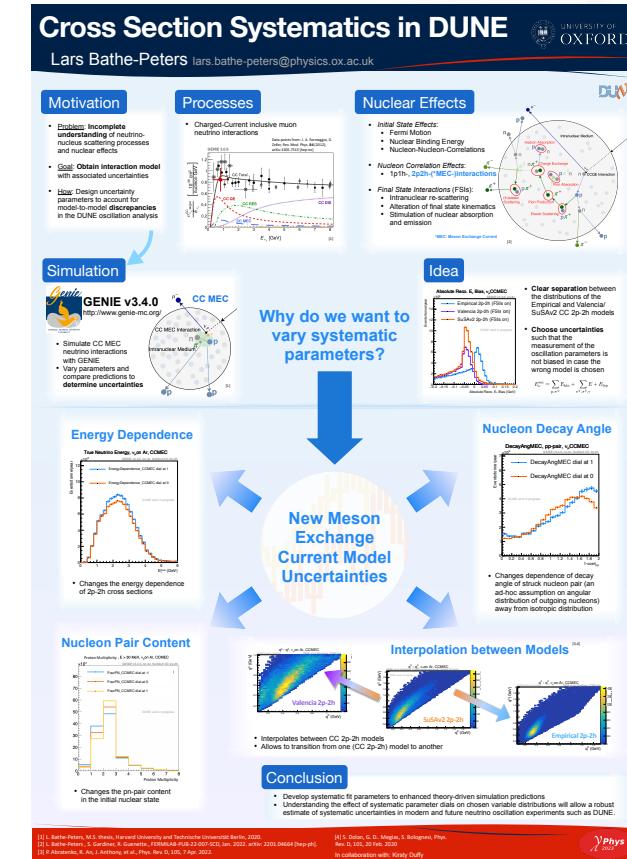
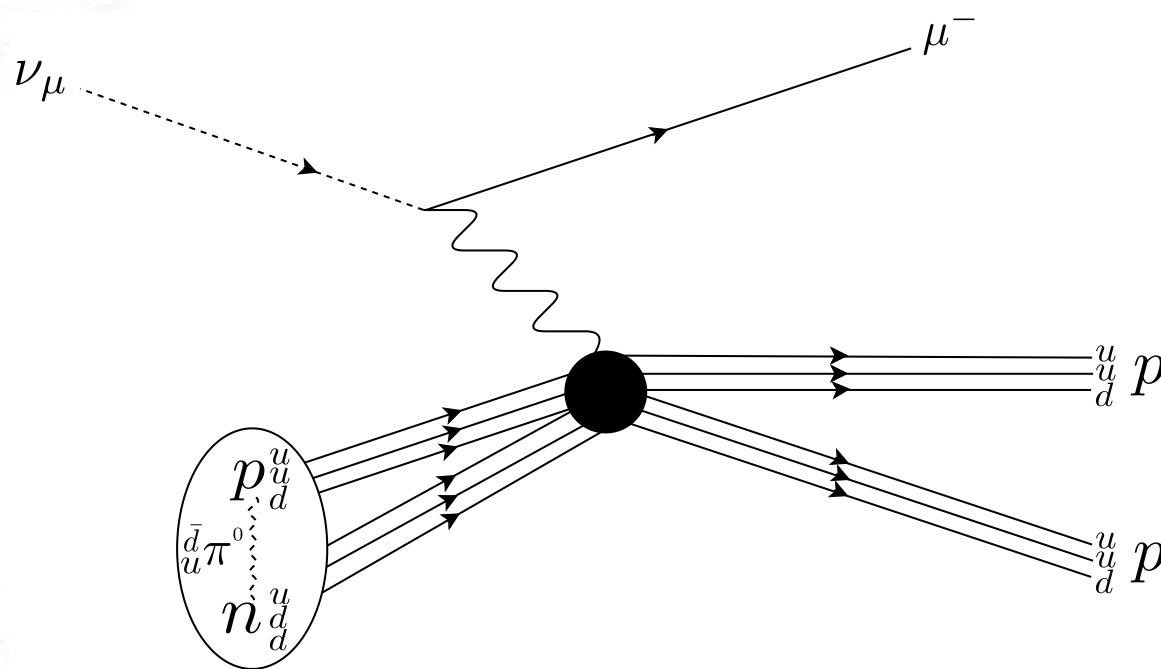


Cross Section Systematics in DUNE

Lars Bathe-Peters lars.bathe-peters@physics.ox.ac.uk

2p2h dials and validations



TH-12

γ Phys
2023

Neutrino Oscillations

Neutrino Oscillations

- Neutrino *mass* and *weak eigenstates* related via PNMS-Matrix:

Usually parametrized by:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

- Neutrino mixing angles: $\theta_{12}, \theta_{23}, \theta_{13}$
- CP-violating phase: δ_{CP}

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- Neutrino event rate:

$$R_{\alpha \rightarrow \beta}(p_{\text{reco}}) = \int_{E_{\min}}^{E_{\max}} \Phi_\alpha(E_{\text{true}}) \cdot \sum_i \sigma_\beta^i(E_{\text{true}}, p_{\text{reco}}) \cdot \sum_j N_j \cdot \varepsilon_\beta(E_{\text{true}}, p_{\text{reco}}) \cdot P_{\alpha \rightarrow \beta}(E_{\text{true}})$$

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Flux

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Flux Cross Section

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Flux Cross Section # of target nuclei

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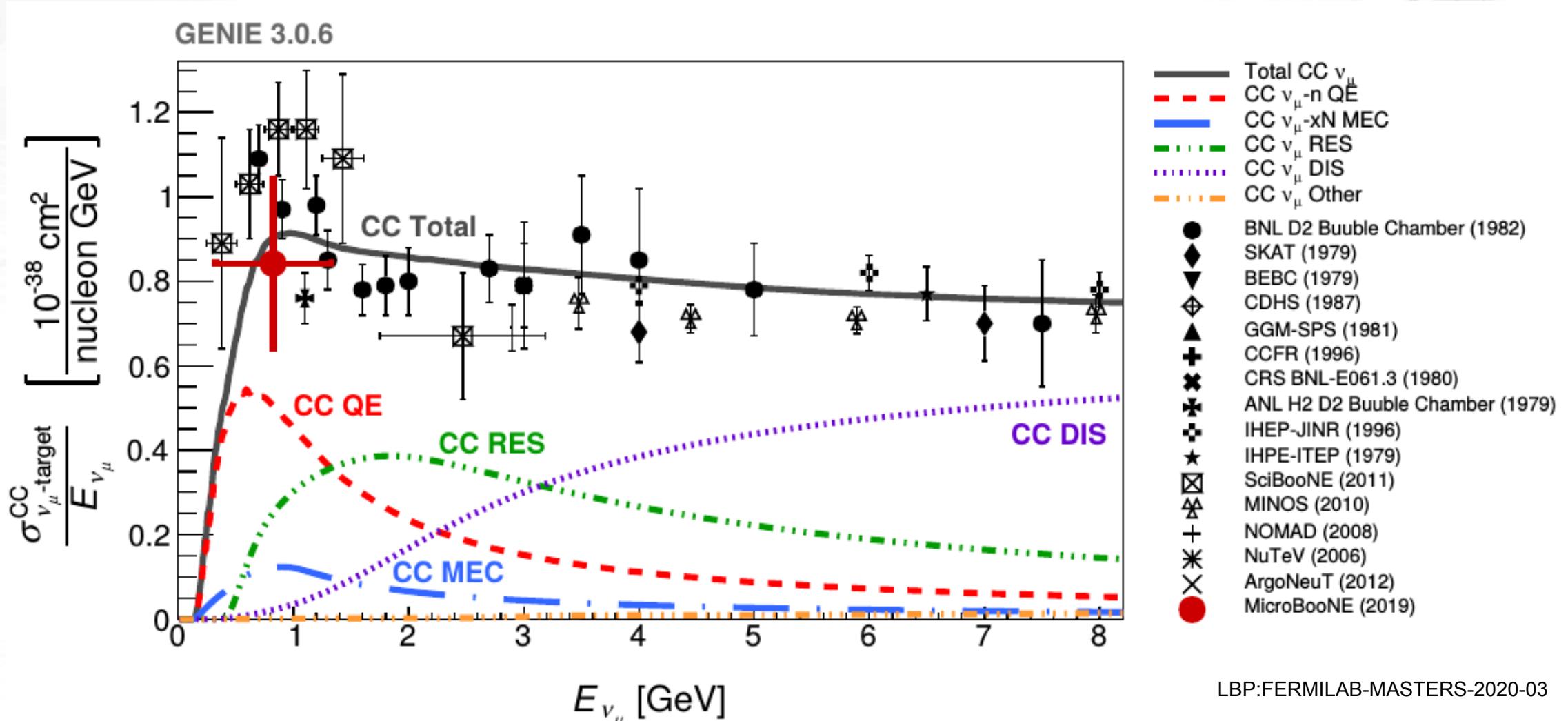
$$R_{\alpha \rightarrow \beta}(p_{\text{reco}}) = \int_{E_{\min}}^{E_{\max}} \Phi_\alpha(E_{\text{true}}) \cdot \sum_i \sigma_\beta^i(E_{\text{true}}, p_{\text{reco}}) \cdot \sum_j N_j \cdot \varepsilon_\beta(E_{\text{true}}, p_{\text{reco}}) \cdot P_{\alpha \rightarrow \beta}(E_{\text{true}})$$

Need to know neutrino energy

Flux	Cross Section	# of target nuclei	Detector Efficiency
$\Phi_\alpha(E_{\text{true}})$	$\sum_i \sigma_\beta^i(E_{\text{true}}, p_{\text{reco}})$	$\sum_j N_j$	$\varepsilon_\beta(E_{\text{true}}, p_{\text{reco}})$

Neutrino-Nucleus Cross Section

Interaction Modes



Nuclear Effects

Initial State Effects

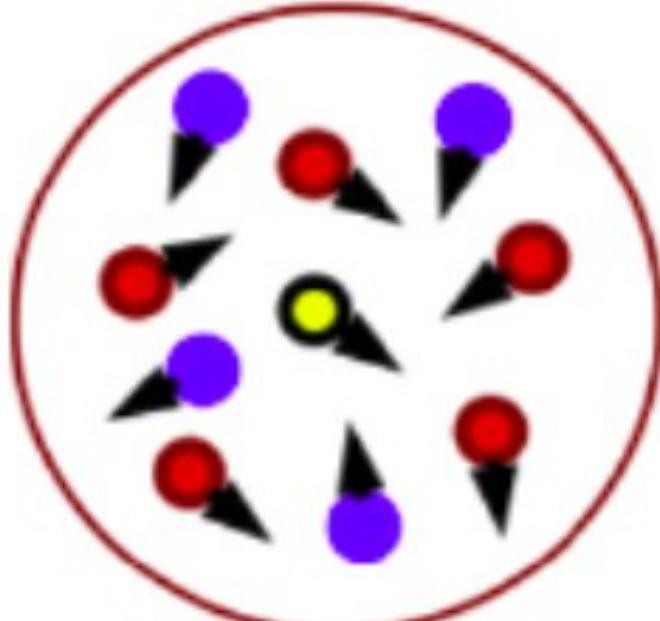
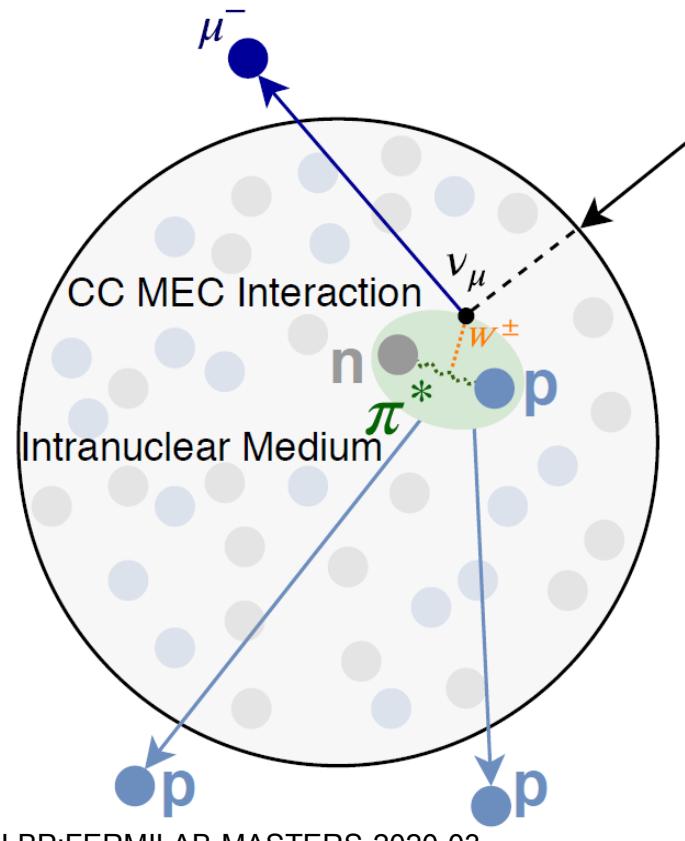
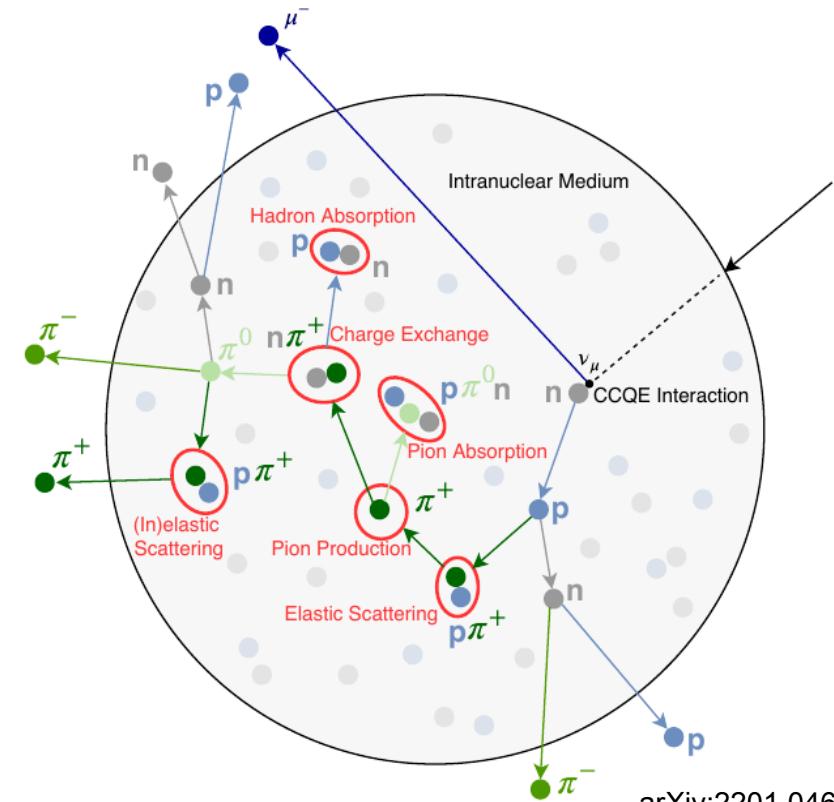


Figure by C. Andreopoulos

Meson Exchange Currents

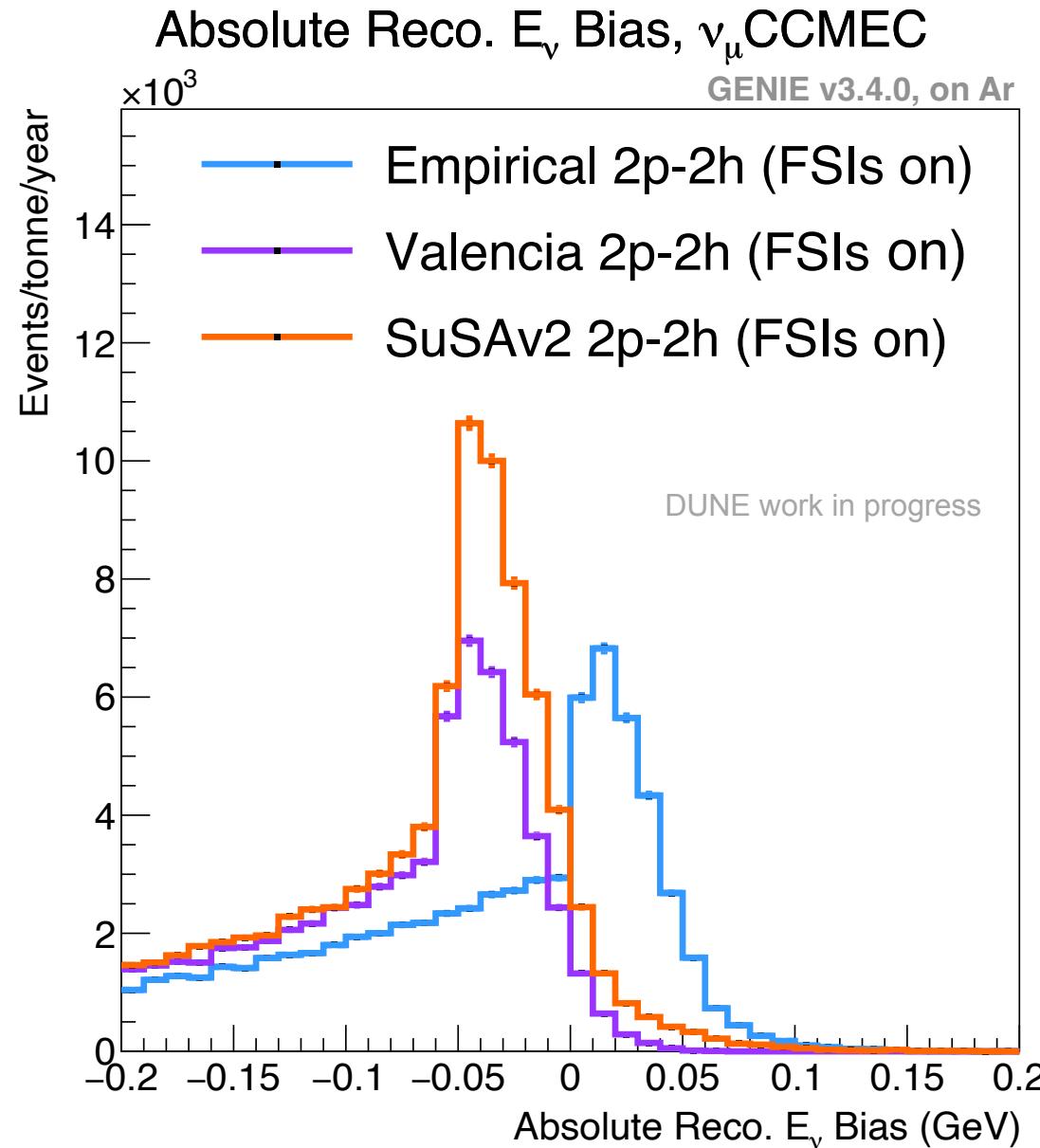


Final State Interactions



2p-2h Model Comparisons

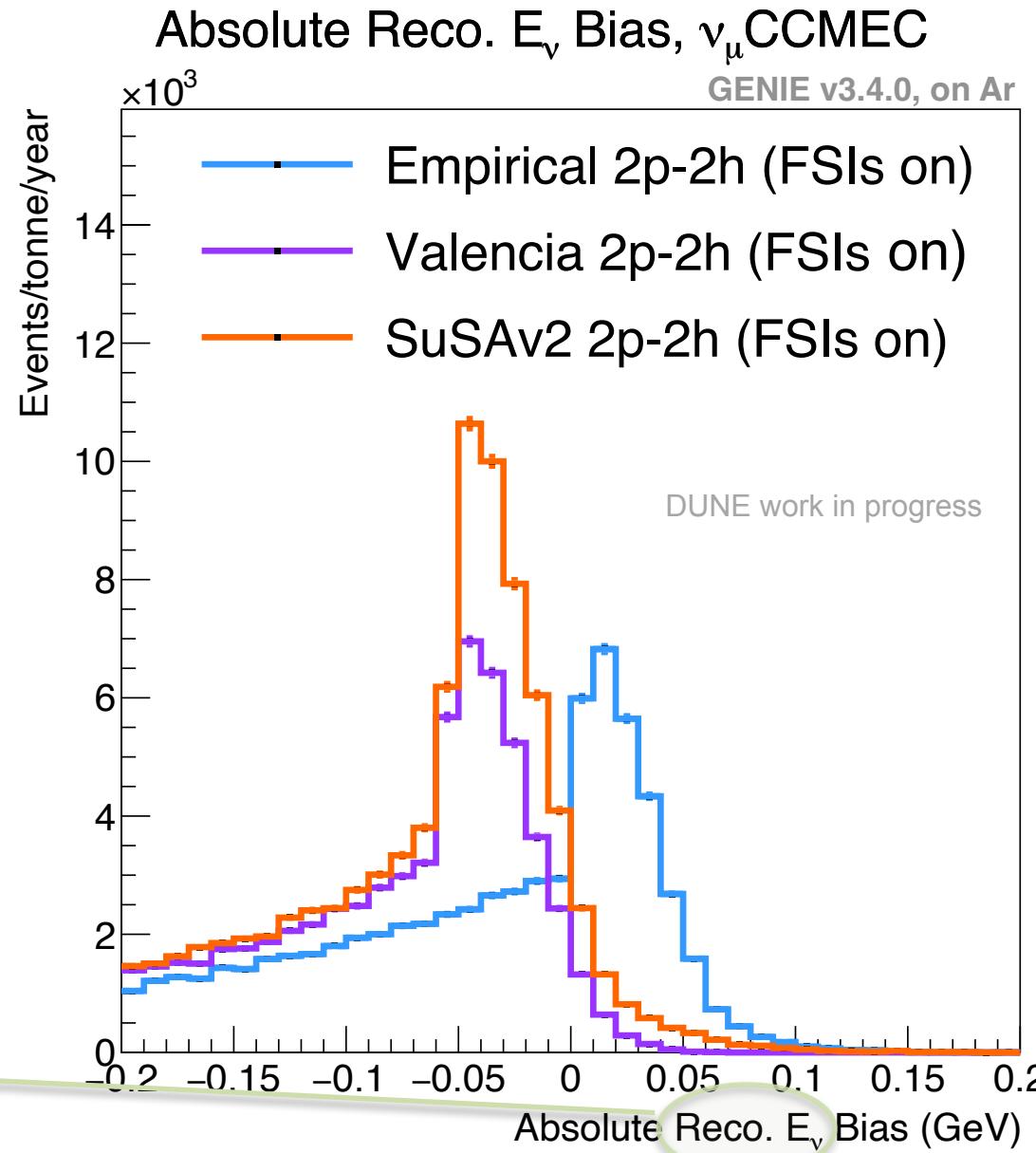
- **Clear separation** between the Empirical and Valencia/SuSAv2 CC 2p-2h models
- **Choose uncertainties** such that the measurement of the oscillation parameters is not biased in case the wrong model is chosen



2p-2h Model Comparisons

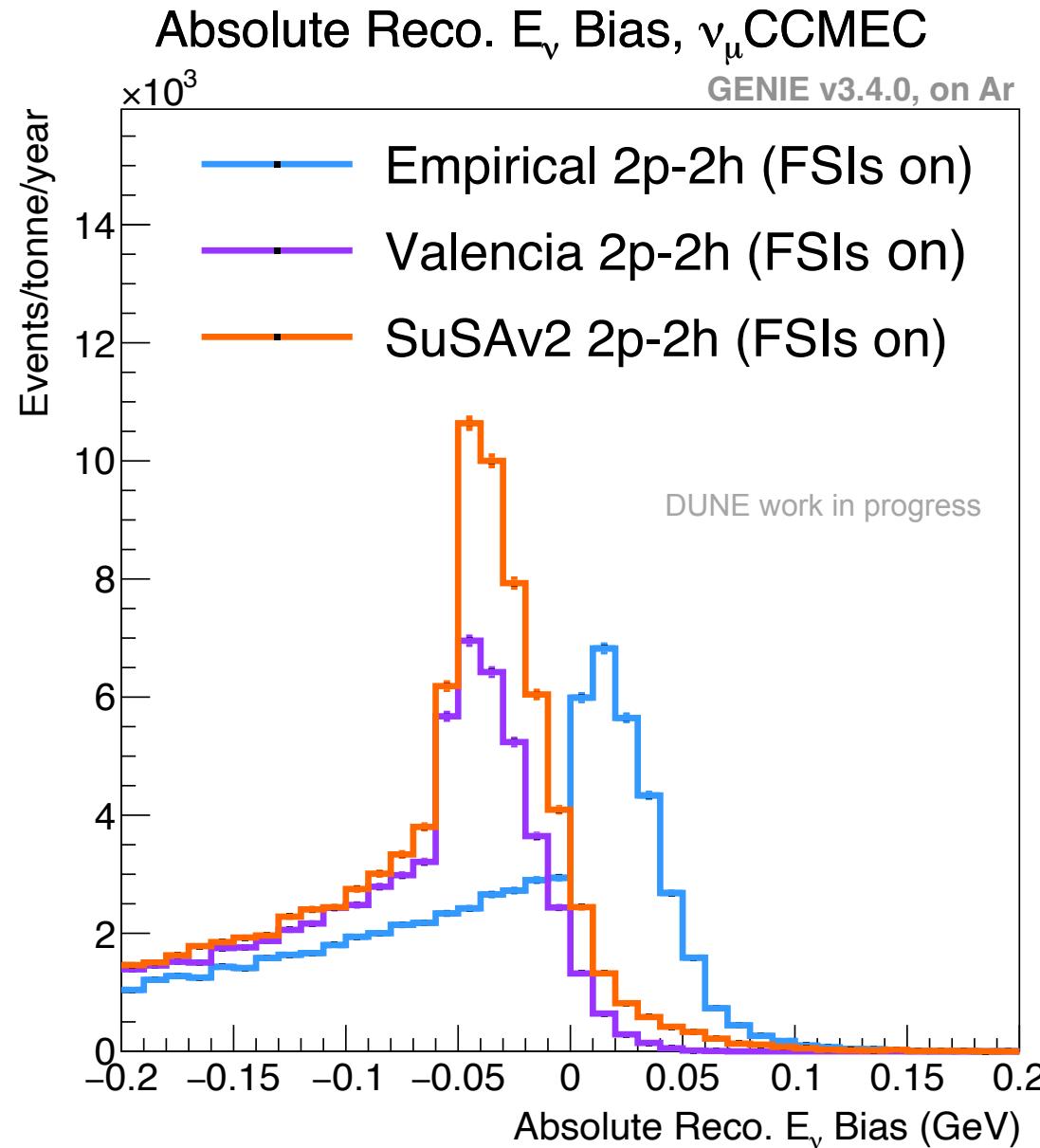
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$$E_\nu^{\text{rec}} = \sum_{\text{p}, \pi^\pm} E_{\text{kin}} + \sum_{e^\pm, \pi^0, \gamma} E + E_{\text{lep}}$$



2p-2h Model Comparisons

- **Clear separation** between the Empirical and Valencia/SuSAv2 CC 2p-2h models
- **Choose uncertainties** such that the measurement of the oscillation parameters is not biased in case the wrong model is chosen



Propagating Uncertainties

Modify physical parameter:

$$P \longrightarrow P' = P \left(1 + x_P \frac{\delta P}{P} \right)$$

Event weight:

$$w_\sigma = \frac{d^n\sigma}{dK^n} \Bigg/ \frac{d^n\sigma'}{dK^n}$$

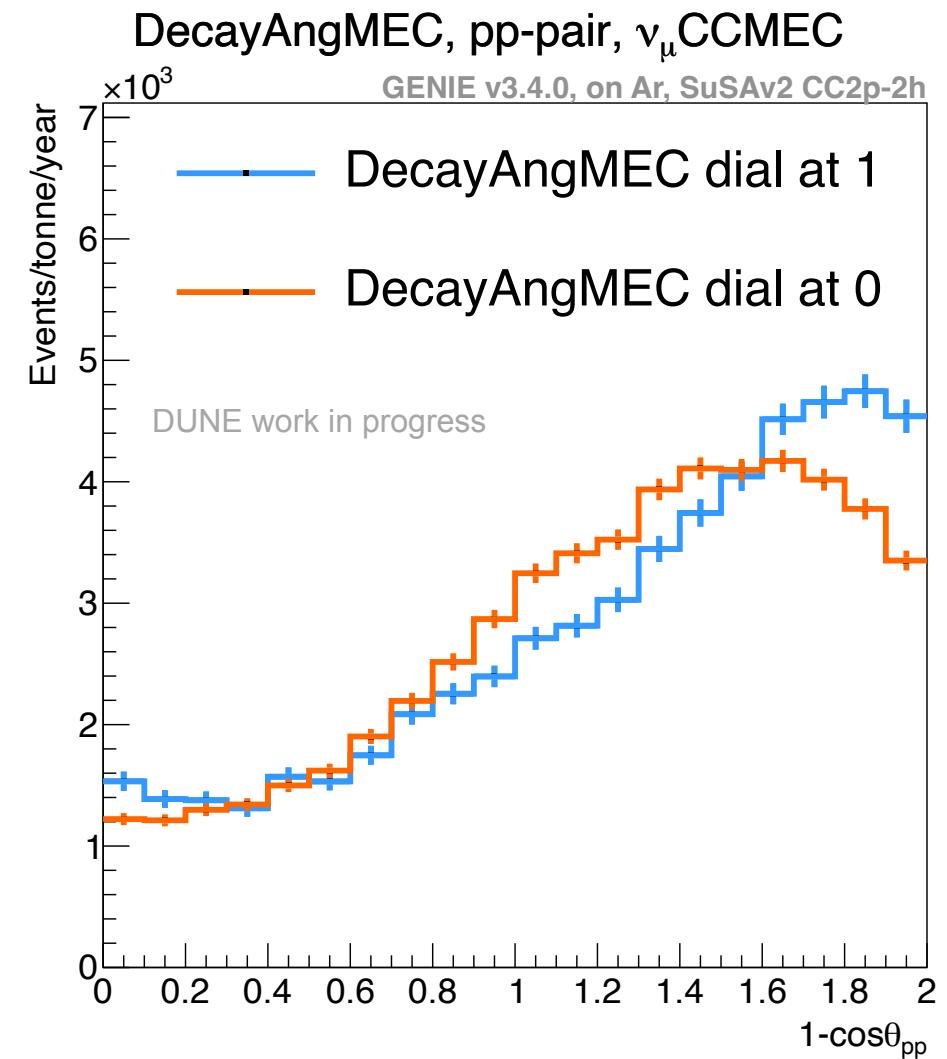
New dials:

- *NormCCMEC*
- *DecayAngMEC*
- *DeltaNotDelta_CCMEC*
- *FracPN_CCMEC*
- *XSecShape_CCMEC*
- *EnergyDependence_CCMEC*

2p-2h Model Uncertainty Dials

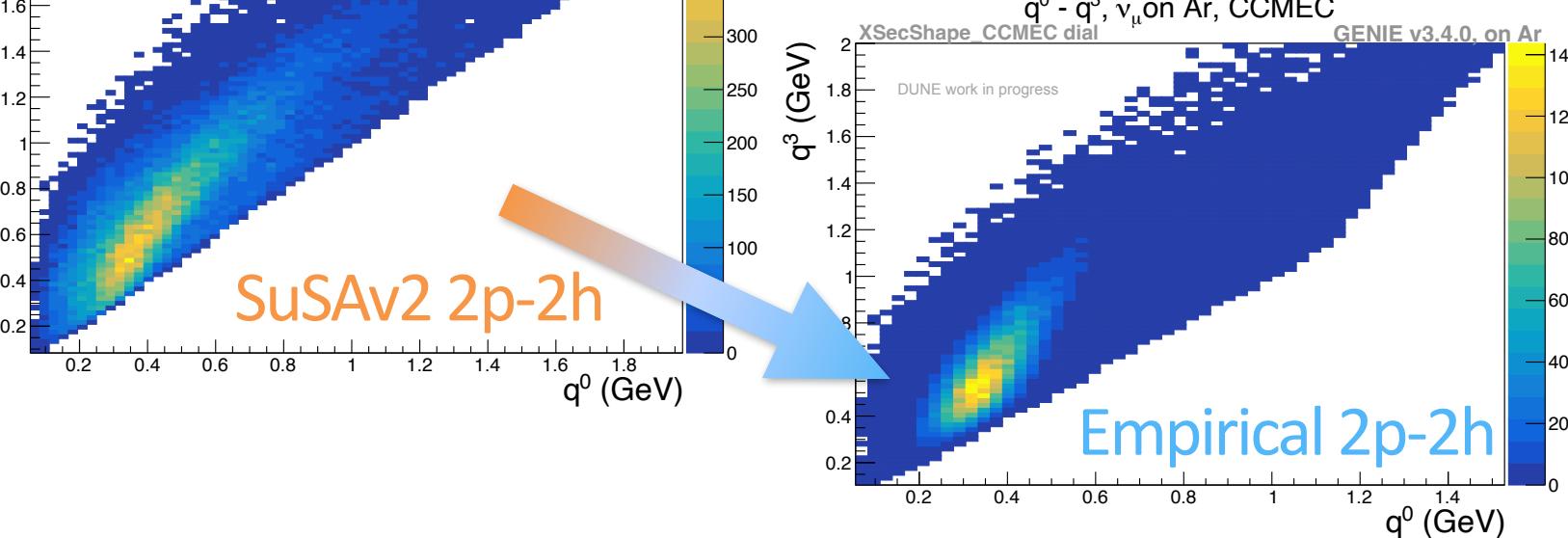
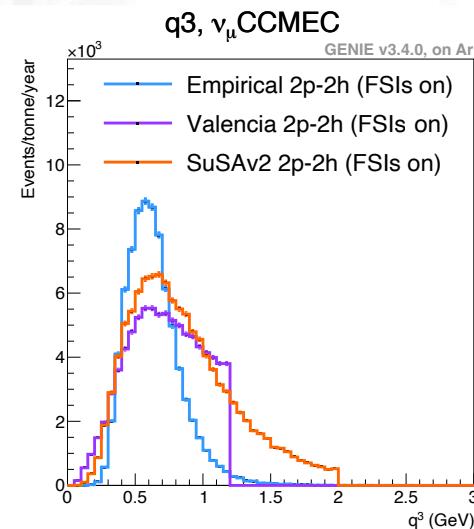
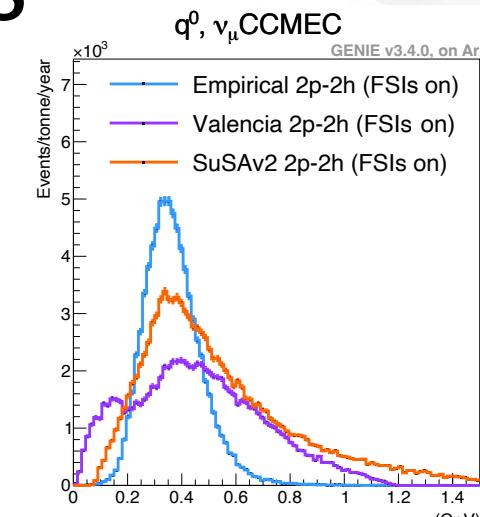
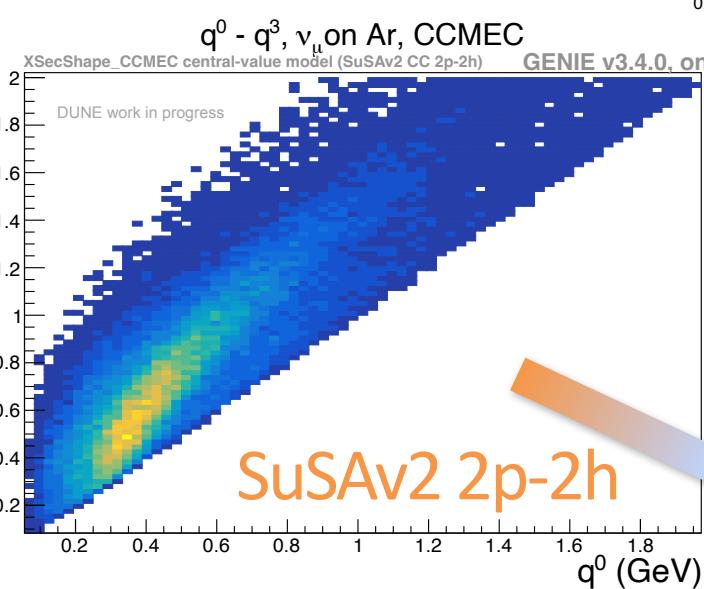
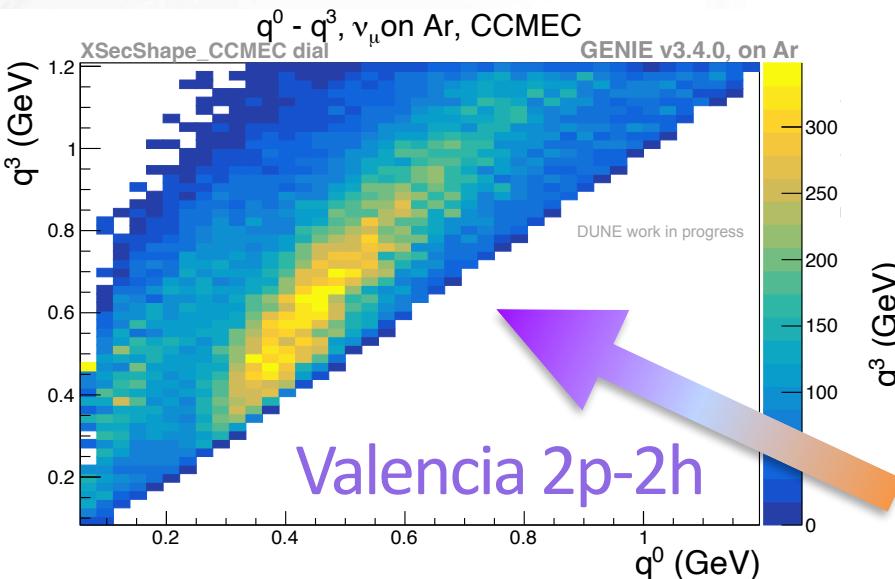
DecayAngMEC - changes angular dependence on struck nucleon pair

- Ad-hoc assumption on **angular distribution** of outgoing nucleons) away from isotropic distribution



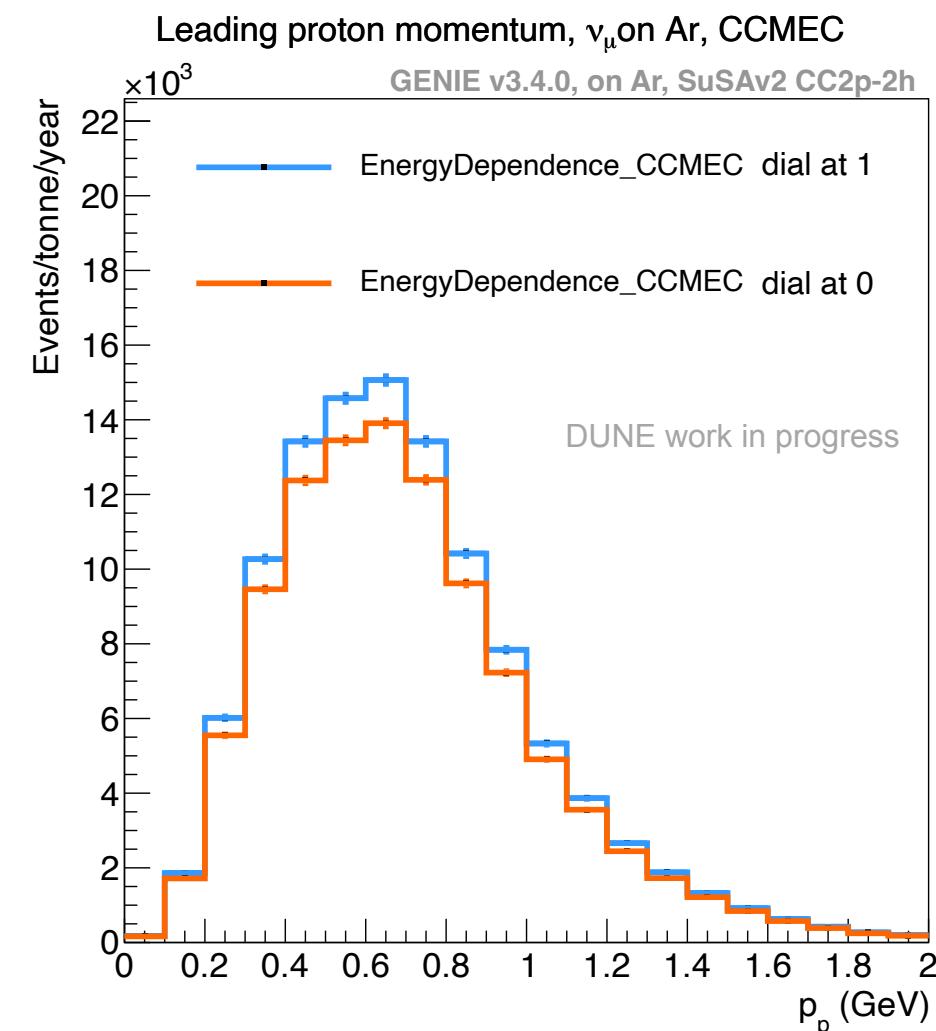
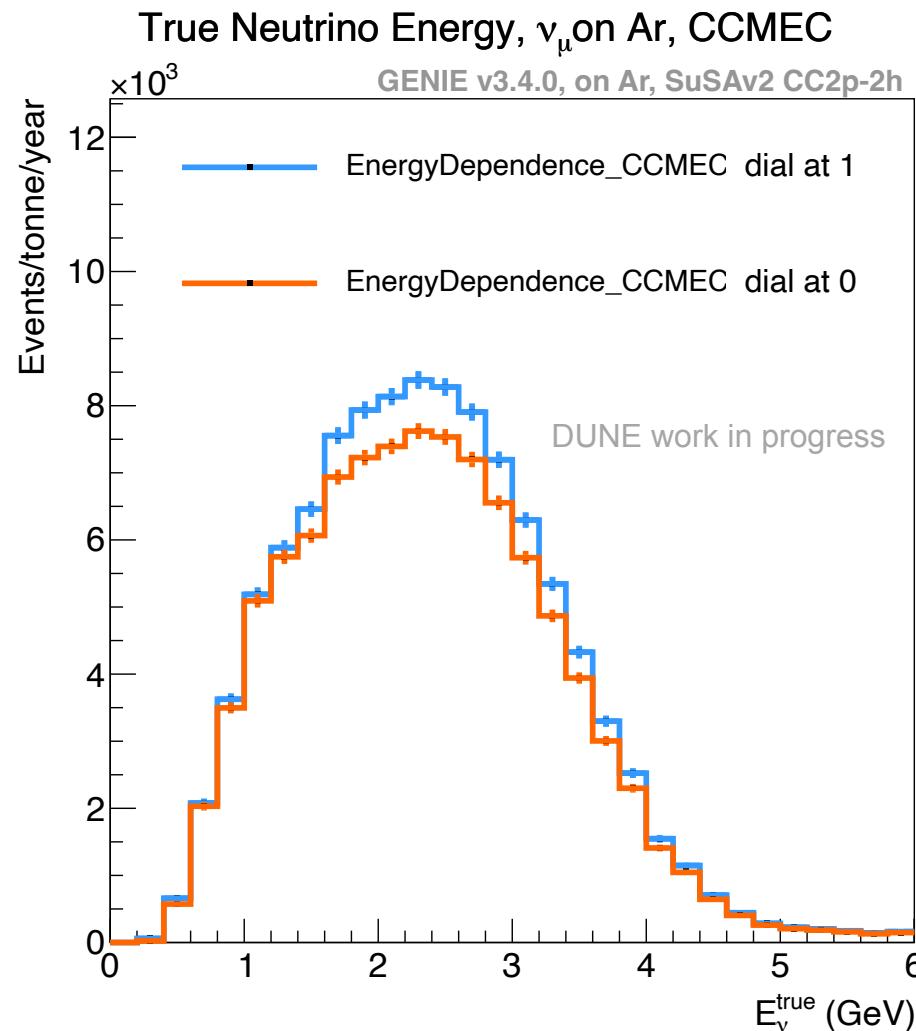
2p-2h Model Uncertainty Dials

XSecShape_CCMEC - interpolates between CC 2p-2h models



2p-2h Model Uncertainty Dials

EnergyDependence_CCMEC - changes the energy dependence of 2p-2h cross sections



Conclusion and Outlook

- Neutrino interactions are complex and necessitate dedicated study
- **Develop systematic fit parameters** to enhanced theory-driven simulation predictions
- Understanding the effect of systematic parameter dials on chosen variable distributions will allow a robust **estimate of systematic uncertainties** in modern and future neutrino oscillation experiments such as DUNE

Thank you!

Poster: TH-12

Cross Section Systematics in DUNE

Lars Bathe-Peters lars.bathe-peters@physics.ox.ac.uk



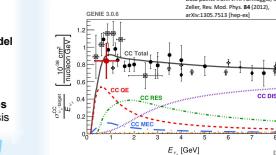
DUNE

Motivation

- Problem: Incomplete understanding of neutrino-nucleus scattering processes and nuclear effects
- Goal: Obtain interaction model with associated uncertainties
- How: Design uncertainty parameters to account for model-to-model discrepancies in the DUNE oscillation analysis

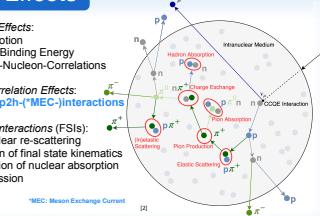
Processes

- Charged-Current inclusive muon neutrino interactions

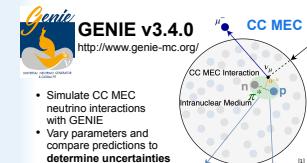


Nuclear Effects

- **Initial State Effects:**
 - Fermi Motion
 - Nuclear Binding Energy
 - Nucleon-Nucleon-Correlations
- **Nucleon Correlation Effects:**
 - $\langle p_{1h}, \langle p_{2h} \rangle \rangle$ -MEC-interactions
- **Final State Interactions (FSIs):**
 - Intranuclear re-scattering
 - Alteration of final state kinematics
 - Stimulation of nuclear absorption and emission



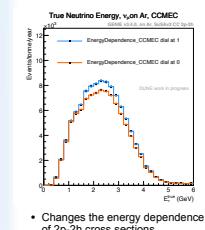
Simulation



- Simulate CC MEC neutrino interactions with GENIE
- Vary parameters and compare predictions to determine uncertainties

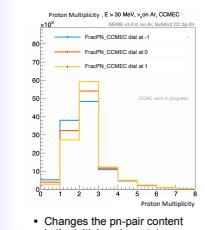
Why do we want to vary systematic parameters?

Energy Dependence



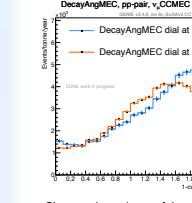
- Changes the energy dependence of 2p-2h cross sections

Nucleon Pair Content



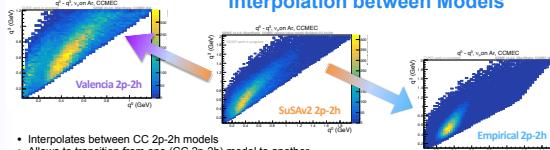
- Changes the pn-pair content in the initial nuclear state

Nucleon Decay Angle



- Changes dependence of decay angle of struck nucleon pair (an ad-hoc assumption on angular distribution of outgoing nucleons) away from isotropic distribution

Interpolation between Models



Conclusion

- Develop systematic fit parameters to enhanced theory-driven simulation predictions
- Understanding the effect of systematic parameter shifts on chosen variable distributions will allow a robust estimate of systematic uncertainties in modern and future neutrino oscillation experiments such as DUNE.

[1] L. Bathe-Peters, M.S. thesis, Harvard University and Technische Universität Berlin, 2020.

[2] L. Bathe-Peters, J. Gardiner, R. Guenette, FERMILAB-PUB-22-007-SCD, Jan. 2022, arXiv:2201.04664 [hep-ph].

[3] P. Abratenko, R. An, J. Anthony, et al., Phys. Rev. D, 105, 7 Apr. 2022.

[4] S. Dolan, G. D. Megias, S. Bolognesi, Phys. Rev. D, 101, 20 Feb. 2020.

In collaboration with: Kirsty Duffy



L 2023



UNIVERSITY OF
OXFORD

Neutrino Physics

ν Cross Sections

Nuclear Effects

Model Comparisons

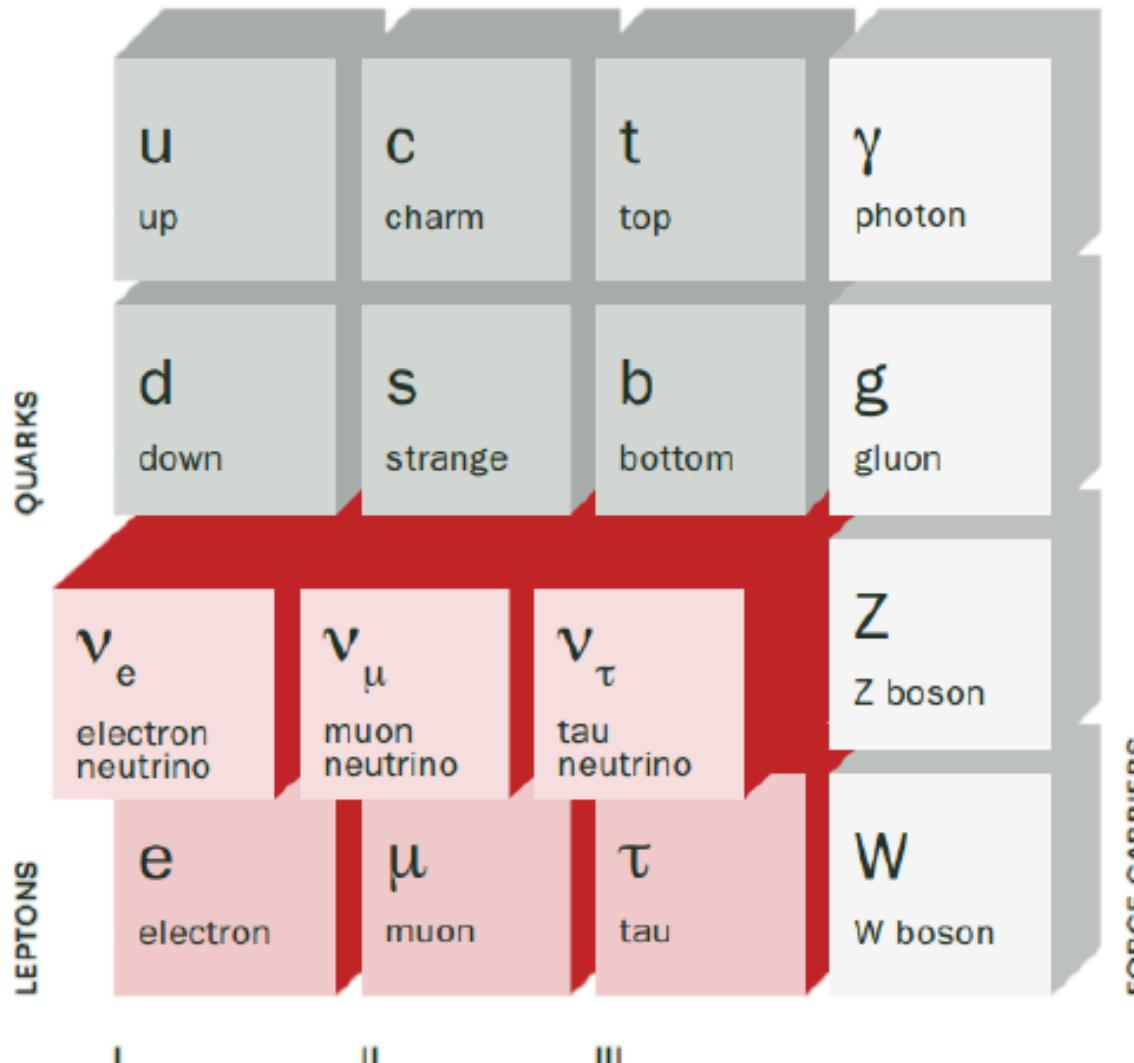
Systematic Parameters

Conclusion and Outlook

Lars Bathe-Peters - Cross Section Systematics in DUNE

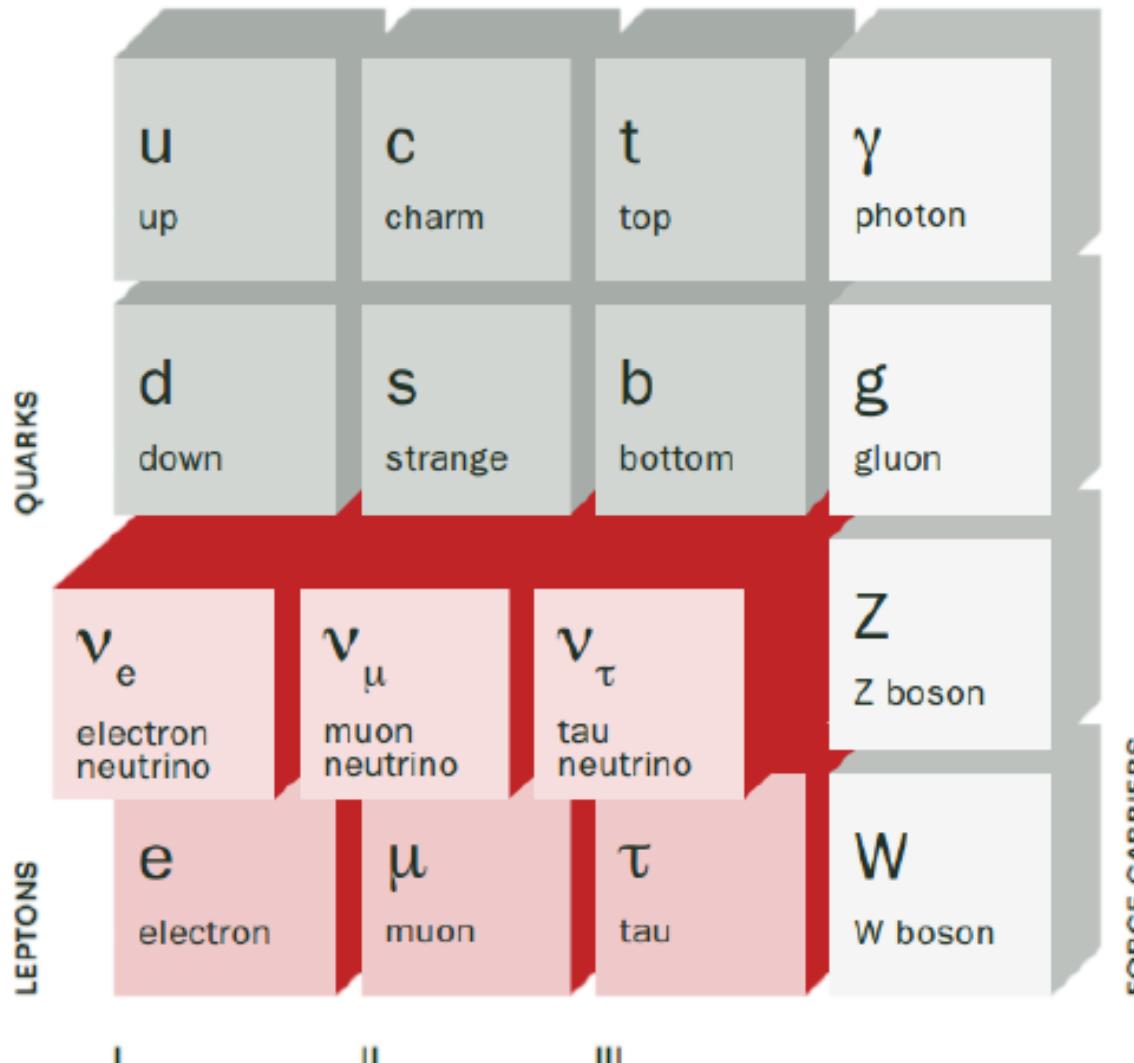
Backup

Neutrinos in the Standard Model



- Three generations (I, II, III) of fermions
- Gauge *bosons* mediate *forces*:
 - Photon \rightarrow electromagnetic
 - Gluon \rightarrow strong
 - W^\pm, Z \rightarrow weak
- Standard Model prediction:
 - 3 *massless* neutrinos (and 3 anti-neutrinos) of 3 different flavors

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Neutrino-Nucleus Interactions

Theoretical Predictions by Neutrino Event Generators

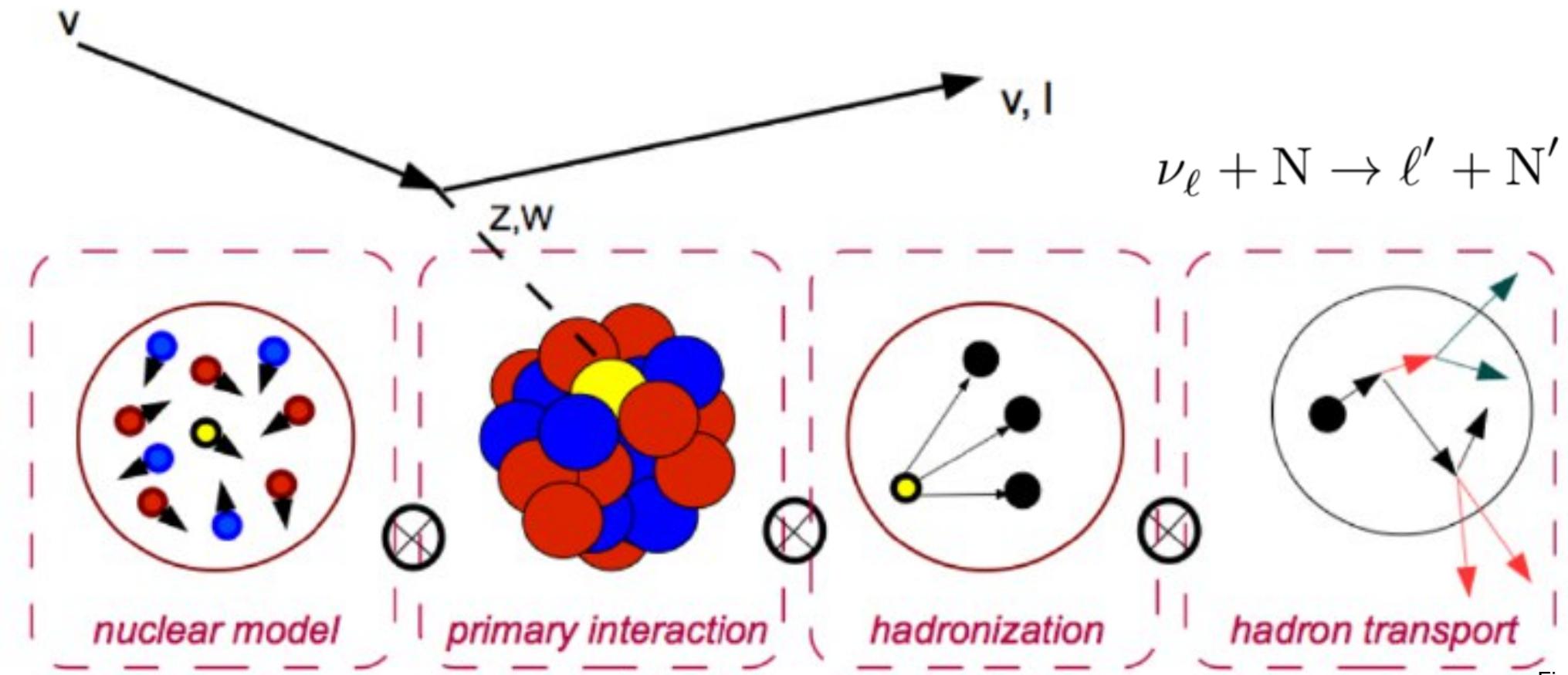
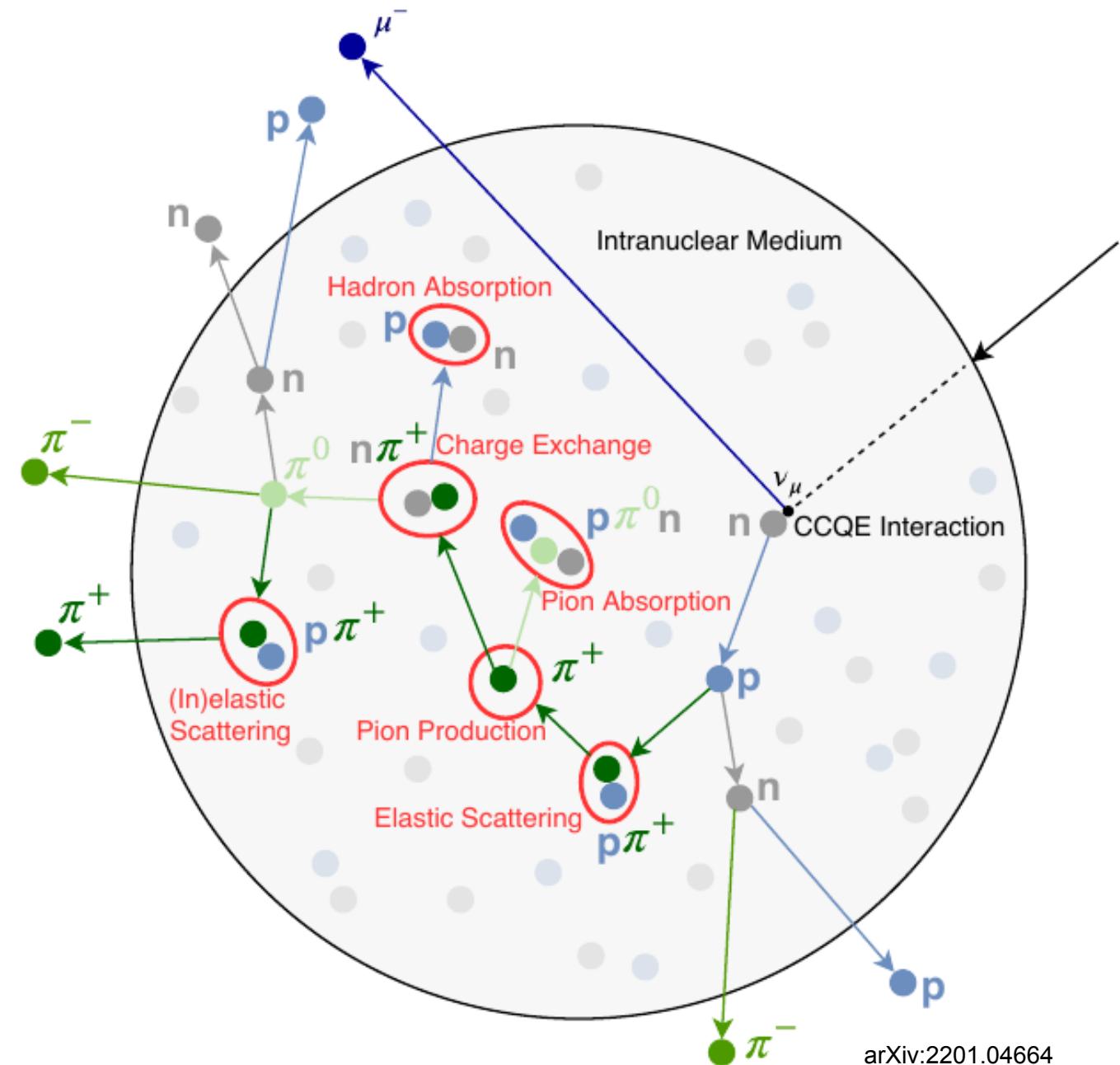
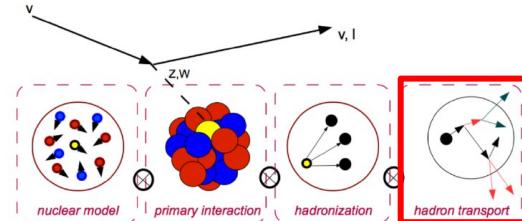


Figure by C. Andreopoulos

Nuclear Effects

Final State Interactions (FSIs)

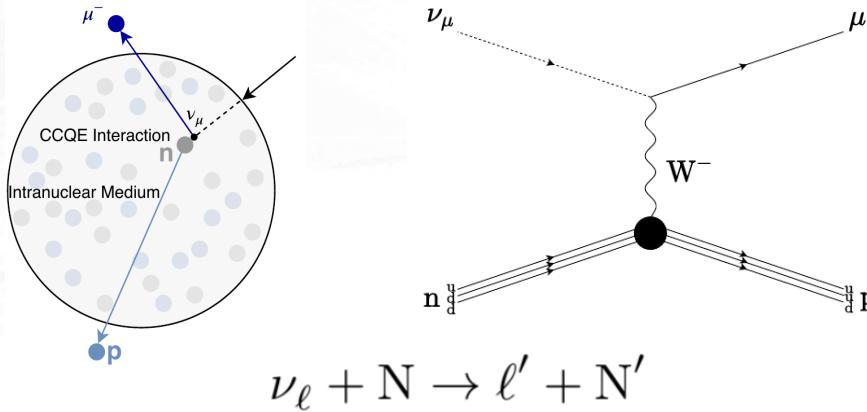
- FSIs inside the nucleus:
 - (In)elastic Scattering
 - Pion Production
 - Absorption
 - Charge Exchange
- Interaction observables, if / once hadron escapes the nucleus.



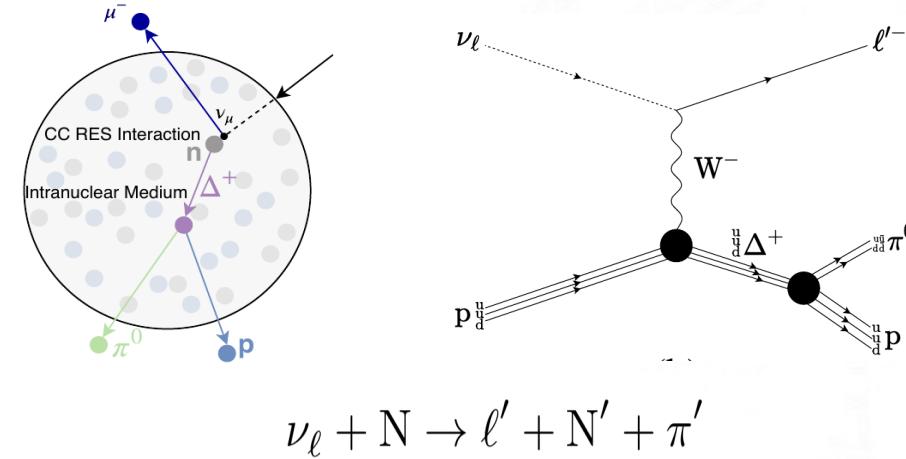
arXiv:2201.04664

Neutrino-Nucleus Interactions

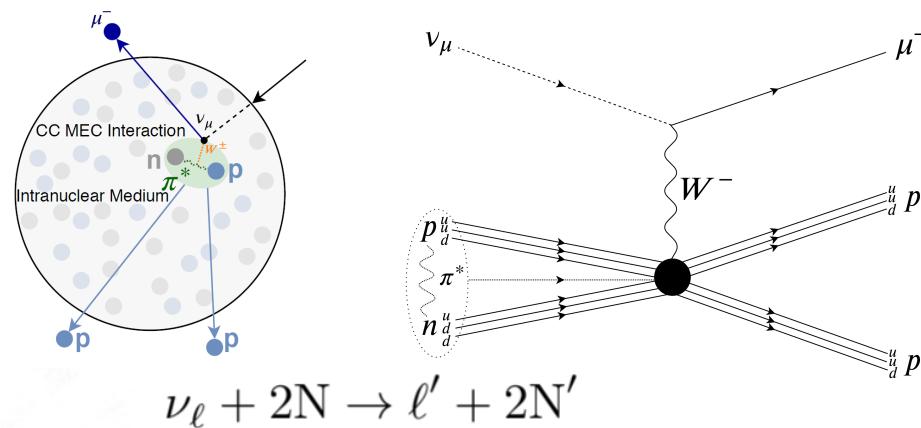
Quasi-Elastic (QE)



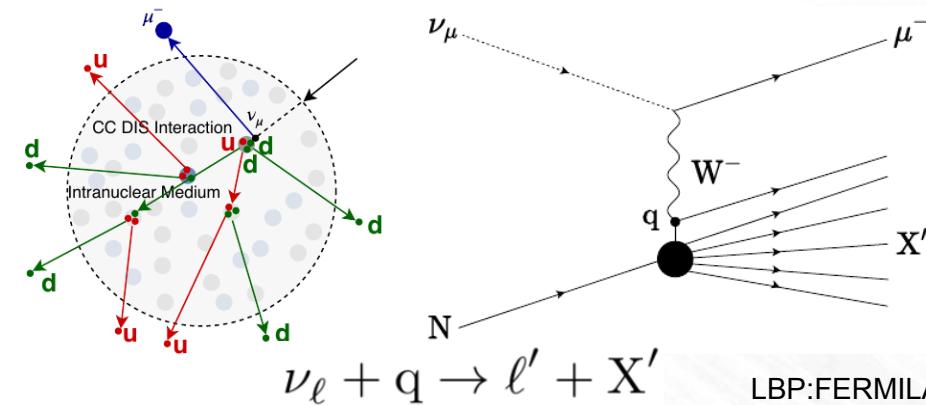
Resonance Excitation (RES)



Multi-Nucleon Processes ($np-nh$)



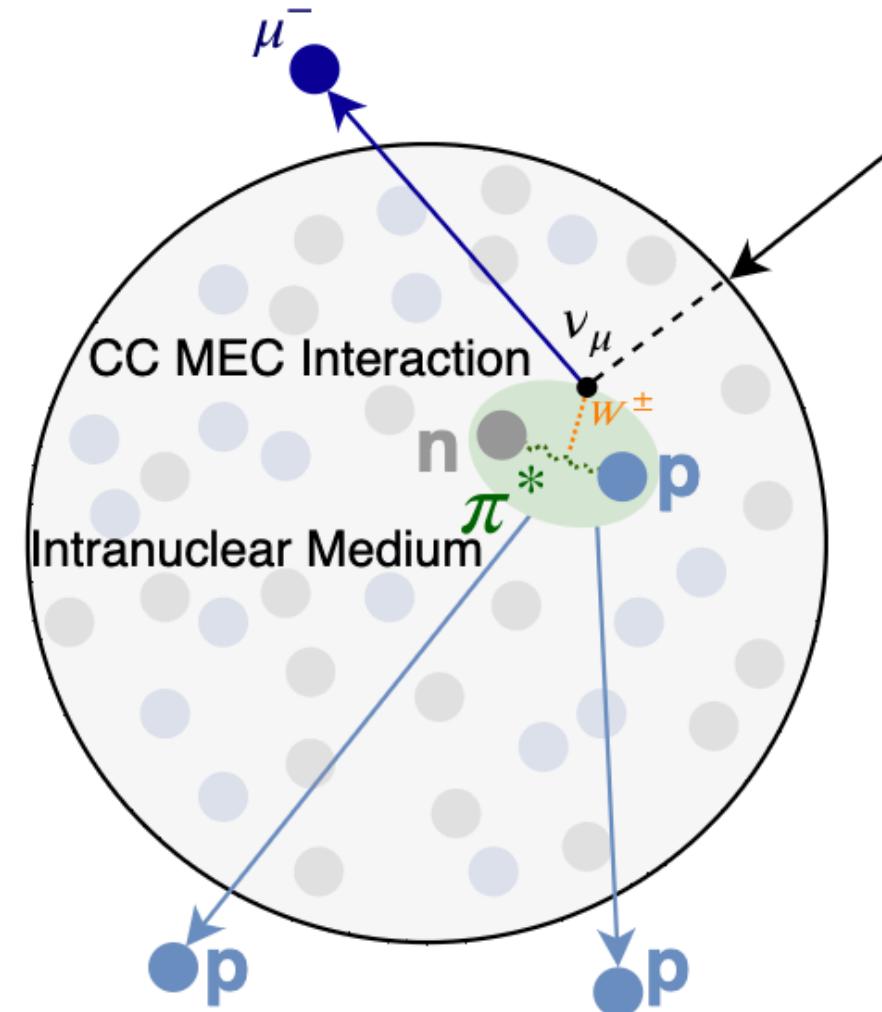
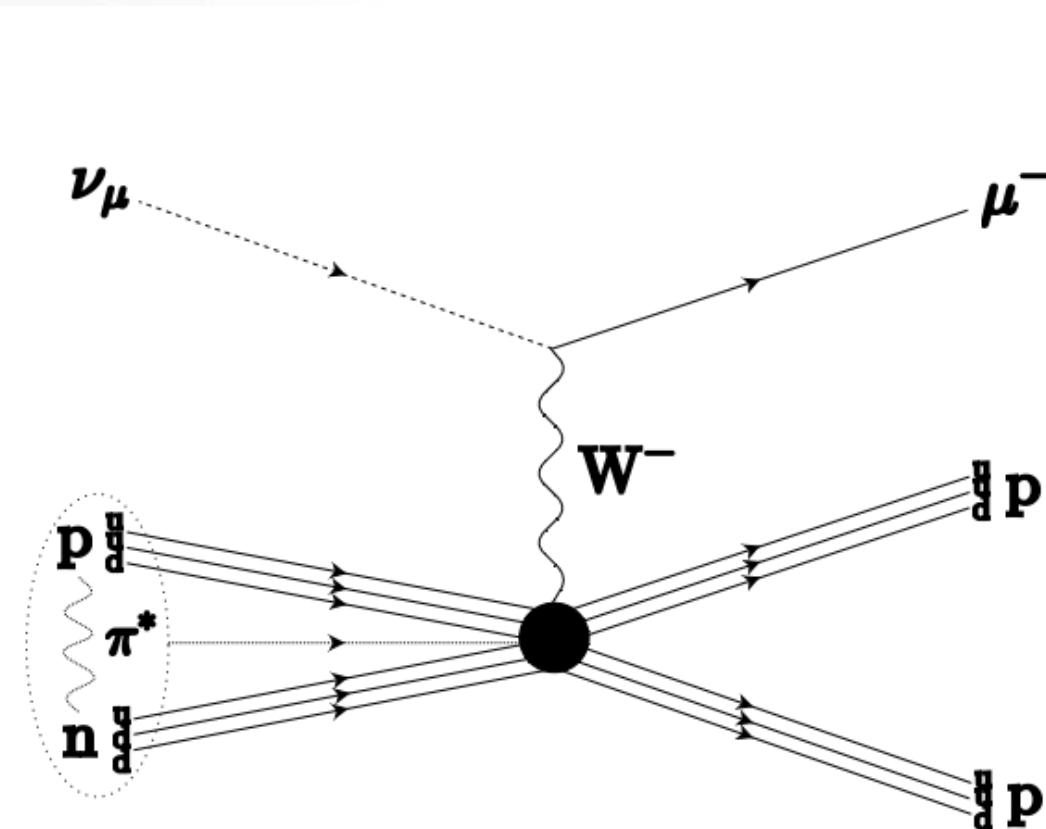
Deep Inelastic Scattering (DIS)



LBP:FERMILAB-MASTERS-2020-03

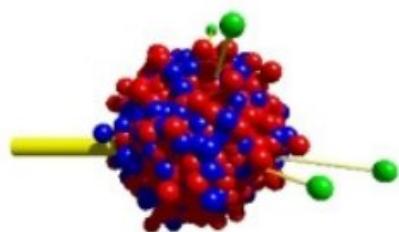


CC 2p-2h Neutrino Interaction



Neutrino Event Generators

- Ambiguous theoretical approach to cross-section calculation
 - Different attempts in cross section predictions
 - Various neutrino event generators to simulate neutrino-nucleus scattering
- Large gap between theory and experiment
 - Need data from experiment



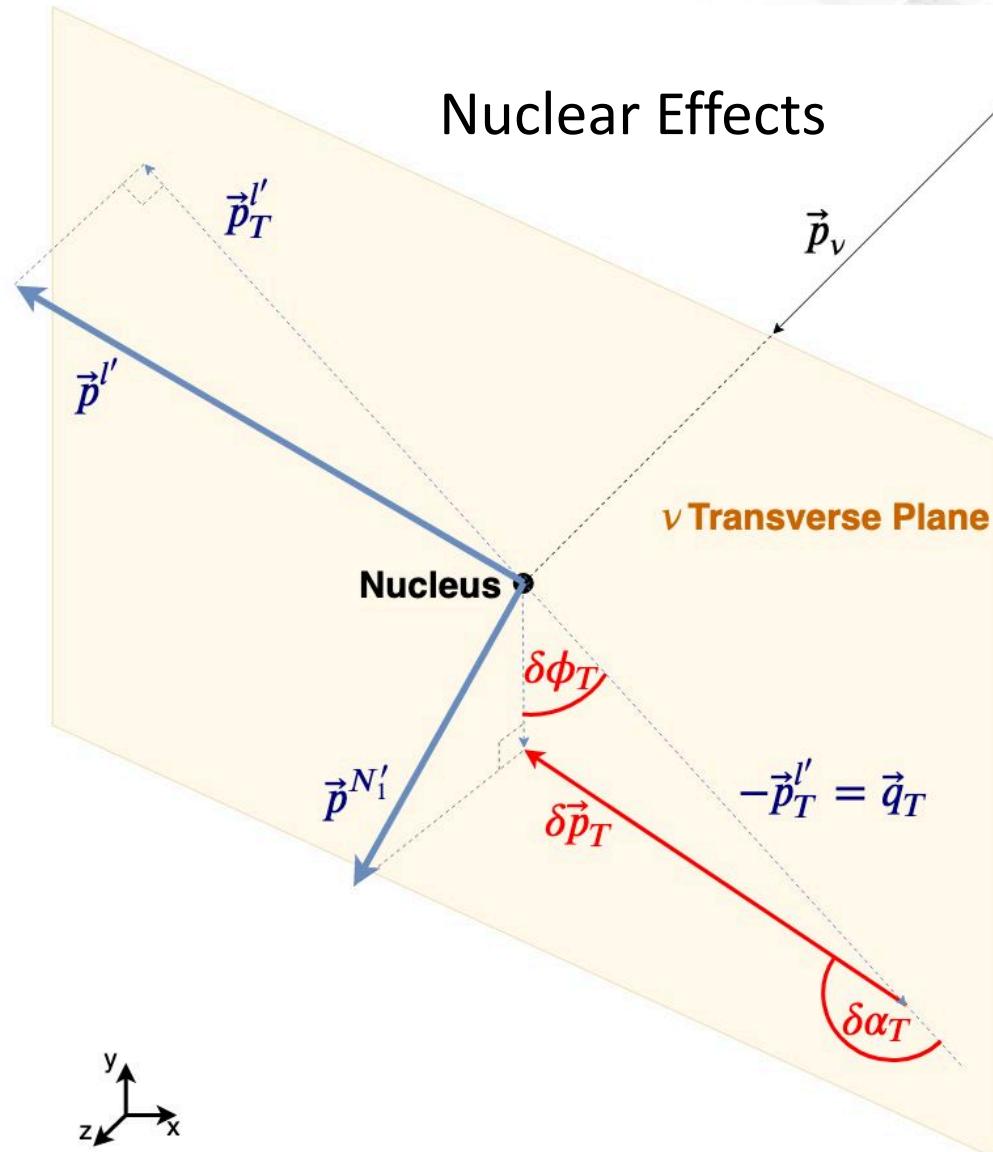
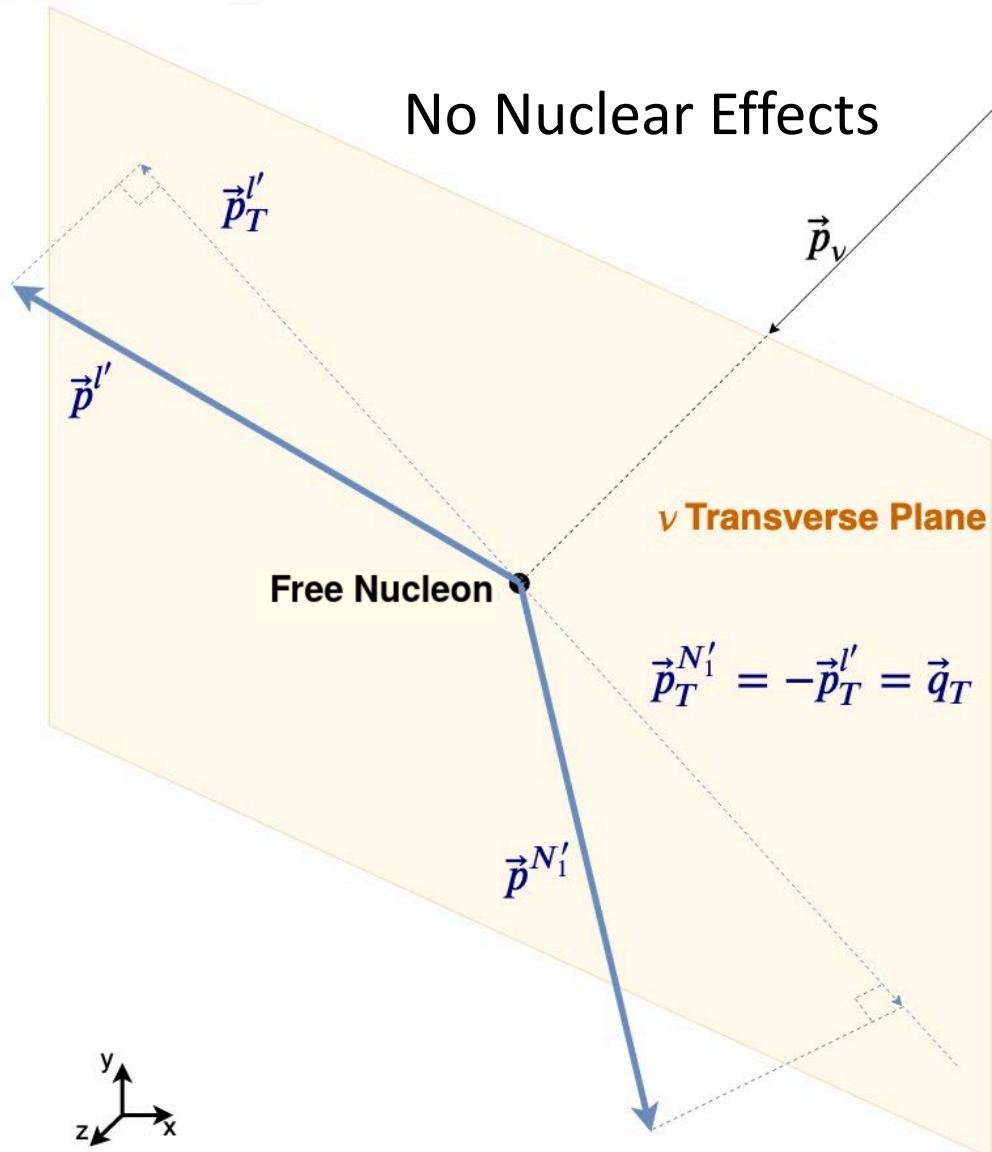
GiBUU
The Giessen Boltzmann-Uehling-Uhlenbeck Project



UNIVERSAL NEUTRINO GENERATOR
& GLOBAL FIT



Transverse Kinematic Imbalance (TKI)



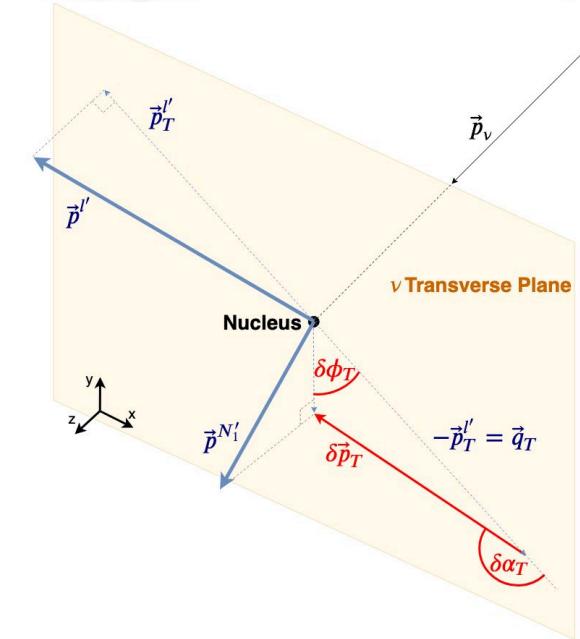
Transverse Kinematic Imbalance Variables

- Any imbalance observed for CCQE interactions between lepton and hadron kinematics is a direct consequence of nuclear effects
- This imbalance (STKI) can be fully characterised by a set of three **Single-Transverse Variables (STVs)**

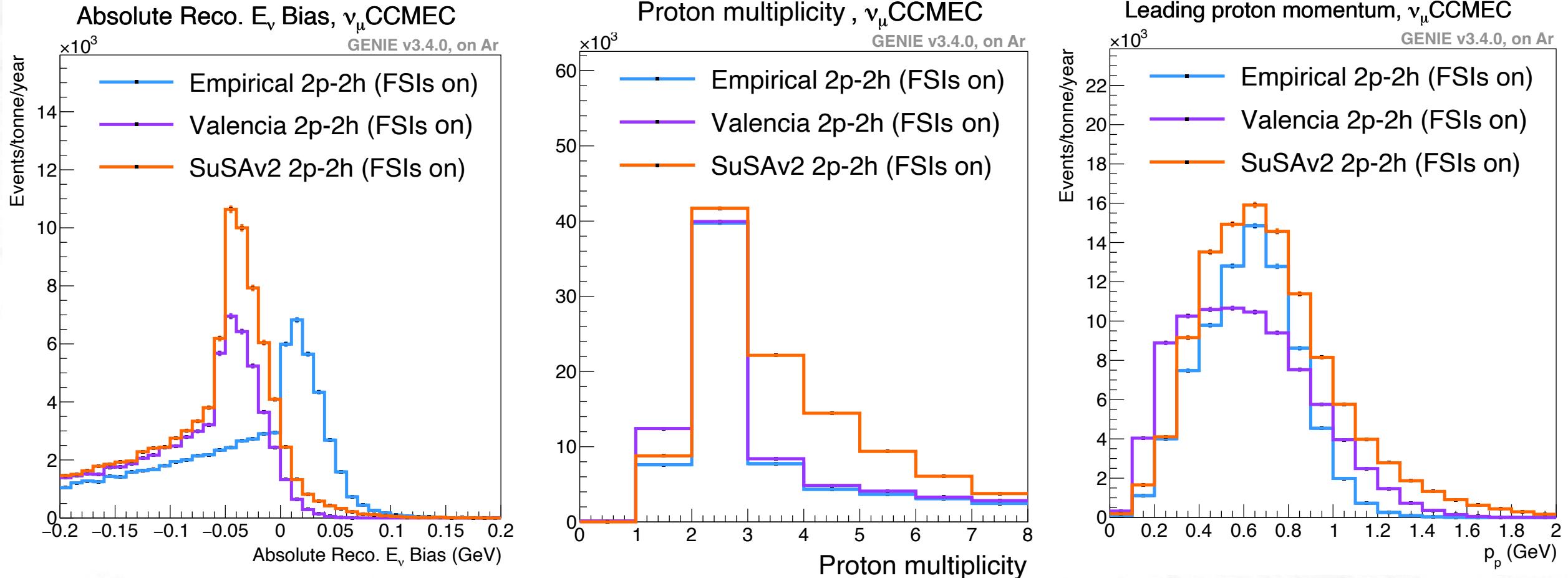
$$\delta \vec{p}_T \equiv \vec{p}_T^{\ell'} + \vec{p}_T^N$$

$$\delta\alpha_T \equiv \arccos \frac{-\vec{p}_T^{\ell'} \cdot \delta \vec{p}_T}{p_T^{\ell'} \delta p_T}$$

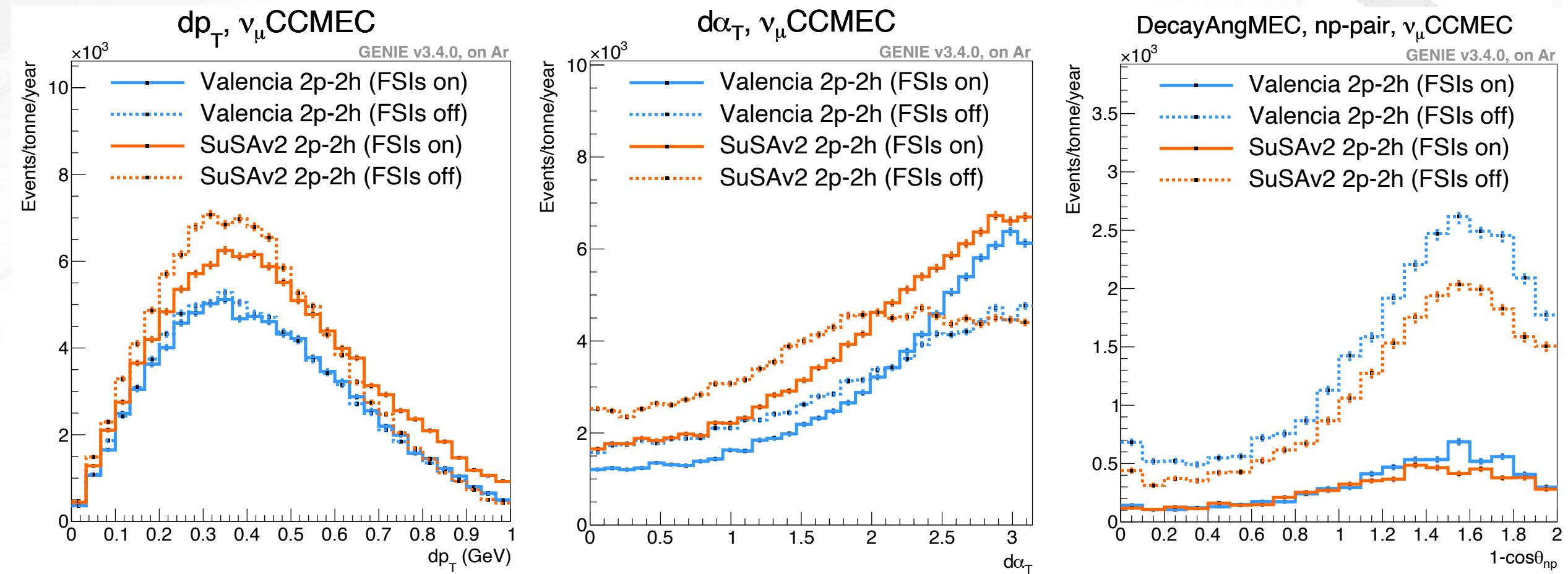
$$\delta\phi_T \equiv \arccos \frac{-\vec{p}_T^{\ell'} \cdot \vec{p}_T^N}{p_T^{\ell'} p_T^N}$$



2p-2h Model Comparisons (FSIs on)

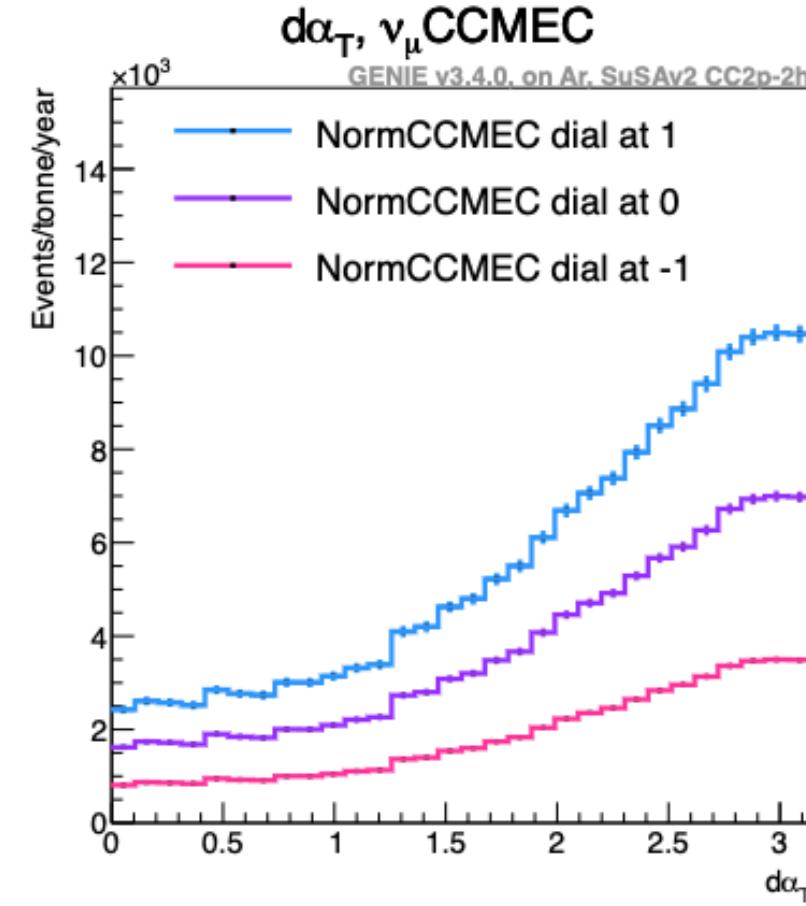
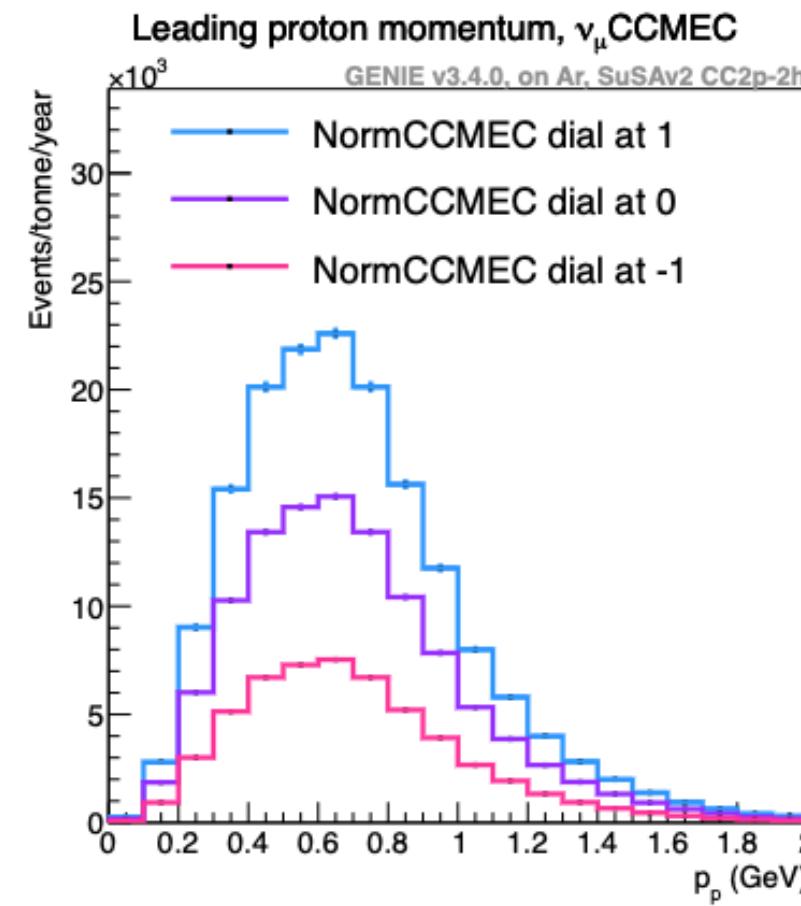
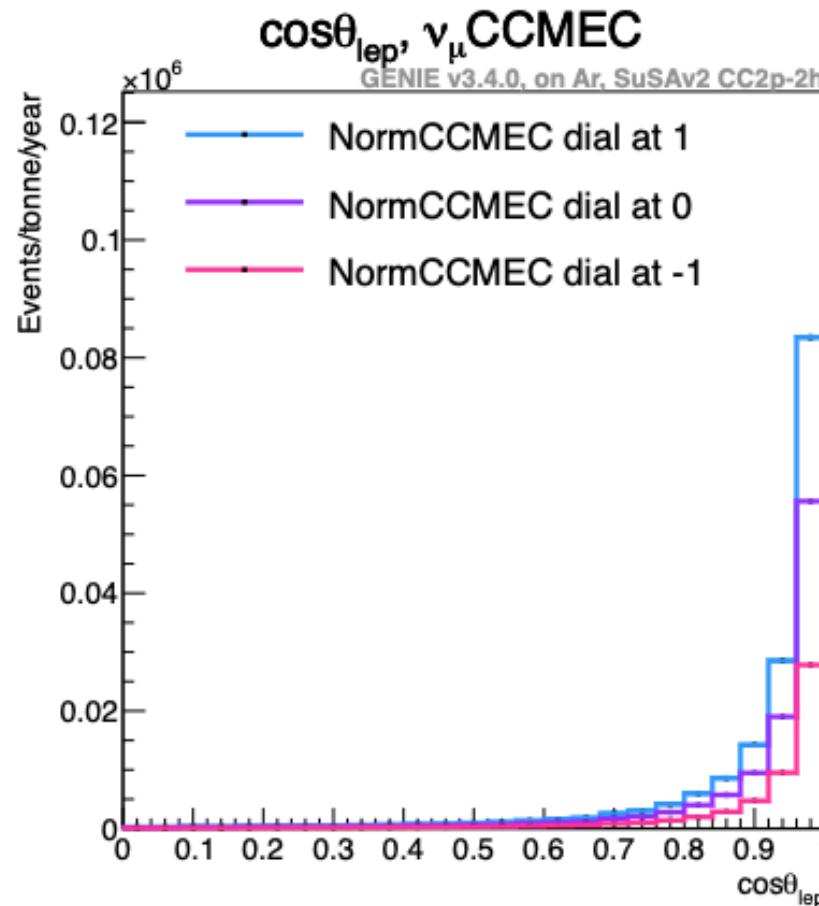


2p-2h Model Comparisons (preFSI vs. postFSI)

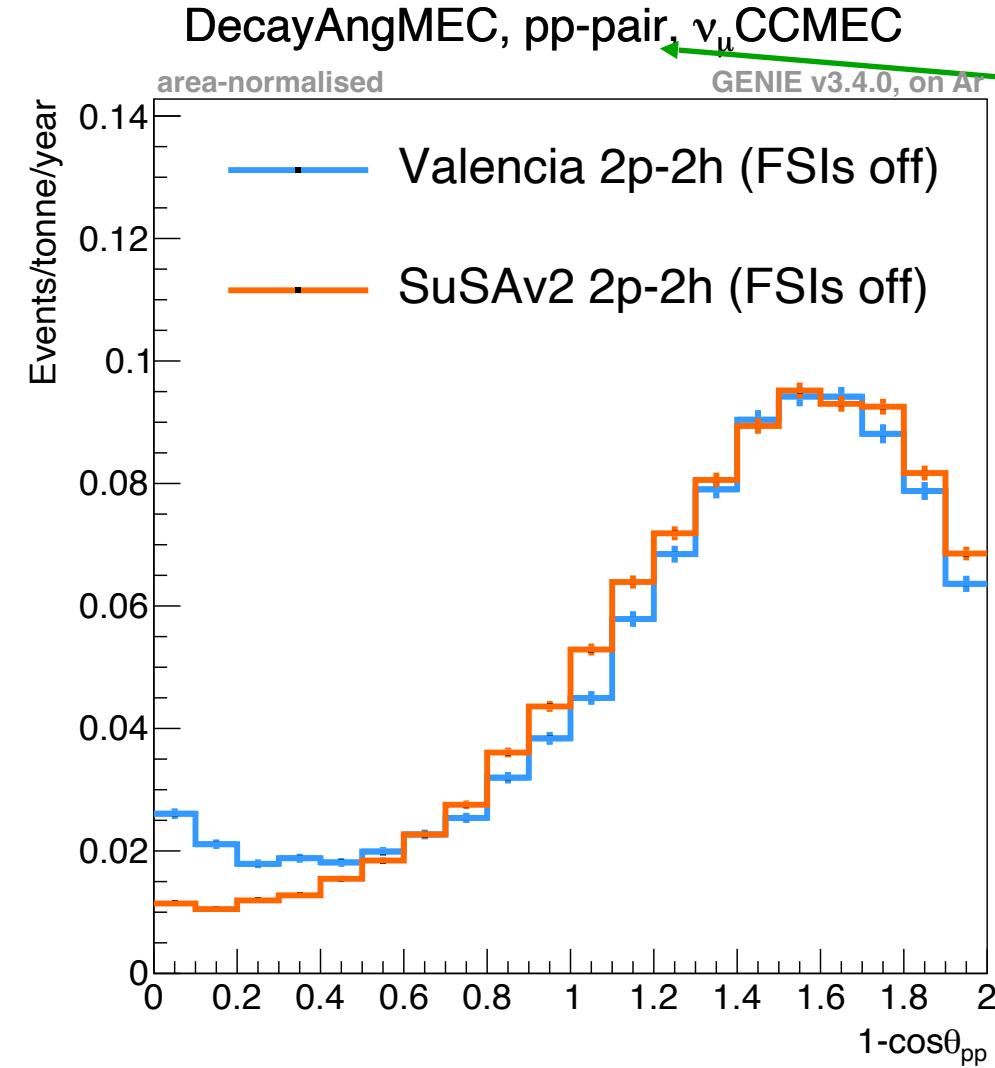
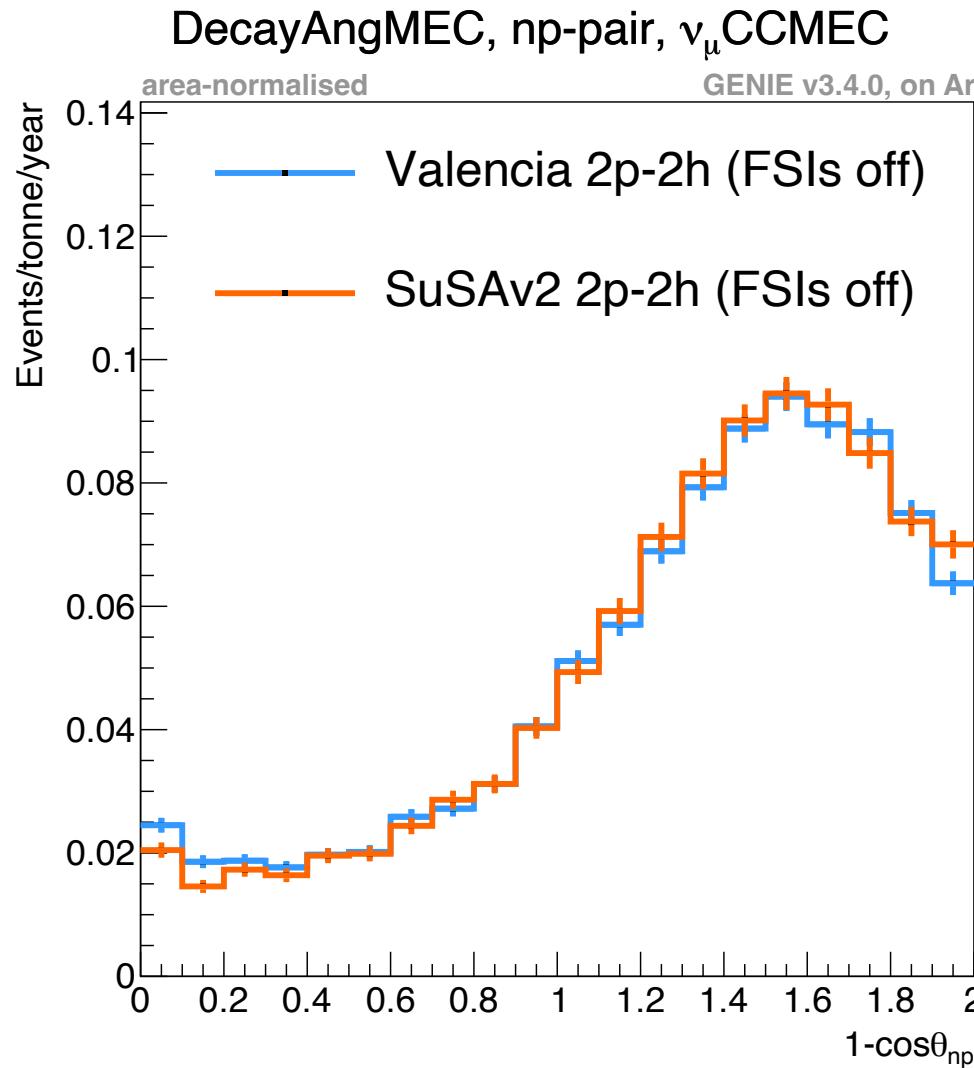


2p-2h Model Uncertainty Dials

NormCCMEC - changes absolute normalisation



2p-2h Model Comparisons (FSIs off)



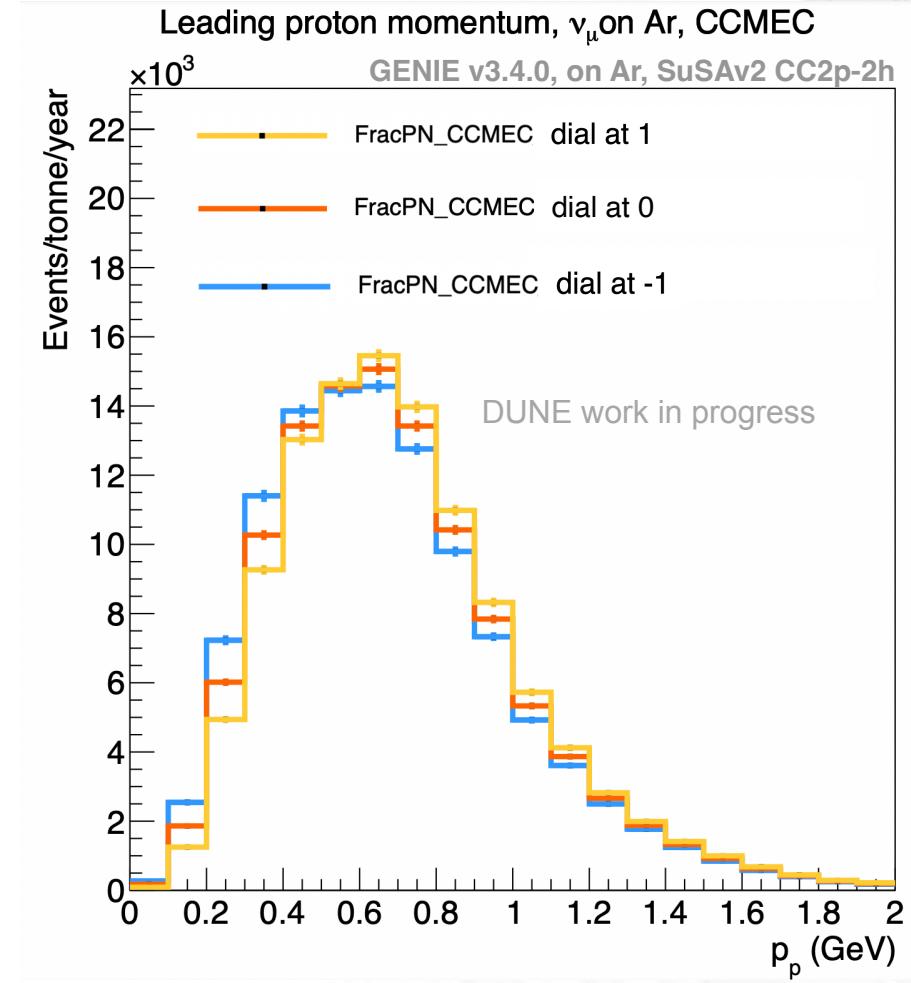
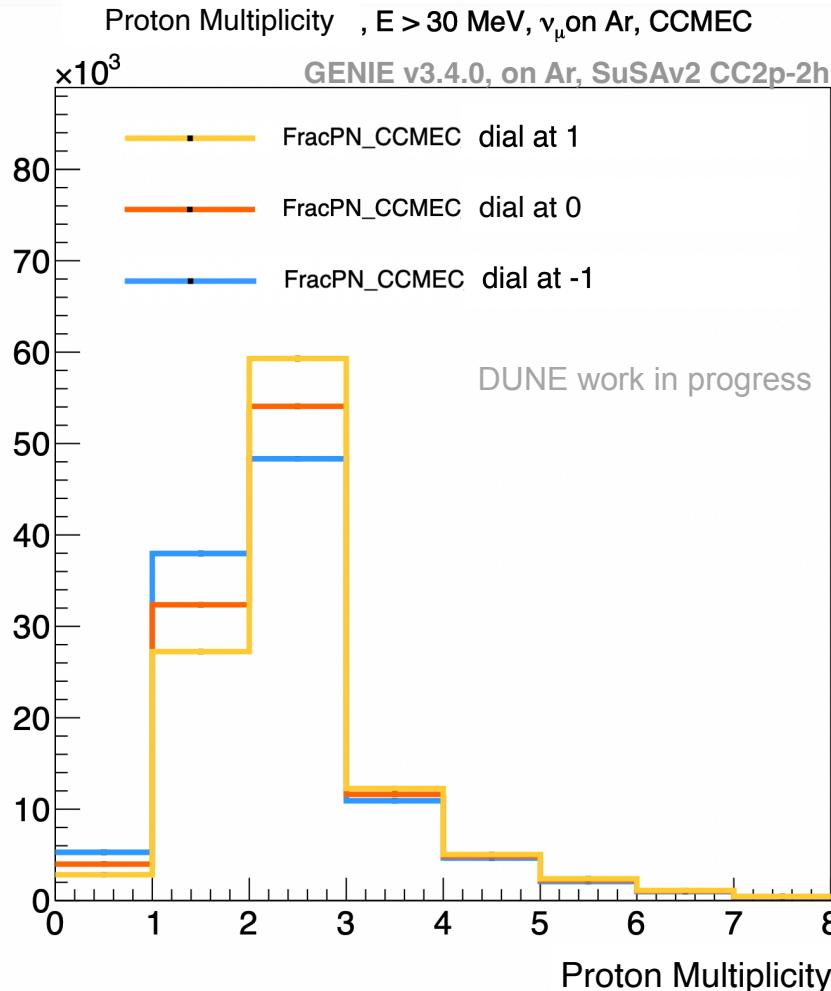
final-state pair!

Muon Neutrino interaction on bound states:

- nn \rightarrow np
- np \rightarrow pp

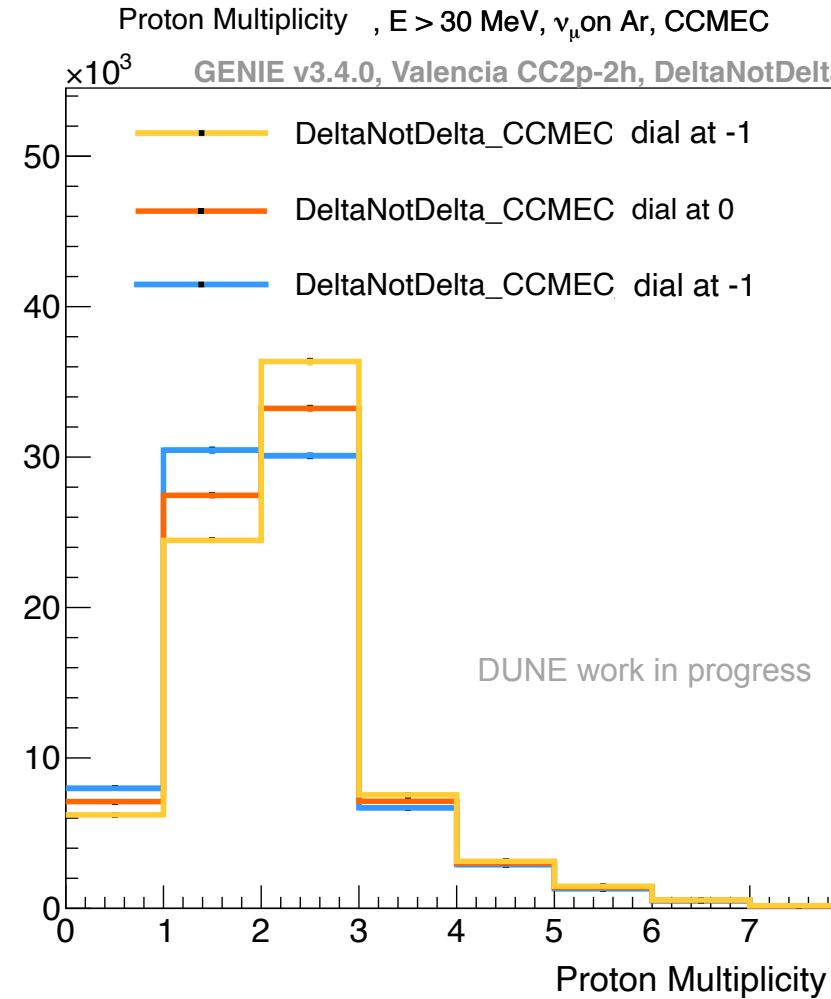
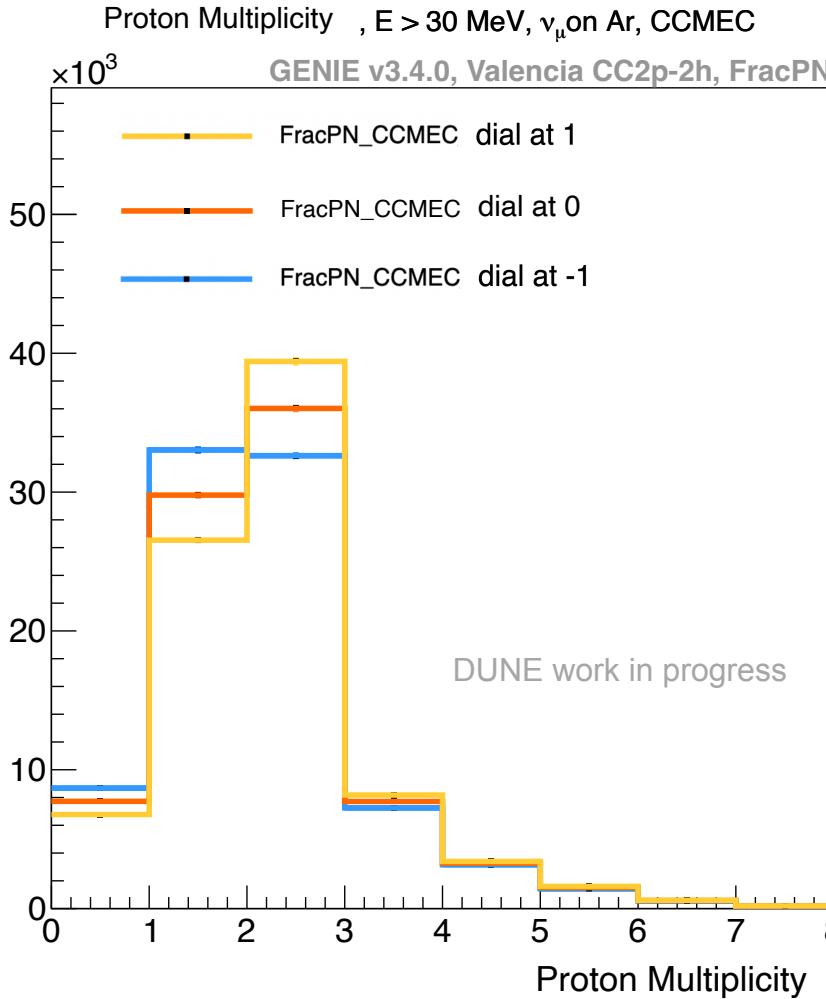
2p-2h Model Uncertainty Dials

FracPN_CCMEC - changes default prediction of initial pair content of nucleons



2p-2h Model Uncertainty Dials

FracPN_CCMEC vs. *DeltaNotDelta_CCMEC* - the effect of Δ -resonances



- *DeltaNotDelta_CCMEC* dial: adjusts relative strength of Δ -resonance contributions to 2p-2h cross sections
- Distinguish four cases
 - !is_pn_event & !is_delta_event
 - !is_pn_event & is_delta_event
 - is_pn_event & !is_delta_event
 - is_pn_event & is_delta_event
- Implemented flagispnevent and flagisdeltaevent variables in NUISANCE

2p-2h Model Uncertainty Dials

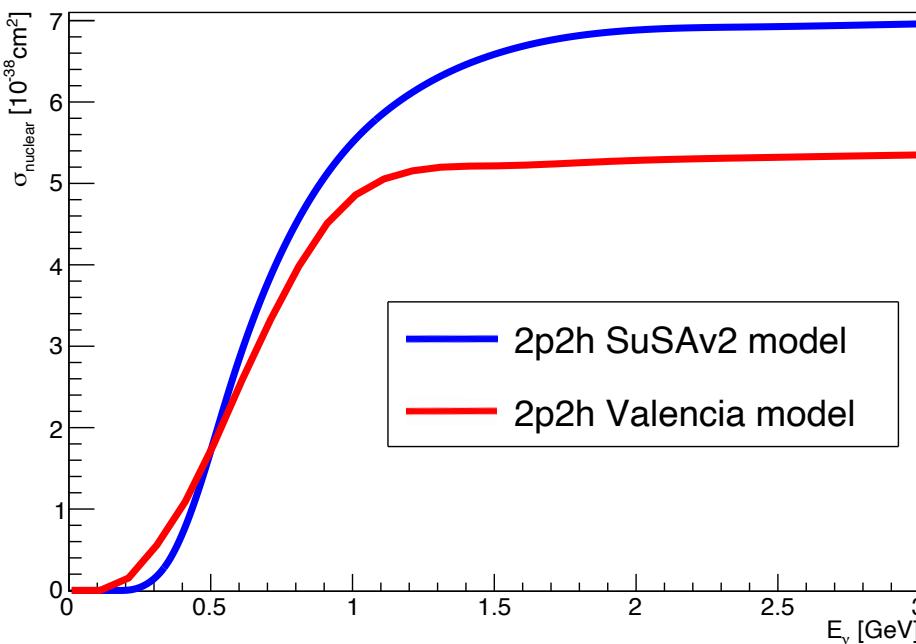
EnergyDependence_CCMEC - changes the energy dependence of 2p-2h cross sections

[T2K-TN-344-v4]

- Parametrise 2p-2h cross section by

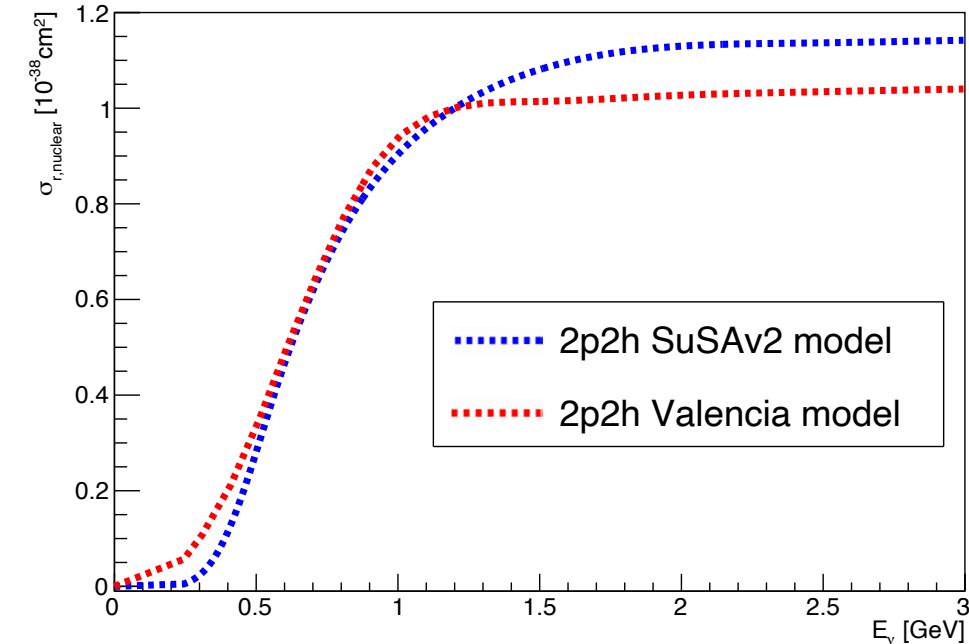
$$\sigma_\nu(E_\nu) = \sigma_\nu^{\text{MC}}(E_\nu) \cdot N_\nu \cdot \left(1 + \frac{1 - a_h}{r_\nu(E_\nu)} \right)$$

GENIE 2p2h cross sections



$$\sigma_r(E_\nu) = \frac{\sigma(E_\nu)}{\sigma(1.2 \text{ GeV})}$$

GENIE 2p2h normalised (1.2 GeV) cross sections



2p-2h Model Uncertainty Dials

EnergyDependence_CCMEC - changes the energy dependence of 2p-2h cross sections

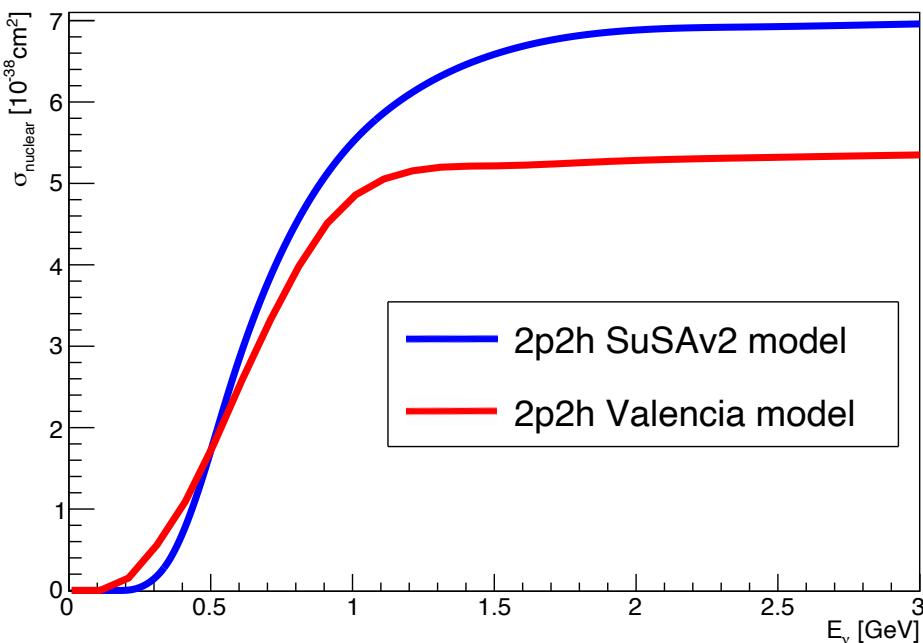
- Parametrise 2p-2h cross section by

$$\sigma_\nu(E_\nu) = \sigma_\nu^{\text{MC}}(E_\nu) \cdot N_\nu \cdot \left(1 + \frac{1 - a_h}{r_\nu(E_\nu)} \right)$$

[T2K-TN-344-v4]

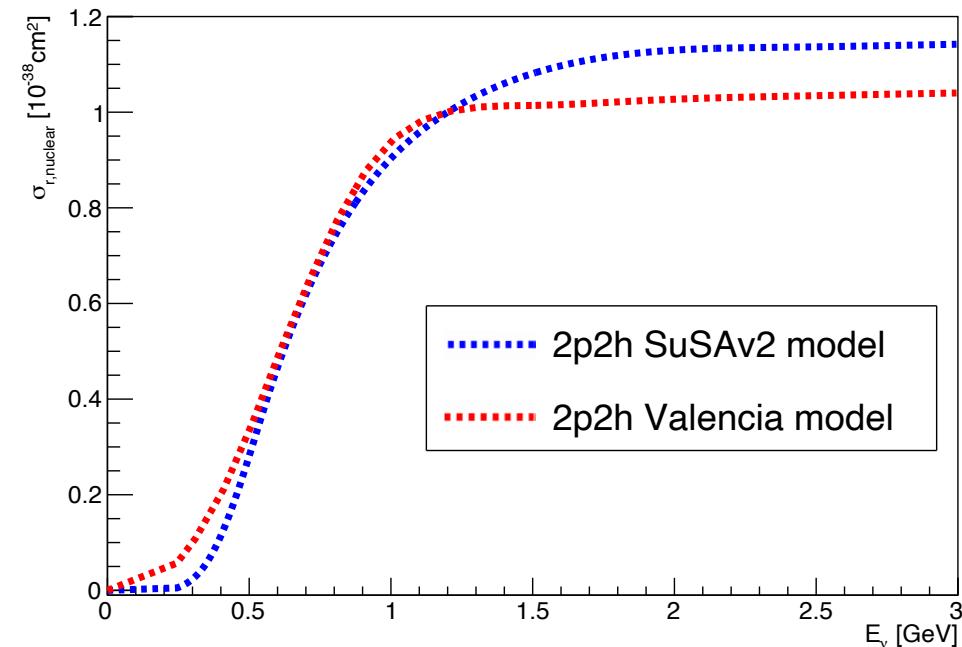
$$r_\nu(E_\nu) = \frac{\sigma_r^{\max}(E_\nu)}{\sigma_r^{\min}(E_\nu)}$$

GENIE 2p2h cross sections



$$\sigma_r(E_\nu) = \frac{\sigma(E_\nu)}{\sigma(1.2 \text{ GeV})}$$

GENIE 2p2h normalised (1.2 GeV) cross sections



2p-2h Model Uncertainty Dials

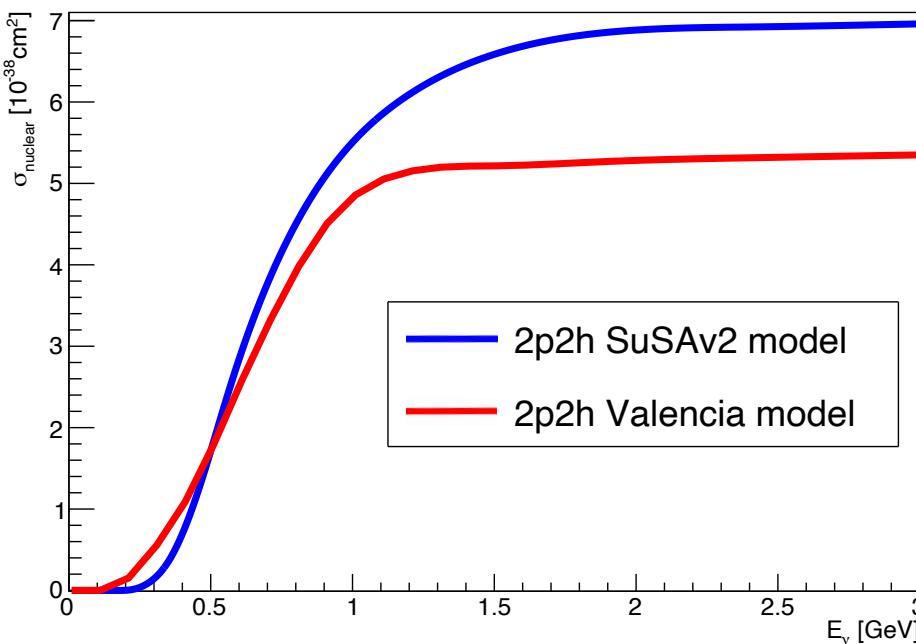
EnergyDependence_CCMEC - changes the energy dependence of 2p-2h cross sections

[T2K-TN-344-v4]

- Parametrise 2p-2h cross section by

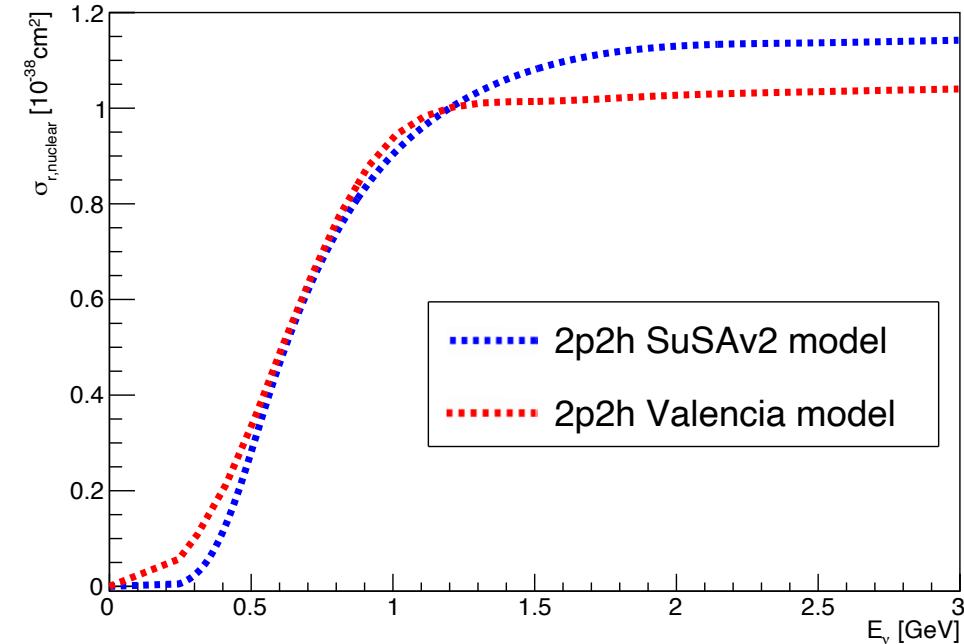
$$\sigma_\nu(E_\nu) = \sigma_\nu^{\text{MC}}(E_\nu) \cdot N_\nu \cdot \left(1 + \frac{1 - a_h}{r_\nu(E_\nu)} \right)$$

GENIE 2p2h cross sections



$$\sigma_r(E_\nu) = \frac{\sigma(E_\nu)}{\sigma(1.2 \text{ GeV})}$$

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2p-2h Model Uncertainty Dials

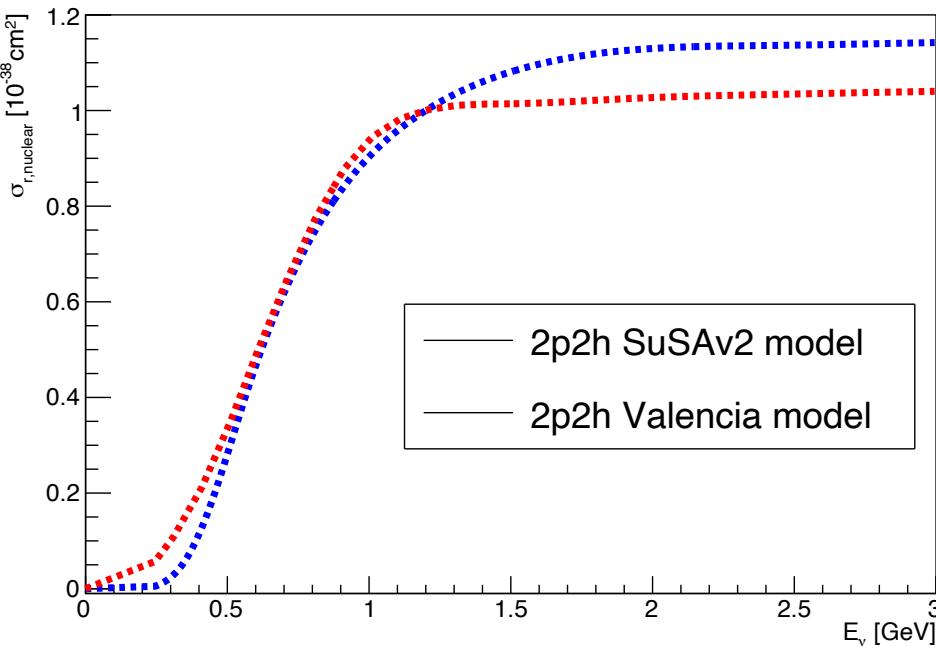
EnergyDependence_CCMEC - changes the energy dependence of 2p-2h cross sections

[T2K-TN-344-v4]

- Parametrise 2p-2h cross section by

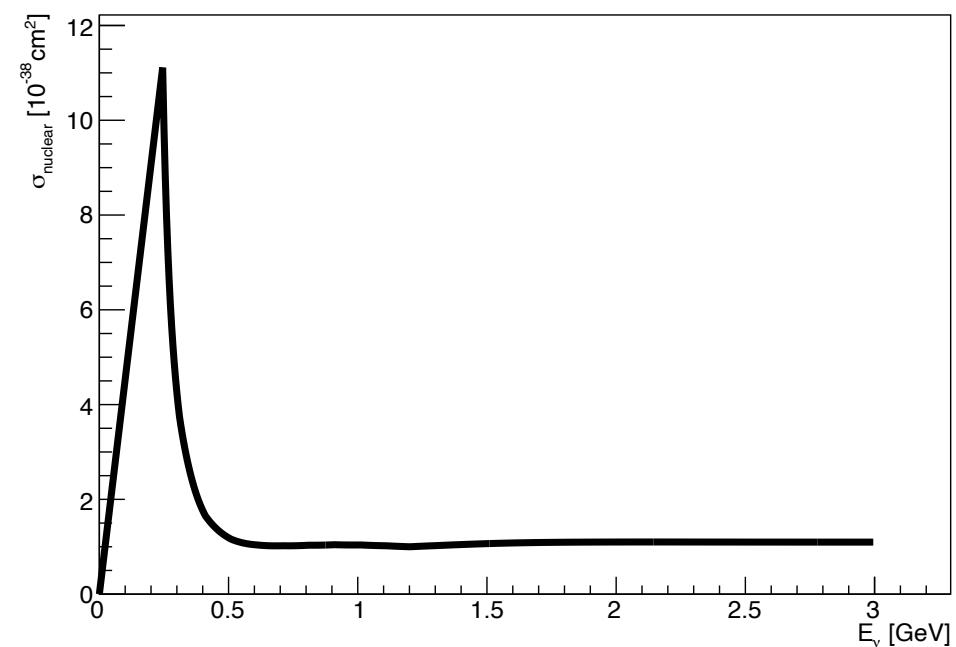
$$\sigma_\nu(E_\nu) = \sigma_\nu^{\text{MC}}(E_\nu) \cdot N_\nu \cdot \left(1 + \frac{1 - a_h}{r_\nu(E_\nu)} \right)$$

GENIE 2p2h normalised (1.2 GeV) cross sections



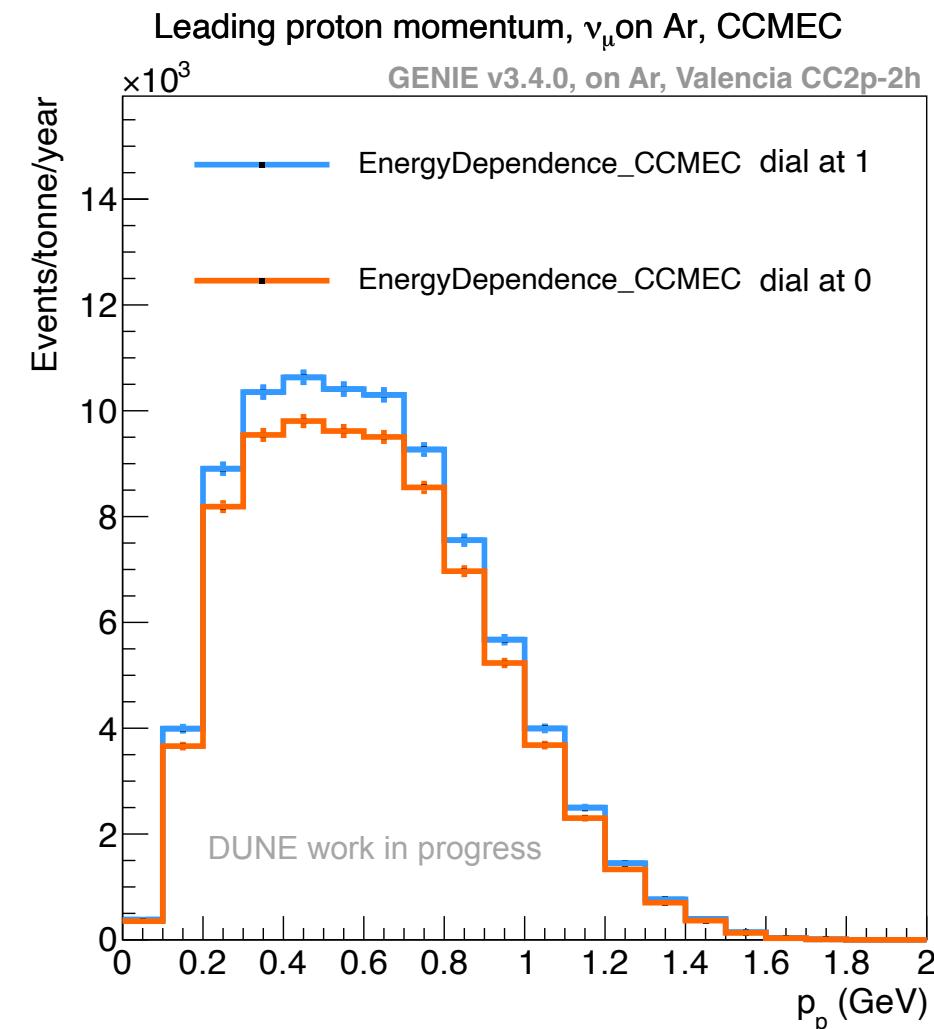
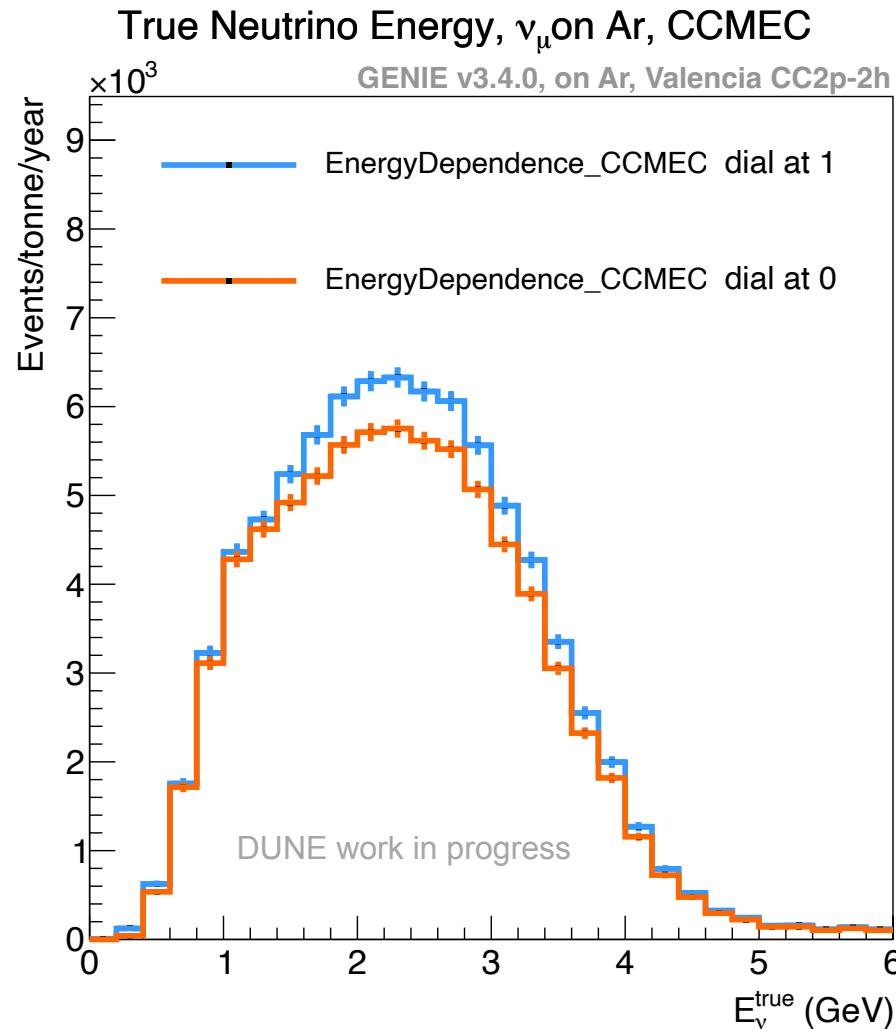
Cross section ratio plot (2p2h SuSAv2 and Valencia)

$$r_\nu(E_\nu) = \frac{\sigma_r^{\max}(E_\nu)}{\sigma_r^{\min}(E_\nu)}$$



2p-2h Model Uncertainty Dials

EnergyDependence_CCMEC - changes the neutrino energy dependence



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