Neutrinos in dense astrophysical environments

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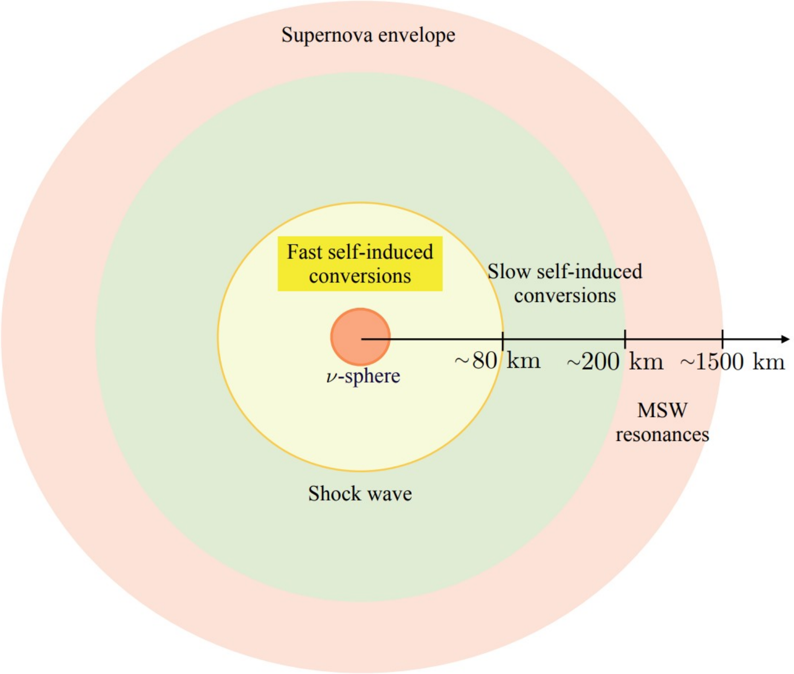
Outline

- Introduction: Why should we care about neutrino flavor transformations in dense astrophysical environments?
- Coherent forward scattering (refraction).
- Quantum kinetic equations
- Progress in numerical solution.
- Challenges and future directions.
- Conclusions

Neutrino flavor evolution in supernovae

- In deep interior, neutrinos are trapped. (Energy ~ O(10) MeV.)
- As they slowly escape the core, the neutrinos change flavor as they emerge.
- The neutrino flavor evolution depends on the medium through which they travel.



Neutrino flavor evolution in Neutron Star Mergers

- Neutron star mergers are known to be sites of r-process nucleosynthesis.
- Electron neutrinos and antineutrinos convert protons to neutrons and vice-versa affecting r-process nucleosynthesis rates.
- Understanding neutrino flavor evolution in neutron star mergers is crucial.

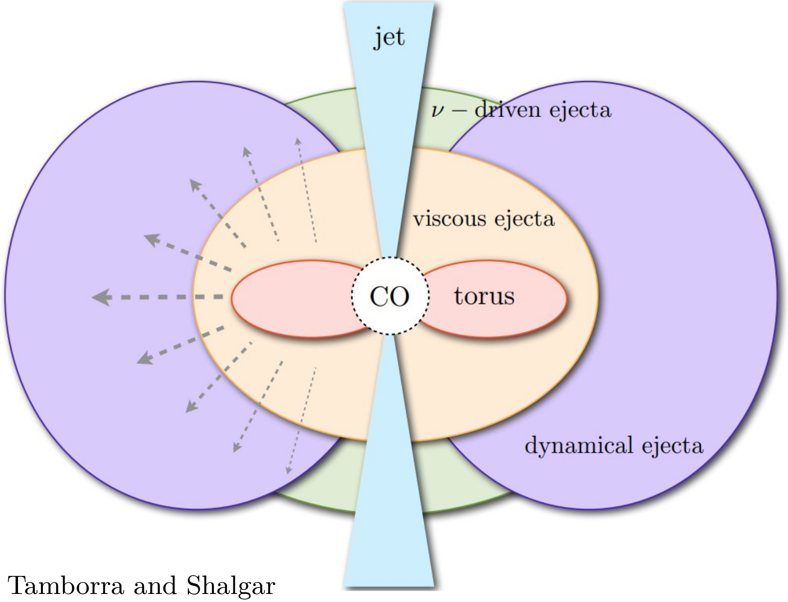


Image: Tamborra and Shalgar
ArXiv: 2011.01948

Role of flavor evolution

- Neutrino energy deposition in core-collapse supernova. This is one of the mechanism by which stalled shock can be revived.

- Neutrino flavor evolution can also affect the rate of nucleosynthesis by changing neutrons to protons and vice versa.

$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Neutrino oscillations

Schrodinger equation

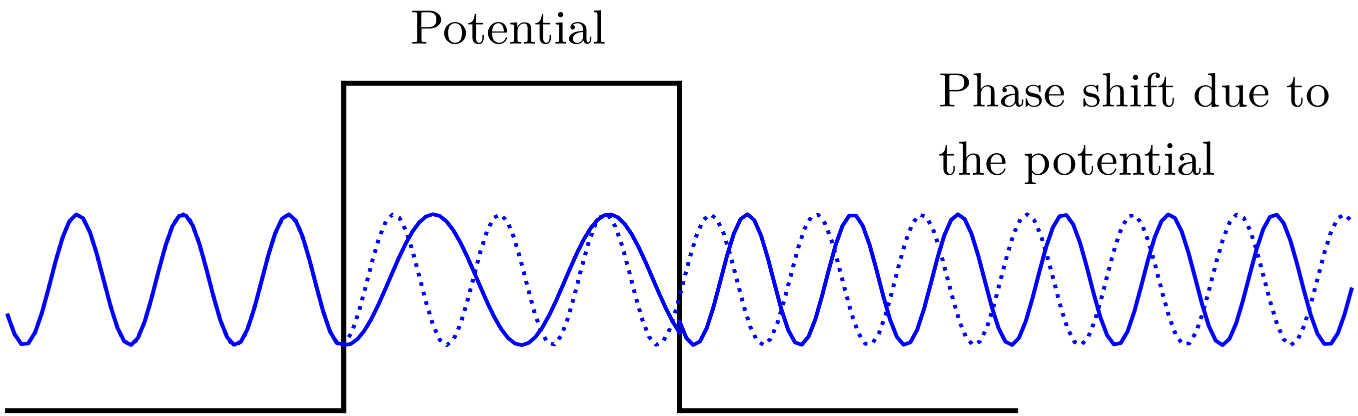
$$i\frac{d\psi(t)}{dt} = H\psi(t)$$

Heisenberg equation

$$i\frac{d\rho(t)}{dt} = [H, \rho(t)]$$

$$\psi = \begin{pmatrix} \langle x|\nu_\alpha\rangle \\ \langle x|\nu_\beta\rangle \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Longleftrightarrow \rho = \begin{pmatrix} aa^* & ab^* \\ a^*b & bb^* \end{pmatrix}$$

Coherent forward scattering of neutrinos



Matter Hamiltonian

Charged current term
due to electrons

$$
\begin{aligned}
H &= \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta_{\mathrm{V}} & \sin 2\theta_{\mathrm{V}} \\ \sin 2\theta_{\mathrm{V}} & \cos 2\theta_{\mathrm{V}} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_{\mathrm{F}}n_e - \sqrt{2}G_{\mathrm{F}}\frac{n_n}{2} & 0 \\ 0 & -\sqrt{2}G_{\mathrm{F}}\frac{n_n}{2} \end{pmatrix} \\
\bar{H} &= -\frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta_{\mathrm{V}} & \sin 2\theta_{\mathrm{V}} \\ \sin 2\theta_{\mathrm{V}} & \cos 2\theta_{\mathrm{V}} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_{\mathrm{F}}n_e - \sqrt{2}G_{\mathrm{F}}\frac{n_n}{2} & 0 \\ 0 & -\sqrt{2}G_{\mathrm{F}}\frac{n_n}{2} \end{pmatrix}
\end{aligned}
$$

Neutral current term
adds a common phase

Hamiltonian of neutrino self-interactions

Number density of neutrinos

Density matrix for neutrinos and antineutrinos

Fermi constant

$$H_{\nu\nu}(\mathbf{v}) = \sqrt{2}G_{\mathrm{F}}n_{\nu}\int \left(\rho(\mathbf{v}') - \bar{\rho}(\mathbf{v}')\right)(1 - \mathbf{v}\cdot\mathbf{v}')d\mathbf{v}'$$

Hamiltonian for neutrino with velocity $\mathbf{v}$

Velocity of neutrino in the medium

Slow Vs Fast collective flavor evolution

- Both have same equations of motion, but different initial angular distributions.

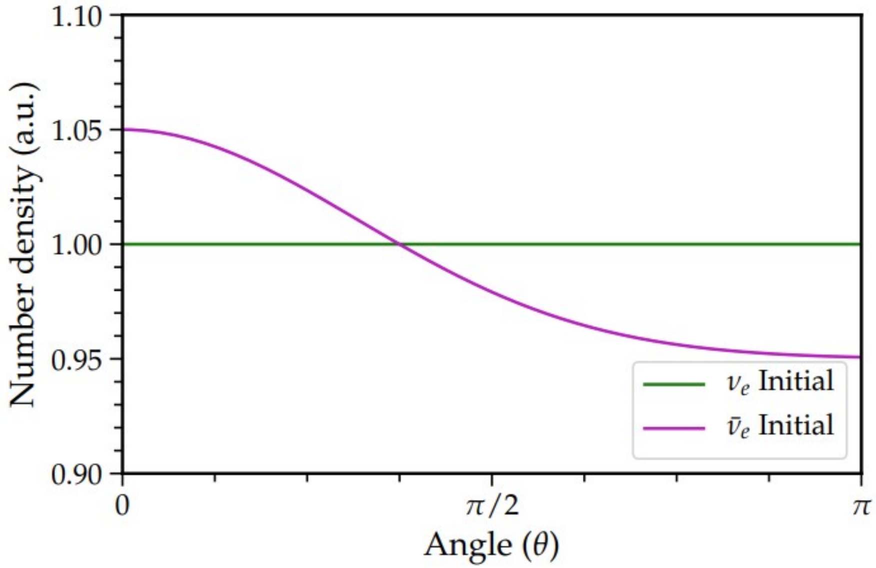
- Slow collective flavor evolution. Requires non-zero vacuum frequency for significant flavor evolution.

- Fast flavor evolution requires crossing in the angular distribution of electron neutrinos and electron anti-neutrinos (ELN-crossing).

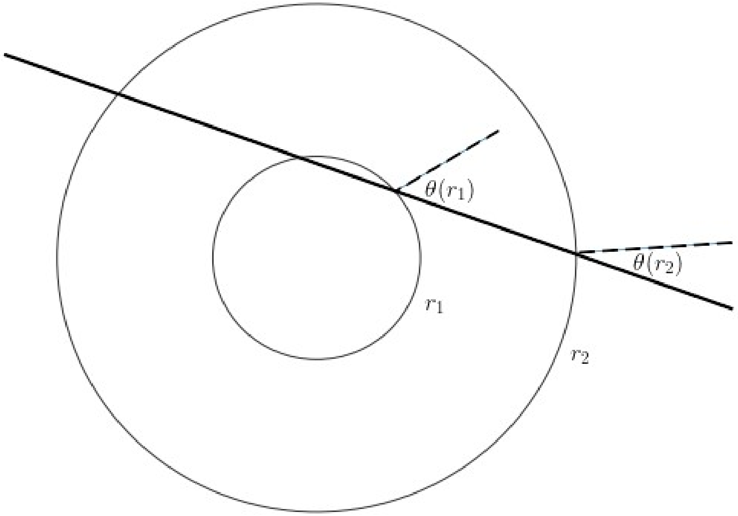
Fast flavor evolution

- Electron lepton number crossing essential for fast flavor evolution.
- No upper limit on the number density of neutrinos.

For a review see Tamborra and Shalgar
ArXiv: 2011.01948



Equations of motion: The spherical geometry.



$$\vec{v}\cdot\vec{\nabla} = \cos\theta\frac{\partial}{\partial r} + \frac{\sin^2\theta}{r}\frac{\partial}{\partial\cos\theta}$$

- A given angle does not represent a neutrino trajectory.
- At each point neutrino can be absorbed, emitted or change momentum.
- Equations that governs this dynamics along with flavor evolution are called **Quantum kinetic equations**.

Quantum kinetic equations

Advection

Does not depend on
neutrinos number density

$$\left(\frac{\partial \rho(\cos\theta, r, t)}{\partial t} + \vec{v}\cdot\vec{\nabla}\rho(\cos\theta, r, t)\right) = \mathcal{C}_{\text{emission}} - \mathcal{C}_{\text{absorb}}\rho(\cos\theta, r, t)$$

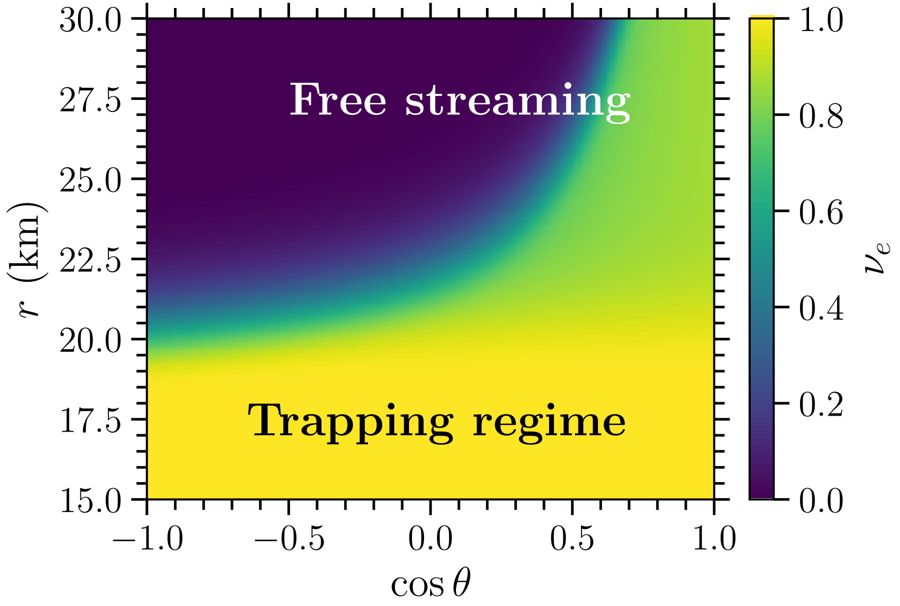
$$+ \quad \int_{-1}^{1} \mathcal{C}_{\text{dir-ch}}\rho(\cos\theta', r, t)d\cos\theta' - \int_{-1}^{1} \mathcal{C}_{\text{dir-ch}}\rho(\cos\theta, r, t)d\cos\theta'$$

$$- \quad i[H(\cos\theta, r, t), \rho(\cos\theta, r, t)]$$

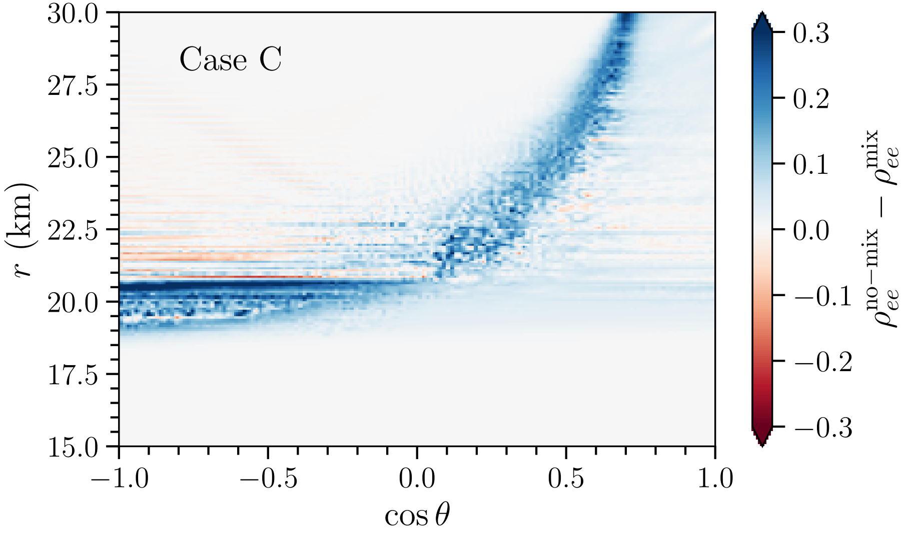
Flavor evolution

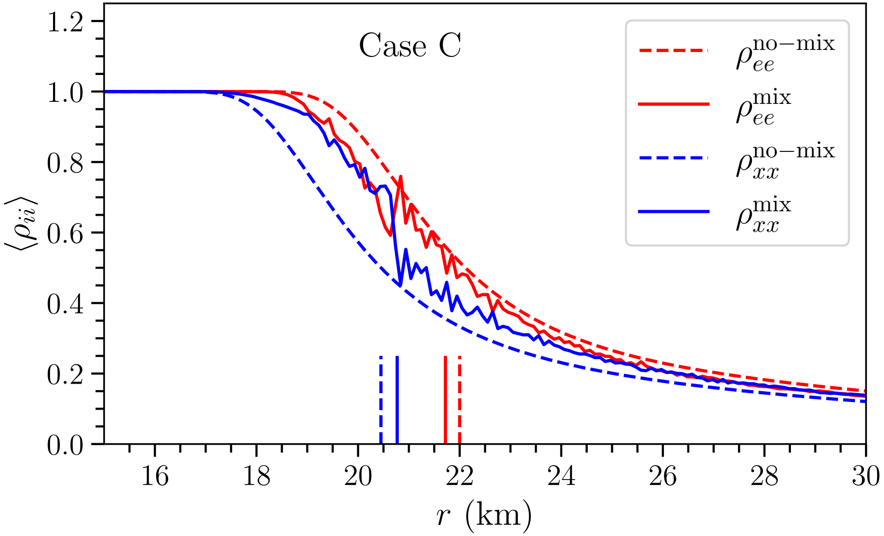
Distributions in absence of flavor evolution

- Depends on the collision strength as a function of radius (due to variation in density, temperature, etc.)
- Depends on the flavor.
- Depends on energy.



Effect of flavor evolution





Shalgar and Tamborra Arxiv:2206.00676 & 2207.04058

Limitations and challenges

- We have assumed spherical symmetry until now.
- But we know that neutrino self-interactions lead to spontaneous breaking of symmetry. This includes the breaking of axial and spherical symmetry.

  (See Duan and Shalgar arXiv:1412.7097)
- For each dimension we need O(100)-O(1000) bins. Which means for the 7-dimensional problem.. We need to solve at least 10^{12} coupled nonlinear equations with time as an independent variable.

Future directions and open questions

Numerical challenges:

- Discretization of partial differential equations implies that we need to solve billions of coupled nonlinear differential equations.
- We need better formulation and numerical techniques to make progress.

Conceptual issues

- In which domain are the quantum kinetic equations valid?

- An important question that has gained a lot of attention recently is the possibility of quantum entanglement due to neutrino self-interactions.

Multiparticle effects (Quantum entanglement)

- Almost all the studies of multiparticle effects rely on investigation flavor evolution of stationary neutrinos in a box; a closed system.
- This is not what happens in astrophysical systems.
- Two neutrinos will encounter each other and probably never meet each other again.
- One cannot gain any insight into the validity of quantum kinetic equations by studying a few stationary particles in a box.

See Shalgar and Tamborra arXiv:2304.13050

Conclusions

- Despite immense progress in recent years, challenges remain in understanding neutrino flavor evolution in dense astrophysical environments.
- The challenges remain on two fronts:
    - 1) Understanding the limitations of the formalism used to derive the Quantum Kinetic equations.
    - 2) Numerical challenges to solve an extremely nonlinear set equations.