On the evaluation of the IBD cross section

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based on

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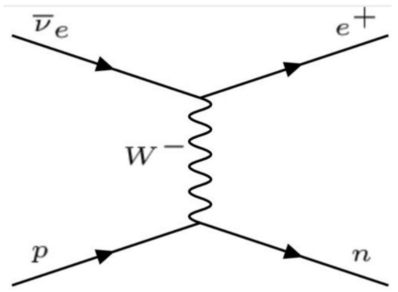
RELEVANCE

Inverse beta decay (IBD)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$



by far the most important mechanism of interaction of low-energy antineutrinos

-all reactor neutrino oscillation studies (Daya Bay, RENO, JUNO…)

-core-collapse SN neutrino detection (SN1987, Kamiokande-II, IMB, …HyperKamiokande)

-geoneutrinos (KamLAND, Borexino…)

In water & scintillator detectors (lots of «free» protons) dominates by orders of magnitude

Precision crucial for high-statistics experiments

LONG STORY SHORT

✓ first cross section prediction 1934 [Bethe & Peierls Nature]

✓ first measurement 1956 [Cowan & Reines, Science]

✓ many improvements since then:

rad. corrections, form factors, V-A structure, hadron mixing

[e.g. 2311.16730.pdf for historical overview]

Most *accurate* cross section estimates available date *about 20 years ago*:

-first order in $\frac{E_\nu}{m_p}$, nucleon recoil [Beacon, Vogel (Phys. Rev. D 1999)]

-all orders, fully relativistic, uncertainty [Strumia, Vissani (Phys. Lett. B, 2003)]

THIS WORK

Estimate *current* IBD cross *section:*
central values and uncertainties assessment

Neutrino interactions with a free nucleon:

- same basic characteristics of lepton scattering
- different hadronic current

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1-\gamma_5) v_e \cdot \quad \bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 + f_3 \frac{q_a}{M} + i g_3 \sigma_{ab} \frac{q^b}{2M} \gamma_5 \right) u_p$$

Full on shell matrix element, precision at low energies

$$\frac{d\sigma}{dt} = \frac{G_F^2 \cos^2\theta_C}{64\pi (s - m_p^2)^2} \overline{|\mathcal{M}^2|}$$

FORM FACTORS

$$\bar{u}_n\left(f_1\gamma_a + g_1\gamma_a\gamma_5 + if_2\sigma_{ab}\frac{q^b}{2M} + g_2\frac{q_a}{M}\gamma_5 + f_3\frac{q_a}{M} + ig_3\sigma_{ab}\frac{q^b}{2M}\gamma_5\right)u_p$$

$$\{f_1, f_2\} = \frac{\{1-(1+\xi)t/4M^2, \xi\}}{(1-t/4M^2)(1-t/M_V^2)^2} \qquad \xi = k_p - k_n = 3.706$$

Form factor linear expansion (at lower energies) *in lieu* of modelling
(double dipole is untested at small E_ν)

$$\frac{F(Q^2)}{F(0)} \equiv 1 - \frac{\langle r^2\rangle\, Q^2}{6} + \mathcal{O}(Q^4)$$

Y.H.Lin, H.W. Hammer and U. G. Meissner, Eur. Phys. J. A 57 (2021) no.8, 255

$$f_1 \approx 1 + \frac{(2.41 \pm 0.02)\ t}{\mathrm{GeV}^2} \qquad f_2 \approx \xi\left(1 + \frac{(3.21 \pm 0.02)\ t}{\mathrm{GeV}^2}\right)$$

Known with precision (negligible source of uncertainty to the cross section)

$$\bar{u}_n\left(f_1\gamma_a + g_1\gamma_a\gamma_5 + if_2\sigma_{ab}\frac{q^b}{2M} + g_2\frac{q_a}{M}\gamma_5 + f_3\frac{q_a}{M} + ig_3\sigma_{ab}\frac{q^b}{2M}\gamma_5\right)u_p$$

SCC

G-parity $G = Ce^{i\pi I_2}$

$GV_\mu G^{-1} = V_\mu\,,\quad GA_\mu G^{-1} = -A_\mu$ First Class

$GV_\mu G^{-1} = -V_\mu\,,\quad GA_\mu G^{-1} = A_\mu$ Second Class

absent if SU(3) symmetry or charge symmetry & time reversal (Ankovski; Giunti; Ivanov, …)

SCC previously neglected for neutrino-nucleon cross section

Are they relevant?

IBD CROSS SECTION EVALUATION

$$\frac{d\sigma}{dt} = \frac{G_F^2 \boxed{\cos^2\theta_C}}{64\pi(s-m_p^2)^2}\overline{|\mathcal{M}^2|}$$

$$\overline{|\mathcal{M}^2|} = A_{\bar\nu}(t) - (s-u)B_{\bar\nu}(t) + (s-u)^2 C_{\bar\nu}(t)$$

$s = (p_\nu + p_p)^2,\ t = q^2 = (p_\nu - p_e)^2 < 0,\ u = (p_\nu - p_n)^2$

$$\Delta = m_n - m_p \approx 1.293\,\mathrm{MeV} \qquad M = \frac{m_n + m_p}{2} \approx 938.9\,\mathrm{MeV}$$

$$\begin{aligned} A_{\bar\nu} = (t-m_e^2)\Big[&8|f_1^2|(4M^2+t+m_e^2) + 8|g_1^2|(-4M^2+t+m_e^2) + 2|f_2^2|(t^2/M^2+4t+4m_e^2) \\ &+8m_e^2 t|g_2^2|/M^2 + 16\mathrm{Re}[f_1^* f_2](2t+m_e^2) + 32m_e^2\mathrm{Re}[g_1^* g_2]\Big] \\ -\Delta^2\Big[&(8|f_1^2| + 2t|f_2^2|/M^2)(4M^2+t-m_e^2) + 8|g_1^2|(4M^2-t+m_e^2) + 8m_e^2|g_2^2|(t-m_e^2)/M^2 \\ &+16\mathrm{Re}[f_1^* f_2](2t-m_e^2) + 32m_e^2\mathrm{Re}[g_1^* g_2]\Big] - 64m_e^2 M\Delta\mathrm{Re}[g_1^*(f_1+f_2)] + A_{SC} \end{aligned}$$

$$B_{\bar\nu} = 32t\mathrm{Re}[g_1^*(f_1+f_2)] + 8m_e^2\Delta(|f_2^2| + \mathrm{Re}[f_1^* f_2 + 2g_1^* g_2])/M + B_{SC}$$

$$C_{\bar\nu} = 8(|f_1^2| + |g_1^2|) - 2t|f_2^2|/M^2 + C_{SC}$$

g_2 only enters the cross section in terms suppressed by m_e^2, or powers of 1/M

For the SCC contribution (not considered in Strumia, Vissani 2003) we find:

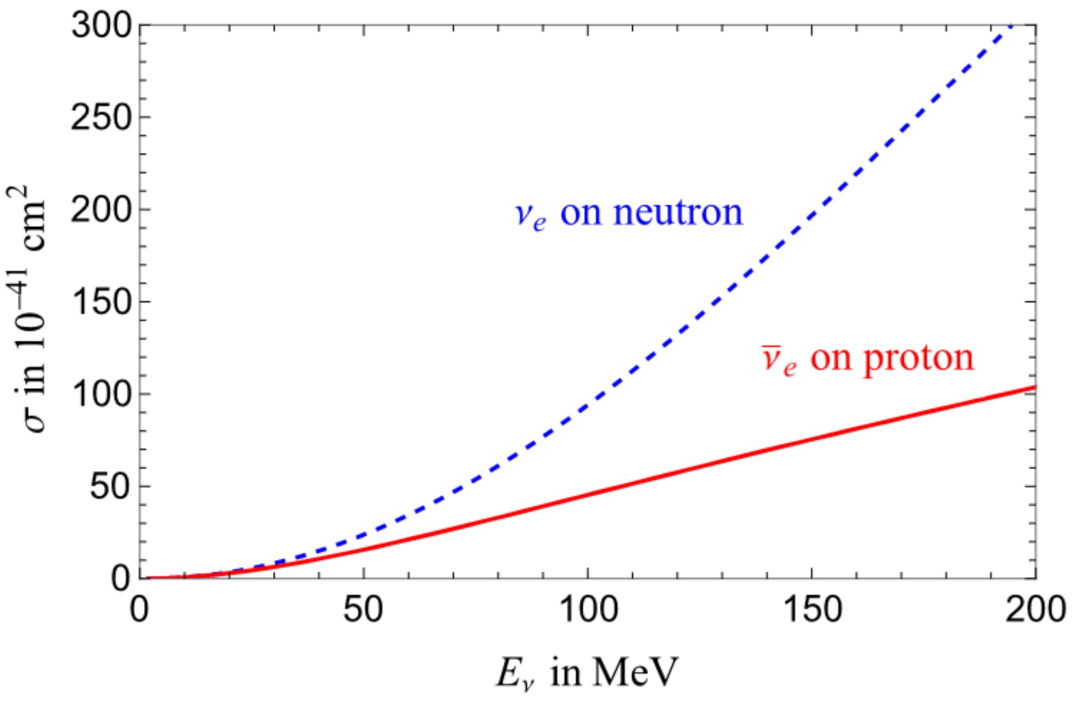
$$A_{SC} = -2\,t\,(4 - t/M^2)\Big[4m_e^2|f_3^2| + |g_3^2|(t - m_e^2) + 8\Delta M \mathrm{Re}[g_3^* g_1] + \Delta^2|g_3^2|\Big] + \mathcal{O}(\Delta^3 M) + \mathcal{O}(\Delta\, m_e^2\, t/M) + \mathcal{O}(m_e^4)$$

$$B_{SC} = 8\, m_e^2\Big[4\,\mathrm{Re}[f_1^* f_3] + \mathrm{Re}[f_2^* f_3]\, t/M^2 + 2\,\mathrm{Re}[g_1^* g_3] + \mathrm{Re}[g_2^* g_3]\, t/M^2 + \Delta|g_3^2|/M\Big] + 16\Delta \mathrm{Re}[(f_1^* + f_2^*)g_3]\, t/M$$

$$C_{SC} = -2|g_3^2|\, t/M^2$$

This suggest we expect SSC dominated by g_3 and suppressed (especially low t)

QE SCATTERING CROSS SECTION (IBD & $\nu_e + n \rightarrow e^- + p$)



We include radiative corrections (A. Kurylov, M. J. Ramsey-Musolf and P. Vogel, PRC 67(2003), 035502)

$$d\sigma(E_\nu, E_e) \rightarrow d\sigma(E_\nu, E_e)\left[1 + \frac{\alpha}{\pi}\left(6.00 + \frac{3}{2}\log\frac{m_p}{2E_e} + 1.2\left(\frac{m_e}{E_e}\right)^{1.5}\right)\right]$$

& final state interactions (Sommerfeld) for the crossed channel reaction $\nu_e + n \rightarrow e^- + p$

$$F(E_e) = \frac{\eta}{1 - \exp(-\eta)}, \text{ with } \eta = \frac{2\pi\alpha}{\sqrt{1 - m_e^2/E_e^2}}$$

REASONS OF UNCERTAINTY AT LOW ENERGY

Cabibbo angle

$$\frac{d\sigma}{dt} = \frac{G_F^2 \cos^2\theta_C}{64\pi(s-m_p^2)^2}\overline{|\mathcal{M}^2|}$$

Super-allowed charged current transitions $\longrightarrow$ $V_{\rm ud}(\text{s.a.}) = 0.9737(3)$

$V_{\rm us} = 0.2245(8)$ $V_{\rm ub} = 3.82(24)\times 10^{-3}$ $\longrightarrow$ $V_{\rm ud}(\text{unit}) = 0.9745(2)$

CKM unitarity

Axial coupling

$$\lambda = -\frac{g_1(0)}{f_1(0)}$$

Direct measure from polarized neutrons

We include the last most precise measure from Perkeo III (2019) and the 8 previous measurements, also the four obtained before 2002 (which have potential systematic problems, but enlarging their errors by a factor of 2)

Neutron lifetime (constraint)

$$\frac{1}{\tau_{\rm n}} = \frac{V_{\rm ud}^2\,(1+3\lambda^2)}{4906.4\pm 1.7\text{s}}$$

$$\tau_{\rm n}(\text{SM}) = 878.38 \pm 0.89\ \text{s}$$

Two methods for τ_n measurements:

1) ultra-cold neutrons are trapped and their number is measured over time (*tot*)
2) using beam neutrons, single channel decay rates are measured (*beam*)

$\delta\sigma(V_{\rm ud}) = 0.66$ ‰

$\delta\sigma(\lambda) = 0.68$ ‰

$\delta\sigma = 0.94$ ‰

$\delta\sigma(V_{\rm ud}) = 0.53$ ‰

$\delta\sigma(\lambda) = 0.55$ ‰

$\delta\sigma = 0.52$ ‰

Strumia, Vissani (2003) estimated δσ = 0.4%

Improvement by at least a factor 4!

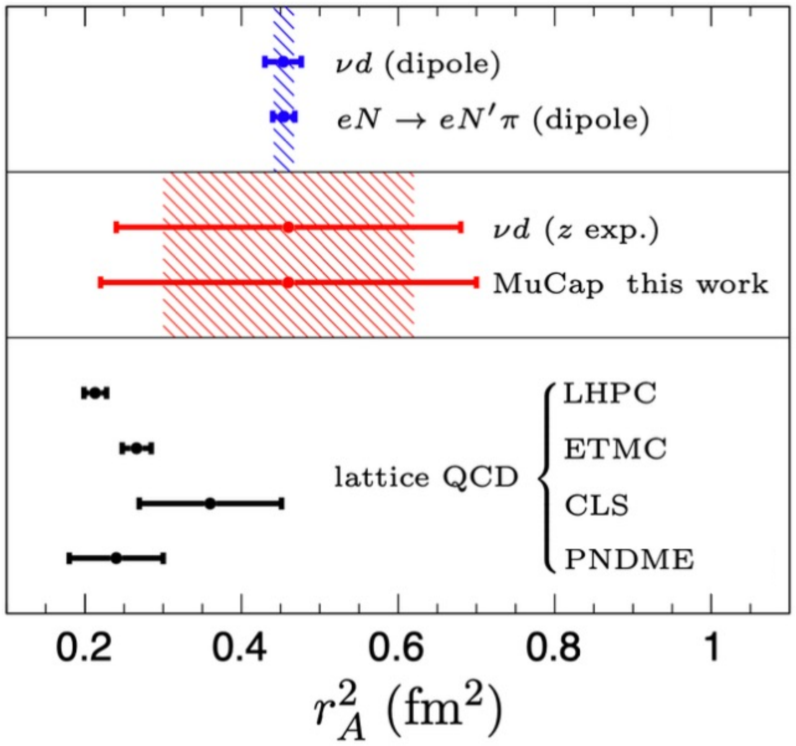
$$\delta\sigma = \sqrt{\vec{\xi}^{\,t}\,\Sigma^2\,\vec{\xi}} \qquad \vec{\xi} = \left(\frac{\partial\sigma}{\partial V_{\rm ud}}\,,\,\frac{\partial\sigma}{\partial\lambda}\right)\Bigg|_{\text{best fit}}$$

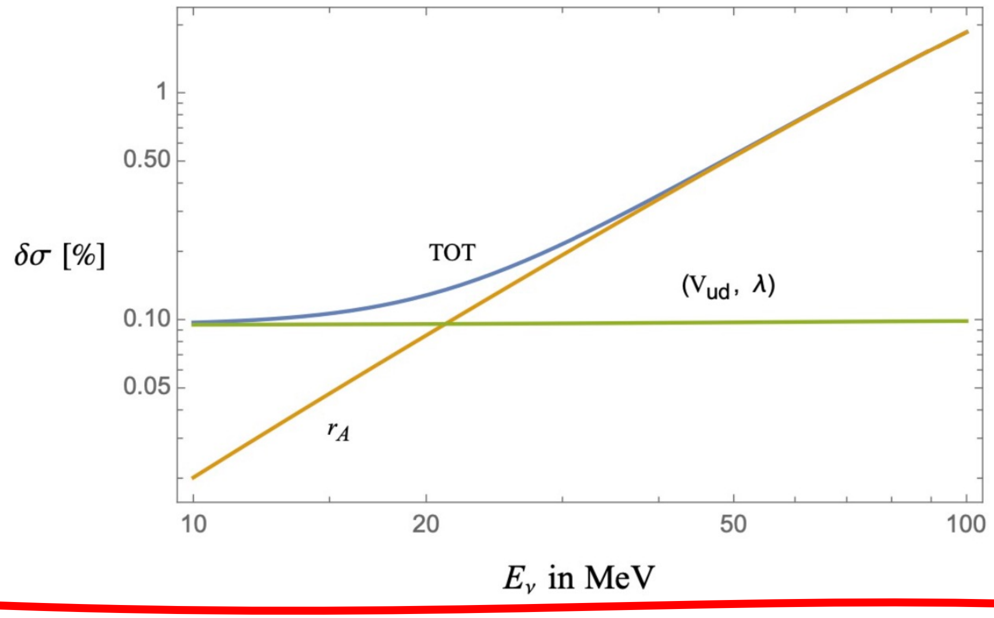
$$\Sigma^2 = \begin{pmatrix} (\delta V_{\rm ud})^2 & , \ \rho\,\delta V_{\rm ud}\,\delta\lambda \\ \rho\,\delta V_{\rm ud}\,\delta\lambda\ , & (\delta\lambda)^2 \end{pmatrix}$$

ρ is the correlation coefficient

Expansion in terms of radii

$$\frac{g_1(Q^2)}{g_1(0)} \equiv 1 - \frac{\langle r_{\mathrm{A}}^2 \rangle \, Q^2}{6} + \mathcal{O}(Q^4)$$





Positron spectrum in Super-Kamiokande

$$\frac{dS_e}{dE_e} = N_p \int_{E_\nu^{max}}^{E_\nu^{min}} dE_\nu \frac{dF}{dE_\nu}(E_\nu) \frac{d\sigma}{dE_e}(E_\nu, E_e) \, \epsilon(E_e)$$

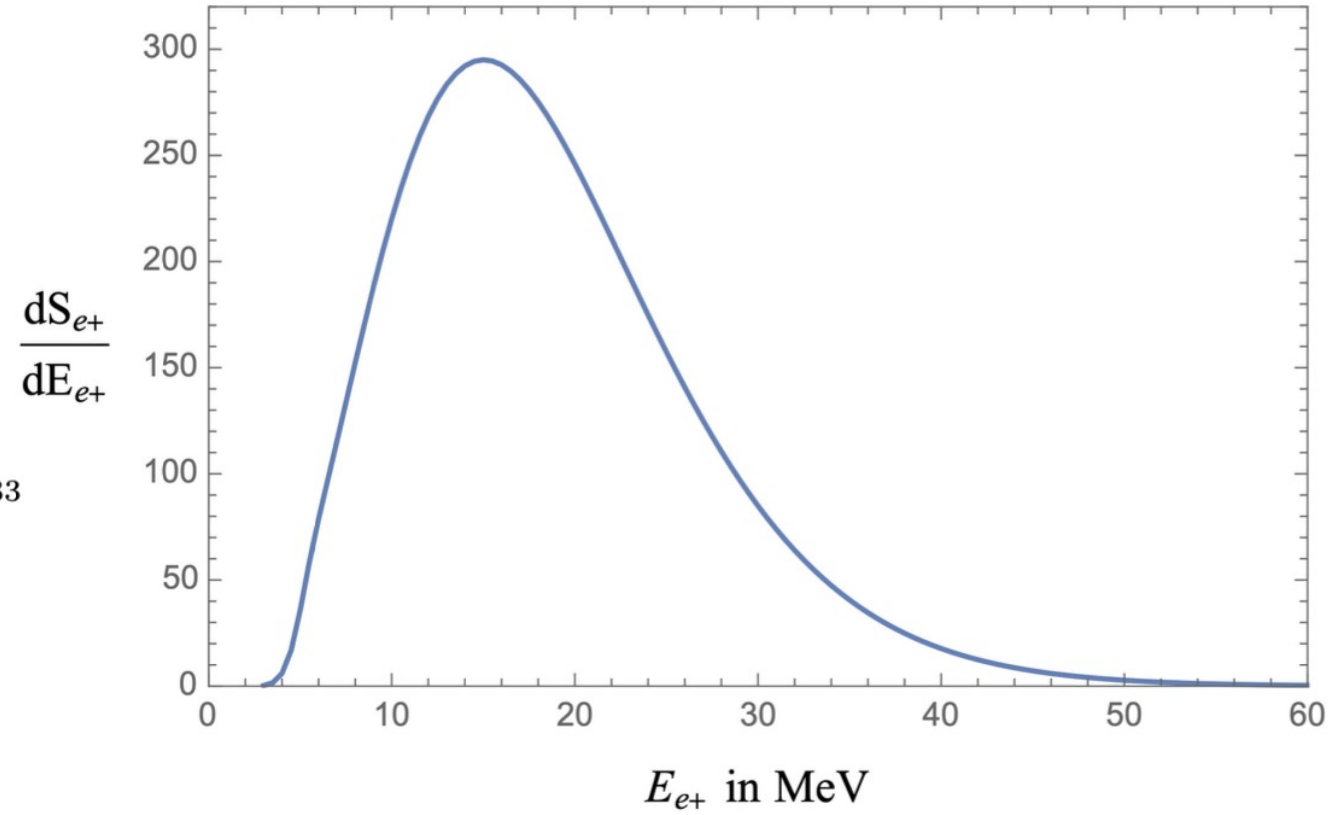
Vissani, J. Phys. G 42, 013001 (2015);
Vissani, Rosso, Symmetry 13, no.10, 1851 (2021)

$$\frac{dF}{dE_\nu} = \frac{\varepsilon}{4\pi D^2} \frac{E_\nu^2 \, e^{-E_\nu/T}}{6T^4}$$

$$\varepsilon = 5 \times 10^{52} \text{ erg and } T = 4 \text{ MeV}$$

$$N_p = 2(1 - \Upsilon_D) \frac{\pi r^2 h \times \rho_{\text{water}}}{m_{\text{H}_2\text{O}}} = 2.167 \times 10^{33}$$

Efficiency ε from Kamiokande II



Conclusions

- We have given an accurate evaluatios of the cross sections for neutrino scattering on nucleons, which we hope to be a useful outcome for current and future neutrino experiments
- Central values of parameters have been updated (since 2003)
- Evaluation of the uncertainties:

  At low energy overall uncertainty at the 0.1 percent level (from Cabibbo angle and axial coupling)

  At higher energies, the uncertainty grows up to the percent level (from the uncertainty associated to the axial form factor)
- We find that the impact of second-class currents on the cross section is negligible at current energies