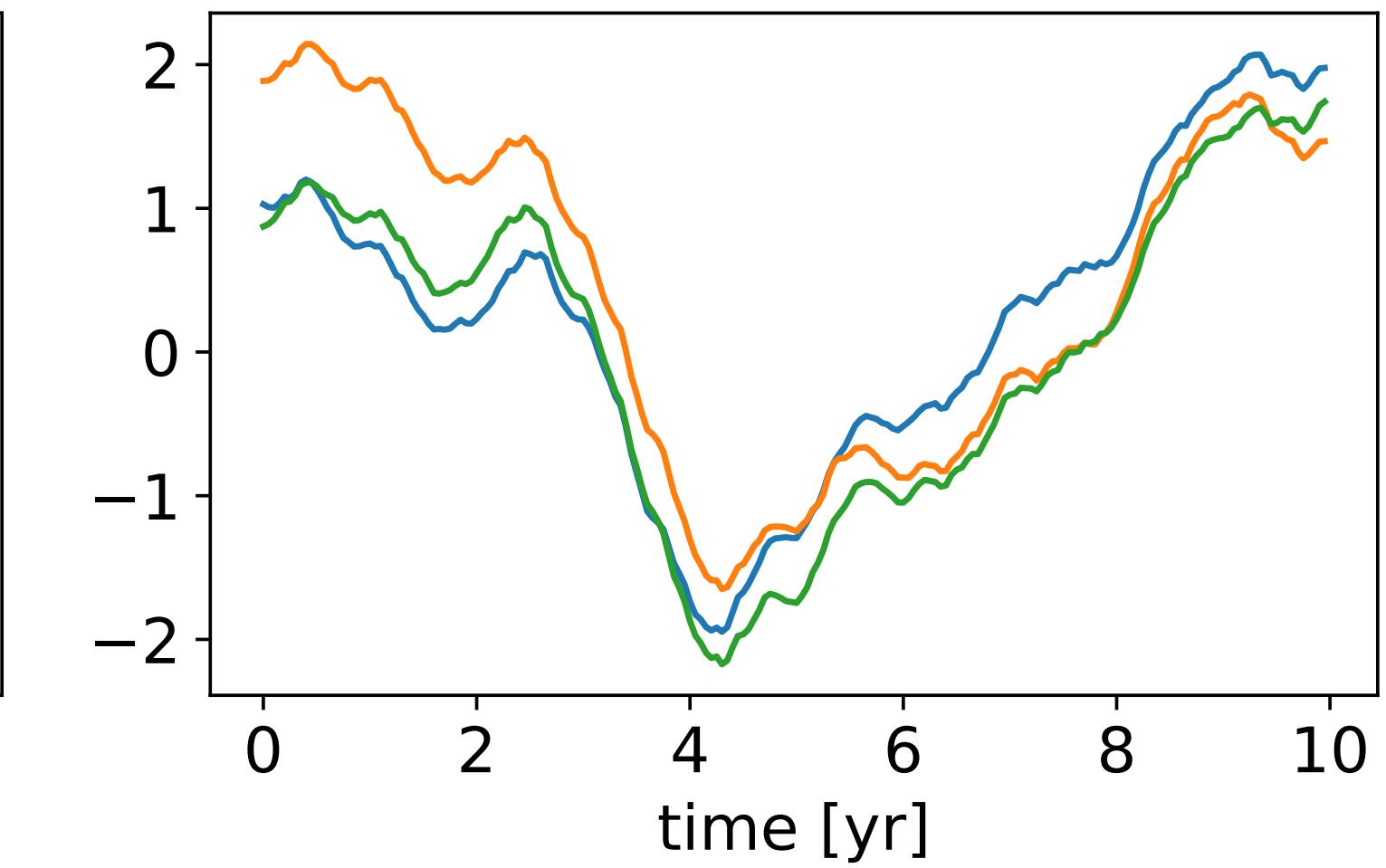
uncorrelated

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix}$$



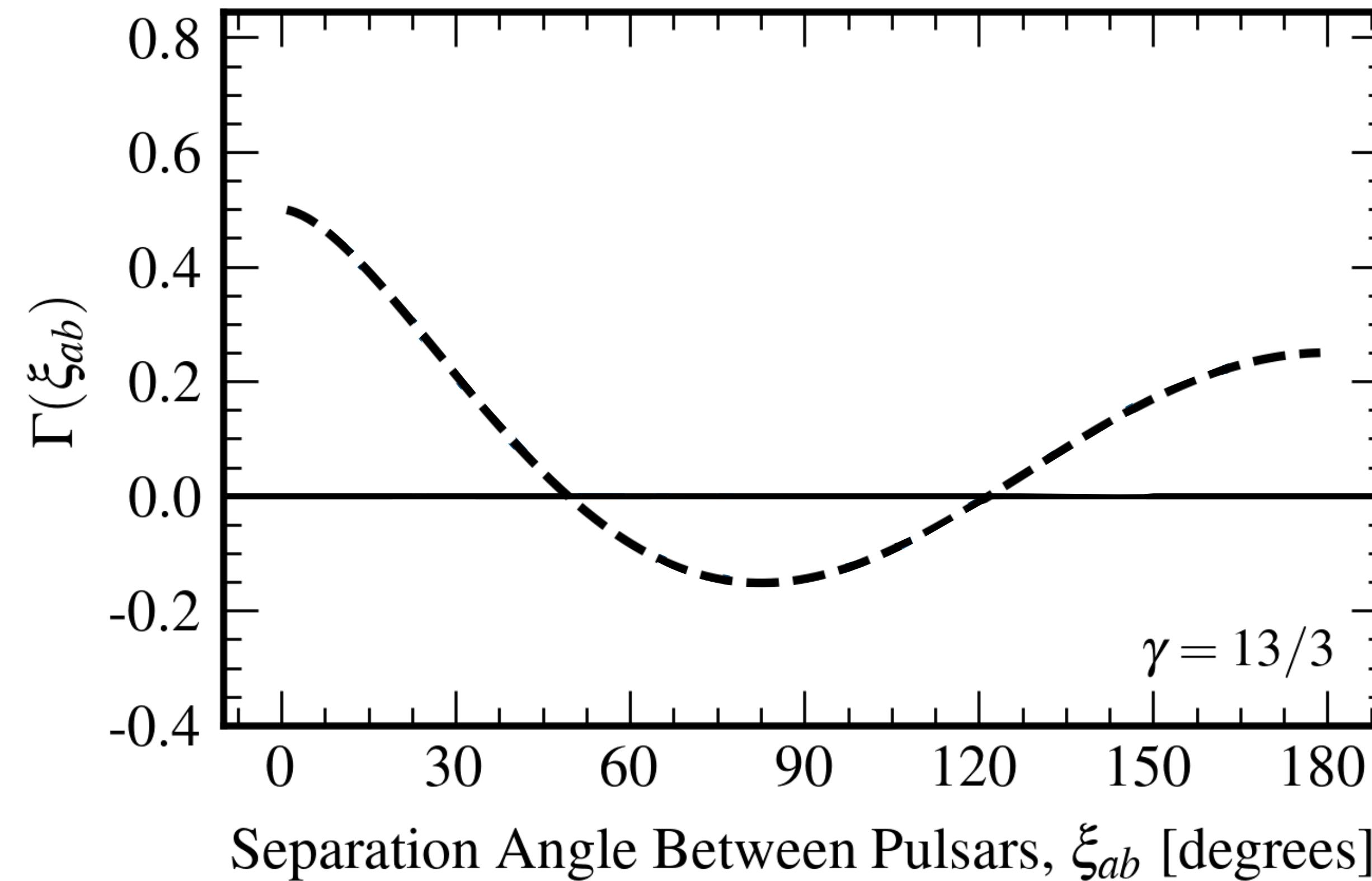
moderately correlated

$$\Gamma_{ab} = \begin{pmatrix} 1 & 0.95 & 0.95 \\ 0.95 & 1 & 0.95 \\ 0.95 & 0.95 & 1 \end{pmatrix}$$

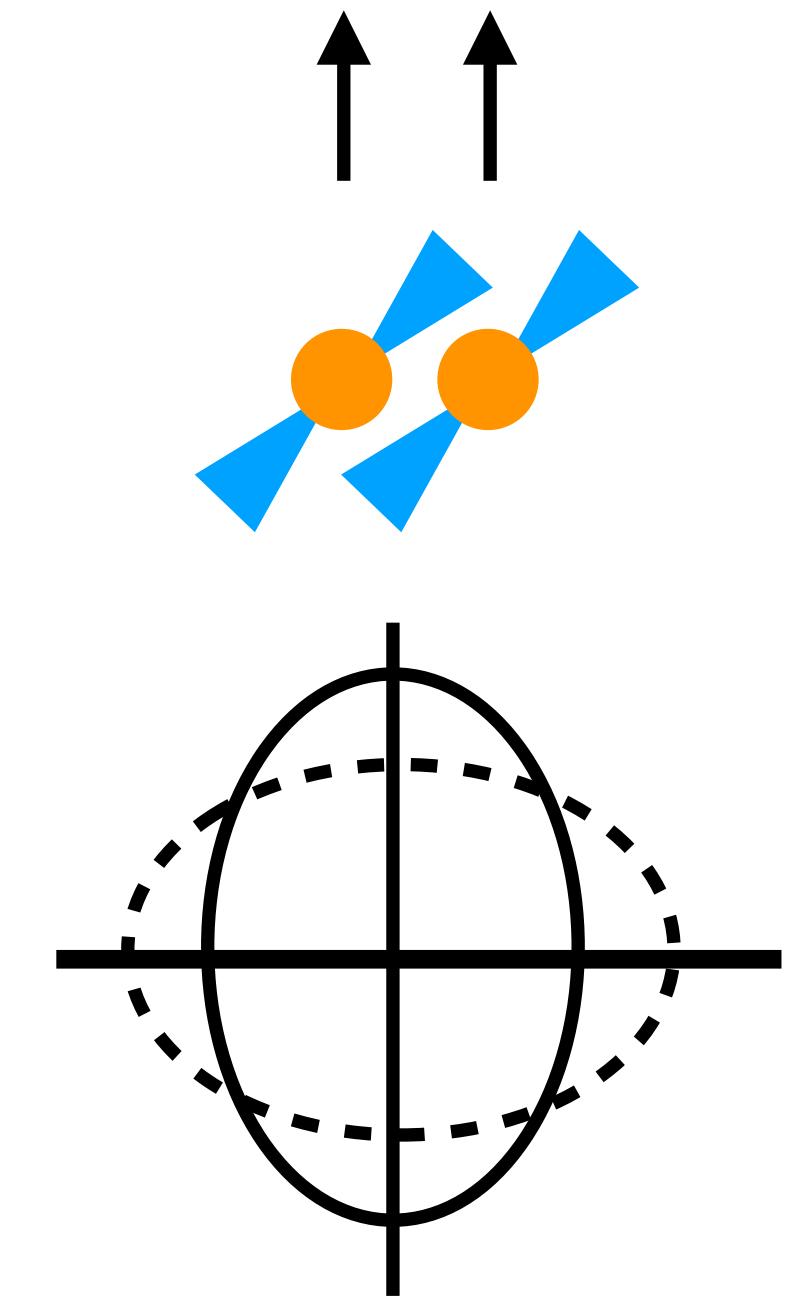
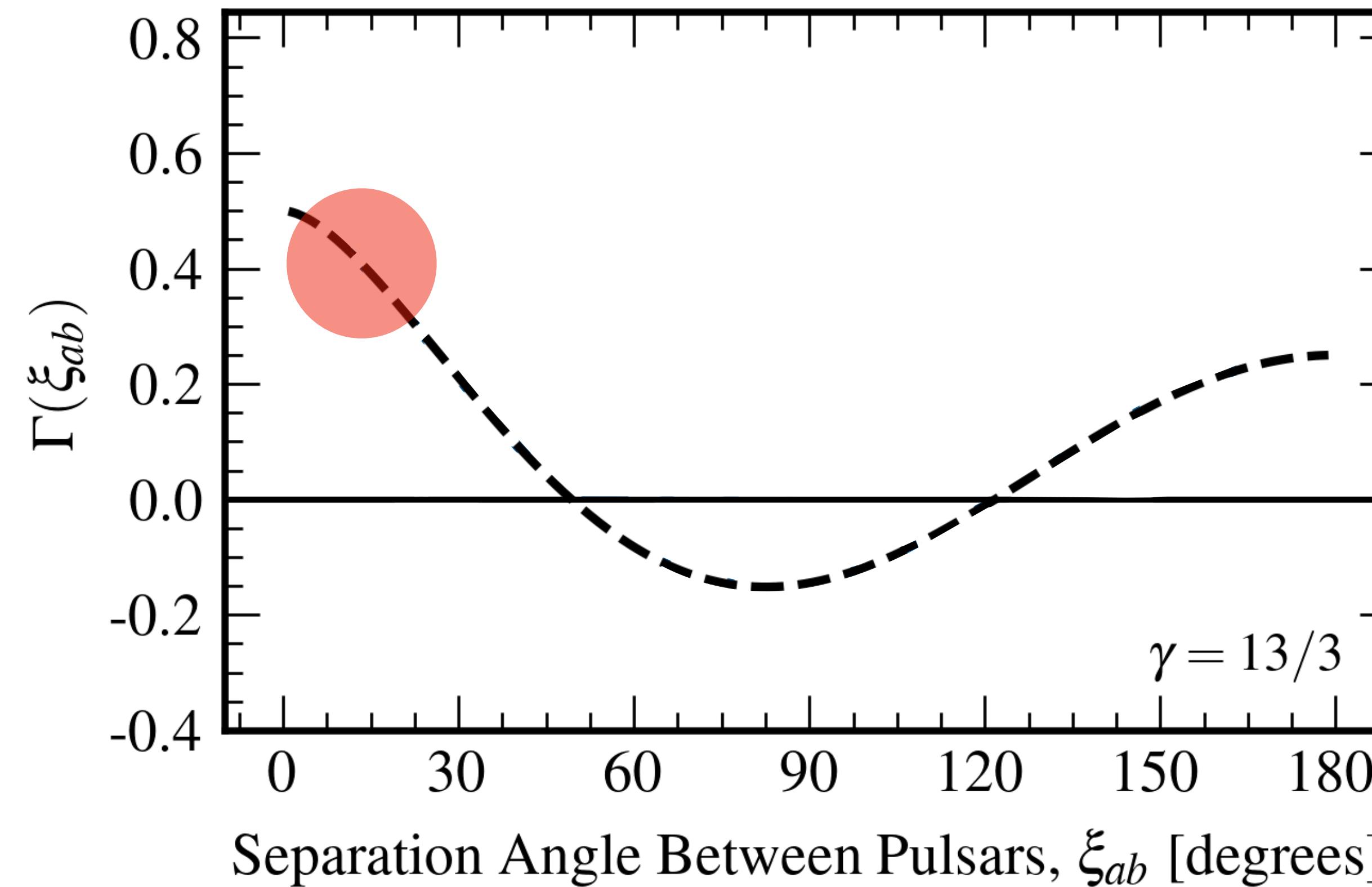


strongly correlated

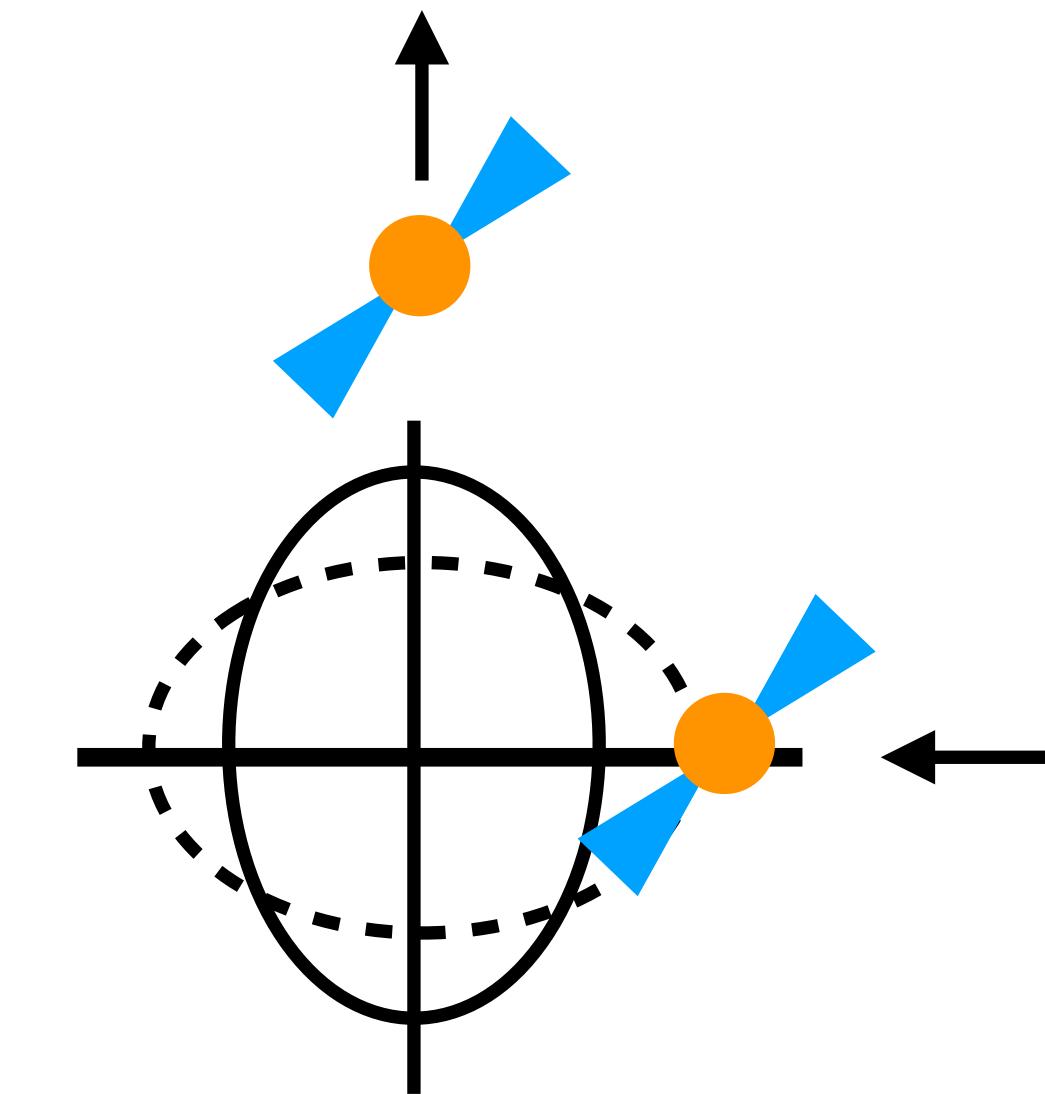
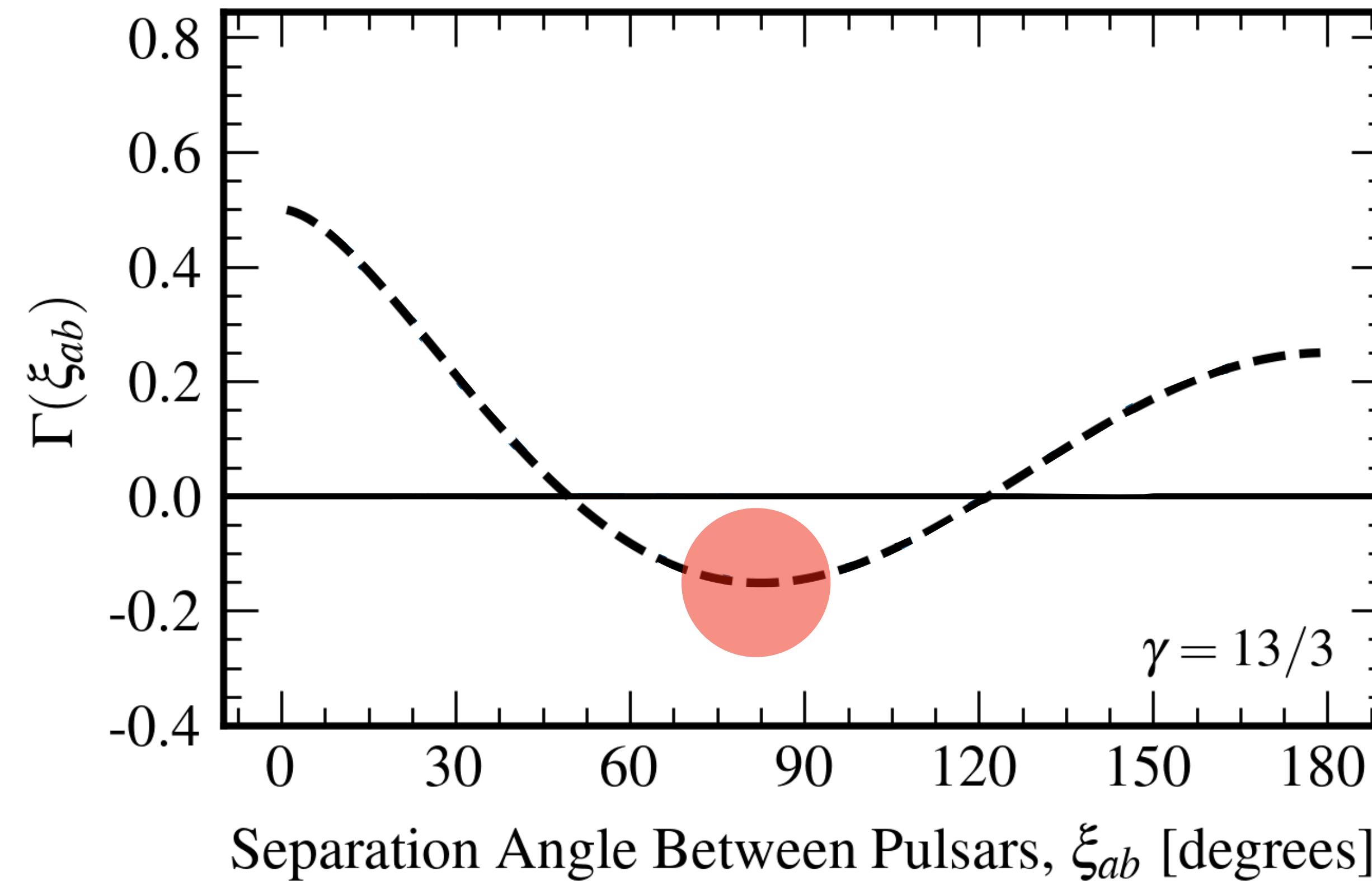
# HELLINGS & DOWNS CURVE



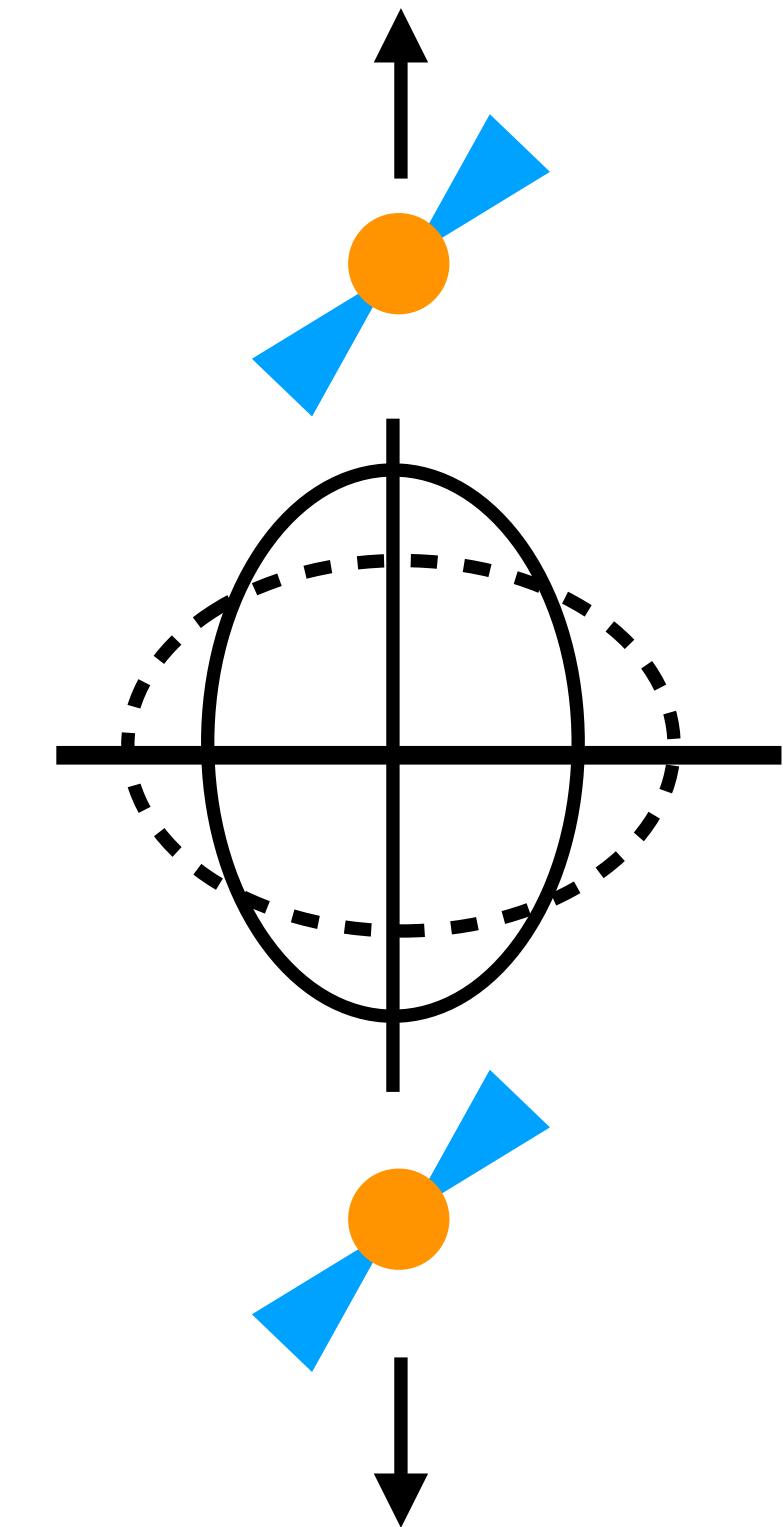
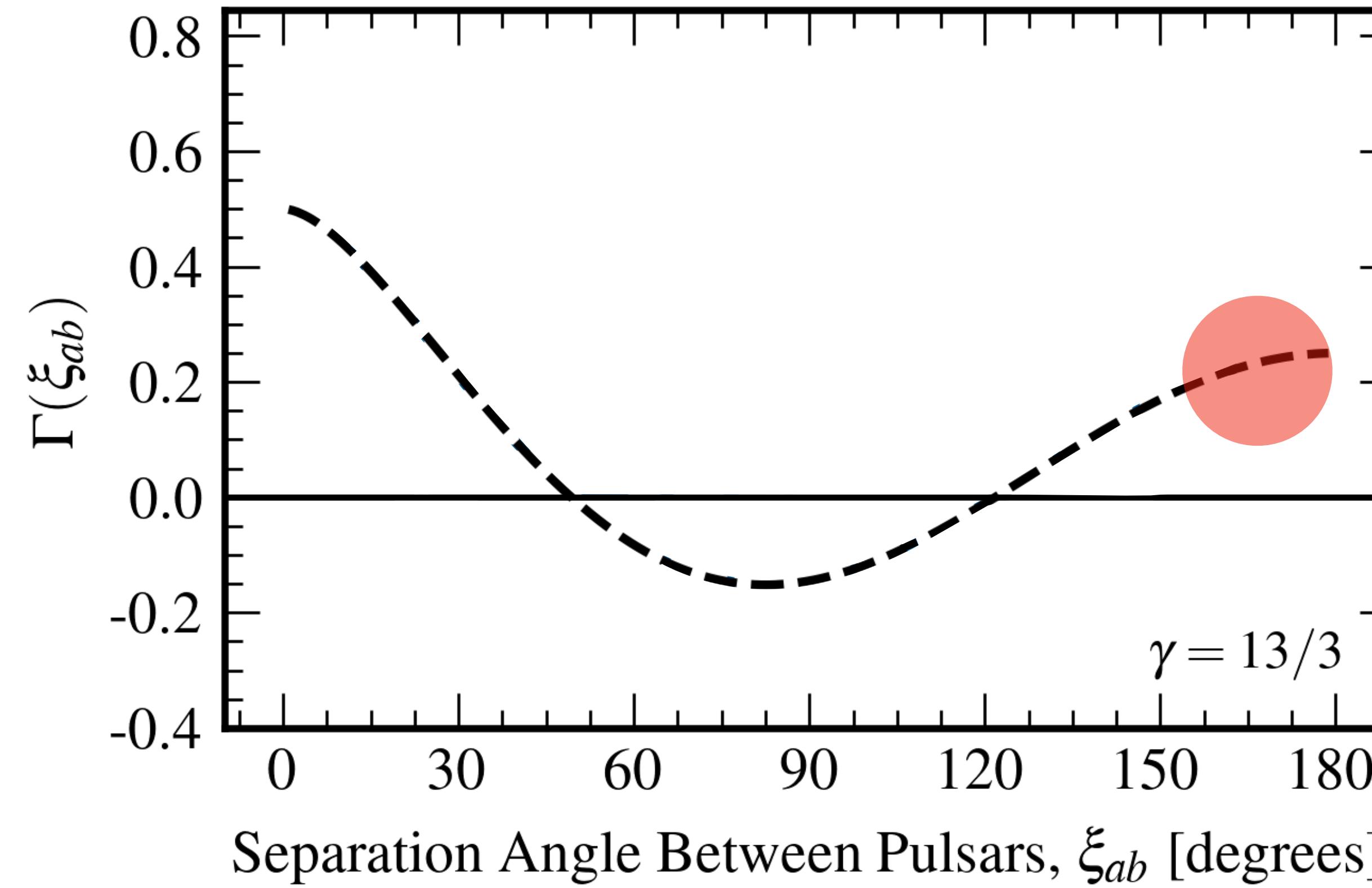
# HELLINGS & DOWNS CURVE



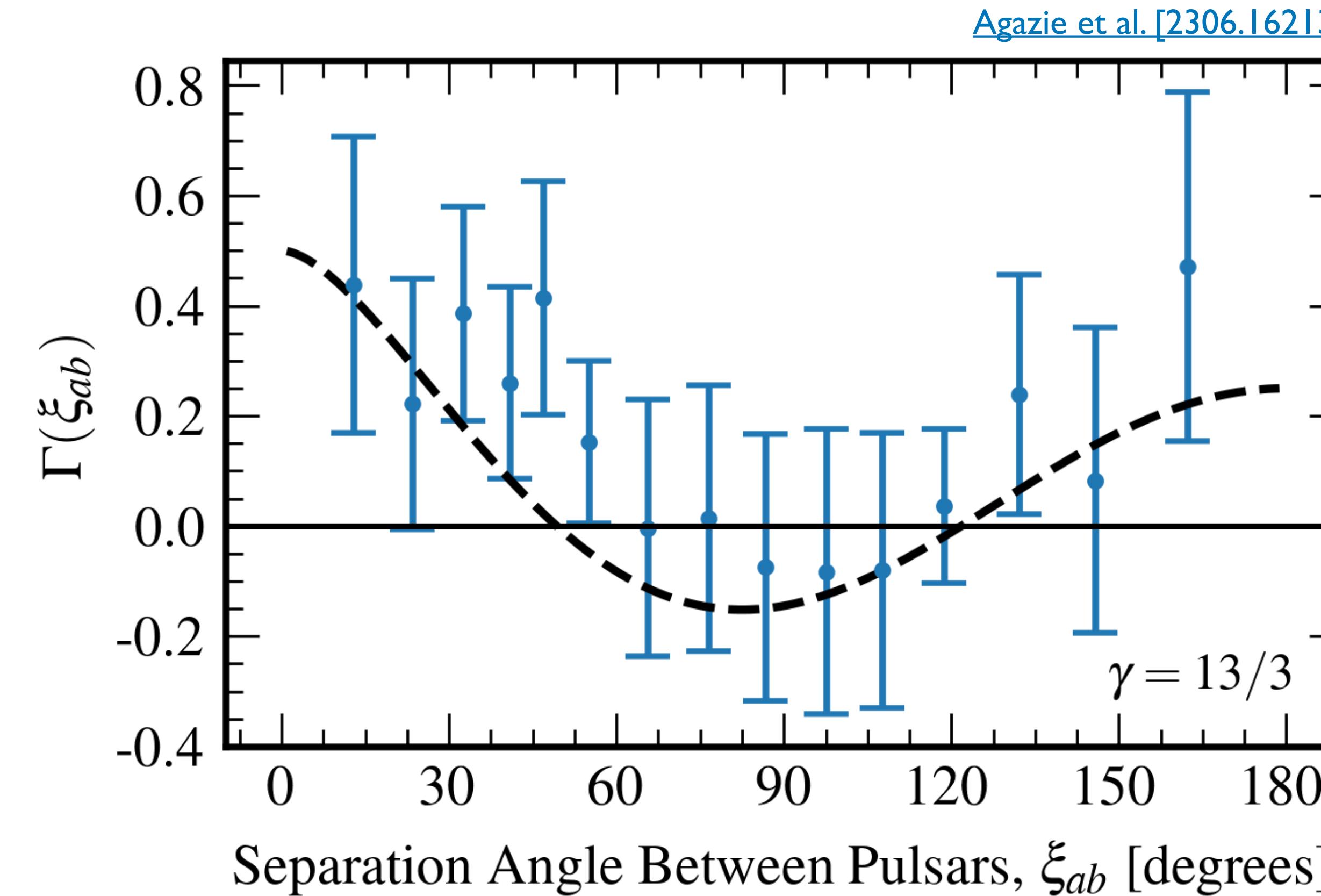
# HELLINGS & DOWNS CURVE



# HELLINGS & DOWNS CURVE

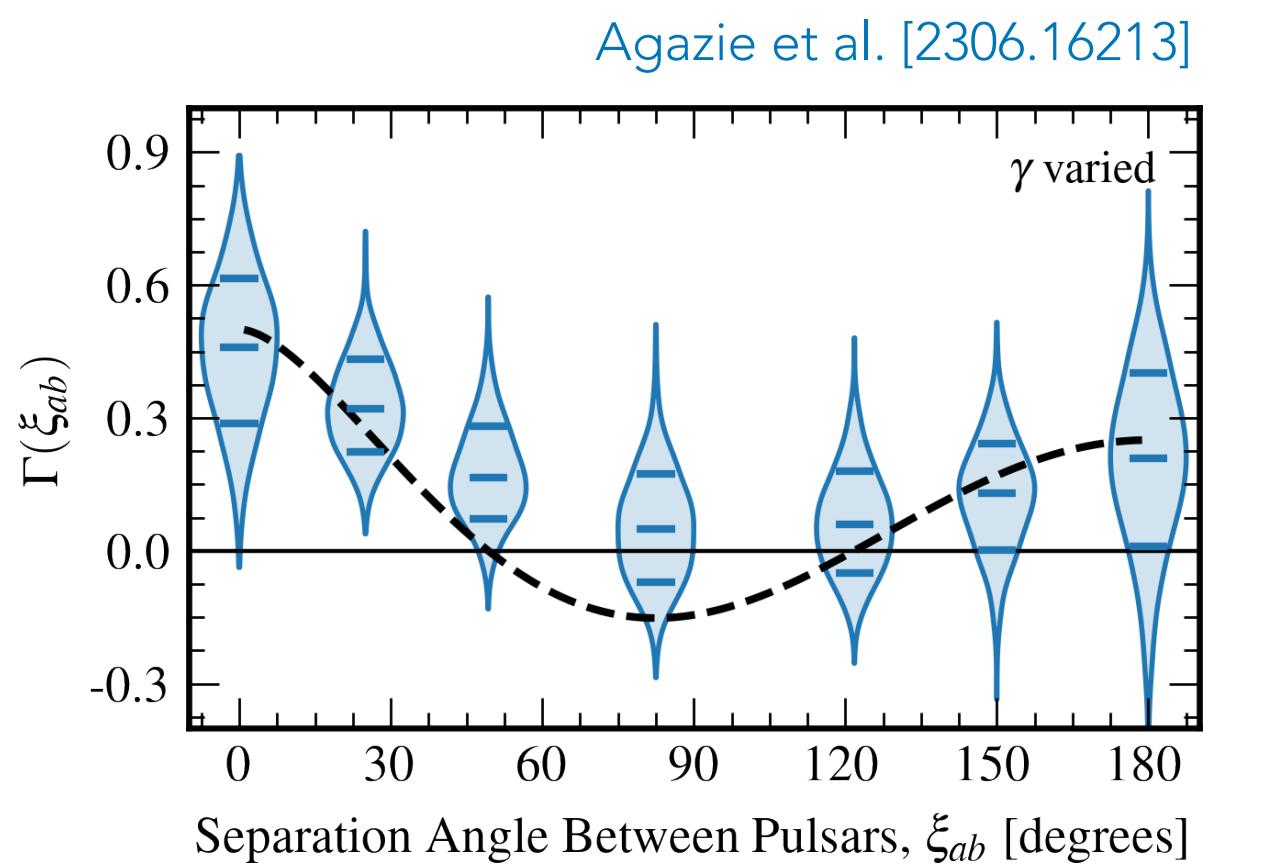


# EVIDENCE FOR GWB

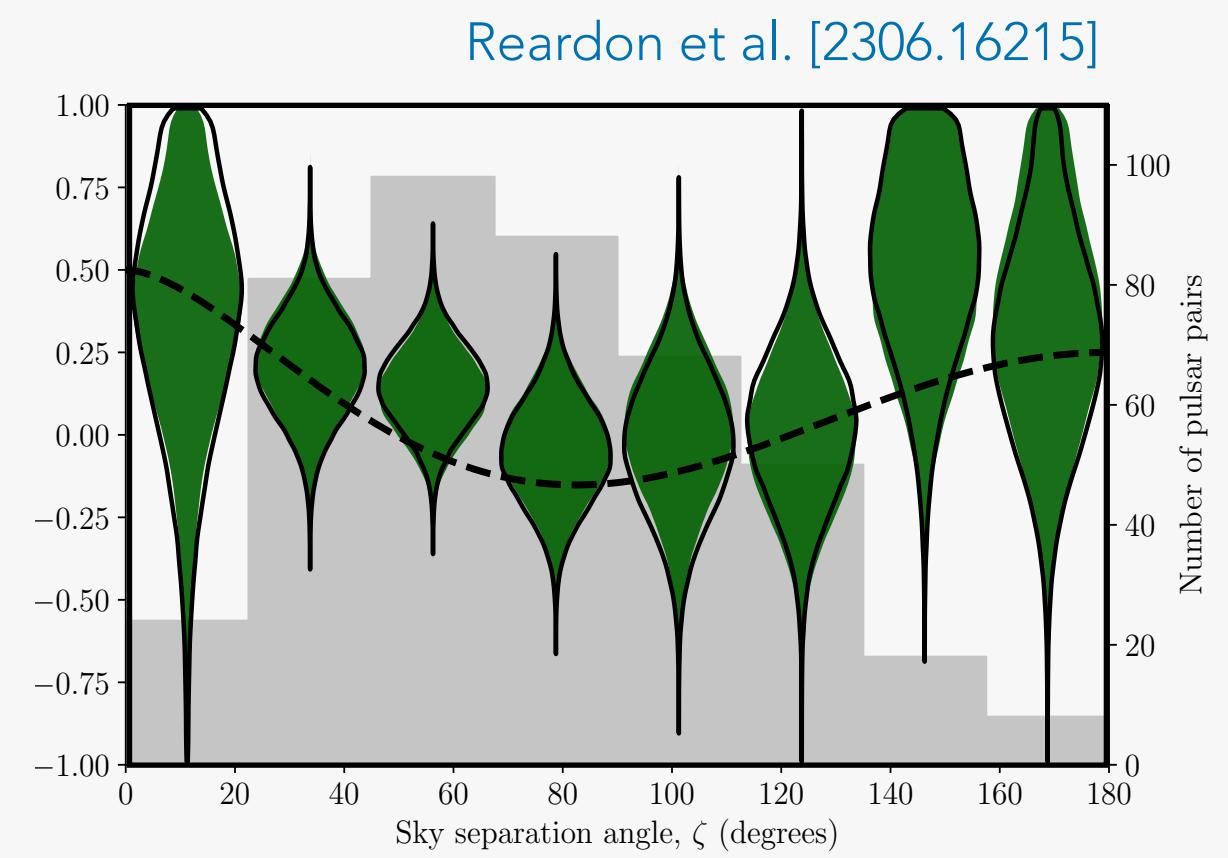


# EVIDENCE FOR GWB

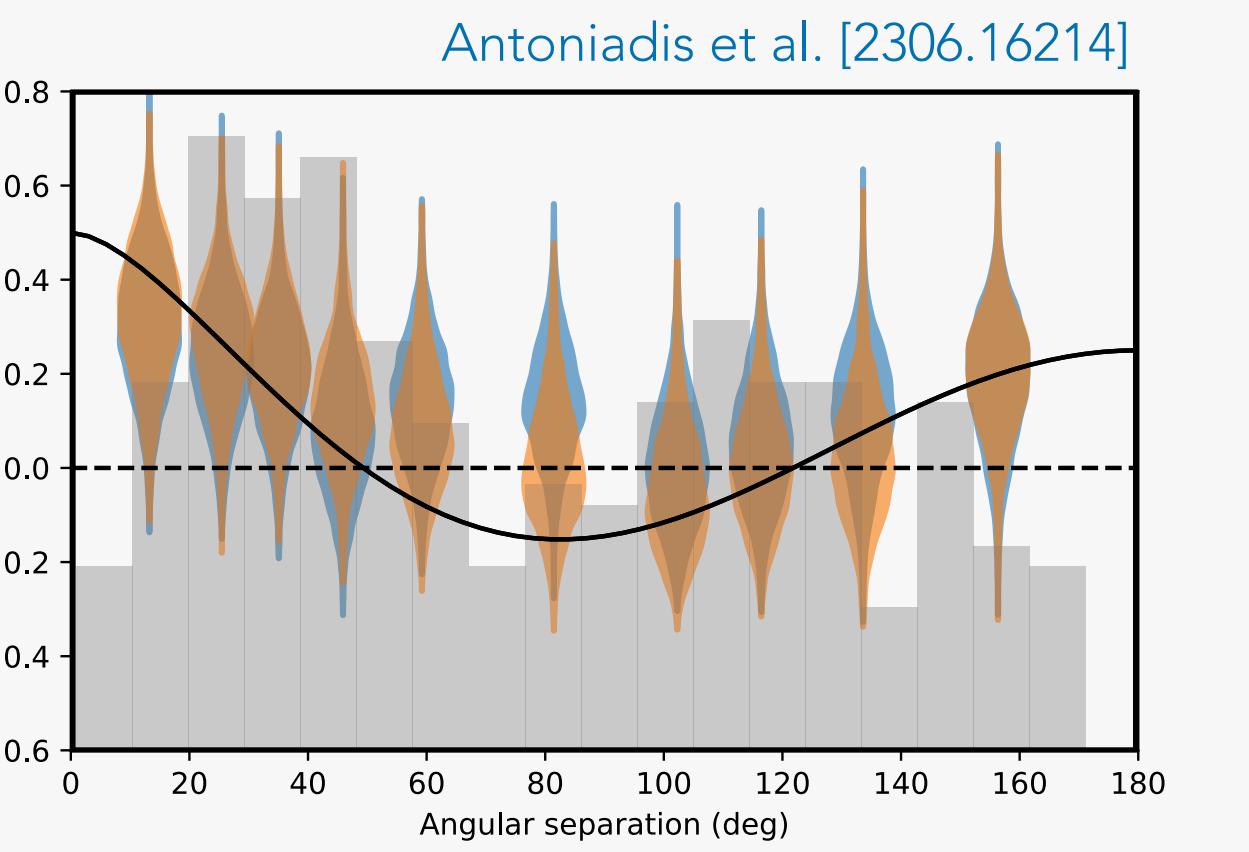
**NANOGrav:**  
68 pulsars, 16yr of data  
~ $3\sigma$  significance



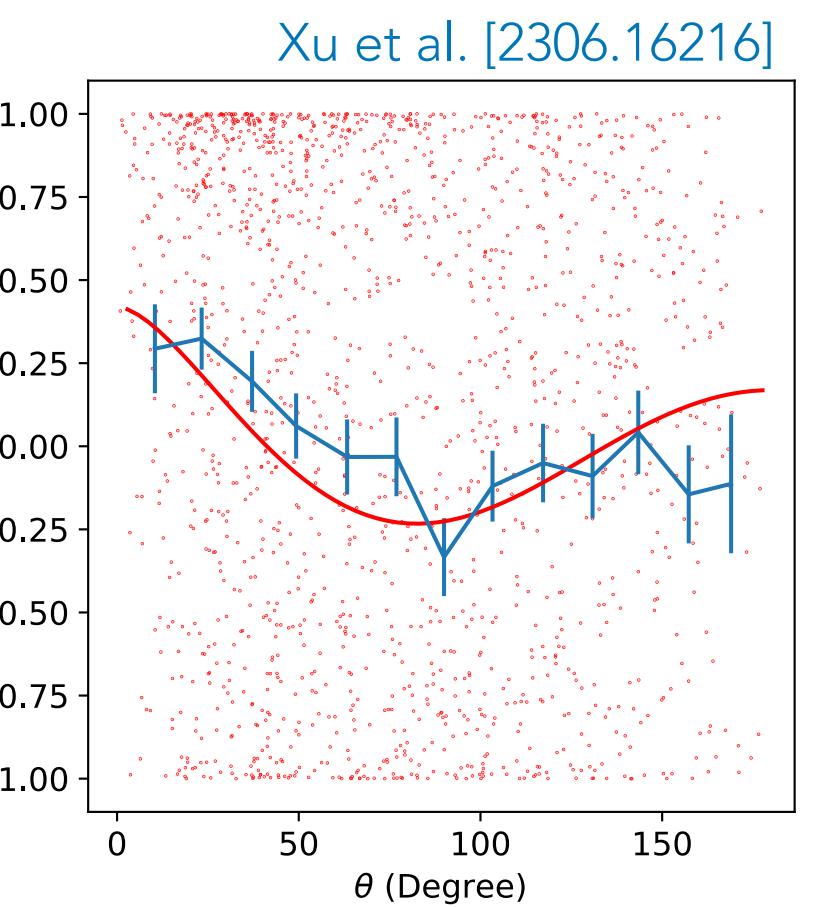
**PPTA:**  
32 pulsars, 18yr of data  
~ $2\sigma$  significance



**EPTA + InPTA:**  
25 pulsars, 24yr of data  
~ $3\sigma$  significance

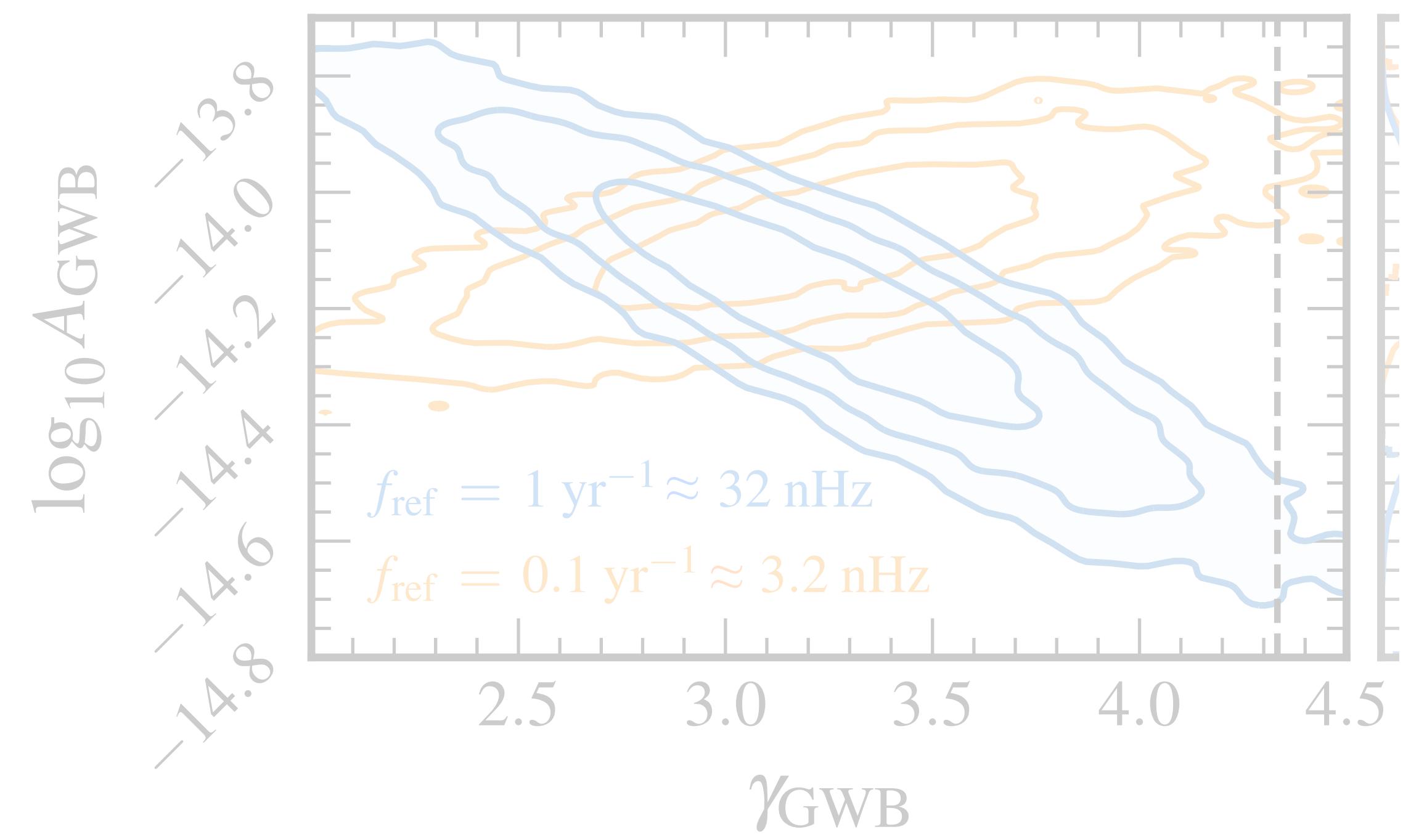
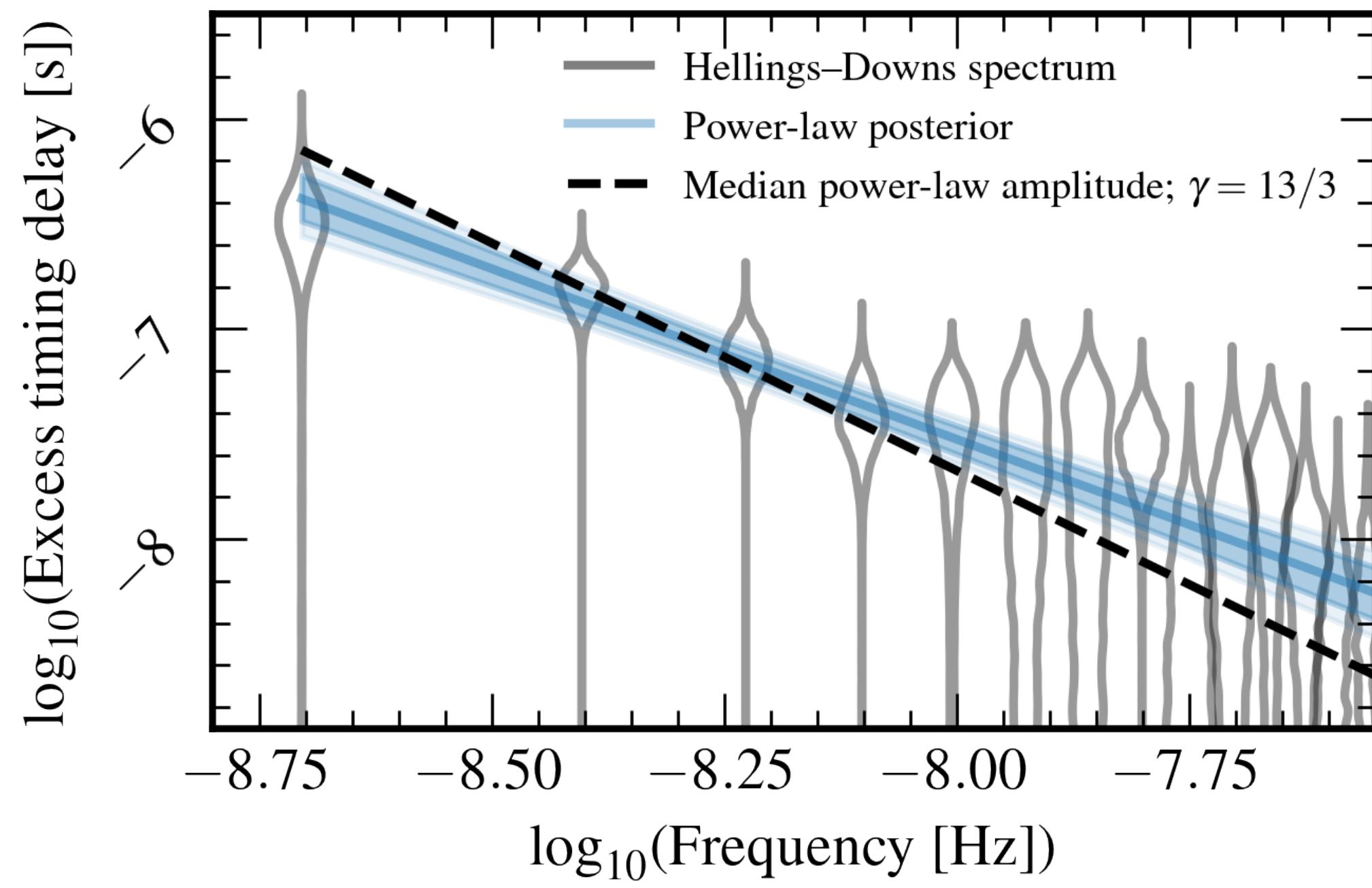


**CPTA:**  
57 pulsars, 3yr of data  
~ $4.6\sigma$  significance



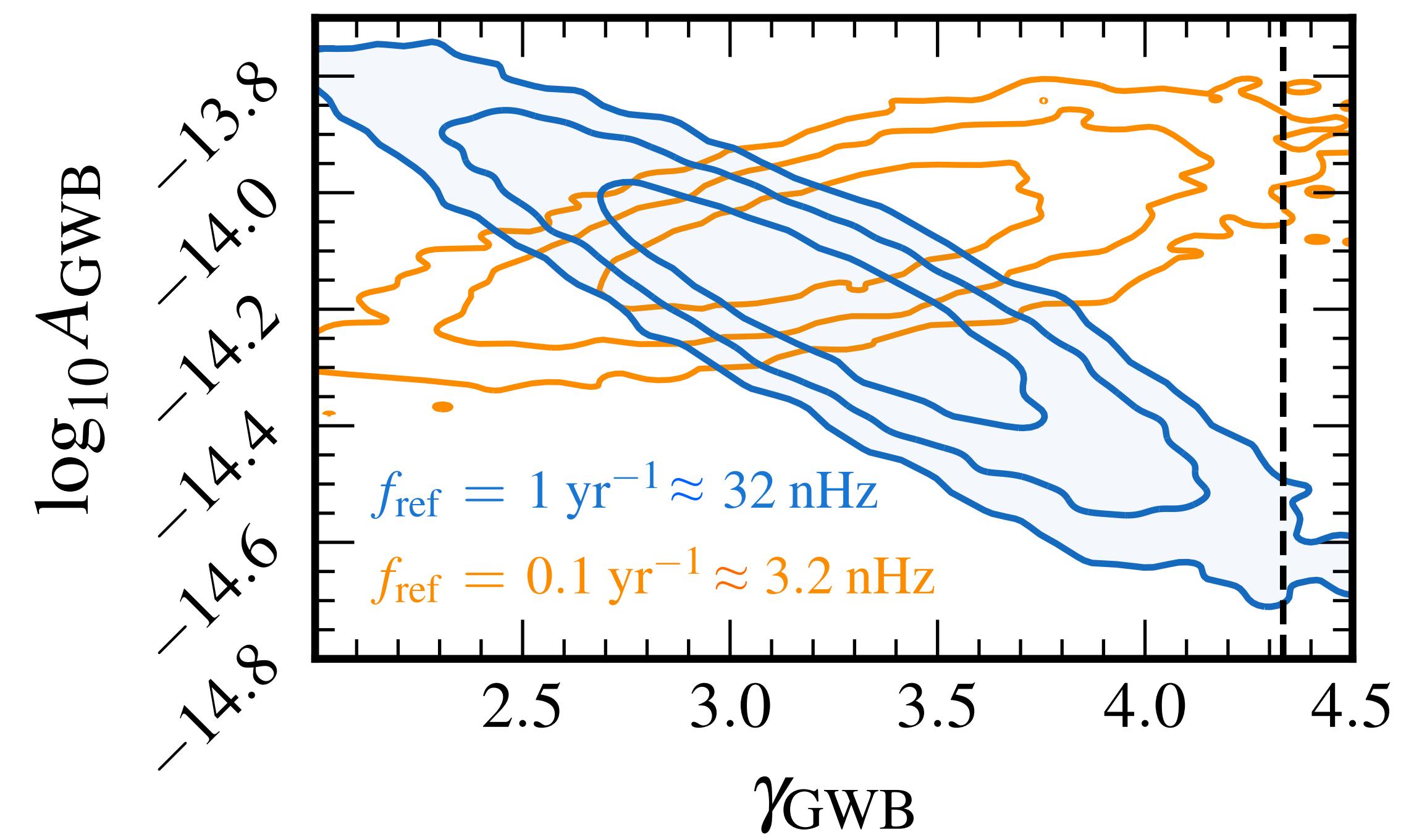
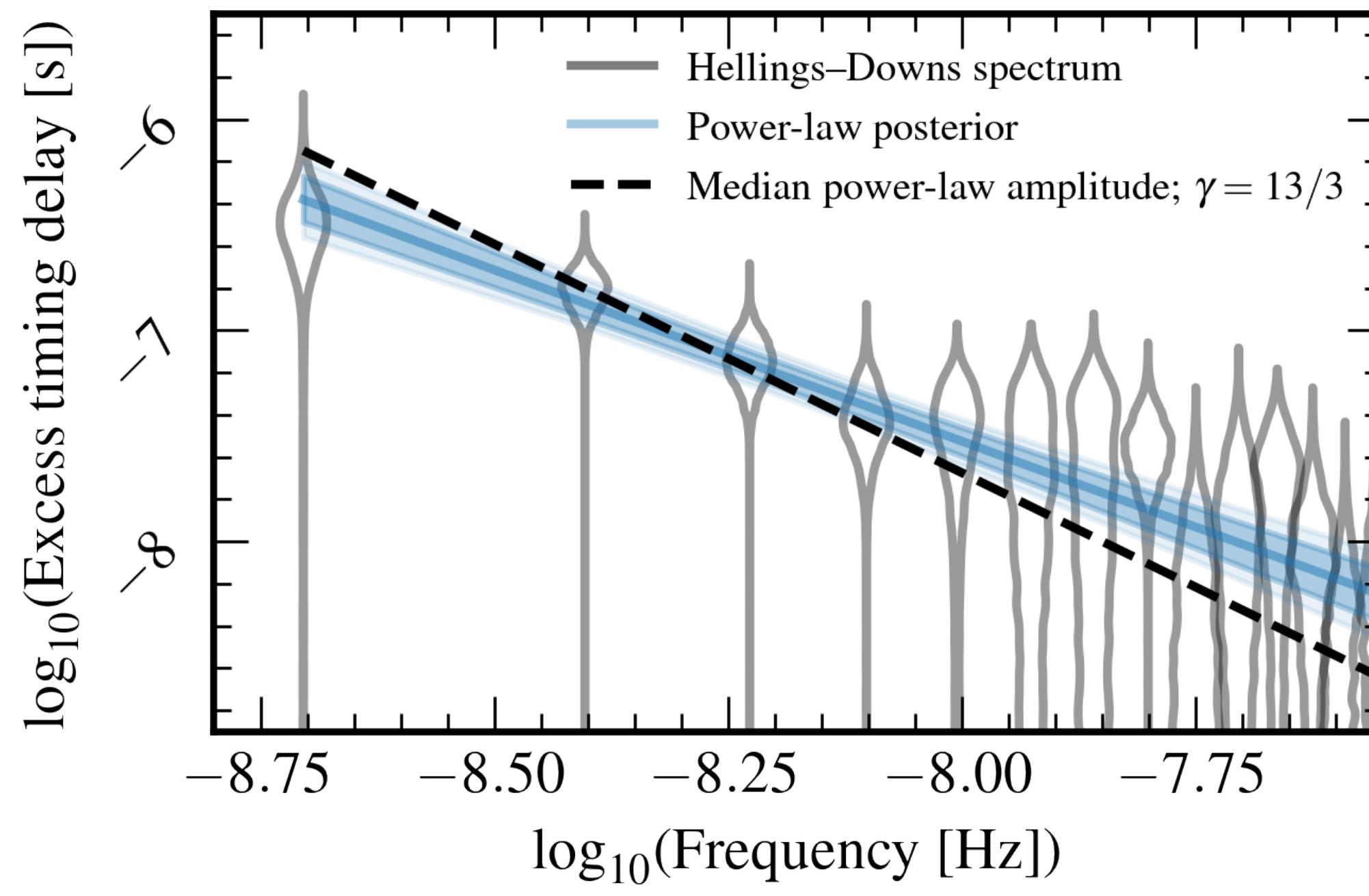
# SPECTRUM

Agazie et al. [2306.16213]



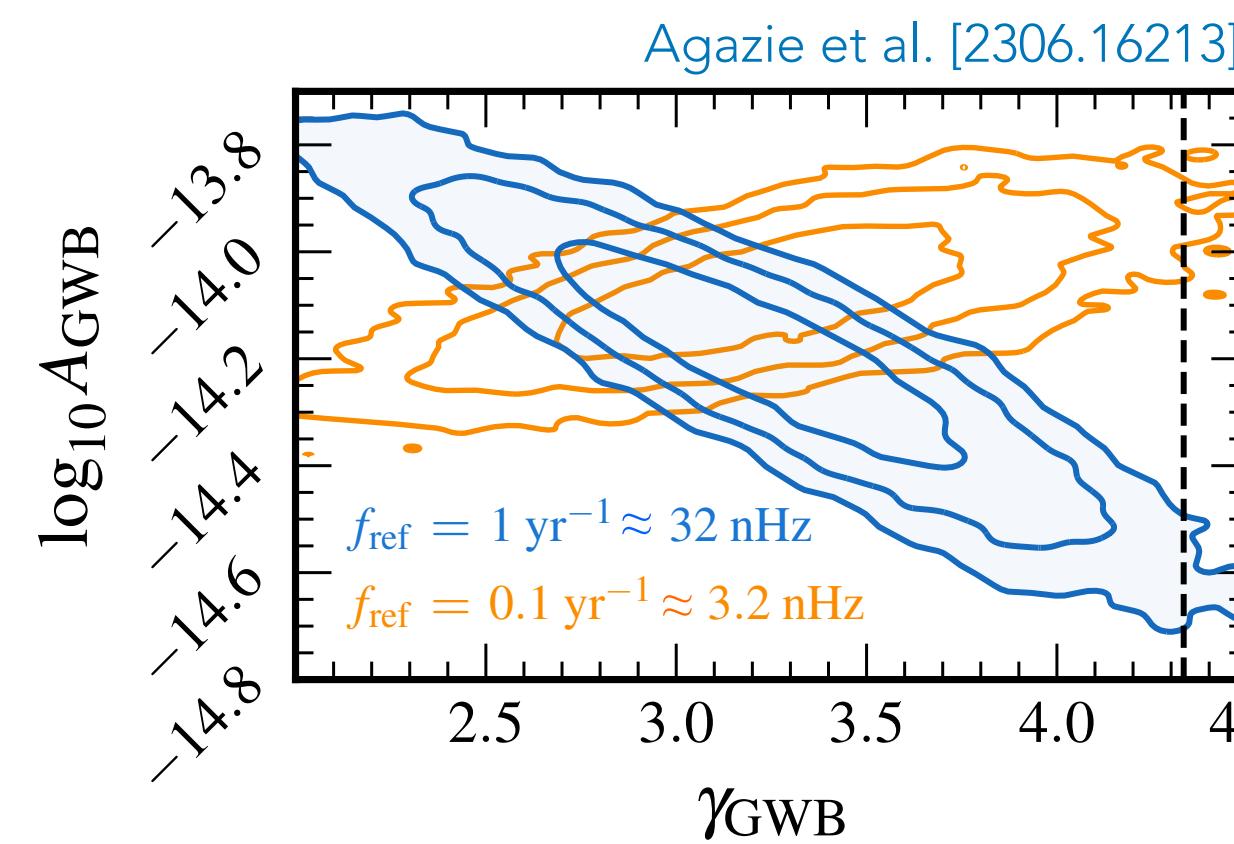
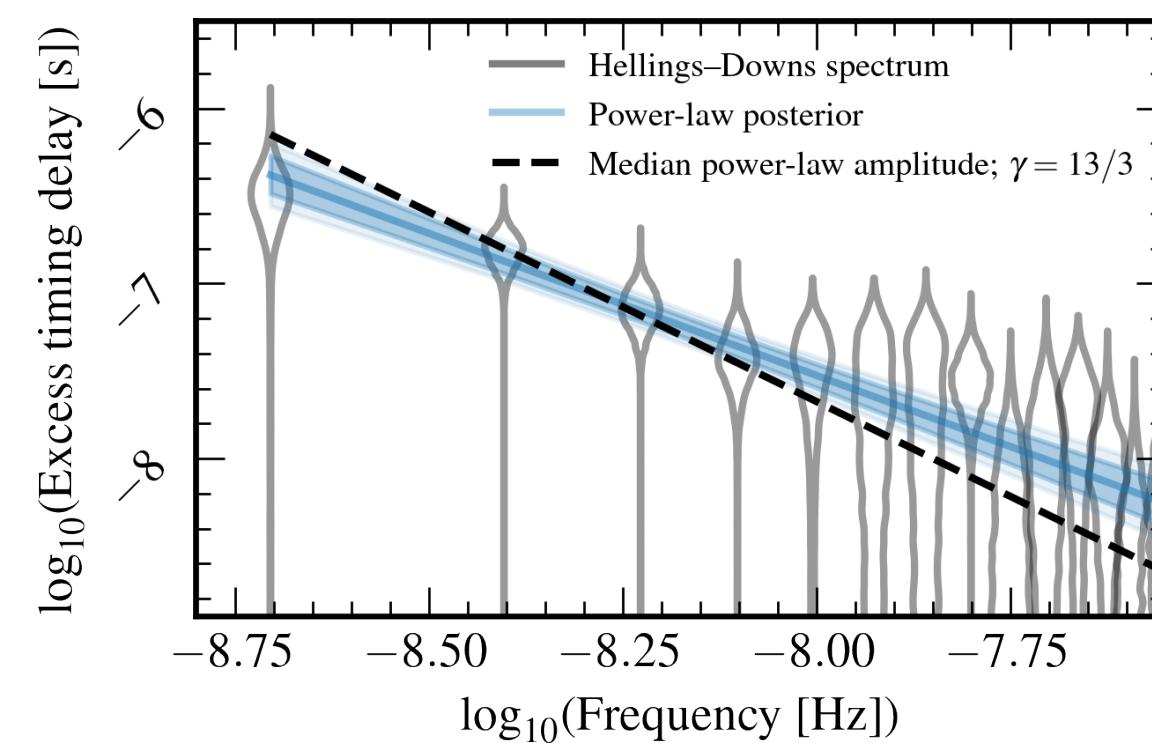
# SPECTRUM

Agazie et al. [2306.16213]

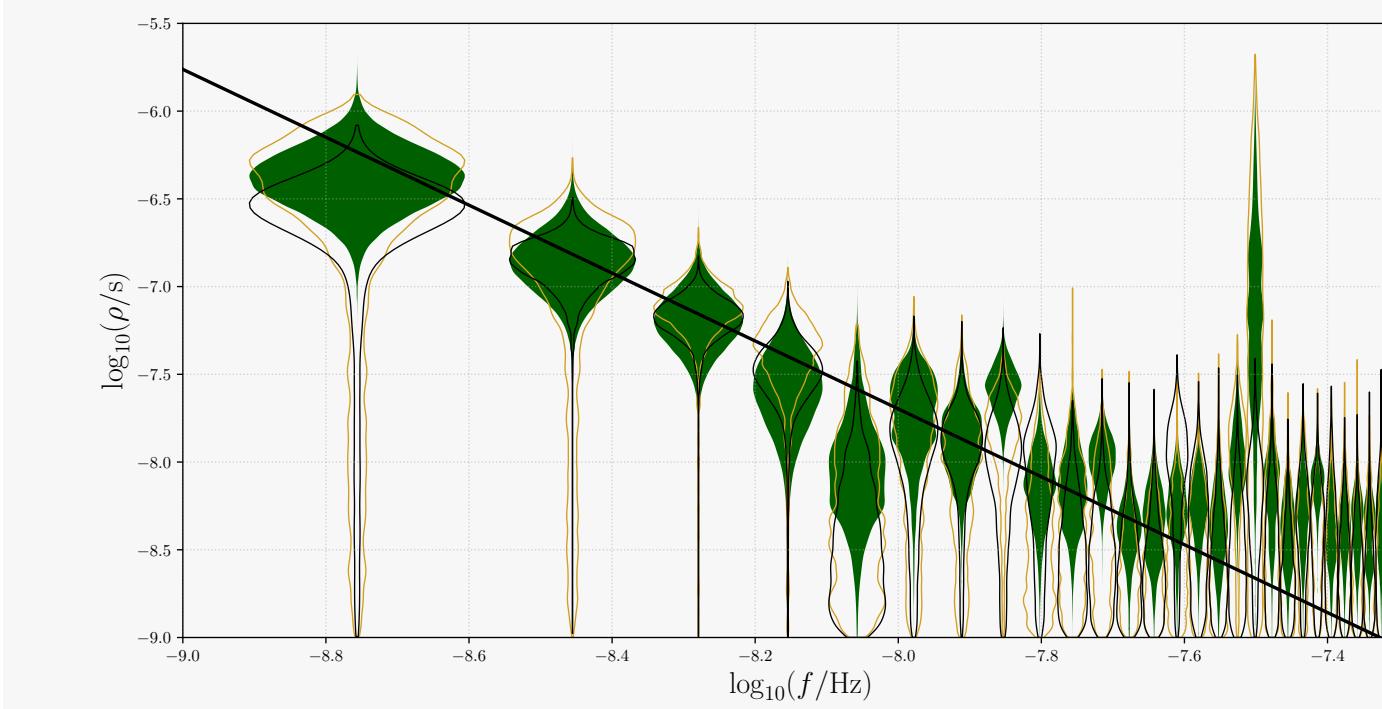


# SPECTRUM

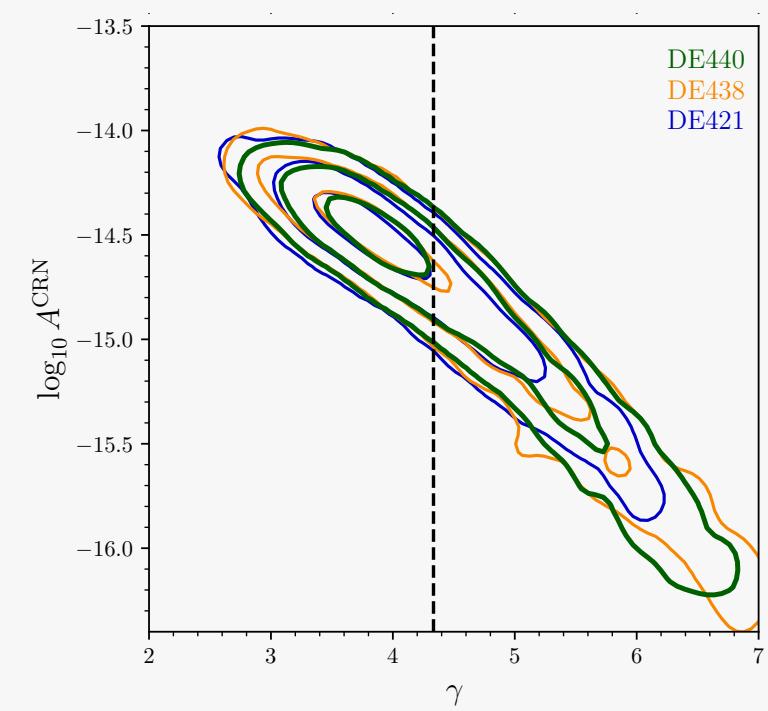
## NANOGrav



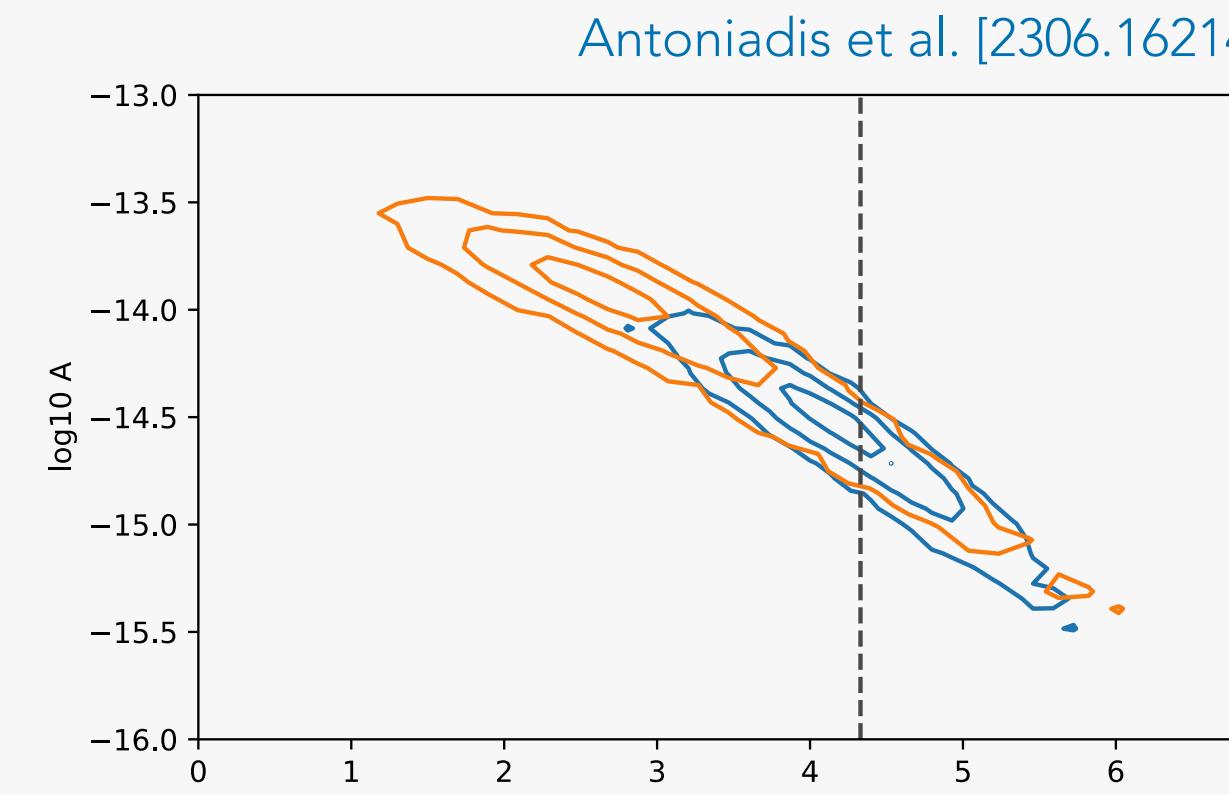
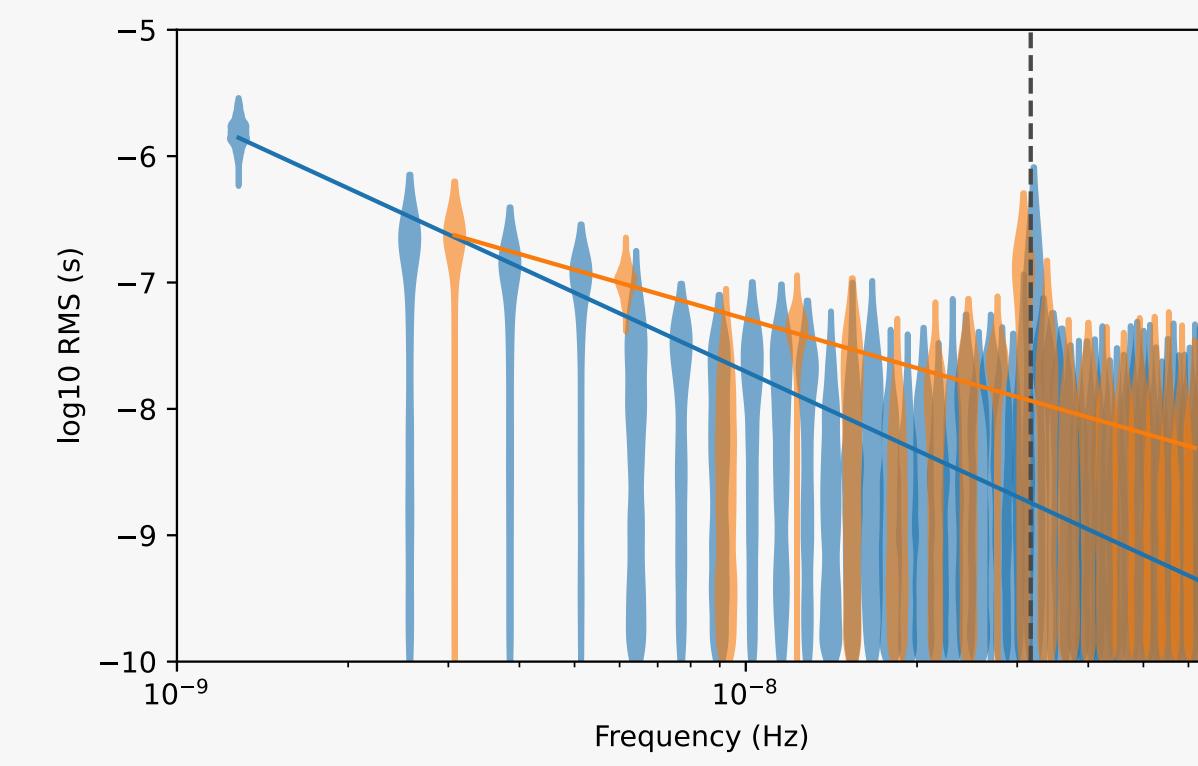
## PPTA



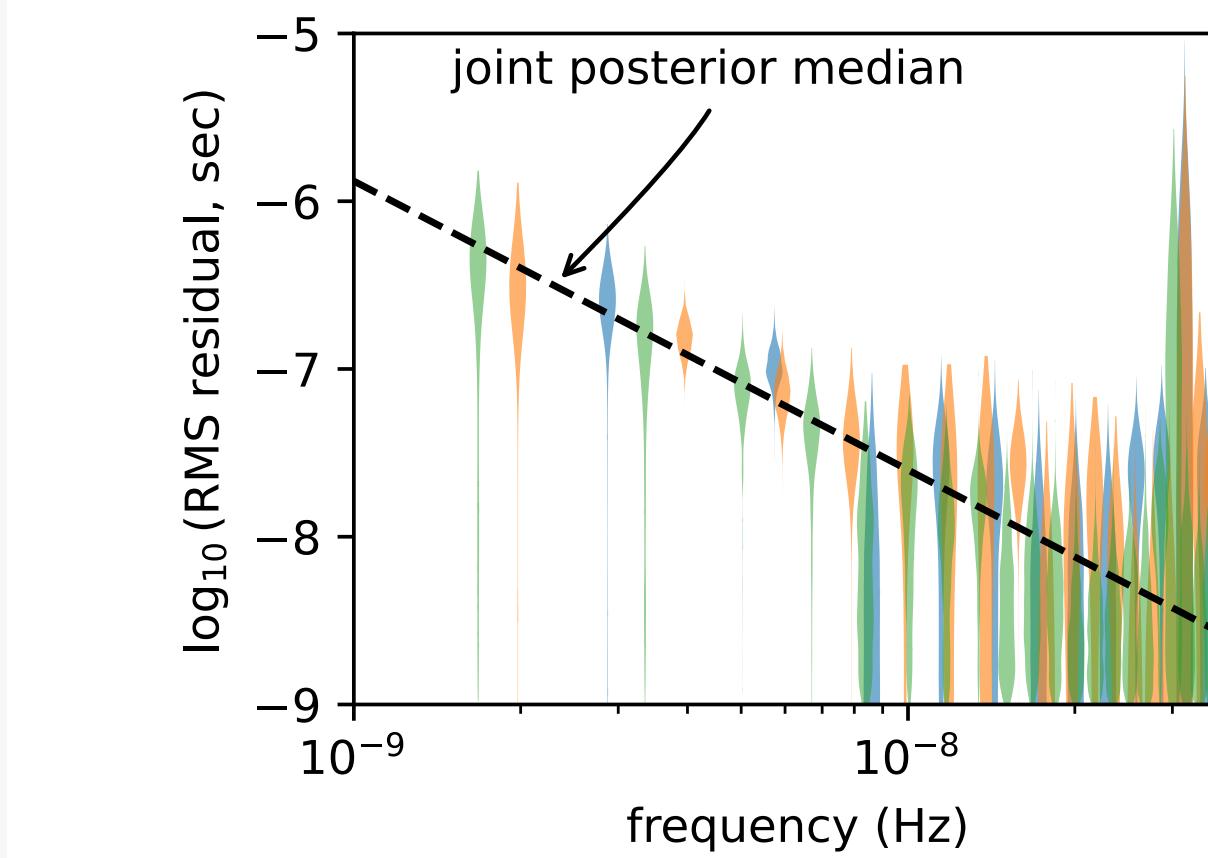
Reardon et al. [2306.16215]



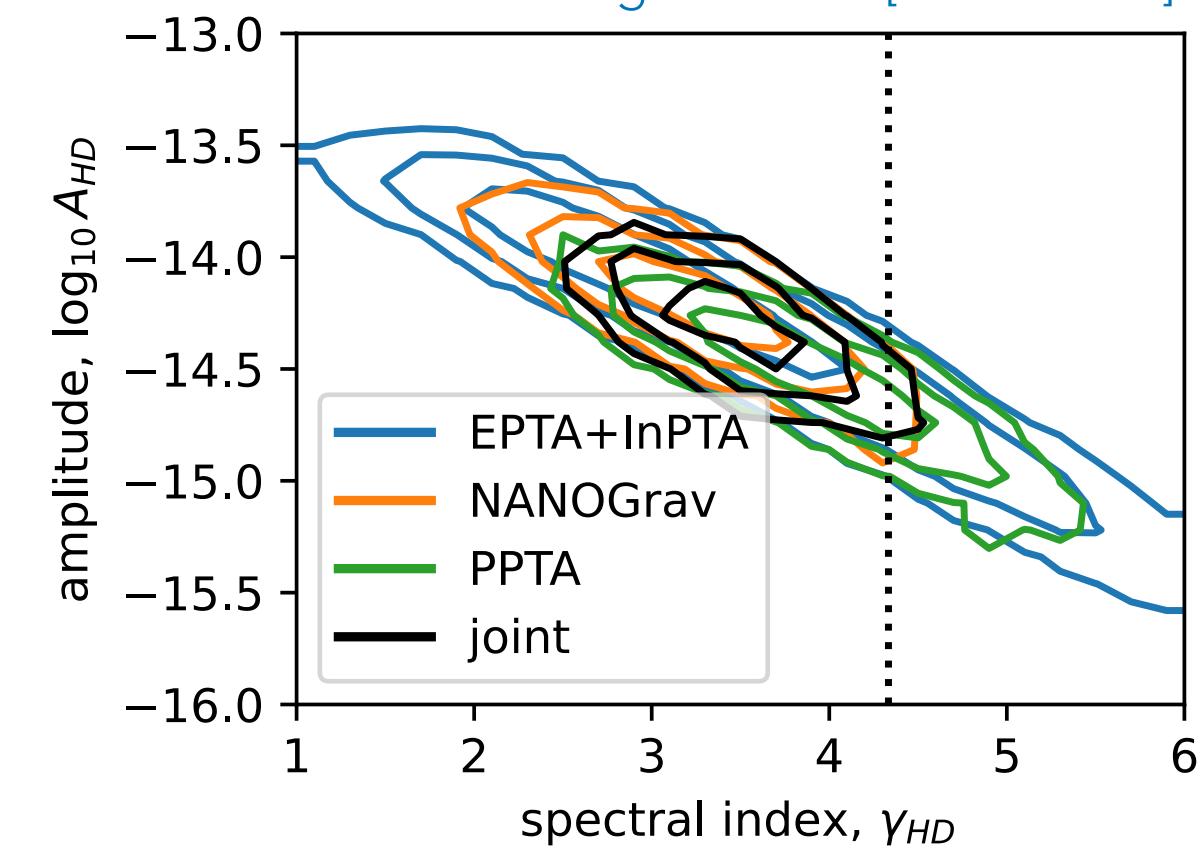
## EPTA + InPTA



## IPTA early data combination



Agazie et al. [2309.00693]

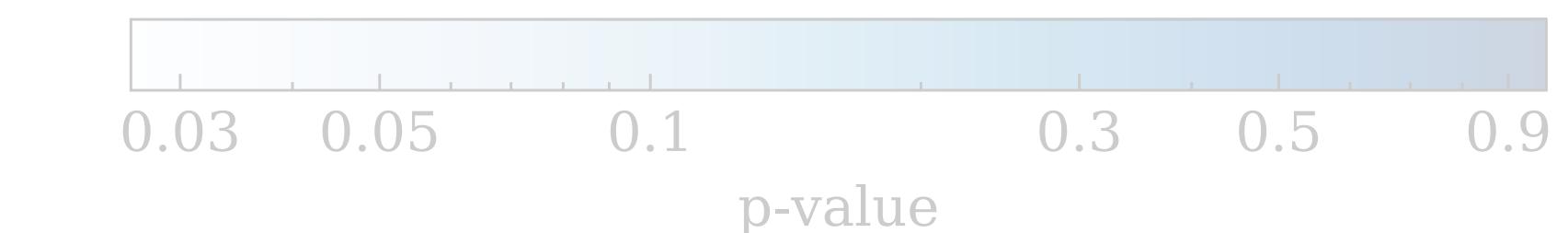
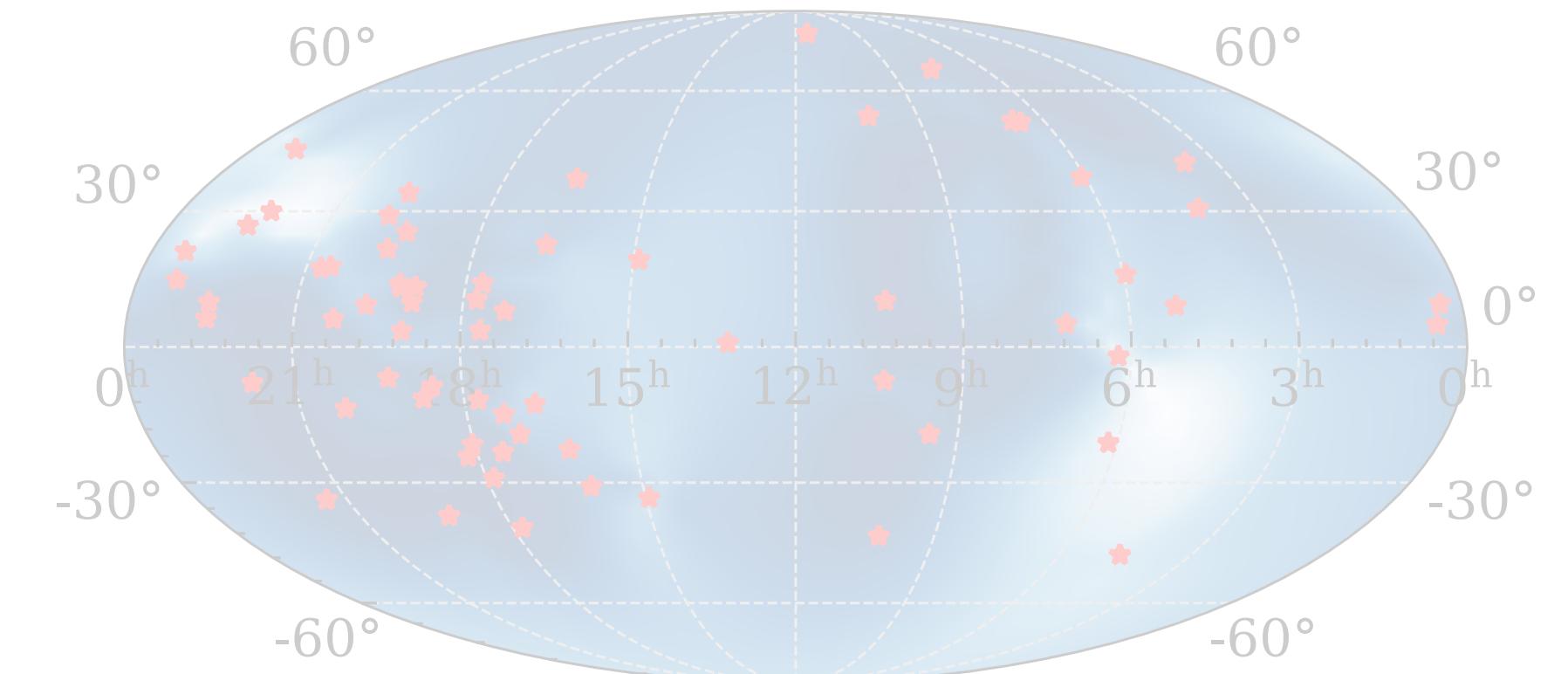
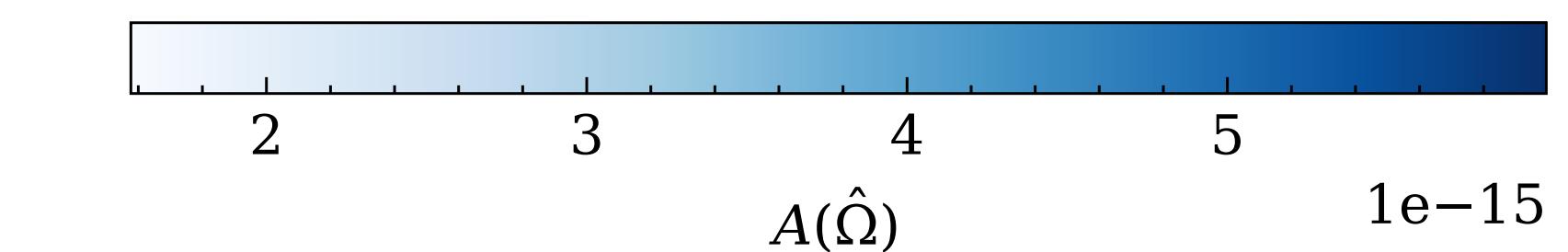
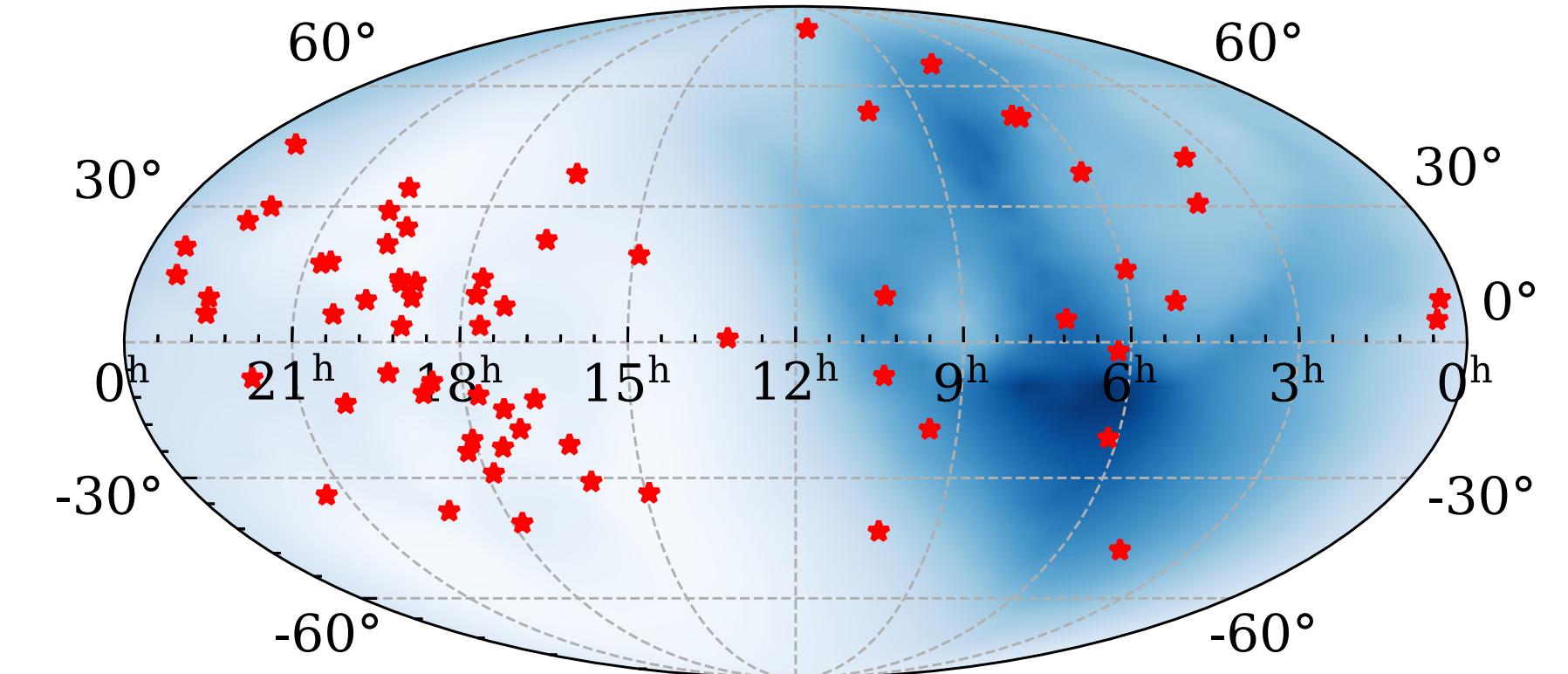


# ANISOTROPIES

$$\Gamma_{ab} \propto \sum_k R_{ab,k} \cdot P_k$$

↑                   ↑                   ↑  
 overlap reduction  
function           PTA response  
function           GWB power

for  $P_k = \text{const}$ ,  $\Gamma_{ab}$  reduces to the HD  
overlap reduction function

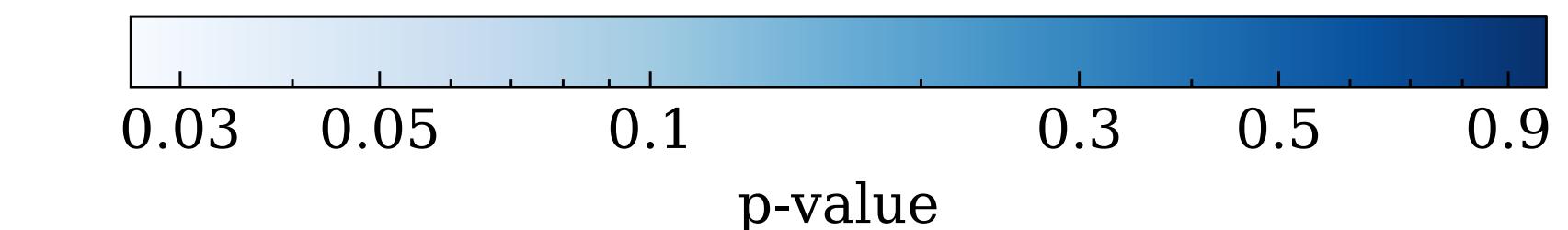
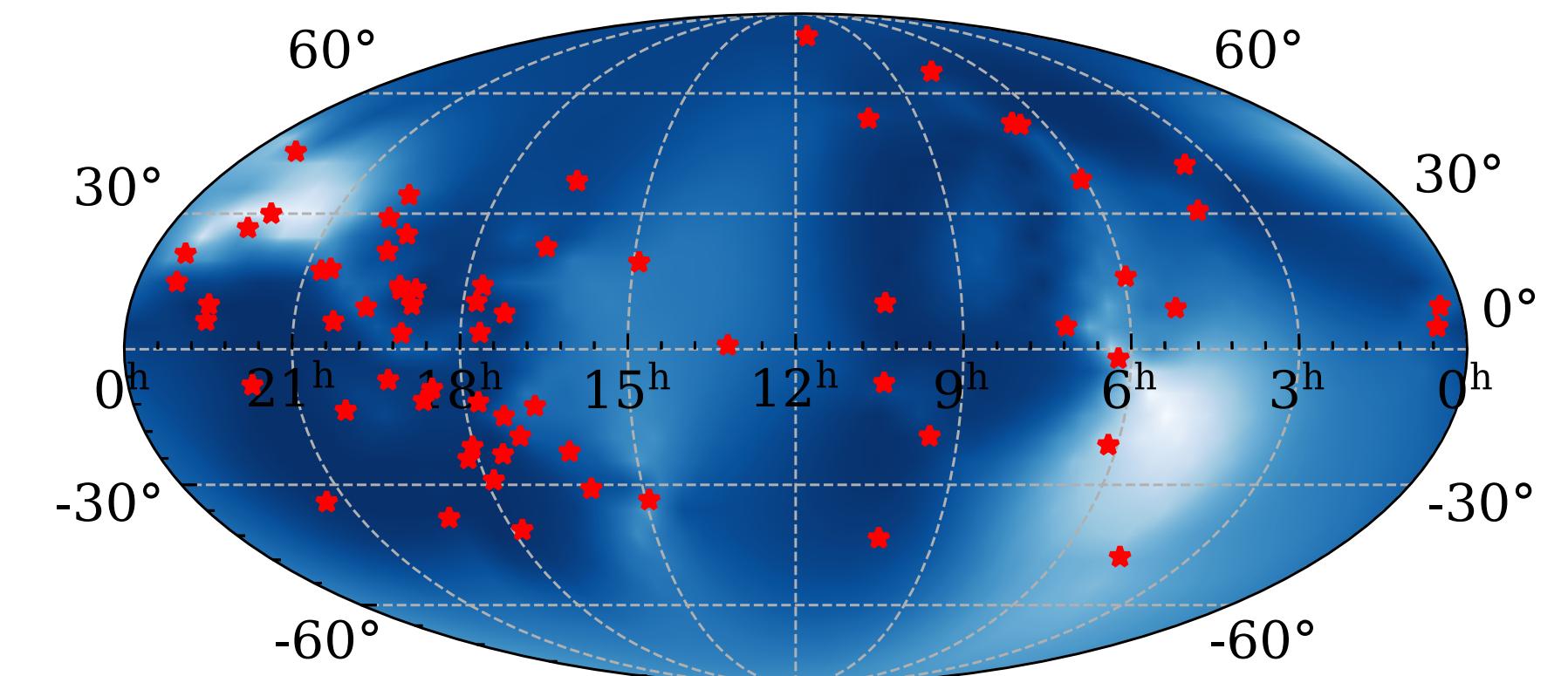
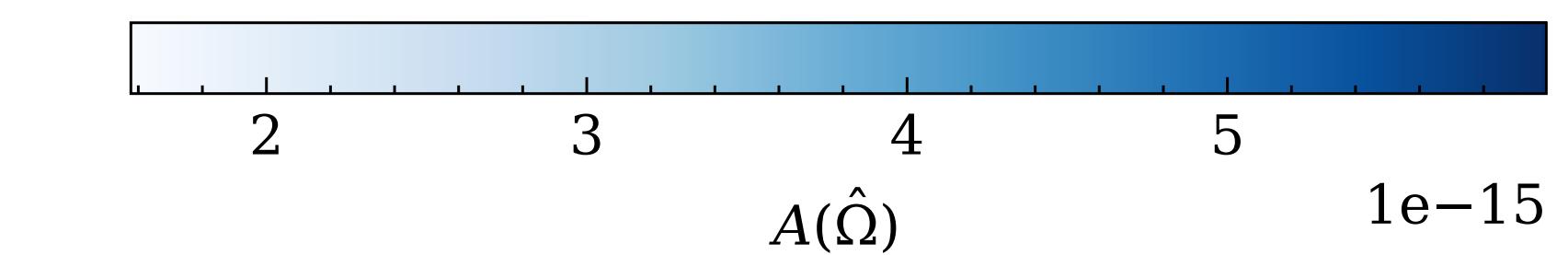
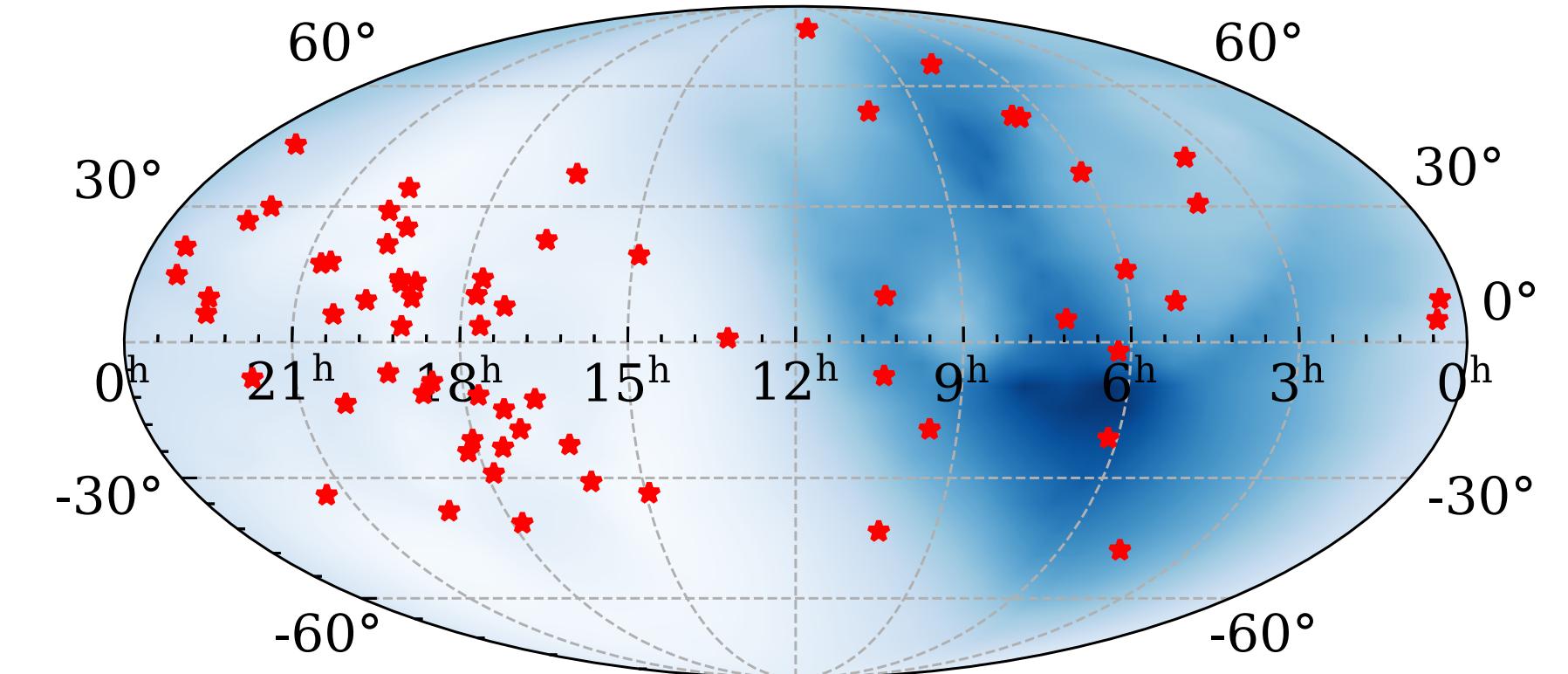


# ANISOTROPIES

$$\Gamma_{ab} \propto \sum_k R_{ab,k} \cdot P_k$$

↑                      ↑  
 overlap reduction function      PTA response function  
 ↓  
 GWB power

for  $P_k = \text{const}$ ,  $\Gamma_{ab}$  reduces to the HD overlap reduction function



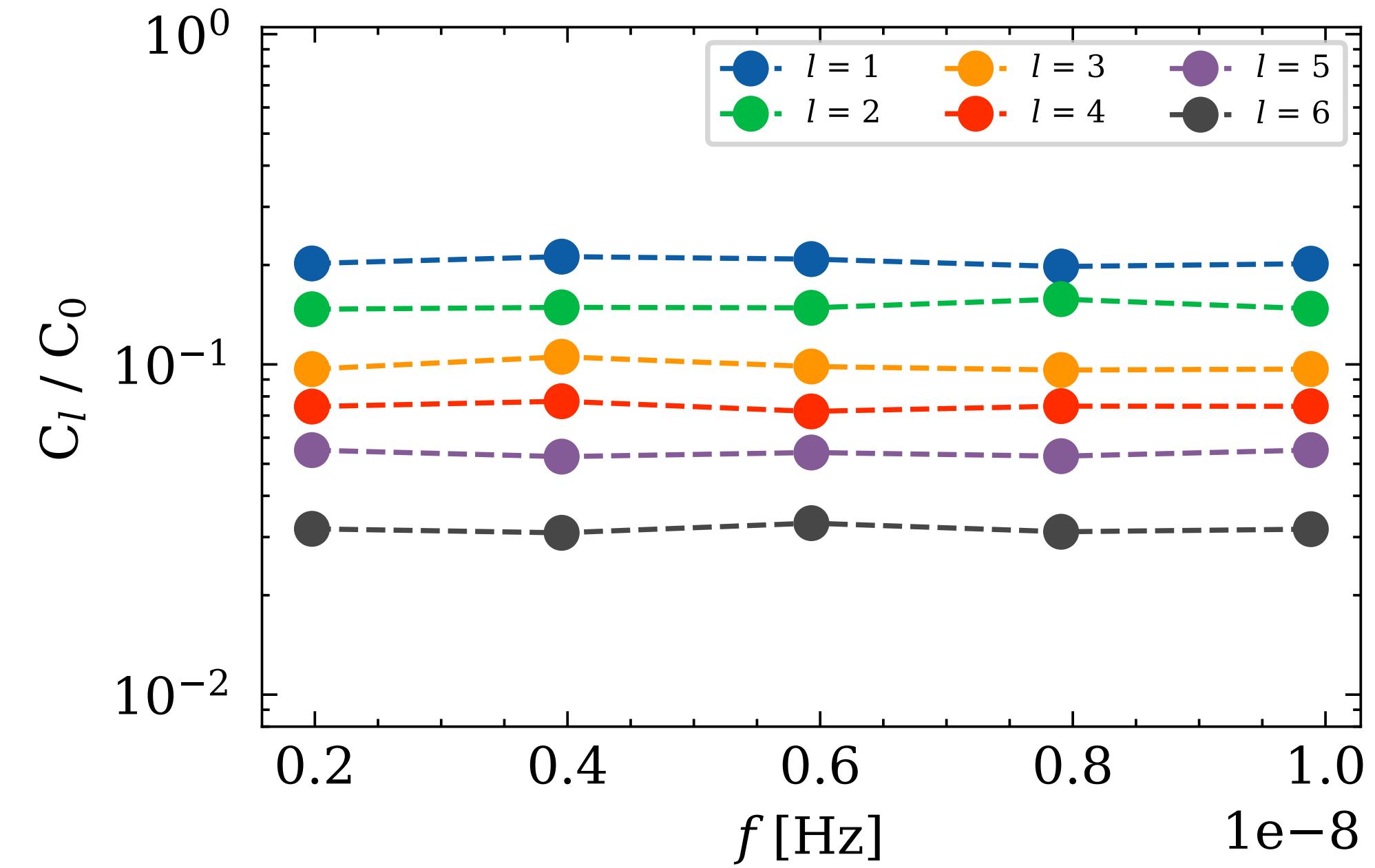
# ANISOTROPIES

$$\Gamma_{ab} \propto \sum_k R_{ab,k} \cdot P_k$$

↑  
 overlap reduction  
 function  
 ↑  
 PTA response  
 function  
 ↑  
 GWB power

for  $P_k = \text{const}$ ,  $\Gamma_{ab}$  reduces to the HD  
overlap reduction function

$$P_k = \sum_{l=0}^{\infty} \sum_{m=-l}^l c_{lm} Y_{lm}(\hat{\Omega}_k) \quad C_l = \frac{1}{2l+1} \sum_{m=-l}^l |c_{lm}|^2$$



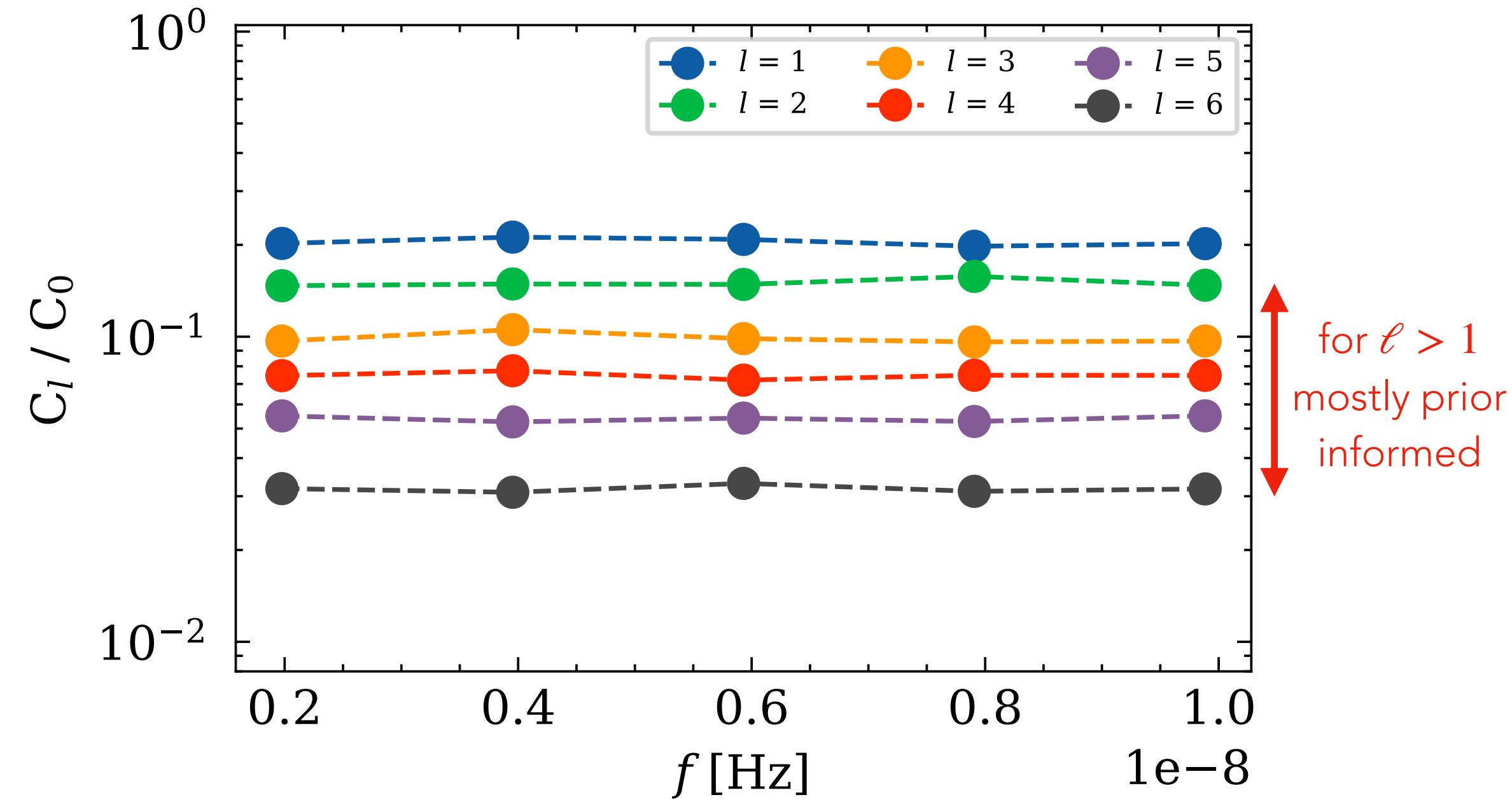
# ANISOTROPIES

$$\Gamma_{ab} \propto \sum_k R_{ab,k} \cdot P_k$$

↑  
overlap reduction  
function  
  
PTA response  
function  
  
GWB power

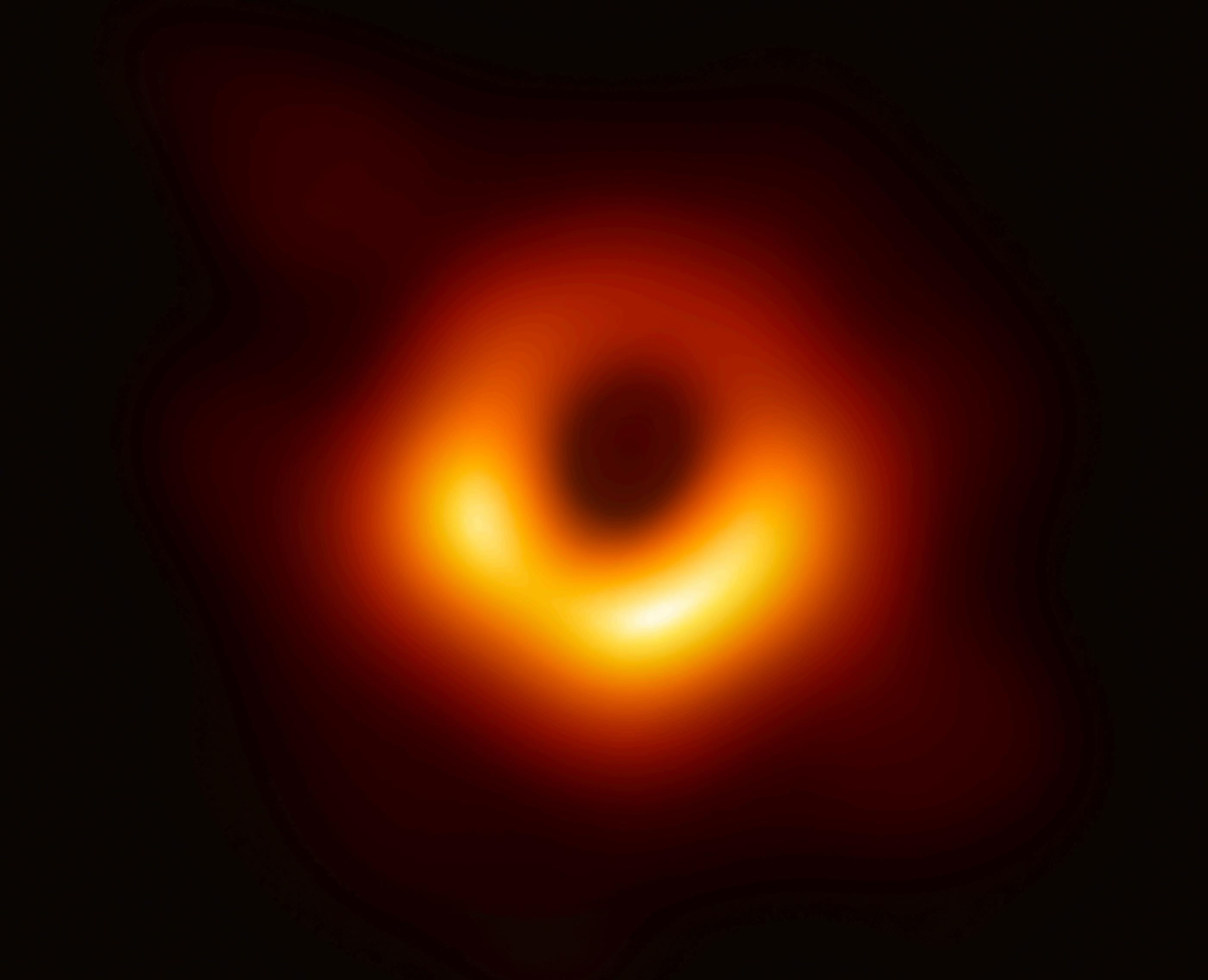
for  $P_k = \text{const}$ ,  $\Gamma_{ab}$  reduces to the HD  
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$$P_k = \sum_{l=0}^{\infty} \sum_{m=-l}^l c_{lm} Y_{lm}(\hat{\Omega}_k) \quad C_l = \frac{1}{2l+1} \sum_{m=-l}^l |c_{lm}|^2$$



what is the source?

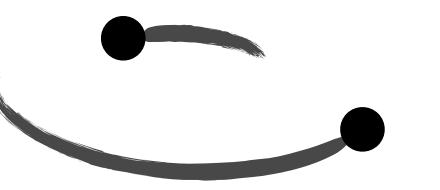
# SUPERMASSIVE BH



A black hole with a bright orange and yellow accretion disk.

most massive galaxies host supermassive black holes at their center

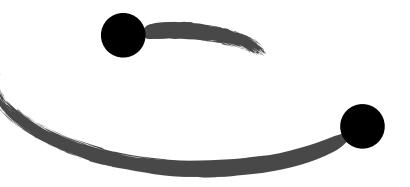
# CONTENDER #1



$$h_c^2(f) = \int dM dq dz \frac{\partial^4 N}{\partial M \partial q \partial z \partial \ln f_p} h_s^2(f_p)$$

Phinney 2001, Wyithe & Loeb 2003

# CONTENDER #1



GW signal from individual SMBHB

$$h_c^2(f) = \int dM dq dz \frac{\partial^4 N}{\partial M \partial q \partial z \partial \ln f_p} h_s^2(f_p)$$

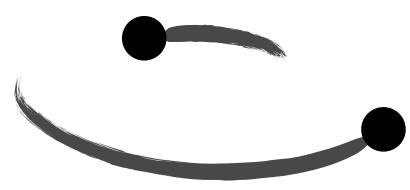
Phinney 2001, Wyithe & Loeb 2003

averaged strain for a circular  
SMBHB

$$h_s^2(f) = \frac{32}{5} \frac{(GM)^{10/3}}{d_c^2} (2\pi f_p)^{4/3}$$

Finn & Thorne 2000

# CONTENDER #1



GW signal from individual SMBHB

$$h_c^2(f) = \int dM dq dz \frac{\partial^4 N}{\partial M \partial q \partial z \partial \ln f_p} h_s^2(f_p)$$

Phinney 2001, Wyithe & Loeb 2003

number density of SMBHB binaries

averaged strain for a circular  
SMBHB

$$h_s^2(f) = \frac{32}{5} \frac{(GM)^{10/3}}{d_c^2} (2\pi f_p)^{4/3}$$

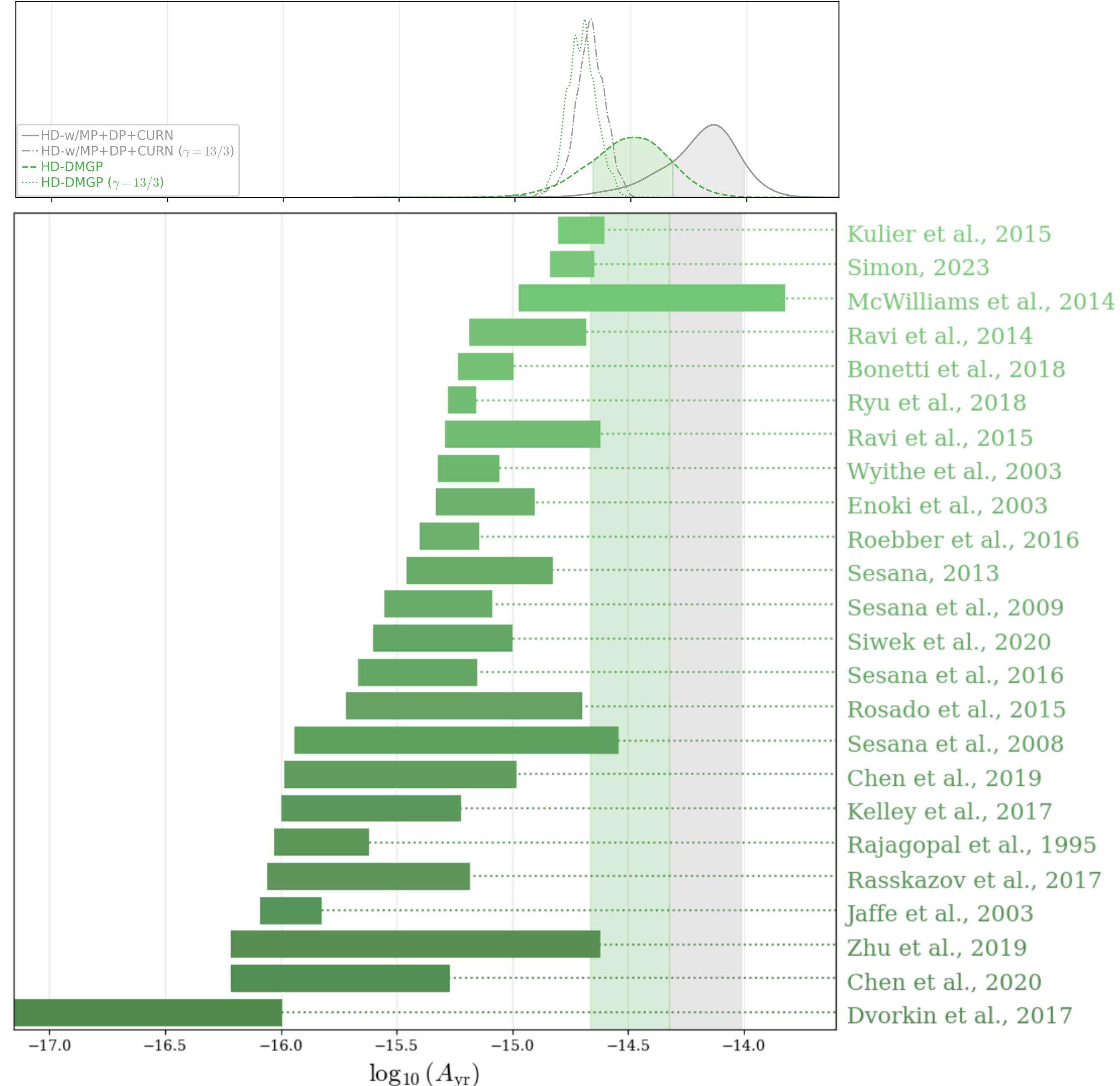
Finn & Thorne 2000

the SMBHB density depends on

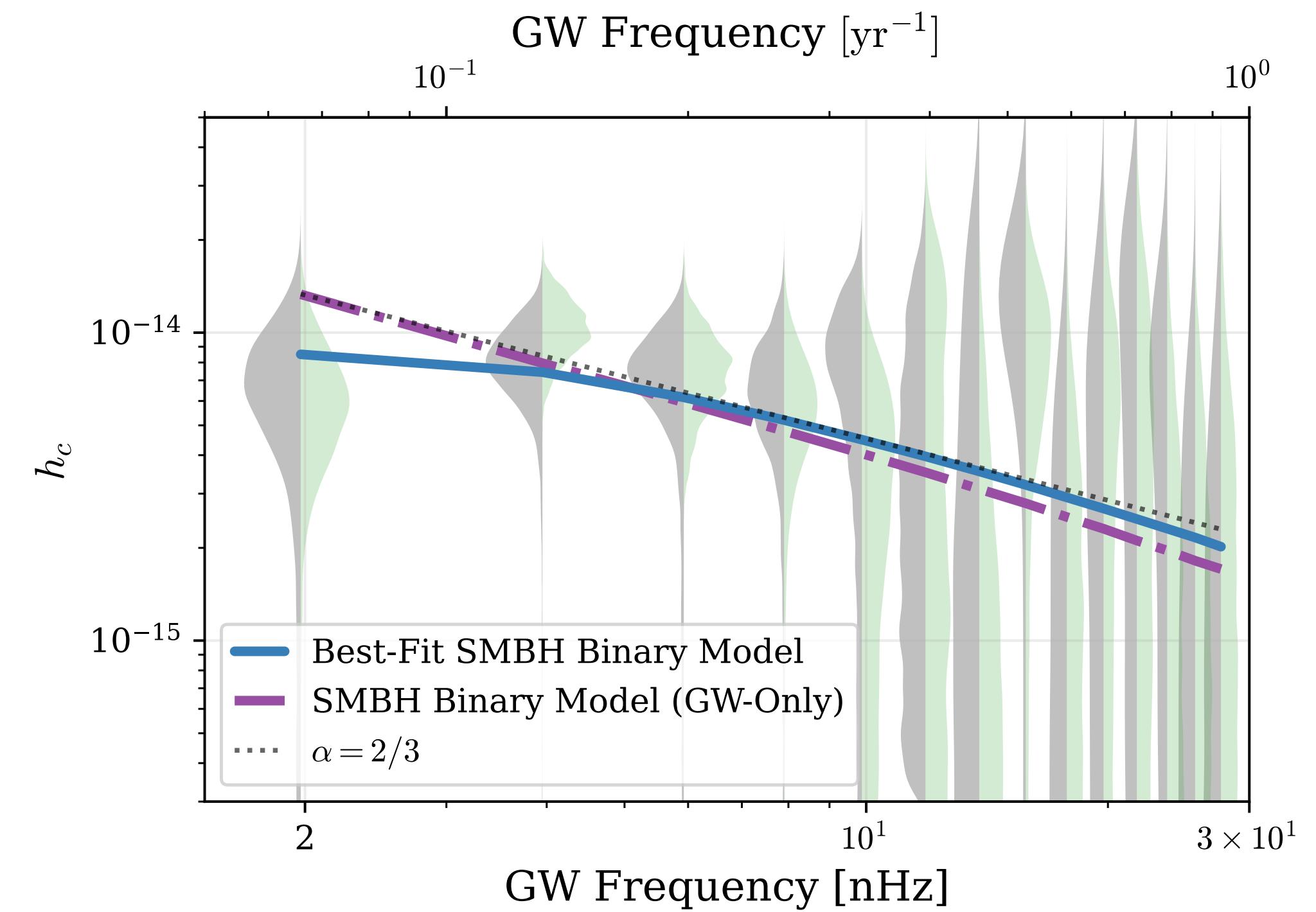
1. galaxies merger rate
2. SMBHB - galaxy mass relation
3. SMBHB binary evolution

# EXPECTATIONS

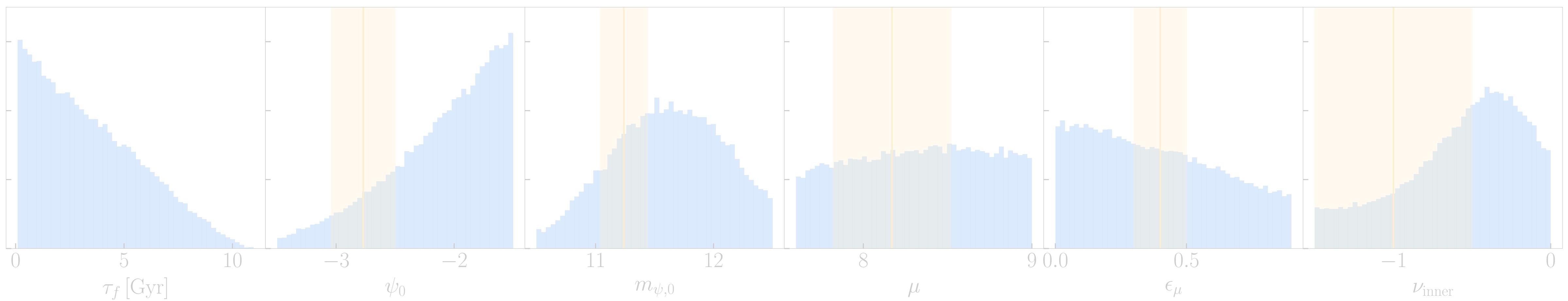
Agazie et al. [2306.16220]



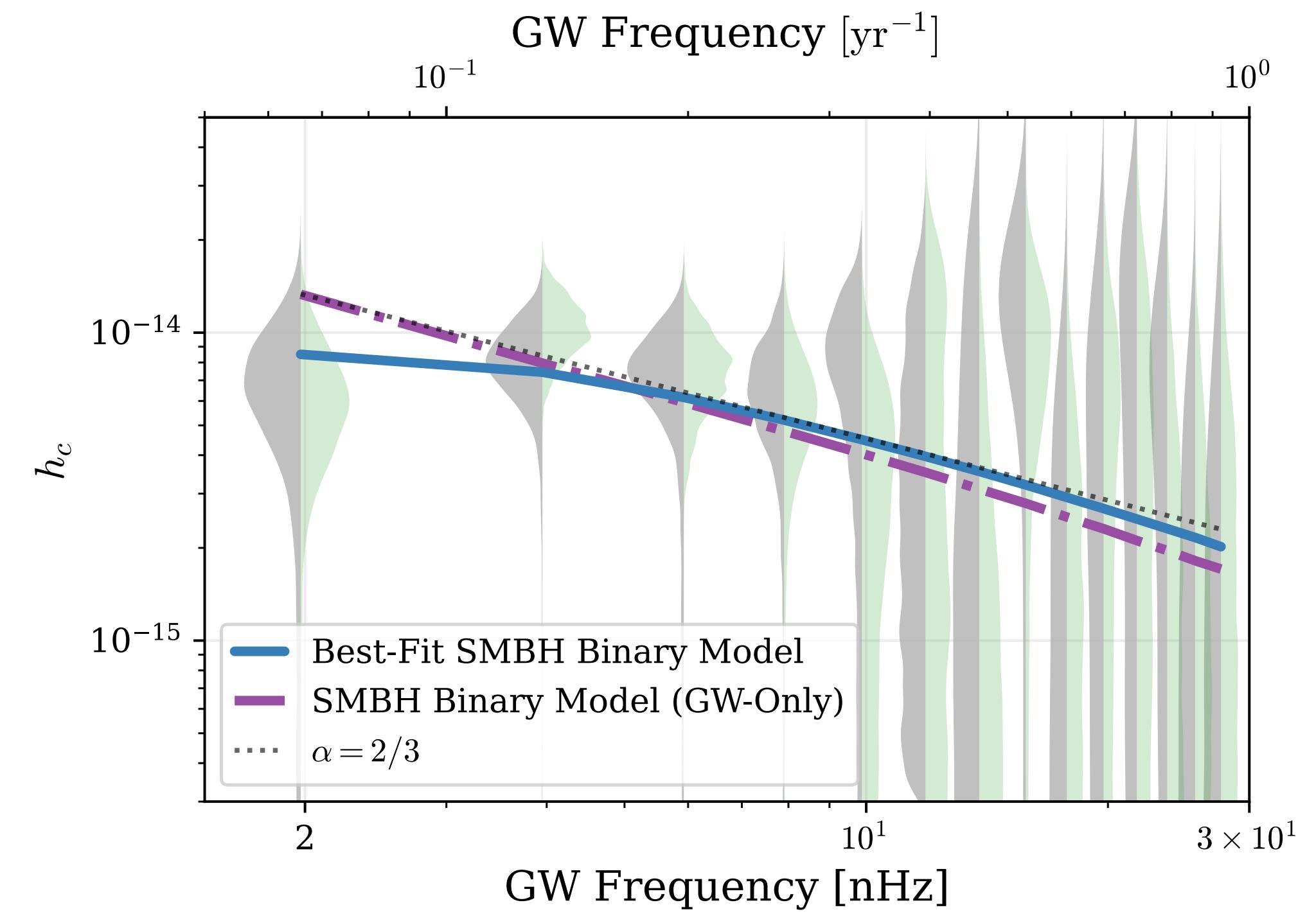
# ADJUSTING EXPECTATIONS



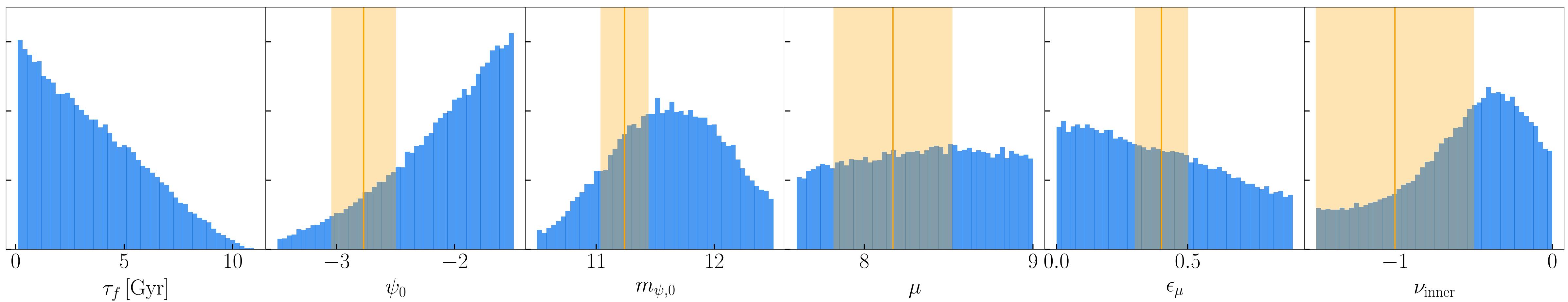
Agazie et al. [2306.16220]



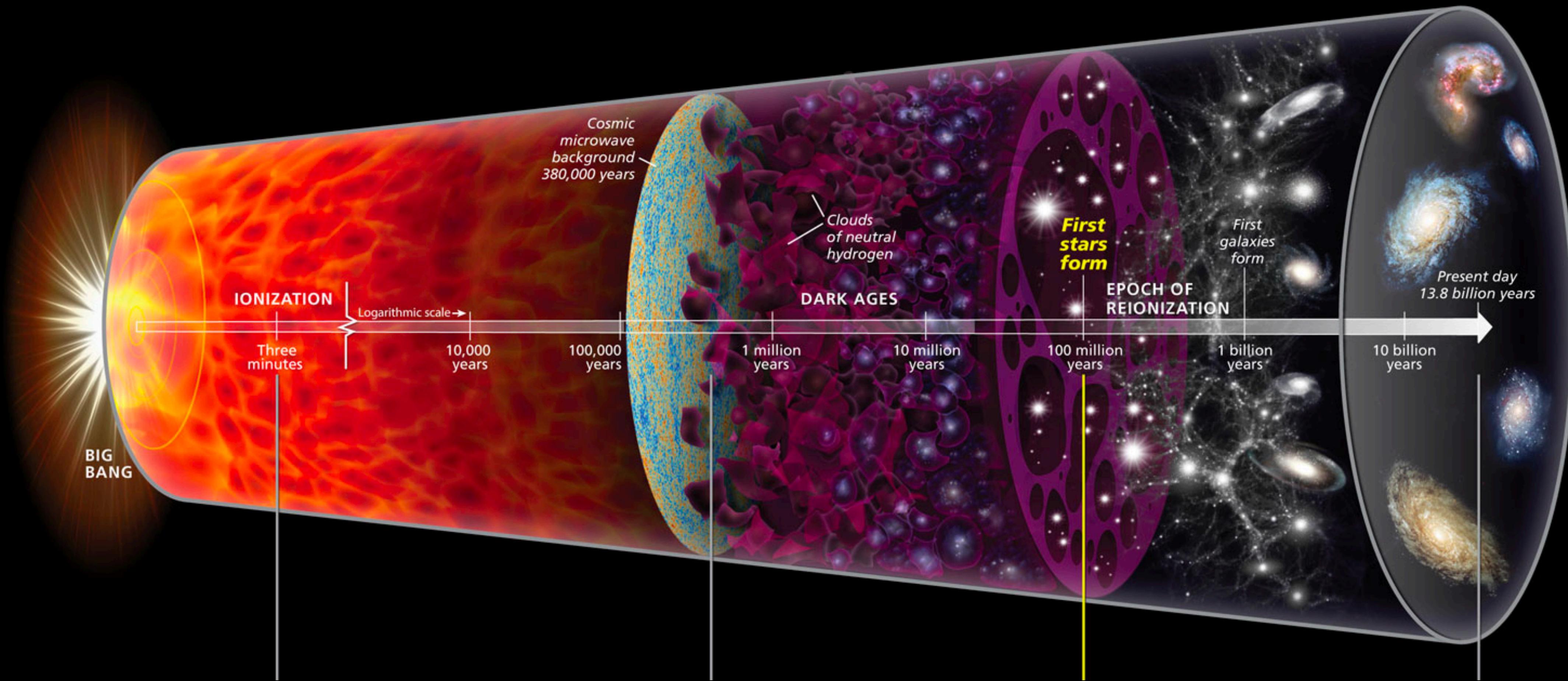
# ADJUSTING EXPECTATIONS



Agazie et al. [2306.16220]



# PRIMORDIAL SOURCES

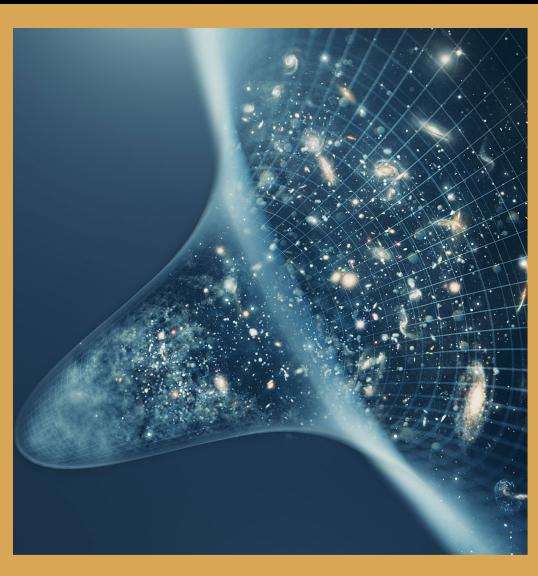


inflation  
baryogenesis  
DM formation?  
???

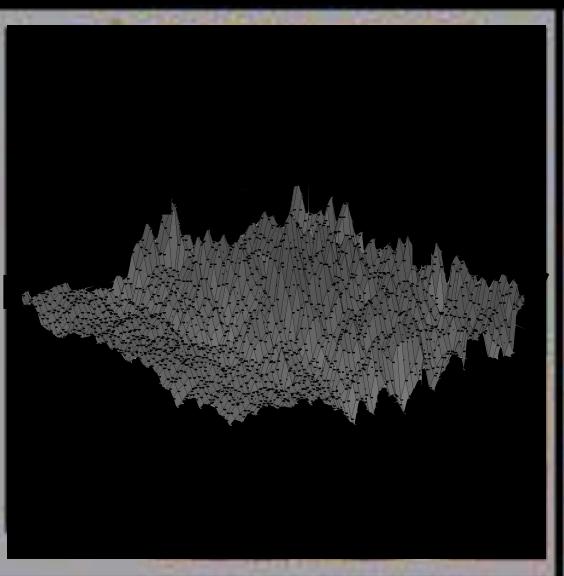
light starts to free stream

# SELECT PLAYER

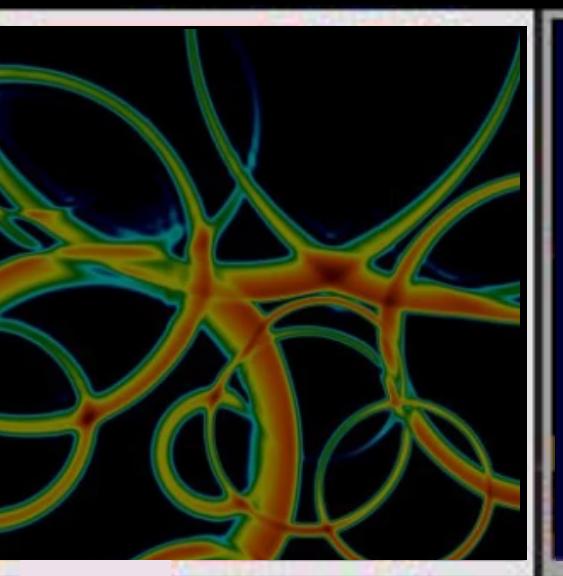
INFLATION



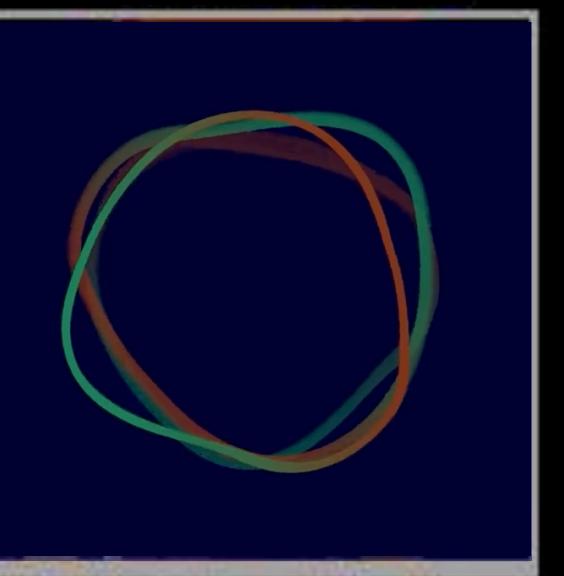
SIGN



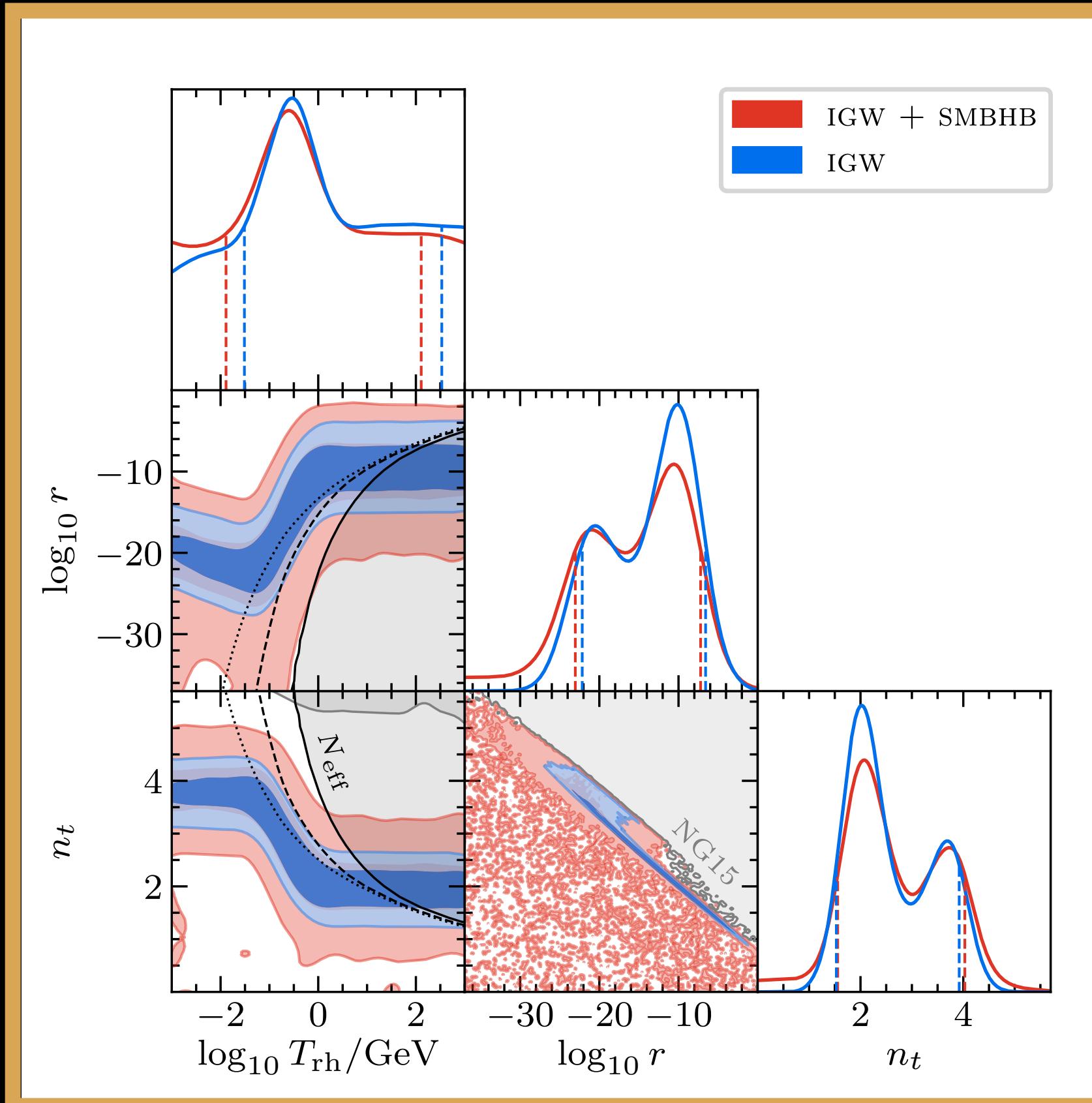
PT



STRINGS

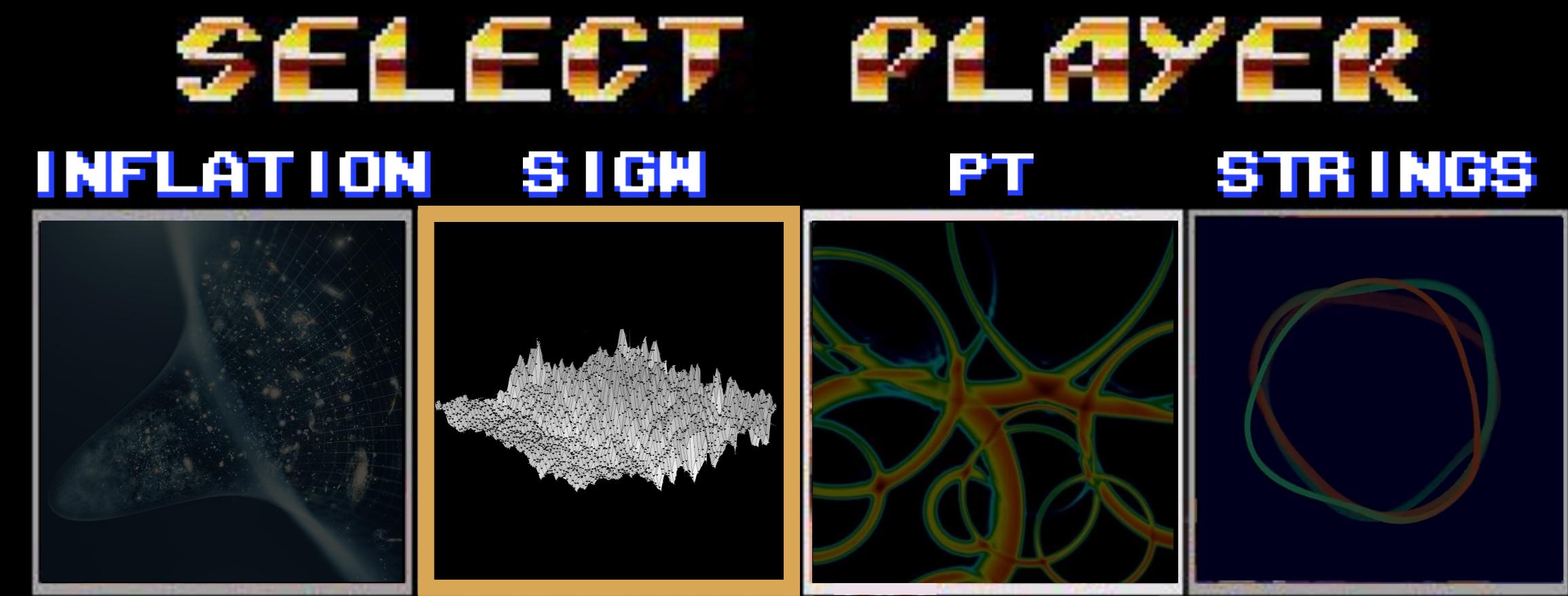


Afzal et al. [2306.16219]

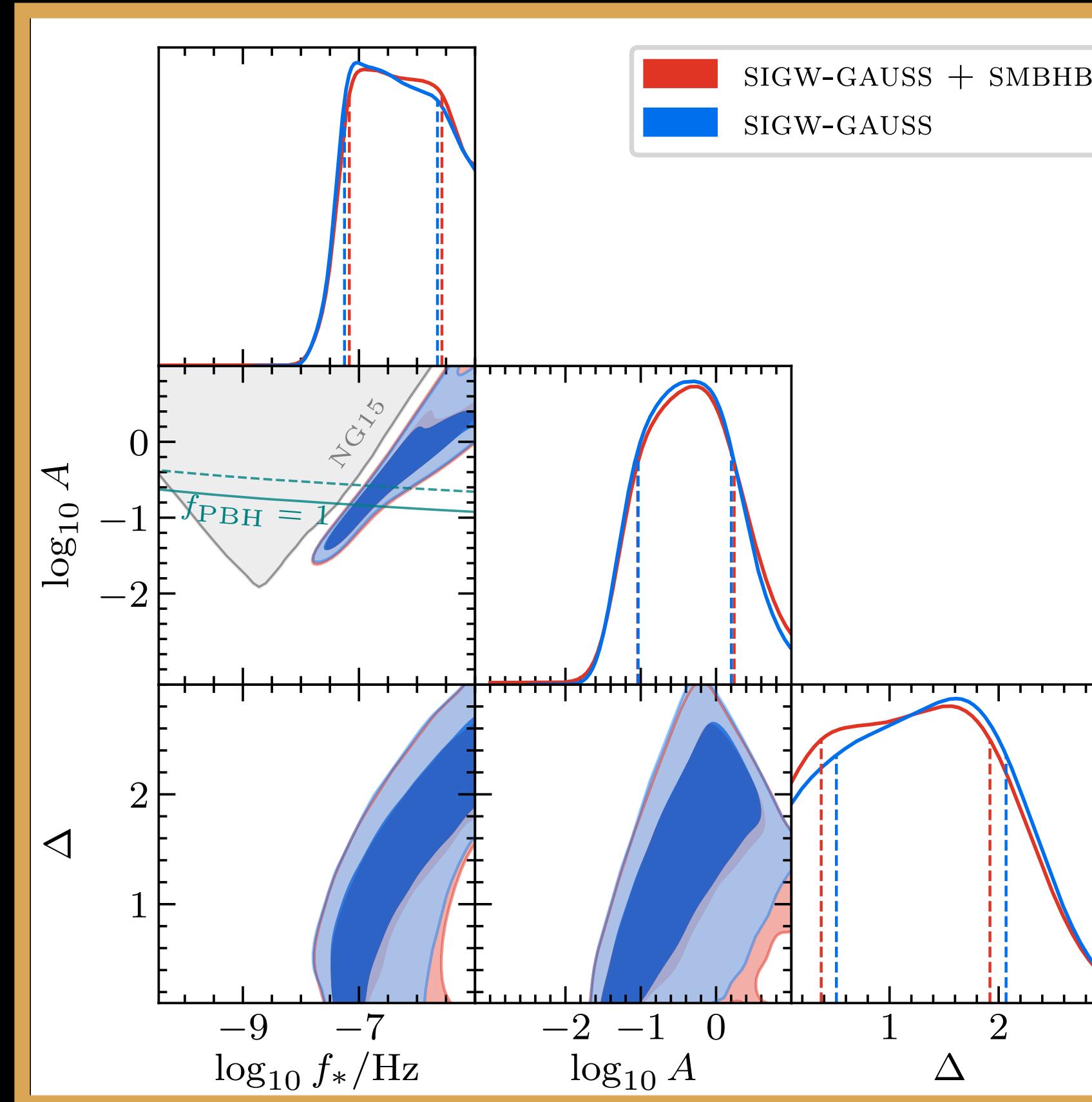


STRONG (RED) SPECTRAL TILT NEEDED

REHEATING TEMP. BELOW 100 GEV AVOIDS NEFF CONST.

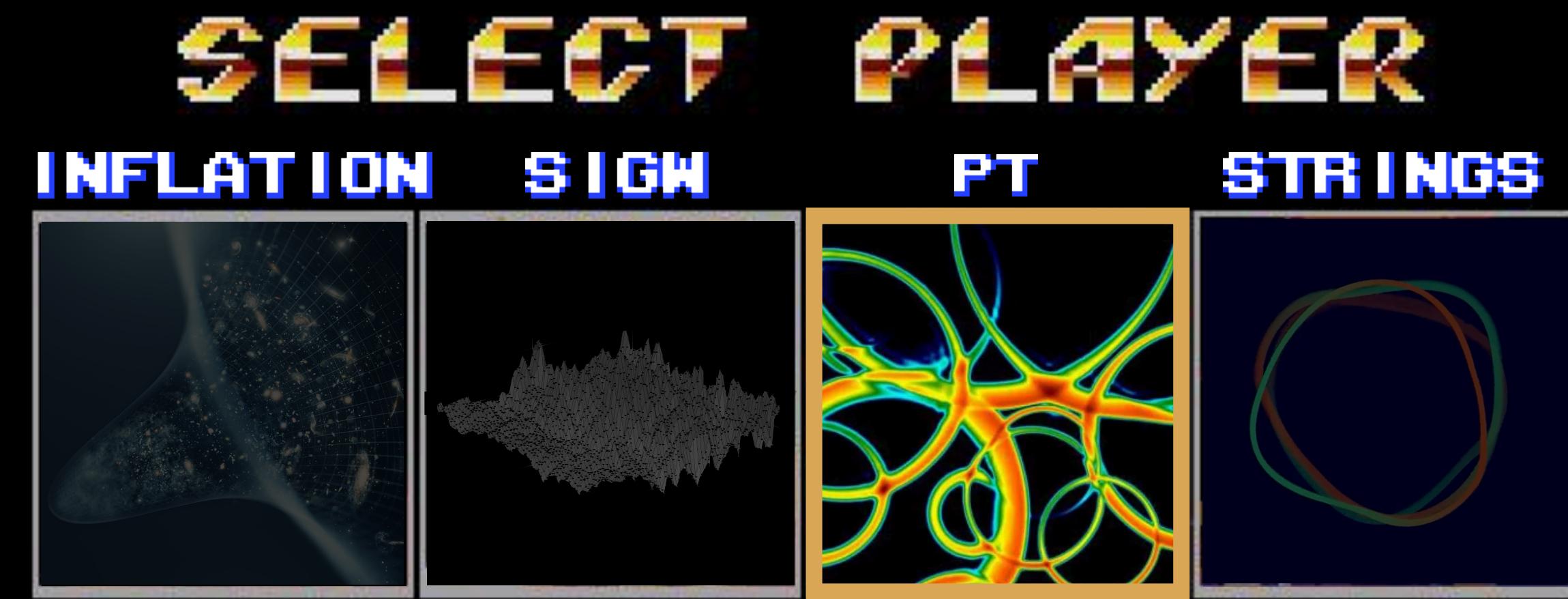


Afzal et al. [2306.16219]

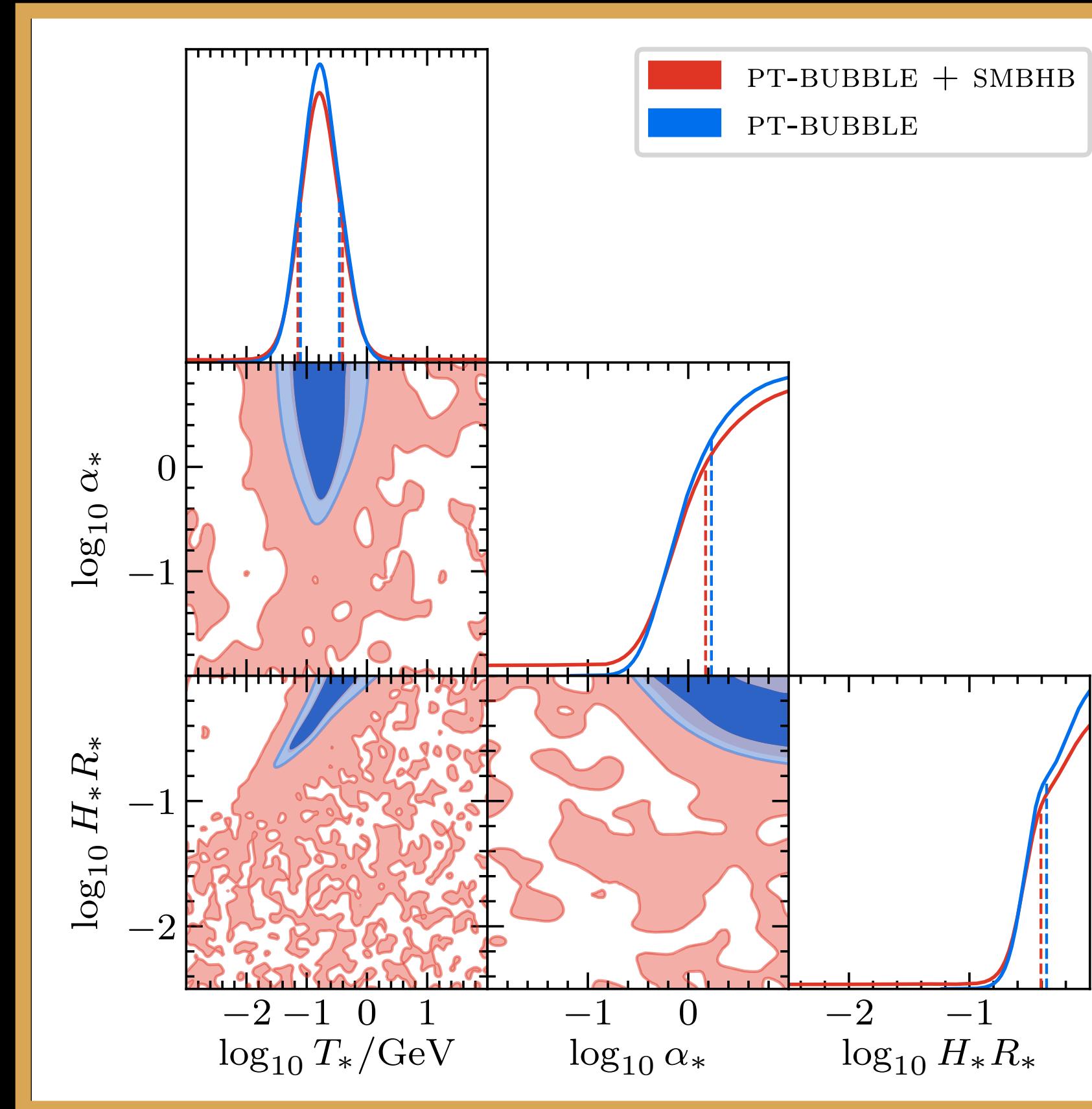


LARGE FEATURE AT MPC SCALES NEEDED

RISK OF OVERPRODUCING PBH



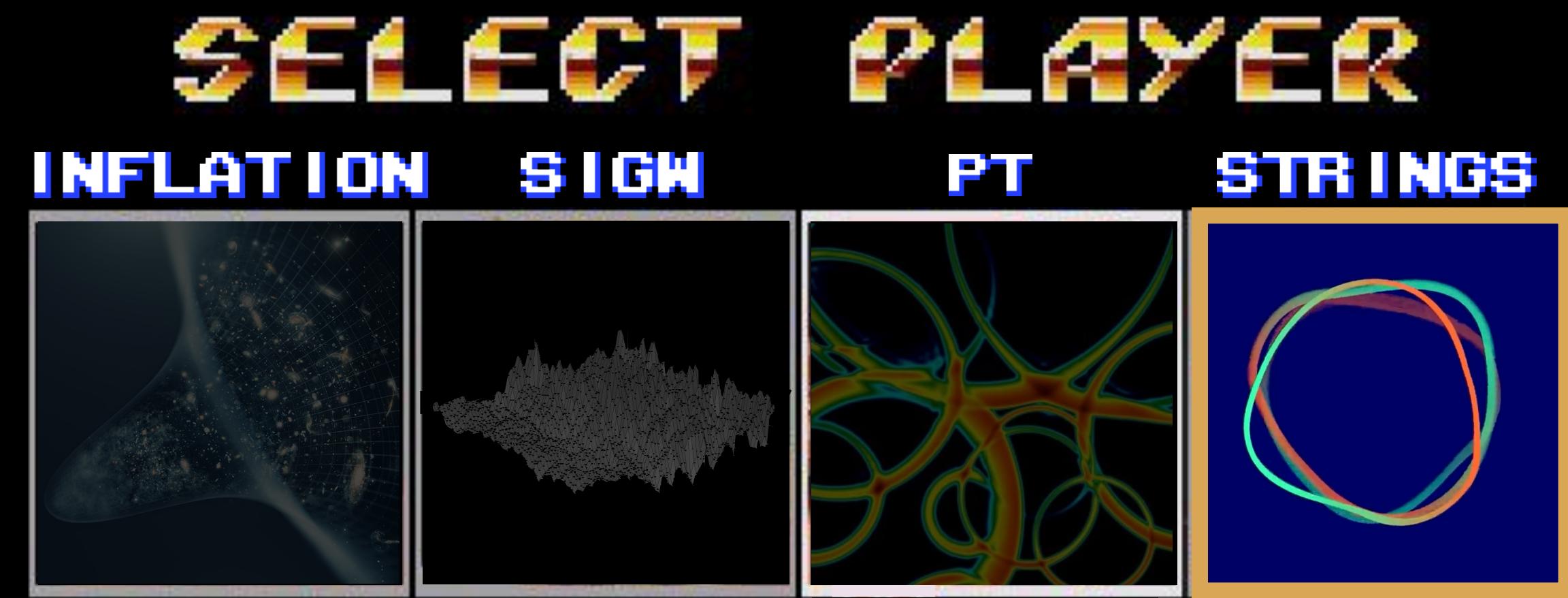
Afzal et al. [2306.16219]



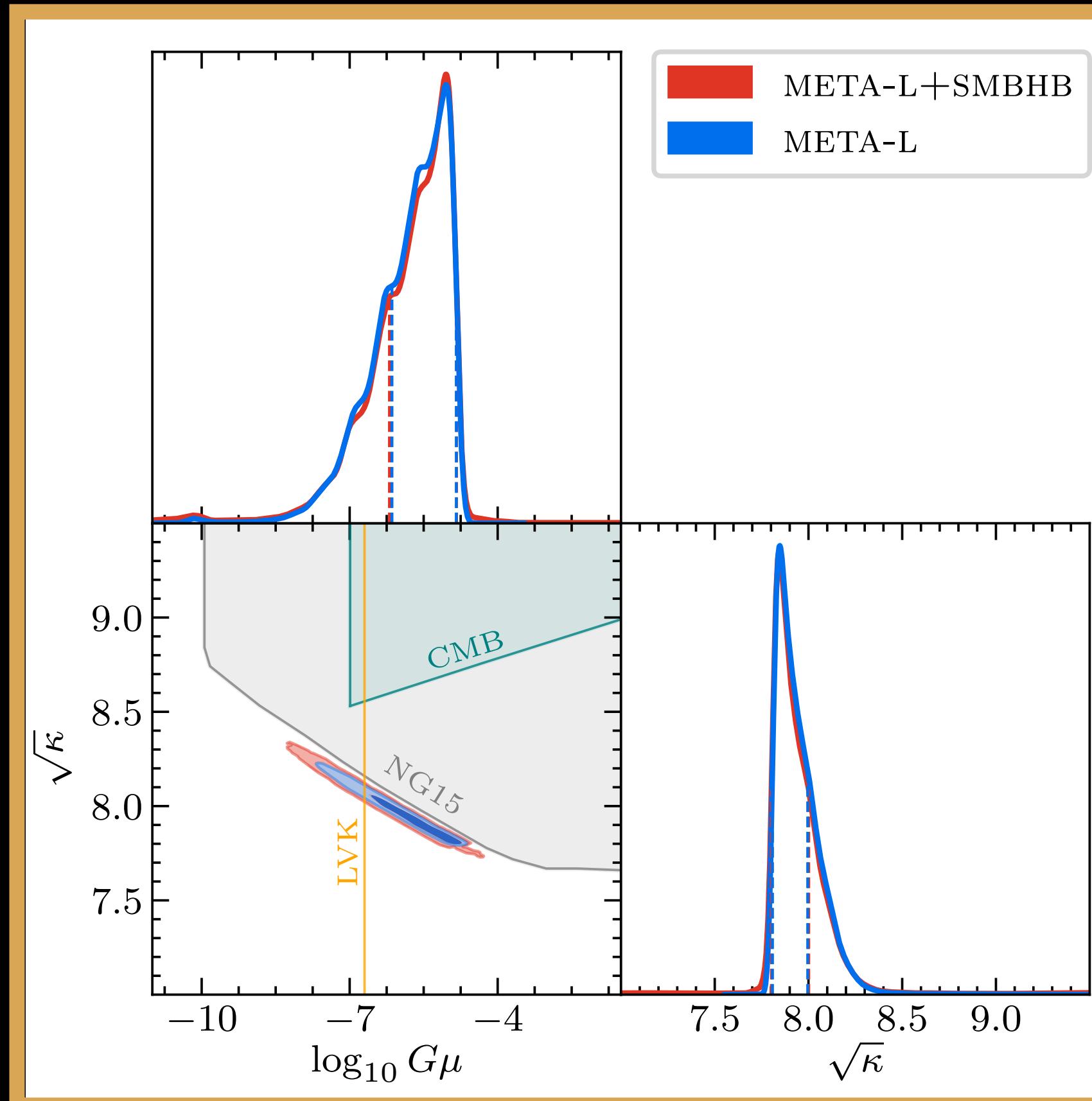
**10–100 MEV TRANSITION TEMP. NEEDED**

**SLOW TRANSITION NEEDED**

**EXTREMELY STRONG TRANSITION NEEDED**



Afzal et al. [2306.16219]



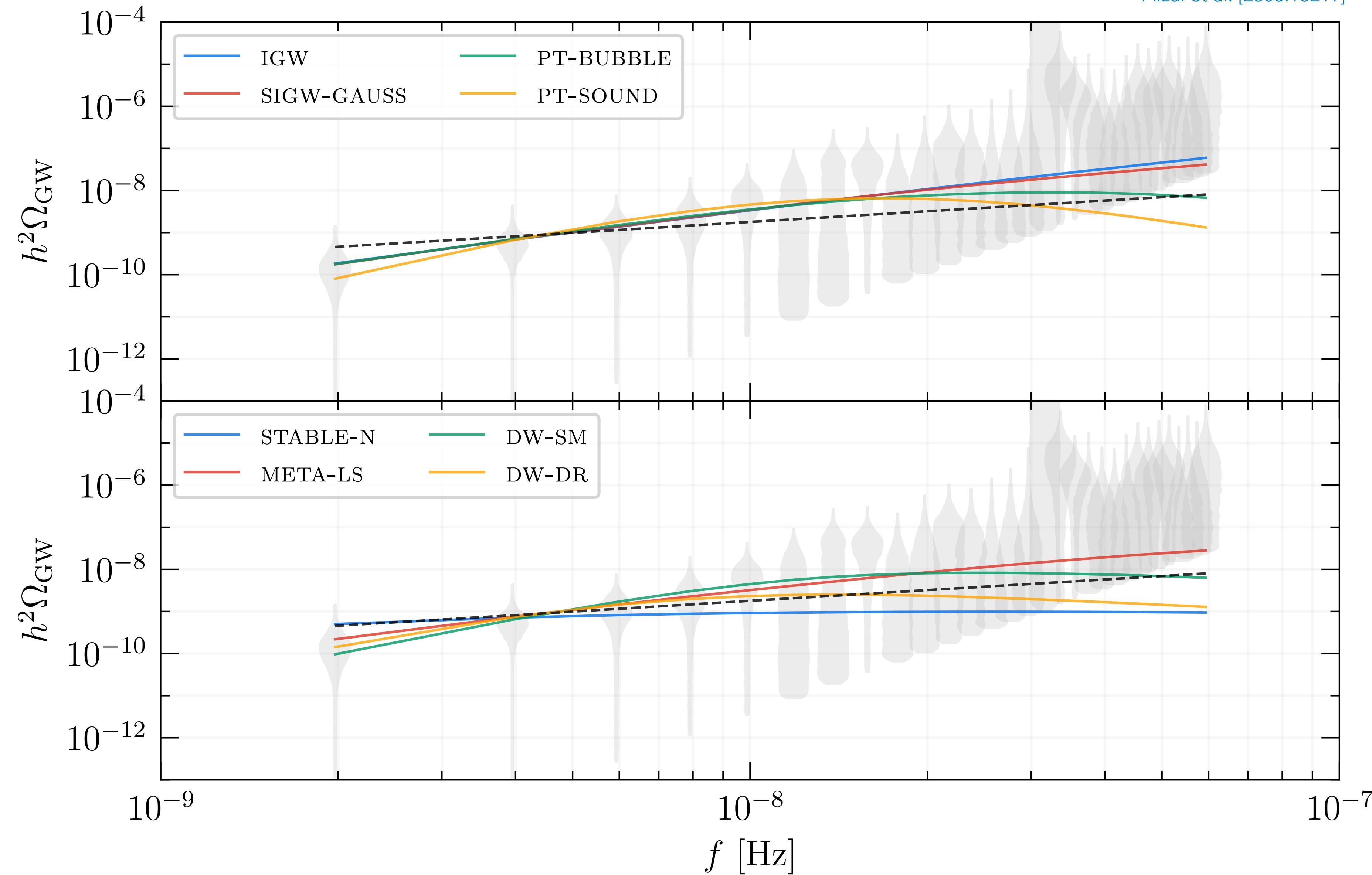
**STABLE STRING DO NOT WORK**

**METASTABLE STRING DO WORK**

**GUT SCALE STRING TENSION COULD WORK**

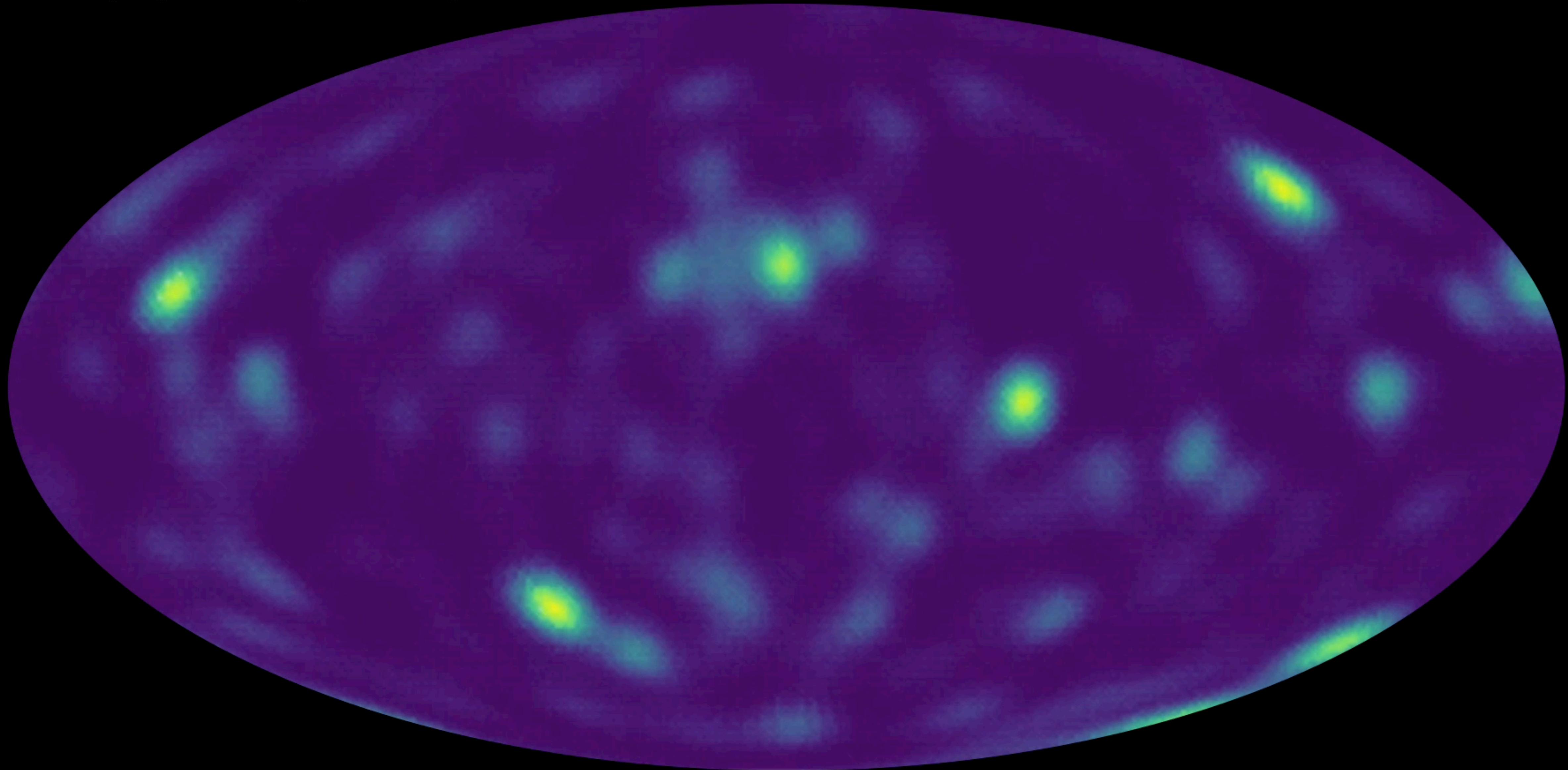
# COSMOLOGICAL SIGNALS

Afzal et al. [2306.16219]



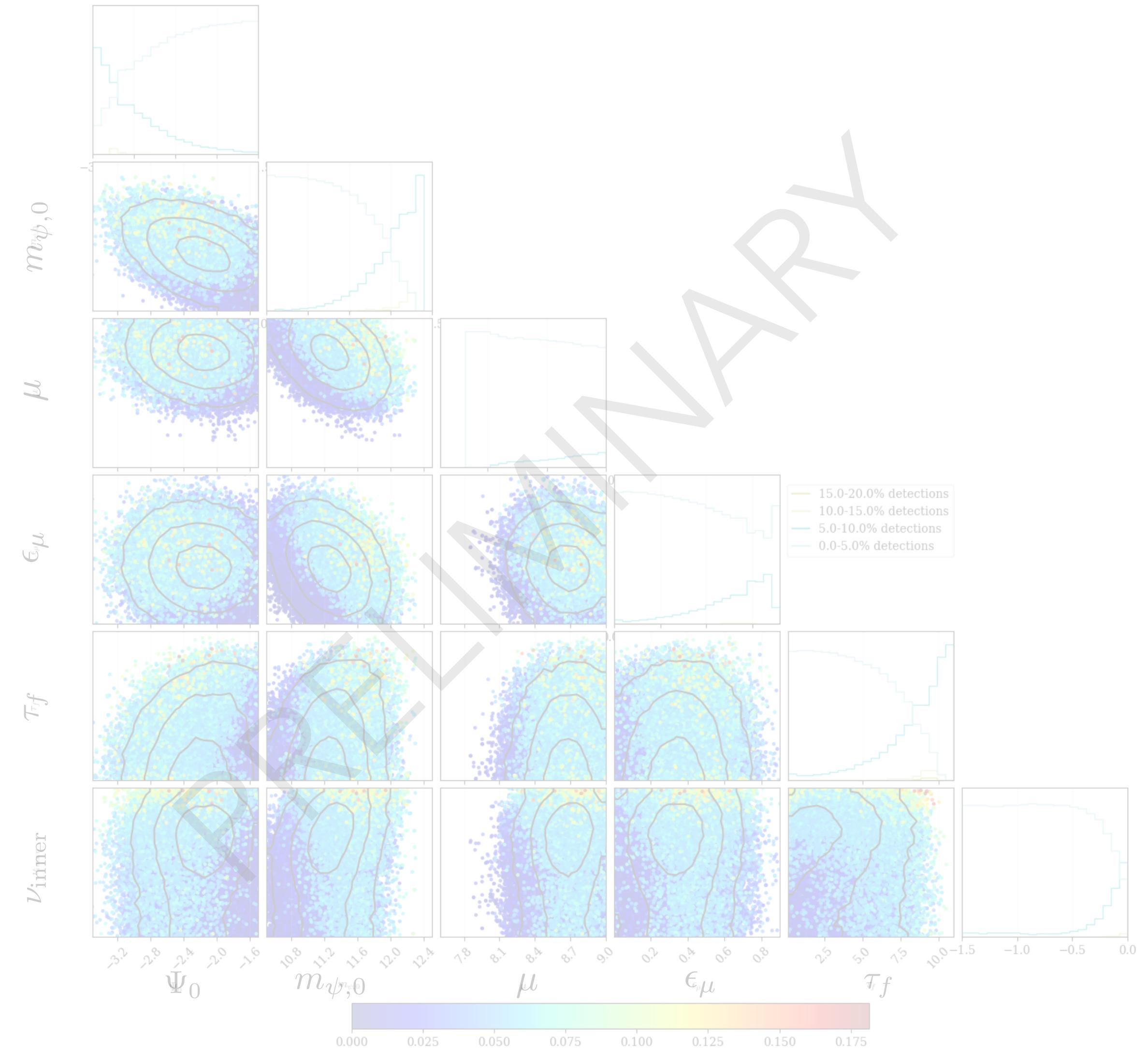
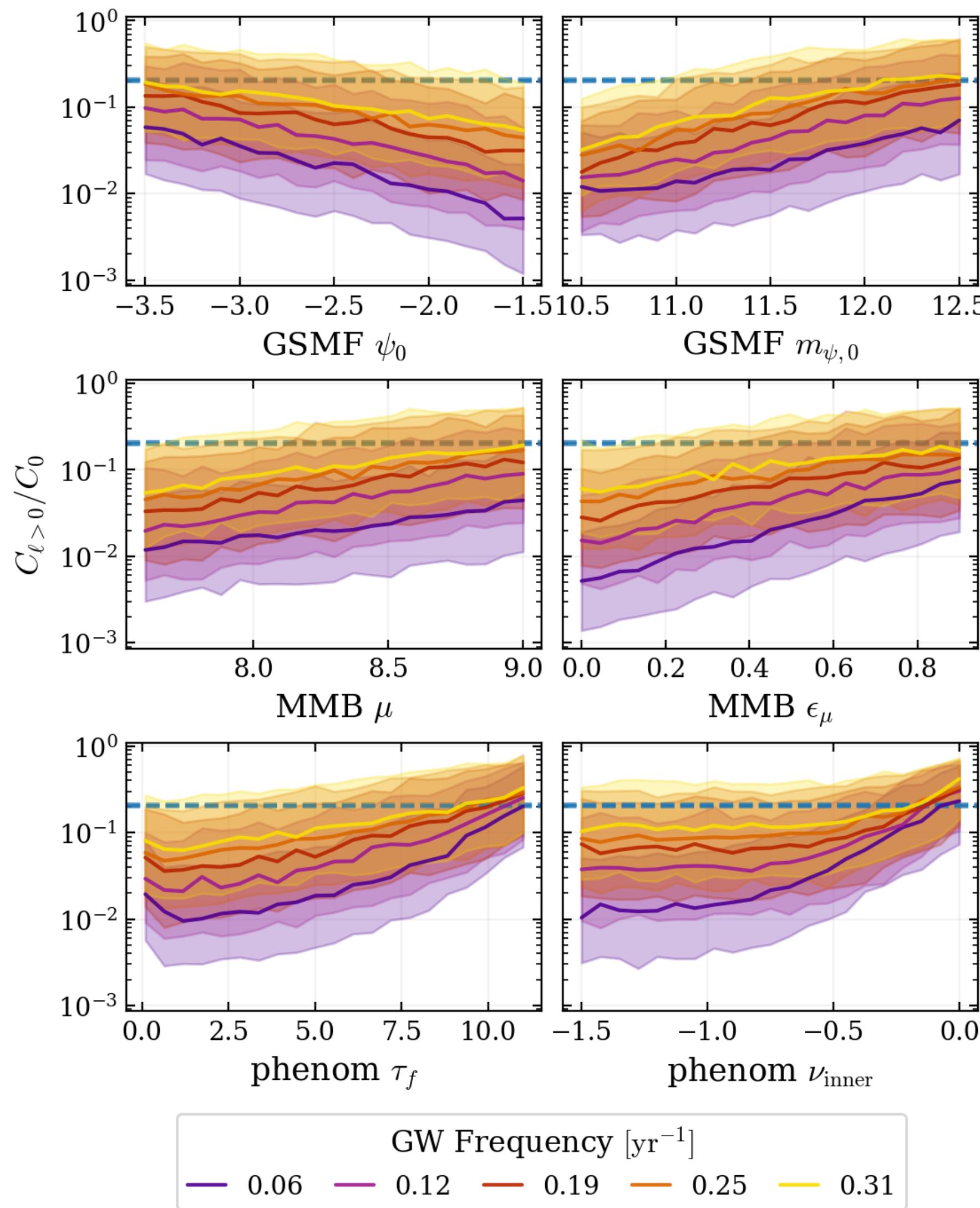
where do we go from here?

# ANISOTROPIES

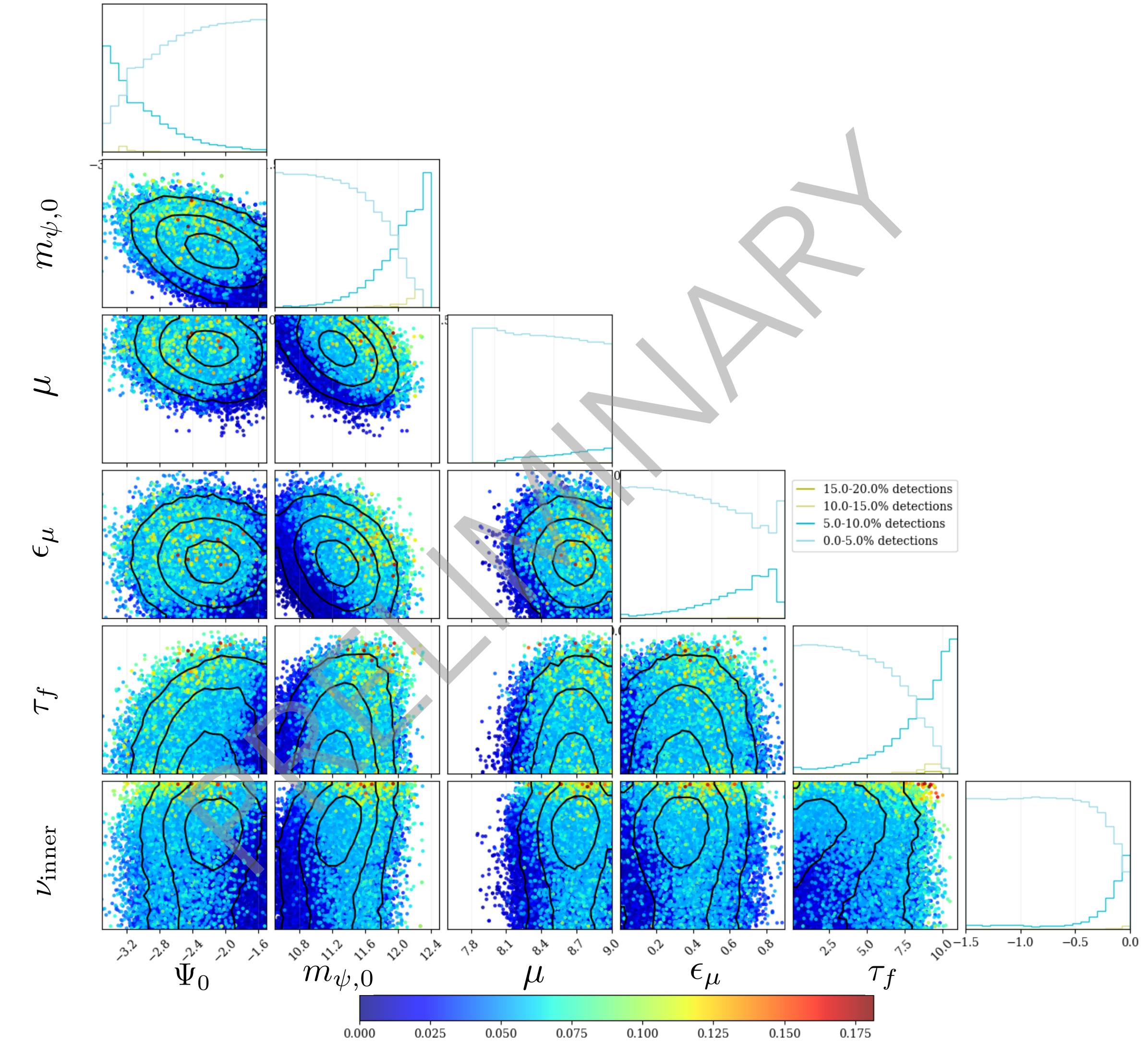
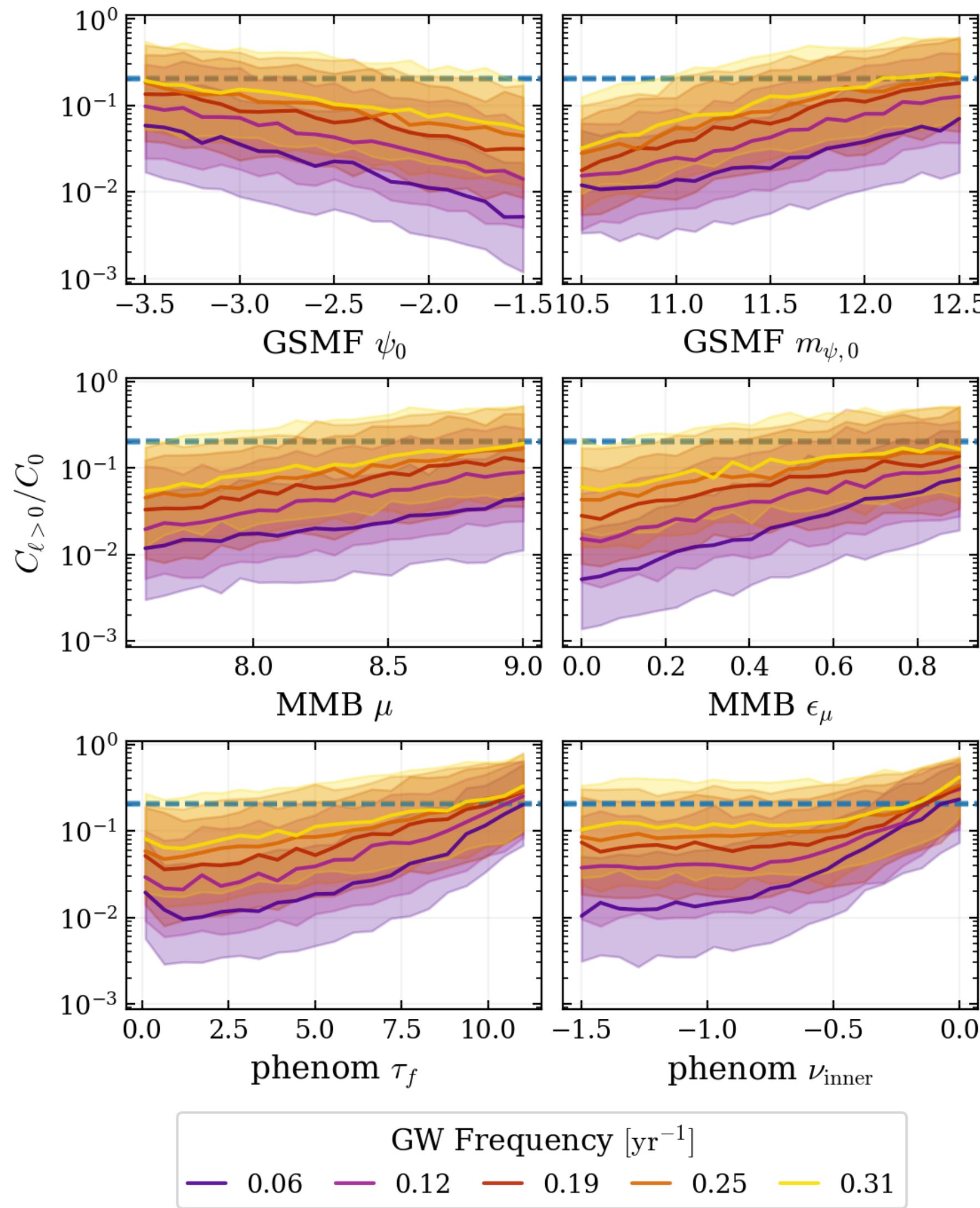


Simulated Data

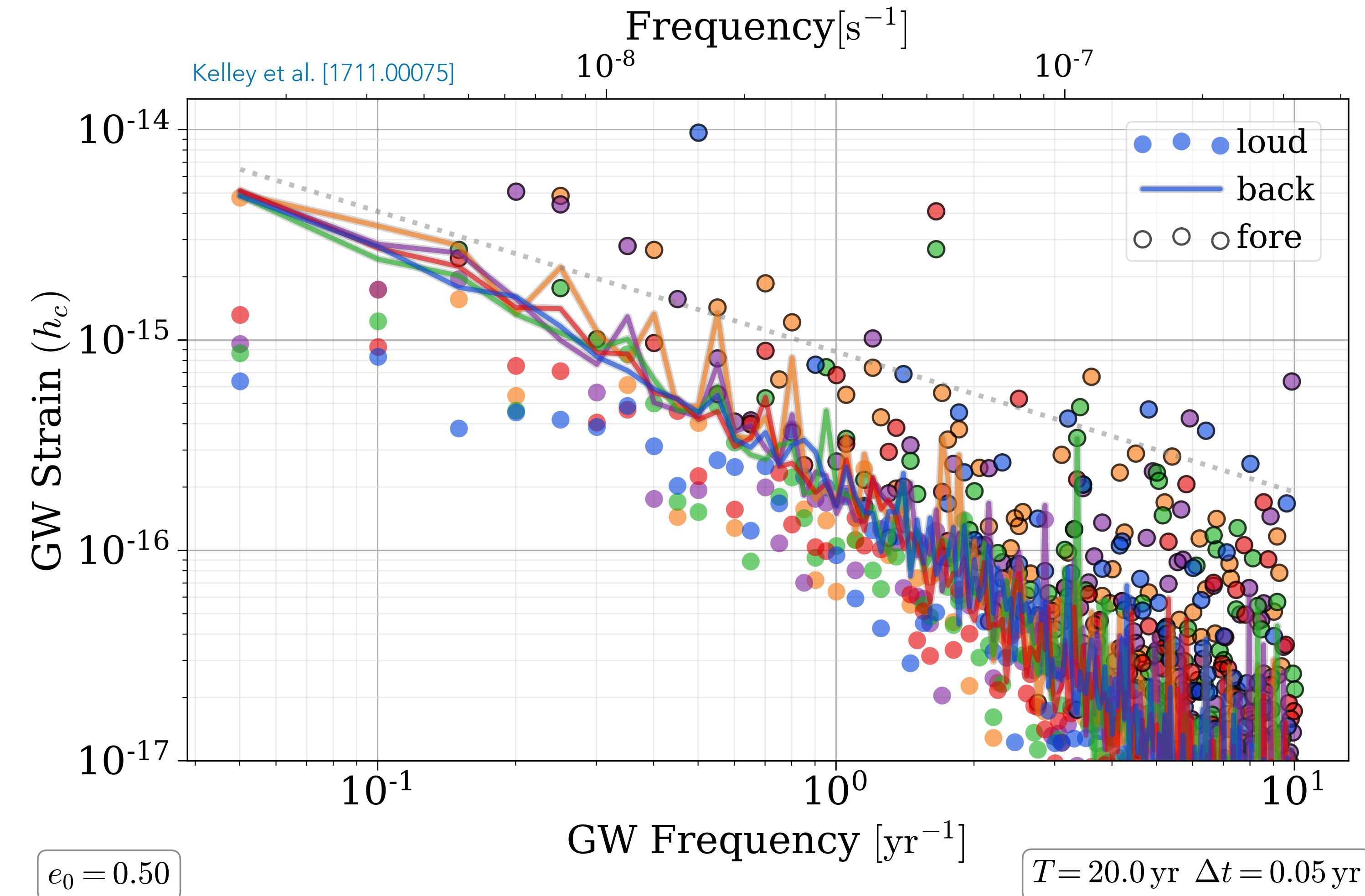
# ANISOTROPIES



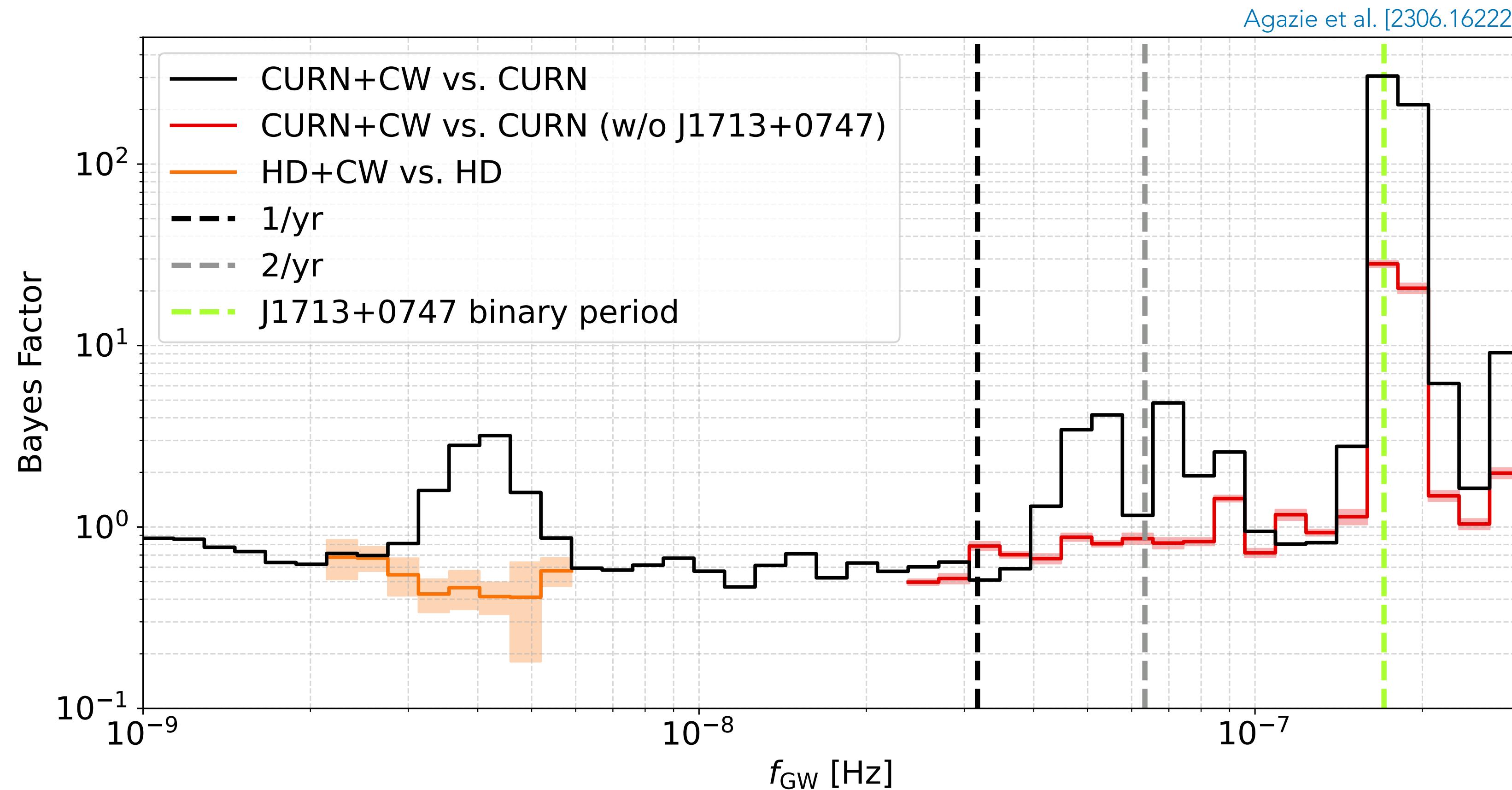
# ANISOTROPIES



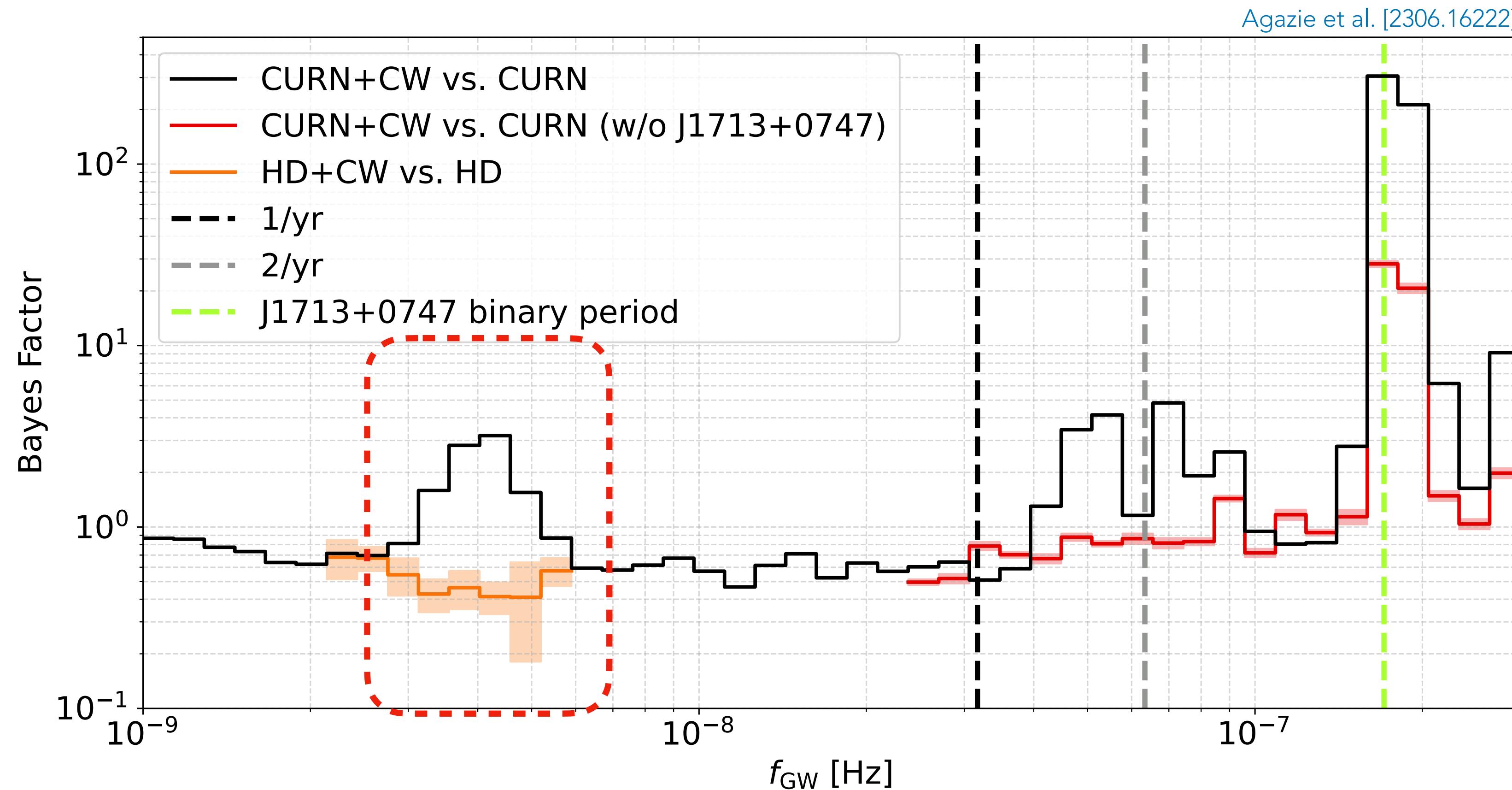
# SINGLE SOURCE



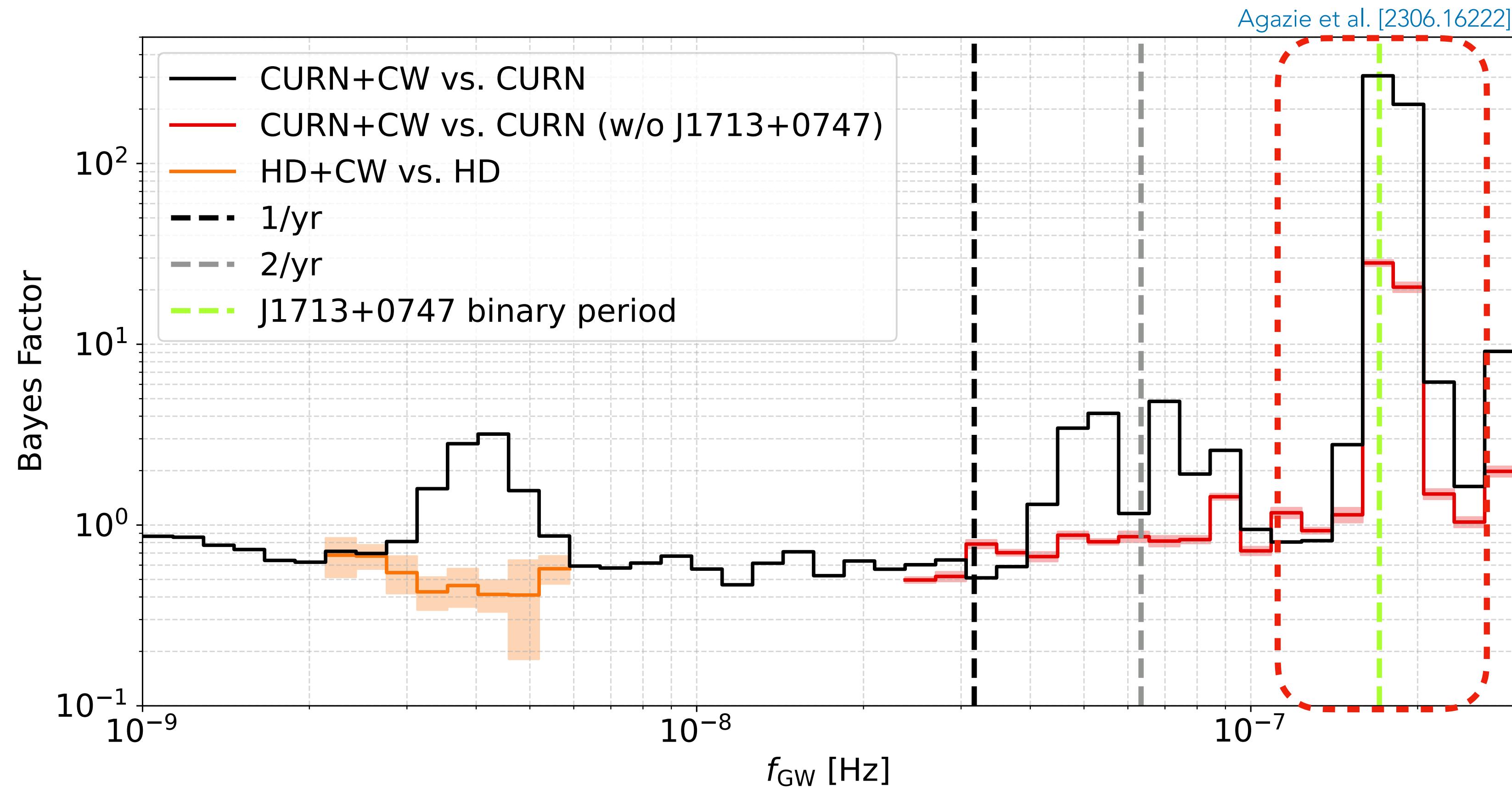
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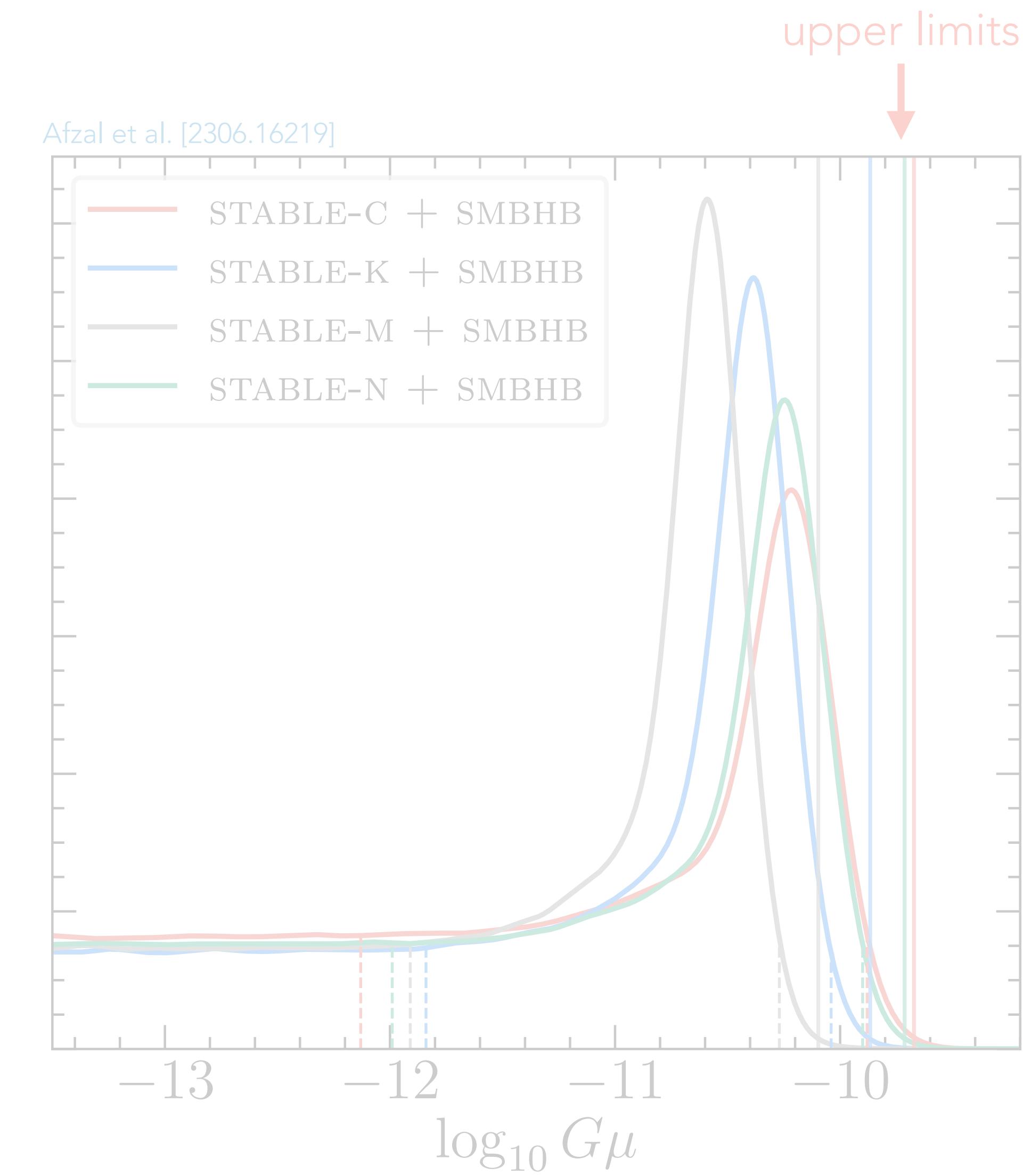
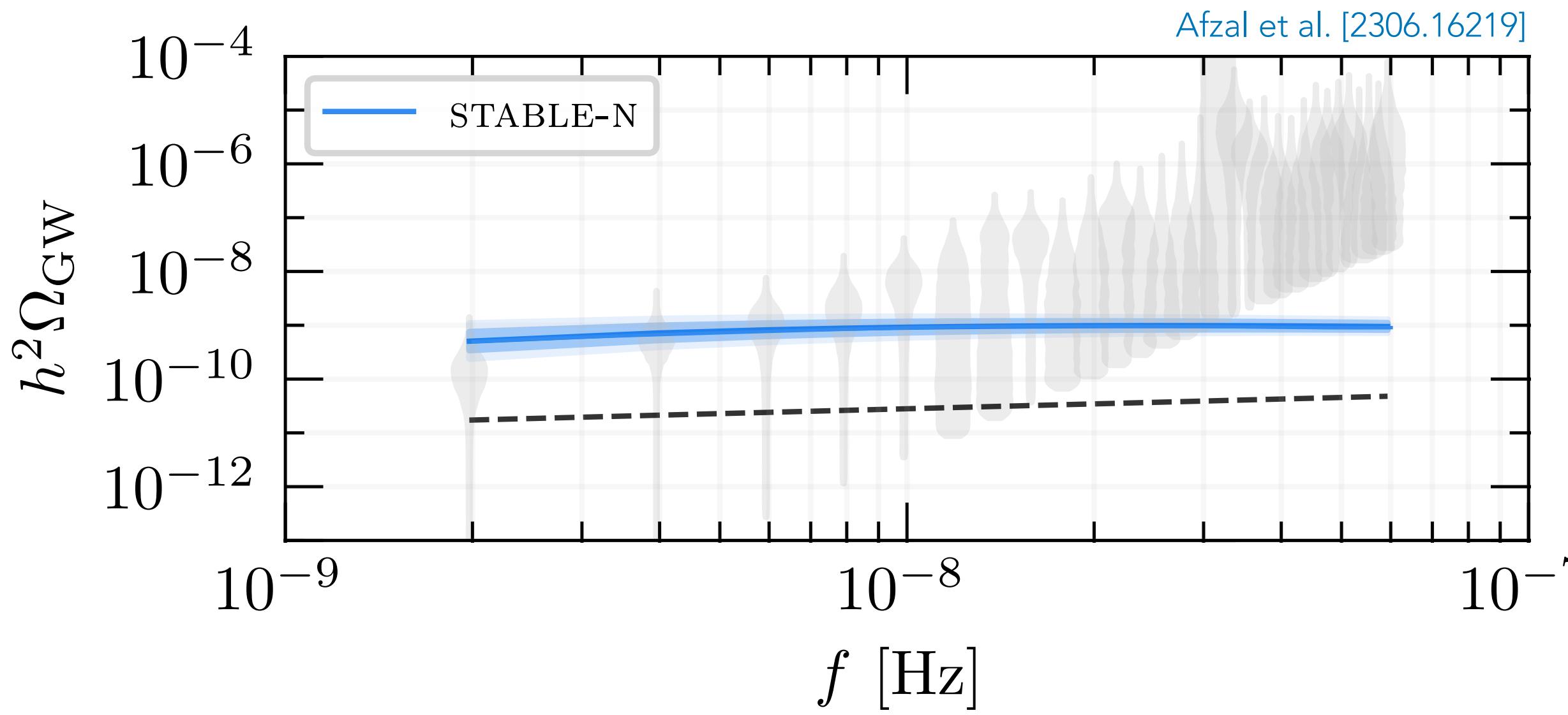
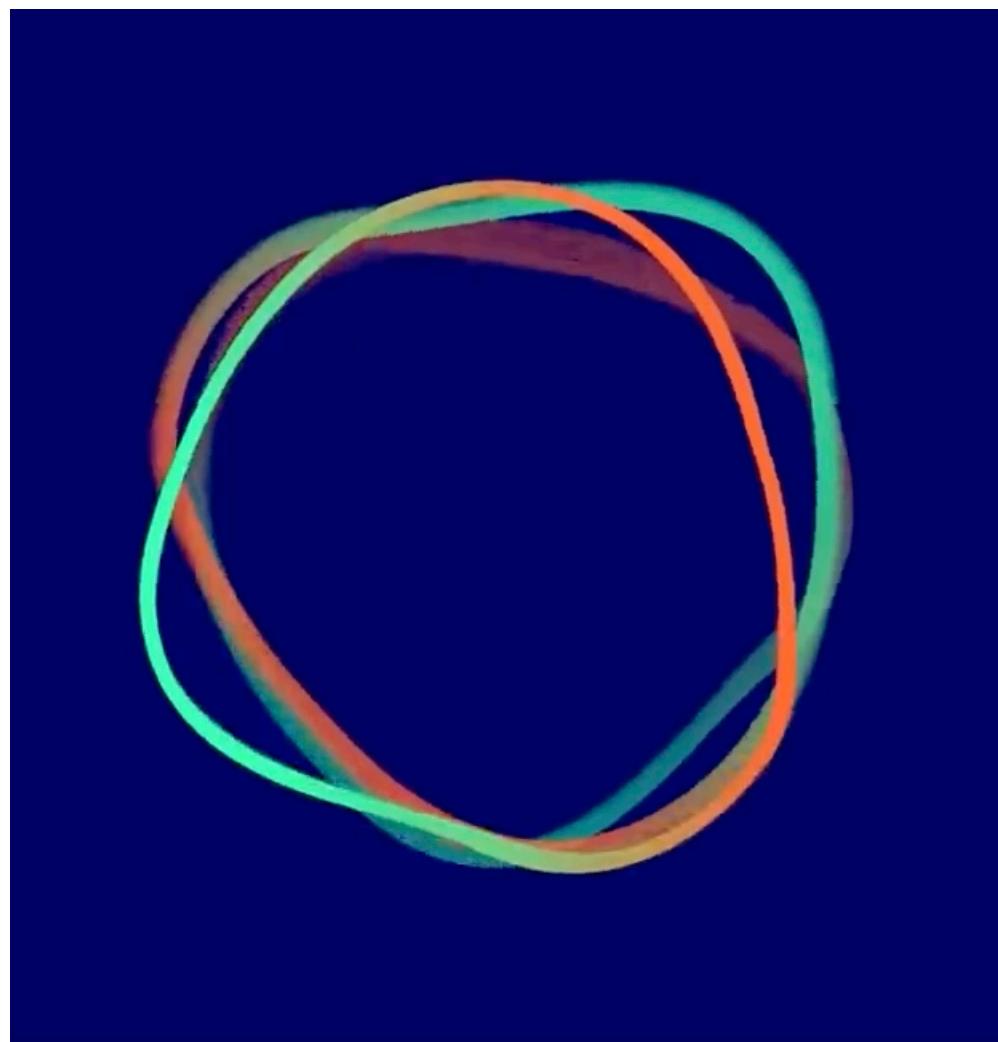


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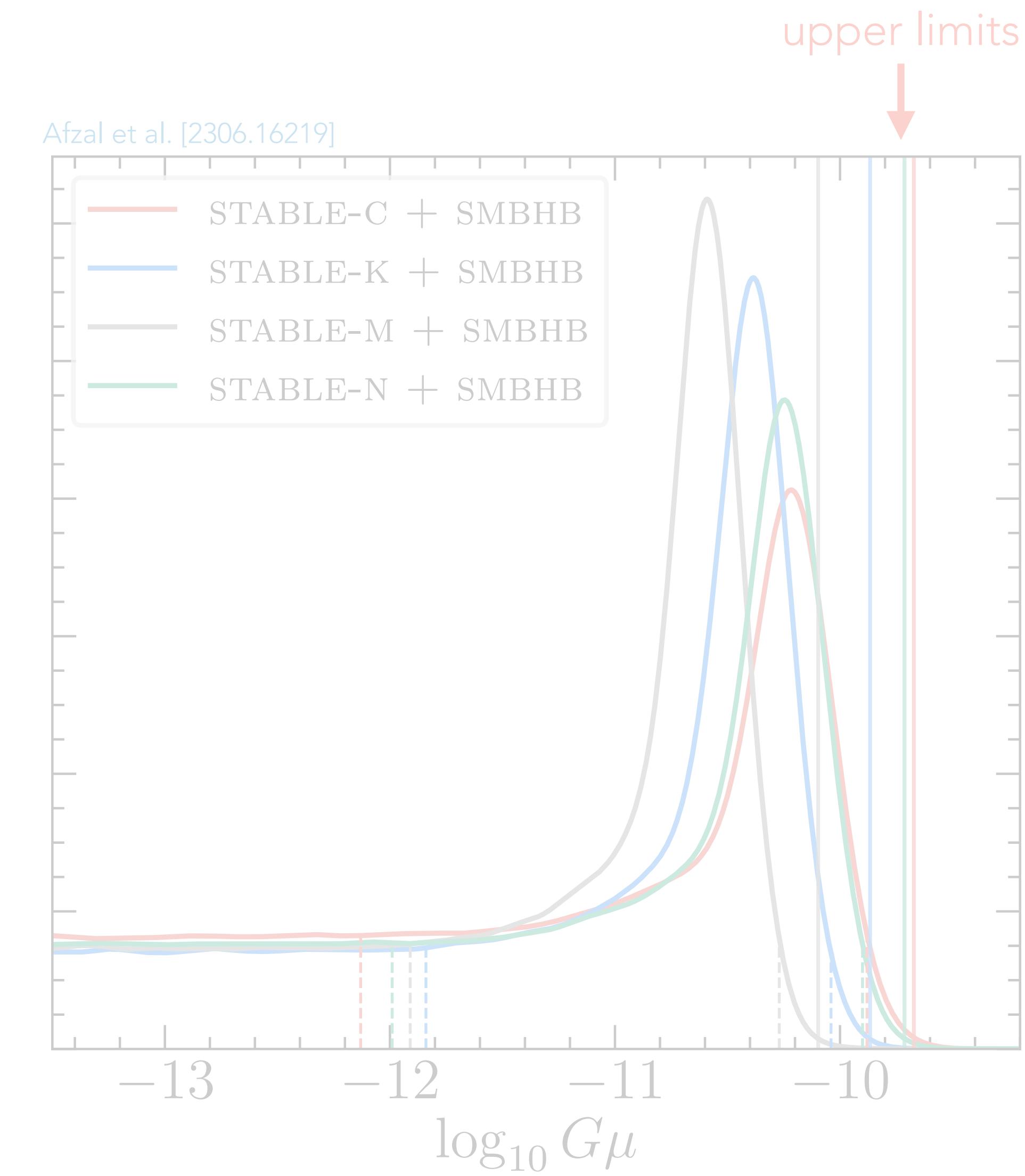
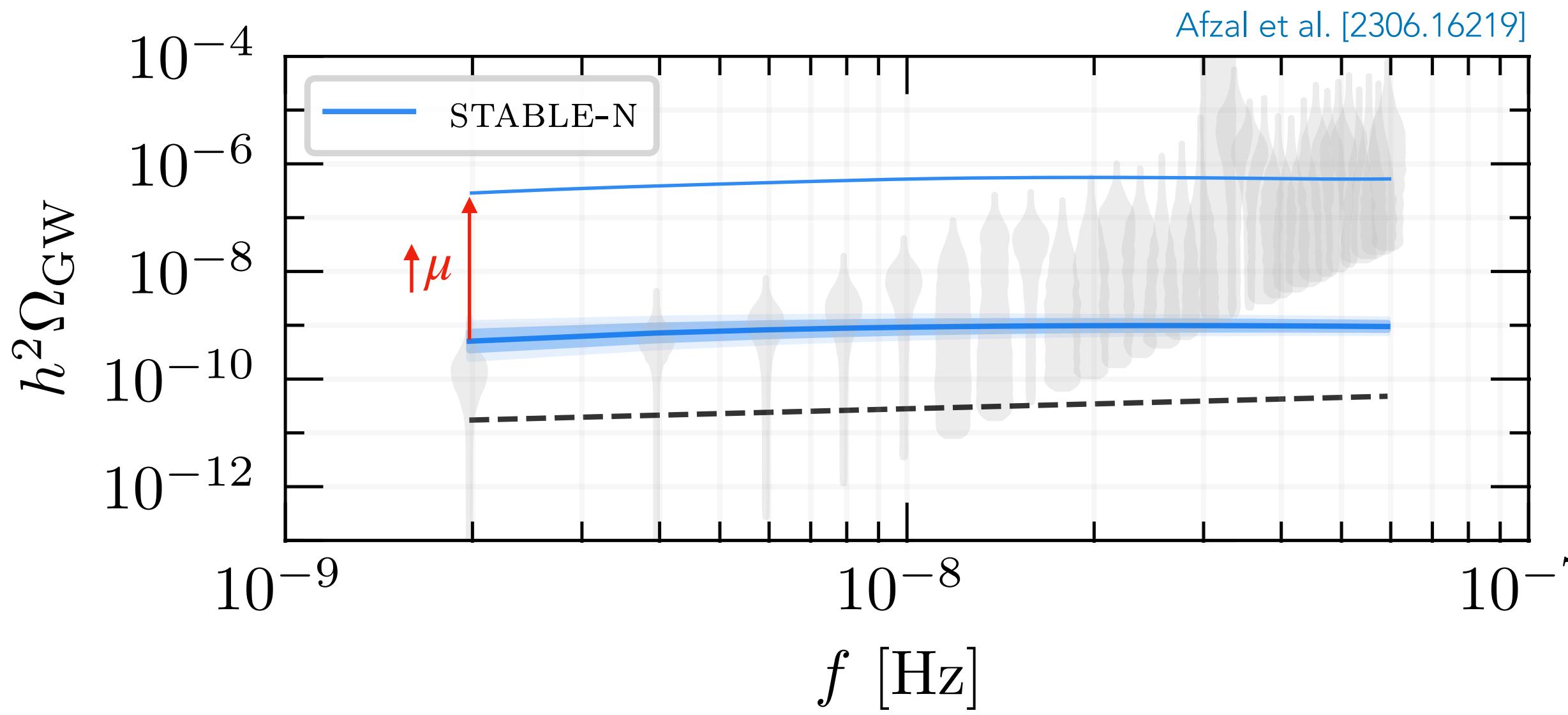
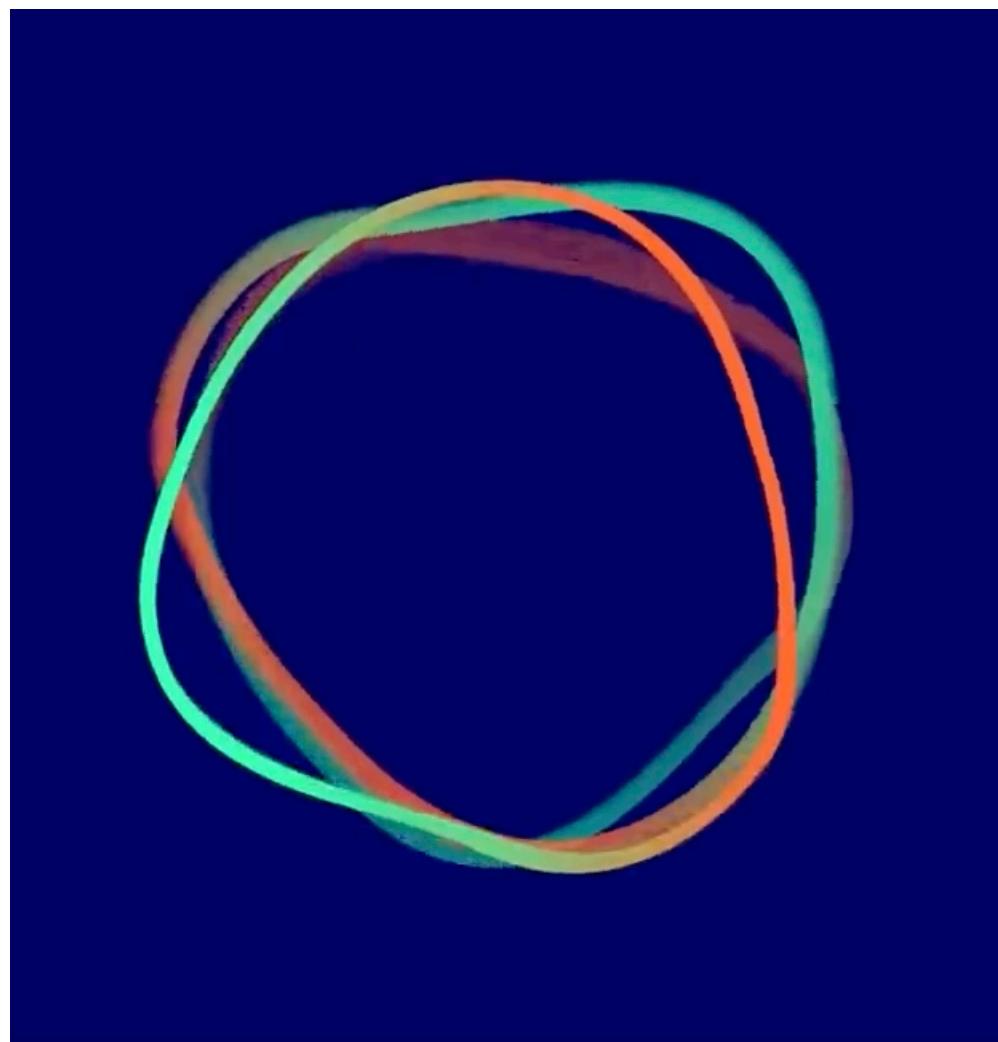


what if it's not new physics

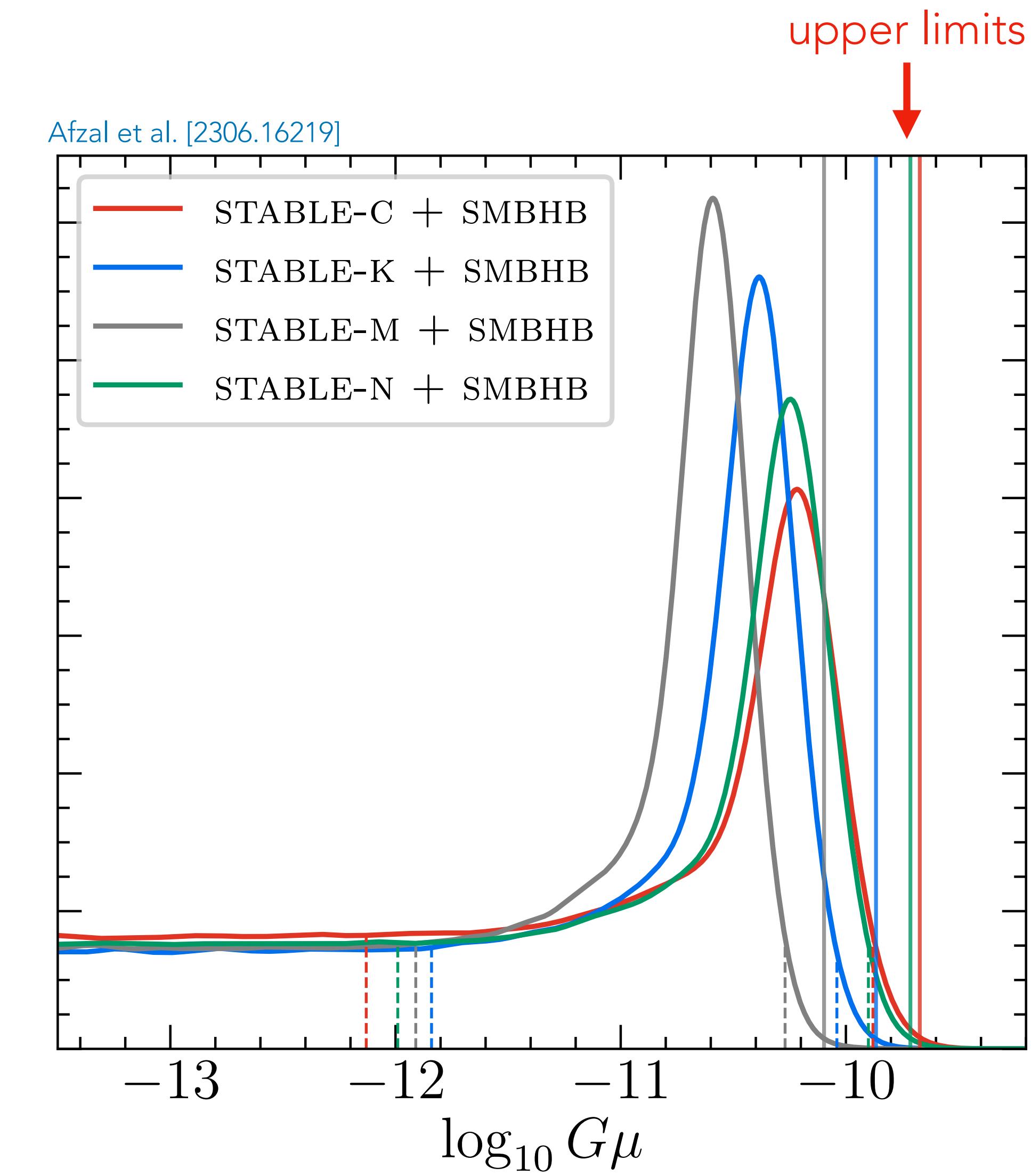
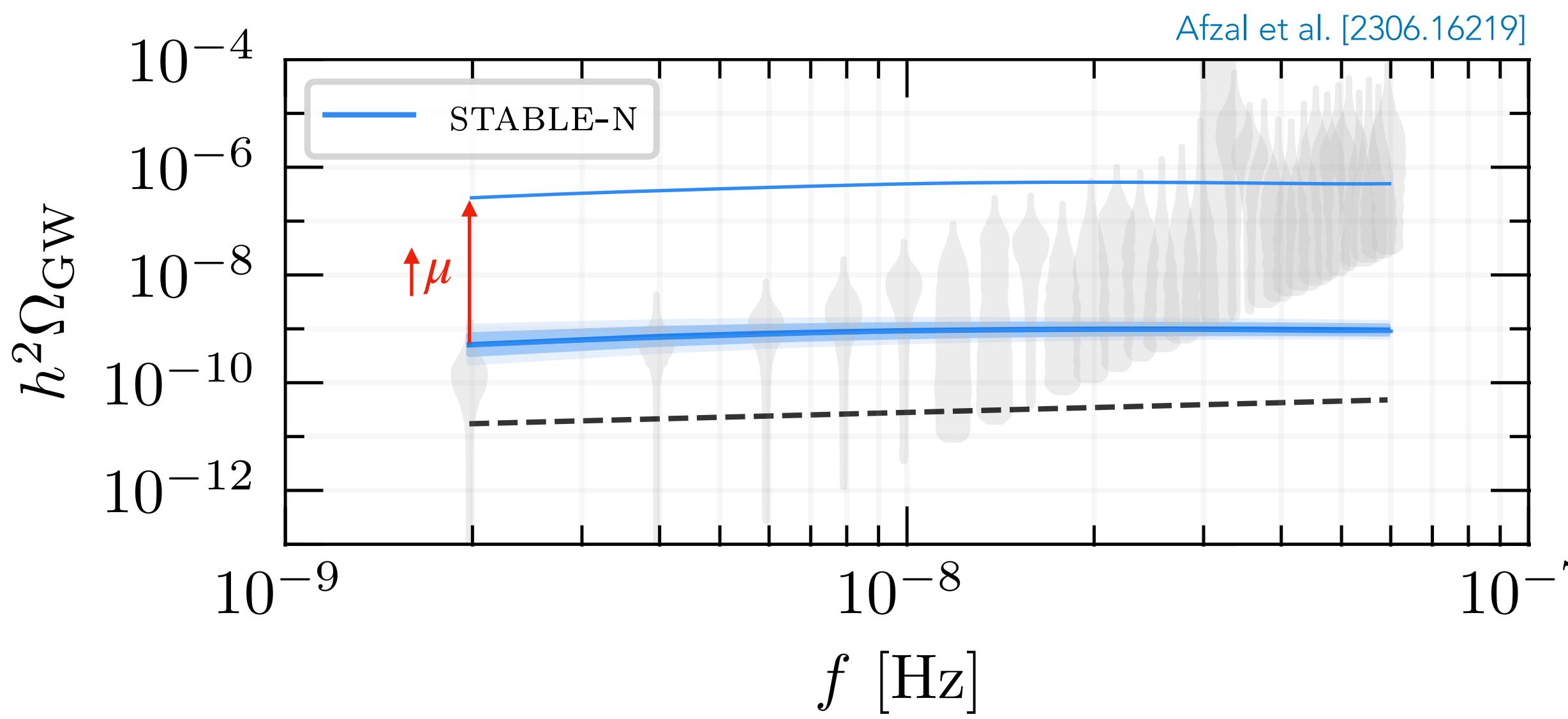
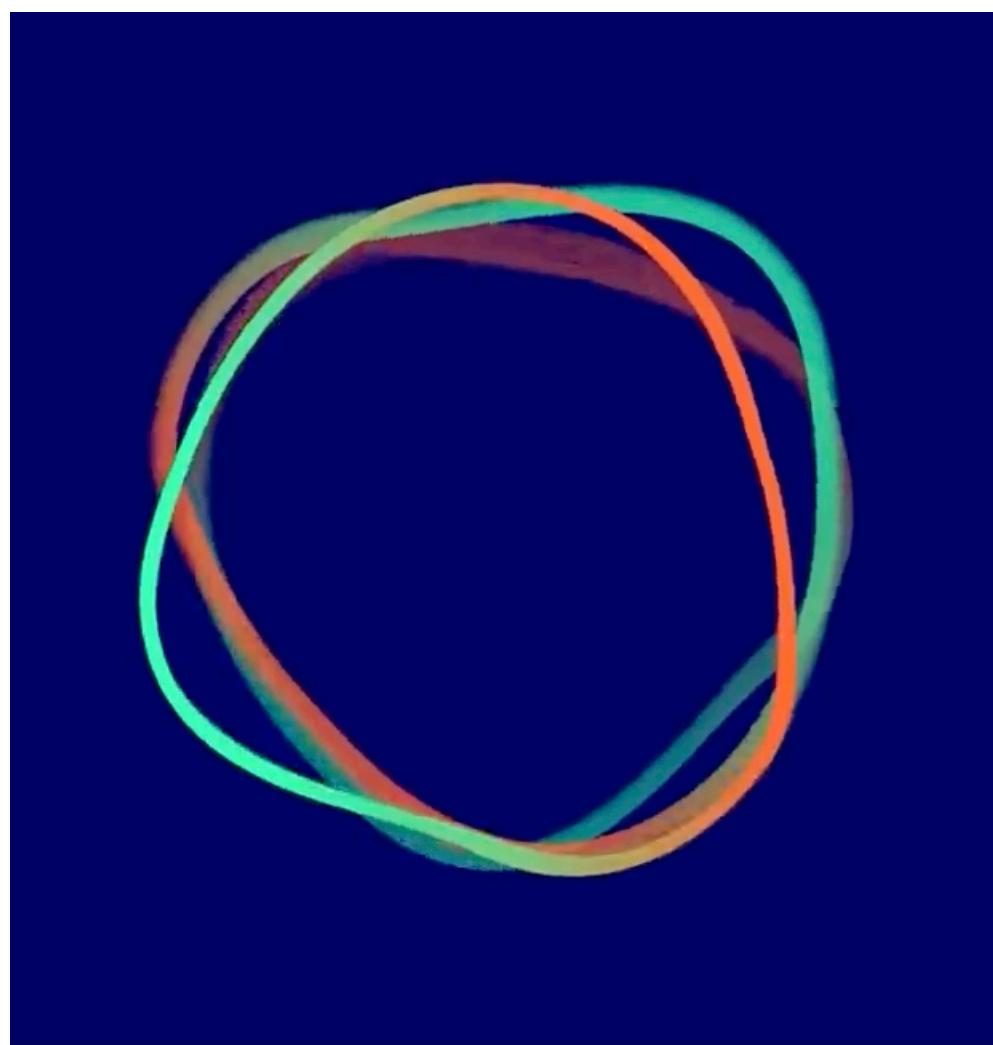
# COSMIC STRINGS



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$$\phi(\vec{x}, t) = \frac{\sqrt{2\rho_\phi}}{m_\phi} \hat{\phi}(\vec{x}) \cos(m_\phi t + \gamma(\vec{x})) + \text{stochastic low-f fluctuations}$$

DM density

$$\phi(\vec{x}, t) = \frac{\sqrt{2\rho_\phi}}{m_\phi} \hat{\phi}(\vec{x}) \cos(m_\phi t + \gamma(\vec{x})) + \text{stochastic low-f fluctuations}$$

DM mass

$$\phi(\vec{x}, t) = \frac{\sqrt{2\rho_\phi}}{m_\phi} \hat{\phi}(\vec{x}) \cos(m_\phi t + \gamma(\vec{x})) + \text{stochastic low-f fluctuations}$$



gravitational signals

$$s(t) \sim \frac{G\rho_\phi}{m_\phi^3} \sin(2m_\phi t)$$

[Khmelnitsky, Rubakov \[1309.5888\]](#)

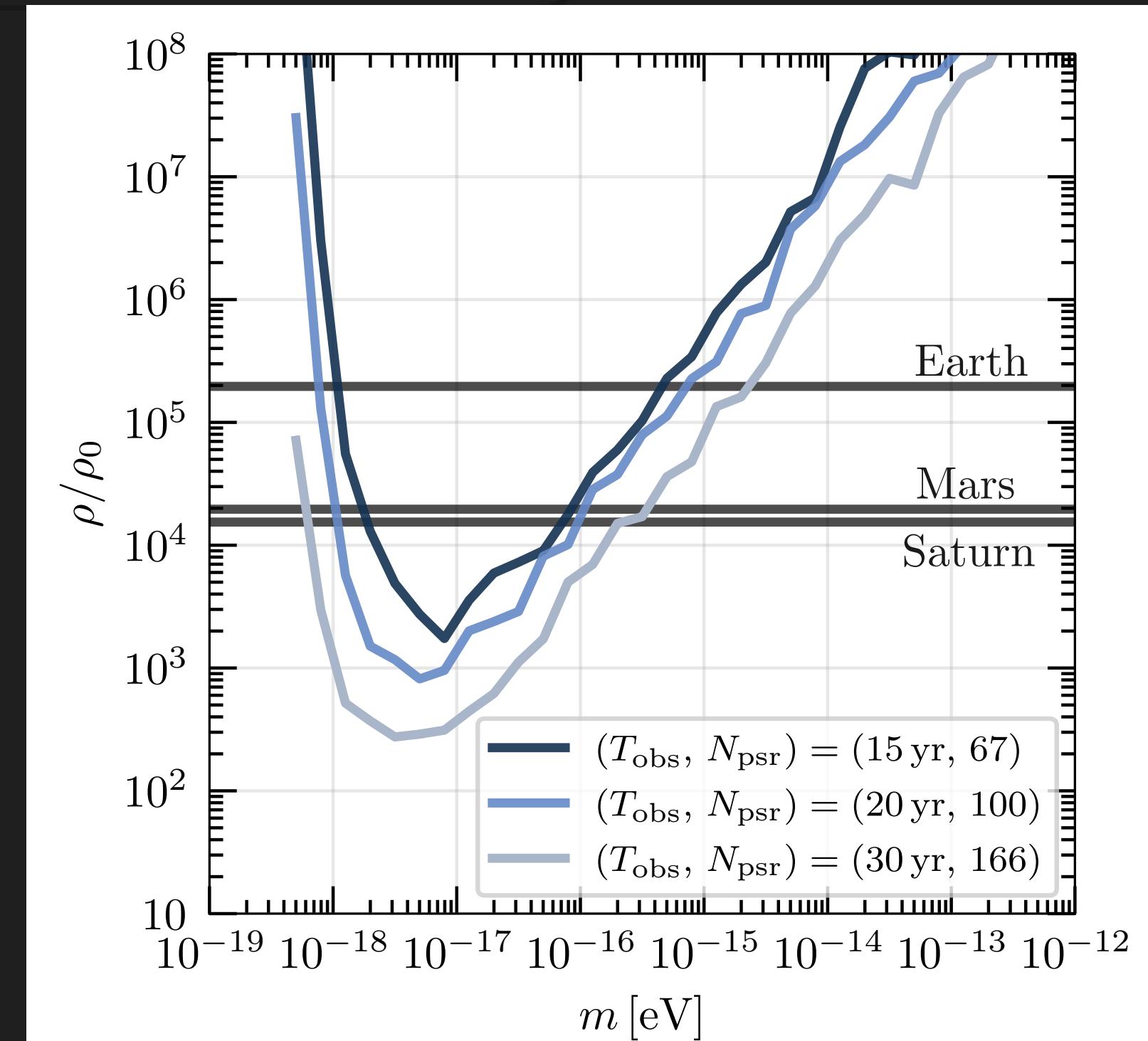
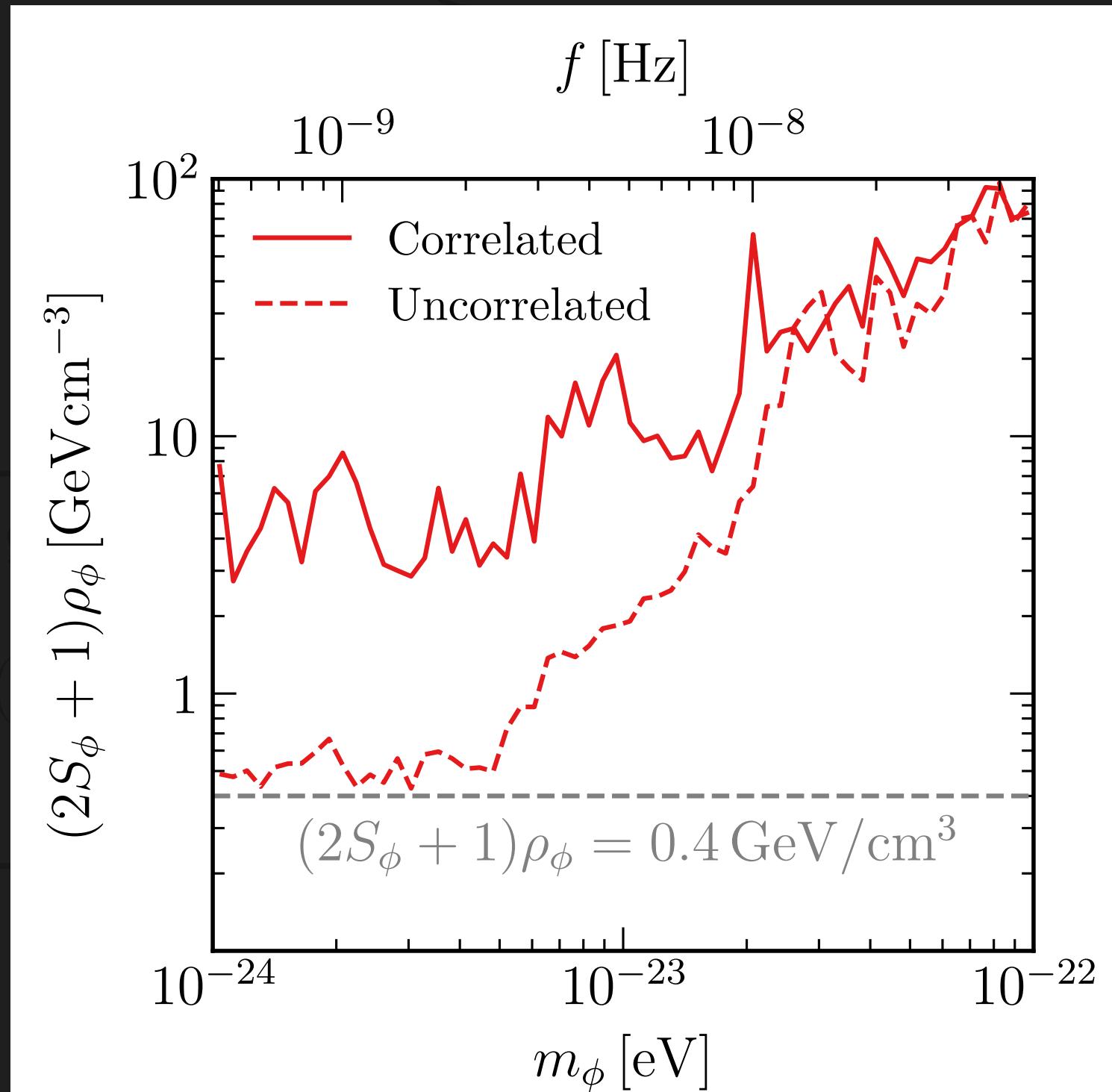
$$\langle ss' \rangle \sim \frac{G^2 \rho^2}{m^3 f^4 \sigma^4} K_0\left(\frac{\omega}{m\sigma^2}\right)$$

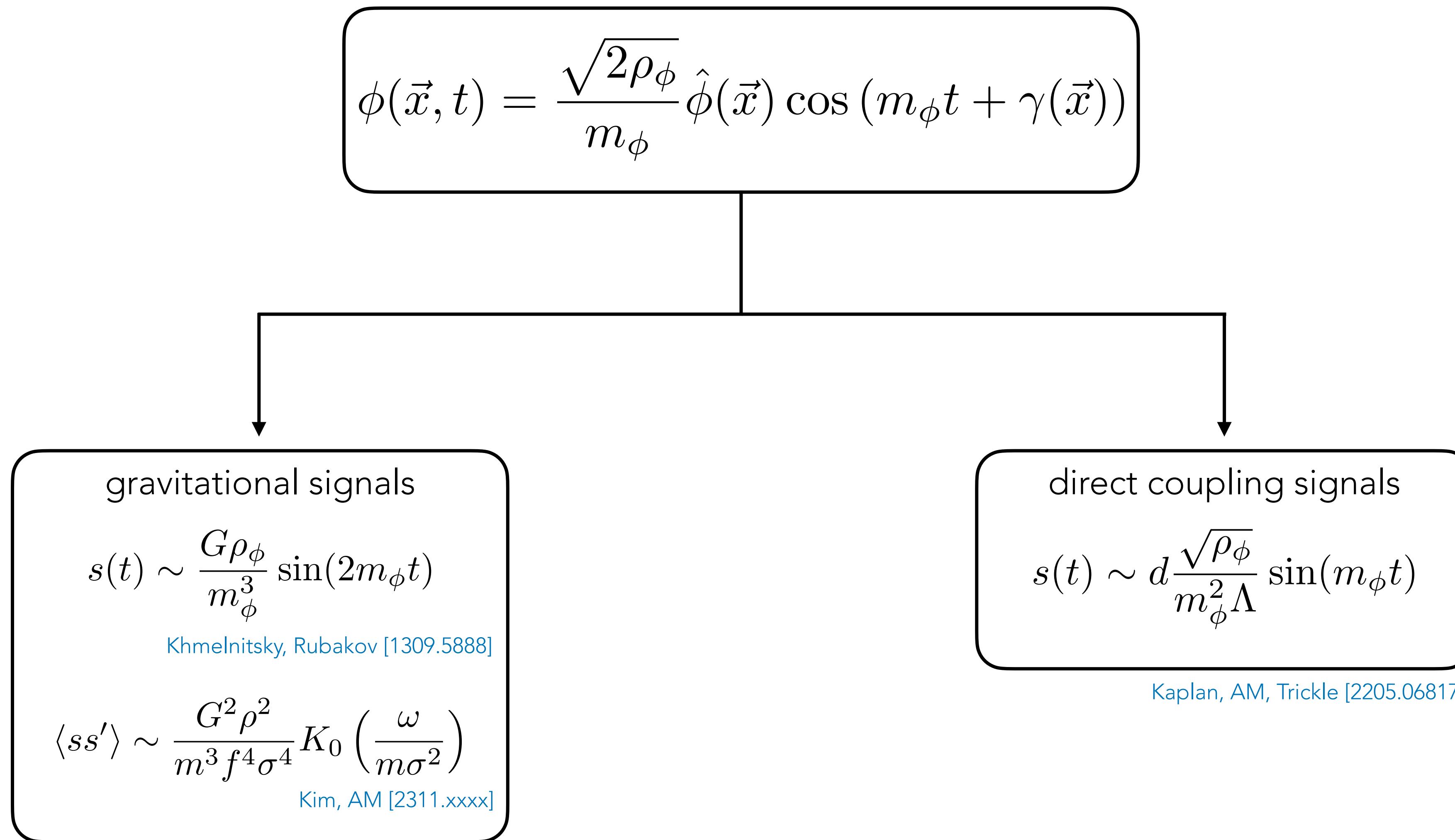
[Kim, AM \[2311.xxxx\]](#)

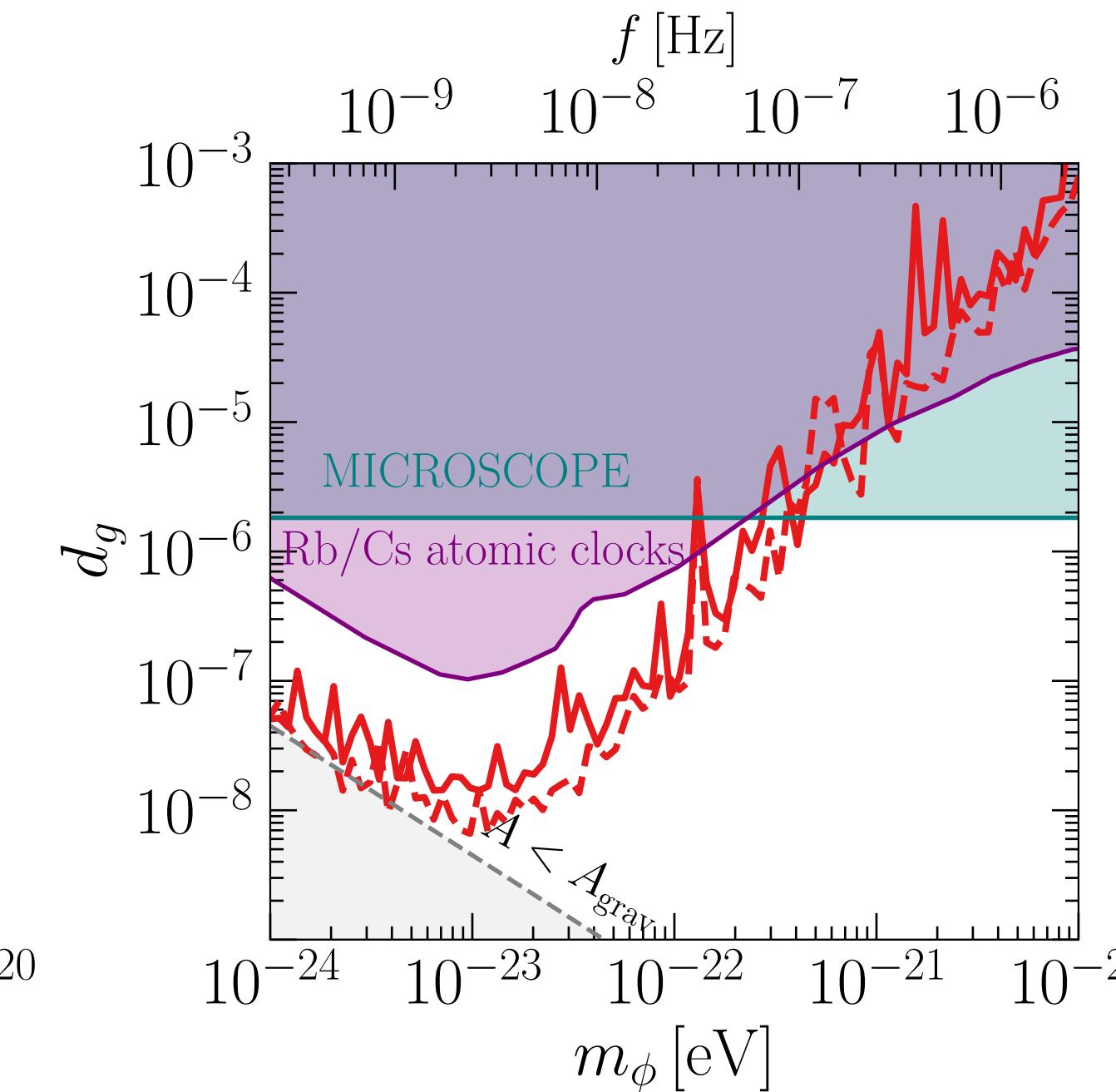
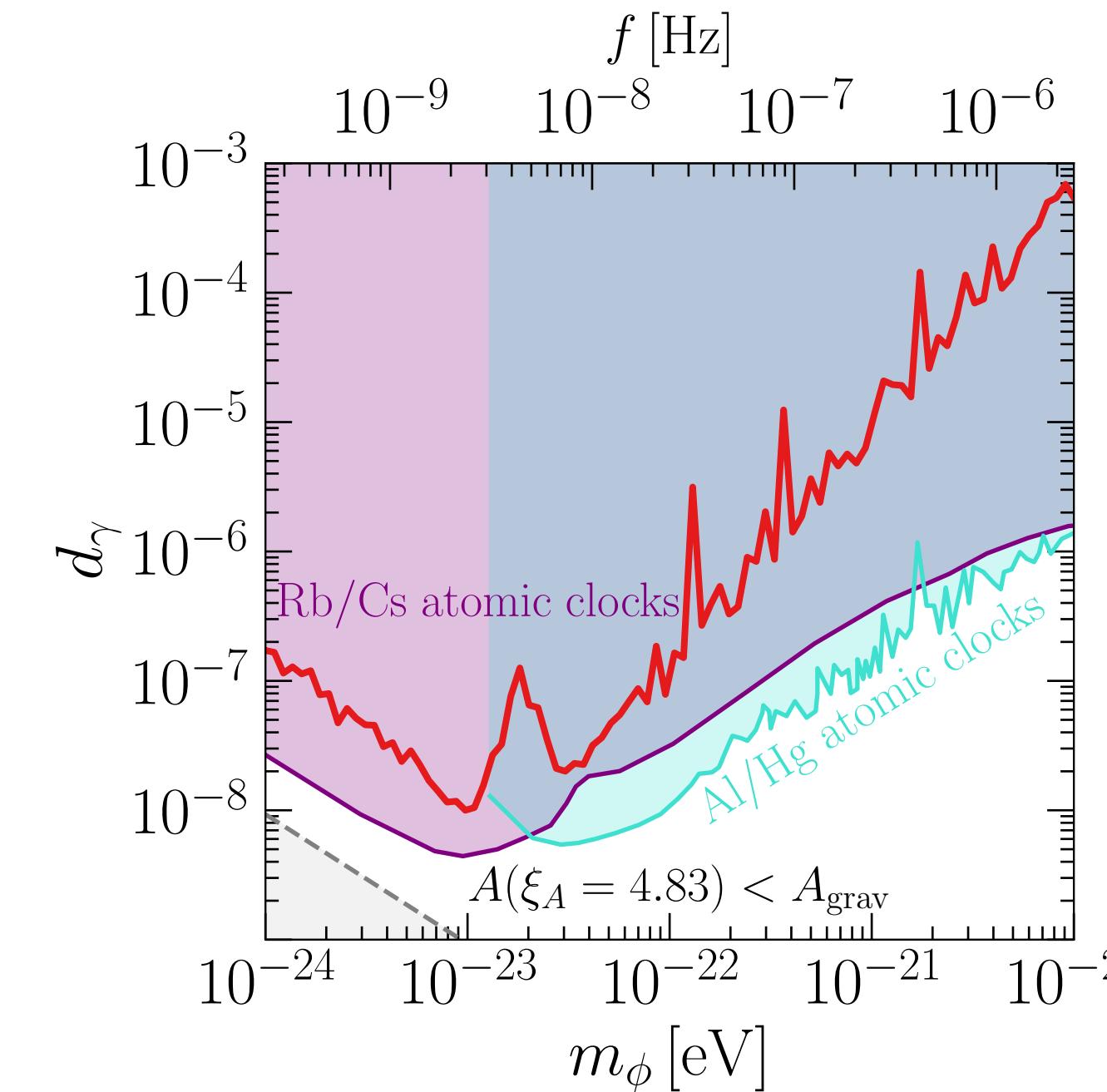
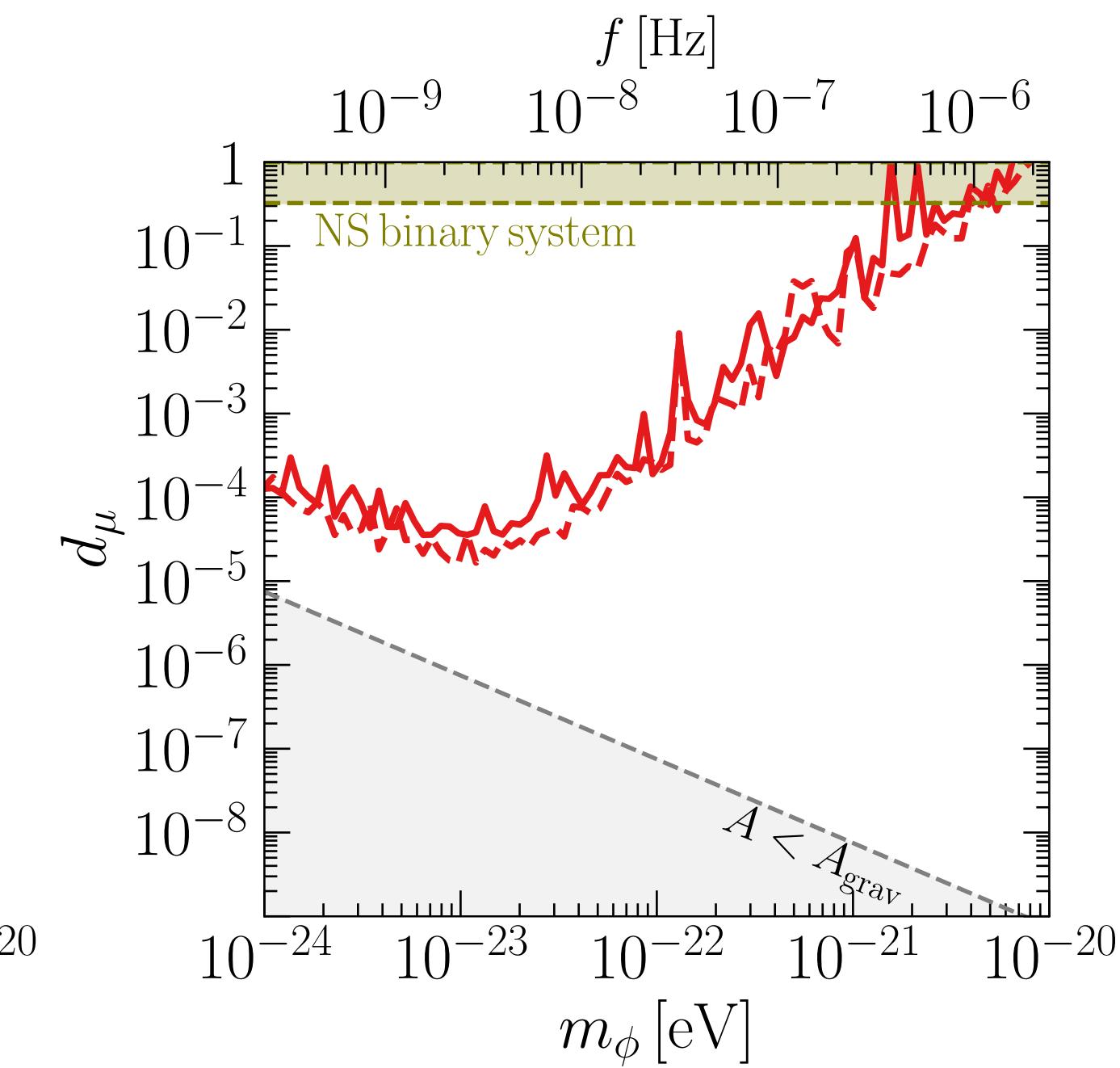
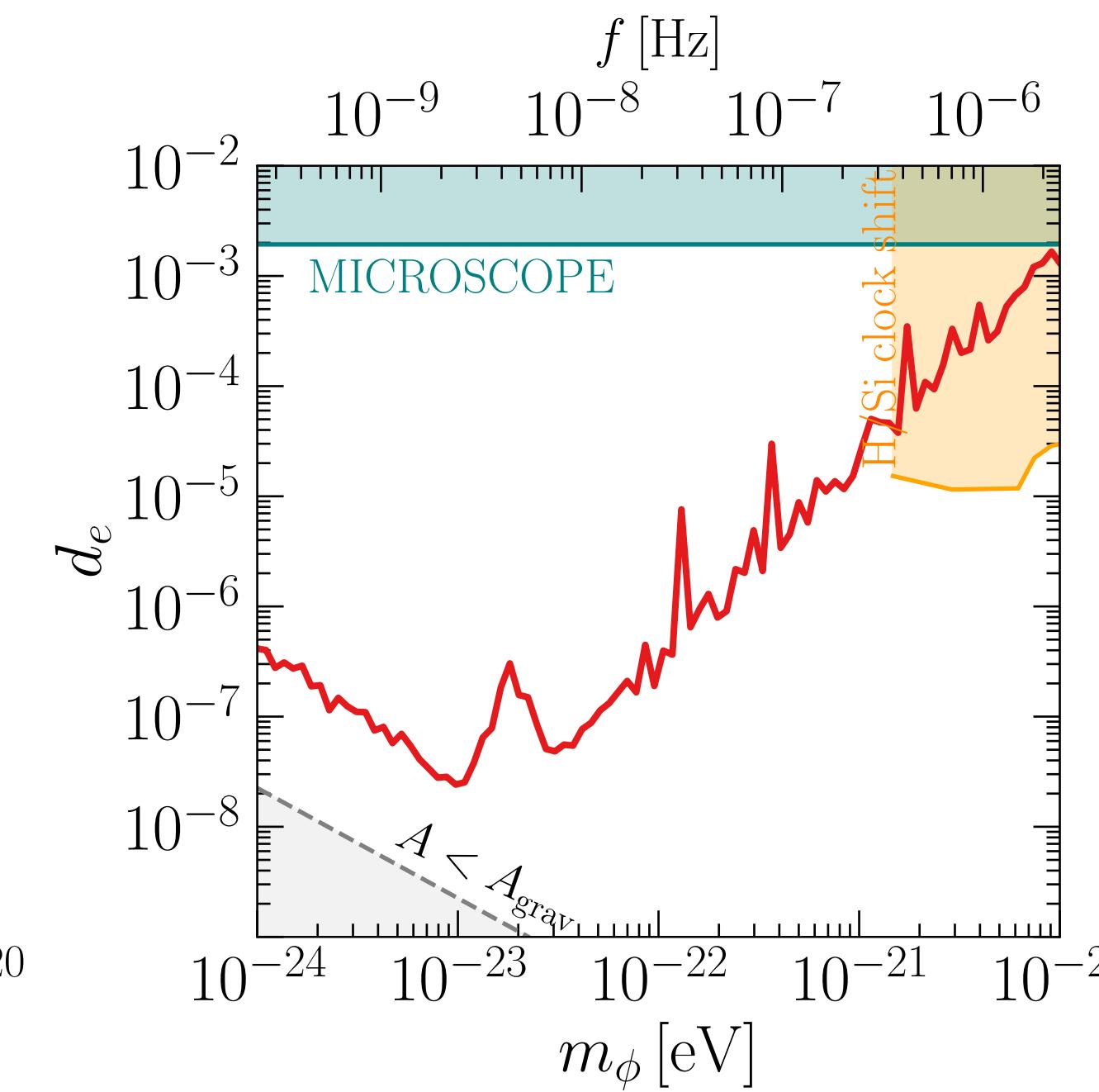
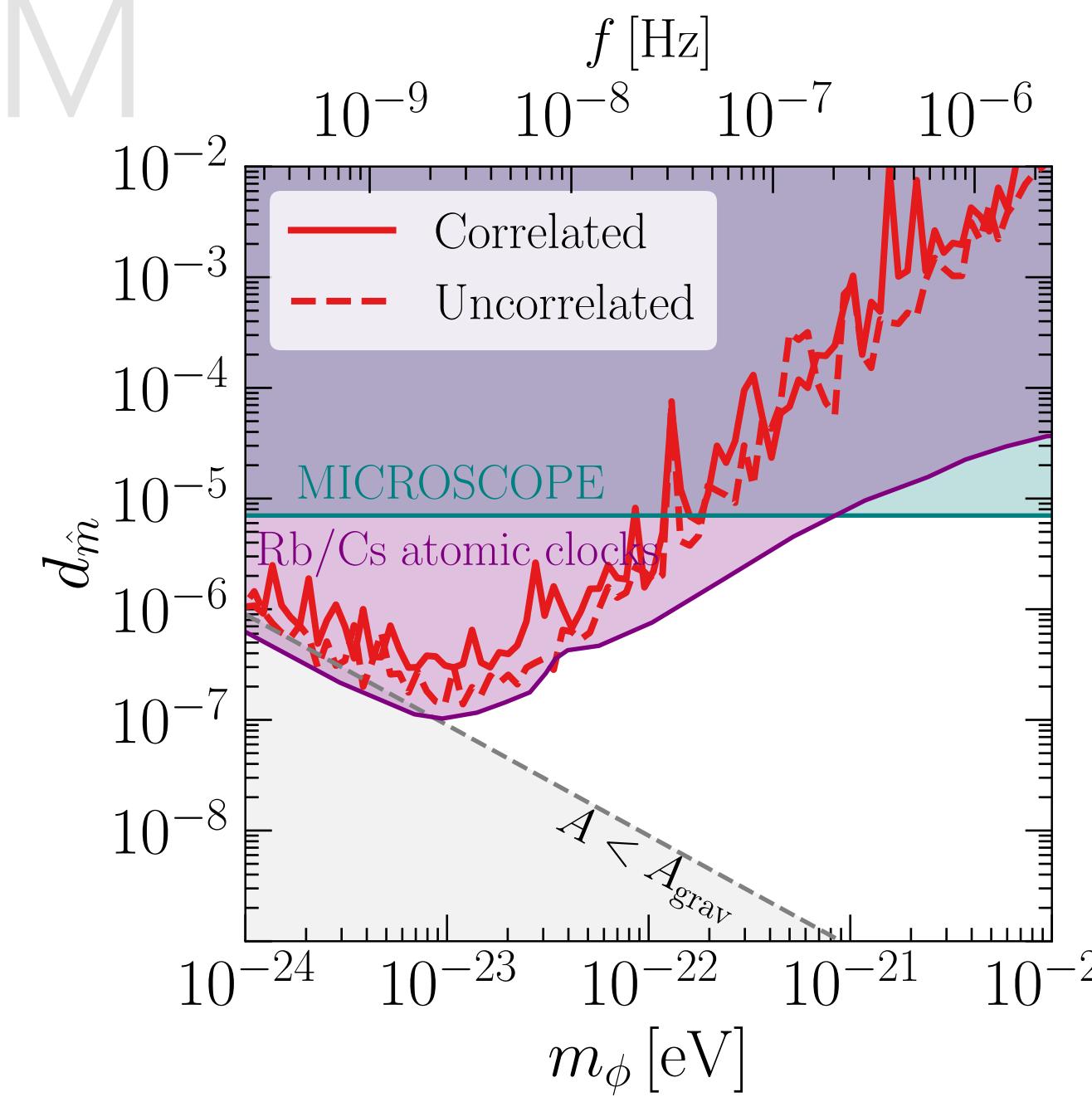
$$\phi(\vec{x}, t) = \frac{\sqrt{2\rho_\phi}}{m_\phi} \hat{\phi}(\vec{x}) \cos(m_\phi t + \gamma(\vec{x}))$$

Afzal et al. [2306.16219]

Kim, AM [2311.xxxx]







**strong evidence for a GWB in the nHz band**

strong evidence for a GWB in the nHz band  
cosmology or astrophysics?

strong evidence for a GWB in the nHz band

cosmology or astrophysics?

CW and anisotropies will help us discriminating

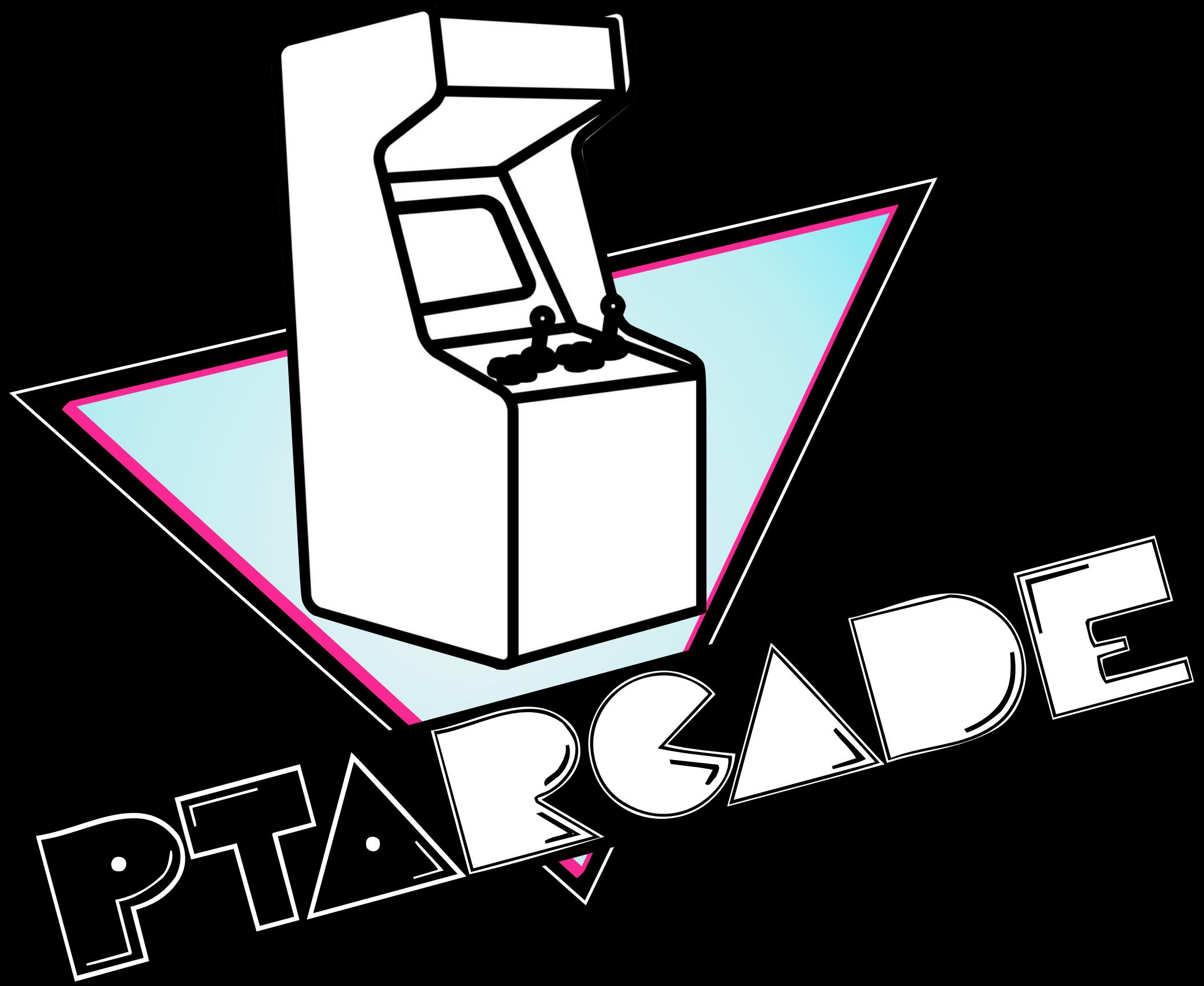
strong evidence for a GWB in the nHz band

cosmology or astrophysics?

CW and anisotropies will help us discriminating

PTA can be used to constrain new physics

backup



have a model you want to test against PTA data?  
say hello to PT Arcade

# PTArcade

## Step 1



```
conda install ptarcade
```

## Step 2



```
from ptarcade.models_utils import prior

parameters = {
    'log_A_star' : prior("Uniform", -14, -6),
    'log_f_star' : prior("Uniform", -10, -6)
}

def S(x):
    return 1 / (1/x + x)

def spectrum(f, log_A_star, log_f_star):
    A_star = 10**log_A_star
    f_star = 10**log_f_star

    return A_star * S(f/f_star)
```

toy model

$$h^2\Omega_{GW}(f) = \frac{A_*}{f/f_* + f_*/f}$$

## Step 3



```
ptarcade -m model.py
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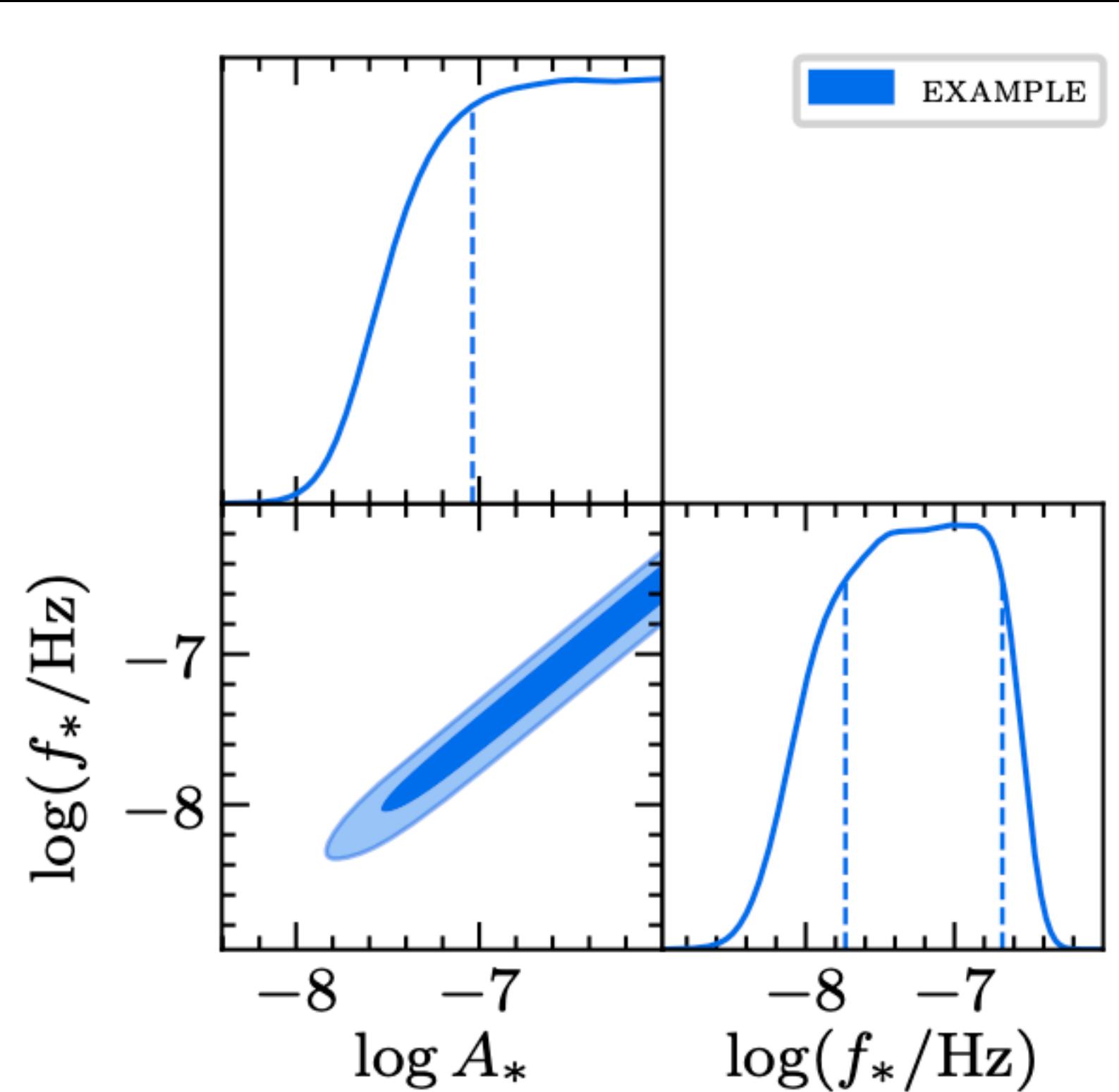


# PTArcade

Step 1

toy model

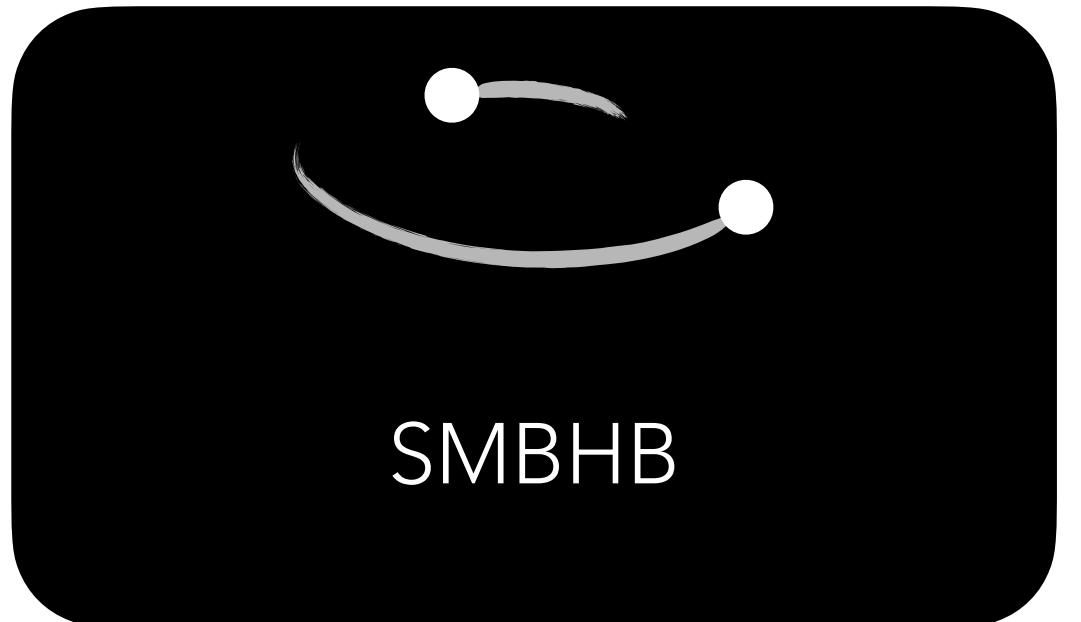
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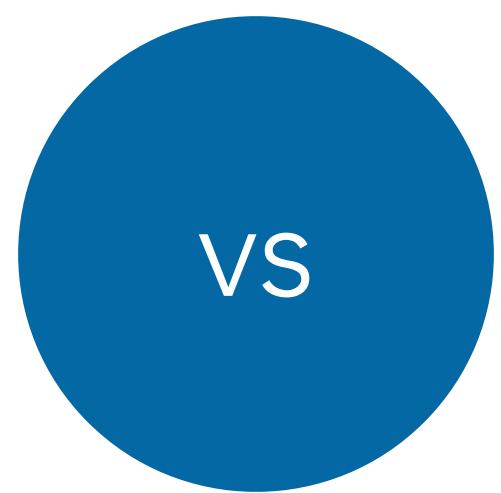
ptarcade -m model.py



# FACE-OFF



SMBHB



VS

inflation

scalar induced GW

phase transitions

cosmic strings

domain walls

# FACE-OFF

$$h^2 \Omega_{\text{GW}} \propto \frac{A^2}{H_0^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{5-\gamma} \text{yr}^{-2}$$

vs

inflation

scalar induced GW

phase transitions

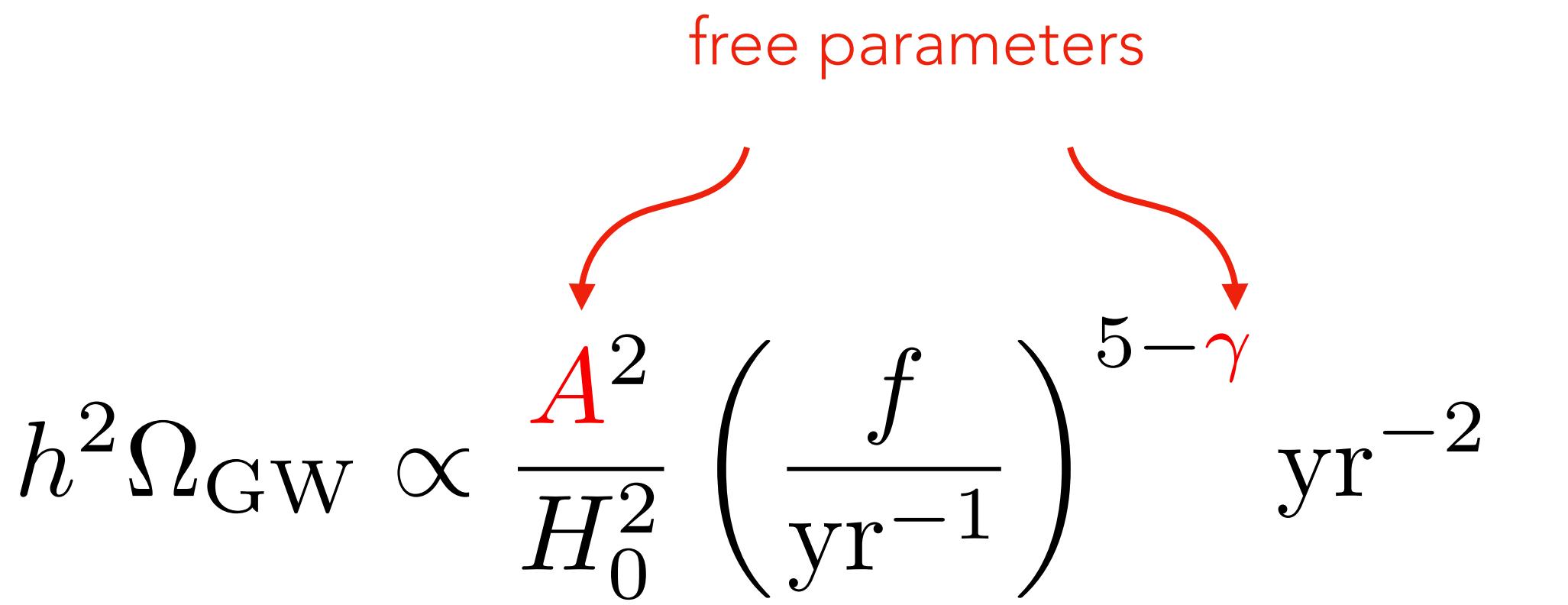
cosmic strings

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$$h^2 \Omega_{\text{GW}} \propto \frac{A^2}{H_0^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{5-\gamma} \text{yr}^{-2}$$

free parameters



vs

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# FACE-OFF

free parameters

$$h^2\Omega_{\text{GW}} \propto \frac{A^2}{H_0^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{5-\gamma} \text{yr}^{-2}$$

vs

free parameters

$$h^2\Omega_{\text{GW}}(f; \Theta)$$

# FACE-OFF

free parameters

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vs

free parameters

$$h^2\Omega_{\text{GW}}(f; \alpha_*, T_*, HR_*)$$

# FACE-OFF

$$\mathcal{B} = \frac{\mathcal{Z}_{\text{NP}}}{\mathcal{Z}_{\text{BHB}}}$$

$$\mathcal{Z} = \int d\Theta \; P(\mathcal{D}|\Theta, \mathcal{H}) \times P(\Theta|\mathcal{H})$$

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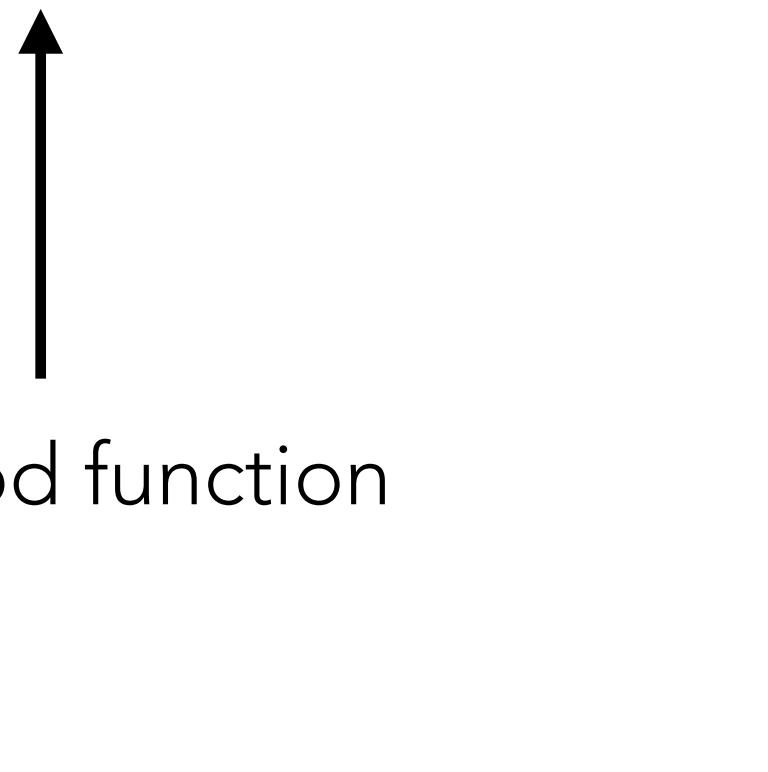


likelihood function

# FACE-OFF

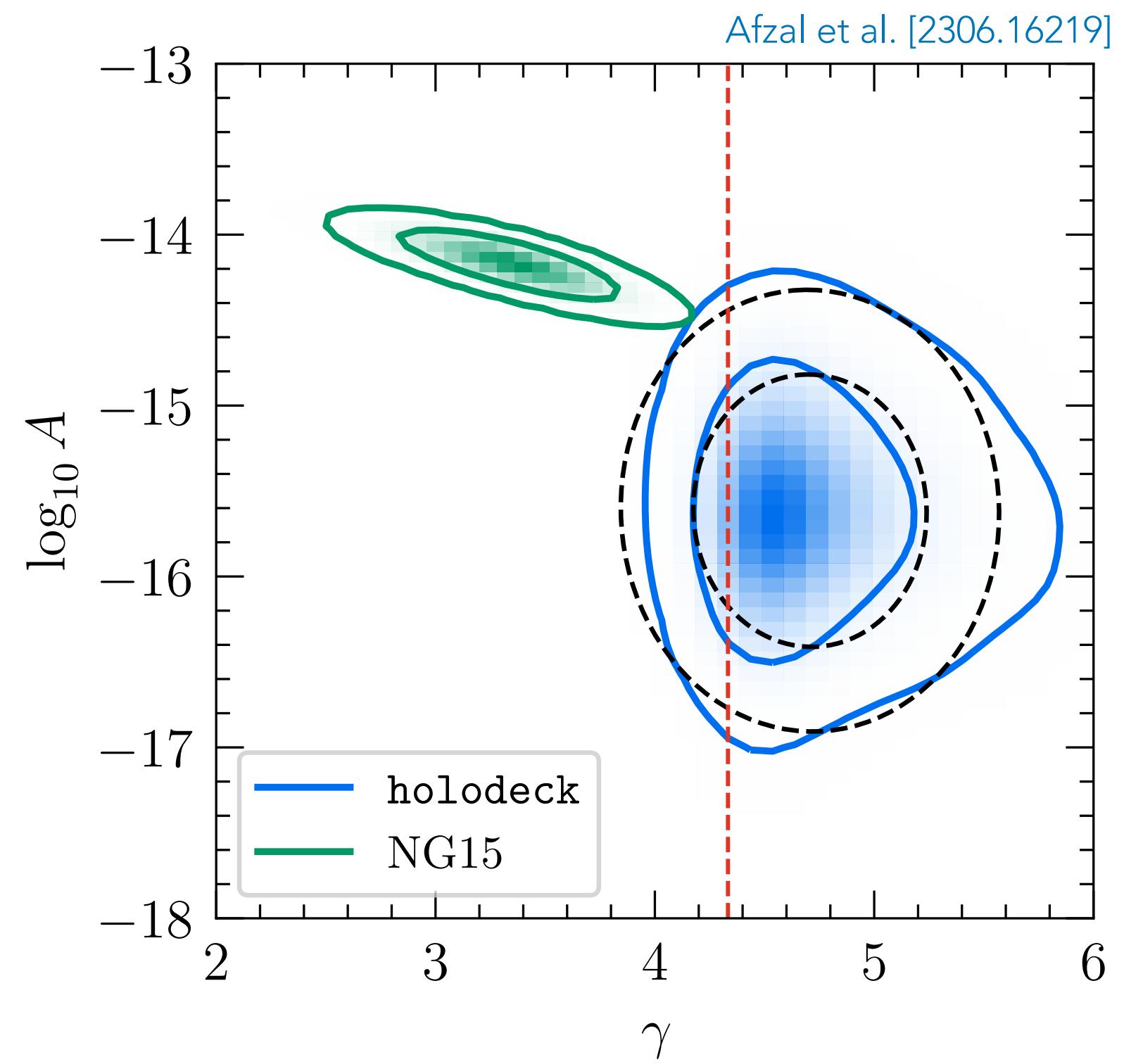
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↑  
likelihood function  
↑  
prior distributions

# FACE-OFF

