Neutrino Oscillations at NOvA

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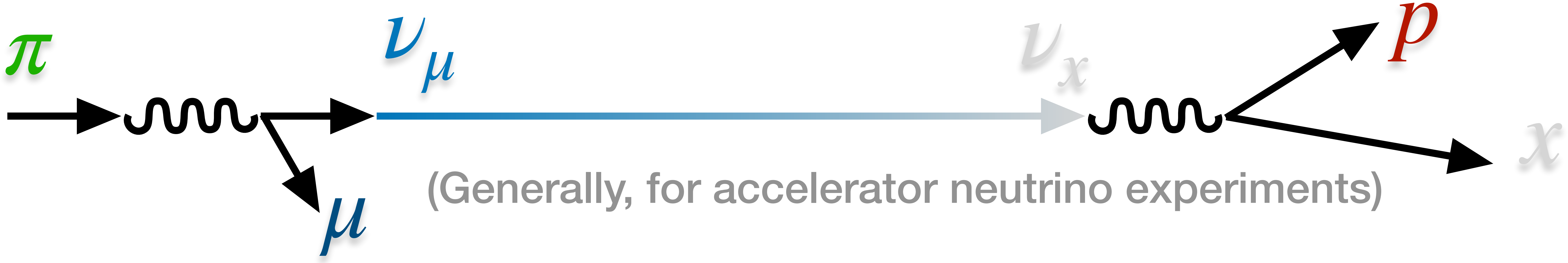
University of Wisconsin - Madison

NuPhys 2023
Kings College London

Neutrinos Oscillate



(Generally, for accelerator neutrino experiments)

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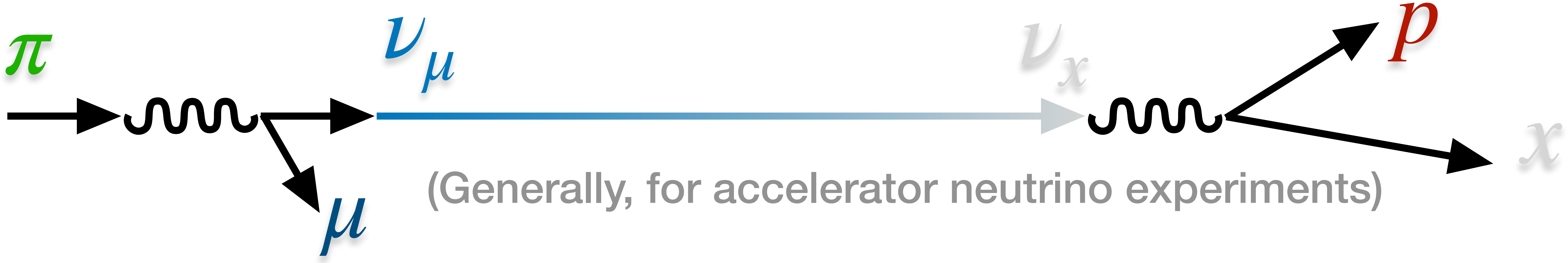


(Generally, for accelerator neutrino experiments)

Oscillation Probability

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_j U^*_{\beta j} U_{\alpha j} \exp\left(-1.27 i \frac{\Delta m^2_{j1} L}{E}\right) \right|^2$$

For 3-Flavour Oscillations, PMNS Mixing Matrix

$$U_{\alpha j} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

6 parameters

$\Delta m^2_{21}, \Delta m^2_{32}$, governs **oscillation frequency**

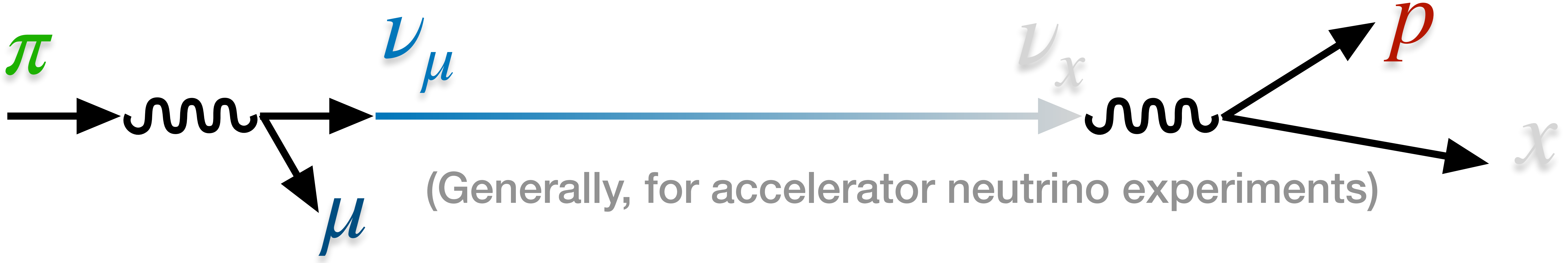
$\theta_{12}, \theta_{13}, \theta_{23}$, governs **oscillation magnitude**

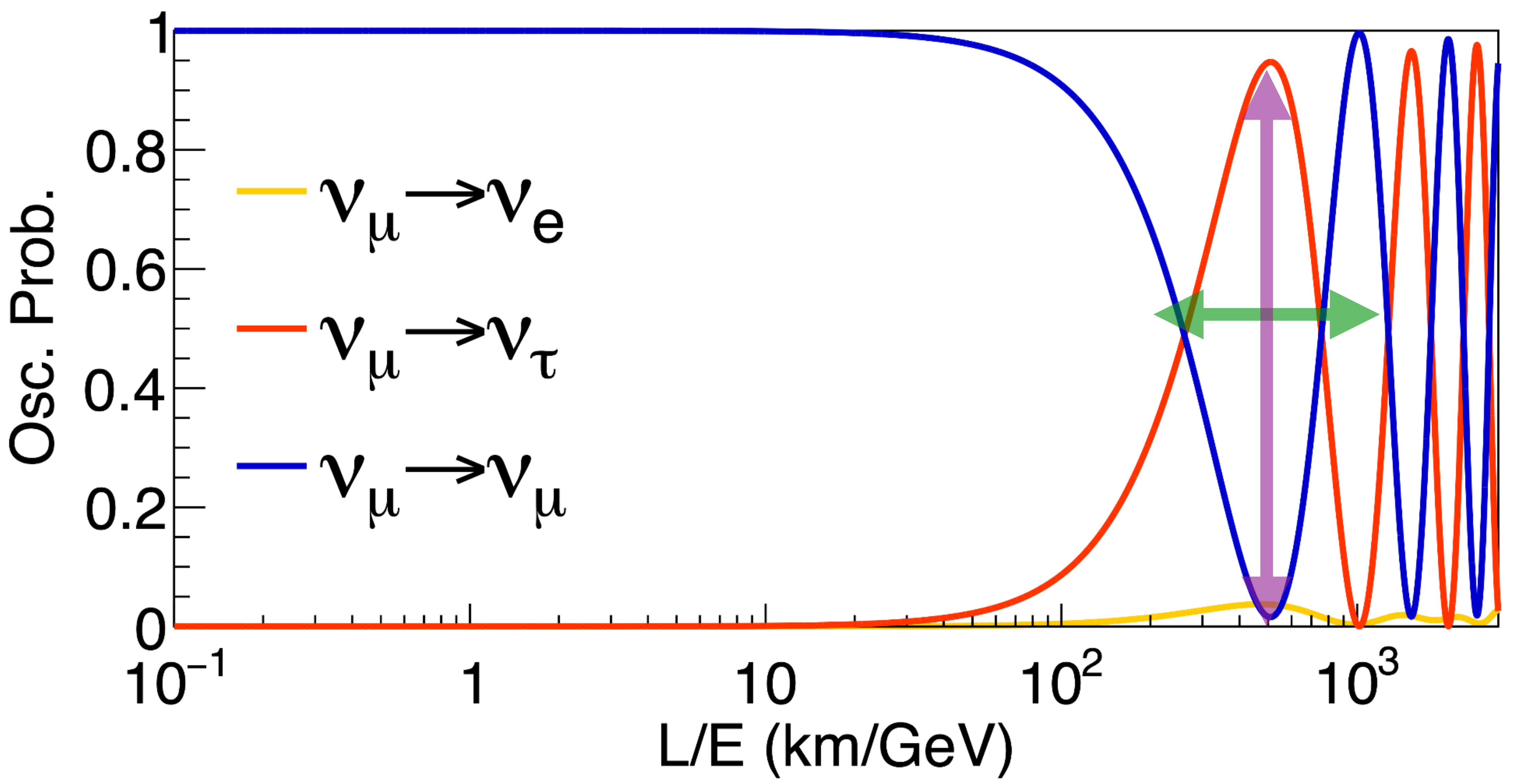
δ_{CP}, governs $\nu - \bar{\nu}$ **differences**

L (baseline), **E** (energy) are **experimental choices**

L/E is characteristic of oscillations

Neutrinos Oscillate





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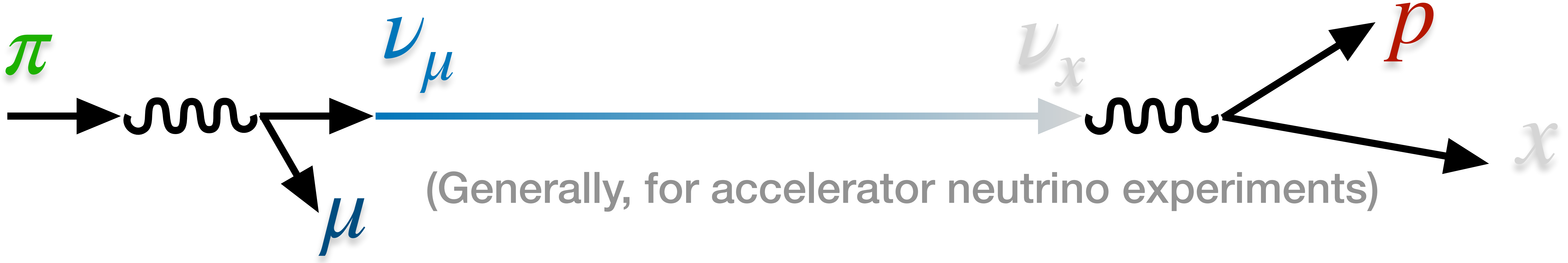
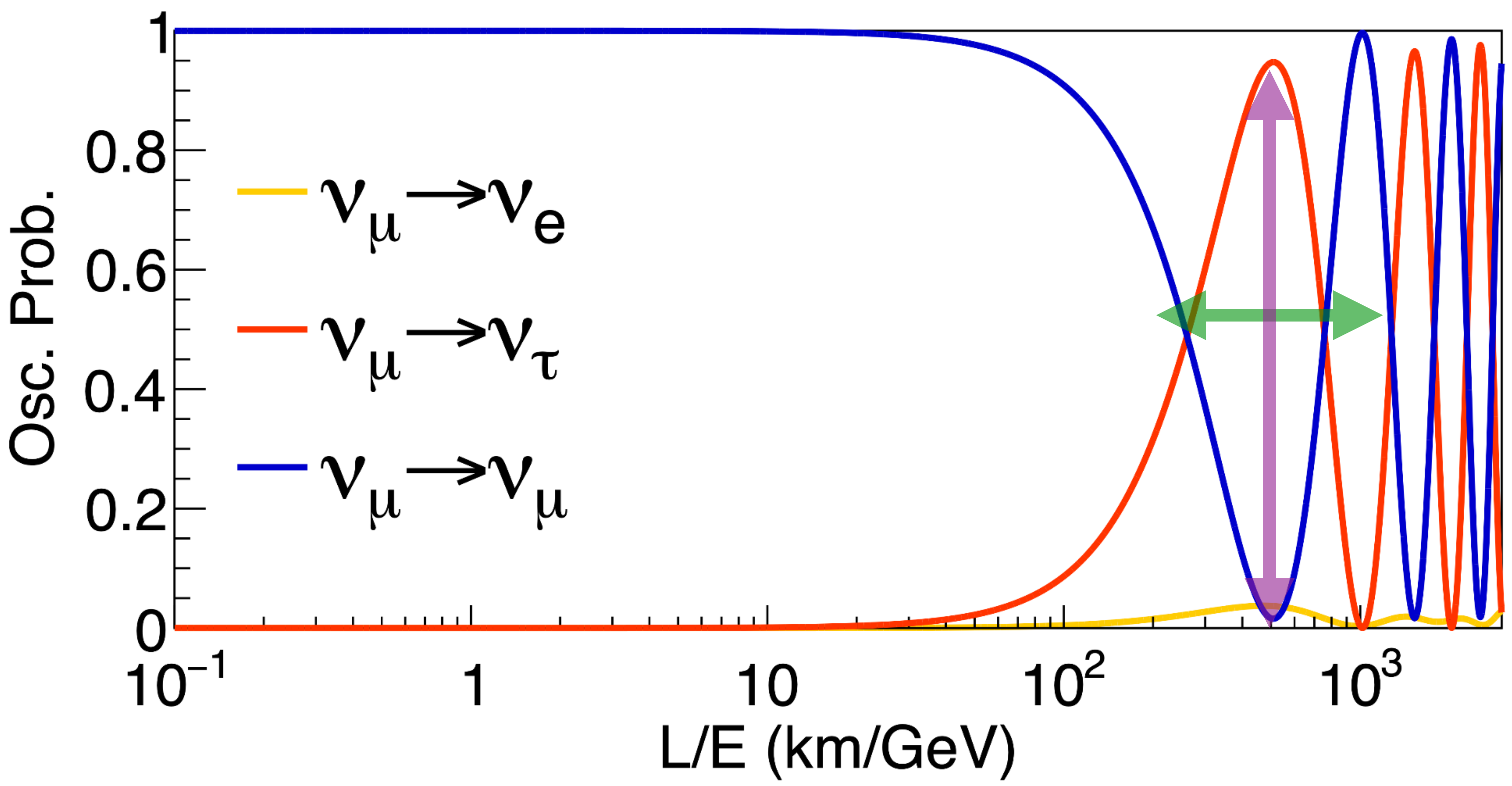
3 Flavour first oscillation maximum around L/E ~500

Δm^2_{32}, θ_{23}, θ_{13}, $\delta_{\rm CP}$ parameters important for these L/E values

Neutrinos Oscillate





Using ν_τ in accelerator neutrino experiments is difficult because of τ mass

3 Flavour first oscillation maximum around L/E ~500

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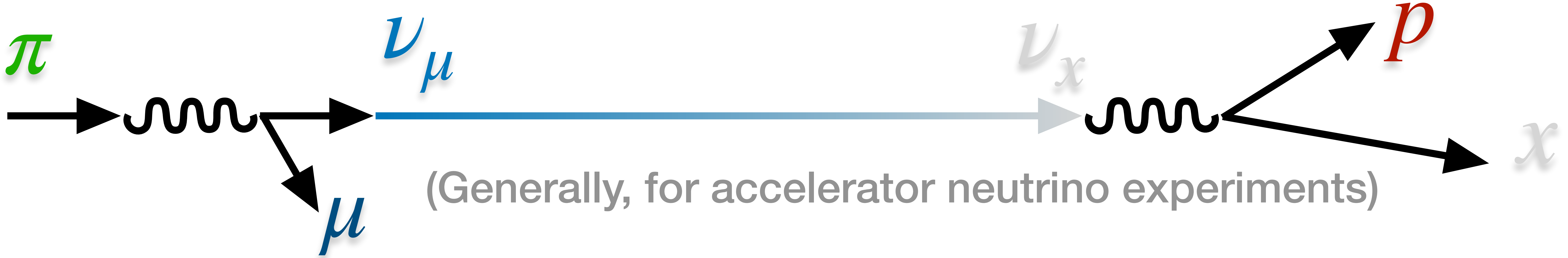
Neutrinos Oscillate



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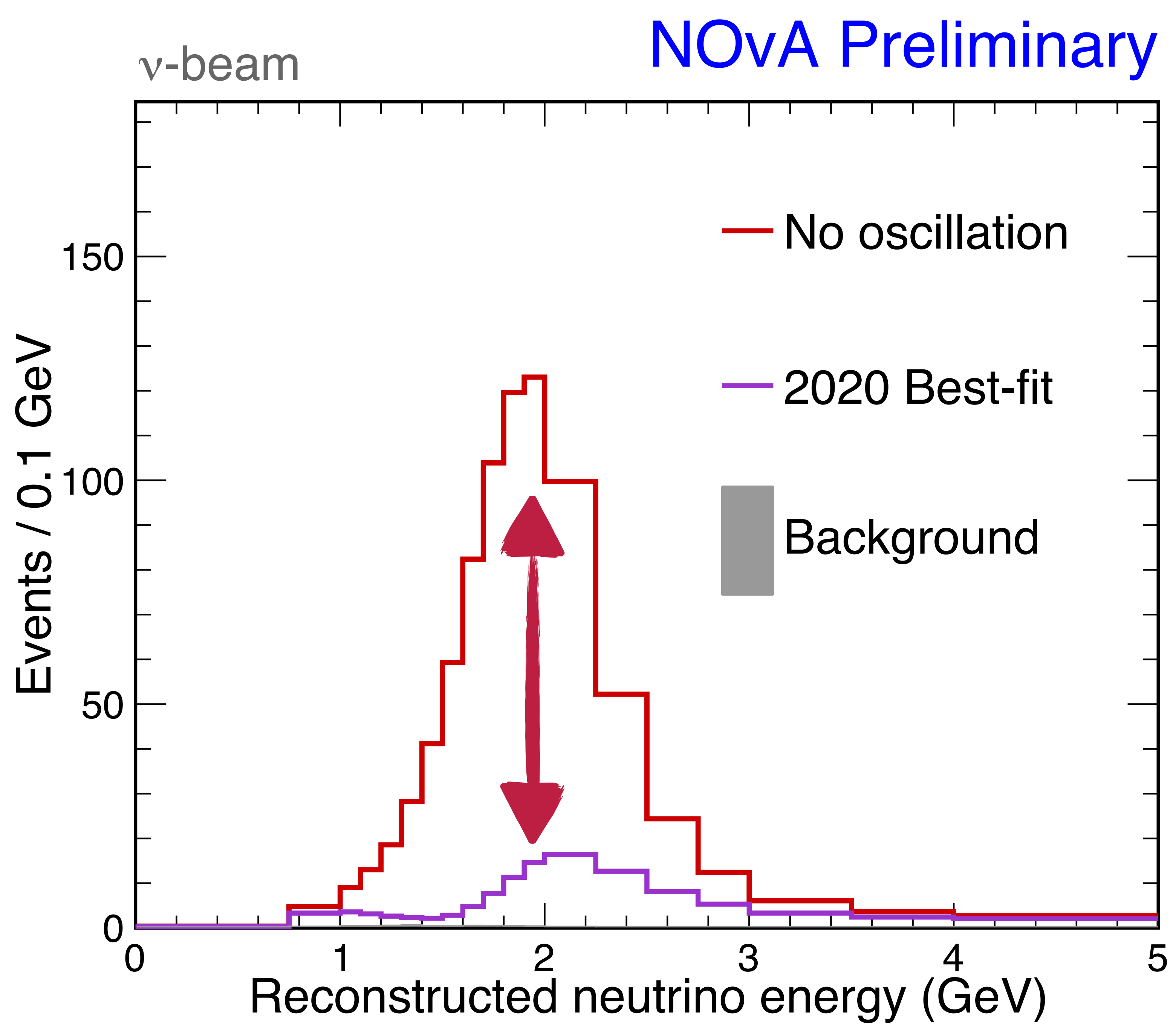
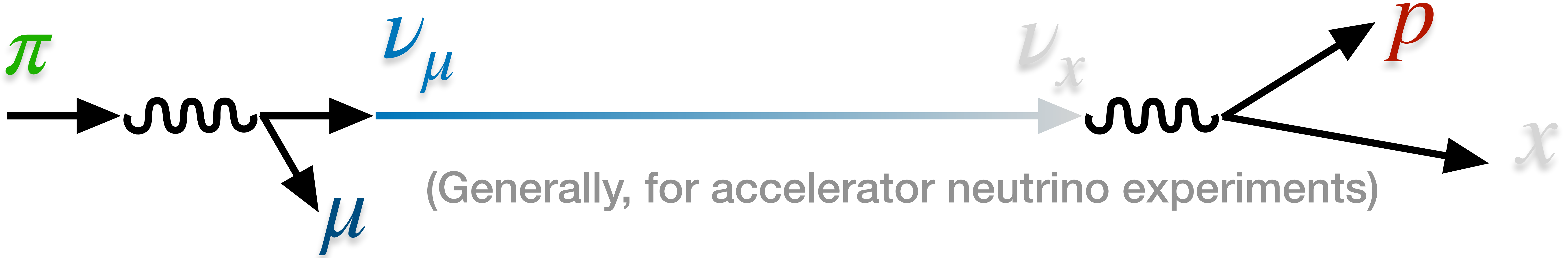


ν_μ Disappearance

deficit of ν_μ interactions
with respect to **prediction**

Neutrinos Oscillate



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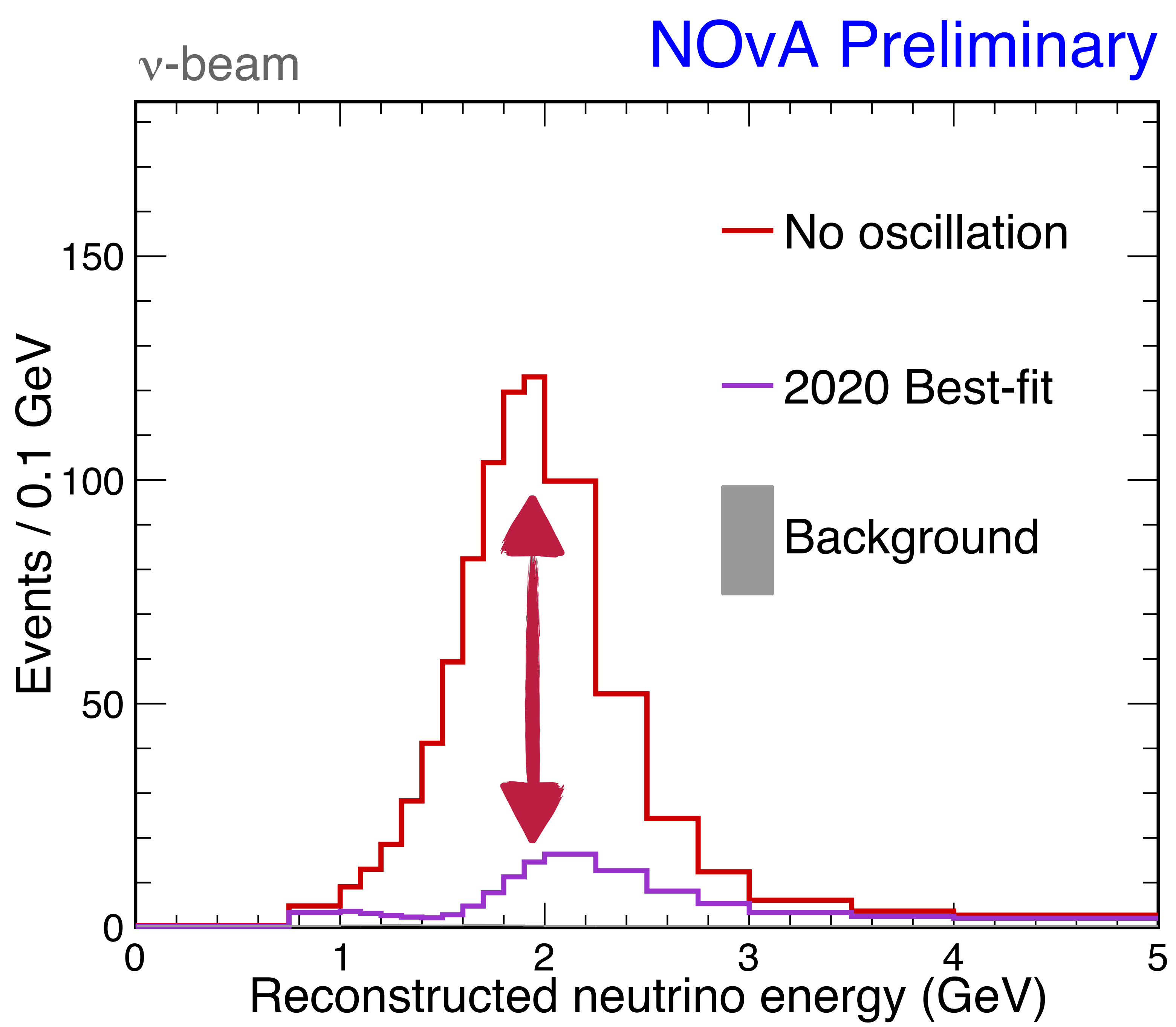
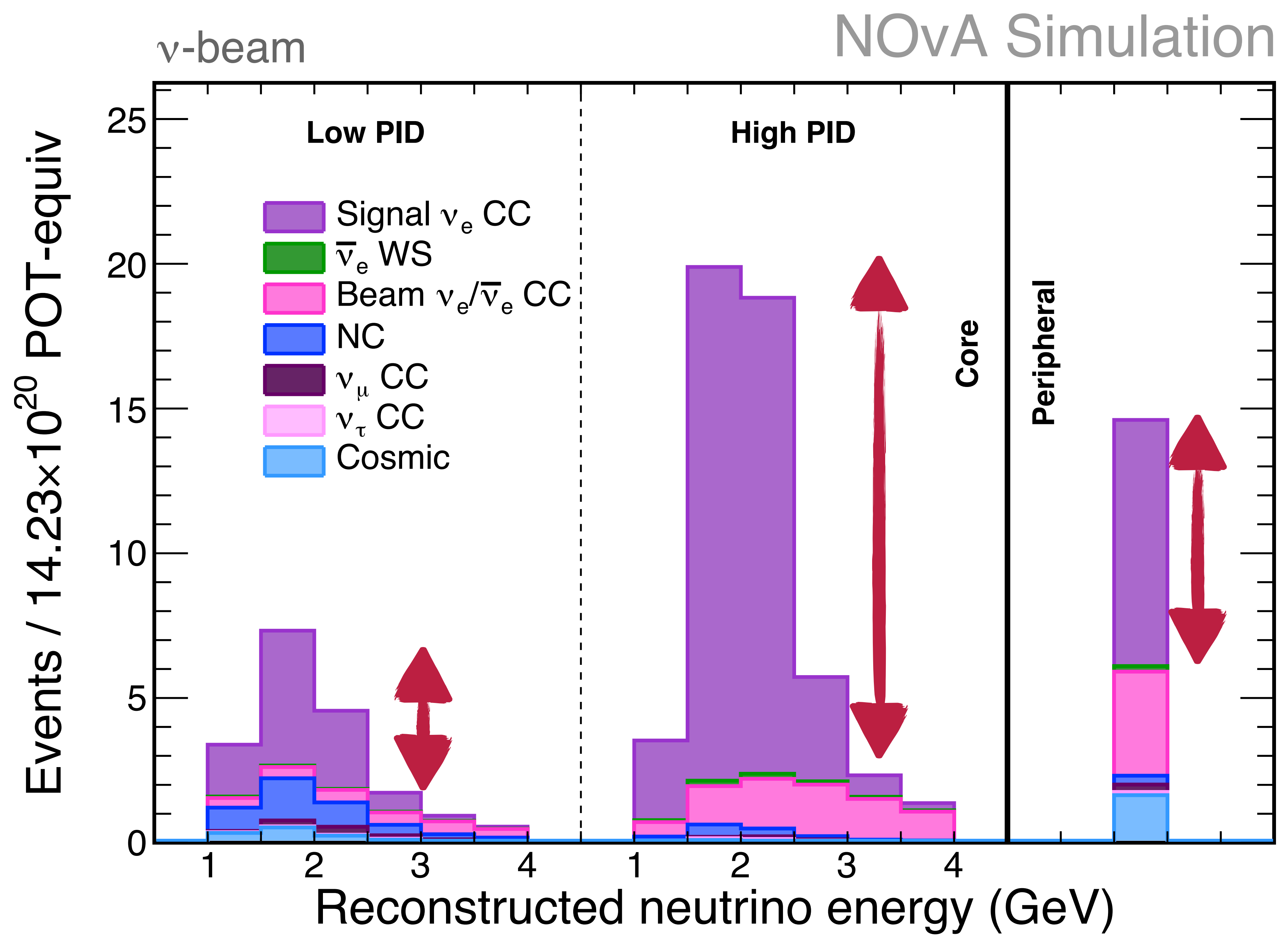


ν_μ **Disappearance**

deficit of ν_μ interactions with respect to **prediction**

ν_e **Appearance**

Excess of ν_e **interactions** with respect to **prediction**



Open Questions

Is three-flavour the full picture?

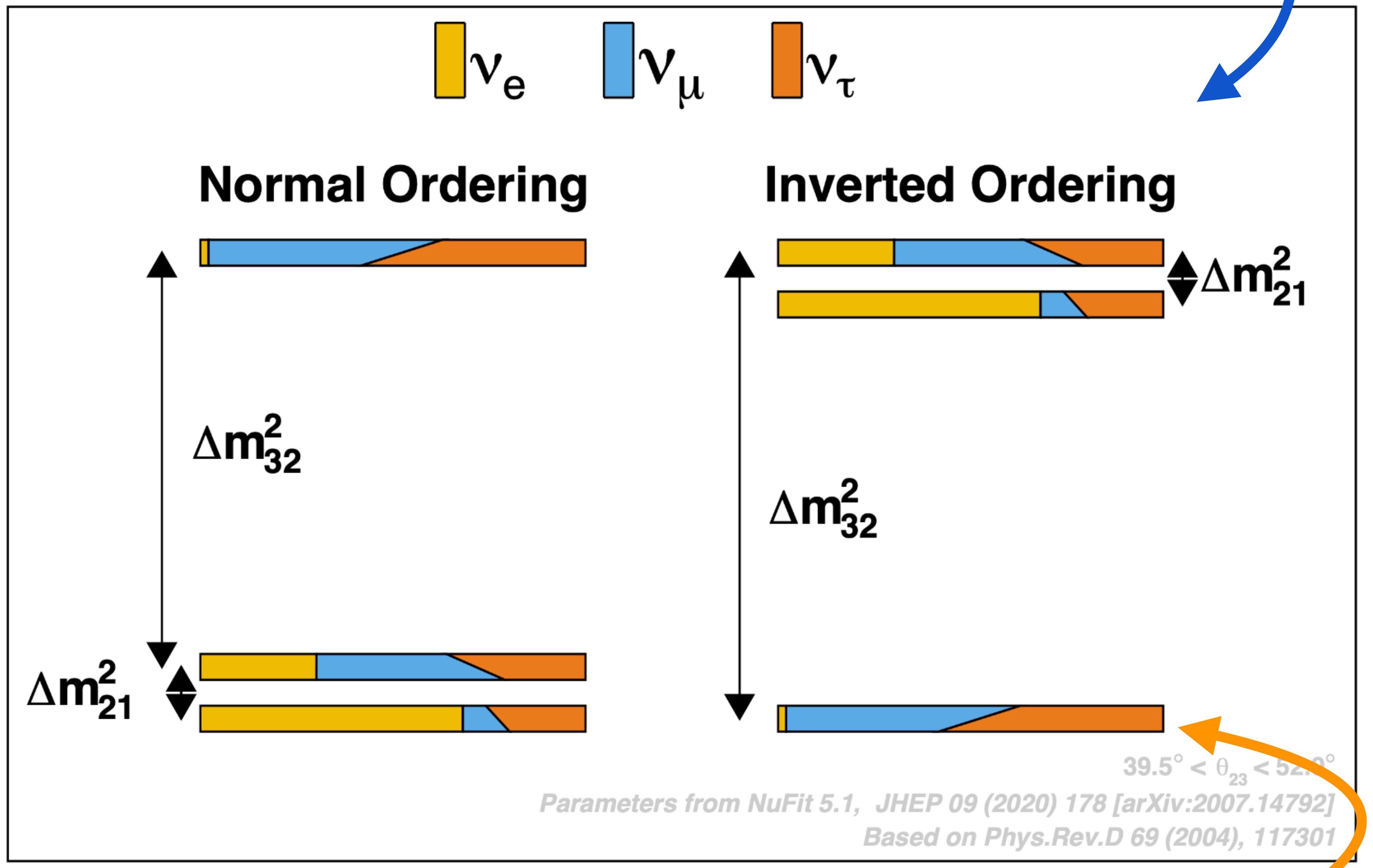
Additional neutrino states?

Non-standard interactions?

What is the neutrino mass ordering?

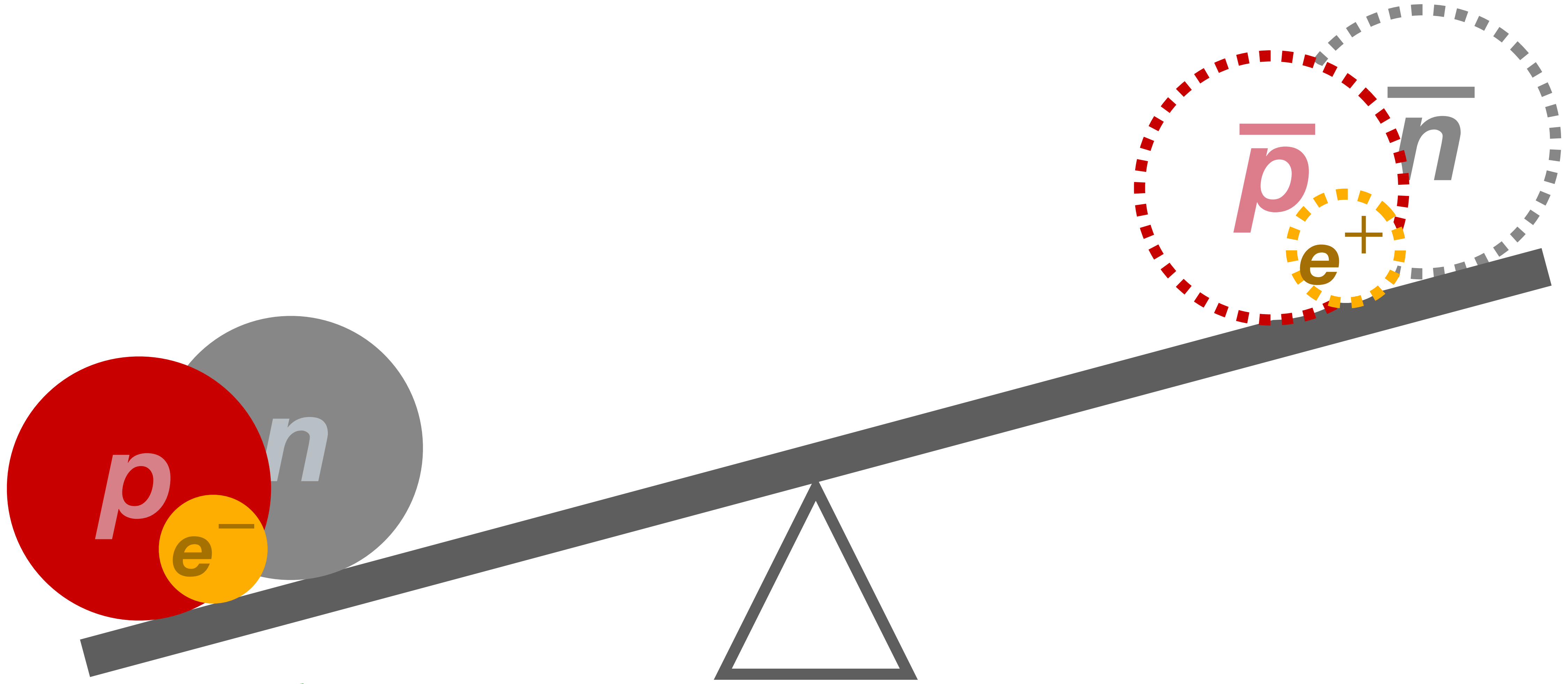
Normal or inverted?

Implications for $0\nu\beta\beta$, cosmology



What is the octant of θ_{23}?

For ν_3, does $\nu_\mu = \nu_\tau$?



Is CP violated?

Non-conservation of CP important for matter-antimatter asymmetry

Open Questions

What is the neutrino mass ordering?
Normal or inverted?
Implications for $0\nu\beta\beta$, cosmology

Is three-flavour the full picture?
Additional neutrino states?
Non-standard interactions?

NOvA's Sterile Neutrino Searches

NOvA's oscillation analyses can probe these open questions!

NOvA's 3 Flavour Analyses

What is the octant of θ_{23}?
For ν_3, does $\nu_\mu = \nu_\tau$?

Is CP violated?
Non-conservation of CP important for matter-antimatter asymmetry

Open Questions

See Luiz Prais' excellent **slides from BSM 2023** for details on NSI

What is the neutrino mass ordering?

Normal or inverted?

Implications for $0\nu\beta\beta$, cosmology

Is three-flavour the full picture?

Additional neutrino states?

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NOvA's 3 Flavour Analyses

What is the octant of θ_{23}?

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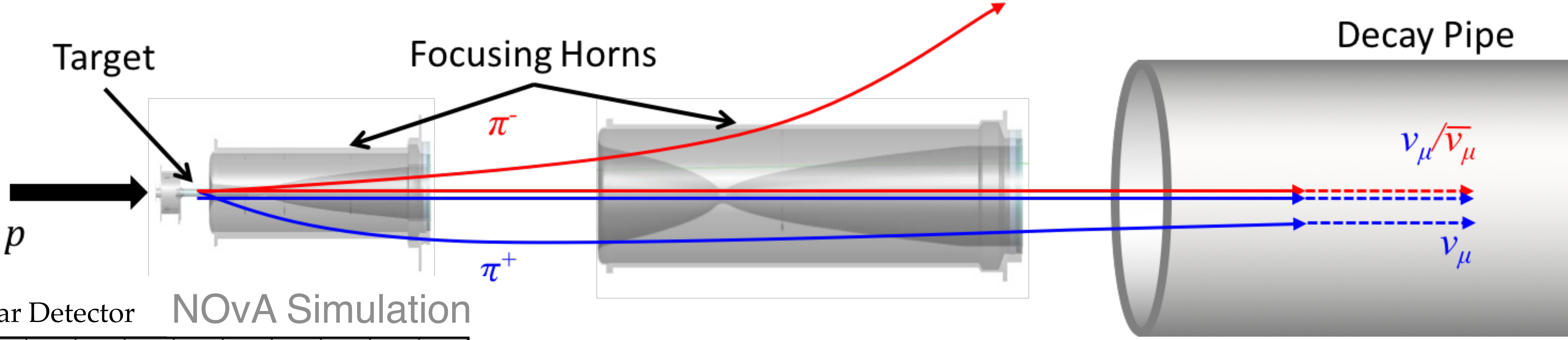
Is CP violated?

Non-conservation of CP important for matter-antimatter asymmetry

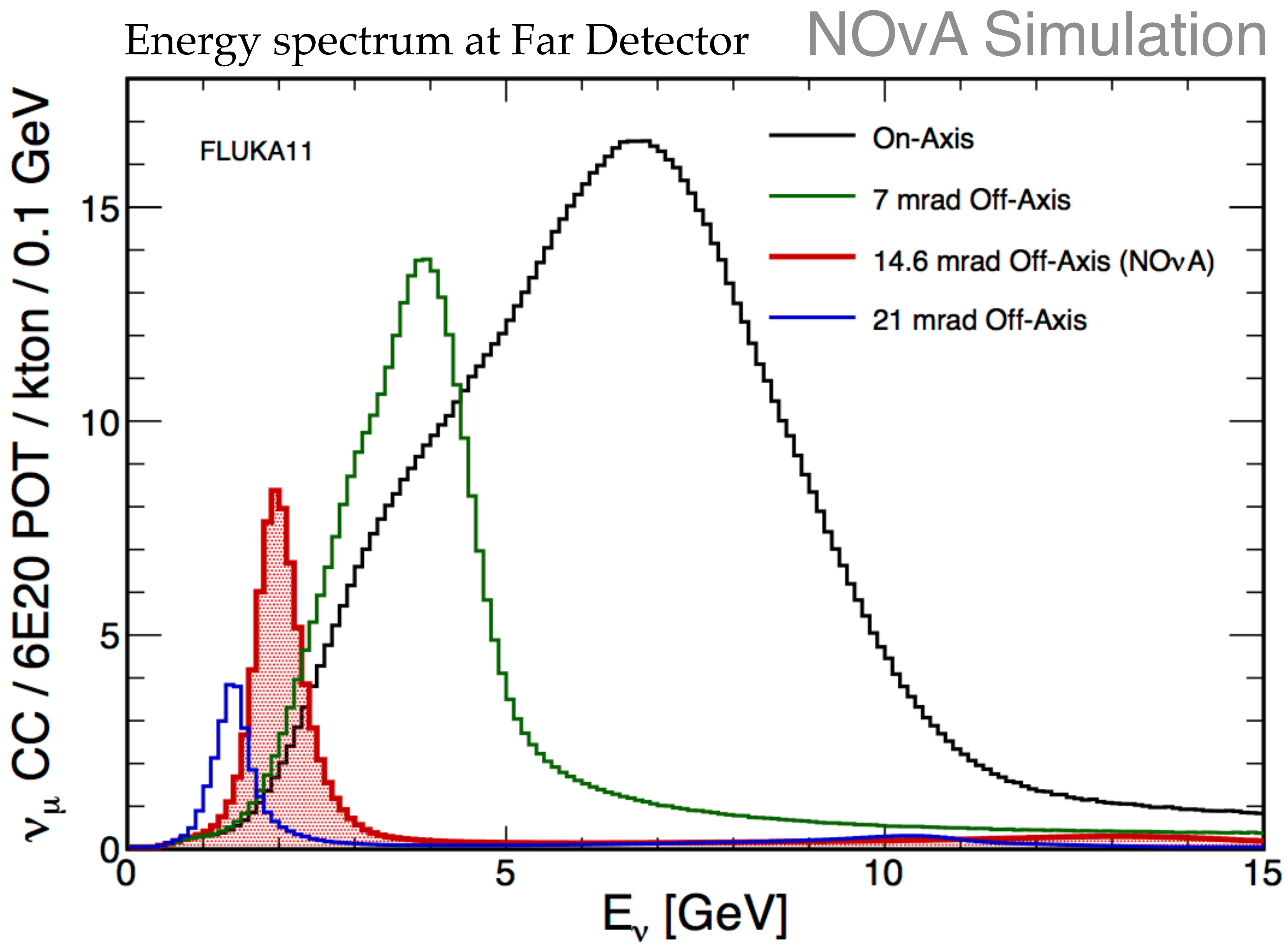
The NuMI Beam

Two focussing horns focus **chosen-sign mesons**, and defocus **opposite-sign mesons**

120 GeV Protons from FNAL's Main Injector strike target producing secondary particles



Energy spectrum at Far Detector



NOvA detectors are placed **14 mrad off-axis of the NuMI beamline**

The off-axis technique reduces the total neutrino flux, **but results in tightly peaked distribution of energies around 2 GeV**

The NOvA Experiment

We fire neutrinos from the NuMI beam through the earth towards Minnesota

Near Detector

~ 1 km from beam source underground @ Fermilab, IL

Provides a measurement of the unoscillated flux

Far Detector

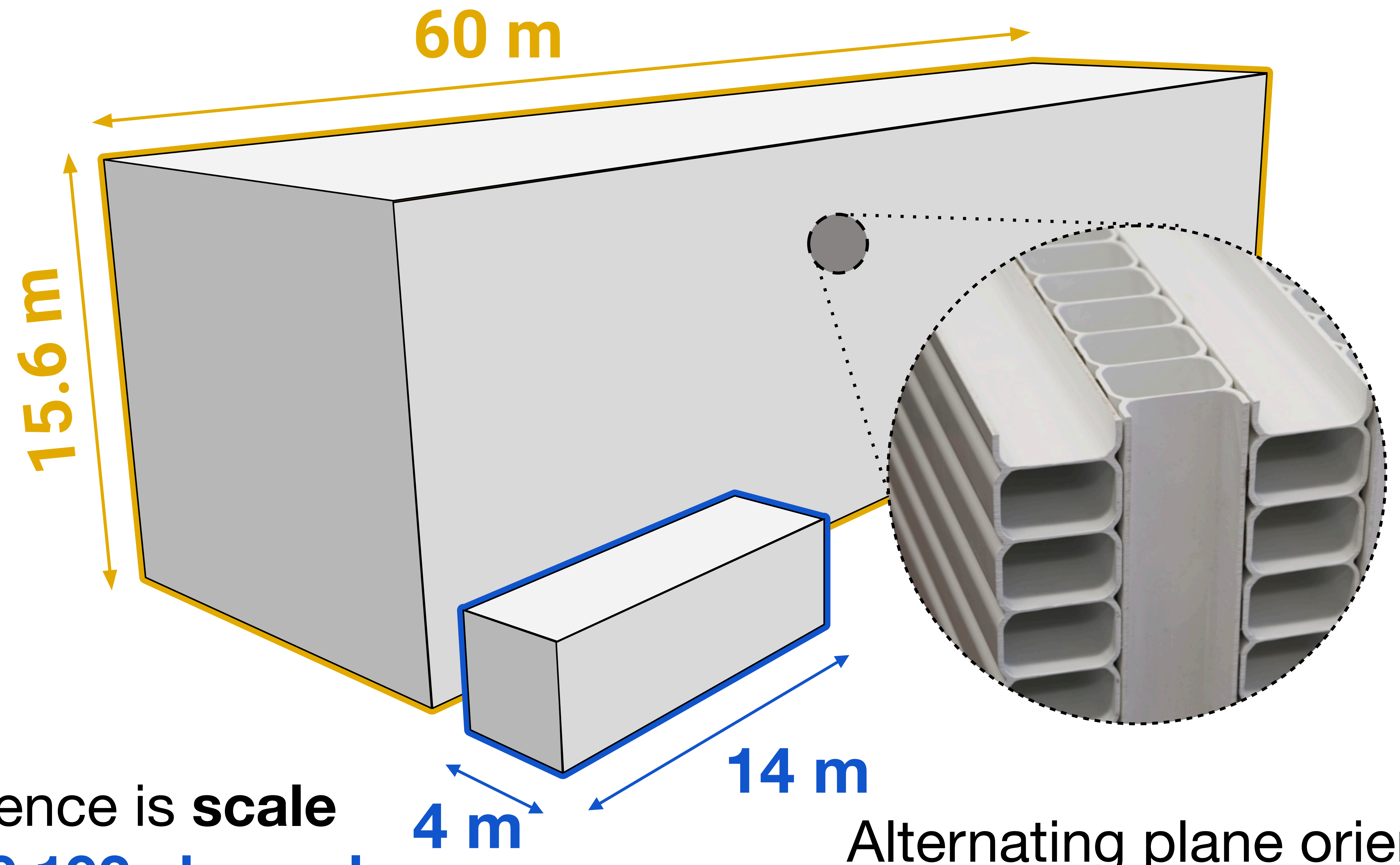
~ 810 km from beam source on surface @ Ash River, MN

Placed near the oscillation maximum

L/E at the Far Detector is around ~400,
Excellent for 3 Flavour oscillations

The NOvA Detectors

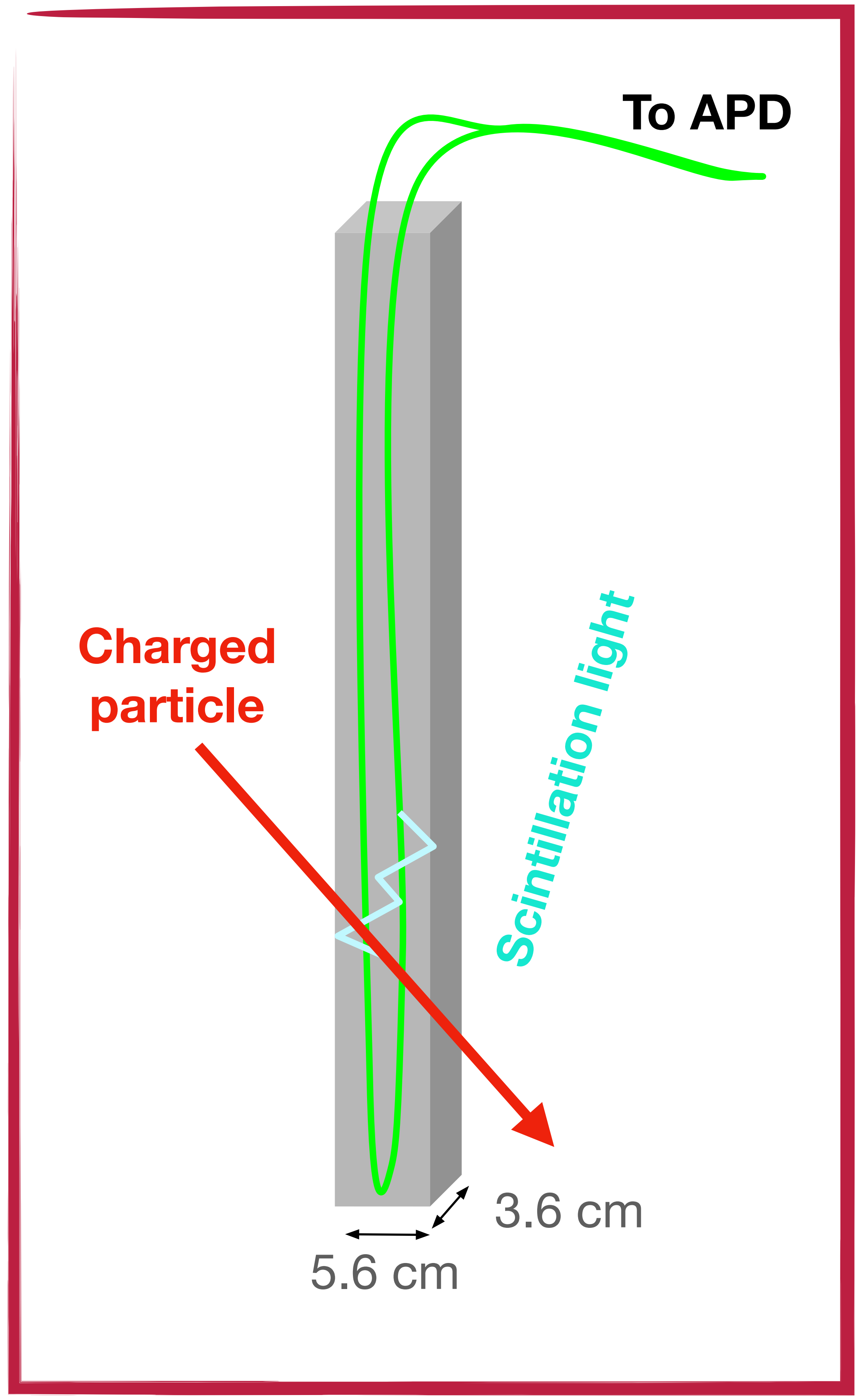
Functionally identical Near and Far detectors
* Segmented tracking calorimeters
* Extruded PVC cells filled with liquid scintillator



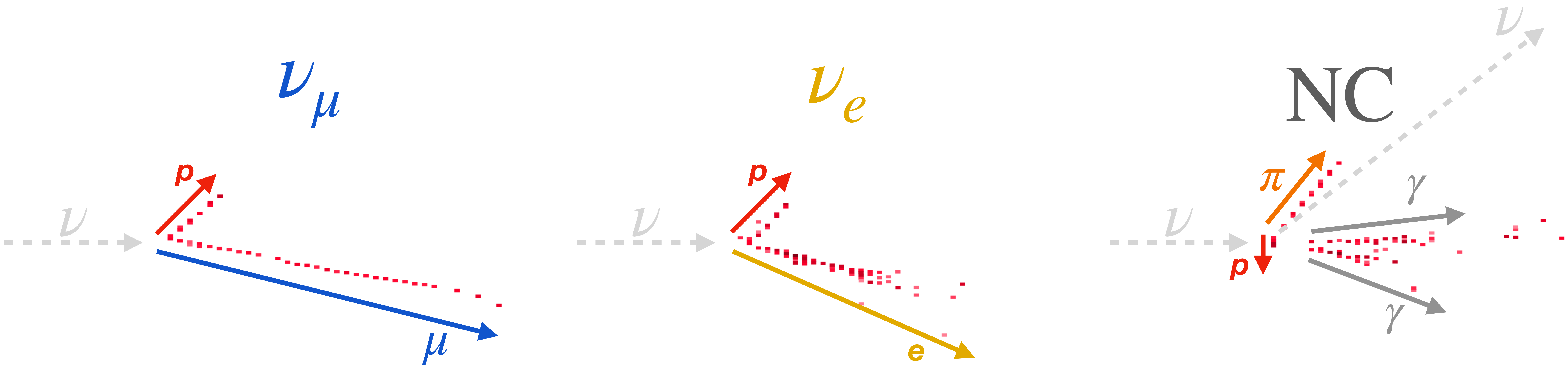
Primary difference is **scale**
ND - 0.3 kT - 20,192 channels
FD - 14 kT - 344,064 channels

Alternating plane orientation
→ **two views per event**

Operating Principle

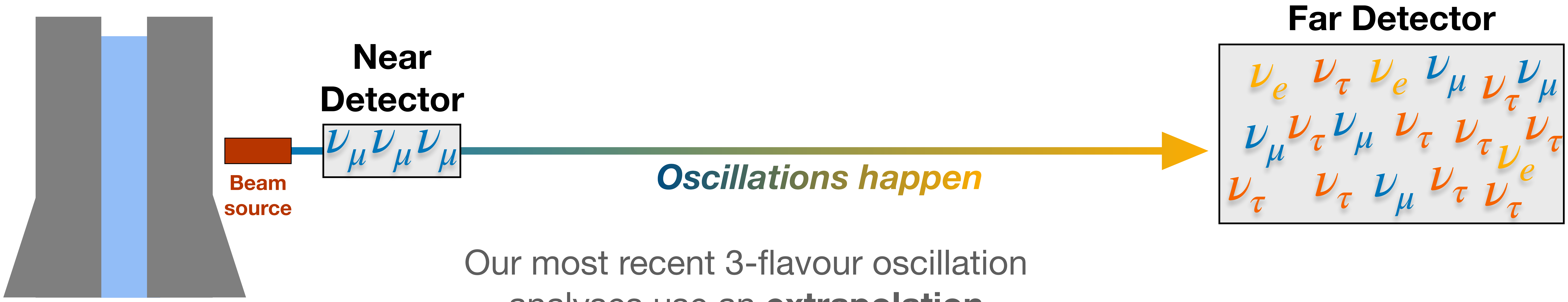


~6 samples per radiation length (~40 cm)
making **electron**-**muon** disambiguation easy



Large distance for π^0 **to photon** conversion helps
disambiguate **electron neutrinos** from **NC interactions**

Three-Flavour Oscillations With NOvA



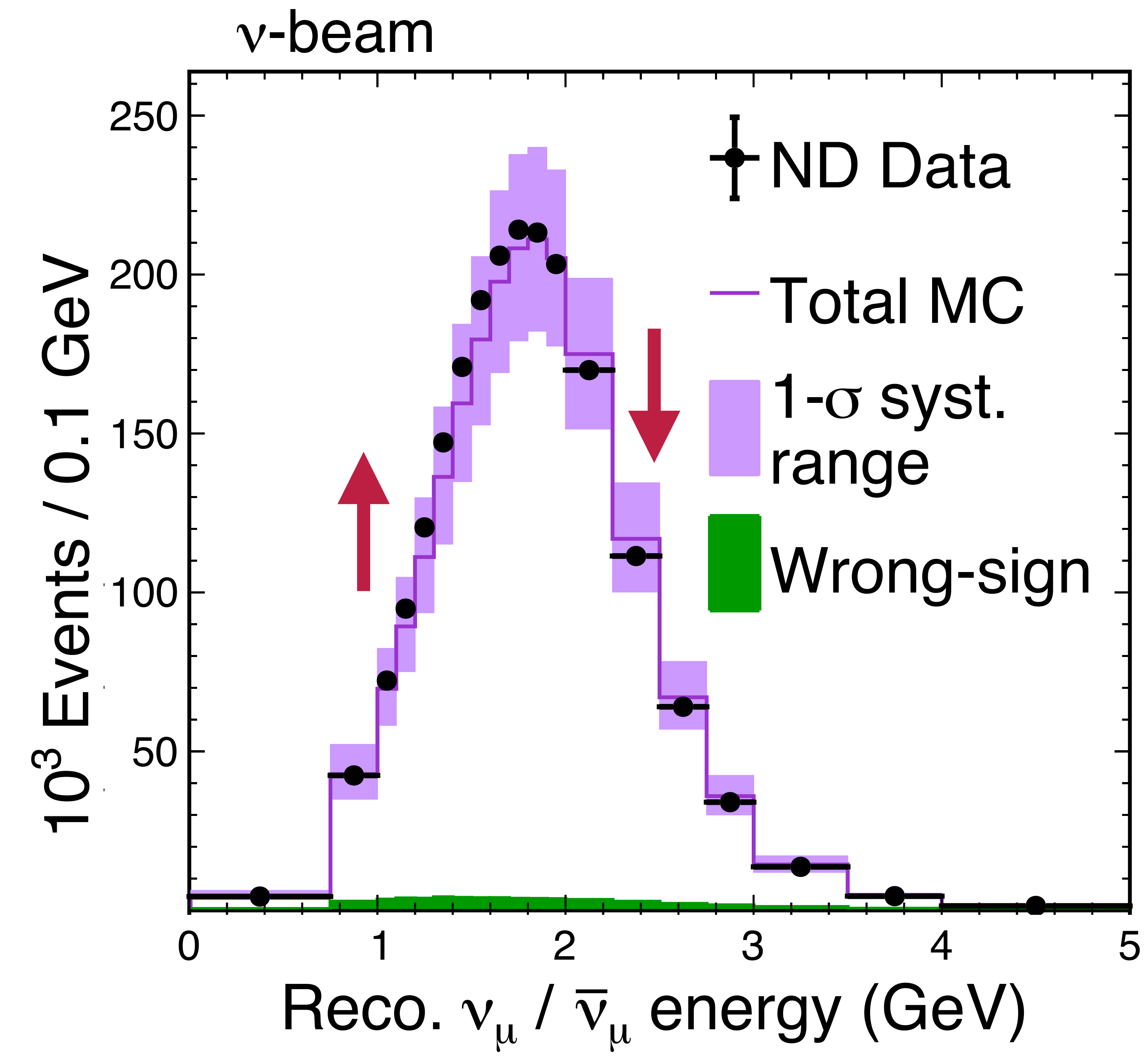
Our most recent 3-flavour oscillation analyses use an **extrapolation technique**

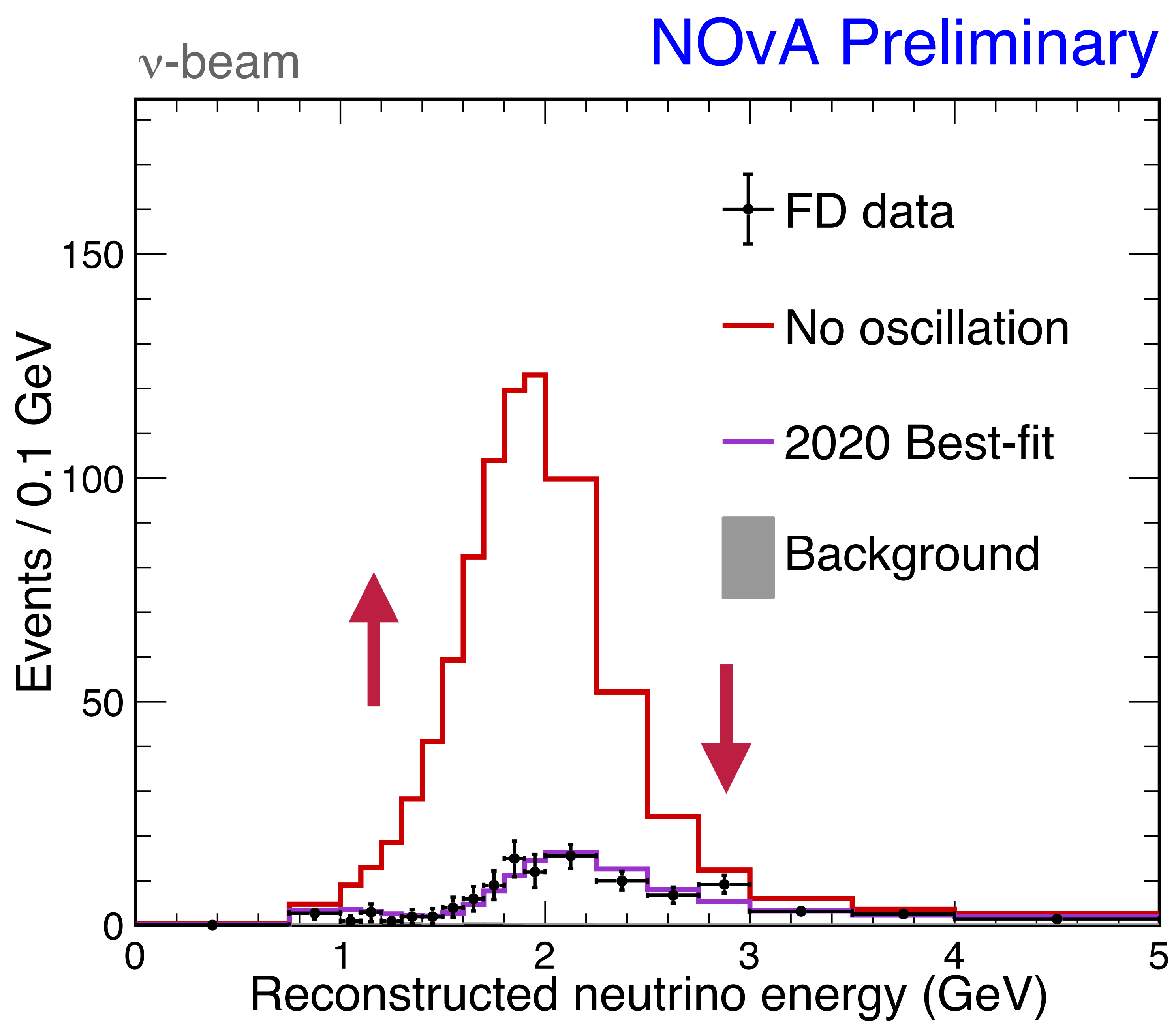
Data/simulation disagreements in the ND are used to predict the unoscillated **FD spectrum**

This analysis uses 2014-2020 data

$$13.6 \times 10^{20} \text{ POT } \nu$$

$$12.5 \times 10^{20} \text{ POT } \bar{\nu}$$

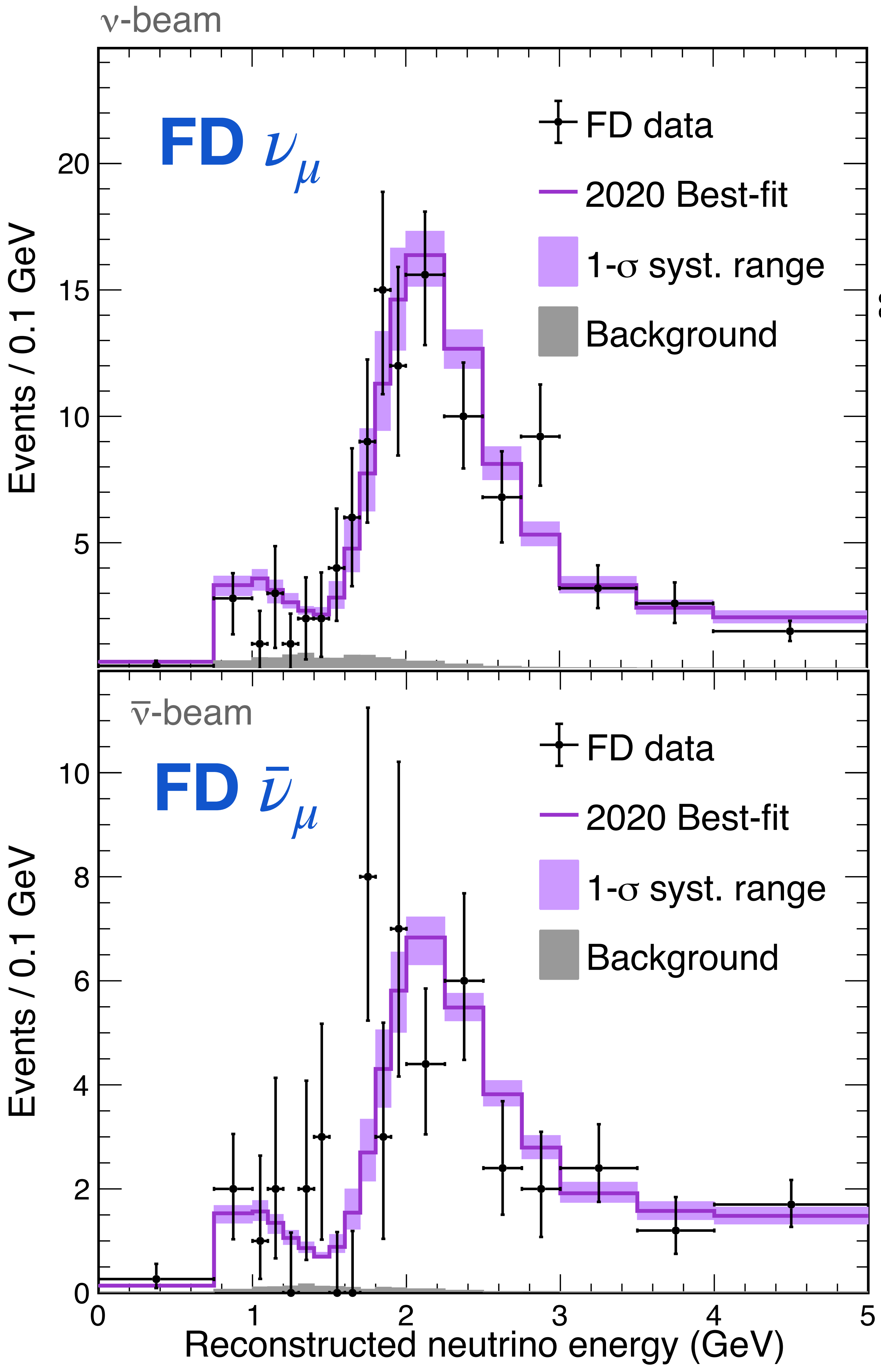


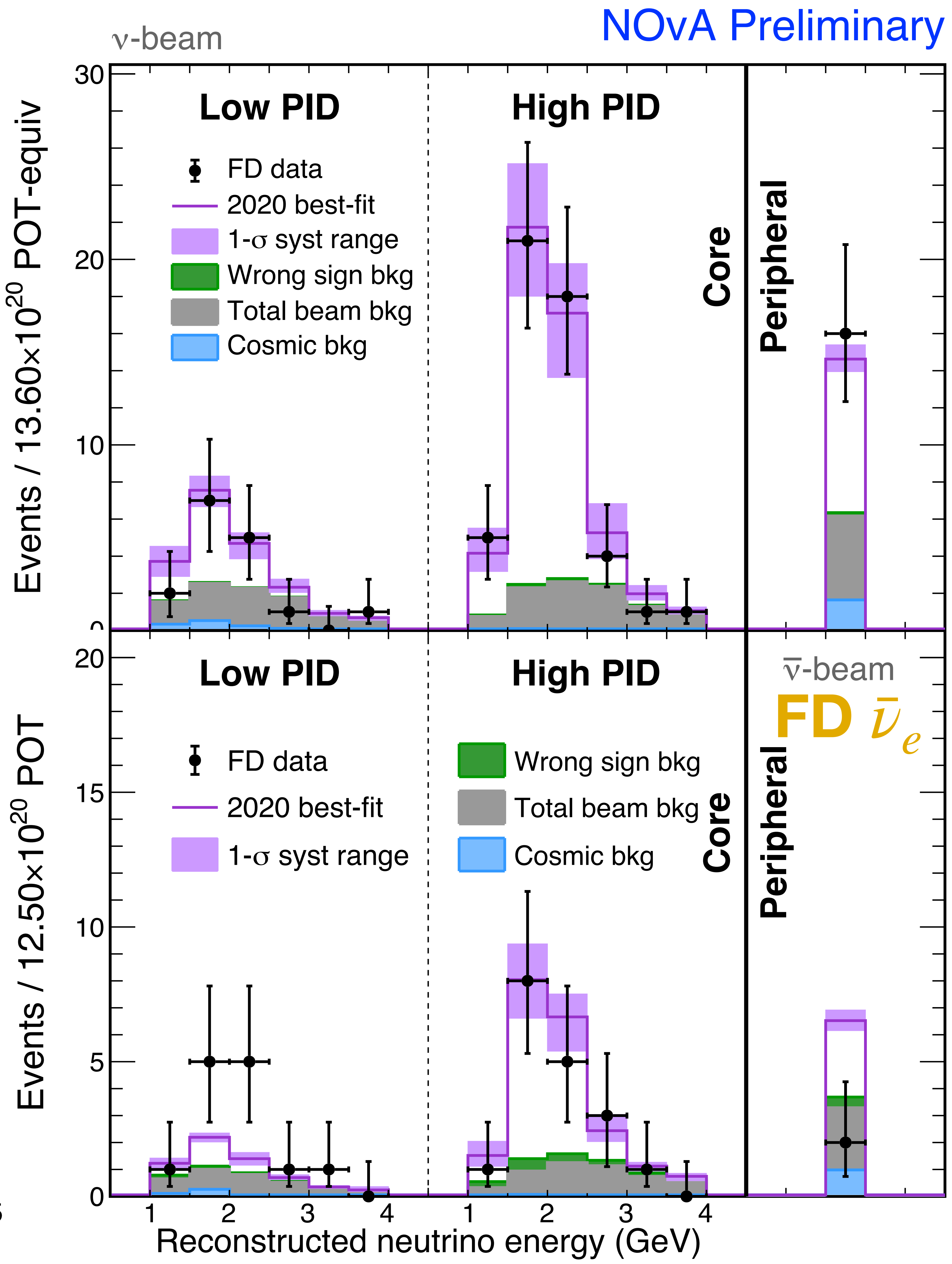


FD Spectrum

Very high ν_μ purity selection in both running modes

	Obs.	BF	Bg.
ν_μ	211	222.3	8.2
$\bar{\nu}_\mu$	105	105.4	2.1





Primary background in ν_e appearance samples **are intrinsic ν_e**

	Obs.	BF	Bg.
ν_e	82	85.8	26.8
$\bar{\nu}_e$	33	33.2	14.0

Two Statistical Treatments

Frequentist Analysis

[Phys.Rev.D 106 (2022) 3, 032004]

Confidence Levels from χ^2 surface

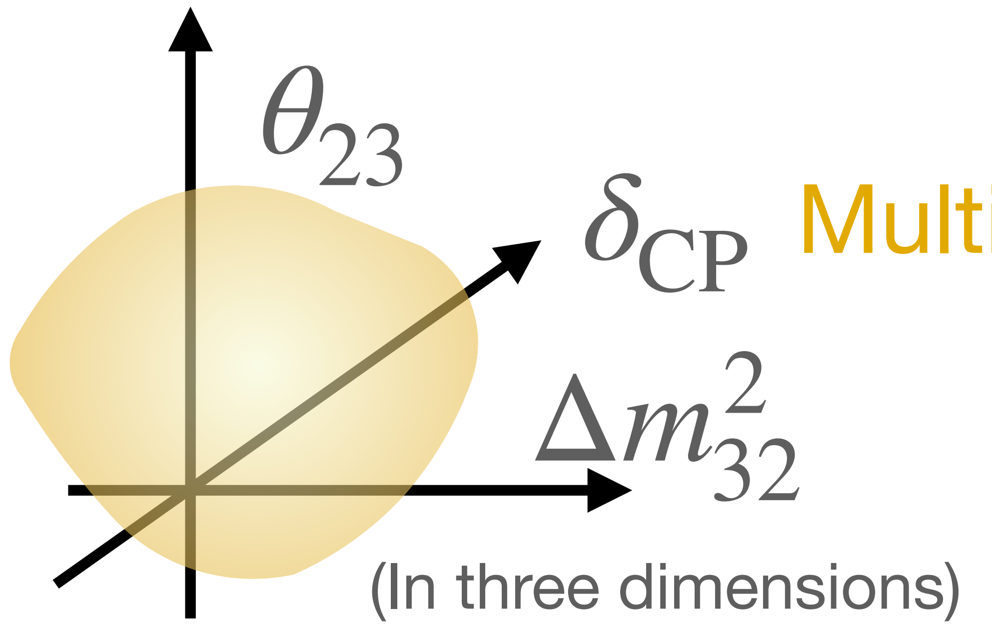
Requires Feldman-Cousins correcting χ^2 surface

Profiles over hidden parameters and choose those that maximise χ^2

Bayesian Analysis

arXiv 2311.07835

Credible Intervals from posterior probability densities



Multi-dimensional probability distribution

Allows quick production of credible intervals!

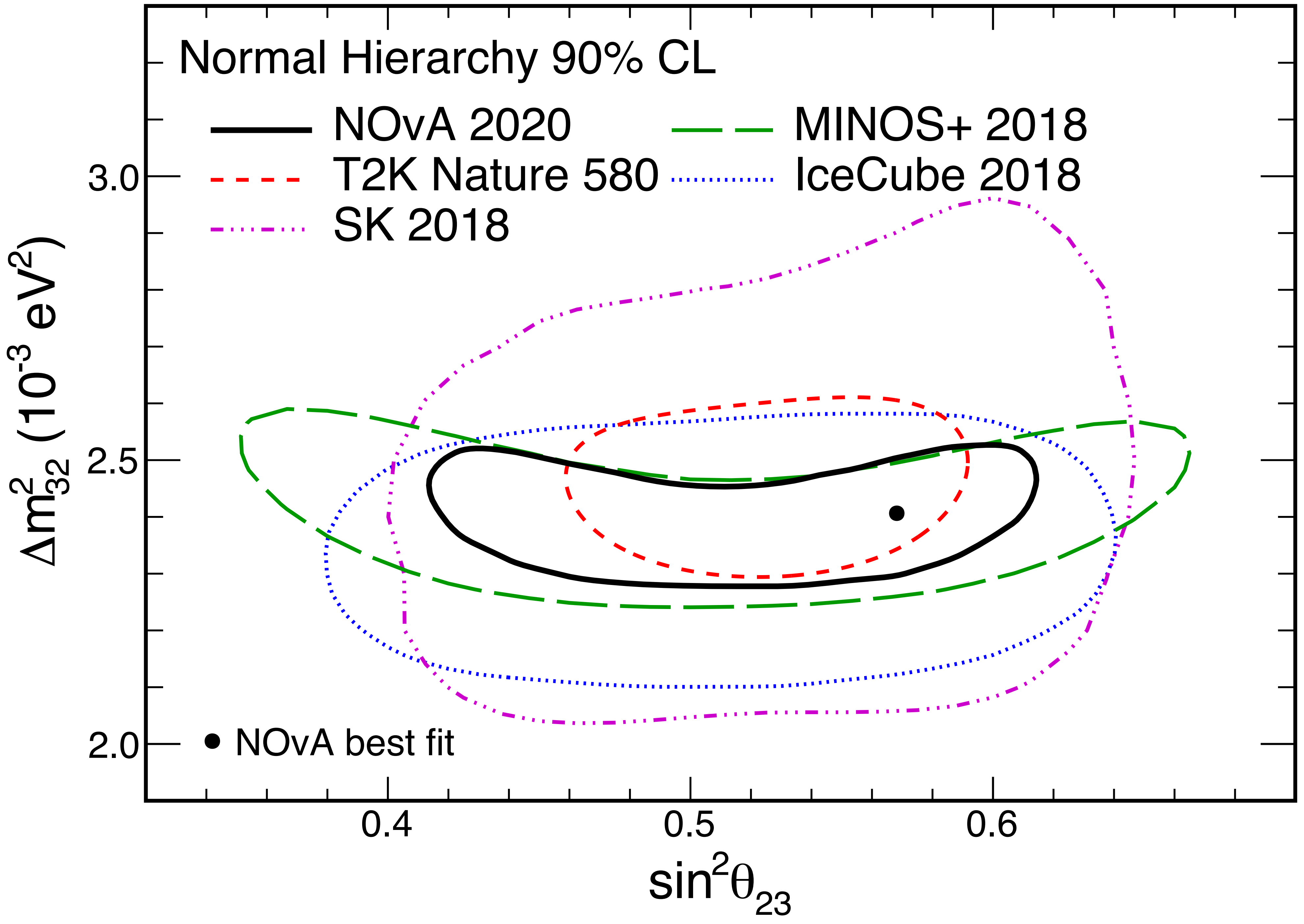
Marginalises over hidden parameters, getting the average contribution to the probability density

Consistent results between the two treatments

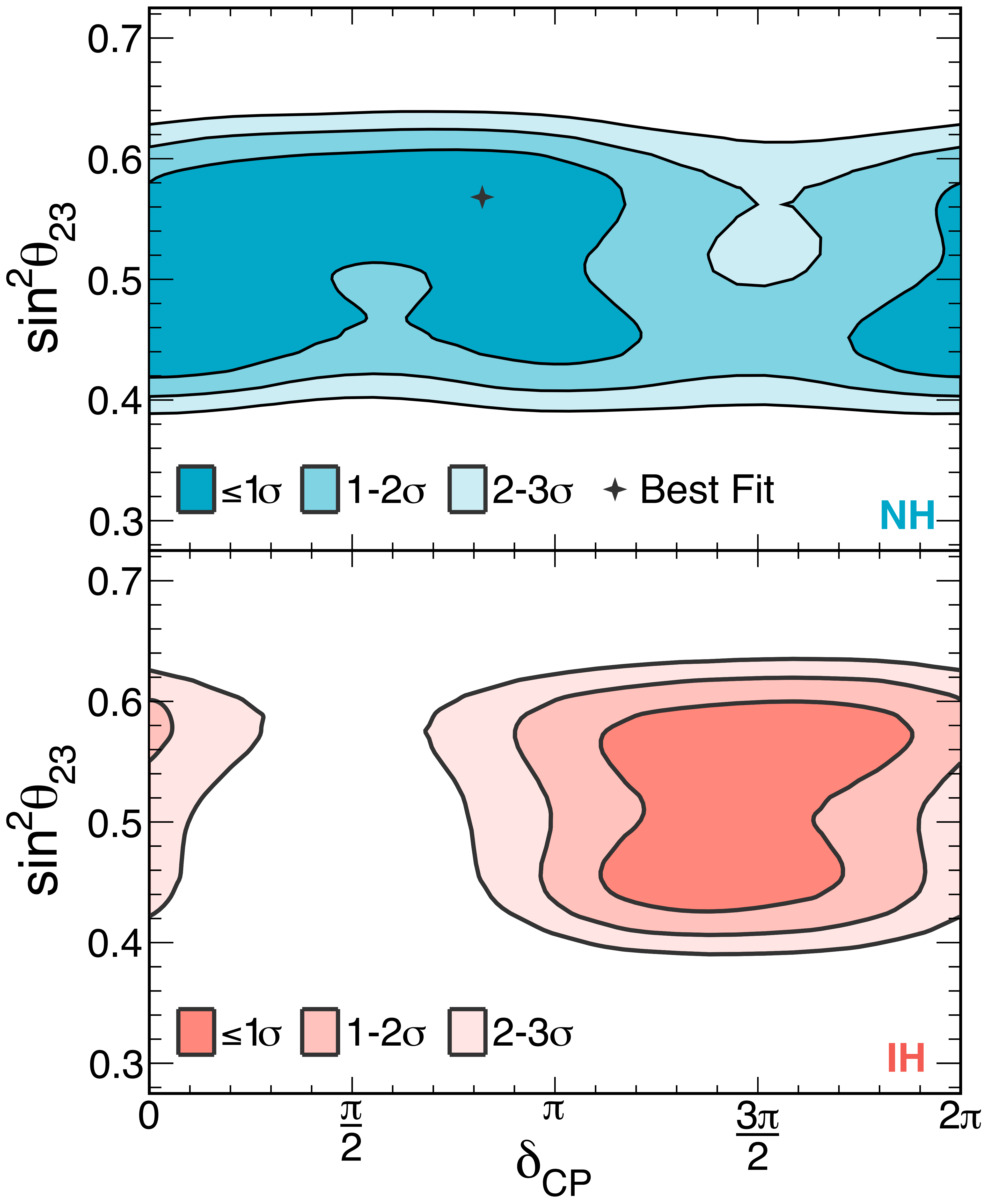
Contours

Phys.Rev.D 106 (2022) 3, 032004

(From 2020 paper, other experiments may be out of date!)



Phys.Rev.D 106 (2022) 3, 032004



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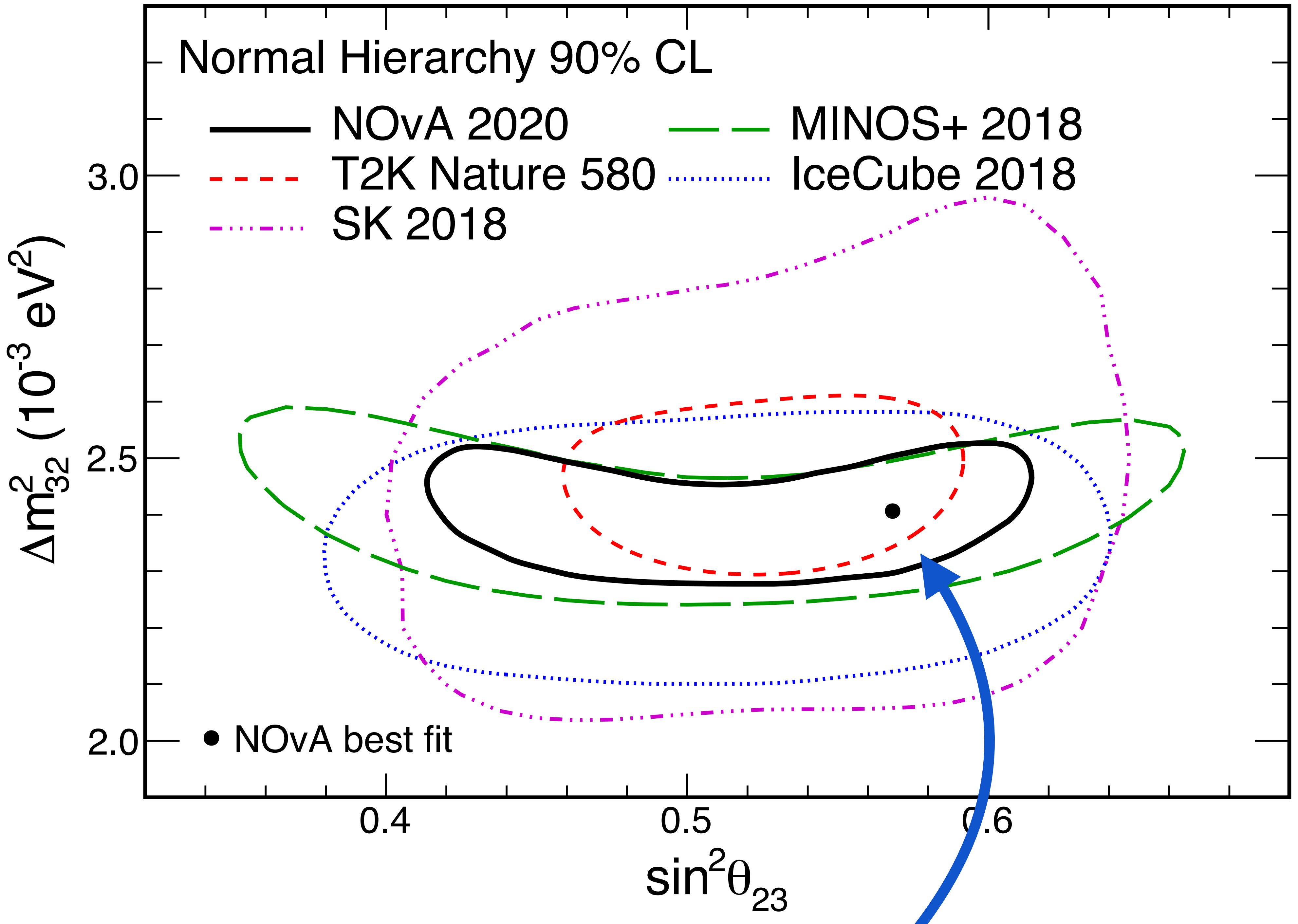
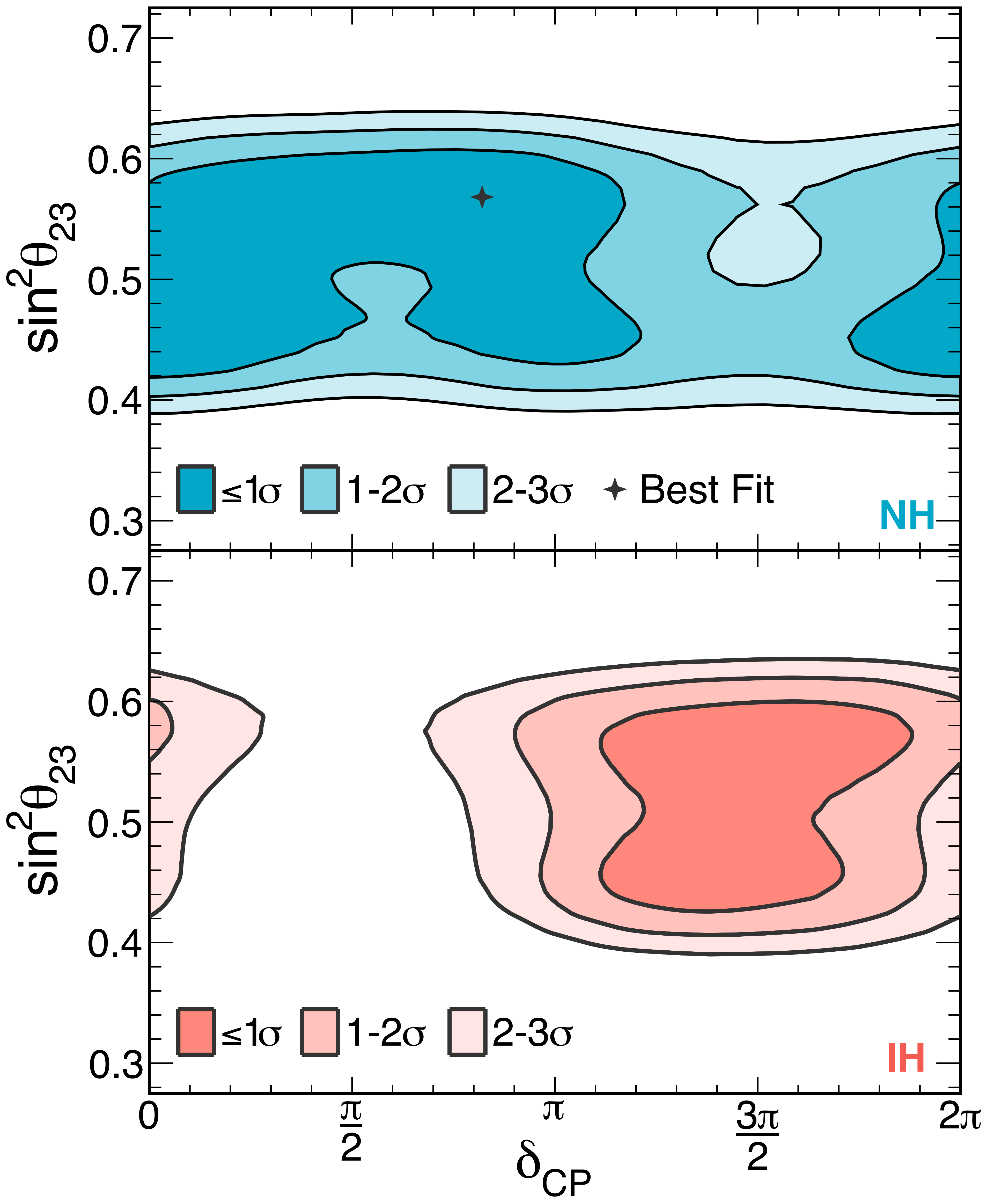


Weak preference for **upper octant**

Phys.Rev.D 106 (2022) 3, 032004



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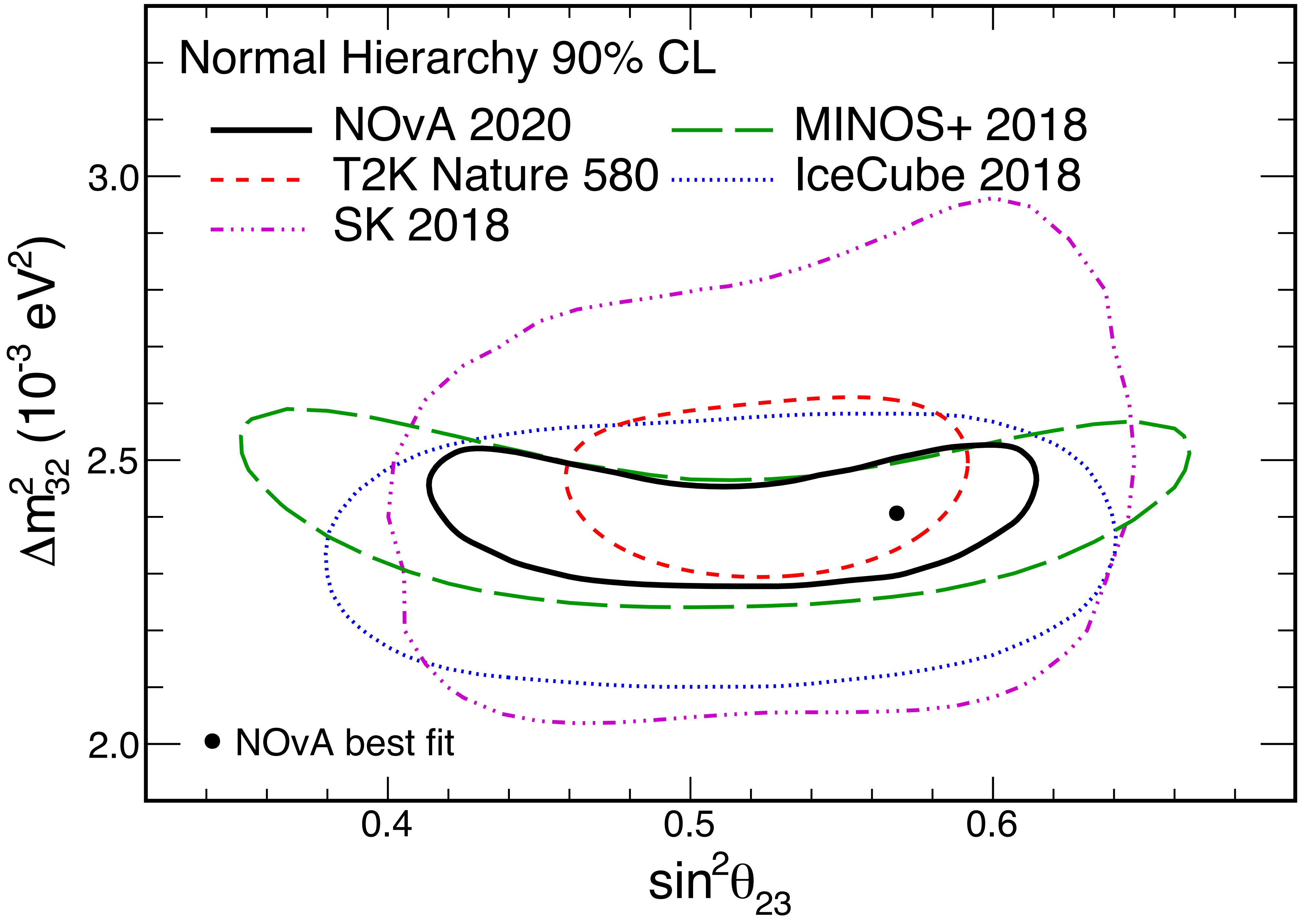
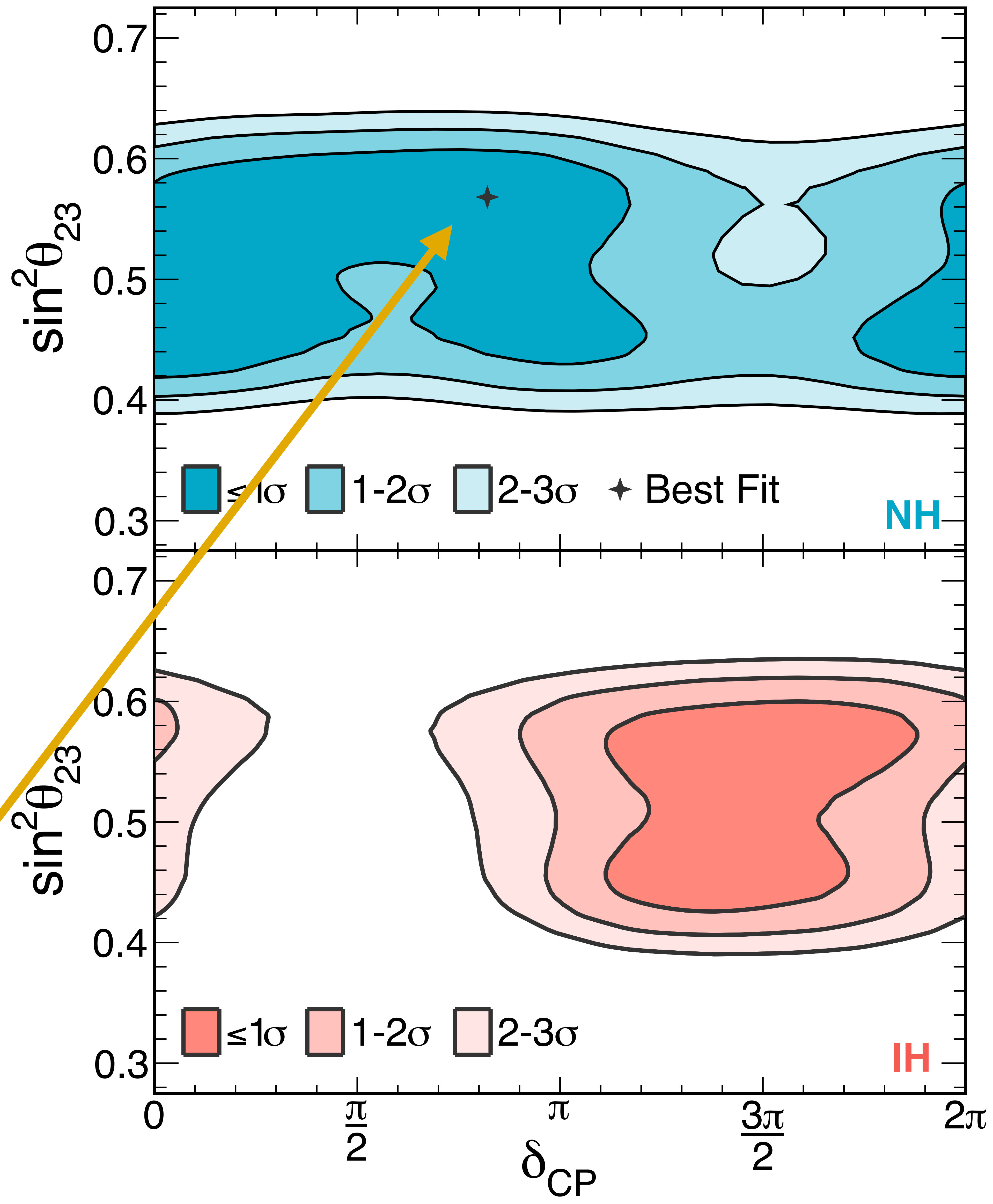


Weak preference for **upper octant**

Weak preference for **Normal Ordering**

Phys.Rev.D 106 (2022) 3, 032004



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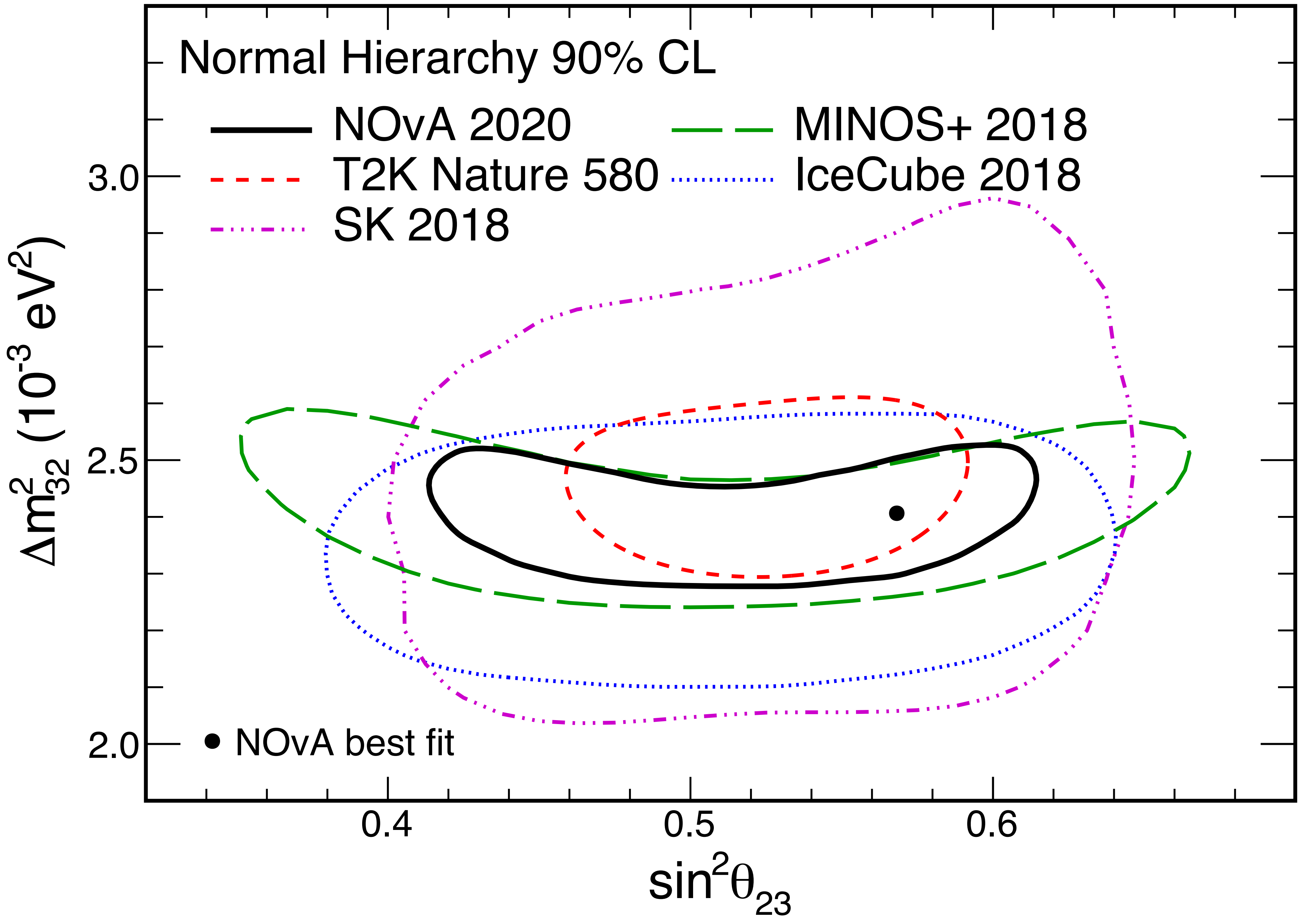
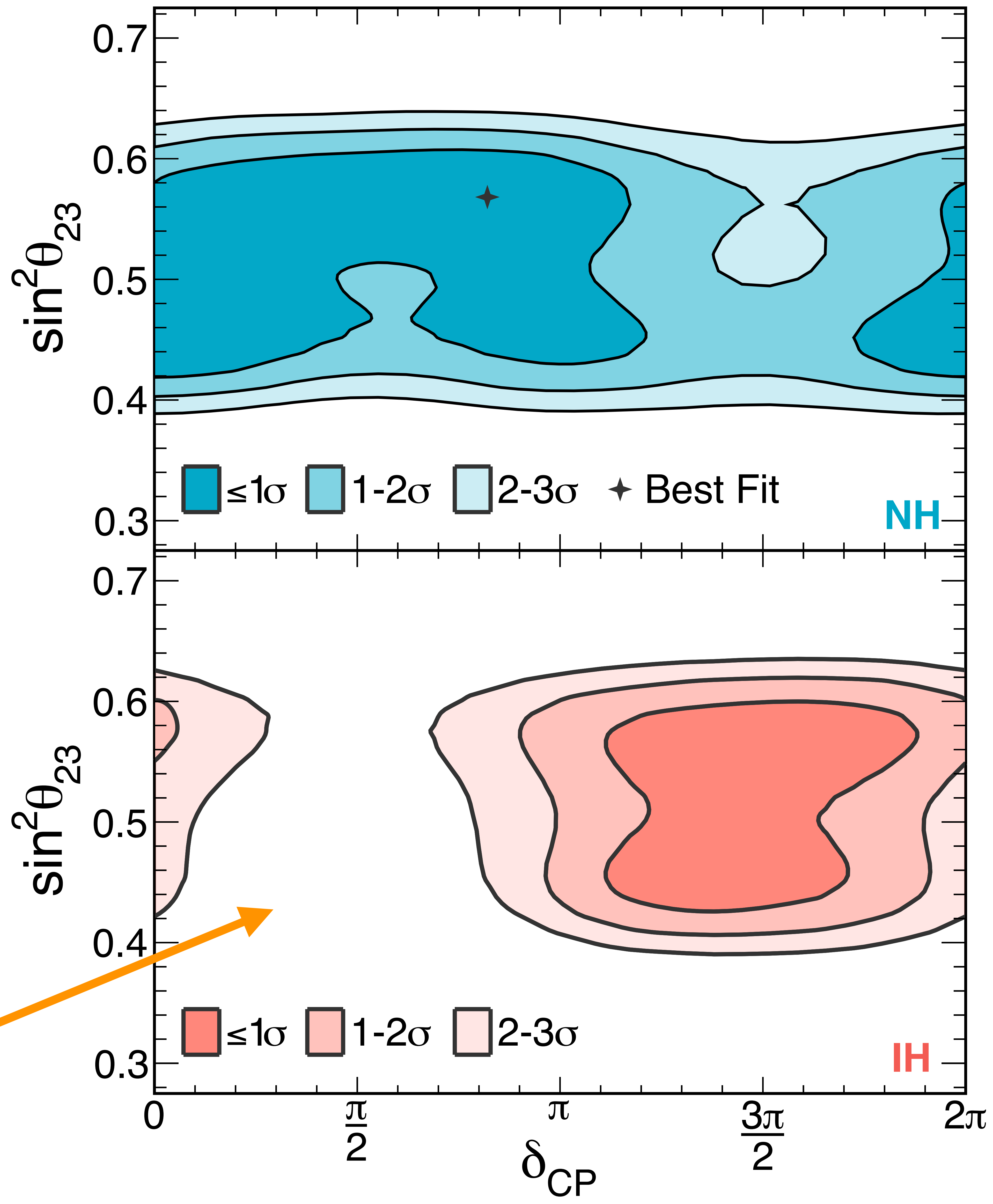


Weak preference for **upper octant**

Weak preference for **Normal Ordering**

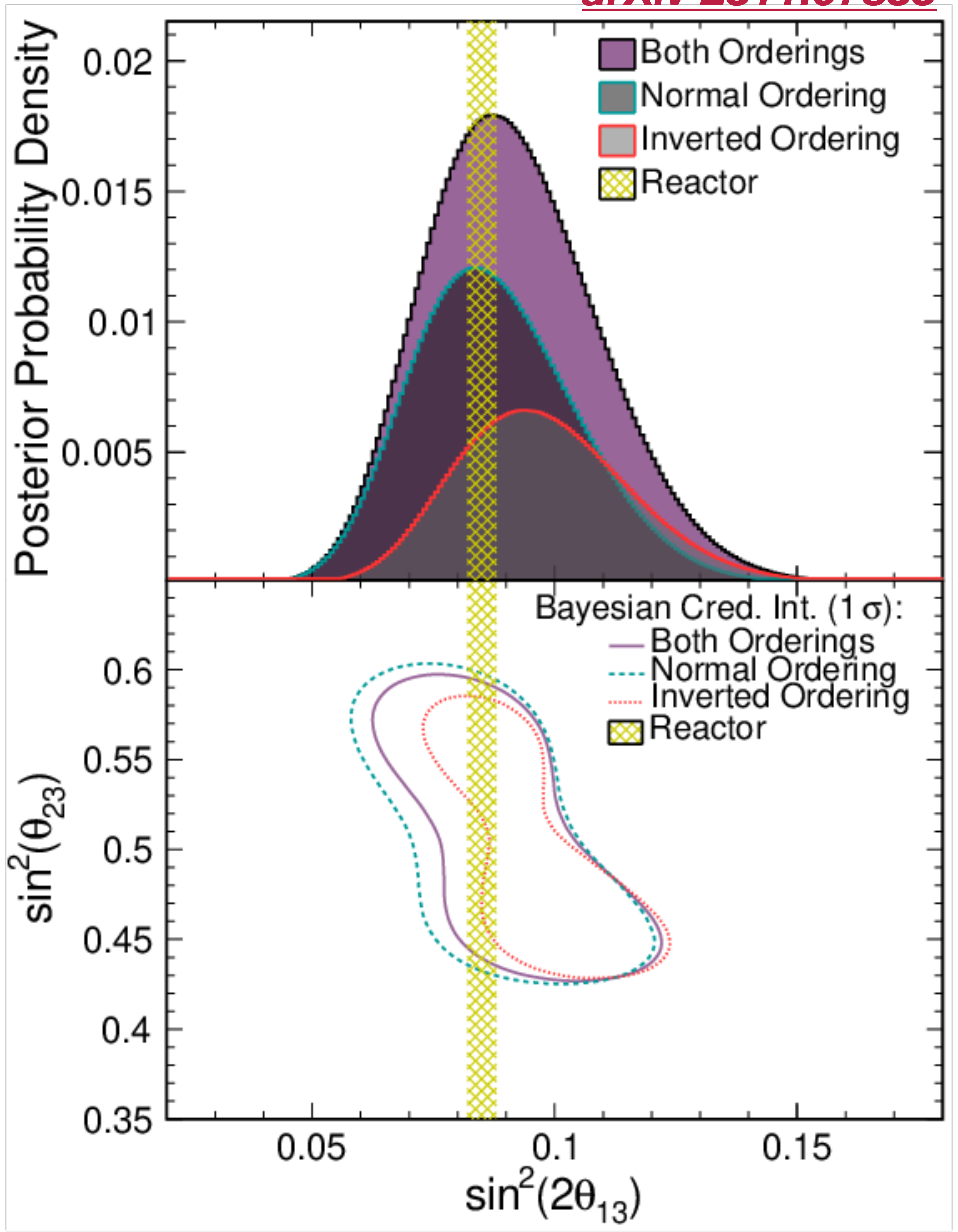
Reject $\delta_{CP} \sim \frac{\pi}{2}$ at 3σ in IH

Phys.Rev.D 106 (2022) 3, 032004



NOvA-Only $\sin^2(2\theta_{13})$

arXiv 2311.07835



Typically our frequentist analyses **use reactor constraint on $\sin^2(2\theta_{13})$**

Bayesian technique allows us to report **measurement of $\sin^2(2\theta_{13})$**

$$\sin^2(2\theta_{13}) = 0.087^{+0.020}_{-0.016}$$

Good agreement with measurement from reactor experiments

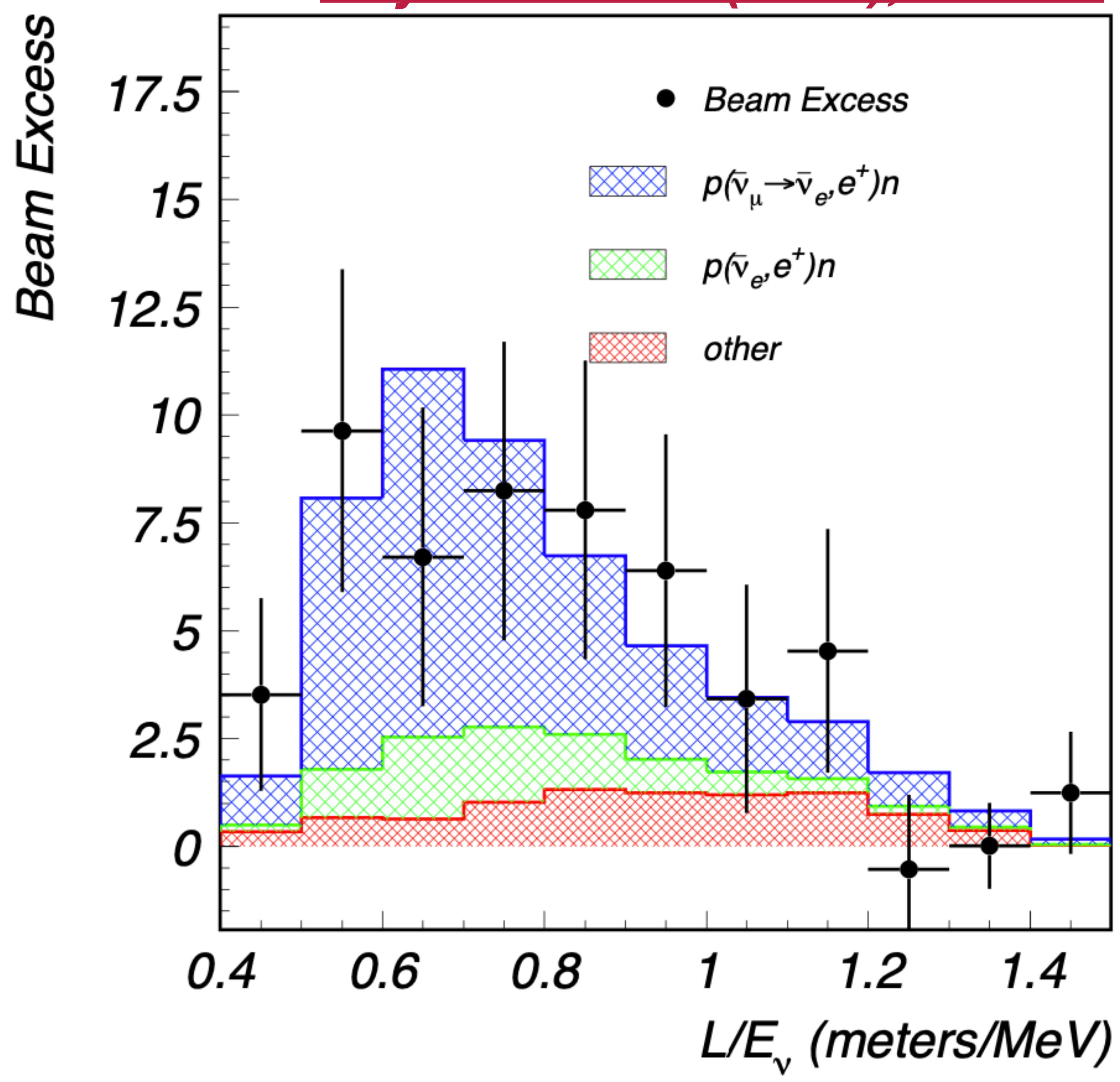
Tests robustness of PMNS model - good agreement at vastly different baselines!

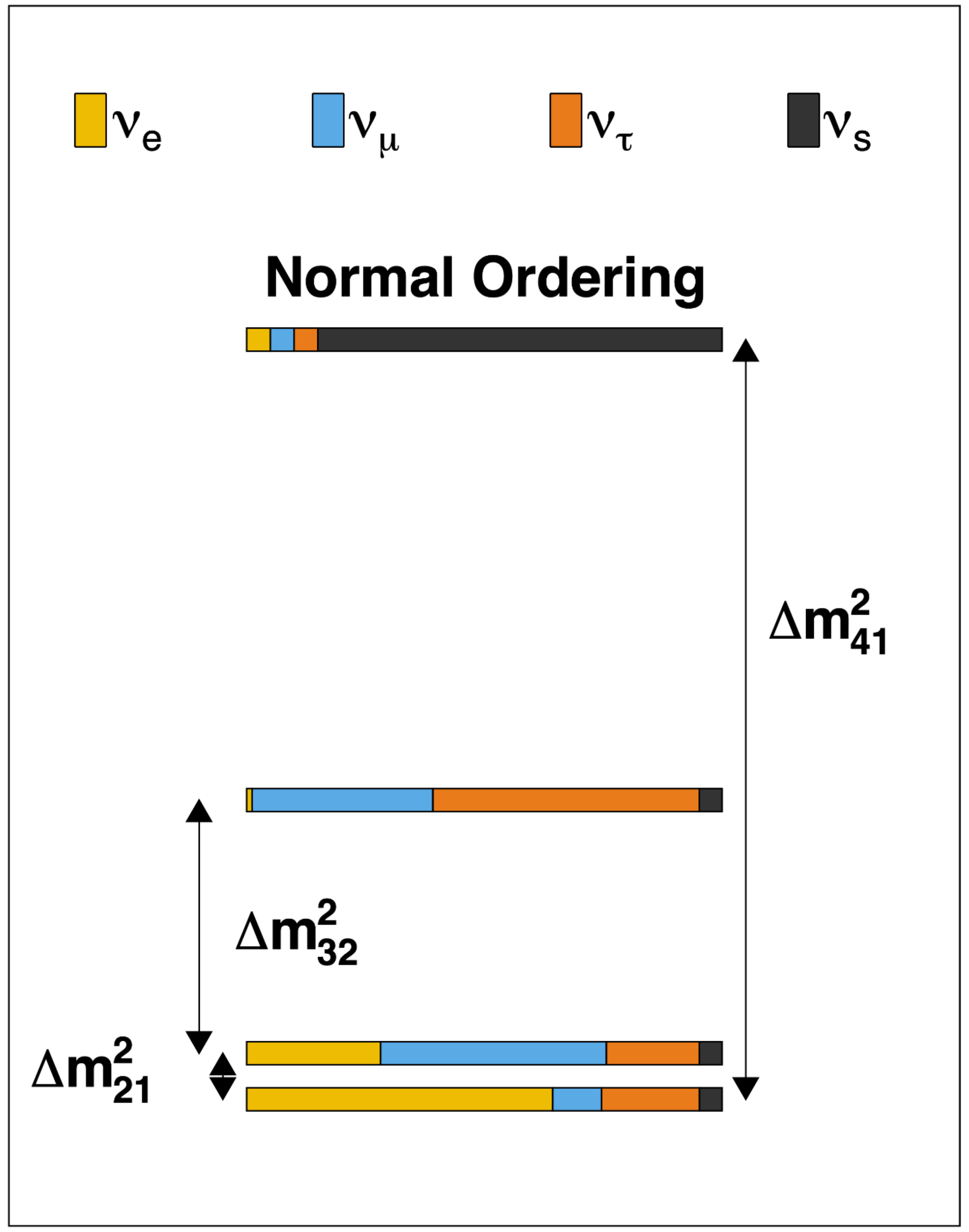
Are 3 Flavours The Full Picture?

Several anomalous results potentially explained by oscillations

$\Delta m^2 >> \Delta m^2_{21}, \Delta m^2_{31}$
(**not predicted by 3-flavour!**)

Phys.Rev.D 64 (2001), 112007





We add a new oscillation frequency, Δm^2_{41}

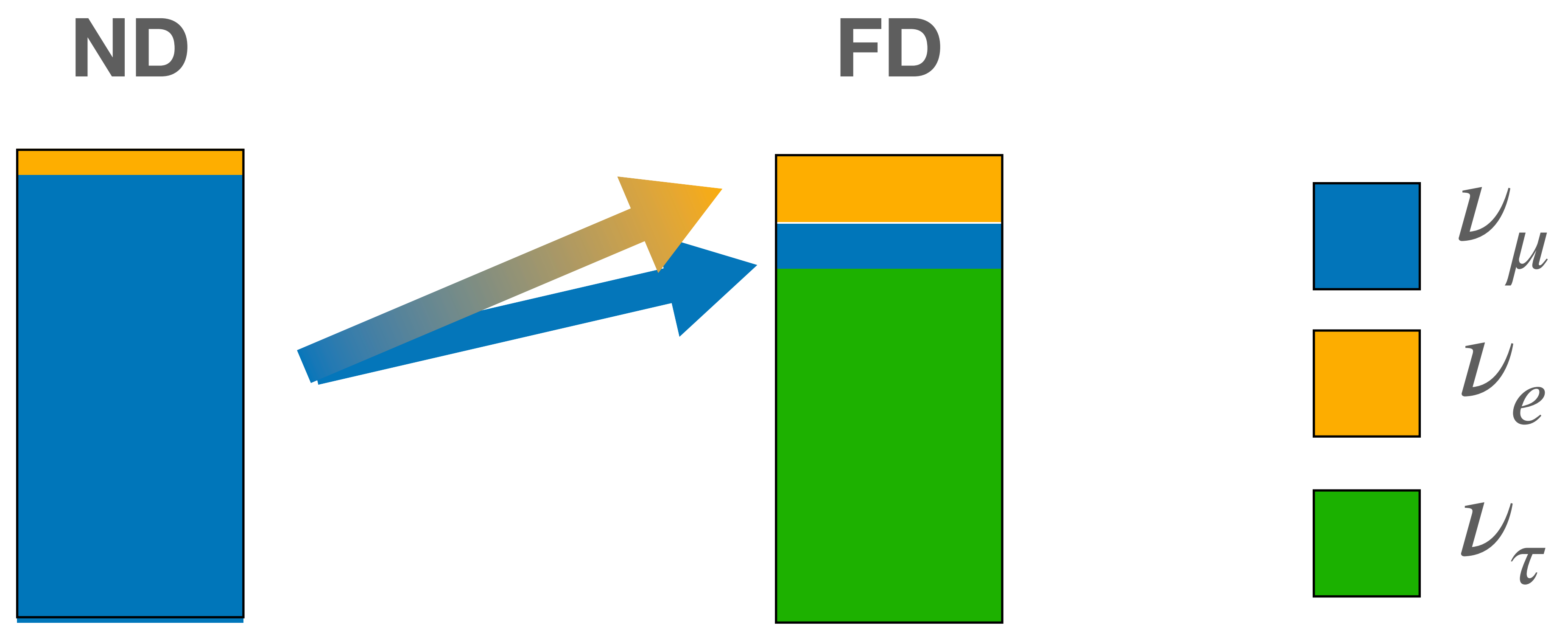
mixing matrix grows: new mixing angles and CP violating phases $\theta_{14}, \theta_{24}, \theta_{34}, \delta_{14}, \delta_{24}, \delta_{34}$

$$U_{\alpha j} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\mu 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

NC Disappearance

For 3-flavour analyses, we're typically looking for

ν_μ **disappearance**, or ν_e **appearance**



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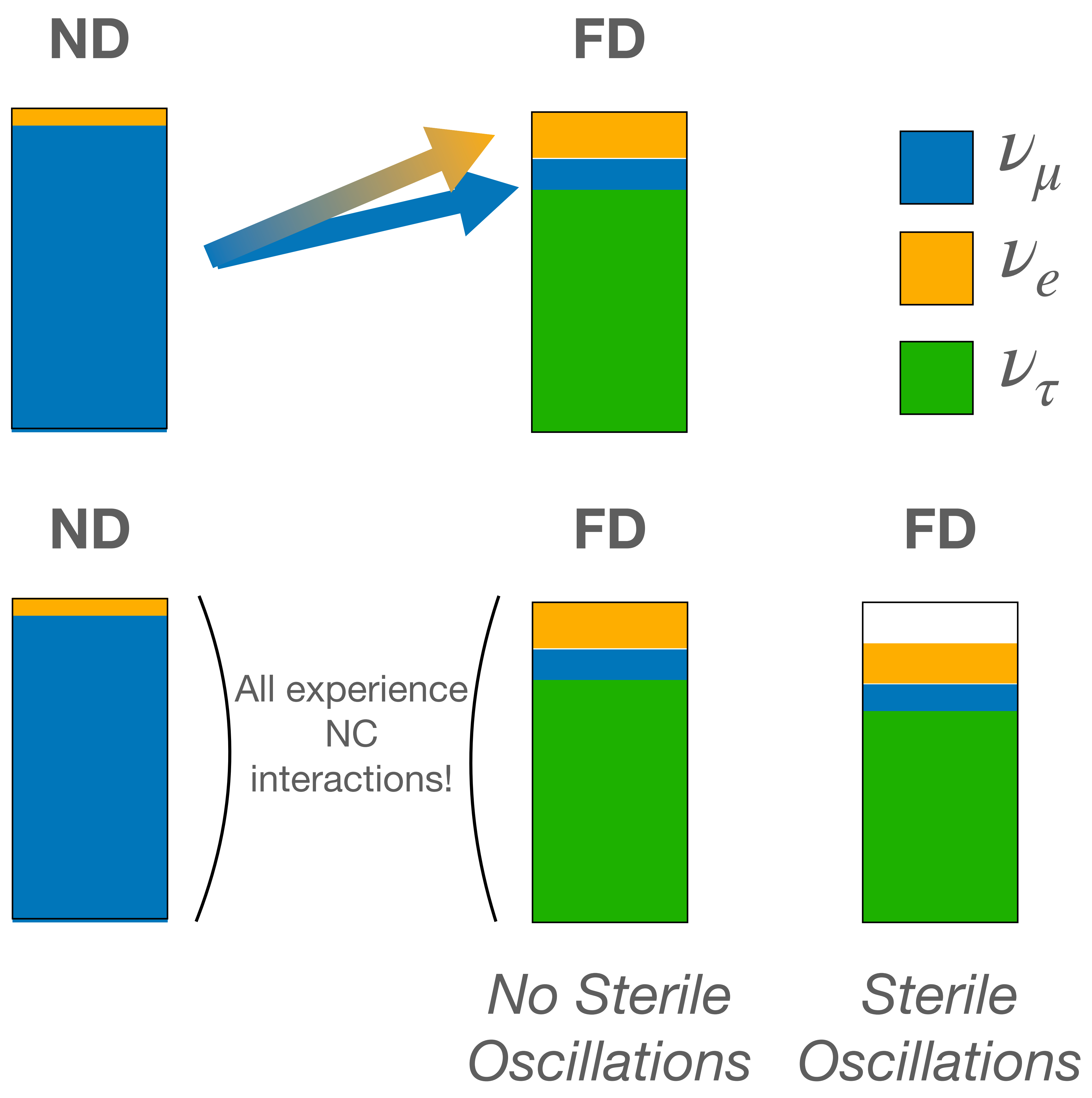
To look for sterile neutrinos, we can look for NC disappearance



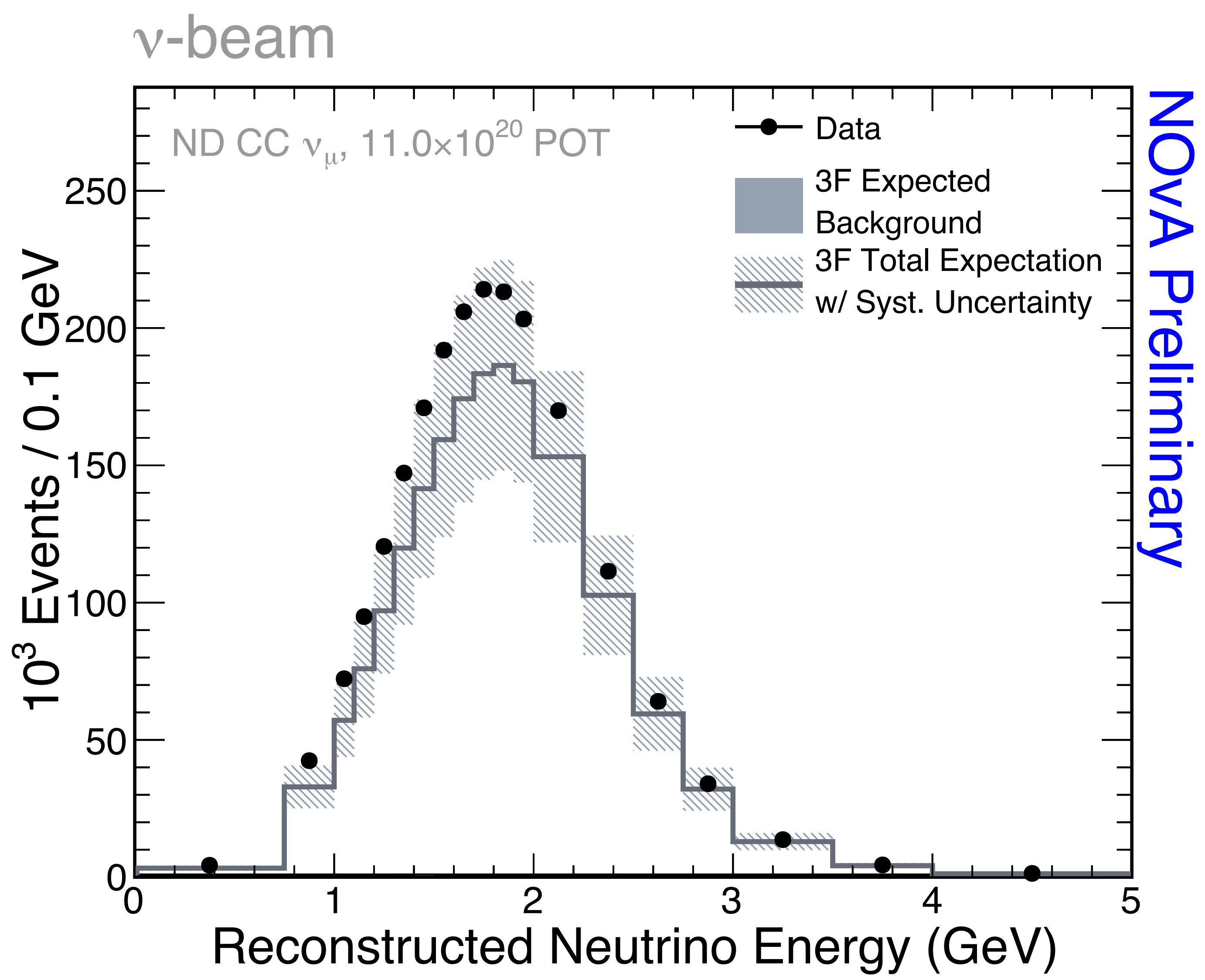
Previous NOvA NC disappearance results

Phys.Rev.D 96 (2017) 7, 072006

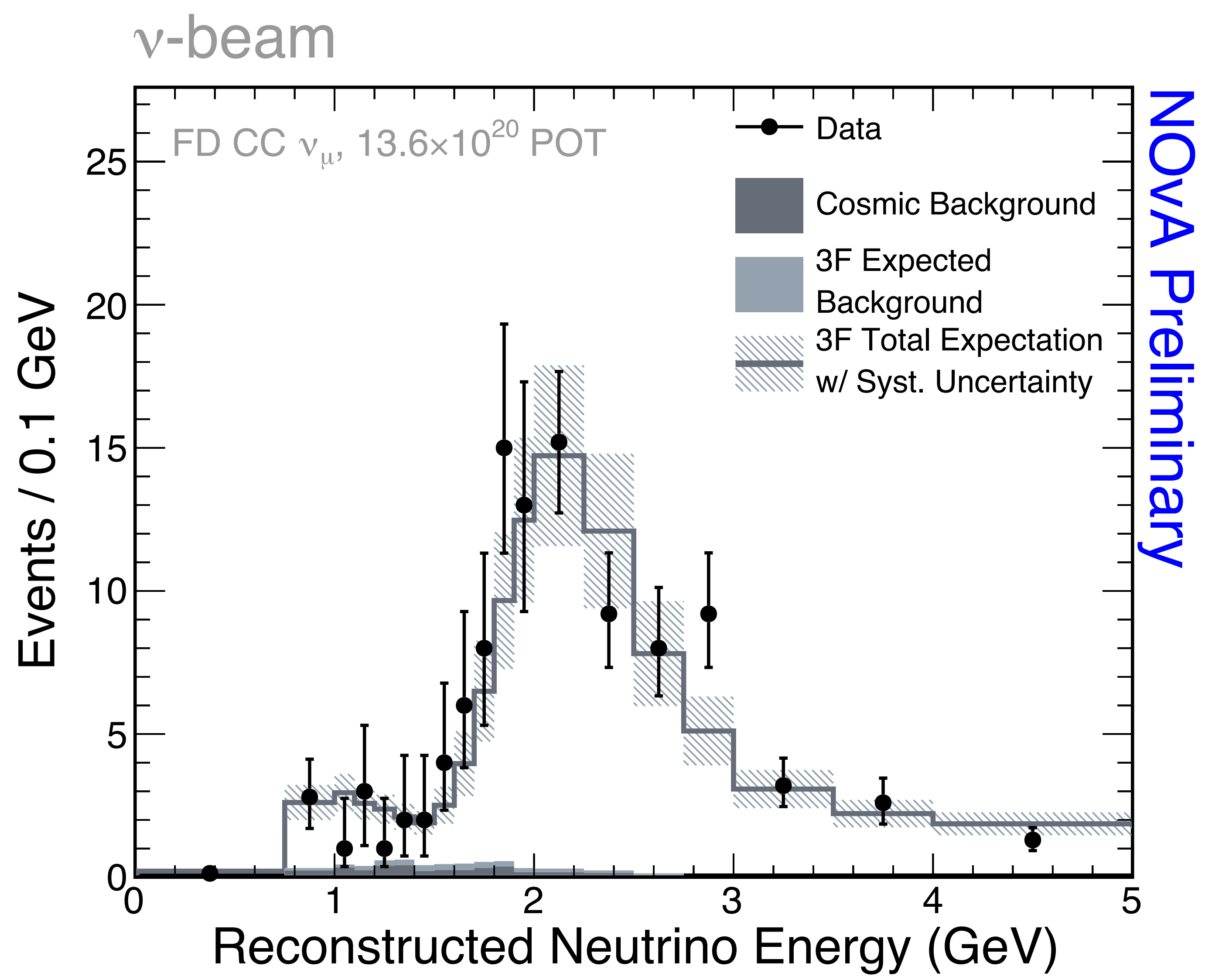
Phys.Rev.Lett. 127 (2021) 20, 201801

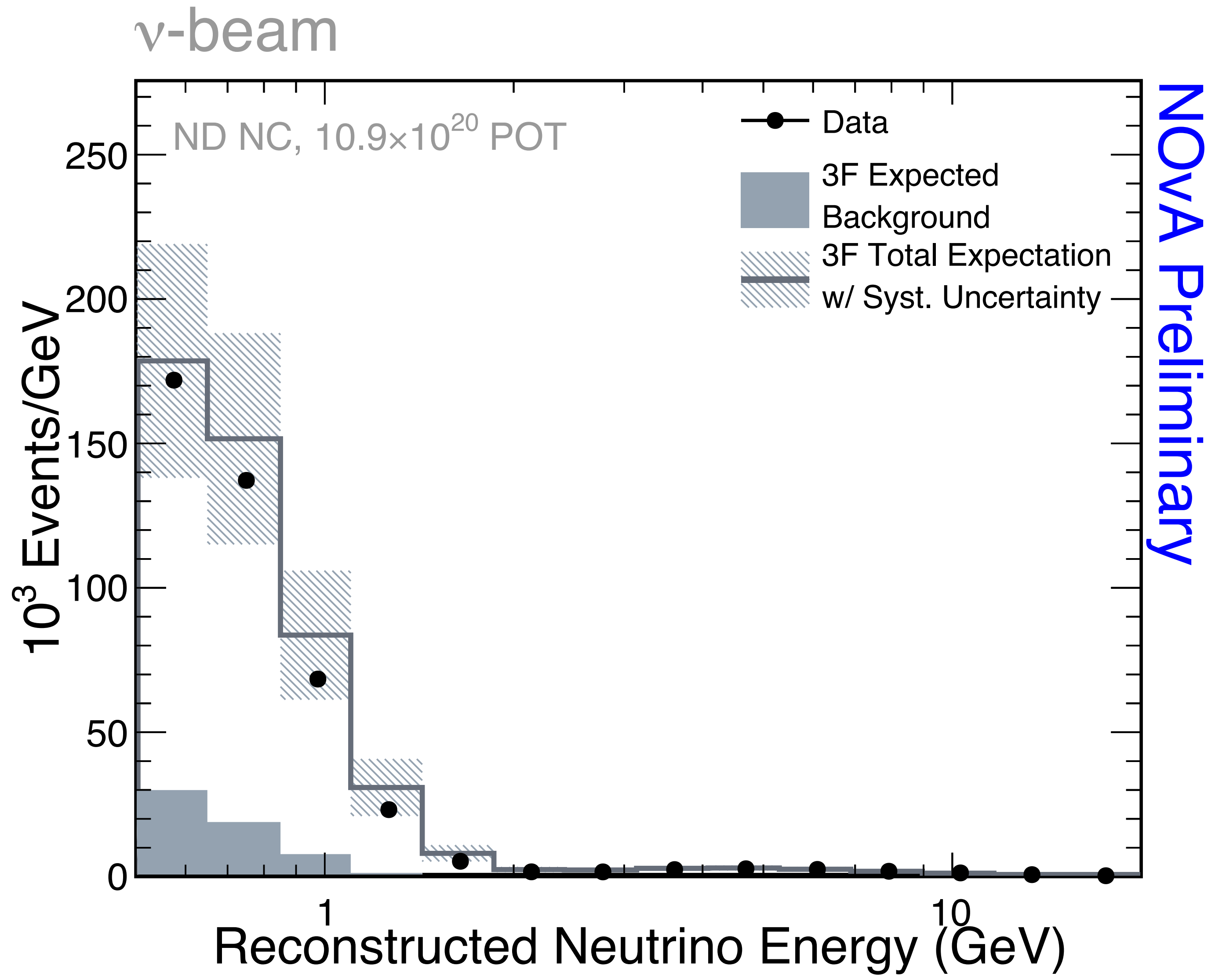
Data Samples

★ **New for this analysis!**

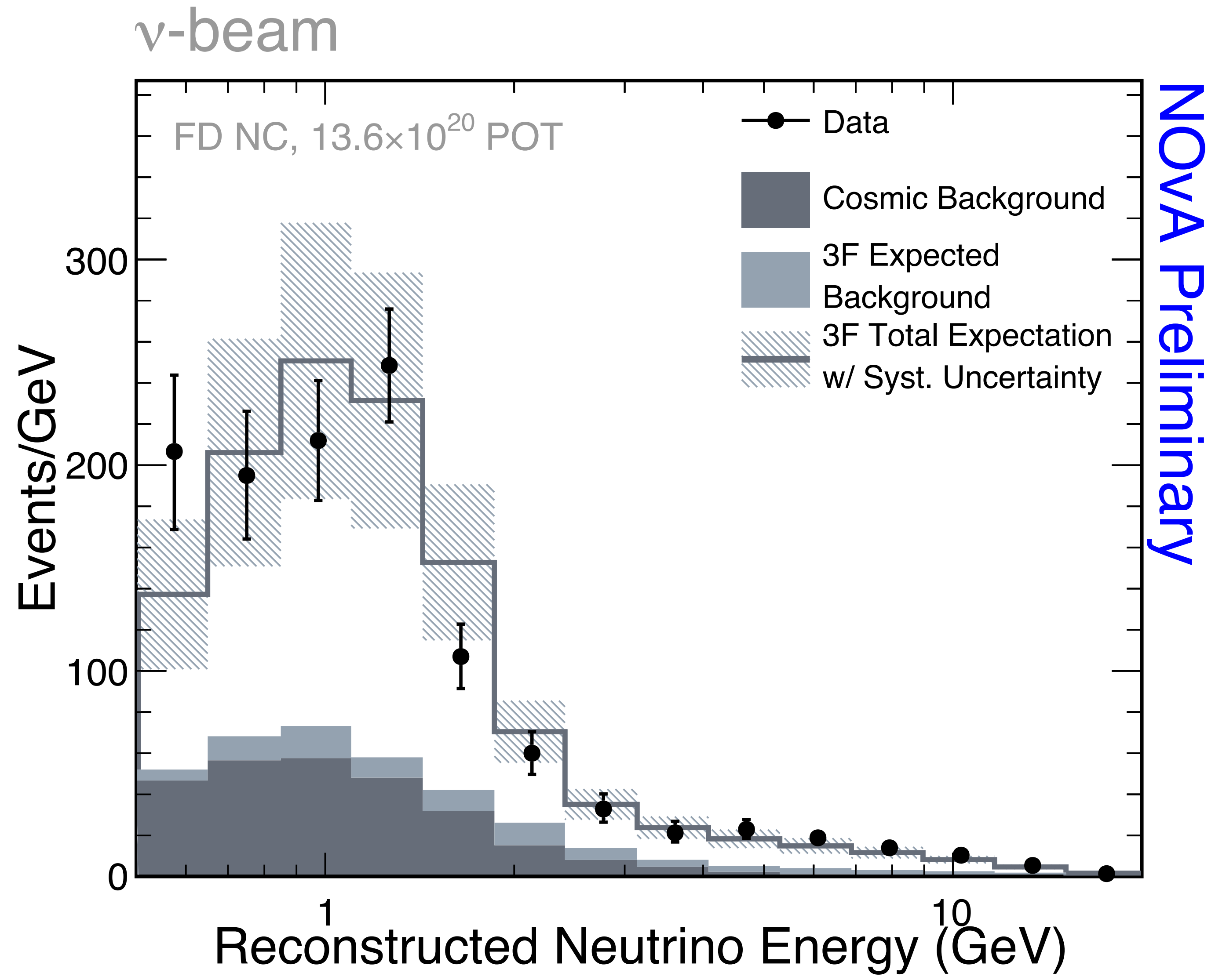


Include neutrino-mode NC samples and ν_μ ★ samples in fit



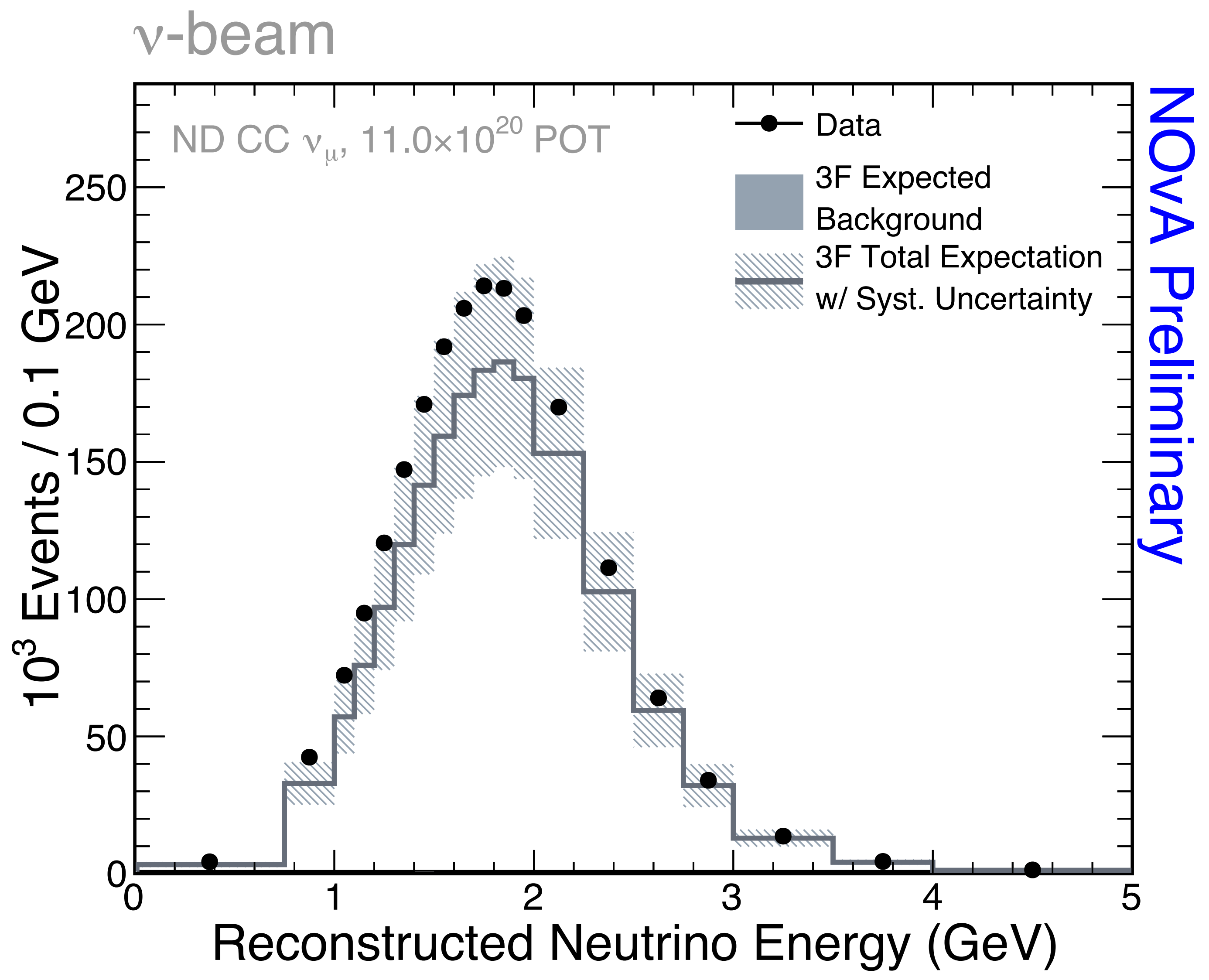


Data agrees with 3-flavour prediction within uncertainties



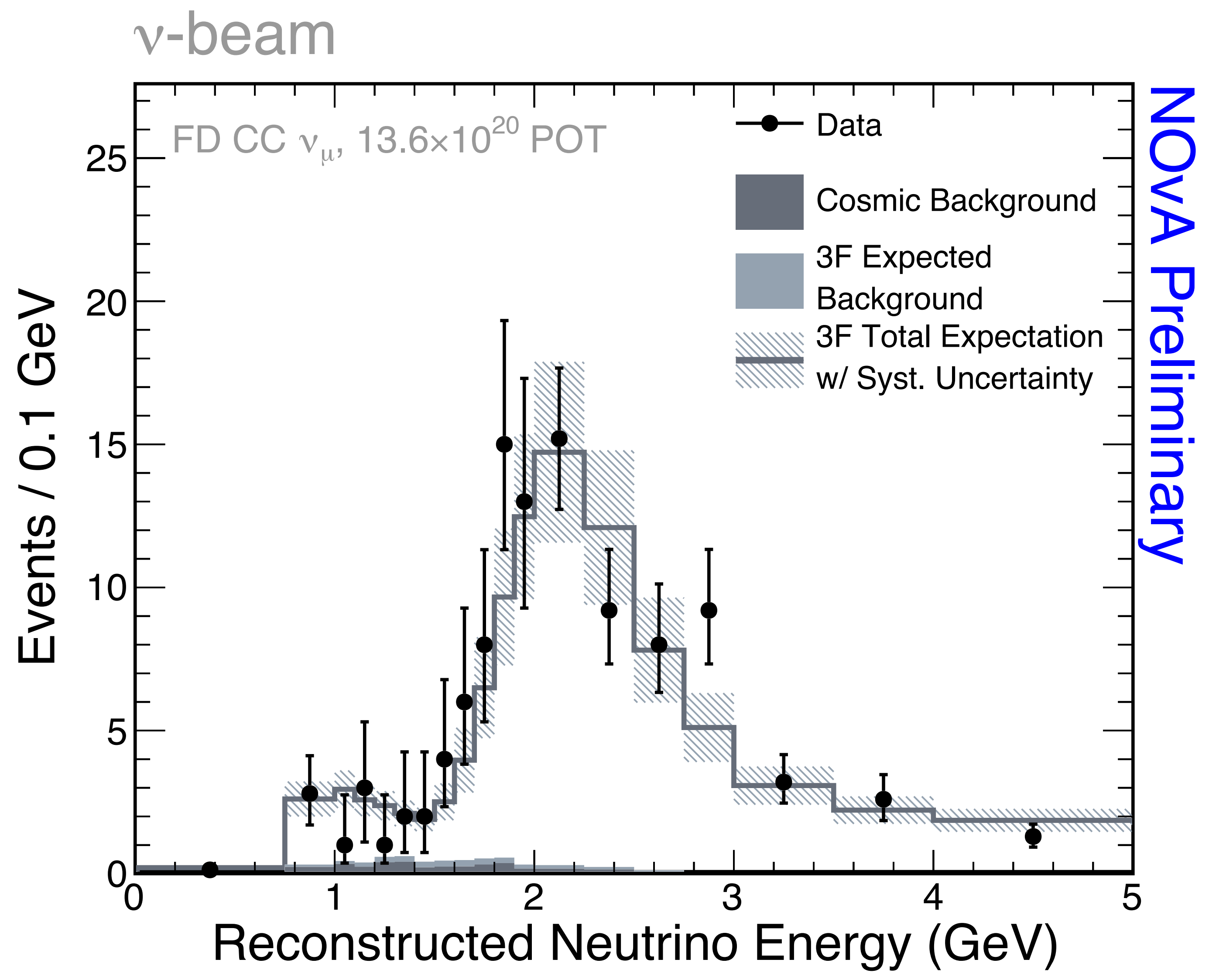
Data Samples

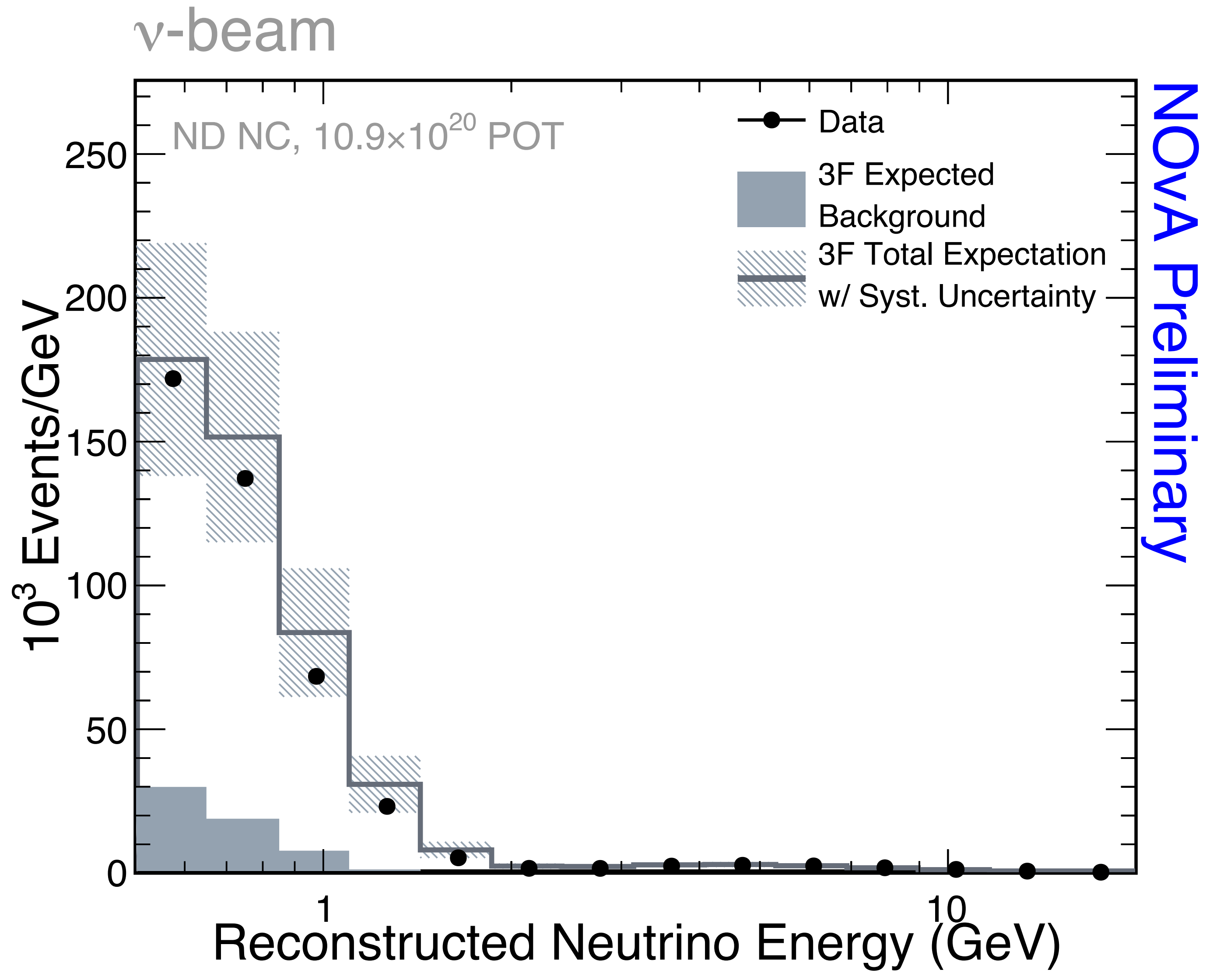
★ **New for this analysis!**

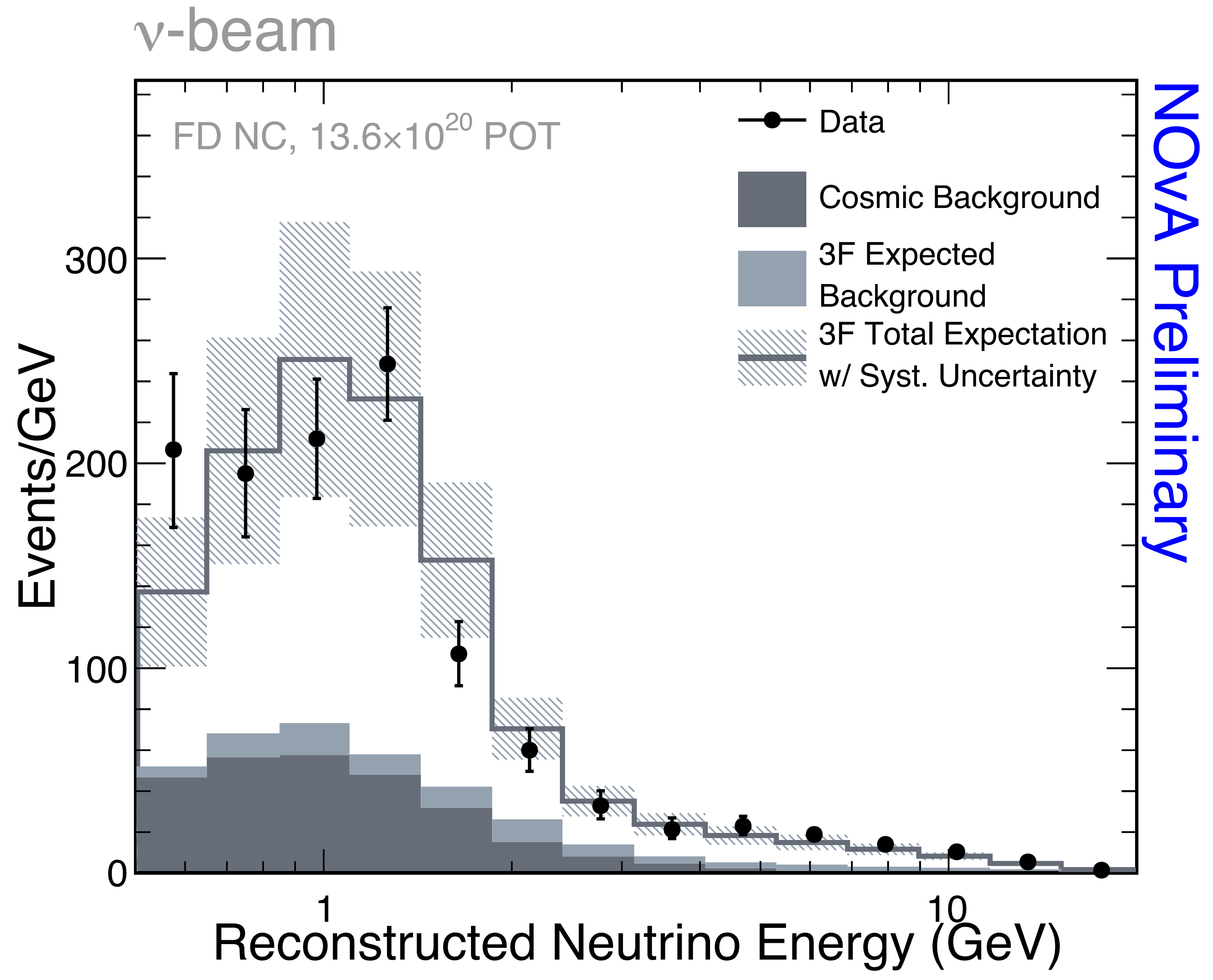


Include neutrino-mode NC samples and ν_μ ★ samples in fit

Simultaneous dual-baseline fit ★ allows sensitivity to higher Δm^2_{41} than previous extrapolation-style analyses







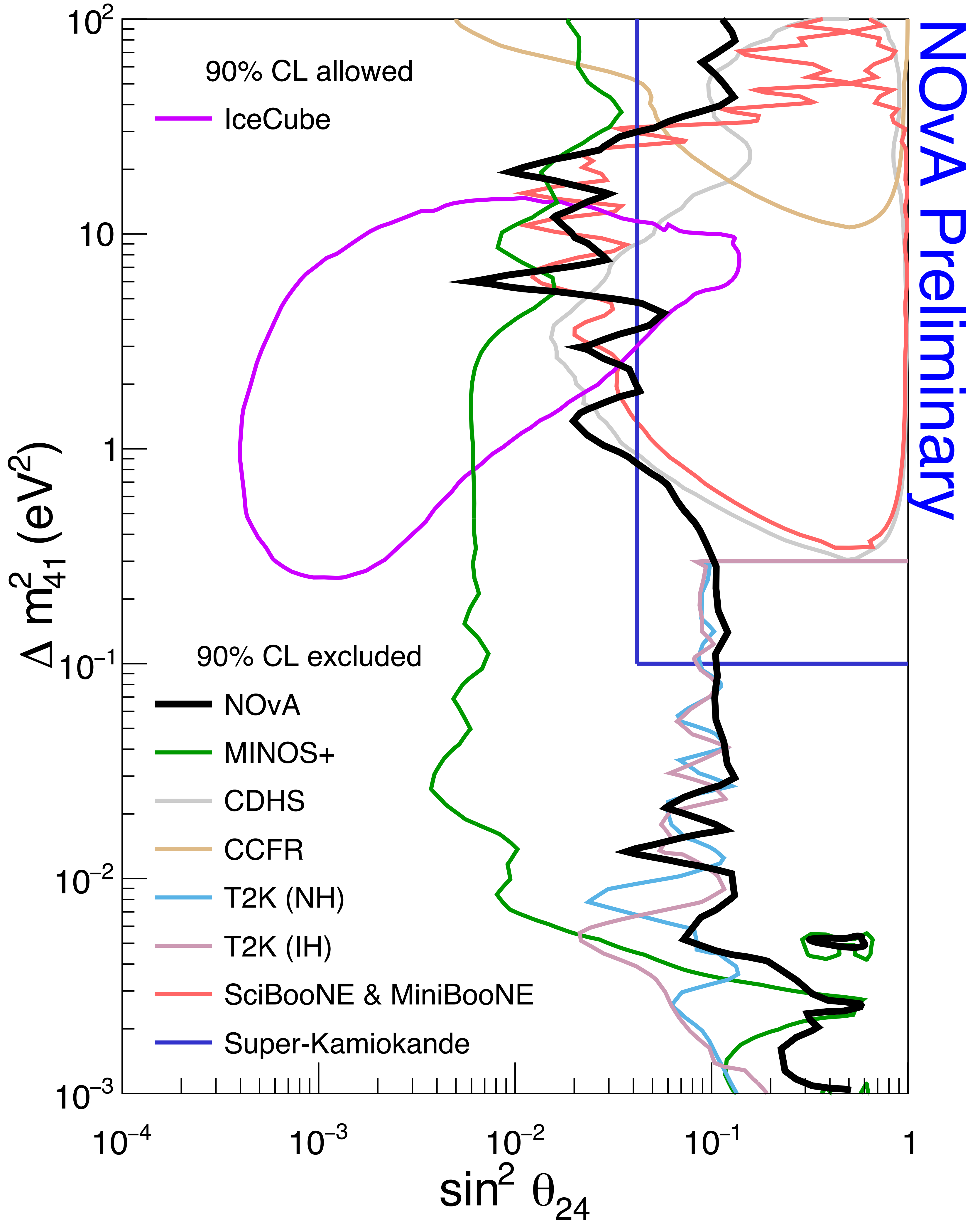
Data agrees with 3-flavour prediction within uncertainties

Fit Results

Sensitivity in high Δm^2_{41} region driven by Near Detector and is systematically limited

Sensitivity at low Δm^2_{41} region driven by FD and is statistically limited

Neutrino Beam (References in backup)



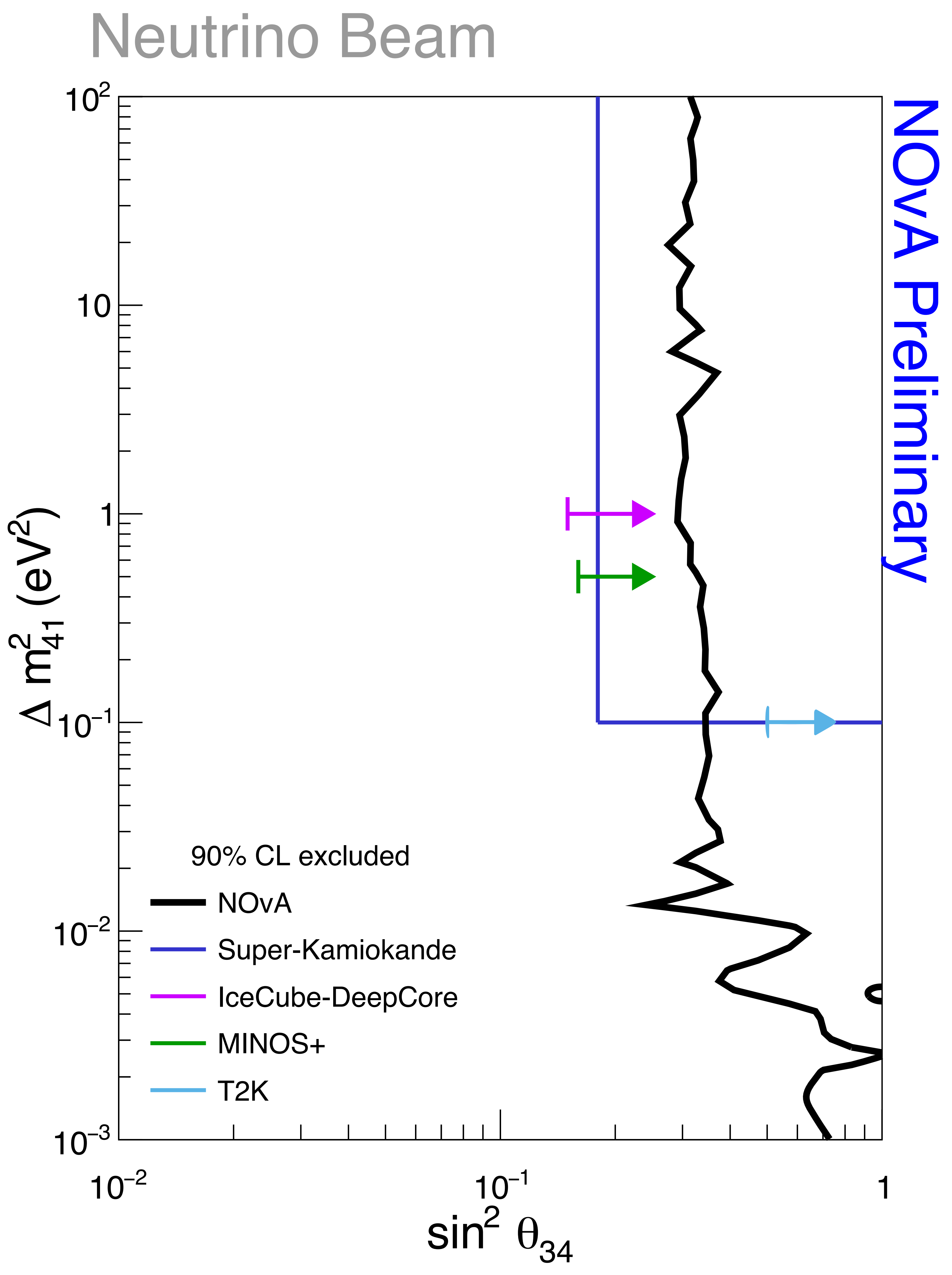
NOvA data shows no evidence for sterile neutrinos under 3+1 model

NOvA contours show constraints on $\sin^2(\theta_{24})$ competitive around $\Delta m^2_{41} = 10\ \mathrm{eV}^2$

Fit Results

NC Disappearance gives access to $\sin^2(\theta_{34})$

For this space, we are more statistically limited across the space



NOvA data shows no evidence for sterile neutrinos under 3+1 model

Note that this does not include data from ν_τ appearance at short baselines, which measure effective mixing angle $\theta_{\mu\tau}$

Summary

NOvA has an extensive physics programme!

NOvA's 3-flavour analysis has slight preference for Upper Octant, Normal Ordering

Bayesian analysis consistent with frequentist analysis, allows looking at data in new ways

NOvA data is consistent with 3-flavour oscillations at the 90% confidence level

Upcoming from NOvA

More data!

Collected ~2x 2020 analysis protons-on-target

Running through 2027

Improving Detector Understanding

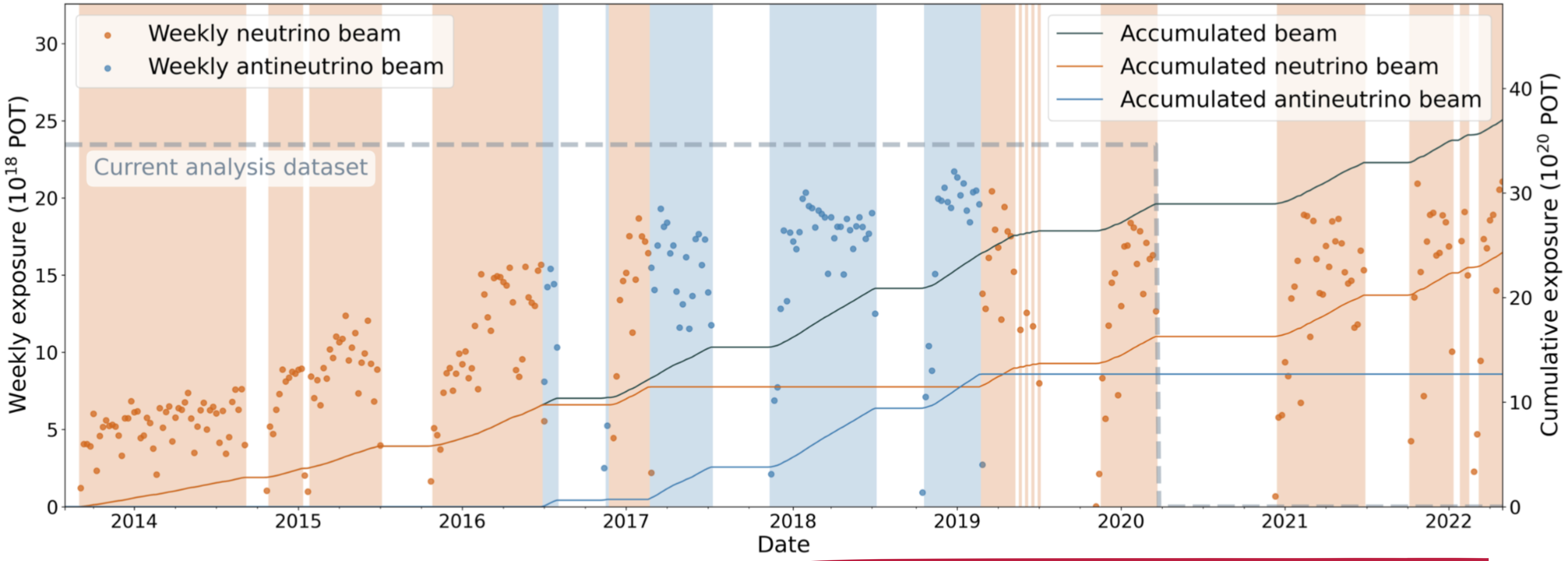
Test Beam run wrapped up, and well into analysis stage

NOvA-T2K Joint Fit

Work is in progress, results expected early next year

Thanks! Any Questions?

Additional Slides



Recorded through 2023

42.1×10^{20} total POT

29.4×10^{20} POT neutrino-beam data

12.7×10^{20} POT neutrino-beam data

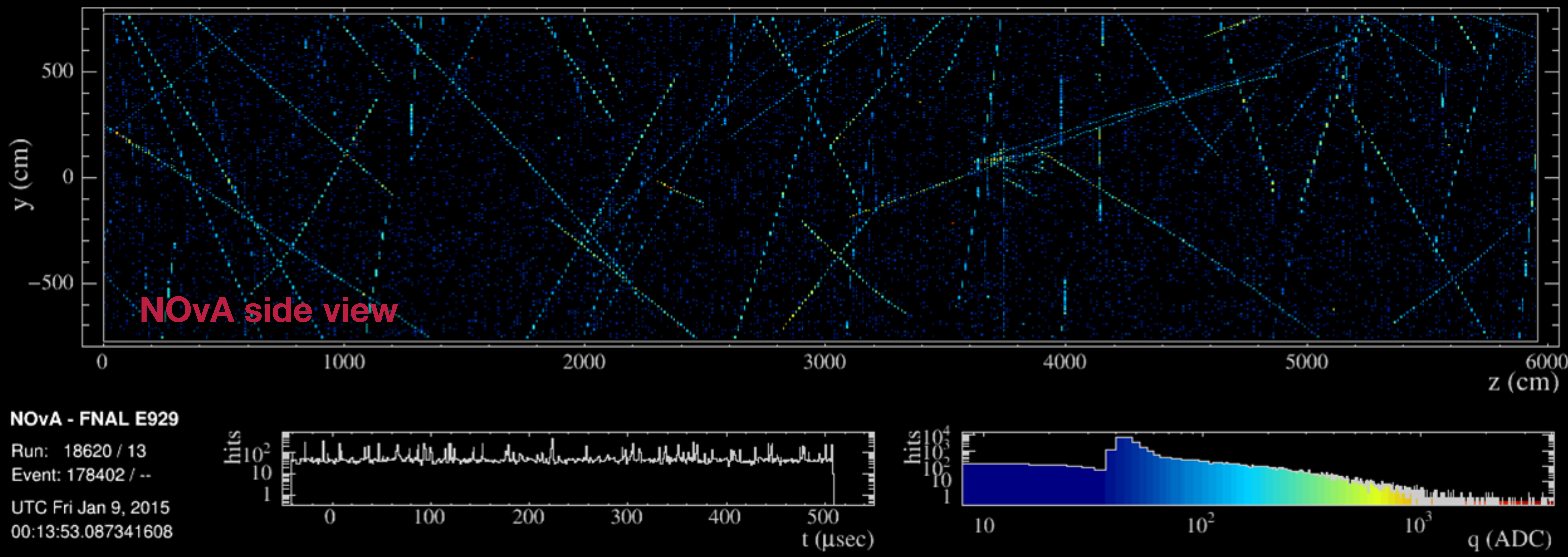
Current analysis dataset

13.6×10^{20} POT neutrino-beam data

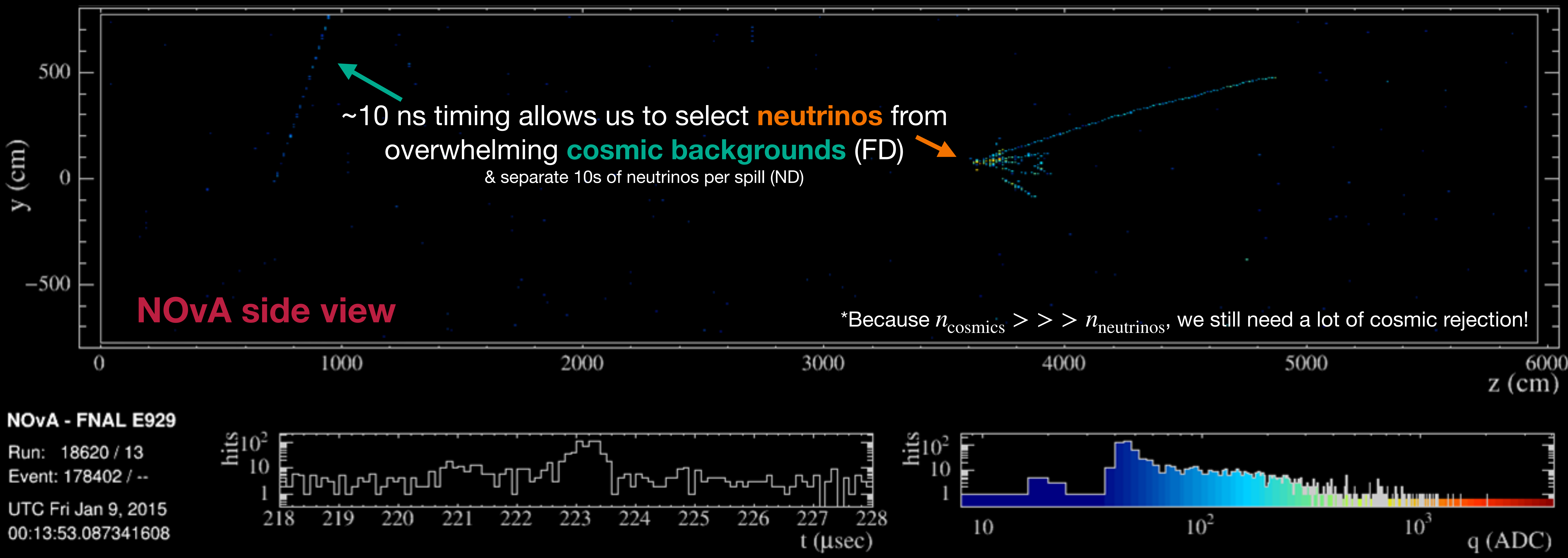
12.5×10^{20} POT antineutrino-beam data

Beam currently down but expected to return February

The NOvA detectors are **optimised** for surface running in a 2 GeV beam!



The NOvA detectors are **optimised** for surface running in a 2 GeV beam!

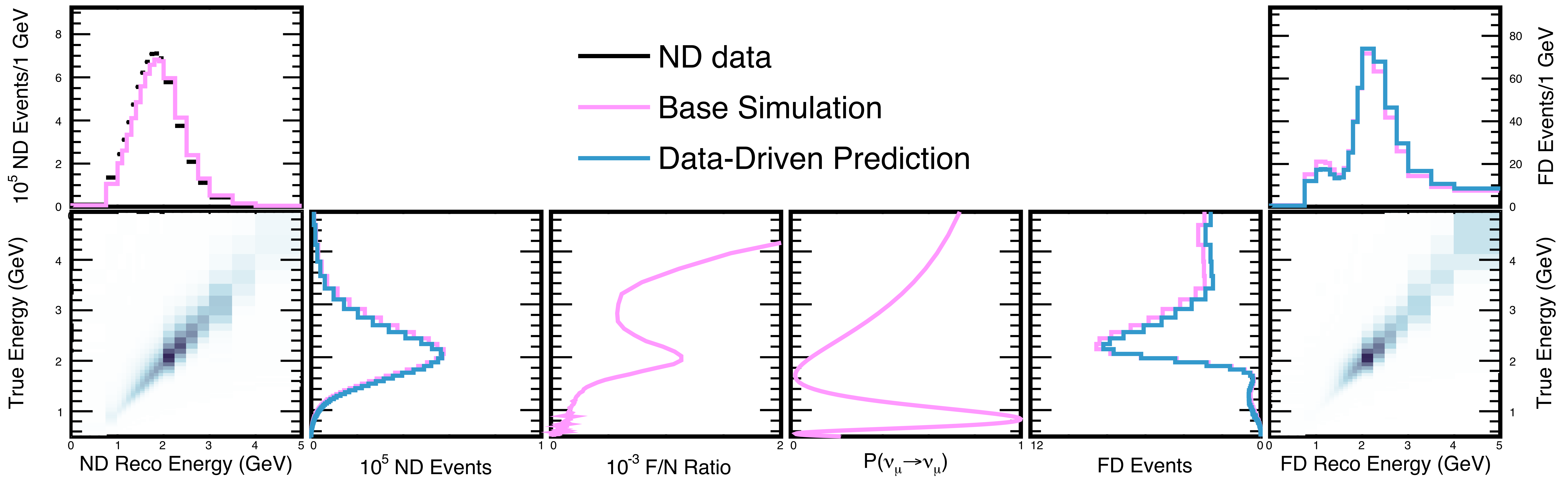


~10 ns timing allows us to select **neutrinos** from overwhelming **cosmic backgrounds** (FD)

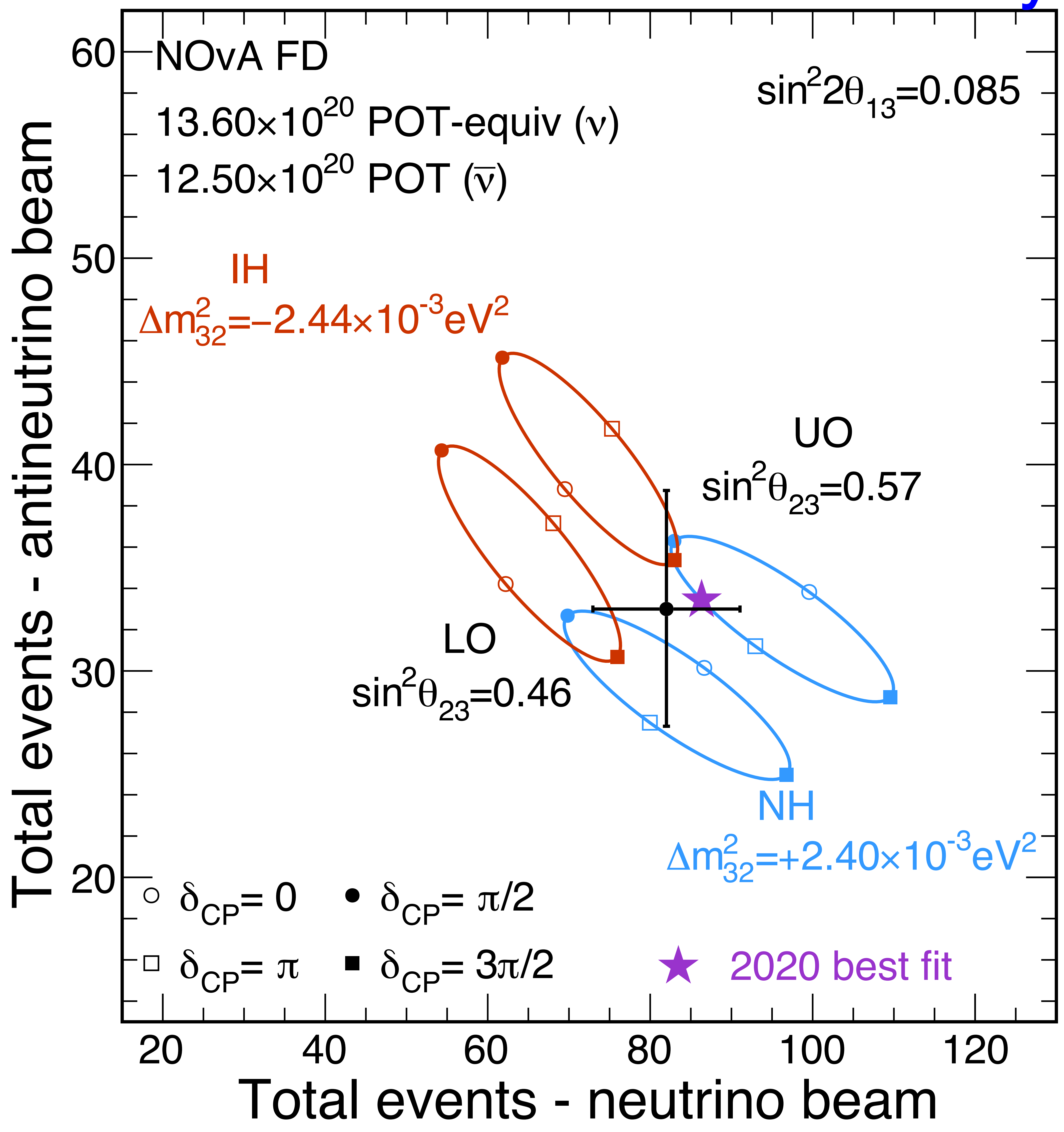
& separate 10s of neutrinos per spill (ND)

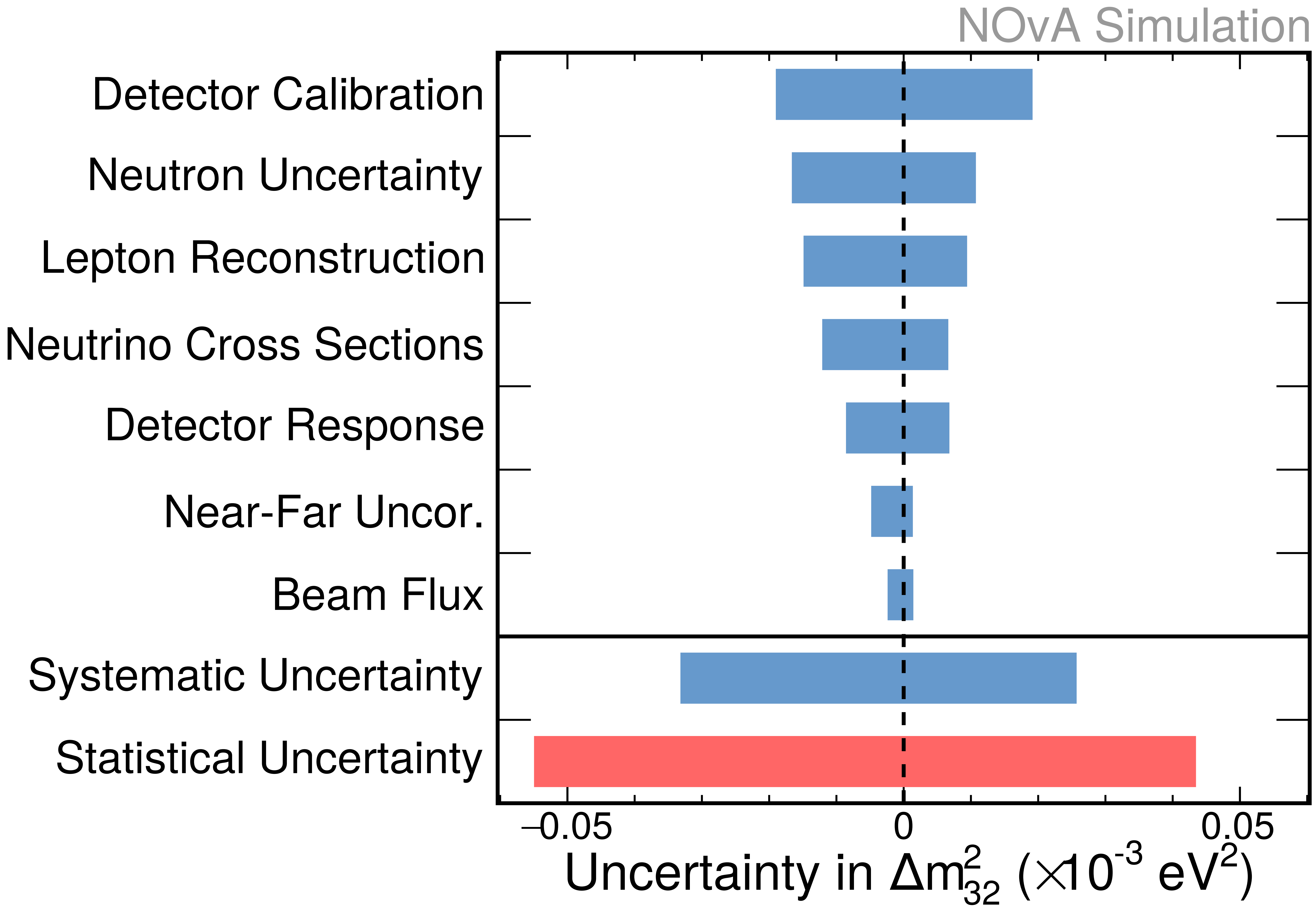
*Because $n_{\text{cosmics}} >>> n_{\text{neutrinos}}$, we still need a lot of cosmic rejection!

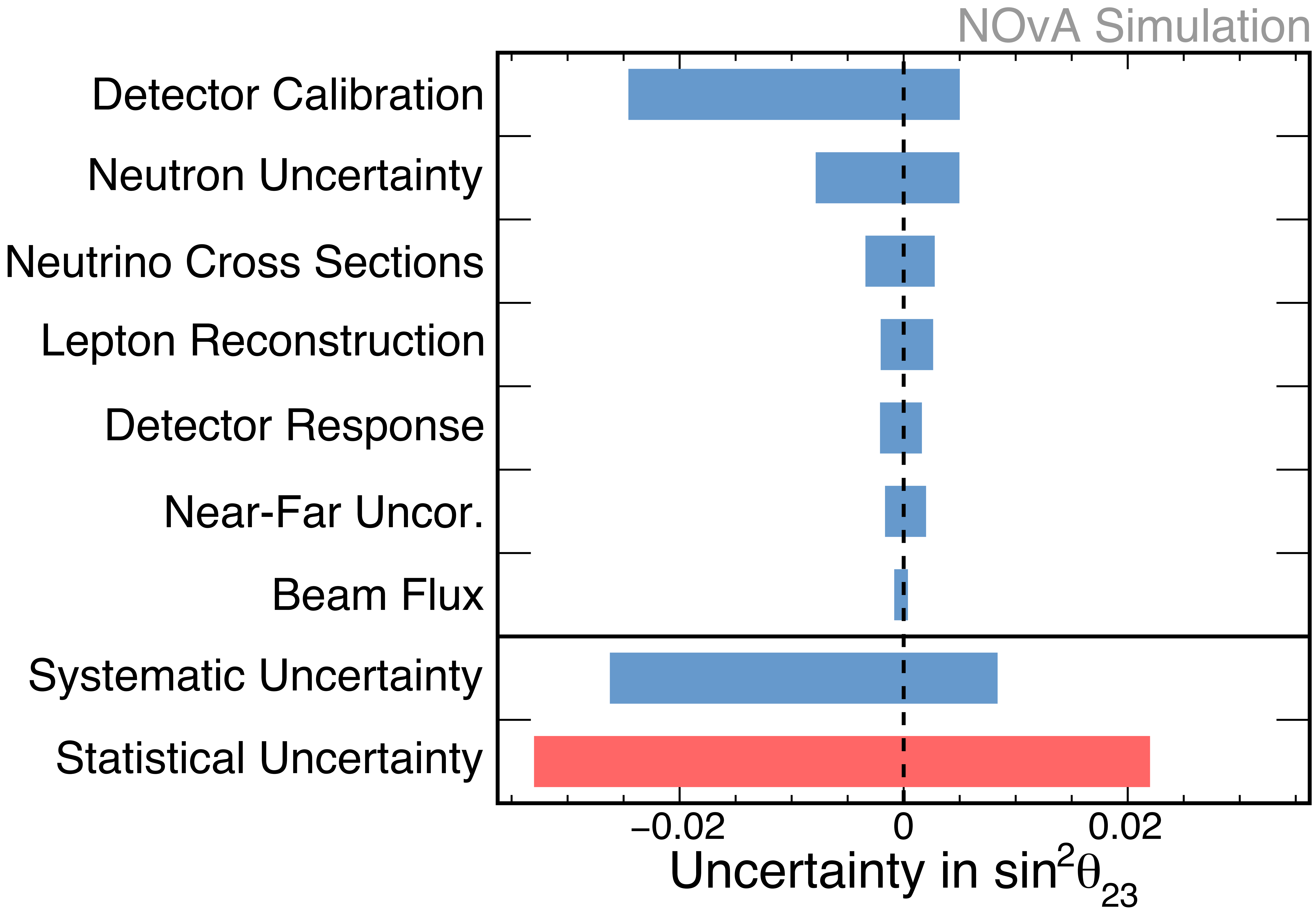
NOvA's Extrapolation Technique



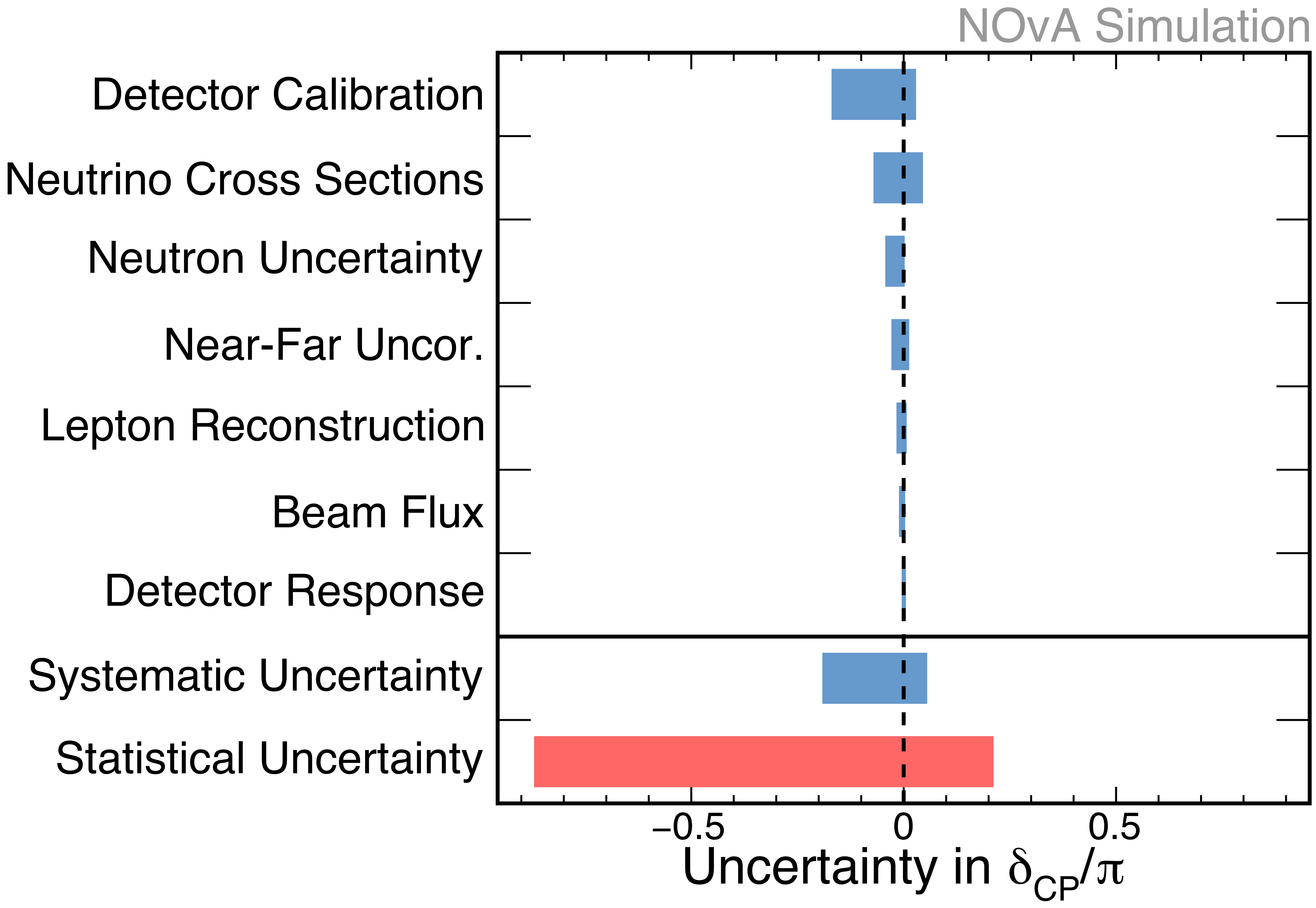
NOvA Preliminary




NOvA Simulation
Detector Calibration
Neutron Uncertainty
Lepton Reconstruction
Neutrino Cross Sections
Detector Response
Near-Far Uncor.
Beam Flux
Systematic Uncertainty
Statistical Uncertainty
−0.05
0
0.05
Uncertainty in Δm^2_{32} ($\times 10^{-3}$ eV^2)



NOvA Simulation
Detector Calibration
Neutron Uncertainty
Neutrino Cross Sections
Lepton Reconstruction
Detector Response
Near-Far Uncor.
Beam Flux
Systematic Uncertainty
Statistical Uncertainty
−0.02
0
0.02
Uncertainty in $\sin^2\theta_{23}$


NOvA Simulation
Detector Calibration
Neutrino Cross Sections
Neutron Uncertainty
Near-Far Uncor.
Lepton Reconstruction
Beam Flux
Detector Response
Systematic Uncertainty
Statistical Uncertainty
−0.5
0
0.5
Uncertainty in δ_{CP}/π

Frequentist v Bayesian BF

Best Fit Point

Frequentist

$$\Delta m^2_{32} = (2.41 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$$

$$sin^2\theta_{23} = 0.57^{+0.03}_{-0.04}$$

$$\delta_{CP} = 0.82^{+0.27}_{-0.87}\pi$$

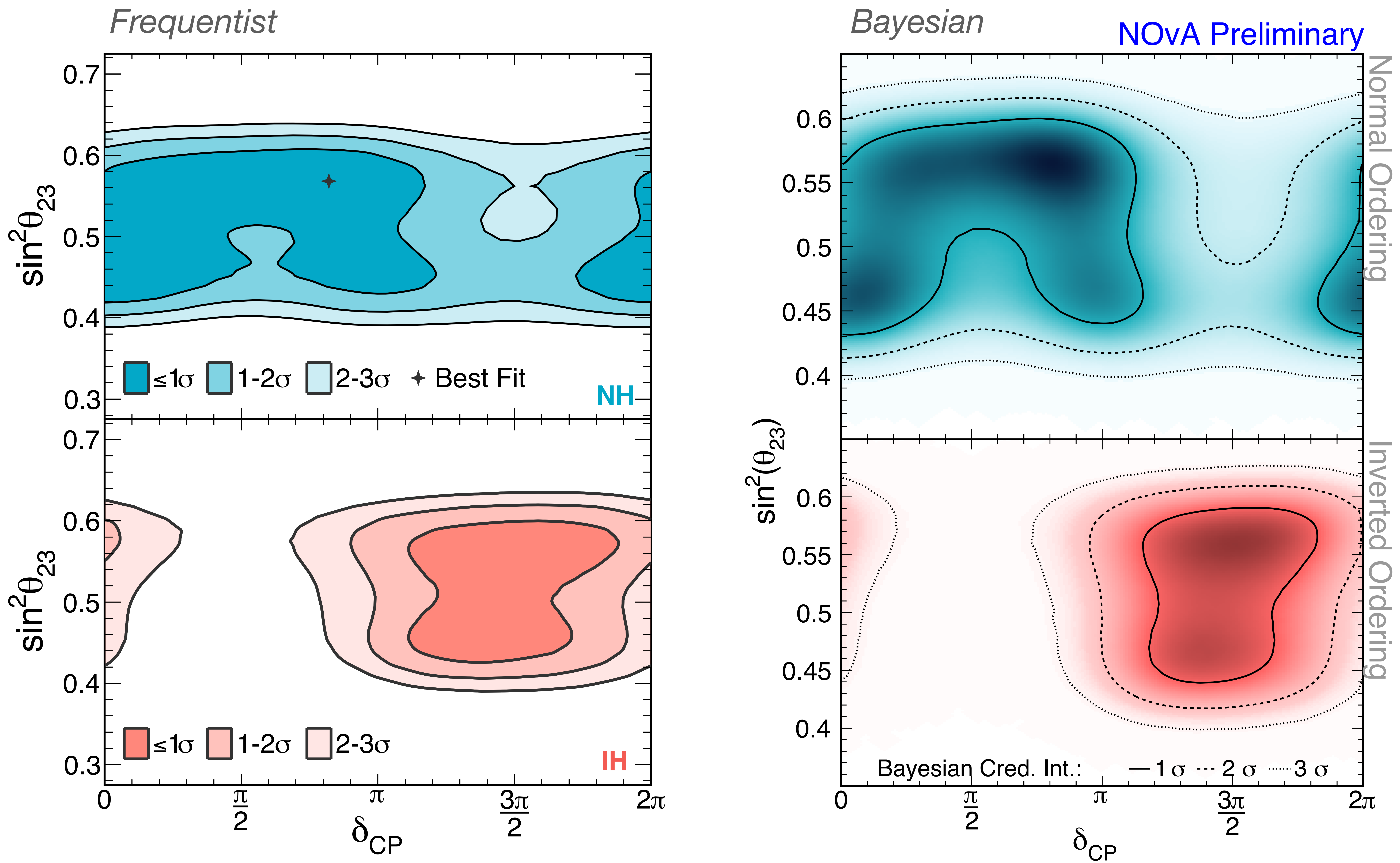
Highest Probability Density

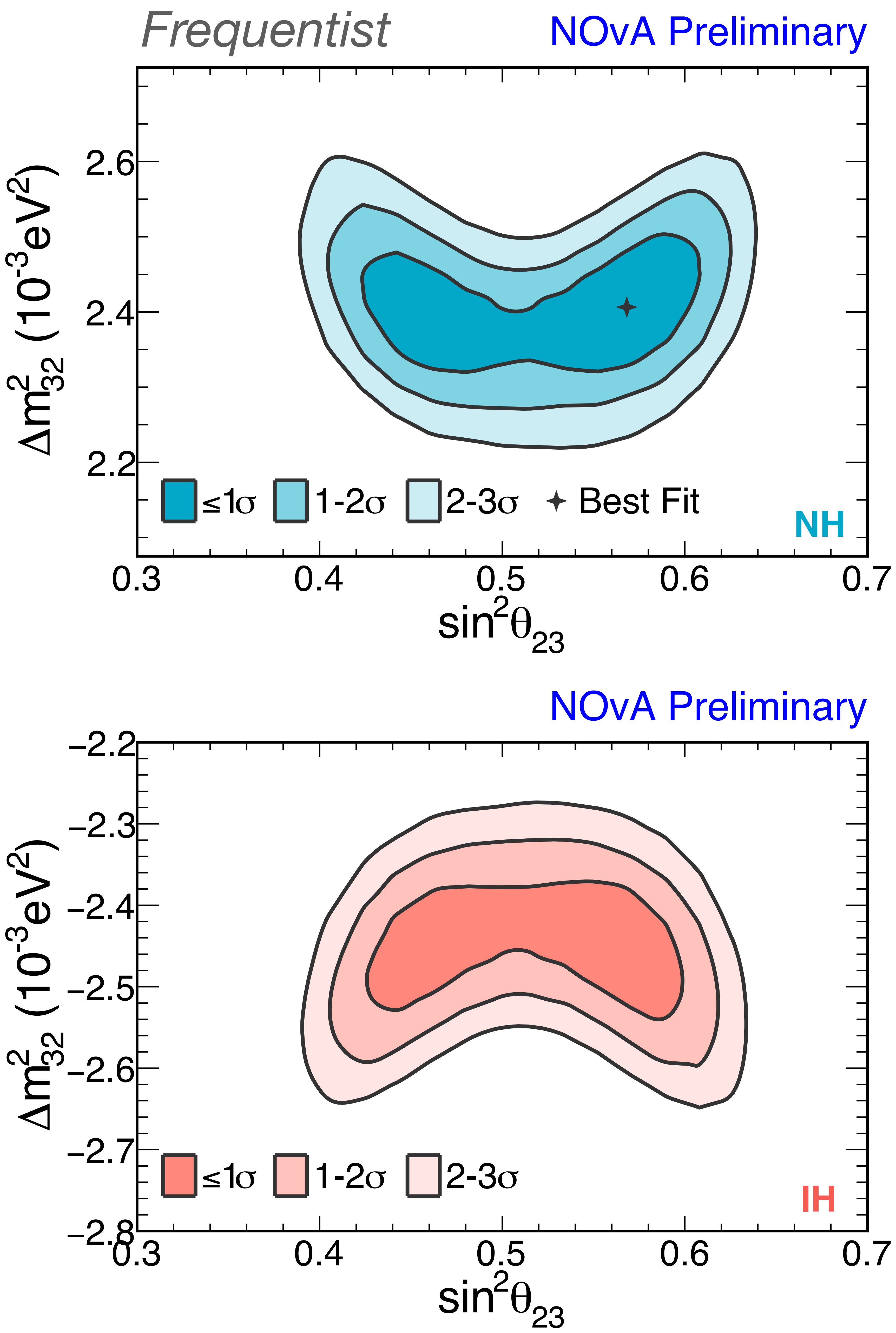
Bayesian

$$\Delta m^2_{32} = (2.39 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2$$

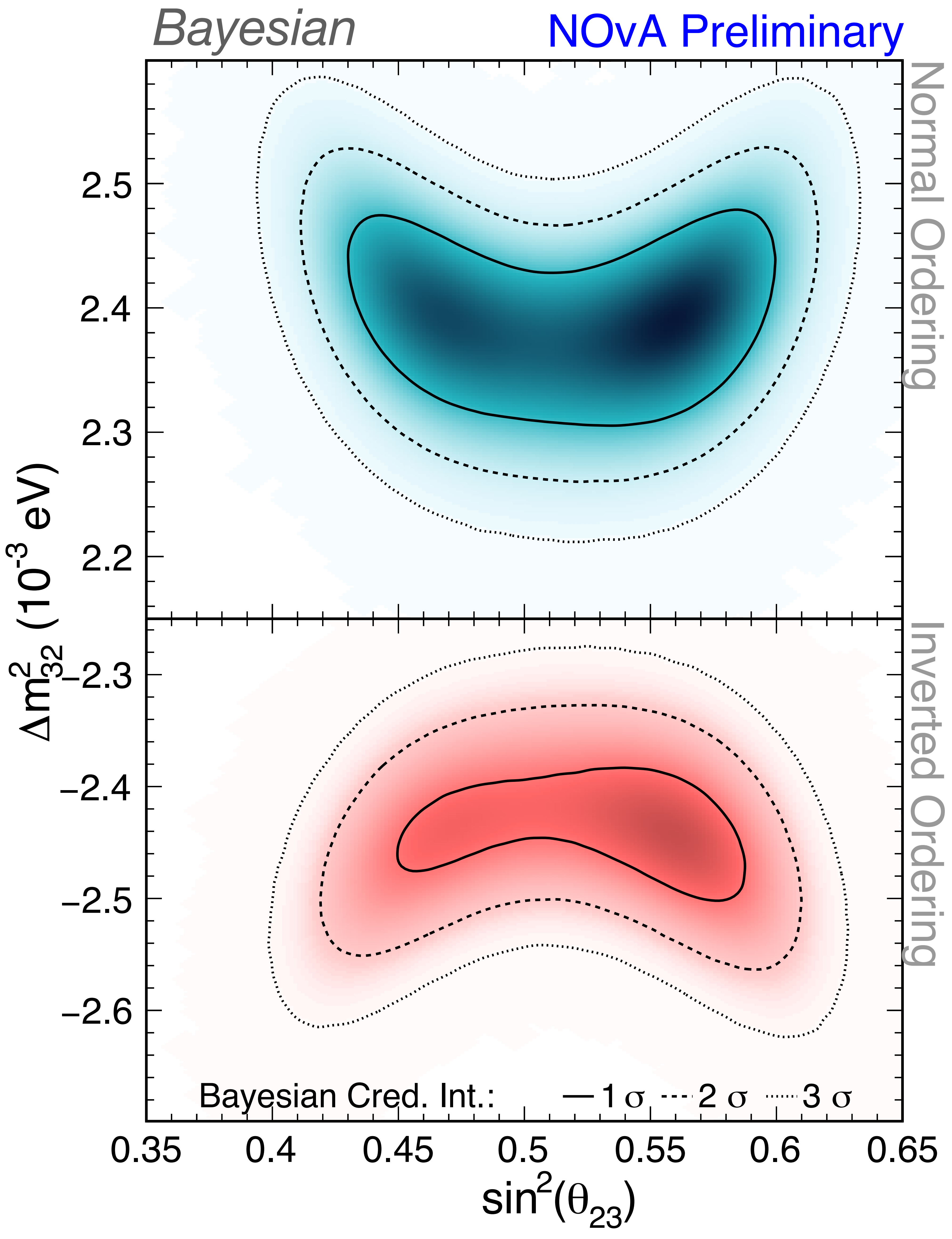
$$sin^2\theta_{23} = 0.56^{+0.03}_{-0.12}$$

$$\delta_{CP} = 0.89\ [0.54,1.07] \cup [1.99,0.48]\pi$$


Frequentist
Bayesian
NOvA Preliminary
≤1σ
1-2σ
2-3σ
Best Fit
NH
IH
Normal Ordering
Inverted Ordering
Bayesian Cred. Int.: 1 σ 2 σ 3 σ
$\sin^2\theta_{23}$
$\sin^2(\theta_{23})$
δ_{CP}
0 $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π
0.3 0.4 0.5 0.6 0.7
0.4 0.45 0.5 0.55 0.6



Frequentist
NOvA Preliminary
2.6
2.4
2.2
0.3
0.4
0.5
0.6
0.7
Δm^2_{32} (10^{-3}eV^2)
$\sin^2\theta_{23}$
≤1σ
1-2σ
2-3σ
Best Fit
NH
NOvA Preliminary
−2.2
−2.3
−2.4
−2.5
−2.6
−2.7
−2.8
IH



Bayesian
NOvA Preliminary
Normal Ordering
Inverted Ordering
2.5
2.4
2.3
2.2
−2.3
−2.4
−2.5
−2.6
Δm^2_{32} (10^{-3} eV)
Bayesian Cred. Int.: 1 σ 2 σ 3 σ
0.35
0.4
0.45
0.5
0.55
0.6
0.65
$\sin^2(\theta_{23})$

A Dual-Baseline Fit

$$1 - P(\nu_\mu \rightarrow \nu_s) \approx 1 - \cos^4\theta_{14}\cos^2\theta_{34}\sin^2 2\theta_{24}\sin^2\Delta_{41}$$
$$- \sin^2\theta_{34}\sin^2 2\theta_{23}\sin^2\Delta_{31}$$
$$+ \frac{1}{2}\sin\delta_{24}\sin\theta_{24}\sin 2\theta_{23}\sin\Delta_{31}$$

Approximate NC Disappearance Probability

(Full calculation used in fit)

At high Δm^2_{41}, ND oscillations give access to shape info + all sterile mixing angles

FD Oscillations at **atmospheric frequency** give us access to $\theta_{24}, \theta_{34}, \delta_{24}$

We also add in an additional sample, ν_μ **disappearance**

$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2 2\theta_{24}\Delta_{41}$$
$$+ 2\sin^2 2\theta_{23}\sin^2\theta_{24}\sin^2\Delta_{31}$$
$$- \sin^2 2\theta_{23}\sin^2\Delta_{31}$$

In ND, sterile frequency oscillations gives independent handle on θ_{24}

FD Oscillations at **atmospheric frequency** also give access to θ_{24}, but mixed up with 3F oscillations

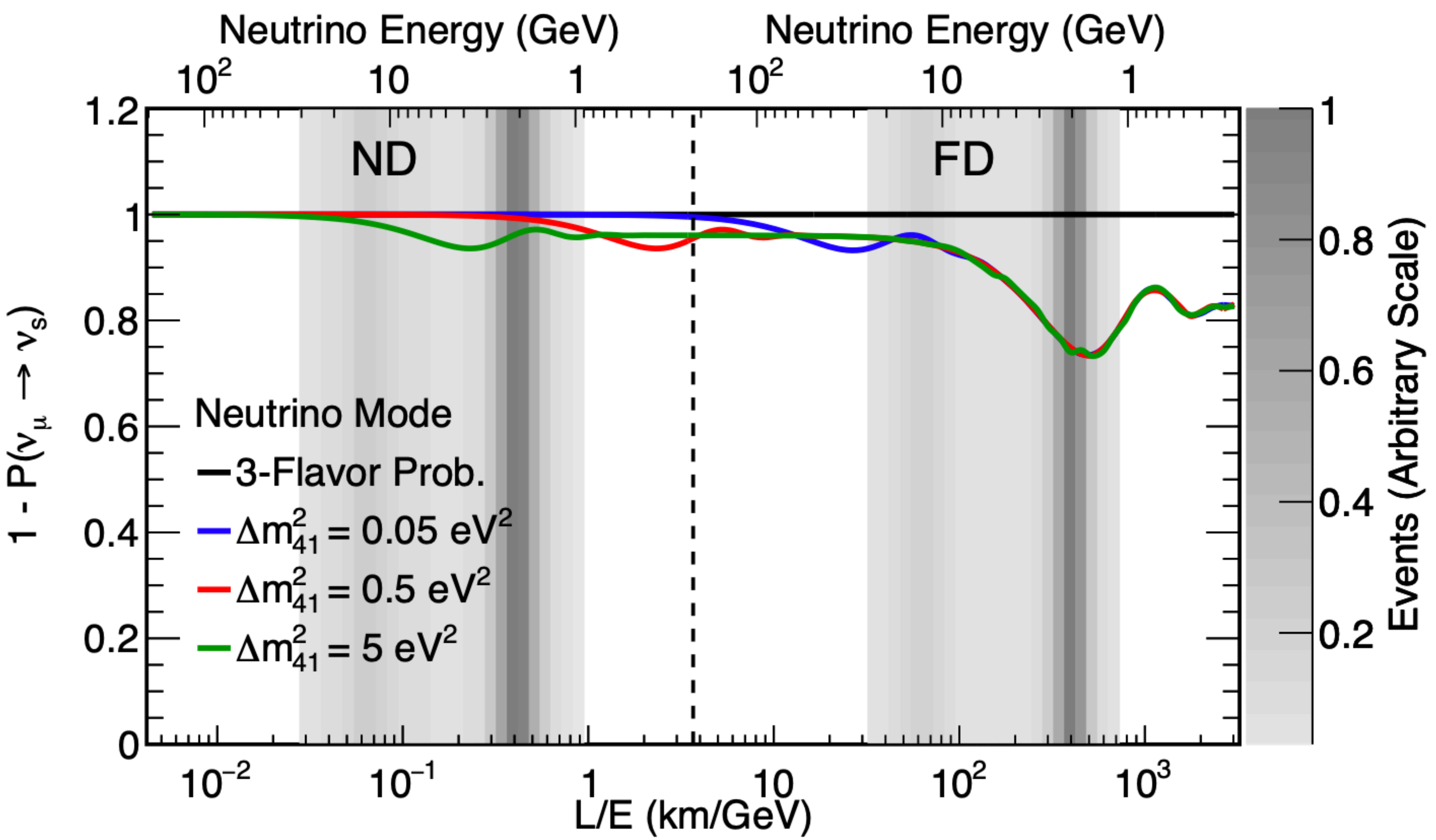
This also helps us to **break degeneracies between sterile-driven oscillations and systematic uncertainties**

Sterile Analysis - A Dual Baseline Fit

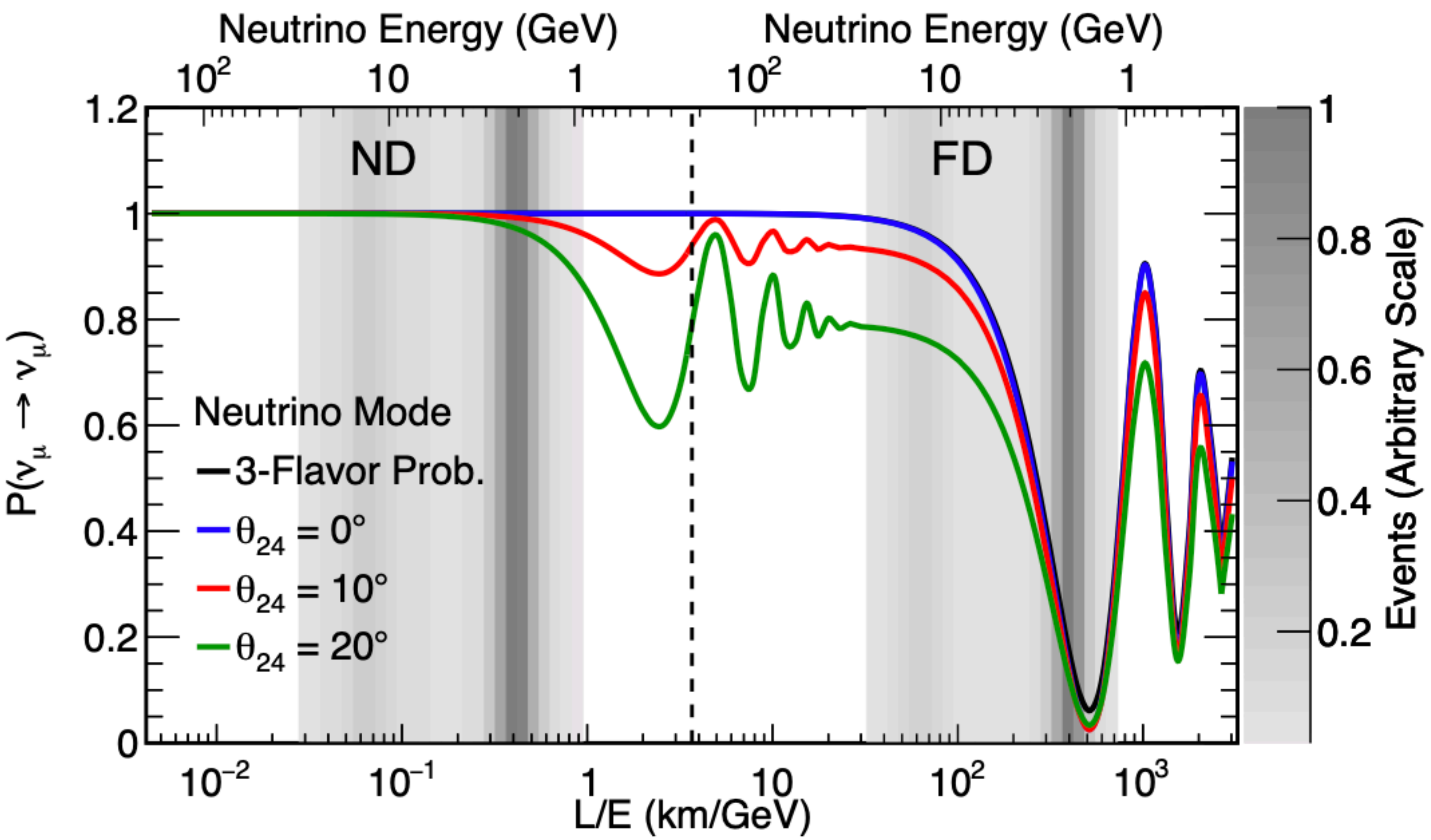
Most interesting region for sterile oscillations is around 1eV^2

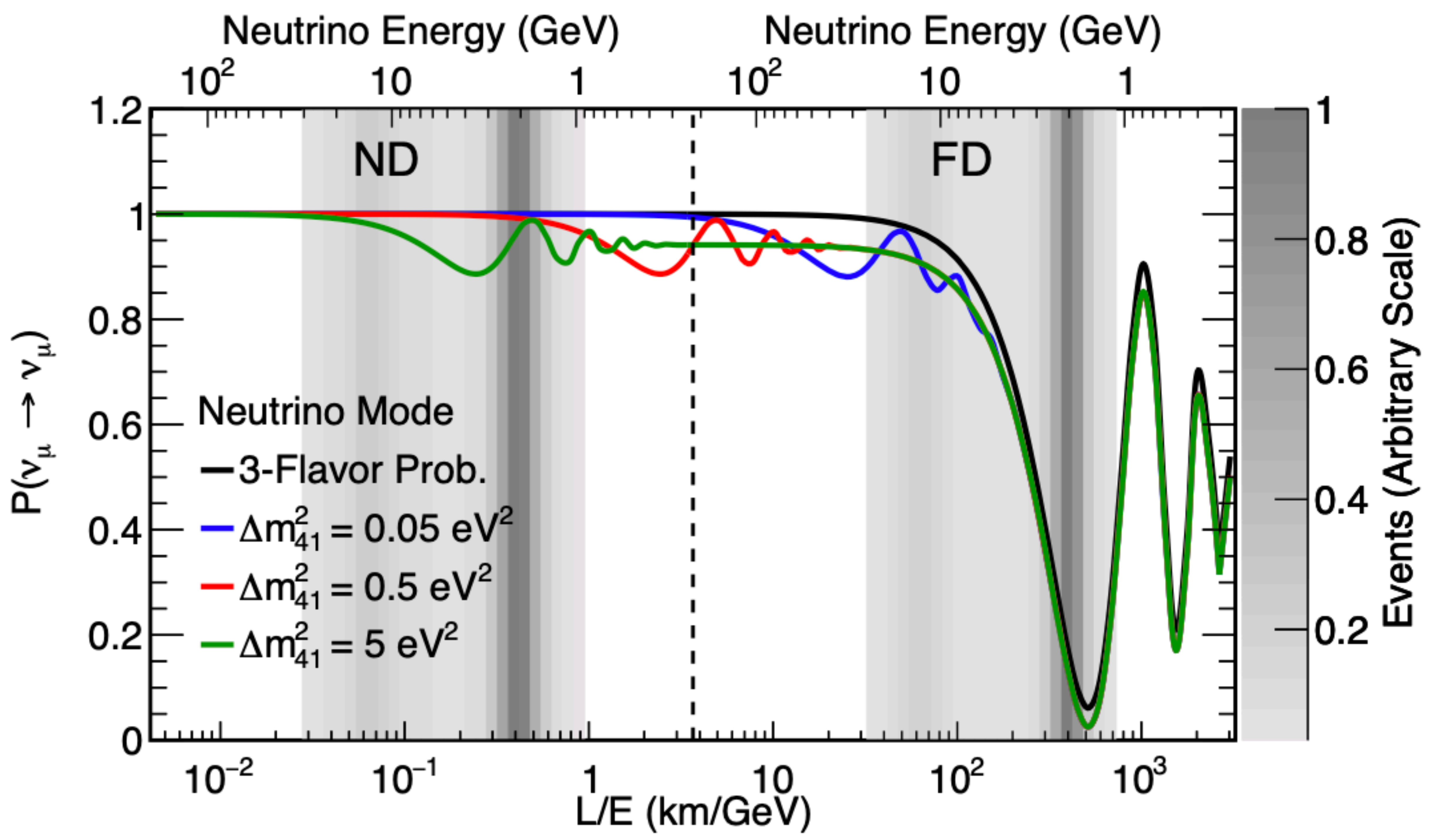
→ sensitivity requires seeing **oscillations in the ND**

Rather than using extrapolation technique, **use a dual-baseline fit**

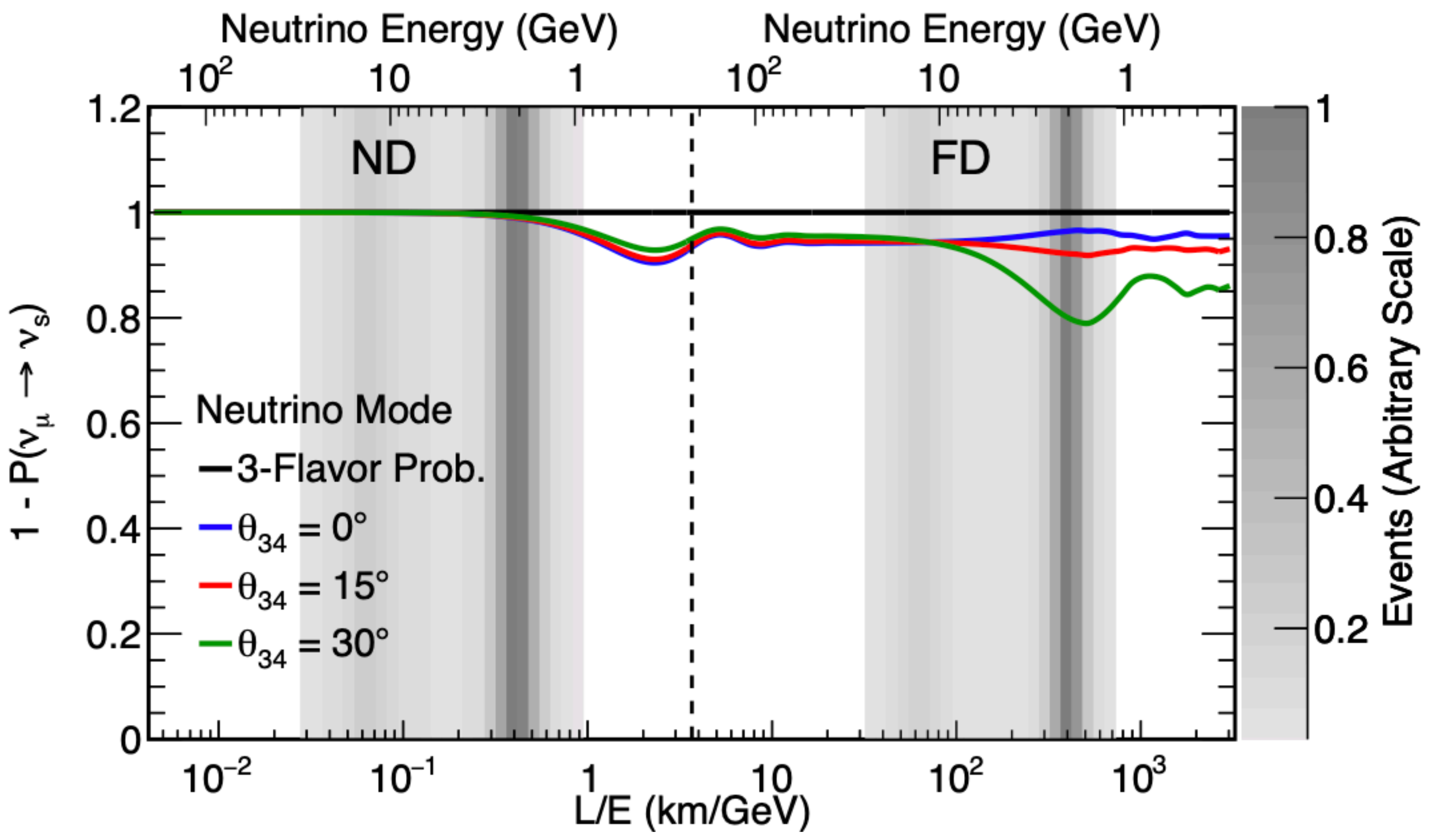


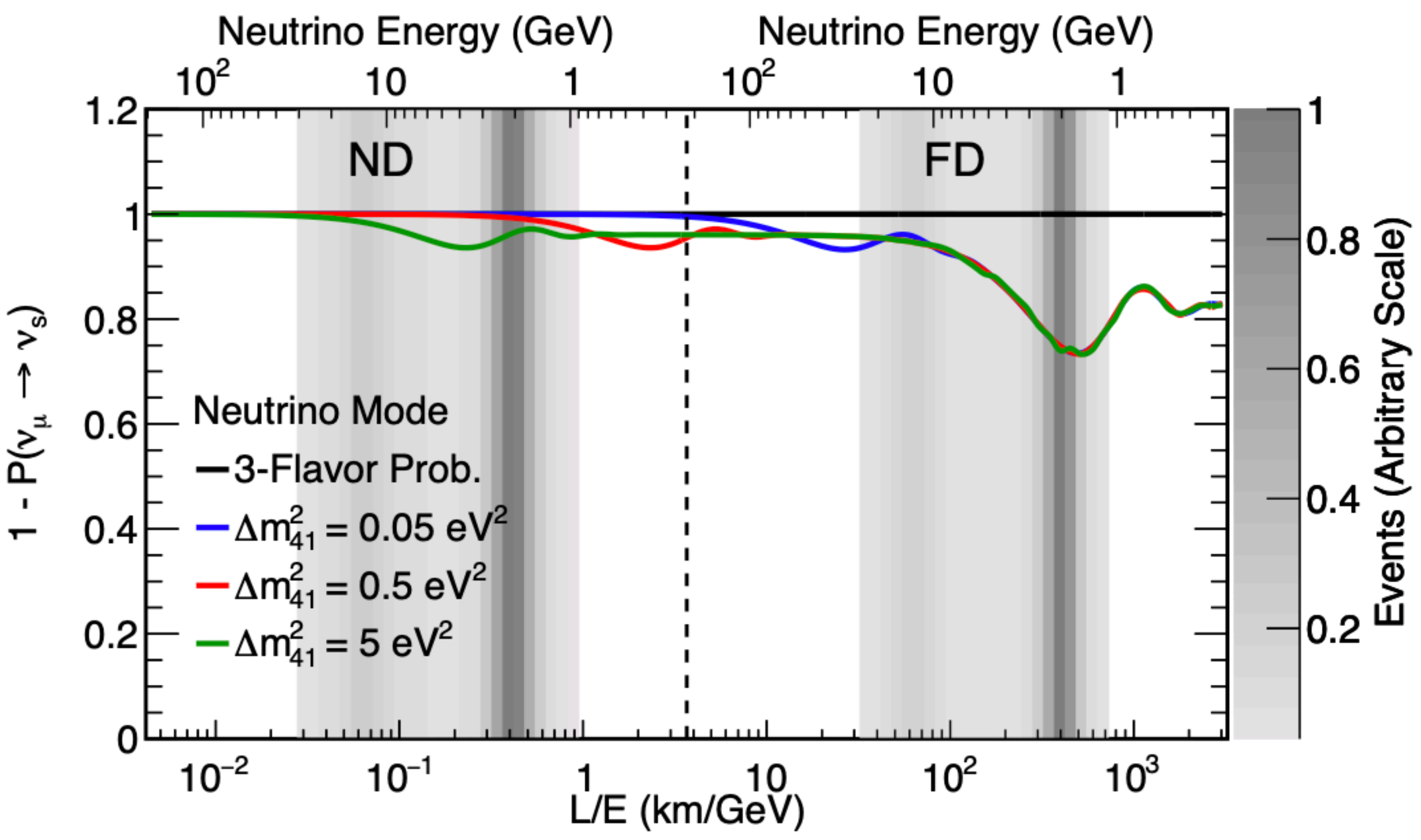
Oscillation Curves, ν_μ CC Disappearance





Oscillation Curves, NC Disappearance



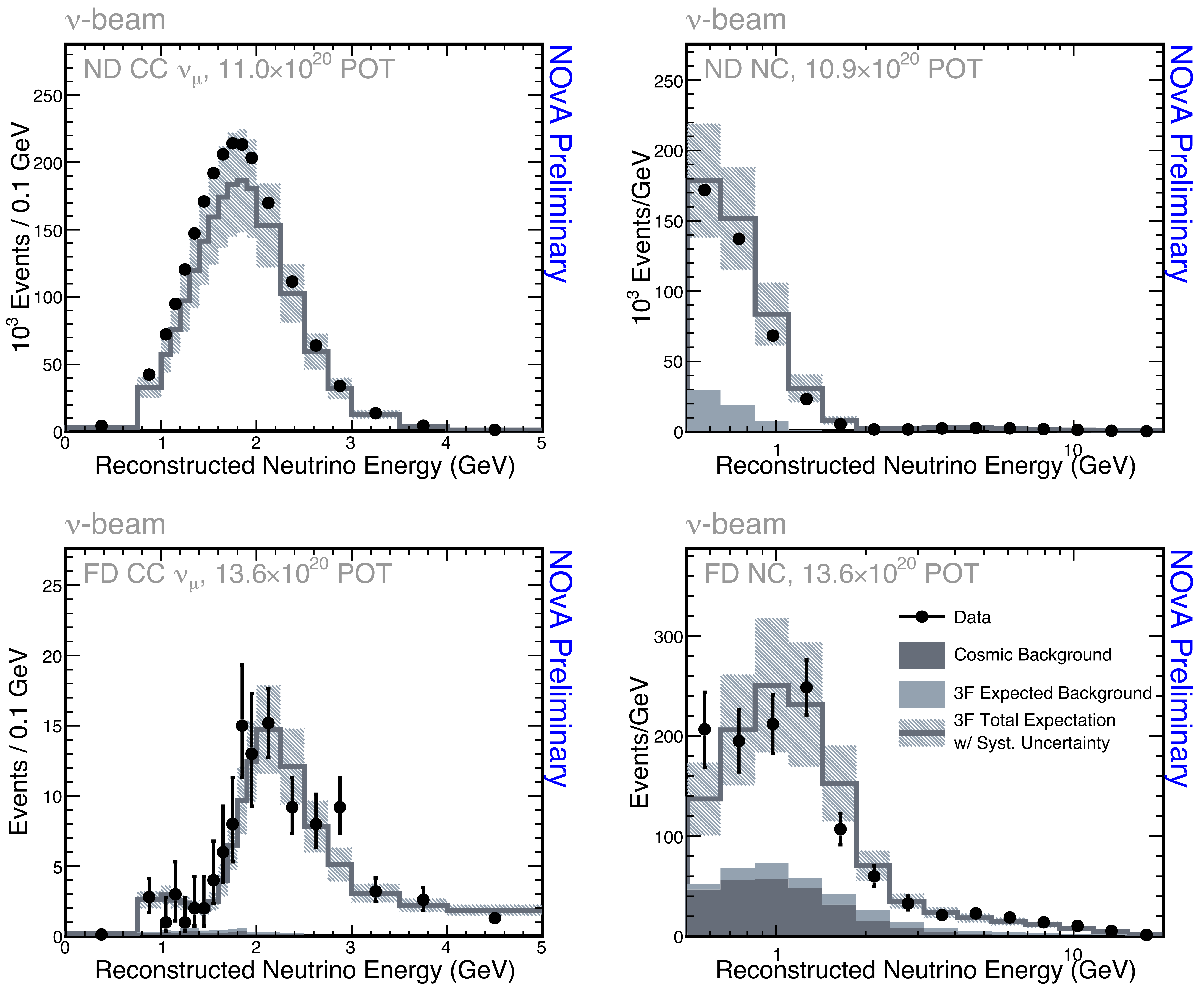


ND ν_μ CC	
Data	2826066
Prediction	2448720 ± 451259
Signal	2436864
Background	11855

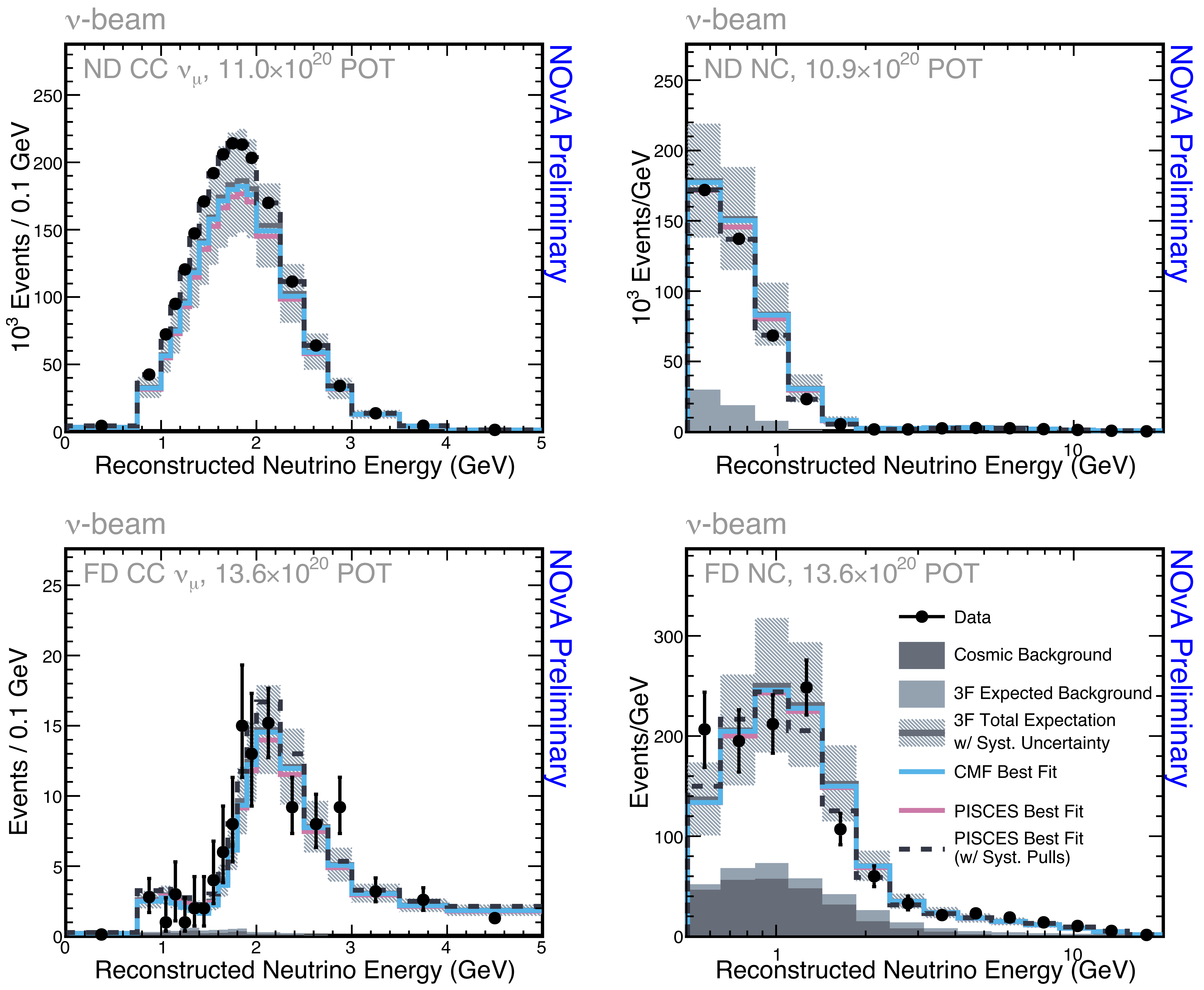
ND NC	
Data	103109
Prediction	115776 ± 25381
Signal	103635
Background	12142

FD ν_μ CC	
Data	209
Prediction	180.55 ± 34.79
Signal	171.88
Background	3.72
Cosmic	4.95

FD NC	
Data	469
Prediction	475.59 ± 30.36
Signal	324.51
Background	63.9
Cosmic	87.13



ν-beam
ND CC νμ, 11.0×10^20 POT
10^3 Events / 0.1 GeV
Reconstructed Neutrino Energy (GeV)
NOvA Preliminary
ν-beam
ND NC, 10.9×10^20 POT
10^3 Events/GeV
Reconstructed Neutrino Energy (GeV)
NOvA Preliminary
ν-beam
FD CC νμ, 13.6×10^20 POT
Events / 0.1 GeV
Reconstructed Neutrino Energy (GeV)
NOvA Preliminary
ν-beam
FD NC, 13.6×10^20 POT
Events/GeV
Data
Cosmic Background
3F Expected Background
3F Total Expectation
w/ Syst. Uncertainty
Reconstructed Neutrino Energy (GeV)
NOvA Preliminary



Two different fitting techniques

CMF

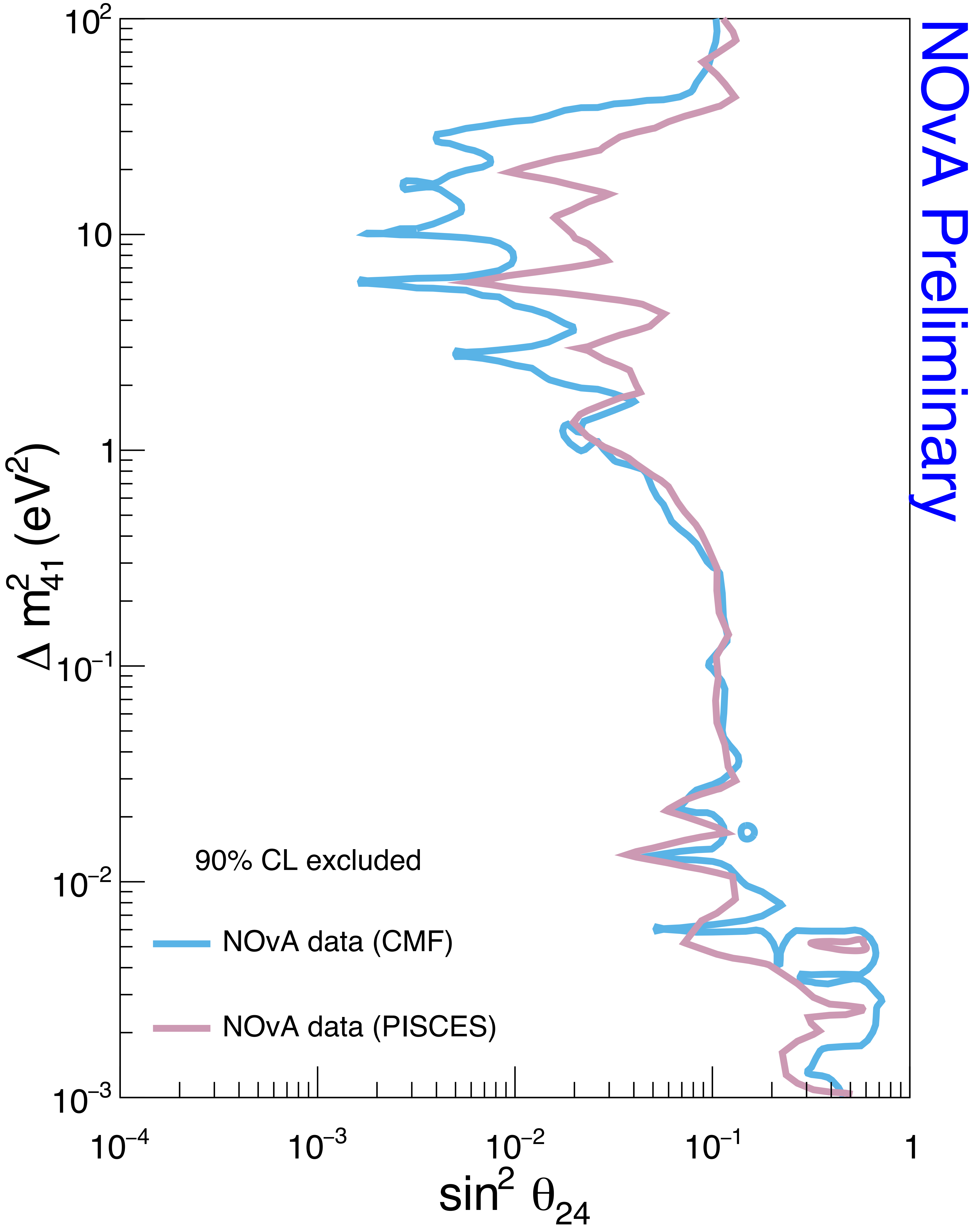
Gaussian Multivariate

Combined Neyman-Pearson χ^2

PISCES

Fits for systematic pulls, then Poisson statistics

Neutrino Beam



Two different fitting techniques

CMF

Gaussian Multivariate

Combined Neyman-Pearson χ^2

PISCES

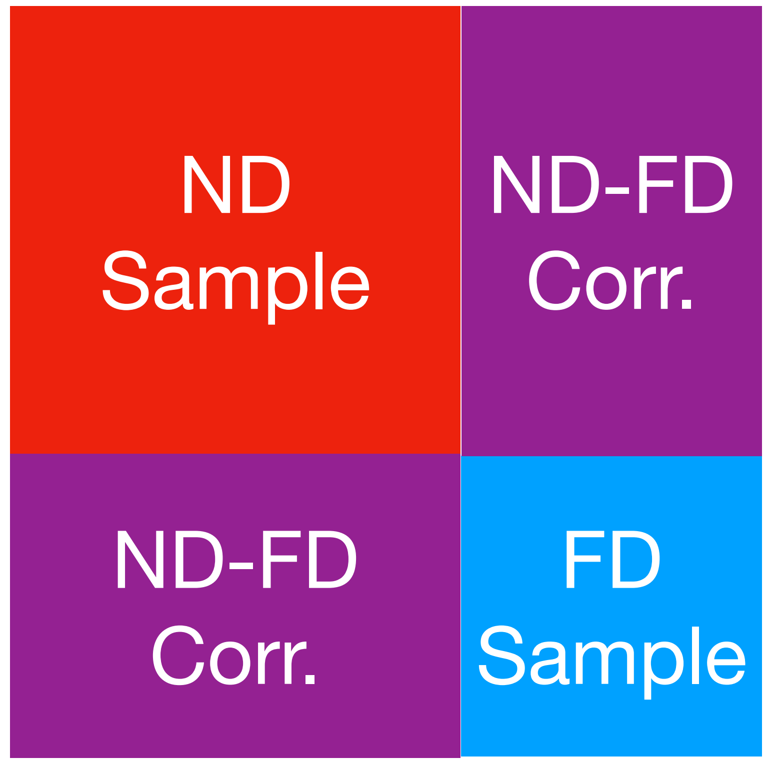
Fits for systematic pulls, then Poisson statistics

Covariance Matrix Fitting With CMF

$$\chi^2 = \sum_{i,j=1} [N_i^{\text{data}} - N_i^{\text{model}}(\Theta)] \times C_{i,j}^{-1} \times [N_j^{\text{data}} - N_j^{\text{model}}(\Theta)]$$

$$C_{i,j} = \frac{1}{U} \sum_{u=1}^{U} [N_i^{CV} - N_i^{u}] \times [N_j^{CV} - N_j^{u}]$$

Covariance matrix bakes in bin-to-bin correlations

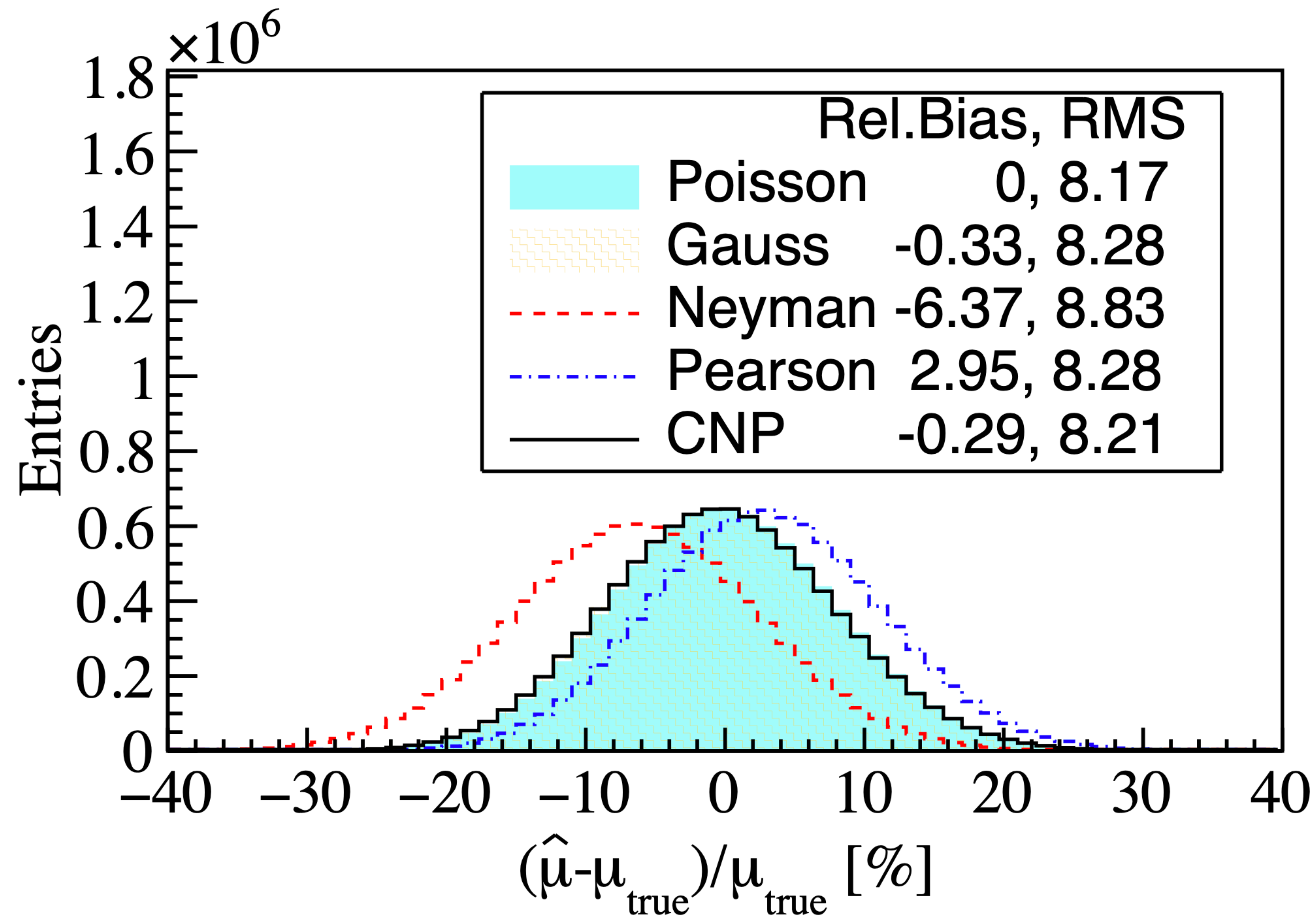


$$C = C_{\text{stat.}} + C_{\text{flux}} + C_{\text{cross-section}} + C_{\dots}$$

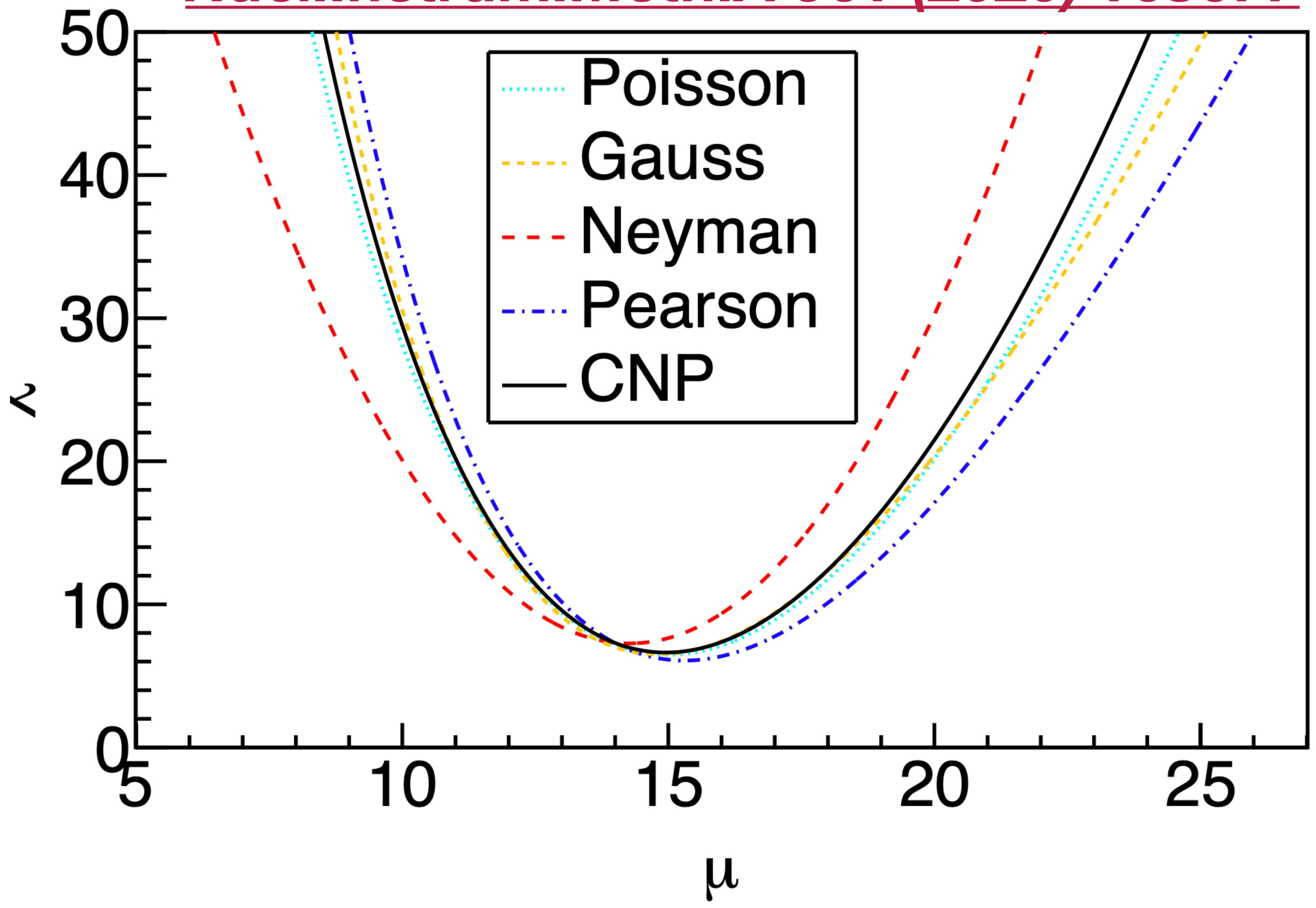
Combined Neyman-Pearson Tchnique

$$\chi^2_{\mathrm{CNP}} \equiv \frac{1}{3}\left(\chi^2_{\mathrm{Neyman}} + 2\chi^2_{\mathrm{Pearson}}\right) = \sum_{i=1}^{n} \frac{(\mu - M_i)^2}{3/(\frac{1}{M_i} + \frac{2}{\mu})}$$

Linear combination of Neyman and Pearson χ^2 give a result that is less biased compared to Poisson statistics and has a more similar RMS



Nucl.Instrum.Meth.A 961 (2020) 163677



Covariance Matrix Fitting With PISCES

$$\chi^2 = \chi^2_{\text{stat}} + \chi^2_{\text{syst}}$$

$$\chi^2_{\text{syst}} = \sum_{ij}^{N} \sum_{\alpha\beta}^{M} (s_{\alpha i} - 1) C_{\alpha i \beta j} (s_{\beta j} - 1)$$

$$\chi^2_{\text{stat}} = 2 \sum_{i}^{N} \left[\left(\sum_{\alpha}^{M} \mu_{\alpha i} s_{\alpha i} \right) - x_i + x_i \log\left(\frac{x_i}{\sum_{\alpha}^{M} \mu_{\alpha i} s_{\alpha i}} \right) \right]$$

i = analysis bin
α = beam component
μ = nominal prediction

s = systematic shift
x = data
C = covariance matrix

Predict spectra for oscillation

Decompose into oscillation channels

Solve for systematic weights

Apply systematic weights by channel

Recompose into systematically shifted spectra

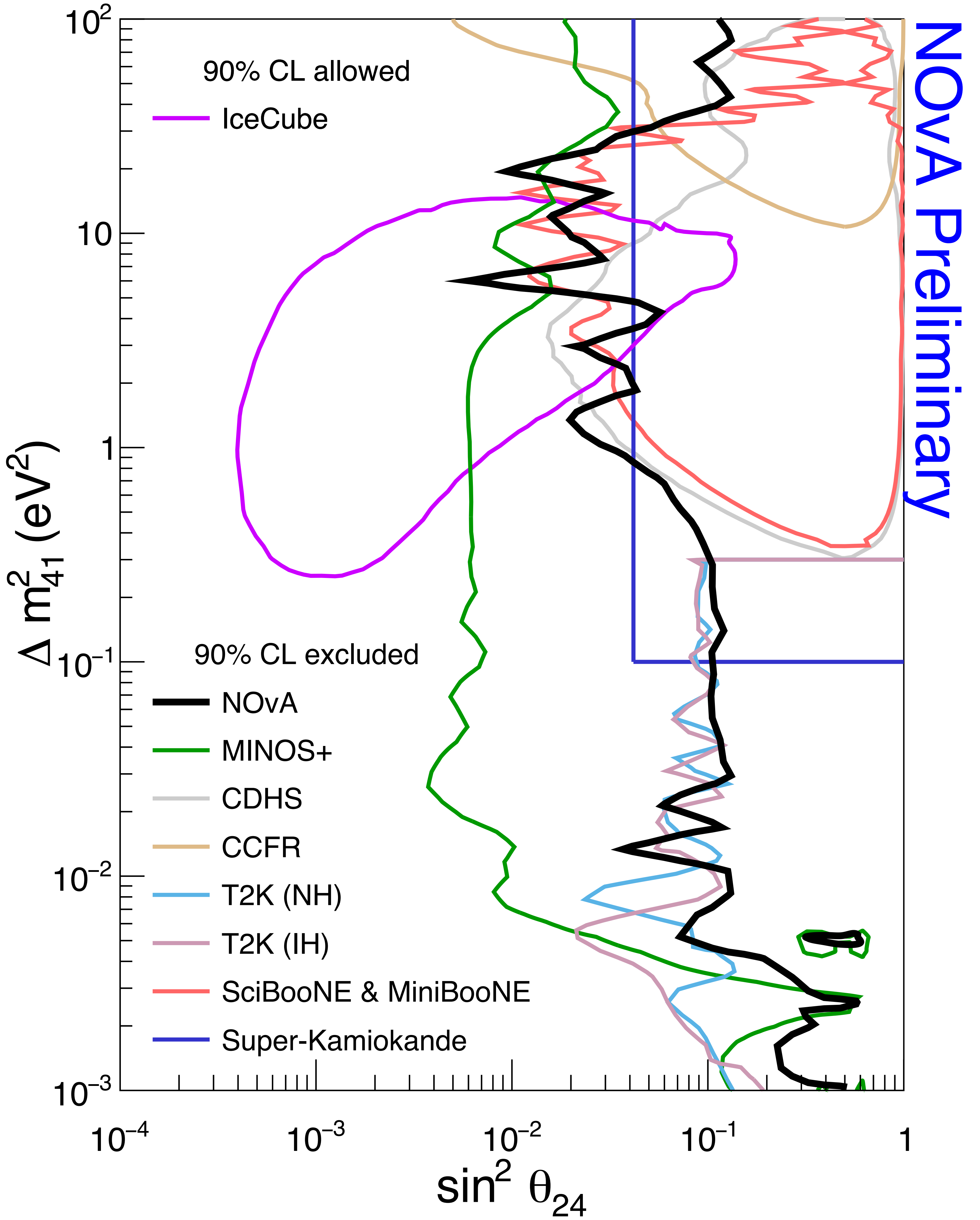
References for "With Friends" Contours

SK: K. Abe et al. (Super- Kamiokande), Phys. Rev. D 91, 052019 (2015)
CDHS: F. Dydak et al. (CDHSW), Phys. Lett. B 134, 281 (1984)
CCFR: I.E. Stockdale et al. (CCFR), Phys. Rev. Lett. 52, 1384 (1984)
SciBooNE: K. B. M. Mahn et al. (SciBooNE, MiniBooNE), Phys. Rev. D 85, 032007 (2012)
MINOS+: P. Adamson et al. (MINOS+) Phys. Rev. Lett. 122, 091803 (2019)
T2K: K. Abe et al. (T2K) Phys. Rev. D 99, 071103(R) (2019)
IceCube: M. G. Aartsen et al. (IceCube), Phys. Rev. Lett. 125, 141801 (2020)

SK Constrains $\sin^2\theta_{24} < 0.041$ @ $\Delta m^2_{41} > 0.1$ eV^2

IceCube allowed region is an exclusion region at 95%

Neutrino Beam



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SK: K. Abe et al. (Super- Kamiokande), Phys. Rev. D 91, 052019 (2015)
MINOS+: P. Adamson et al. (MINOS+) *Phys.Rev.Lett. 15, 151803* 117 (2016)
T2K: K. Abe et al. (T2K) Phys. Rev. D 99, 071103(R) (2019)
IceCube: M. G. Aartsen et al. (IceCube), *Phys.Rev.D* 95 11, 112002 (2017)

MINOS+ - constrains $\sin^2(\theta_{34}) < 0.2$ @ $\Delta m^2_{41} = 0.5$ eV^2

T2K - constrains $|U_{\tau4}|^2 < 0.5$ @ $\Delta m^2_{41} = 0.1$ eV^2

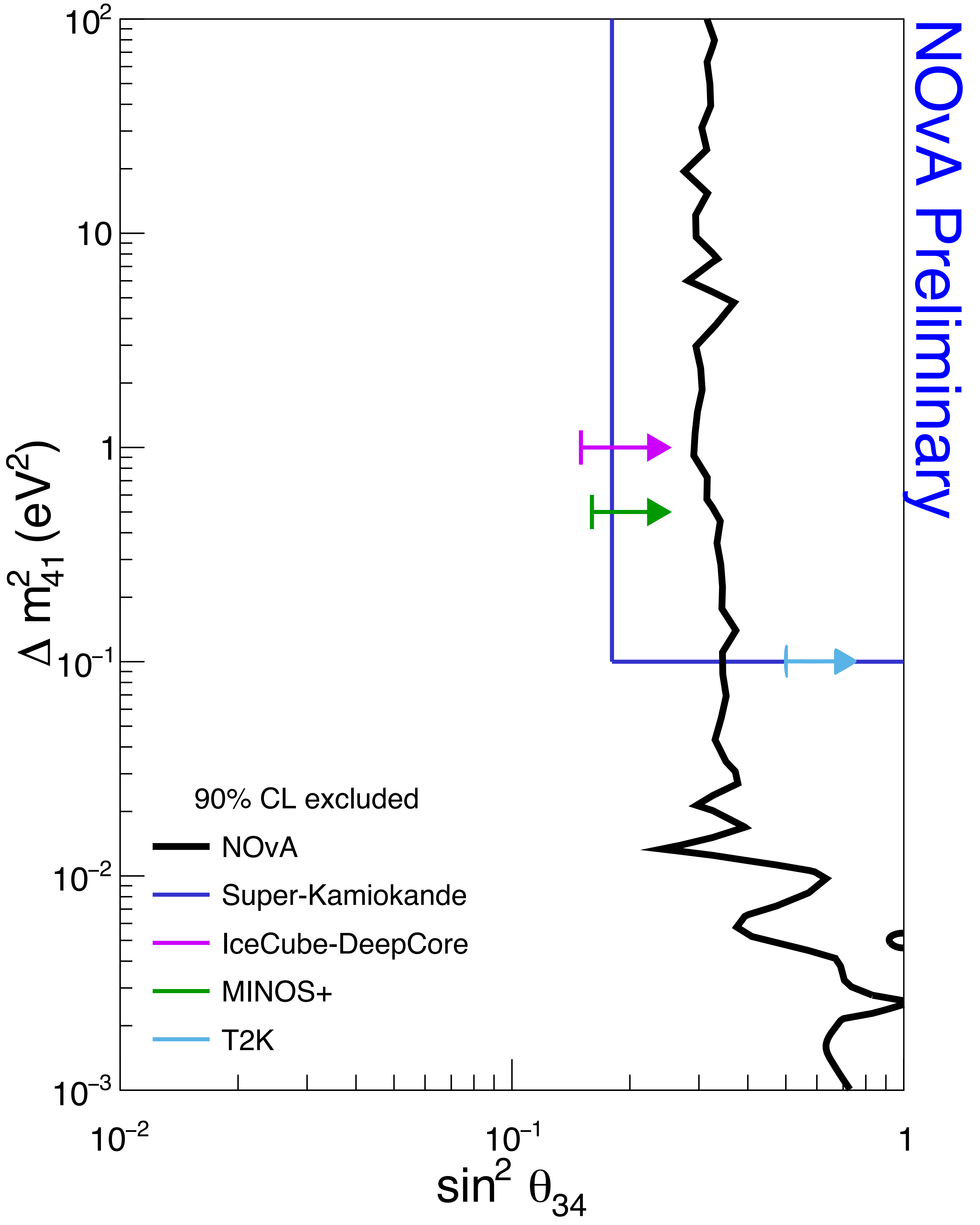
IceCube - constrains $|U_{\tau4}|^2 < 0.15$ @ $\Delta m^2_{41} = 1$ eV^2

SK - constrains $|U_{\tau4}|^2 < 0.18$ @ $\Delta m^2_{41} > 0.1$ eV^2

$$|U_{\tau4}|^2 = \cos^2\theta_{14}\cos^2(\theta_{24})\sin^2(\theta_{34})$$

Generally set to zero

Neutrino Beam



References for "With Friends" Contours

CDHS: F. Dydak et al. (CDHSW), Phys. Lett. B 134, 281 (1984)
CCFR: I.E. Stockdale et al. (CCFR), Phys. Rev. Lett. 52, 1384 (1984)
E531: N. Ushida et al. *Phys.Rev.Lett.* 57 (1986) 2897-2900
CHORUS: R. Tsenov et al. *Balk.Phys.Lett.* 17 (2009) 191-200
NOMAD: P. Astier et al. *Nucl.Phys.B* 611 (2001) 3-39
OPERA: N. Agafonova et al. *Phys.Rev.D* 100 (2019) 5, 051301

$$\sin^2(2\theta_{\mu\tau}) = \sin^2(2\theta_{24})\sin^2(\theta_{34})$$